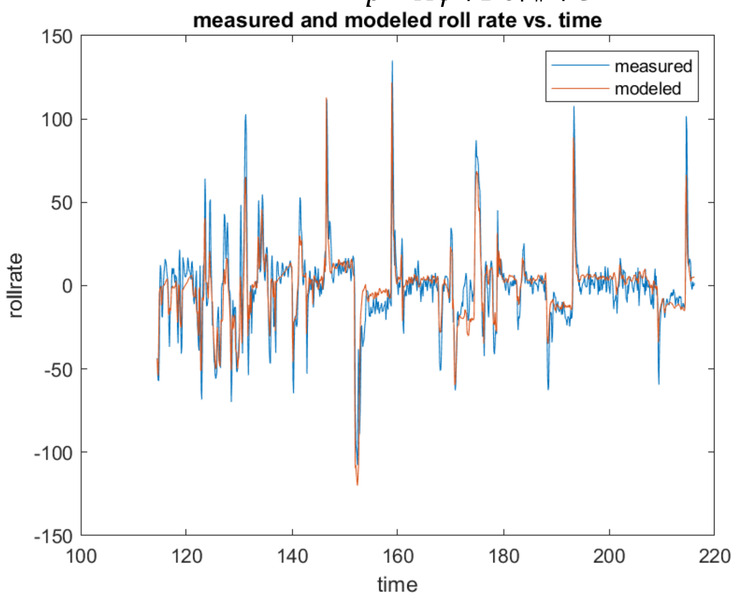
**ME 401/5501 – Robotics Systems Identification**

**HW #1: Due Wednesday August 31st 2022**

1) Using the aircraft data on Canvas, estimate the parameters given the model structure shown below. After estimating the parameters, use the model and the flight data to estimate the roll rate. Provide the estimated parameters as well as a plot showing the measured roll rate vs. time and the model roll rate vs. time (on a single figure).

$$p = A\varphi + B\delta_{!"\#} + C$$




```
ME_494_HW_1_1
the estimated parameters are

T_hat =

    0.4625
  148.0395
    7.8825
```

% p = A * phi + B * ail + C

load hw1_timber.mat

t = timber.t; %measured data%

```
phi = timber.roll;  %measured data%

ail = timber.aileron;  %measured data%

p = timber.rollrate;  %measured data%

x = [phi, ail, ones(1001,1)];

T_hat =(x'*x)\x'*p;

disp('the estimated parameters are')

T_hat

p_m = T_hat(1) * phi + T_hat(2) * ail + T_hat(3);

plot(t,p,t,p_m)

title('measured and modeled roll rate vs. time')

xlabel('time')

ylabel('rollrate')

legend('measured','modeled')
```

2) Consider the model y(t) = tan(u*0.9*pi/2) evaluated for the inputs u = linspace(0,1,N). Use the model structure:

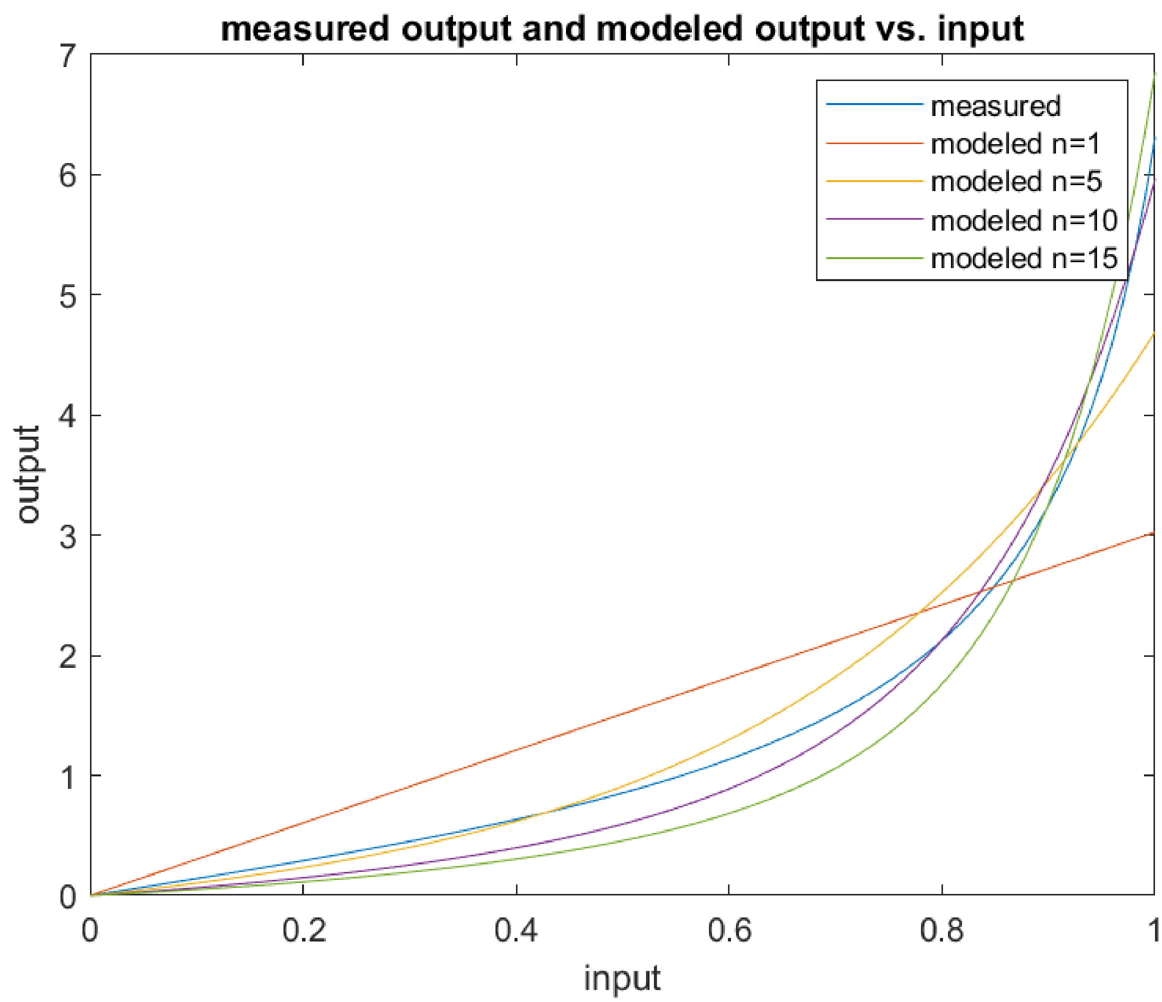
   $ $y^* =$

   $+ \theta_{"} u^{"}$

   "%&

   to describe these data. Note that this is a nonlinear model that is linear-in-theparameters $\theta$.

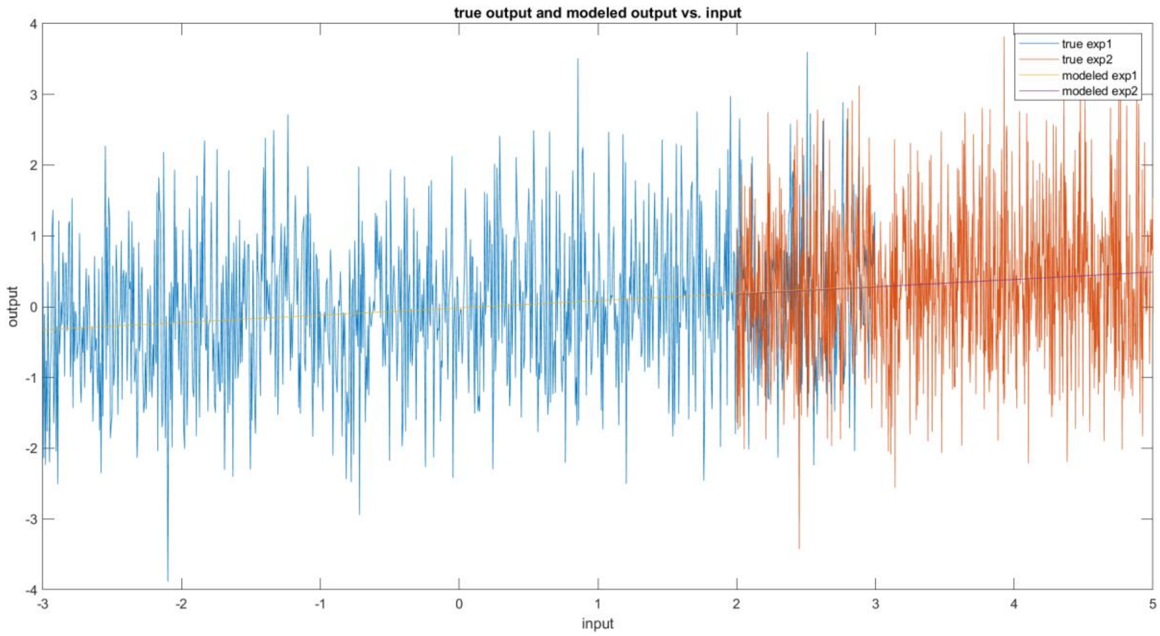
   - Generate a dataset of y based on the model. N = 100.
   - Create a Matlab script that solves for varying orders of the model structure (n varies from 1 to 20). Make sure to save the different parameters (n = 1 will have 2 parameters in theta, n = 2 will have 3, and so on).
   - Plot the estimated output $y^*$ for n = 1, 5, 10 and 15 as well as the true output (from the model) on a **single** plot.

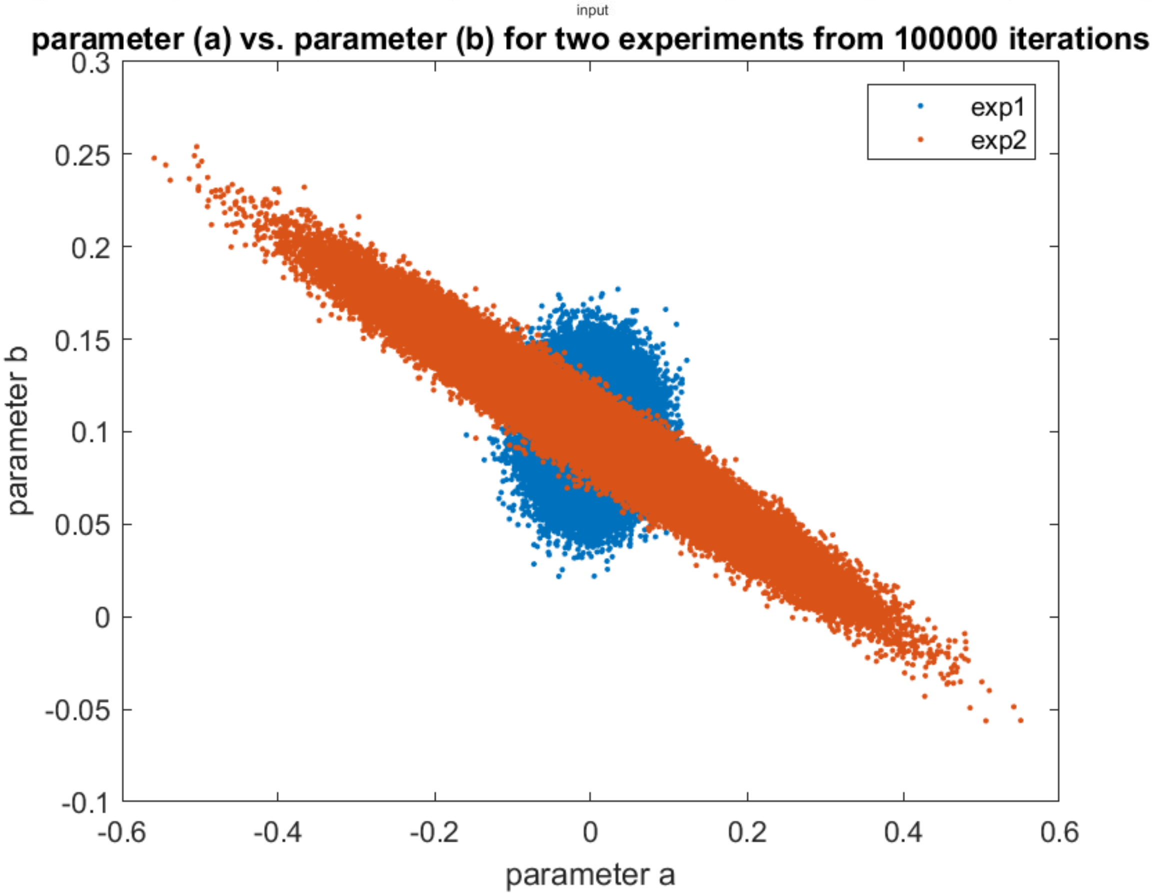



```
u = linspace(0,1,100);

y = tan(u*0.9*pi/2);

h = y';

t = u';

x = 0;

Y_hat_list = [];

for n = [1:20]

    x = x + t.^n;

    T_hat = (x'*x)\x'*h;

    Y_hat = T_hat * x;
```

```
    Y_hat_list = [Y_hat_list, Y_hat];
end
figure(1)
plot( u, y, u, Y_hat_list(:,1), u, Y_hat_list(:,5), u, Y_hat_list(:,10), u, Y_hat_list(:,15))
title('measured output and modeled output vs. input')
xlabel('input')
ylabel('output')
legend('measured','modeled n=1','modeled n=5','modeled n=10','modeled n=15')
```

3) Consider the model $y(t) = n_t + D*u$. Two experimental scenarios are to be investigated with $u_1$=linspace(-3,3,N) and $u_2$=linspace(2,5,N).
   - $D = 0.1$, $N = 1000$, and $n_t \sim N(0,1)$
   - Use the model structure $y = a + b*u$, and estimate the parameters *a* and *b* using ordinary least squares.
   - Repeat each experiment 100000 times. Make sure to store the estimated parameters for each trial (I would suggest creating an extra 2D array that can store experiment #1's parameters and a 2D array for experiment #2).
   - For the final run for each experiment plot the true output and the model output on a single plot.
   - Plot the *a* estimates versus the *b* estimates for the two experiments on a single plot. The x-axis should be the bias (a) and the y-axis should be the slope term (b). Discuss what you find in the figure.




Discussion: For experiment 1 both the a and b parameters are centered about (0, .1). For experiment 1 the a parameter approximately ranges from -.2 to .2 and the b parameter approximately ranges from 0 to .2. For experiment 2 the a and b parameter point cloud has a stretched oval shape and follows a negative slope. For experiment 2 the a parameter approximately ranges from -.6 to .6 and the b parameter approximately ranges from -.1 to .3.

```
D = .1;
N = 1000;
u1 = linspace(-3,3,N);
u1t = u1';
u2 = linspace(2,5,N);
u2t = u2';

P_hat1_list = [];
P_hat2_list = [];

% ordinary least squares %
for n = [1:100000]
   nt =randn(1000,1);
   nt = nt';
   y1 = nt + D*u1; % true output experiment1 %
   y1t = y1';
   y2 = nt + D*u2; % true output experiment2 %
   y2t = y2';
   x1 = [ones(1000,1),u1t];
   P_hat1 = (x1'*x1)\x1'*y1t;
   Y_hat1 = P_hat1(1)+ P_hat1(2)*u1;
   P_hat1_list = [P_hat1_list, P_hat1];
   x2 = [ones(1000,1),u2t];
   P_hat2 = (x2'*x2)\x2'*y2t;
   Y_hat2 = P_hat2(1)+ P_hat2(2)*u2;
   P_hat2_list = [P_hat2_list, P_hat2];
end
figure(1)
plot(u1, y1, u2, y2, u1, Y_hat1, u2, Y_hat2)
title('true output and modeled output vs. input')
xlabel('input')
ylabel('output')
legend('true exp1','true exp2','modeled exp1','modeled exp2')
figure(2)
plot(P_hat1_list(1,:), P_hat1_list(2,:), '.', P_hat2_list(1,:), P_hat2_list(2,:), '.')
title('parameter (a) vs. parameter (b) for two experiments from 100000 iterations')
xlabel('parameter a')
ylabel('parameter b')
legend('exp1','exp2')
```