

```
In [1]: 1 import numpy as np
        2 import matplotlib.pyplot as plt
        3 from scipy.stats import norm
        4 import pandas as pd
        5 from scipy.integrate import odeint
        6 from scipy.integrate import solve_ivp
```

Opening predator-prey dataset

```
In [2]: 1 df = pd.read_csv('predator-prey-data.csv', index_col=False)
        2 df.head()
```

```
Out[2]:
```

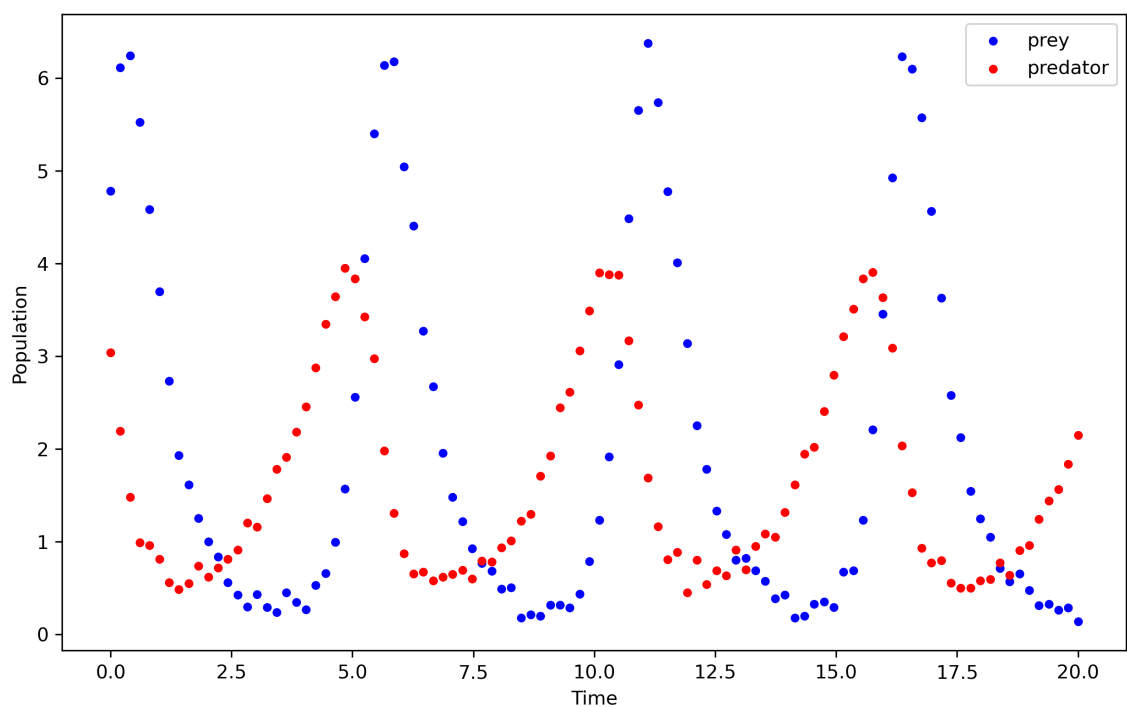
	Unnamed: 0	t	x	y
0	0	0.000000	4.781635	3.035257
1	1	0.202020	6.114005	2.189746
2	2	0.404040	6.238361	1.478907
3	3	0.606061	5.520524	0.989836
4	4	0.808081	4.582546	0.957827

```

In [3]: 1 # Loading data into read-only numpy arrays
2 data = df[['t', 'x', 'y']].values
3 # data[1], data[2] = data[2].copy(), data[1].copy()
4 data.flags.writeable = False
5
6
7 # Plotting
8 plt.figure(dpi =300, figsize=(10, 6))
9 point_width = 13
10 plt.scatter(data[:,0], data[:,1], label = 'prey', color = 'blue', s =pc
11 plt.scatter(data[:,0], data[:,2], label = 'predator', color = 'red', s=
12 plt.ylabel('Population')
13 plt.xlabel('Time')
14 plt.legend()
15

```

Out[3]: <matplotlib.legend.Legend at 0x13fc52f10>



Objective functions

Defining volterra equations function

```

In [4]: 1 def predator_pre_y_odes(initial_conditions,time ,alpha, beta, delta, gamma)
2     x = initial_conditions[0] # initial predator population
3     y = initial_conditions[1] # initial prey population
4     dxdt = (alpha * x) - (beta * x * y) # Prey ODE
5     dydt = (delta * x * y) - (gamma * y) # Predator ODE
6     return [dxdt, dydt]
7

```

```
In [5]: 1 #Function that will return the data for predator and prey for a given s
2 def predator_prey_integration(time,initial_conditions,parameters):
3     alpha,beta,delta,gamma = parameters
4     results = odeint(predator_prey_odes,initial_conditions, time, args=
5     predator_values,prey_values = results[:,0], results[:,1]
6     return np.array([predator_values,prey_values]).T
7
8
```

Defining objective functions

```
In [101]: 1 def weighted_sse(actual, predicted):
2     '''Weighted sum of squared erros'''
3
4     sd = 0.01
5
6     sd_list = sd*actual #List of estimated standard deviations
7
8     inv_sd = 1/sd_list #List of inverted standard deviations from sd_li
9
10    weighted_sse = np.sum(sd_list*((actual - predicted)**2))
11
12    #weighted_sse =
13
14    return weighted_sse
15
16
17 def MAE(actual, predicted):
18     '''Mean absolute error'''
19     return np.mean(np.abs(actual - predicted))
20
21
22
23 def MSE(actual, predicted):
24     '''Mean squared error'''
25     return np.mean((actual - predicted)**2)
```

```
In [7]: 1 test = np.array([1,2,3])
2
3 mult = 3*test
4 print(mult)
5
6 sum = np.sum(mult*test)
7
8 print(sum)
```

```
[3 6 9]
42
```

Algorithms & Optimisation

Simulated Annealing

```

In [8]: 1 def random_walk_annealing(parameters): #A random walk designed for anne
2         lst = [parameter + np.random.normal(0, 0.5) for parameter in paramet
3         # Ensure all elements are positive
4         while any(x <= 0 for x in lst):
5             for indx in range(len(lst)):
6                 if lst[indx] <= 0:
7                     lst[indx] = max(0, parameters[indx] + np.random.normal(
8
9         return lst
10
11
12
13 def simulated_annealing(initial_temp,cooling_constant, data, time, init
14
15     temp = initial_temp #Scaling factor for random movement. We square
16     start = parameters #Initial starting parameters
17     x_n = start
18     scores = [] #A score is just the value of the objective function ev
19
20     current_est = predator_prey_integration(time, initial_conditions, >
21     current_score = objective(data, current_est) #The current value of
22     scores.append(current_score) #Keeping track of the values of the ob
23
24     #cur = function(x) #The function value of the current x solution
25     history = [x_n] #Stores previously searched x values
26
27     for i in range (max_iterations):
28
29         proposal = random_walk_annealing(x_n) #A new proposal for the p
30         new_est = predator_prey_integration(time, initial_conditions, p
31         new_score = objective(data, new_est) #Calculate new value of ob
32
33         delta = new_score - current_score #Difference in objective func
34
35         #if proposal < 0 or proposal > 1:
36             #proposal = x_n # Reject proposal by setting it equal to pre
37
38         acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate
39
40         #if delta < 0:
41             #x_n = proposal ##Accept proposal
42             #current_score = new_score
43
44         if np.random.rand() < acceptance_probability: #else if it is no
45             x_n = proposal #Accept proposal
46             current_score = new_score
47
48         scores.append(current_score)
49         temp = cooling_constant**i * initial_temp #Cool temperature
50         #print(temp)
51         history.append(x_n) #Add to history
52
53     return x_n, scores

```

In [90]:

```

1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4
5 #Taking random draw for initial parameters (initial guess)
6 alpha = np.random.uniform(0,1)
7 beta = np.random.uniform(0,1)
8 delta = np.random.uniform(0,1)
9 gamma = np.random.uniform(0,1)
10 parameters = [alpha, beta, delta, gamma]
11 initial_temp = 20
12 cooling_constant = 0.10
13
14 # Using MSE
15 x_best, scores = simulated_annealing(initial_temp,cooling_constant, inp

```

```

/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in exp
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability

```

In [184]:

```

1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4
5 #Taking random draw for initial parameters (initial guess)
6 alpha = np.random.uniform(0,1)
7 beta = np.random.uniform(0,1)
8 delta = np.random.uniform(0,1)
9 gamma = np.random.uniform(0,1)
10 parameters = [alpha, beta, delta, gamma]
11 initial_temp = 20
12 cooling_constant = 0.10
13
14 # Using MSE
15 x_best, scores = simulated_annealing(initial_temp,cooling_constant, inp

```

```

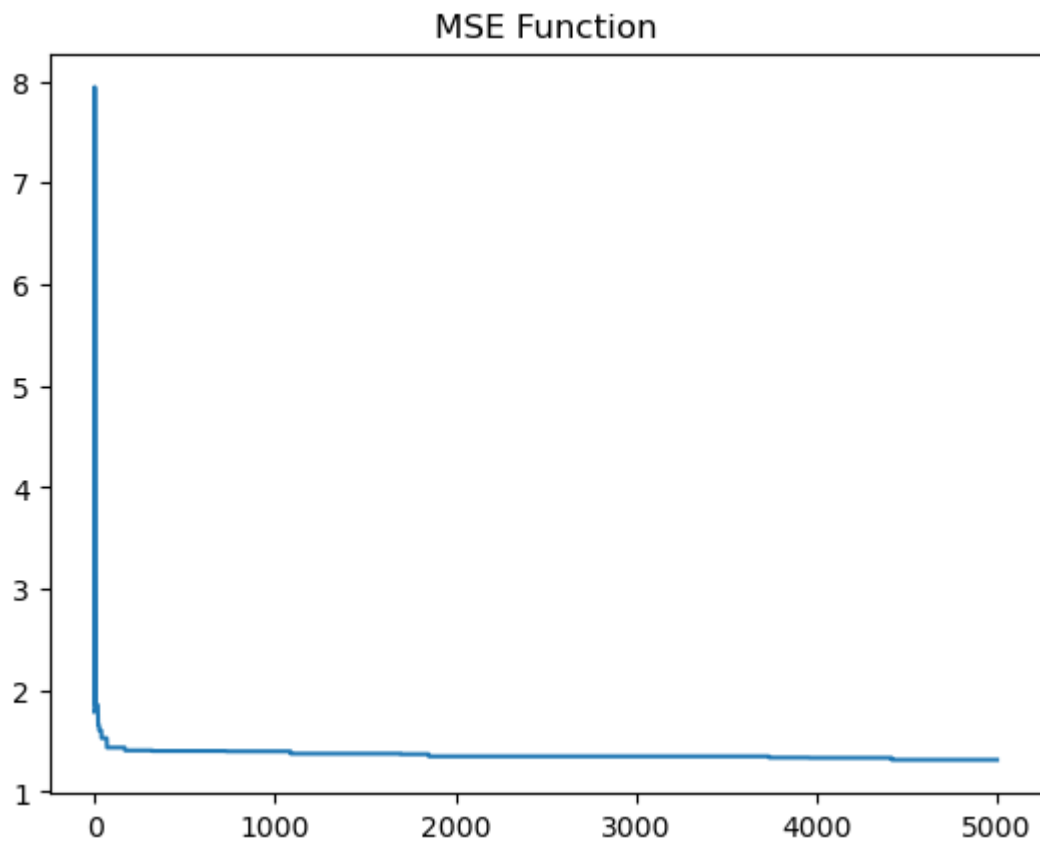
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in exp
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability

```

Solution discovery over iterations

```
In [185]: 1 xvalues = [x for x in range(0,len(scores))]  
2  
3  
4  
5 plt.plot(xvalues,scores)  
6 plt.title("MSE Function")  
7  
8 print("LOWEST MSE: " + str(scores[-1]))
```

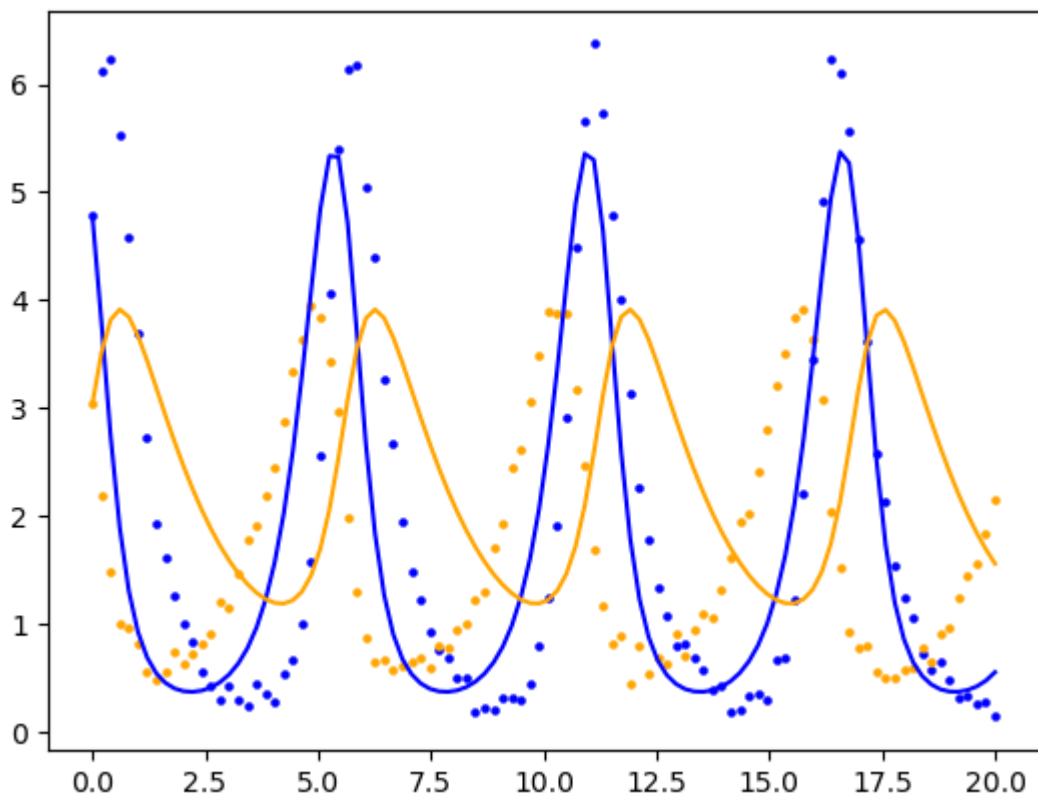
LOWEST MSE: 1.3145243092280952



Curve fit

```
In [186]: 1 # t, x, y = data
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 scaling = 2
5
6 parameters = x_best
7
8 # Using MSE
9 x = predator_prey_integration(t, initial_conditions, parameters)
10
11 plt.figure(dpi=300, figsize=(6, 5))
12 point_width = 13
13 plt.plot(t, x[:,0], color="b")
14 plt.plot(t, x[:,1], color="orange")
15
16 plt.scatter(t, data[:,1], color='blue', s=5)
17 plt.scatter(t, data[:,2], color='orange', s=5)
18
19 #mse_prey = MSE(data[:,1], x[:,0]) #MSE for fitted curve
20 #mse_predator = MSE(data[:,2], x[:,1])
21 #mse_total = mse_prey + mse_predator
22 print("MAE: " + str(scores[-1]))
```

MAE: 1.3145243092280952



Calculation of mean and variance

```
In [96]: 1 ### Distribution of parameters for multiple runs
2
3 def multiple_runs_annealing(initial_temp,cooling_constant,input_data,t,
4
5     mse_total_list = []
6
7
8     for i in range(n_runs):
9
10         x_best, scores = simulated_annealing(initial_temp,cooling_const
11
12         mse = scores[-1]
13
14         mse_total_list.append(mse) #Add total MSE for this simulation t
15
16     return mse_total_list
17
```

```
In [106]: 1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 #Taking random draw for initial parameters (initial guess)
5 alpha = np.random.uniform(0,1)
6 beta = np.random.uniform(0,1)
7 delta = np.random.uniform(0,1)
8 gamma = np.random.uniform(0,1)
9 parameters = [alpha, beta, delta, gamma]
10 parameters = [alpha, beta, delta, gamma]
11
12 initial_temp = 20
13 cooling_constant = 0.10
14
15 mse_total_list = multiple_runs_annealing(initial_temp,cooling_constant,
16
17 #print(mse_total_list)
```

```
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_58892/385309489
9.py:38: RuntimeWarning: overflow encountered in exp
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_58892/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_58892/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/Users/alex_1/anaconda3/lib/python3.11/site-packages/scipy/integrate/_odep
ack_py.py:248: ODEintWarning: Excess work done on this call (perhaps wrong
Dfun type). Run with full_output = 1 to get quantitative information.
    warnings.warn(warning_msg, ODEintWarning)
```

```
In [107]: 1 print(mse_total_list)

[2.6473227546360665, 2.7464957827676564, 2.8201901313461293, 2.68480725254
8282, 2.6589224841462658, 2.5022908556330266, 2.4583578426621084, 2.439019
9669793287, 2.460643513633285, 2.543829159819853]
```


In [108]:

```

1
2 mean_mse_annealing = np.mean(mse_total_list)
3 std_mse_annealing = np.std(mse_total_list)
4
5 print("Average MSE = " + str(mean_mse_annealing))
6 print("Standard deviation of MSE = " + str(std_mse_annealing))

```

Average MSE = 2.5961879744172007

Standard deviation of MSE = 0.12680792857239243

FOR WEIGHTED SSE

In [189]:

```

1
2
3 input_data = data[:,1:3]
4 initial_conditions = [input_data[0][0], input_data[0][1]]
5 t = data[:,0]
6 #Taking random draw for initial parameters (initial guess)
7 alpha = np.random.uniform(0,1)
8 beta = np.random.uniform(0,1)
9 delta = np.random.uniform(0,1)
10 gamma = np.random.uniform(0,1)
11 parameters = [alpha, beta, delta, gamma]
12 parameters = [alpha, beta, delta, gamma]
13
14 initial_temp = 20
15 cooling_constant = 0.10
16
17 mse_total_list = multiple_runs_annealing(initial_temp,cooling_constant,
18

```

```

/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in exp

```

```

    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability

```

```

/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide

```

```

    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability

```

```

/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide

```

```

    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability

```

In [190]:

```

1 print(mse_total_list)
2 print(np.std(mse_total_list))

```

```

[1.4283946885408745, 1.3067741402910857, 1.4442398586927112, 1.28940092054
71197, 1.4204590248705167, 1.4440930616053458, 1.3031423818937582, 1.35896
96758657763, 1.3465599587990993, 1.3043258816810026]
0.06051240976134452

```

Comparison of cooling schedules

```
In [221]: 1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 #Taking random draw for initial parameters (initial guess)
5 alpha = np.random.uniform(0,1)
6 beta = np.random.uniform(0,1)
7 delta = np.random.uniform(0,1)
8 gamma = np.random.uniform(0,1)
9 parameters = [alpha, beta, delta, gamma]
10 parameters = [alpha, beta, delta, gamma]
11
12 initial_temp = 20
13
14 cooling_constants = np.arange(0.10,1,0.10)
15
16 mean_mse_per_simulation = [] #The mean MSE per simulation
17 sd_mse_per_simulation = [] #The standard deviation per simulation
18
19 for constant in cooling_constants:
20
21     mse_total_list = multiple_runs_annealing(initial_temp,constant,input_data)
22     mean_mse_annealing = np.mean(mse_total_list)
23     std_mse_annealing = np.std(mse_total_list)
24     mean_mse_per_simulation.append(mean_mse_annealing)
25     sd_mse_per_simulation.append(std_mse_annealing)
26
27
```

```
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in exp
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability
/Users/alex_1/anaconda3/lib/python3.11/site-packages/scipy/integrate/_odepack_py.py:248: ODEintWarning: Excess work done on this call (perhaps wrong Dfun type). Run with full_output = 1 to get quantitative information.
  warnings.warn(warning_msg, ODEintWarning)
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability
```

```
In [222]: 1 mean_mae_per_simulation = []
2 sd_mae_per_simulation = []
3
4 for constant in cooling_constants:
5
6     mae_total_list = multiple_runs_annealing(initial_temp, constant, input_data)
7     mean_mae_annealing = np.mean(mae_total_list)
8     std_mae_annealing = np.std(mae_total_list)
9     mean_mae_per_simulation.append(mean_mae_annealing)
10    sd_mae_per_simulation.append(std_mae_annealing)
11
```

```
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in exp
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptance probability
```

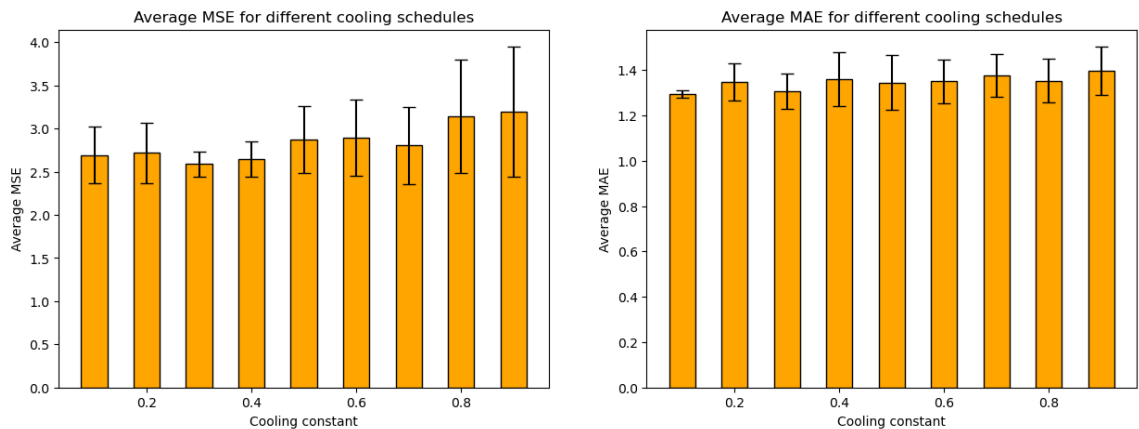
In [223]:

```

1  fig, axes = plt.subplots(1, 2, figsize=(15, 5))
2
3
4  bar_width = 0.05
5
6  axes[0].bar(cooling_constants, mean_mse_per_simulation, yerr = sd_mse_per_simulation)
7  axes[1].bar(cooling_constants, mean_mae_per_simulation, yerr = sd_mae_per_simulation)
8
9  axes[0].set_xlabel("Cooling constant")
10 axes[1].set_xlabel("Cooling constant")
11
12 axes[0].set_ylabel("Average MSE")
13 axes[1].set_ylabel("Average MAE")
14
15 axes[0].set_title("Average MSE for different cooling schedules")
16 axes[1].set_title("Average MAE for different cooling schedules")
17
18
19
20 #plt.bar(cooling_constants, mean_per_simulation, yerr = sd_per_simulation)
21 #plt.ylabel("Average MSE")
22 #plt.xlabel("Cooling constant")
23 #plt.title("Average MSE for different cooling schedules (30 simulations)")
24
25 #plt.show()
26

```

Out[223]: Text(0.5, 1.0, 'Average MAE for different cooling schedules')



Hill climbing

In [238]:

```

1
2
3 def random_walk(parameters):
4     lst = [parameter + np.random.normal(0, 0.5) for parameter in paramet
5     # Ensure all elements are positive
6     while any(x <= 0 for x in lst):
7         for indx in range(len(lst)):
8             if lst[indx] <= 0:
9                 lst[indx] = max(0, parameters[indx] + np.random.normal(
10
11     return lst
12
13 def hill_climbing(data, time, initial_conditions, parameters, objective
14     '''Tries to find the best solution using random walker'''
15     # Initialize starting parameter state
16     scores = []
17     x_n = parameters
18
19     current_est = predator_prey_integration(time, initial_conditions, x
20     current_score = objective(data, current_est)
21     scores.append(current_score)
22
23     for _ in range(max_iterations):
24         # Generate a random walk for parameters
25         x_n_1 = random_walk(x_n)
26
27         # Calculate the current and next estimations
28         current_est = predator_prey_integration(time, initial_conditions, x
29         new_estimation = predator_prey_integration(time, initial_conditions, x
30         new_score = objective(data, new_estimation)
31
32         # If the next estimation is better, update the parameters
33         if new_score < current_score:
34             current_score = new_score
35             x_n = x_n_1
36             scores.append(current_score)
37
38     return x_n, scores
39

```

In [239]:

```

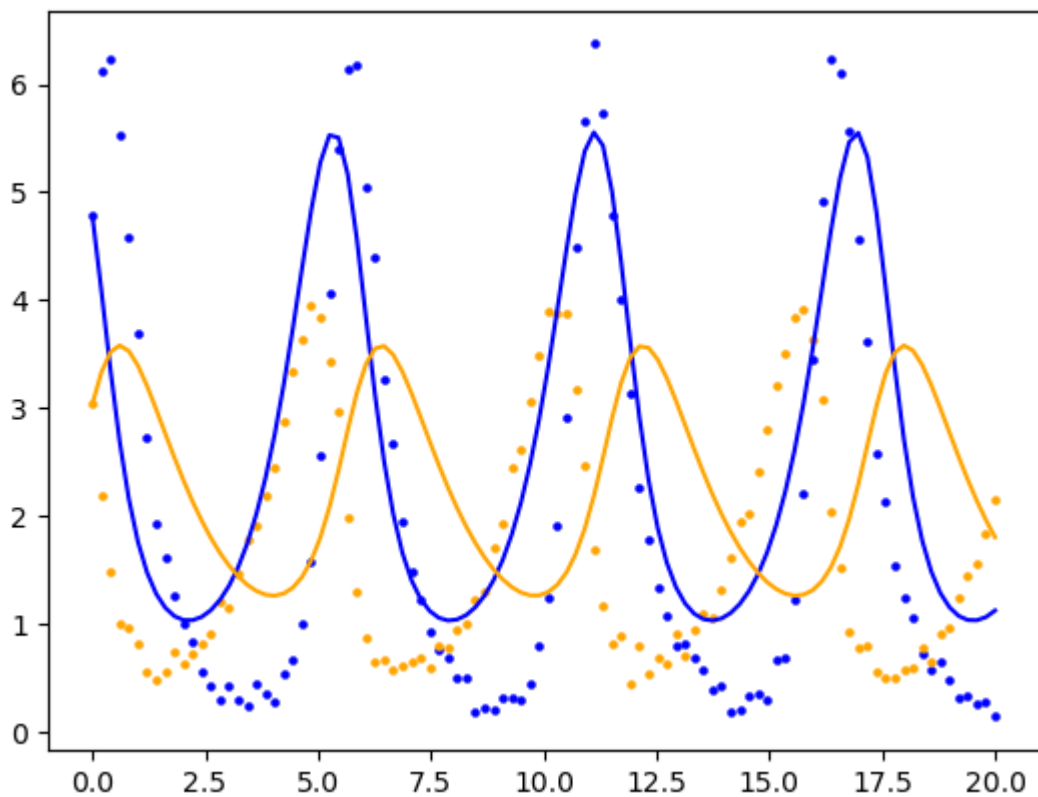
1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 #Taking random draw for initial parameters (initial guess)
5 alpha = np.random.uniform(0,1)
6 beta = np.random.uniform(0,1)
7 delta = np.random.uniform(0,1)
8 gamma = np.random.uniform(0,1)
9 parameters = [alpha, beta, delta, gamma]
10
11 # Using MSE
12 x_best,scores = hill_climbing(input_data, t, input_data[0], parameters,

```

Curve fit

```
In [241]: 1 # t, x ,y = data
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4
5 parameters = x_best
6
7 # Using MSE
8 x = predator_prey_integration(t,initial_conditions,parameters)
9
10
11
12 point_width = 13
13 plt.plot(t, x[:,0],color = "b")
14 plt.plot(t, x[:,1],color = "orange")
15
16 plt.scatter(t, data[:,1], color= 'blue', s = 5)
17 plt.scatter(t, data[:,2], color= 'orange', s = 5)
18
19 mse_prey = MSE(data[:,1],x[:,0]) #MSE for fitted curve
20 mse_predator = MSE(data[:,2],x[:,1])
21 mse_total = mse_prey + mse_predator
22 print("Mean Square Error: " +str(mse_total))
23
```

Mean Square Error: 5.385424572357688



```
In [195]: 1 def multiple_runs_hill_climbing(input_data, t, initial_conditions, para
2
3     mse_total_list = []
4
5     for i in range(n_runs):
6
7         x_best = hill_climbing(input_data, t, input_data[0], parameters
8
9         x = predator_prey_integration(t, initial_conditions, x_best)
10
11         mse_prey = MSE(data[:,1], x[:,0])
12         mse_predator = MSE(data[:,2], x[:,1])
13         mse_total = mse_prey + mse_predator
14
15         mse_total_list.append(mse_total) #Add total MSE for this simul
16
17     return mse_total_list
18
```

```
In [32]: 1 mean_mse_hill_climbing = np.mean(mse_total_list)
2 std_mse_hill_climbing = np.std(mse_total_list)
3
4 print("Average MSE = " + str(mean_mse_hill_climbing))
5 print("Standard deviation of MSE = " + str(std_mse_hill_climbing))
```

Average MSE = 7.020538795747848

Standard deviation of MSE = 0.525718488225756

Comparison of Optimisation Algorithms

MSE objective function

```
In [228]: 1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 #Taking random draw for initial parameters (initial guess)
5 alpha = np.random.uniform(0,1)
6 beta = np.random.uniform(0,1)
7 delta = np.random.uniform(0,1)
8 gamma = np.random.uniform(0,1)
9 parameters = [alpha, beta, delta, gamma]
10 parameters = [alpha, beta, delta, gamma]
11
12 initial_temp = 20
13 cooling_constant = 0.10
14
15 mse_total_list_annealing = multiple_runs_annealing(initial_temp,cooling
16 mean_mse_annealing = np.mean(mse_total_list_annealing)
17 std_mse_annealing = np.std(mse_total_list_annealing)
18
```

```
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in exp
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/Users/alex_1/anaconda3/lib/python3.11/site-packages/scipy/integrate/_odep
ack_py.py:248: ODEintWarning: Excess work done on this call (perhaps wrong
Dfun type). Run with full_output = 1 to get quantitative information.
  warnings.warn(warning_msg, ODEintWarning)
```

```
In [229]: 1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 #Taking random draw for initial parameters (initial guess)
5 alpha = np.random.uniform(0,1)
6 beta = np.random.uniform(0,1)
7 delta = np.random.uniform(0,1)
8 gamma = np.random.uniform(0,1)
9 parameters = [alpha, beta, delta, gamma]
10
11
12 mse_total_list_hill = multiple_runs_hill_climbing(input_data, t, initia
13 mean_mse_hill_climbing = np.mean(mse_total_list_hill)
14 std_mse_hill_climbing = np.std(mse_total_list_hill)
```

MAE objective function

In [230]:

```

1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 #Taking random draw for initial parameters (initial guess)
5 alpha = np.random.uniform(0,1)
6 beta = np.random.uniform(0,1)
7 delta = np.random.uniform(0,1)
8 gamma = np.random.uniform(0,1)
9 parameters = [alpha, beta, delta, gamma]
10 parameters = [alpha, beta, delta, gamma]
11
12 initial_temp = 20
13 cooling_constant = 0.10
14
15 mae_total_list_annealing = multiple_runs_annealing(initial_temp,cooling
16 mean_mae_annealing = np.mean(mae_total_list_annealing)
17 std_mae_annealing = np.std(mae_total_list_annealing)

```

```

/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in exp
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: overflow encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
/var/folders/rc/tn2ys5g55157vhhlmn0_vfwr0000gq/T/ipykernel_70535/385309489
9.py:38: RuntimeWarning: divide by zero encountered in scalar divide
    acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability

```

In [231]:

```

1 input_data = data[:,1:3]
2 initial_conditions = [input_data[0][0], input_data[0][1]]
3 t = data[:,0]
4 #Taking random draw for initial parameters (initial guess)
5 alpha = np.random.uniform(0,1)
6 beta = np.random.uniform(0,1)
7 delta = np.random.uniform(0,1)
8 gamma = np.random.uniform(0,1)
9 parameters = [alpha, beta, delta, gamma]
10
11
12 mae_total_list_hill = multiple_runs_hill_climbing(input_data, t, initia
13 mean_mae_hill_climbing = np.mean(mae_total_list_hill)
14 std_mae_hill_climbing = np.std(mae_total_list_hill)

```

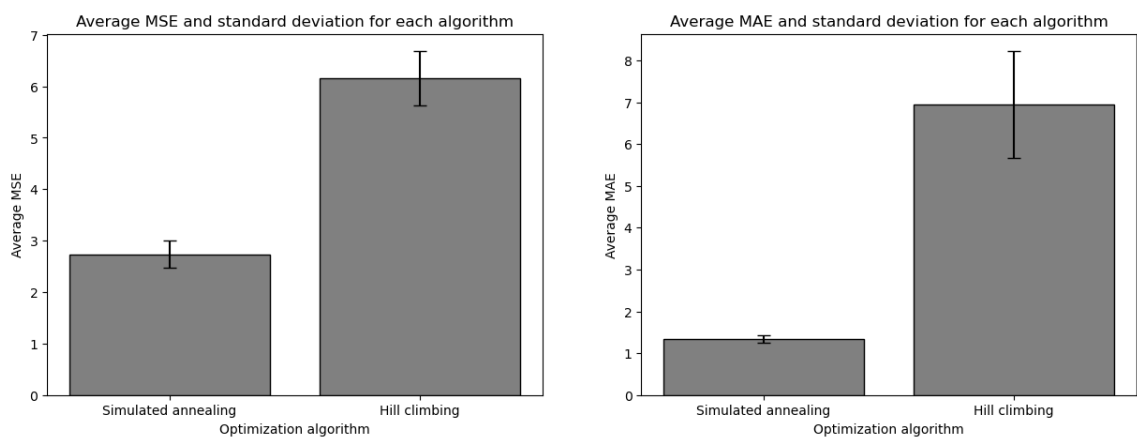
In [237]:

```

1  ##PLOTTING
2
3  fig, axes = plt.subplots(1, 2, figsize=(15, 5))
4
5
6  x = ["Simulated annealing", "Hill climbing"]
7  mse_list = [mean_mse_annealing, mean_mse_hill_climbing] #Stores the mean
8  std_mse_list = [std_mse_annealing, std_mse_hill_climbing]
9
10 mae_list = [mean_mae_annealing, mean_mae_hill_climbing] #Stores the mean
11 std_mae_list = [std_mae_annealing, std_mae_hill_climbing]
12
13 axes[0].bar(x, mse_list, yerr = std_mse_list, color = "grey", ec = "black")
14 axes[1].bar(x, mae_list, yerr = std_mae_list, color = "grey", ec = "black")
15
16
17
18 axes[0].set_xlabel("Optimization algorithm")
19 axes[1].set_xlabel("Optimization algorithm")
20
21 axes[0].set_ylabel("Average MSE")
22 axes[1].set_ylabel("Average MAE")
23
24 axes[0].set_title("Average MSE and standard deviation for each algorithm")
25 axes[1].set_title("Average MAE and standard deviation for each algorithm")
26
27
28
29 #plt.bar(x, mse_list, yerr = std_list, color = "grey", ec = "black", ecol
30 #plt.title("Bar plot comparing the means and errors of the MSE")
31 #plt.ylabel("MSE")
32 #plt.xlabel("Optimization algorithm")

```

Out[237]: Text(0.5, 1.0, 'Average MAE and standard deviation for each algorithm')



In []:

1

In []:

1