In [2]:
```
import numpy as np
import matplotlib.pyplot as plt
from scipy.stats import norm
import pandas as pd
from scipy.integrate import odeint
import time
import statistics
import random
import scipy.stats

```

Opening predator-prey dataset

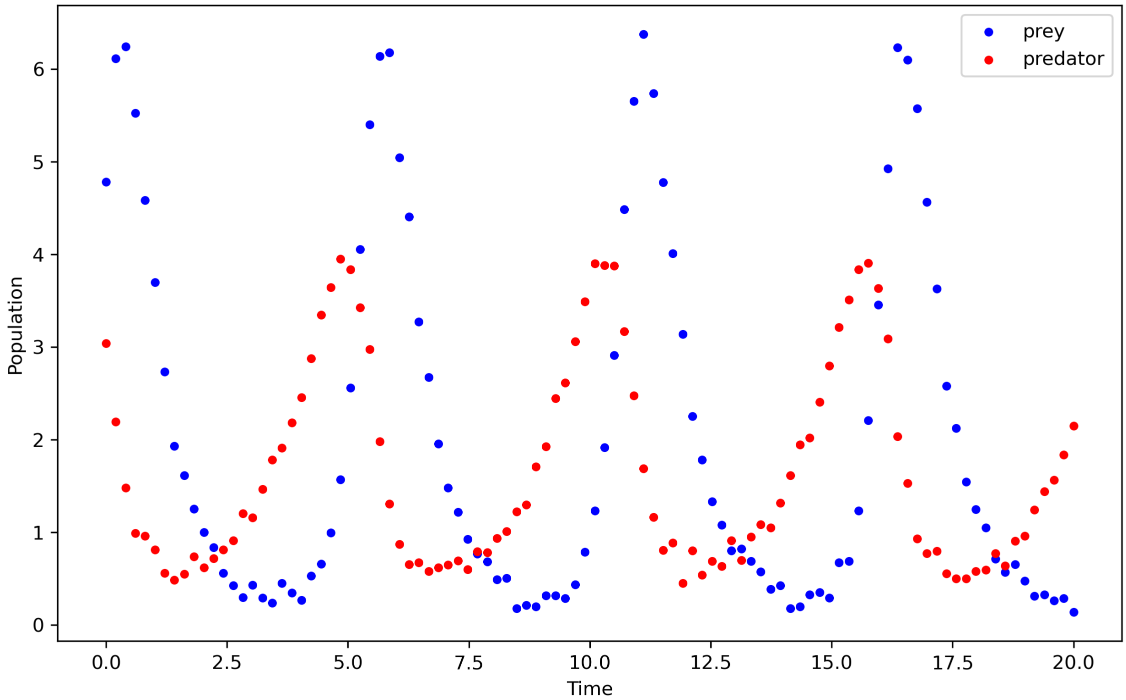
In [3]:
```
df = pd.read_csv('predator-prey-data.csv', index_col=False)
df.head()

```

Out[3]:

	Unnamed: 0	t	x	y
0	0	0.000000	4.781635	3.035257
1	1	0.202020	6.114005	2.189746
2	2	0.404040	6.238361	1.478907
3	3	0.606061	5.520524	0.989836
4	4	0.808081	4.582546	0.957827

In [4]:
```
# Loading data into read-only numpy arrays
data = df[['t','x','y']].values
# data[1], data[2] = data[2].copy(), data[1].copy()
data.flags.writeable = False


# Plotting
plt.figure(dpi =300, figsize=(10, 6))
point_width = 13
# X should be prey
plt.scatter(data[:,0], data[:,1], label = 'prey', color = 'blue', s =po
plt.scatter(data[:,0], data[:,2], label = 'predator', color = 'red', s=
plt.ylabel('Population')
plt.xlabel('Time')
plt.legend()

```

Out[4]:
```
<matplotlib.legend.Legend at 0x14c151ad390>
```



Objective functions

Defining volterra equations function

In [5]:

```
def predator_prey_odes(initial_conditions,time ,alpha, beta, delta, gam
    x = initial_conditions[0]  # initial predator population
    y = initial_conditions[1]  # initial prey population
    dxdt = (alpha * x) - (beta * x * y)  # Predator ODE
    dydt = (delta * x * y) - (gamma * y)  # Predator ODE
    return [dxdt, dydt]

#Function that will return the data for predator and prey for a given s
def predator_prey_integration(time,initial_conditions,parameters):
    alpha,beta,delta,gamma = parameters
    #odeint is now used as part of this function which returns the # of
    results = odeint(predator_prey_odes,initial_conditions, time, args=
    predator_values,prey_values = results[:,0], results[:,1]
    return np.array([predator_values,prey_values]).T

```

In [ ]:

```



```

Defining objective functions

In [6]:
```
# modulo linear error
def MSE(actual, predicted):
    '''Mean squared error'''
    return np.mean((actual - predicted)**2)

def MSE2(actual, predicted):
    '''Mean squared error, handles nan values'''
    x1, y1 = actual[:, 0], actual[:, 1]

    # Getting useful indexes
    indx_x = np.where(~np.isnan(x1))
    indx_y = np.where(~np.isnan(y1))
    x2, y2 = predicted[:, 0], predicted[:, 1]

    err1 = (x1[indx_x] - x2[indx_x])**2
    err2 = (y1[indx_y] - y2[indx_y])**2

    # Concatenate the arrays before calculating the mean
    errors = np.concatenate([err1, err2])

    # Use np.nanmean to handle NaN values during the mean calculation
    return np.nanmean(errors)


def MAE(actual, predicted):
    '''Calculate Mean Absolute Error (MAE) for multidimensional data.''
    mae = np.mean(np.abs(actual - predicted))
    return mae

def MAE2(actual, predicted):
    '''Calculate Mean Absolute Error (MAE) for multidimensional data, h
    x1, y1 = actual[:, 0], actual[:, 1]

    # Getting useful indexes
    indx_x = np.where(~np.isnan(x1))
    indx_y = np.where(~np.isnan(y1))

    x2, y2 = predicted[:, 0], predicted[:, 1]

    err1 = np.abs(x1[indx_x] - x2[indx_x])
    err2 = np.abs(y1[indx_y] - y2[indx_y])

    mae = np.nanmean(np.concatenate([err1, err2]))

    return mae


```

Algorithms & Optimisation

Defining minimization algorithms

In [7]:

```

def random_walk(parameters, variance = 0.5):
    lst = [parameter + np.random.normal(0, 1) for parameter in paramete
    # Ensure all elements are positive
    while any(x <= 0 for x in lst):
        for indx in range(len(lst)):
            if lst[indx] <= 0:
                while lst[indx] < 0:
                    lst[indx] = parameters[indx] + np.random.normal(0,
    return lst


def hill_climbing(data, time, initial_conditions, parameters, objective
    '''Tries to find the best solution using random walker'''
    # Initialize starting parameter state
    scores = []
    x_n = parameters
    all_scores = []

    current_est = predator_prey_integration(time, initial_conditions, x
    current_score = objective(data, current_est)
    scores.append(current_score)
    number_iterations= 1

    for k in range(max_iterations):
        # Generate a random walk for parameters
        x_n_1 = random_walk(x_n, variance)

        # Calculate the current and next estimations
        current_est = predator_prey_integration(time, initial_conditior
        new_estimation = predator_prey_integration(time, initial_condit

        new_score = objective(data, new_estimation)

        # If the next estimation is better, update the parameters
        if new_score < current_score:
            number_iterations = k
            current_score = new_score
            x_n = x_n_1
            scores.append(current_score)

    return x_n, scores, number_iterations
```

In [8]:
```
def simulated_annealing(initial_temp,cooling_constant, data, time, init

    temp = initial_temp #Scaling factor for random movement. We square
    start = parameters #Initial starting parameters
    x_n = start
    scores = [] #A score is just the value of the objective function ev

    current_est = predator_prey_integration(time, initial_conditions, x
    current_score = objective(data, current_est) #The current value of
    scores.append(current_score) #Keeping track of the values of the ob

    #cur = function(x) #The function value of the current x solution
    history = [x_n] #Stores previously searched x values

    for i in range (max_iterations):
        proposal = random_walk(x_n) #A new proposal for the parameters
        new_est = predator_prey_integration(time, initial_conditions, p
        new_score = objective(data, new_est) #Calculate new value of ob

        delta = new_score - current_score #Difference in objective func

        #if proposal < 0 or proposal > 1:
           #proposal = x_n # Reject proposal by setting it equal to pre

        acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate

        #if delta < 0:
           #x_n = proposal ##Accept proposal
            #current_score = new_score

        if np.random.rand() < acceptance_probability: #else if it is no
            x_n = proposal #Accept proposal
            current_score = new_score

        scores.append(current_score)
        temp = cooling_constant**i * initial_temp #Cool temperature
        #print(temp)
        history.append(x_n) #Add to history

    return x_n, scores
```

Multiple run algorithms

In [9]:
```python
def uniform_draw_g(lower_bound, upper_bound):
    while True:
        yield np.random.uniform(lower_bound, upper_bound)

def multiple_runs_annealing(initial_temp,cooling_constant,input_data,t,

    mse_total_list = []
    all_all_best = []

    for i in range(n_runs):

        x_best, scores = simulated_annealing(initial_temp,cooling_const
        all_all_best.append(x_best)

        x = predator_prey_integration(t,initial_conditions,x_best)
        mse_prey = MSE(data[:,1],x[:,0])
        mse_predator = MSE(data[:,2],x[:,1])
        mse_total = mse_prey + mse_predator

        mse_total_list.append(mse_total) #Add total MSE for this simula

    return np.array(all_all_best) , mse_total_list



def multi_run_hill_climbing(data, objective, nruns = 50, nsamples=100,
    initial_conditions = data[0][1:3]
    time = data[:,0]

    # Defining generators for variables
    alpha = uniform_draw_g(0,1)
    beta = uniform_draw_g(0,1)
    delta = uniform_draw_g(0,1)
    gamma = uniform_draw_g(0,1)

    # Lists for storing values
    parameter_list = []
    best = []
    best_score = float('inf')
    best_param = None
    num_iterations = []

    # Running simulation for
    for __ in range(nruns):

        parameters = [next(alpha), next(beta), next(delta), next(gamma)
        params, score, iterations = hill_climbing(data[:,1:3], time, ir
        parameter_list.append(params)
        num_iterations.append(iterations)
        scores.append(score)

        #Saving best parameter combination
        if score[-1] < best_score:
            best_score = score[-1]
            best_param = params

    parameter_list = np.array(parameter_list)

    return parameter_list, best_param, scores, best_score, num_iteratic
```

```
60
```

Plotting hill climbing

In [10]:

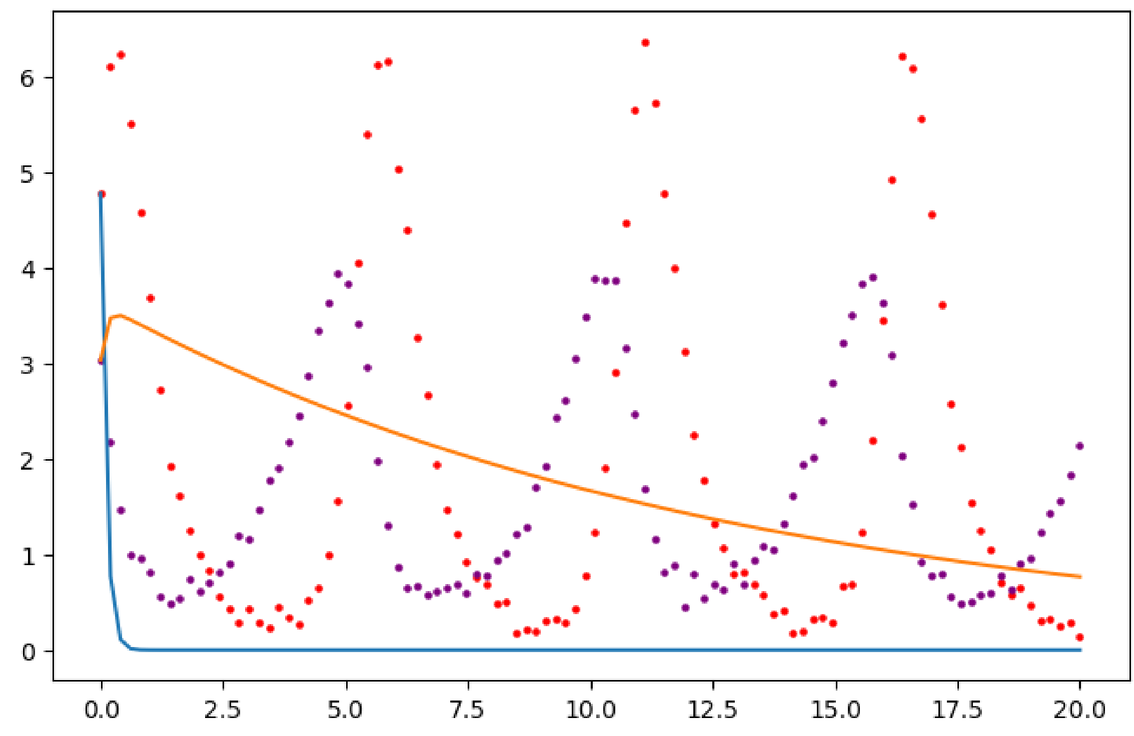
```
input_data = data[:,1:3]
# t =
initial_conditions = [input_data[0][0], input_data[0][1]]

alpha = np.random.uniform(0.5, 2)
beta = np.random.uniform(0.5, 2)
delta = np.random.uniform(0.5, 2)
gamma = np.random.uniform(0.5, 2)
parameters = [alpha, beta, delta, gamma]

# Using MSEx
x_best, scores, num_iterations = hill_climbing(input_data, data[:,0], i
print(x_best,scores,num_iterations)
```

```
[2.058470826384955, 3.3348226913782497, 0.33391386324627437, 0.07775331273
010042] [1.7841476511911036, 1.7630633552849264, 1.7463166480090968, 1.741
6628363579707, 1.7024628624472102, 1.6549539797197672, 1.541264871748595]
16
```

Plotting hill climbing results

In [11]:
```python
# t, x ,y = data
initial_conditions = [input_data[0][0], input_data[0][1]]
t = data[:,0]

parameters = x_best
# Using MSE
x = predator_prey_integration(t,initial_conditions,parameters)

# Increase the figure size
plt.figure(figsize=(8, 5))

plt.plot(t, x[:,0])
plt.plot(t, x[:,1])

plt.scatter(t, data[:,1], color= 'red', s =5)
plt.scatter(t, data[:,2], color= 'purple', s=5)

plt.figure(figsize=(10, 8))
```

Out[11]: <Figure size 1000x800 with 0 Axes>



<Figure size 1000x800 with 0 Axes>

Running simulation for different random walker variance

In [ ]:

```
# parameter_list = np.array(parameter_list)
# # Create a figure with 3x3 subplotshttp://localhost:8888/notebooks/De
# fig, axes = plt.subplots(3, 3, figsize=(12, 12), sharex=True)

# # Plot histograms on each subplot using for loops with the same color
# color = 'blue'
# titles = ['variance = 0.1', 'variance=0.25', 'variance=0.5']
# x_titles = ['alpha', 'beta', 'delta', 'omega']

# for i in range(3):
#     for j in range(3):
#         ax = axes[i, j]
#         ax.hist(parameter_list[j][:,i])
#         ax.set_xlabel(x_titles[i])
#         ax.set_ylabel('Frequency')
#         ax.set_title(titles[j])

# # Adjust layout to prevent overlapping
# plt.tight_layout()

# # Show the plot
# plt.show()

```

Running multi run for hill climbing

In [12]:

```python

# We save the parameter estimation we will use as ground truth for test
parameter_list, reference_param, scores, reference_score, num_iteratior

# Integrating with best guess
results = predator_prey_integration(t,initial_conditions,reference_parã

# Increase the figure size
plt.figure(figsize=(8, 5))

plt.plot(data[:,0], results[:,0])
plt.plot(data[:,0], results[:,1])

plt.scatter(t, data[:,1], color= 'red', s =5)
plt.scatter(t, data[:,2], color= 'purple', s=5)

plt.show()

```

```
C:\Users\Aleks\AppData\Local\anaconda3\Lib\site-packages\scipy\integrate\_
odepack_py.py:248: ODEintWarning: Excess work done on this call (perhaps w
rong Dfun type). Run with full_output = 1 to get quantitative information.
  warnings.warn(warning_msg, ODEintWarning)
```

```
---------------------------------------------------------------------------
KeyboardInterrupt                         Traceback (most recent call last)
Cell In[12], line 2
      1 # We save the parameter estimation we will use as ground truth for testing
----> 2 parameter_list, reference_param, scores, reference_score, num_iterations = multi_run_hill_climbing(data, MSE,nruns = 200)
      4 # Integrating with best guess
      5 results = predator_prey_integration(t,initial_conditions,reference_param)

Cell In[9], line 47, in multi_run_hill_climbing(data, objective, nruns, nsamples, variance)
     44 for __ in range(nruns):
     46     parameters = [next(alpha), next(beta), next(delta), next(gamma)]
---> 47     params, score, iterations = hill_climbing(data[:,1:3], time, initial_conditions, parameters, objective, max_iterations=nsamples,variance=variance)
     48     parameter_list.append(params)
     49     num_iterations.append(iterations)

Cell In[7], line 30, in hill_climbing(data, time, initial_conditions, parameters, objective, max_iterations, variance)
     28 # Calculate the current and next estimations
     29 current_est = predator_prey_integration(time, initial_conditions, x_n)
---> 30 new_estimation = predator_prey_integration(time, initial_conditions, x_n_1)
     32 new_score = objective(data, new_estimation)
     34 # If the next estimation is better, update the parameters

Cell In[5], line 12, in predator_prey_integration(time, initial_conditions, parameters)
     10 alpha,beta,delta,gamma = parameters
     11 #odeint is now used as part of this function which returns the # of infected in the model
---> 12 results = odeint(predator_prey_odes,initial_conditions, time, args=(alpha,beta,delta,gamma))
     13 predator_values,prey_values = results[:,0], results[:,1]
     14 return np.array([predator_values,prey_values]).T

File ~\AppData\Local\anaconda3\Lib\site-packages\scipy\integrate\_odepack_py.py:242, in odeint(func, y0, t, args, Dfun, col_deriv, full_output, ml, mu, rtol, atol, tcrit, h0, hmax, hmin, ixpr, mxstep, mxhnil, mxordn, mxords, printmessg, tfirst)
    240 t = copy(t)
    241 y0 = copy(y0)
--> 242 output = _odepack.odeint(func, y0, t, args, Dfun, col_deriv, ml, mu,
    243                          full_output, rtol, atol, tcrit, h0, hmax, hmin,
    244                          ixpr, mxstep, mxhnil, mxordn, mxords,
    245                          int(bool(tfirst)))
    246 if output[-1] < 0:
    247     warning_msg = _msgs[output[-1]] + " Run with full_output = 1 to get quantitative information."
```

```
KeyboardInterrupt:
```

Kaya's code section: Points removal

In [13]:

```python
def point_removal(time, input_data, points_removed, Focus = 'both'):
    '''removes points randomly'''

    #We set the seed for removing points
    random.seed(123)

    prey = input_data.T[0].copy()
    predator = input_data.T[1].copy()

    # initialize set up for removing points randomly given the bounds
    removal_options = np.arange(0,len(time))

    # choose points to be removed randomly
    if points_removed > len(removal_options):
        points_removed = len(removal_options)
        print('WARNING: Maximum number of points that can be removed ha
    removed_points_indices = random.choices(removal_options, k = points

    # remove points based on choices for points to be removed
    if Focus == 'both':
        for i in removed_points_indices:
            prey[i] = None
            predator[i] = None

    elif Focus == 'prey':
        for i in removed_points_indices:
            prey[i] = None

    elif Focus == 'predator':
        for i in removed_points_indices:
            predator[i] = None

    return np.array([time, prey, predator]).T


```

In [14]:

```
def extrema_removal(time, input_data, points_removed, Focus = 'both'):
    '''Removes points in extrema'''
    prey = input_data.T[0].copy()
    predator = input_data.T[1].copy()

    # Calculate mean and variance to set regions for data
    mean_prey_population, mean_predator_population = np.mean(prey), np.
    variance_prey, variance_predator = statistics.variance(prey), stati

    # set upper bound and lower bound for point removals
    ub_prey, lb_prey = mean_prey_population + 1.645*variance_prey/len(t
    ub_predator, lb_predator = mean_predator_population + 1.645*varianc

    # initialize set up for removing points randomly given the bounds
    prey_options = []
    predator_options = []
    # enumerate through list of stored points
    for index, prey_count in enumerate(prey):
        # check if they are in specified region
        if prey_count > ub_prey or prey_count < lb_prey:
            prey_options.append([index, prey_count, predator[index]])
    for index, predator_count in enumerate(predator):
        if predator_count > ub_predator or predator_count < lb_predator
            predator_options.append([index, prey[index], predator_count

    # remove points from list depending on which focus is set
    removal_options = []
    if Focus == 'both':
        removal_options = removal_options + prey_options + predator_opt
    elif Focus == 'prey':
        removal_options = removal_options + prey_options
    elif Focus == 'predator':
        removal_options = removal_options + predator_options
    else:
        print('Error: Removal option not known. Try either both, prey,

    # choose points to be removed randomly
    if points_removed > len(removal_options):
        points_removed = len(removal_options)
        print('WARNING: Maximum number of points that can be removed ha
    removed_points_indices = random.choices(np.array(removal_options).T

    # turn the list into integers so we can remove them based on the in
    integer_array = []
    for counter in range(len(removed_points_indices)):
        integer_array.append(int(removed_points_indices[counter]))

    # update the lists based on points we wanted to remove
    if Focus == 'both':
        for i in integer_array:
            prey[i] = None
            predator[i] = None

    elif Focus == 'prey':
        for i in integer_array:
            prey[i] = None

    elif Focus == 'predator':
        for i in integer_array:
            predator[i] = None

```

```
    return np.array(time), np.array(prey), np.array(predator)

# extrema_removal(t, input_data, 5, Focus = 'both')
```

In [15]:

```
def midpoint_removal(time, input_data, points_removed, Focus = 'both'):
    '''Removes points close to the mean'''
    prey = input_data.T[0].copy()
    predator = input_data.T[1].copy()

    # Calculate mean and variance to set regions for data
    mean_prey_population, mean_predator_population = np.mean(prey), np.
    variance_prey, variance_predator = statistics.variance(prey), stati

    # set upper bound and lower bound for point removals
    ub_prey, lb_prey = mean_prey_population + 1.645*variance_prey/len(t
    ub_predator, lb_predator = mean_predator_population + 1.645*varianc

    # initialize set up for removing points randomly given the bounds
    prey_options = []
    predator_options = []
    # enumerate through list of stored points
    for index, prey_count in enumerate(prey):
        # check if they are in specified region
        if prey_count <= ub_prey or prey_count >= lb_prey:
            prey_options.append([index, prey_count, predator[index]])
    for index, predator_count in enumerate(predator):
        if predator_count <= ub_predator or predator_count >= lb_predat
            predator_options.append([index, prey[index], predator_count

    # remove points from list depending on which focus is set
    removal_options = []
    if Focus == 'both':
        removal_options = removal_options + prey_options + predator_opt
    elif Focus == 'prey':
        removal_options = removal_options + prey_options
    elif Focus == 'predator':
        removal_options = removal_options + predator_options
    else:
        print('Error: Removal option not known. Try either both, prey,

    # choose points to be removed randomly
    if points_removed > len(removal_options):
        points_removed = len(removal_options)
        print('WARNING: Maximum number of points that can be removed ha
    removed_points_indices = random.choices(np.array(removal_options).T

    # turn the list into integers so we can remove them based on the in
    integer_array = []
    for counter in range(len(removed_points_indices)):
        integer_array.append(int(removed_points_indices[counter]))

    # update the lists based on points we wanted to remove
    if Focus == 'both':
        for i in integer_array:
            prey[i] = None
            predator[i] = None

    elif Focus == 'prey':
        for i in integer_array:
            prey[i] = None

    elif Focus == 'predator':
        for i in integer_array:
            predator[i] = None

```

```
    return np.array([time, prey, predator]).T

```

In [ ]:

```
# 1. Run multi run for different size datasets save best parameters
# 2.Calculate MSE for each run for best parameters
# 2. Do this for 2x, one only for predator, other for prey
# 4. Plot error relative to best solution of y axis
# 5. On x axis should be relative number points

```

Hypothesis testing random removal points (Aleks section)

Duplicating code for the functions I use in case they are different

In [ ]:

```
def random_walk(parameters, variance = 0.5):
    lst = [parameter + np.random.normal(0, 1) for parameter in paramet
    # Ensure all elements are positive
    while any(x <= 0 for x in lst):
        for indx in range(len(lst)):
            if lst[indx] <= 0:
                while lst[indx] < 0:
                    lst[indx] = parameters[indx] + np.random.normal(0,
    return lst


def hill_climbing(data, time, initial_conditions, parameters, objectiv
    '''Tries to find the best solution using random walker'''
    # Initialize starting parameter state
    scores = []
    x_n = parameters
    all_scores = []

    current_est = predator_prey_integration(time, initial_conditions, 
    current_score = objective(data, current_est)
    scores.append(current_score)
    number_iterations= 1

    for k in range(max_iterations):
        # Generate a random walk for parameters
        x_n_1 = random_walk(x_n, variance)

        # Calculate the current and next estimations
        current_est = predator_prey_integration(time, initial_conditio
        new_estimation = predator_prey_integration(time, initial_condi

        new_score = objective(data, new_estimation)

        # If the next estimation is better, update the parameters
        if new_score < current_score:
            number_iterations = k
            current_score = new_score
            x_n = x_n_1
            scores.append(current_score)

    return x_n, scores, number_iterations

def simulated_annealing(initial_temp,cooling_constant, data, time, ini

    temp = initial_temp #Scaling factor for random movement. We square
    start = parameters #Initial starting parameters
    x_n = start
    scores = [] #A score is just the value of the objective function e

    current_est = predator_prey_integration(time, initial_conditions, 
    current_score = objective(data, current_est) #The current value of
    scores.append(current_score) #Keeping track of the values of the o

    #cur = function(x) #The function value of the current x solution
    history = [x_n] #Stores previously searched x values

    for i in range (max_iterations):
        proposal = random_walk(x_n) #A new proposal for the parameters
        new_est = predator_prey_integration(time, initial_conditions, 
        new_score = objective(data, new_est) #Calculate new value of o

```

```
        delta = new_score - current_score #Difference in objective fun
        
        #if proposal < 0 or proposal > 1:
           #proposal = x_n # Reject proposal by setting it equal to pr
        
        acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculat
        
        #if delta < 0:
           #x_n = proposal ##Accept proposal
            #current_score = new_score
        
        if np.random.rand() < acceptance_probability: #else if it is n
            x_n = proposal #Accept proposal
            current_score = new_score
        
        scores.append(current_score)
        temp = cooling_constant**i * initial_temp #Cool temperature
        #print(temp)
        history.append(x_n) #Add to history
    
    return x_n, scores


def uniform_draw_g(lower_bound, upper_bound):
    while True:
        yield np.random.uniform(lower_bound, upper_bound)

def multiple_runs_annealing(initial_temp,cooling_constant,input_data,t

    mse_total_list = []
    all_all_best = []

    for i in range(n_runs):

        x_best, scores = simulated_annealing(initial_temp,cooling_cons
        all_all_best.append(x_best)

        x = predator_prey_integration(t,initial_conditions,x_best)
        mse_prey = MSE(data[:,1],x[:,0])
        mse_predator = MSE(data[:,2],x[:,1])
        mse_total = mse_prey + mse_predator

        mse_total_list.append(mse_total) #Add total MSE for this simul

    return np.array(all_all_best) , mse_total_list



def multi_run_hill_climbing(data, objective, nruns = 50, nsamples=100,
    initial_conditions = data[0][1:3]
    time = data[:,0]

    # Defining generators for variables
    alpha = uniform_draw_g(0,1)
    beta = uniform_draw_g(0,1)
    delta = uniform_draw_g(0,1)
    gamma = uniform_draw_g(0,1)

    # Lists for storing values
    parameter_list = []
    best = []
```

```
    best_score = float('inf')
    best_param = None
    num_iterations = []

    # Running simulation for
    for __ in range(nruns):

        parameters = [next(alpha), next(beta), next(delta), next(gamma
        params, score, iterations = hill_climbing(data[:,1:3], time, i
        parameter_list.append(params)
        num_iterations.append(iterations)
        scores.append(score)

        #Saving best parameter combination
        if score[-1] < best_score:
            best_score = score[-1]
            best_param = params

    parameter_list = np.array(parameter_list)

    return parameter_list, best_param, scores, best_score, num_iterati


def point_removal(time, input_data, points_removed, Focus = 'both'):
    '''removes points randomly'''

    #We set the seed for removing points
    random.seed(123)

    prey = input_data.T[0].copy()
    predator = input_data.T[1].copy()

    # initialize set up for removing points randomly given the bounds
    removal_options = np.arange(0,len(time))

    # choose points to be removed randomly
    if points_removed > len(removal_options):
        points_removed = len(removal_options)
        print('WARNING: Maximum number of points that can be removed ha
    removed_points_indices = random.choices(removal_options, k = point

    # remove points based on choices for points to be removed
    if Focus == 'both':
        for i in removed_points_indices:
            prey[i] = None
            predator[i] = None

    elif Focus == 'prey':
        for i in removed_points_indices:
            prey[i] = None

    elif Focus == 'predator':
        for i in removed_points_indices:
            predator[i] = None

    return np.array([time, prey, predator]).T
```

Getting distribution of averages of best guesses for hill climbing (reference dataset)

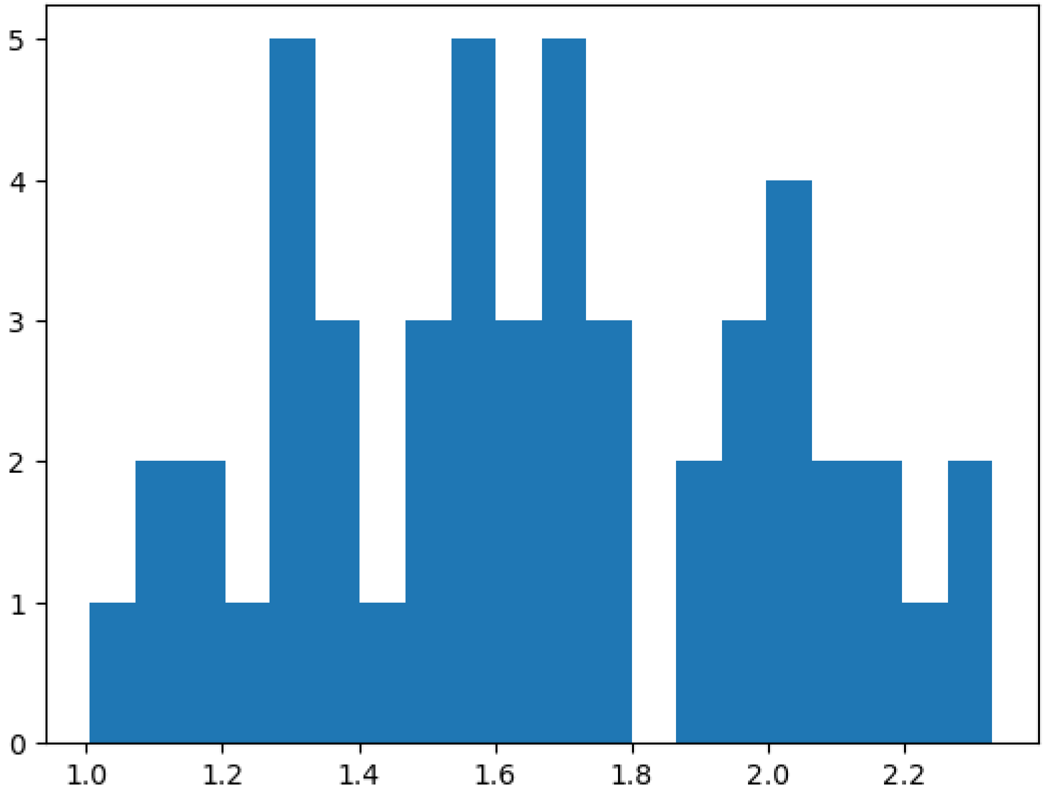
In [22]:
```
# We get the reference distribution for testing

# Timing your code
start_time = time.time()

# Reference distribution of averages
ref_average1 = []
for k in range(50):
    parameter_list, best_param, scores, best_score, num_iterations = mu
    # Appending average
    ref_average1.append(np.mean(parameter_list, axis=0))

ref_average1 = np.array(ref_average1)

end_time = time.time()

# Calculating and printing the total time
total_time = end_time - start_time
print(f"Total time taken: {total_time} seconds")

# param_distribution, reference_param, scores, reference_score, num_ite
```

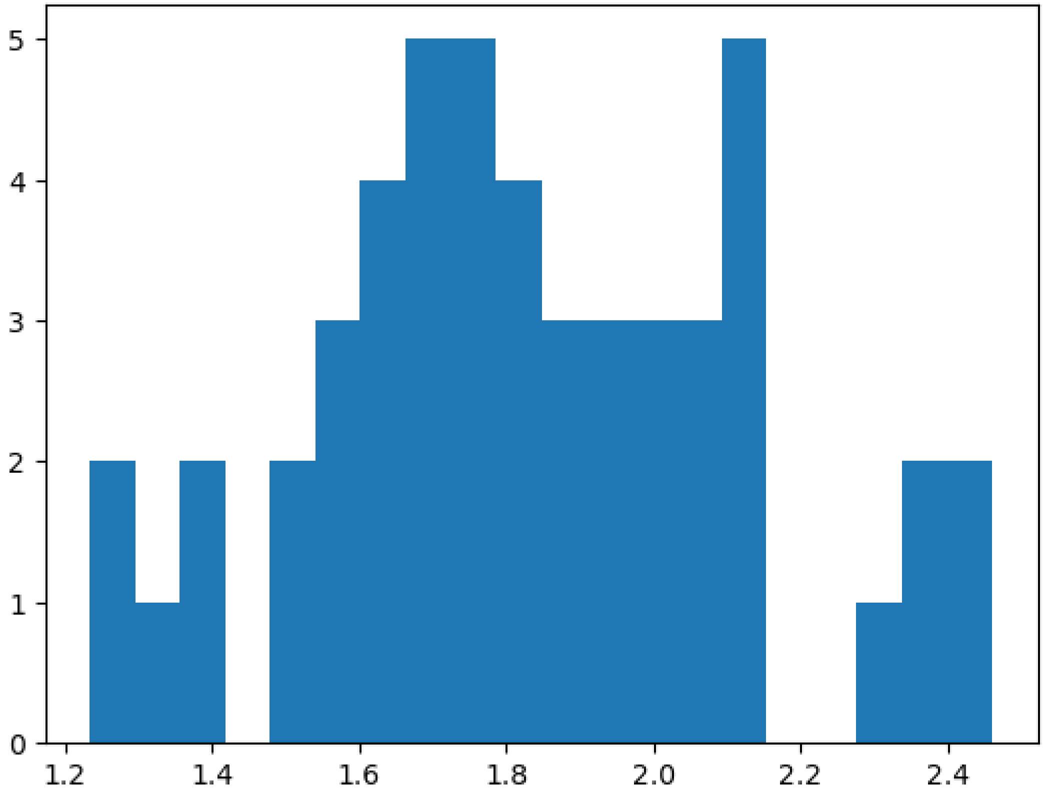
```
Total time taken: 439.93707609176636 seconds
```

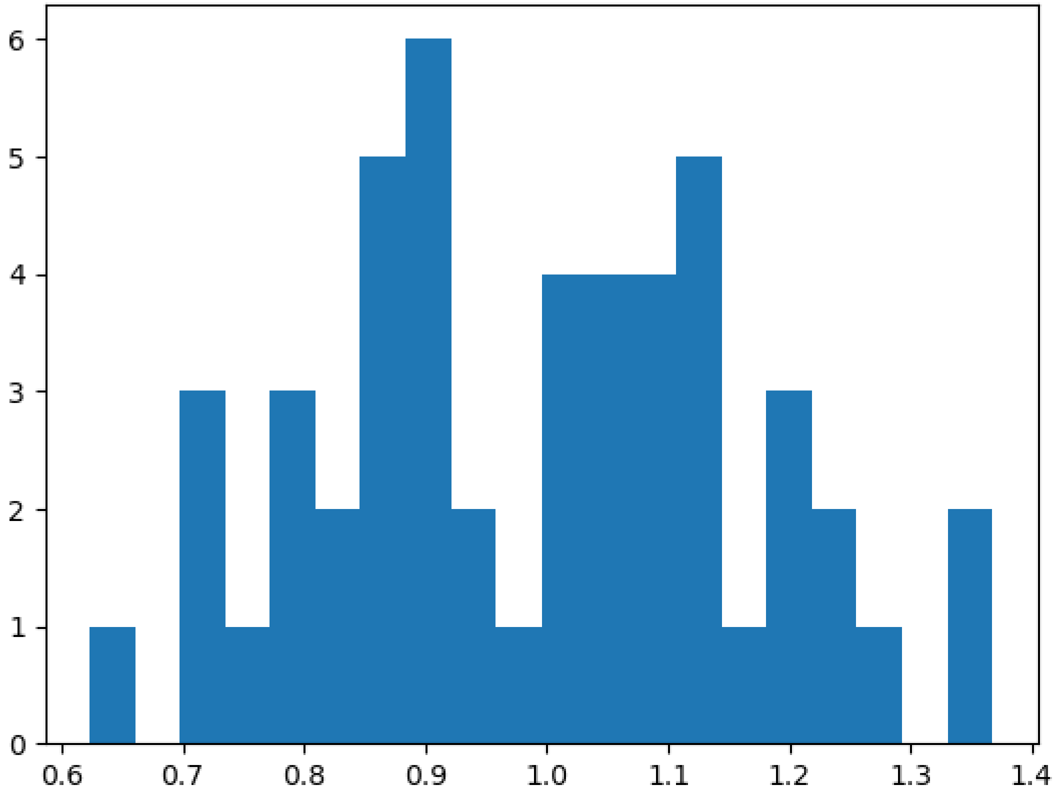
In [ ]:
```

```

These histograms plots are optional (I dont think im adding them to the report)

In [24]:
```
# num_iterations
# print(ref_average)
# for k in range(3):
#         plt.hist(ref_average1[:,k], bins = 20)
#         plt.show()

# print(np.var(ref_average1,axis=0))
```








```
[0.1170307  0.08449044 0.02953349 0.11417836]
```

In [25]:
```
# # print(param_distribution)
# plt.axvline(x=reference_param[0], color='r', linestyle='--', label='V
# plt.hist(param_distribution[:,0], bins =30)
```

```
---------------------------------------------------------------------------
NameError                                 Traceback (most recent call las
t)
Cell In[25], line 2
      1 # print(param_distribution)
----> 2 plt.axvline(x=reference_param[0], color='r', linestyle='--', label
='Vertical Line at x=2.5')
      3 plt.hist(param_distribution[:,0], bins =30)

NameError: name 'reference_param' is not defined
```

Running welch test between reference distribution of averages and incomplete time series (hill climbing)

In [27]:
```
def random_point_ttests(data,ref_distribution, points_removed, focus_ch
    '''runs welch test for multi run of hill climbing for every paramet
    param_distribution = ref_distribution
#     focus_choices = ['prey', 'predator', 'both']
#     focus_choices = ['prey', 'predator', 'both']
    scores = [[],[]]
    p_values = {'prey': [], 'predator': [], 'both': []}

    for indx, choice in enumerate(focus_choices):
        print(choice)
        for npoints in points_removed:
            print(npoints)
    #         print(f"Points removed: {npoints}")
            limited_data = point_removal(data[:,0], data[:,1:3], npoint
            #Getting distribution of averages
            average_distribution = []
            for k in range(30):
                parameter_list, best_param, scores, best_score, num_ite
                #Appending average
                average_distribution.append(np.mean(parameter_list, axi

            average_distribution = np.array(average_distribution)
            t_stat1, p_value1 = scipy.stats.ttest_ind(ref_distribution[
            t_stat2, p_value2 = scipy.stats.ttest_ind(ref_distribution[
            t_stat3, p_value3 = scipy.stats.ttest_ind(ref_distribution[
            t_stat3, p_value4 = scipy.stats.ttest_ind(ref_distribution[
            p_values[choice].append([p_value1, p_value2, p_value3, p_va

    return p_values


```

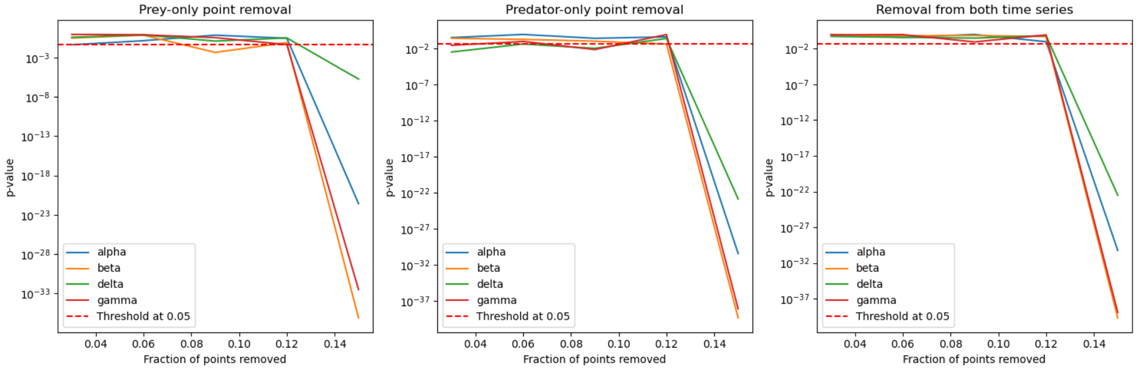
In [28]:

```
start_time = time.time()
p_values_hill_climbing =random_point_ttests(data,ref_average1, np.arang

end_time = time.time()

# Calculating and printing the total time
total_time = end_time - start_time
print(f"Total time taken: {total_time} seconds")
```

```
prey
3
6
9
12
15

C:\Users\Aleks\AppData\Local\anaconda3\Lib\site-packages\scipy\integrate\_
odepack_py.py:248: ODEintWarning: Excess accuracy requested (tolerances to
o small). Run with full_output = 1 to get quantitative information.
  warnings.warn(warning_msg, ODEintWarning)

predator
3
6
9
12
15

C:\Users\Aleks\AppData\Local\anaconda3\Lib\site-packages\scipy\integrate\_
odepack_py.py:248: ODEintWarning: Illegal input detected (internal error).
Run with full_output = 1 to get quantitative information.
  warnings.warn(warning_msg, ODEintWarning)
C:\Users\Aleks\AppData\Local\anaconda3\Lib\site-packages\scipy\integrate\_
odepack_py.py:248: ODEintWarning: Run terminated (internal error). Run wit
h full_output = 1 to get quantitative information.
  warnings.warn(warning_msg, ODEintWarning)

both
3
6
9
12
15
Total time taken: 1060.7844800949097 seconds
```

Visualizing p-values from welch test for hill climbing

In [43]:
```

# Fraction points removed
fraction_points = np.arange(3,16,3) / 100
p_values_prey = np.array(p_values_hill_climbing['prey'])
p_values_predator = np.array(p_values_hill_climbing['predator'])
p_values_both = np.array(p_values_hill_climbing['both'])

# Creating subplots
fig, axes = plt.subplots(1, 3, figsize=(15, 5))

# Plotting p-values when only prey points are removed
axes[0].plot(fraction_points, p_values_prey[:, 0], label='alpha')
axes[0].plot(fraction_points, p_values_prey[:, 1], label='beta')
axes[0].plot(fraction_points, p_values_prey[:, 2], label='delta')
axes[0].plot(fraction_points, p_values_prey[:, 3], label='gamma')
axes[0].axhline(y=0.05, color='r', linestyle='--', label='Threshold at
axes[0].set_ylabel('p-value')
axes[0].set_xlabel('Fraction of points removed')
axes[0].set_yscale('log')
axes[0].set_title('Prey-only point removal')  # Add title to the first
axes[0].legend()

# Plotting p-values when only predator points are removed
axes[1].plot(fraction_points, p_values_predator[:, 0], label='alpha')
axes[1].plot(fraction_points, p_values_predator[:, 1], label='beta')
axes[1].plot(fraction_points, p_values_predator[:, 2], label='delta')
axes[1].plot(fraction_points, p_values_predator[:, 3], label='gamma')
axes[1].axhline(y=0.05, color='r', linestyle='--', label='Threshold at
axes[1].set_ylabel('p-value')
axes[1].set_xlabel('Fraction of points removed')
axes[1].set_yscale('log')
axes[1].set_title('Predator-only point removal')
axes[1].legend()

# Plotting p-values when both prey and predator points are removed
axes[2].plot(fraction_points, p_values_both[:, 0], label='alpha')
axes[2].plot(fraction_points, p_values_both[:, 1], label='beta')
axes[2].plot(fraction_points, p_values_both[:, 2], label='delta')
axes[2].plot(fraction_points, p_values_both[:, 3], label='gamma')
axes[2].axhline(y=0.05, color='r', linestyle='--', label='Threshold at
axes[2].set_ylabel('p-value')
axes[2].set_xlabel('Fraction of points removed')
axes[2].set_yscale('log')
axes[2].set_title('Removal from both time series')
axes[2].legend()

# Adjusting layout
plt.tight_layout()
plt.savefig('welch_tests_hill_climbing', dpi = 300)
plt.show()

```


Prey-only point removal
Predator-only point removal
Removal from both time series
p-value
Fraction of points removed
alpha
beta
delta
gamma
Threshold at 0.05

Getting distribution of averages of best guesses for simulated annealing (reference dataset)

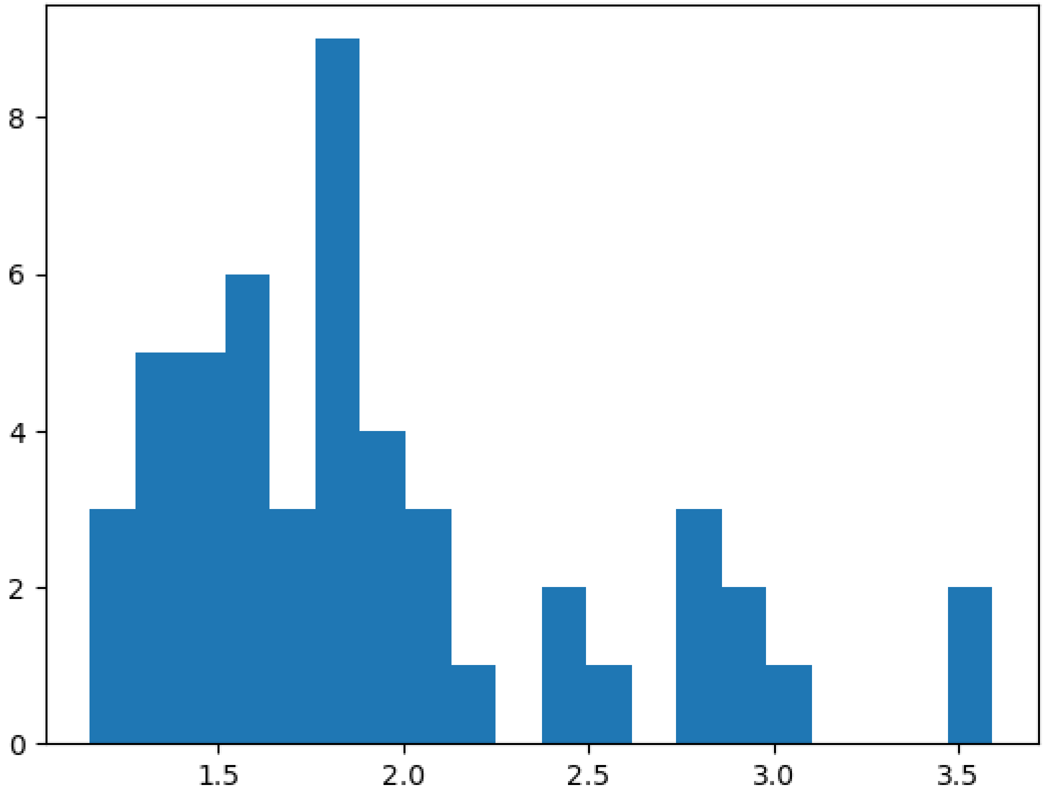
In [40]:
```
# We get the reference distribution for testing

# Timing your code
start_time = time.time()

initial_temp = 20
cooling_constant = 0.10

#Taking random draw for initial parameters (initial guess)
alpha = np.random.uniform(0,1)
beta = np.random.uniform(0,1)
delta = np.random.uniform(0,1)
gamma = np.random.uniform(0,1)
parameters = [alpha, beta, delta, gamma]
parameters = [alpha, beta, delta, gamma]

# Reference distribution of averages
ref_average2 = []
for k in range(50):
#     parameter_list, best_param, scores, best_score, num_iterations =
    parameter_list, scores= multiple_runs_annealing(initial_temp,coolin
    # Appending average
    ref_average2.append(np.mean(parameter_list, axis=0))

ref_average2 = np.array(ref_average2)
end_time = time.time()

# Calculating and printing the total time
total_time = end_time - start_time
print(f"Total time taken: {total_time} seconds")
```

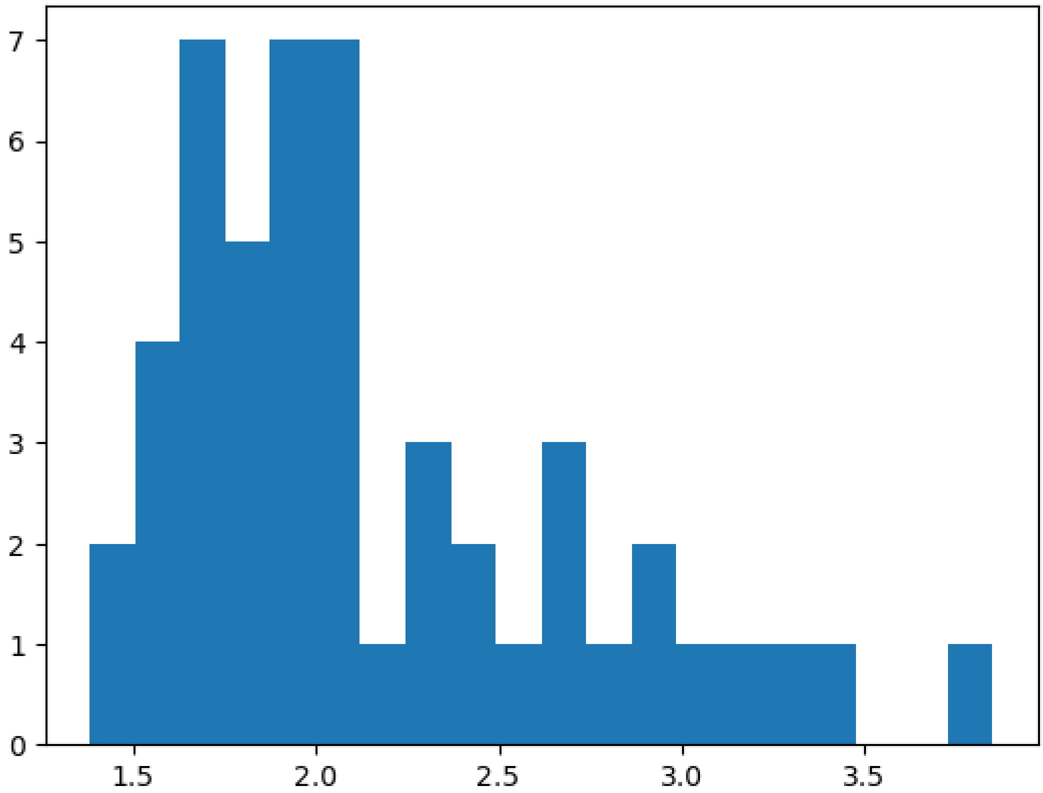
```
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\951072704.py:25: RuntimeWa
rning: overflow encountered in exp
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\951072704.py:25: RuntimeWa
rning: overflow encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\951072704.py:25: RuntimeWa
rning: divide by zero encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability

Total time taken: 1028.2772996425629 seconds
```

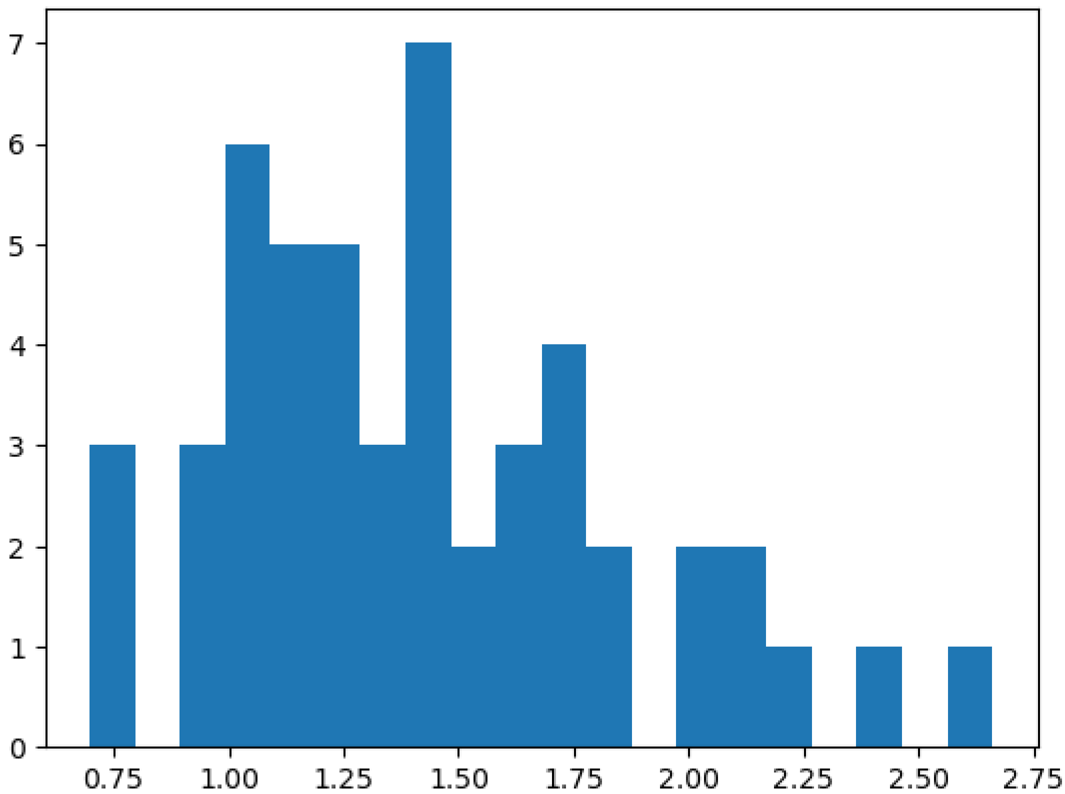
In [41]:

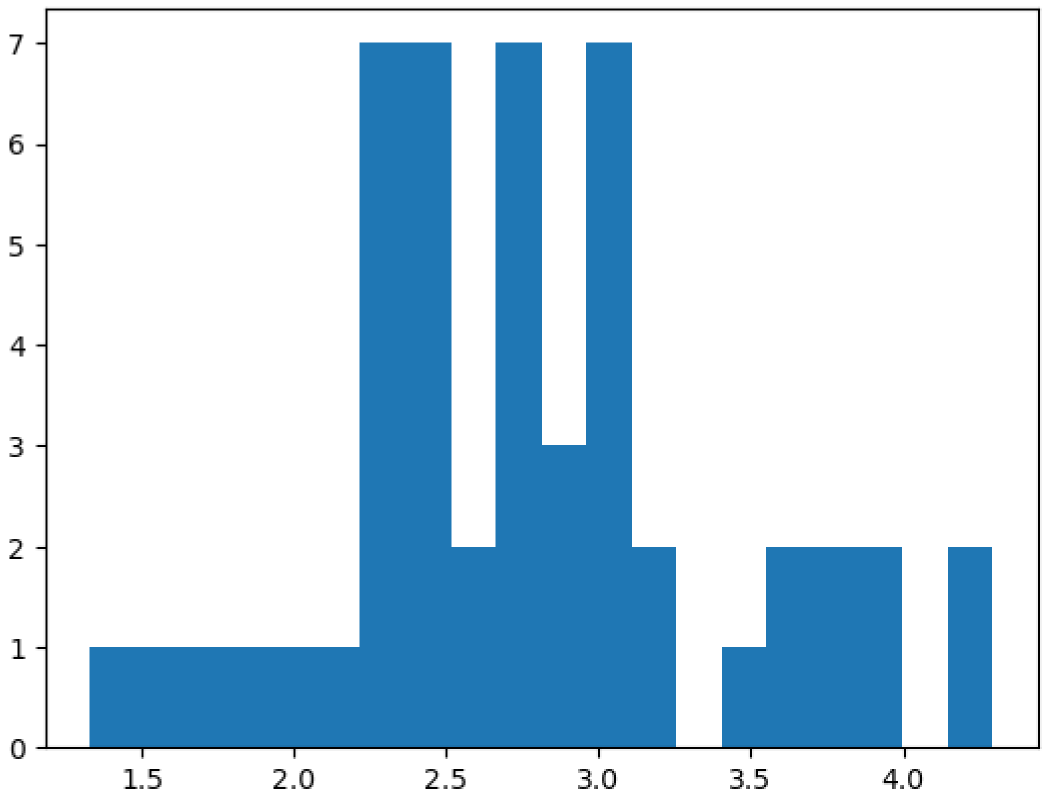
```
# print(np.var(ref_average1,axis=0))
print(np.var(ref_average2,axis=0))

for k in range(4):
        plt.hist(ref_average2[:,k], bins = 20)
        plt.show()

```

```
[0.35481283 0.29947082 0.18320262 0.4135741 ]
```






7
6
5
4
3
2
1
0
0.75
1.00
1.25
1.50
1.75
2.00
2.25
2.50
2.75


7
6
5
4
3
2
1
0
1.5
2.0
2.5
3.0
3.5
4.0

In [20]:

```
def random_point_ttests2(data,ref_distribution, points_removed, focus_c
    '''runs welch test for multi run of simulated annealing for every p
    param_distribution = ref_distribution
#     focus_choices = ['prey', 'predator', 'both']
#     focus_choices = ['prey', 'predator', 'both']
    scores = [[],[]]
    p_values = {'prey': [], 'predator': [], 'both': []}
    initial_temp = 20
    cooling_constant = 0.10

    for indx, choice in enumerate(focus_choices):
        print(choice)
        #We Iteratively increase the amount of points we remove
        for npoints in points_removed:
            print(f"points: {npoints}")
    #         print(f"Points removed: {npoints}")
            limited_data = point_removal(data[:,0], data[:,1:3], npoint
            #Getting distribution of averages
            average_distribution = []
            for k in range(30):
                parameter_list, best_score = multiple_runs_annealing(ir
                #Appending average
                average_distribution.append(np.mean(parameter_list, axi

            average_distribution = np.array(average_distribution)
            t_stat1, p_value1 = scipy.stats.ttest_ind(ref_distribution[
            t_stat2, p_value2 = scipy.stats.ttest_ind(ref_distribution[
            t_stat3, p_value3 = scipy.stats.ttest_ind(ref_distribution[
            t_stat3, p_value4 = scipy.stats.ttest_ind(ref_distribution[
            p_values[choice].append([p_value1, p_value2, p_value3, p_va

    return p_values
```

In [44]:
```python
start_time = time.time()

# Calculating p-values for t-test
points_removed_annealing = np.arange(3,16,3)
p_values_annealing = random_point_ttests2(data,ref_average2,points_remo

end_time = time.time()
# Calculating and printing the total time
total_time = end_time - start_time
print(f"Total time taken: {total_time/60} min")
```

```
prey
points: 3

C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\951072704.py:25: RuntimeWa
rning: overflow encountered in exp
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\951072704.py:25: RuntimeWa
rning: overflow encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\951072704.py:25: RuntimeWa
rning: divide by zero encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability

points: 6
points: 9
points: 12
points: 15

C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\951072704.py:25: RuntimeWa
rning: invalid value encountered in scalar divide
  acceptance_probability = min(np.exp(-(delta/temp)),1)#Calculate acceptan
ce probability
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\264107711.py:4: RuntimeWar
ning: overflow encountered in scalar multiply
  dxdt = (alpha * x) - (beta * x * y)  # Predator ODE
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\264107711.py:5: RuntimeWar
ning: overflow encountered in scalar multiply
  dydt = (delta * x * y) - (gamma * y)  # Predator ODE
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\2008888099.py:4: RuntimeWa
rning: overflow encountered in square
  return np.mean((actual - predicted)**2)
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\2008888099.py:15: RuntimeW
arning: overflow encountered in square
  err1 = (x1[indx_x] - x2[indx_x])**2

predator
points: 3
points: 6
points: 9
points: 12
points: 15

C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\2008888099.py:16: RuntimeW
arning: overflow encountered in square
  err2 = (y1[indx_y] - y2[indx_y])**2
```

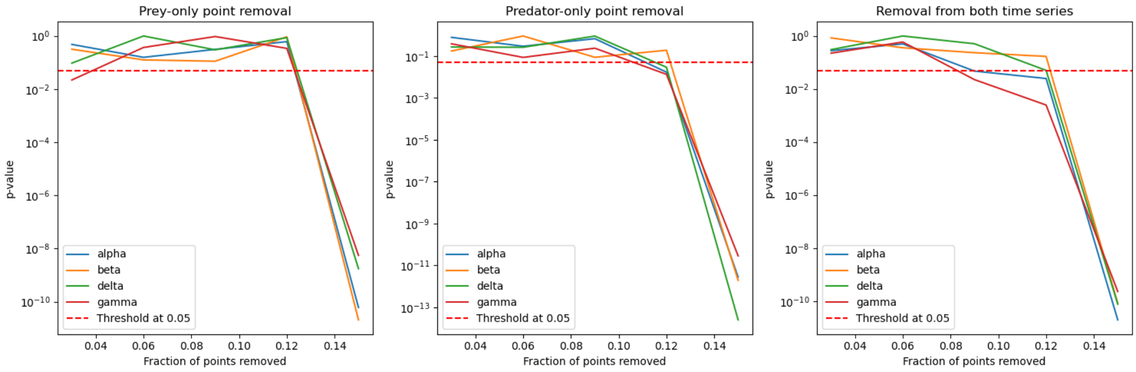
```
both
points: 3
points: 6
points: 9
points: 12
points: 15
```

```
C:\Users\Aleks\AppData\Local\Temp\ipykernel_764\264107711.py:4: RuntimeWar
ning: invalid value encountered in scalar subtract
  dxdt = (alpha * x) - (beta * x * y)  # Predator ODE
```

```
Total time taken: 63.41379015445709 min
```

In [46]:

```

# Fraction points removed

# Fraction points removed
fraction_points = np.arange(3,16,3) / 100
p_values_prey = np.array(p_values_annealing['prey'])
p_values_predator = np.array(p_values_annealing['predator'])
p_values_both = np.array(p_values_annealing['both'])

# Creating subplots
fig, axes = plt.subplots(1, 3, figsize=(15, 5))

# Plotting p-values when only prey points are removed
axes[0].plot(fraction_points, p_values_prey[:, 0], label='alpha')
axes[0].plot(fraction_points, p_values_prey[:, 1], label='beta')
axes[0].plot(fraction_points, p_values_prey[:, 2], label='delta')
axes[0].plot(fraction_points, p_values_prey[:, 3], label='gamma')
axes[0].axhline(y=0.05, color='r', linestyle='--', label='Threshold at
axes[0].set_ylabel('p-value')
axes[0].set_xlabel('Fraction of points removed')
axes[0].set_yscale('log')
axes[0].set_title('Prey-only point removal')  # Add title to the first
axes[0].legend()

# Plotting p-values when only predator points are removed
axes[1].plot(fraction_points, p_values_predator[:, 0], label='alpha')
axes[1].plot(fraction_points, p_values_predator[:, 1], label='beta')
axes[1].plot(fraction_points, p_values_predator[:, 2], label='delta')
axes[1].plot(fraction_points, p_values_predator[:, 3], label='gamma')
axes[1].axhline(y=0.05, color='r', linestyle='--', label='Threshold at
axes[1].set_ylabel('p-value')
axes[1].set_xlabel('Fraction of points removed')
axes[1].set_yscale('log')
axes[1].set_title('Predator-only point removal')
axes[1].legend()

# Plotting p-values when both prey and predator points are removed
axes[2].plot(fraction_points, p_values_both[:, 0], label='alpha')
axes[2].plot(fraction_points, p_values_both[:, 1], label='beta')
axes[2].plot(fraction_points, p_values_both[:, 2], label='delta')
axes[2].plot(fraction_points, p_values_both[:, 3], label='gamma')
axes[2].axhline(y=0.05, color='r', linestyle='--', label='Threshold at
axes[2].set_ylabel('p-value')
axes[2].set_xlabel('Fraction of points removed')
axes[2].set_yscale('log')
axes[2].set_title('Removal from both time series')
axes[2].legend()

plt.savefig('welch_test_annealing', dpi=300)

# Adjusting layout
plt.tight_layout()
plt.show()
```


Prey-only point removal
Predator-only point removal
Removal from both time series
p-value
Fraction of points removed
alpha
beta
delta
gamma
Threshold at 0.05

In [ ]:
```
1
```