University of New Mexico
Department of Computer Science
Exam II
CS261: Mathematical Foundations of Computer Science

Name:_____
Email:_____

Instructions:

1. Write your name and email address legibly in the space provided above.
2. Write your name legibly at the upper right hand corner on each page.
3. There are 4 problems in the exam.
4. Show the steps of your solutions.
5. This is a close-book exam. You must not discuss the questions with anyone except the professor in charge.
6. You are allowed to use a one page double-sided handwritten "cheating sheet" that you have brought to the exam and a "dumb" calculator. Nothing else permitted.
7. Write your answers legibly.
8. Don't spend too much time on any single problem. All questions are weighted equally. If you get stuck, move on to something else and get back later.
9. Good luck and enjoy the exam!

1. (Euclid's Algorithm) Answer the following questions:

(a) Use Extended Euclid's Algorithm to calculate gcd(127, 38) and the integers s, t such that gcd(127, 38)=127s + 38t.

(b) Use the Chinese Remainder's Theorem to find ALL solutions to the system of congruence:

$$\begin{cases} x \bmod 13 = 5 \\ x \bmod 14 = 3 \end{cases}$$

2. (Arbitrary Base Numbers) Answer the following questions and show the detailed steps of your solutions:

(a) Find the hexadecimal representation of the decimal number 224.

(b) Find the base 5 representation of the decimal number 270.

(c) Find the decimal representation of the octal (i.e., based 8) number $(2740)_8$.

Calculate (d), (e) for two binary numbers without converting the numbers to decimal.

(d) $(1101)_2 + (1010)_2$

(e) $(1011)_2 \cdot (1001)_2$

3. (Logic) Answer the following questions:

(a) Construct the truth table for $(p \to r) \vee (q \to r) \leftrightarrow (p \vee q) \to r$

(b) Is the following argument valid?

$$\begin{array}{rc} & (r \wedge \neg s) \vee (q \wedge \neg s) \wedge \\ & \neg s \to \big((p \wedge r) \to t\big) \\ & t \to (s \wedge \neg r) \\ \therefore & p \to r \end{array}$$

Hint: consider the definition of valid argument.

4. (Number Theory) The natural numbers (i.e., positive integers) can be divided into 3 categories, the ***abundant***, the ***perfect***, and the ***deficient***. A natural number d is a ***proper divisor*** of a natural number n if $d|n$ and $d < n$. A natural number n is perfect if n is equal to the sum of all of its proper divisors; n is abundant if n is less than the sum of all of its proper divisors; n is deficient if it is larger than the sum of all its proper divisors.

Answer the following questions:

(a) Which categories do the numbers 1, 2, 6, 12 belong to, i.e., abundant, perfect or deficient? Explain why?

(b) Which categories do the prime numbers belong to, abundant, perfect or deficient? Explain why?

(c) Let p be an arbitrary prime number, and n a natural number. Which category does p^n belong to, i.e., abundant, perfect or deficient? Explain why?

(d) Let n be an abundant number, and k be a positive integer, which category does kn, the product of k and n, belong to, i.e., abundant, perfect or deficient? Explain why?