

Homework 3: Due Oct 6 (Thursday)

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
 - (a) Albuquerque is the capital of New Mexico.
 - (b) Texas is the largest state in terms of area in the US.
 - (c) In the 1930s, the average price for a plane ticket is \$2, while the average price for a train ticket is \$0.25.
 - (d) $1 + 1 = 2$.
 - (e) $1 \div 0 = 1$.
 - (f) $x + 1 = 2$.
2. Let p , q , and r be the propositions:
 p = "You have the flu."
 q = "You miss the final examination."
 r = "You pass the course."
Express each of these propositions as an English sentence.
 - (a) $p \rightarrow q$
 - (b) $\neg q \leftrightarrow r$
 - (c) $q \rightarrow \neg r$
 - (d) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$
 - (e) $\neg q \rightarrow \neg p$
3. Construct the truth tables for the following compound propositions:
 - (a) $p \wedge \neg p$
 - (b) $p \vee \neg p$
 - (c) $(p \vee \neg q) \rightarrow q$
 - (d) $(p \rightarrow q) \leftrightarrow (\neg q \rightarrow \neg p)$
 - (e) $(p \rightarrow q) \rightarrow (q \rightarrow p)$
 - (f) $(p \vee q) \rightarrow (p \oplus q)$
 - (g) $(p \oplus q) \rightarrow (p \wedge q)$
 - (h) $(p \vee q) \oplus (p \wedge q)$
4. Show that if p , q , and r are compound propositions such that p and q are logically equivalent and q and r are logically equivalent, then p and r are logically equivalent.
5. Translate the following English sentences into logic expressions:

- (a) You can see a rated R movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of p : “You can see a rated R movie”, q : “You are over 18 years old”, and r : “You have the permission of a parent”.
 - (b) To use the wireless network in Los Angeles International airport you must pay the daily fee unless you are an AT&T customer. Express your answer in terms of p : “You can use the wireless network in Los Angeles International airport”, q : “You pay the daily fee”, and r : “You are an AT&T customer”.
6. Construct a NAND gate only digital circuit to realize the logic expression $p \leftrightarrow q$.
7. Use De Morgan’s laws to find the negation of each of the following statements.
- (a) CS students are smart and happy.
 - (b) To get into CS, you need to be good at math or good at programming.
8. Determine if the following statements are tautologies:
- (a) $(p \wedge q) \rightarrow p$
 - (b) $p \rightarrow (p \vee q)$
 - (c) $\neg p \rightarrow (p \rightarrow q)$
 - (d) $(p \wedge q) \rightarrow (p \rightarrow q)$
 - (e) $\neg(p \rightarrow q) \rightarrow p$
 - (f) $\neg(p \rightarrow q) \rightarrow \neg q$
9. Let $P(x)$ be the statement “the word x contains the letter a ”. What are the truth values for the following?
- (a) $P(\text{orange})$
 - (b) $P(\text{lemon})$
 - (c) $P(\text{true})$
 - (d) $P(\text{false})$
10. Translate each of these statements into logical expressions by using quantifiers and predicates with one or two variables.
- (a) A student in our discrete math class has lived in Florida.
 - (b) There is a student in our discrete math class who got the perfect grade in Midterm I.
 - (c) Everyone in our class loves discrete math.
 - (d) There is a student in our class who has been to every state in the US.
 - (e) There is a student in our class who has been to every city of at least one state in the country.

- (f) Everyone in our class has been to at least one state in the country.
11. Argue that $(\exists x P(x)) \wedge (\exists x Q(x))$ and $\exists x (P(x) \wedge Q(x))$ are not logically equivalent.
 12. What rule of inference is used in each of these arguments?
 - (a) Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
 - (b) Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
 - (c) If it is rainy, then the pool will be closed. It is rainy. Therefore, the pool is closed.
 - (d) If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
 - (e) If I go swimming, then I will stay in the sun too long. If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn.
 13. What rule of inference is used in each of these arguments?
 - (a) Kangaroos live in Australia and are marsupials. Therefore, kangaroos are marsupials.
 - (b) It is either hotter than 100 degrees today or the air pollution is dangerous. It is less than 100 degrees outside today. Therefore, the air pollution is dangerous.
 - (c) Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore, Linda can work as a lifeguard.
 - (d) Steve will work at a computer company this summer. Therefore, this summer Steve will work at a computer company or he will be a beach bum.
 - (e) If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, then I will understand the material.
 14. Use rules of inference to show that the hypotheses “If it does not rain or if it is not foggy, then the sailing race will be held and the lifesaving demonstration will go on”, “If the sailing race is held, then the trophy will be awarded”, and “The trophy was not awarded” imply the conclusion “It rained”.
 15. What rules of inference are used in this argument? “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”
 16. For each of these arguments determine whether the argument is correct or incorrect and explain why.
 - (a) All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.

- (b) Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics. Therefore, Natasha is a computer science major.
 - (c) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.
 - (d) Everyone who eats granola everyday is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.
17. Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?
- (a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.
 - (b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$.
 - (c) If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.
18. Is the following argument valid? Babies are illogical. Nobody is despised who can manage a crocodile. Illogical persons are despised. Babies cannot manage crocodiles.