University of New Mexico
Department of Computer Science
Midterm Exam I
CS261: Mathematical Foundations of Computer Science

Name:______________________________
Email:_______________________________

Instructions:

1. Write your name and email address legibly in the space provided above.
2. Write your name legibly at the upper right hand corner on each page.
3. There are 4 problems in the exam.
4. This is a close-book exam. You must not discuss the questions with anyone except the professor in charge.
5. You are only allowed to use a one page double-sided handwritten “cheating sheet” that you have brought to the exam. Nothing else permitted.
6. Write your answers legibly.
7. Don’t spend too much time on any single problem. All questions are weighted equally. If you get stuck, move on to something else and get back later.
8. Good luck and enjoy the exam!

1. (10pt) (Set Theory) Answer the following questions:

(a) Let $A = \{a, b, c\}$ and $B = \{d, e\}$. What is $A \cup B$, $A \cap B$, $A - B$, $A \oplus B$, and 2^A (the power set of A)?

(b) In class, we know that in general $A \times B$ is not the same as $B \times A$. Under what situation, will $A \times B = B \times A$? Explain why? What is $A \times \emptyset$, where ϕ is the empty set.

(c) Does $C - A = C - B$ imply $A = B$? Explain why?

(d) Does $A - C = B - C$ imply $A = B$? Explain why?

2. (10pt) (Sequences and Recurrence Relations) Answer the following questions with brief explanations:

(a) Find a simple analytical form for the sum $\sum_{j=0}^{n}\left(\frac{10}{11}\right)^{j}$. What happens when n goes to infinity?

(b) Find a simple analytical form for the sum $\sum_{j=0}^{n}(2j+1)$.

(c) Consider the following recurrence relation, which is called the Stern's diatomic series:

$$\begin{cases} f_0 = 0 \\ f_1 = 1 \\ f_{2n} = f_n \\ f_{2n+1} = f_n + f_{n+1} \end{cases}$$

(c.1) Write the terms f_2, f_3, f_4, f_5, f_6, f_7, f_8, f_9, f_{10}.

(c.2) Consider the Calkin–Wilf sequence defined as $g_n = \frac{f_n}{f_{n+1}}$.

Write the terms g_0, g_1, g_2, g_3, g_4, g_5, g_6, g_7, g_8, g_9.

Can you guess what the Calkin–Wilf sequence produce?

3. (10pt) (Logic) Answer the following questions:

(a) Are $\neg\big((p \to q) \land (p \to r)\big)$ and $p \land \neg q \land \neg r$ logically equivalent?

(b) Translate the following sentence into logic using nested quantifiers:

"For every rational number x, there exists two integers a and b, such that $x = \frac{a}{b}$.

(c) Consider the following sentence:

"If the streets are wet, then it has rained."

"The streets are wet."

"Therefore, it has rained."

(c.1) Translate the above sentence into logic using the proposition variables:

p = "the streets are wet"

q = "it has rained"

(c.2) Is the argument valid? Is the argument correct?

4. (10pt) (Cardinality)

Let $\mathbb{N} = \{1, 2, 3,4, \dots\}$ be the set of national numbers. Is $\mathbb{N}\times\mathbb{N}\times\mathbb{N}$ countable? Prove your claim?