Sept 20, 2016

Argument: a list of premises followed by a conclusion.

Logic form: $(p_1 \wedge p_2 \wedge \cdots \wedge p_k) \longrightarrow q$

Argument Form:

$\because$ because

$$
\begin{array}{l}
p_1 \\
p_2 \\
\vdots \\
p_n \\
\hline
\therefore \; q
\end{array}
$$

therefore

All whales are mammals

Mammals are warm blooded

Therefore all whales are warm blooded.

Translation 1

$$\begin{array}{l} \quad p \\ \quad q \\ \hline \therefore \; r \end{array}$$

$P(x)$: x is a whale

$Q(x)$: x is a mammal

$R(x)$: x is warm blooded

$$\begin{array}{l} \forall x\,(P(x) \rightarrow Q(x)) \\ \forall x\,(Q(x) \rightarrow R(x)) \\ \hline \therefore \forall x\,(P(x) \rightarrow R(x)) \end{array}$$

All computer science students are good at math

I'm not good at math

therefore I'm not a CS student

Introduce

$P(x)$: x is a CS student

$Q(x)$: x is good at math

$\cancel{R(x)}$ $\forall x \; (P(x) \rightarrow Q(x))$

$\cancel{\neg Q(x)}$ $\neg Q(I)$

$\therefore \quad \neg P(I)$

An argument is valid if it is impossible for the conclusion to be false when all the premises are True, i.e., the underlying implication is a tautology.

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_k) \rightarrow q$$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If $\underbrace{\text{Sean travels to a conference}}_{p}$

then $\underbrace{\text{261 lecture will be replace by an exam.}}_{q}$

261 Lecture is replaced by an exam

therefore Sean travels to a conference

$$\begin{array}{l} p \to q \\ q \\ \hline \therefore\ p \end{array}$$

$$((p \to q) \wedge q) \to p$$

p	q	$p \to q$	$(p \to q) \wedge q$	$(p \to q) \wedge q \to p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

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if I travel, lecture ⇐ exam

lecture is Not replaced by an exam

therefore I didn't travel

$p \to q$

$\neg q$

$\therefore \neg p$

$((p \to q) \wedge \neg q) \to \neg p$

p	$\neg p$	q	$p \to q$	$\neg q$	$(p \to q) \wedge \neg q$	$((p \to q) \wedge \neg q) \to \neg p$
T	F	T	T	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	F	T
F	T	F	T	T	T	T

Rules of Inference

$$\begin{array}{l} p \\ p \to q \\ \hline \therefore q \end{array}$$

Modus Ponens

$$\begin{array}{l} p \to q \\ \neg q \\ \hline \therefore \neg p \end{array}$$

Modus Tollens

Pf:

$$\begin{array}{ll} p \to q & \\ \neg q \to \neg p & \text{by contrapositive logic equivalence} \\ \neg q & \text{modus ponens} \\ \hline \therefore \neg p & \end{array}$$

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

hypothetic syllogism

~~p q r $p \Rightarrow q$ $q \Rightarrow r$ $(p \Rightarrow q) \Rightarrow (q \Rightarrow r)$~~

p	q	r	$p \Rightarrow q$	$q \Rightarrow r$	$(p \Rightarrow q) \wedge (q \Rightarrow r)$	$p \Rightarrow r$	$(\ ) \Rightarrow (\ )$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Disjunctive Syllogism

$p \vee q$

$\neg p$

$\therefore \; q$

q is T

p or q must be T

p is F

Assuming both premises $p \vee q$ and $\neg p$ are true, check to see if it's possible for q to be F.

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p

$\therefore \; p \vee q$

addition

~~$p \wedge q$~~

p
q

$\therefore \; p$

simplification

$p \wedge q$

$\therefore \; p$

Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore\ p \wedge q \end{array}$$

Resolution,

$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \hline \therefore\ q \vee r \end{array}$$

Determine if the following argument is valid

$\neg p \wedge q$

$r \to p$

$\neg r \to s$

$s \to t$

$\therefore \quad t$

Pf:

$\neg p \wedge q$

$\neg p$ simplification

$r \to p$ modus Tollens

$\neg r$

$\neg r \to s$ modus ponens

s

$s \to t$ modus ponen

$\therefore \quad t$

$p \to q \longrightarrow \neg q \to \neg p$

$\cancel{p \to q}$

$\neg p \to r$

$r \to s$ — hypothetic syllogism — $\neg p \to s$

$\therefore \neg q \to s$ $\qquad$ $\therefore \neg q \to s$

$(p \wedge q) \vee r$

$r \to s$

$\therefore p \vee s$

$p \to q \equiv q \vee \neg p$

$(p \wedge q) \vee r$

$\neg(\neg(p \wedge q)) \vee r$ — logic equivalence

$\neg(p \wedge q) \to r$ — logic equivalen

$r \to s$ — hypothetical syllogism

$\neg(p \wedge q) \to s$ — logic equivalen

$\neg(\neg(p \wedge q)) \vee s$

$(p \wedge q) \vee s$

$(p \vee s) \wedge (q \wedge s)$ — distributive law

$\therefore p \vee s$ — simplification

Rules of Inference with Quantifiers

Universal Instantiation

$$\frac{\forall x\, P(x)}{\therefore P(c)}$$

c is a particular instance/individual

Universal Generalization

$$\frac{P(c) \text{ for arbitrary instance } c}{\therefore \forall x\, P(x)}$$

Existential Instantiation:

$$\frac{\exists x\, P(x)}{\therefore P(c) \text{ for some particular instance}}$$

Existential Generalization

$$\frac{P(c) \text{ for some particular } c}{\therefore \exists x\, P(x)}$$

All CS students are good at math.

I'm not good at ~~[illegible]~~ math

therefore I'm not CS.

$P(x)$: x is CS

$Q(x)$: x is good at math

$$\forall x \, (P(x) \rightarrow Q(x))$$

$$\neg Q(\text{Sean})$$

$$\therefore \neg P(\text{Sean})$$

$$\forall x \, (P(x) \rightarrow Q(x))$$

universal instantiation

$$P(\text{Sean}) \rightarrow Q(\text{Sean})$$

modus tollens

$$\neg Q(\text{Sean})$$

$$\neg P(\text{Sean})$$

Logic and Proof

Recall an odd number n can be written as

$2k+1$ for integer k. (n is odd $\Leftrightarrow n = 2k+1$)

Prove that if n is odd, then n^2 is odd

$P(n)$: n is odd

$Q(n)$: n^2 is odd

$$\forall n \; (P(n) \rightarrow Q(n))$$

Need.

$$\frac{P(c) \rightarrow Q(c) \text{ for abitrary } c}{\therefore \; \forall n \; (P(n) \rightarrow Q(n))}$$

In other words wee need to show

~~If~~ For abitray c if c is odd then c^2 is odd

c is odd

$c = 2k+1$

$c^2 = \cancel{4k^2}\,(2k+1)^2 = 4k^2 + 4k + 1 = \underline{2(2k^2+2k)+1}$

c^2 is odd,