Homework 2: Due Sept 20 (Tuesday)

1. What are the terms a_0, a_1, a_2, and a_3 of the following sequences?

   (a) $a_n = (-1)^n$.

   (b) $a_n = 3$.

   (c) $a_n = 7 + 4^n$.

   (d) $a_n = 2^n + (-2)^n$.

   (e) $a_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}$.

2. List the first 5 terms of each of these sequences.

   (a) The sequence obtained by starting with 100 and obtaining each term by subtracting 5 from the previous term.

   (b) The sequence whose n^{th} term is the sum of the squares of the first n positive integers.

   (c) The sequence whose n^{th} term is $\lfloor \sqrt{n} \rfloor$.

   (d) The sequence whose first two terms are 1 and 2 and each succeeding term is the sum of the two previous terms.

3. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list. Assuming that your formula or rule is correct, determine the next three terms of the sequence.

   (a) 1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, ...

   (b) 1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, ...

   (c) 1, 0, 2, 0, 4, 0, 8, 0, 16 , 0, ...

   (d) 3, 6, 12, 24, 48, 96, 192, ...

   (e) 15, 8, 1, -6, -13, -20, -27, ...

   (f) 3, 5, 8, 12, 17, 23, 30, 38, 47, ...

4. Show that $\sum_{j=1}^{n} \frac{1}{j(j+1)} = \frac{n}{n+1}$.

5. Calculate the following sums. Show the steps of your calculation.

   (a) $\sum_{j=100}^{200} j$.

   (b) $\sum_{j=10}^{20} 2^j$.

6. Find the solution to each of these recurrence relations and prove the correctness of your solution using mathematical induction.

(a) $a_n = -a_{n-1}$, $a_0 = 5$.

(b) $a_n = a_{n-1} + 3$, $a_0 = 1$.

(c) $a_n = a_{n-1} - n$, $a_0 = 4$.

(d) $a_n = (n+1)a_{n-1}$, $a_0 = 2$.

(e) $a_n = 2a_{n-1} - a_{n-2}$, $a_1 = 1$, $a_2 = 1$.

(f) $a_n = 2a_{n-1} + a_{n-2} + 1$, $a_1 = 1$, $a_2 = 1$.

7. Find a simple analytical form for the sum S_n of the following sequence, and use mathematical induction to prove the correctness: $a_n = 2n - 1$ and $S_n = a_1 + a_2 + ...$

8. Use the characterisitic equation nethod to solve the following recurrent relations:

$$\begin{cases} a_n = 3a_{n-1} - a_{n-2} & \text{for } n \geq 2 \\ a_0 = 1 \\ a_1 = 1 \end{cases}$$

9. Use the generating function method to solve the following recurrent relations:

$$\begin{cases} S_n = S_{n-1} + n & \text{for } n \geq 1 \\ a_0 = 0 \end{cases}$$