Sept 8ch

Characteristic Equation
$$F_n = F_{n-1} + F_{n-2}$$

$$F_i = 1$$

$$F_o = 1$$

$$F_i = 1$$

$$F_0 = 1$$

$$\gamma^n = \gamma^{n-1} + \gamma^{n-2}$$

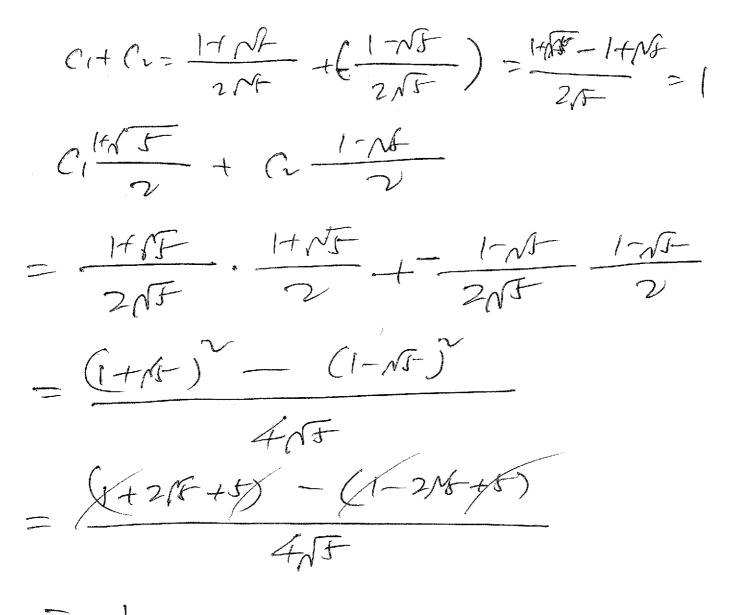
Factor out
$$\gamma^{n-2}$$
 we get $\gamma^2 = \gamma + 1$

$$\gamma^{2}-\gamma-1=0 \qquad \gamma=\frac{1\pm\sqrt{5}}{2}$$

$$\gamma_{1}=\frac{1+\sqrt{5}}{2} \qquad \gamma_{2}=\frac{1-\sqrt{5}}{2}$$

Observe that if & satisfies the recurrence relation Vn= yn-1+ yn-2 than orn also satisfies the rowsian for some contact C +0 (cr) = (cr) + (cr)Fa Fa-c Fa-z The solutions of the recession is of the former Fa= arin+arin $= C_1 \left(\frac{(1455)}{2} \right)^n + C_1 \left(\frac{1-N5}{2} \right)^n$) C1 -1+15 + a 1-1/2 = 1

 $\begin{cases} C_1 = \frac{1+NF}{2NF} \\ C_{Y2} = \frac{1-NF}{2NF} \end{cases}$



$$\int a_{n} = 3a_{n-1} - 2a_{n-2}$$

$$Q_{i} = 1$$

$$Q_{0} = 0$$

Solution.

Use characteristic equation motherd

Assume On= rh of v to

 $\gamma^{n} = 3 \gamma^{n-1} - 2 \gamma^{n-2}$

Farer v^{n-2} , we ger v=3r-2

7-38+2=0 = Inscoritic equation

(r-1)(r-2)=0

ri=1 r=2

The solution is of the form FD

 $Qn = C_{11}^{n} + C_{12}^{n}$

C1+2C2=1

C1+a=0

1 C=+1

$$Q_n = Q_{n-1} + 2$$

$$Q_{n}-Q_{n-1}=2$$

 $\begin{array}{l}
Q_{1} = 2Q_{1-1} - Q_{1-2} \\
Q_{1} = 4 \\
Q_{0} = 2
\end{array}$

Solution. characteristic equation:

r=2r-1

~-2r+1=0 (r-1)=0

71,2= 1

Assume the solution is if the form

Assume the solution is of the form

Cathain

$$\begin{cases} G_n = 2G_{n-1} + 1 \\ G_0 = 1 \end{cases}$$

Observation:

$$G_{n}(G_{n})_{i} = 2a_{n-1} - 2a_{n-2}$$

$$Q_{in} = 3Q_{in-1} - 2Q_{in-2}$$

$$Q_{i} = 2Q_{o} + 1 = 3$$

$$Q_{o} = 1$$



S an= 2an-1+1 Cao= €0

The generative function is for the squence an is clefined as

G(x) = aotaix+aix+aix+aix+aix+

$$= aot$$

$$\frac{2a_0x}{x} + \frac{2a_1x^3}{t} + \frac{2a_2x^3}{t} + \frac{1}{x^3} + \dots$$

$$+2x\left(\frac{\alpha_{0}+\alpha_{1}x+\alpha_{1}x^{2}+\cdots\right)}{+x+x^{2}+x^{3}+\cdots}$$

$$= 2x + 4$$

$$+ 2x + 4$$

$$+ x + x + x + - -$$

$$G = 2xG + \frac{x}{1-x}$$

 $(1-2x)G = \frac{x}{1-x}$ $G = \underbrace{(1-2x)}_{1-x} \frac{x}{1-x}$ To get du an, we do du Taylor expansion of

$$f(x) \propto f(x) + \frac{f(x)}{1!} \times + \frac{f'(x)}{2!} \times + \frac{f'(x)}{3!} \times^{3}$$

$$+ \frac{f''(x)}{n!} \times + \frac{f''(x)}{n!} \times + \frac{f''(x)}{3!} \times^{3}$$

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$$f^{(n)}(x) = n! (+\infty)$$

$$\frac{1}{1-x} = 1 + x + x + x^2 + \cdots$$

$$\chi_{+\chi'_{+}\chi'_{+}\chi'_{+}} = \chi \left(1 + \chi + \chi'_{+} - \frac{\chi}{1 - \chi}\right) = \frac{\chi}{1 - \chi}$$

Let
$$y = 2x$$

$$\frac{1}{1-2x} = \frac{1}{1-y} = ++y+y^2+y^3+ -$$

 $(1x+2x)+3x^3+4x^6)$ $(4x+3x^2+2x^3+1x^6)$ What i the coefficient for x^6 $2x^2-1x^6+3x^3+2x^3+4x^6.3x^2$ $=(2+6+1x)x^6=4xx^6$

Bach to Gb)
$$G(b) = (1-2x) \cdot \frac{x}{1-x}$$

$$= (1+2x+2x+2x+2x)$$

$$= (1+2x+2x+2x+-$$

$$= 0x^{0} + 7x^{1} + 7x^{2} + \cdots + 7x^{n} +$$

$$=2^{n-1}$$

$$\begin{cases} Q_n = 2Q_{n-1} + 1 \\ Q_0 = 0 \end{cases}$$

From Generary Tundien, we found On=2-1 Prove that this is correct for all n.

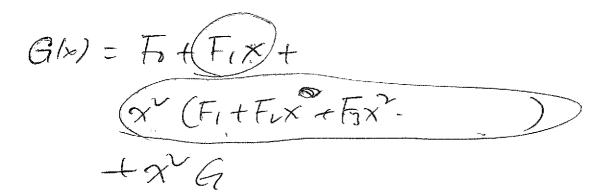
Prof by Industion.

Basis no

$$a_0 = 2^0 - 1 = 0$$

 $a_k = 2^k - 1$ Assume for n=k, Need to show April 2 k+1-1 rusy the inducer hypothesis

apri = 2ap+ 1 = 2(2 =1)+1 = 2.2 2+1 = 2k+1-1



5 FOGRA

= Fot Fix + x G

+x (Fix+Fix+ Fix+.

= Fot Fixt x'G

+x (G-Fo)

G= Fi+Fix+xG+XGG-Fo)