Sept 15, 2016

"If you don't $\underbrace{\text{clean up}}_{p}$, then no $\underbrace{\text{ice crem}}_{q}$!"

Does ~~this~~ imply ~~th~~ after you clean up ~~there~~ will be ice cream?

$$\neg p \rightarrow \neg q \stackrel{?}{\equiv} p \rightarrow q$$

p	q	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$	$p \rightarrow q$
T	T	F	F	T	T
T	F	F	T	T	F
F	T	T	F	F	T
F	F	T	T	T	T

proposition

Logic connectives $\neg, \wedge, \vee, \oplus, \underset{\text{iff}}{\leftrightarrow}, \rightarrow$

Logic equivalence

$$p \equiv \neg p \rightarrow F \equiv \neg p \rightarrow (q \wedge \neg q)$$

$$\underbrace{\text{Real numbers are } \cancel{\text{no}}\text{uncountable}}_{p}$$

If you assume real numbers are countable, then there will be a contradiction

$$\neg p \rightarrow F \qquad \neg p \rightarrow (q \wedge \neg q)$$

$$\underbrace{\sqrt{2} \text{ is irrational}}_{p} \equiv \neg p \rightarrow F$$

if $\sqrt{2}$ is rational, then there is a contradiction.

Pf by contradiction

Assume $\sqrt{2}$ is rational,

~~there there exists~~ $p, q \in \mathbb{Z}$ <u>co-prime</u>

such that

$$\sqrt{2} = \frac{p}{q}$$

$$\boxed{\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12}}$$

$$p^2 = 2q^2$$

p is even $\qquad p = 2k$

$$p^2 = (2k)^2 = 2q^2$$

~~then~~ $4k^2 = 2q^2$, hence $2k^2 = q^2$

q is even

$(p \to q) \equiv (\neg q \land p \to F)$

Recall $p \to q$ and $(\neg q \land p) \to F$ are logically equivalent if $(p \to q) \leftrightarrow ((\neg q \land p) \to F)$ is a tautology.

p	q	$p \to q$	$\neg q$	$\neg q \land p$	$(\neg q \land p) \to F$	$\leftrightarrow$
T	T	T	F	F	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

De Morgan's Law

$$\neg(p \land q) \equiv \neg p \lor \neg q$$

p	q	$p \land q$	$\neg(p \land q)$	$\neg p$	$\neg q$	$\neg p \lor \neg q$	$\leftrightarrow$
T	T	T	F	F	F	F	T
T	F	F	T	F	T	T	T
F	T	F	T	T	F	T	T
F	F	F	T	T	T	T	T

$\cancel{\overline{A \cup B} = \overline{A} \cap}$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$\overline{A \cap B} = \{x \mid \text{it's not the case "} x \in A \text{ and } x \in B \text{"}\}$

$= \{x \mid x \notin A \text{ or } x \notin B\} = \overline{A} \cup \overline{B}$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

$\neg(p \to q)$

Recall $p \to q \equiv q \vee \neg p$

$\neg(p \to q) \equiv \neg(q \vee \neg p) \equiv \neg q \wedge \neg(\neg p)$

$\equiv p \wedge \neg q$

Prove that

$p \wedge q \equiv \neg(p \to \neg q)$

$p \to (\neg q) \equiv \neg p \vee (\neg q)$

$\neg(p \to \neg q) \equiv \neg(\neg p \vee \neg q)$

$\equiv \neg(\neg p) \wedge \neg(\neg q)$

$= p \wedge q$

Connection between logic and computers

Using logic to design a 1-bit adder

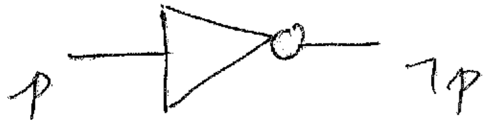
carry

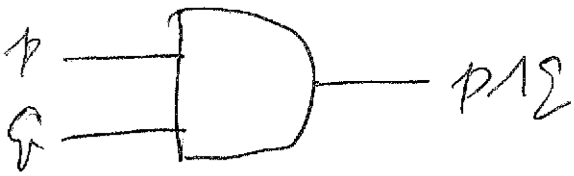
$$\begin{array}{ccccc} 1 & + & 1 & = & 10 \\ 1 & + & 0 & = & 01 \\ 0 & + & 1 & = & 01 \\ 0 & + & 0 & = & 00 \\ a & & b & & c\ s \end{array}$$

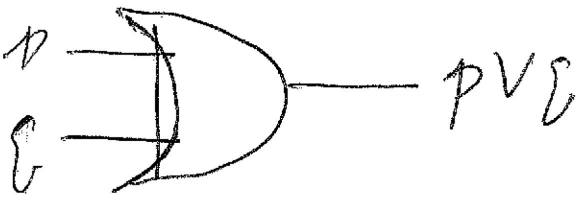
a	b	s	c
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	F

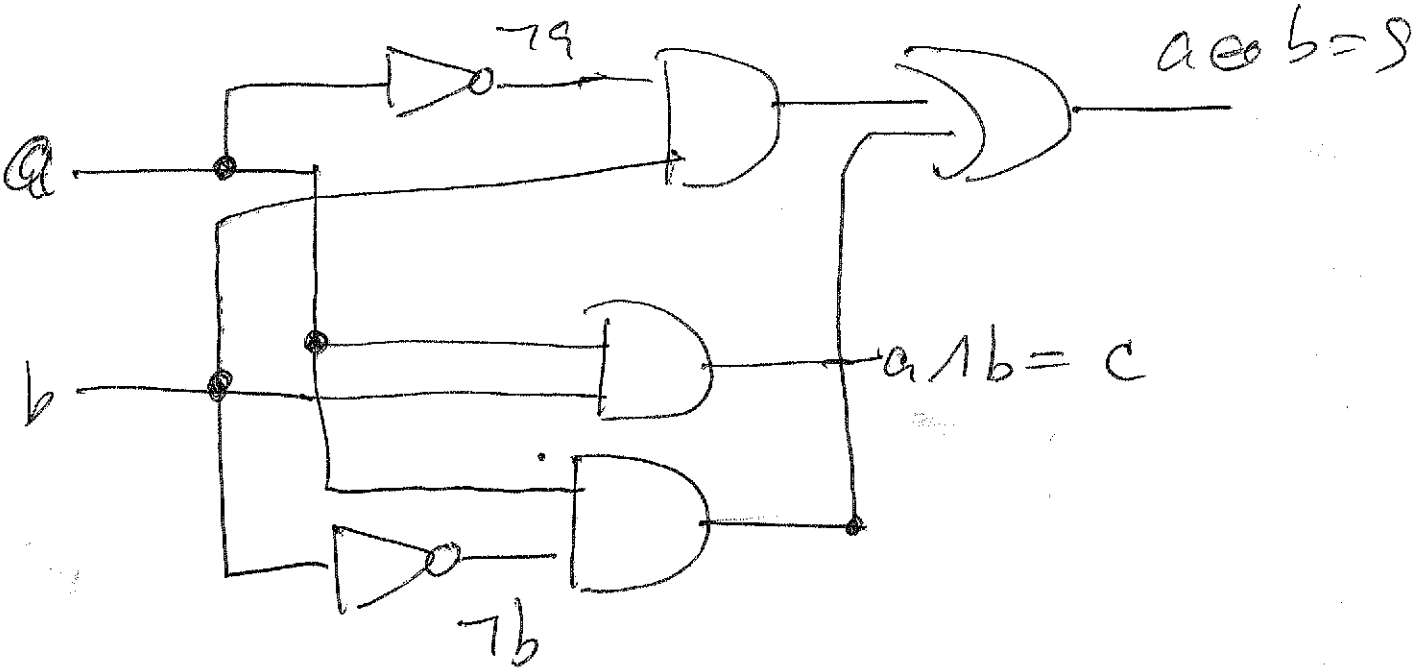
$s = a \oplus b = (a \wedge \neg b) \vee (\neg a \wedge b)$

$C = a \wedge b$

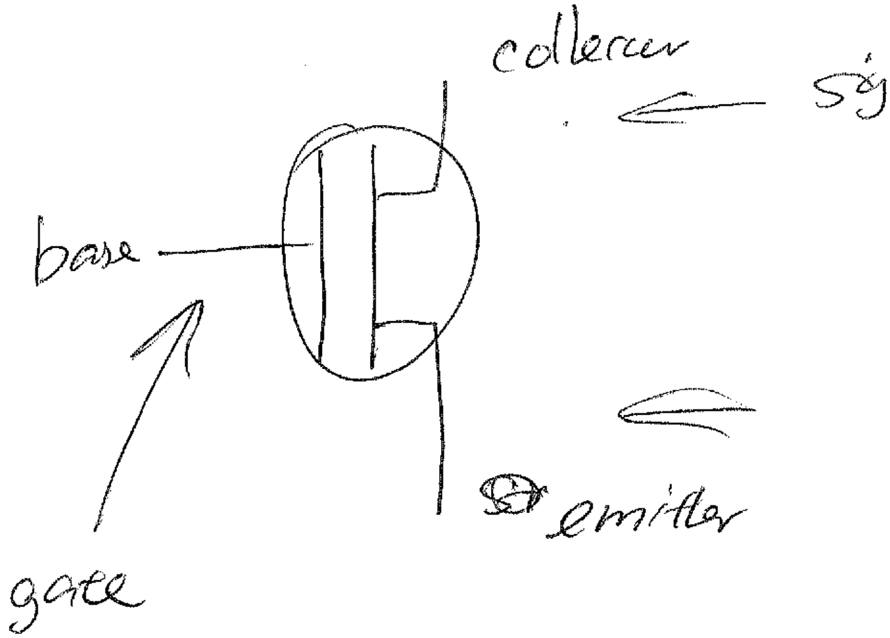

p
¬p


p
q
p∧q


p
q
p∨q


a
b
¬a
a⊕b=s
a∧b=c
¬b

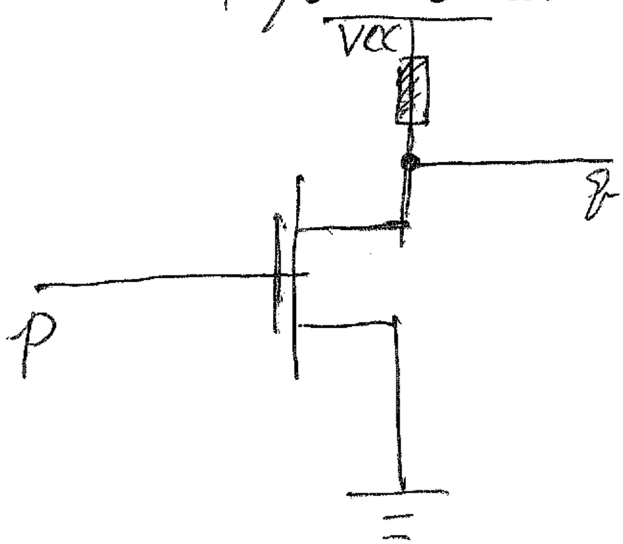
Transistor



When a high voltage ≥ 1.0 volt is applied to base, the collecor and emitter is connected

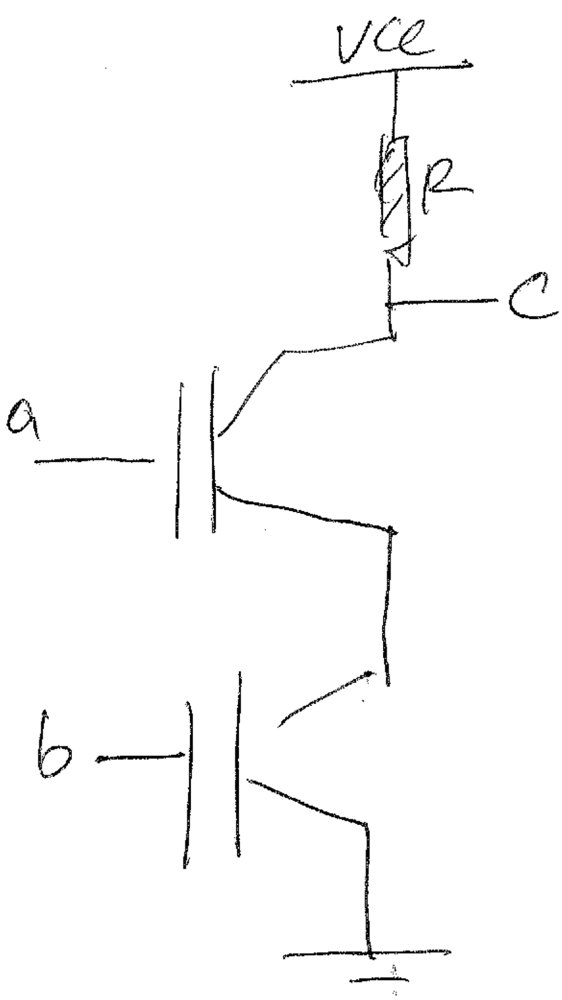
When a low voltage 0 volt is applied to the base, the collector and the emitter is disconnected.

in order to use a trasistor
we have to first map T/1 to high voltase
and F/0 to low voltase



p=1 q=0

p=0 p=1



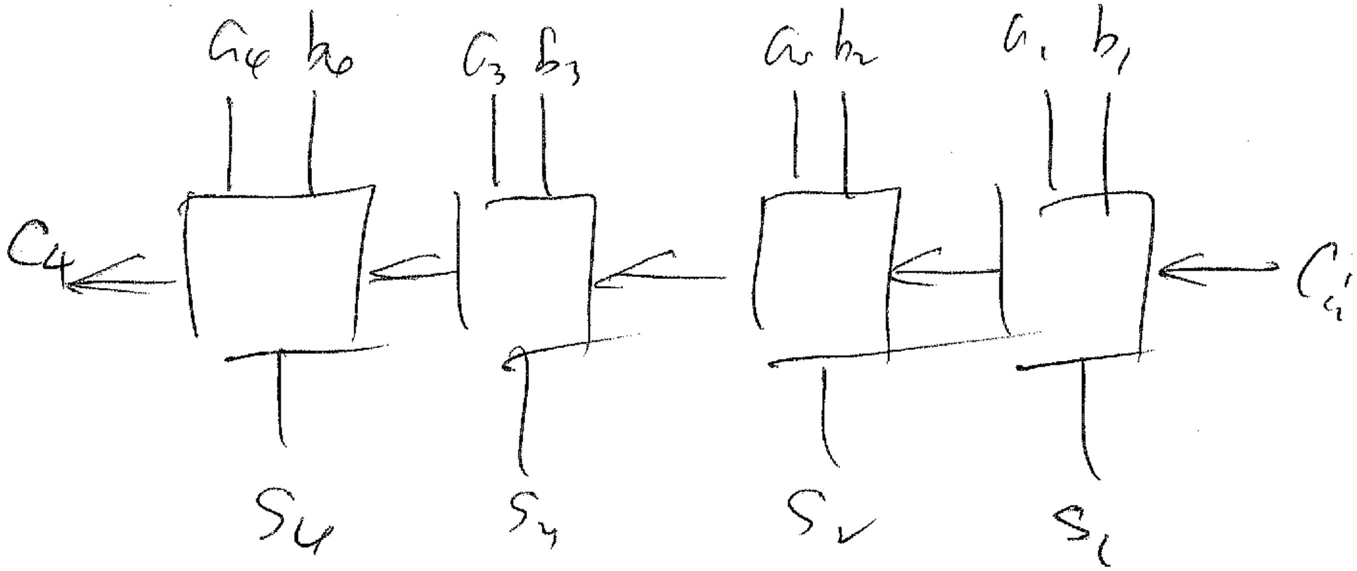
a=1 b=1 c=0

a=1 b=0 c=1

a=0 b=1 c=1

a=0 b=0 c=1

$c = \neg(a \wedge b)$


a4 b4
a3 b3
a2 b2
a1 b1
C4
Cin
S4
S3
S2
S1

Propositional Functions and Logic Quantifiers

Def A predicate is a function such that when applied to the individual becomes a proposition.

$P(x): \ x \geq 3.$

Logic quatifiers.

universal quantifier: $\forall x P(x)$

for every x, $P(x)$ is T.

existential quantifier $\exists x P(x)$

there exists an x such that $P(x)$ is T.

$x+y=0$

$\forall x \exists y \quad x+y=0$

For every x, there exists a y, such that $x+y=0$

Negation of quantifiers

$\neg(\forall x\, P(x)) \equiv \exists x\, (\neg P(x))$

$\neg(\exists x\, P(x)) \equiv \forall x\, (\neg P(x))$

$\neg\left(\forall x\ \underbrace{\exists y\ P(x,y)}\right)$

Argument and Rules of Inference

Def: an argument consists of a list of statements followed by a conclusion

premises.

Example.

All whales are ~~mm~~ mammals (p)

Mammals are warm blooded (q)

} premises

Therefore whales are warm blooded. (r)

conclusion

$$p \wedge q \rightarrow r$$

Argument Form:

$$\begin{array}{ll} & p_1 \\ & p_2 \\ & p_3 \\ & \vdots \\ & p_k \\ \hline \therefore & q \end{array}$$

logic Form

$$(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \to q$$