

Section 3.

Sept 13, 2016

Logic

the discipline that studies the method of reasoning

George Boole and Augustus De Morgan

Algebra

Numbers.

Algebraic operations

$+$, $-$, \times , \div

log, exponential

Logic

Propositions: a declarative sentence that is either True (T) or False (F) but not both..
usually denoted by p, q, r, s, t .

Logic connectors

$\wedge \vee \neg \oplus \leftrightarrow \rightarrow, -$

the earth is round.

T

Today's temp is 67°F
high.

$$2+3=4$$

F

Non propositions

$$3-x=5.$$

Do you speak spanish?

We use

proposition, statement, claim interchangeably

An argument is a sentence aiming to persuade someone to accept a particular conclusion.

A proof: a sequence of arguments to establish the truth of a proposition.

An axiom or postulate is a proposition that is accepted to be true without proof.

A theorem is a proposition that has been proven to be true.

A ~~cor~~ lemma is "baby" theorem

A corollary is a proposition that follows with little or no proof from a theorem.

A ~~scientific~~ scientific law is proposition based on experimental observations.

Negation: \neg

$\neg p$ 'not p' "It's not the case of p"
~~that is false~~

p	$\neg p$
T	F
F	T

Conjunction: \wedge "and"

$p \wedge q$ "p and q"

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction. "V" "or"

$p \vee q$ "p or q"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive or. \oplus

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	\oplus T
F	F	F

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Calculate the truth table for
 ~~$(p \wedge (\neg q)) \vee ((\neg p) \wedge q)$~~

~~$(p \wedge (\neg q)) \vee ((\neg p) \wedge q)$~~

$p \wedge q$ and $q \wedge p$

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

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$$(p \wedge \neg q) \vee (q \wedge \neg p)$$

p	q	$\neg p$	$\neg q$	$p \wedge \neg q$	$q \wedge \neg p$	$\textcircled{\text{X}} \vee \textcircled{\text{X}}$
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

$$A \cap B = \{x \mid \cancel{x \in B} \quad \underline{x \in A} \quad \text{and} \quad \underline{x \in B}\}$$

$$A \cup B = \{x \mid x \in A \quad \overset{p \wedge q}{\text{or}} \quad x \in B\}$$

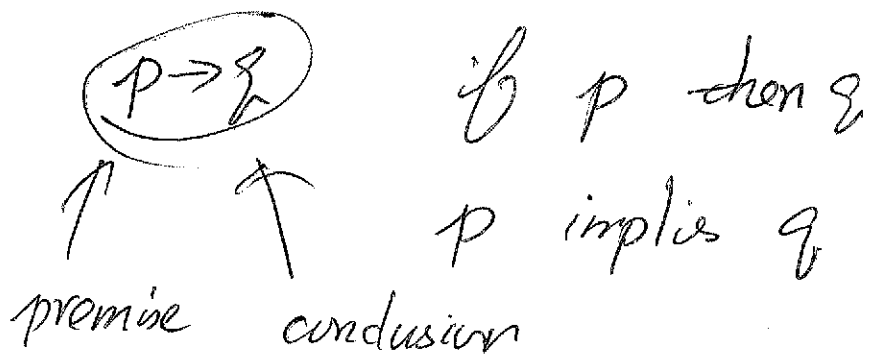
$$A \oplus B = (A - B) \cup (B - A) \quad \checkmark$$

$$= \{x \mid \underline{x \in A} \text{ and } x \notin B\} \cup \{x \mid x \in B \text{ and } x \notin A\}$$

$$(p \wedge (\neg q)) \vee (q \wedge (\neg p))$$

$$A - B = A \cap \overline{B}$$

implication \rightarrow



p	q	$p \rightarrow q$	
T	T	T	✓
T	F	F	✓
F	T	T	
F	F	T	

if you're Shag then I'm MS.

$p \rightarrow q$

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Calculate the Truth Table for
 $q \vee \neg p$.

p	q	$\neg p$	$q \vee \neg p$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Thus $q \vee \neg p$ is equivalent to $p \Rightarrow q$

Ways to express $p \Rightarrow q$

if p then q

p implies q

p is sufficient for q

q unless $\neg p$

if not

p only if q

$$q \vee (\neg p)$$

Biconditional. \iff if and only if

$$p \iff q$$

p if and only if q
iff

~~$p \iff q$~~

p	q
T	T
T	F
F	T
F	F

$p \iff q$
T
F
F
T

$p \oplus q$
F
T
T
F

Calculate the truth table for

$$(p \rightarrow q) \wedge (q \rightarrow p)$$

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p	q	$(p \rightarrow q)$	$(q \rightarrow p)$	$(p \rightarrow q) \wedge (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$$(p \rightarrow q) \wedge (q \rightarrow p)$$

if p then q and if q then p

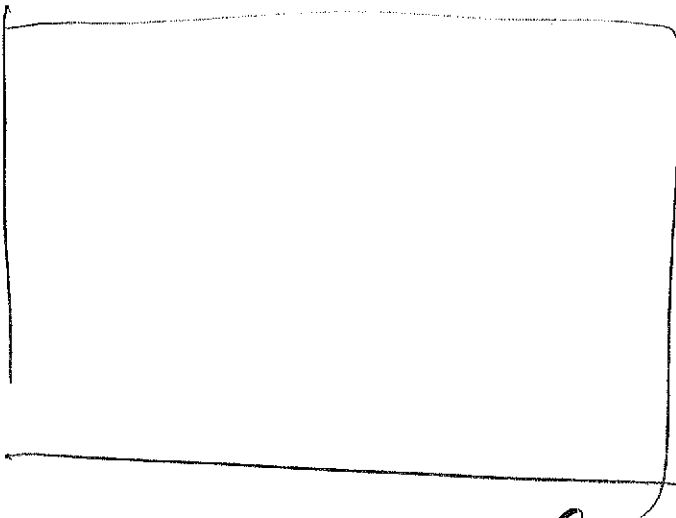
p only if q

p if q and only if q

ff was invented Paul Halmos

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Def:



Q.E.D.



quod erat demonstrandum
what to be

$$p \rightarrow q$$

$$(\neg q) \rightarrow (\neg p)$$

$$q \rightarrow p$$

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p	q	$p \rightarrow q$	$\neg p$	$\neg q$	$(\neg q) \rightarrow (\neg p)$	$q \rightarrow p$
T	T	T	F	F	T	T
T	F	F	F	T	F	F
F	T	T	T	F	T	T
F	F	T	T	T	T	T

if your cumulative $\geq 80\%$, then you get an A

if you ^{didn't} get an A then your cum $< 80\%$

if you get an A then your cum $\geq 80\%$

Precedence of Logic connectors.

$$\neg \quad \textcircled{\wedge} \quad \textcircled{\vee} \rightarrow \quad \textcircled{\leftrightarrow}$$

$$\cdot \quad +$$

$$((\neg p) \vee q) \rightarrow (r \wedge s) \leftrightarrow t$$

Propositional Equivalence

A compound proposition that's always T is called a tautology.

Two \neq compound propositions p, q are logically equivalent if $p \leftrightarrow q$ is a tautology.

~~$$(p \wedge (p \rightarrow q)) \rightarrow q$$~~

~~$$p \rightarrow q \quad p \rightarrow q \quad (p \rightarrow q) \rightarrow (p \rightarrow q)$$~~

$$(p \wedge (p \Rightarrow q)) \Rightarrow q$$

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p	q	$p \Rightarrow q$	$p \wedge (p \Rightarrow q)$	$(p \wedge (p \Rightarrow q)) \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

~~Example~~

Example

$$x + 3 = 5$$

$$x + 3 = 5 \wedge x + 3 + (-2) = 5 + (-2) \Rightarrow x = 2$$

When a compound proposition is always F ,
then it is called a contradiction

$$p \wedge (\neg p)$$

p	$\neg p$	$p \wedge (\neg p)$
T	F	F
F	T	F

$$p \Leftrightarrow (\neg p \rightarrow (q \wedge \neg q))$$

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p	q	$\neg p$	$\neg q$	$q \wedge \neg q$	$\neg p \rightarrow (q \wedge \neg q)$	\Leftrightarrow
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	F	T	T

↑