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Logic

the discipline that studies the method of reasoning

George Boole and Augustus De Morgan

Algebra	Logic
Numbers.	Propositions: a declarative sentence that is either True (T) or False (F) but not both.. usually denoted by p, q, r, s, t.
Algebraic operations	Logic connectors
$+, -, \times, \div$	$\wedge \quad \vee \quad \neg \quad \oplus \quad \Leftrightarrow \quad \rightarrow, -$
log, exponential	

the earth is round. T

Todays high temp is 67°F.

$2+3=4$ F

Non propositions

$3-x=5$.

Do you speak spanish?

We use

proposition, statement, claim interchangeably

An argument is a sentence aiming to persuade someone to accept a particular conclusion.

A proof: a sequence of arguments to establish ~~the~~ truth of a proposition

An axiom or postulate is a proposition that is accepted to be true without proof.

A theorem is a proposition that has been proven to be true.

A ~~cor~~ Lemma is "baby" theorem

A corollary is a proposition that follows with little or no proof from a theorem

A ~~scientific~~ scientific law is proposition based on experimental observations

Negation : $\neg$

$\neg p$ "not p" "It's not the case of p"

~~that~~ ~~B is true~~

p	$\neg p$
T	F
F	T

Conjunction : $\wedge$ "and"

$p \wedge q$ "p and q"

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjuction. "$\vee$" "or"

$p \vee q$ "p or q"

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive or. $\oplus$

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Calculate the truth table for

$$\cancel{(p \wedge (\neg q)) \vee ((\neg p) \wedge q)}$$

$$\cancel{(p \wedge (\neg q)) \vee (}$$

$p \wedge q$ and $q \wedge p$

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$(p \land (\neg q)) \lor (q \land (\neg p))$

p	q	$\neg p$	$\neg q$	$p \land (\neg q)$	$q \land \neg p$	$\lor$
T	T	F	F	F	F	F
T	F	F	T	T	F	T
F	T	T	F	F	T	T
F	F	T	T	F	F	F

$A \cap B = \{x \mid \cancel{x \in B}\ \underline{x \in A} \text{ and } \underbrace{x \in B}_{q}\}$

$p \wedge q$

$A \cup B = \{x \mid x \in A \underbrace{\text{ or }}_{\vee} x \in B\}$

$A \oplus B = (A - B) \cup (B - A)$

$= \{x \mid \underline{x \in A} \text{ and } x \notin B\} \cup \{x \mid x \in B \text{ and } x \notin A\}$

$(p \wedge (\neg q)) \vee (q \wedge (\neg p))$

$A - B = A \cap \overline{B}$

implication $\rightarrow$

$p \rightarrow q$ if p then q

premise conclusion

p implies q

p	q	$p \rightarrow q$	
T	T	T	✓
T	F	F	✓
F	T	T	
F	F	T	

if you're Shaq then I'm MJ.

$p \rightarrow q$

Calculate the Truth Table for

$q \vee \neg p$.

p	q	$\neg p$	$q \vee \neg p$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

Thus $q \vee \neg p$ is equivalent to $p \supset q$

Ways ~~to~~ express $p \Rightarrow q$

if p then q

p implies q

p is sufficient for q

q unless ~~(p)~~ not p $\qquad q \vee (\neg p)$

if not

p only if q

Biconditional. $\Leftrightarrow$ if and only if

$p \Leftrightarrow q$ p if and only if q

iff

$p \Leftrightarrow q$

p	q	$p \Leftrightarrow q$	$p \oplus q$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

Calculate the truth table for

$(p \to q) \land (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \land (q \to p)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

$(p \to q) \land (q \to p)$

if p then q and if q then p

p only if q

p if q and only if q

"iff" was invented Paul Halmos

Pf:

Q.E.D. $\blacksquare$ $\square$ $\square$ $\square$

quod erat demonstratum

what to be

$p \to q$ $\qquad$ $(\neg q) \to (\neg p)$ $\qquad$ $q \to p$

p	q	$p \to q$	$\neg p$	$\neg q$	$\neg q \to \neg p$	$q \to p$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	F	T	T	T	T	T

if your cumulative $\geq 90\%$, then you get an A

if you didn't get an A then your exam $< 90\%$

if you get an A then your exam $\geq 90\%$

Precedence of Logic connectors.

$$\neg \quad \wedge \quad \vee \quad \rightarrow \quad \leftrightarrow$$

$$\oplus$$

$$\cdot \quad +$$

$$(((\neg p) \vee q) \rightarrow (r \wedge s)) \leftrightarrow t$$

Propositional Equivalance

A compound proposition that's always T is called a <u>tautology</u>

Two ~~q~~ compound propositions p, q are logically equivalent if $p \leftrightarrow q$ is a tautology.

$\cancel{(p \land (p \to q)) \to q}$

~~p q $p \to q$~~

$(p \wedge (p \to q)) \to q$

p	q	$p \to q$	$p \wedge (p \to q)$	$(p \wedge (p \to q)) \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Example

$$x + 3 = 5$$

$$x+3=5 \ \wedge \ x+3+(-2) = 5+(-2) \Rightarrow x=2$$

When a compound proposition is always F,
then it is called a contradiction

$p \wedge (\neg p)$

p	$\neg p$	$p \wedge (\neg p)$
T	F	F
F	T	F

$p \Longleftrightarrow (\neg p \to (q \wedge \neg q))$

p	q	$\neg p$	$\neg q$	$q \wedge \neg q$	$\neg p \to (q \wedge \neg q)$	$\Longleftrightarrow$
T	T	F	F	F	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	F	T