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# CS261 Discrete Math

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Office Hours: T R 9:30-10:30am

TAs 1.5 TBA

Textbook Rosen's
Discrete Mathematics and its Applications 7/E

Topics:
- Set Theory
- Logic
- sequences
- relations / graphs
- Probability

Exam Schedule

3 Exams

Exam 1 Sept 29th (Thursday)

Exam 2 Nov 3rd (Thursday)

Exam 3 Dec 8th (Thursday)

You'll be dropped if you miss any of the exams.

Grading:

HW (25%) + Exam 1 (25%) + E2 (25%) + E3 (25%)
= 100%

| | |
|---|---|
| $\geq 90\%$ | A |
| $\geq 60\%$ | C |
| Highest | A+ |

UNM Learn. HW# / online

Homework

1. Always due at the lecture on due date

2. Up to 24 hour late  2% penalty

   $> 24$ hr  0.

3. Grade change  $< 7$ days

Academic Honesty

Theory: algorithm design

radiosurgery treatment planning


Lately: LDR

# Set Theory

A set is a well-defined unordered collection of objects.

well-defined: it is possible to distinguish one object from another object in the set

it is possible to determine whether a given object is in a set or not.

unordered: order is not important

Representation: Roster method

$A = \{ C, \text{Java}, \text{Python}, \text{Ruby}, \}$

Usually we use capital letters to represent a set.

A, B, C, ..

we use lower case letters to represent the objects in a set. a, b, c, ..

elements

$A = \cancel{\{a, a, b, c\}}$

$\{a, b, c\}$ $\{c, b, a\}$

When an element $a$ belongs to a set $A$, we write $a \in A$, otherwise we write $a \notin A$

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The set of Natural Numbers

$\{1, 2, 3, 4, \cdots\}$

$\{1, 2, 3, 4, \cdots, 1{,}000{,}000\}$

Set Builder Notation:

$\mathbb{N} = \{ x \mid x \text{ is a natural number}\}$

$2.5 \notin \mathbb{N}$

$\mathbb{Z} \leftarrow$ integer

$\mathbb{Q} \leftarrow$ rational number

$\mathbb{R} \leftarrow$ real number

$\mathbb{C} \leftarrow$ complex number

$$\mathbb{Q} = \left\{ \frac{p}{q} \;\middle|\; p, q \in \mathbb{Z},\ q \neq 0 \right\}$$

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Relations between Sets

(1) equality: two sets $A$, $B$ are equal if they contain the same collection of objects or elements

(2) $A$ is a subset of $B$ if all elements in $A$ are also in $B$, we write $A \subseteq B$ $\qquad \leq$

$A = \{a, b, c\}$ $\quad$ $B = \{d, c, b, a\}$ $\quad A \subseteq B$ $\qquad <$

Note $A \subseteq A$

A set $A$ is a proper subset of $B$ if $A \subseteq B$ and there exists at least one element in $B$ that's not in $A$. and we write $A \subset B$

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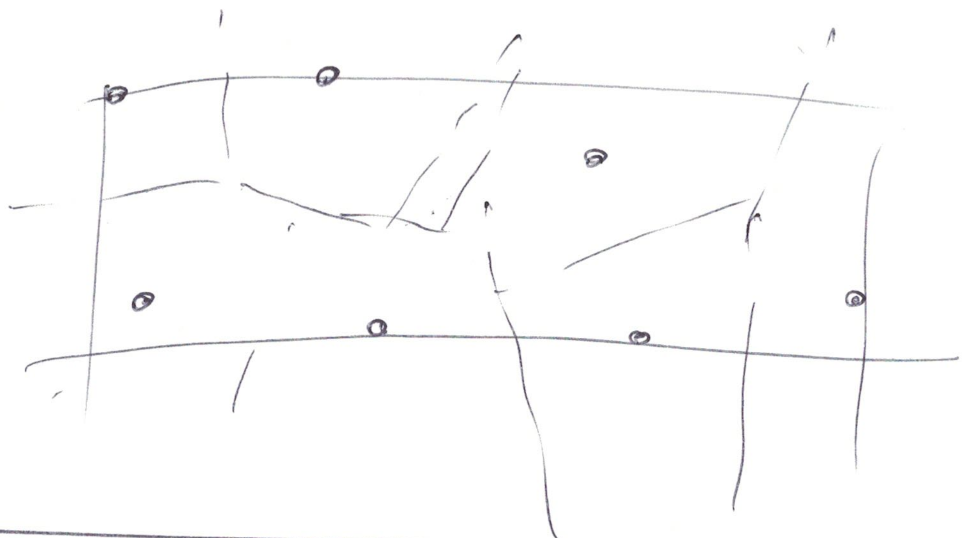
(3) $B$ is a superset of $A$ if $A \subseteq B$

$B$ is a proper superset of $A$ if $A \subset B$

Universal set $U$: contains everything in the context.

Empty set $\phi = \{ \}$

For any set $A$. $\phi \subseteq A \subseteq U$



~~Note~~ ~~$\phi$~~

Let $A$ be a set, the power set of $A$, denoted by $2^A$ or $P(A)$, is the set that contains all the subsets of $A$.

$A = \{a, b, c\}$

$$2^A = \{\phi, A, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{a,c\}\}$$

$$2^\phi = \{\phi\}$$

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Set operations

$$A \cap B = \{x \mid x \in A \overset{\text{and}}{,} x \in B\}$$

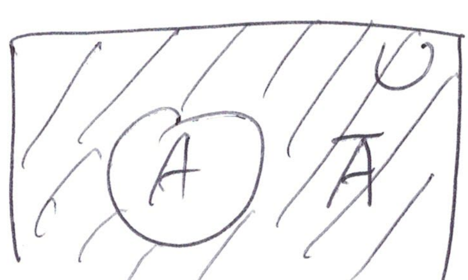
$$A \cup B = \{x \mid x \in A, \text{or } x \in B\}$$

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$


$A-B$

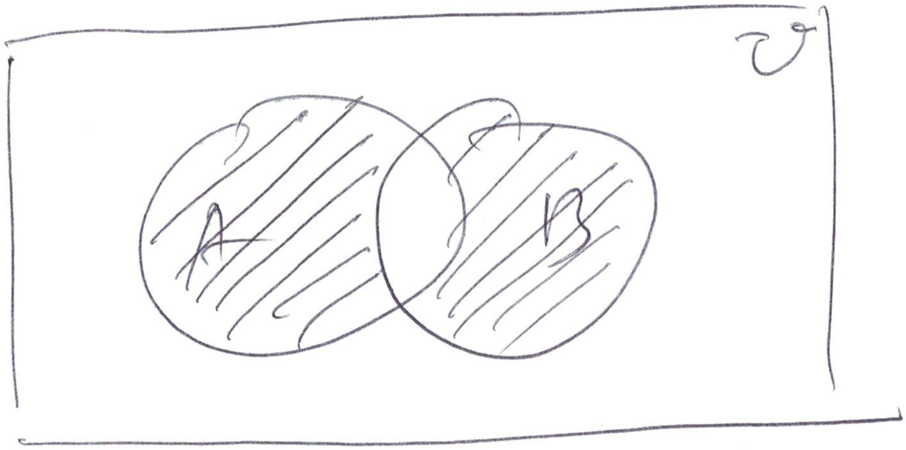
$$A - B = A - A \cap B$$

$$\bar{A} = U - A = A^c$$



symmetric difference

$$A \oplus B = (A-B) \cup (B-A)$$




Cartesian Product

$$A \times B = \{ (a,b) \mid a \in A,\ b \in B \}$$

ordered pair

$$(a,b) \neq (b,a)$$

$$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \cdots, a_n) \mid a_j \in A_j\}$$

$A = \{a, b\}$, $B = \{1, 2\}$

$$A \times B = \{ (a,1), (a,2), (b,1), (b,2) \}$$

Example 1

Does $A - C = B - C$ imply $A = B$ ?

No.

$A = \{a, b, c\}$

$B = \{a, b, c, d\}$

$C = \{c, d, e\}$

$A - C = \{a, b\} = B - C$

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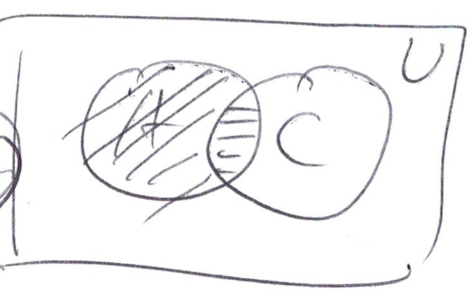
Example 2

Does $A \cup C = B \cup C$ and $A \cap C = B \cap C$ imply $A = B$ ?

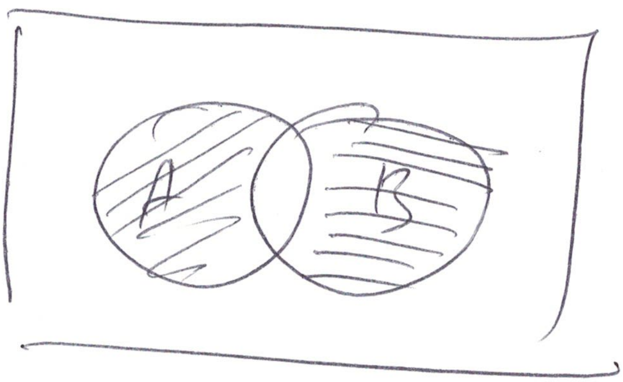
$A = ((A \cup C) - C) \cup (A \cap C)$




$B = ((B \cup C) - C) \cup (B \cap C)$

Example 3

if $A - B = B - A$, does this imply $A = B$?




$$A = (A - B) \cup (A \cap B)$$

$$\shortparallel$$

$$B = (B - A) \cup (B \cap A)$$