April 5

Exam 2 Thursday

5 problems

P1 Conditional Probability

P2 Shortest path

P3 BFS.

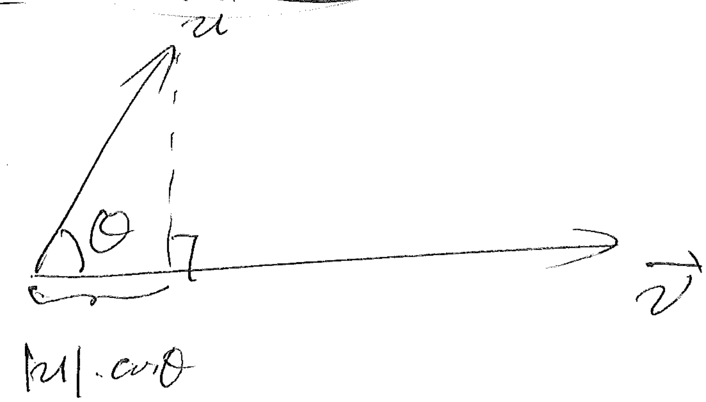
P4.

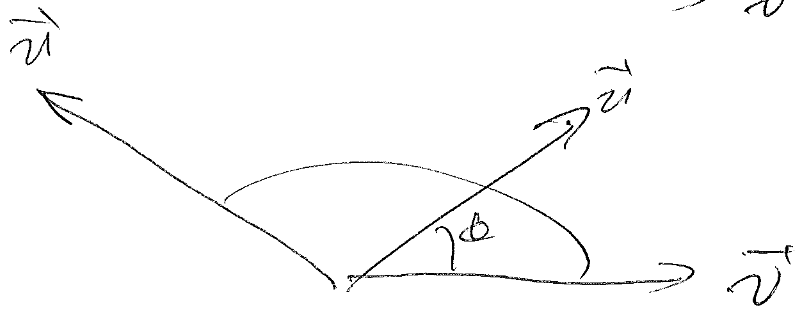
P5 Randomized Algorithm

Markov Inequality

12:20 — 1:50 pm

$$\langle u, v\rangle = |u| \cdot |v| \cos\theta$$

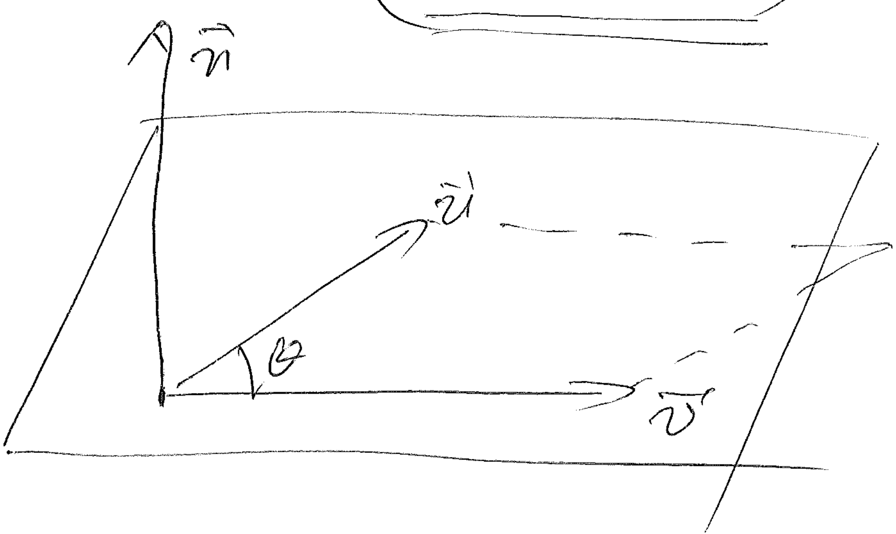


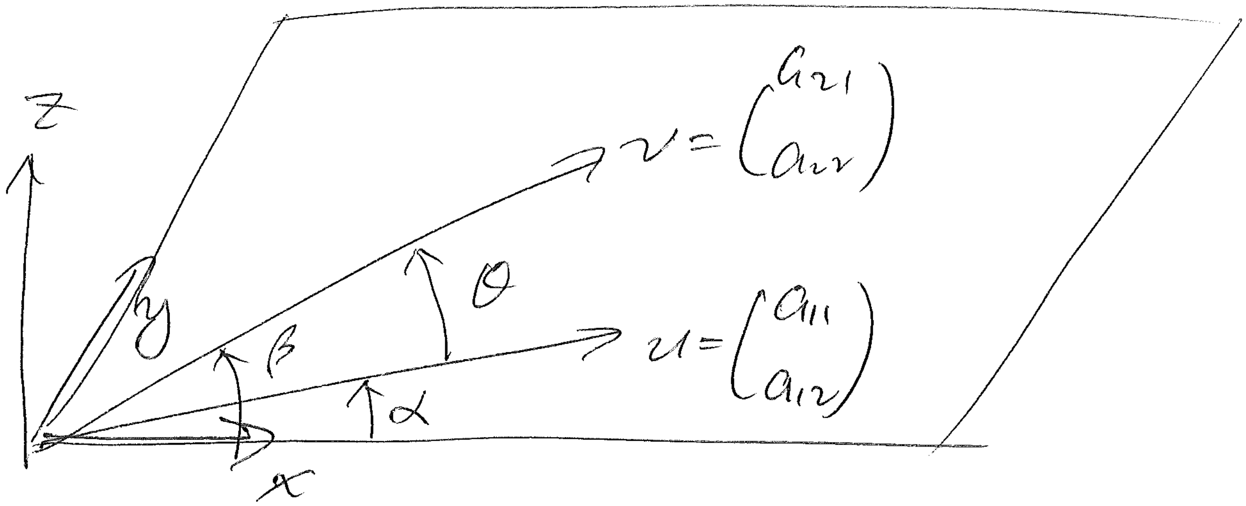


---

cross produce

$$\vec{u} \times \vec{v} = \left(|u| \cdot |v| \cdot |\sin\theta| \right) \vec{n}$$



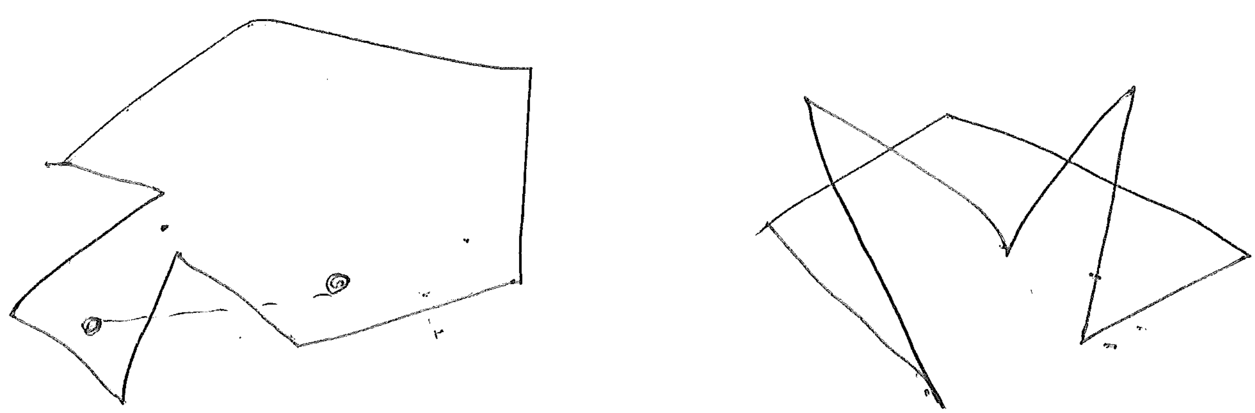


$$|u \times v| = \det \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix}$$

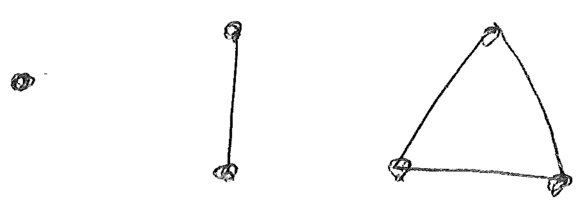
$$\det \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix} = |u| \cdot |v| \cdot \sin(\beta - \alpha)$$

· convex polygon :

a simple polygon is convex if the line segment connecting any two points inside the polygon is also contained in the polygon

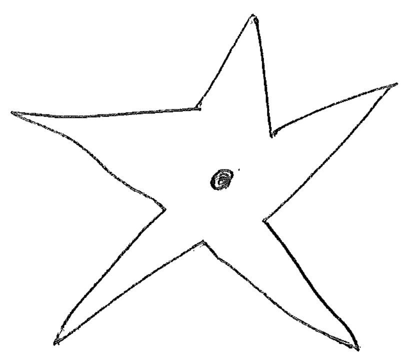


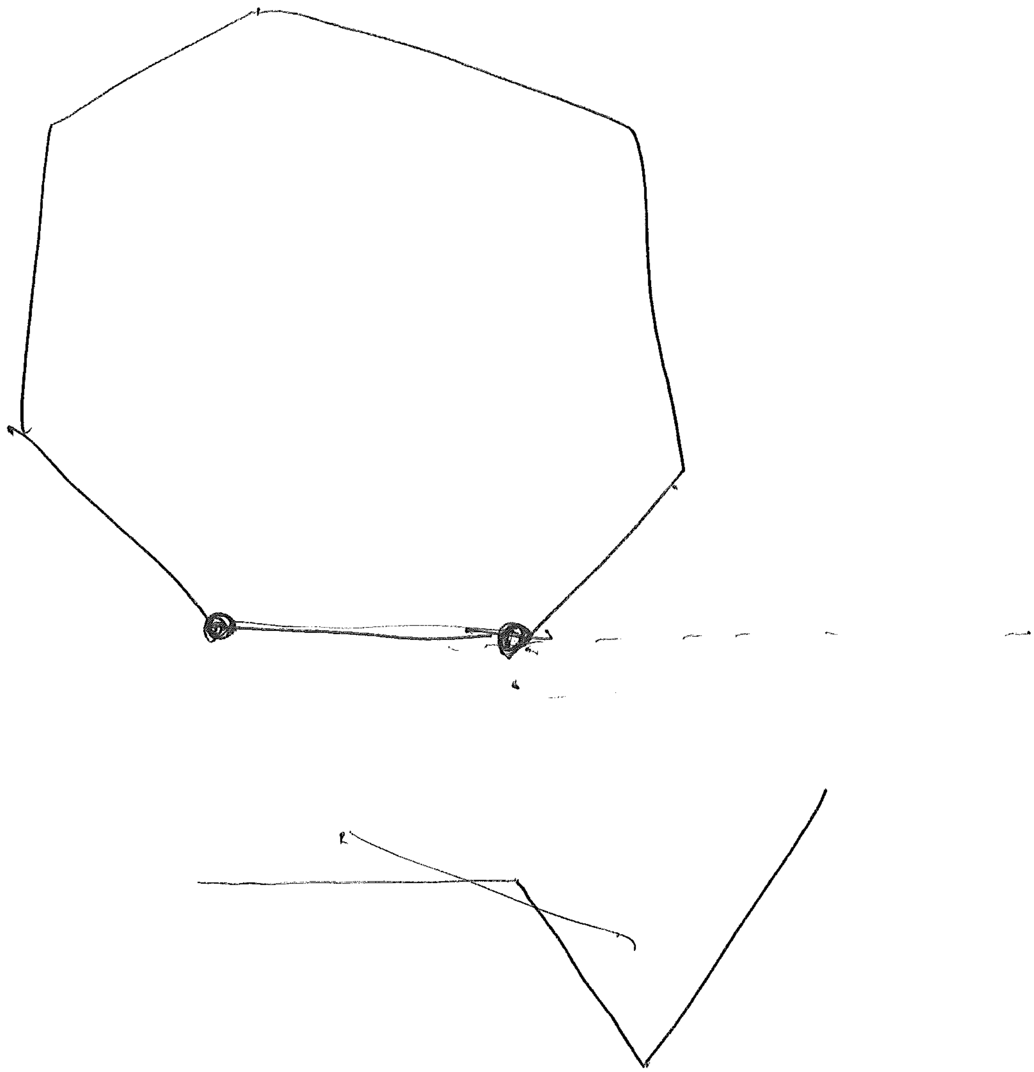
a polygon is simple if its edges ~~ar~~ do not intersect each other



Star-shaped polygon

a simple polygon is star-shaped iff there exists a point such that the line segment between this point to all other points are contained in the polygon





Tailor's Problem

A tailor has

16 ~~units~~ ~~feet~~ yard of material A

11 ~~feet~~ yard of materia B

15 ~~feet~~ yard of material C

a suit will use 2 yards of A, 1 yard of B, 1 yard of C ← \$30

a dress uses 1 yard of A, 2 yard of B, 3 yards of C ← \$50

Maximize profit

Let x_1 be the # suits, x_2 be the # dresses

$$\max\ 30x_1 + 50x_2$$

s.t.

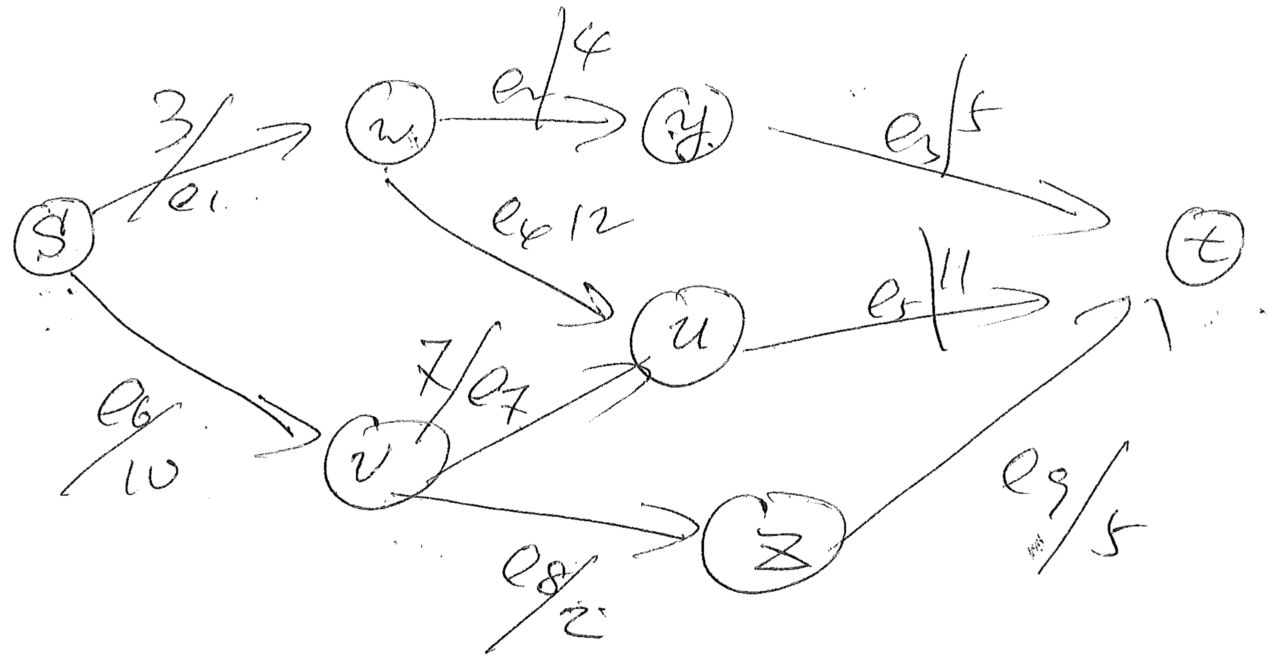
$$2x_1 + x_2 \le 16 \leftarrow A$$

$$x_1 + 2x_2 \le 11 \leftarrow B$$

$$x_1 + 3x_2 \le 15 \leftarrow C$$

$$\cancel{x_1, x_2}\ x_1 \ge 0,\ x_2 \ge 0$$

$$x_1, x_2 \in \mathbb{Z}^+ \cup \{0\}$$



~~Can we~~

What's the most amount of oil we can send from s to t?

---

introduce variable x_j for e_j to denote the amount of oil sent through e_j.

$$\max \quad x_1 + x_6$$

$$\text{s.t.} \quad 0 \le x_1 \le 3 \quad \leftarrow e_1$$

$$\vdots$$

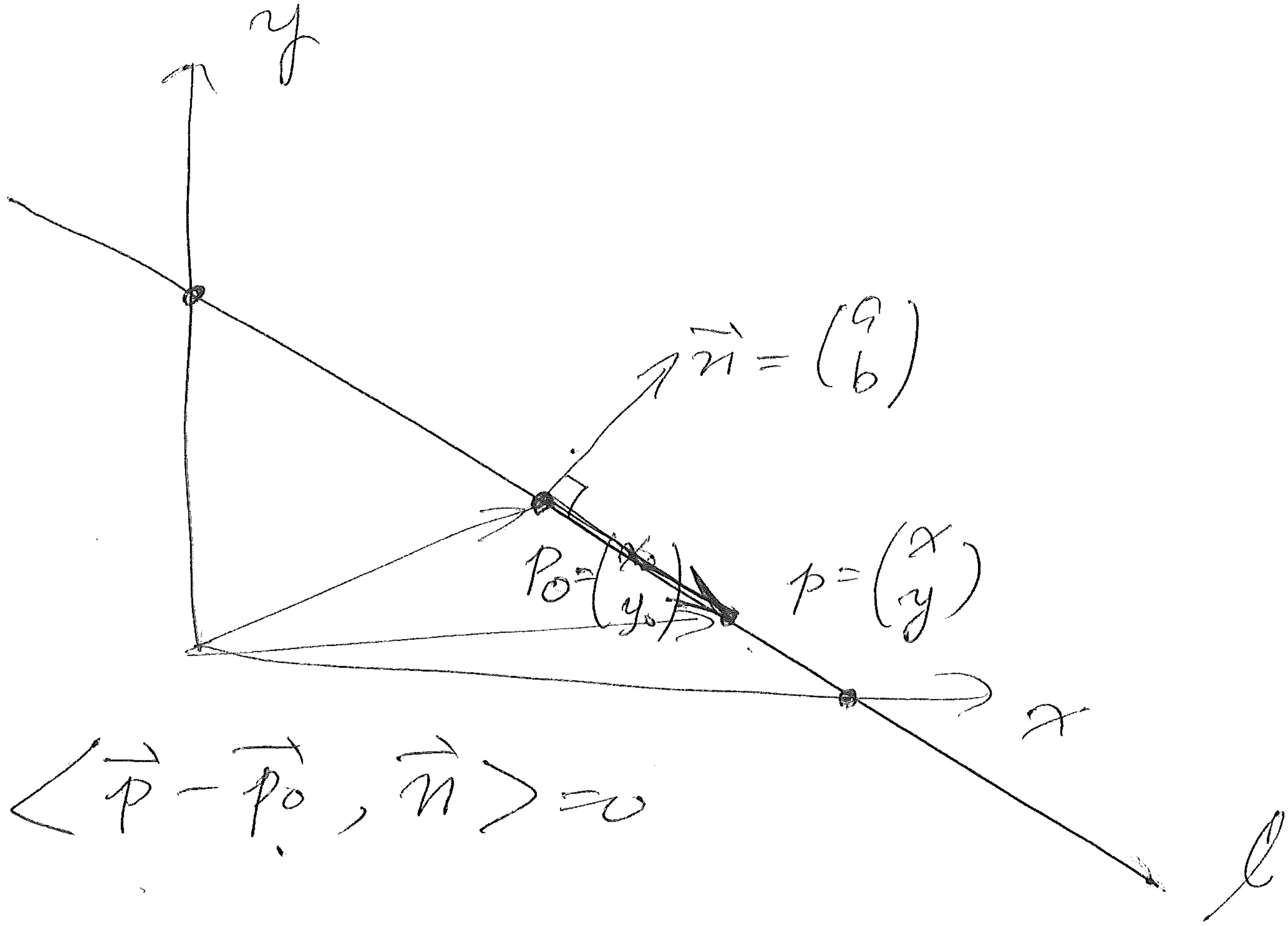
$$\leftarrow e_9$$

$$x_6 = x_7 + x_8 \quad \leftarrow v$$

$$\vdots$$

The meaning of LP constraint.

Line function



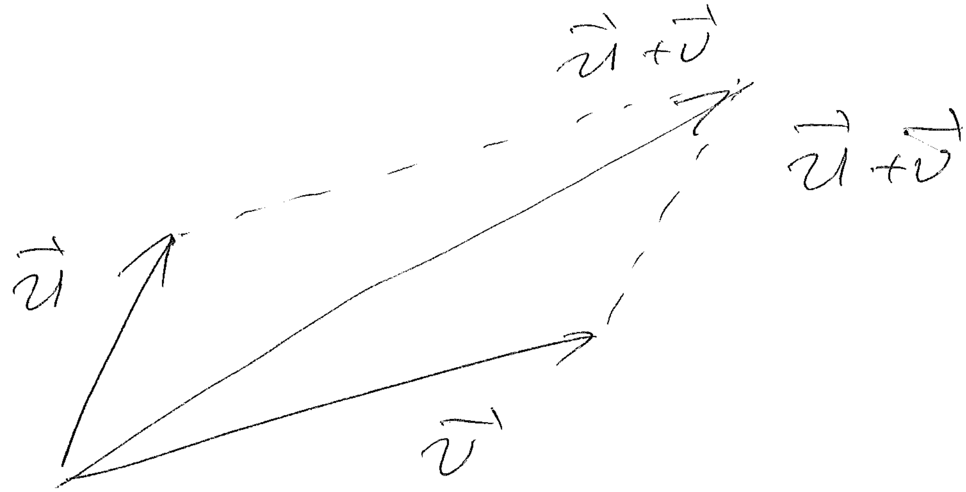
$$\langle \vec{p} - \vec{p_0}, \vec{n} \rangle = 0$$

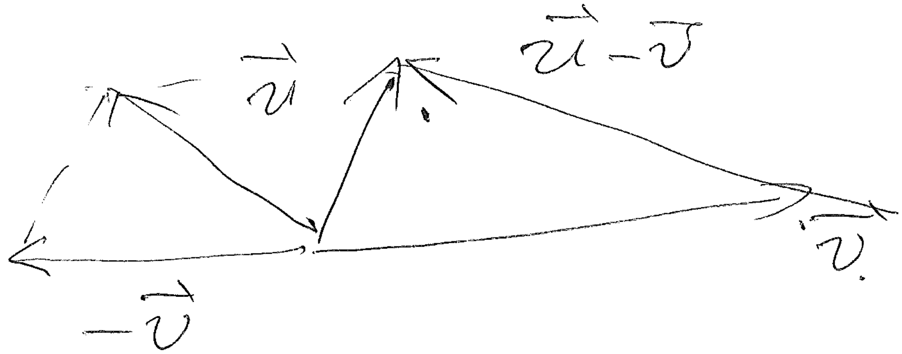
$$\left\langle \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix} \right\rangle$$

$$= \left\langle \begin{pmatrix} x - x_0 \\ y - y_0 \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix} \right\rangle$$

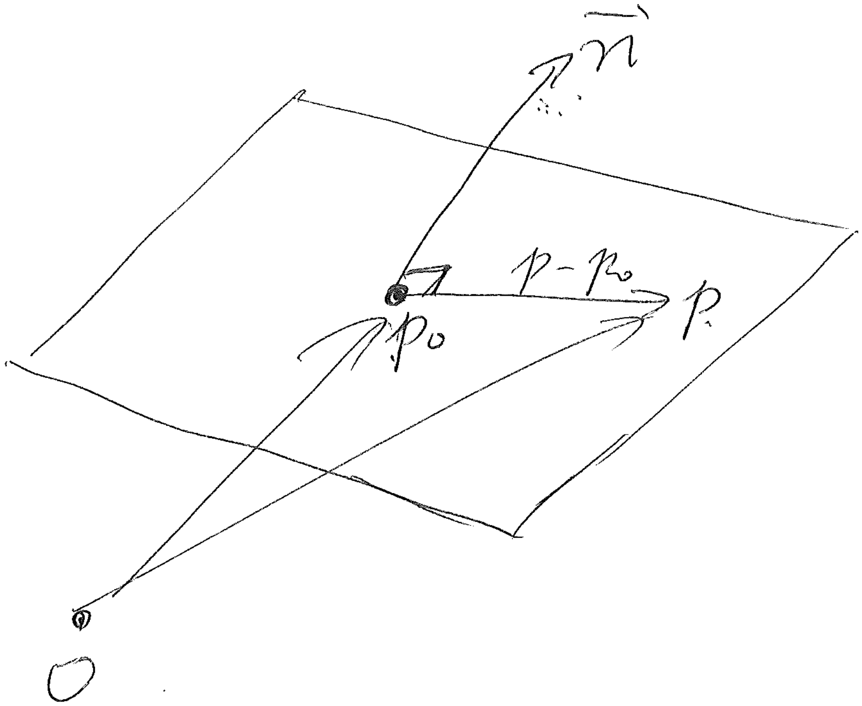
$$= a(x - x_0) + b(y - y_0) = 0$$

$$\underline{\underline{ax}} + \underline{\underline{by}} - ax_0 - by_0 = 0$$





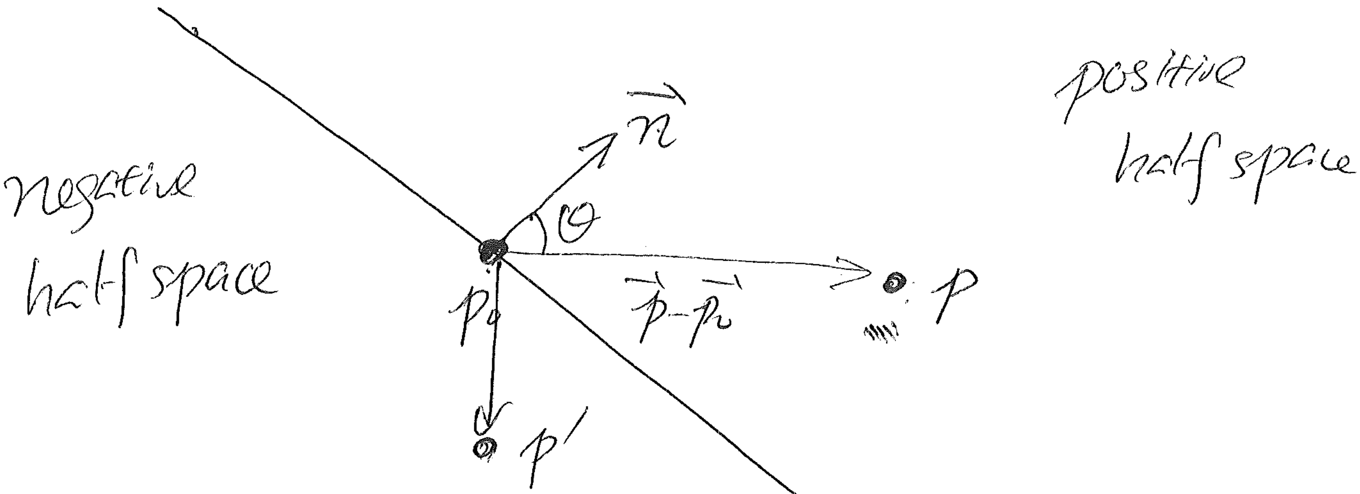
$$\vec{u}-\vec{v}$$
$$=\vec{u}+(-\vec{v})$$



$$\langle p - p_0 , \vec{n} \rangle = 0$$

$$\left\langle \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} - \begin{pmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_n^0 \end{pmatrix} , \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix} \right\rangle = 0$$

$$a_1 x_1 + a_2 x_2 + \cdots + a_n x_n - a_1 x_1^0 - a_2 x_2^0 - \cdots a_n x_n^0 = 0$$



$$\langle \vec{p}-\vec{p_0}\,,\,\vec{n}\rangle > 0$$

$$\left\langle \begin{pmatrix} x-x_0 \\ y-y_0 \end{pmatrix}, \begin{pmatrix} a \\ b \end{pmatrix} \right\rangle > 0$$

$$a(x-x_0)+b(y-y_0)>0$$

$$ax+by > ax_0+by_0$$

16

$$f(x+\Delta x) = f(x) + \nabla f^{T} \cdot \Delta x$$

$$f(x+\Delta x) = f(x) + f'(x) \cdot \Delta x + \cancel{\frac{f''(0)}{2!} \Delta x^2}$$

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$$

$$f(x) = 3x_1^2 + 4x_2^2$$

min f

$$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{pmatrix} =$$

$$f(x) = (x-3)^2$$

min f $\quad \cancel{2(x-3)}$

$$x_0 = 0$$

$$f(x_0 + \Delta x) = f(x_0) + f'(x_0) \cdot \Delta x$$

$$= 9 + (-6) \cdot \Delta x$$

in order for $f(x_0 + \Delta x) < f(x_0)$

$$(-6)\,\Delta x < 0$$

$$\Delta x > 0$$