Homework 3
Due: March 8 (Tuesday) 3:30pm

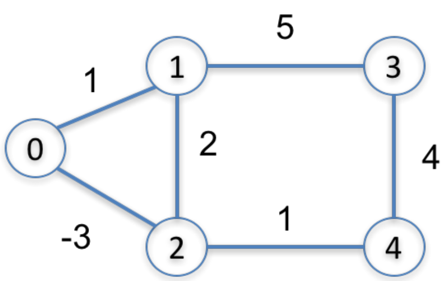
1. Consider the following instance of the knapsack problem.

   There are 6 items. Each item is represented as an ordered pair, where the first integer indicates its size and the second integer indicates its value. The items are: $(6, 2)$, $(3, 6)$, $(4, 3)$, $(1, 2)$, $(8, 17)$, $(5, 9)$.

   The knapsack size is 12.

   Build the dynamic programming table for solving the problem. Each entry in the table should have 2 attributes, where the first attributes the total value of the items, and the second entry is binary, indicating whether the current last item is taken or not.

2. Simulating Bellman-Ford, Floyd-Washall, and Dijkstra's algorithm using the following graph. In the case of Bellman-Ford and Dijkstra's algorithm, use vertex 0 as the source.



3. The Bellman-Ford algorithm for single-source shortest paths as discussed in class has a running time of $O\left(|V|^3\right)$, where $|V|$ is the number of vertices in the given graph. Describe how you can improve this running time to $O\left(|V||E|\right)$, where $|E|$ is the number of edges.

4. Let $G(V, E)$ be an undirected graph such that each vertex has an even degree. Design an $O(|V| + |E|)$ time algorithm to direct the edges of G such that, for each vertex, the outdegree is equal to the indegree.

5. A Hamiltonian path is a simple path that includes all the vertices of the graph. Design an efficient algorithm to determine if a given DAG exhibits a Hamiltonian path.

6. Let $G(V, E)$ be an undirected weighted graph, and let T be the MST of G. Suppose that we increase the weights of all the edgesin G by a constant c. Is T still the MST of G? Explain why?

7. Let $G(V, E)$ be an undirected weighted graph, and let T be an MST of G. Suppose that we add a new vertex v to G, together with some weighted edges from v to some vertices of G. Design a linear-time algorithm to find the new MST of G after adding v.

8. Arbitrage is the use of discrepancies in currency exchange rates to transform one unit of currency into more than one unit of the same currency. Hypothetically, suppose that 1 US dollar buys 46.4 Indian Rupees. 1 Indian Rupee buys 2.5 Japanese Yens, and 1 Japanese Yen buys 0.0091 US dollars. Then by converting currencies, a trader can start with 1 US dollar and buy 46.4

Rupees, and then use the Rupees to buy $46.4 \times 2.5 = 116$ Yens, and then use the Yens to buy $116 \times 0.0091 = 1.0556$ US Dollars. Thus turning a profit of 5.56 cents. Suppose you are given an $n \times n$ currency exchange table. Design an efficient algorithm to discover if there is an arbitrage.

9. Design an algorithm for updating an MST after deleting a node or an edge.