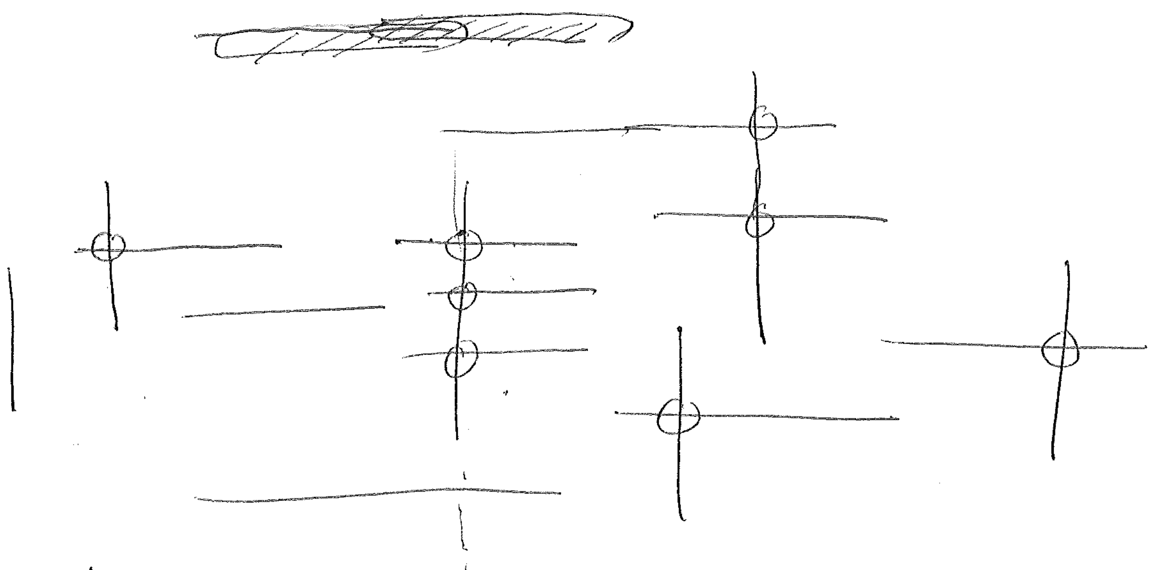
Feb 18, 2016

Exam 1 Thursday Feb 25.

Recall Line segment Intersection Problem

Given n vertical and n horizontal line segments

Find all intersections.



Plane sweeping

① Sort all vertical line segments based on their x-coordinates from left to right

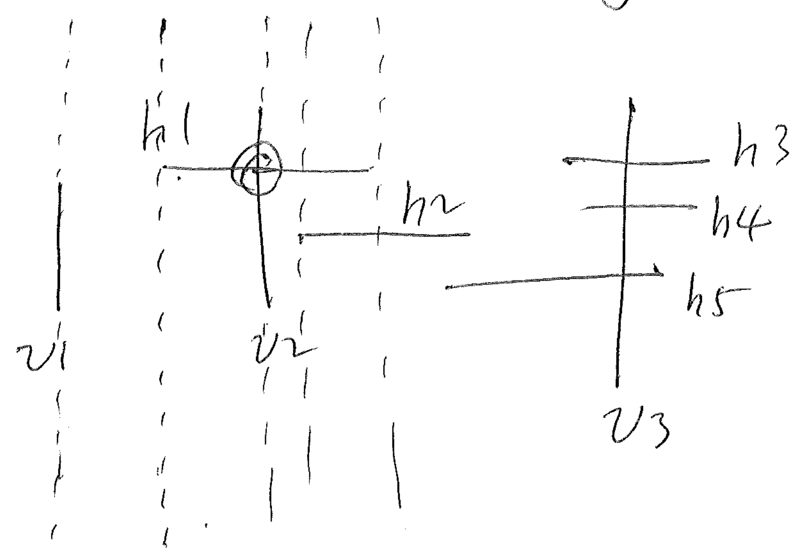
② For each vertical line segment from left to right

Find all intersections on it

Alg V2.

Sort all vertical linesegments from left to right and place them in an array A.[1..n]

initialized a BST storing all horizontal linesegments intersecting the current sweepline.



A [v1 | v2 | v3] BST = ϕ

A [~~v1~~ | v2 | v3] BST = {h1

A [~~v1~~ | ~~v2~~ | v3] output intersection

A [~~v1~~ | ~~v2~~ | v3] BST = {h1, h2)

Alg V3

Sort the leftends, rightends of horizontal linesegments and the x-coordinates of the vertical linesegments from left to right, and put those in an event array $A[1..3n]$

For $j = 1$ to $3n$

~~$A[j]$ case~~

Switch $A[j]$:

case 1 leftend of a horizontal line segment

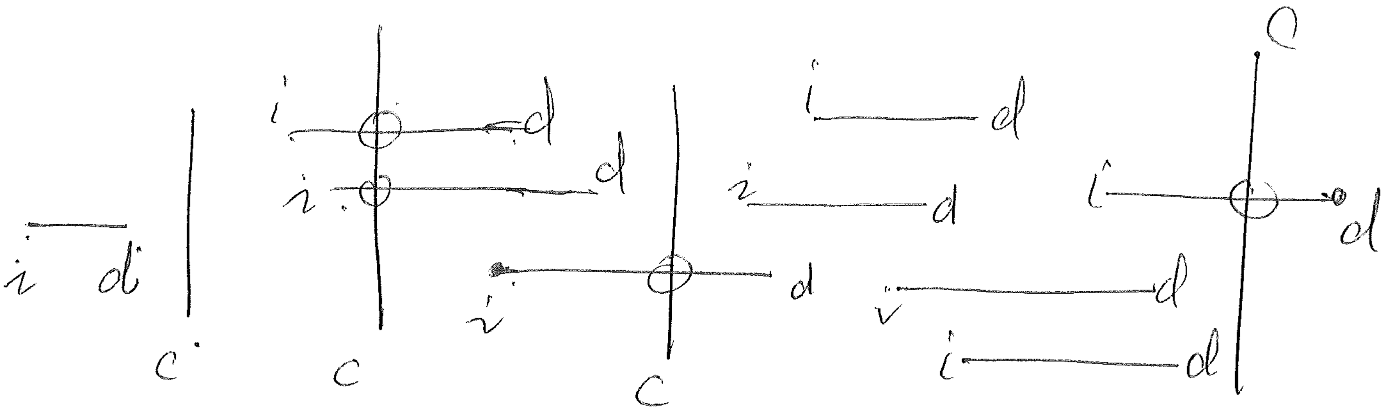
insert the corresponding line segment into the BST

case 2 rightend of a horizontal line segment

delete from the BST

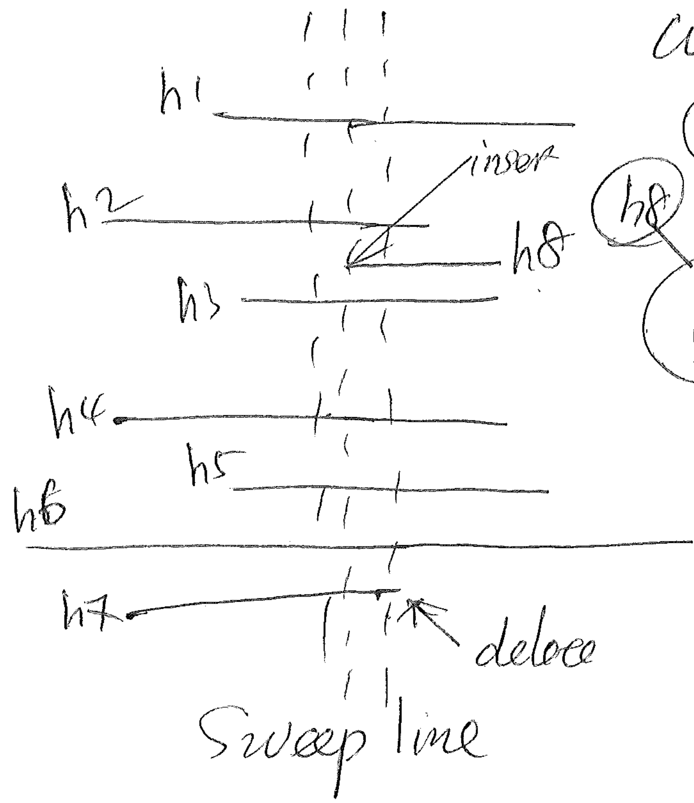
case 3 vertical line segment

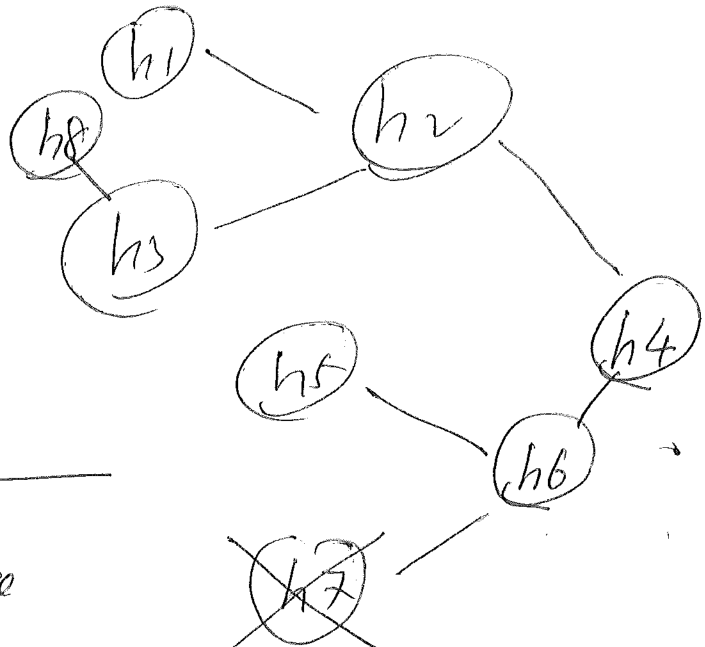
output intersections



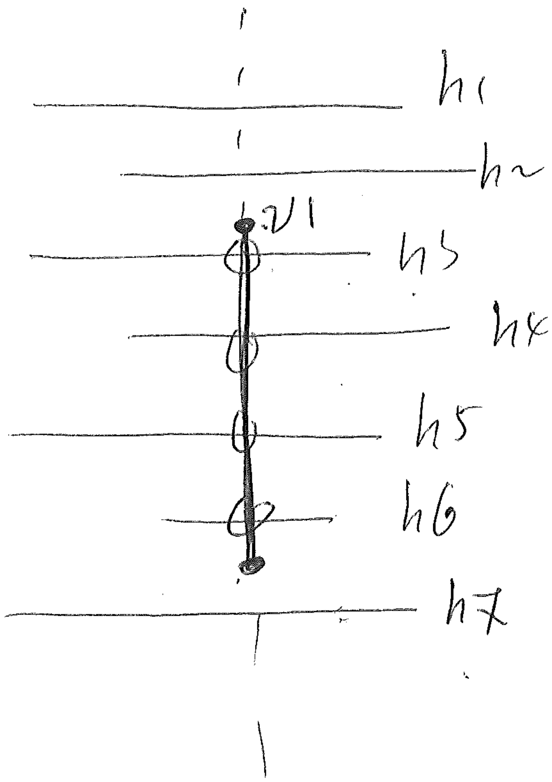
Q1 How BST is organized, i.e., how the horizontal line segments are stored in the BST

We will store based on y-coord.





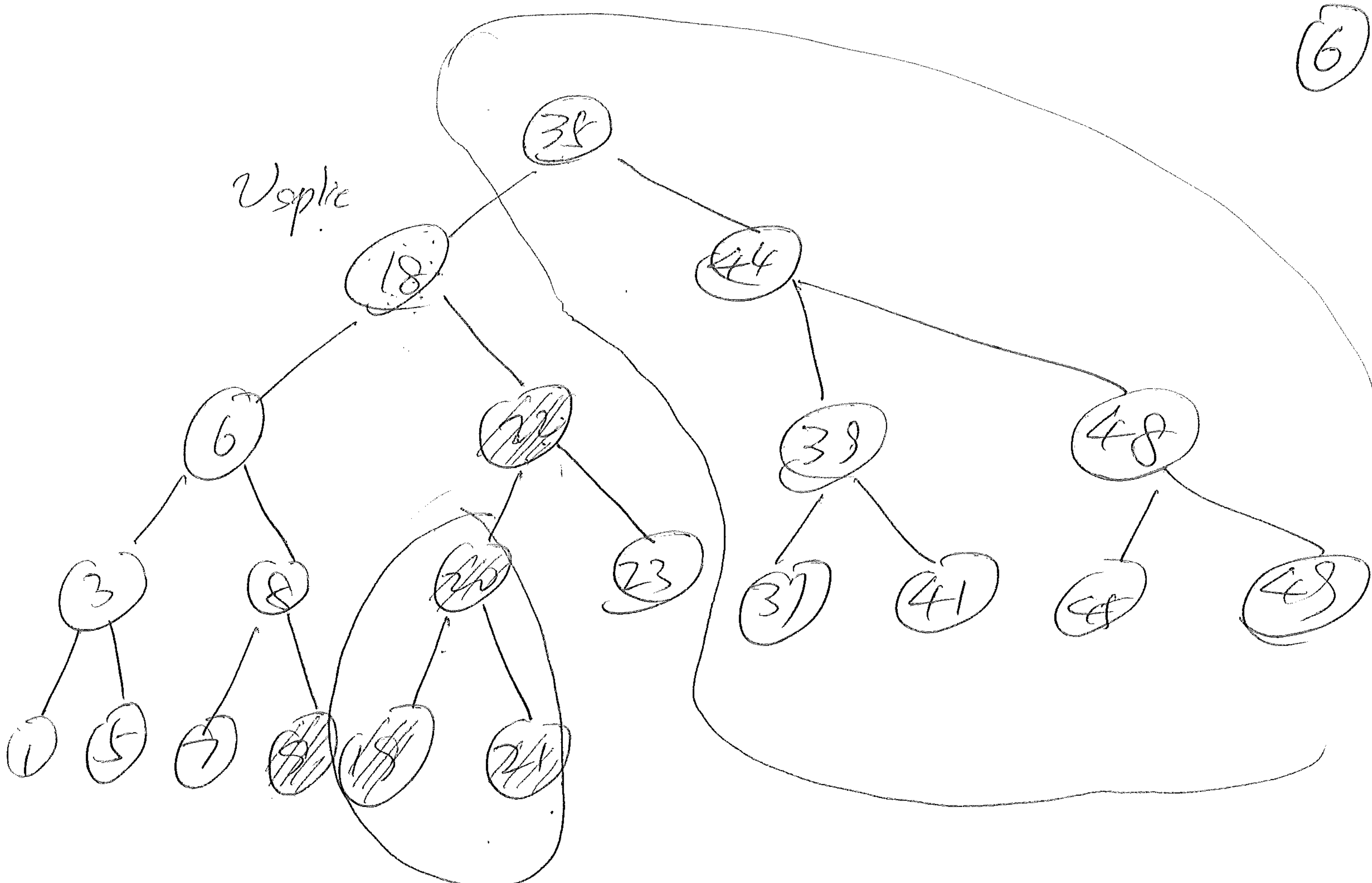
Q2 how to calculate intersections?



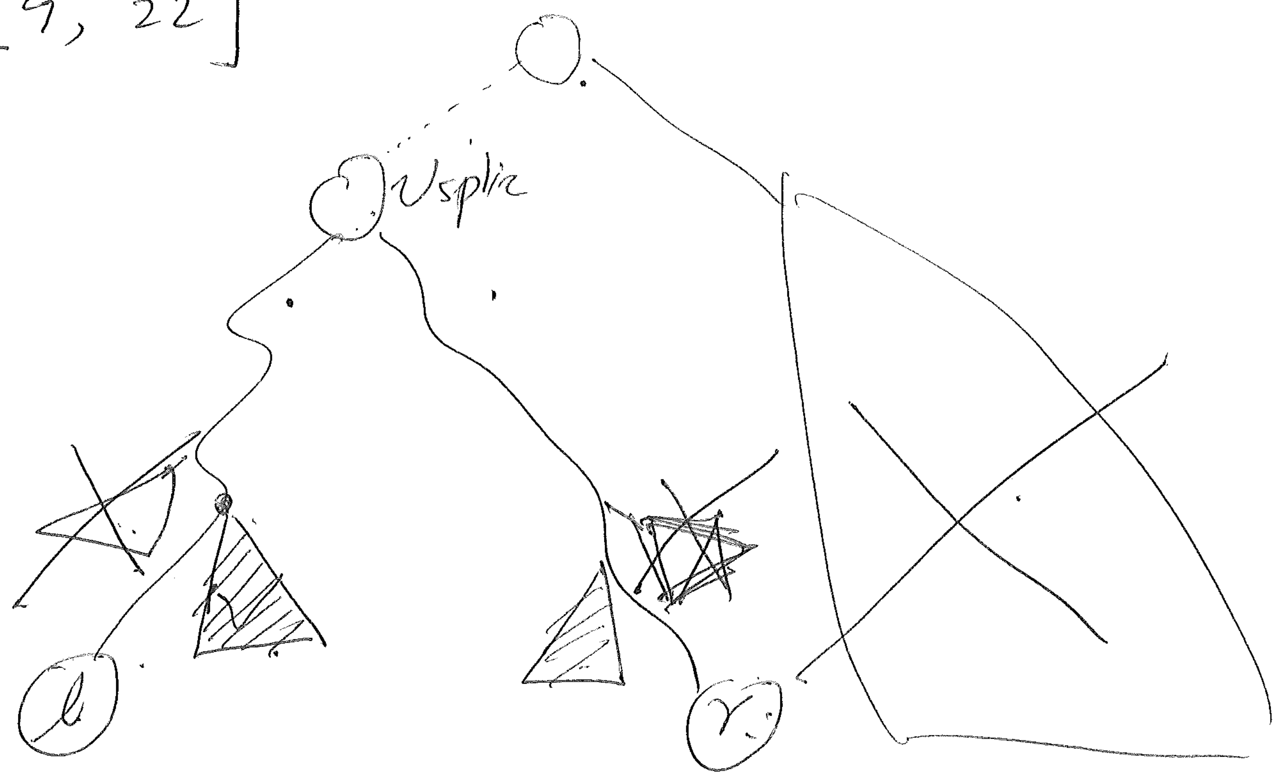
Observation: every horizontal line segments whose y-coordinate is with the y-range of the vertical line segment will generate an intersection

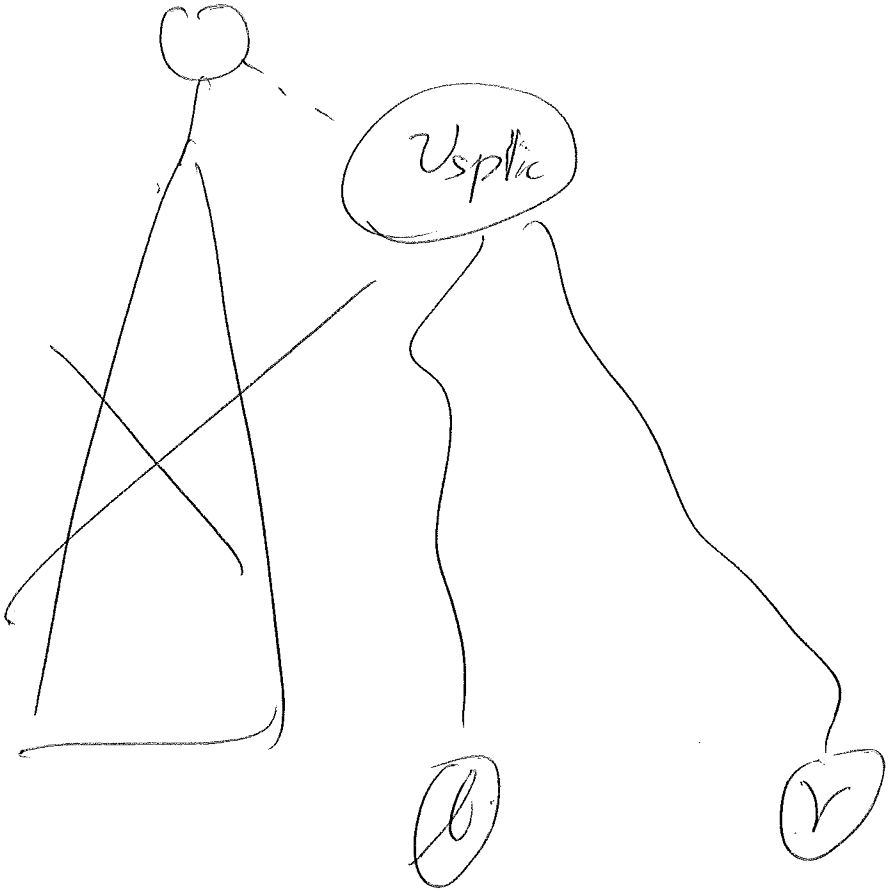
The Problem:

Given a BST. T and a query range $[l, r]$, output all ~~key~~ nodes of T whose key is within $[l, r]$



Query. [9, 22]





Range Query

input BST T, a query range $[l, r]$

output all nodes whose keys are within $[l, r]$

Find v_{split}

Going down from Vsplit to l, output all right subtrees

Going down from Vsplit to r, output all left subtrees

For every node on the paths from Vsplit to l and r output if within $[l, r]$

Running Time

$O(\log n + k)$, where k is the number of nodes within the query range

This type of running time is called output sensitive.

Sort the leftend points, rightend points and vertical line segments from left to right in an event array $A[1..3n]$.

For $j=1$ to $3n$

switch $A[j]$

case 1: leftend point, insert into BST (base y-coord)

case 2: rightend point, delete from BST

case 3: vertical line segment

rangequery $[y_B, y_T]$ on the BST

output intersections

Running Time: $\underbrace{n\log n}_{\text{sorting}} + \sum_{j=1}^{3n}(\log n + k_j)$

$= n\log n + \sum_{j=1}^{3n}\log n + \sum_{j=1}^{3n}k_j$

$= O(n\log n + K)$

where K is the number of intersections

$$A_{4\times 6}\left(B_{6\times 2}\, C_{2\times 9}\right)$$

$$\left(A_{4\times 6}\, B_{6\times 2}\right) C_{2\times 9}$$

$$\cancel{4} \quad A_{4\times 6}\, B_{6\times 2} = \left(\quad\right)_{4\times 2}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & \\ & & & & & \end{pmatrix} \begin{pmatrix} 0 & \\ 0 & \\ 0 & \\ 0 & \\ 0 & \\ 0 & \end{pmatrix} \qquad 4\times 6\times 2 = 48$$

$$A_{4\times 6}\,\underbrace{\left(B_{6\times 2}\, C_{2\times 9}\right)}_{D_{6\times 9}} \qquad 6\times 2\times 9 + 4\times 6\times 9 = 324$$

$$\underbrace{\left(A_{4\times 6}\, B_{6\times 2}\right)}_{D'_{4\times 2}} C_{2\times 9} \qquad 4\times 6\times 2 + 4\times 2\times 9 = 48 + 72 = 120$$

The Problem how to multiply

$$A_1 \times A_2 \times \cdots \times A_n \quad ?$$

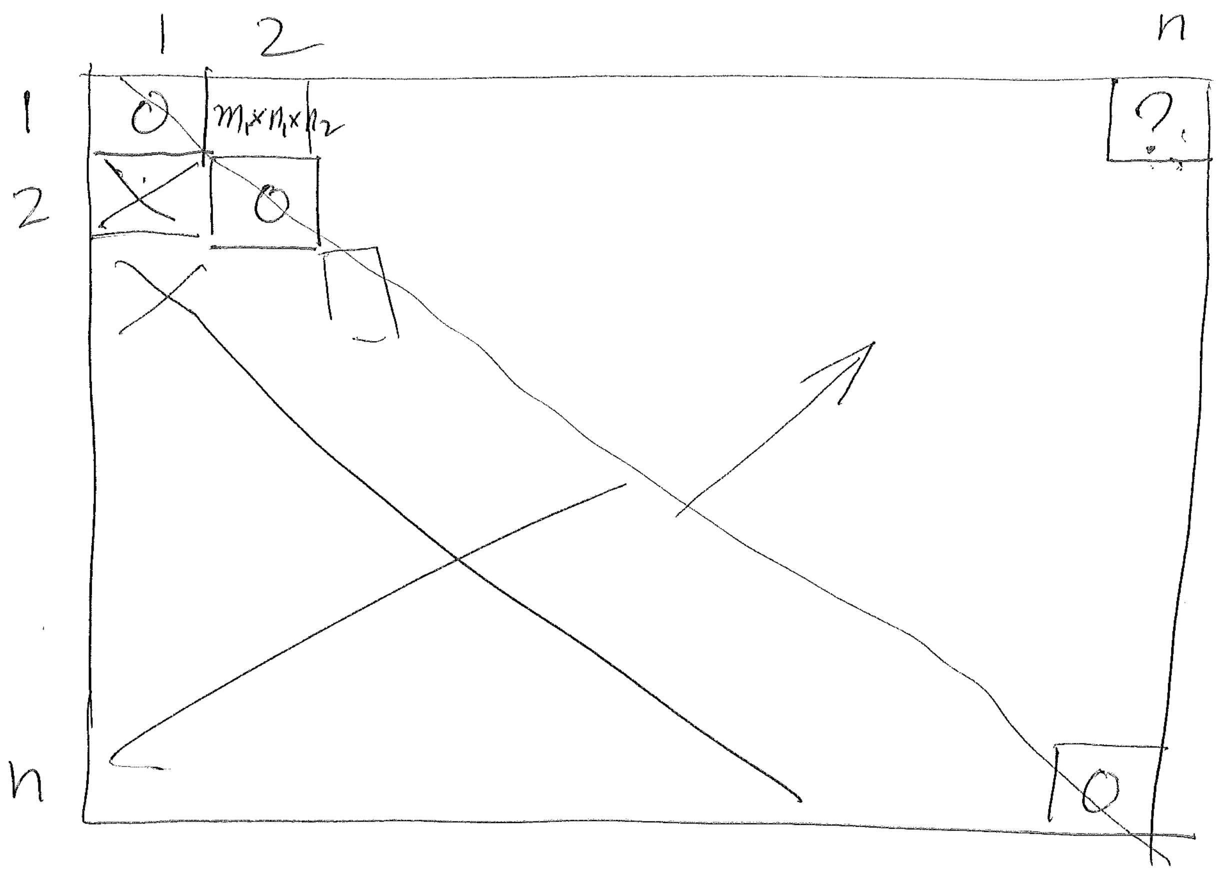
A_j is $m_j \times n_j$

$n_{j-1} = m_j$

$$A_j \times A_{j+1}$$

$$m_j \times n_j \quad m_{j+1} \times n_{j+1}$$

$$(A_1 \times \cdots \times A_j)(A_{j+1} \times \cdots \times A_n)$$



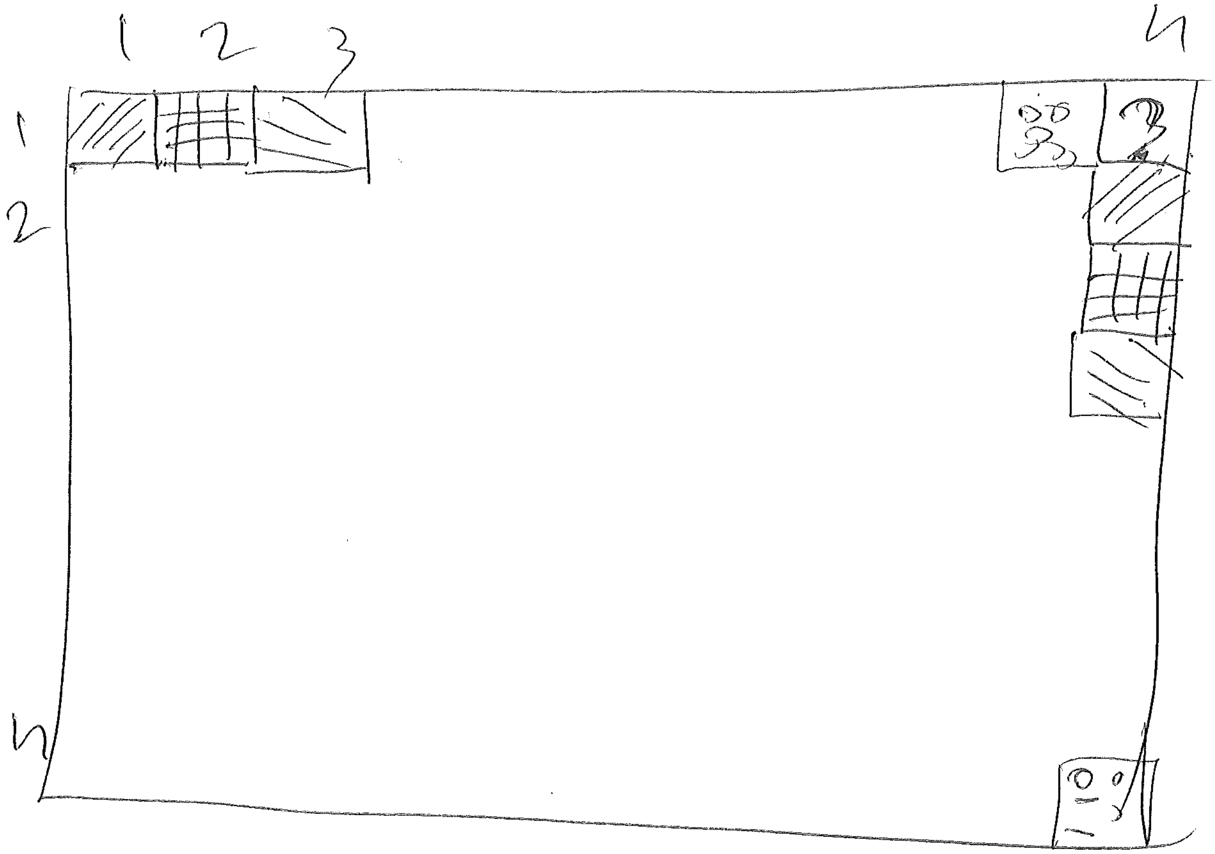
Each entry (i,j) means the optimal ways to multiply ~~A~~ $A_i A_{i+1} \cdots A_j$

$(A_1 A_2 A_3 A_4) A_5$

$(A_1 A_2 A_3) A_4 A_5$

$(A_1 A_2)(A_3 A_4 A_5)$

$A_1 (A_2 A_3 A_4 A_5)$



$A_1 \; A_2 \; \cdots \; A_n$

$O(n^3)$