Feb 16, 2016

Exam I. Thursday (Feb 25th)

Dynamic Programming

Building a table and record the process of induction

Converting recursion to iteration

$$\begin{cases} F_n = \cancel{F_{n-1}} + F_{n-2} \\ F_1 = 1 \\ F_0 = 1 \end{cases}$$

$F_{10} = ?$

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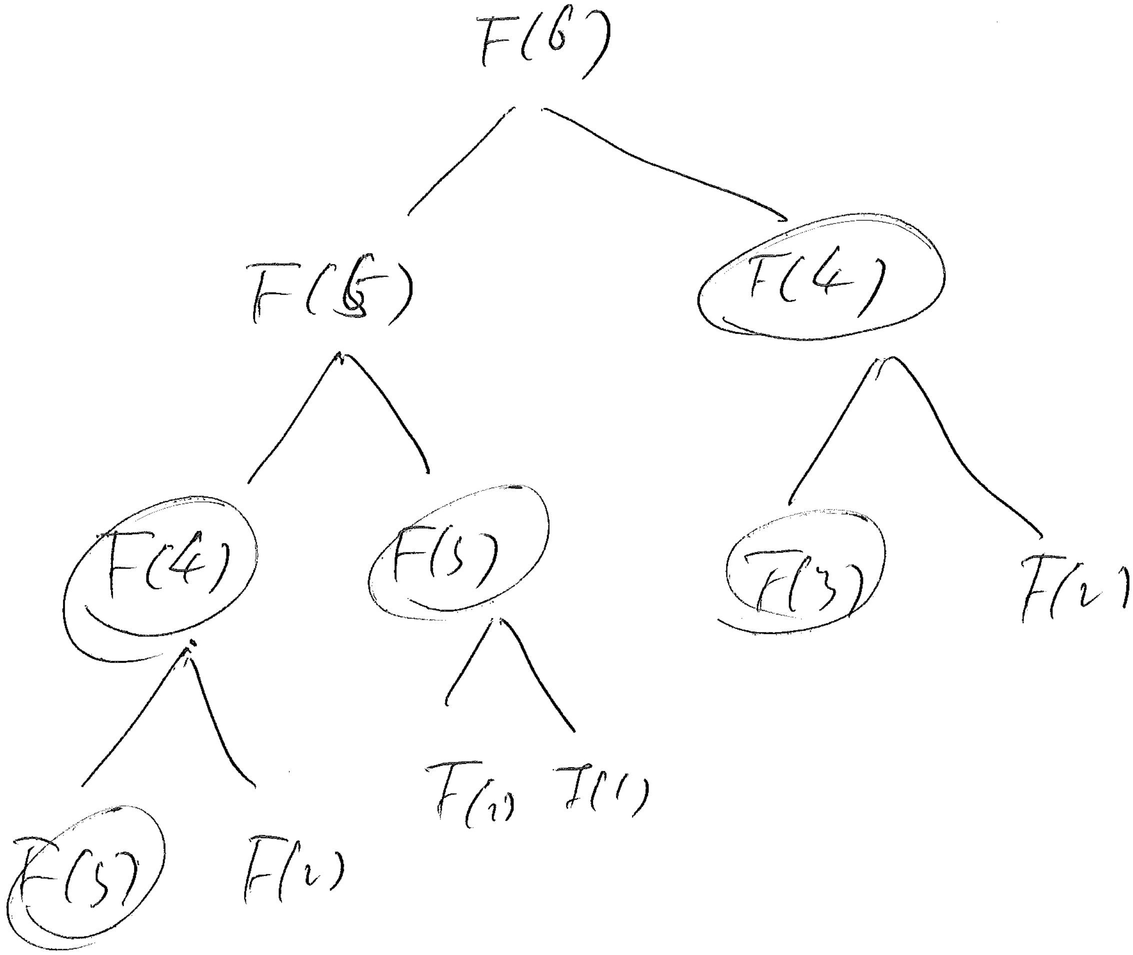
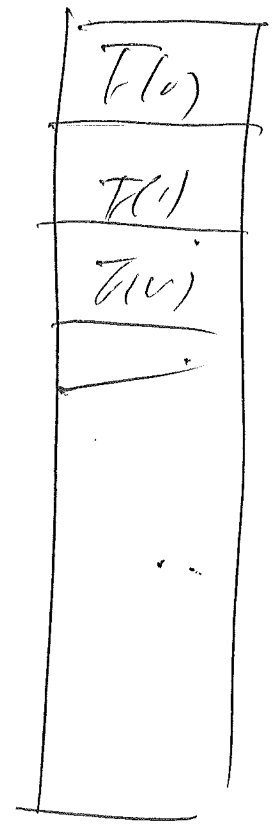
②

```
F (int x)
{
  if x=0 or x=1, return 1
  else return
      F(x-1) + F(x-2);
}

F(int x)
{
  initialize an array F[0..x]
  ~~For j=2~~
  F[0] = 1 , F[1] = 1
  For j=2 to x
      F[j] = F[j-1] + F[j-2]
  return F[x];
}
```


F(6)
F(5)
F(4)
F(4)
F(3)
F(3)
F(2)
F(3)
F(2)
F(2)
F(1)


F(0)
F(1)
F(2)
...

Knapsack Problem.

Given n items $S = \{s_1, s_2, \cdots, s_n\}$

each item s_j has an integer size $k_j > 0$

and an integer value $c_j > 0$

You are given a knapsack of ~~integer~~ size

$K \geq 0$

Find a subset of items $S^* \subseteq S$

such that:

(1) $\sum_{s \in S^*} k_s \leq K$

(2) $\sum_{s \in S^*} c_s$ is maximized.

$$\max \sum_{s \in S^*} c_s$$

$$\text{s.t.} \sum_{s \in S^*} k_s \leq K$$

integer linear programming.

introducing x_j as indicator variable for item S_j, $x_j \in \{0, 1\}$

if $x_j = 1$, the item S_j is chosen

if $x_j = 0$ the item S_j is not.

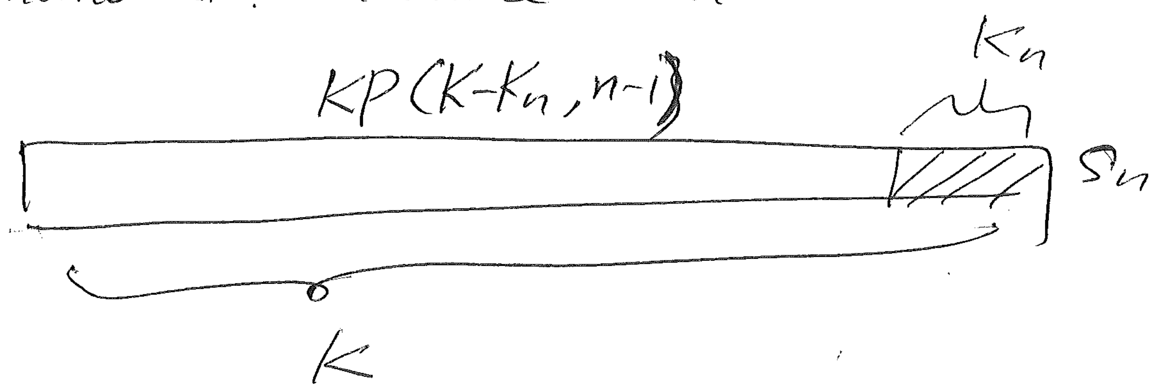
$$\max \sum_{j=1}^{n} x_j C_j$$

$$\text{s.t.} \quad \sum_{j=1}^{n} x_j K_j \leq K$$

$$x_j \in \{0, 1\}$$

Let the solution be $\underbrace{KP(K, n)}$

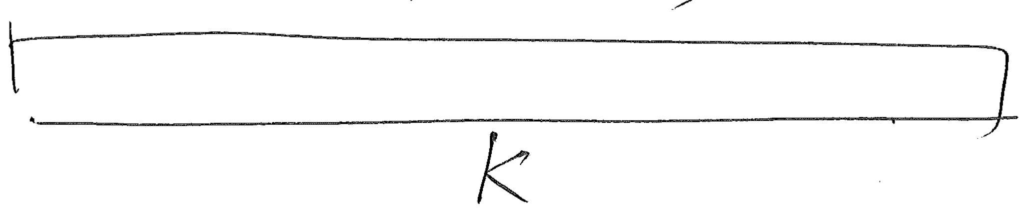
Scenario 1. assume $S_n \in S'^{*}$



$$KP(K-K_n, n-1) + C_n$$

Scenario 2. S_n is not included in <u>any</u> optimal solution

$$KP(K, n-1)$$



$$KP(K, n) = \max\{\; KP(K-K_n,\, n-1) + C_n,\;\; KP(K,\, n-1) \;\}$$

First n-1 items

First n-1 items

$$KP(0, n) = 0 \quad , \quad KP(K, 0) = 0$$

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iterative implementation

n \ k	0	1	2	$k-k_m$	k	$K-1$	K
ϕ	0	0	0			0	0
$\{S_1\}$	0						
$\{S_1, S_2\}$	0						
$\{S_1 \cdots S_{m-1}\}$							
$\{S_1 \cdots S_m\}$					KP(k,m)		
$\{S_1, S_2 \cdots S_n\}$	0						?

$S_j(k_j, c_j)$

$S_3(3, 13)$

$S_1(2, 6)$ $S_2(4, 2)$ ~~$S_3(2, 10)$~~

$S_4(2, 8)$

$K = 4$

K	1		2		3		4	
$\{S_1\}$	0	0	6	1	6	1	6	1
$\{S_1, S_2\}$			~~8~~ 6	0				
$\{S_1, S_2, S_3\}$			6 ~~8~~	0				
$\{S_1, S_2, S_3, S_4\}$							14	1

Running time $O(nK)$ pseudo-polynomial

$KP(1, 1) = \max\{$

infeasible

$KP(1-2, 0\} + 6$

$KP(1, 0\}$

$\}$

$KP(2, 1) = \max\{$

$KP(2-2, 0) + 6,$

$KP(2, 0)$

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