Assignment 1

Alex Baker

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CS 362

Problem 1:

a.)

**True**,

Proof:

b.)

**True**,

Proof:

c.)

**True**,

Proof:

From parts a and b with , which directly implies that .

d.)

**False**,

Proof:

Let and , if then for all

It must be . However, which is a contradiction

e.)

**True**,

Proof:

Polynomial >> linear

f.)

**False**,

Proof:

Let and , if then for all

It must be . However, which is a contradiction

Problem 2:

The following functions are given in order from smallest to largest growth rates:

1. This has a growth rate of 0 and is the smallest constant
2. This is a constant as well but larger that the first one
3. taking extra logs will always make the function slower
4. slightly slower than a normal log function
5. Same growth rate as log but the big base gives smaller values
6. Same growth rate as #3 but has a smaller base
7. sqrt(n) grows fast than log(n)
8. Grows slightly faster than log but still slow
9. linear is faster than log but not as fast as quadratic
10. quadratic is faster than linear and log
11. exponential is faster than quadratic
12. grows at same rate as #11 but has a larger base
13. exponent grows linear instead of log
14. same as #13 but has a larger base
15. faster than exponential since both exponent and base increase even though at a slow pace
16. same as #15 but with a larger base
17. This will oscillate between and zero, but still grows the fastest

Problem 3:

a.)

Let the root of the short tree be at level 0. For the given short tree, there are 256 leaf nodes with distinct integer keys. The root by definition then has children, which make up level 1. Level 2 then has nodes, and level 3 has . By definition, each non-leaf node has at least 2 leafs, a min and max key. So the level with 2 nodes per parent is the last level in a short tree. This short tree has a height of 3.

b.)

For a short tree with n nodes, let the vertex be level 0. Level 1 will then have , level 2 will have , and so on. This pattern repeats until a node only has two children, and the number of repetitions represents the height of the short tree. This pattern can be generalized as follows:

Let n be the level a node v is in the short tree, and x be the total number of leafs in the short tree, then the number of leafs at node v can be given as:

The level in the short tree has at most 2 leaf nodes is the bottom level, so this value is at most 2 (since this will be lower if the number of leafs are not a power of 2):

(raise both sides by )

So the height of a short tree of n keys is

Problem 4:

If n is the number of lines used to partition the plane into distinct regions, then f(n) is the number of regions formed by the n lines. Then f(n) is bounded by 2n, or f(n) = O(2n).

Proof: Induction

Base:

Let n = 1, then f(n) = 2(1) = 2 which is true for one line segment.

Induction Step:

Assume f(n) = 2n is true, need to show f(n+1) = O(2n)

f(n+1) = 2(n+1) = 2n + 2 = O(2n)

Therefore, the number of distinct regions is O(2n)

Problem 5:

Proof:

Problem 6:

Guess:

Proof: Induction

Base:

Induction Step:

Assume

So,