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2. Tires 91
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How to get the most from the seminar

- Ask questions
- Look at the screen (animations)
- Write notes – lots of them
- Revisit the slides and your notes
- Focus
- Discuss the topics, especially the one you do not fully understand, with your colleagues

- You know that this seminar is about engineering methodologies, not about recipes
- You are curious and pragmatic
- You want to learn and you are open to different ideas
- You do not like jargon and big theories
- You don't take yourself, vehicle dynamics and race car engineering too seriously; these disciplines can be boring…
- You understand I won't hesitate to make jokes at my expense (or someone else's expense) or use example that make point through comic means
- You do not have to agree with me; I could be wrong

Not Fast and Furious; Fast and…*Curious*

Some unrelated but important perspectives

"Human beings, who are almost unique (among animals) in having the ability to learn fr the experience of others, are also remarkable for their apparent disinclination to do so"

Douglas Adar

"If you don't know your weaknesses, you don't know your strengths. If you don't know w you win you won't know why you lose."

"Racing is part science, part art and (a little) part of luck. "

"Win. Know why. Win again."

"The chain is as strong as the weakest link. "

3 questions you need to answer:
Why?
How?
By how much?

"The definition of the problem will be the focal point of all your problem-solving effort."

"Knowledge shared is knowledge[2]"

"There are too many parameters (car, driver, track, environmental conditions…) to be spot in your calculation; ***you work in delta, trend and sensitivities. "***

Some important lessons I learnt about problem solving

"We cannot solve our problems with the same thinking we used when we created them"

Einstein

"If I had an hour to save the world. I would spend 59 minutes defining the problem and one minute finding solutions. And I find in most organizations people are running around spending sixty minutes finding solutions to problems that don't matter"

Einstein

"Intelligence is the ability to find a solution to a problem you never encountered before."

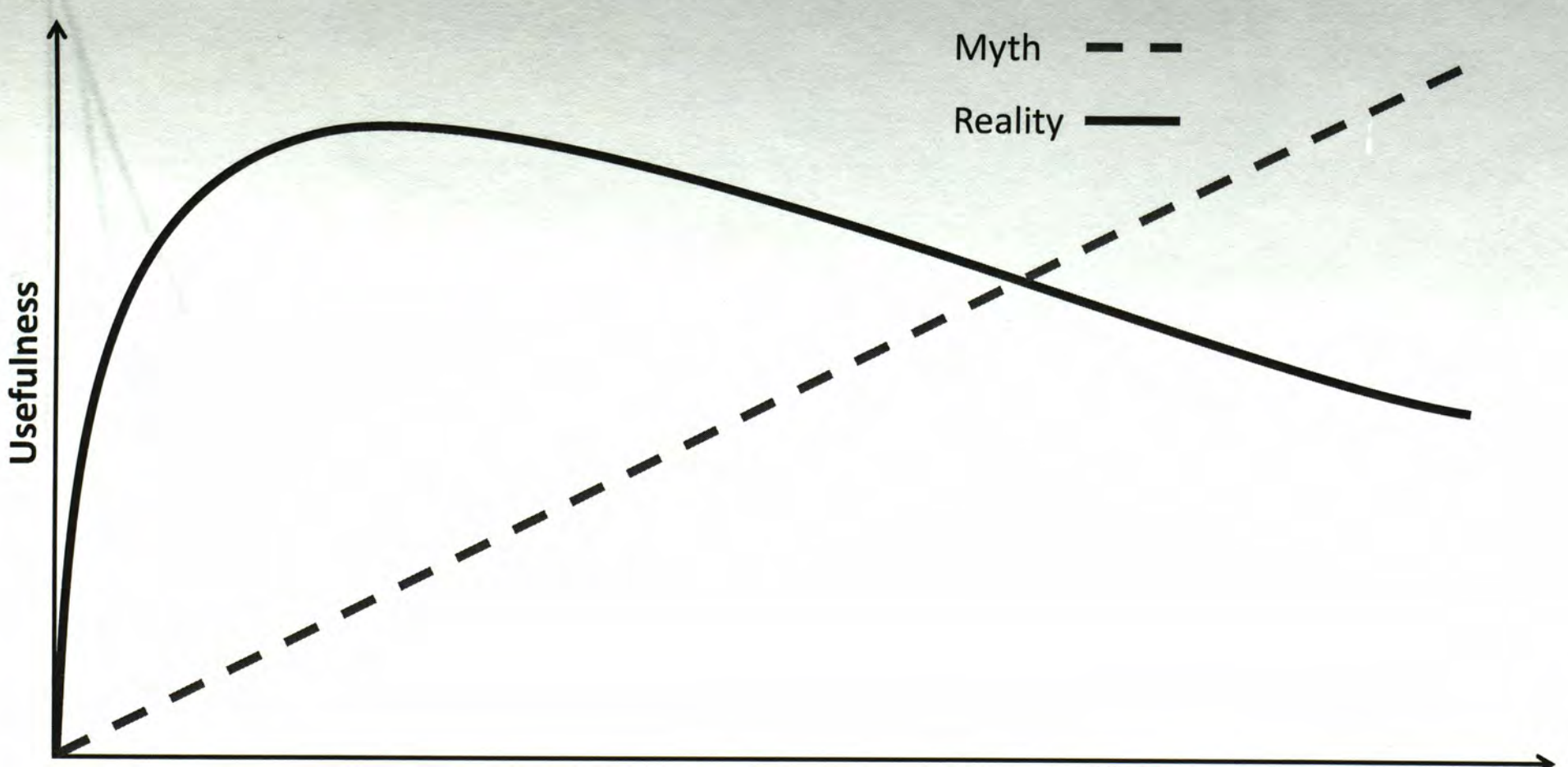
"Insanity is doing the same thing over and over again and expecting different results."

Einstein

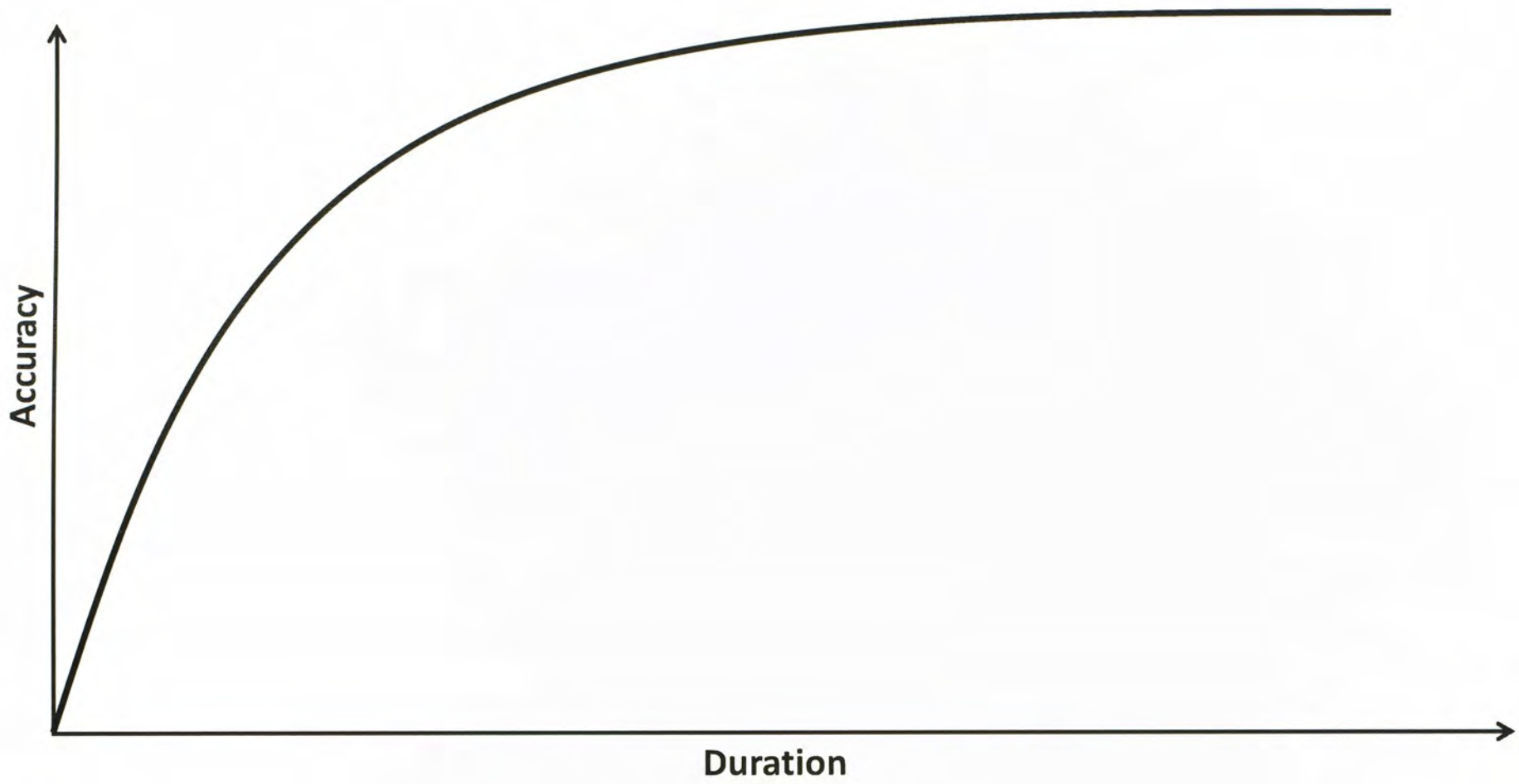
4 words I learnt the importance of:

- *Accuracy*
- *Usefulness*
- *Meaningfulness*
- *Repeatability*

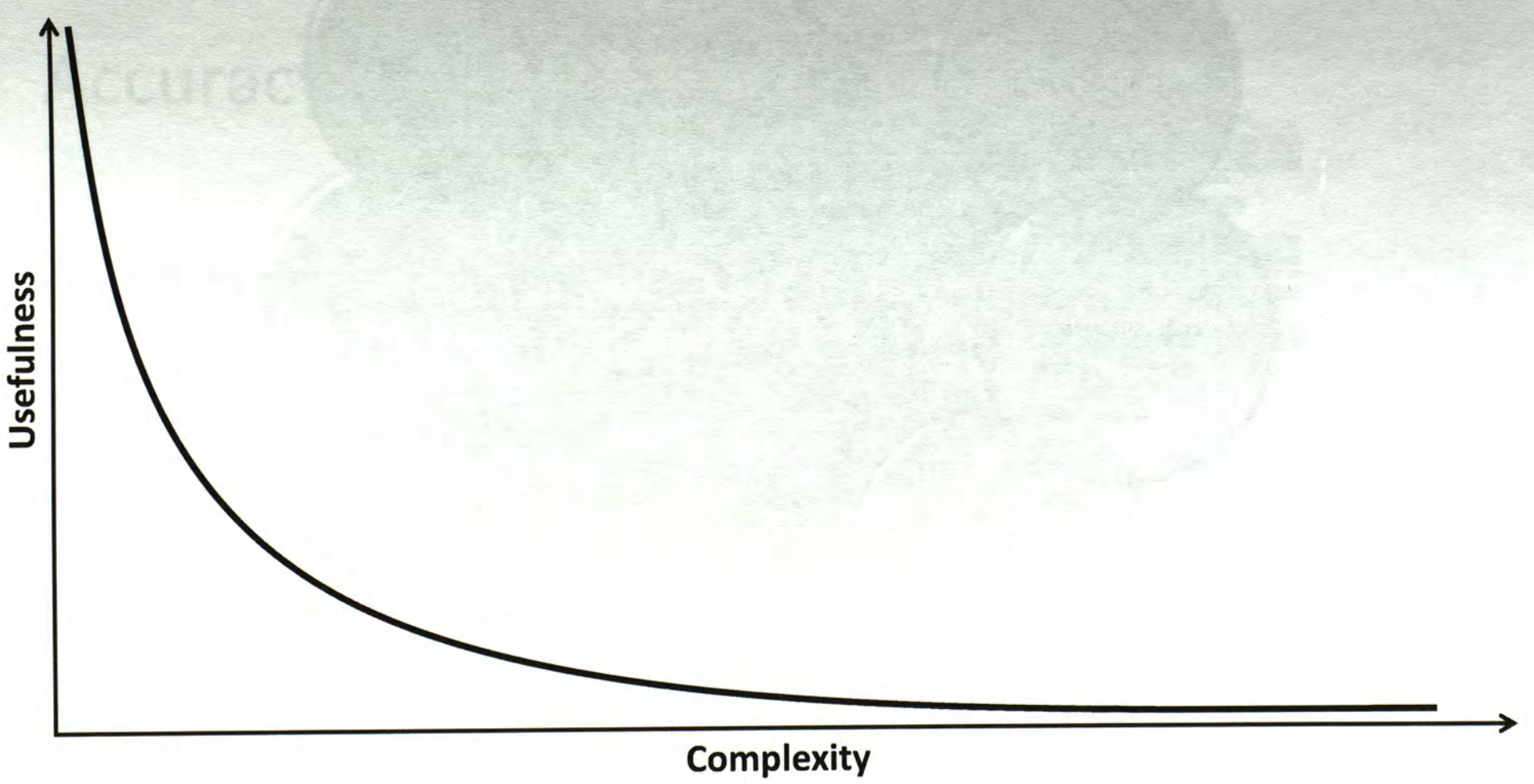
Myth and Reality



Accuracy and Duration



Usefulness and Complexity

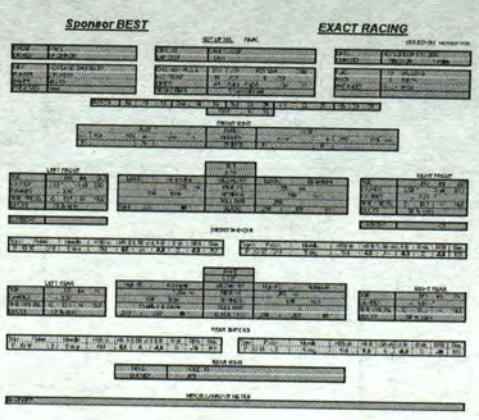


Racing Team

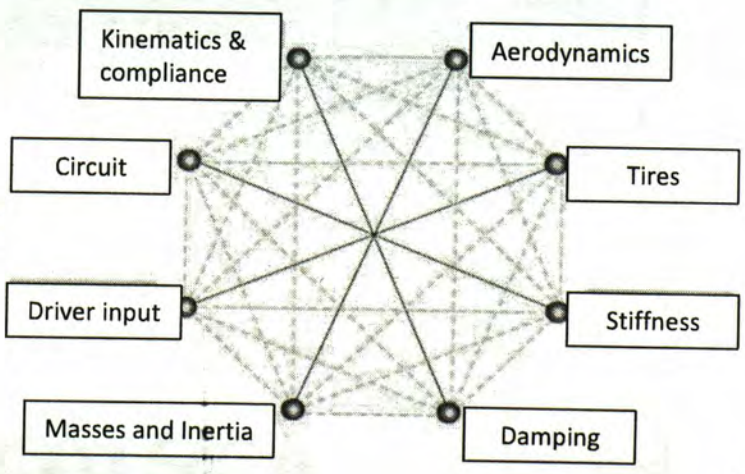
Three Essential Criteria to Master to Capture Vehicle Dynamics and Race Car Engineering Complexity

Accuracy

Numbers of parameters



Parameters interdependence



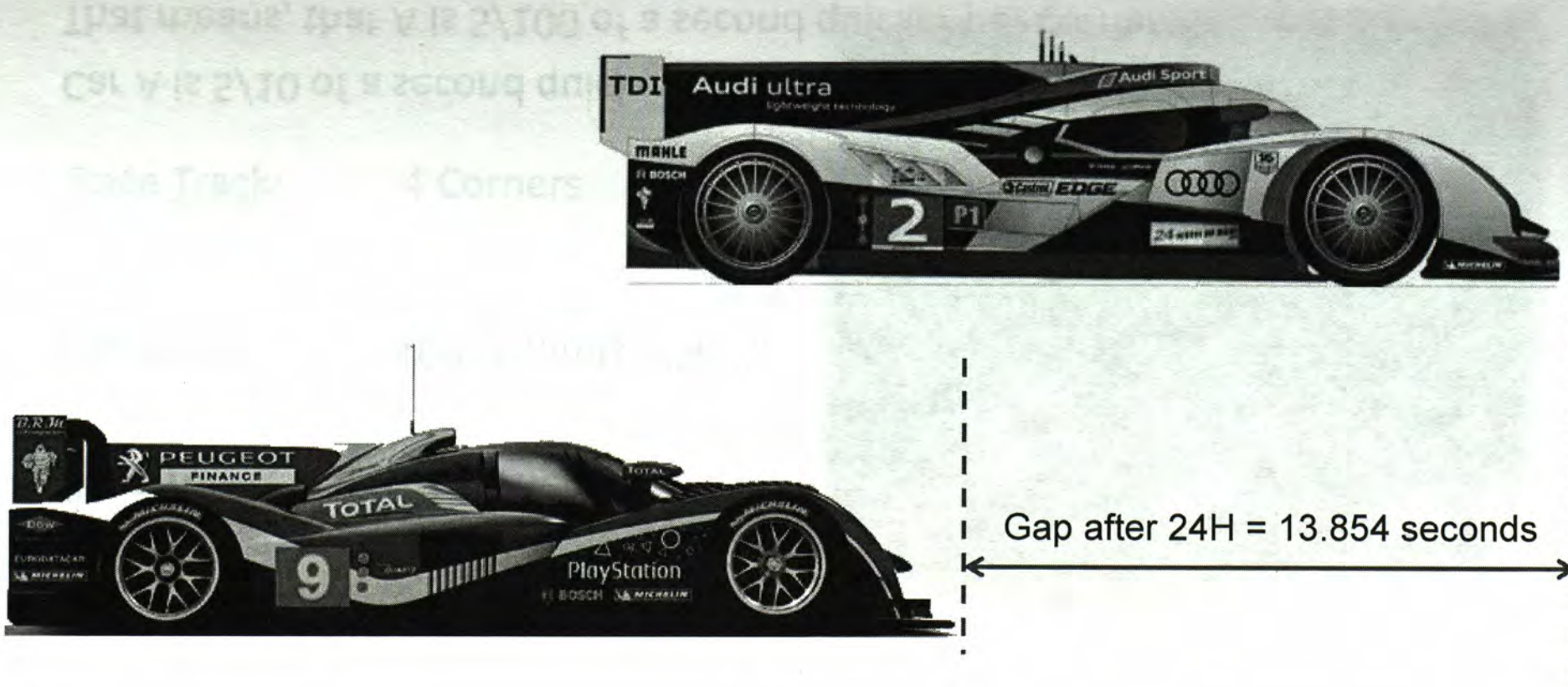
The Need of Accuracy

2012 Formula 1 Chinese Grand Prix 13-15 April 2012								
Pos	No	Driver	Team	Q2	Time Difference	% Difference		
1	2	Mark Webber	Red Bull Racing-Renault	01:35.700	-	-		
2	8	Nico Rosberg	Mercedes	01:35.725	0.025	0.03%		
3	7	Michael Schumacher	Mercedes	01:35.794	0.094	0.10%		
4	15	Sergio Perez	Sauber-Ferrari	01:35.831	0.131	0.14%		
5	14	Kamui Kobayashi	Sauber-Ferrari	01:35.853	0.153	0.16%		
6	4	Lewis Hamilton	McLaren-Mercedes	01:35.902	0.202	0.21%	0.29%	
7	10	Romain Grosjean	Lotus-Renault	01:35.903	0.203	0.21%		
8	9	Kimi Räikkönen	Lotus-Renault	01:35.921	0.221	0.23%		
9	3	Jenson Button	McLaren-Mercedes	01:35.942	0.242	0.25%		
10	5	Fernando Alonso	Ferrari	01:35.982	0.282	0.29%		
11	1	Sebastian Vettel	Red Bull Racing-Renault	01:36.031	0.331	0.35%		
12	6	Felipe Massa	Ferrari	01:36.255	0.555	0.58%		
13	18	Pastor Maldonado	Williams-Renault	01:36.283	0.583	0.61%		
14	19	Bruno Senna	Williams-Renault	01:36.289	0.589	0.62%		
15	11	Paul di Resta	Force India-Mercedes	01:36.317	0.617	0.64%		
16	12	Nico Hulkenberg	Force India-Mercedes	01:36.745	1.045	1.09%		
17	16	Daniel Ricciardo	STR-Ferrari	01:36.956	1.256	1.31%		1.31%

2012 NSCS Daytona 500 Qualifying Results 27th February 2012									
Pos	No.	Driver	Team	Time	Speed	Difference	% Difference		
1	99	Carl Edwards	Fastenal Ford	46.216	194.738	0	-		
2	16	Greg Biffle	3M Ford	46.371	194.087	0.155	0.08%		
3	88	Dale Earnhardt Jr.	Diet Mountain Dew/National Guard Chevrolet	46.385	194.028	0.169	0.09%		
4	9	Marcos Ambrose	Stanley Ford	46.392	193.999	0.176	0.09%		
5	13	Casey Mears	GEICO Ford	46.429	193.844	0.213	0.11%		
6	24	Jeff Gordon	Drive to End Hunger Chevrolet	46.439	193.803	0.223	0.11%		
7	56	Martin Truex Jr.	NAPA Auto Parts Toyota	46.472	193.665	0.256	0.13%		
8	6	Ricky Stenhouse Jr.	Ford EcoBoost Ford	46.472	193.665	0.256	0.13%		
9	21	Trevor Bayne	Motorcraft/Quick Lane Tire & Auto Center Ford	46.484	193.615	0.268	0.14%	0.17%	
10	14	Tony Stewart	Office Depot/Mobil 1 Chevrolet	46.486	193.607	0.270	0.14%		
11	55	Mark Martin	Aaron's Toyota	46.511	193.503	0.295	0.15%		
12	48	Jimmie Johnson	Lowe's Chevrolet	46.524	193.449	0.308	0.16%		
13	43	Aric Almirola	Smithfield Helping Hungry Homes Ford	46.540	193.382	0.324	0.17%		0.26%
14	27	Paul Menard	Menards/Peak Chevrolet	46.542	193.374	0.326	0.17%		
15	34	David Ragan	Scorpion Truck Bedliners Ford	46.572	193.249	0.356	0.18%		
16	17	Matt Kenseth	Best Buy Ford	46.573	193.245	0.357	0.18%		
17	39	Ryan Newman	U.S. ARMY/Quicken Loans Chevrolet	46.578	193.224	0.362	0.19%		
18	22	AJ Allmendinger	Shell Pennzoil Dodge	46.603	193.121	0.387	0.20%		
19	2	Brad Keselowski	Miller Lite Dodge	46.634	192.992	0.418	0.21%		
20	29	Kevin Harvick	Budweiser Chevrolet	46.653	192.914	0.437	0.22%		
21	20	Joey Logano	The Home Depot Toyota	46.664	192.868	0.448	0.23%		
22	31	Jeff Burton	Caterpillar Chevrolet	46.686	192.777	0.470	0.24%		
23	42	Juan Pablo Montoya	Target Chevrolet	46.729	192.600	0.513	0.26%		
24	5	Kasey Kahne	Farmers Insurance Chevrolet	46.733	192.583	0.517	0.27%		
25	26	Tony Raines	Front Row Motorsports Ford	46.745	192.534	0.529	0.27%		
26	30	David Stremme	Inception Motorsports Toyota	46.884	191.963	0.668	0.34%		
27	18	Kyle Busch	M&M's Brown Toyota	46.906	191.873	0.690	0.35%		
28	1	Jamie McMurray	Bass Pro Shops/Tracker Boats Chevrolet	46.914	191.840	0.698	0.36%		
29	10	Danica Patrick	GoDaddy.com Chevrolet	46.939	191.738	0.723	0.37%		
30	9	Kenny Wallace	American Ethanol Toyota	46.981	191.567	0.765	0.39%		
31	32	Terry Labonte	C&J Energy Ford	46.992	191.522	0.776	0.40%		
32	36	Dave Blaney	Ollie's Bargain Outlet Chevrolet	46.996	191.506	0.780	0.40%		
33	51	Kurt Busch	Hendrickcars.com Chevrolet	47.031	191.363	0.815	0.42%		
34	33	Elliott Sadler	General Mills/Kroger Chevrolet	47.054	191.270	0.838	0.43%		
35	40	Michael Waltrip	Aaron's Dream Machine Toyota	47.076	191.180	0.860	0.44%		
36	87	Joe Nemechek	AM FM Energy Wood & Pellet Stoves Toyota	47.081	191.160	0.865	0.44%		
37	11	Denny Hamlin	FedEx Express Toyota	47.089	191.127	0.873	0.45%		
38	78	Regan Smith	Furniture Row Racing Chevrolet	47.105	191.063	0.889	0.46%		
39	98	Michael McDowell	No.98 KLOVE/Curb Records Ford Ford	47.123	190.990	0.907	0.47%		
40	83	Landon Cassill	Burger King Toyota	47.218	190.605	1.002	0.51%		0.51%
41	38	David Gilliland	MHP/Power Pak Pudding Ford	47.357	190.046	1.141	0.59%		
42	47	Bobby Labonte	Kroger Toyota	47.363	190.022	1.147	0.59%		
43	97	Bill Elliott	AM FM Energy Wood & Pellet Stoves Toyota	47.381	189.950	1.165	0.60%		
44	37	Mike Wallace	Poynt.com Ford	47.405	189.853	1.189	0.61%		
45	93	David Reutimann	Burger King Toyota	47.560	189.235	1.344	0.69%		
46	23	Robert Richardson	North Texas Pipe Toyota	47.761	188.438	1.545	0.79%		
47	7	Robby Gordon	MAPEI/Menards/SPEED Energy Dodge	47.814	188.229	1.598	0.82%		
48	49	JJ Yeley	America Israel Racing Toyota	47.884	187.954	1.668	0.86%		

Formula Student UK 2012														
Sequence	Place	Car Num	Team	Penalty	Cost Score	Presentation Score	Design Score	Acceleration Score	Skid Pad Score	Sprint Score	Endurance Score	Economy Score	Total Score	% Diff.
1	1	4	Chalmers University of Technology		77.4	70.0	141	67.5	47.8	121.8	300.0	25.0	850.5	
2	2	6	Delft University of Technology		60.5	62.8	148	71.1	43.9	110.5	251.2	100.0	848.1	0.28%
3	3	56	Monash University		78.7	65.4	146	34.9	50.0	120.3	299.2	46.3	840.7	1.15%
4	4	2	TU Munich	-2	31.5	35.1	116	60.9	44.8	150.0	297.9	50.0	784.2	
5	5	1	University of Stuttgart		79.4	63.1	102	40.9	37.6	120.5	284.7	50.4	778.6	
6	6	49	Tallinn University of Applied Sciences	-35	71.3	57.4	128	59.6	44.3	119.5	256.5	19.0	720.5	
7	7	92	Oxford Brookes University		62.8	69.1	120	51.6	50.0	125.8	194.5	44.8	718.6	
8	8	23	UAS Amberg-Weiden		69.2	42.9	127	34.8	36.6	76.3	220.9	57.6	665.4	
9	9	96	UAS Zwickau	-2	59.7	73.5	117	49.4		57.9	214.9	91.2	661.5	
10	10	5	TU Graz		68.2	44.7	108	17.5	44.2	48.1	182.5	77.4	590.5	
11	11	74	University of Erlangen-Nuremburg		63.4	31.7	117	56.5	31.1	77.5	132.8	78.0	587.9	
12	12	64	Liverpool John Moores University		54.3	38.3	86	37.1	45.0	58.2	180.2	45.2	544.4	
13	13	33	ETH Zurich		69.6	49.5	145	75.0	48.9	127.6	6.0		521.5	
14	14	42	RWTH Aachen	-2	48.7	52.9	58	44.0	34.9	60.8	181.2	20.4	498.8	
15	15	3	University of Hertfordshire		68.6	68.3	147	48.3	42.6	93.1	22.0		489.9	
26	26	58	Institut Supérieur de l'Automobile et des Transports		61.9	48.0	85	54.4	24.1	77.4	14.0		364.7	
27	27	40	Eindhoven University of Technology		76.8	55.3	116	72.0	38.3				358.5	
28	28	51	Lund University	-2	60.2	33.5	76	18.3		7.5	130.0	30.0	353.4	
29	29	54	Karlstad University		49.3	56.2	66	28.7	34.0	72.4	16.0		322.5	
30	30	112	Swansea University		58.0	40.9	116	42.8	22.1	28.0	12.0		319.7	
31	31	20	University of Maribor	-2	46.3	34.8	94	42.2	34.3	40.7	22.0		312.3	
32	32	43	Brunel University		76.7	57.8	85	30.0	36.2	7.5	4.0		297.1	
33	33	66	Østfold University College			23.1	89.5	6.1	22.4	7.5	127.2	18.1	293.9	
34	34	13	Lancaster University		65.9	53.3	91	38.2	18.0	7.5	12.0		285.9	
35	35	80	University of Warwick		76.7	70.0	102			7.5	21.0		277.1	

Le Mans 24H 2011



$$Gap\ in\ percentage = \frac{13.854}{24 \times 60 \times 60} \times 100 = 0.02\%!!!$$

Understanding the need of accuracy

Race Track: 10 Corners

Lap time: 100 seconds [1'40'']

Car A is 5/10 of a second quicker than Car B which is 0.5 % quicker.
That means, that A is 5/100 of a second quicker per corner

That means, as an average, that A is 25/1000 of a second going in and 25/1000 of a second going out of each corner.

Understanding the need of accuracy

ace Track: 4 Corners

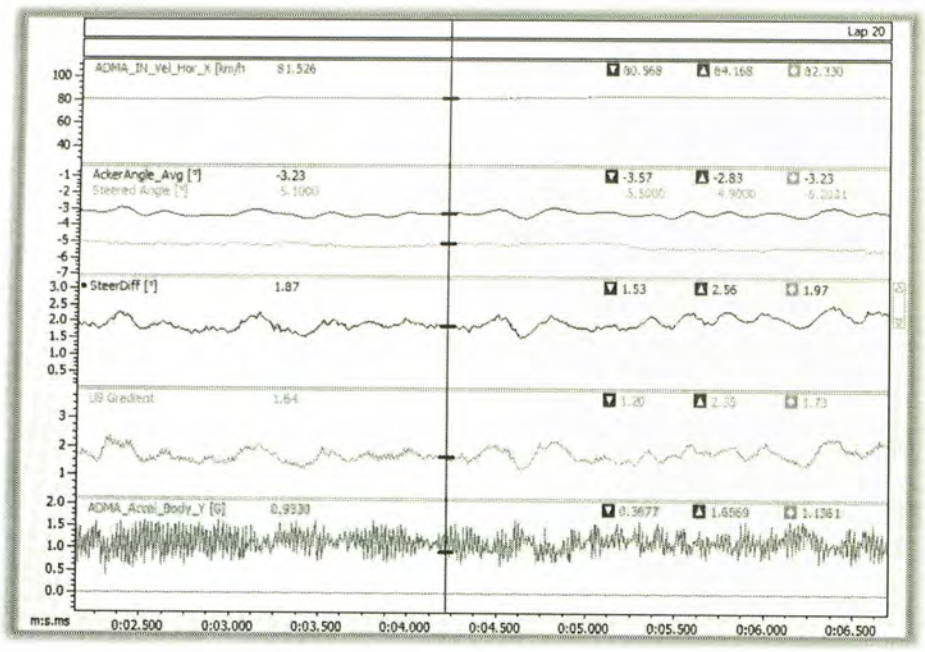
ap time: 24 seconds

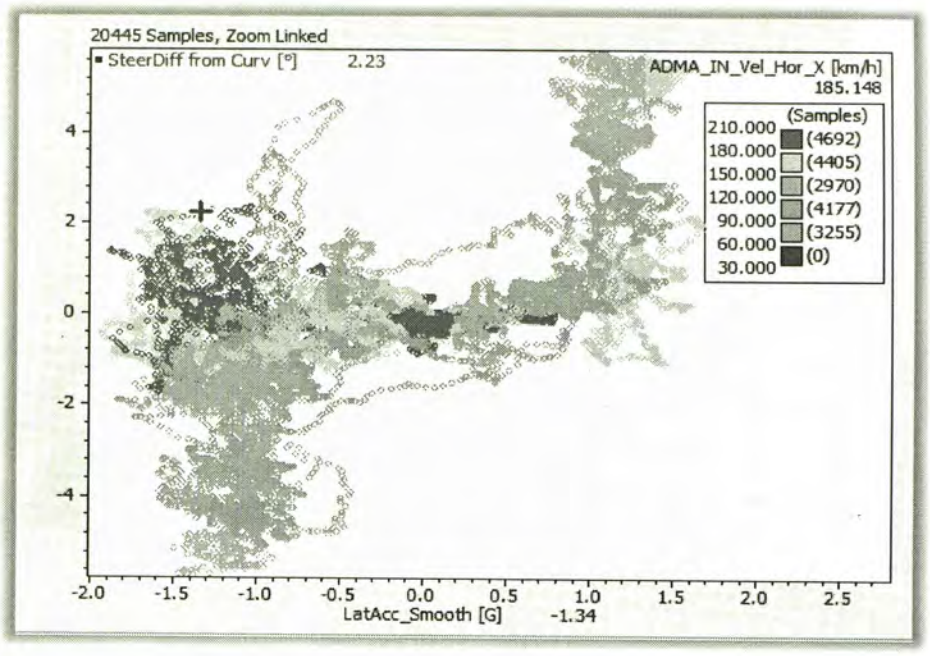
ar A is 12/100 second quicker than Car B which is 0.5 % quicker.
hat means, that A is 3/100 of a second quicker per corner

hat means, as an average, that A is 15/1000 of a second going in and 5/1000 of a second going out of each corner.

Understanding the need of accuracy

Will you see the difference in the car handling and driver input (steering, throttle, brake, clutch gear) if you log at 10 Hz ?





Number of Parameters

Sponsor BEST ***EXACT RACING***

SET UP NO. FINAL ISSUED ON: ########

EVENT	RACE
DRIVER	Mr DRIVER
DIFF	HEWLAND SALISBURY
PLATES	2 PLATES
RAMPS	45 / 80
PRE-LOAD	none

CIRCUIT	VANCOUVER
LAP DIST	1.648
ENG. NO / MILES	080 358 REV LIM 7390
M/C FRONT	Ft .750 Rr .750
DISCS	AP Solid PADS CM 93
BRAKE BIAS	T from full front

DATE	SEPTEMBER 1 ST 2000
CHASSIS	T00/20-28 ? miles
FUEL	10 GALLONS
RACK	6 TEETH
RAD INLET	FULLY OPEN

RATIOS	1st	15 : 31	2nd	18 : 30	3rd	21 : 29	4 th	22 : 25	5th	23 : 24
					CWP	10 : 31				

FRONT WING

26.00 °					ANGLE	27.00 °					
out.	.750	.750	in	none	in	GURNEY	none	in.	.750	.750	out.
Ft			Rr			SKIRTS	Ft		Rr		

LEFT FRONT					TILT 0.00				RIGHT FRONT			
TOE	.060 ins OUT								TOE	.060 ins OUT		
CASTER	5.50 °	Trail STD			GEOMETRY	Low RC	No antidive		CASTER	5.50 °	Trail STD	
CAMBER	- 3.50 °				RIDE HT	1.225 ins			CAMBER	- 3.00 °		
TIRE PRESS.	C 12.5	H 19.0			SPRINGS	500 lb/in			TIRE PRESS.	C 13.5	H 19.0	
DUCTS	50 % open				ROLL BAR	.590			DUCTS	50 % open		
					BLADE	.250 x .150 90 °						
X WEIGHT									X WEIGHT		+20	

FRONT SHOCKS

Type	Piston	Needle	HSB sh	HS B	LSB sh	LS B	R sh.	REB	Gas
P	D 10 D14	5 deg	Std	4.0	a+	-6.0	C	-0.5	150

Type	Piston	Needle	HSB sh	HS B	LSB sh	LS B	R sh.	REB	Gas
P	D 10 D14	5 deg	Std	4.0	A+	-6.0	C	-0.5	150

LEFT REAR					RAKE 0.725				RIGHT REAR			
TOE	.120 ins IN								TOE	.080 ins IN		
CAMBER	- 2.20 °				GEOMETRY	High RC	Antisquat		CAMBER	- 1.80 °		
TIRE PRESS.	C 12.0	H 18.0			RIDE HT	1.950 ins			TIRE PRESS.	C 12.0	H 18.0	
DUCTS	50 % open				SPRINGS	800 lb/in			DUCTS	50 % open		
					ROLL BAR	Double Adjustable						
					BLADE / ADJ	.350 x .250 45 °						

REAR SHOCKS

Type	Piston	Needle	HSB sh	HS B	LSB sh	LS B	R sh.	REB	Gas
P	D 16 L 2	5 deg	Std	5.0	B	-6.0	D	-18	180

Type	Piston	Needle	HSB sh	HS B	LSB sh	LS B	R sh.	REB	Gas
P	D 16 L 2	5 deg	Std	5.0	B	-6.0	D	-18	180

REAR WING

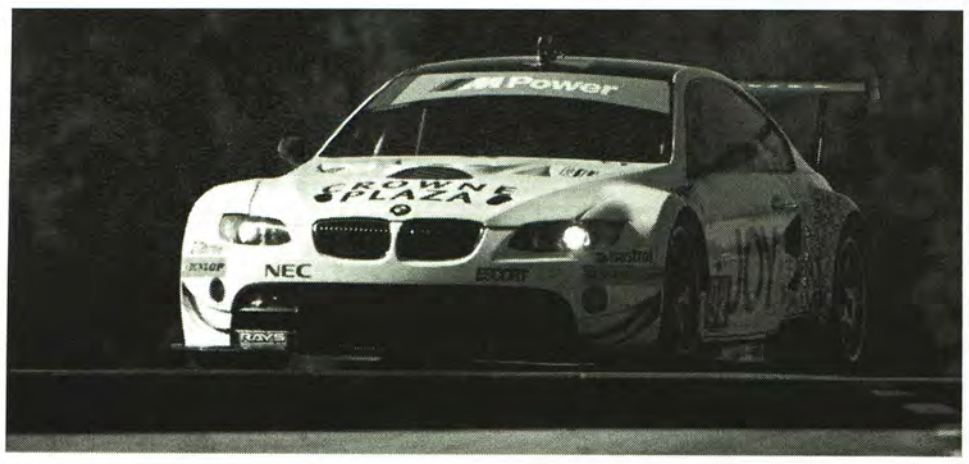
HOLE	HOLE 11
GURNEY	.875

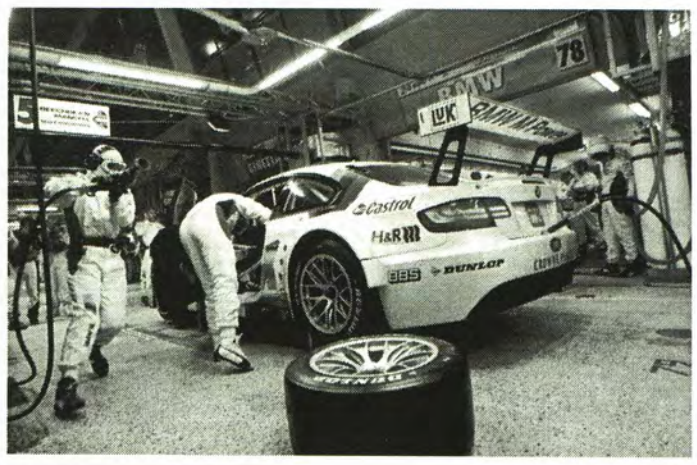
MISCELLANEOUS NOTES

New FWEP

Setup Possibilities

Racing today requires the knowledge of a lot of parameters influencing the car performance.





1. Front Springs [soft, medium, stiff]
2. Rear Springs [soft, medium, stiff]

$3^2 = 9$ possibilities

Setup Possibilities

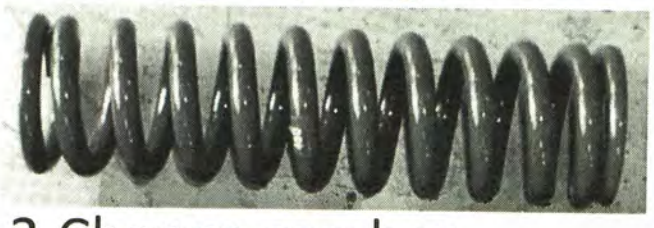
1. Ride Height
2. Caster
3. Camber
4. Toe
5. Springs
6. Damper Low Speed Bump
7. Damper High Speed Bump
8. Damper Low Speed Rebound
9. Damper High Speed Rebound
10. Damper Piston
11. Damper Pressure
12. Damper Bump Shimming
13. Damper Needle
14. Damper Rebound Shimming
15. Roll Center
16. Ant Dive / Anti Lift /Anti Squat
17. Wings Setting
18. Gurney flaps
19. Tire pressure
20. Antiroll Bar
21. Antiroll Bar Blades
22. Antiroll Bar Blades Position
23. Bump Rubber
24. Brake Master Cylinders

$3^{2x24} = 3^{48} =$ 79,766,443,076,872.5 Trillions possibilities....

Parameter Interdependence

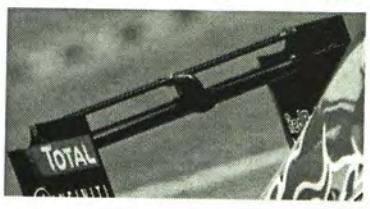
If you change…

1.Change spring rate



2.Change camber

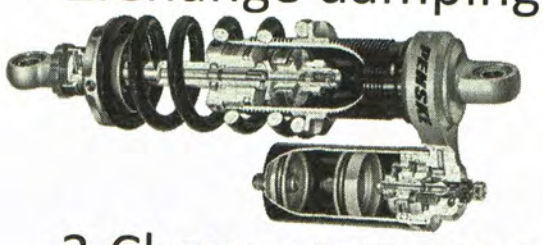
3.Change wing angle



4.Change Ackerman

Do you need to change…

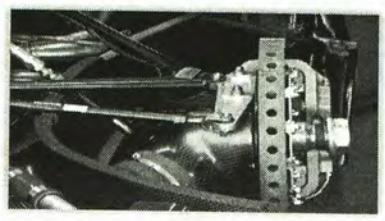
1.Change damping



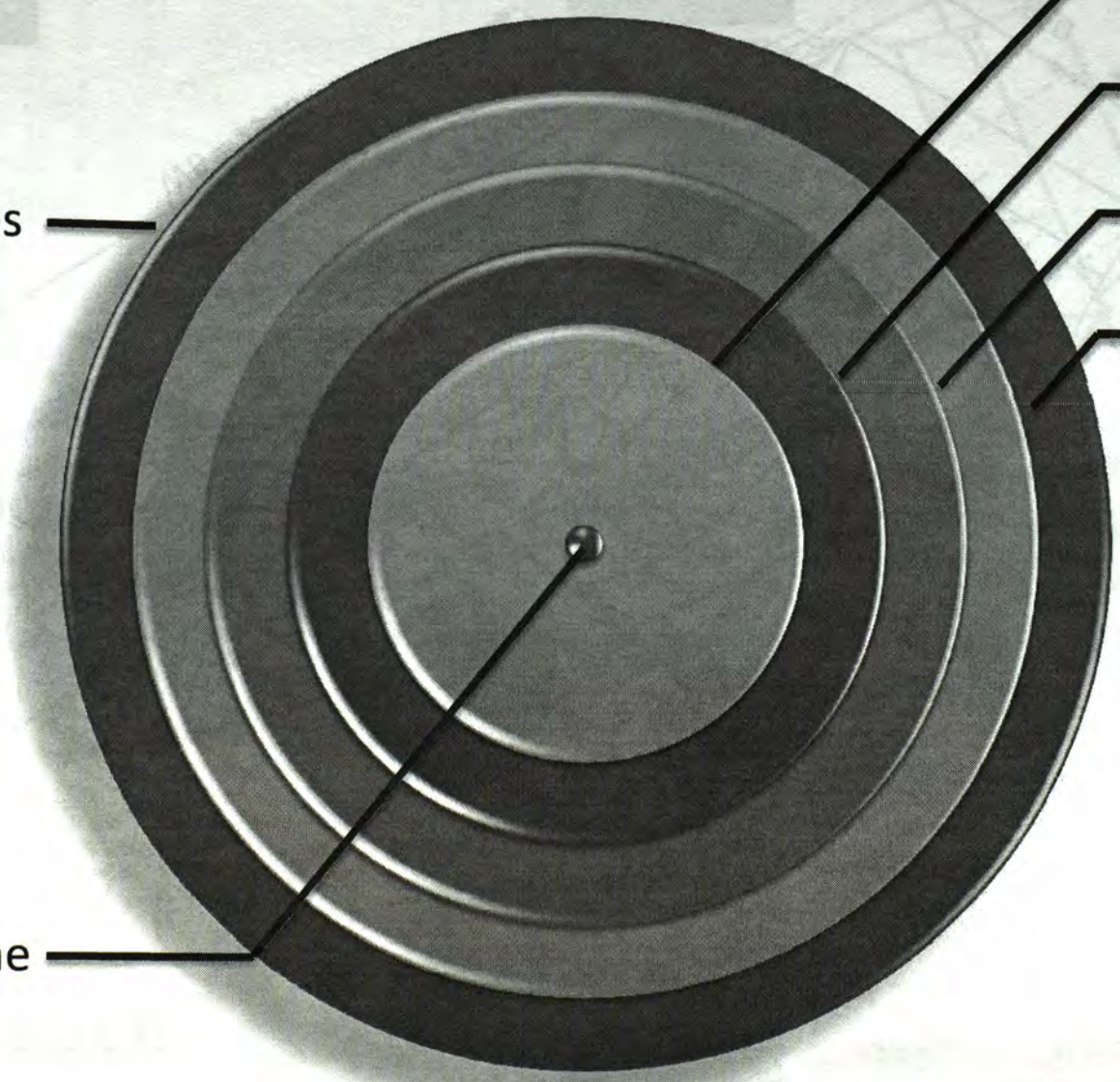
2.Change pressure

3.Change ride height

4.Change toe



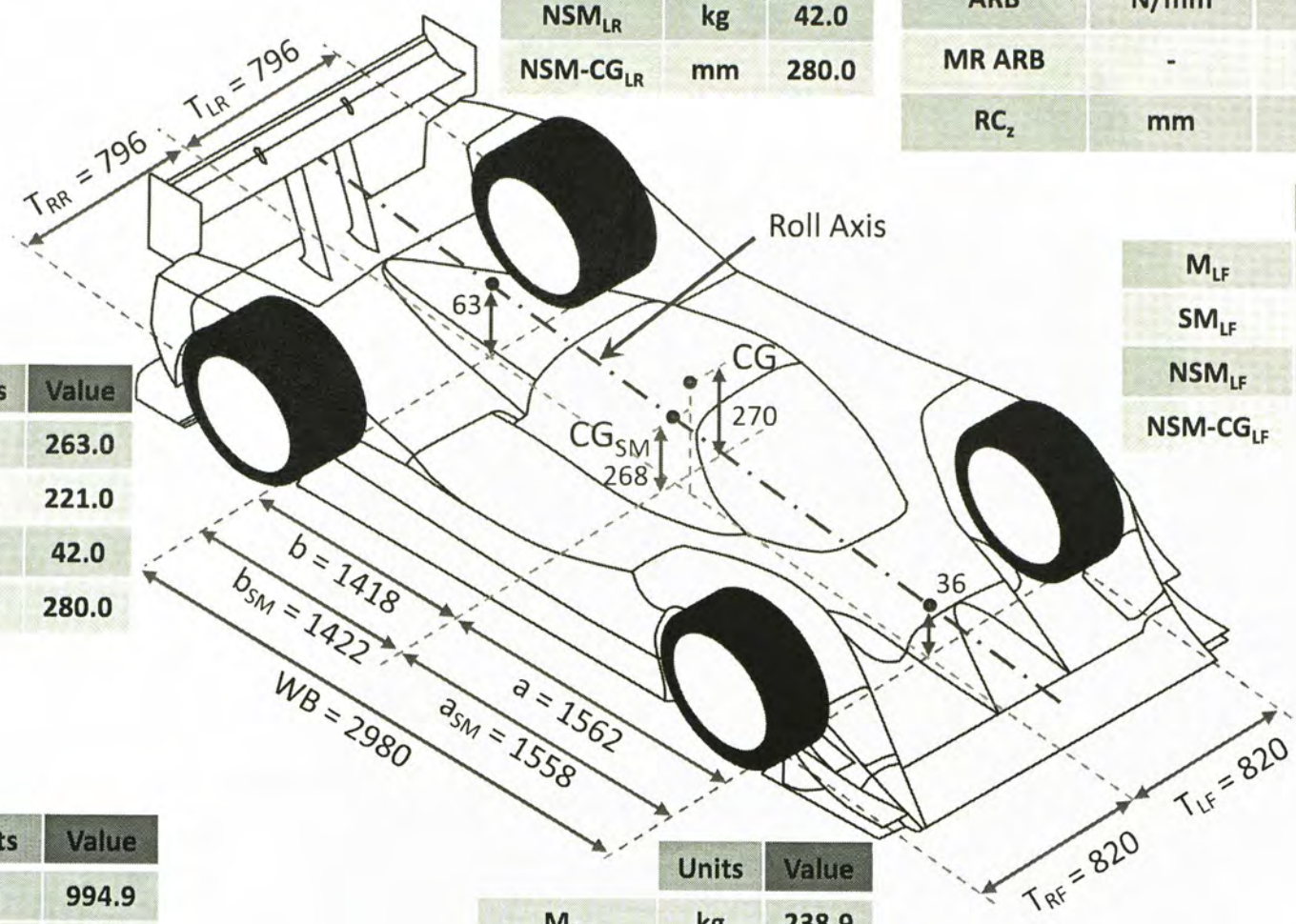
Reasoning Path



Reference

	Units	Value
M_{LR}	kg	258.5
SM_{LR}	kg	216.5
NSM_{LR}	kg	42.0
$NSM\text{-}CG_{LR}$	mm	280.0

	Units	Front	Rear
Springs	N/mm	169.4	138.3
MR springs	-	1.03	1.38
ARB	N/mm	97.8	169.4
MR ARB	-	0.65	0.75
RC_z	mm	36	63



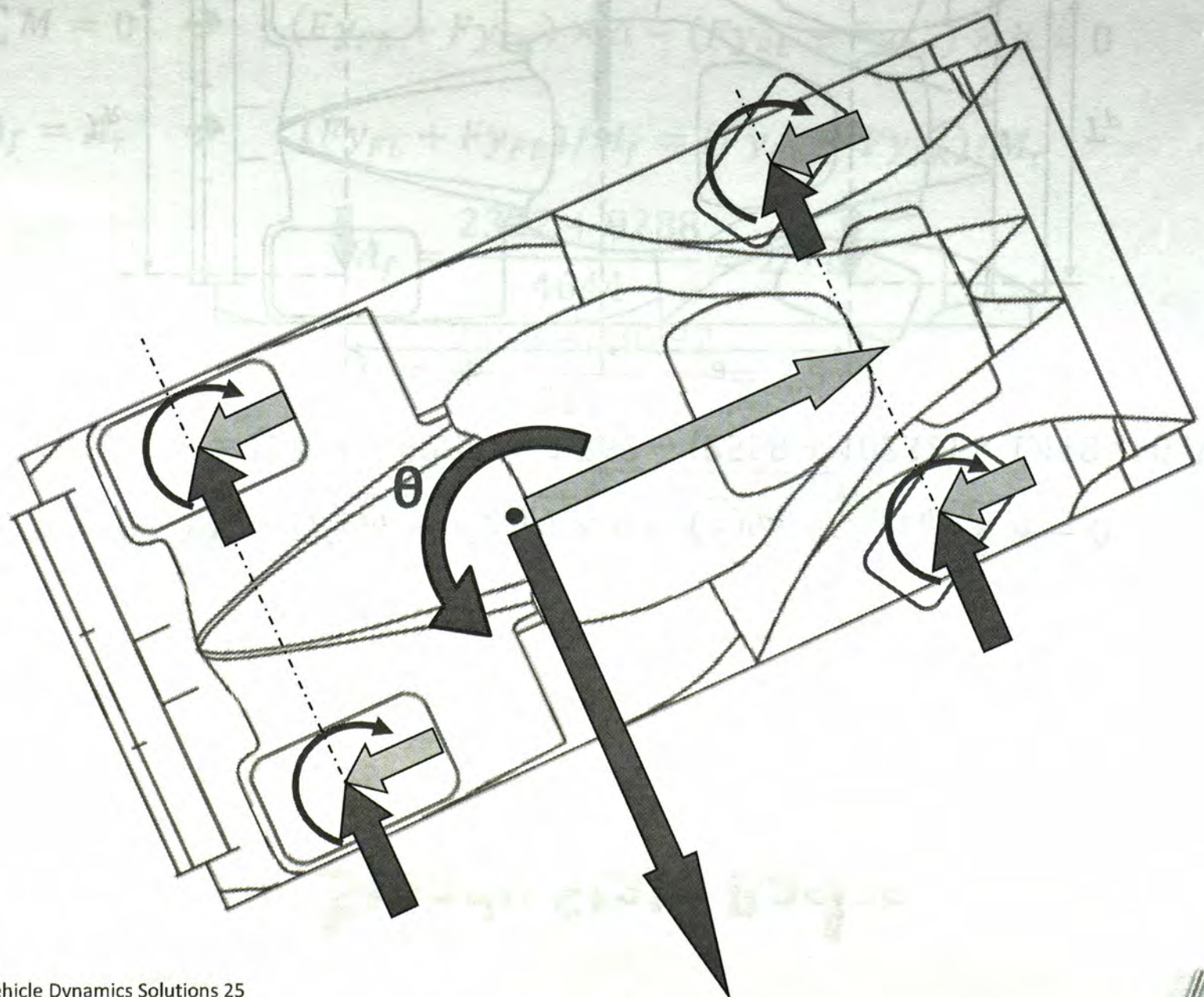
	Units	Value
M_{LF}	kg	234.5
SM_{LF}	kg	197.5
NSM_{LF}	kg	37.0
$NSM\text{-}CG_{LF}$	mm	280.0

	Units	Value
M_{RR}	kg	263.0
SM_{RR}	kg	221.0
NSM_{RR}	kg	42.0
$NSM\text{-}CG_{RR}$	mm	280.0

	Units	Value
M_{Total}	kg	994.9
CG_z	m	0.270
CG_{SMz}	m	0.268

	Units	Value
M_{RF}	kg	238.9
SM_{RF}	kg	201.9
NSM_{RF}	kg	37.0
$NSM\text{-}CG_{RF}$	mm	280.0

1. Vehicle Dynamics Basics



Steady State Basics

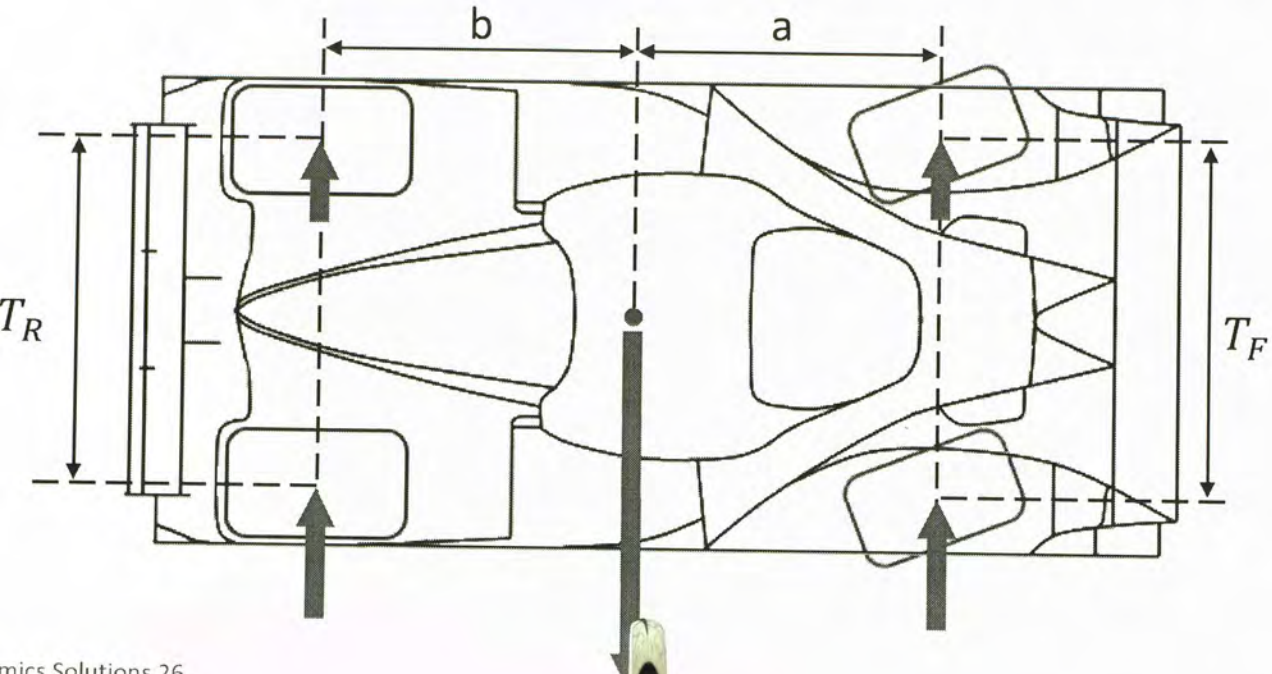
➔ $Fy_{FL} + Fy_{FR} + Fy_{RL} + Fy_{RR} = Mass \times Lateral\ Acc.$

$= (Fy_{FL} + Fy_{FR} + Fy_{RL} + Fy_{RR})/Mass = Lateral\ Acc.$

$= (2322 + 9288 + 2558 + 10232)/9760 = 2.5G$

$= 0$ ➔ $(Fy_{FL} + Fy_{FR}) \times a - (Fy_{RL} + Fy_{RR}) \times b = 0$

$= (2323 + 9288) \times 1.562 - (2558 + 10232) \times 1.418 = 0\ Nm$



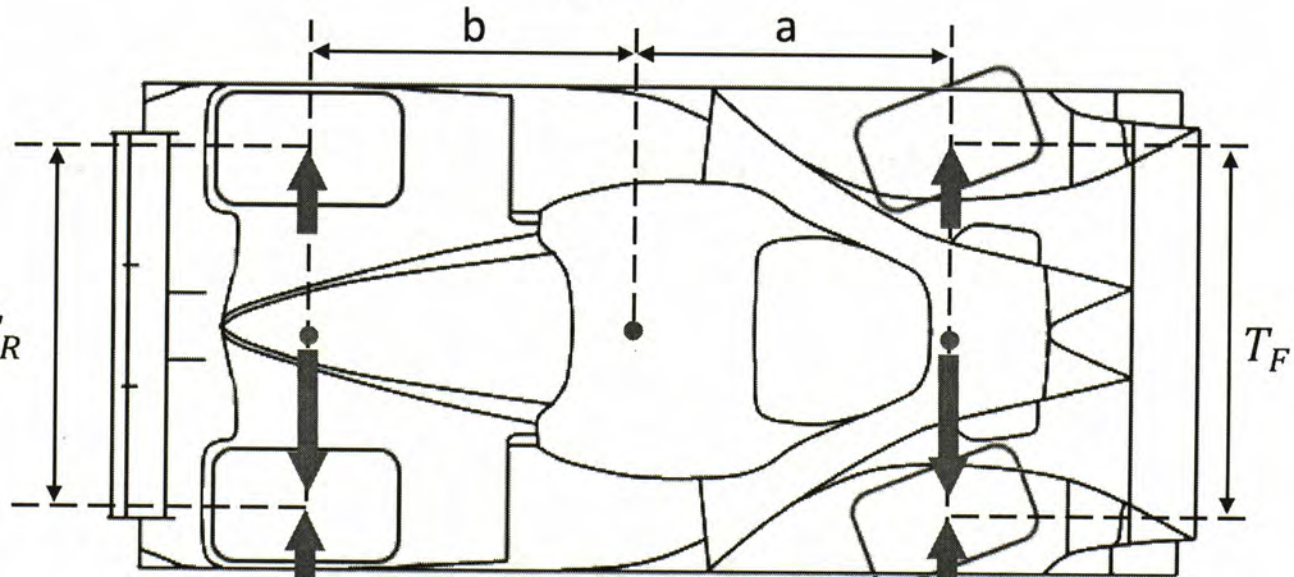
Steady State Basics

$= 0$ ➔ $(Fy_{FL} + Fy_{FR}) \times a - (Fy_{RL} + Fy_{RR}) \times b = 0$

A_r ➔ $(Fy_{FL} + Fy_{FR})/M_f = (Fy_{RL} + Fy_{RR})/M_r$

$$A_f = \frac{2322 + 9288}{4644} = 2.5\ G$$

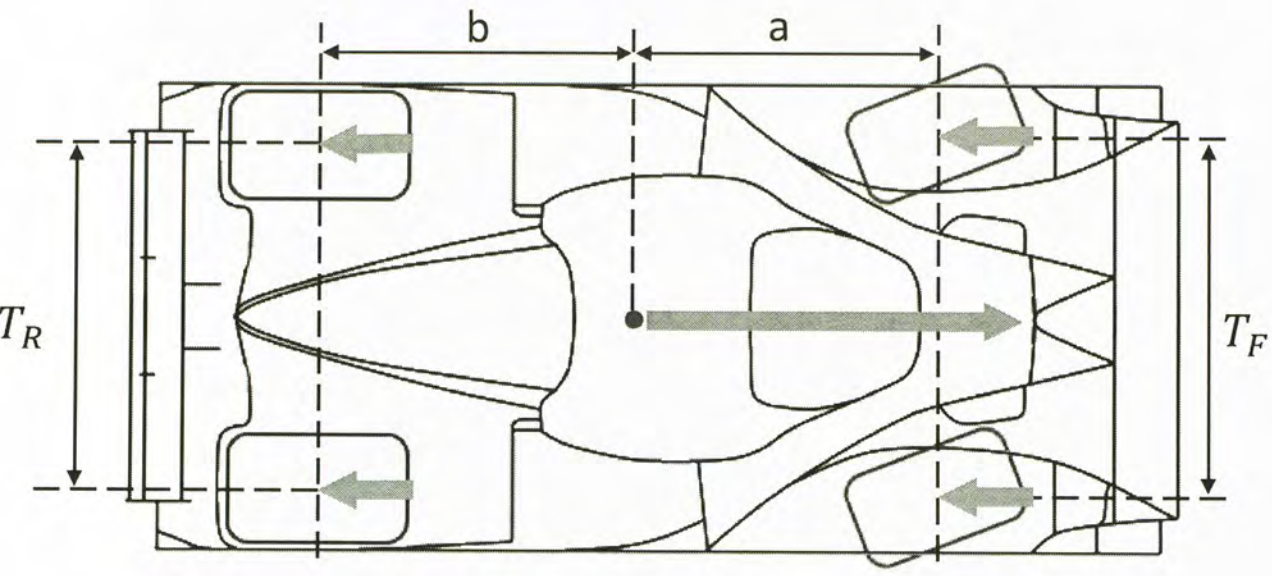
$$A_r = \frac{2558 + 10232}{5116} = 2.5\ G$$



Steady State Basics

➔ $Fx_{FL}+Fx_{FR}+Fx_{RL}+Fx_{RR}=Mass\times Longitudinal\ Acc.$

0 ➔ $\left(Fx_{FR}\times\frac{T_F}{2}+Fx_{RR}\times\frac{T_R}{2}\right)-\left(Fx_{FL}\times\frac{T_F}{2}+Fx_{RL}\times\frac{T_R}{2}\right)=0$



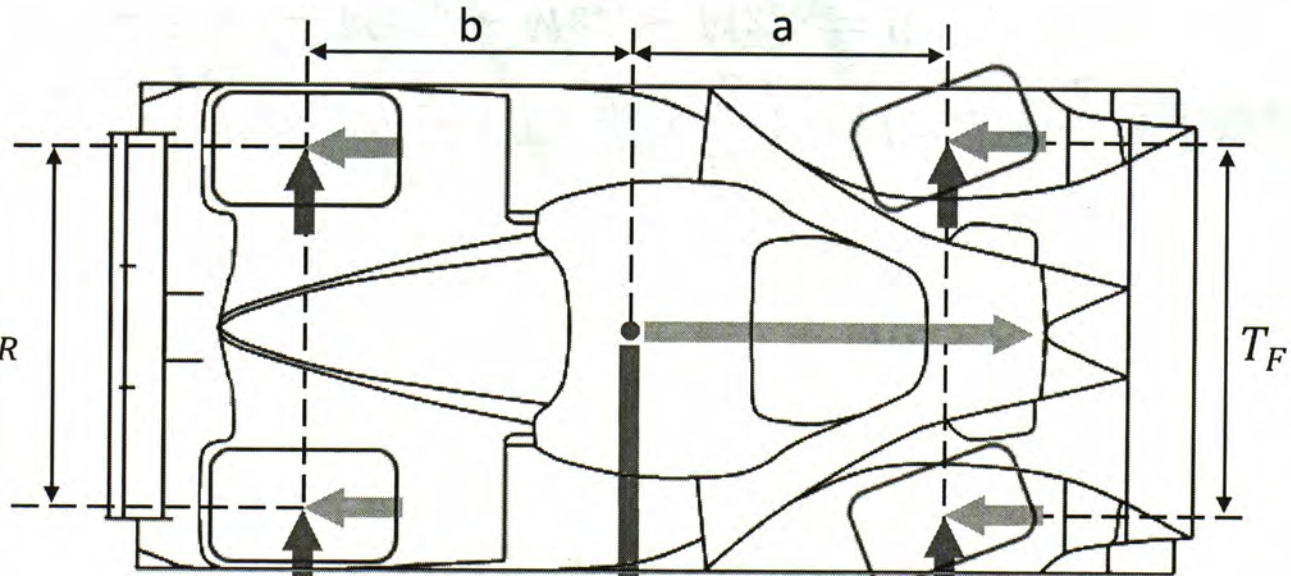
Steady State Basics

➔ $Fy_{FL}+Fy_{FR}+Fy_{RL}+Fy_{RR}=Mass\times Lateral\ Acc.$

➔ $Fx_{FL}+Fx_{FR}+Fx_{RL}+Fx_{RR}=Mass\times Longitudinal\ Acc.$

➔ $[a\times(Fy_{FL}+Fy_{FR})-b\times(Fy_{RL}+Fy_{RR})]$

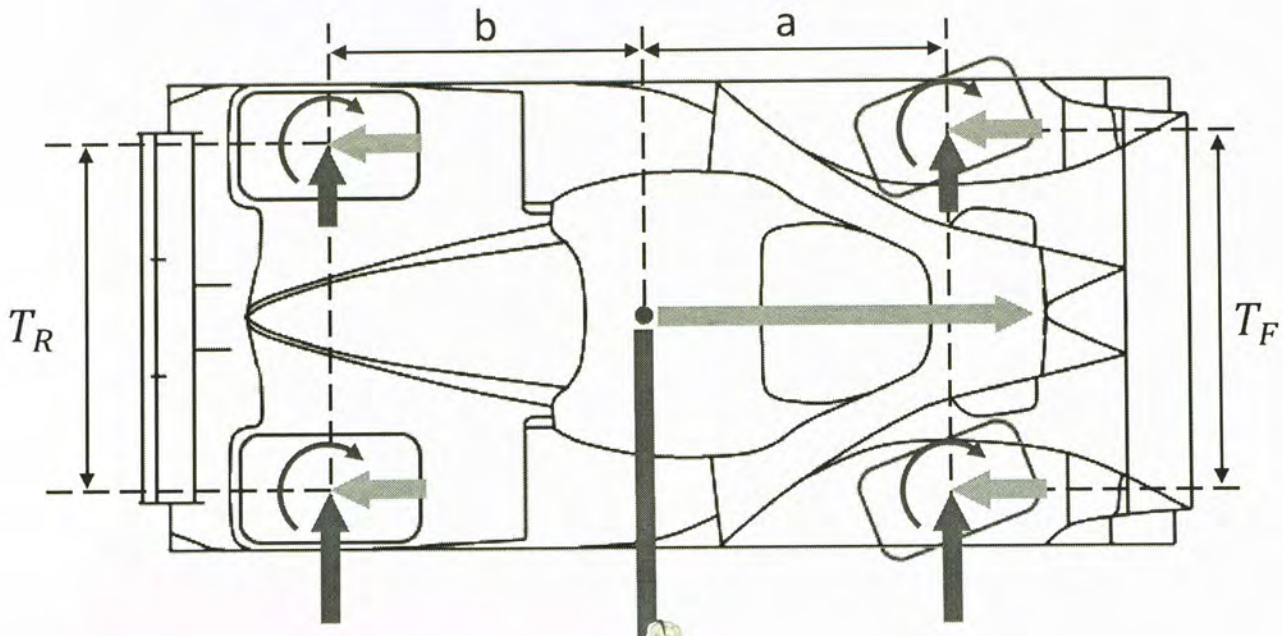
$+\left[\left(Fx_{FR}\times\frac{T_F}{2}+Fx_{RR}\times\frac{T_R}{2}\right)-\left(Fx_{FL}\times\frac{T_F}{2}+Fx_{RL}\times\frac{T_R}{2}\right)\right]=0$



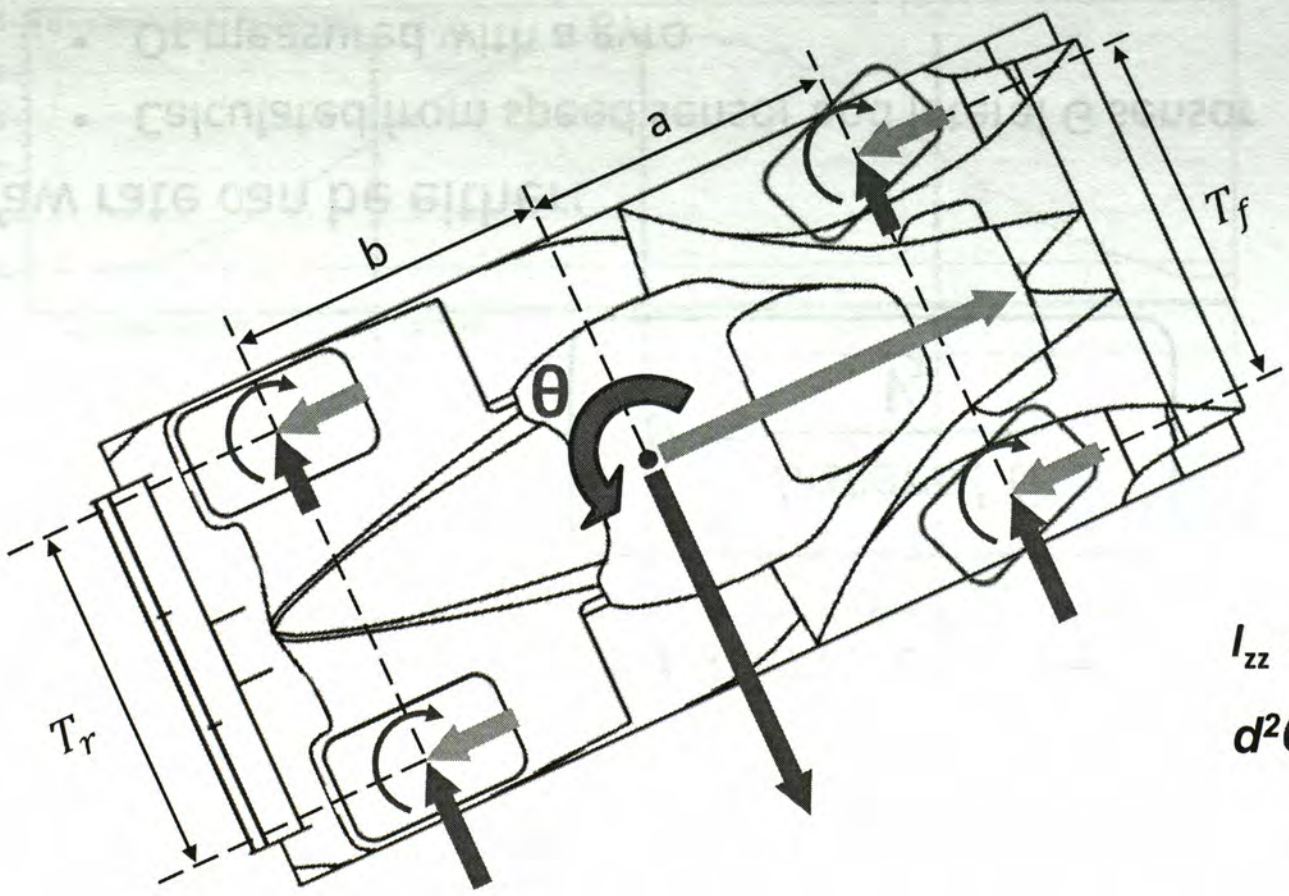
$$\sum Fy \rightarrow Fy_{FL} + Fy_{FR} + Fy_{RL} + Fy_{RR} = Mass \times Lateral\ Acc.$$

$$\sum Fx \rightarrow Fx_{FL} + Fx_{FR} + Fx_{RL} + Fx_{RR} = Mass \times Longitudinal\ Acc.$$

$$\sum M = 0 \rightarrow [a \times (Fy_{FL} + Fy_{FR}) - b \times (Fy_{RL} + Fy_{RR})] + \left[\left(Fx_{FR} \times \frac{T_F}{2} + Fx_{RR} \times \frac{T_R}{2}\right) - \left(Fx_{FL} \times \frac{T_F}{2} + Fx_{RL} \times \frac{T_R}{2}\right)\right] - Mz_{fl} - Mz_{fr} - Mz_{rl} - Mz_{rr} = 0$$



Transient Basics: Yaw Moment Equation



I_{zz} = Moment of Inert

$d^2\theta/dt^2$ = Yaw accele
= d(yaw rate
= d(gyro)/dt

$$[(Fx_{FL}\frac{T_F}{2} + Fx_{RL}\frac{T_R}{2}) - (Fx_{FR}\frac{T_F}{2} + Fx_{RR}\frac{T_R}{2})] + [(Fy_{FL} + Fy_{FR}) \times a - (Fy_{RL} + Fy_{RR}) \times b] - M_{zFR} - M_{zFL} - M_{zRR} -$$

$$= I_{ZZ}\ d^2\theta / dt^2$$

Vehicle Dynamics Basics

$$V_x = \dot{\theta} R \quad and \quad Lateral\ G = \frac{V^2}{R}$$

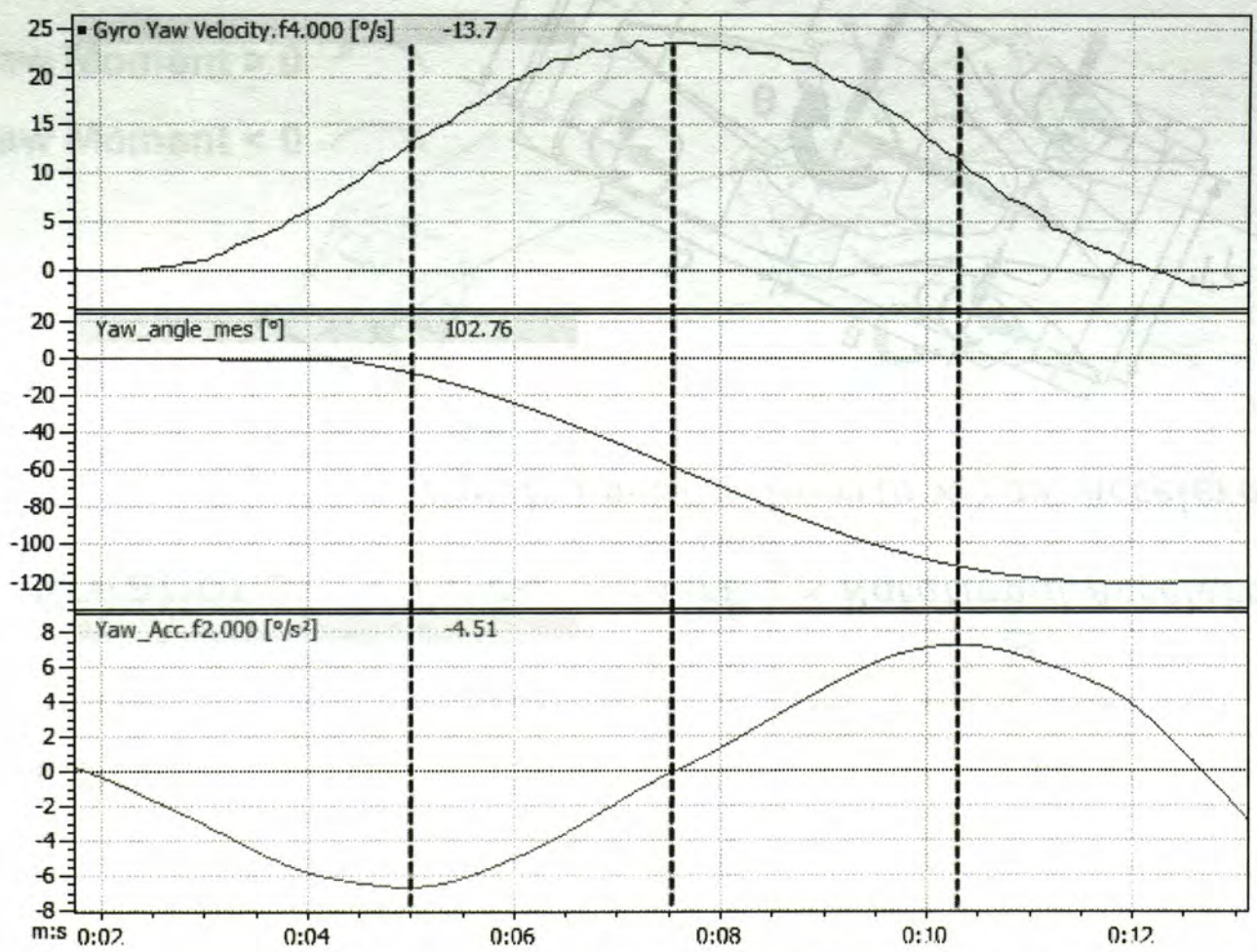
$$\rightarrow Lateral\ G = \frac{\dot{\theta} V^2}{V}$$

$$\boxed{\dot{\theta} = \frac{Lateral\ G}{V_x}}$$

Yaw rate can be either:

- Calculated from speed sensor and lateral G sensor
- Or measured with a gyro

Example of Gyro data in a corner



Yaw Velocity

Yaw Angle

$$\int Yaw\ Velocity\ dt$$

Yaw Acceleration

$$\frac{d(Yaw\ Velocity)}{dt}$$

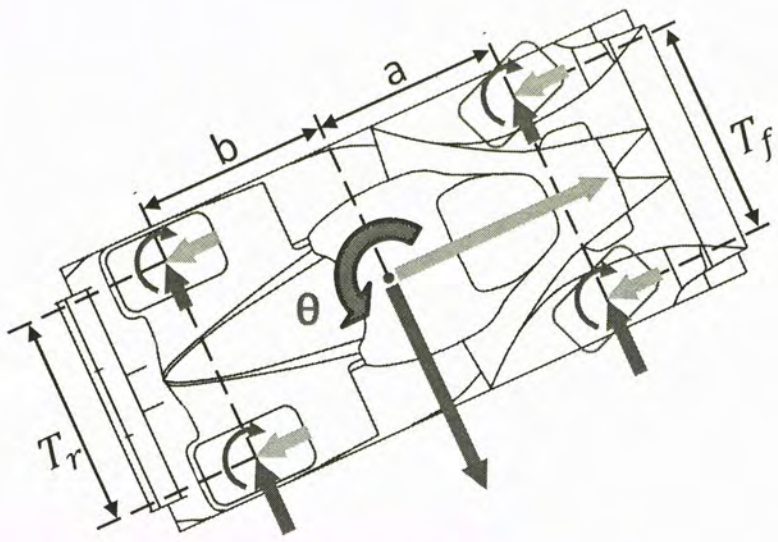
Vehicle Dynamics Basics

Newton second law

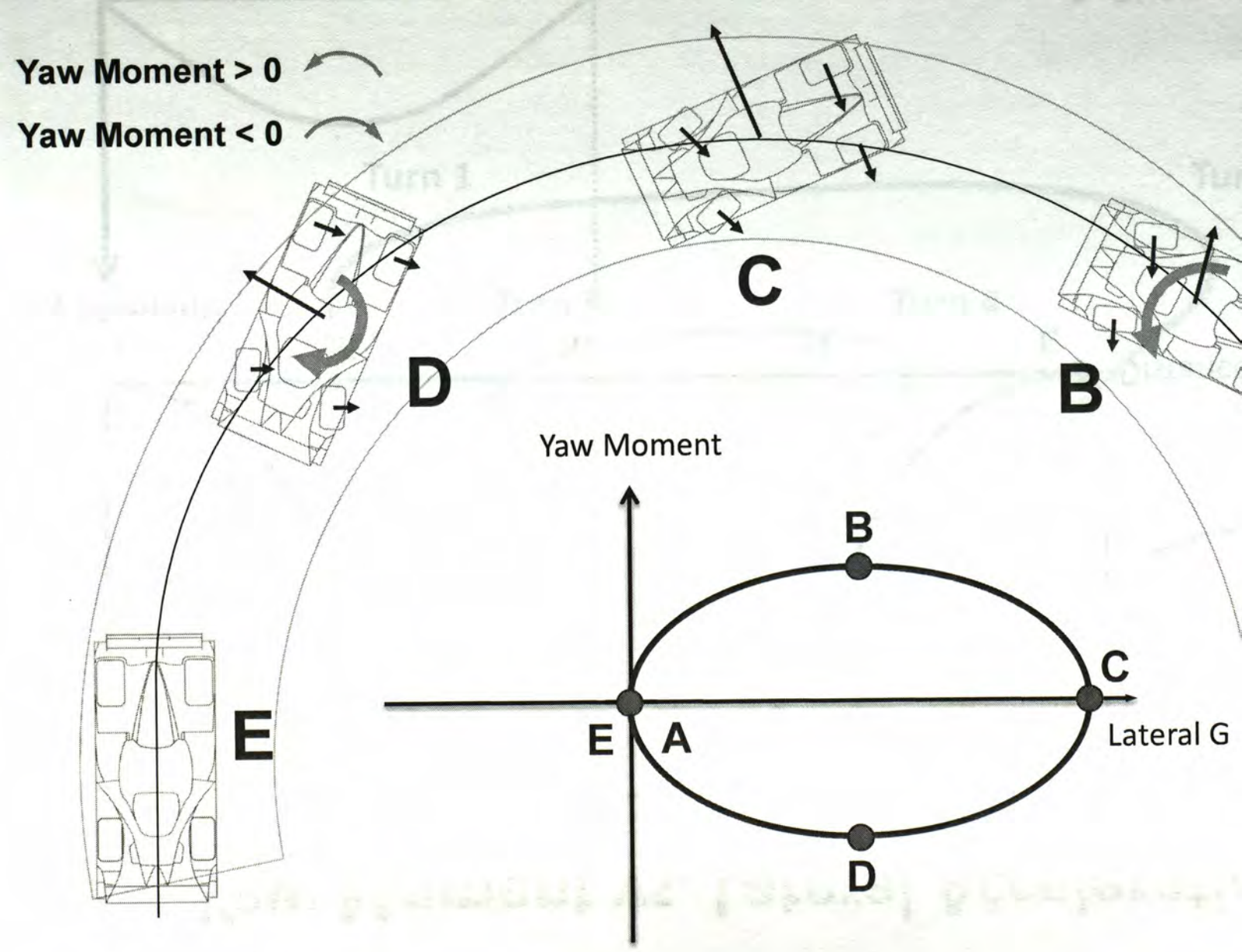
Linear $Force = Mass \times Linear\ Acceleration$

Rotational $Torque = Inertia \times Rotational\ Acceleration$

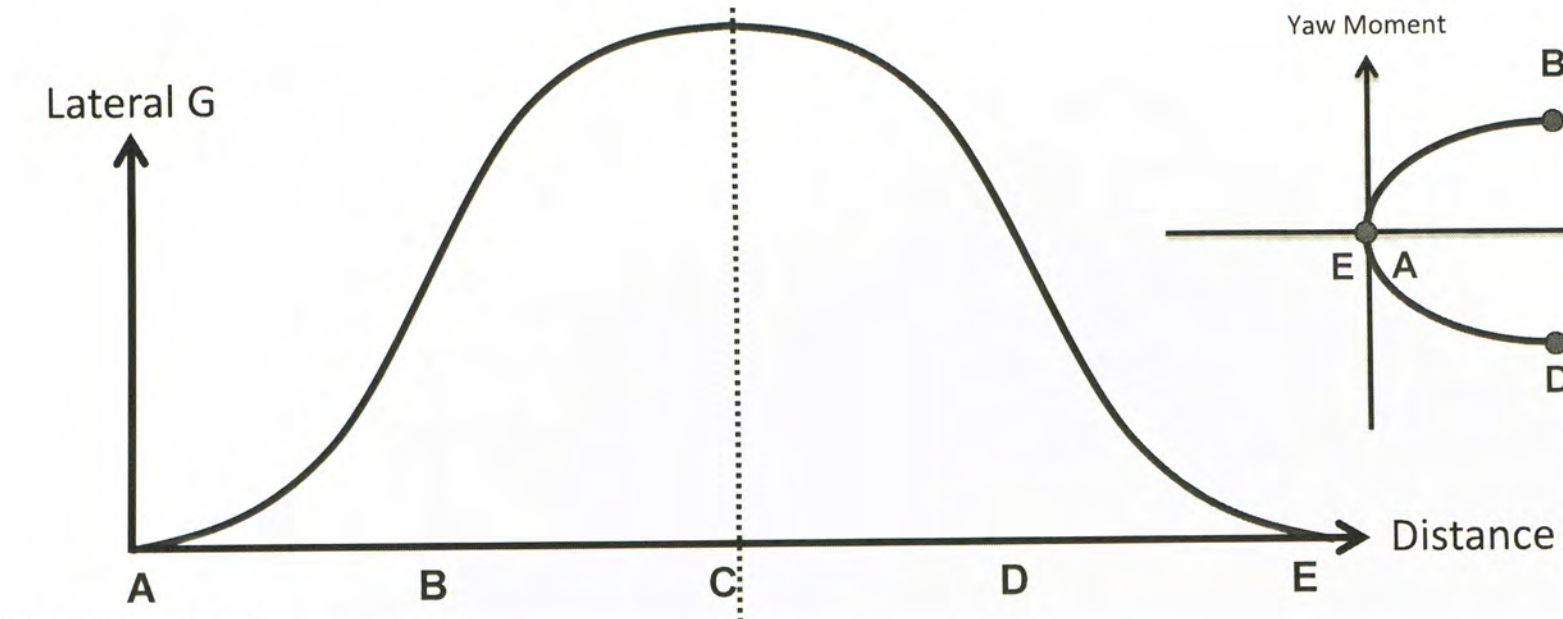
$Yaw\ Moment = Inertia \times Yaw\ Acceleration$

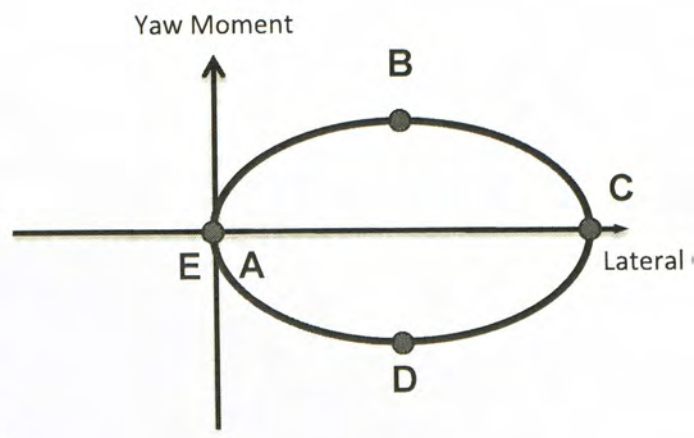


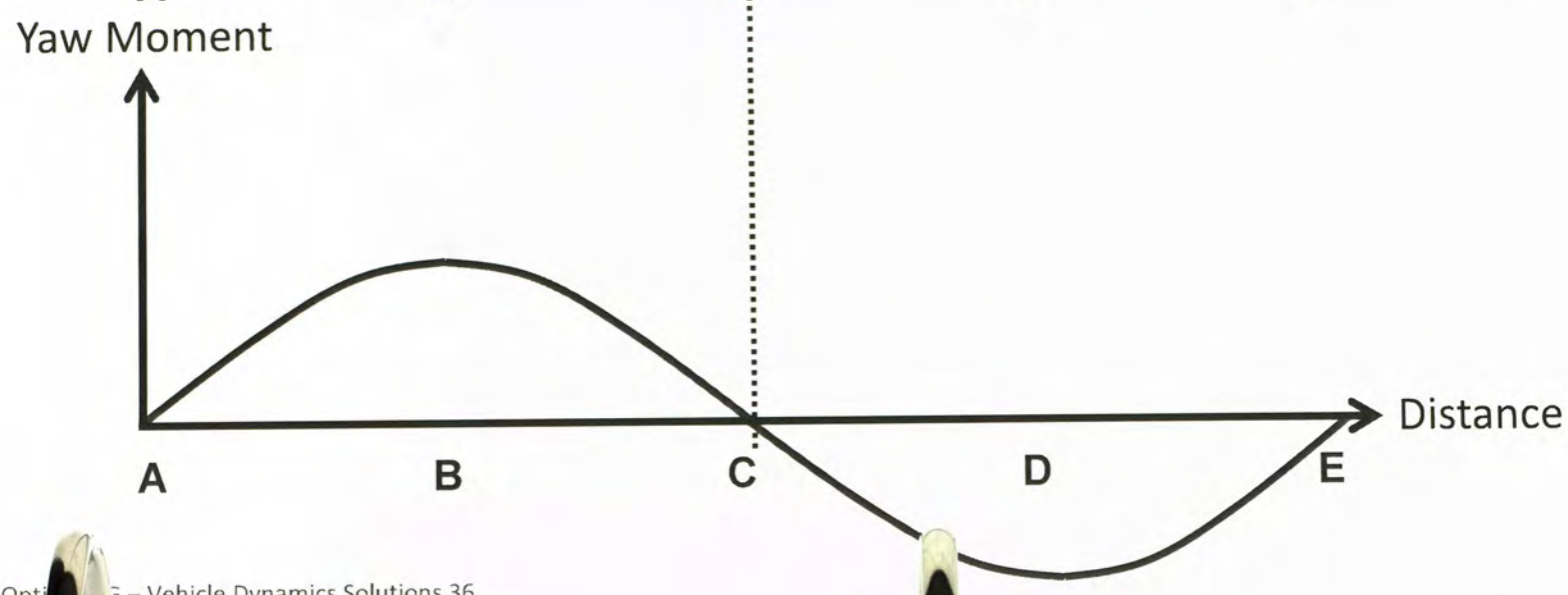
Yaw Moment vs. Lateral Acceleratio



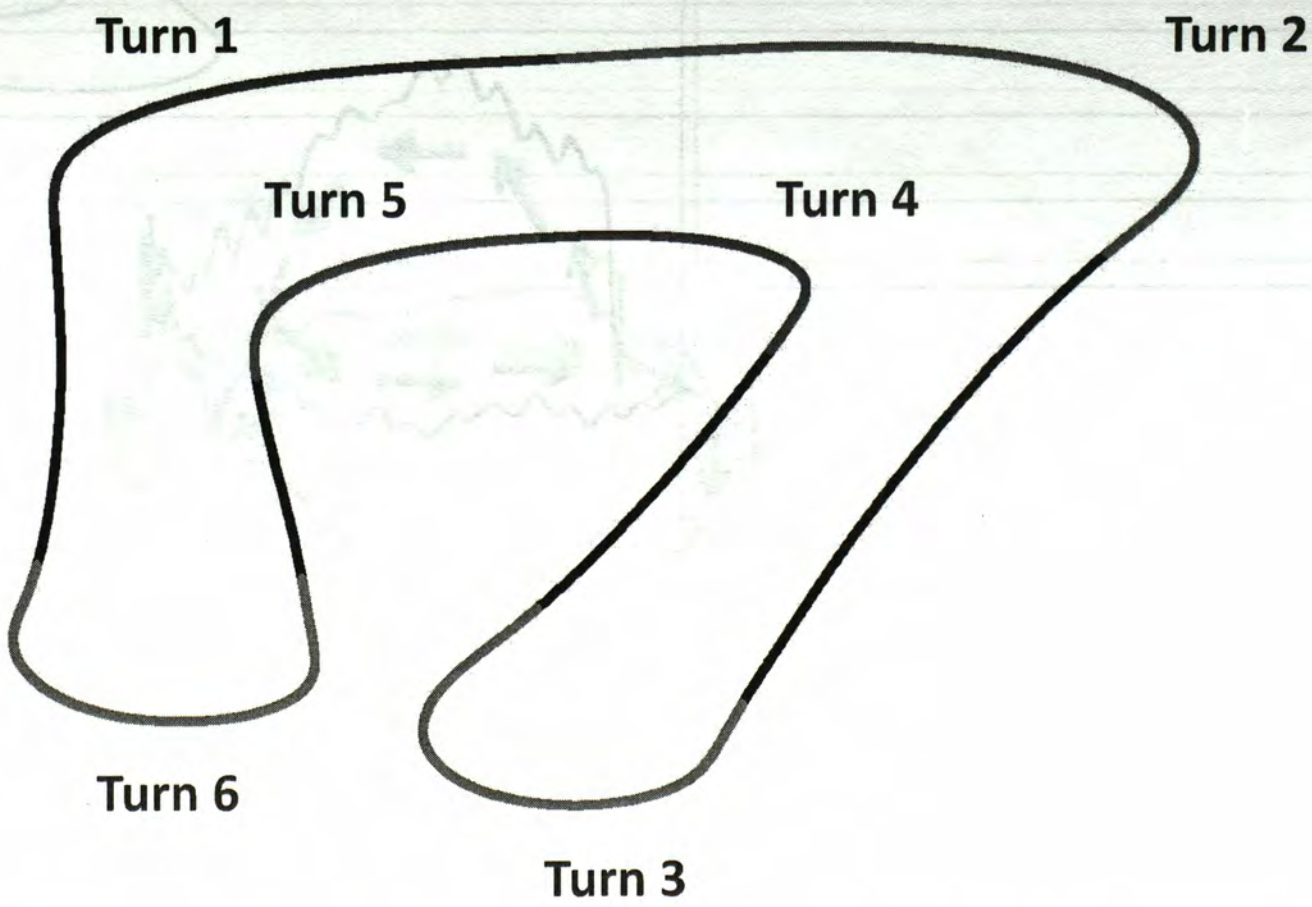
Yaw Moment vs. Lateral Acceleration



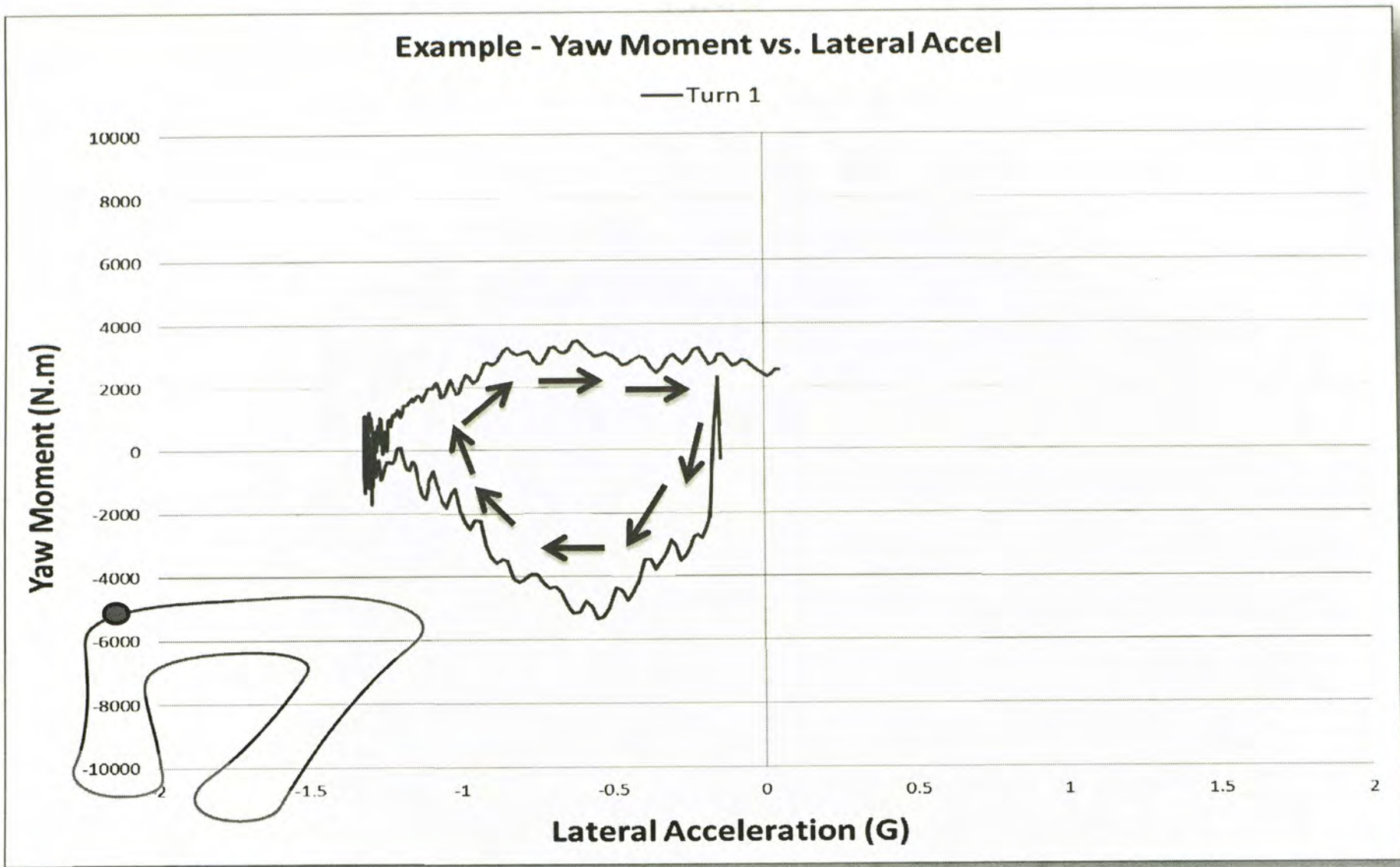




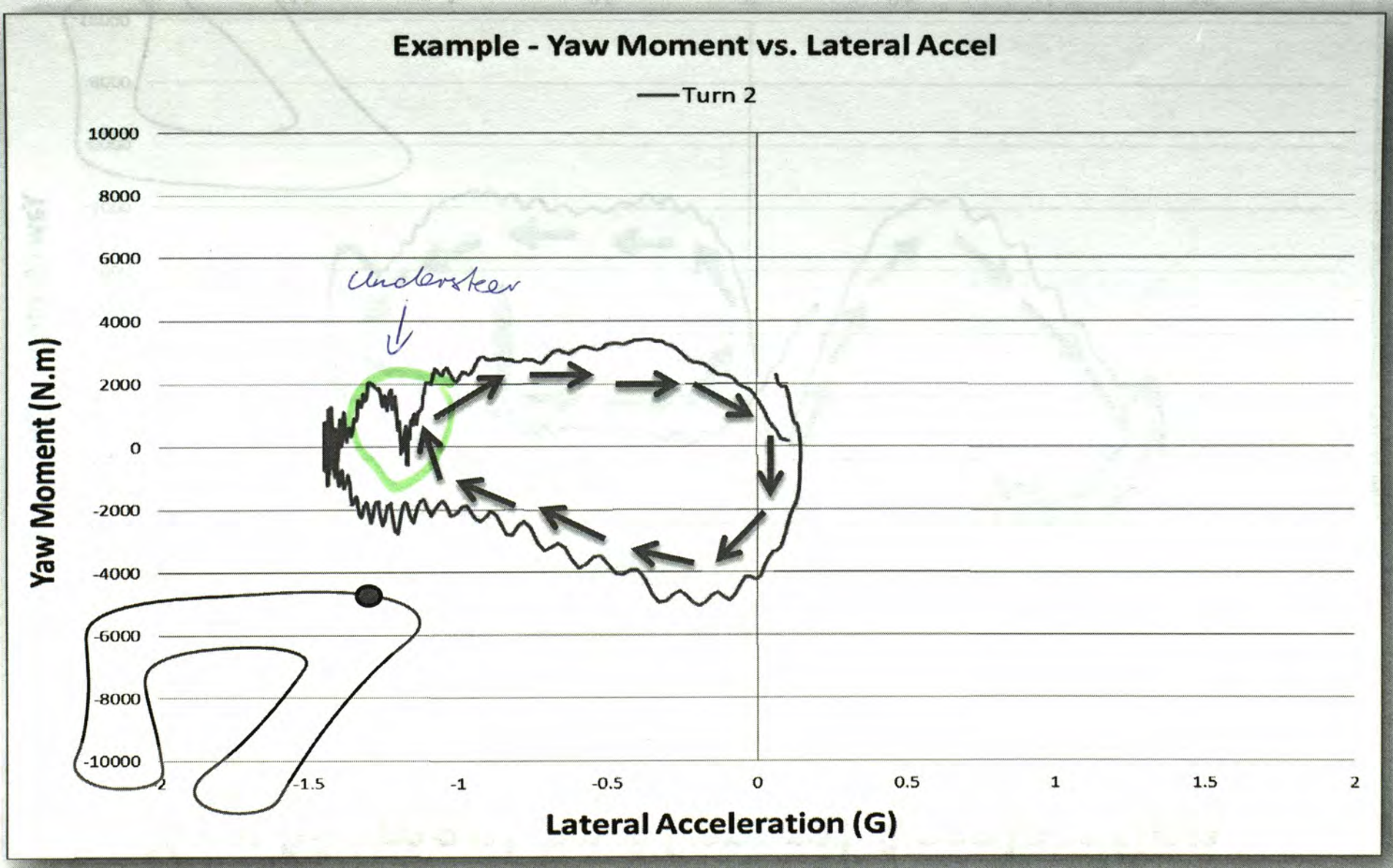
Yaw Moment and Lateral Acceleration



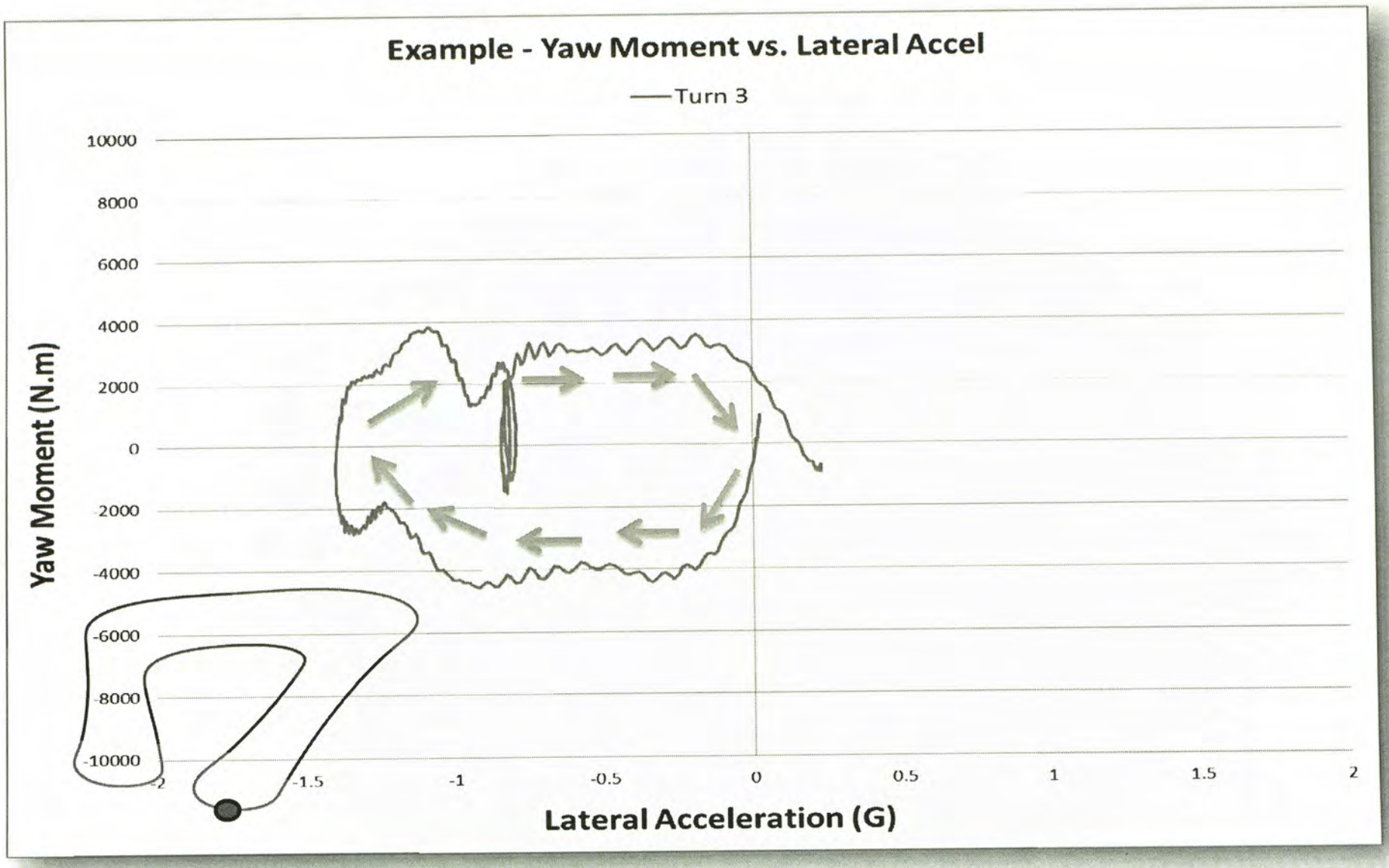
Yaw Moment and Lateral Acceleration



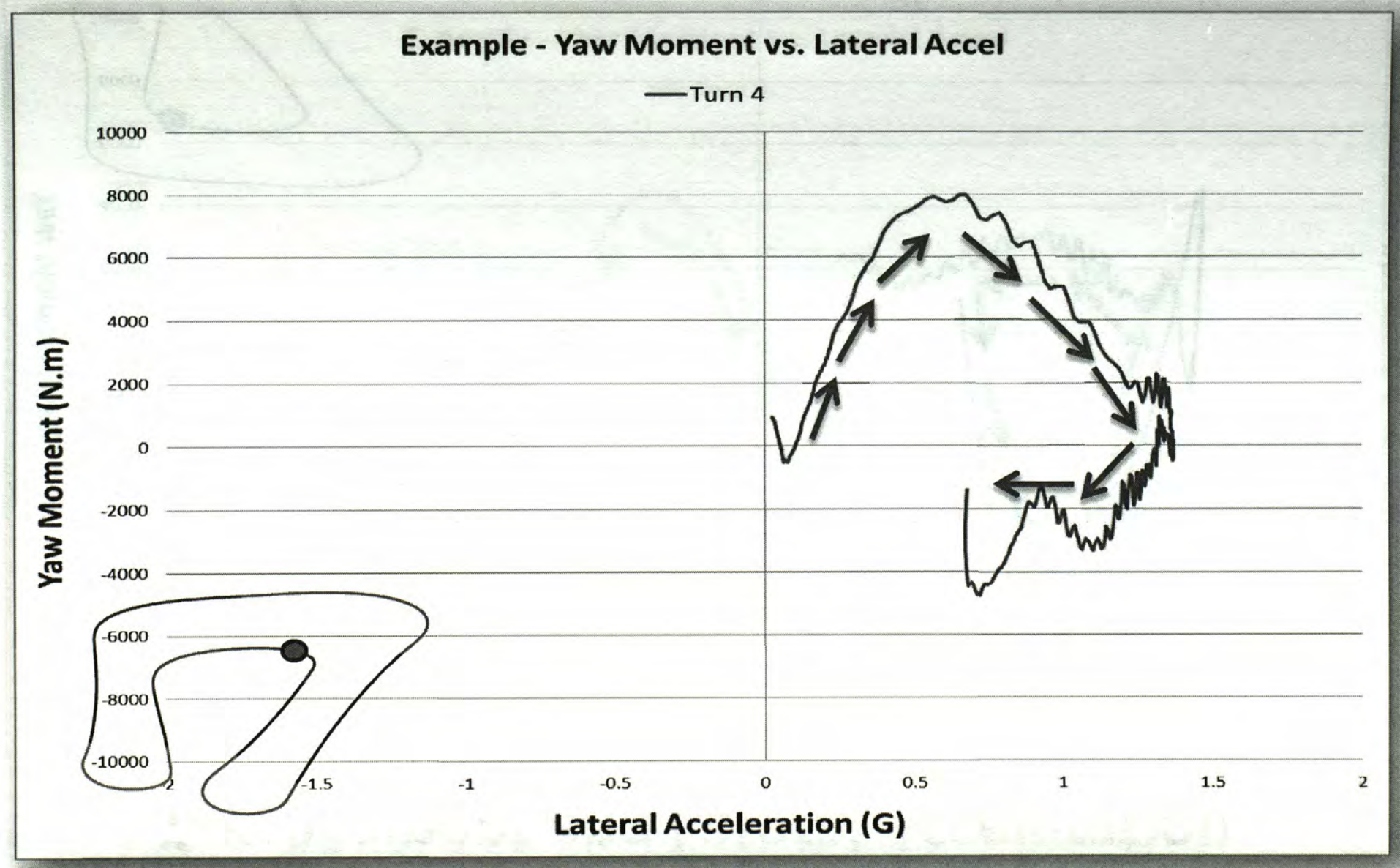
Yaw Moment and Lateral Acceleration



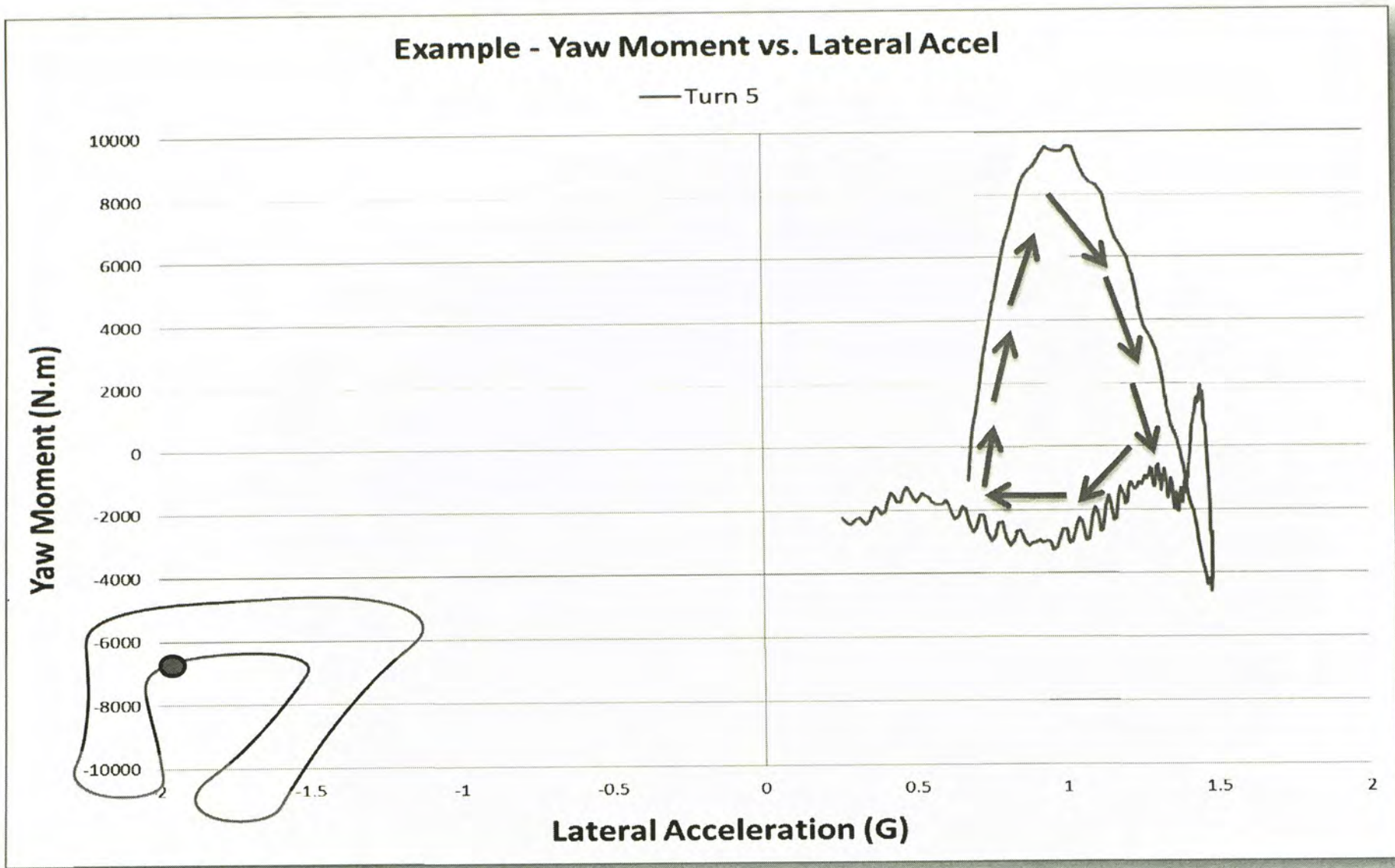
Yaw Moment and Lateral Acceleration



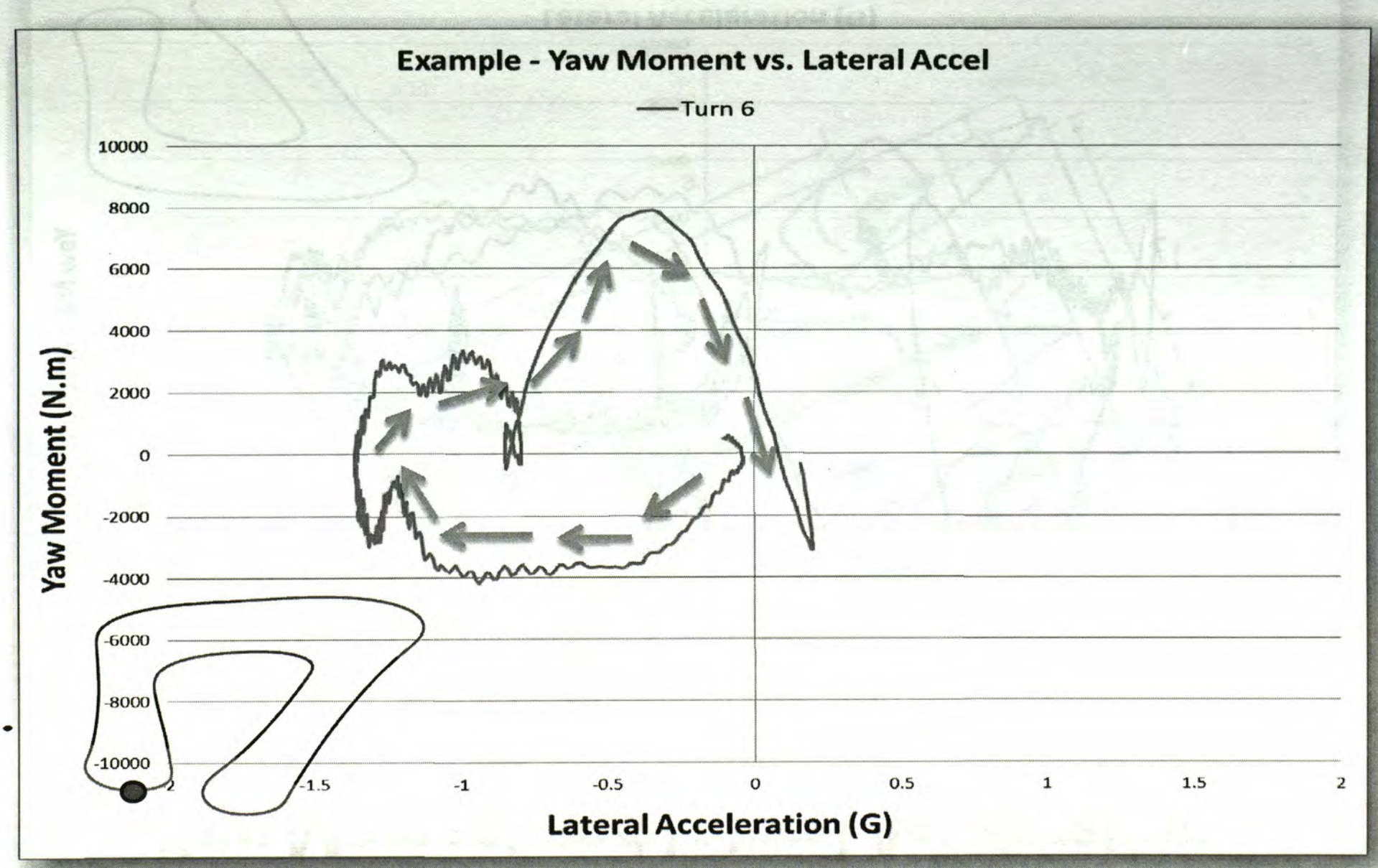
Yaw Moment and Lateral Acceleration



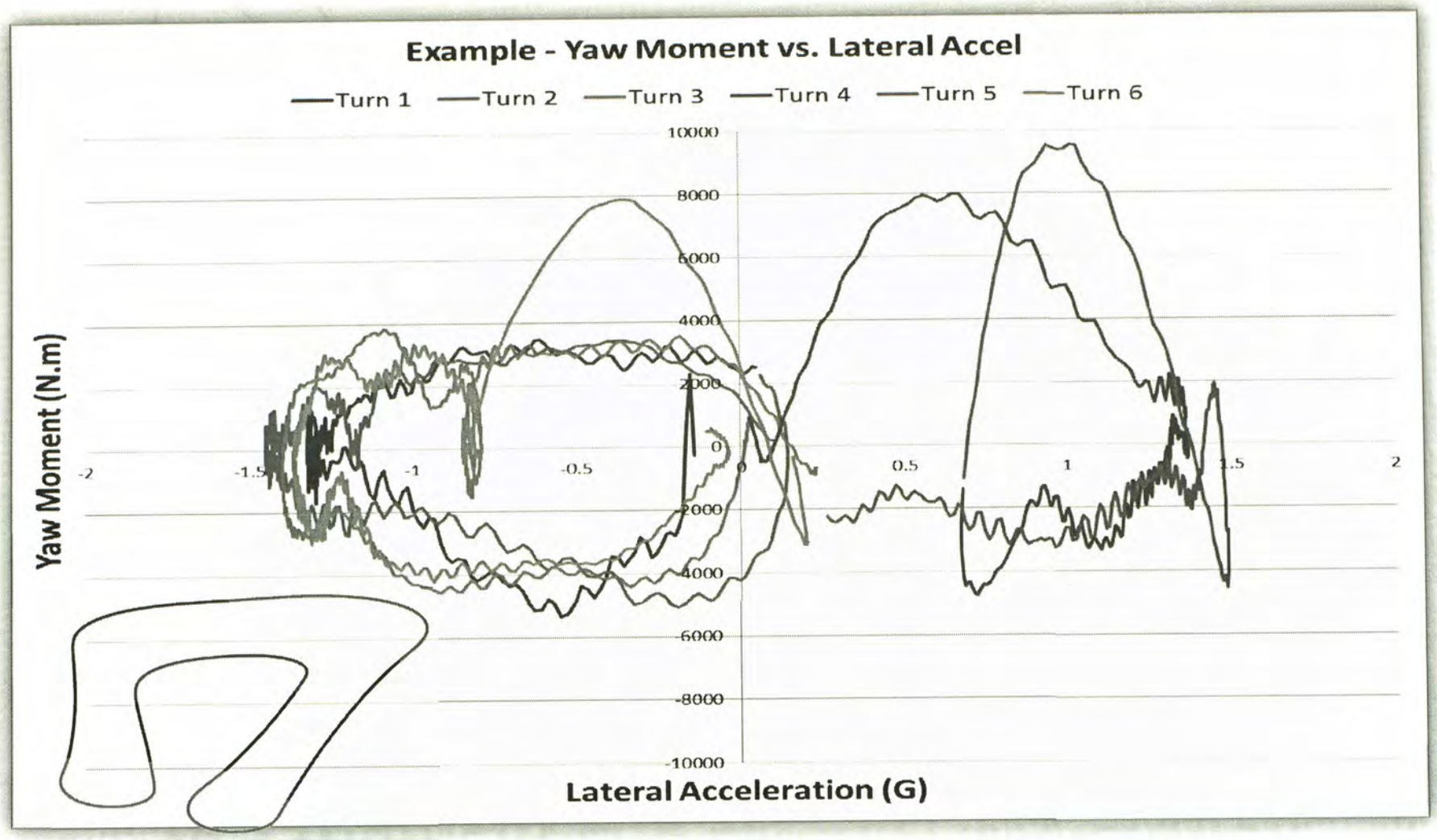
Yaw Moment and Lateral Acceleration



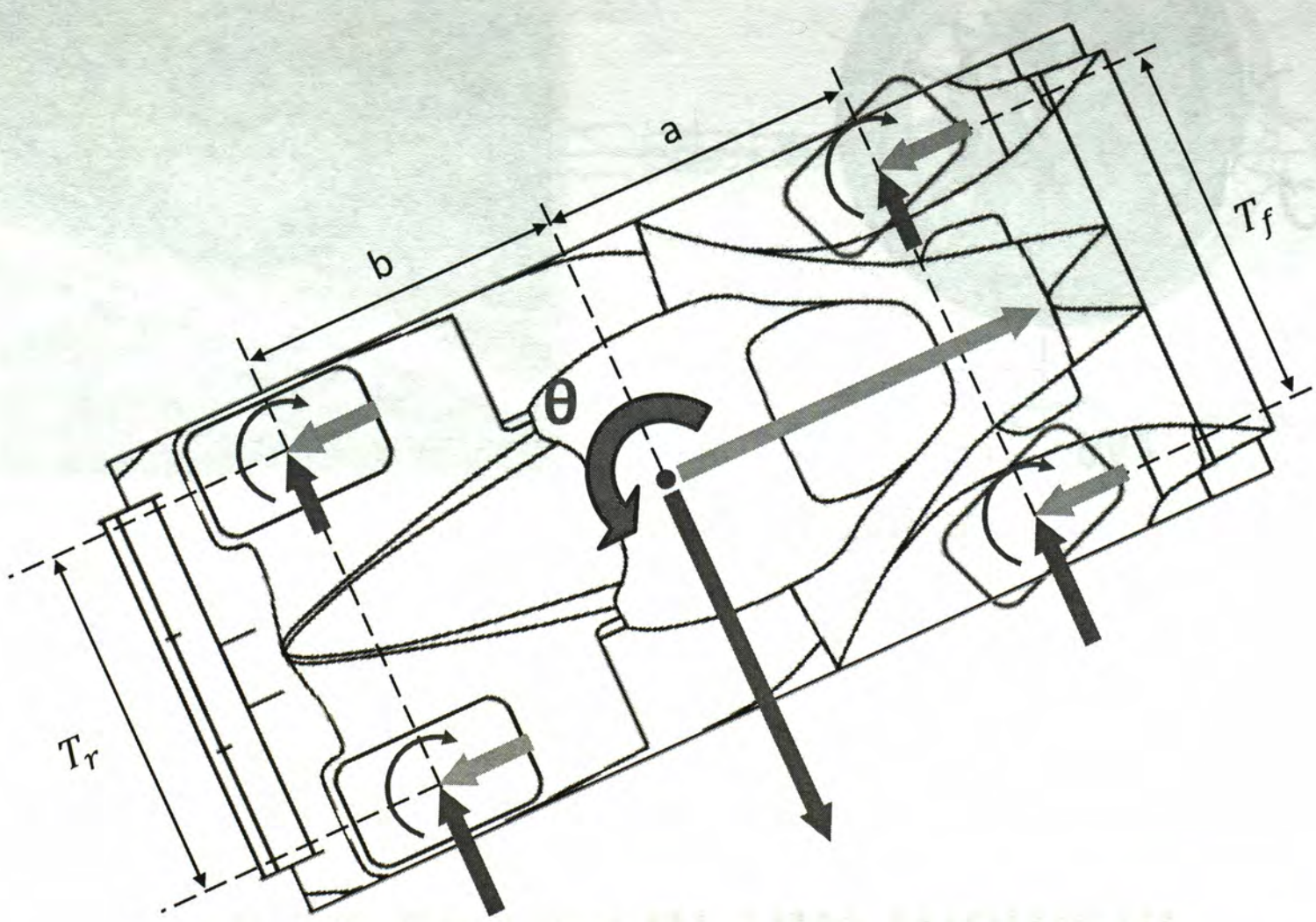
Yaw Moment and Lateral Acceleration



Yaw Moment and Lateral Acceleration



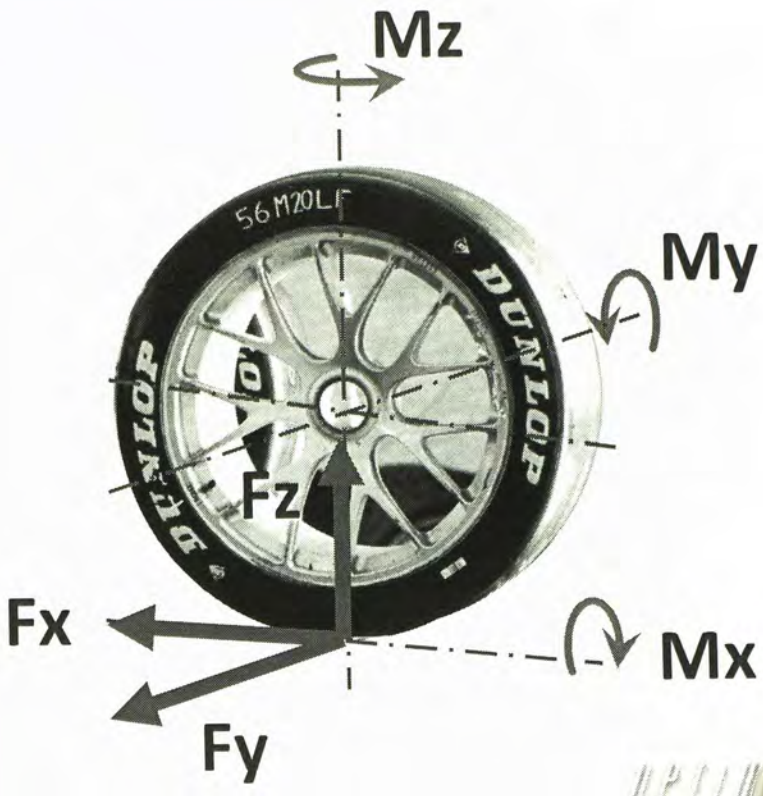
The 12 Causes of Yaw Moment



The 12 Causes of Yaw Moment

Wheel force transducers measure all tire forces at the wheel center

The yaw moment contribution from each tire can be calculated from these forces



The 12 Causes of Yaw Moment

FL R
FR G
RL B
RR Y

Yaw Moment from Fx (longitudinal)

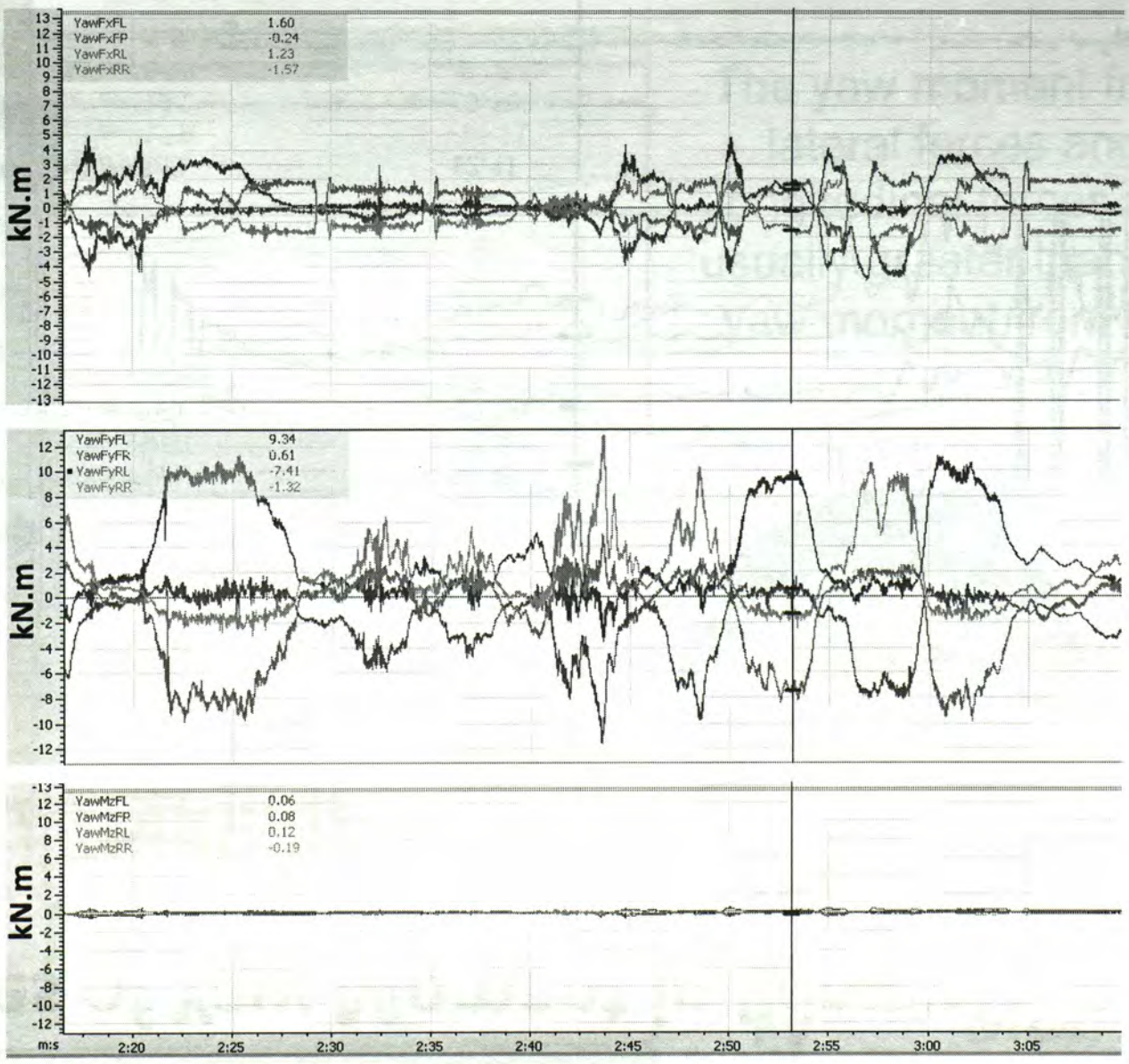
+

Yaw Moment from Fy (lateral)

+

Yaw Moment from Mz (tire self aligning torque)

Steering Wheel Feedback



The 12 Causes of Yaw Moment in Percentage

Total Yaw Moment

=

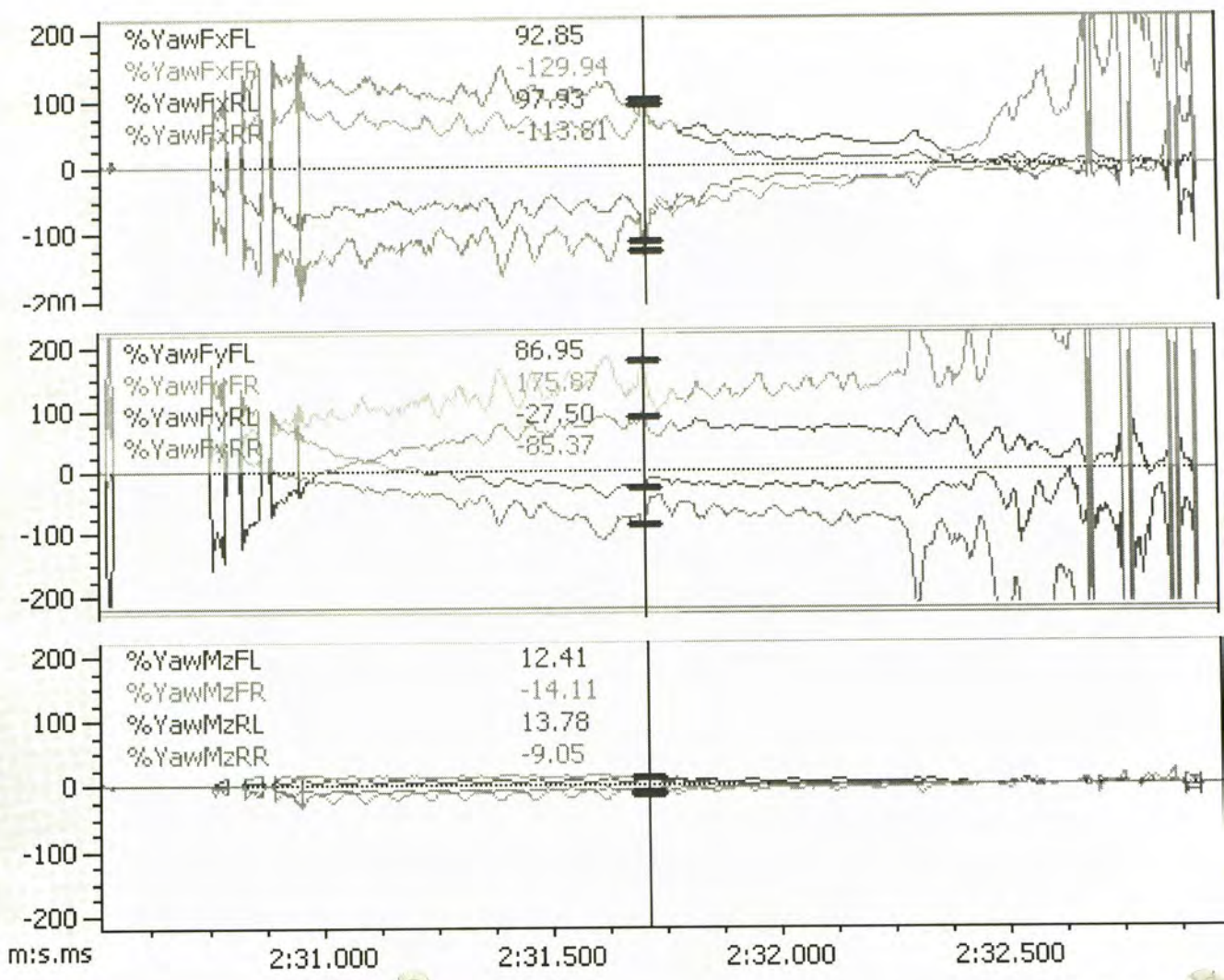
Yaw Moment from Fx (longitudinal)

+

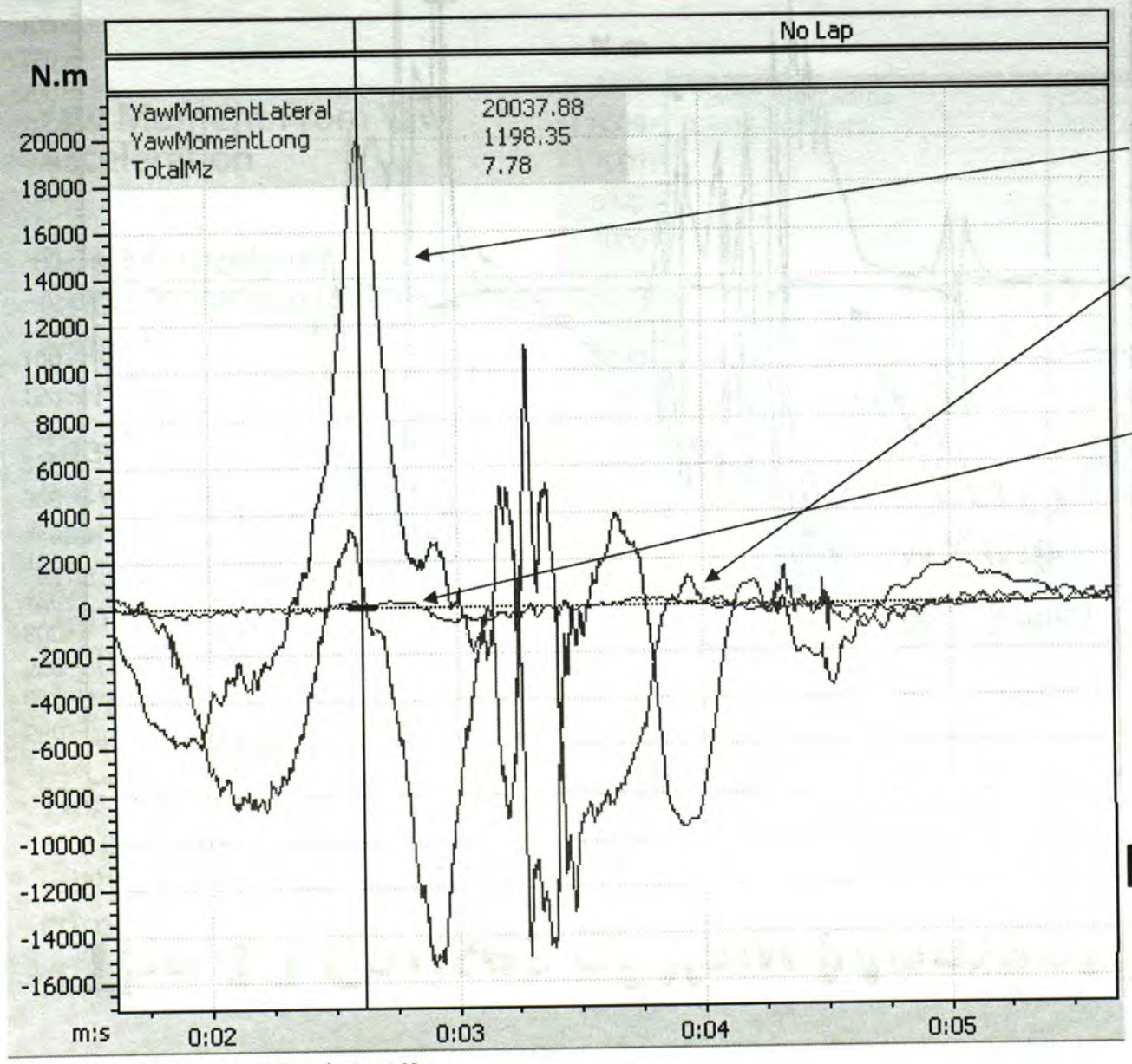
Yaw Moment from Fy (lateral)

+

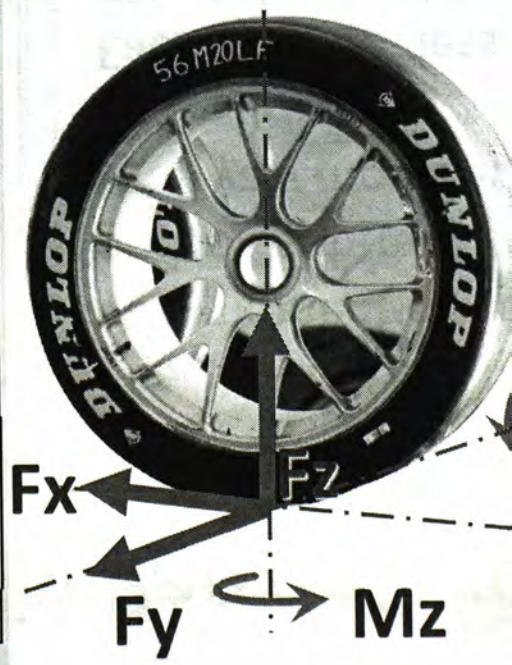
Yaw Moment from Mz (tire self aligning torque)



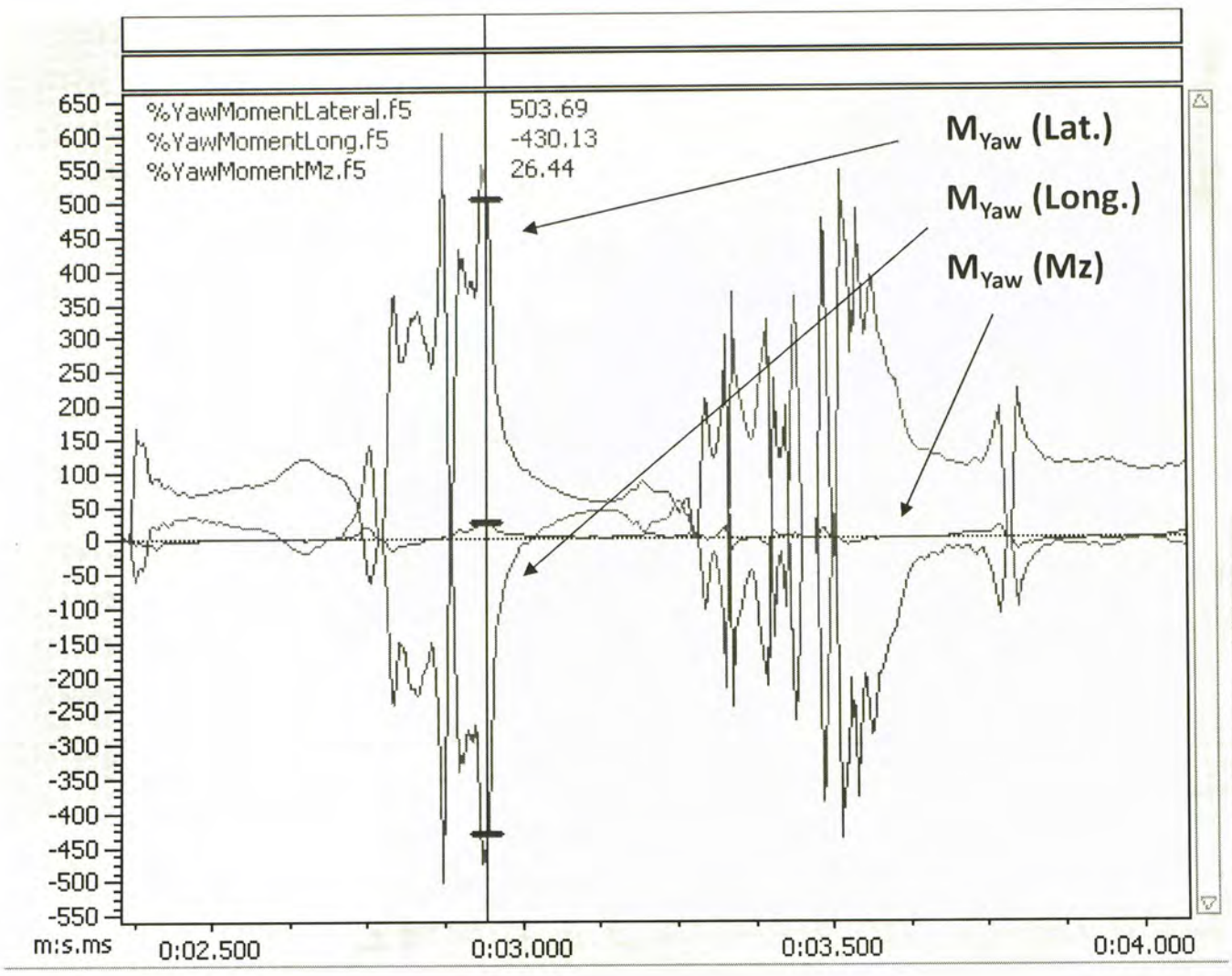
The 12 Causes of Yaw Moment



The yaw moment f
lateral forces an
longitudinal forces
usually greater than
yaw moment from



The 12 Causes of Yaw Moment in Percentage



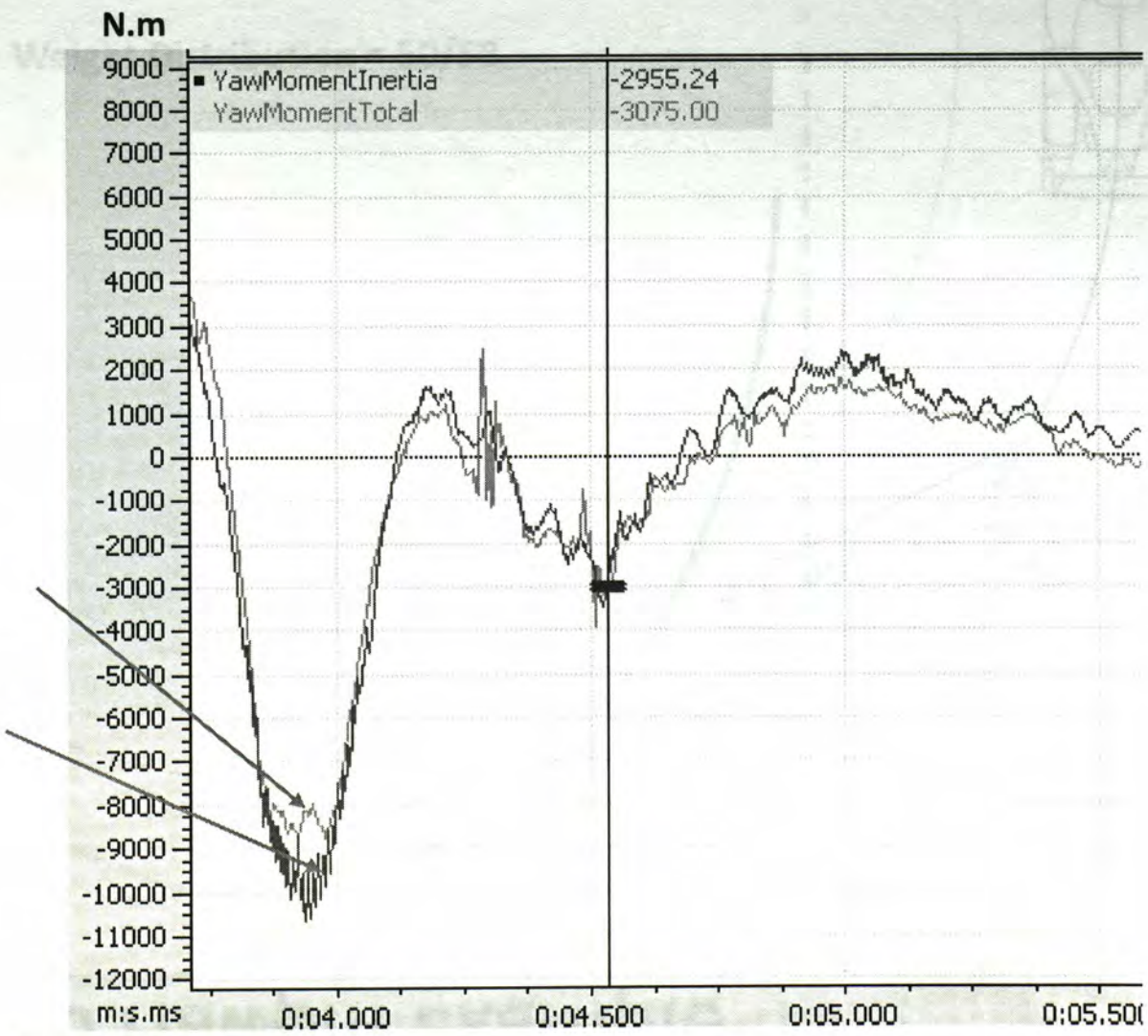
The 12 causes of the yaw moment can either add or subtract to the total yaw moment.

Each can be less than 0% of the total or more than 100% of the total.

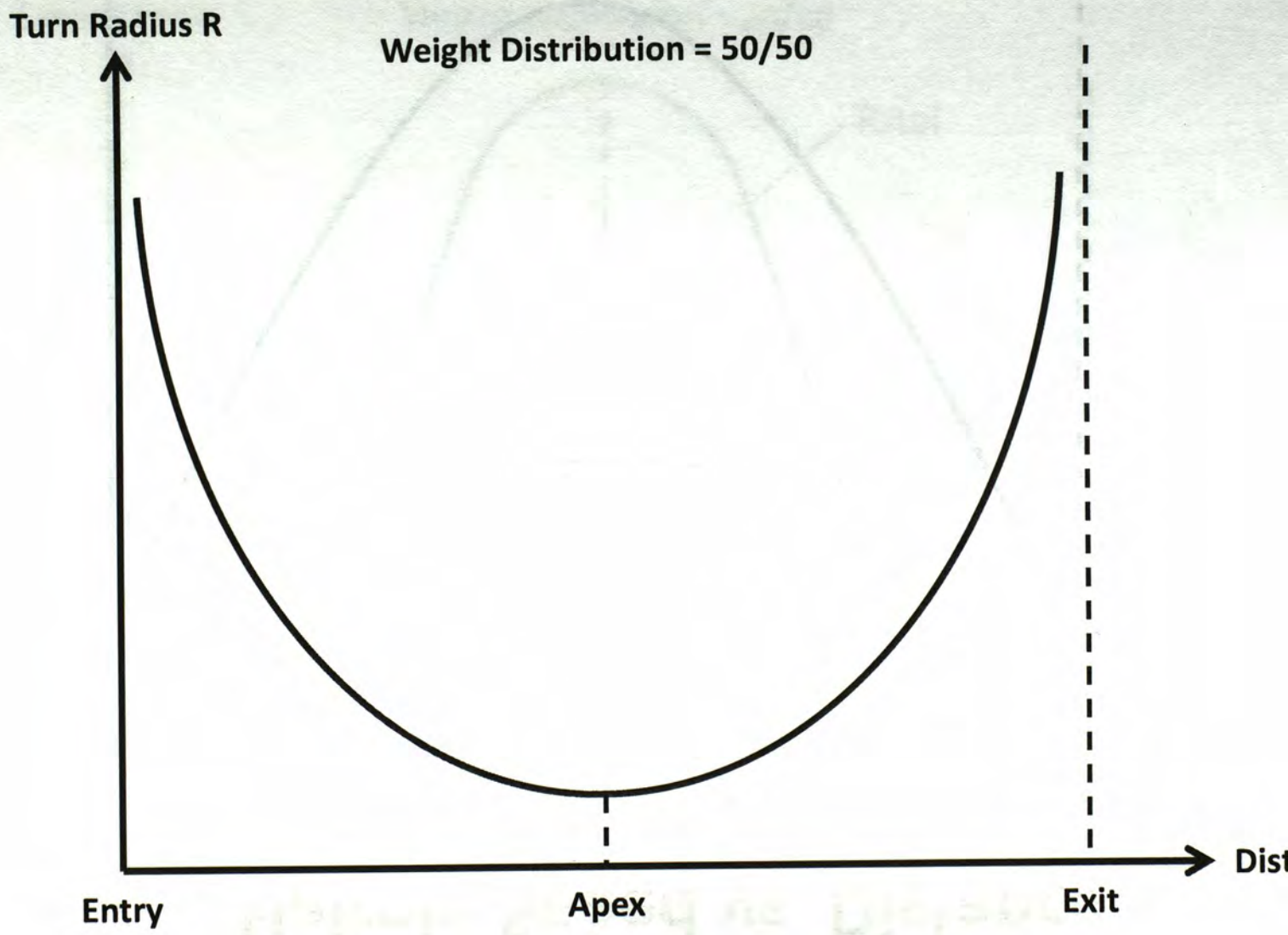
Yaw Moment Measurement

with Kistler Wheel Force Transducer

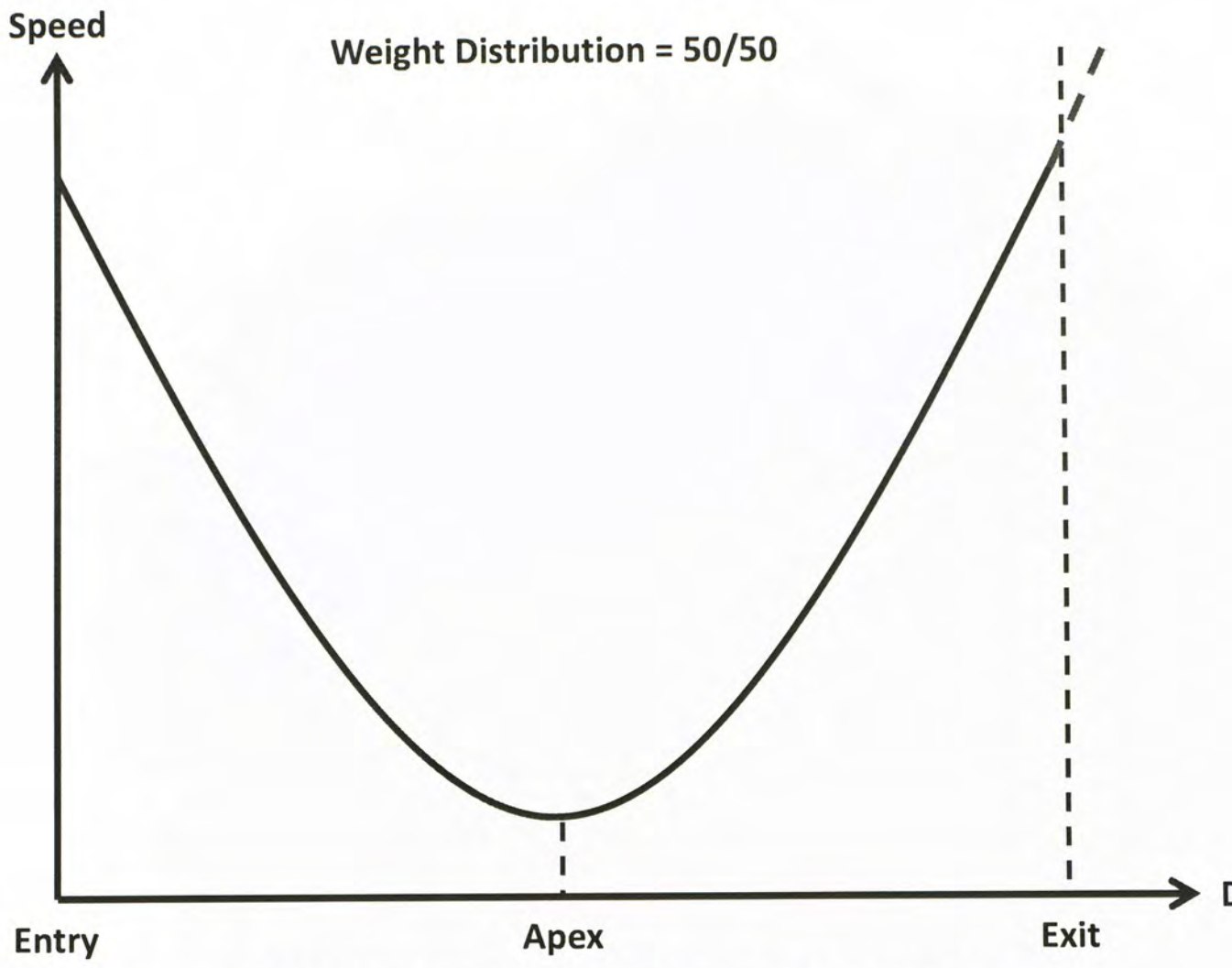
- Yaw Moment From Yaw Acceleration
- Two Methods of Calculating Yaw Moment
- There is a difference between the yaw moment calculated with the wheel forces transducers and the yaw moment calculated with the gyro
- This difference is exaggerated on cars with a lot of suspension compliance



Hairpin Turn Radius vs. Distance

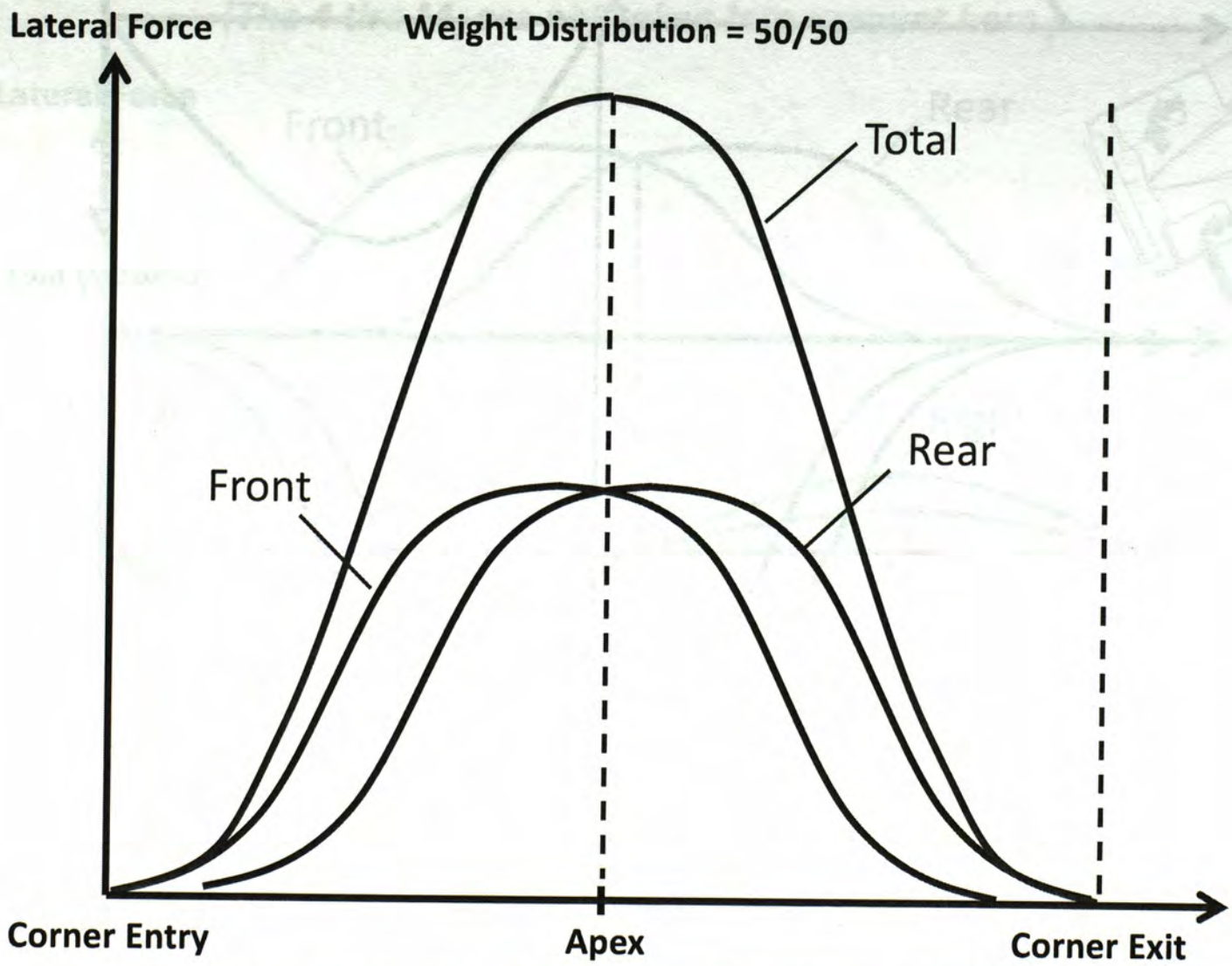


Hairpin Speed vs. Distance

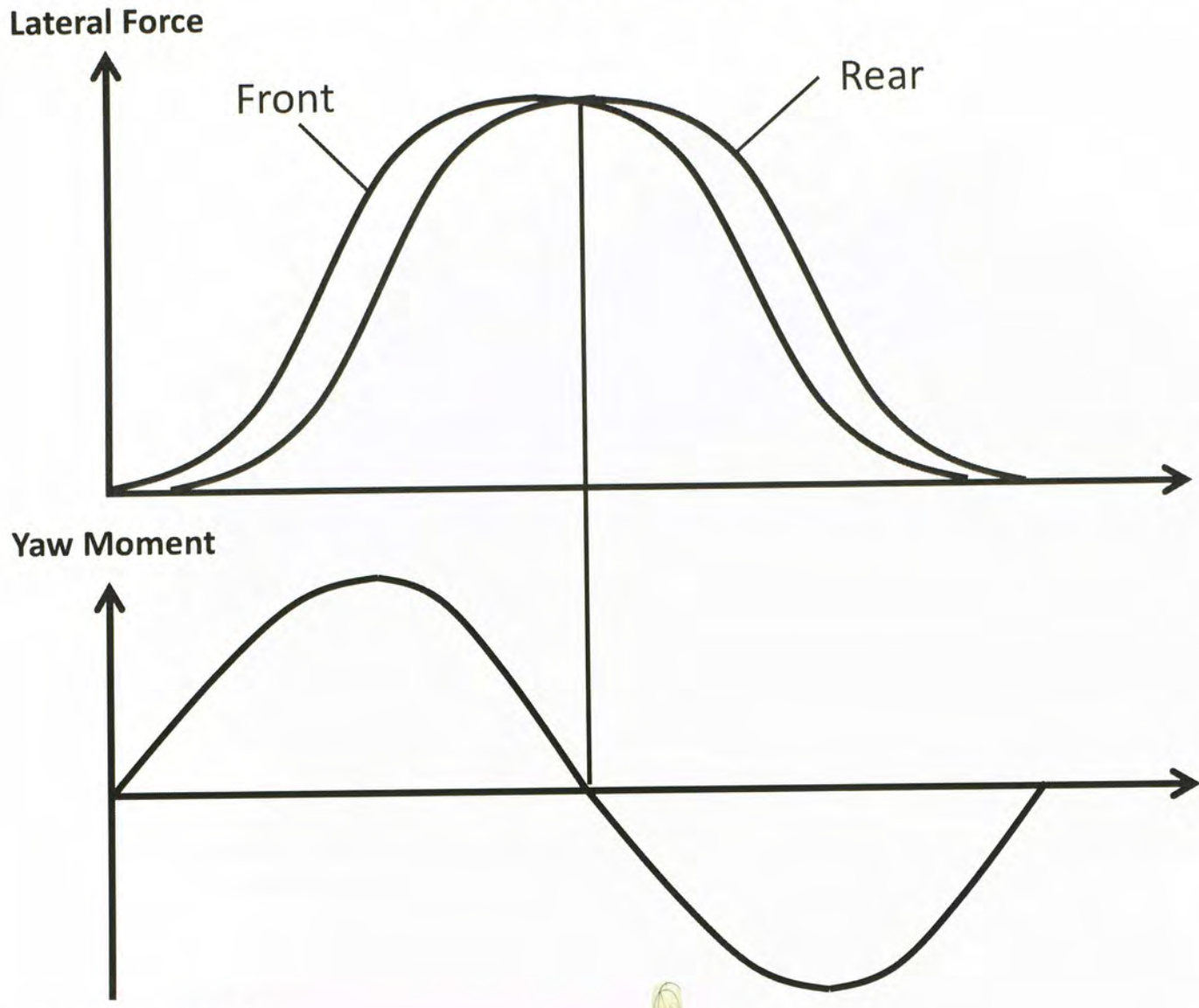


We assume the car accelerates as fast as it decelerates!

Hairpin Lateral Force Component



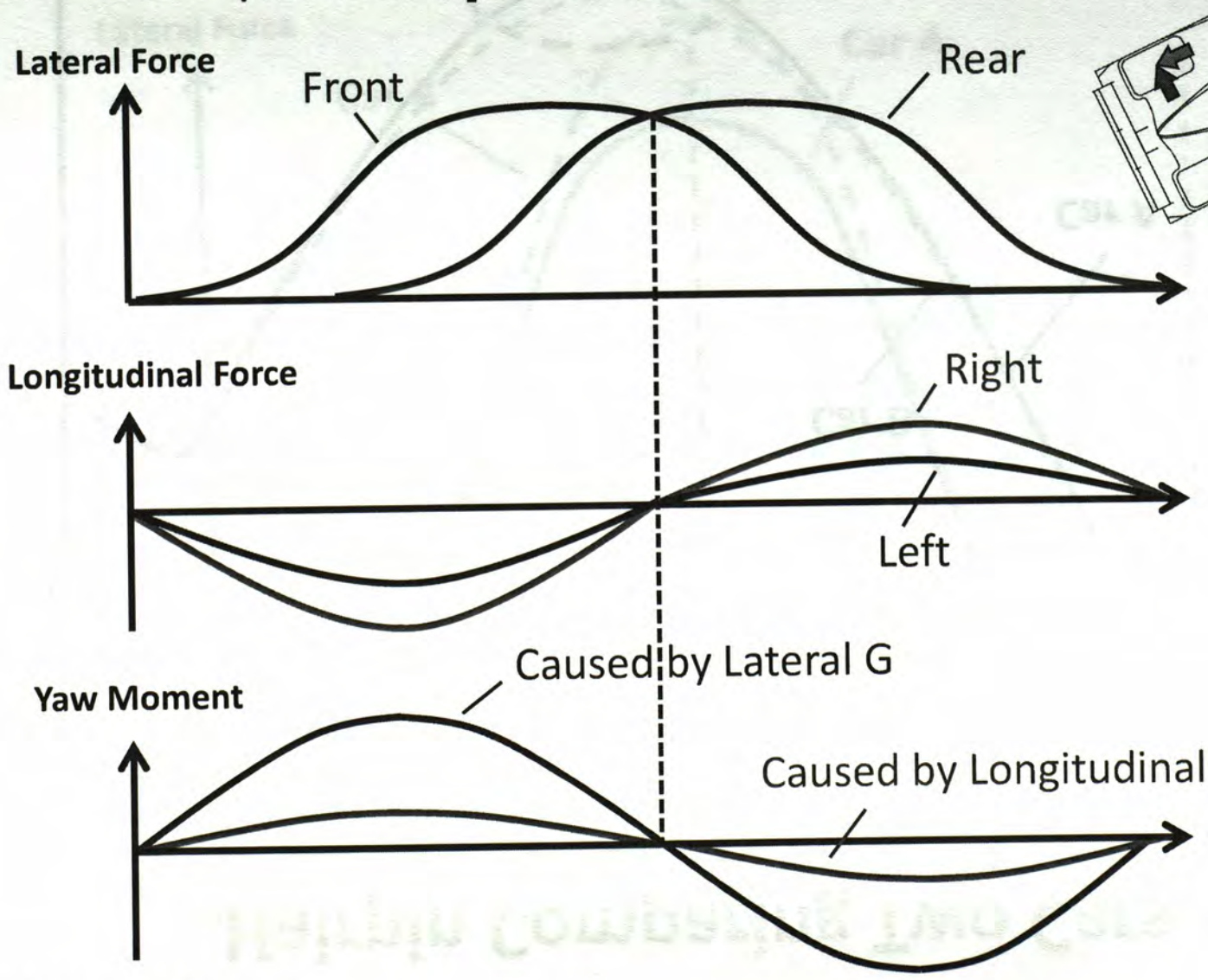
Hairpin Yaw Moment Sequence due to Tire Fy



Hairpin Yaw Moment Sequence

Yaw Moment due to Tire Lateral and Longitudinal

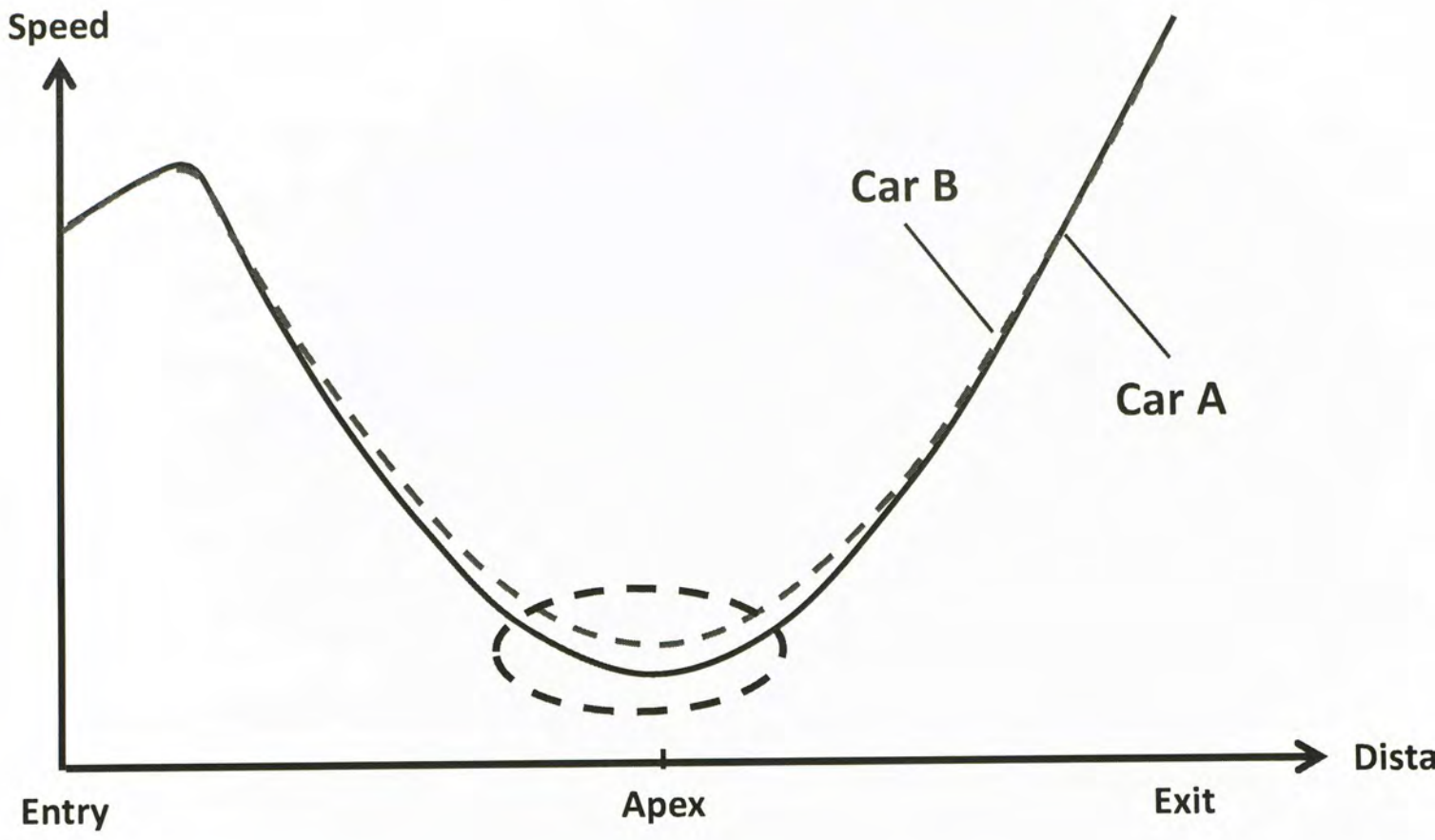
(The 4 tire M_z are not taken into account here)



Hairpin Comparing Two Cars

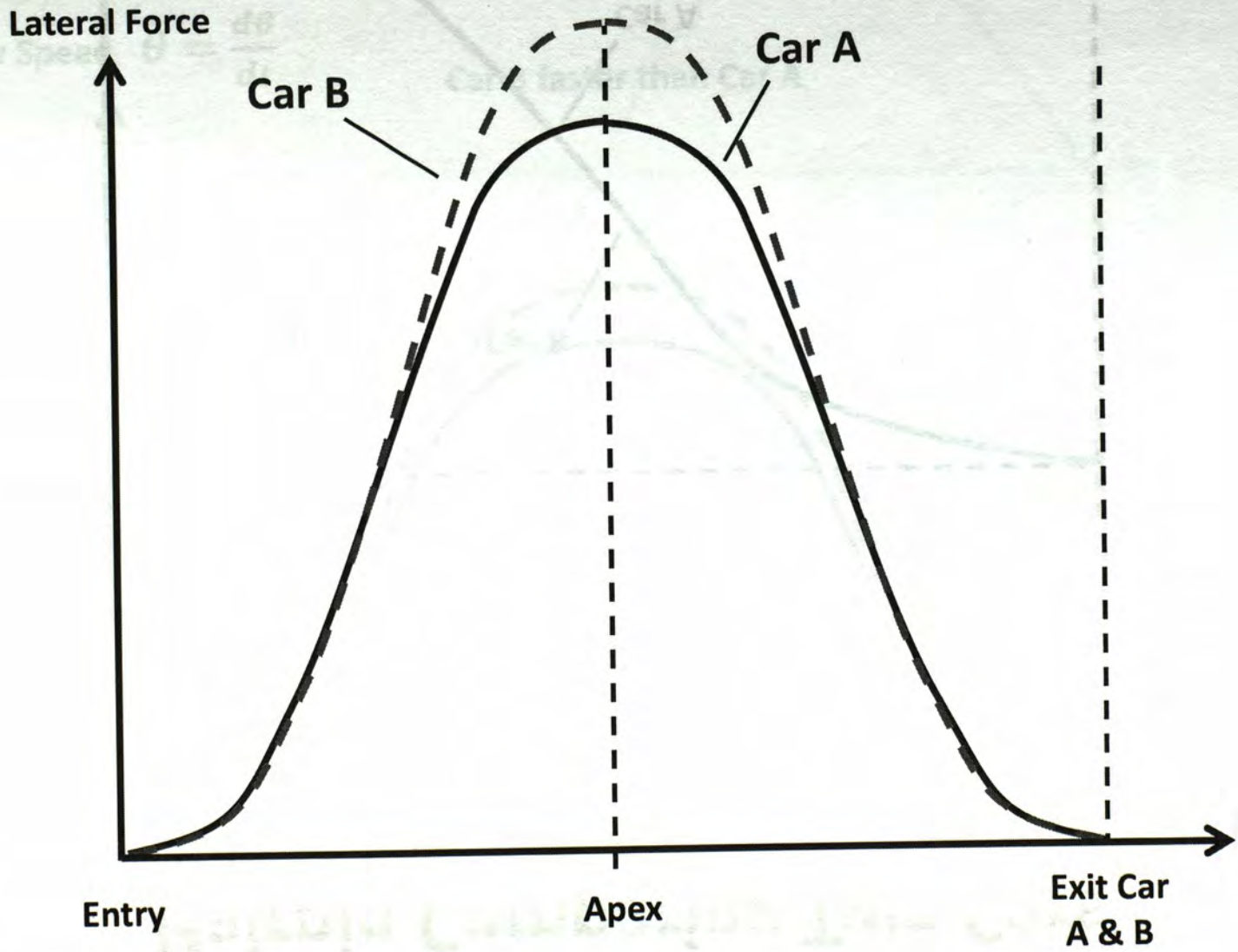
Speed vs. Distance

For the same turn radius R: We have Car B faster than Car A

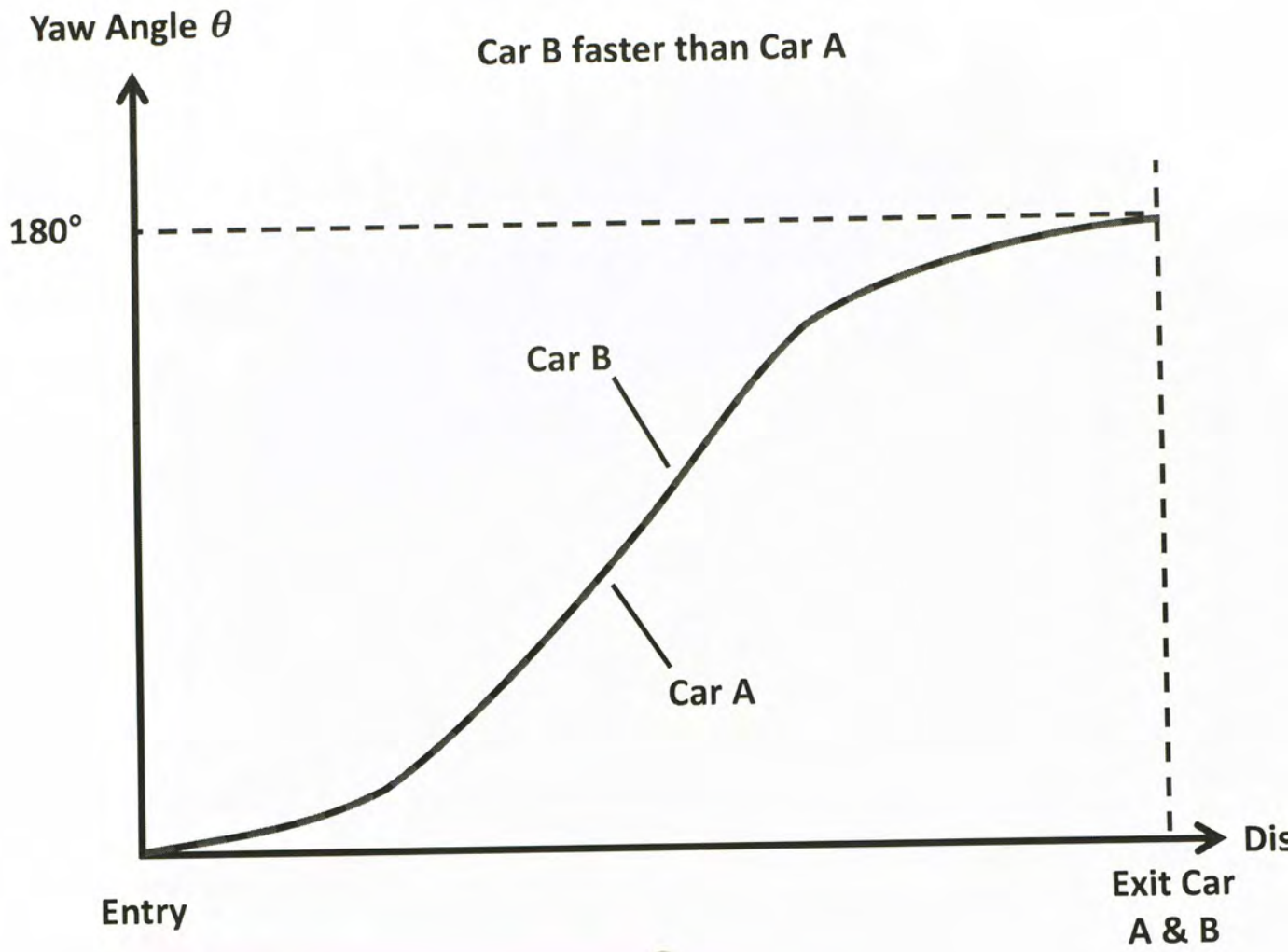


Hairpin Comparing Two Cars

Lateral Force Components vs. Distance

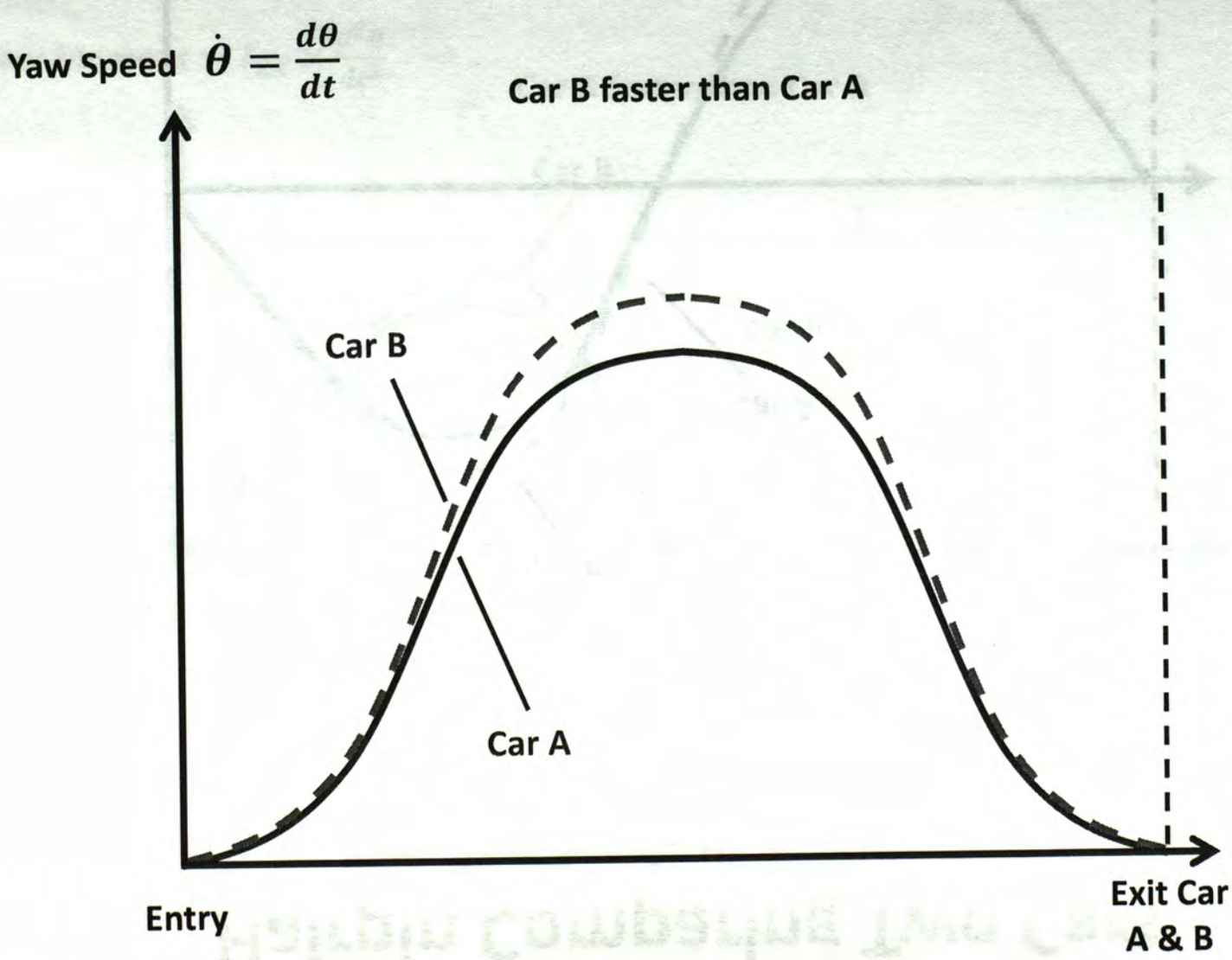


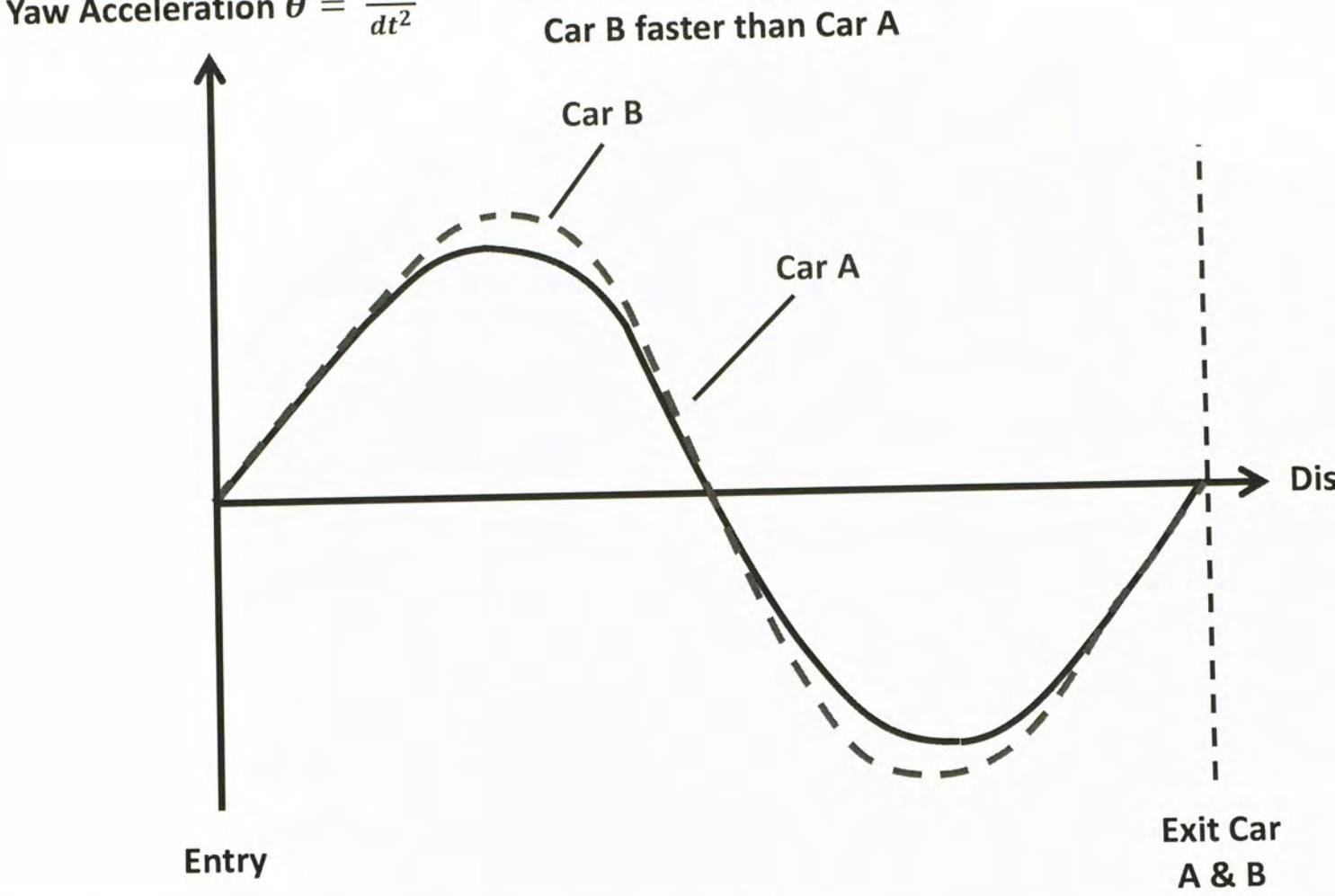
Lateral Force Components vs. Distance



Hairpin Comparing Two Cars

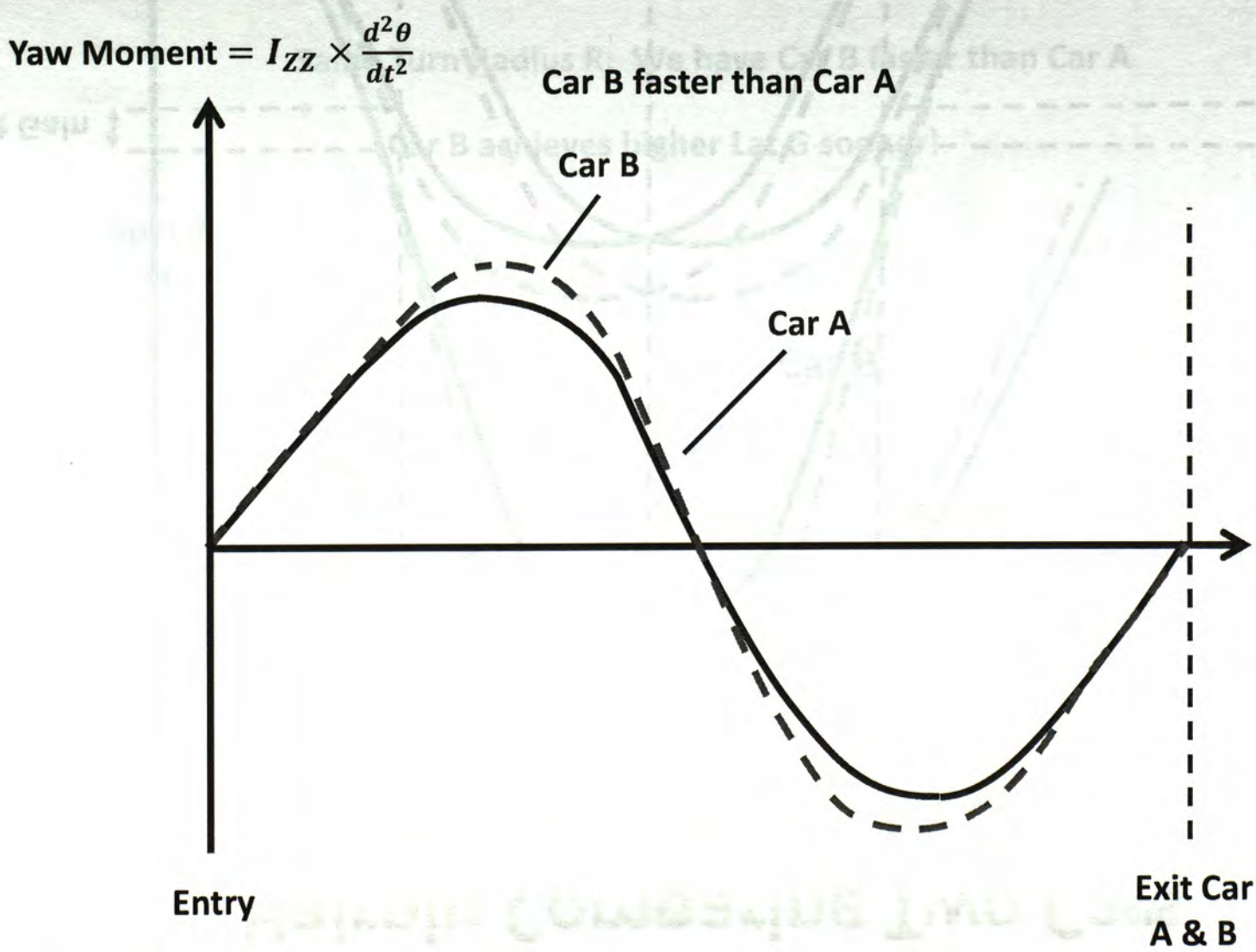
Yaw Speed vs. Distance



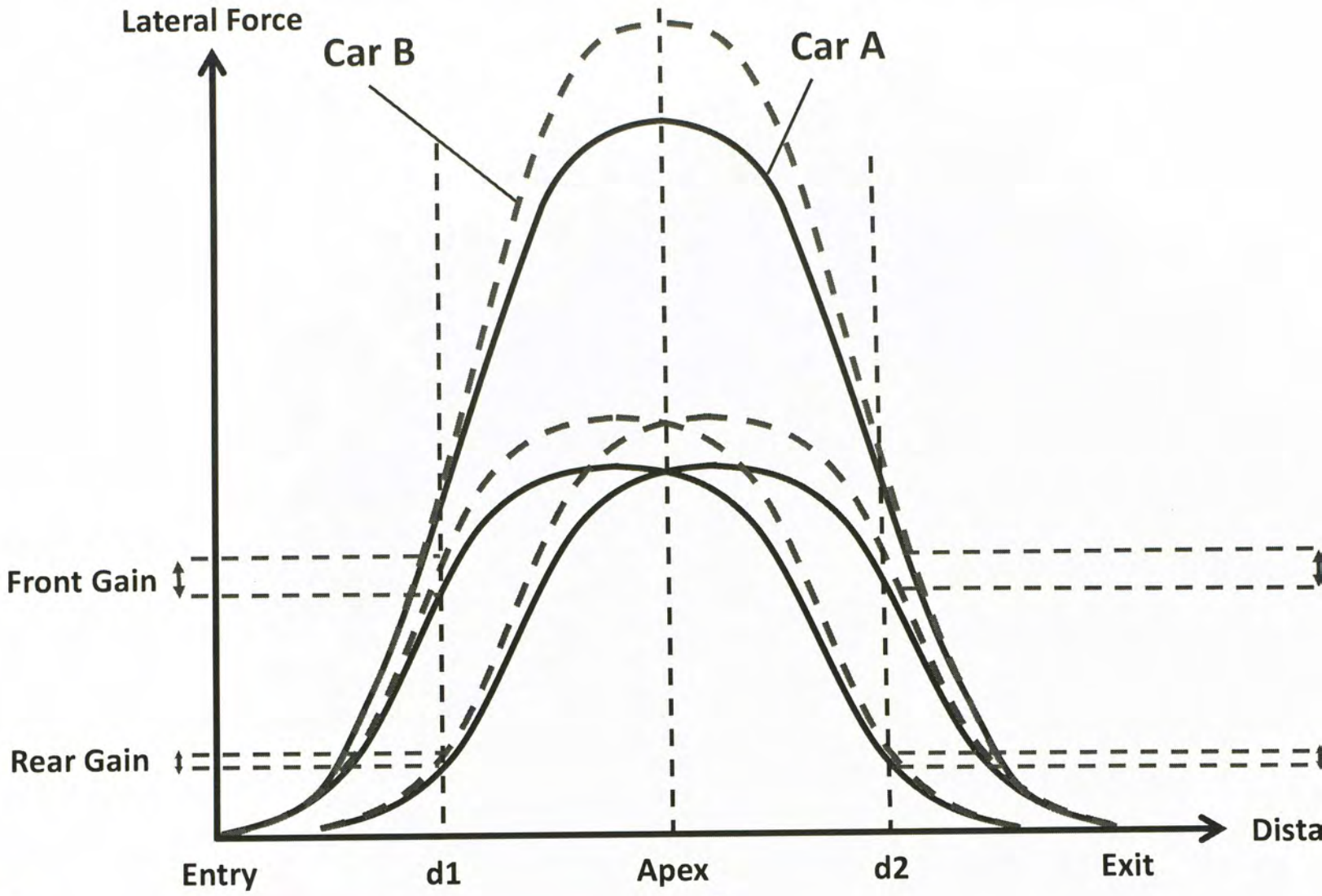


Hairpin Comparing Two Cars

Yaw Moment vs. Distance



Lateral Force Components vs. Distance

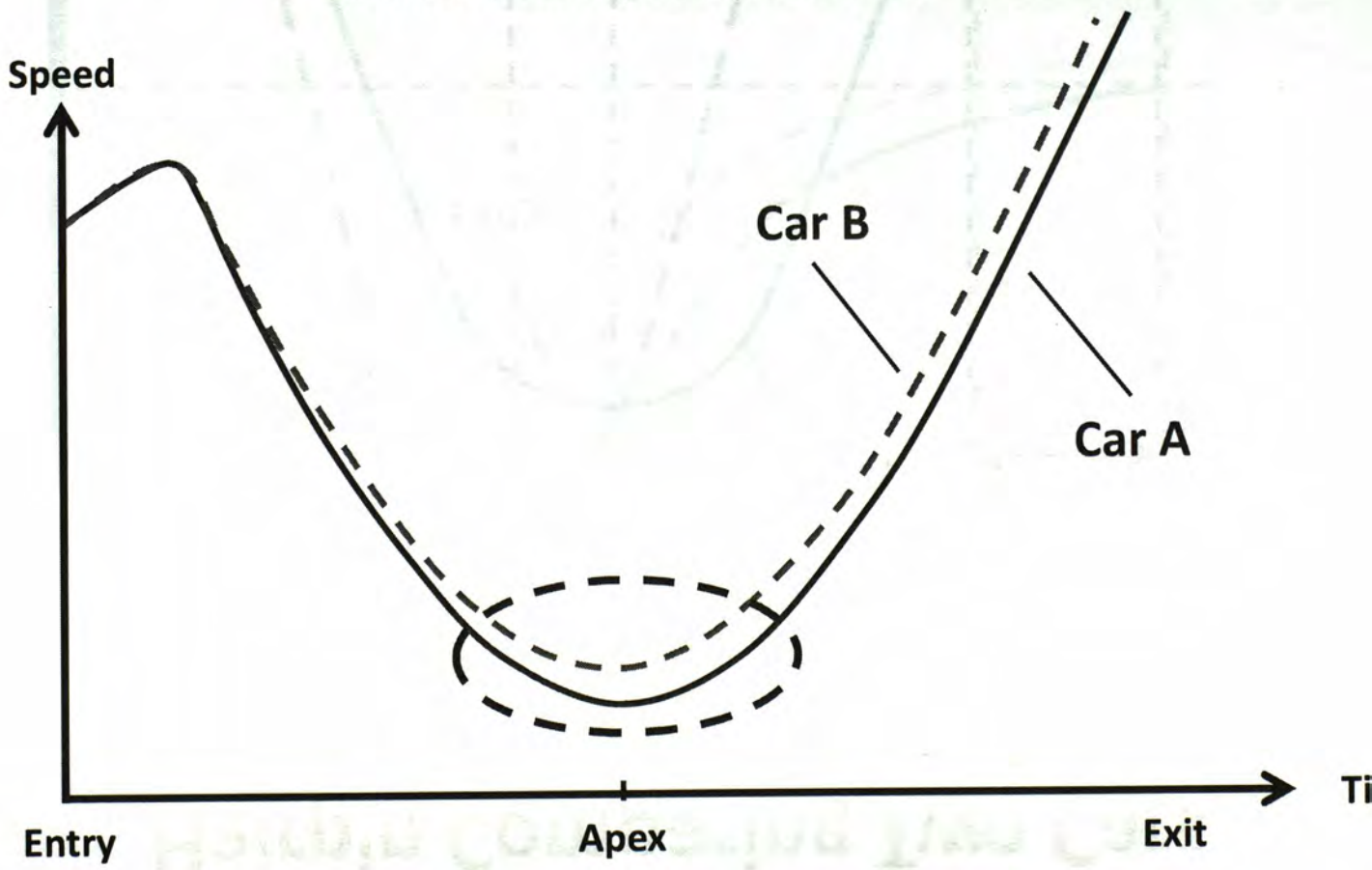


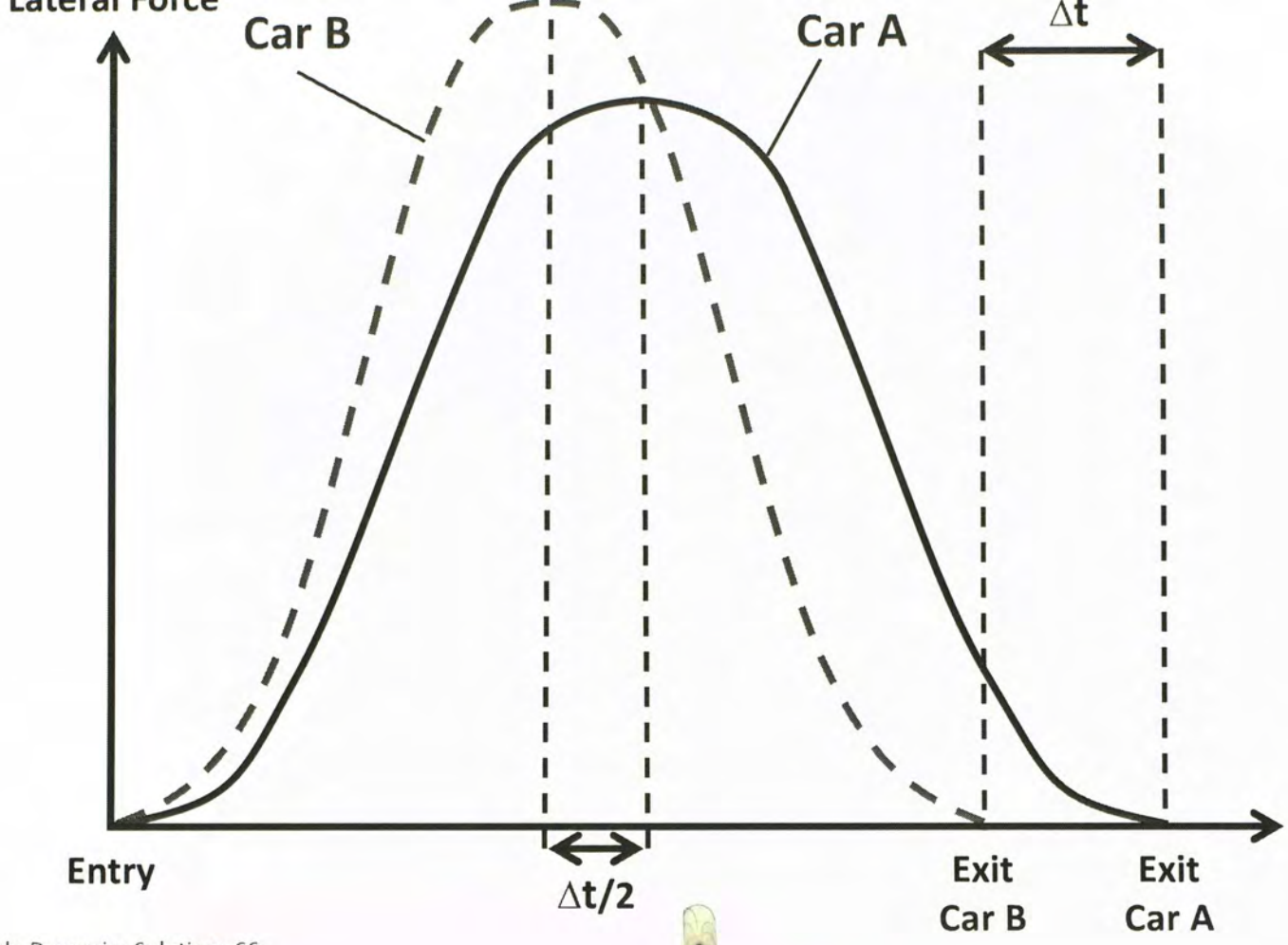
Hairpin Comparing Two Cars

Speed vs. Time

Same Turn Radius R: We have Car B faster than Car A

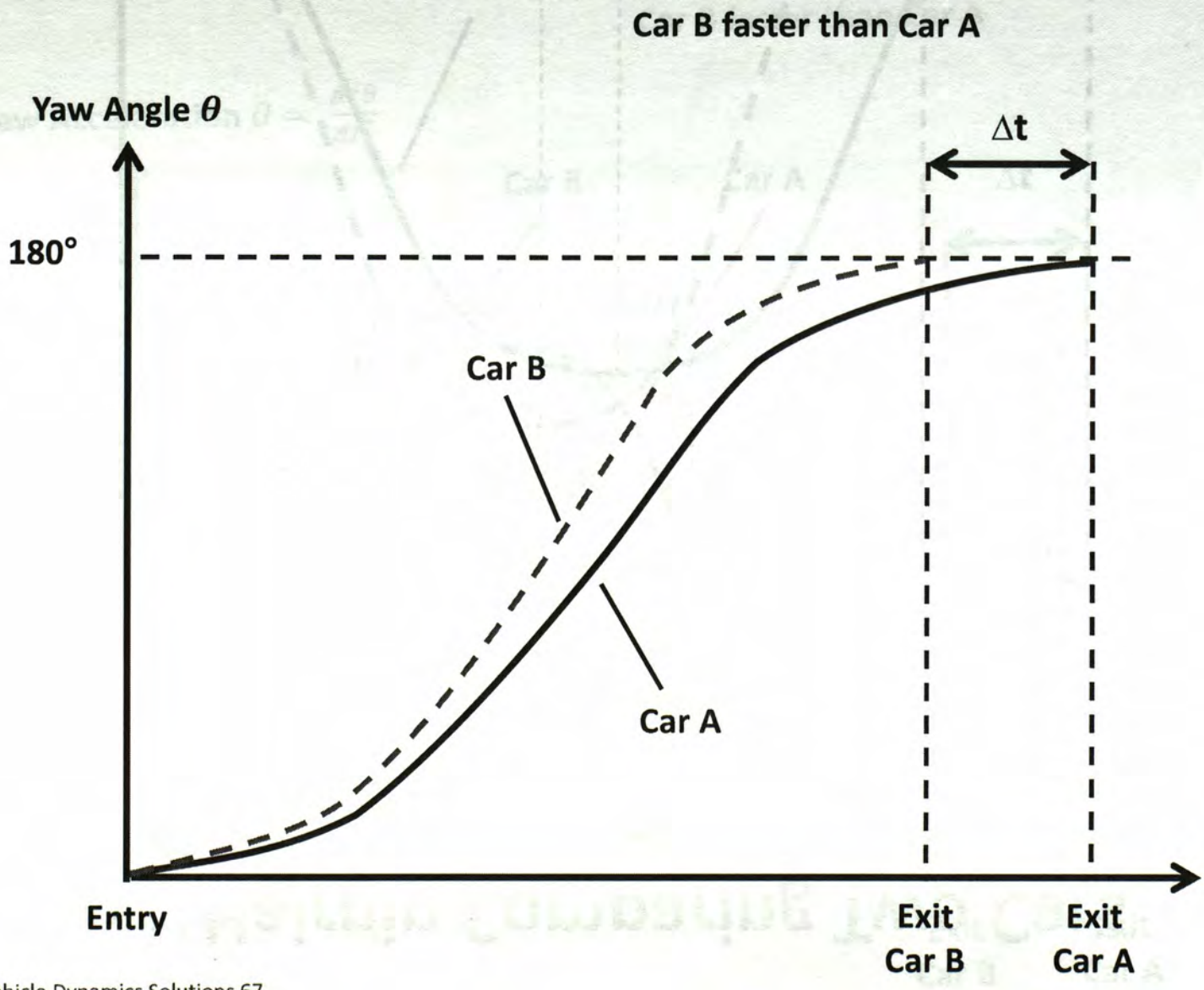
Car B achieves higher Lat G sooner!

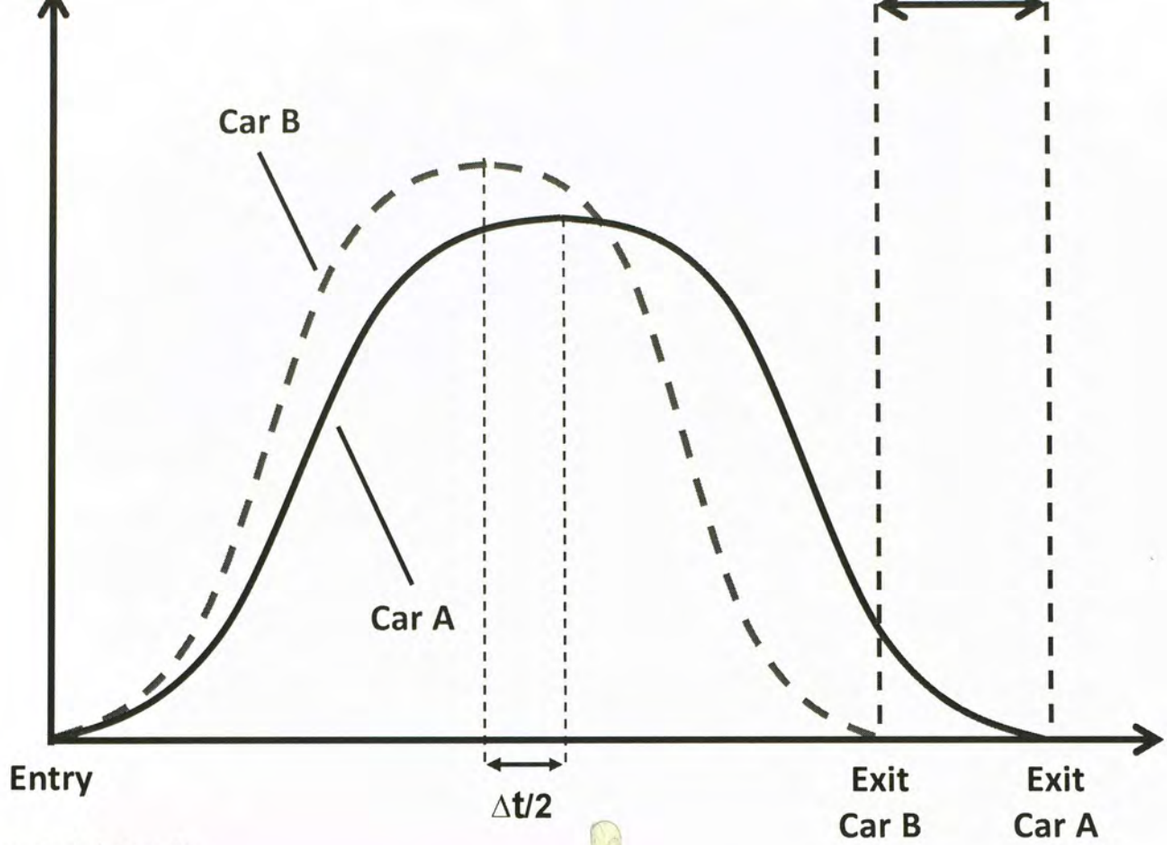




Hairpin Comparing Two Cars

Body Yaw Angle vs. Time



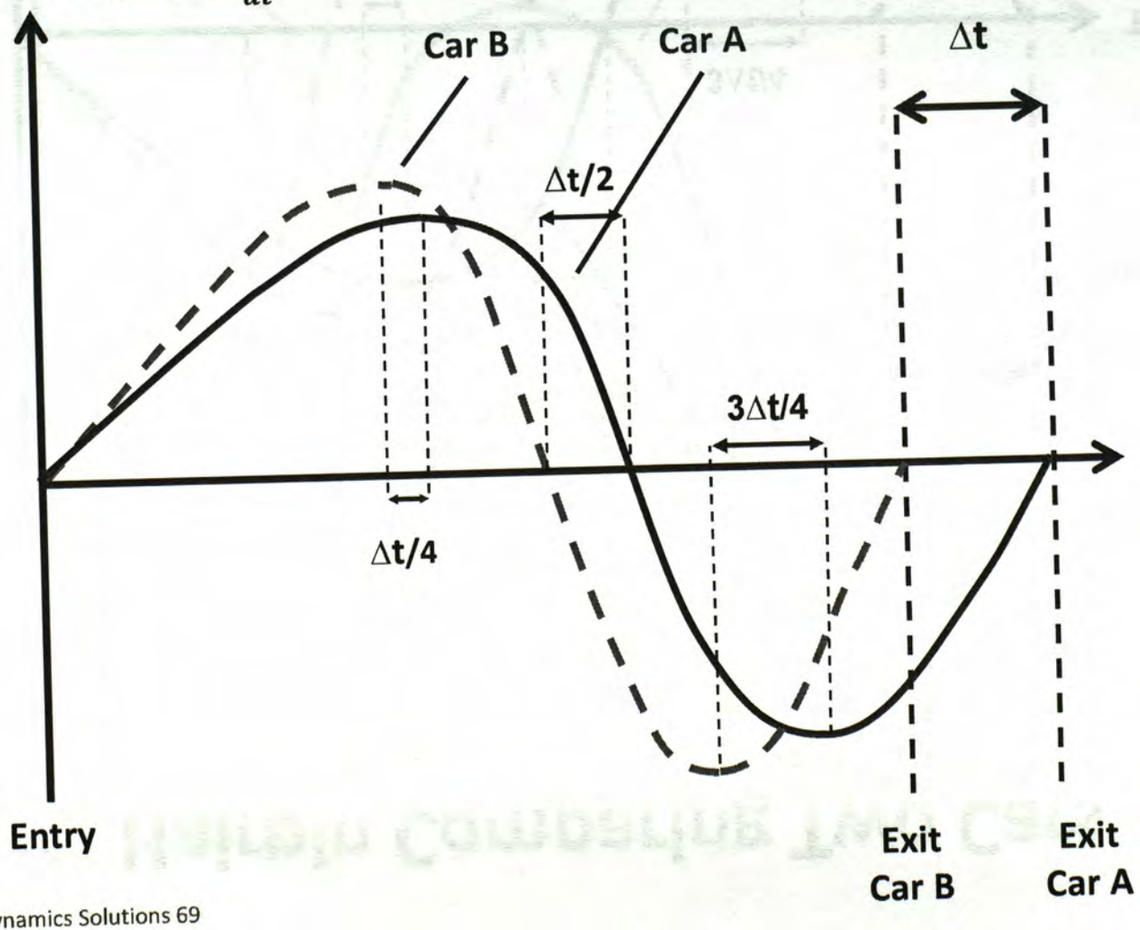


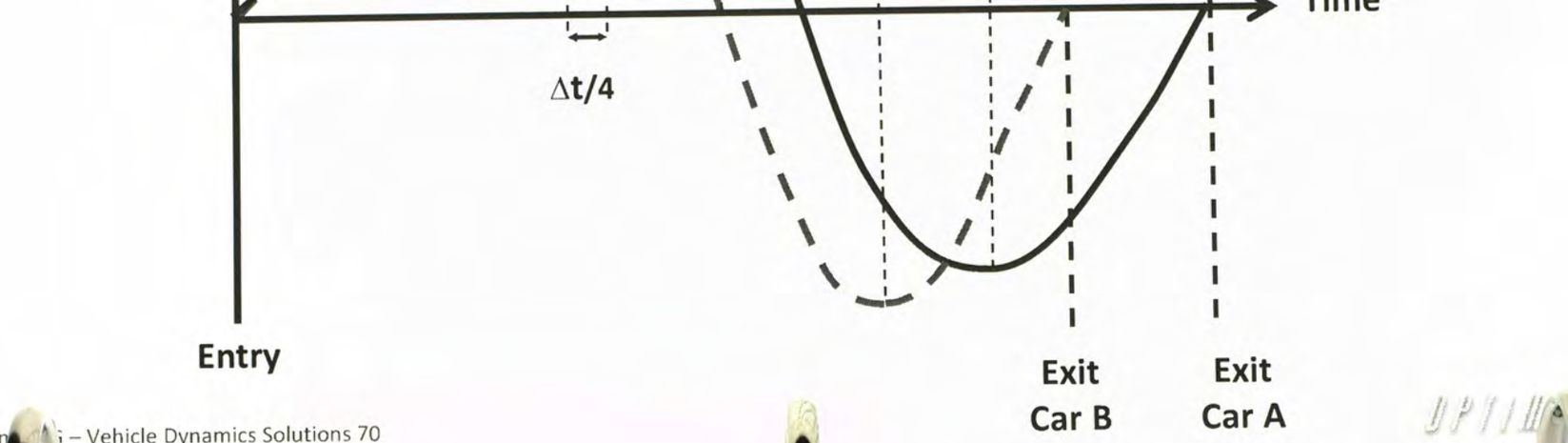
Hairpin Comparing Two Cars

Yaw Acceleration vs. Time

Car B faster than Car A

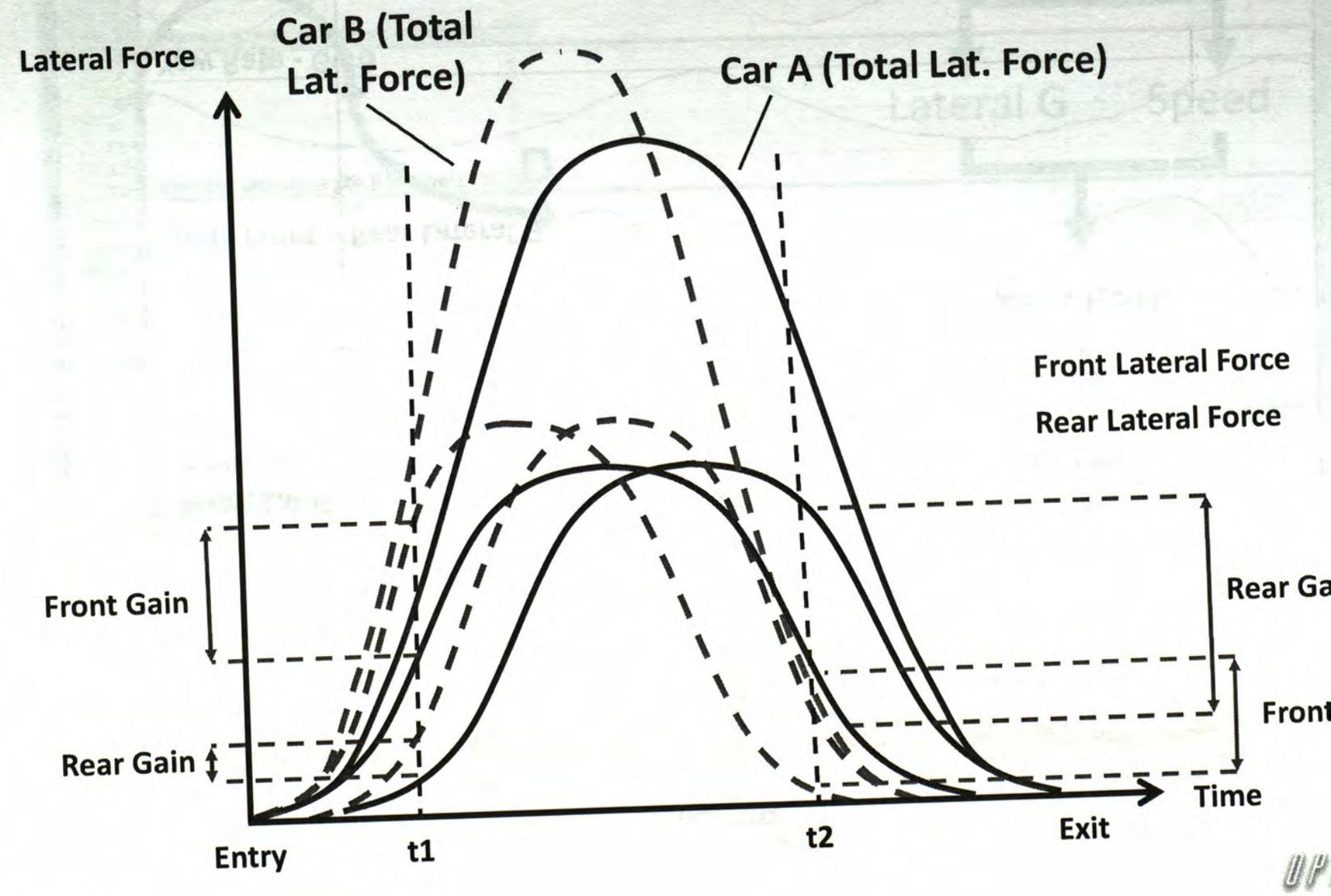
Yaw Acceleration $\ddot{\theta} = \frac{d^2\theta}{dt^2}$



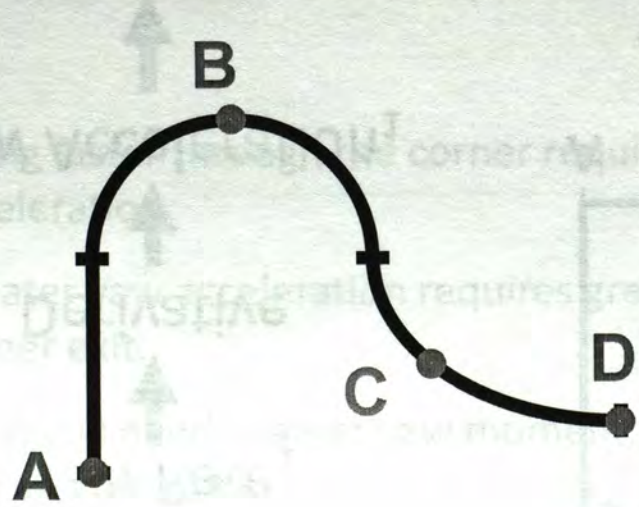


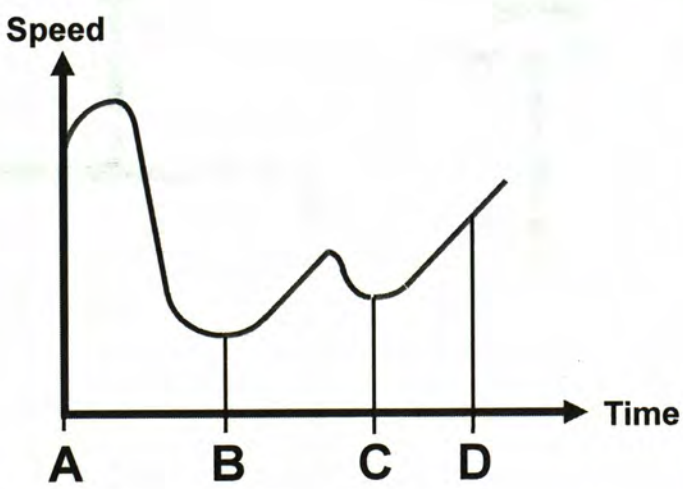
Hairpin Comparing Two Cars

Lateral Force Components vs. Time



Yaw Moment Track Analysis





Data Acquisition

Lateral G | Speed

Yaw Rate

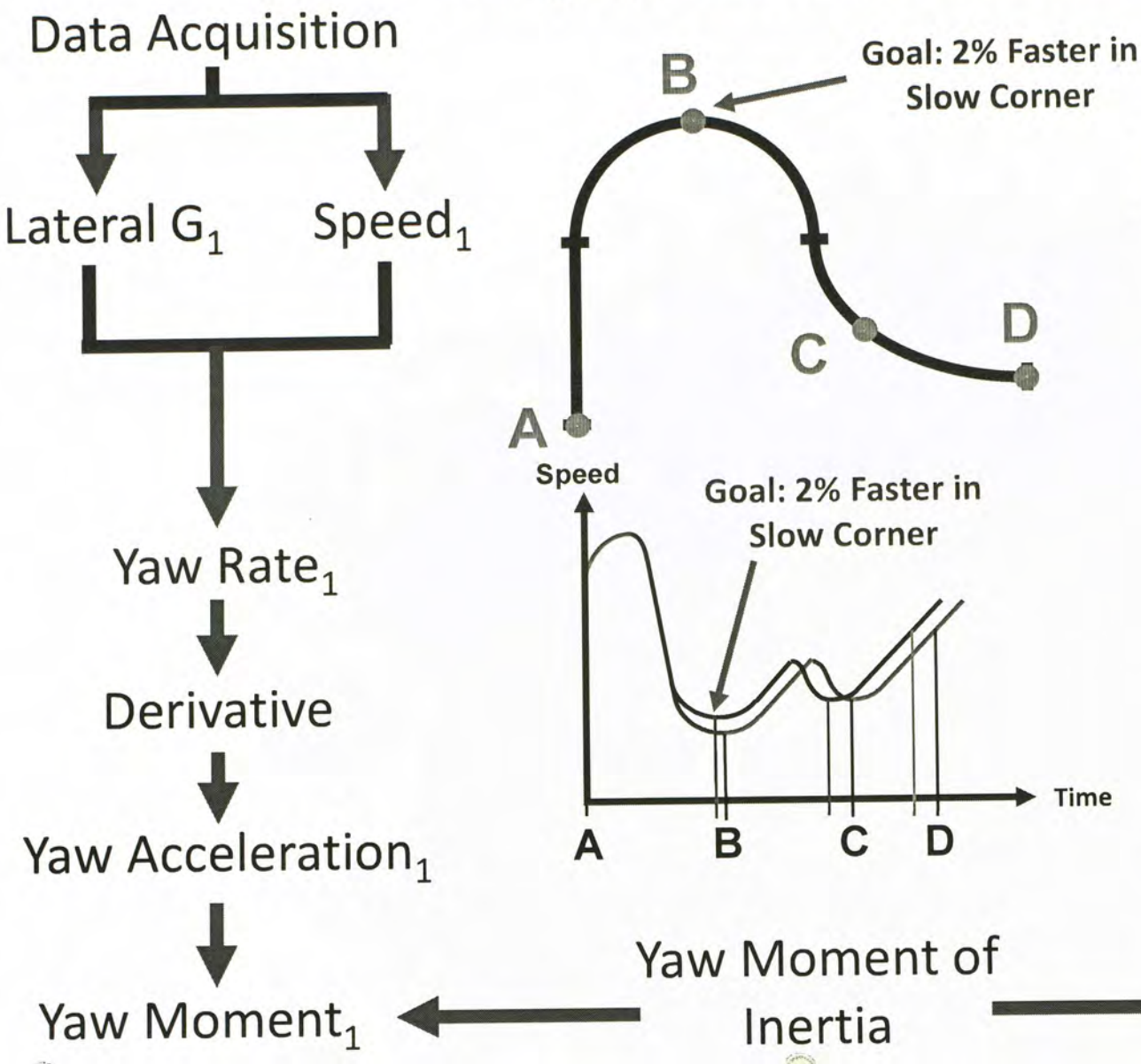
Derivative

Yaw Acceleration

Yaw Moment of Inertia → Yaw Moment

Yaw Moment Track Analysis

Using acquired data for simulation



Hairpin Comparing Two Cars

Conclusion

Going faster through the corner requires faster yaw velocity in less time, thus g
acceleration.

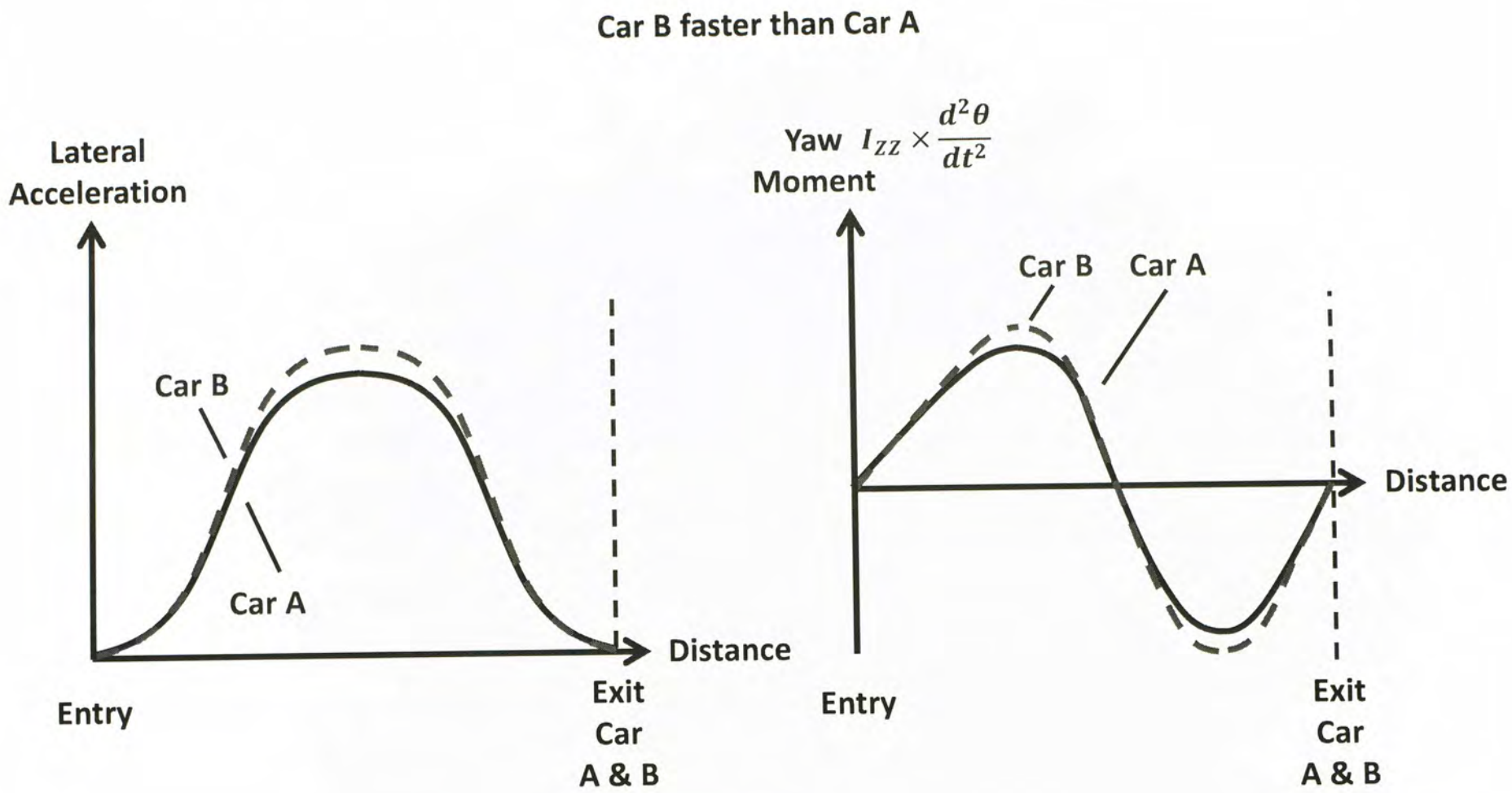
Greater yaw acceleration requires greater yaw moment - positive for corner en
corner exit.

Car B will need a larger yaw moment, thus more front grip relative to rear grip
and vice versa for exit.

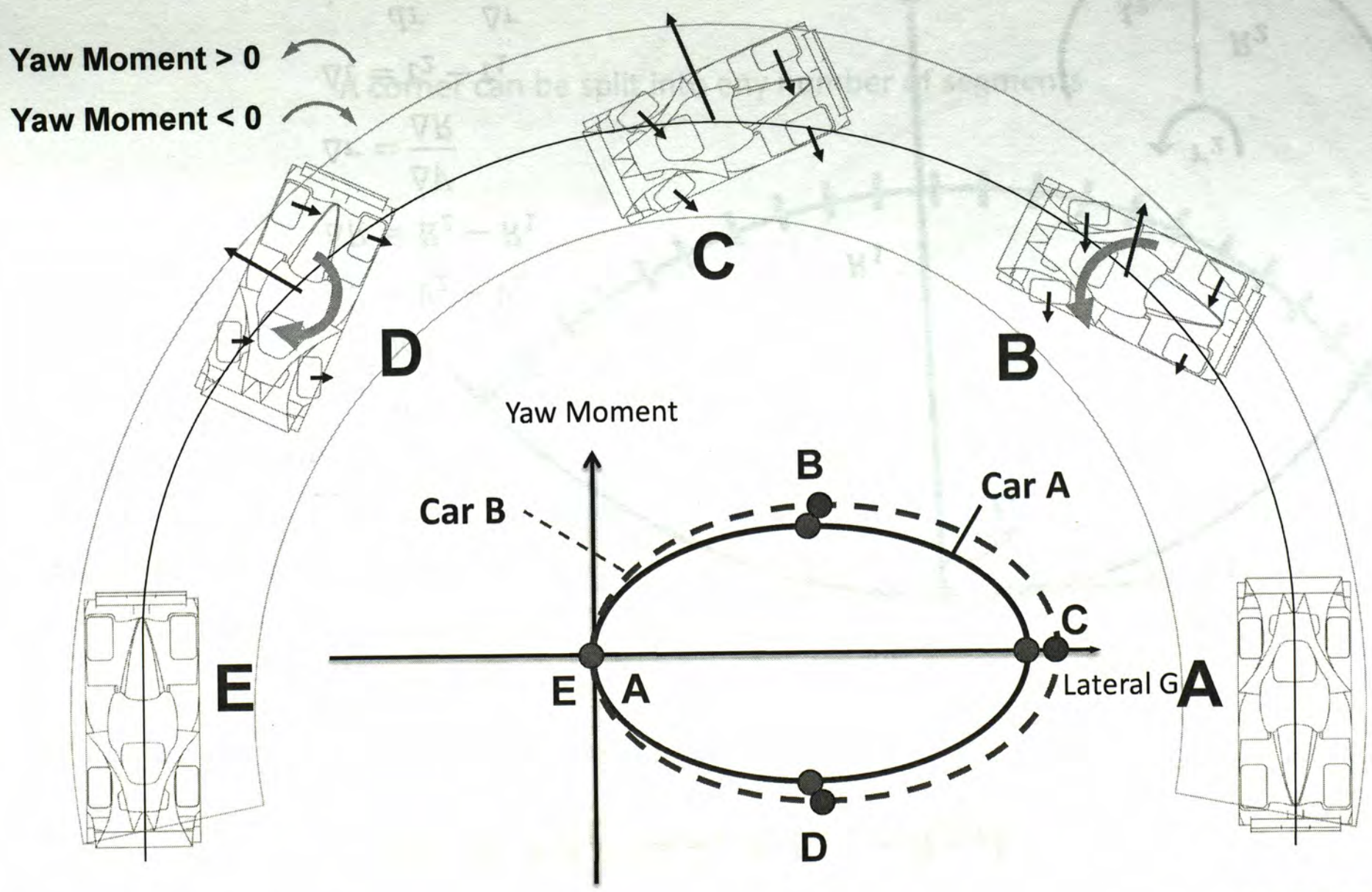
Entry	→	Yaw Moment = $(F_{yFL} + F_{yFR})a - (F_{yRL} +$
		+2.5% +1.5
Apex	→	Yaw Moment = $(F_{yFL} + F_{yFR})a - (F_{yRL} +$
		+4.0% +4.0
Exit	→	Yaw Moment = $(F_{yFL} + F_{yFR})a - (F_{yRL} +$
		+1.5% +2.5

% of change in grip are a qualitative example. Do not take these as real
You should go through your own simulation / calculation

Lateral Acceleration & Yaw Moment



Yaw Moment vs. Lateral Acceleration



Lateral acceleration is assumed to be maximum possible

Lateral acceleration and radius are given
Speed is the output

$$V = \sqrt{A_y \times Radius}$$

$$r = \frac{V}{R}$$

Yaw Speed

$$\Delta V = V_2 - V_1$$

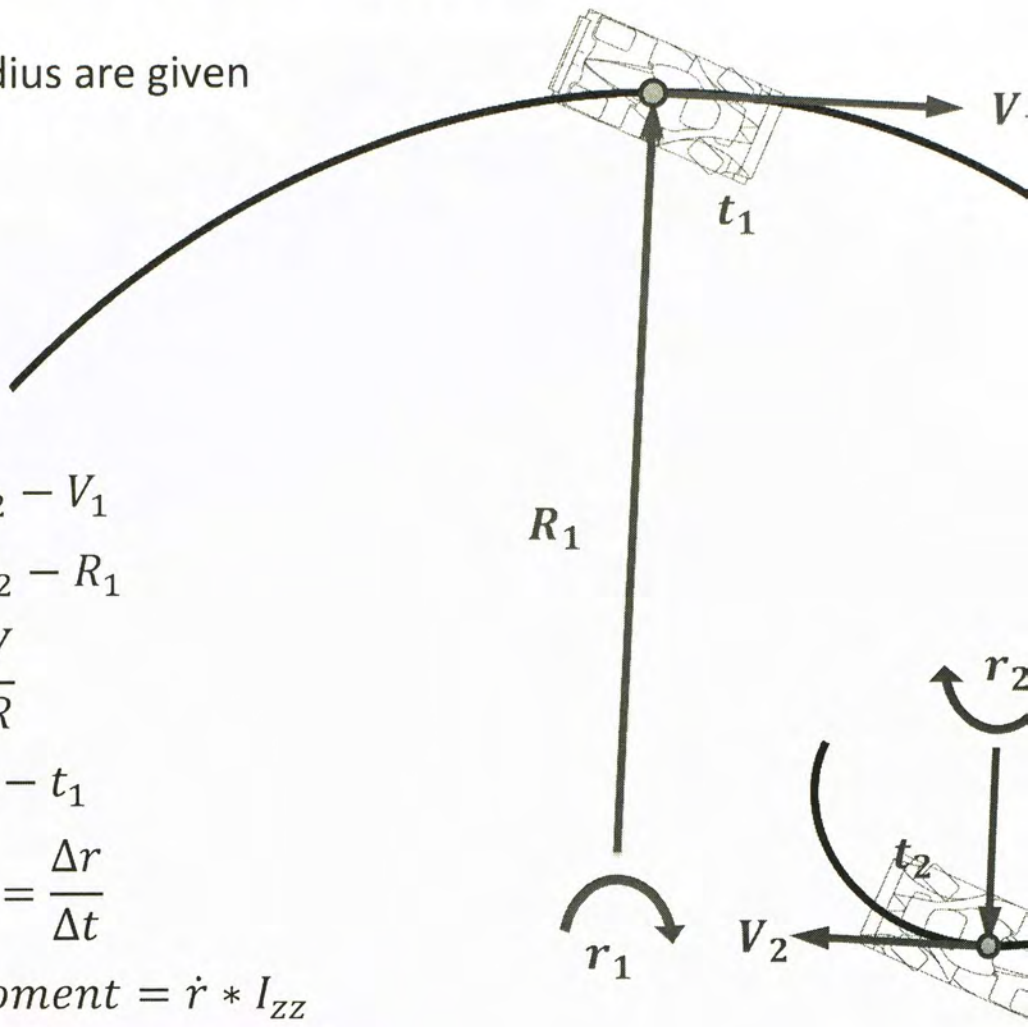
$$\Delta R = R_2 - R_1$$

$$\Delta r = \frac{\Delta V}{\Delta R}$$

$$\Delta t = t_2 - t_1$$

$$\dot{r} = \frac{dr}{dt} = \frac{\Delta r}{\Delta t}$$

$$Yaw\ Moment = \dot{r} * I_{zz}$$



Yaw Moment Needed

A corner can be split into any number of segments

$$\Delta V = V_{x+1} - V_x$$

$$\Delta R = R_{x+1} - R_x$$

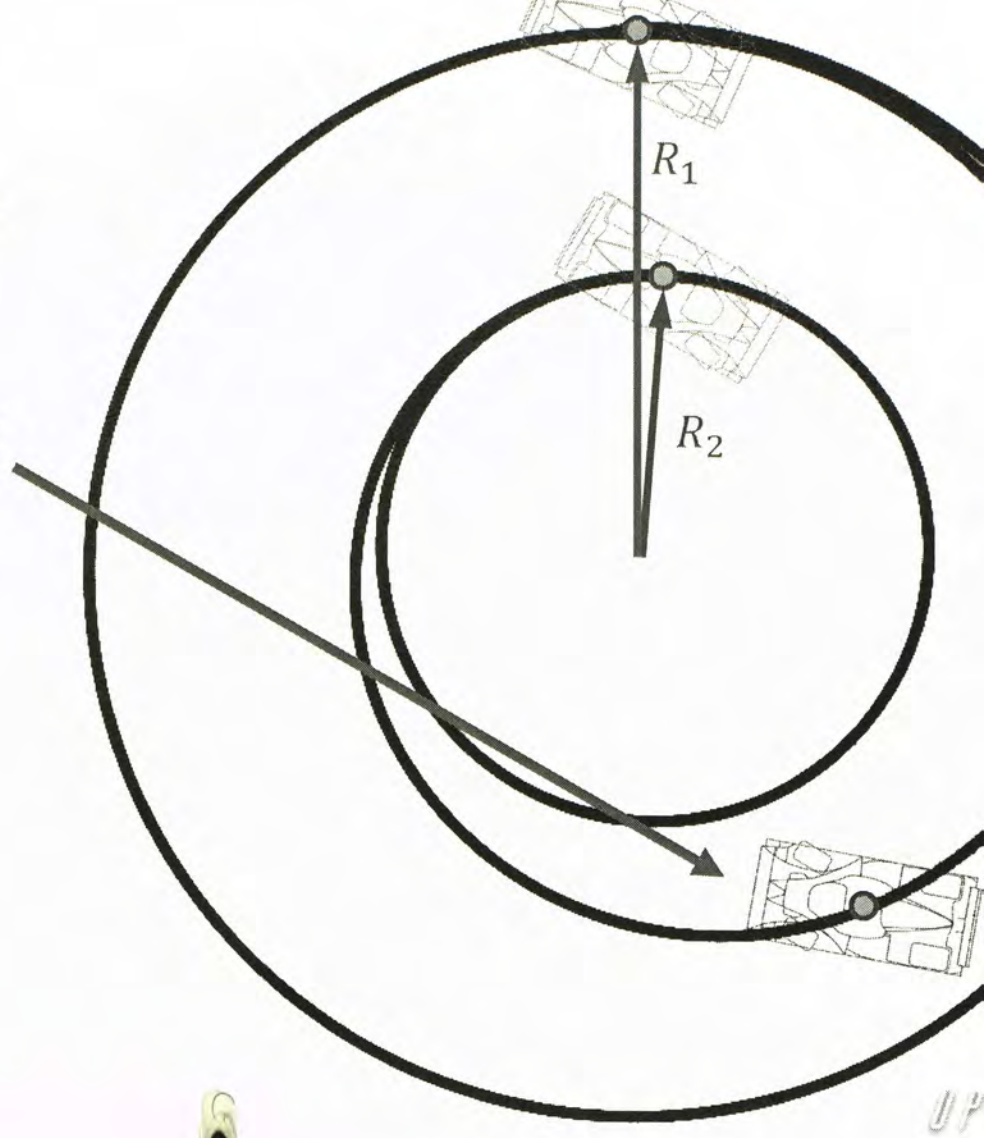
$$\Delta r = \frac{\Delta V}{\Delta R}$$

$$\Delta t = t_{x+1} - t_x$$

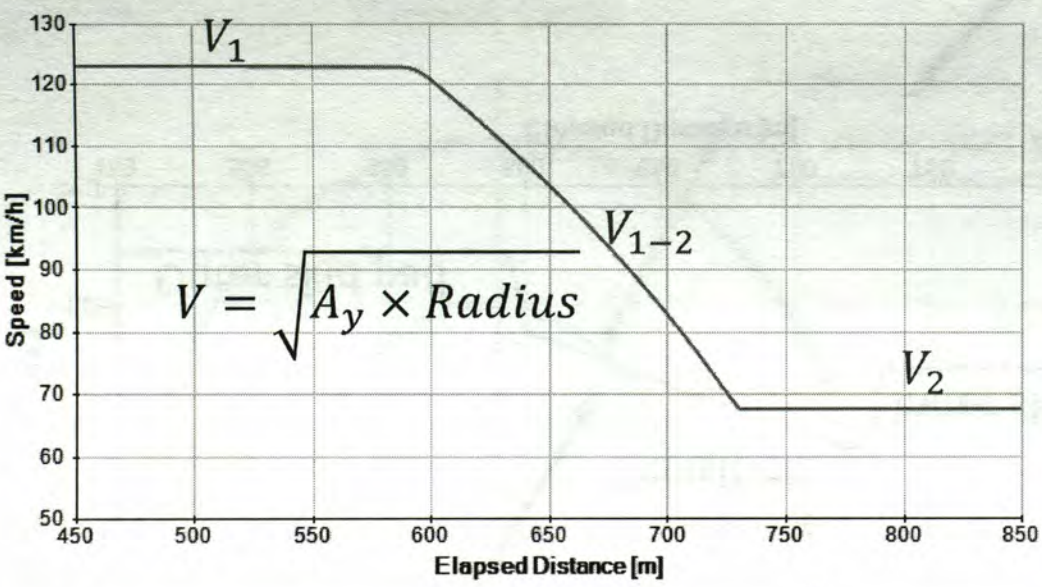
$$\dot{r} = \frac{dr}{dt} = \frac{\Delta r}{\Delta t}$$

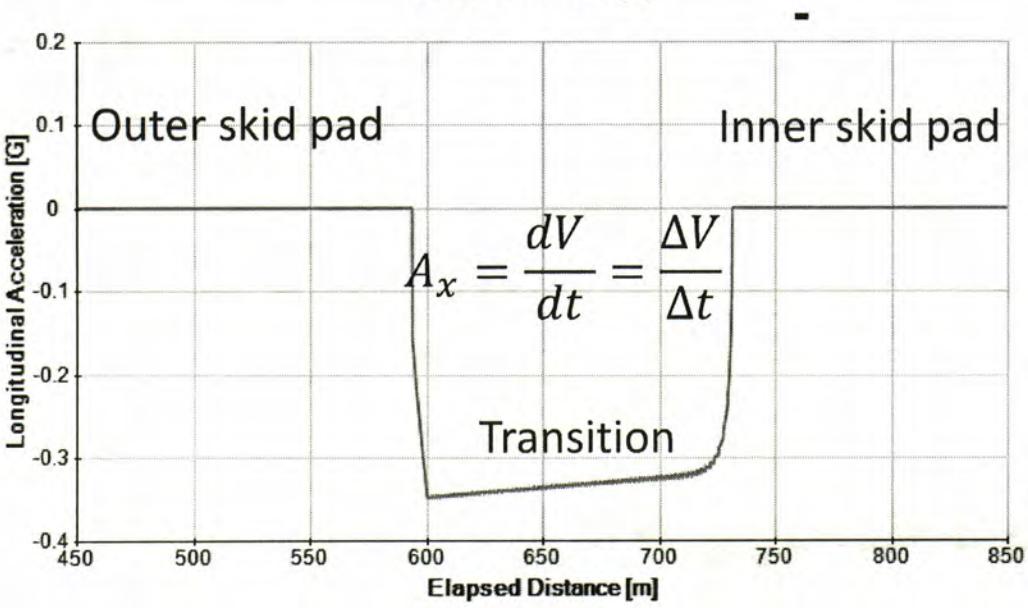
$$Yaw\ Moment = \dot{r} * I_{zz}$$

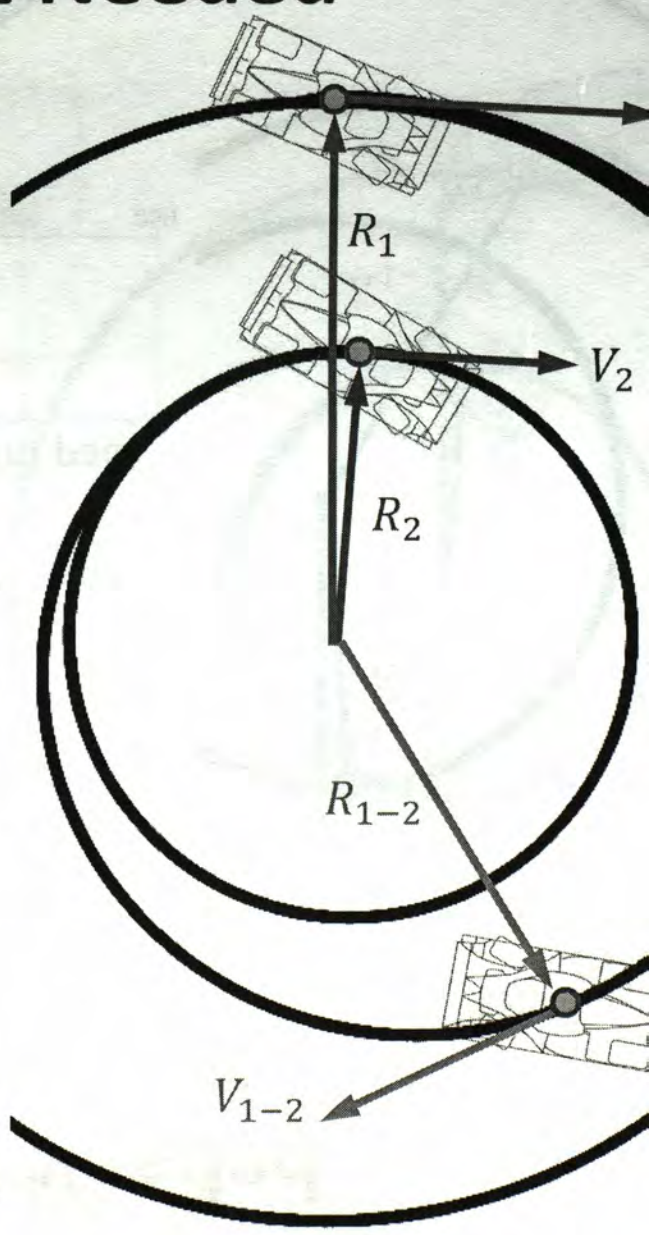
Go from the larger skid pad to the smaller skid pad as fast as possible without over or understeer

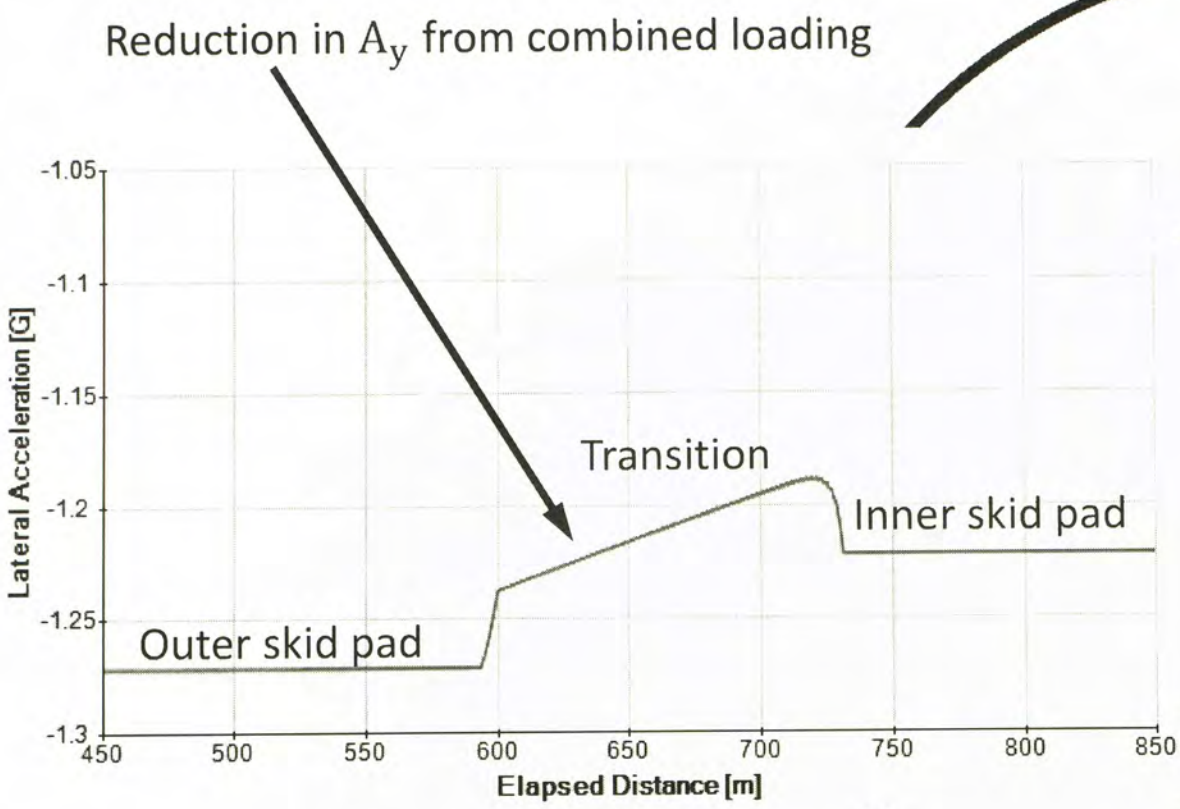


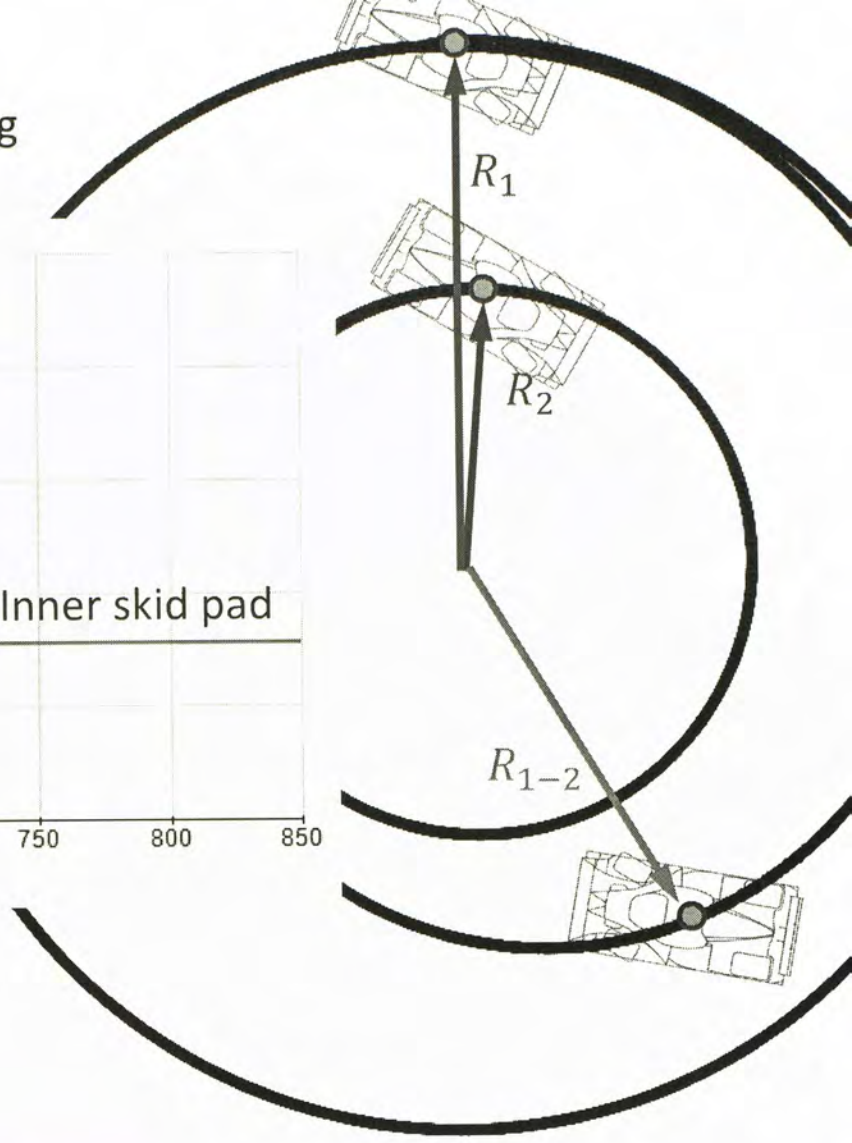
Yaw Moment Needed



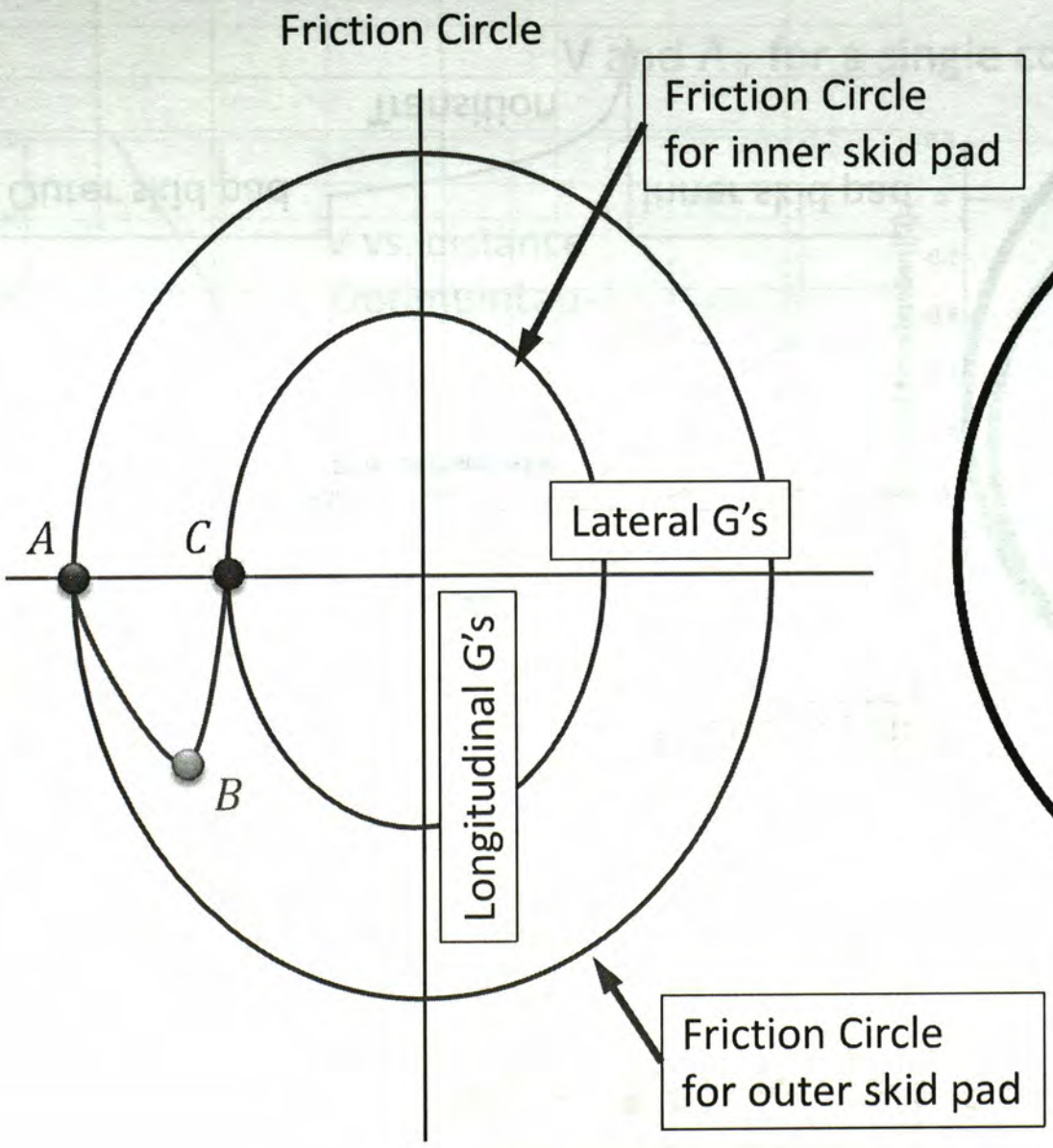


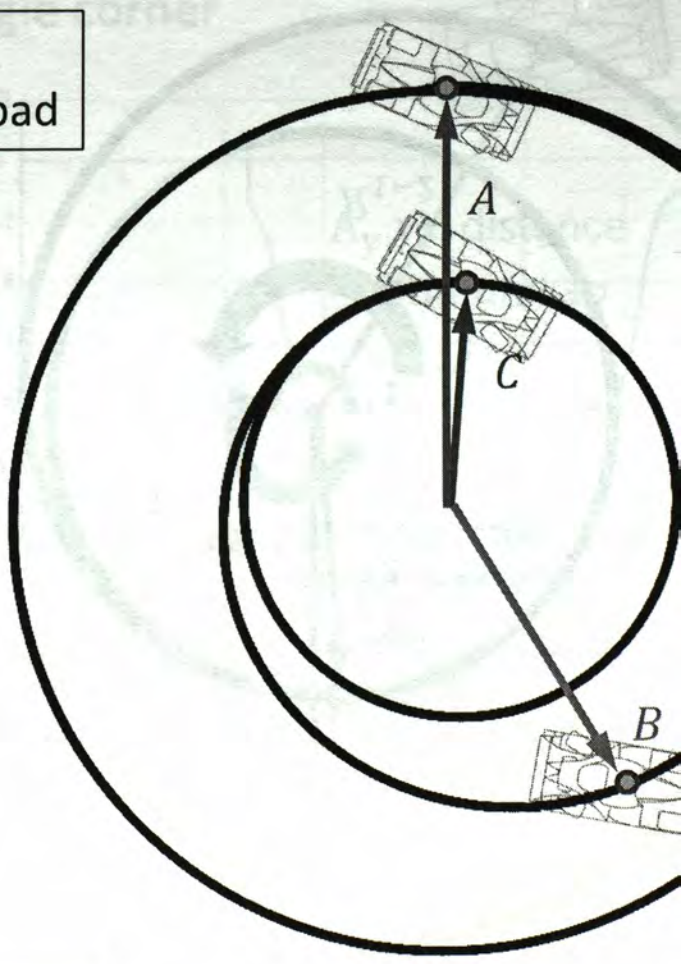


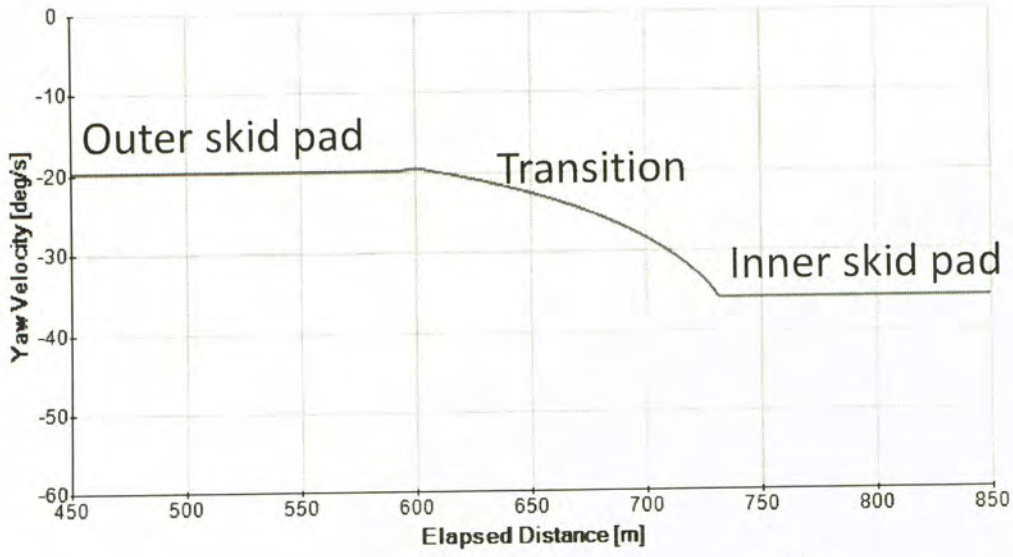


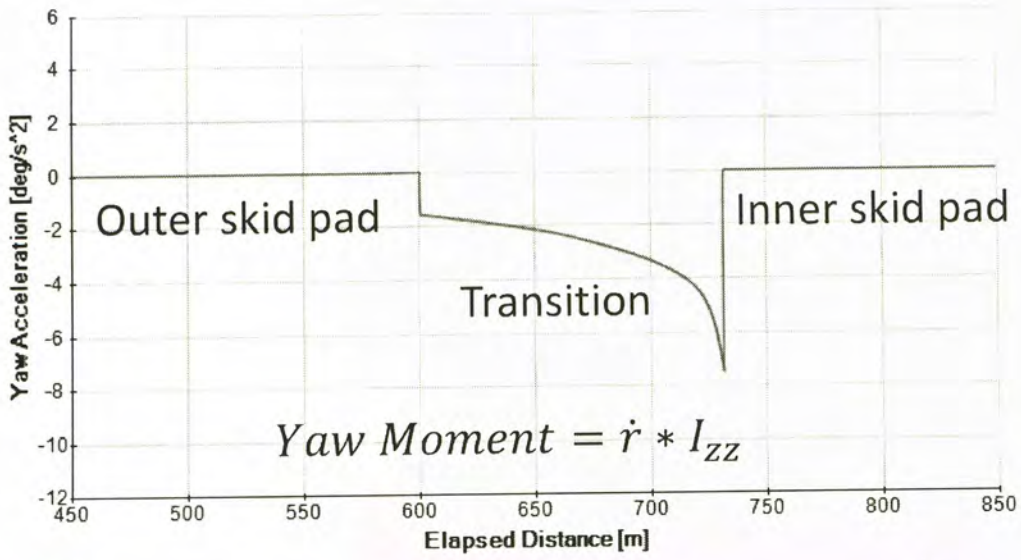


Yaw Moment Needed

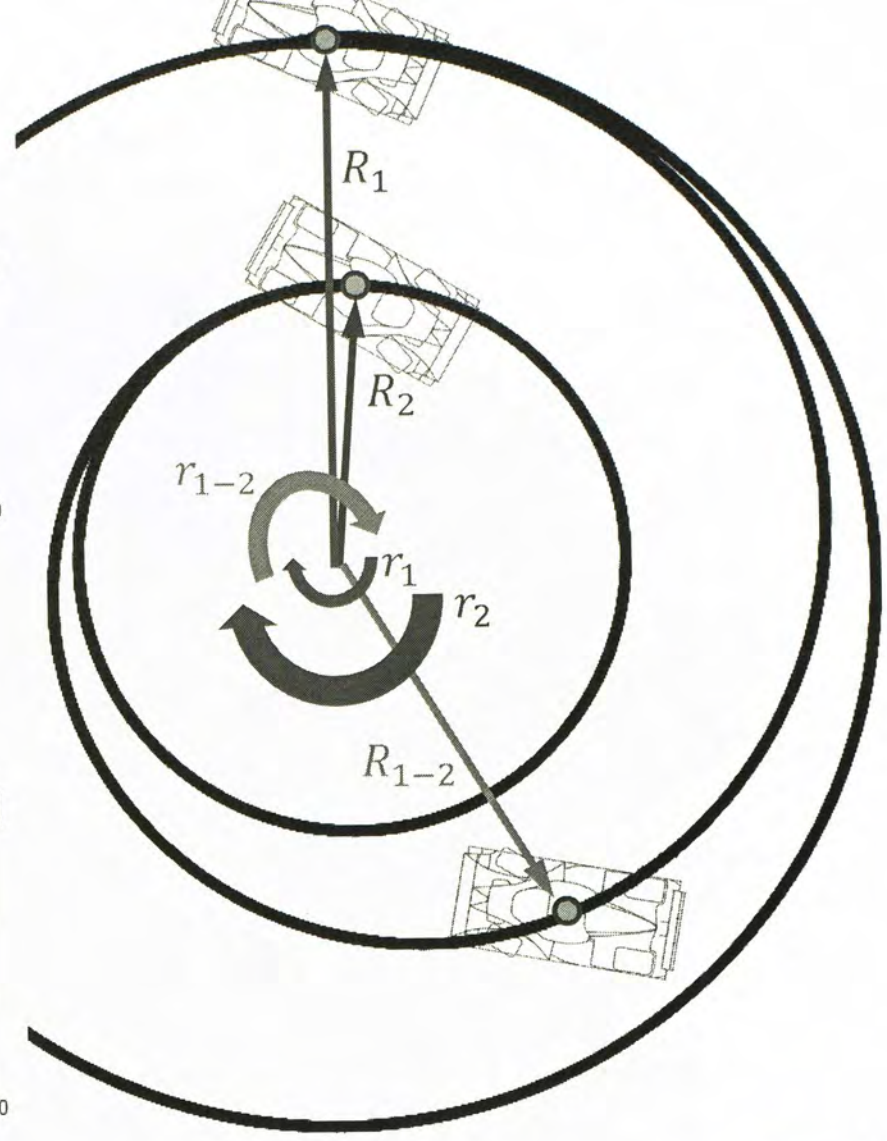






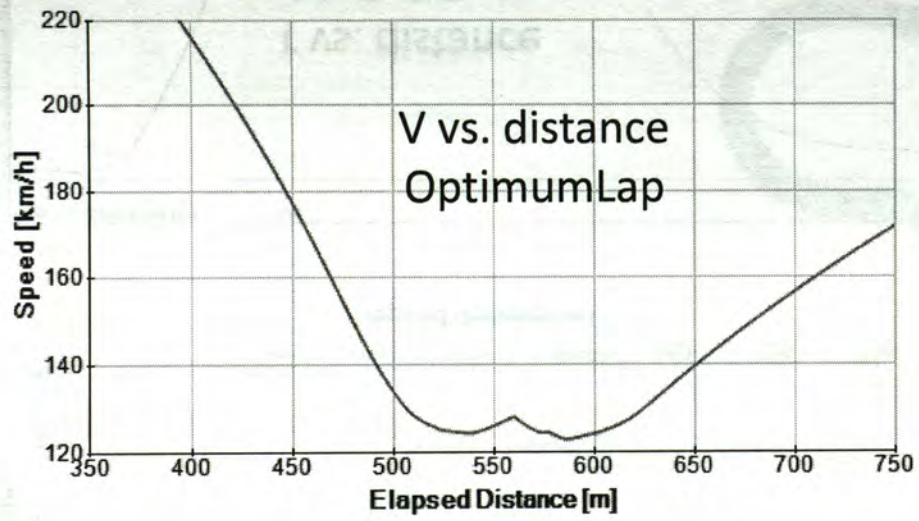


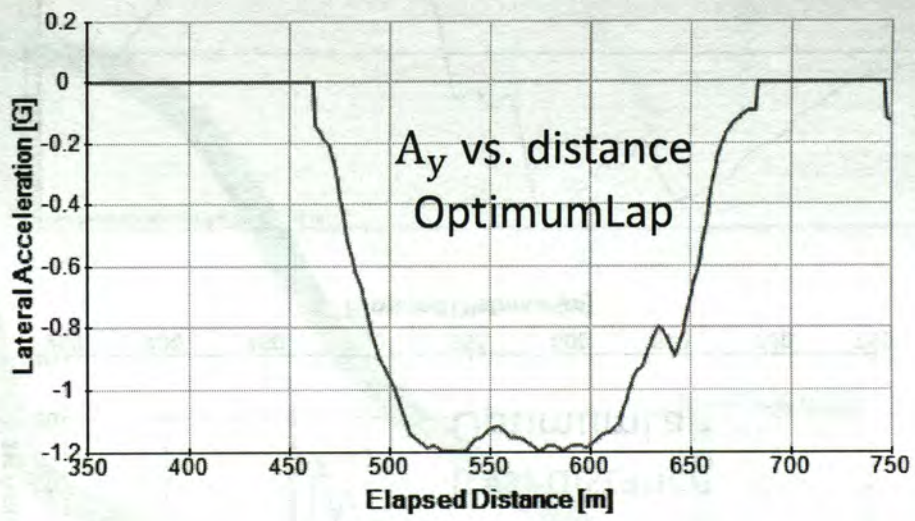
$$Yaw\ Moment = \dot{r} * I_{zz}$$

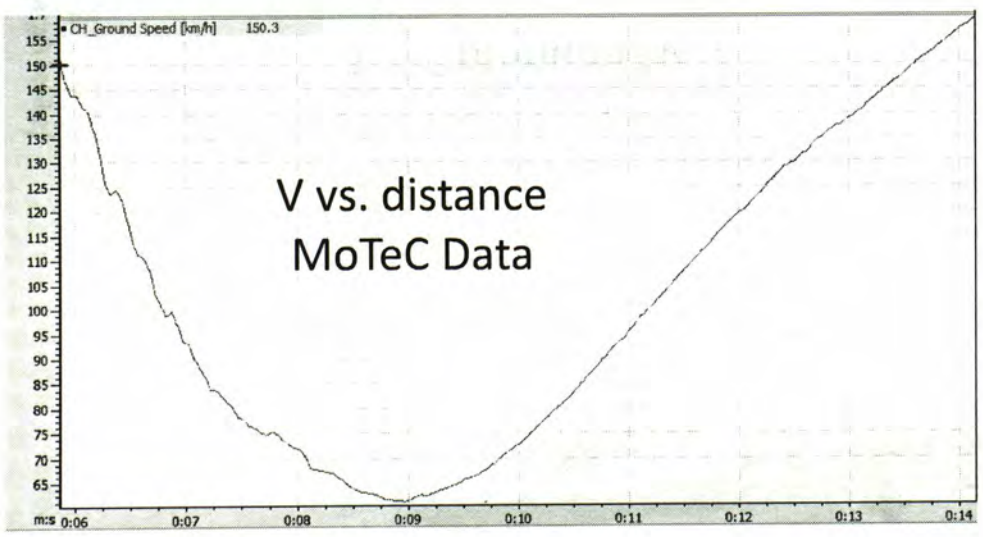


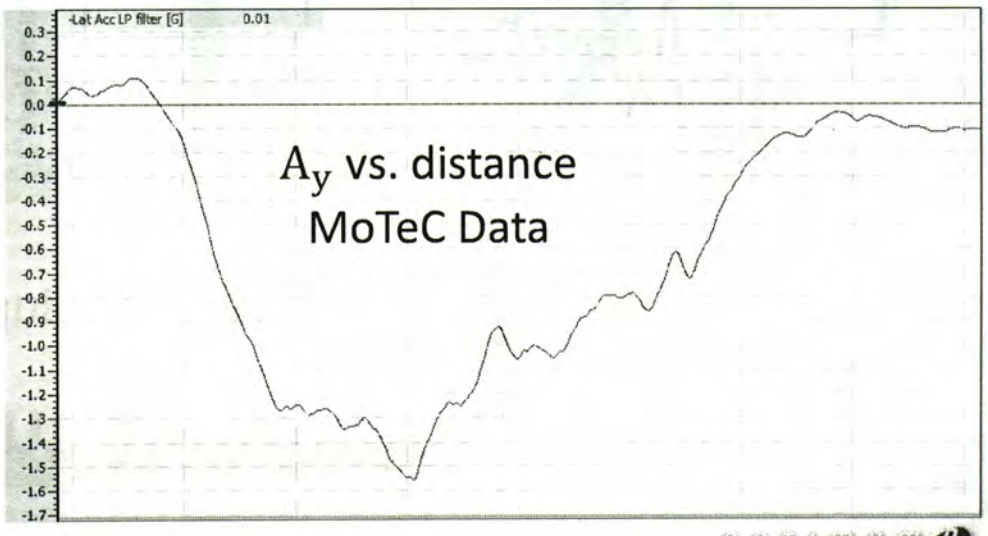
Yaw Moment Needed

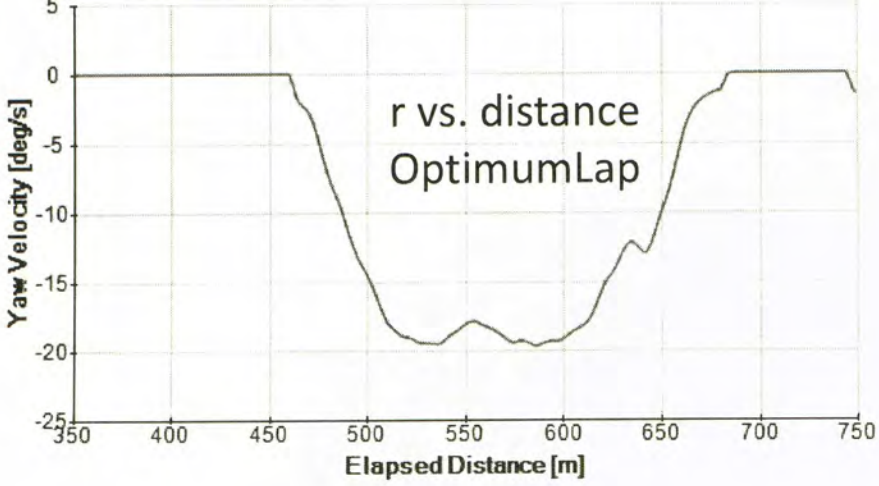
V and A_y for a single corner

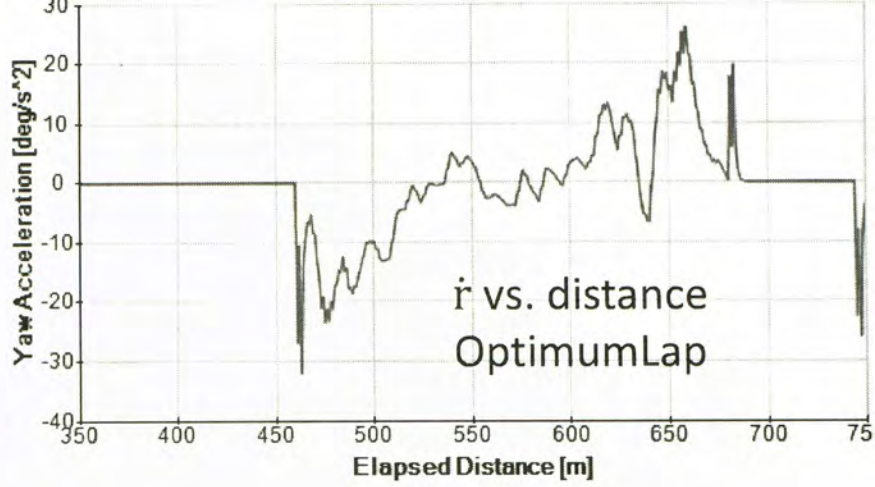


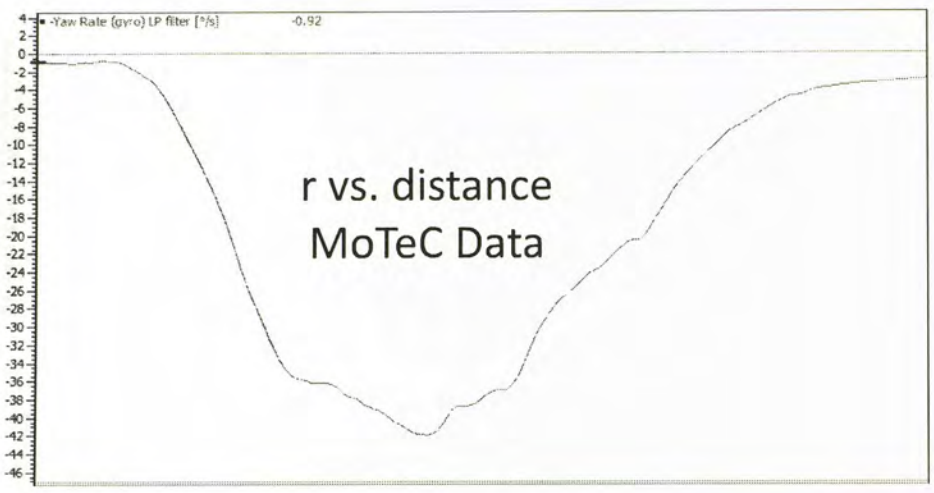


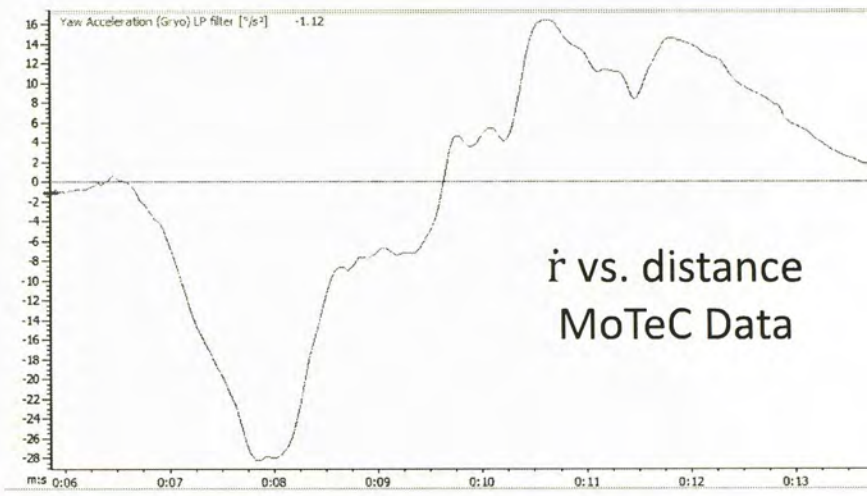








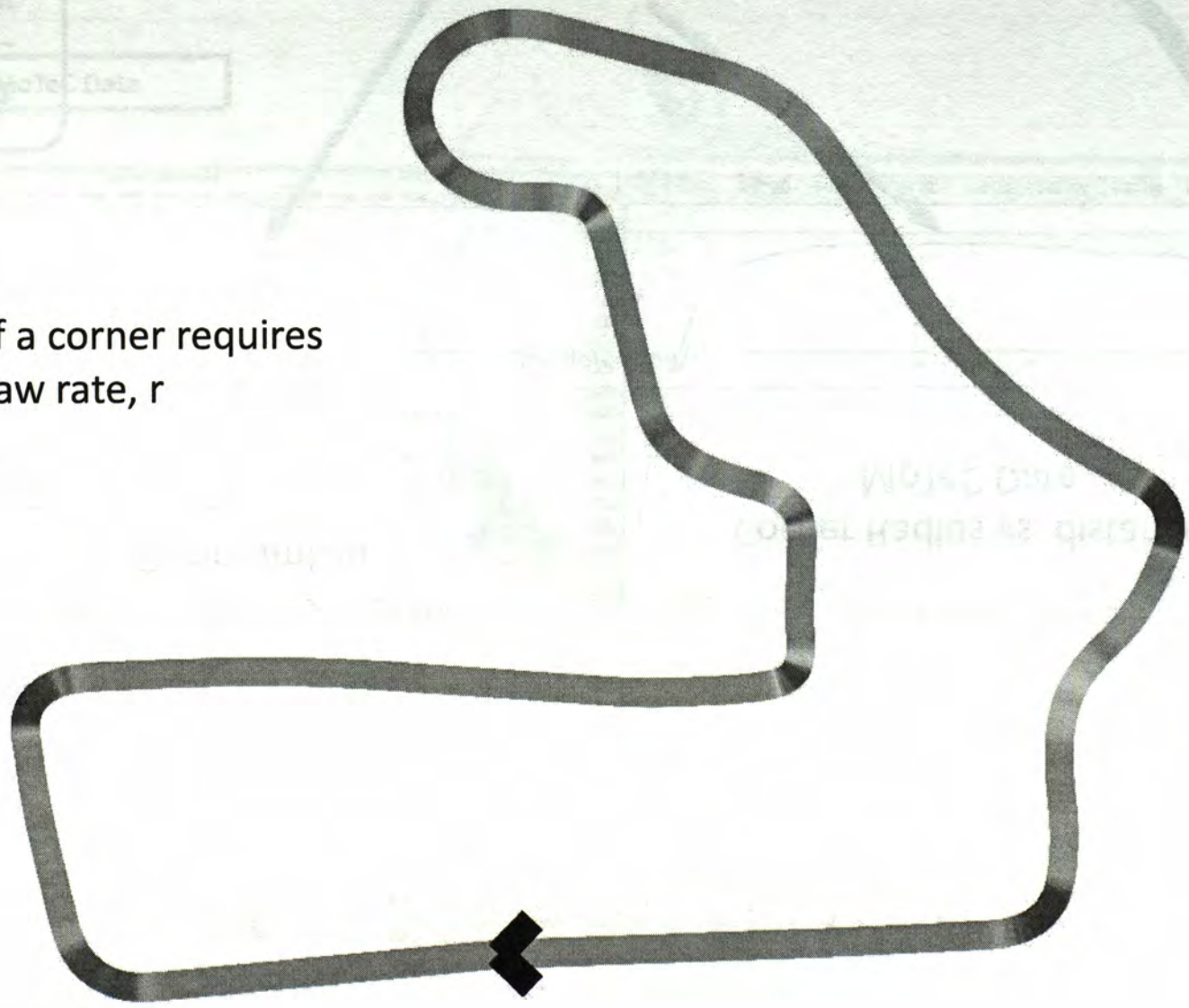




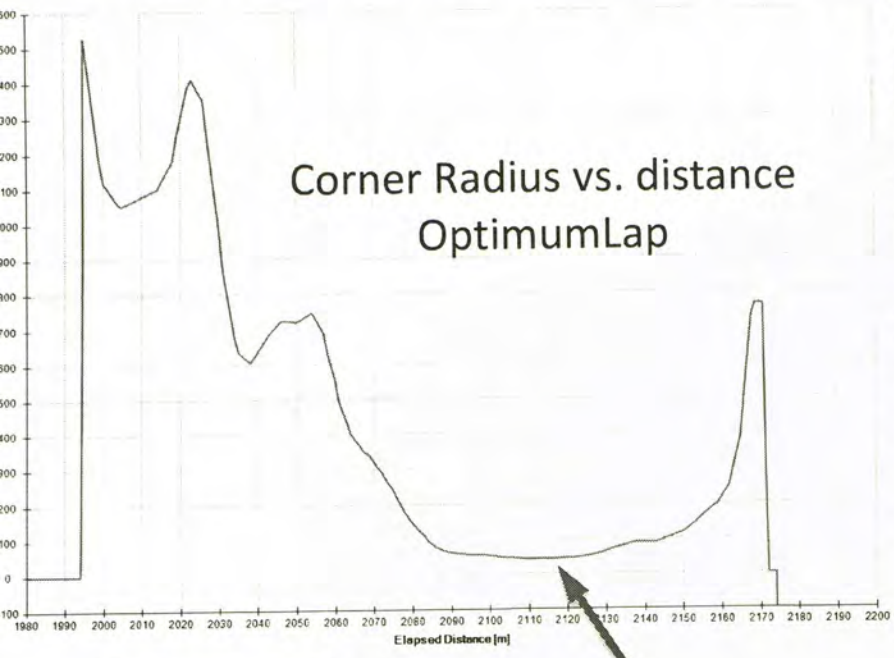
Yaw Moment Needed

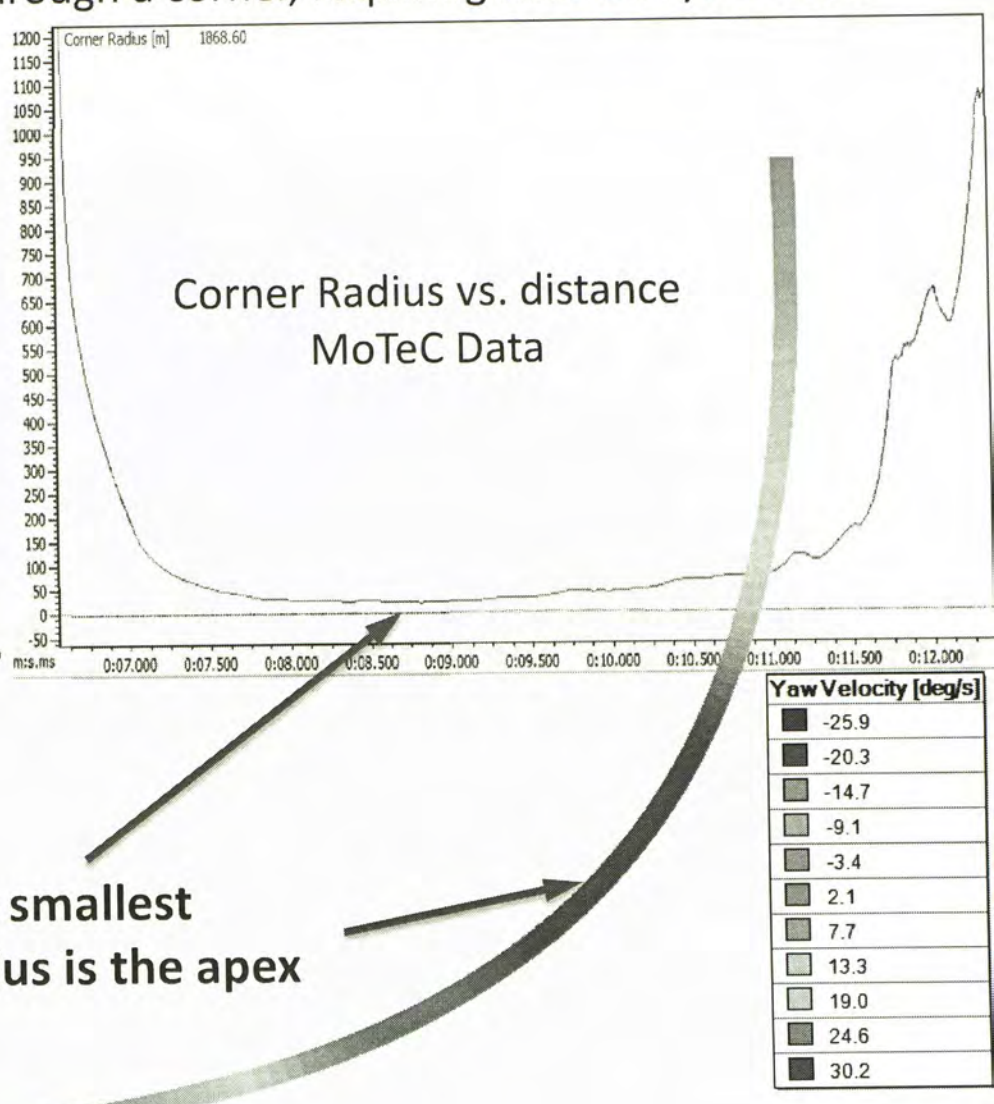
$$r = \frac{Lat\ Accel}{Speed} = \frac{Speed^2/Radius}{Speed} = \frac{Speed}{Radius}$$

Every part of a corner requires a different yaw rate, r



A car will travel a path of varying radius through a corner, requiring different yaw rates



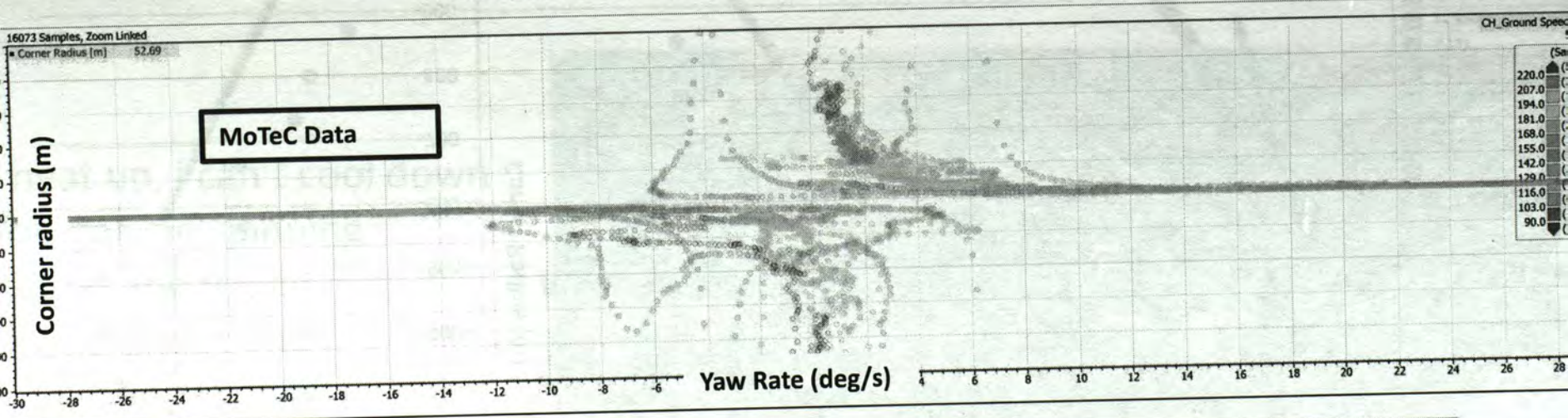


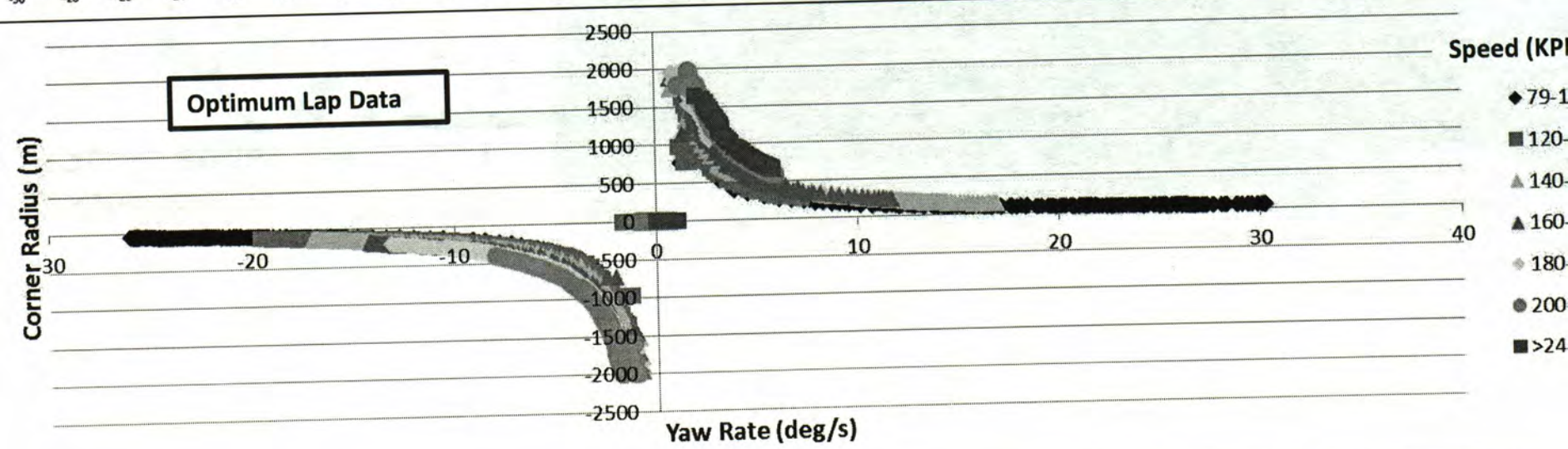
$$R = \frac{V^2}{A_y}$$

The smallest radius is the apex

Yaw Moment Needed

Smaller radiuses require higher yaw rates



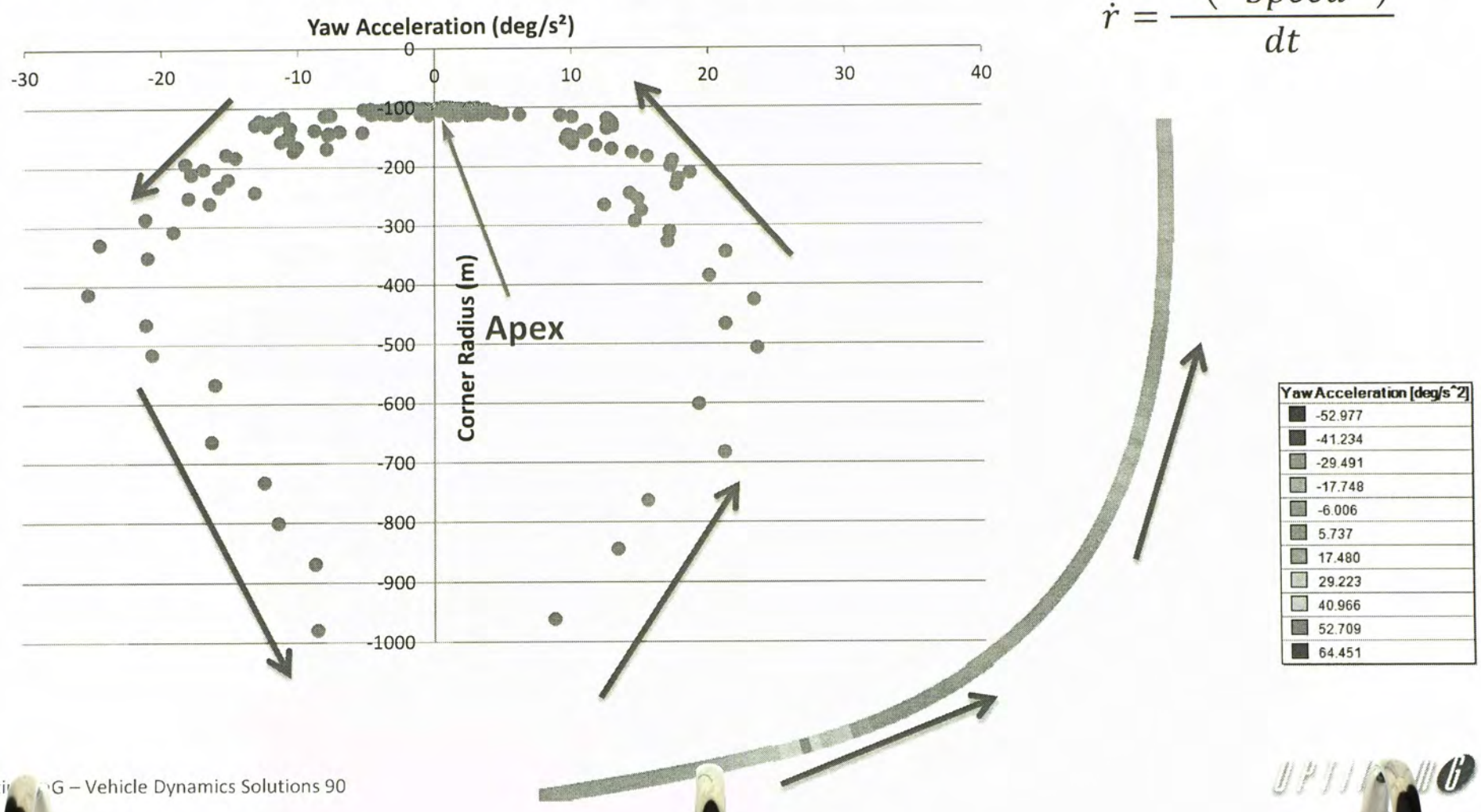


To change a yaw rate you must have yaw acceleration!

Every part of a corner requires a different yaw rate
Therefore, a different yaw acceleration is also needed

$$Yaw\ Moment = (Yaw\ acceleration) * I_{zz}$$

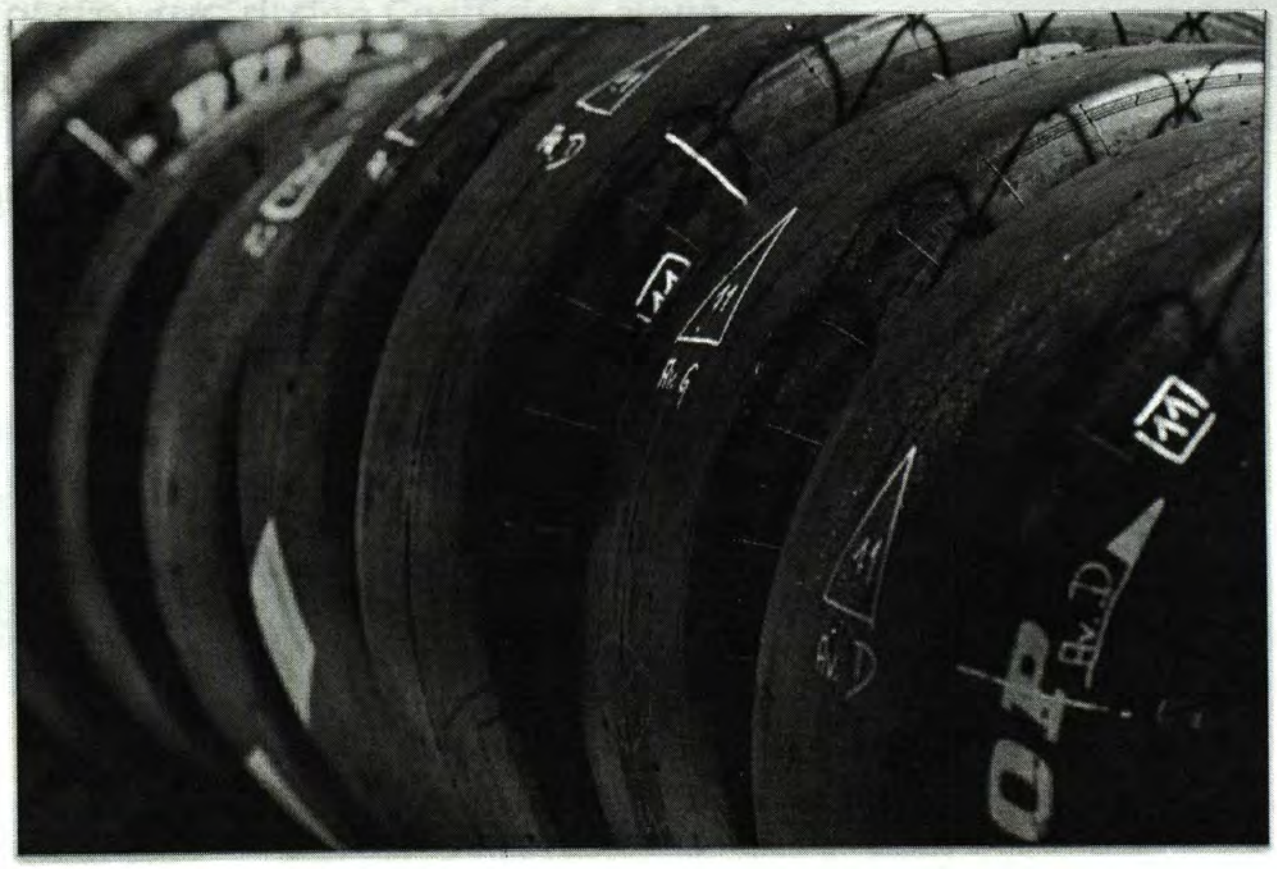
$$\dot{r} = \frac{d\left(\frac{Lat\ Accel}{Speed}\right)}{dt}$$

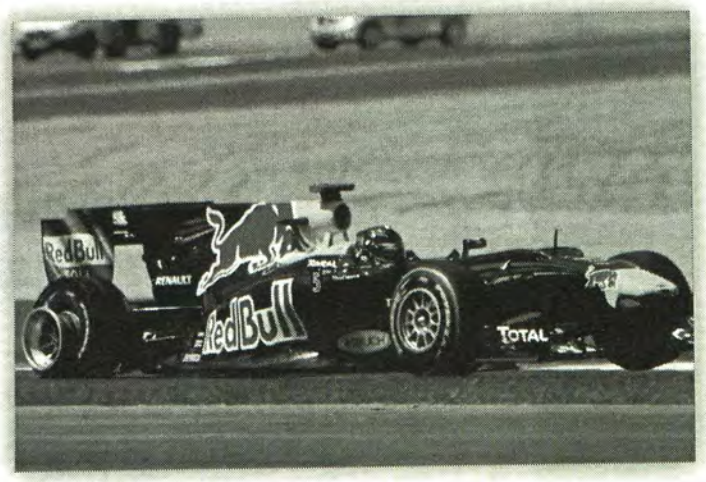


2. Tires

I heat up, I can't cool down
You got me spinning
Round and round
Round and round
And round it goes
Where it stops nobody knows

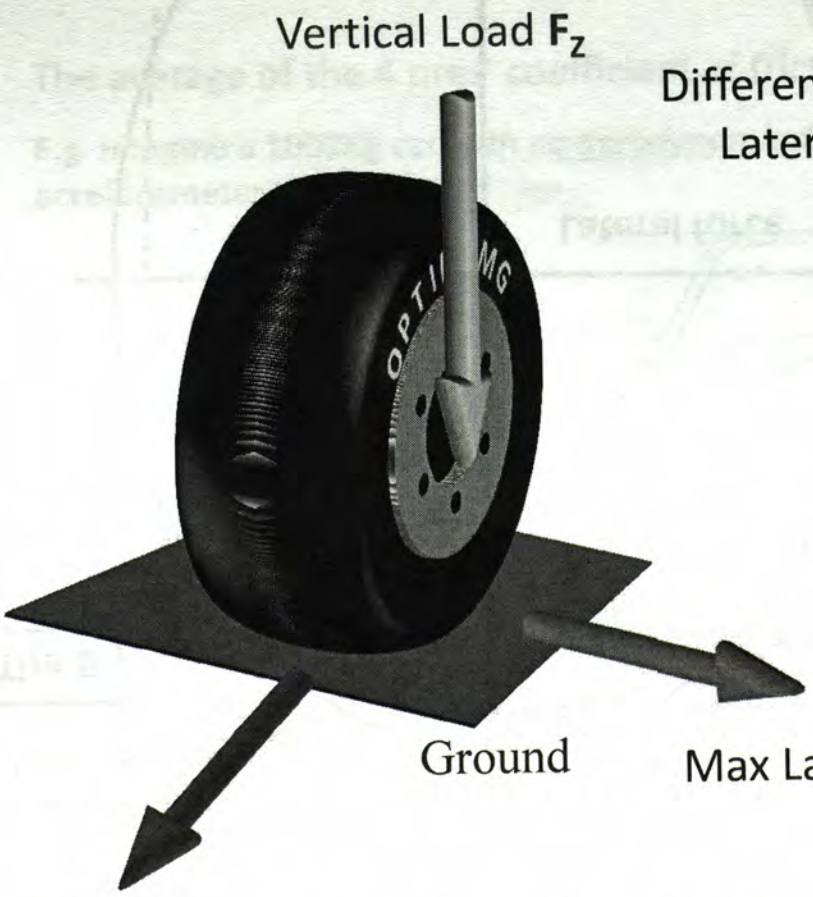
Steve Miller
Abracadabra

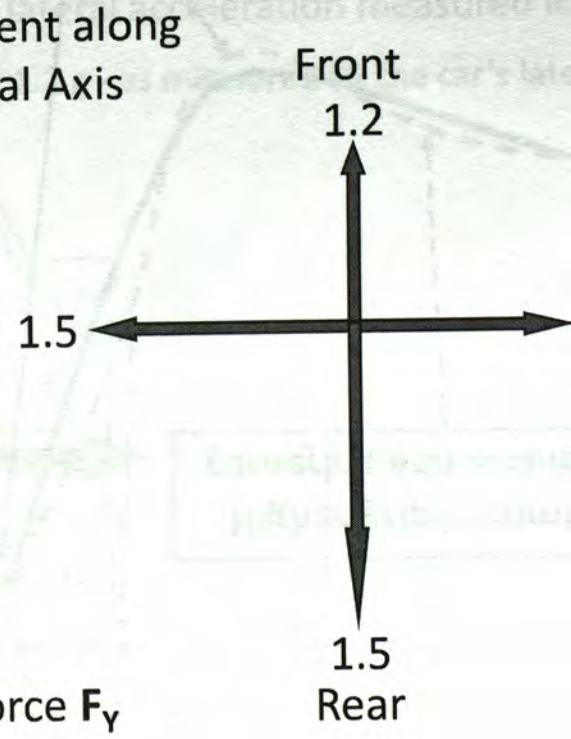




- The tire is the most important component of the vehicle
- It is the only link with the earth (hopefully!)
- All other sections suspension, aerodynamics, drive train, are there maximize the use of the tire
- Gives the driver his/her only real feedback

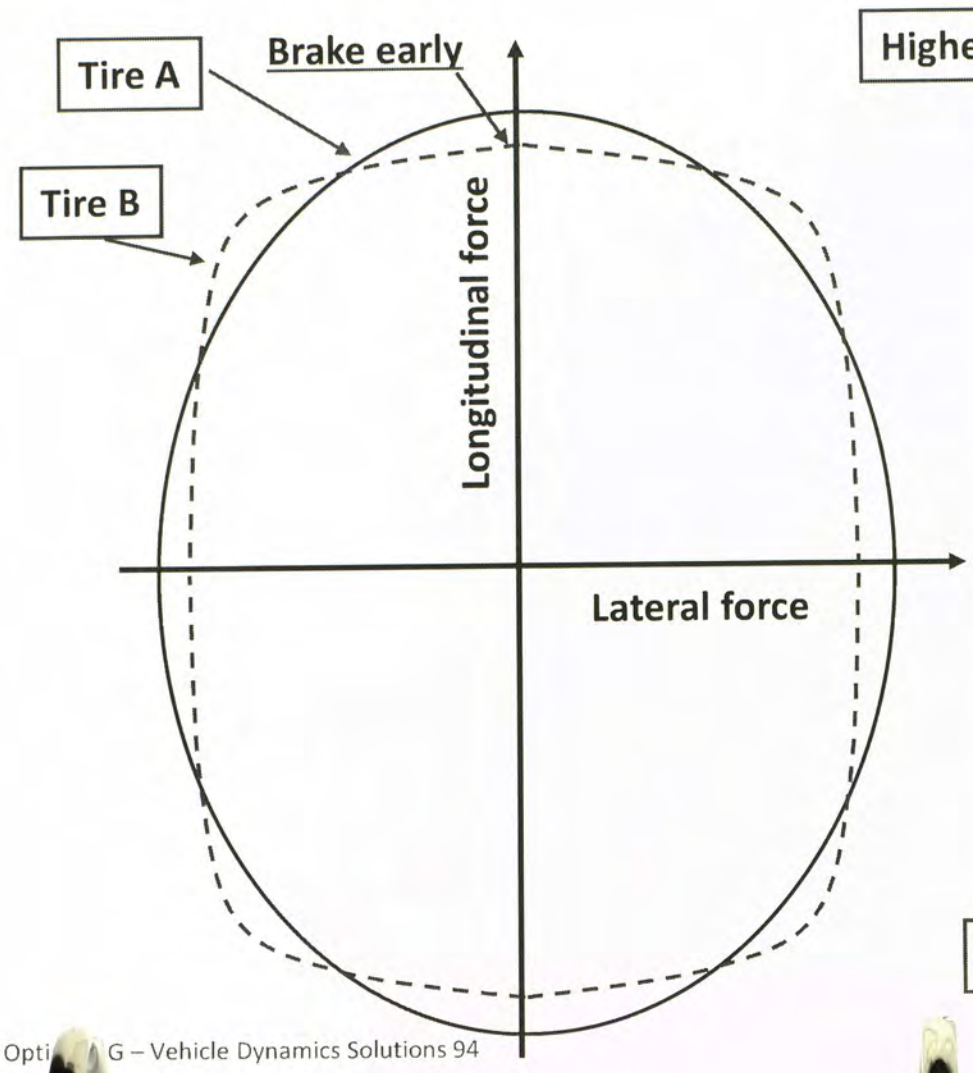
Friction Coefficient Definitions

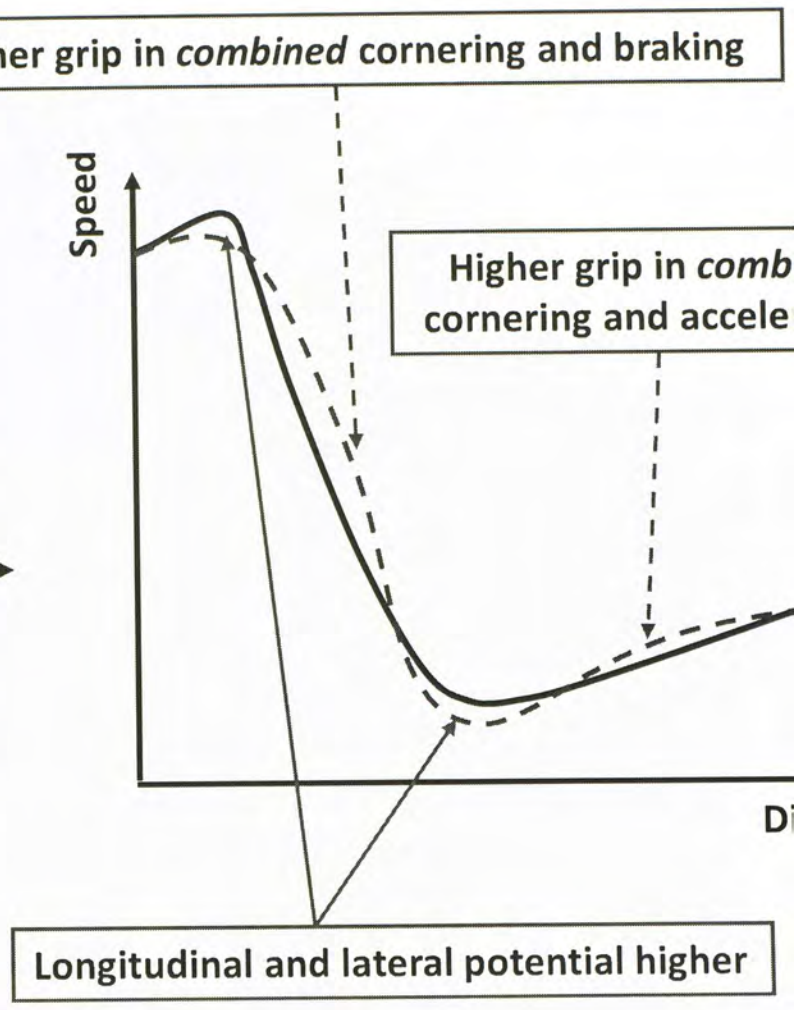




$$\mu_X = \frac{F_X}{F_Z} \qquad \mu_Y = \frac{F_Y}{F_Z}$$

Comparing Different Tires Using Friction Ellipses

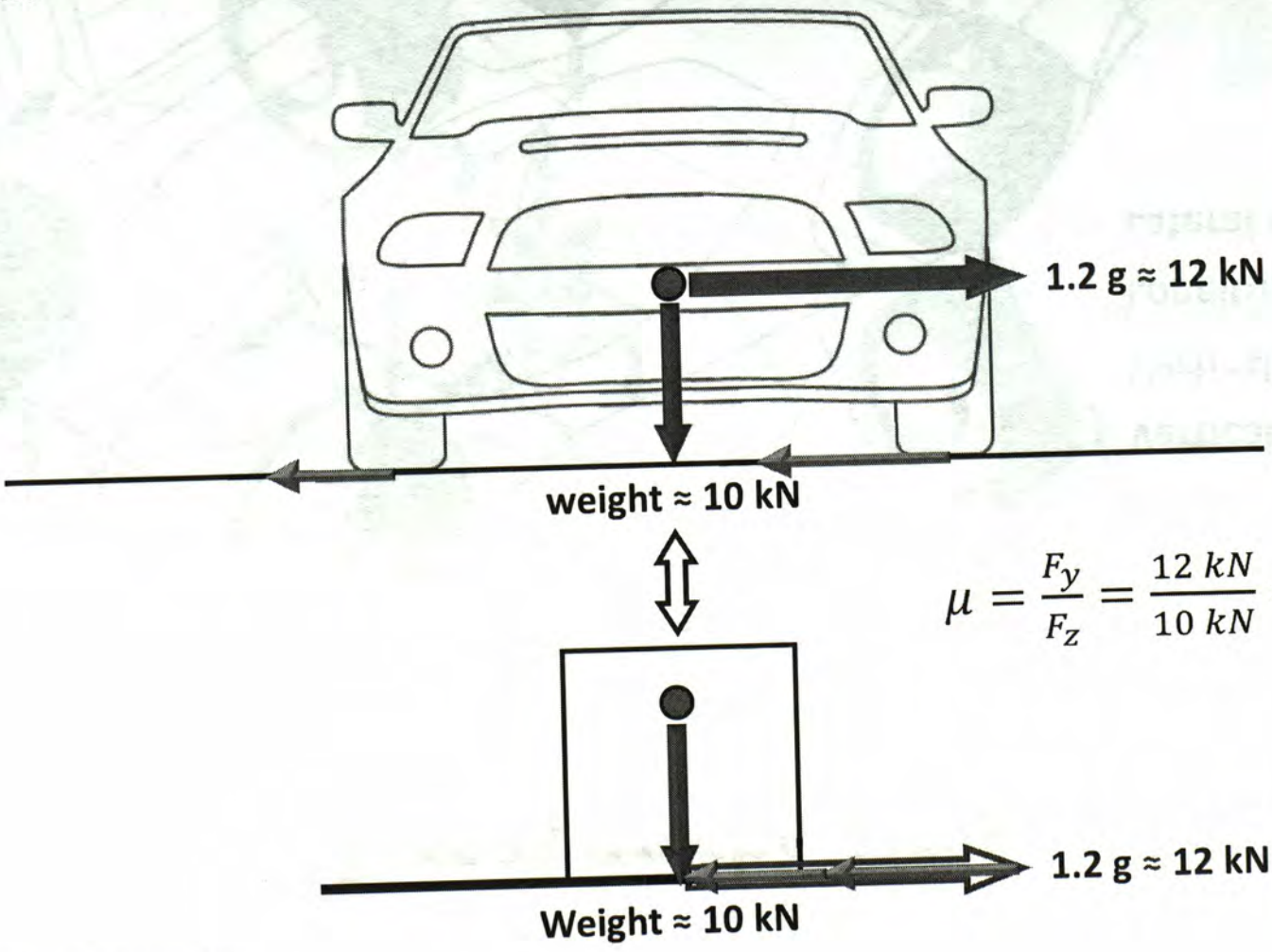




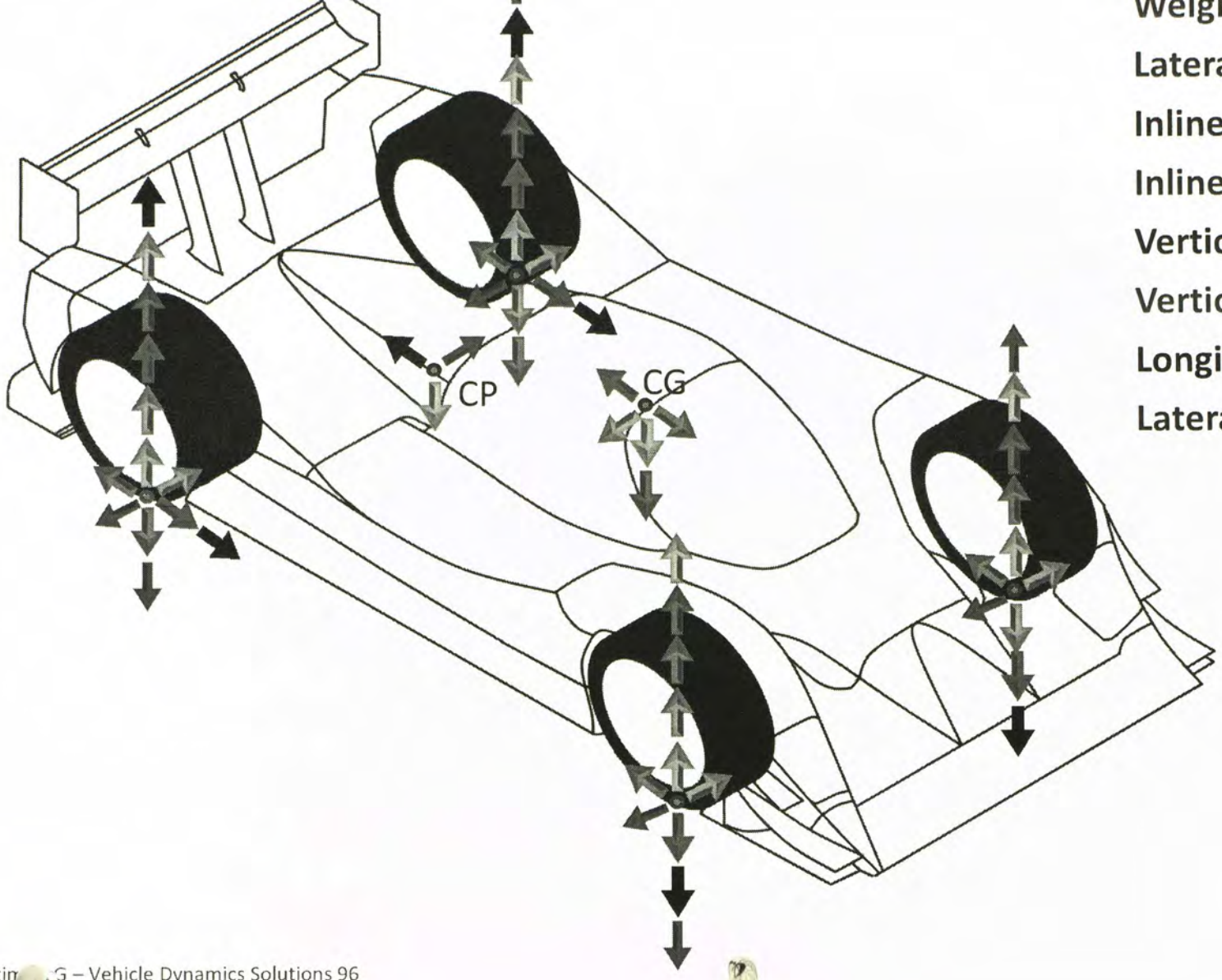
The Tire Coefficient of Friction

The average of the 4 tires' coefficient of friction equals the car's lateral acceleration measured

E.g. Imagine a 1000Kg car *with no aerodynamic downforce*, cornering at 1.2 g - as measured by the car's la accelerometer:



$$\mu = \frac{F_y}{F_z} = \frac{12\ kN}{10\ kN} = 1.2$$



Weight

Lateral G

Inline G (Braking)

Inline G (Acceleration)

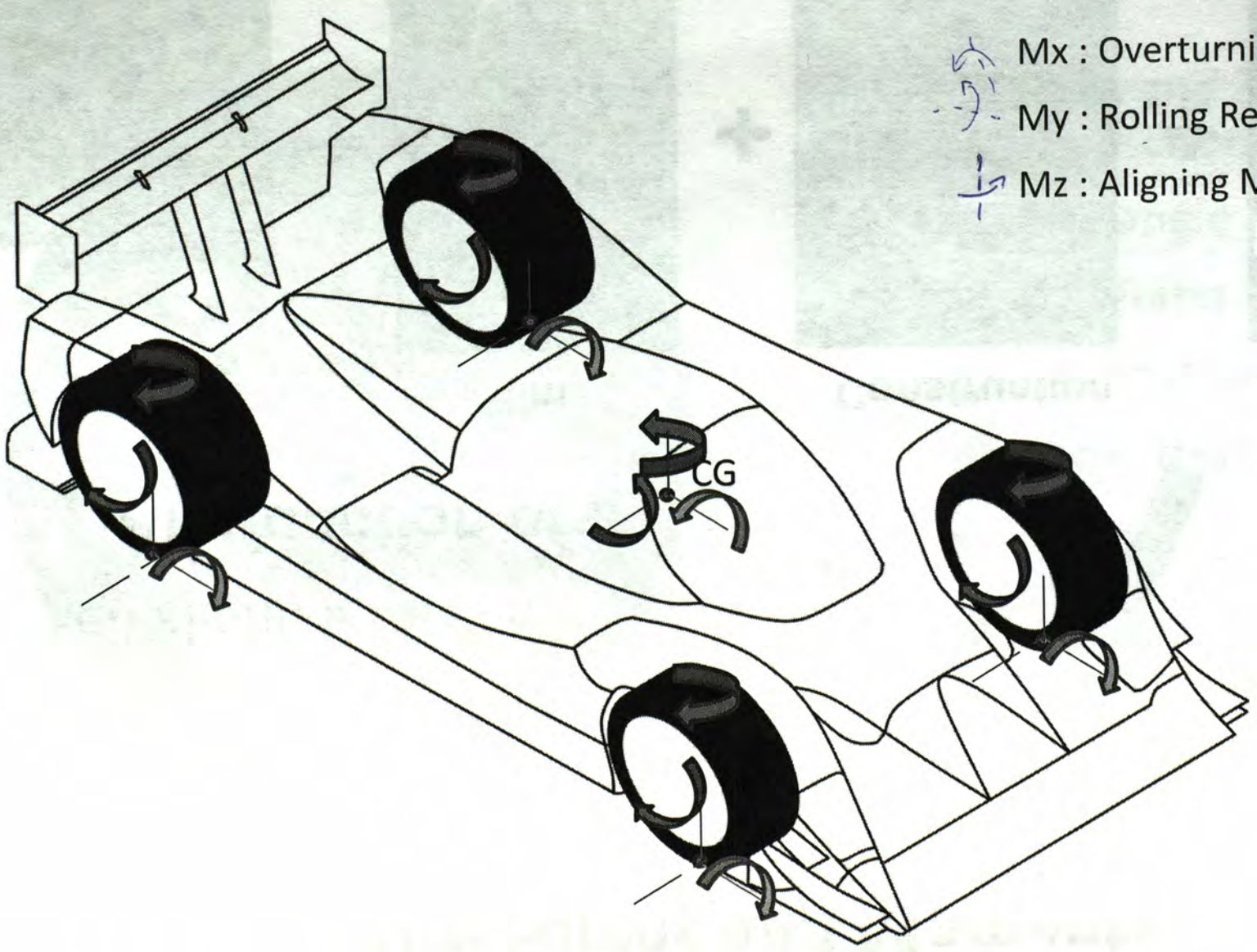
Vertical G

Vertical Aero

Longitudinal Aero

Lateral Aero

Moments on the Tires



Mx : Overturning Moment

My : Rolling Resistance

Mz : Aligning Moment

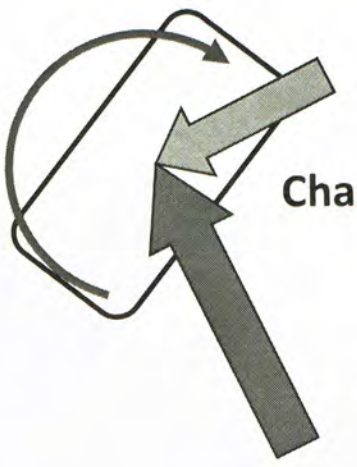
Tire Forces and Moments

Lateral force

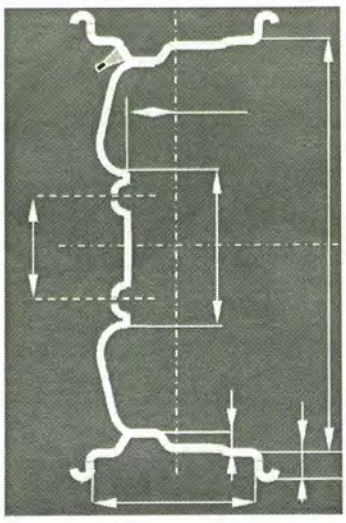
Longitudinal force

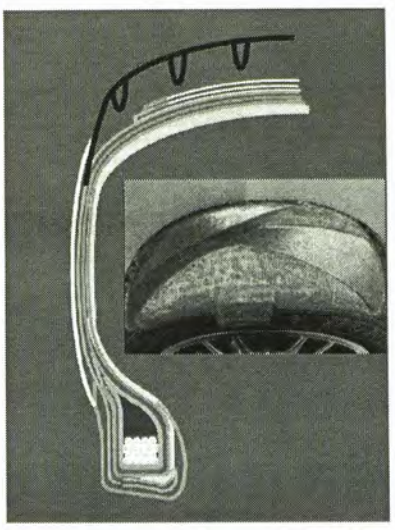
Self aligning torque

Are function of .....

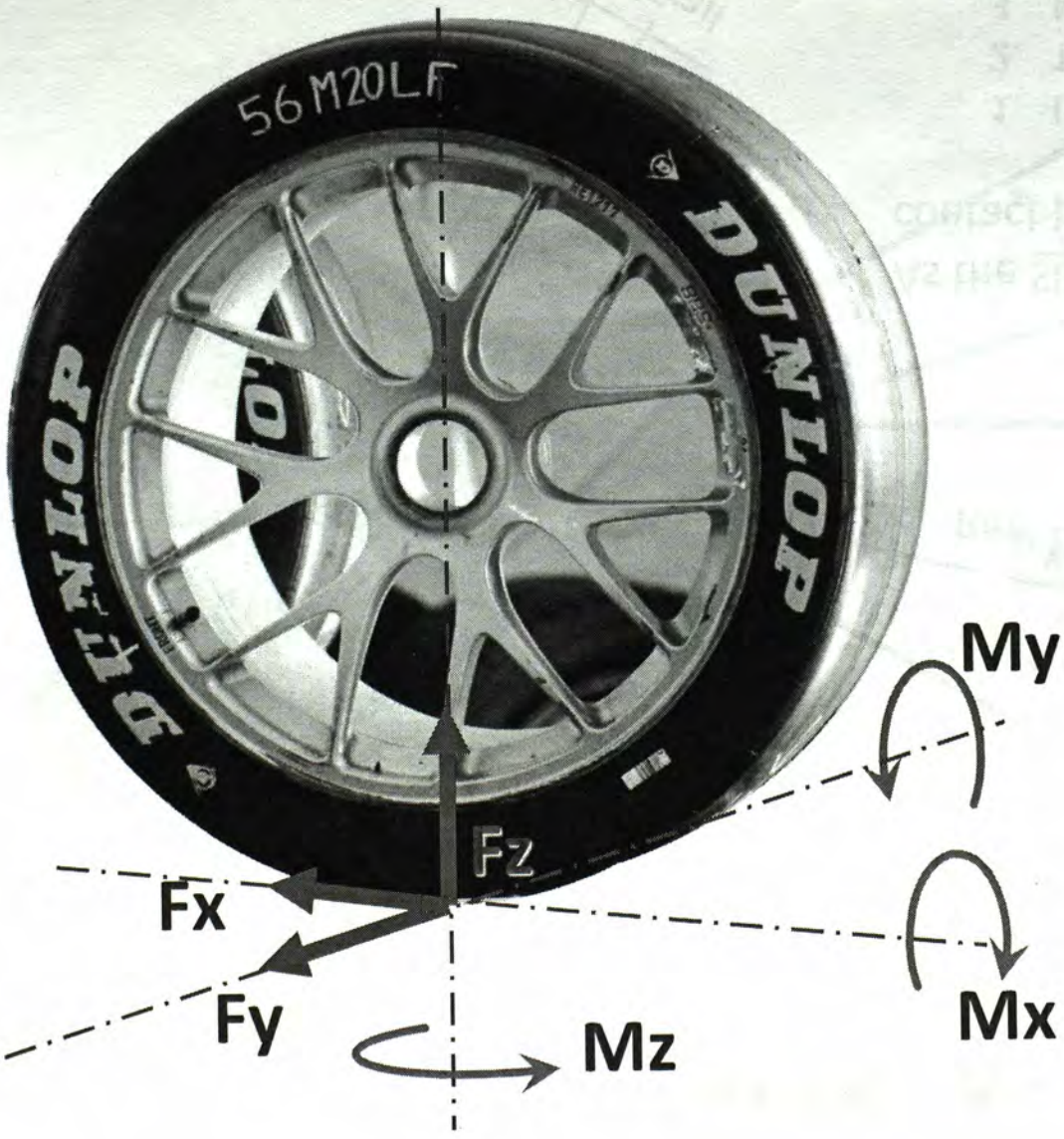


Ground + **Rim** + **Construction** + **Con**





Tire Forces and Moments



2 Forces and 3 M
are a function of:

a) Slip angle

b) Slip Ratio

c) Vertical Load

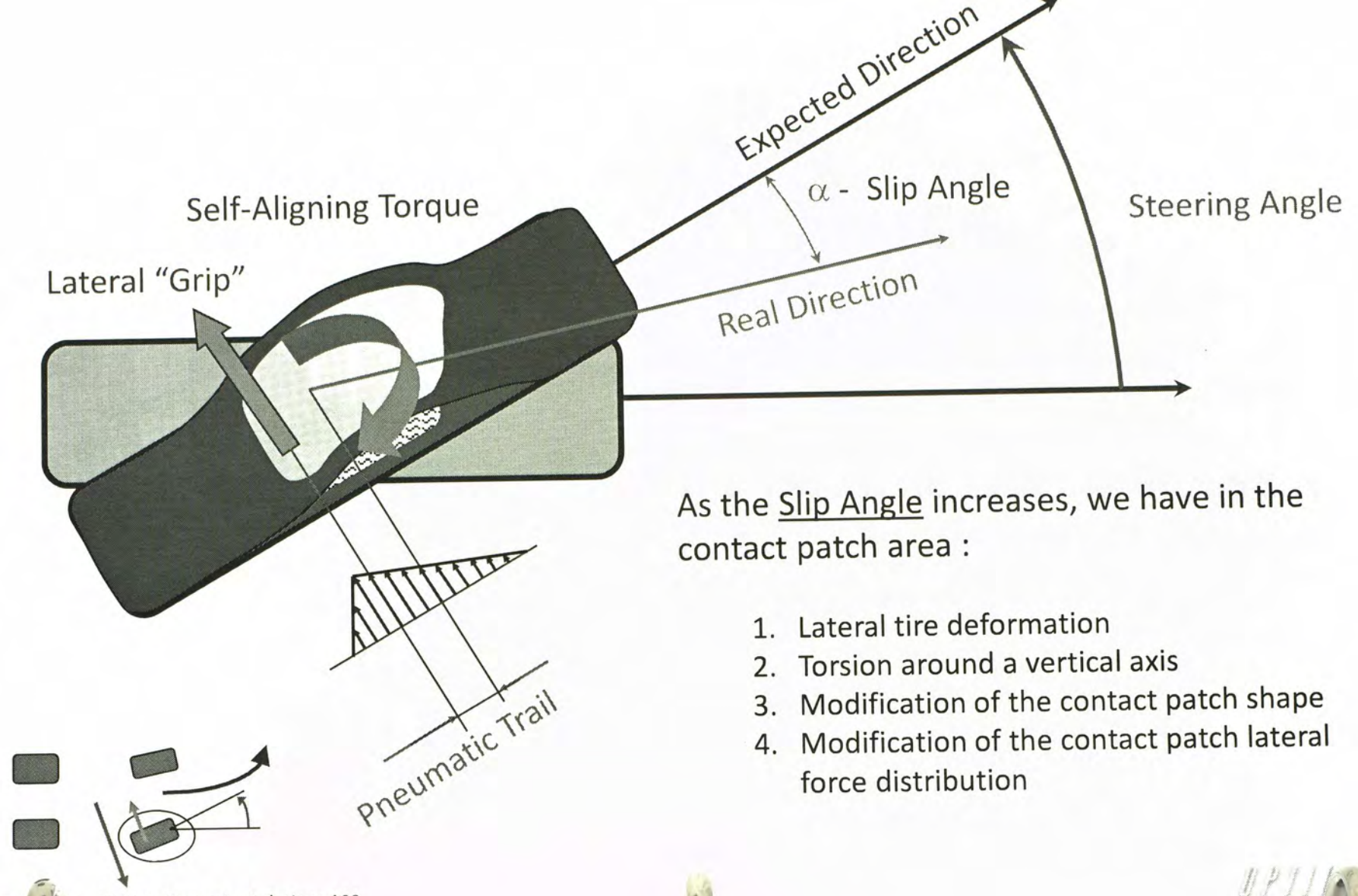
d) Inclination Ar

e) Speed

f) Pressure

g) Temperature

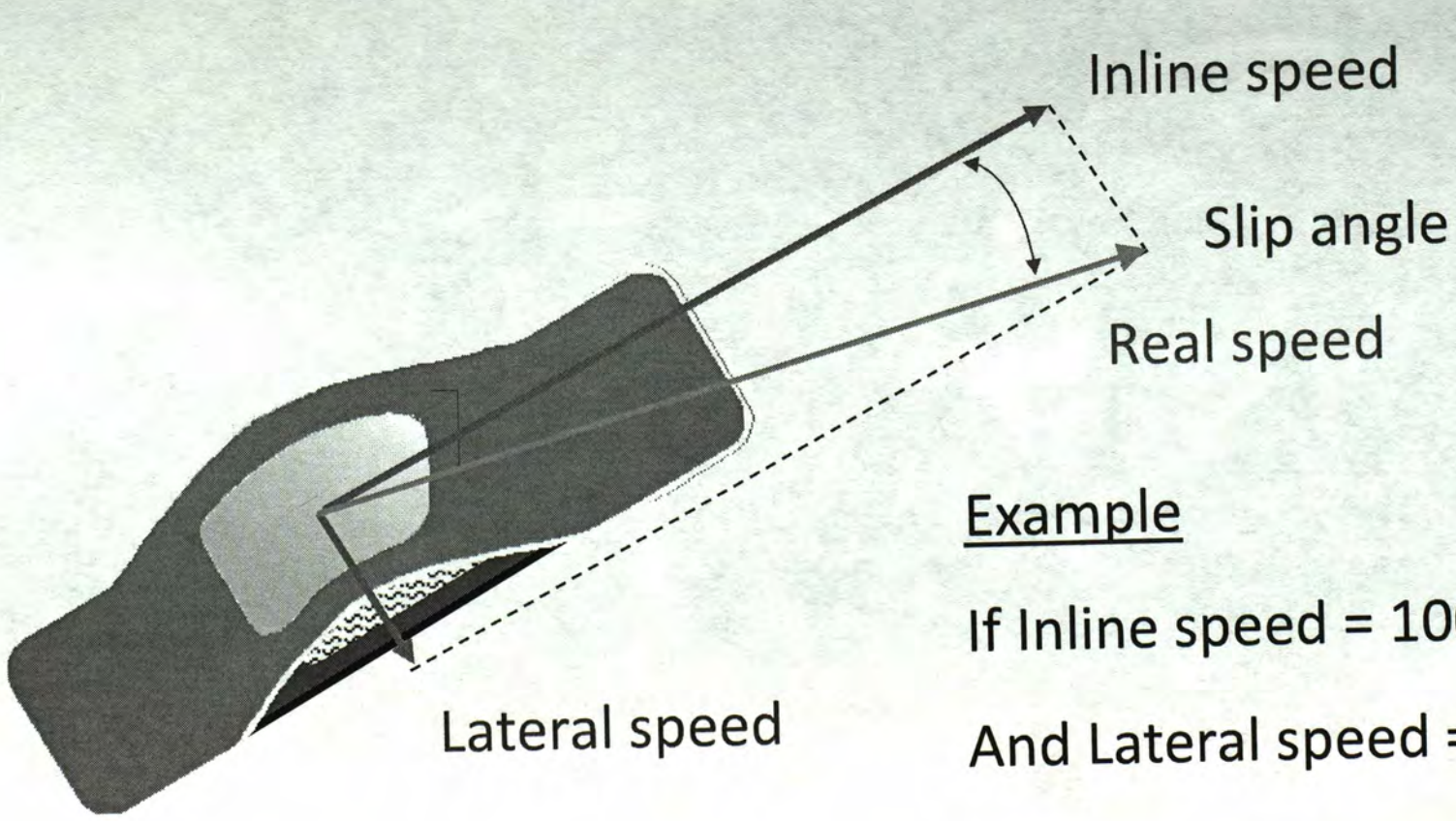
h) Rim Design



As the Slip Angle increases, we have in the contact patch area :

1. Lateral tire deformation
2. Torsion around a vertical axis
3. Modification of the contact patch shape
4. Modification of the contact patch lateral force distribution

Calculating Slip Angle



Example

If Inline speed = 100 km/h

And Lateral speed = 10 km/h

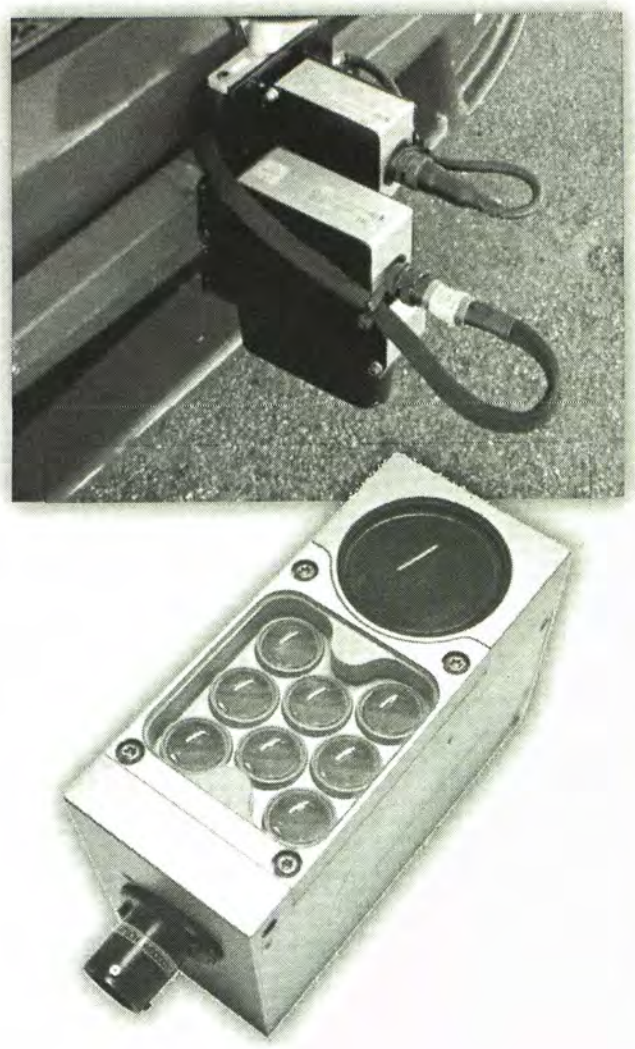
Real Speed

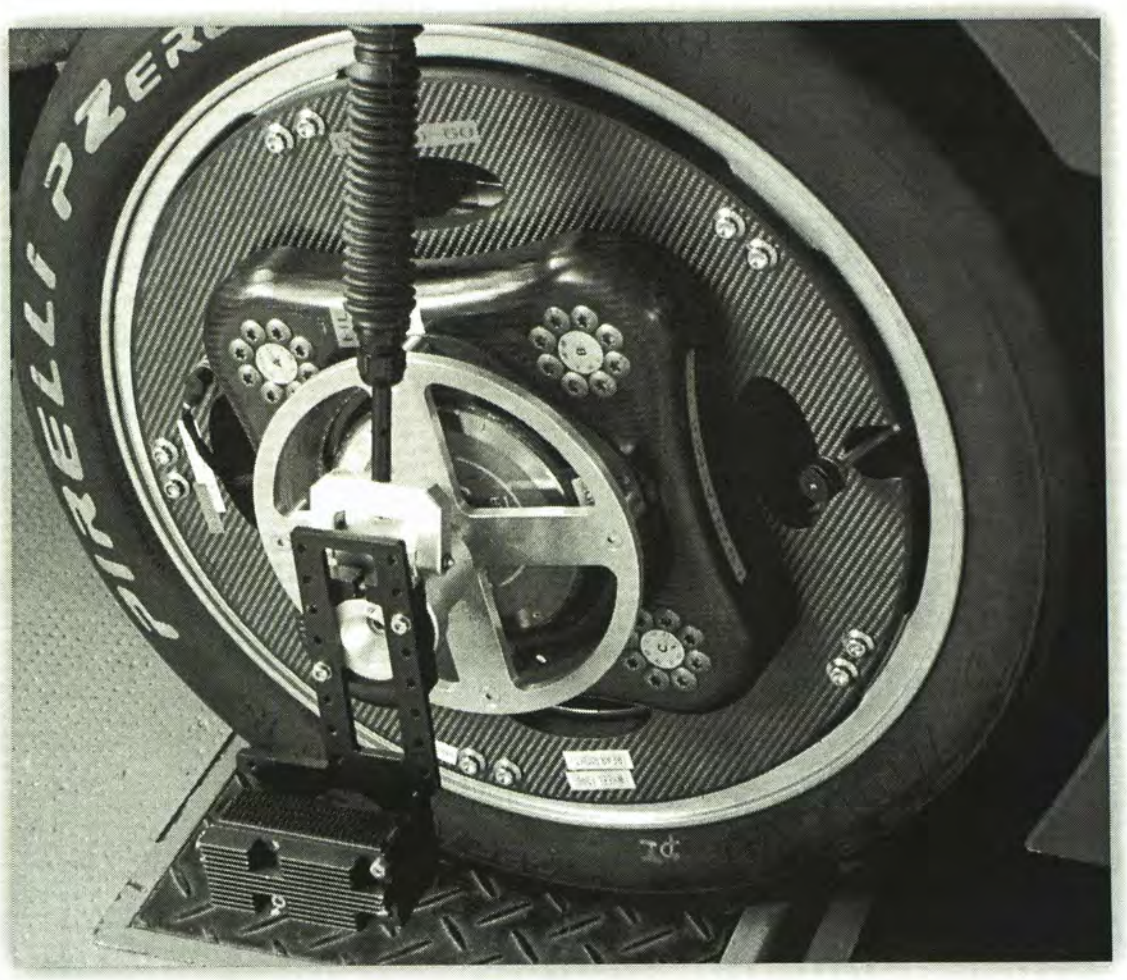
$= \sqrt{10^2 + 100^2} = 100.5$ km/h

Slip angle = arc tan (10/100)

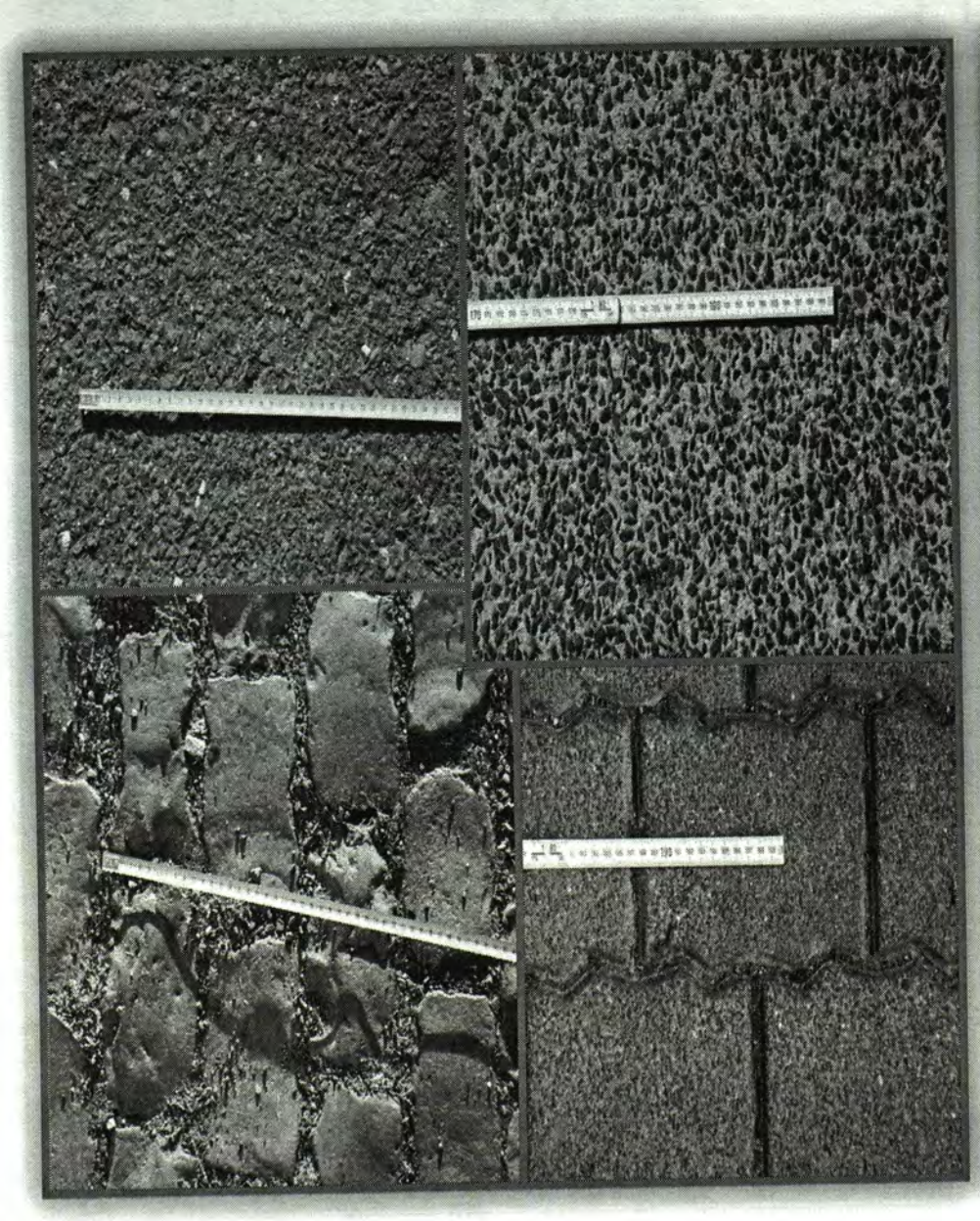
= 5.71 º

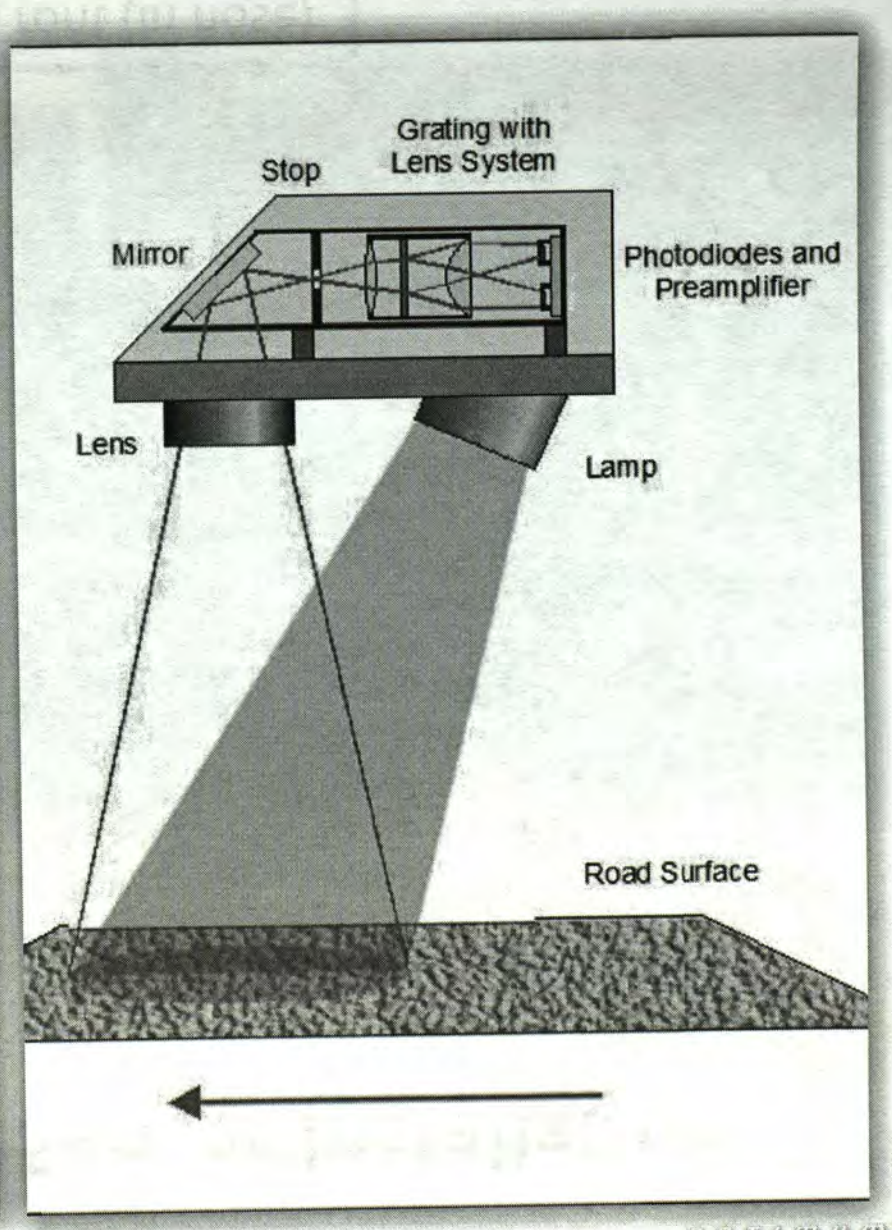
What is a Slip Angle Sensor?



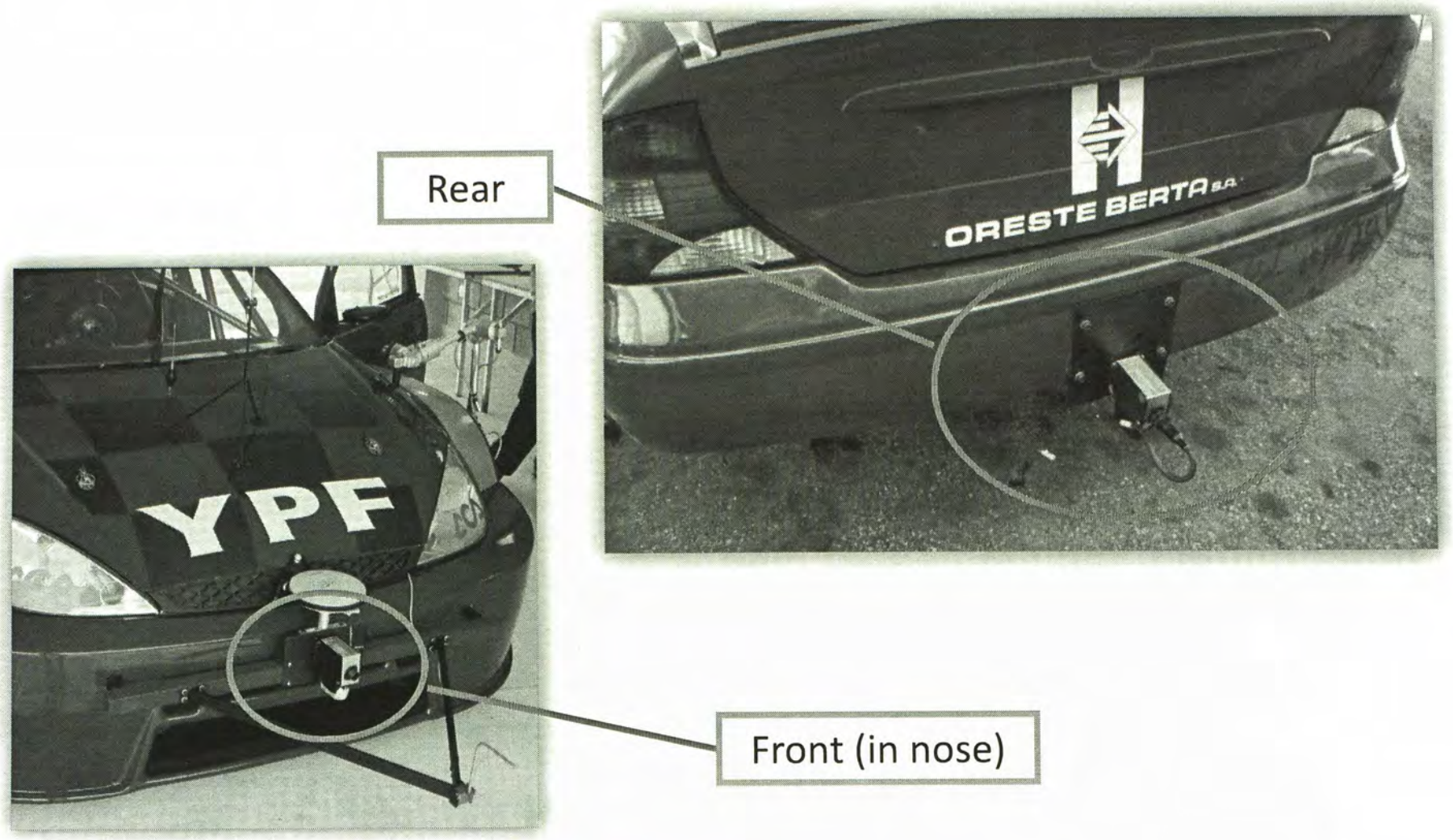


Road Surface

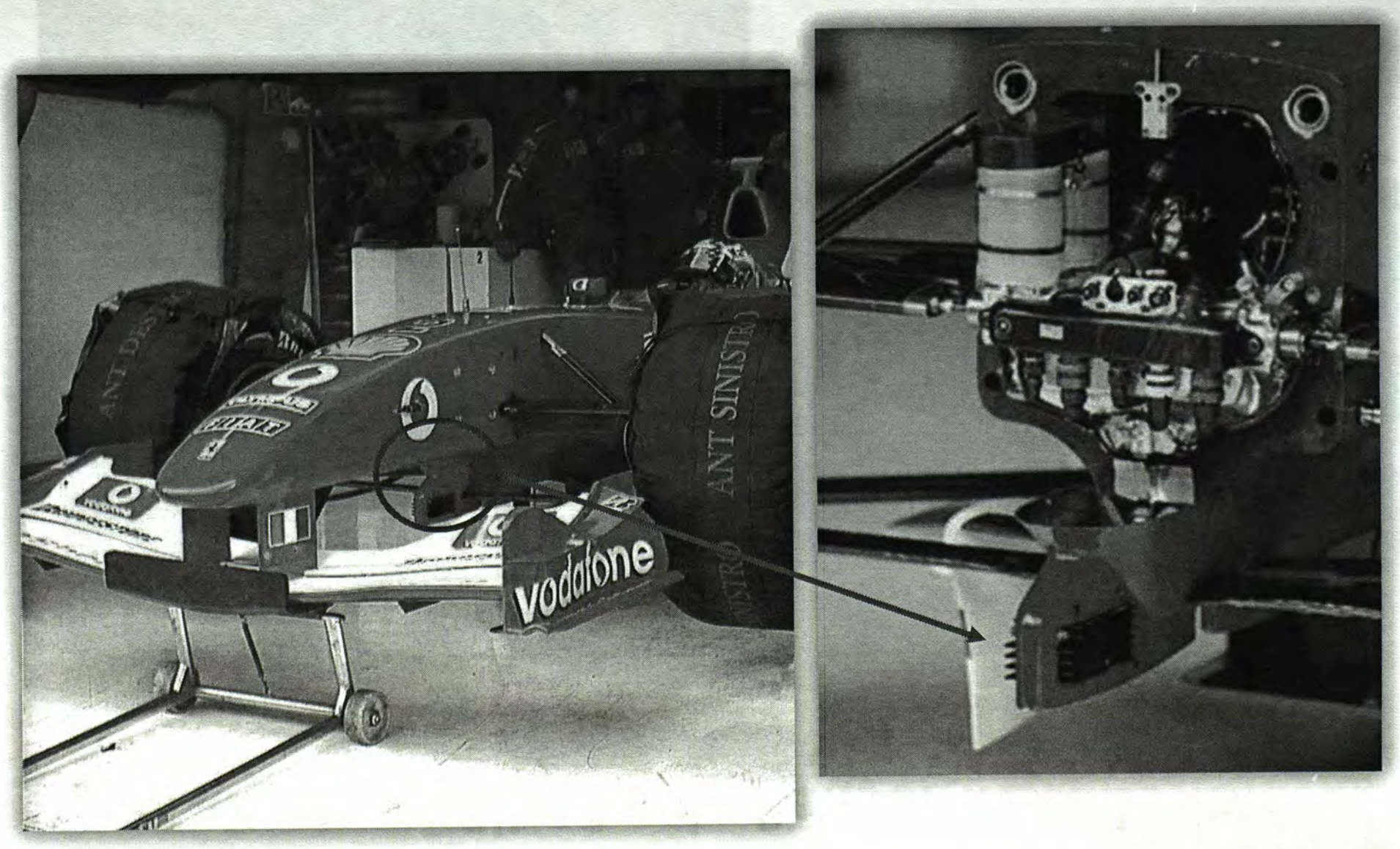




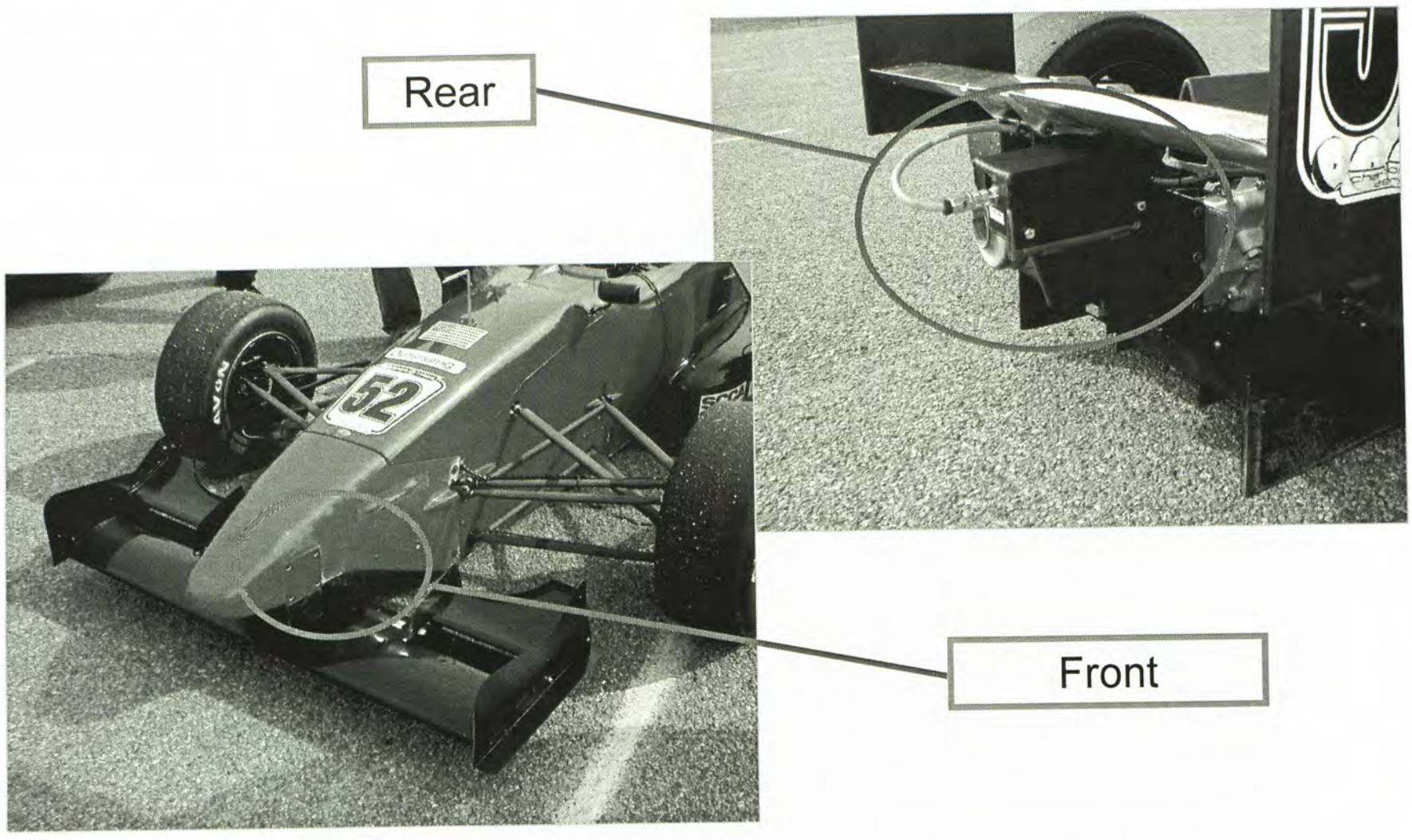
Example of Slip Angle Sensor Installation



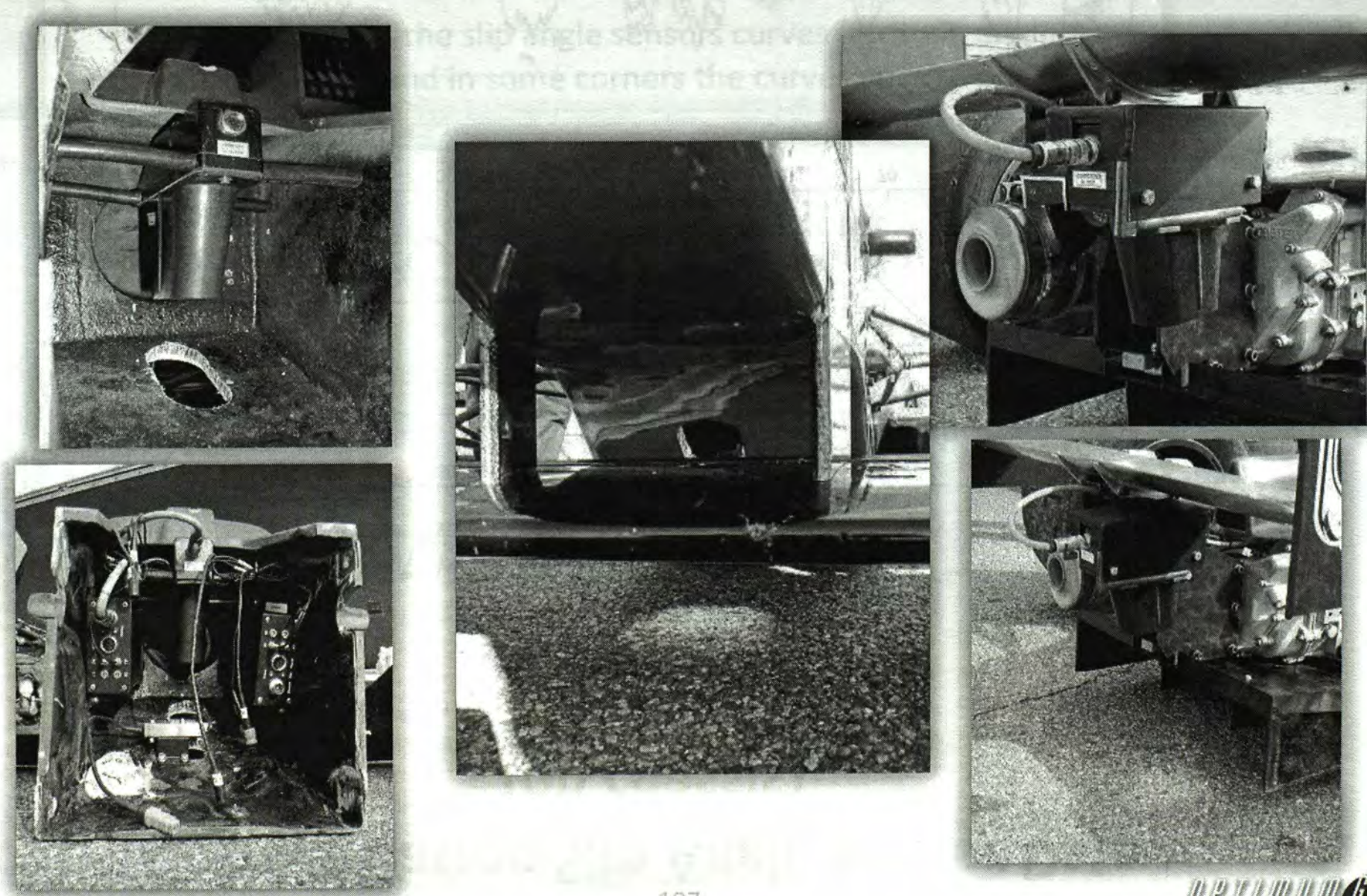
Example of Slip Angle Sensor Installation



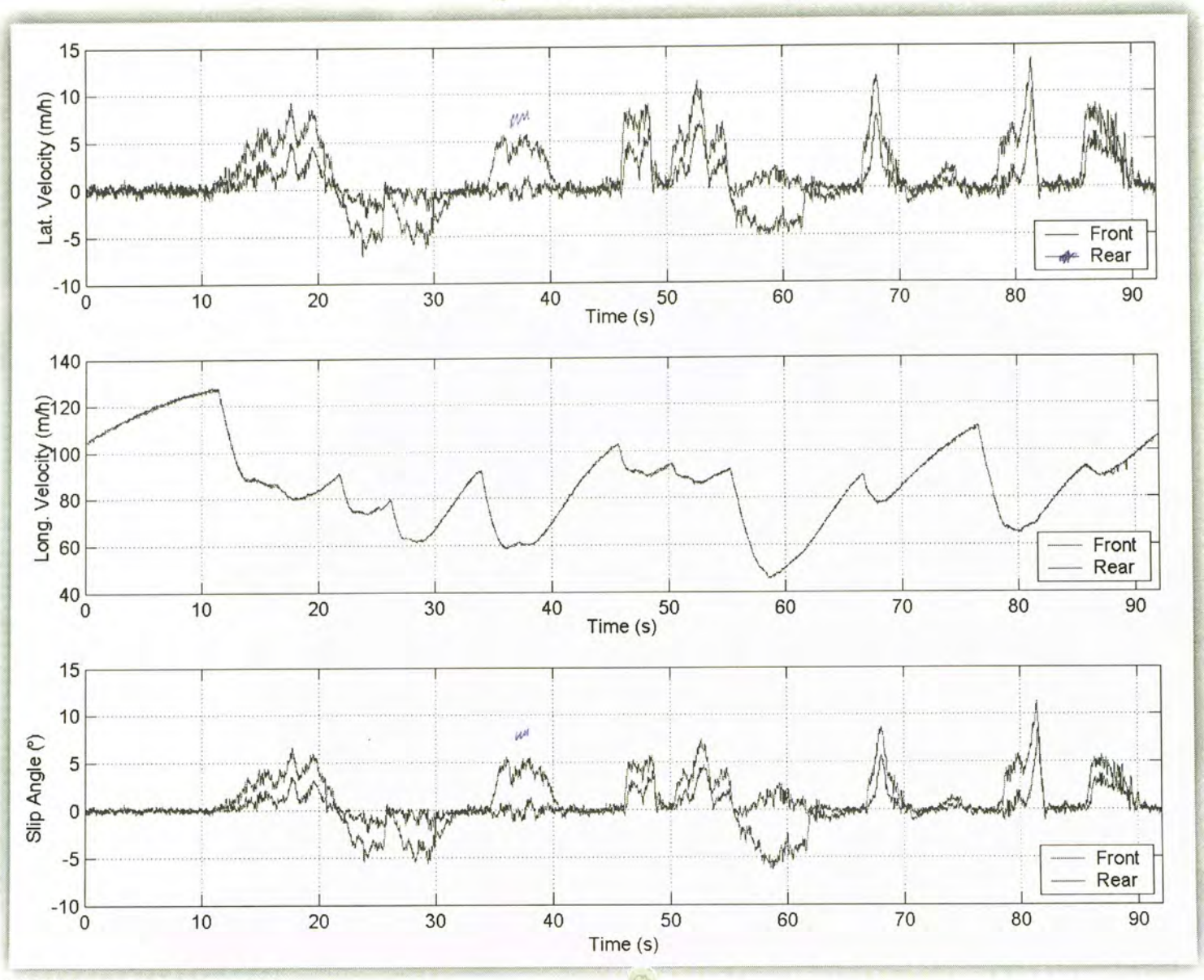
Example of Slip Angle Sensor Installation



Example of Slip Angle Sensor Installation

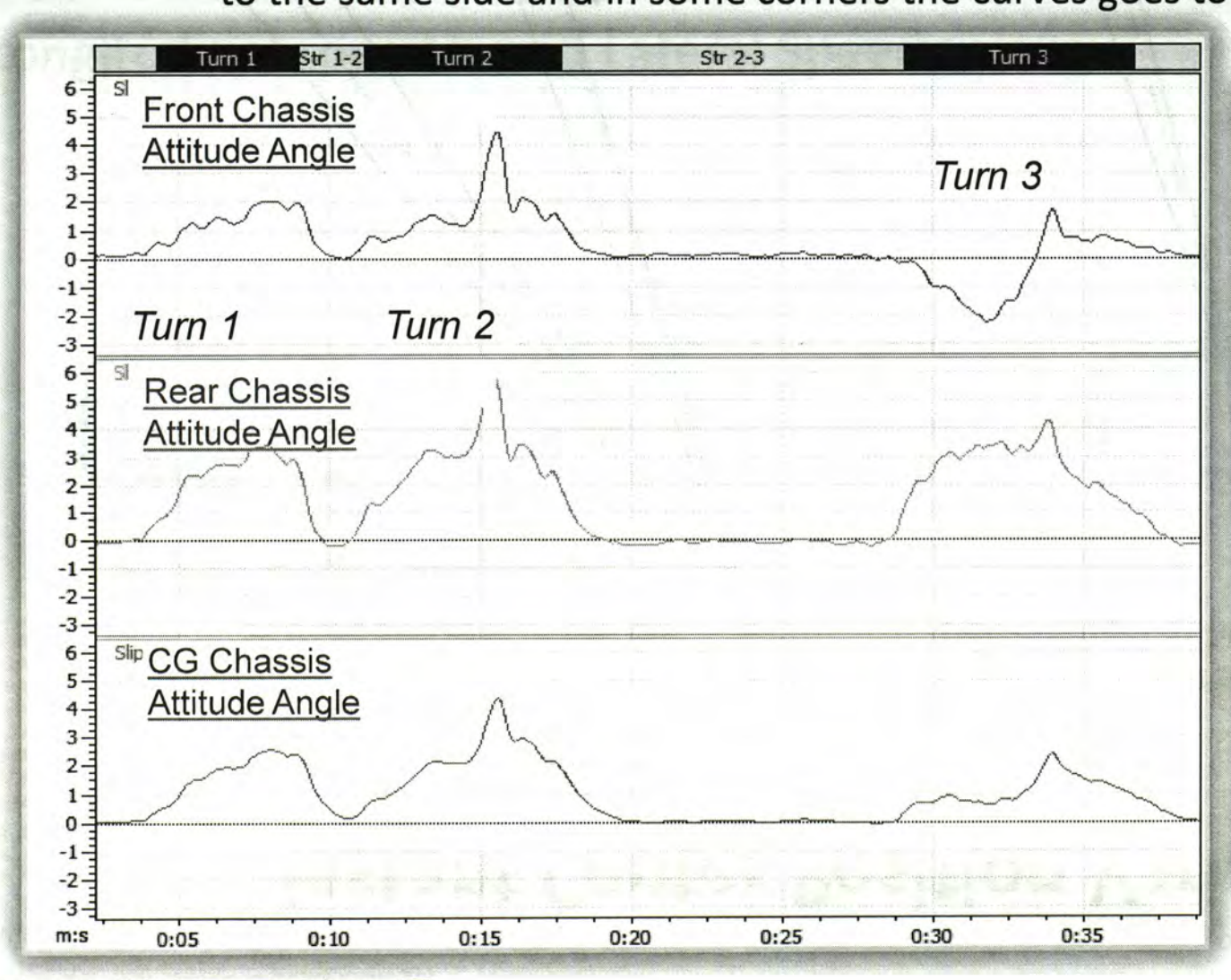


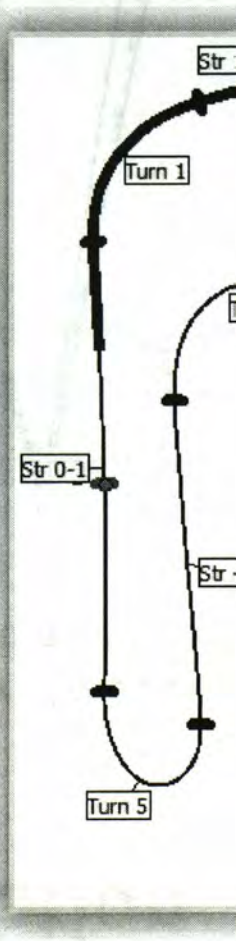
Corrsys-Datron Slip Angle Sensor Data (on chassis)



Example of Slip Angle Sensor Data

In some corners the slip angle sensors curves (attitude angle) goes
to the same side and in some corners the curves goes to opposite sid

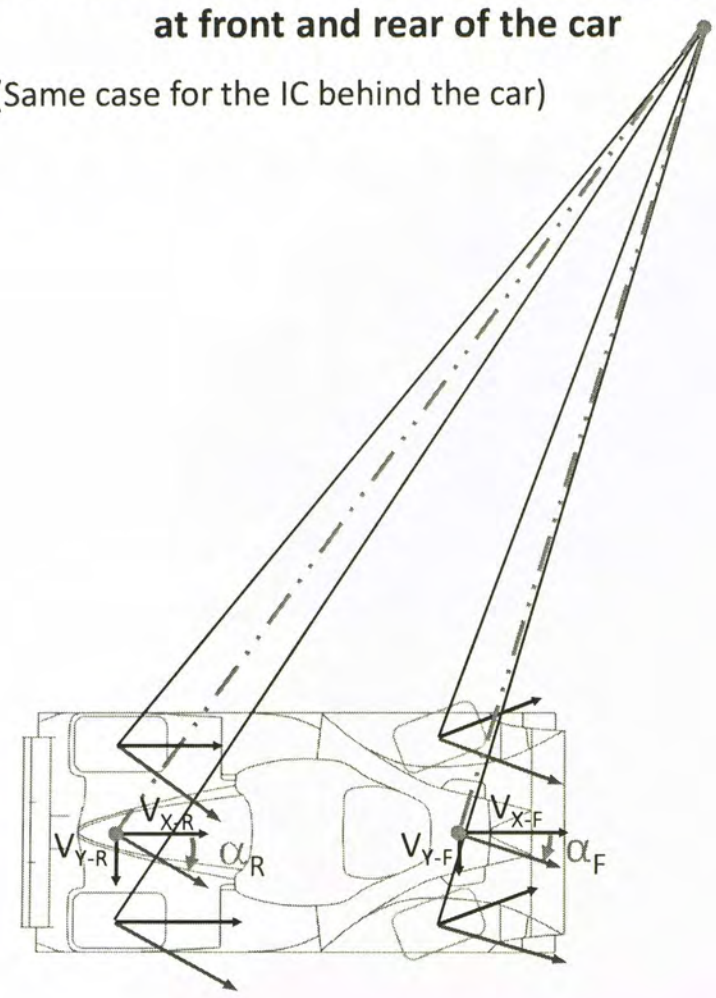




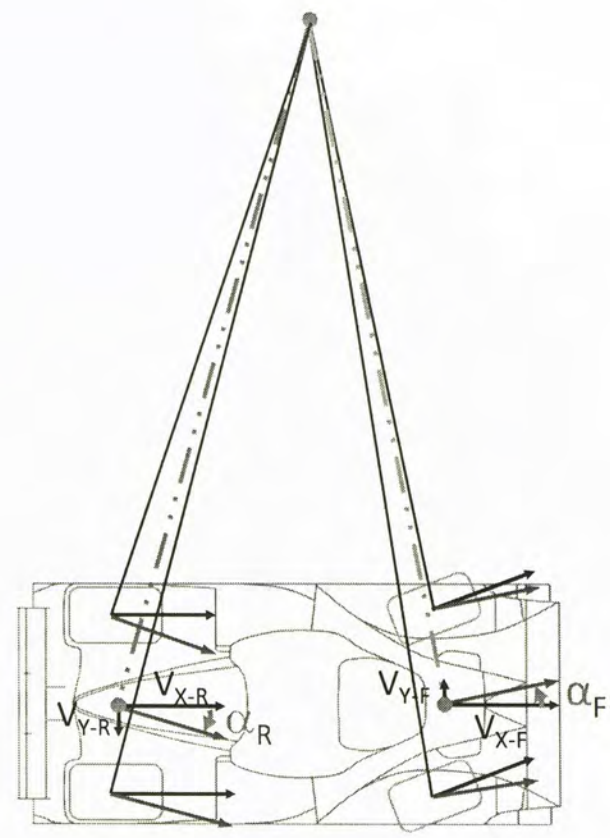
Instant Center Position Variation

In front of the car – slip angle sign the same at front and rear of the car

(Same case for the IC behind the car)

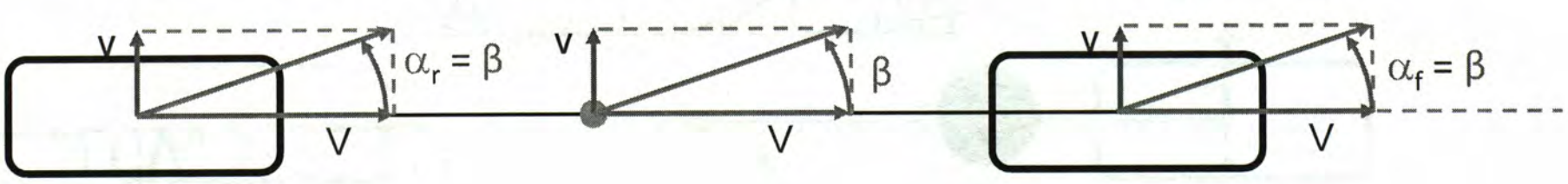


Close to the center of the car – slip angle sign different at front and rear of the car



Where is the slip angle coming from?
CG Slip Angle β

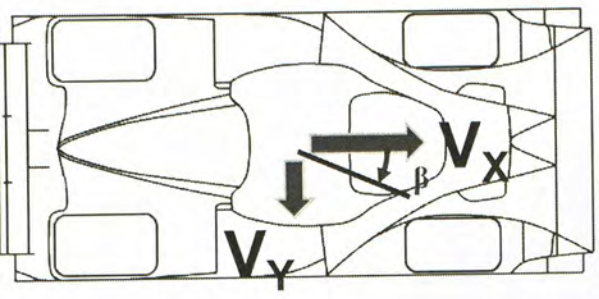
Longitudinal speed V and Lateral Speed v



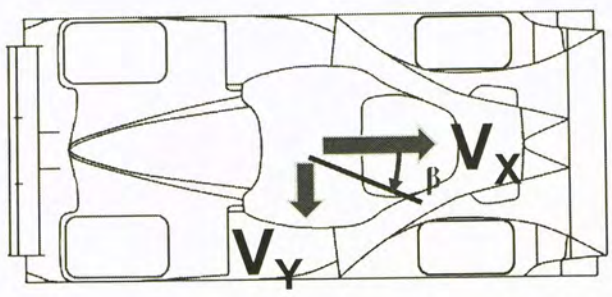
Slip angle without yaw velocity

1

Imagine you are driving straight and a gust of wind makes your car move in the Y-direction, but you did not input any steering.



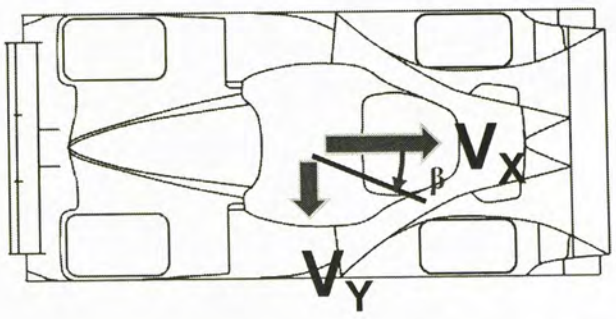
2



3

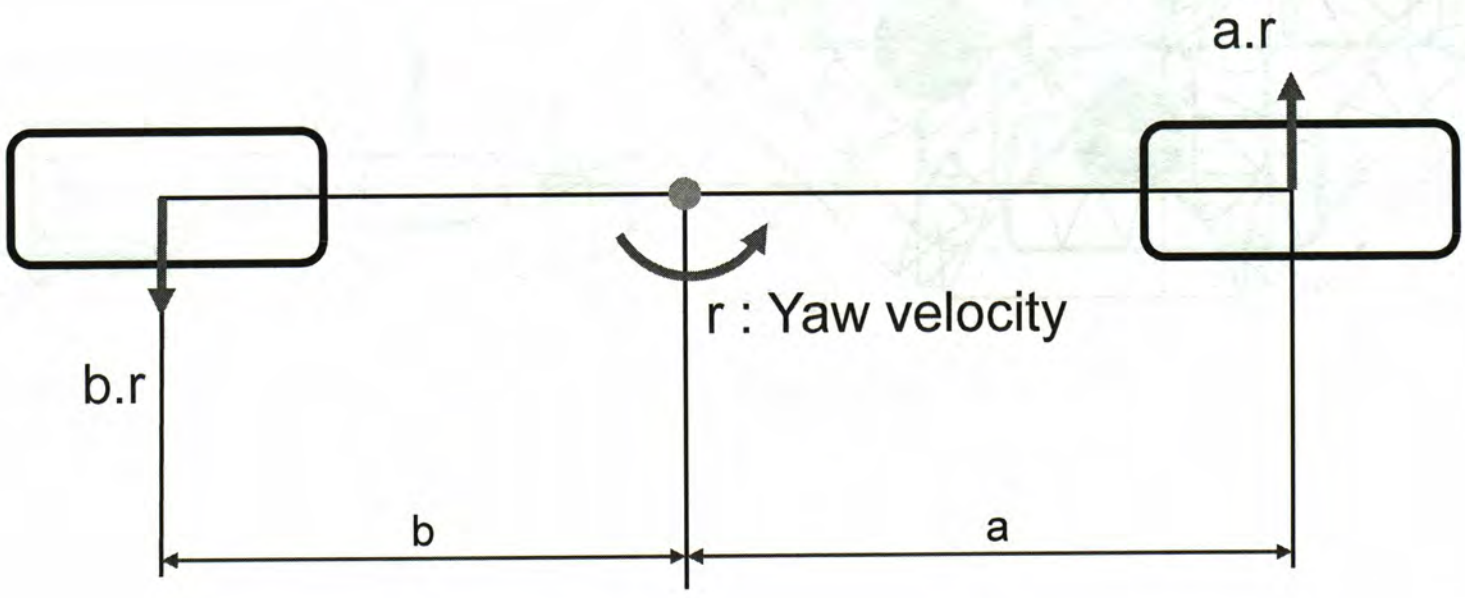
ote that if V_Y change, there will be a lateral celeration and still no signal from the gyro.

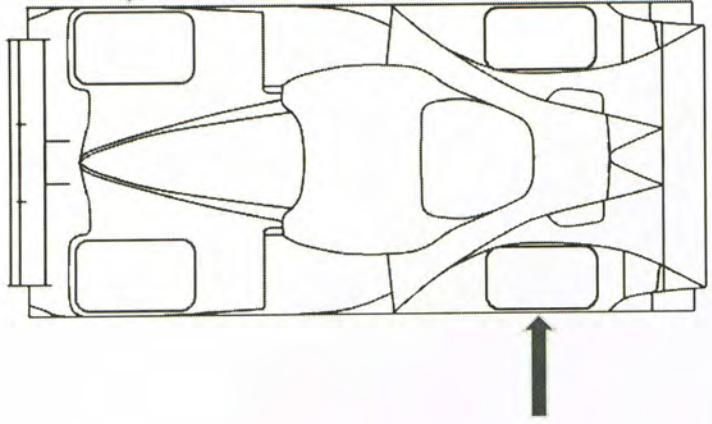
ote that it possible to have a lateral acceleration thout being in a corner!

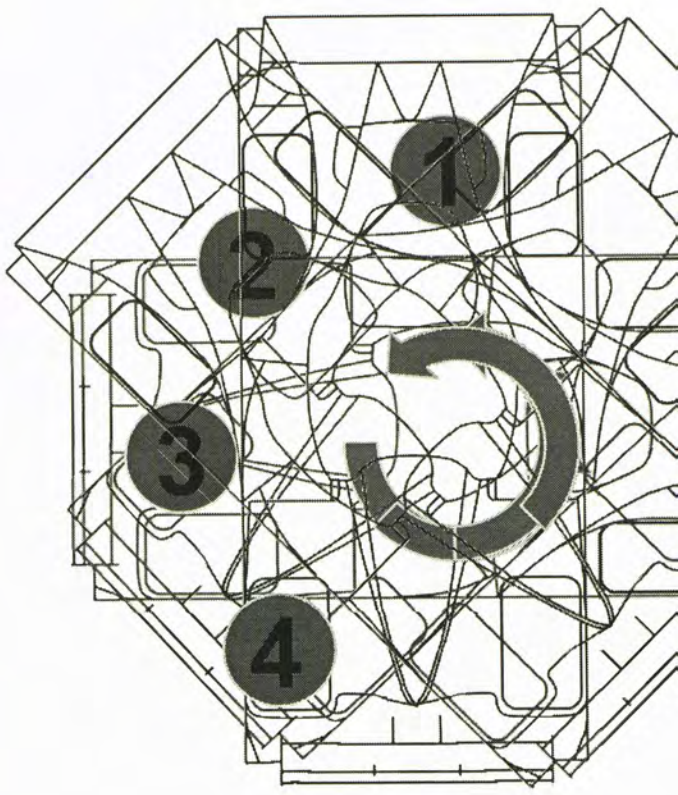


Where is the slip angle coming from? Yaw Velocity r

aw velocity r

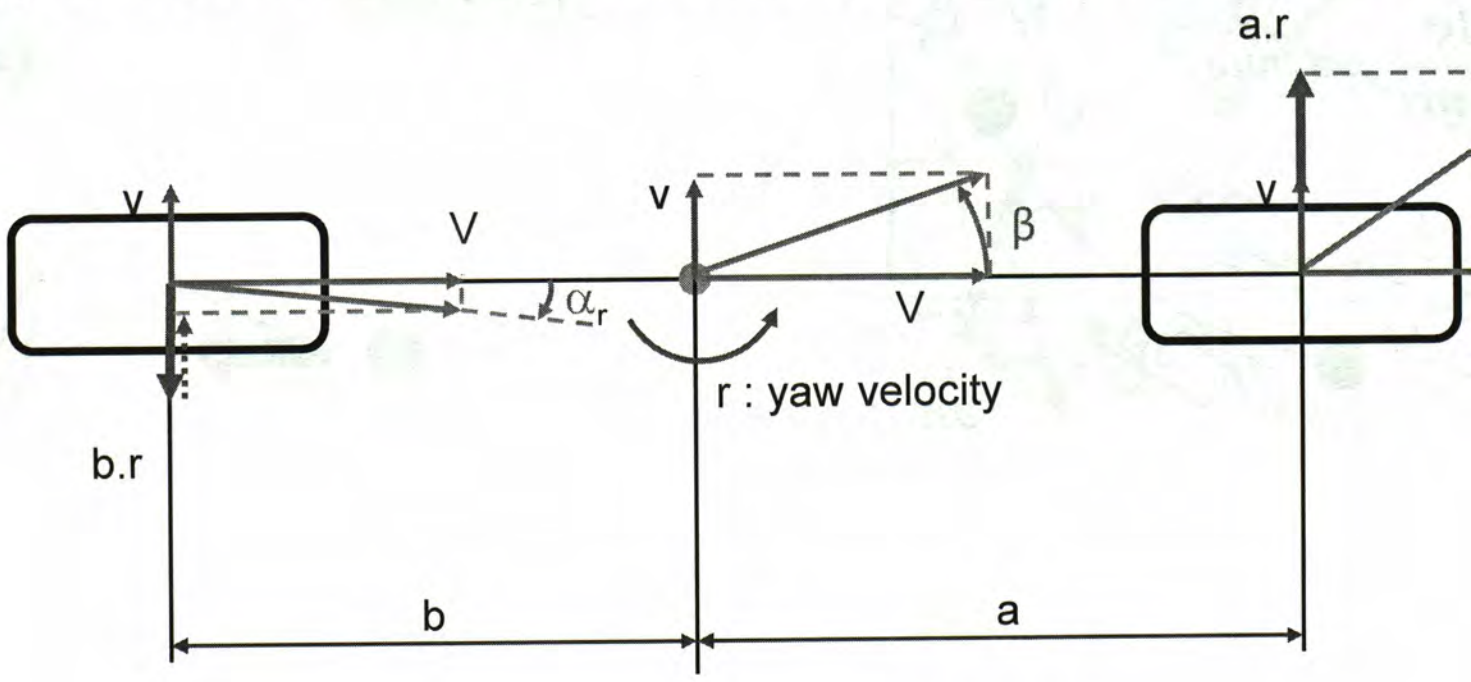




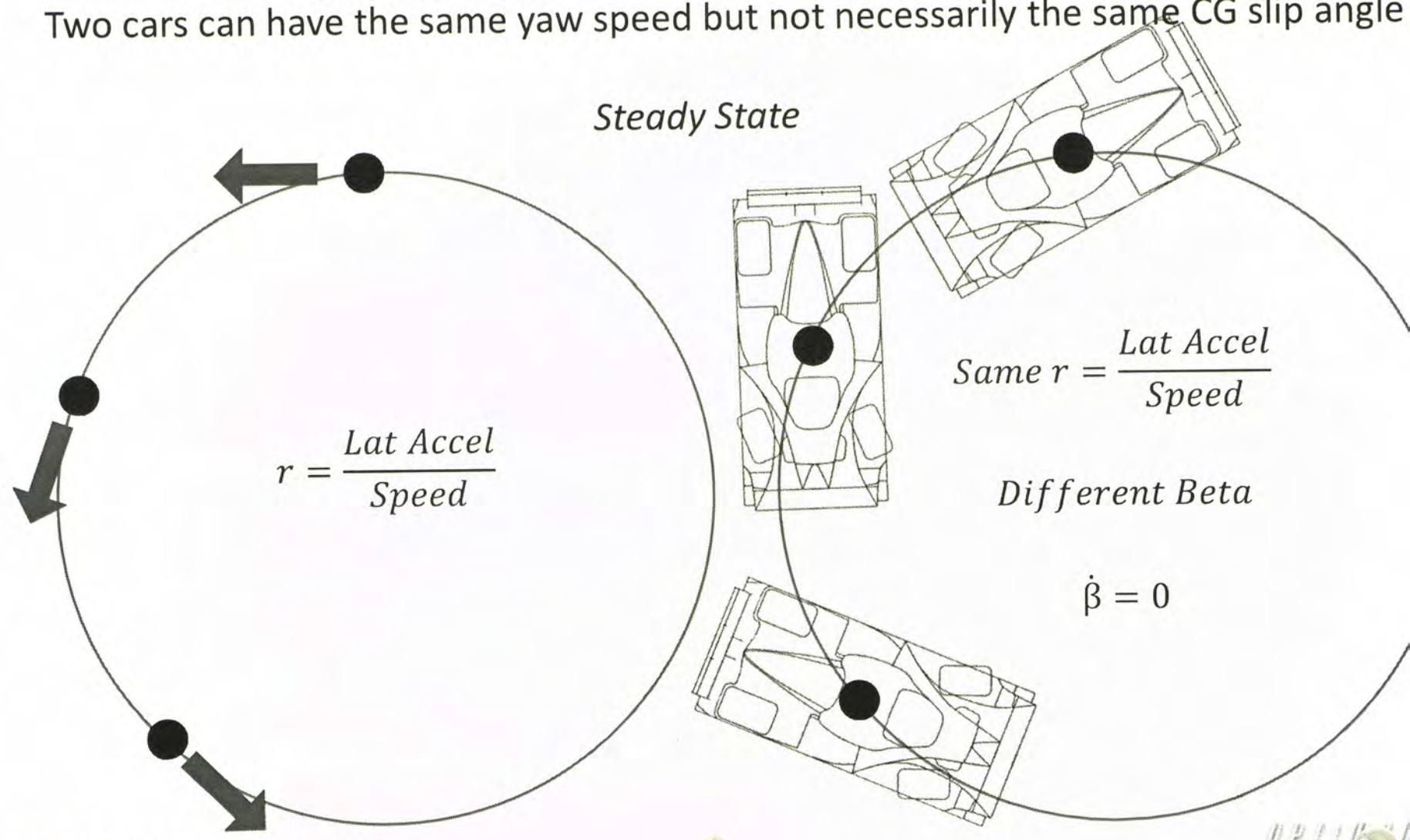


Where is the slip angle coming fro
CG Slip Angle β and Yaw Velocity

Longitudinal Velocity V, Lateral Velocity v and Yaw Velocity

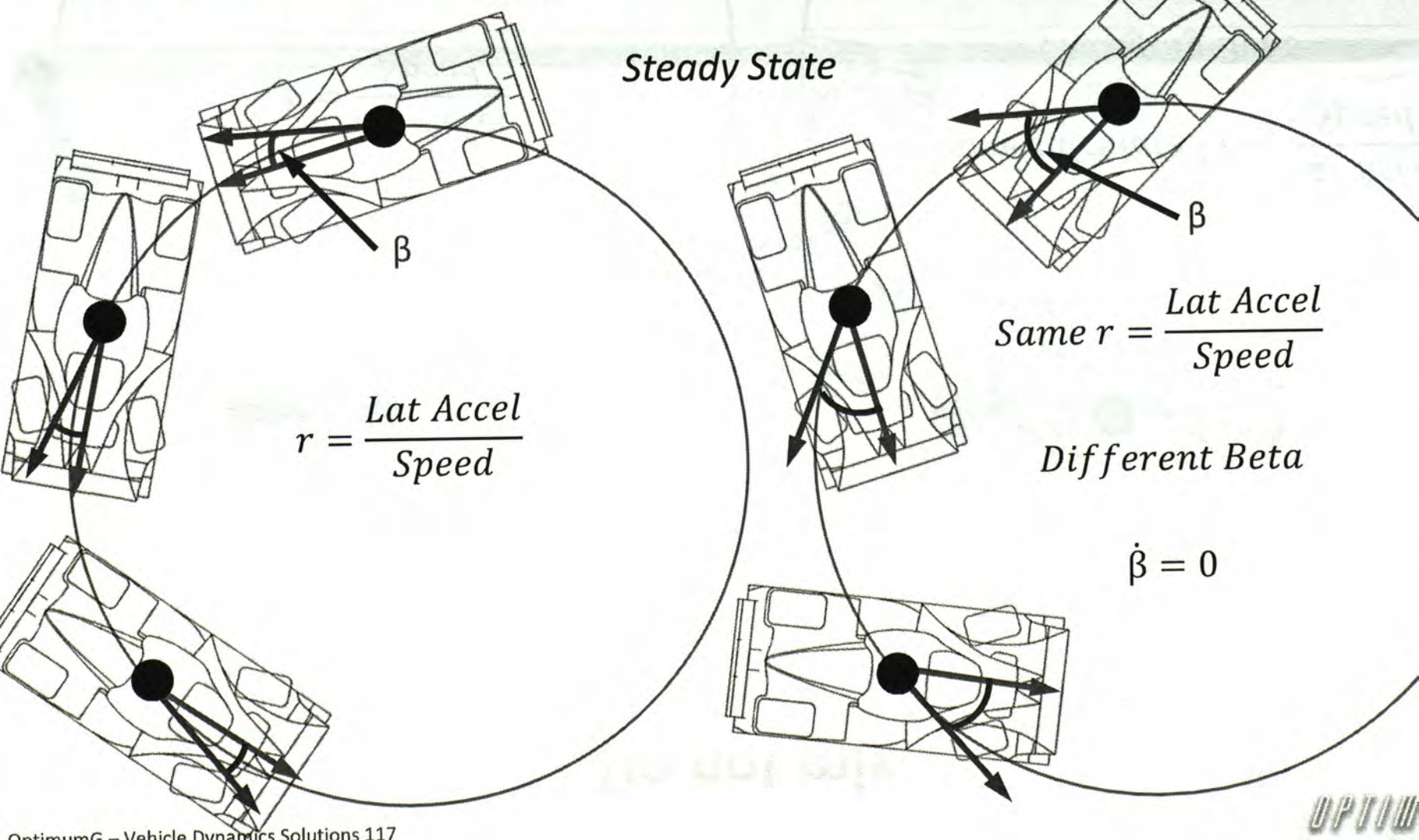


Two cars can have the same yaw speed but not necessarily the same CG slip angle

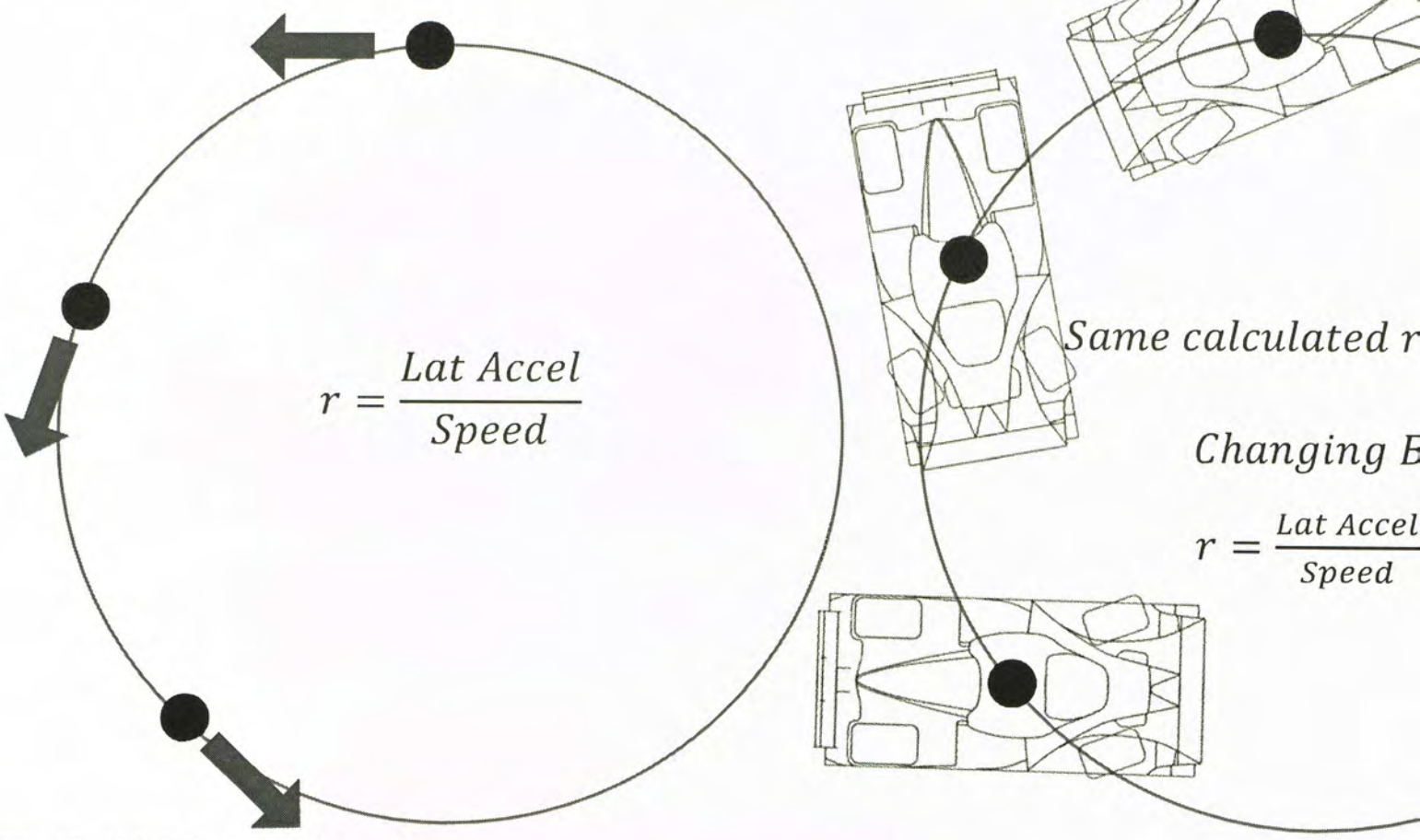


Do not mix
Yaw Velocity r and CG Slip Angle β

Two cars can have the same yaw speed but not necessarily the same CG slip ang

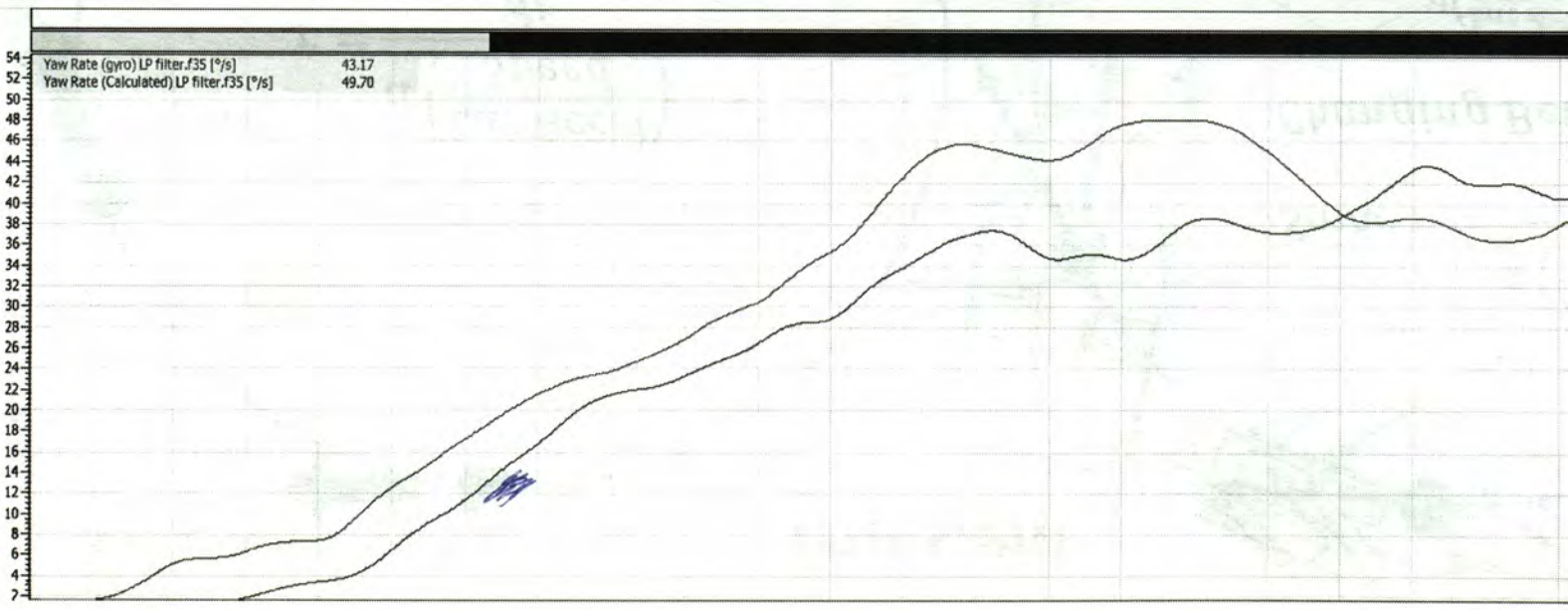


Two cars can have the same yaw speed but not necessarily the same



Do not mix
Yaw Velocity r and CG Slip Angle β

Two cars can have the same yaw speed but not necessarily the same



$$r = \frac{Lat\ Accel}{Speed}$$

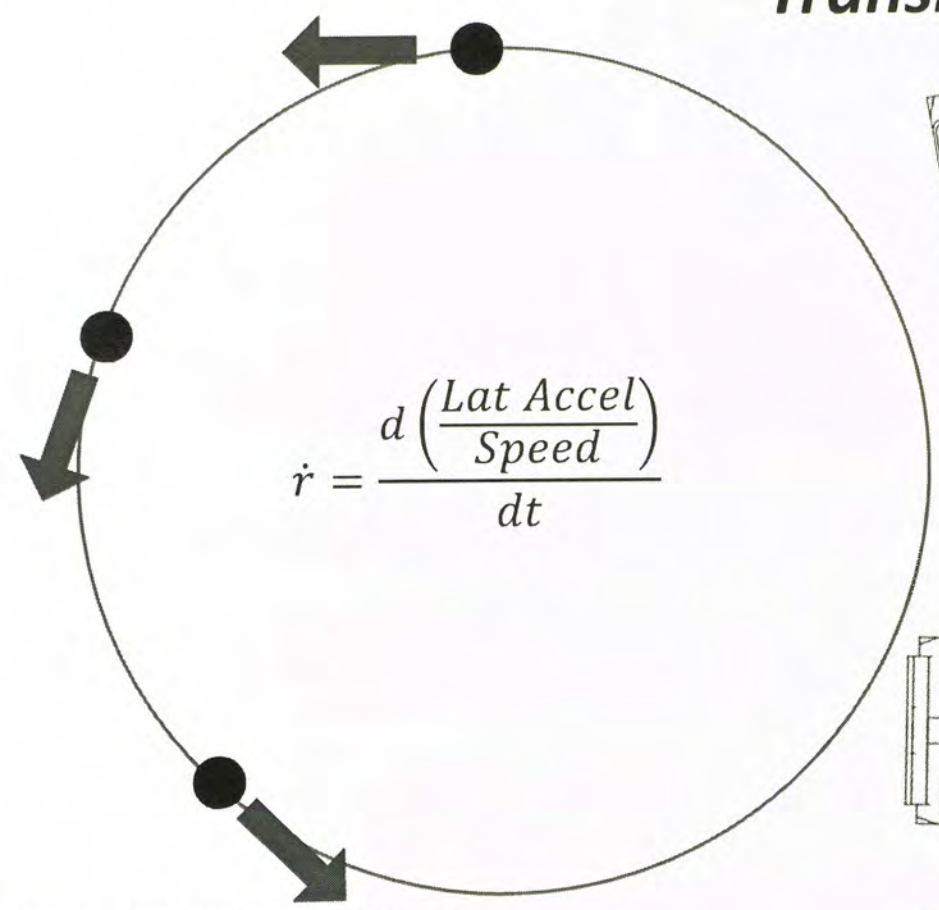
Calculated

$$r = \frac{Lat\ Accel}{Speed} + \dot{\beta}$$

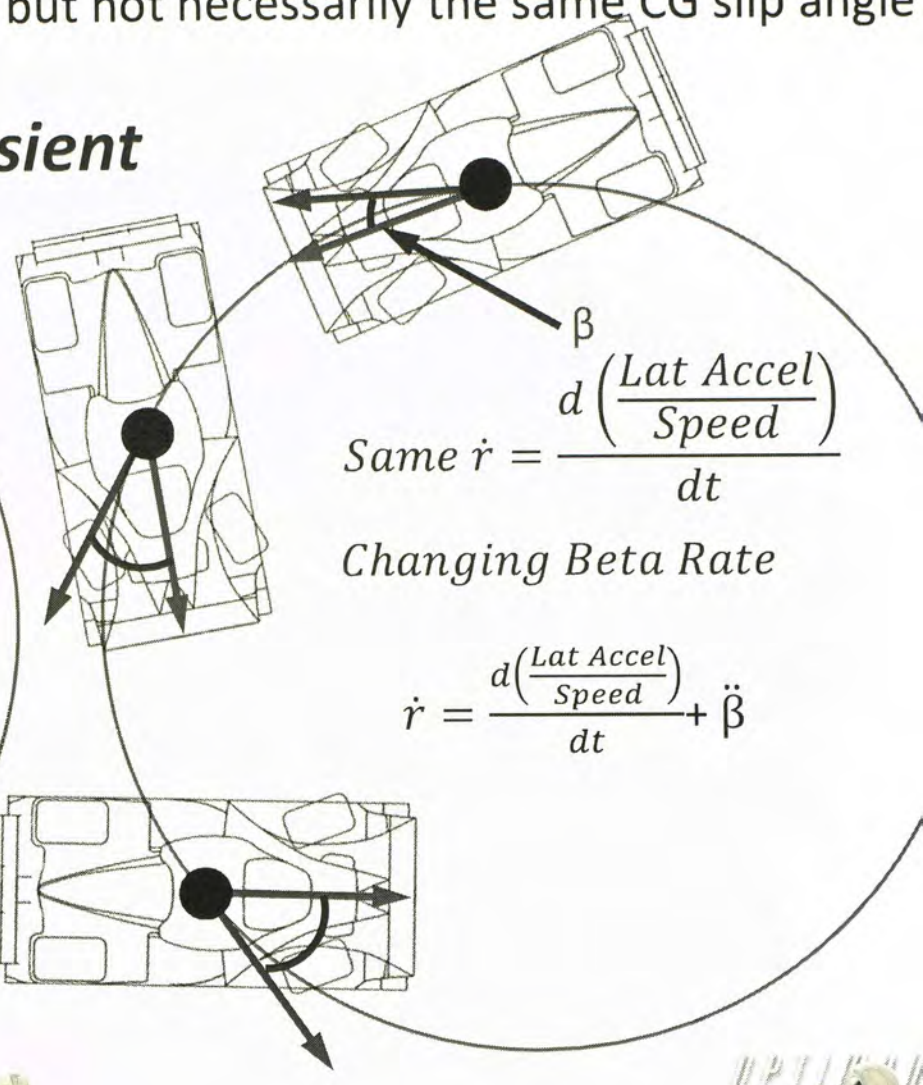
Gyro

Two cars can have the same yaw speed but not necessarily the same CG slip angle

Transient



$$\dot{r} = \frac{d\left(\frac{Lat\ Accel}{Speed}\right)}{dt}$$



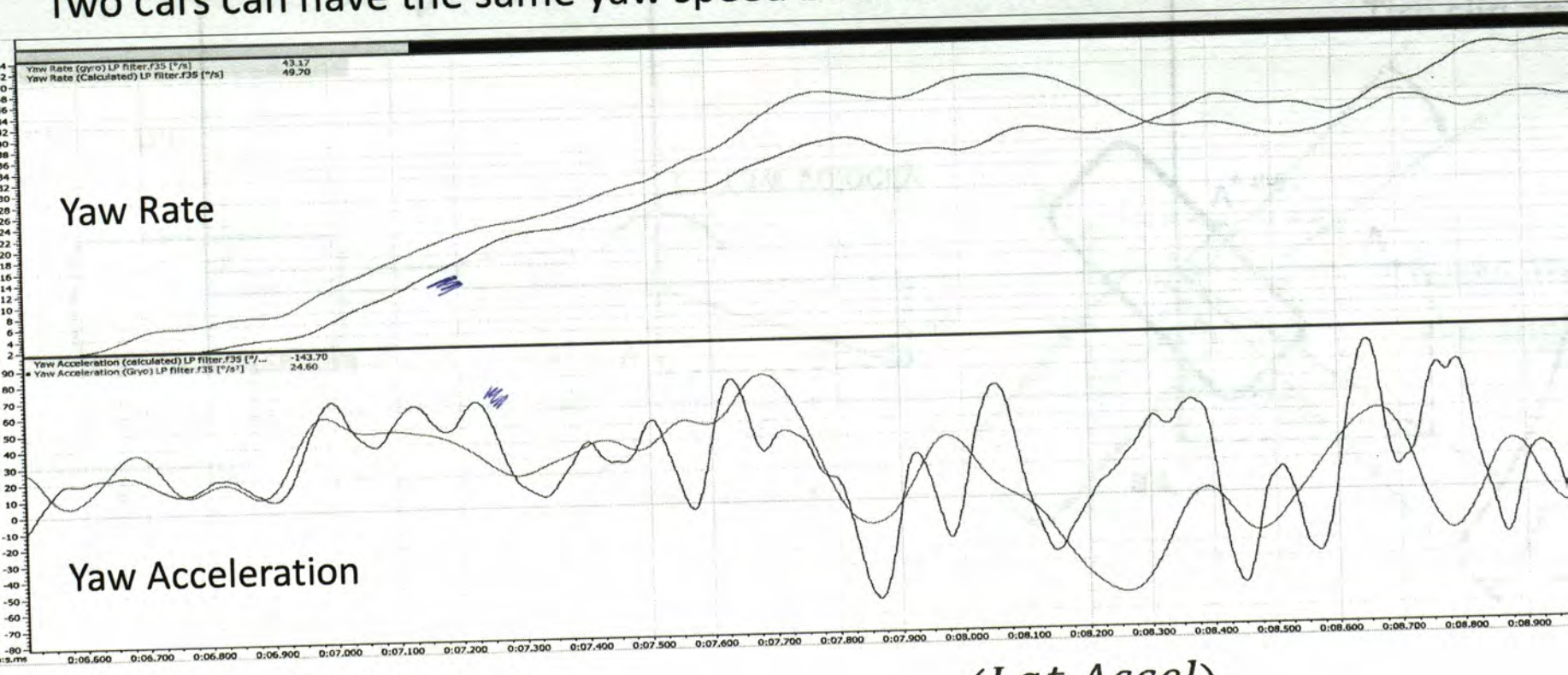
$$Same\ \dot{r} = \frac{d\left(\frac{Lat\ Accel}{Speed}\right)}{dt}$$

Changing Beta Rate

$$\dot{r} = \frac{d\left(\frac{Lat\ Accel}{Speed}\right)}{dt} + \ddot{\beta}$$

Do not mix
Yaw Velocity r and CG Slip Angle β

Two cars can have the same yaw speed but not necessarily the same CG slip ang



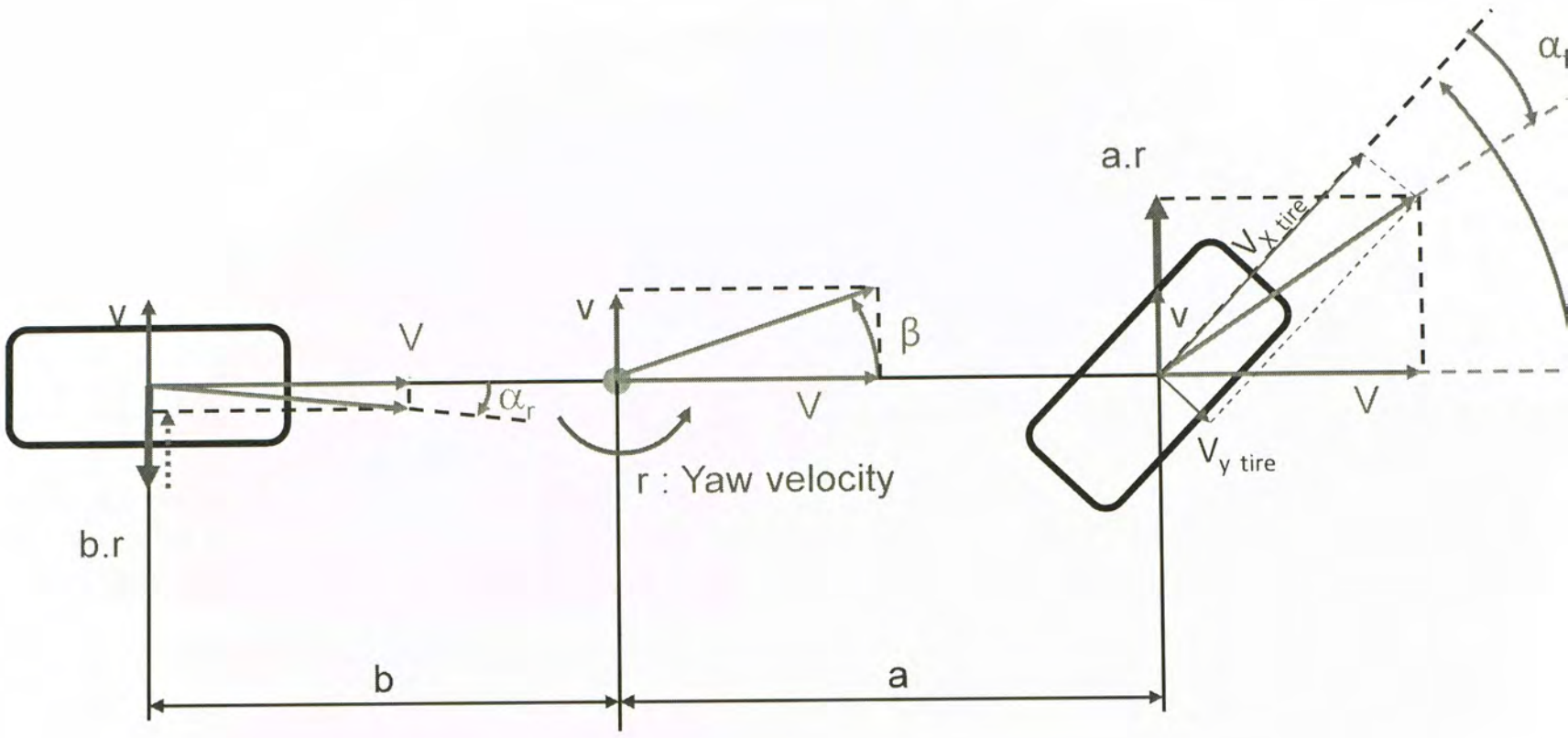
$$\dot{r} = \frac{d\left(\frac{Lat\ Accel}{Speed}\right)}{dt}$$

Calculated

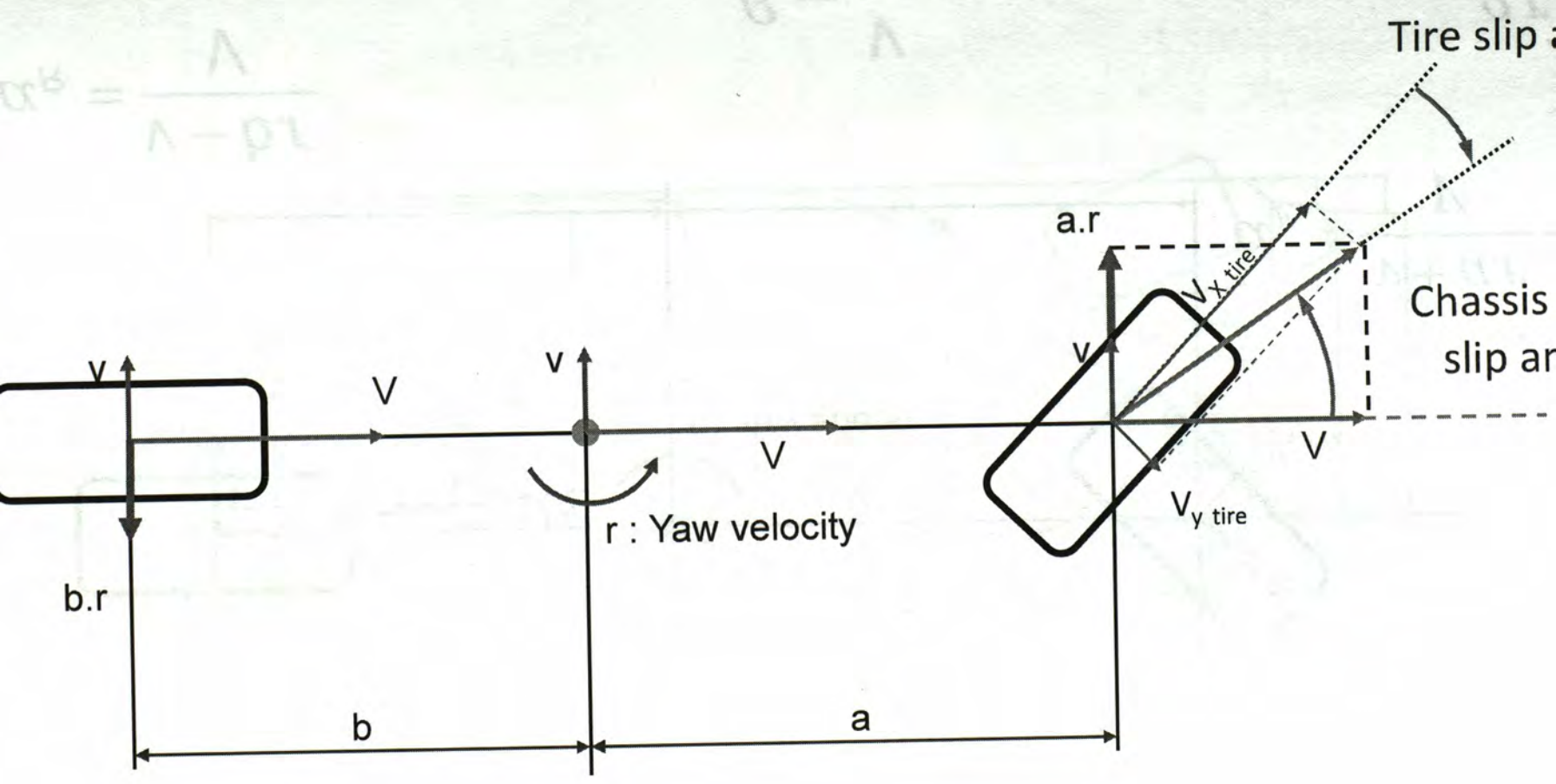
$$\dot{r} = \frac{d\left(\frac{Lat\ Accel}{Speed}\right)}{dt} + \ddot{\beta}$$

Derivate of Gyro

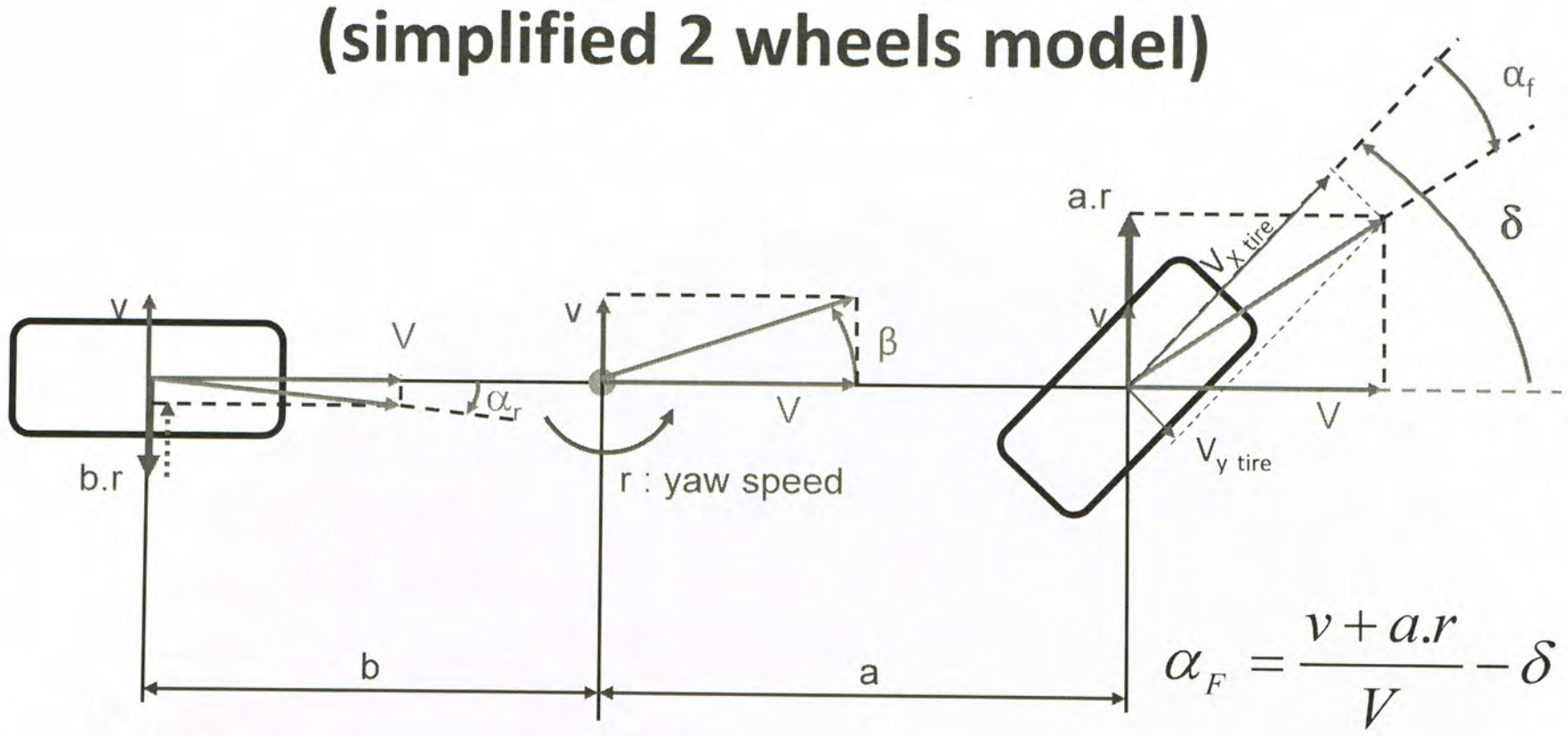
Where is the slip angle coming from?
CG Slip Angle β, Yaw Velocity r and Steering δ



Do not mix
Chassis Slip Angle and Tire Slip Angle



Tire slip angle calculation (simplified 2 wheels model)



$$\alpha_F = \frac{v + a.r}{V} - \delta$$

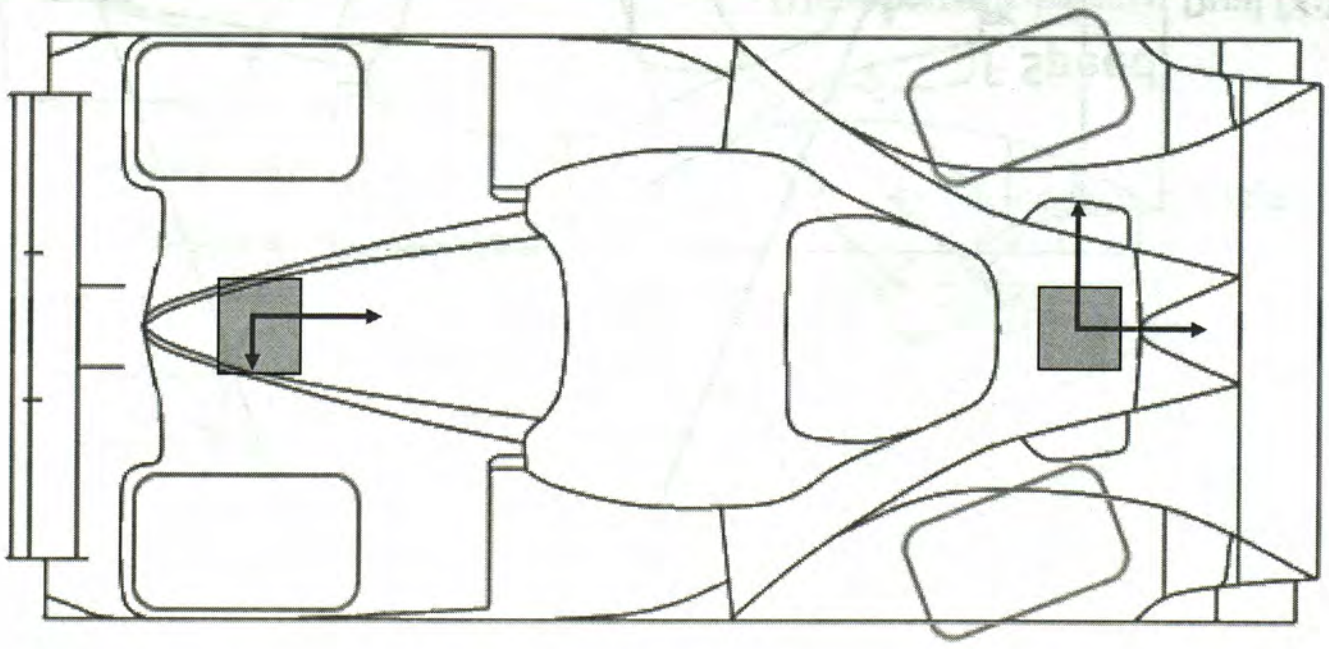
$$\alpha_R = \frac{v - b.r}{V}$$

$$\beta = \frac{v}{V}$$

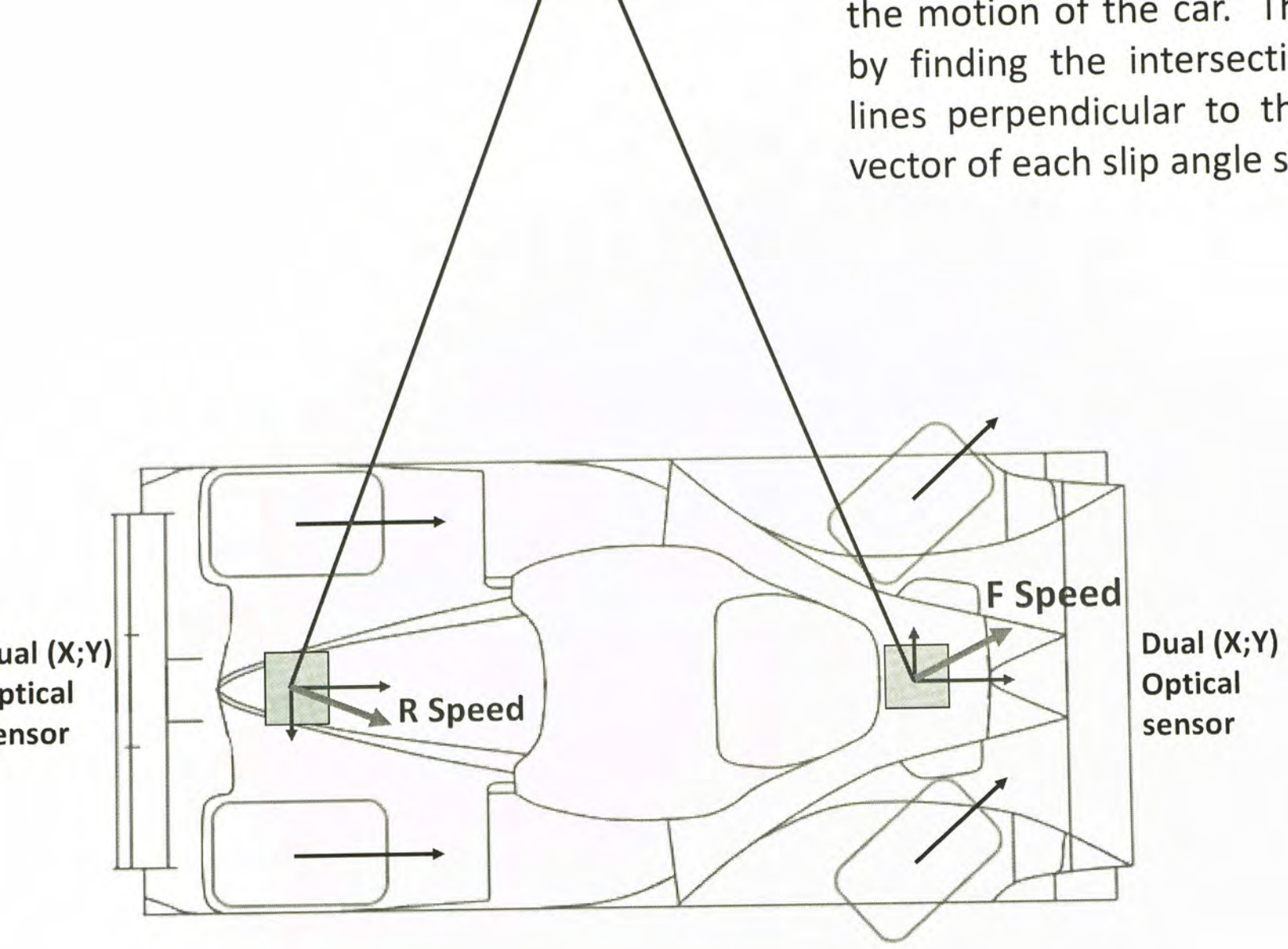
$$\alpha_F = \beta + \frac{a.r}{V} - \delta$$

$$\alpha_R = \beta - \frac{b.r}{V}$$

How to Calculate the Four Tire Slip Angles Using Two Slip Angle Sensors

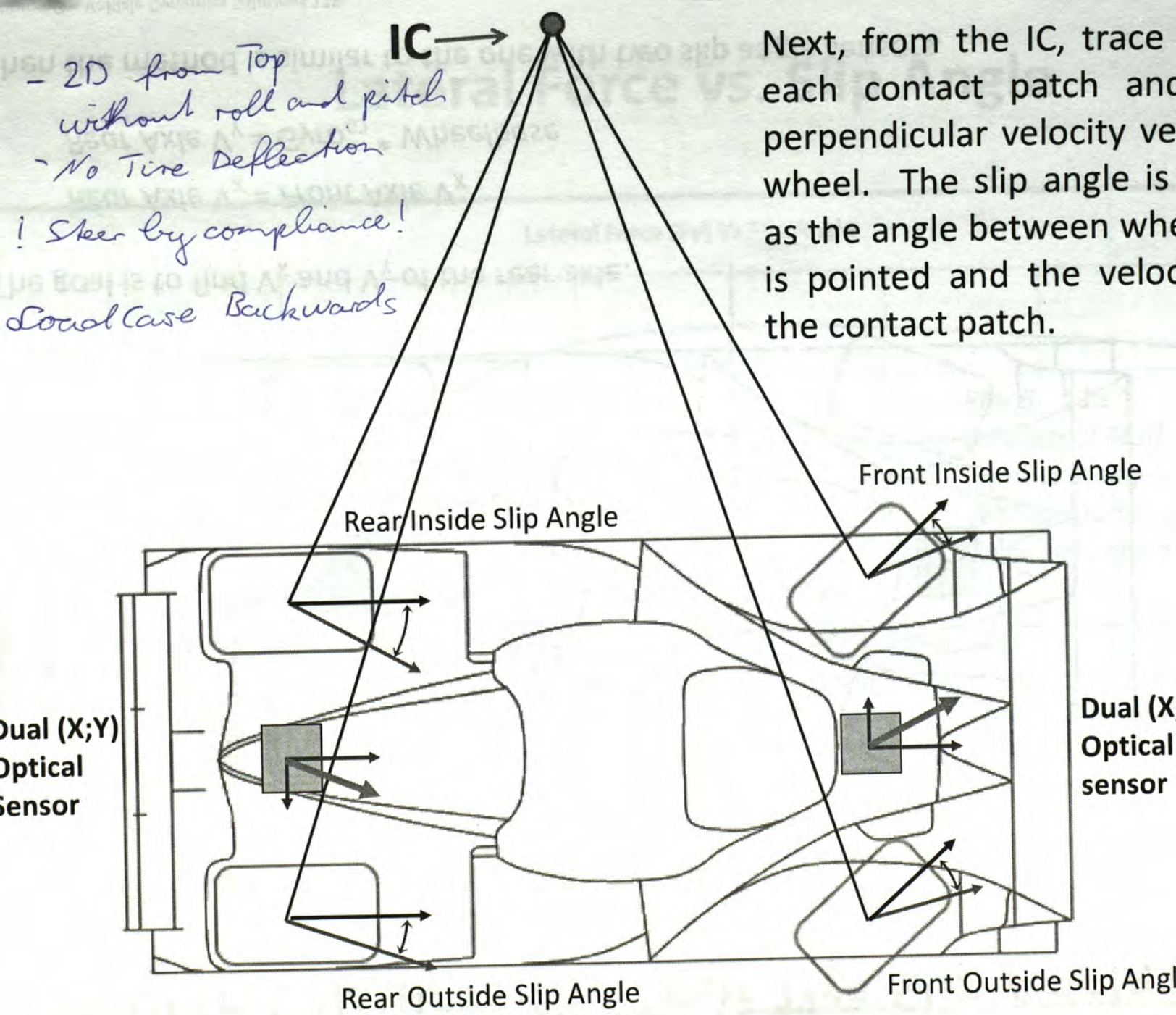


the motion of the car. This
by finding the intersectio
lines perpendicular to the
vector of each slip angle se

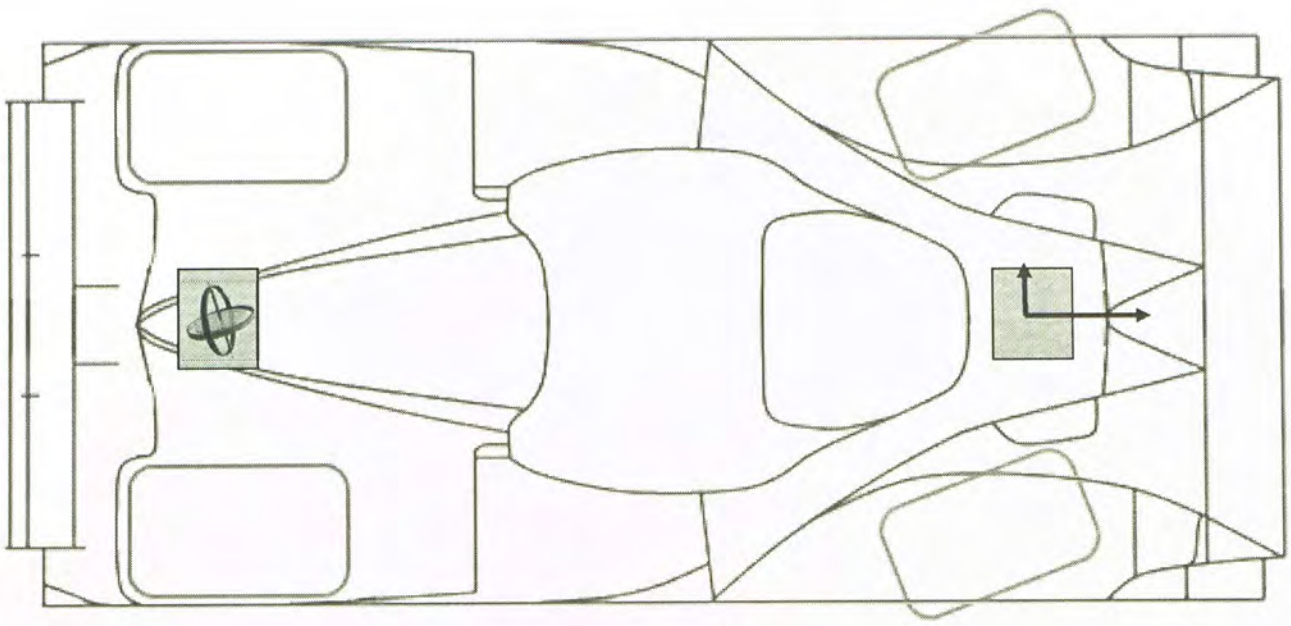


- 2D from Top
without roll and pitch
- No Tire Deflection
! Steer by compliance!
Local Case Backwards

Next, from the IC, trace li
each contact patch and
perpendicular velocity vect
wheel. The slip angle is t
as the angle between wher
is pointed and the velocit
the contact patch.



How to Calculate the Four Tire Slip Angles Using One Slip Angle Sensor and One Gyro



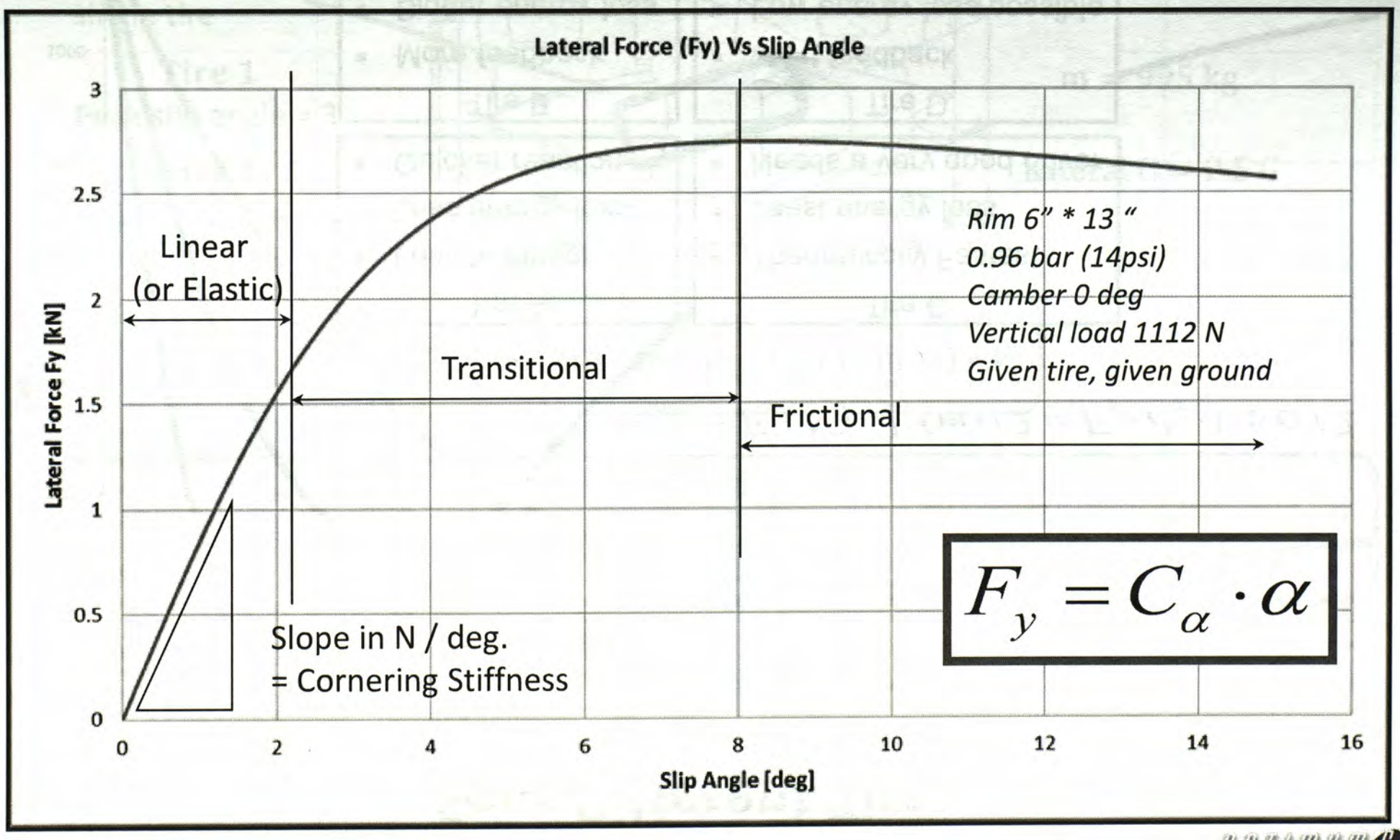
The goal is to find V_X and V_Y of the rear axle.

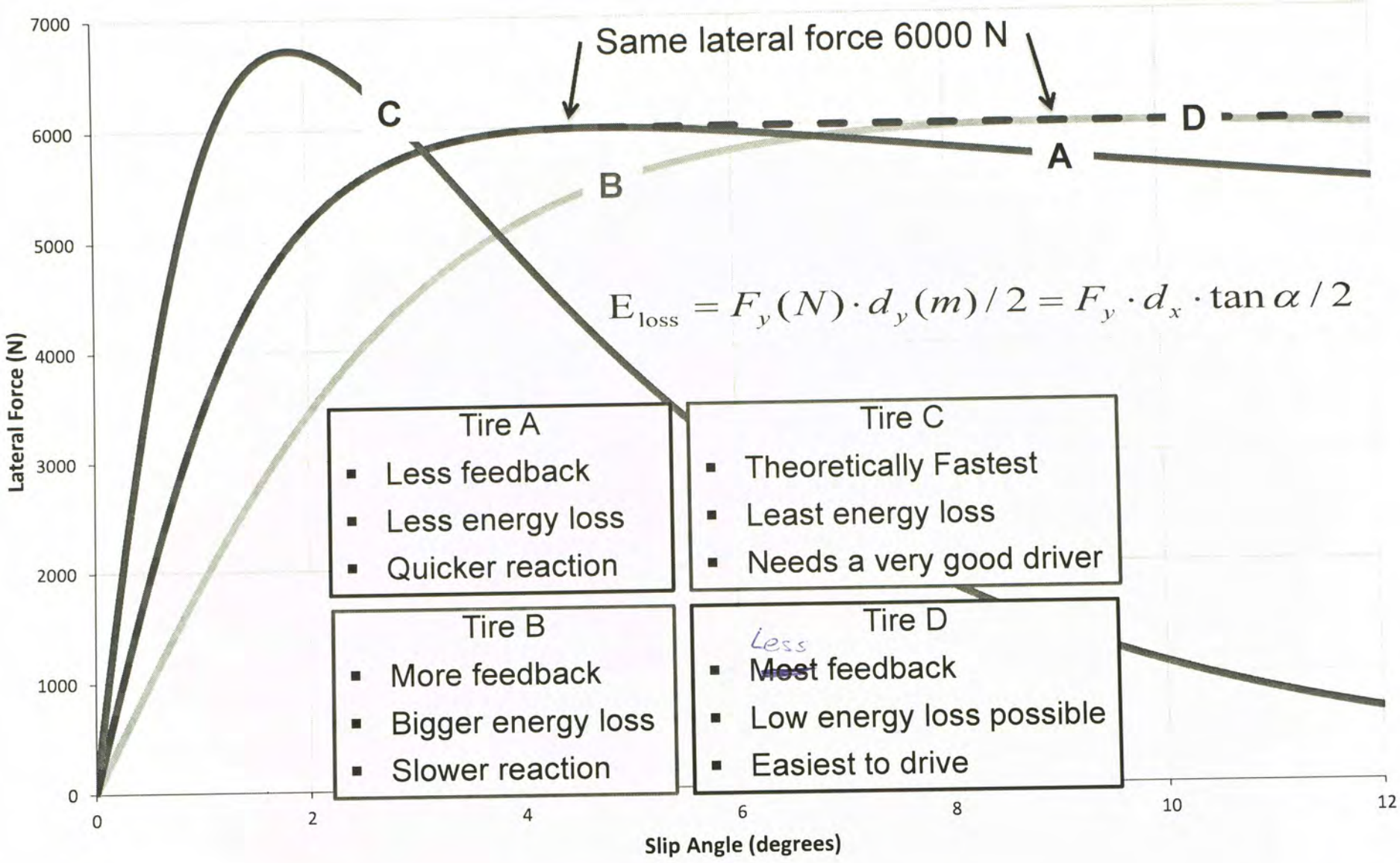
Rear Axle V_X = *Front Axle* V_X

Rear Axle V_Y = $Gyro_{\omega}$ * *Wheelbase*

Then the method is similar to the one with two slip angle sensors.

Lateral Force vs. Slip Angle





Power Loss

Imagine the car runs on one tire

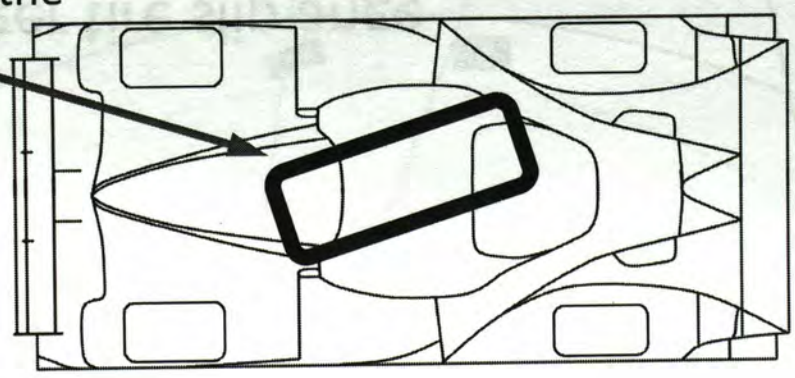
All force is produced by the single tire

Tire 1

Peak slip angle = 3

Tire 2

Peak slip angle = 9



$V = 50m/s = 180kph$

$m = 995\ kg$

$Lateral\ G = 2.5\ G$

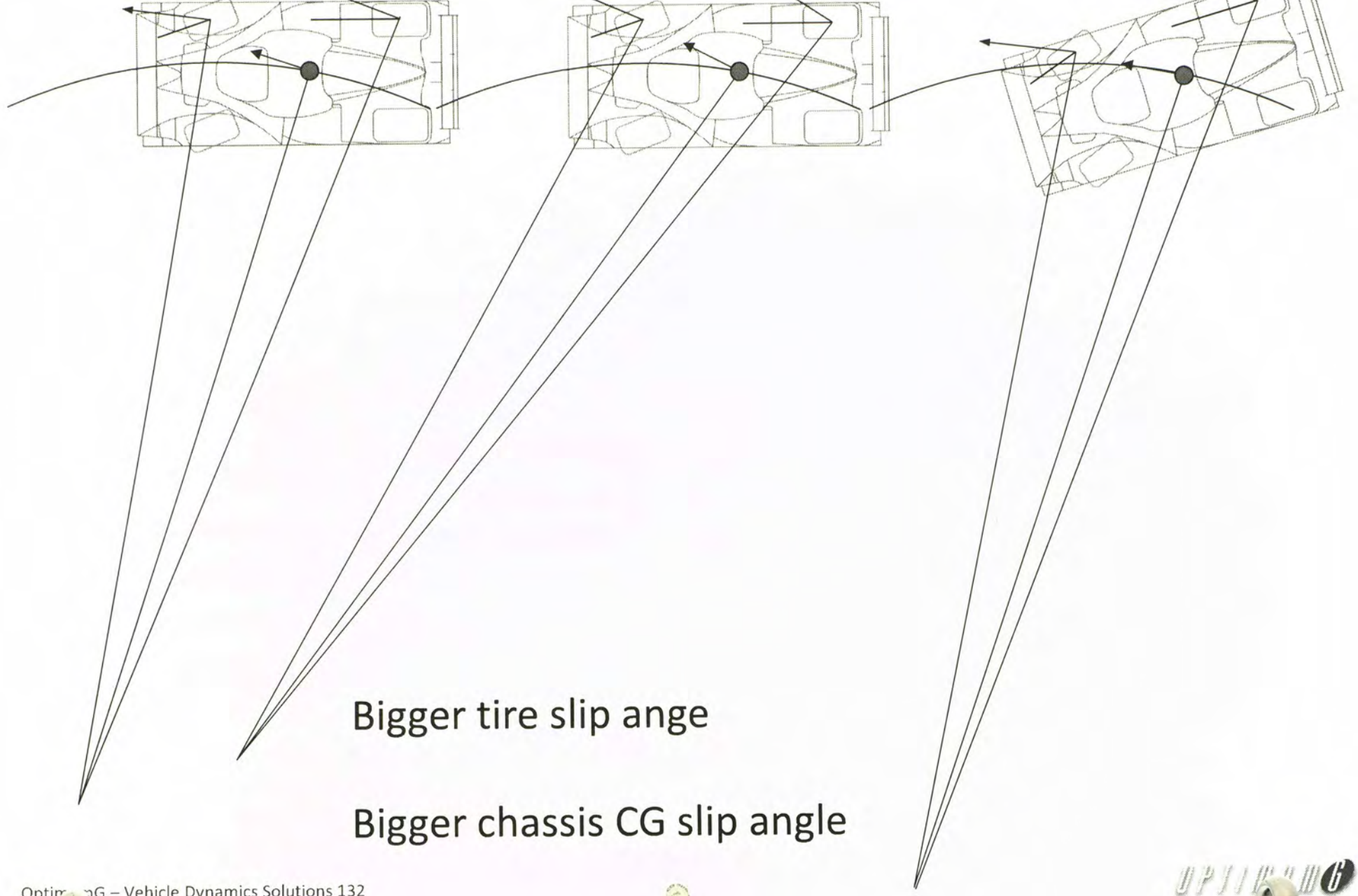
$$E_{loss} = (m * lateral\ g * g) * \sin(\propto) * V$$

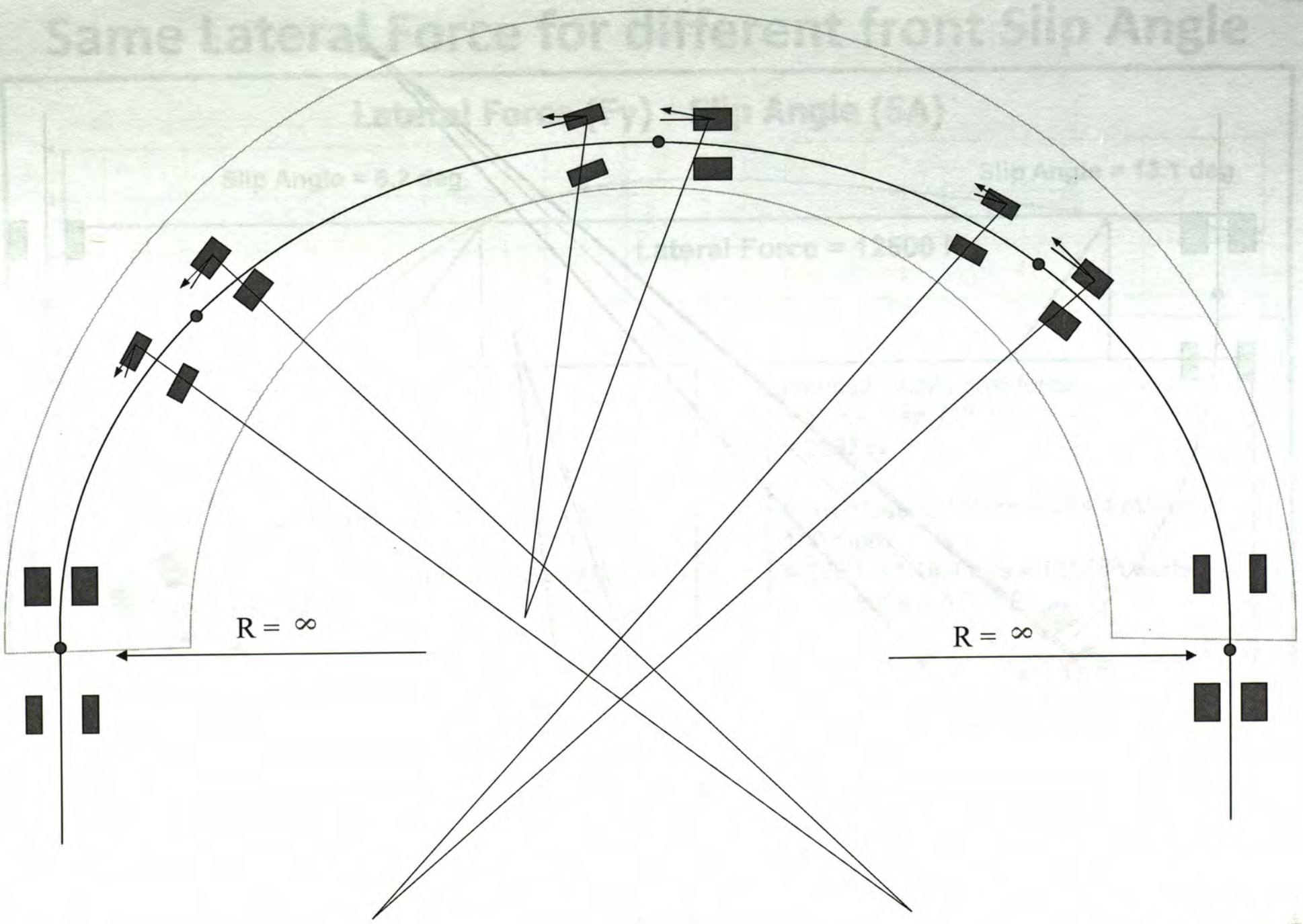
Tire 1

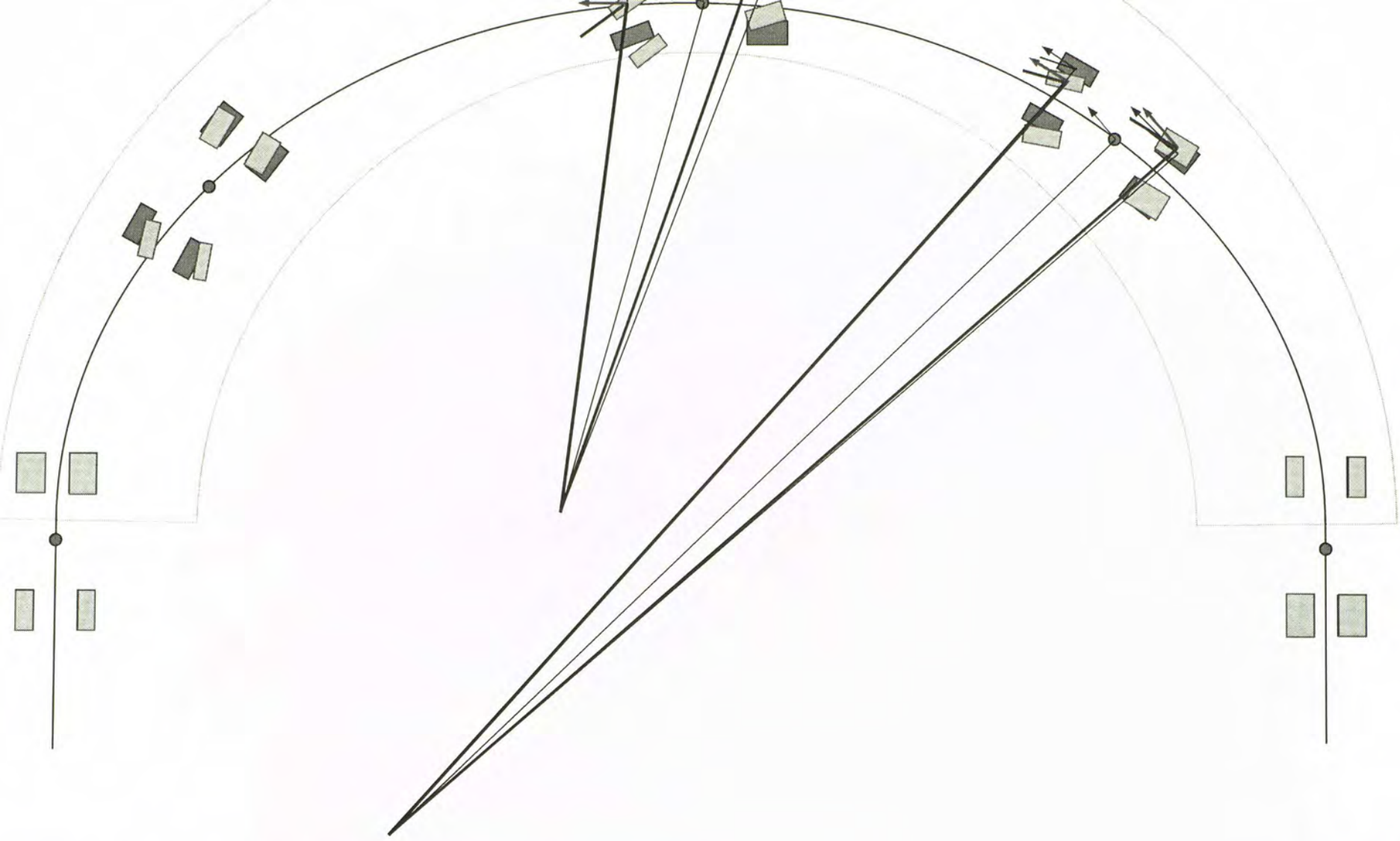
$$E_{loss} = (950 * 2.5 * 9.81) * \sin\left(3 * \frac{\pi}{180}\right) * 50 = 60968.1\ W = 61.0\ KW = 81.7\ hp$$

Tire 2

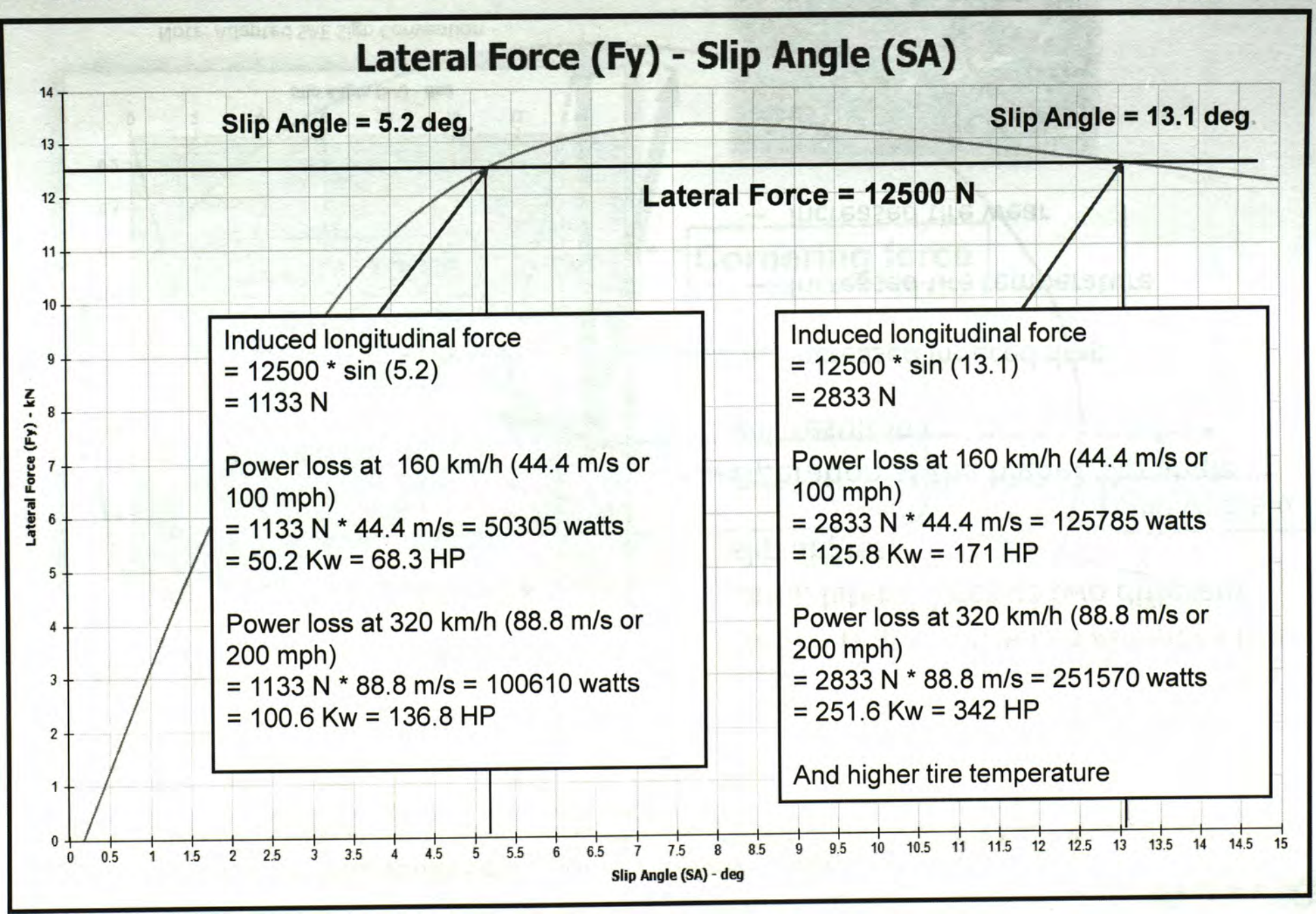
$$E_{loss} = (950 * 2.5 * 9.81) * \sin\left(9 * \frac{\pi}{180}\right) * 50 = 182236.4\ W = 182.2\ KW = 244.3\ hp$$


Bigger tire slip ange
Bigger chassis CG slip angle


R = ∞
R = ∞



Same Lateral Force for different front Slip Angle



Induced longitudinal force
= 12500 * sin (5.2)
= 1133 N

Power loss at 160 km/h (44.4 m/s or 100 mph)
= 1133 N * 44.4 m/s = 50305 watts
= 50.2 Kw = 68.3 HP

Power loss at 320 km/h (88.8 m/s or 200 mph)
= 1133 N * 88.8 m/s = 100610 watts
= 100.6 Kw = 136.8 HP

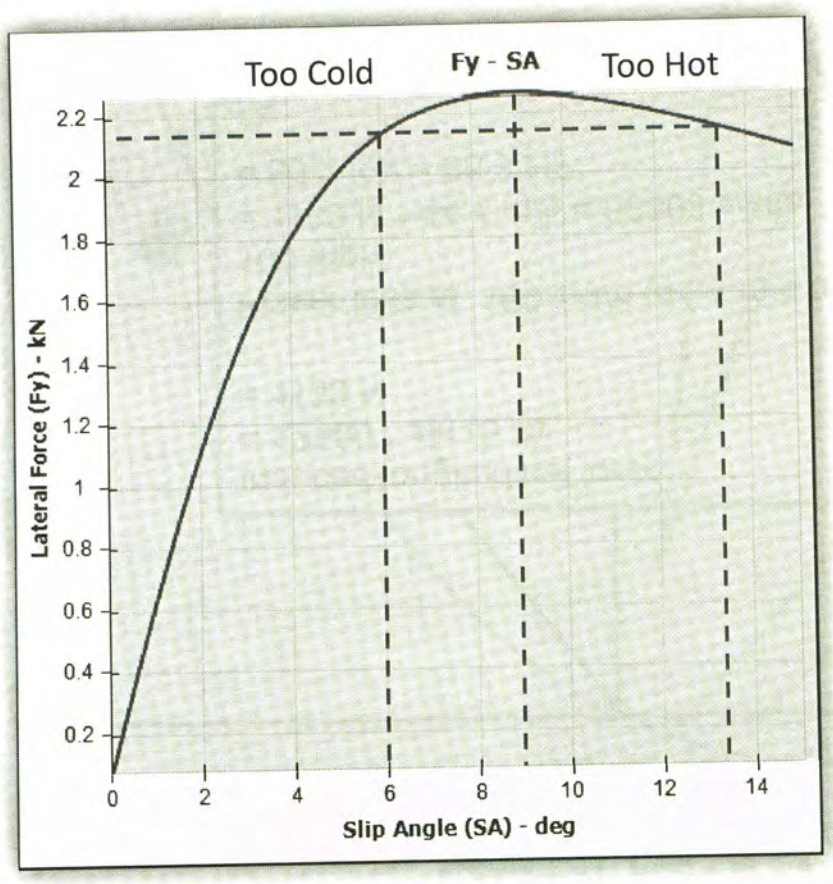
Induced longitudinal force
= 12500 * sin (13.1)
= 2833 N

Power loss at 160 km/h (44.4 m/s or 100 mph)
= 2833 N * 44.4 m/s = 125785 watts
= 125.8 Kw = 171 HP

Power loss at 320 km/h (88.8 m/s or 200 mph)
= 2833 N * 88.8 m/s = 251570 watts
= 251.6 Kw = 342 HP

And higher tire temperature

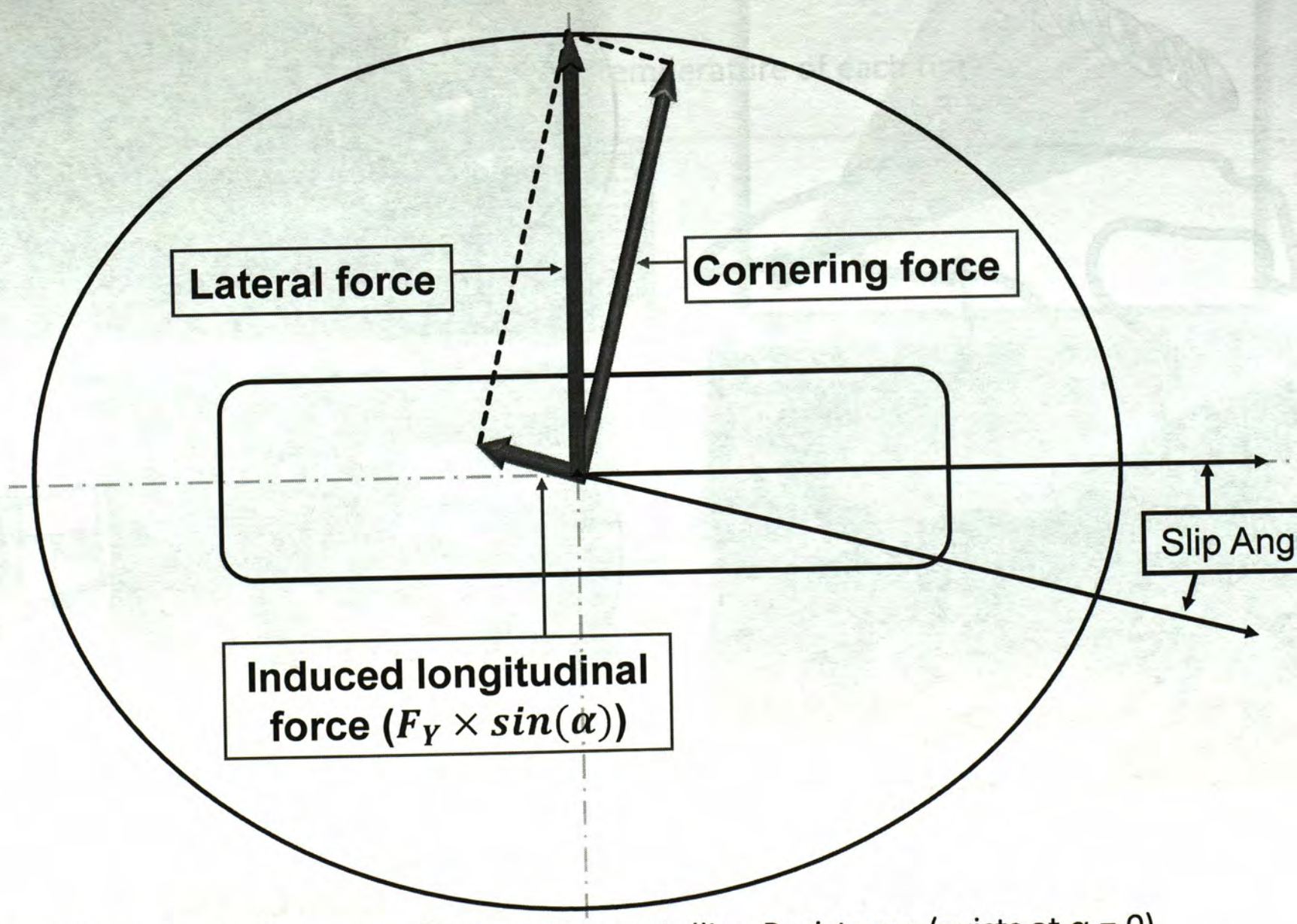
Same Lateral Force But Different Slip Angle



Note: Adapted SAE Sign Convention

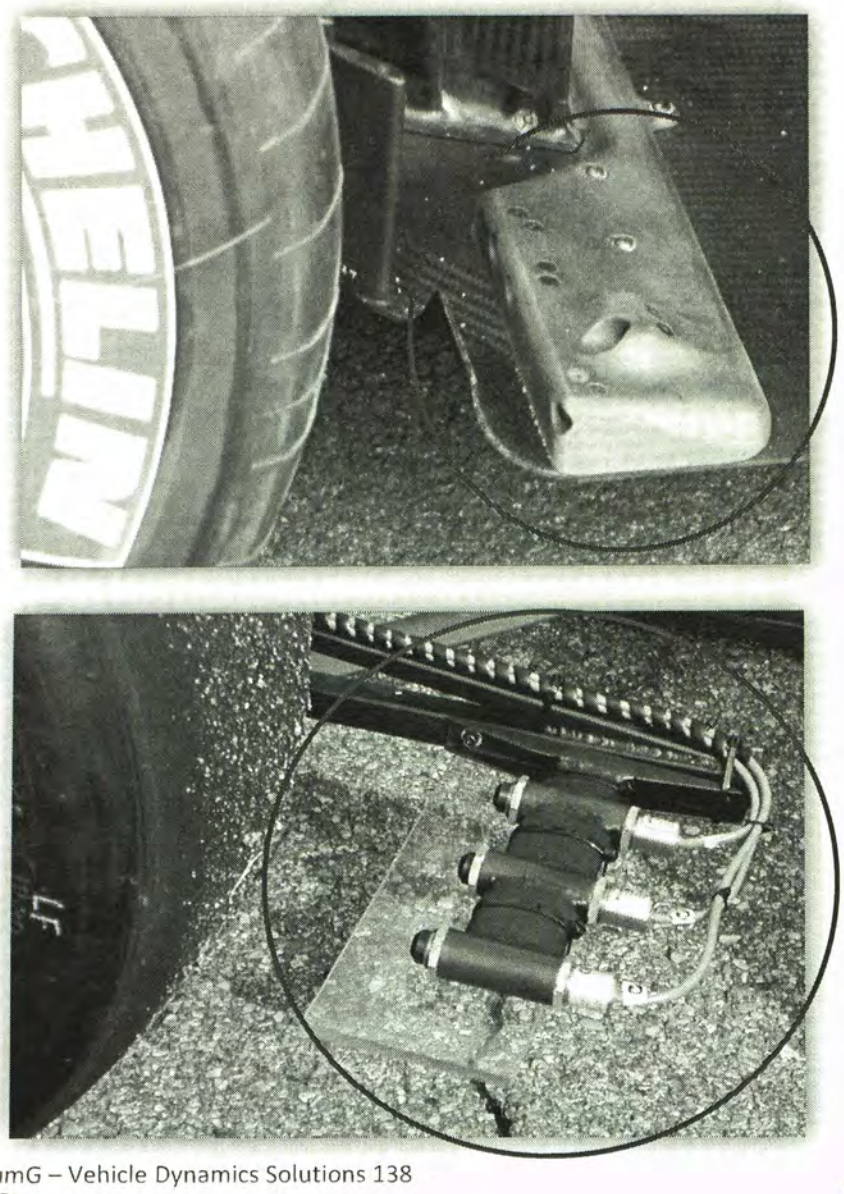
- Notice that the tire can generate the same lateral force at two different slip angles
- Operation at the higher slip angle will result in:
  - Increased induced drag
  - Increased tire temperature
  - Increased tire wear

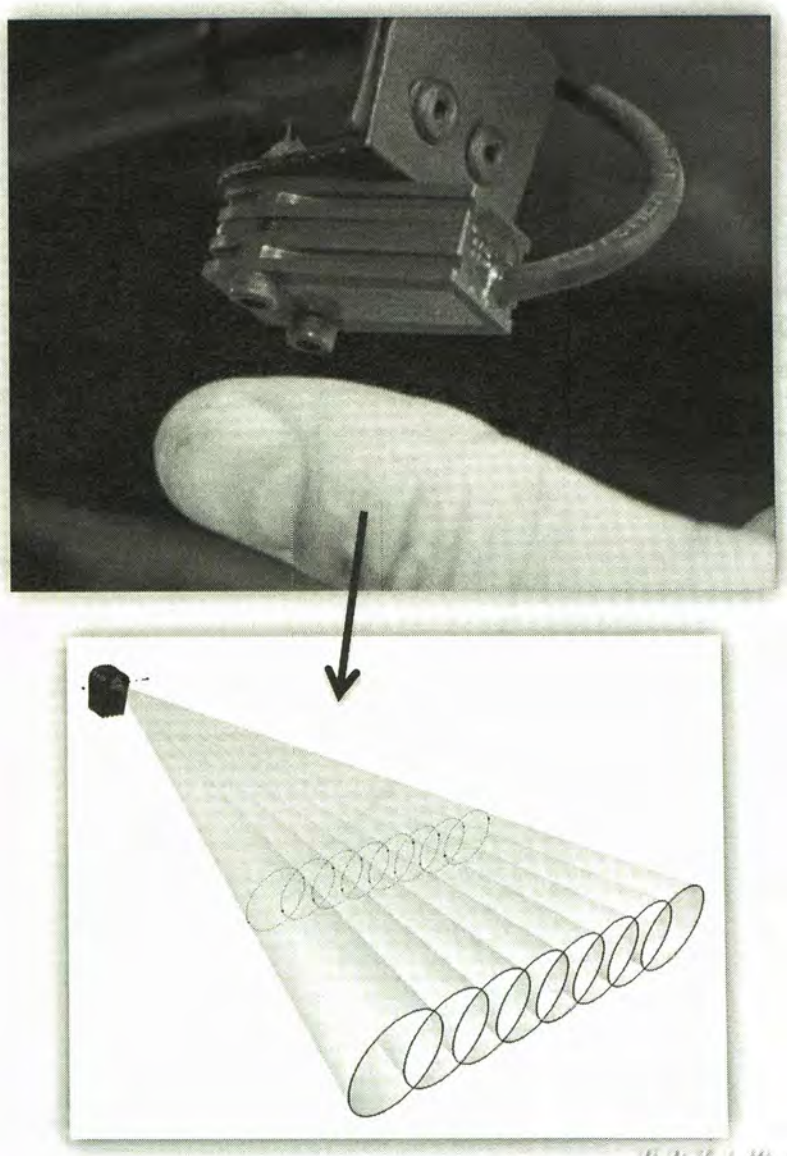
Tire Lateral Force and Cornering Force



Drag at the tire = Induced longitudinal force + Rolling Resistance (exists at $\alpha = 0$)

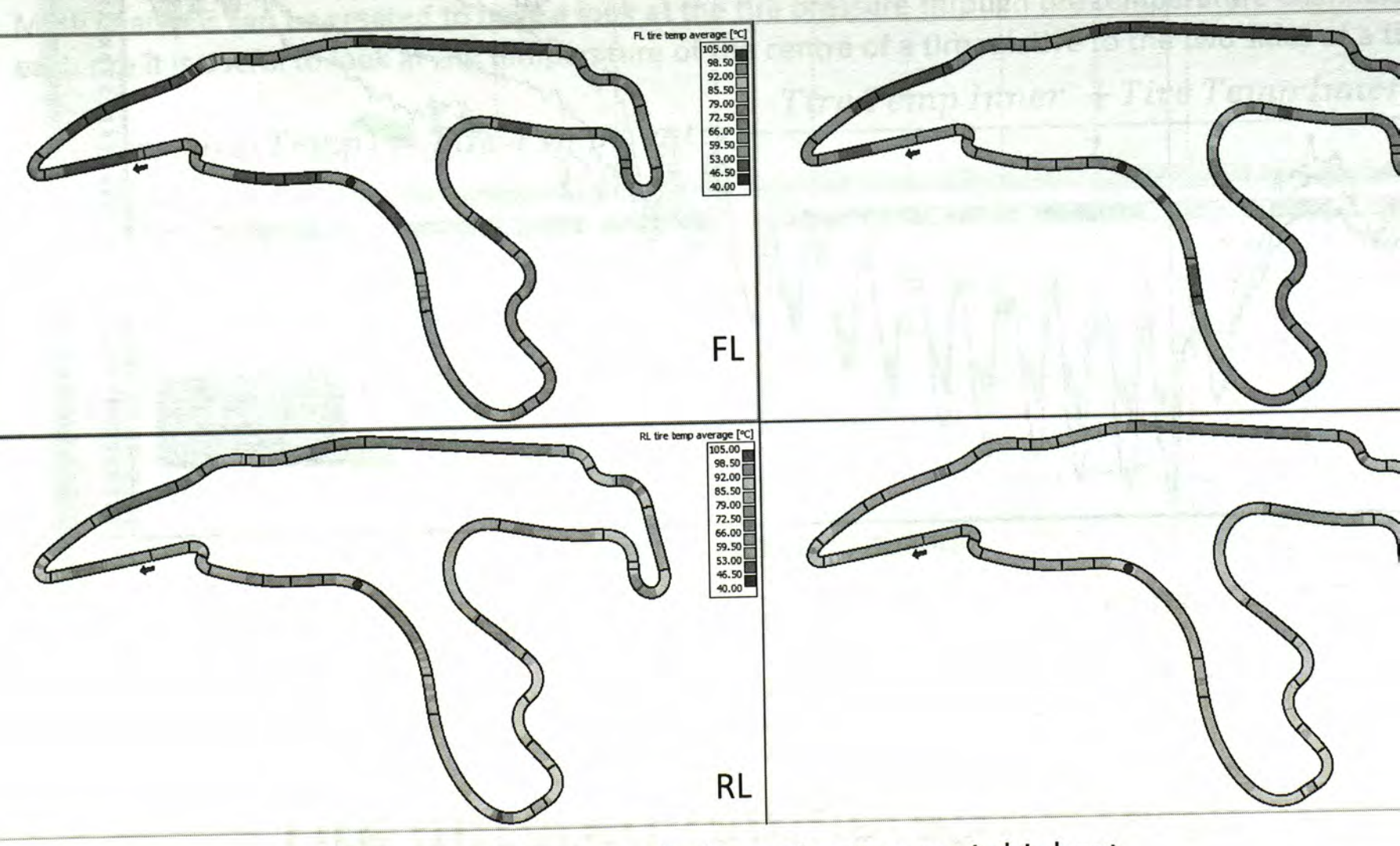
Infrared Tire Temperature Sensors





Tire Infrared Temperature Data

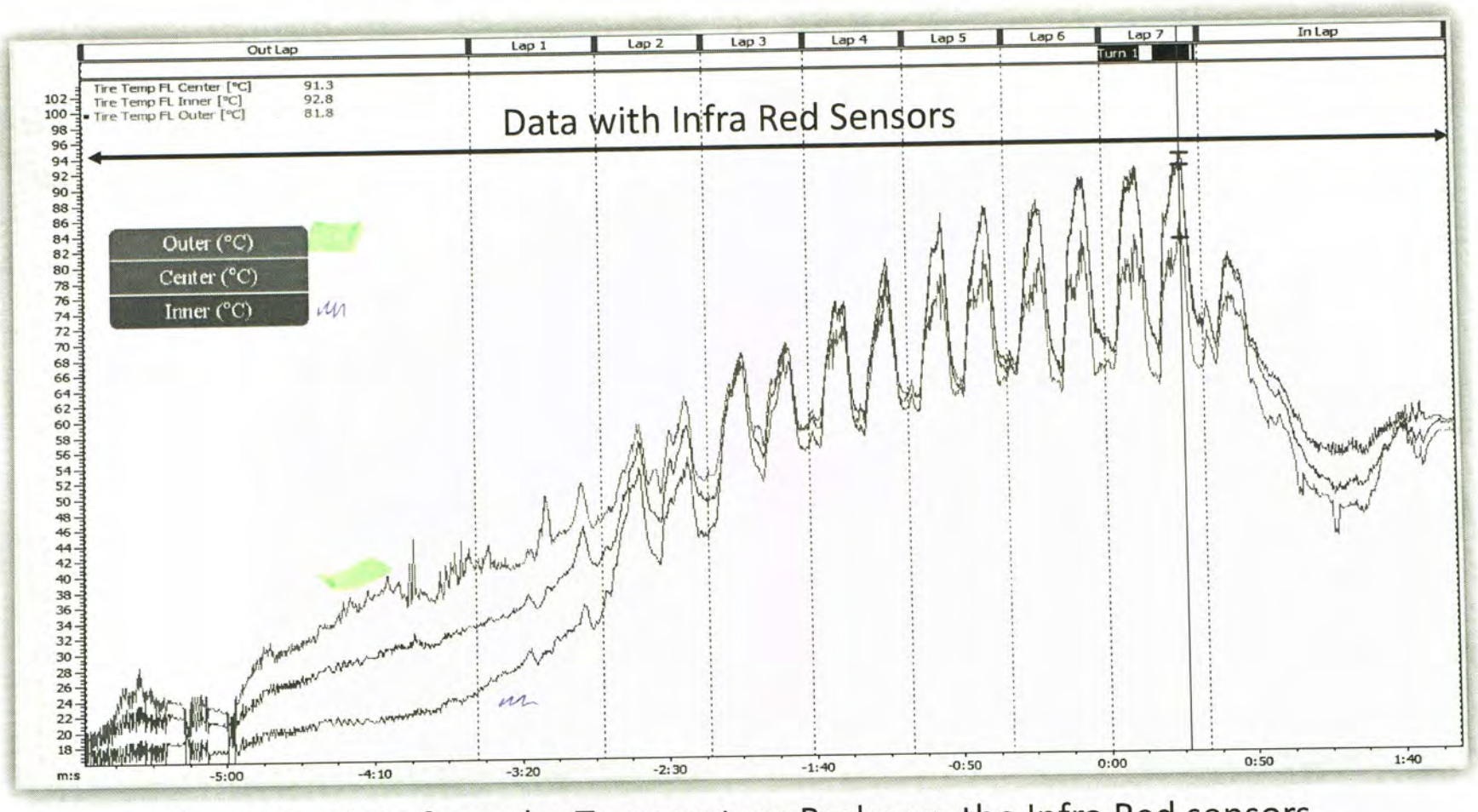
One Lap: Average temperature of each tire



Visualization of where the tire use is highest

Tire Infrared Temperature Data

Entire outing on an Oval: Front Left Tire

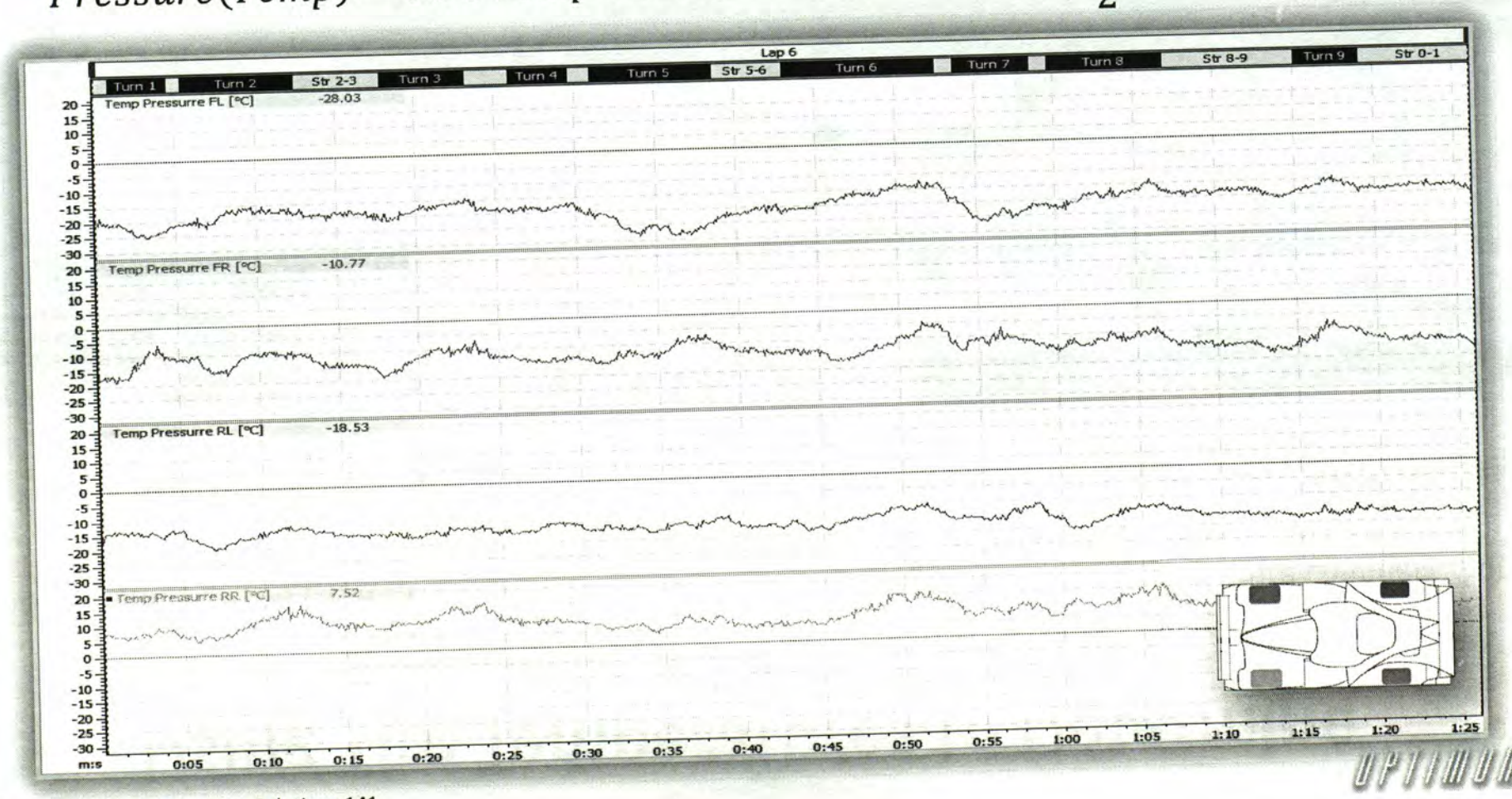


Given the data from the Temperature Probe vs. the Infra Red sensors, would you change your setup differently?

Using Tire Temperatures to Determine Appropriate Tire Pressure

Math channels can be created to have a look at the tire pressure through tire temperature channels. For each tire it is useful to look at the temperature of the centre of a tire relative to the two sides of a tire.

$$Pressure(Temp) = Tire\ Temp\ Center - \frac{Tire\ Temp\ Inner + Tire\ Temp\ \overset{Outer}{\cancel{Inner}}}{2}$$



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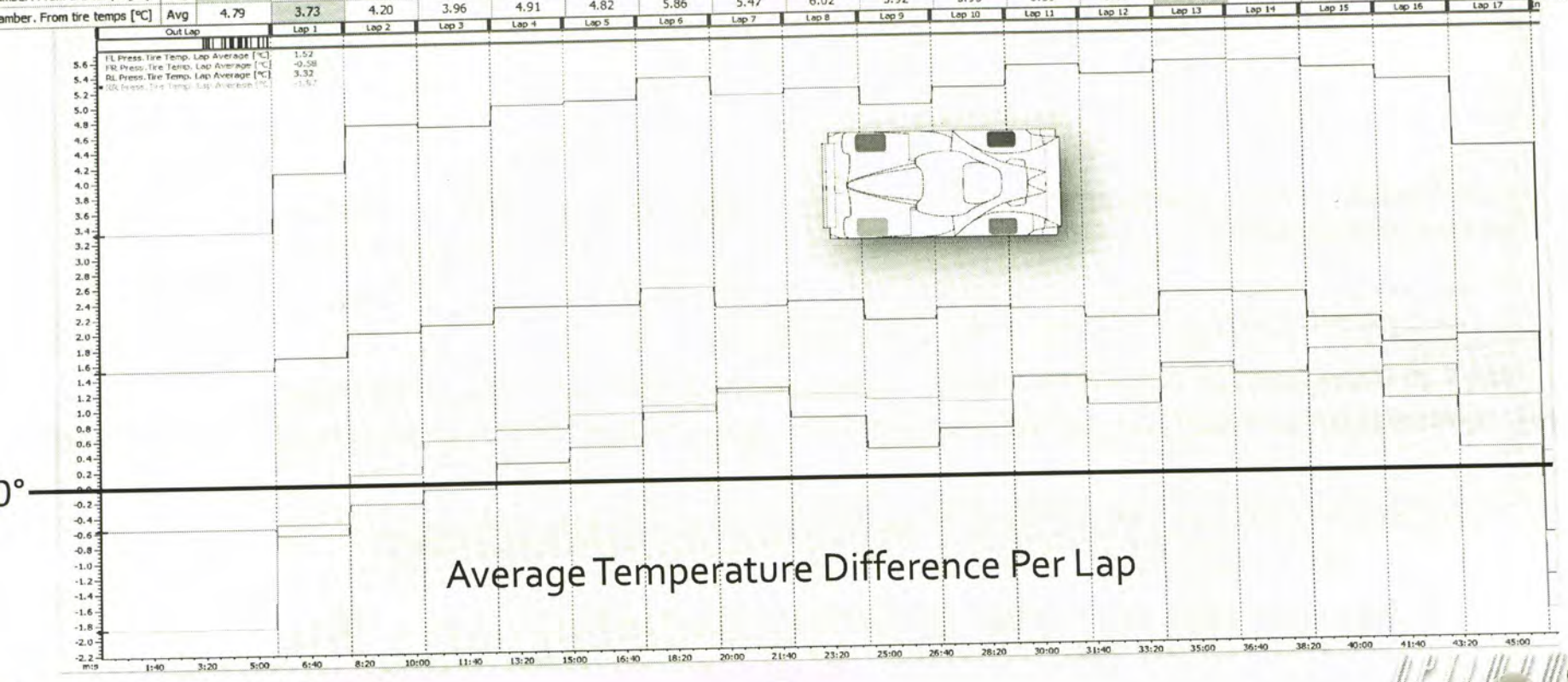
$$Pressure(Temp) = Tire\ Temp\ Center - \frac{Tire\ Temp\ Inner + Tire\ Temp\ Outer}{2}$$

		Out Lap [5:33.428]	Lap 1 [2:21.237]	Lap 2 [2:20.396]	Lap 3 [2:20.796]	Lap 4 [2:20.399]	Lap 5 [2:21.326]	Lap 6 [2:21.116]	Lap 7 [2:21.271]	Lap 8 [2:24.928]	Lap 9 [2:20.137]	Lap 10 [2:22.266]	Lap 11 [2:21.759]	Lap 12 [2:21.940]	Lap 13 [2:21.926]	Lap 14 [2:21.525]	Lap 15 [2:22.673]	Lap 16 [2:23.880]	Lap 17 [2:38.953]	In Lap [0:24.044]
mber. From tire temps [°C]	Avg	1.47	4.67	4.66	4.86	4.44	5.37	4.82	5.75	5.17	6.17	6.40	7.00	7.31	7.38	7.47	6.90	7.43	5.94	5.12
mber. From tire temps [°C]	Avg	1.39	1.66	1.21	0.60	1.11	0.72	1.40	0.45	2.07	2.49	2.73	1.83	2.49	1.94	2.26	2.09	1.93	3.46	2.60
mber. From tire temps [°C]	Avg	3.91	5.61	5.92	6.90	7.25	7.47	7.39	8.31	8.04	7.76	8.27	8.81	8.59	8.82	8.69	9.22	9.57	8.44	8.42
mber. From tire temps [°C]	Avg	4.79	3.73	4.20	3.96	4.91	4.82	5.86	5.47	6.02	5.92	5.95	6.39	6.30	6.62	6.53	6.15	6.28	5.62	5.12



Using Tire Temperatures to Determine Appropriate Camber

- Math channels can be created to have a look at the camber through tire temperature channels
- For each tire it is useful to look at the temperature of inside of the tire compared to the outside

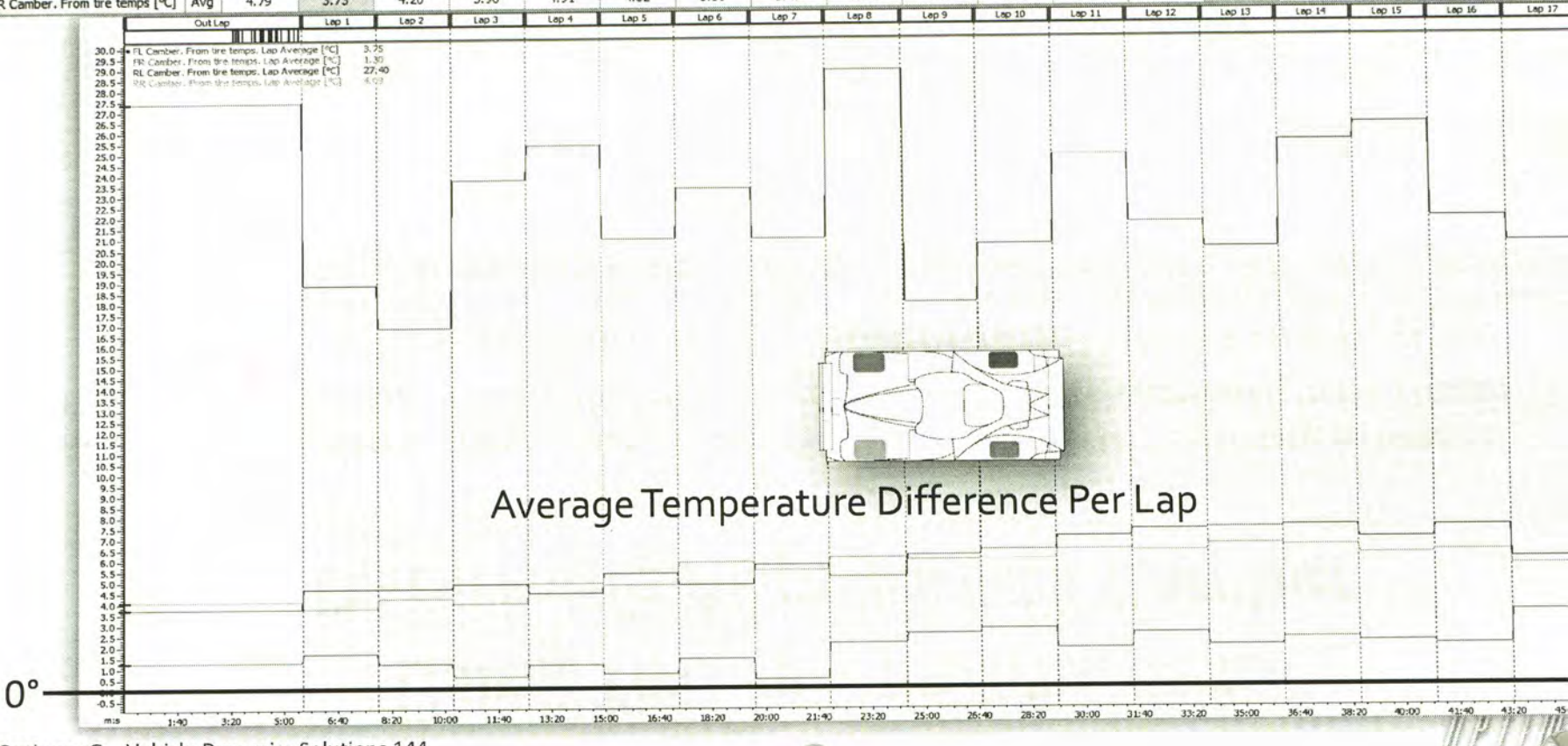
$$Camber\ FL(Temperature) = Tire\ Temp\ FL\ Inner - Tire\ Temp\ FL\ Outer$$

Using Tire Temperatures to Determine Appropriate Camber

- Math channels can be created to have a look at the camber through tire temperature channels
- For each tire it is useful to look at the temperature of inside of the tire compared to the outside

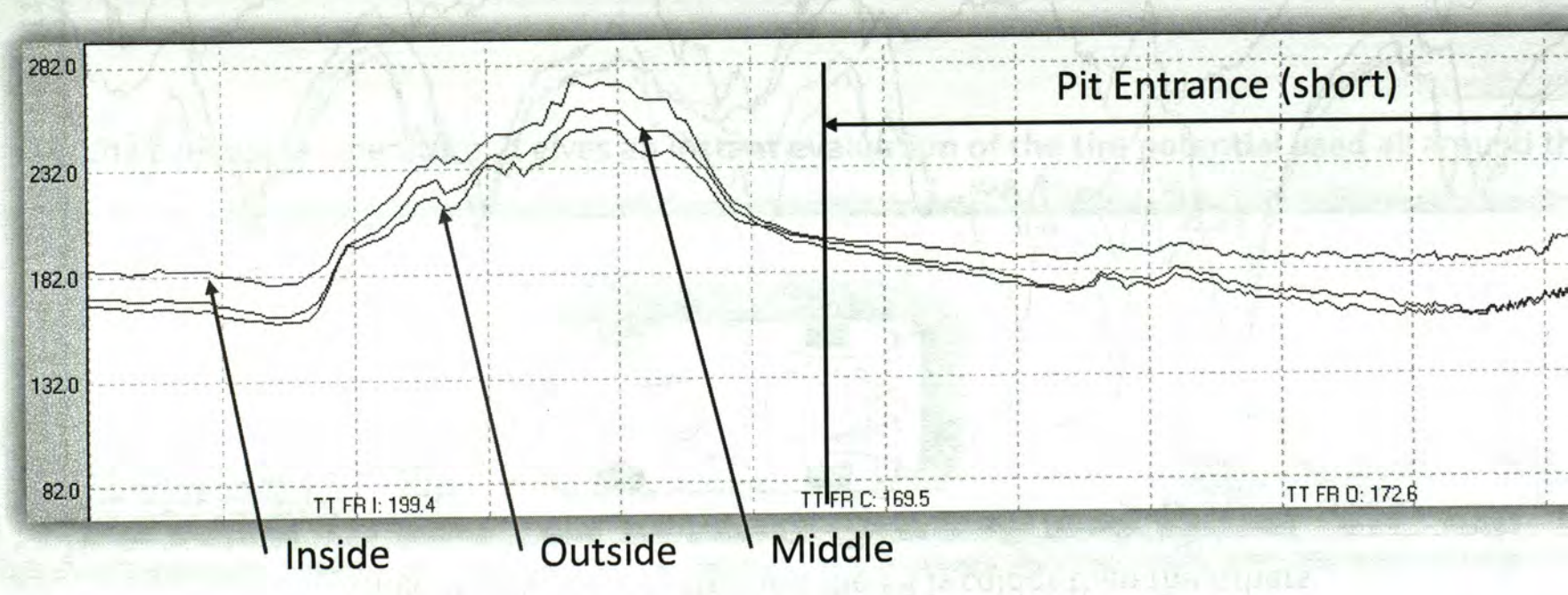
$$Camber\ FL(Temperature) = Tire\ Temp\ FL\ Inner - Tire\ Temp\ FL\ Outer$$

		Out Lap [5:33.428]	Lap 1 [2:21.237]	Lap 2 [2:20.396]	Lap 3 [2:20.796]	Lap 4 [2:20.399]	Lap 5 [2:21.326]	Lap 6 [2:21.116]	Lap 7 [2:21.271]	Lap 8 [2:24.928]	Lap 9 [2:20.137]	Lap 10 [2:22.266]	Lap 11 [2:21.759]	Lap 12 [2:21.940]	Lap 13 [2:21.926]	Lap 14 [2:21.525]	Lap 15 [2:22.673]	Lap 16 [2:23.880]	Lap 17 [2:38.953]
L Camber. From tire temps [°C]	Avg	1.47	4.67	4.66	4.86	4.44	5.37	4.82	5.75	5.17	6.17	6.40	7.00	7.31	7.38	7.47	6.90	7.43	5.94
R Camber. From tire temps [°C]	Avg	1.39	1.66	1.21	0.60	1.11	0.72	1.40	0.45	2.07	2.49	2.73	1.83	2.49	1.94	2.26	2.09	1.93	3.46
L Camber. From tire temps [°C]	Avg	3.91	5.61	5.92	6.90	7.25	7.47	7.39	8.31	8.04	7.76	8.27	8.81	8.59	8.82	8.69	9.22	9.57	8.44
R Camber. From tire temps [°C]	Avg	4.79	3.73	4.20	3.96	4.91	4.82	5.86	5.47	6.02	5.92	5.95	6.39	6.30	6.62	6.53	6.15	6.28	5.62



IR Temperature Data vs. Temperature Probe

Oval Example: Front Right Tire (Static Negative Camber)



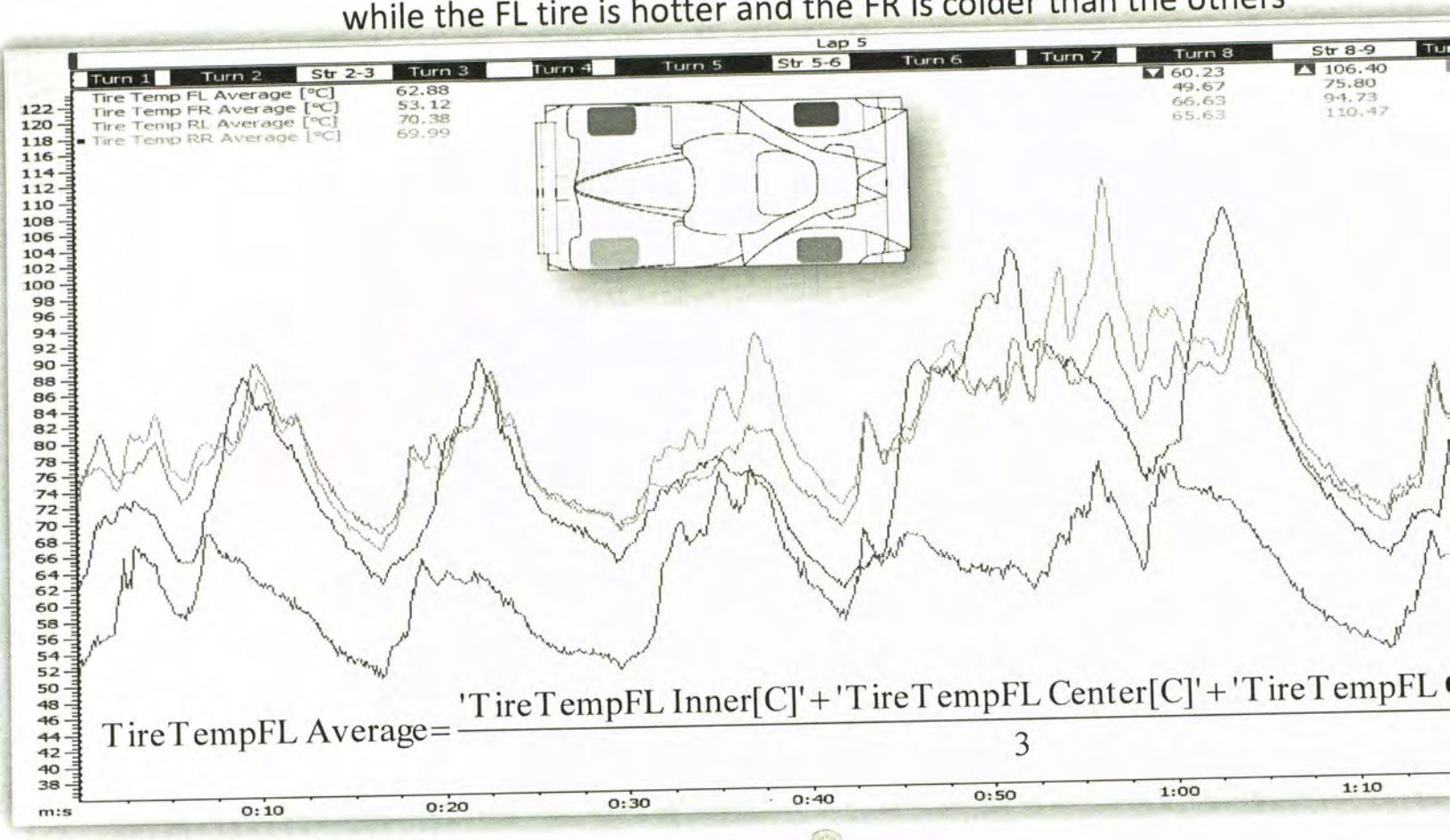
If the temperatures were measured immediately the vehicle stopped it would appear that the tire needs a little less negative camber.

In fact it is close to perfect in the corner.

Using Tire Temperatures to Determine the Average Temperature of each Tire

Looking at it gives an **instant evaluation of the tire potential used** all around the ca

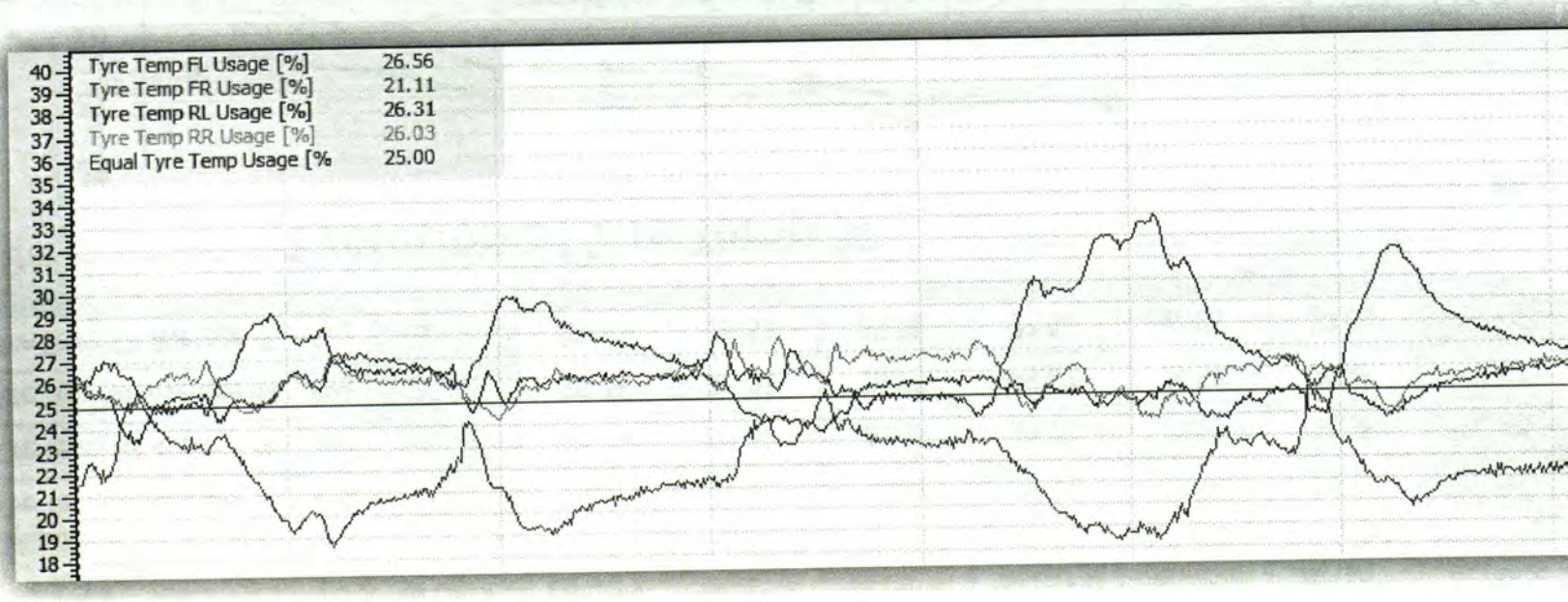
For this outing, the RR and RL tires seems to be used similarly, while the FL tire is hotter and the FR is colder than the others



$$\text{TireTempFL Average} = \frac{\text{'TireTempFL Inner[C]'} + \text{'TireTempFL Center[C]'} + \text{'TireTempFL}}{3}$$

Using Tire Temperatures to Determine the Usage of each Tire

Like the average temperature, it gives an **instant evaluation of the tire potential used** all



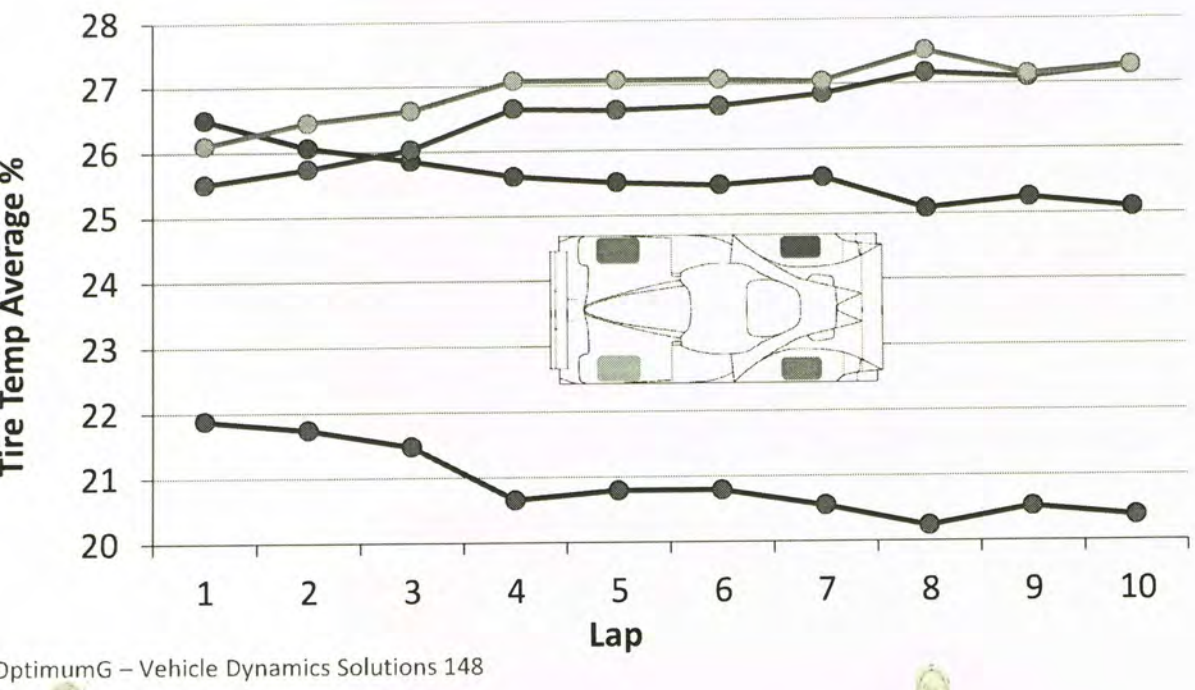
$$Tire\ Temp\ FL\ Usage = \frac{Tire\ Temp\ FL\ Inner + Tire\ Temp\ FL\ Center + Tire\ Tem}{Sum\ of\ all\ tire\ temperatures}$$

Tire Temperatures - % of each Tire vs. Total

Evolution During a Run

		M Lap 1 [1:25.017]	O Lap 2 [1:24.456]	1 Lap 3 [1:24.323]	2 Lap 4 [1:25.083]	3 Lap 5 [1:25.249]	4 Lap 6 [1:25.605]	5 Lap 7 [1:25.961]	6 Lap 8 [1:25.765]	7 Lap 9 [1:27.529]	8 Lap 10 [1:26.007]
Tire Temp FL Average %	Avg	26.50	26.07	25.86	25.61	25.51	25.46	25.57	25.08	25.25	25.09
Tire Temp FR Average %	Avg	21.88	21.74	21.48	20.65	20.79	20.79	20.55	20.23	20.52	20.38
Tire Temp RL Average %	Avg	25.51	25.74	26.03	26.65	26.62	26.67	26.85	27.17	27.09	27.26
Tire Temp RR Average %	Avg	26.10	26.45	26.63	27.08	27.08	27.08	27.04	27.51	27.14	27.27

Evolution of Tire Temp %

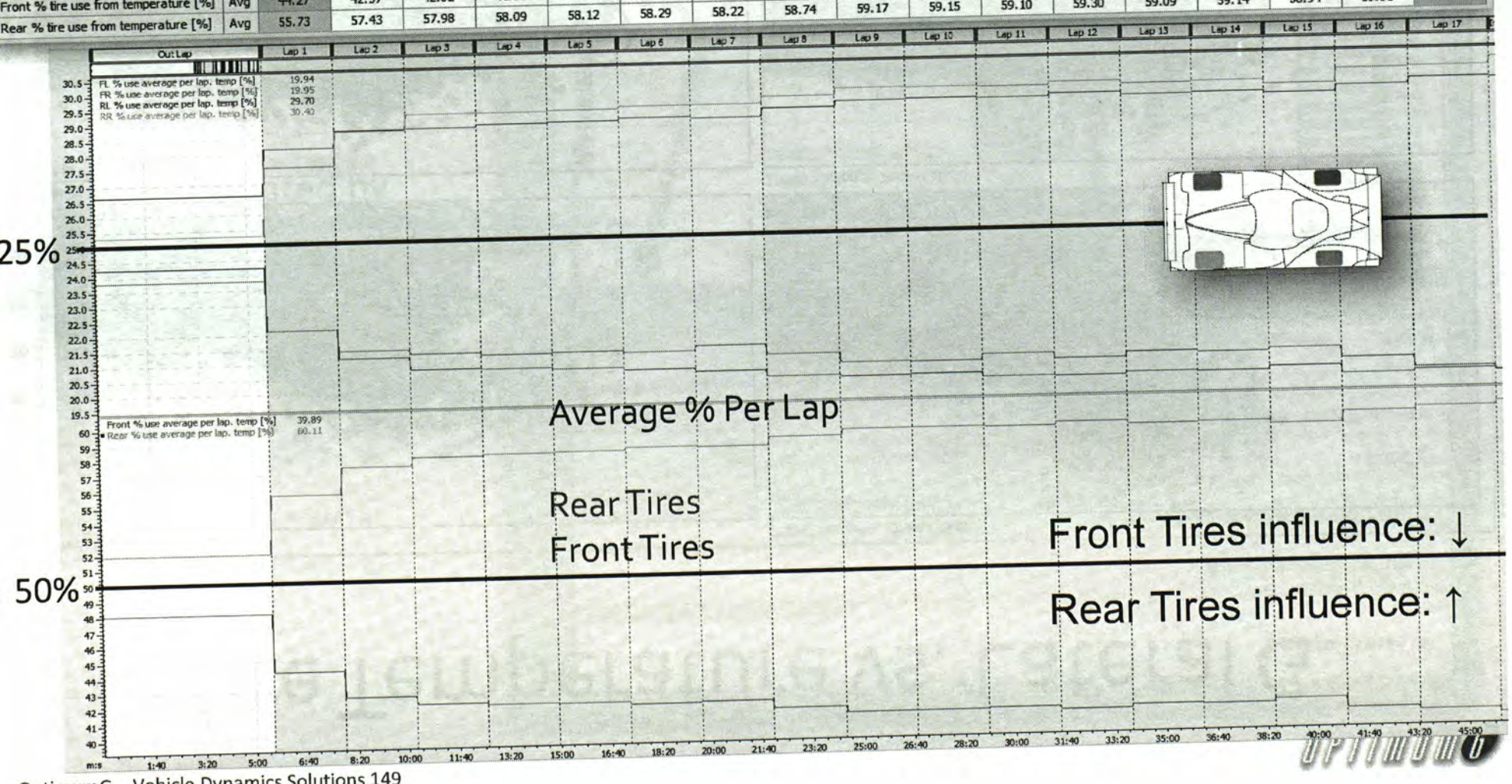


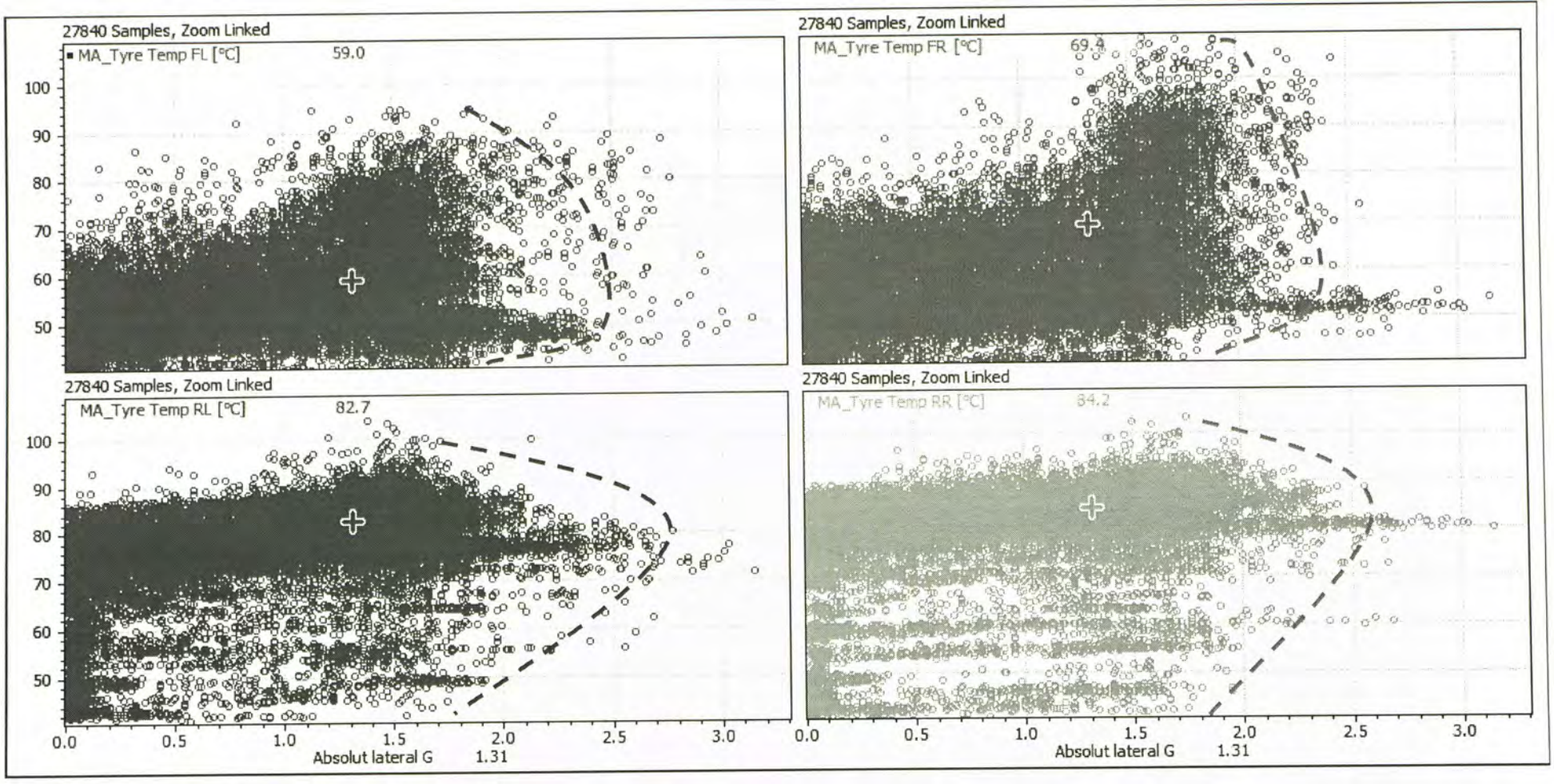
Front Tires influence: ↓

Rear Tires influence: ↑

Tire Temperatures - % of each Tire vs. Total

		Lap 1 [2:21.237]	Lap 2 [2:20.396]	Lap 3 [2:20.796]	Lap 4 [2:20.399]	Lap 5 [2:21.326]	Lap 6 [2:21.116]	Lap 7 [2:21.271]	Lap 8 [2:24.928]	Lap 9 [2:20.137]	M Lap 10 [2:22.266]	Lap 11 [2:21.759]	Lap 12 [2:21.940]	Lap 13 [2:21.926]	Lap 14 [2:21.525]	Lap 15 [2:22.673]	Lap 16 [2:23.880]	Lap 17 [2:38.953]
FL % tire use from temperature [%]	Avg	22.13	21.16	20.74	20.73	20.67	20.57	20.46	20.25	20.18	20.23	20.07	20.06	20.13	20.11	20.22	19.92	20.02
FR % tire use from temperature [%]	Avg	22.14	21.41	21.28	21.18	21.21	21.15	21.33	21.00	20.65	20.63	20.83	20.64	20.79	20.74	20.84	20.47	20.11
RL % tire use from temperature [%]	Avg	28.16	28.73	28.74	28.85	28.85	28.95	28.89	29.18	29.37	29.39	29.37	29.45	29.37	29.38	29.31	29.52	29.73
RR % tire use from temperature [%]	Avg	27.56	28.70	29.24	29.24	29.27	29.33	29.33	29.56	29.80	29.76	29.74	29.85	29.72	29.76	29.63	30.09	30.13
Front % tire use from temperature [%]	Avg	44.27	42.57	42.02	41.91	41.88	41.71	41.78	41.26	40.83	40.85	40.90	40.70	40.91	40.86	41.06	40.39	40.14
Rear % tire use from temperature [%]	Avg	55.73	57.43	57.98	58.09	58.12	58.29	58.22	58.74	59.17	59.15	59.10	59.30	59.09	59.14	58.94	59.61	59.86



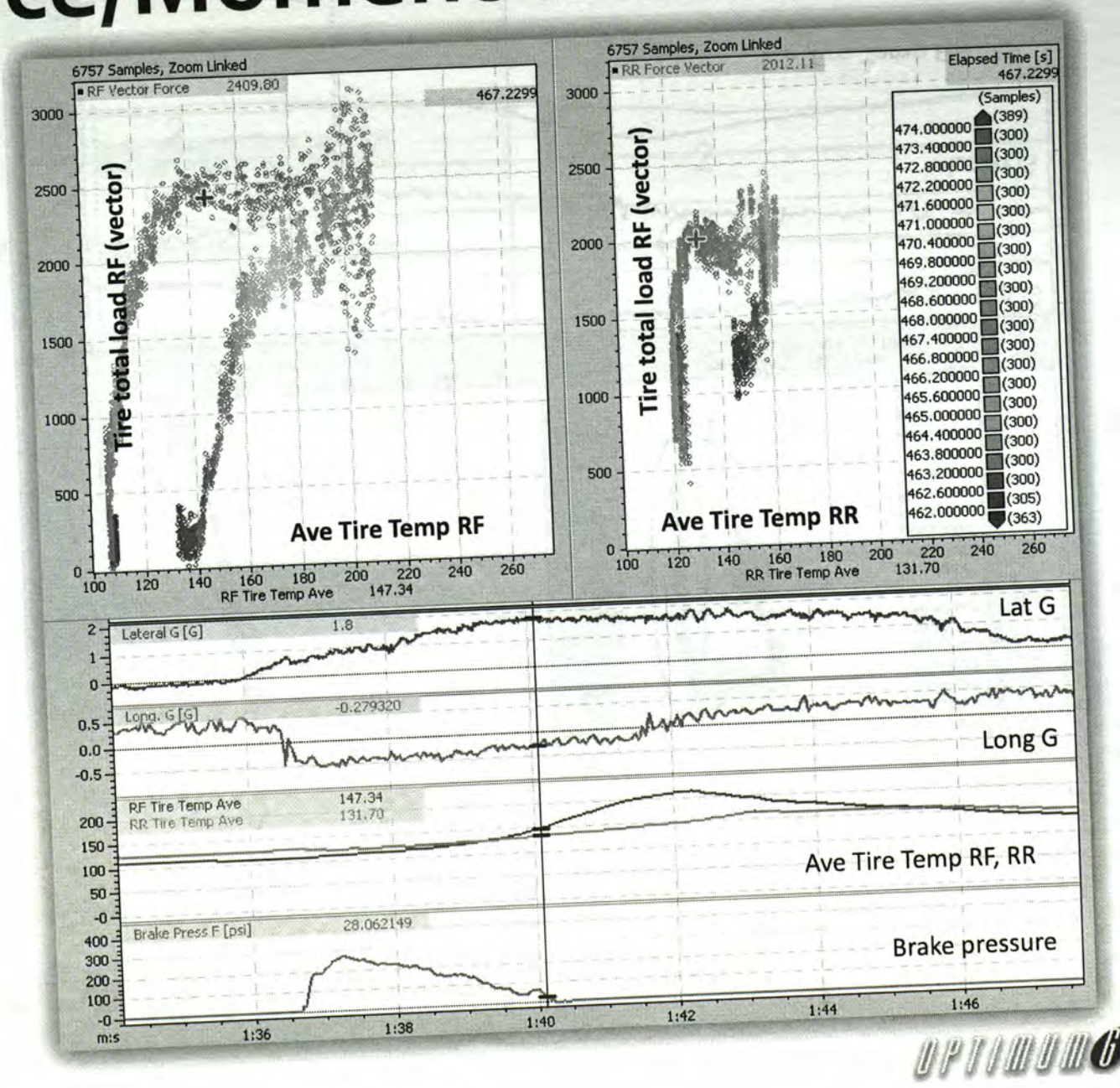


Wheel Force/Moment Transducer

By looking **at tire forces** a better understanding of **temperature build-up** can be realized

Data points colored by corner progress (blue = entry, red = exit)

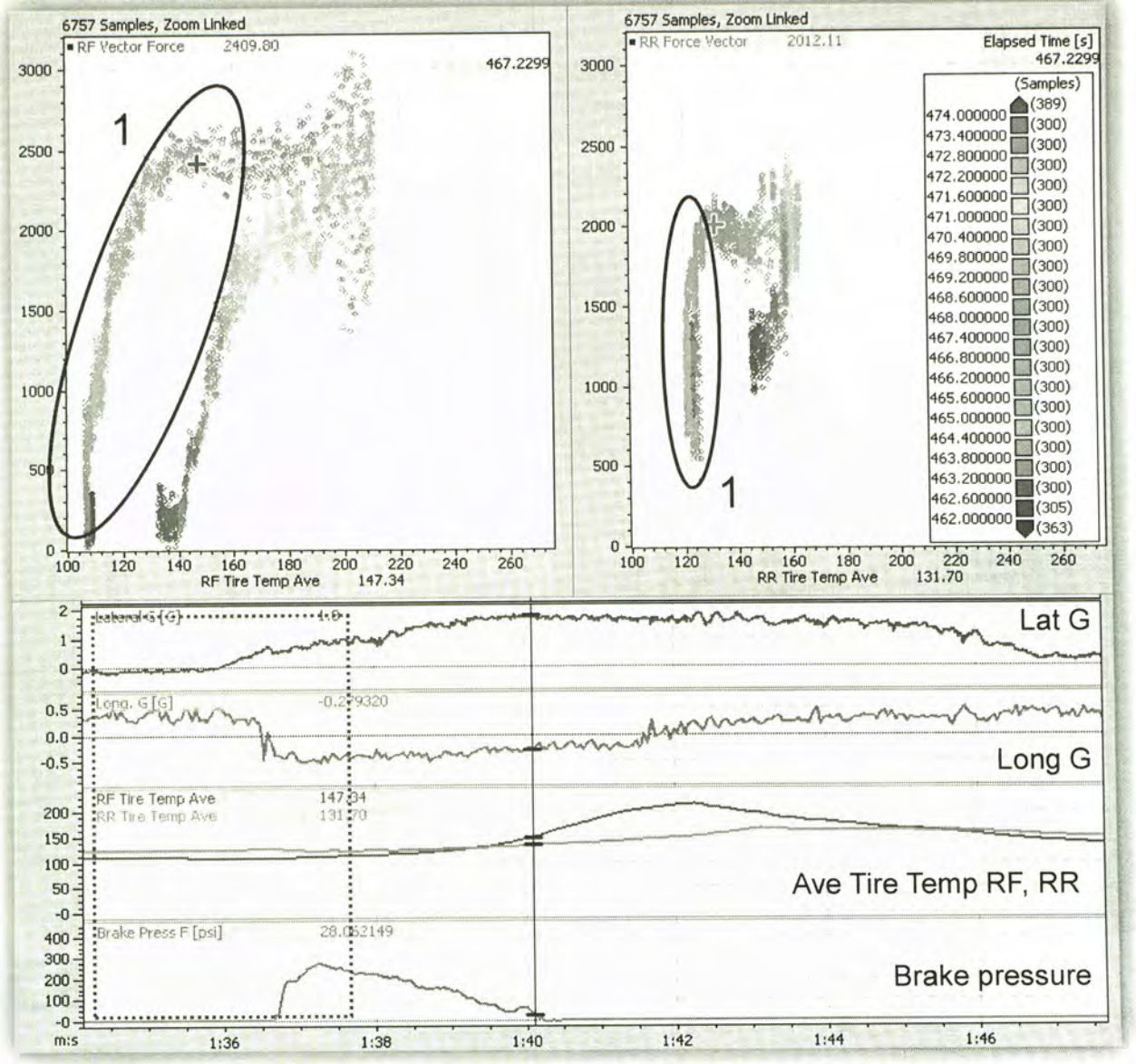
$y = \sqrt{F_x^2 + F_y^2}$



By looking at tire forces a better understanding of temperature build-up can be realized

Example – First corner on oval:

Braking, fronts build temperature faster



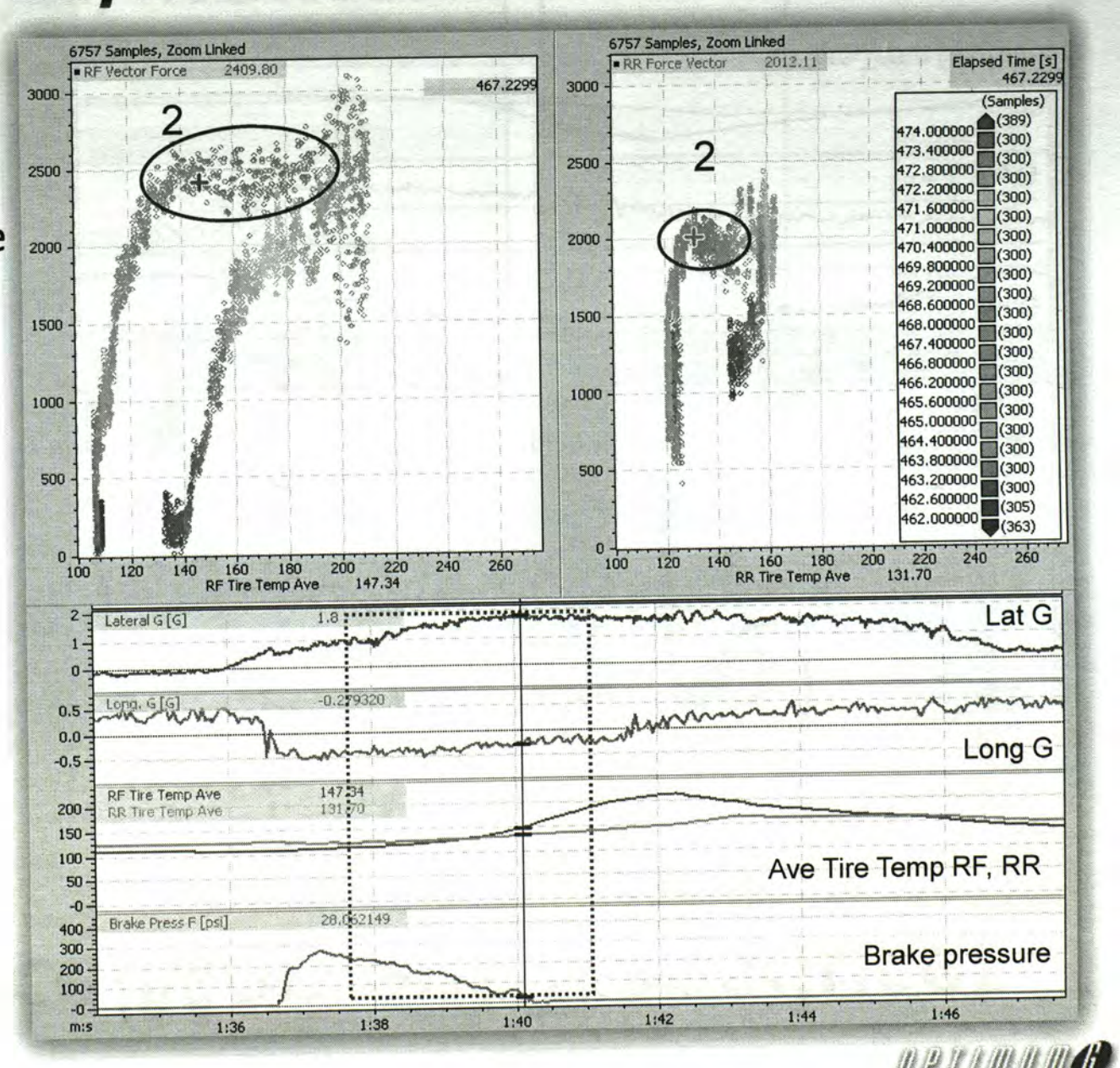
Wheel Force/Moment Transducer

By looking at tire forces a better understanding of temperature build-up can be realized

Example – First corner on oval:

Braking, fronts build temperature faster

Turn-in, rear start to build



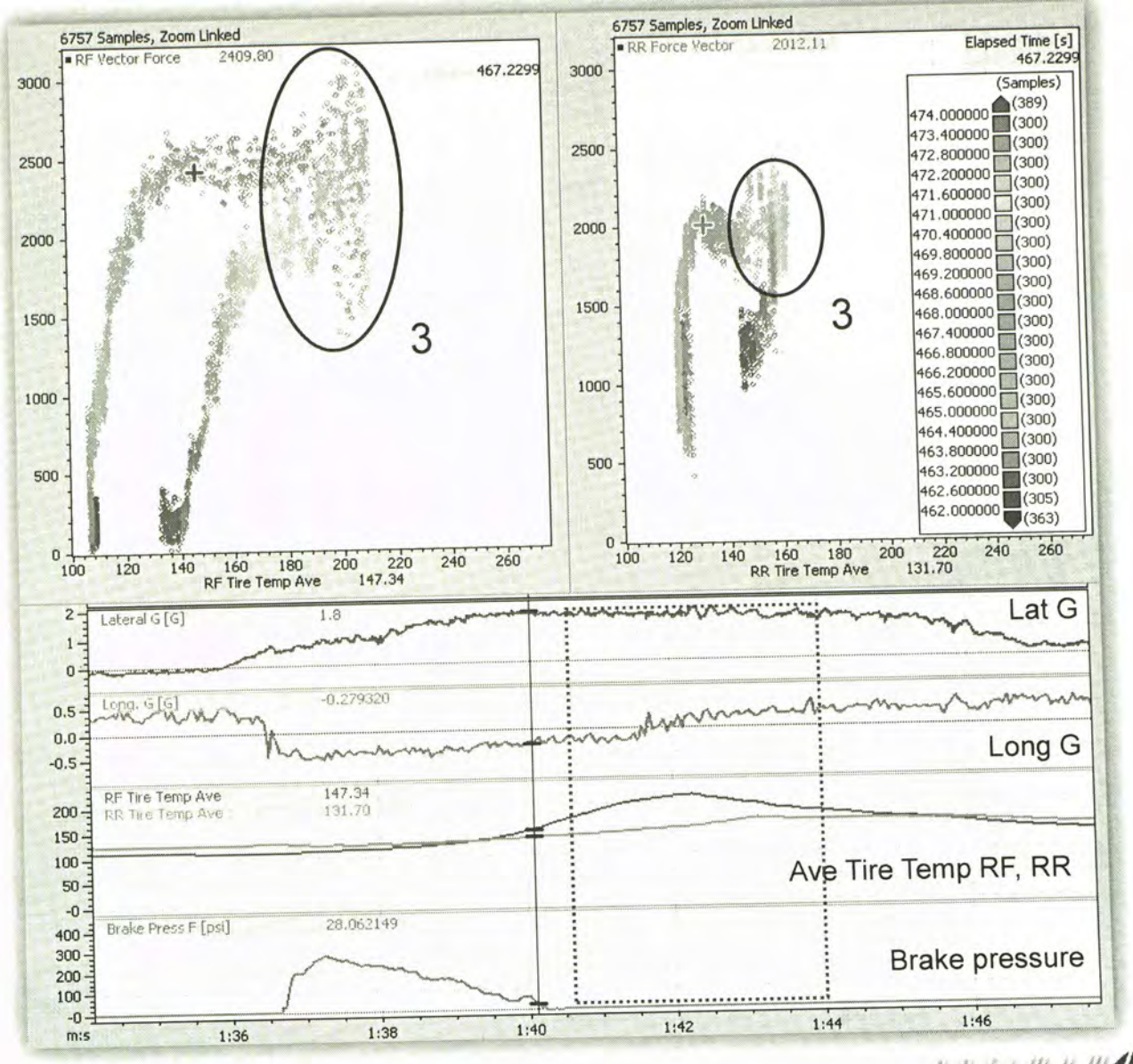
By looking at tire forces a better understanding of temperature build-up can be realized

Example – First corner on oval:

Braking, fronts build temperature faster

Turn-in, rear start to build

Mid corner, tire loads increase as temperature increases



Wheel Force/Moment Transducer

By looking at tire forces a better understanding of temperature build-up can be realized

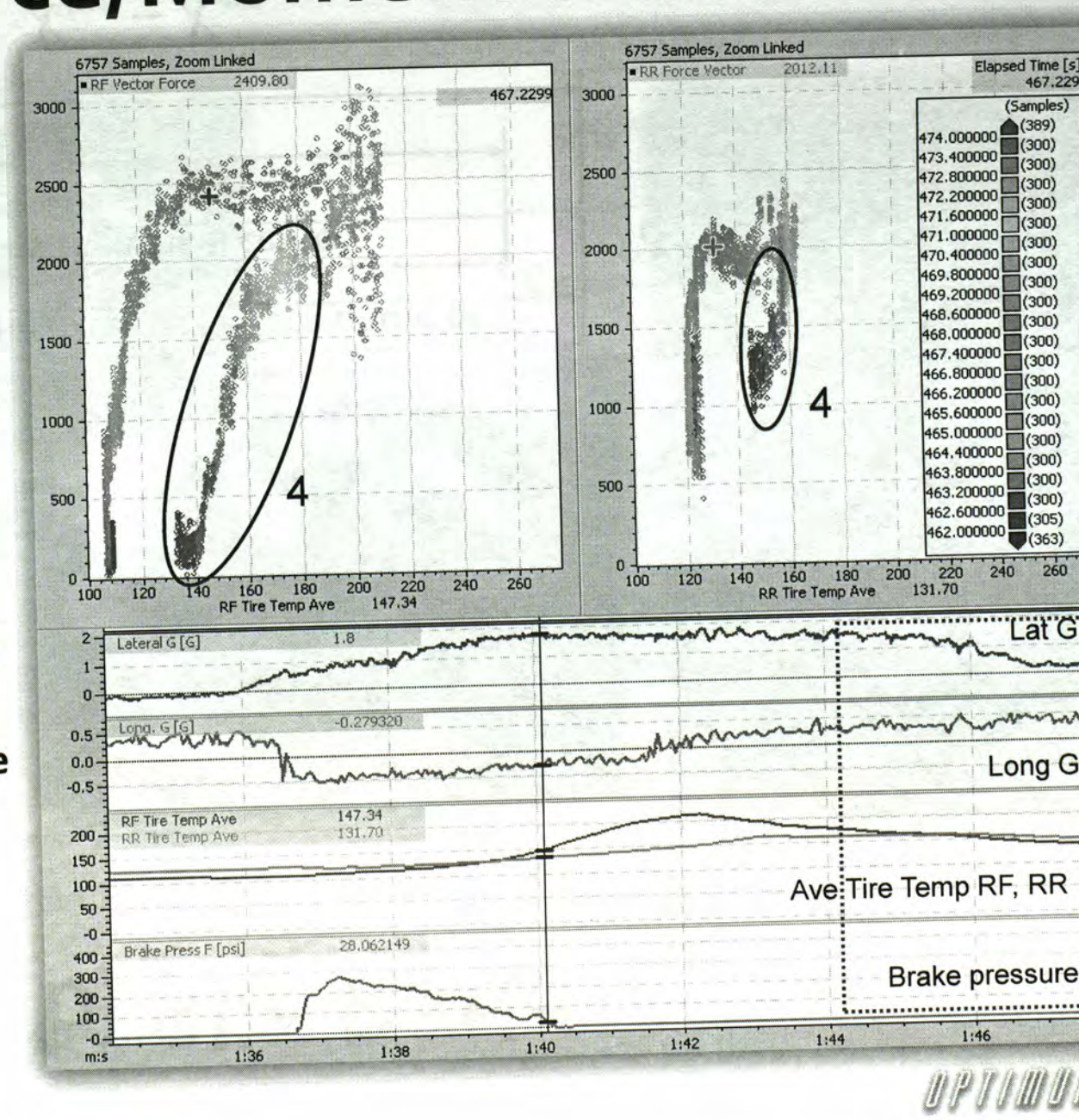
Example – First corner on oval:

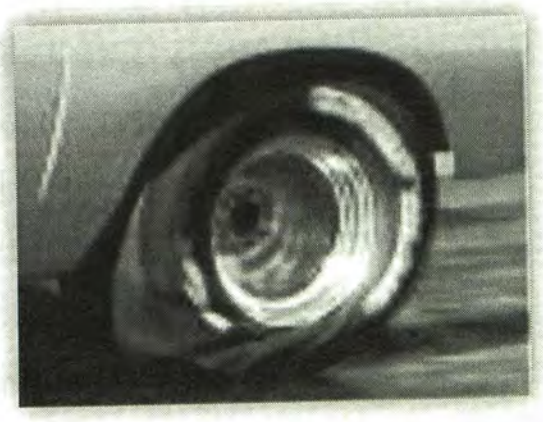
Braking, fronts build temperature faster

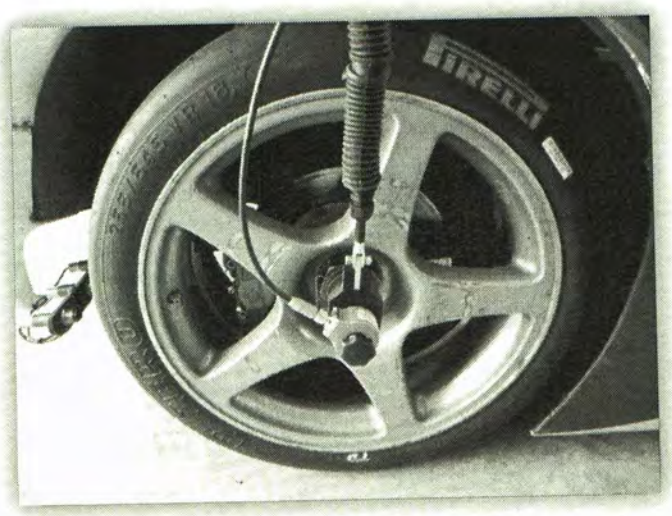
Turn-in, rear start to build

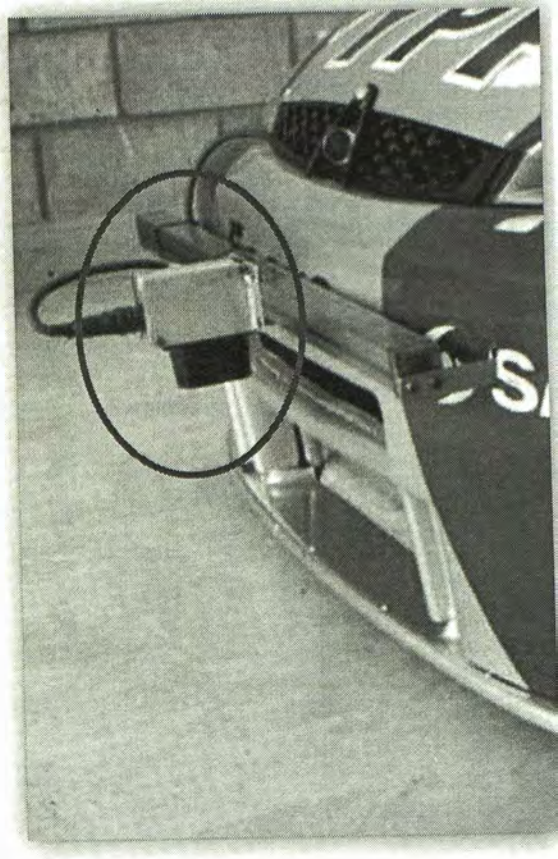
Mid corner, tire loads increase as temperature increases

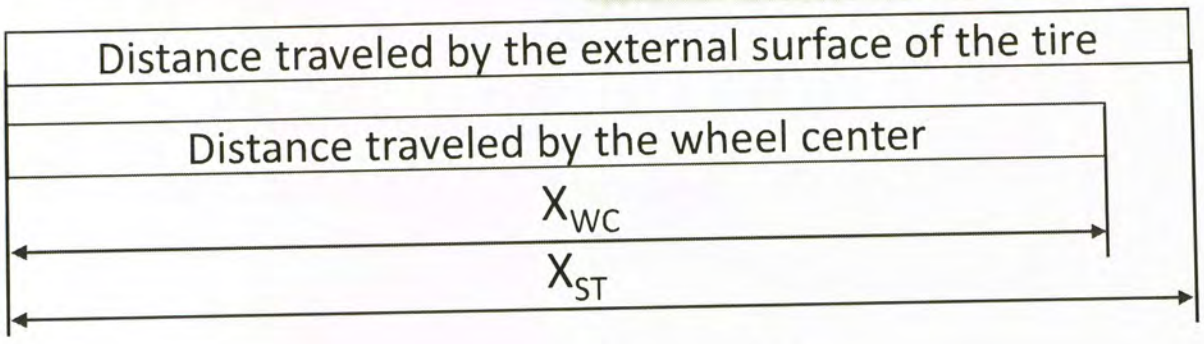
Corner exit, front temperature falls, rear stabilizes due to drive torque





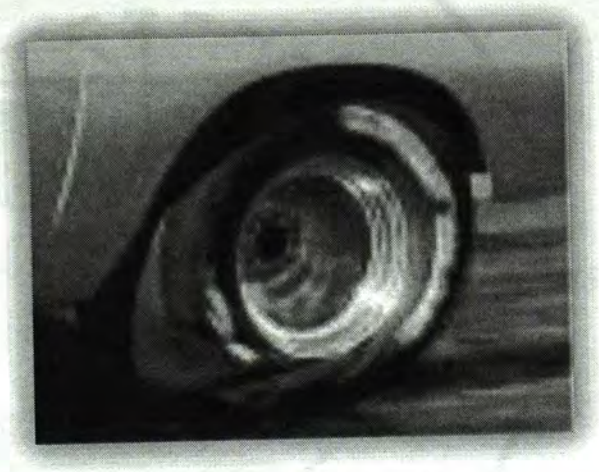






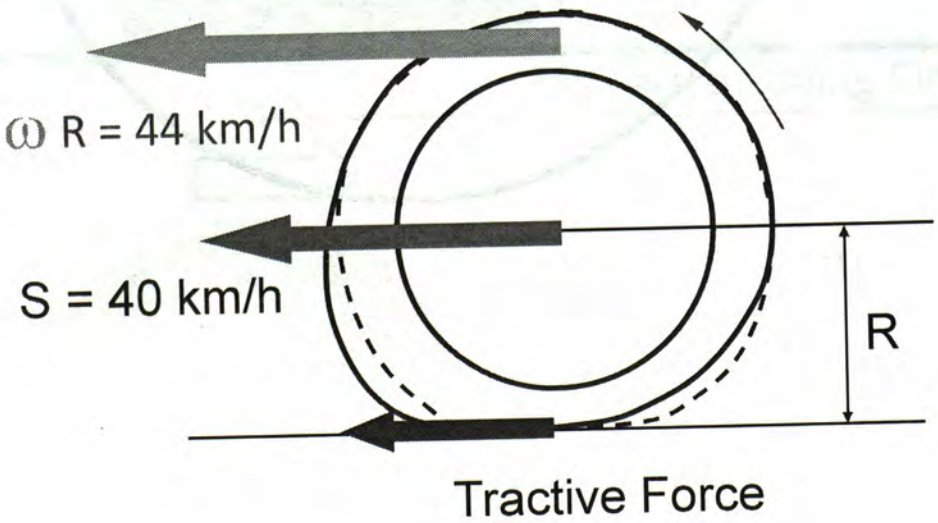
$$Sr = \frac{X_{WC} - X_{ST}}{X_{WC}}$$

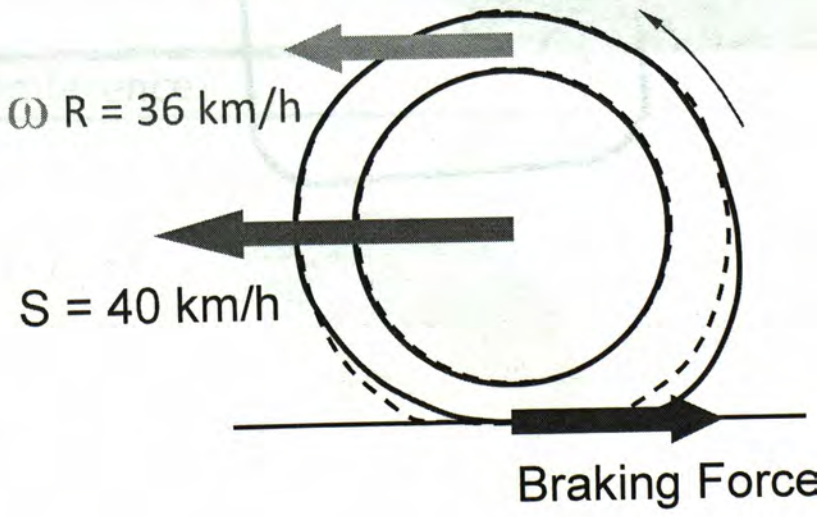
Slip Ratio



Sr is 'Slip Ratio' :

$$Sr = \frac{S - \omega \cdot R_e}{S}$$





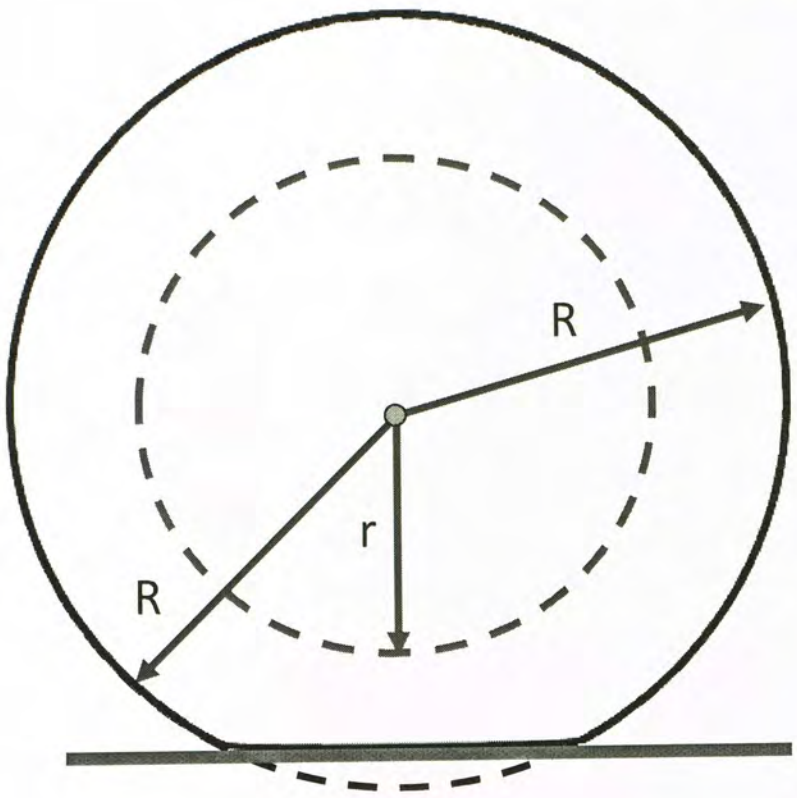
Sr = (44 - 40) / 40 = 10 %

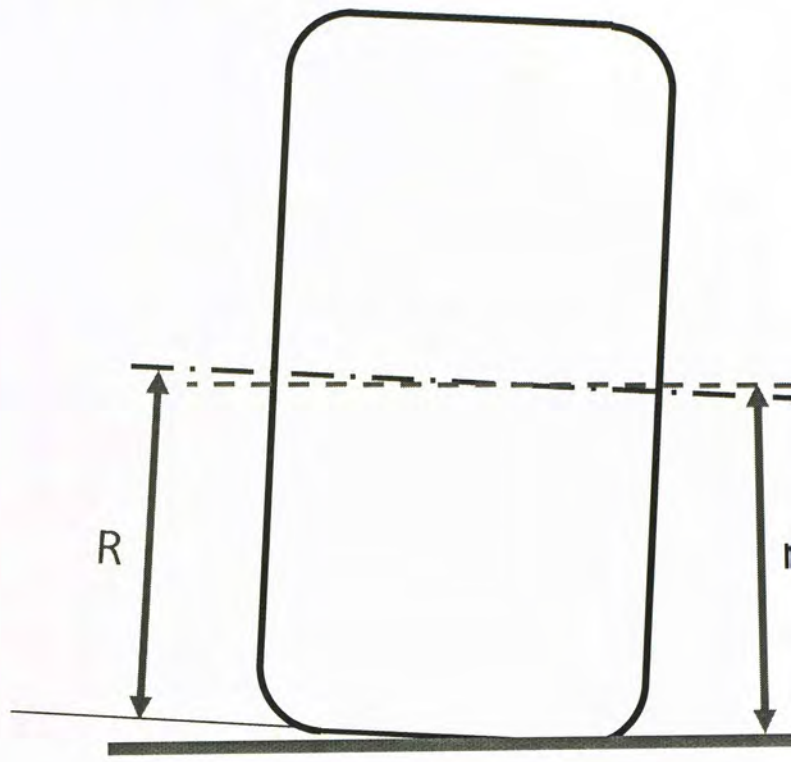
Sr = (40 - 36) / 40 = -10 %

Don't make the mistake between rolling radius and loaded radius !!

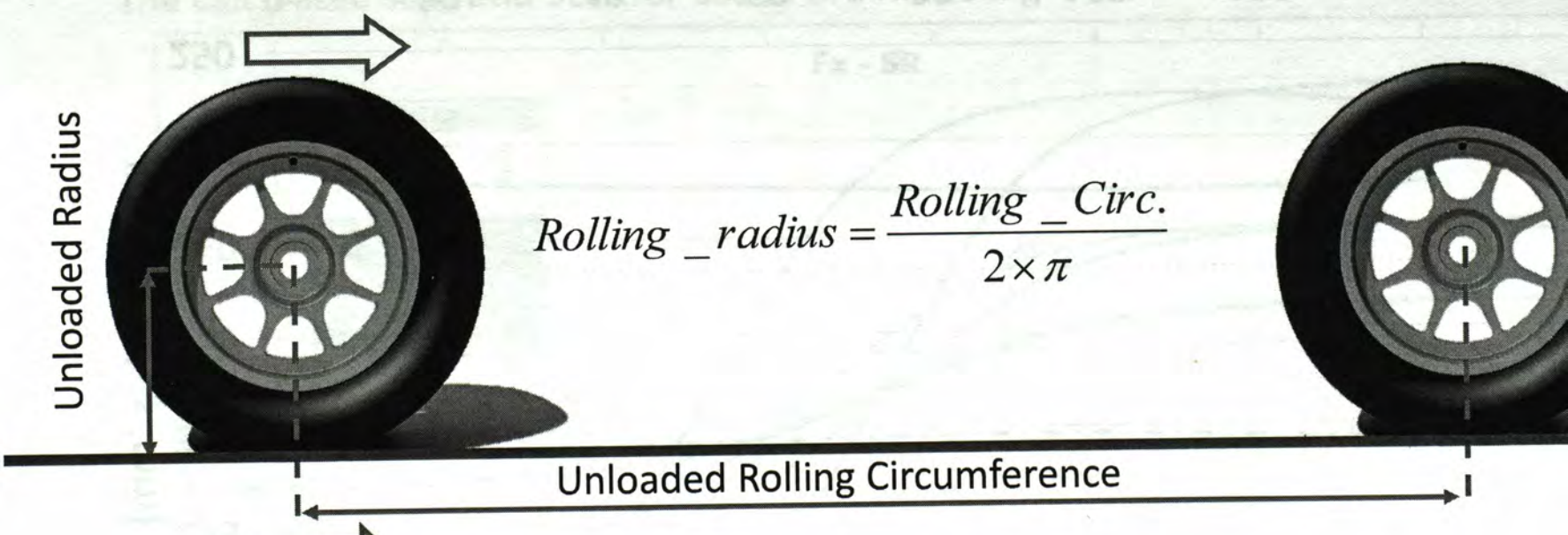
R: rolling radius

r: loaded radius

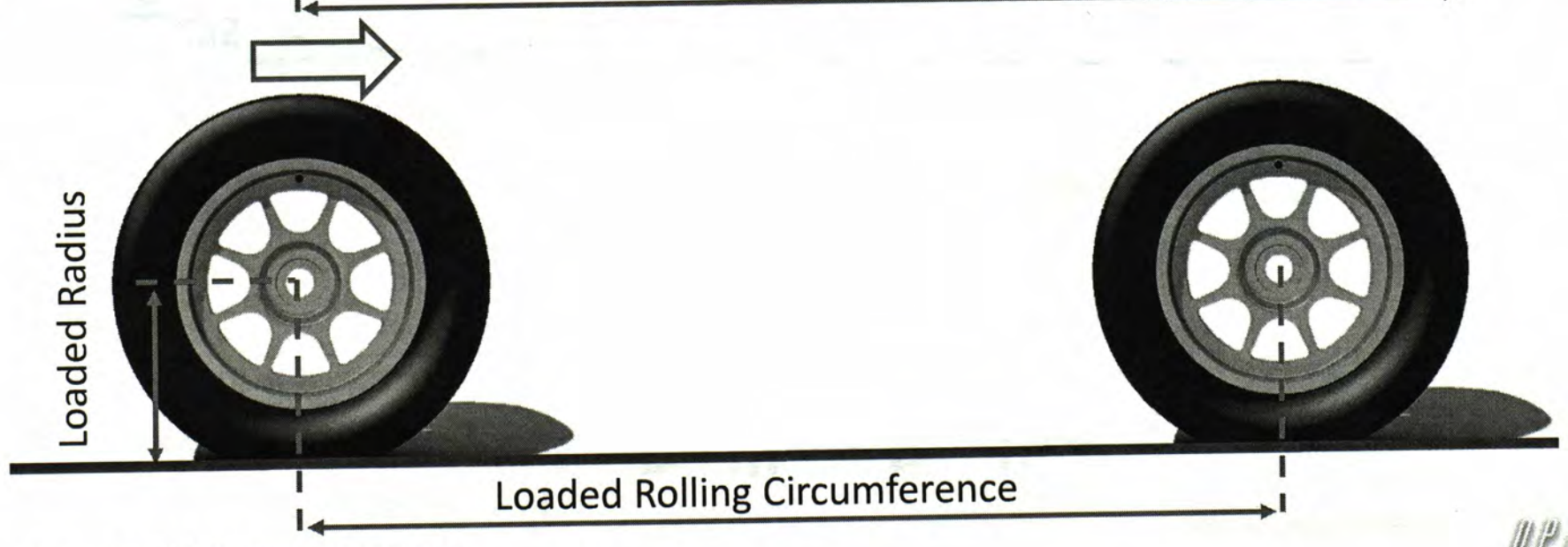




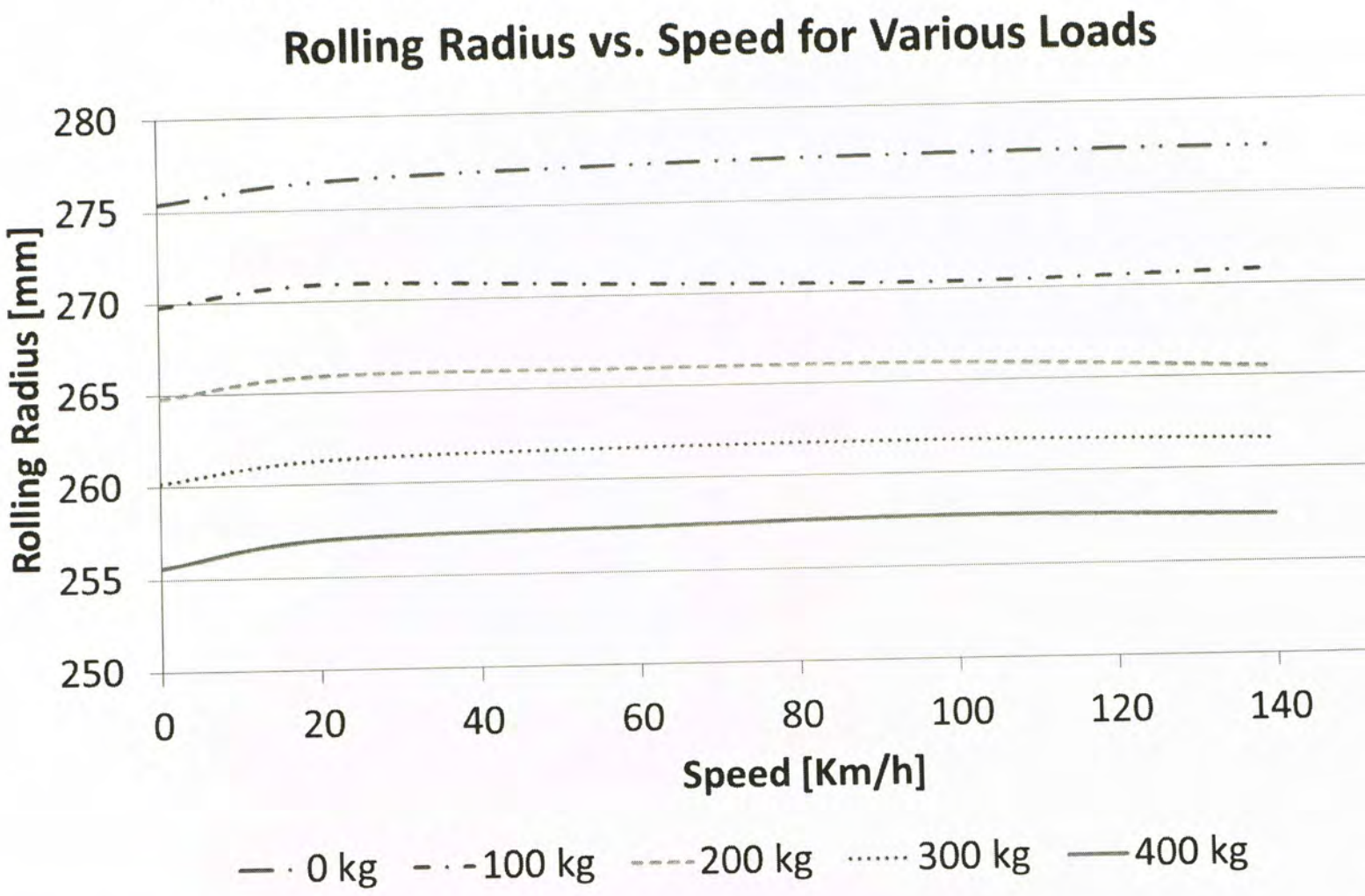
Rolling Radius



$$Rolling_radius = \frac{Rolling_Circ.}{2 \times \pi}$$

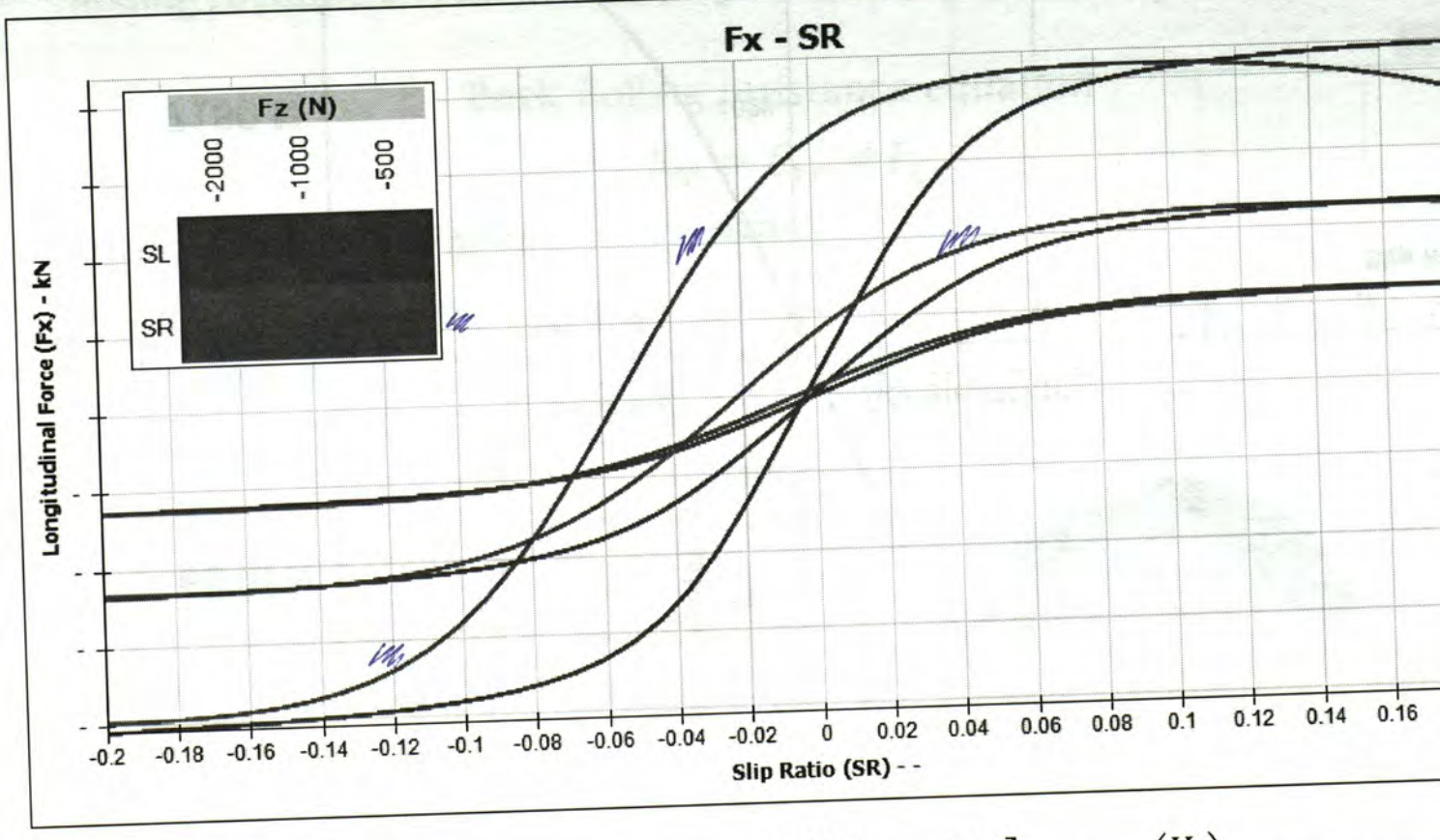


Rolling radius increases with speed and decreases with load



Slip Ratio

Tire test machines will use the calculated Slip Ratio SR during testing.
The calculated slip ratio SL is for use in tire modeling



$$SR = \left[\frac{N \times RL}{k \times V \times \cos(SA)}\right] - 1$$

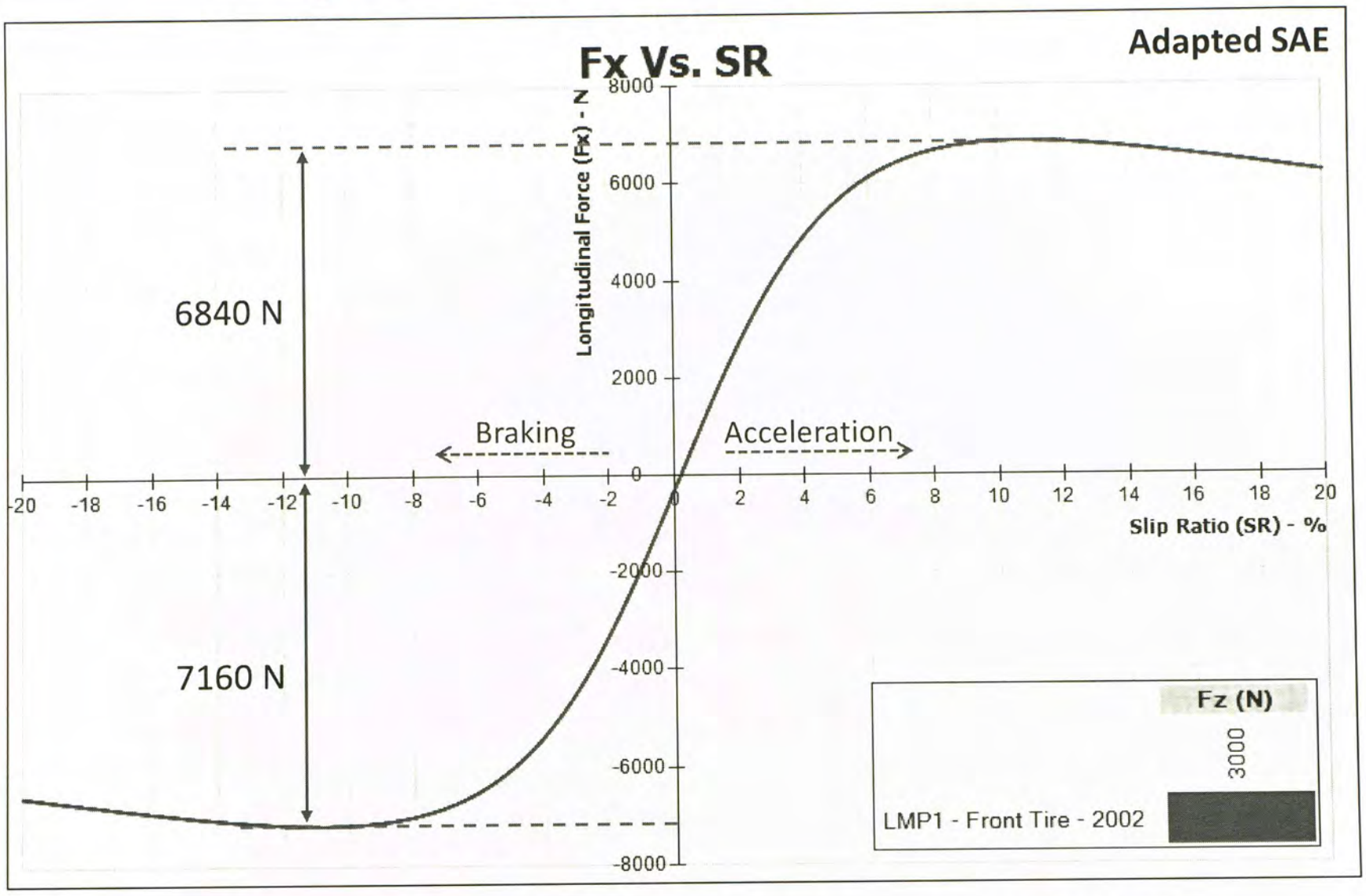
$$SL = \left[\left(\frac{V_1}{N_1}\right) \times \left(\frac{N}{V}\right)\right] - 1$$

$\left(\frac{V_1}{N_1}\right)$ *is at free ro*

k = 168.07 in English units or 265.26 in S.I. units
N = Wheel RPM

RL = Loaded Ra
V = Road Speed
SA = Slip Angle

Longitudinal Force Vs. Slip Ratio



A tire is not necessarily longitudinally symmetrical! (More grip in braking)

Rolling Resistance

Rolling resistance is measured as a force more commonly than a moment

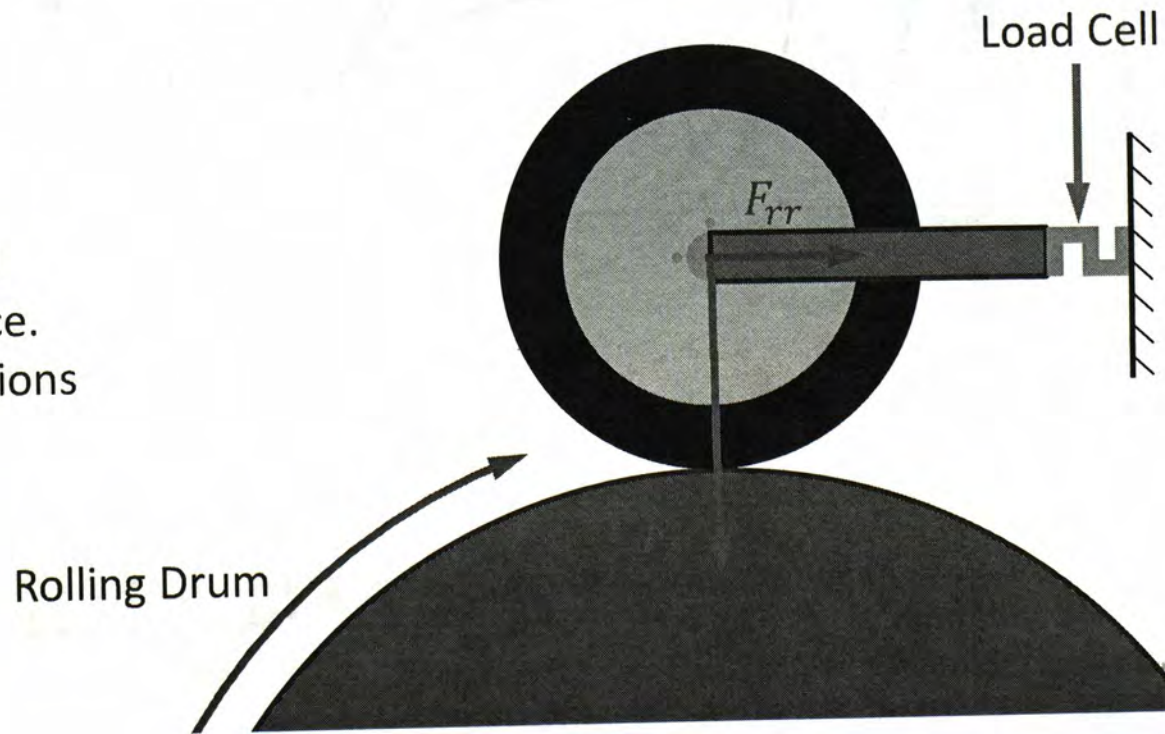
Basic Rolling resistance equation

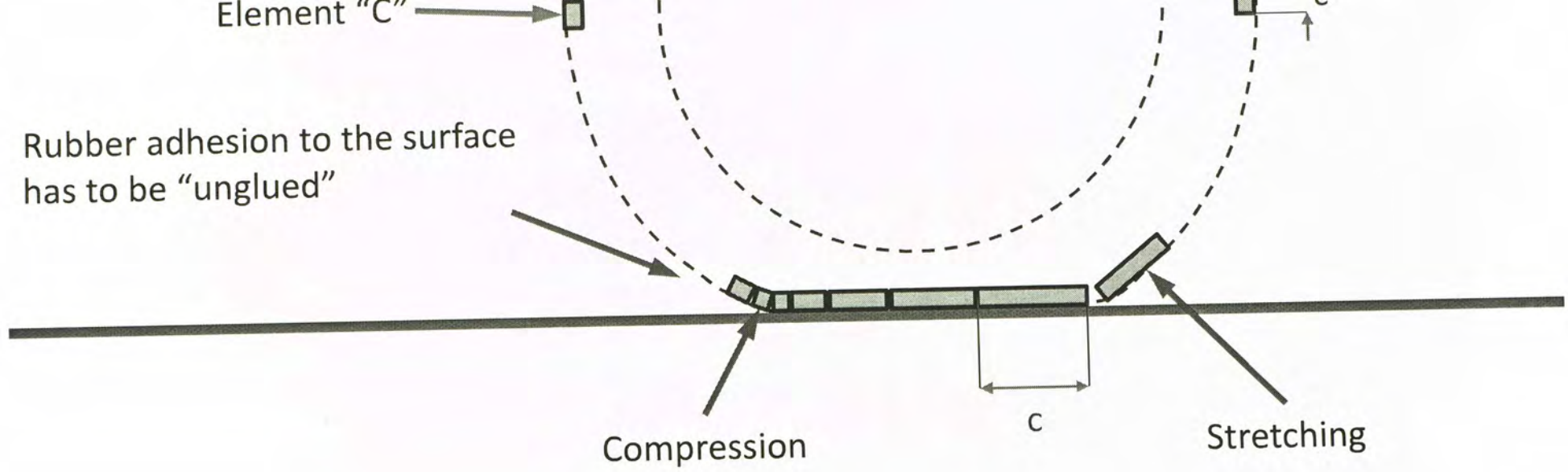
$$F_{rr} = C_{rr} \times F_z$$

$F_{rr} = Rolling\ Resistance\ Force$

$C_{rr} = Rolling\ Resistance\ Coefficient$

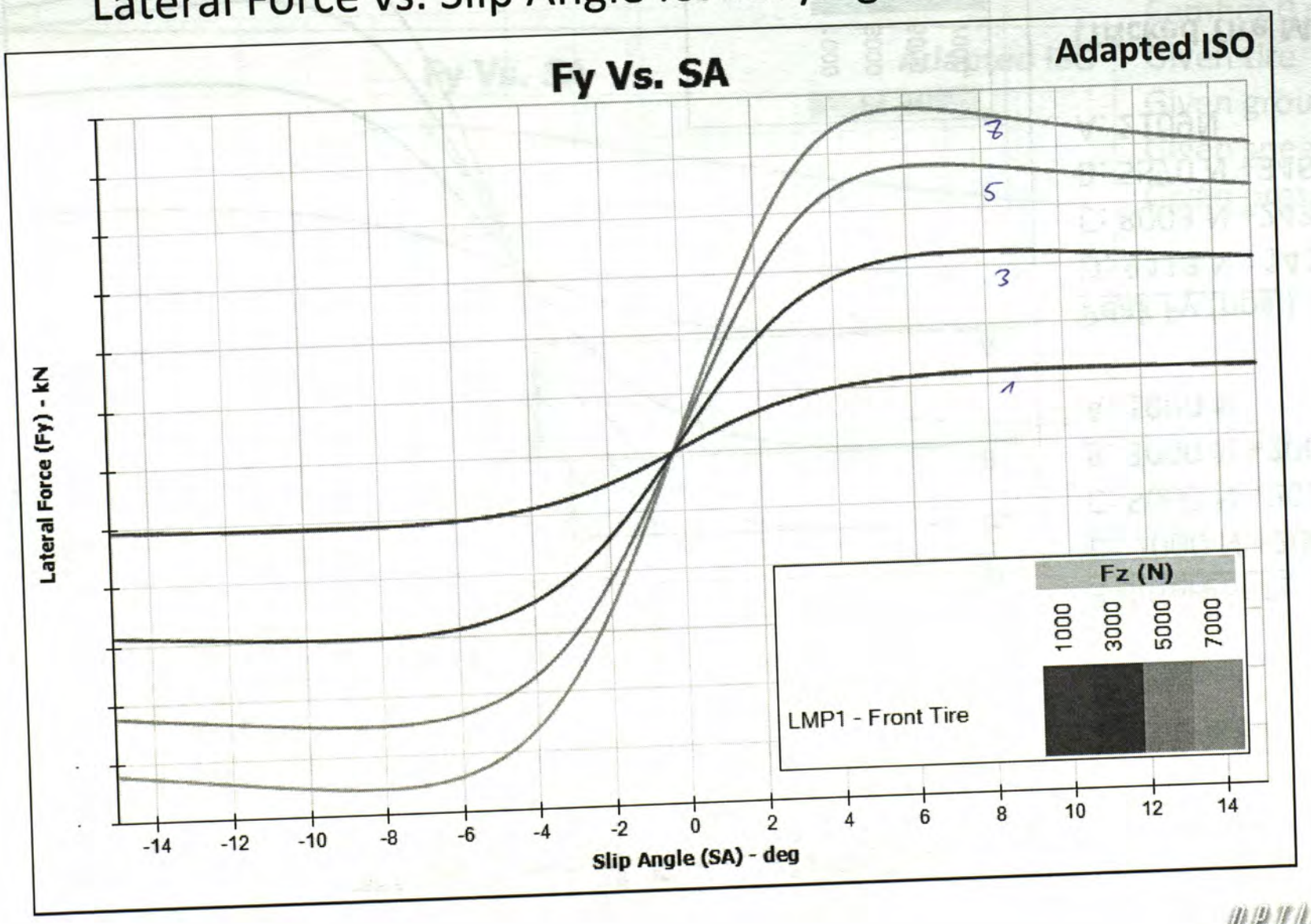
There are ISO and SAE standard test to measure rolling resistance. Each use their own set of equations



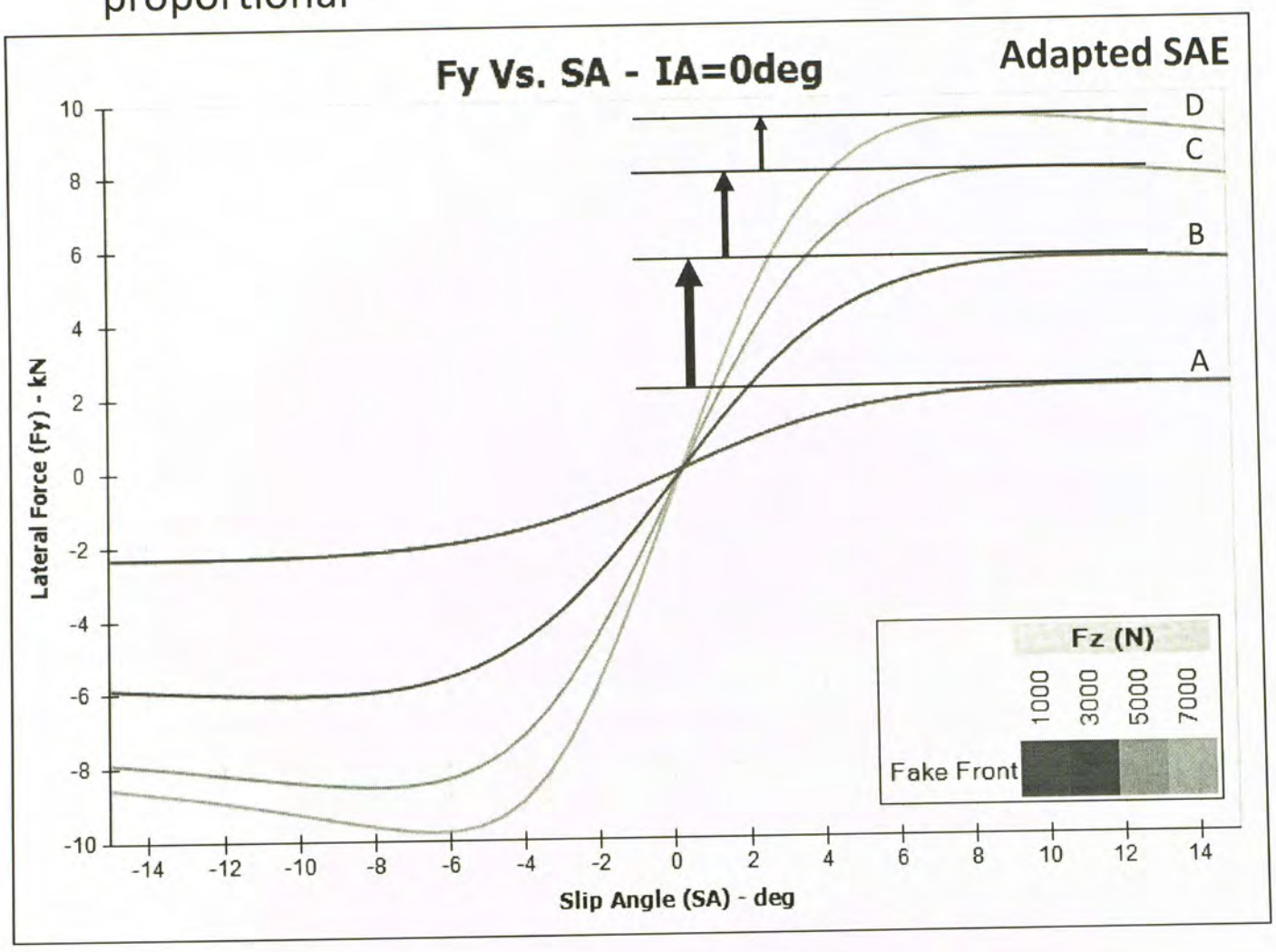


c) Vertical Load

Lateral Force vs. Slip Angle for varying Vertical Loads



- When increasing vertical load on a tire, the increase in lateral force is not proportional

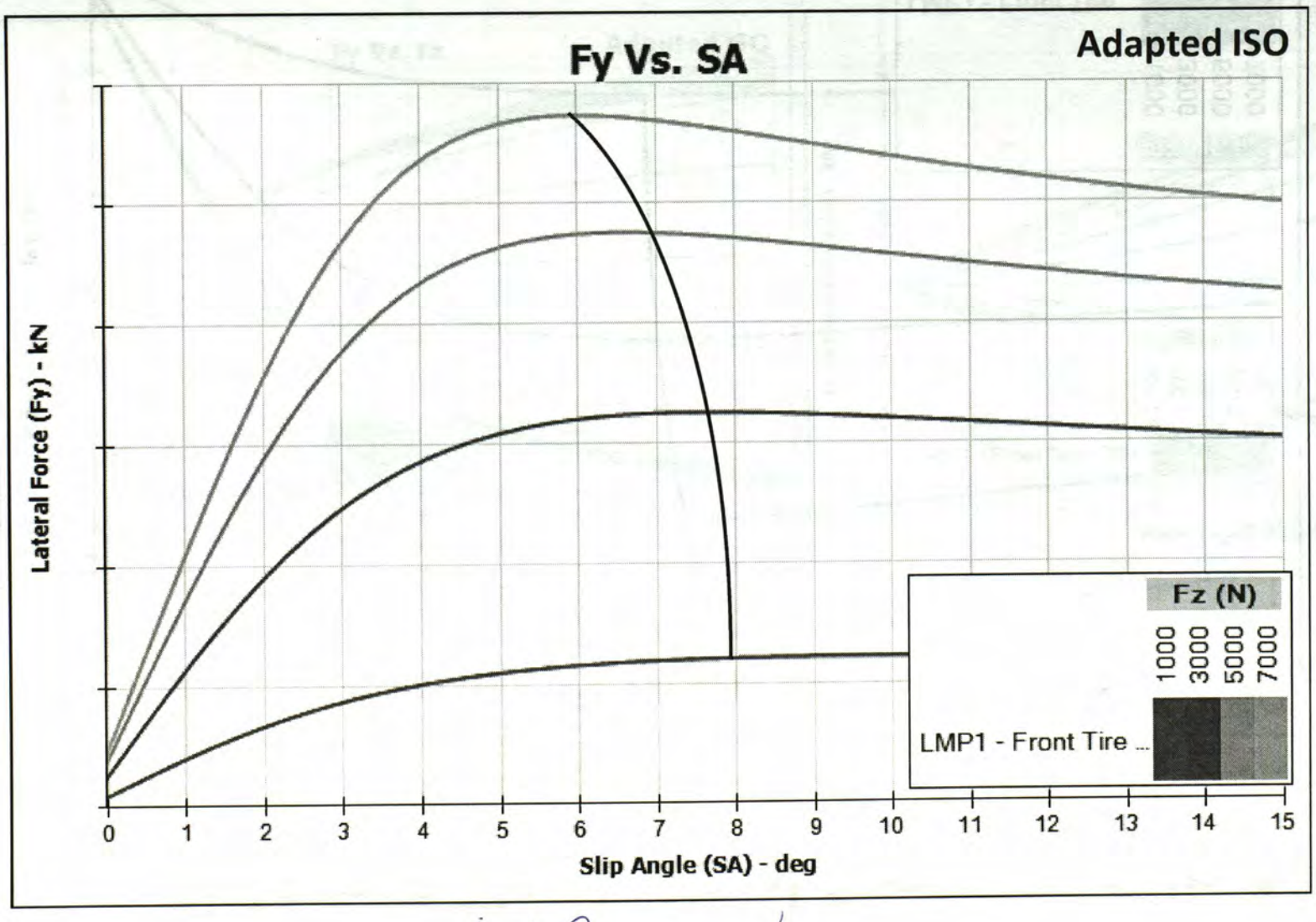


3 different Fz
D: 7000 N +20
C: 5000 N +20
B: 3000 N +20
A: 1000 N

Peak Fy (pos.
D: 9418 N +1
C: 8003 N +2
B: 5570 N +3
A: 2109N

(Tricked Tire

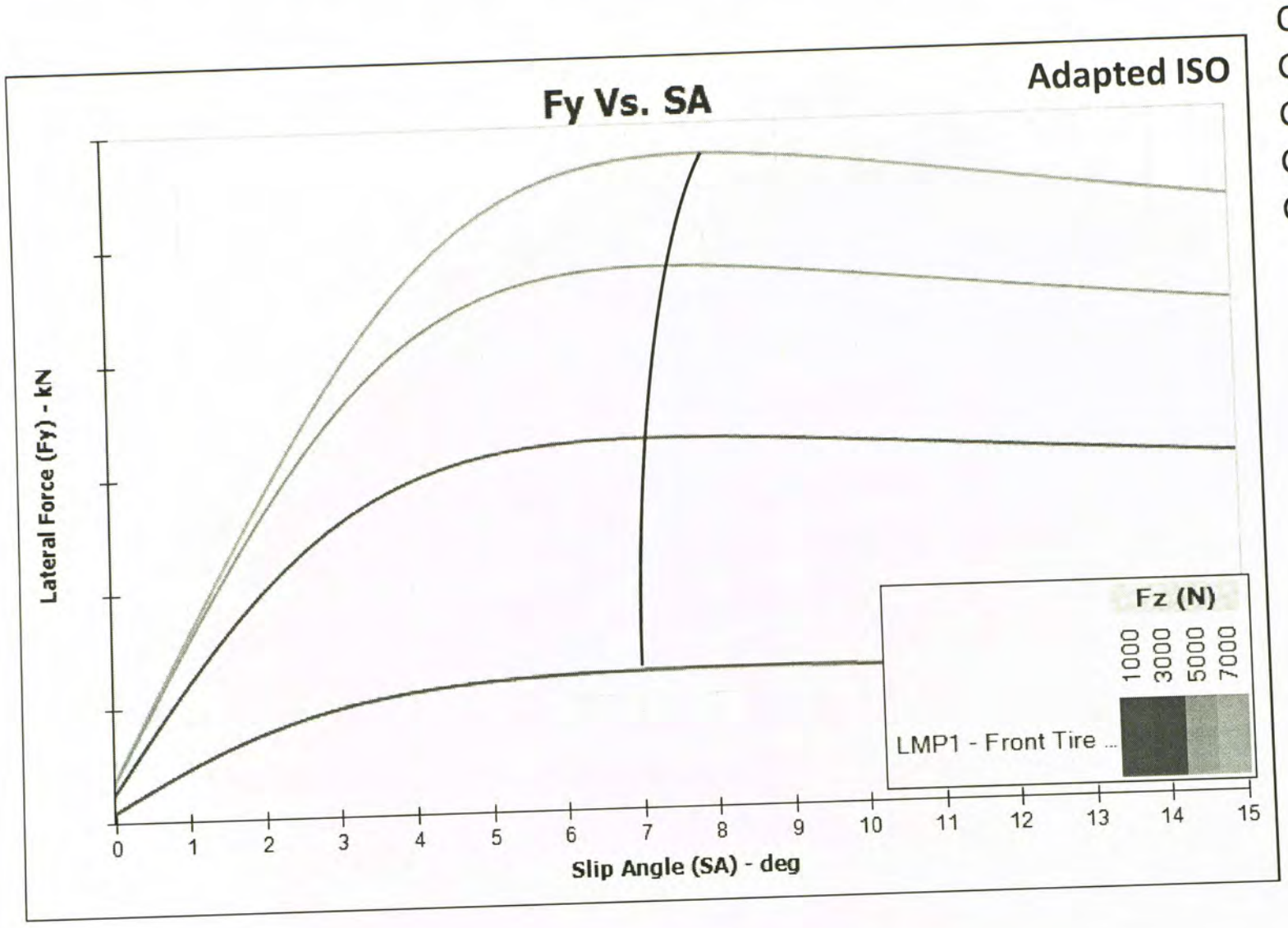
Tire Lateral Force vs. Slip Angle



Cambe
Given
Given
Given
Given

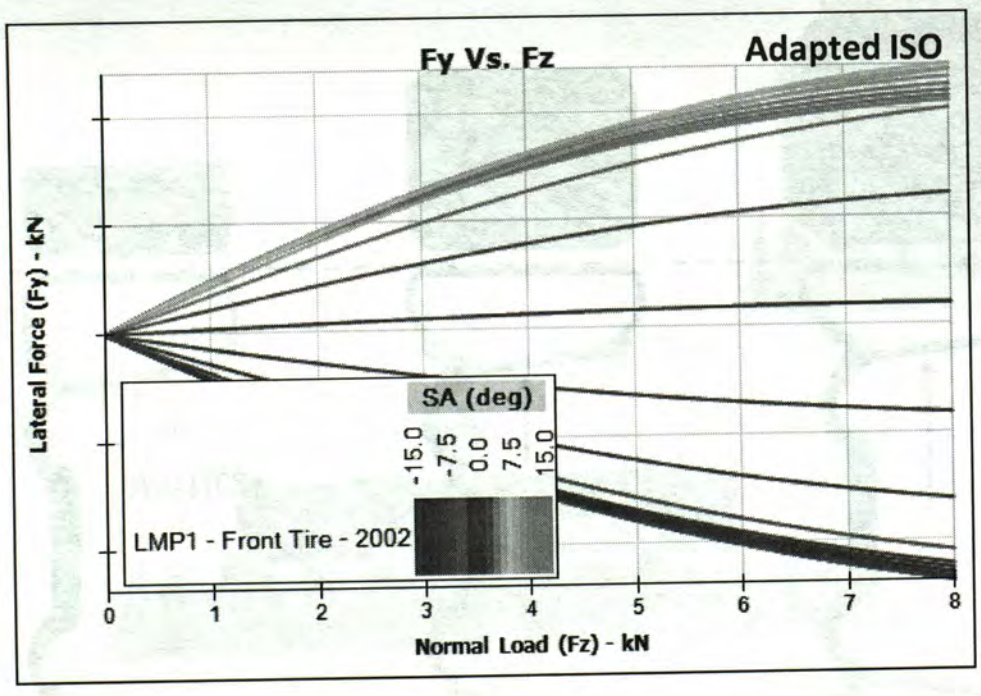
! Ackermann !

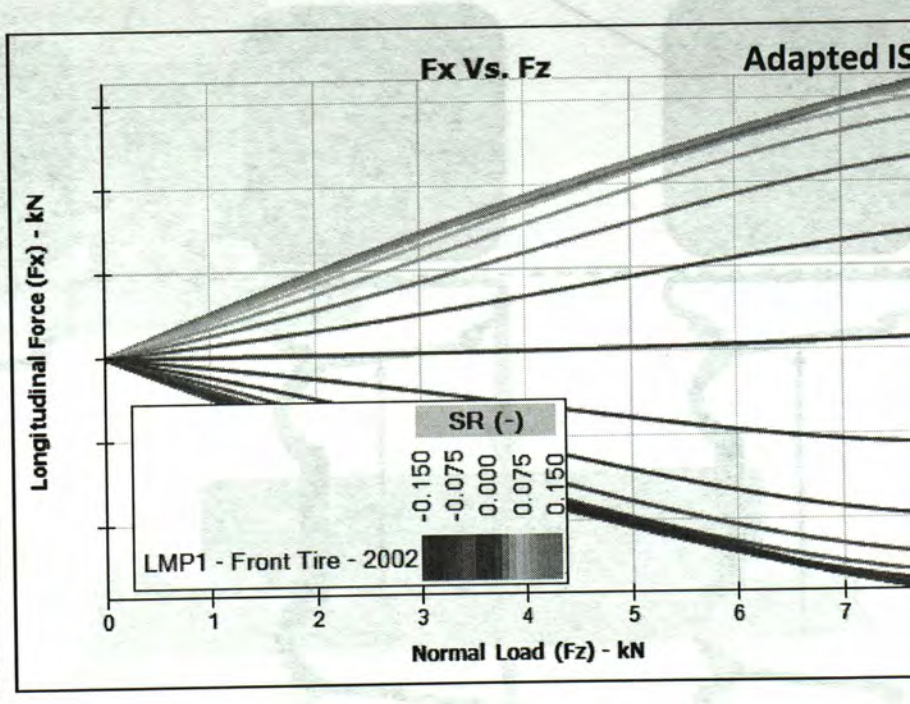
Different Tire Lateral Force vs. Slip Angle



Camber 0 deg
Given tire
Given ground
Given speed
Given pressure

Tire Force vs. Vertical Force



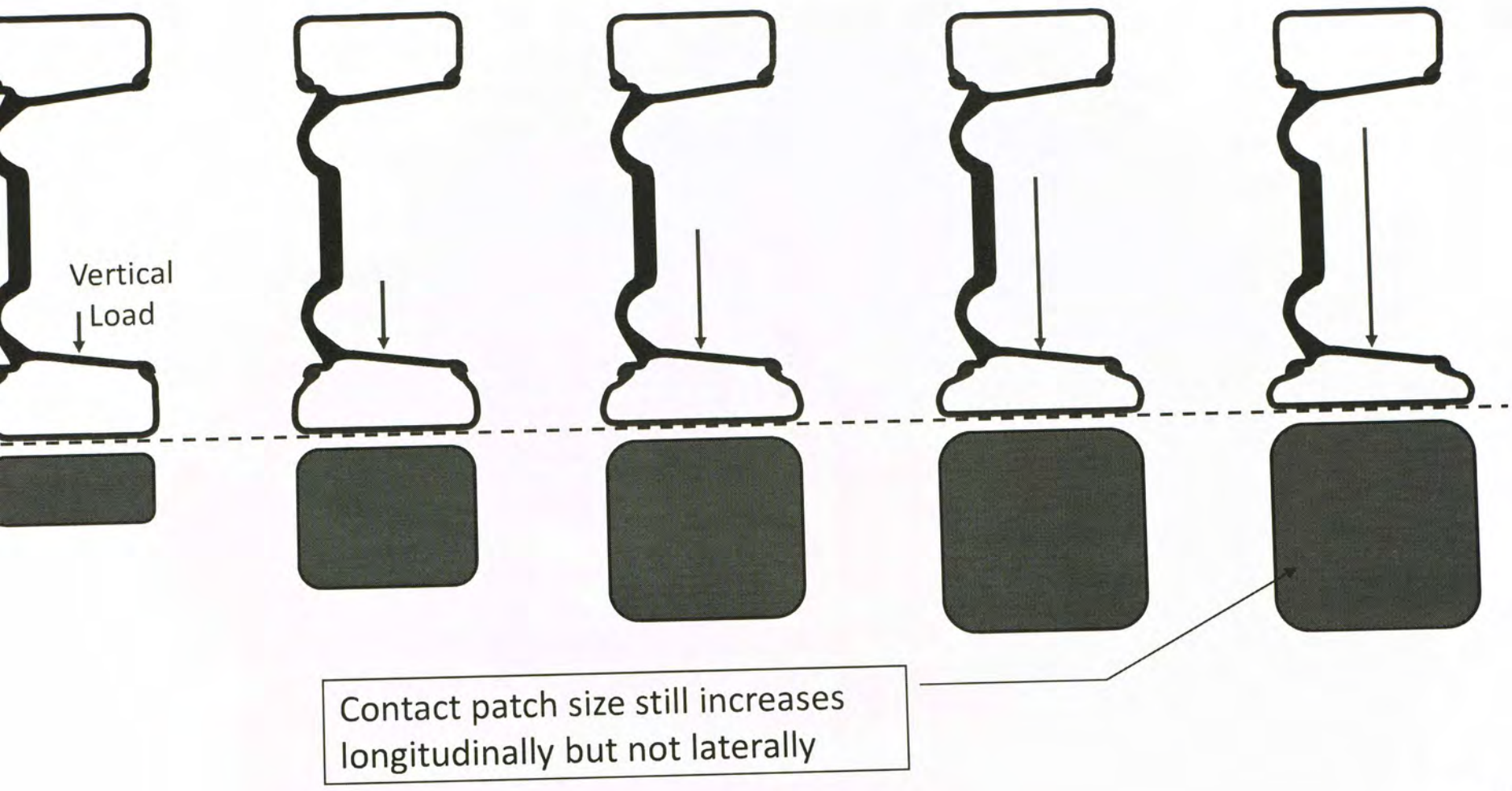


This is a good way to evaluate the quality of your model over a wide range of vertical loads

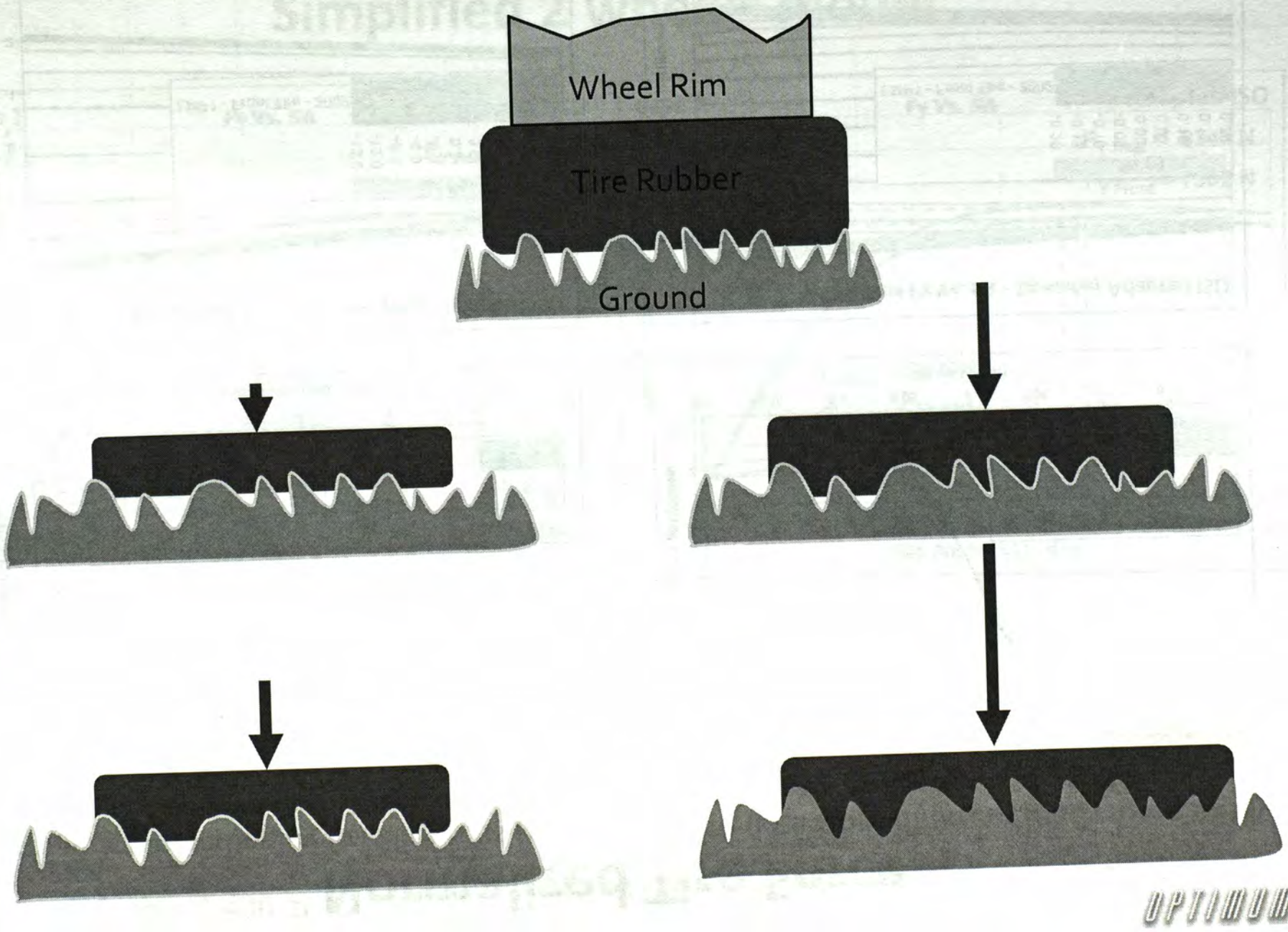
Not uncommon that models will diverge outside tested range – you have to be careful with extrapolation

Tire Contact Patch Analysis - Micro

Patch area increases with vertical load

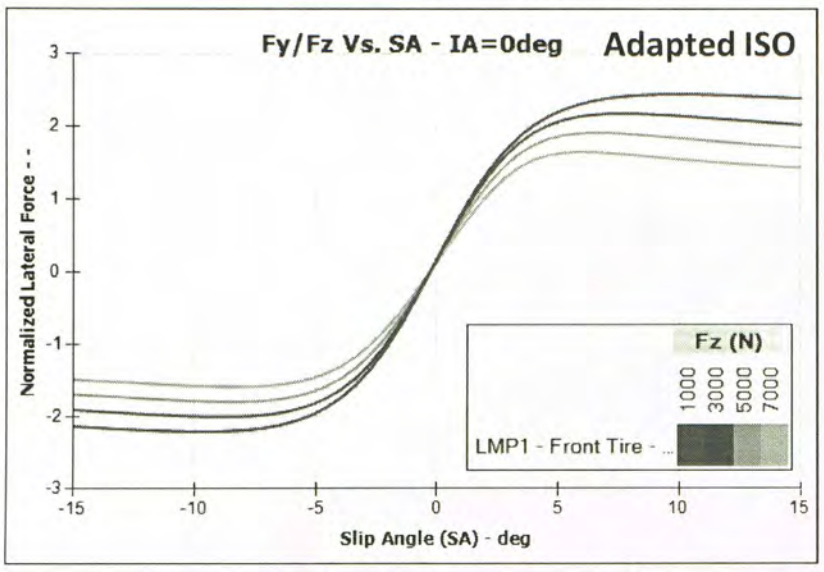


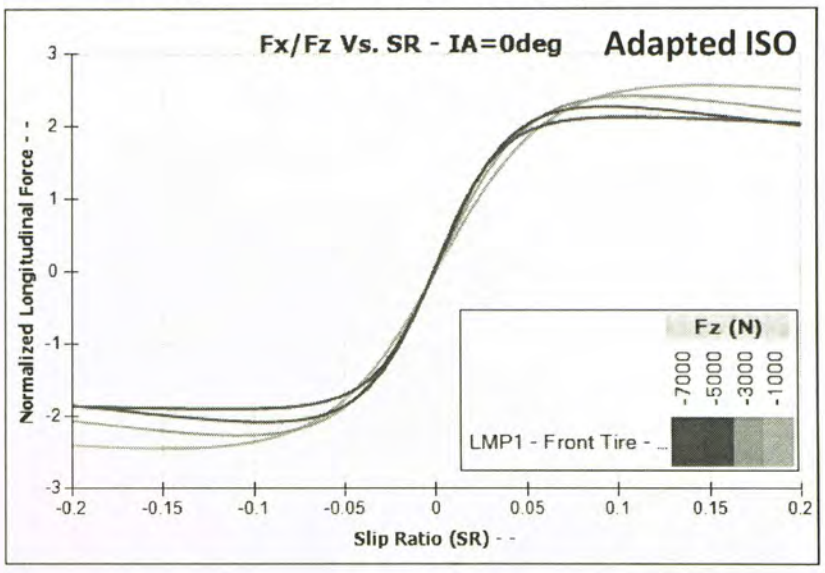
Tire Contact Patch Analysis - Micro

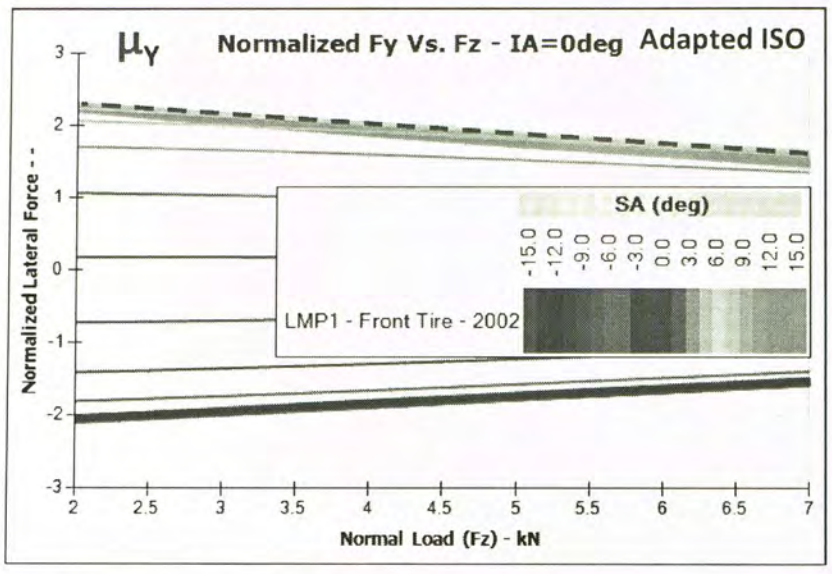


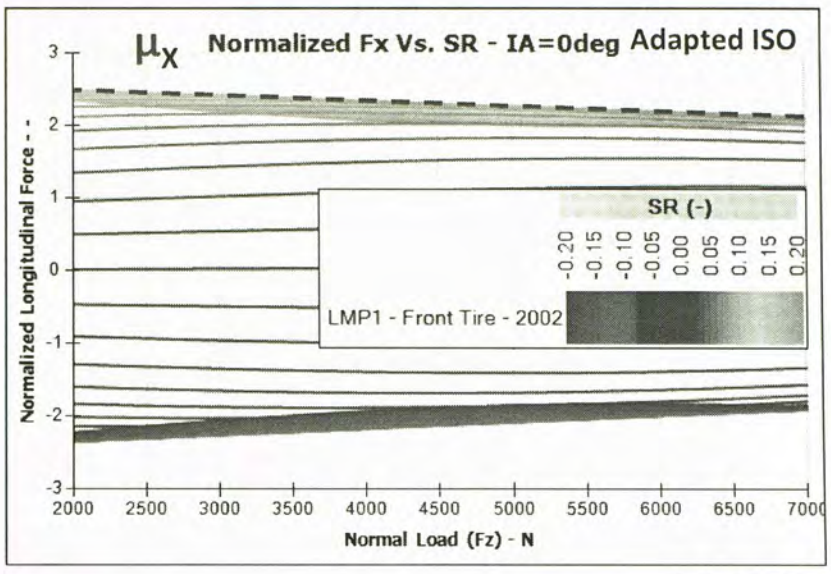
Normalizing (dividing by Fz) allows us to compare tire data from different loads more easily.

This is similar to coefficient of friction which is the peak value for a given condition (Fz, IA, V, P) = μ

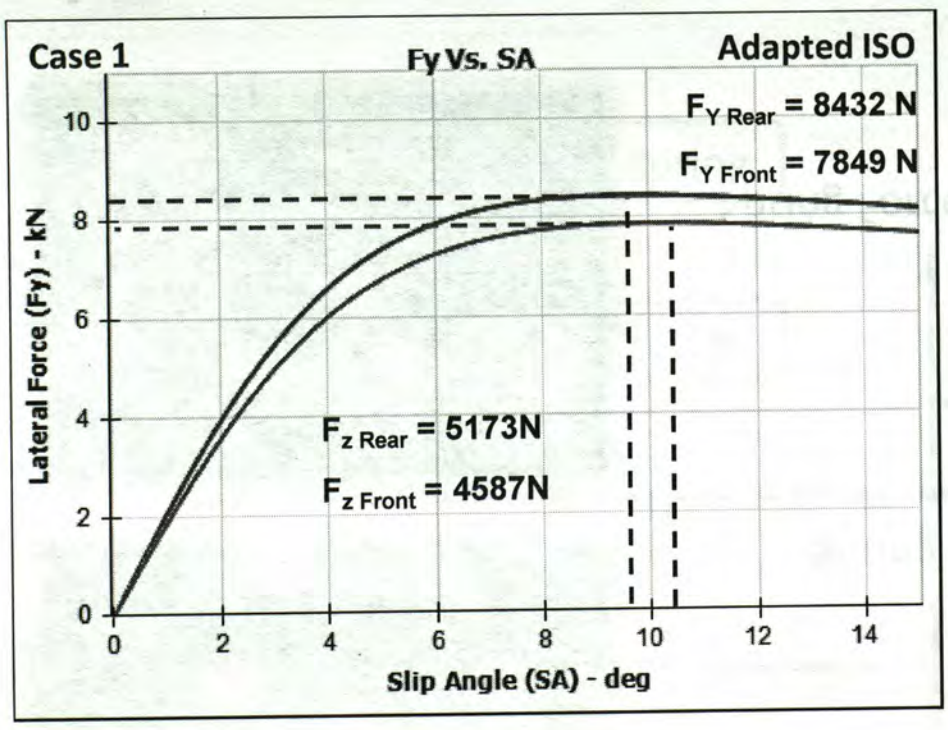


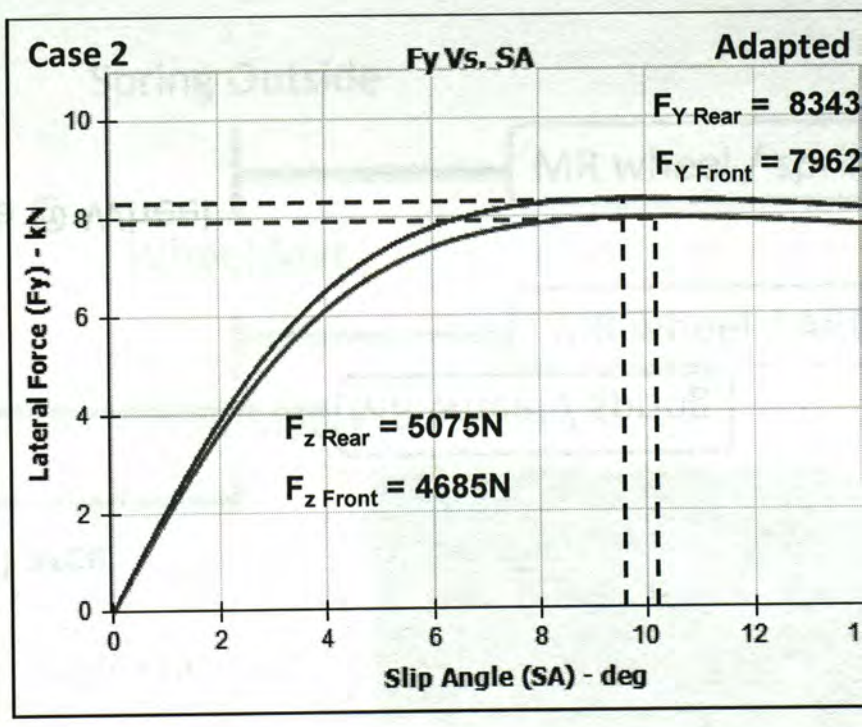






Example of weight distribution influence. Simplified 2 wheels model.





Wheelbase = 2.980m

Case 1: Weight Dist. = 47%

W_F = 4587N a = 1.579m

W_R = 5173N b = 1.401m

$$Yaw\ Moment = (F_{Y_{Front}} \times a) - (F_{Y_{Rear}} \times b)$$

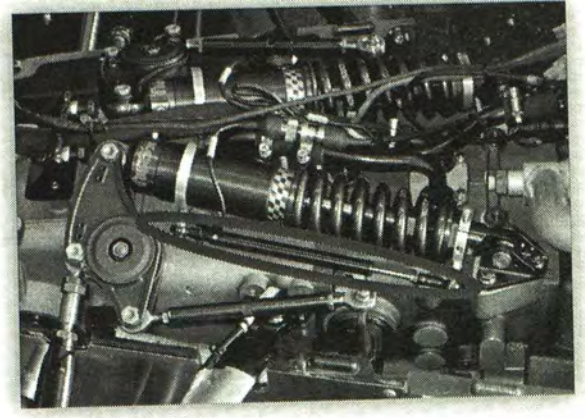
$$Yaw\ Moment = (7849 \times 1.579) - (8432 \times 1.401) = 580.3N.$$

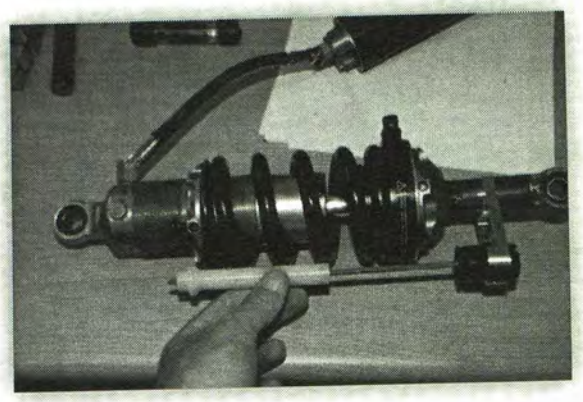
Case 2: Weight Dist. = 48%

W_F = 4685N a = 1.550 m

W_R = 5075N b = 1.430 m

$$Yaw\ Moment = (7962 \times 1.550) - (8343 \times 1.430) = 410.6N.$$





Spring Mvt

Spring Stiffness

Spring Force

MR wheel / spring

Spring Force @ Wheel

! Doesn't take into account : - ARB
- Damper

Motion Ratio and Forces

Spring Inside

MR wheel / spring

Wheel Mvt

MR wheel / ARB

ARB Mvt

Spring Outside

MR wheel / spring

Wheel Mvt

MR wheel / ARB

ARB Mvt

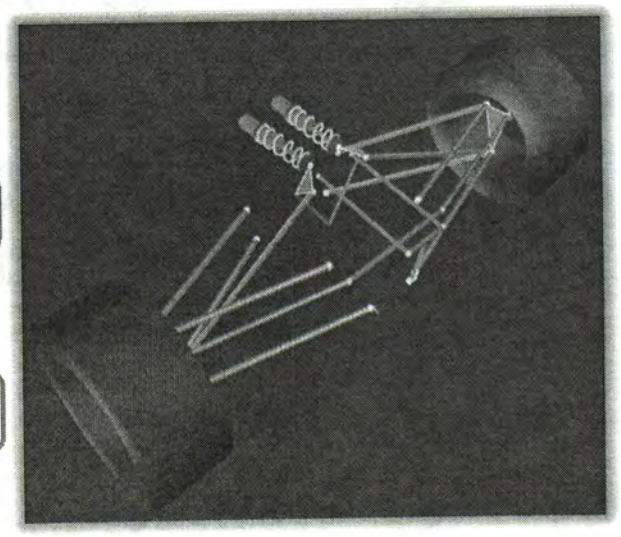
ARB Torsion

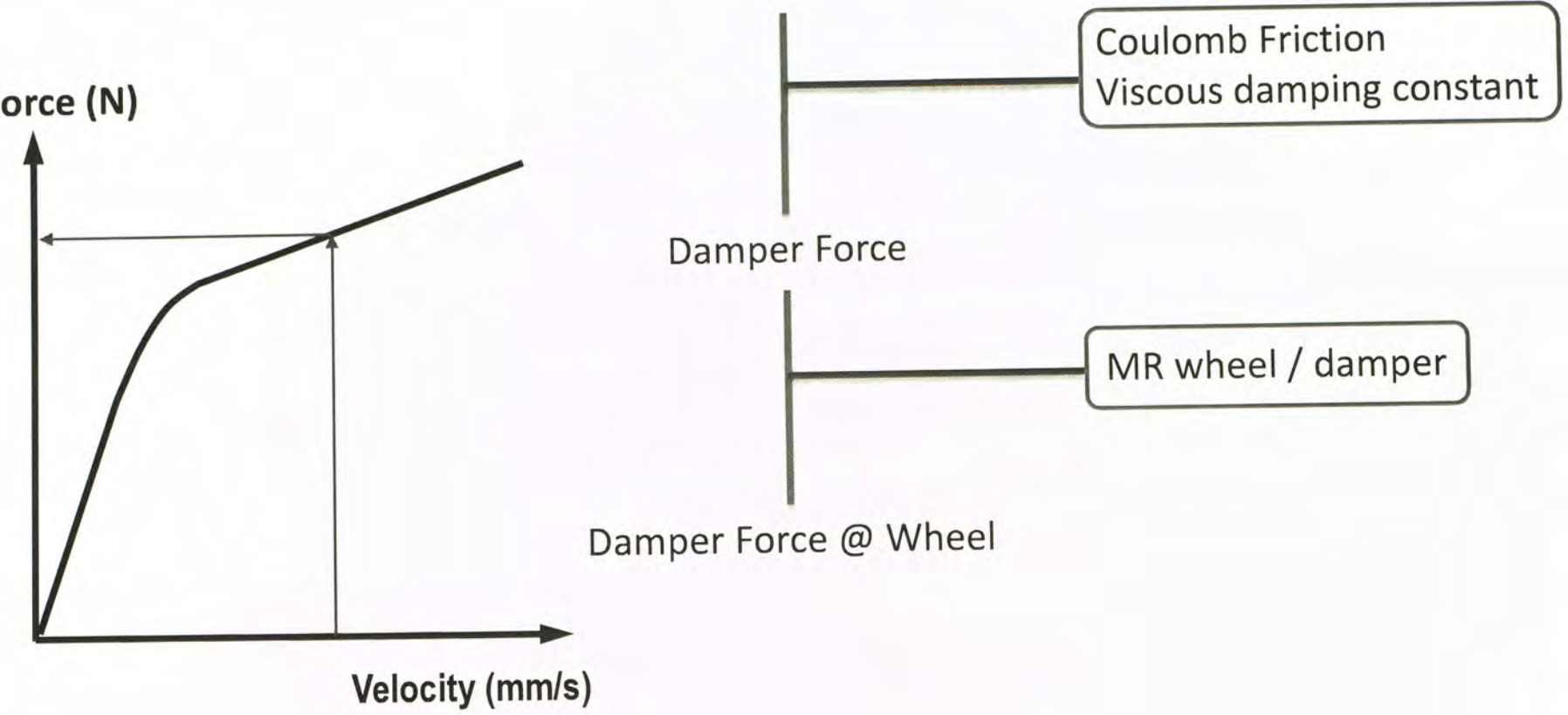
ARB Stiffness

ARB Force

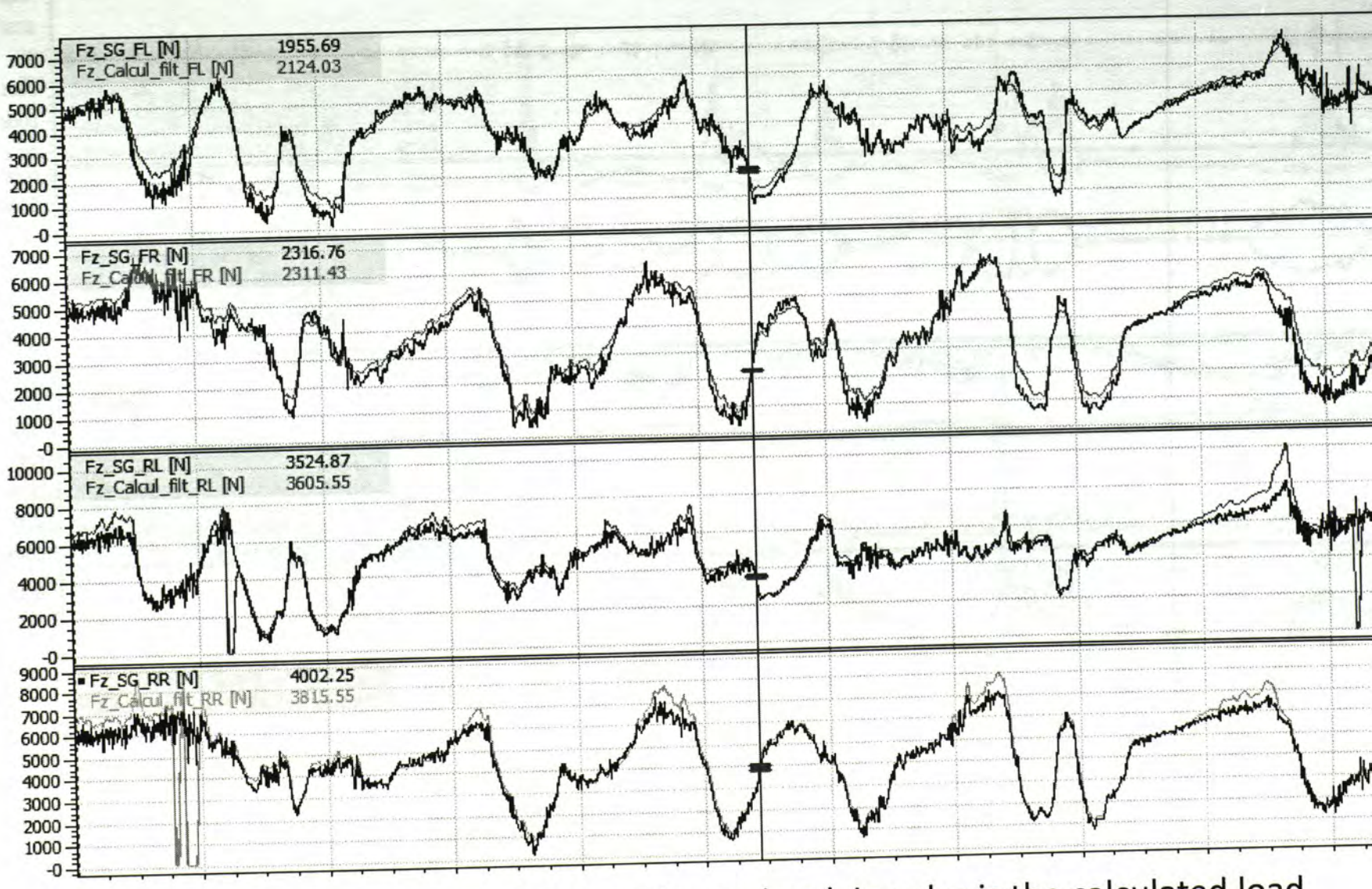
MR wheel / ARB

ARB Force @ Wheel

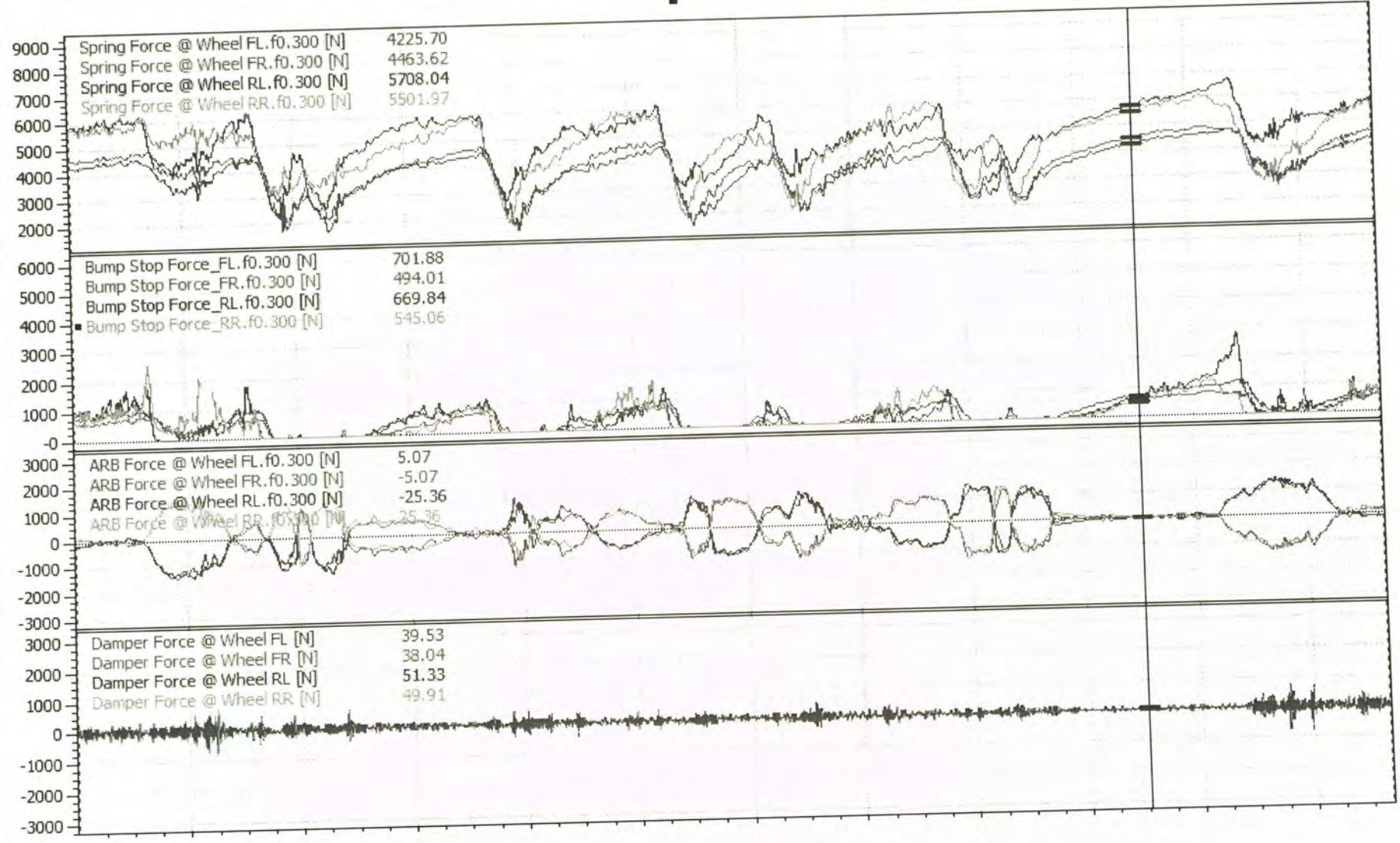




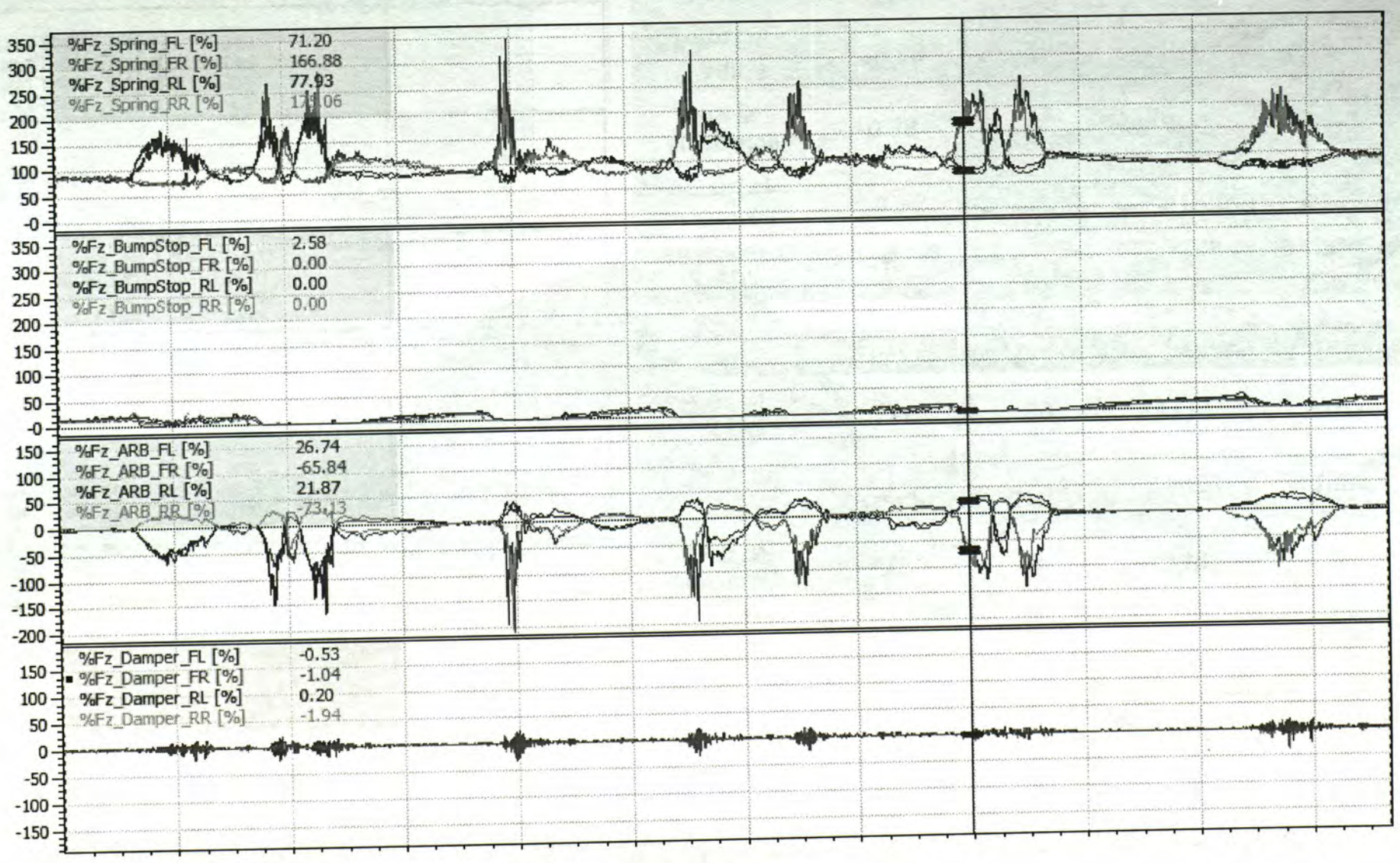
Strain Gauge Measure & Calculation



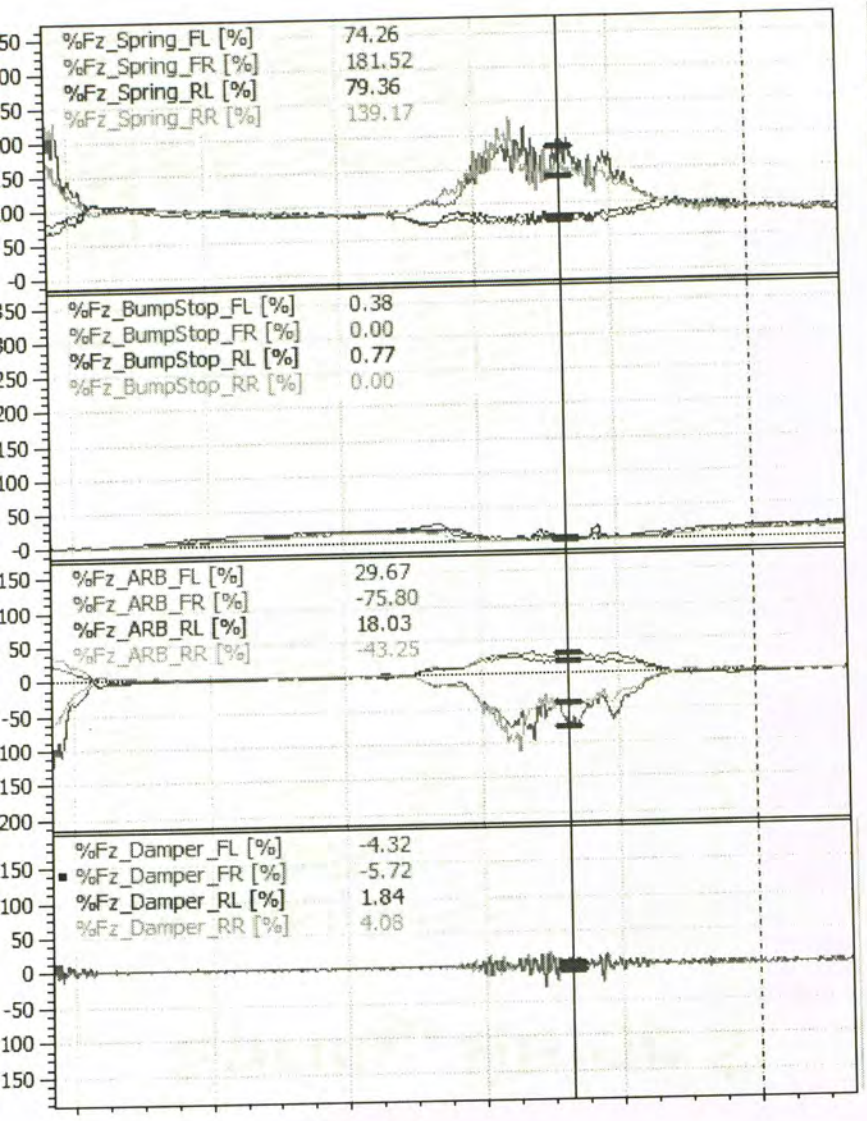
In black is the Strain gauge measure on the pushrod, in color is the calculated load.



Spring, Bump Stop, ARB and Damper %FZ



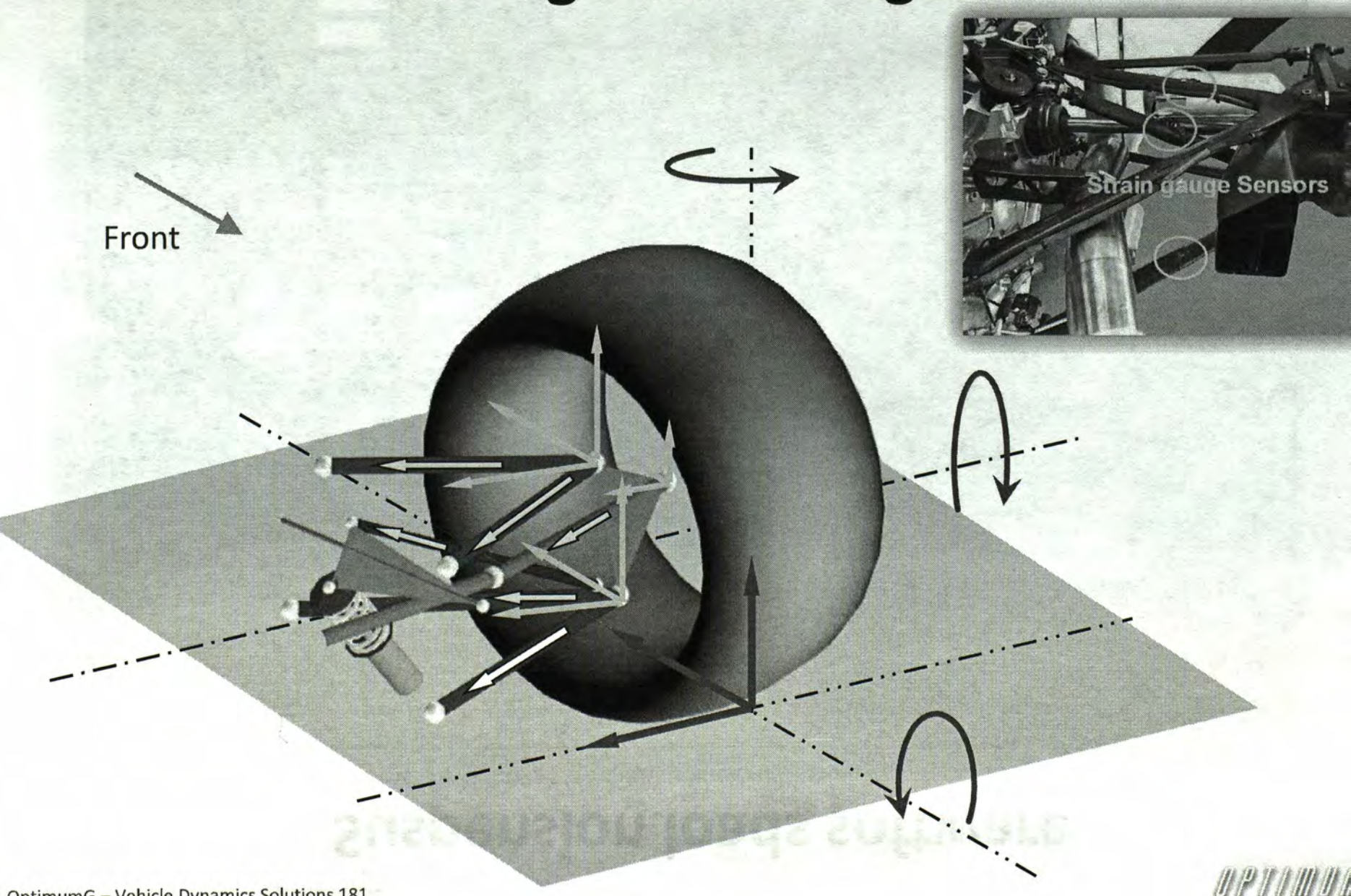
Spring, Bump Stop, ARB and Dampers Forces



@ the Wheel	FL	FR	RL	RR
Spring Force [N]	+2951	+2823	+4792	+3504
Bump Stop Force [N]	+15	0	+46	0
ARB Force [N]	+1179	-1179	+1089	-1089
Damper Force [N]	-171	-89	+111	+102
Total [N]	3974	1555	6038	2517

@ the Wheel	FL	FR	RL	RR
Spring Force [%]	+74.26	+181.52	+79.36	+139.17
Bump Stop Force [%]	+0.38	0.00	+0.77	0.00
ARB Force [%]	+29.68	-75.80	+18.03	-43.25
Damper Force [%]	-4.32	-5.72	+1.84	+4.08
Total [%]	100	100	100	100

Measurement of Tire Forces on a Car using Strain Gauges



Suspension loads software

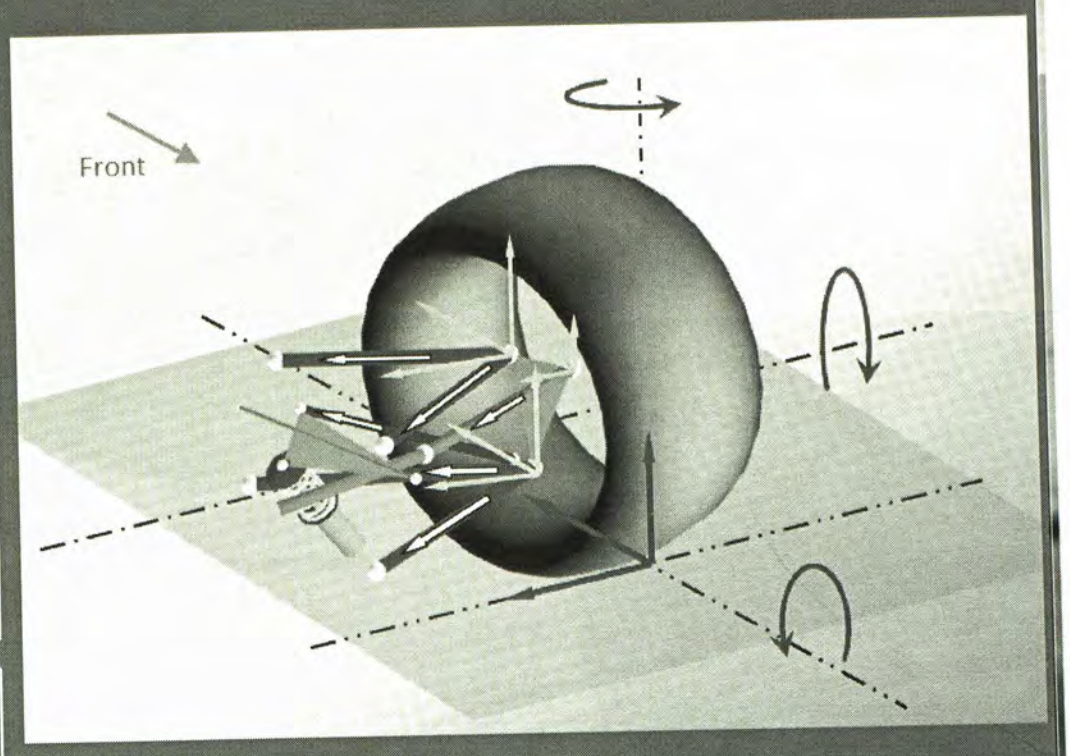
File Calculate ?

Suspension Geometry

	X (mm)	Y (mm)	Z (mm)
Lower A Arm			
Chasis Fore	125	261.5	177.5
Chasis Aft	-282.5	367.3	177.5
Upright	4	749.5	185
Upper A Arm			
Chasis Fore	83.47	320.908	376
Chasis Aft	-261.92	406.14	398
Upright	-15	710	398
Steering Tie-Rod			
Rack	160	338.5	379
Upright	105	740	400
Pushrod			
Rocker	-4	111	445
Attachment	1.5	706.252	212.306

Wheel Geometry (mm)

Half Track	819.483
Longitudinal offset	0
Vertical offset	0
Tire Diameter	680



Suspension forces calculation from contact patch loads

Contact Patch Loads (N, N-m)

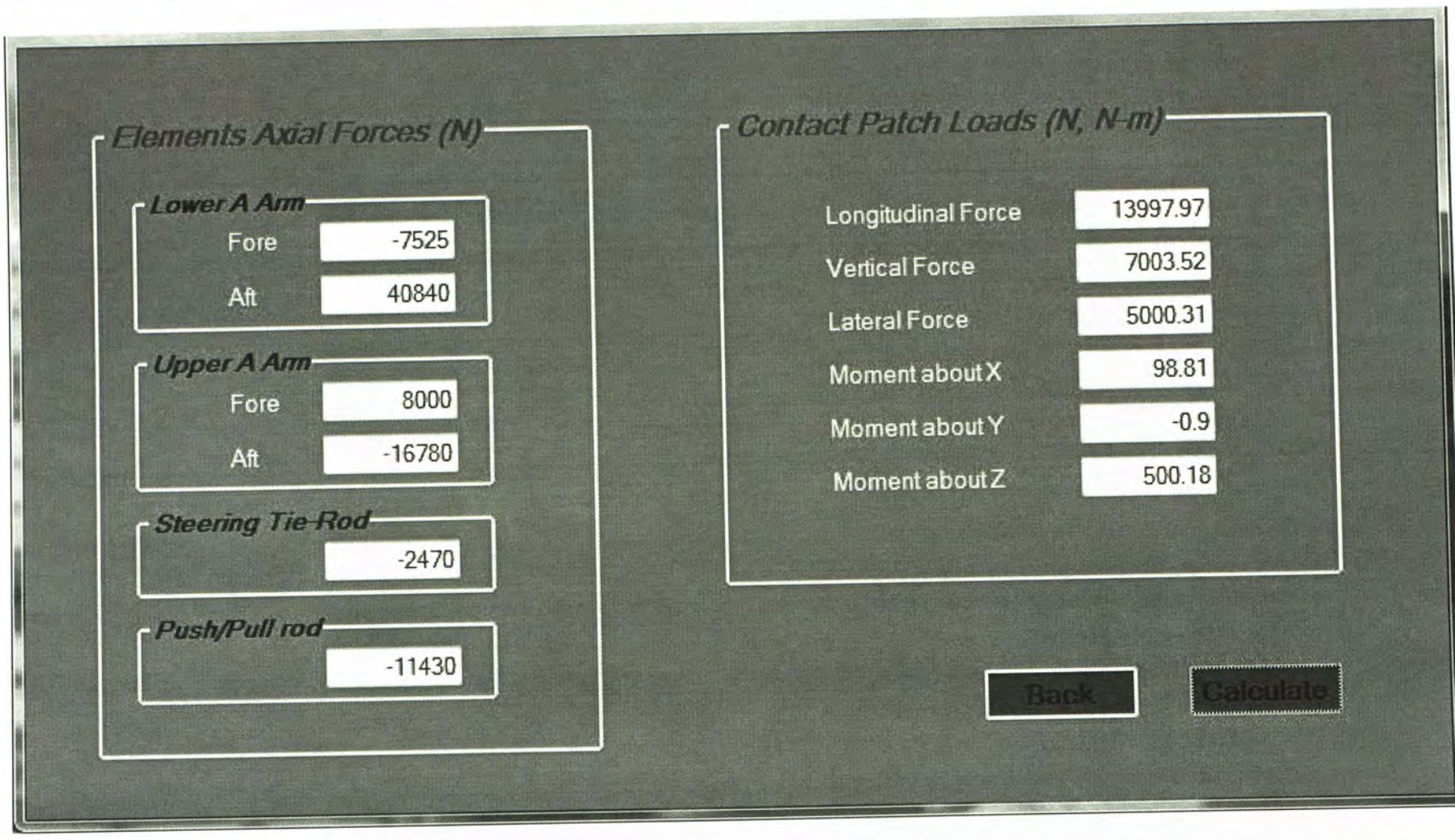
Longitudinal Force	14000
Vertical Force	7000
Lateral Force	5000
Moment about X	100
Moment about Y	0
Moment about Z	500

Back Calculate

Elements Axial Forces (N)

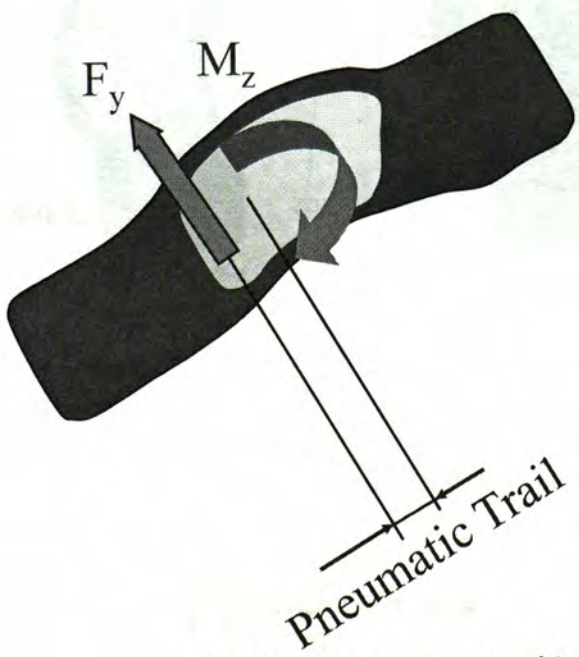
Lower A Arm	
Fore	-7525.62
Aft	40839.11
Upper A Arm	
Fore	7997.54
Aft	-16777.54
Steering Tie-Rod	
	-2471.9
Push/Pull rod	
	-11429.84

Contact patch loads calculation from suspension forces



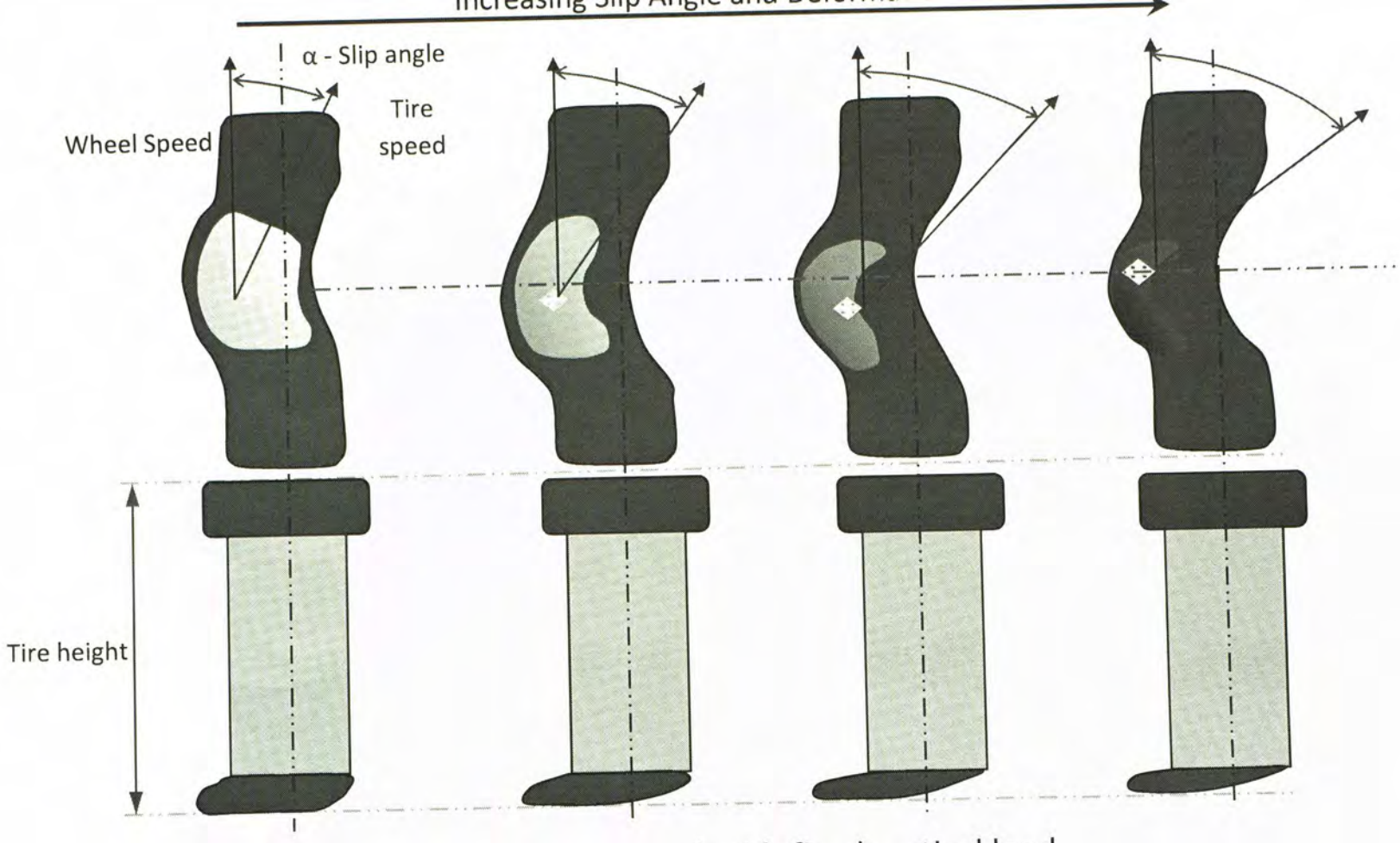
Pneumatic Trail vs. Slip Angle

- In order to understand pneumatic trail, we must understand why M_z is generated
- M_z is generated because the center of action of F_y is not directly on the ax due to elastic deformation of the tire footprint
- This distance is referred to as the pneumatic trail, which can be found usir



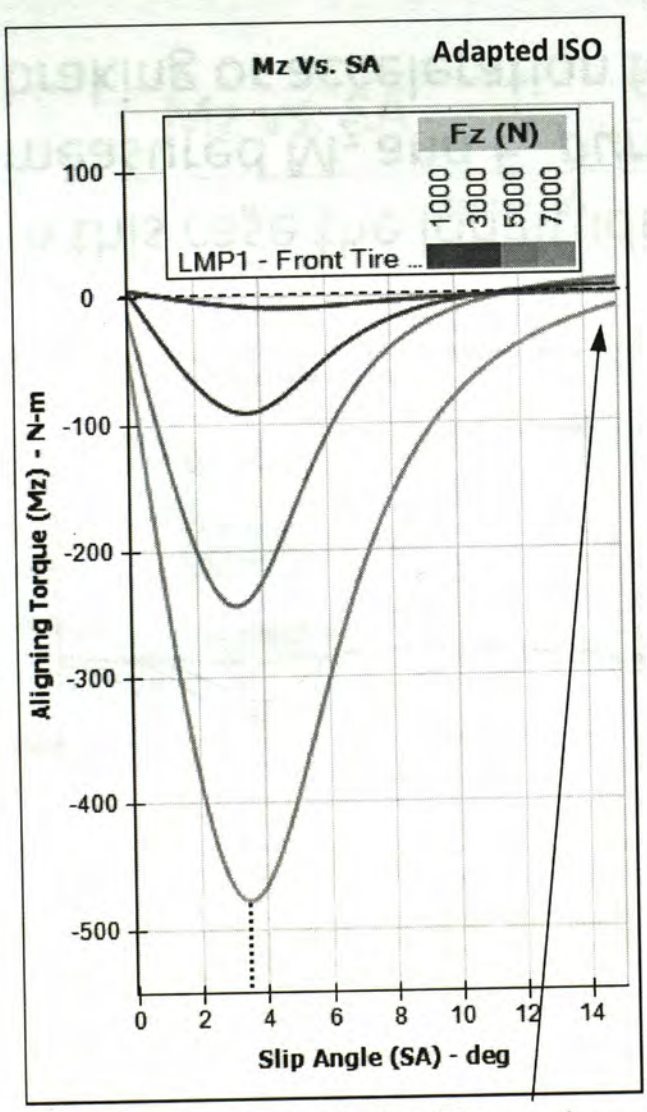
$$trail = \frac{M_z}{F_y}$$

(Note: All comments on this slide assume no effects from mechanical trail)

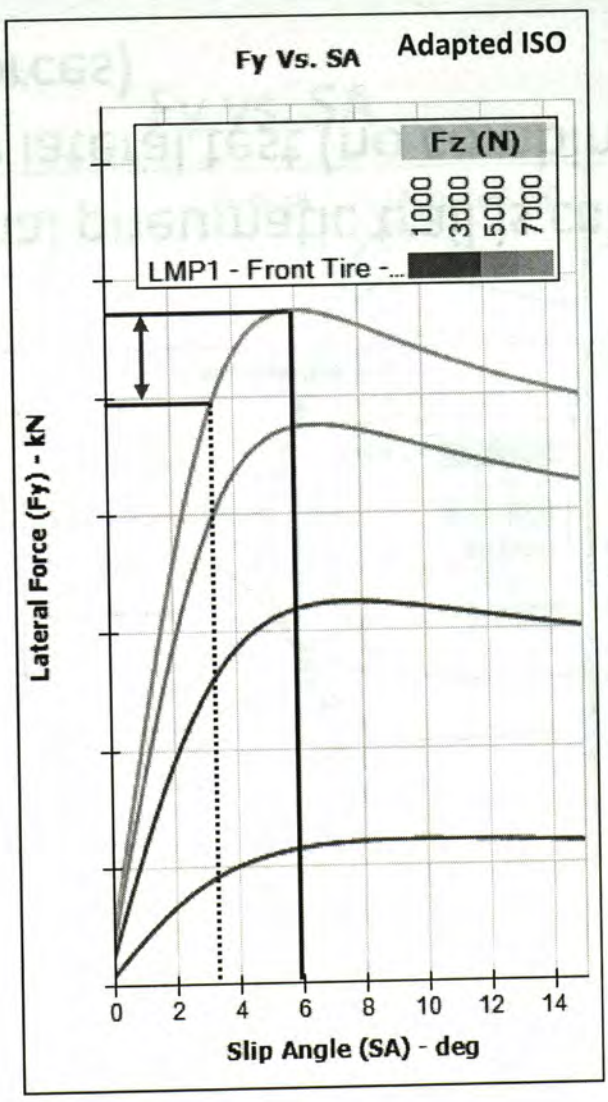


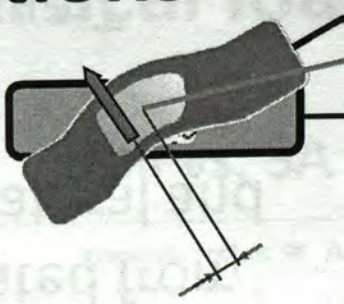
Fixed inclination angle, 0°; fixed vertical load

Contact Patch Area Deformations



Note that at high slip angles the Mz can change its sign.





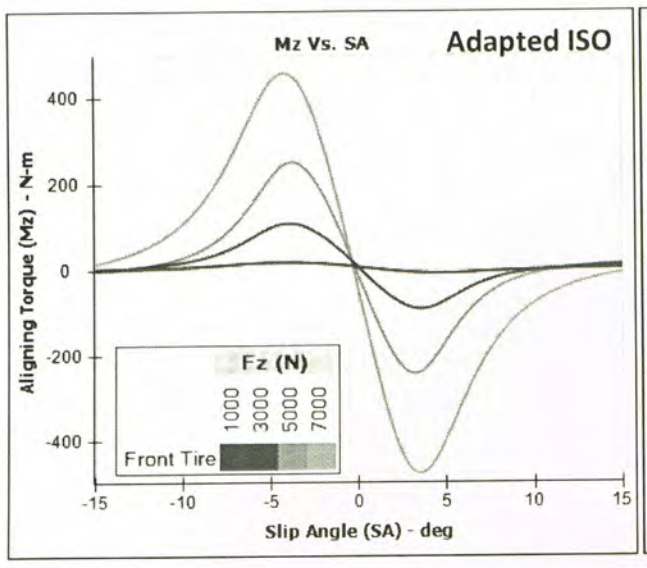
Once the peak of the ti
alignment torque has b
reached , anymore ado
driver steering input (a
therefore any additiona
angle) will decrease th
steering wheel torque.

The slip angle at which t
maximum self alignmen
is reached is *always* sma
the slip angle at which t
maximum lateral grip is

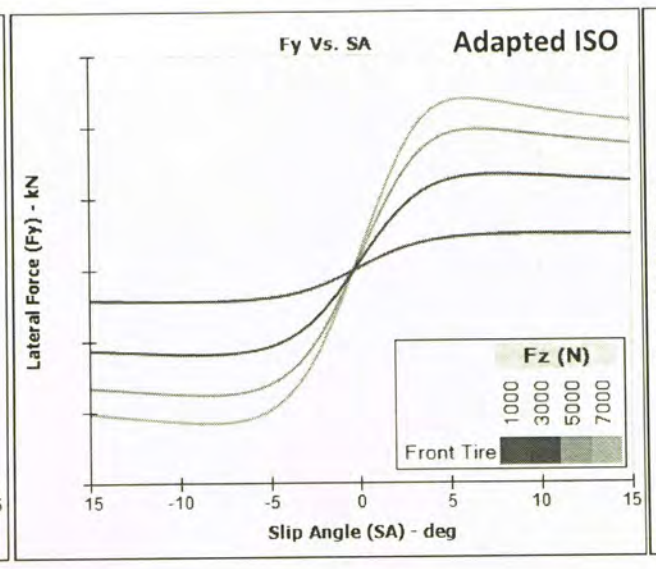
Practically that means th
driver will feel a decreas
steering wheel torque *b*
will reach the maximum
of the tire lateral force.

Pneumatic Trail for different tire vertical loads

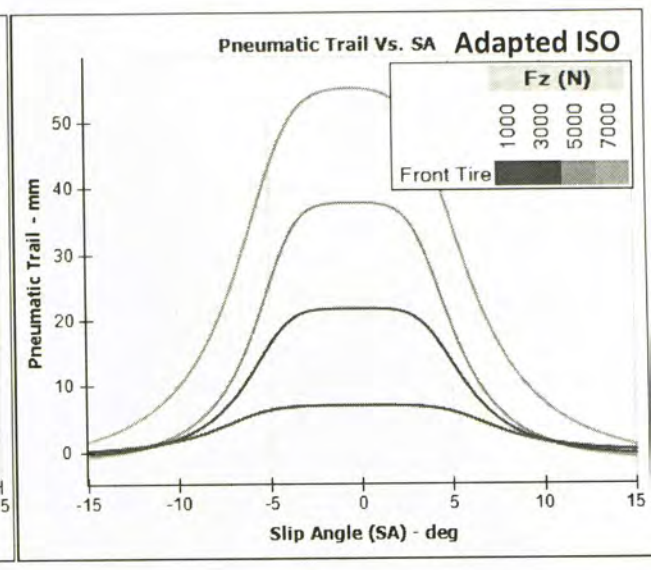
Mz vs. SA



Fy vs. SA



Trail vs. SA

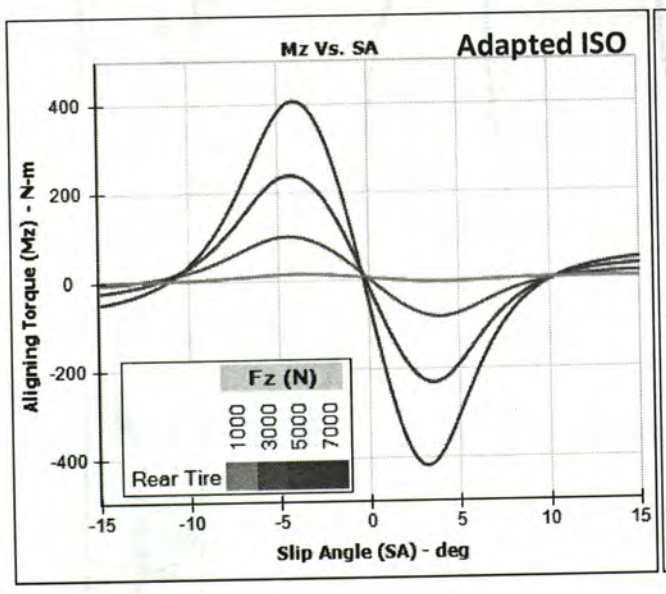


In this case the longitudinal pneumatic trail is calculated from measured M_Z and F_Y *pure* lateral test (no combined lateral and braking or acceleration forces)

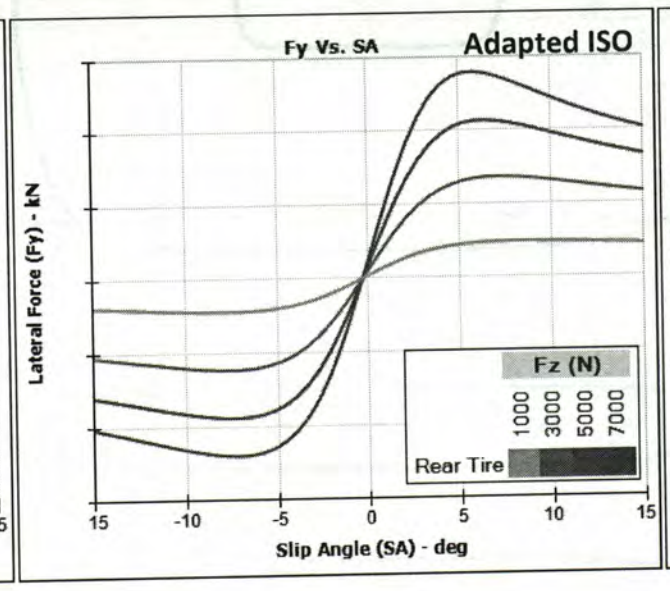
Tire data curves generated using OptimumT

Pneumatic Trail for different tire vertical loads

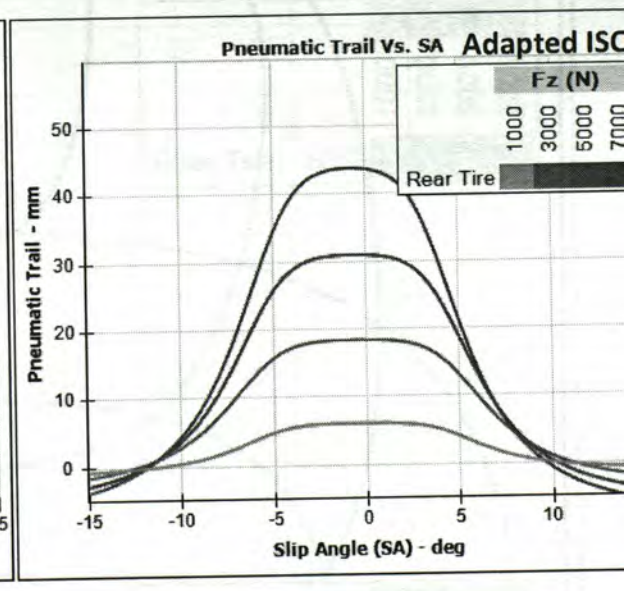
Mz vs. SA



Fy vs. SA



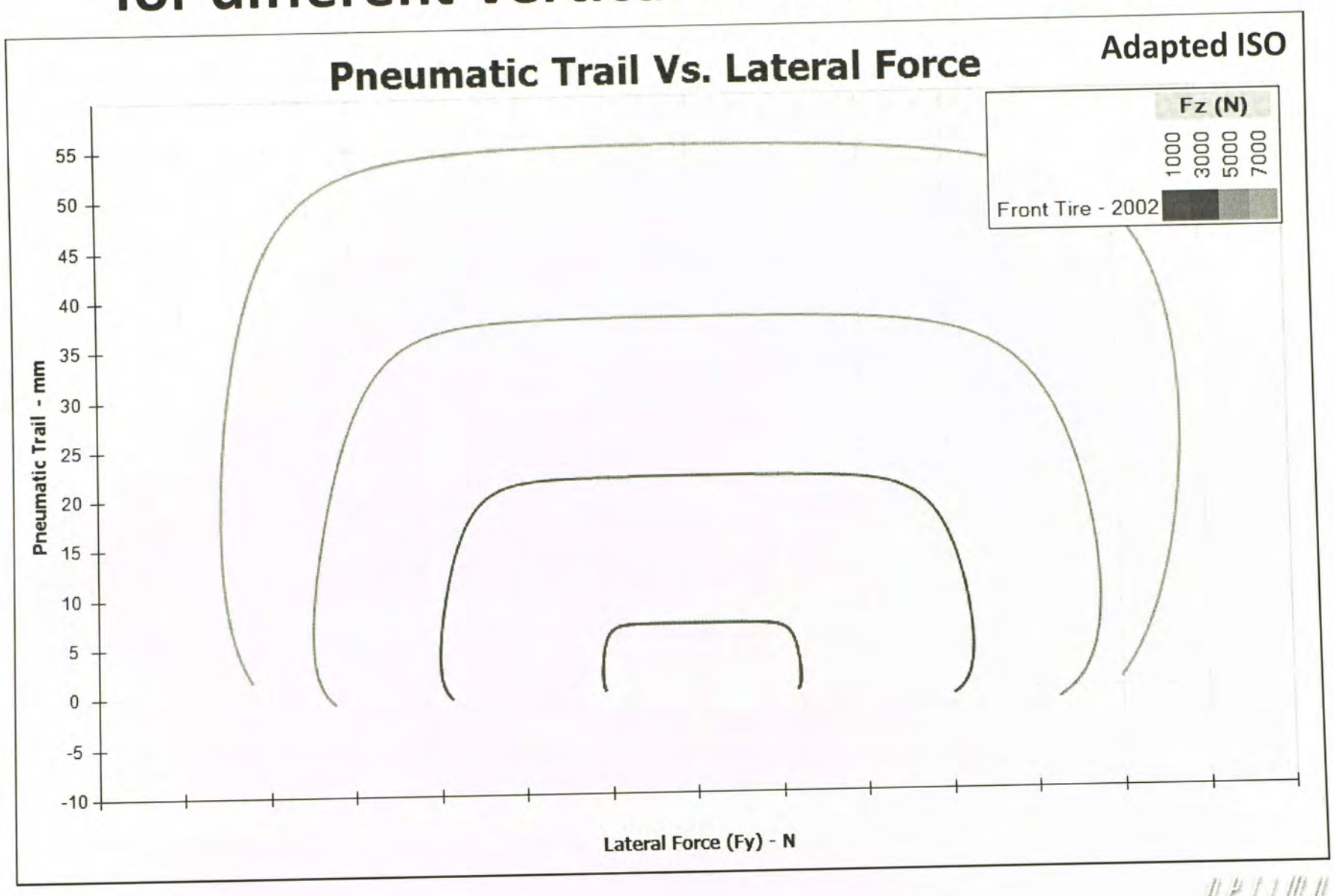
Trail vs. SA



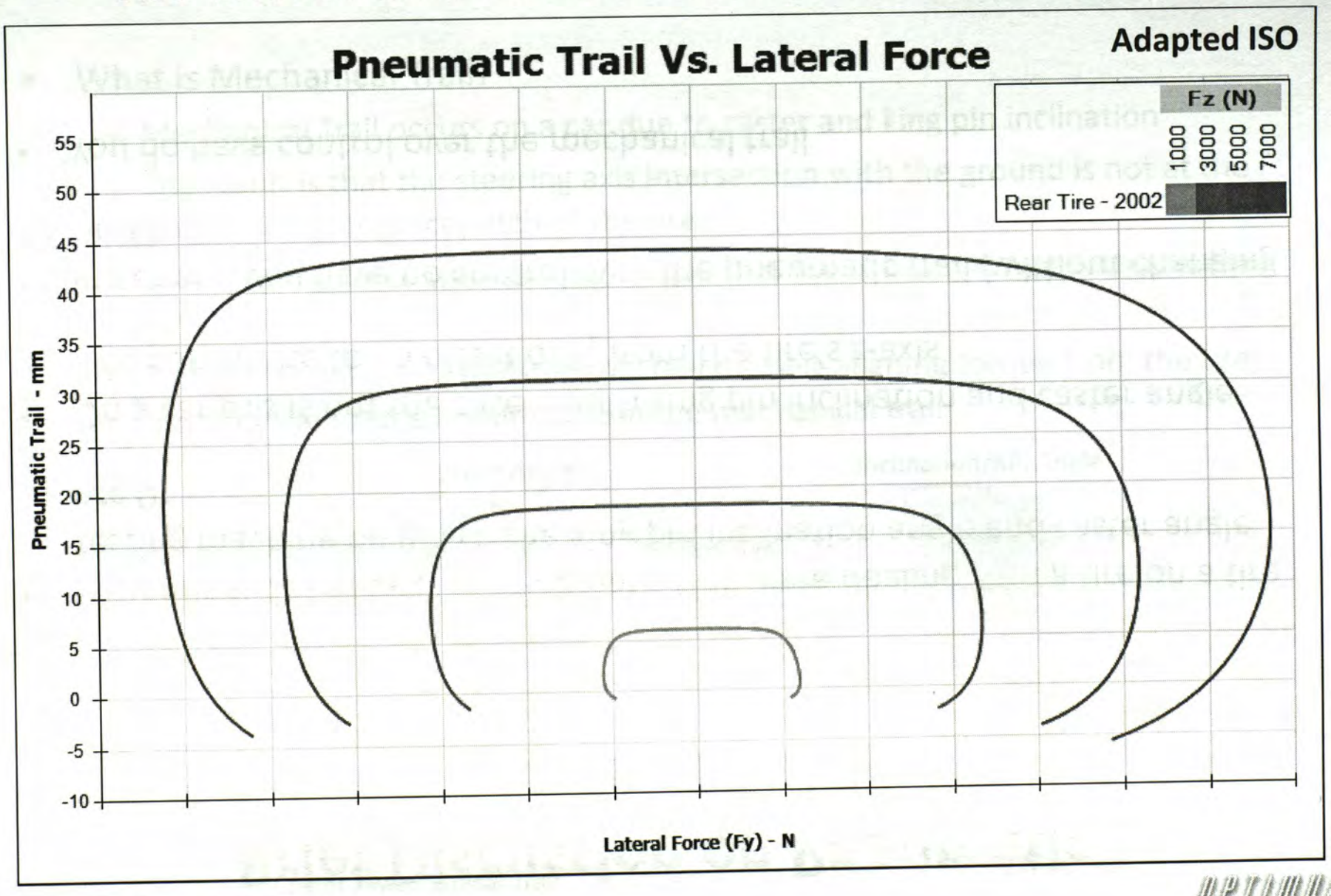
In this case the longitudinal pneumatic trail is calculated from measured M_Z and F_Y *pure* lateral test (no combined lateral and braking or acceleration forces)

Tire data curves generated using OptimumT

for different Vertical Loads and Camber



Pneumatic Trail vs. Fy
for different Vertical Loads and Camber



Brief Discussion on Pneumatic and Mechanical Trail

Previous slides regarding pneumatic trail were dealing with a tire on
testing machine on which the king pin inclination angle and caster a
are 0°

On a car this is not the case – both king pin inclination and caster an
also contribute to an offset of F_y from the tire's z-axis

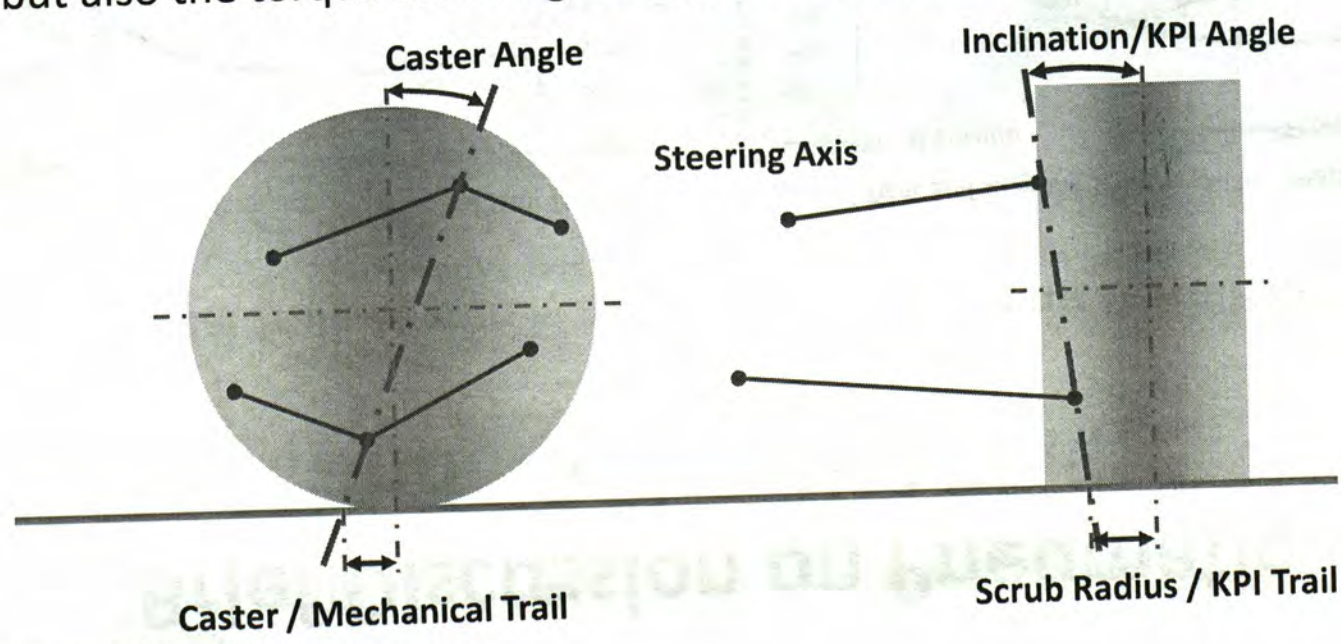
In general you have no control over the pneumatic trail (without cha
tires)

You do have control over the mechanical trail

Brief Discussion on Pneumatic and Mechanical Trail

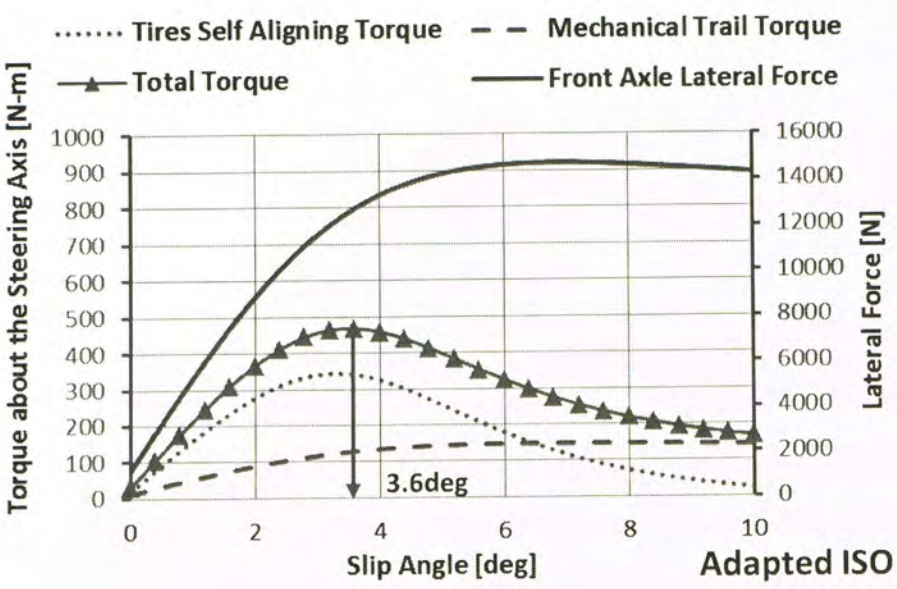
What is Mechanical Trail?

- Mechanical Trail occurs on a car due to caster and king pin inclinatio
- The result is that the steering axis intersection with the ground is no
  center of the contact patch of the tire
- This contributes to the resisting torque that the driver will feel in the
  wheel
- Therefore, the driver doesn't only feel the self-aligning torque from
  but also the torque resulting from the mechanical trail

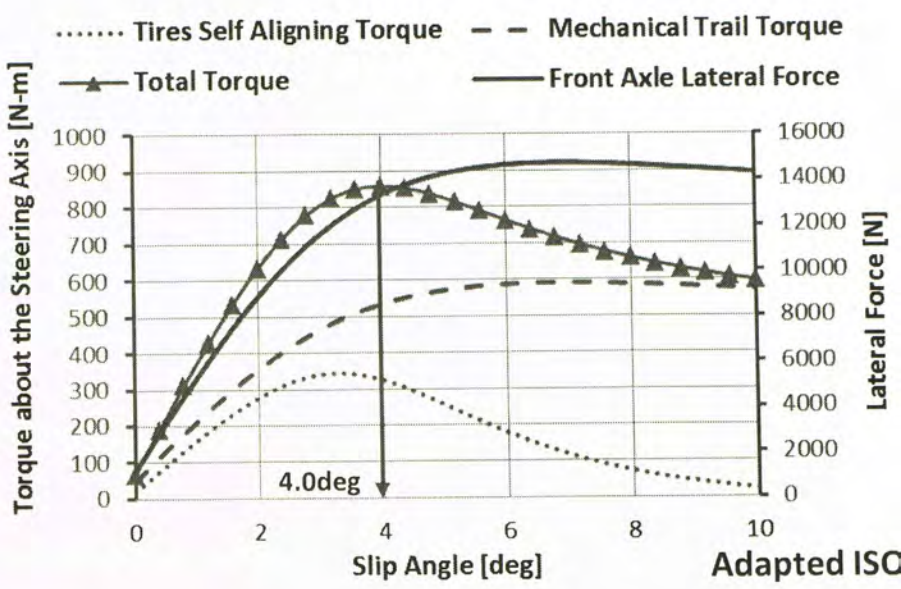


Brief Discussion on Pneumatic and Mechanical Trail

Mechanical Trail: 10 mm



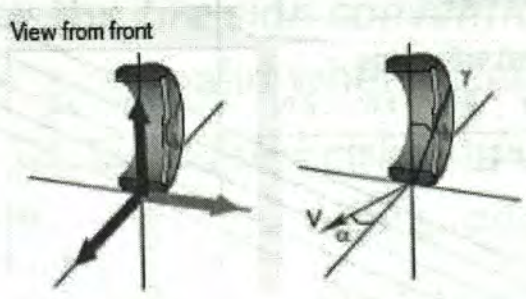
Mechanical Trail: 40 mm

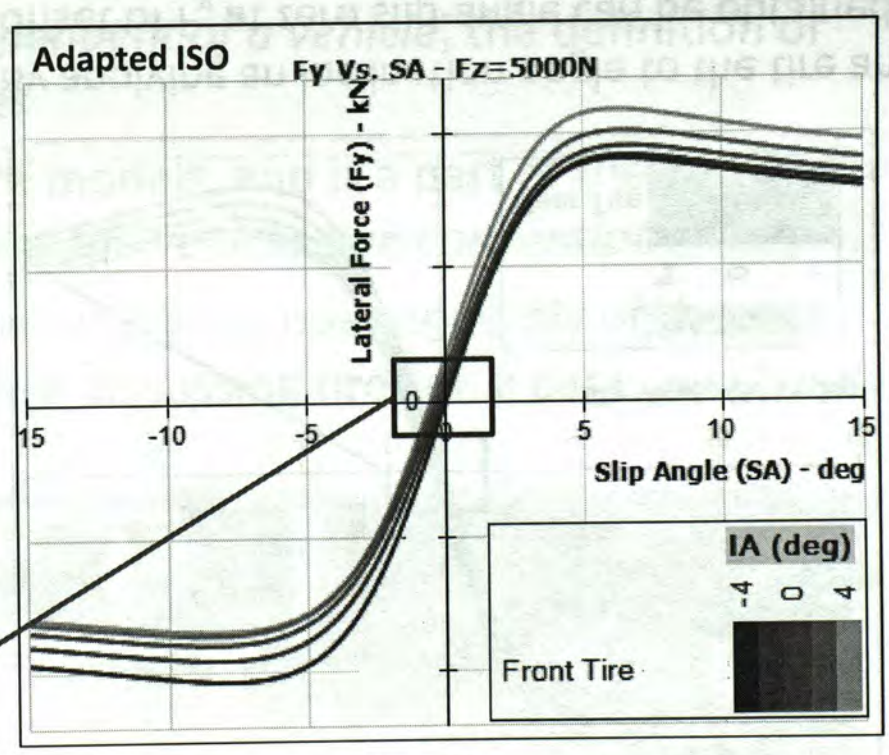


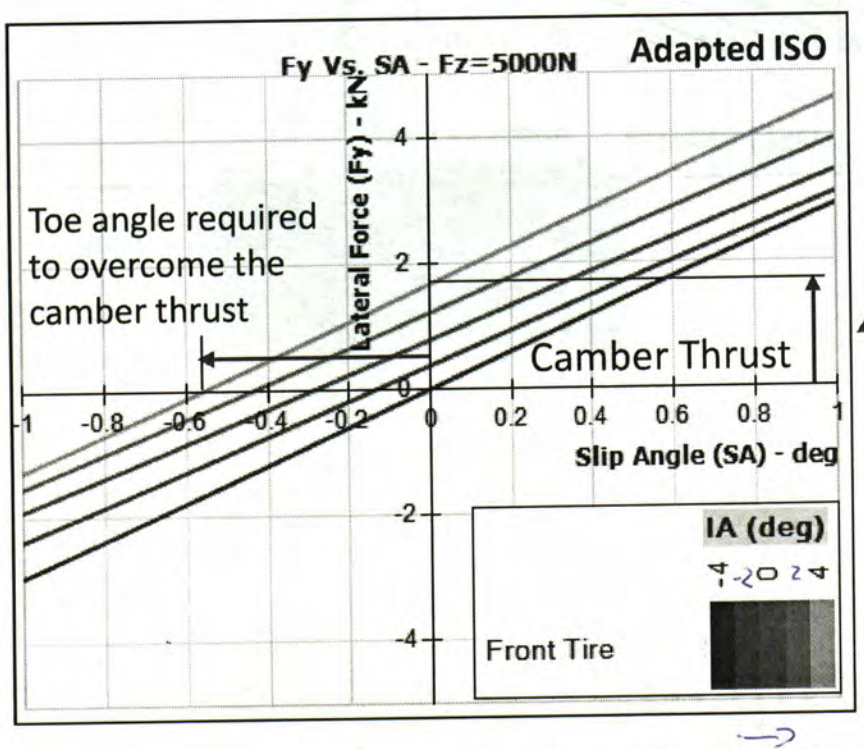
Does not take into account the static torque due to Right/Left Jacking effect.
Assume Fz_{FR} = 5600N and Fz_{FL} = 2000N continuously.

d) Inclination Angle

Lateral Force vs. Slip Angle for varying Inclination Angles





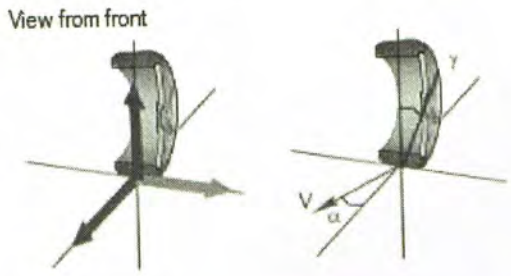


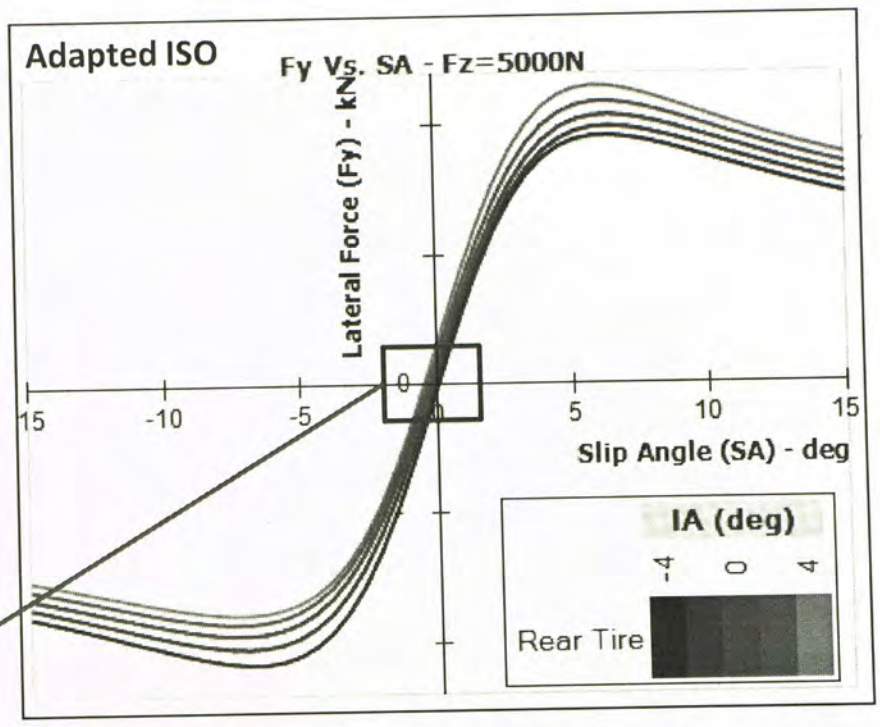
By applying an inclination angle to the tire ar
offset of F_y at zero slip-angle can be obtained

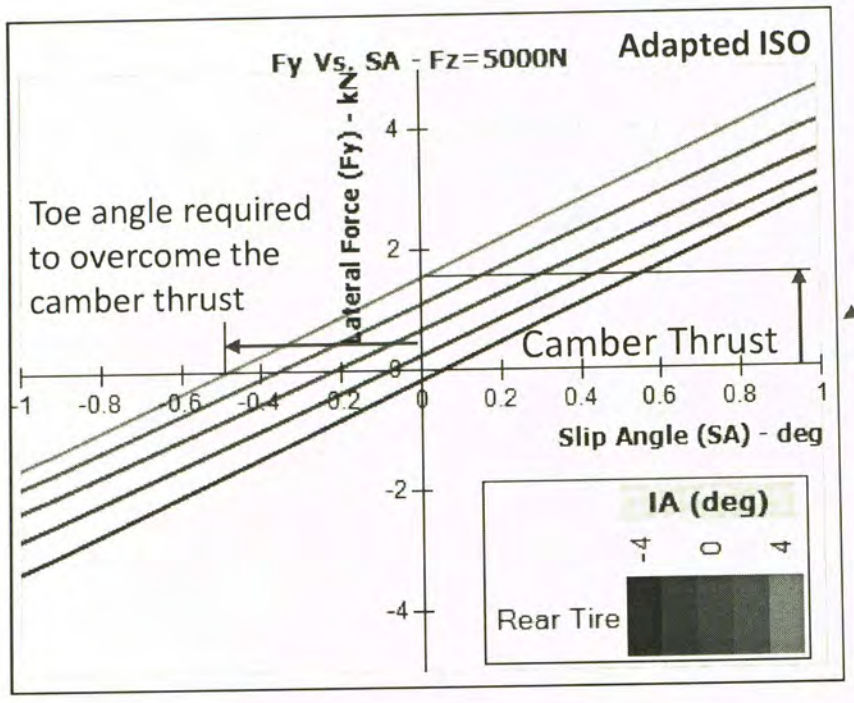
For this example tire, by adding 4° of inclination angle the F_y at zero slip-angle has increased in magnitude from 120 N to 400 N

Note: 1 Fz, 5 IA

Lateral Force vs. Slip Angle for varying Inclination Angles







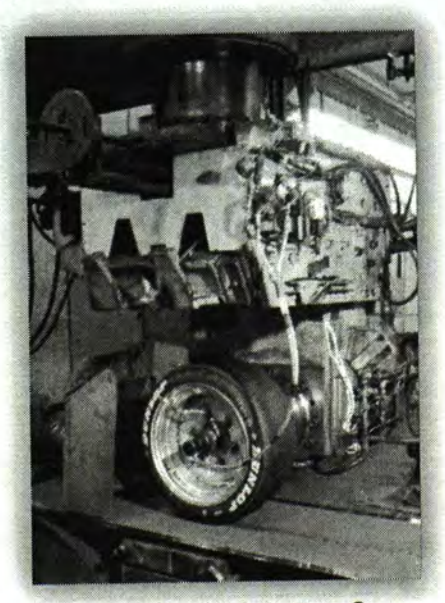
By applying an inclination angle to the tire an offset of F_y at zero slip-angle can be obtained

For this example tire, by adding 4° of inclination angle the F_y at zero slip-angle has increased in magnitude from 120 N to 400 N

Note: 1 Fz, 5 IA

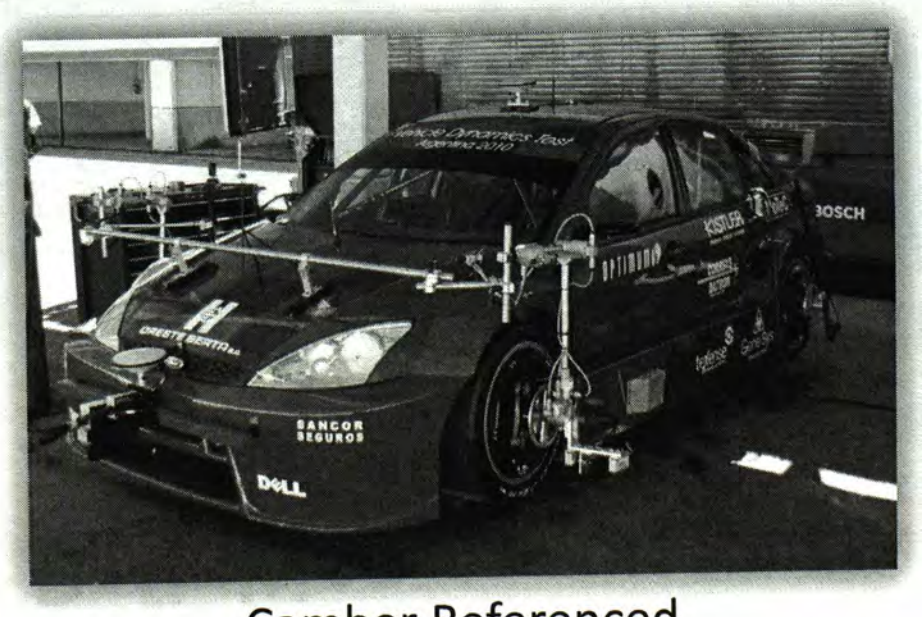
Inclination Angle vs. Camber Angle

- Because the *tire sign convention is independent of a vehicle*, the definition of camber is not valid when discussing tires
- Therefore inclination angle is used in tire models, and is a part of the tire sign convention, and therefore *independent of the vehicle sign convention*
- In this presentation when we are dealing with tires independently of the car we will use inclination angle, when we are discussing tires on a car we will use camber

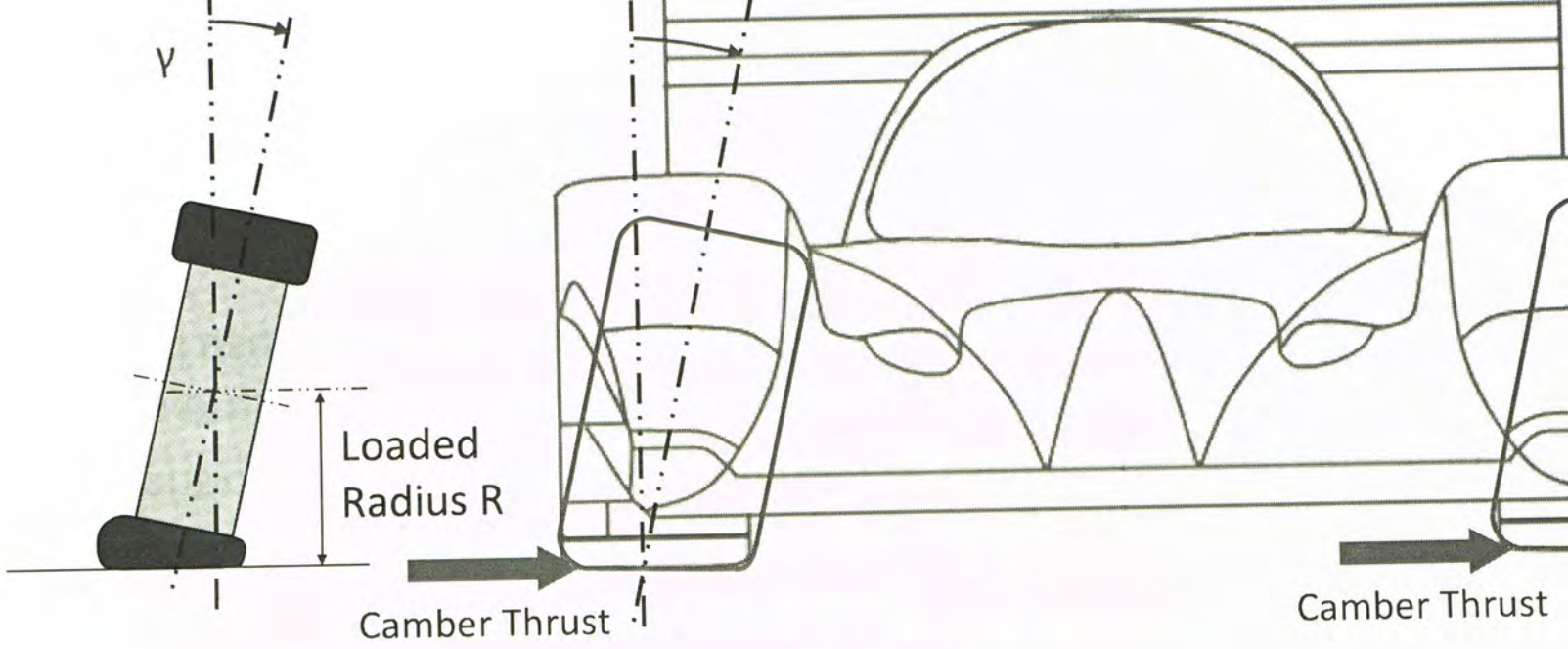


Inclination Angle Referenced

VS.



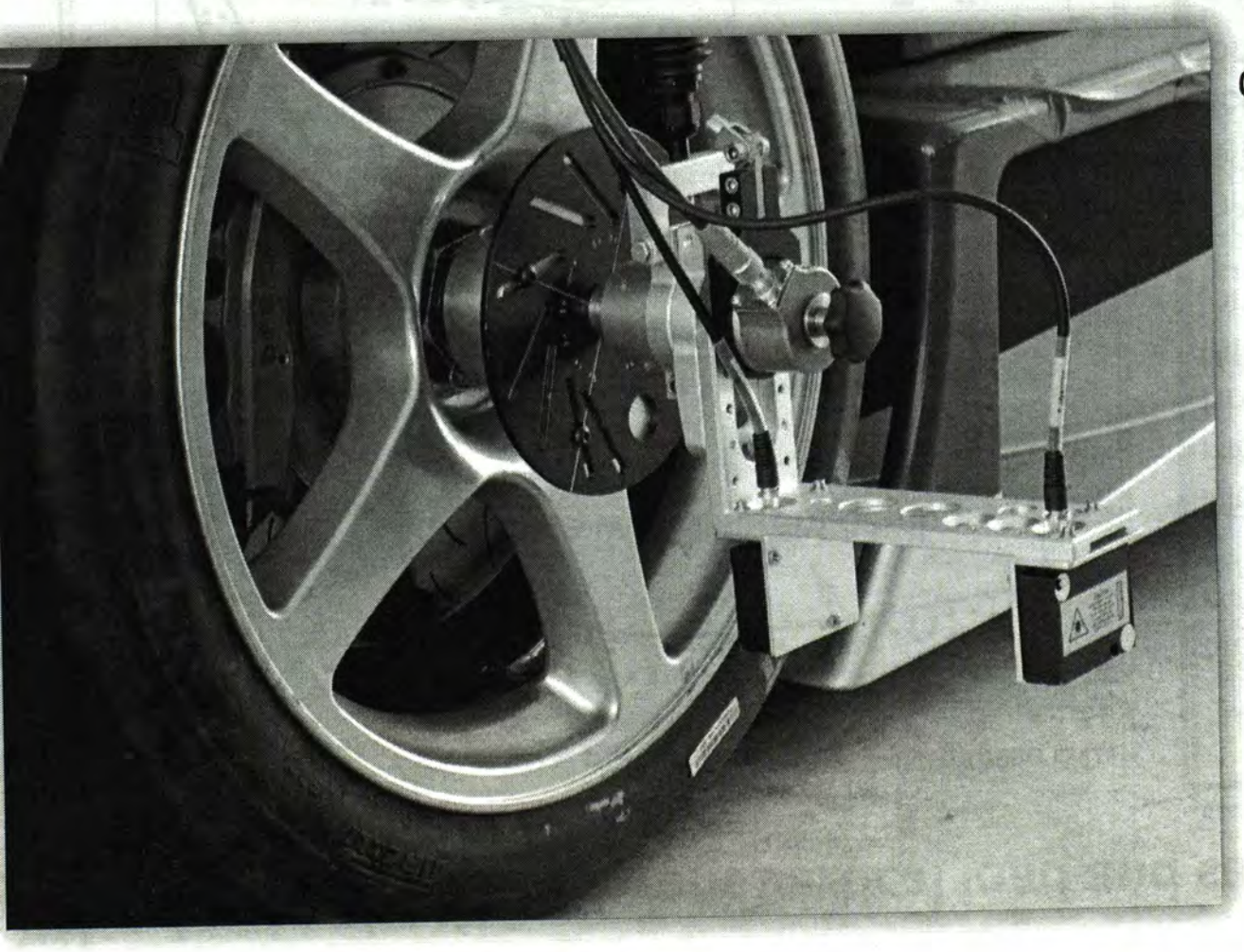
Camber Referenced



When the top of the wheel is oriented toward the inner side of the car, th
camber is defined as < 0.

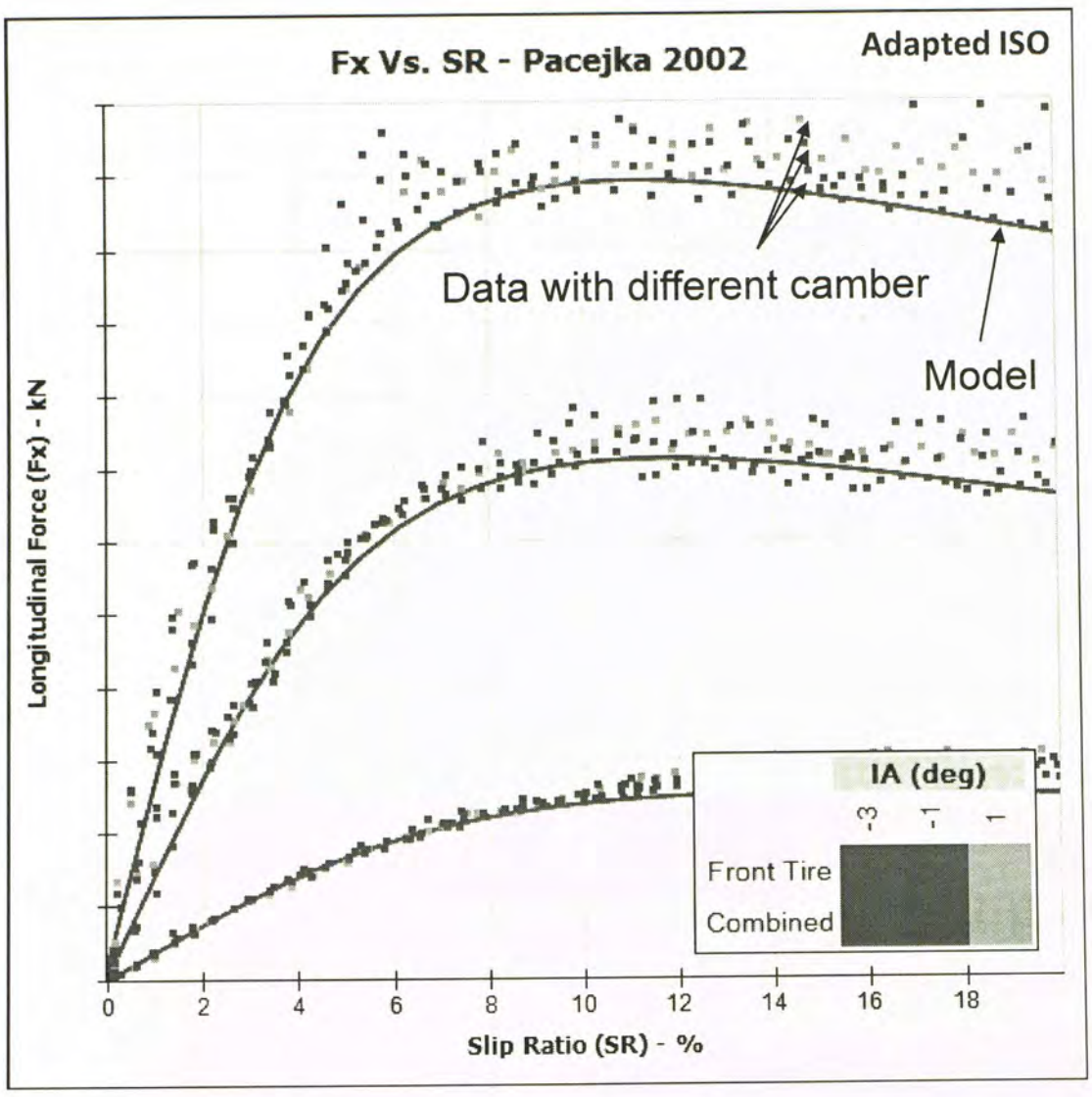
Camber

Measured Using Camber Sensor



Could be useful

for different vertical load and camber



Attention!!!!

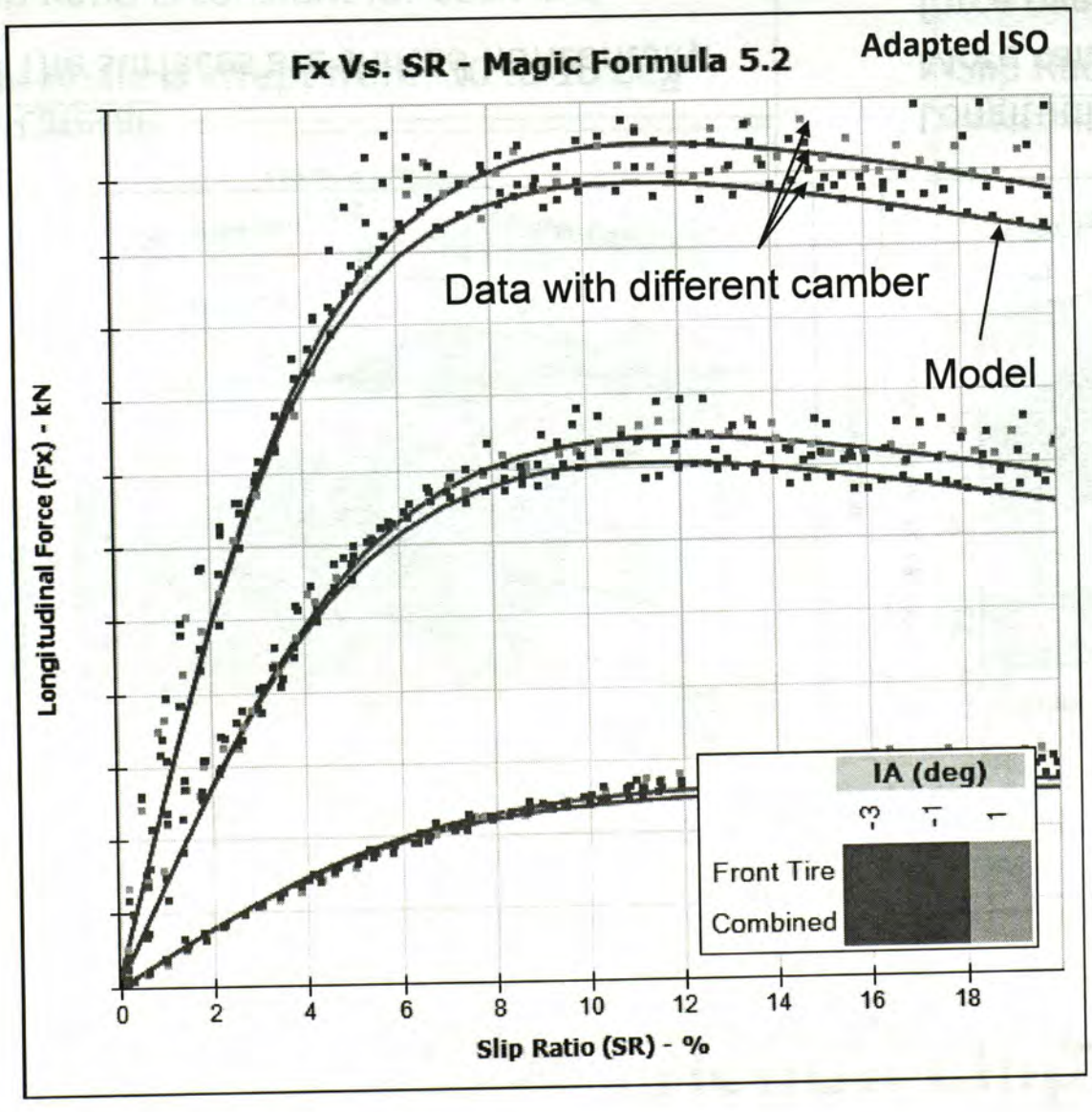
Note the difference between model and data!!

Most of the Pacejka tire models (99, 2002 etc..) do **not** take into account the influence of camber on longitudinal forces!

Only the Pacejka 5.2 and 6.1 version takes the camber into account for longitudinal tire force.

Tire longitudinal force vs. slip ratio

for different vertical load and camber

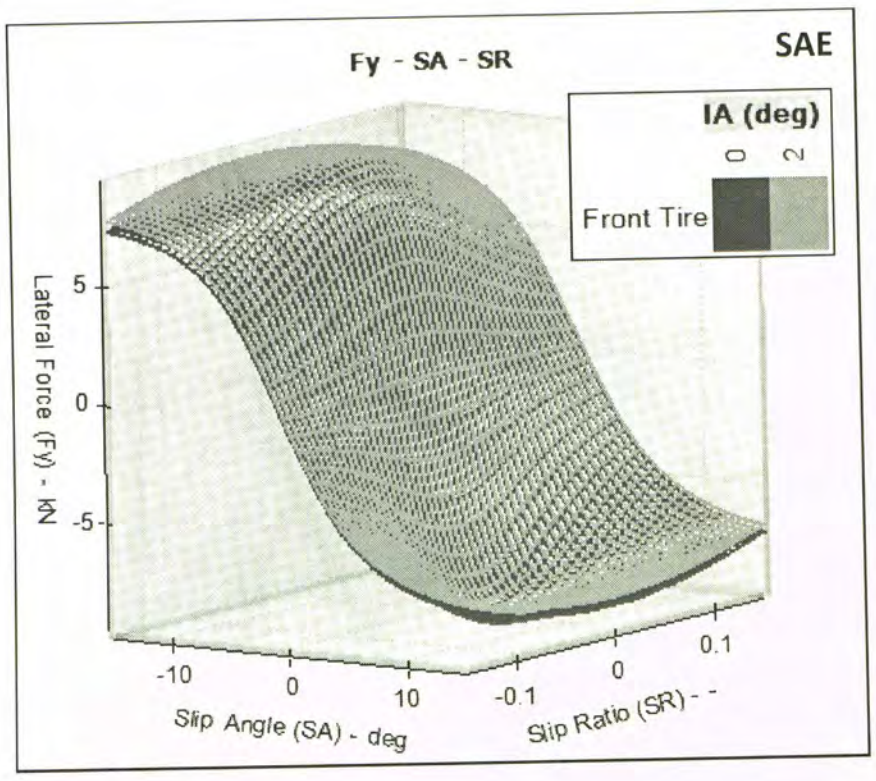


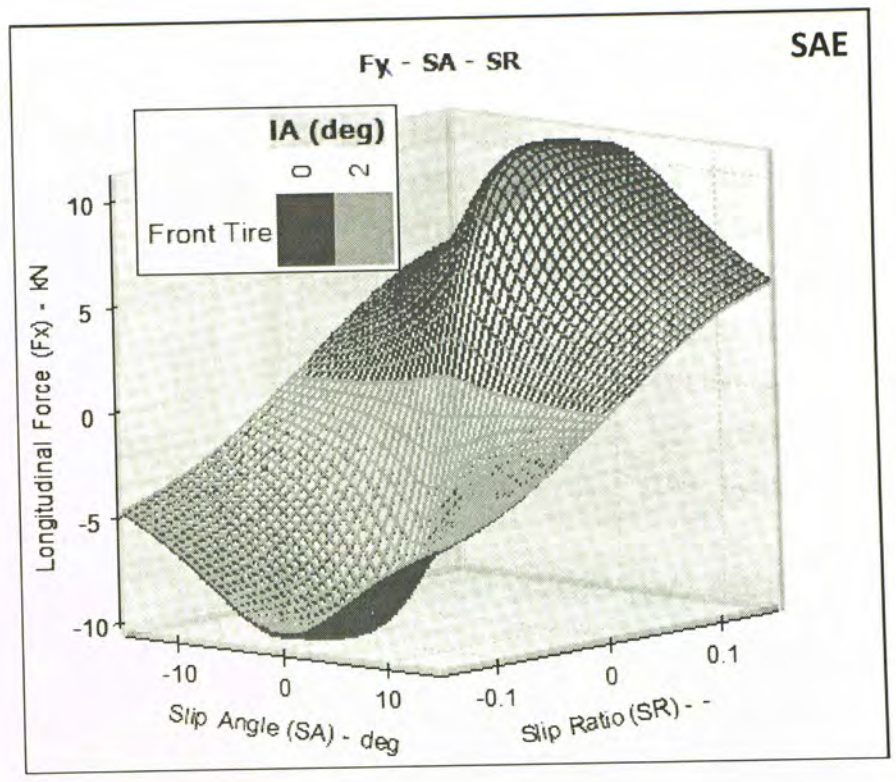
Magic Formula 5.2 & 6.1 Model

Notice that this model does show the change in Longitudinal Force (Fx) as the Inclination Angle (IA) varies.

Notice that even the Pacejka 5.2 and 6.1 has its flaws. It shows the Longitudinal Force (Fx) as being the same for an Inclination Angle (IA) of 2 and -2 degrees. Since tires are asymmetrical this is an error in the model.

Inclination Angle Effects





Lateral:
The surfaces are shifted horizontally.

Longitudinal:
More camber gives us less longitudinal force over the entire range.

Note: Must use Pacejka MF5.2 or newer to see these effects.

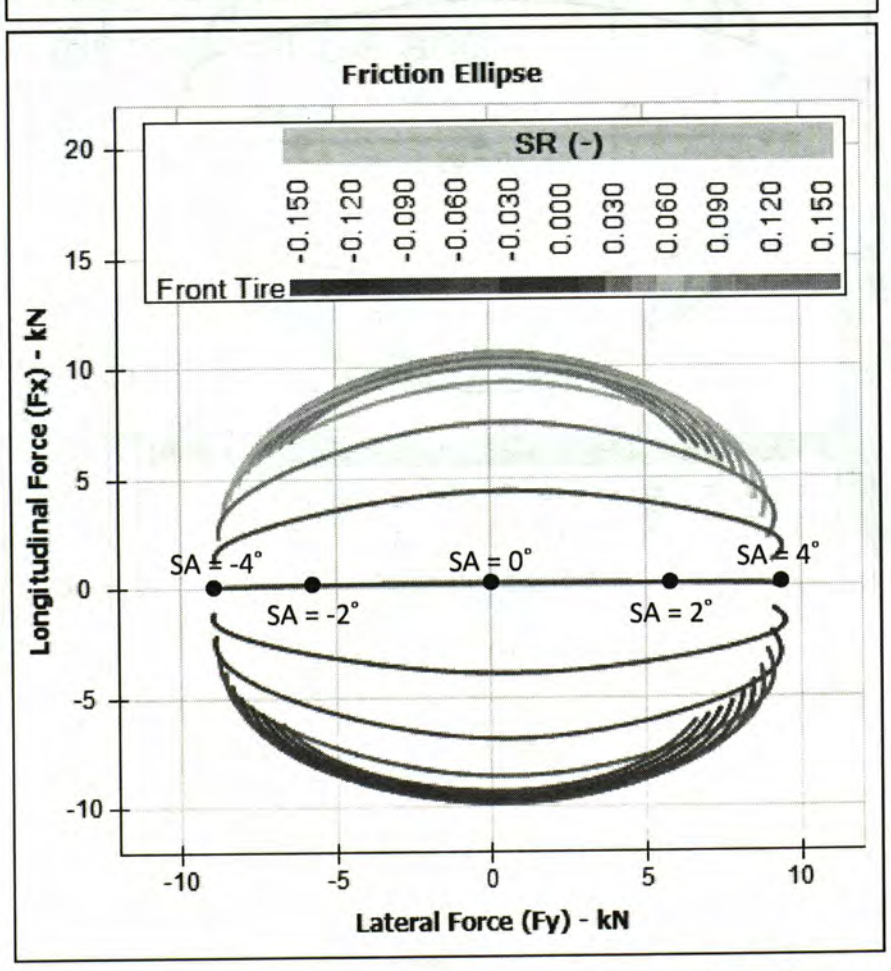
Tire Forces and Moments

Slip Ratio is constant for each line

Slip Angle is swept from -10 to 10 deg

Slip Angle is constant for each line

Slip Ratio is swept from -0.15 to 0.15



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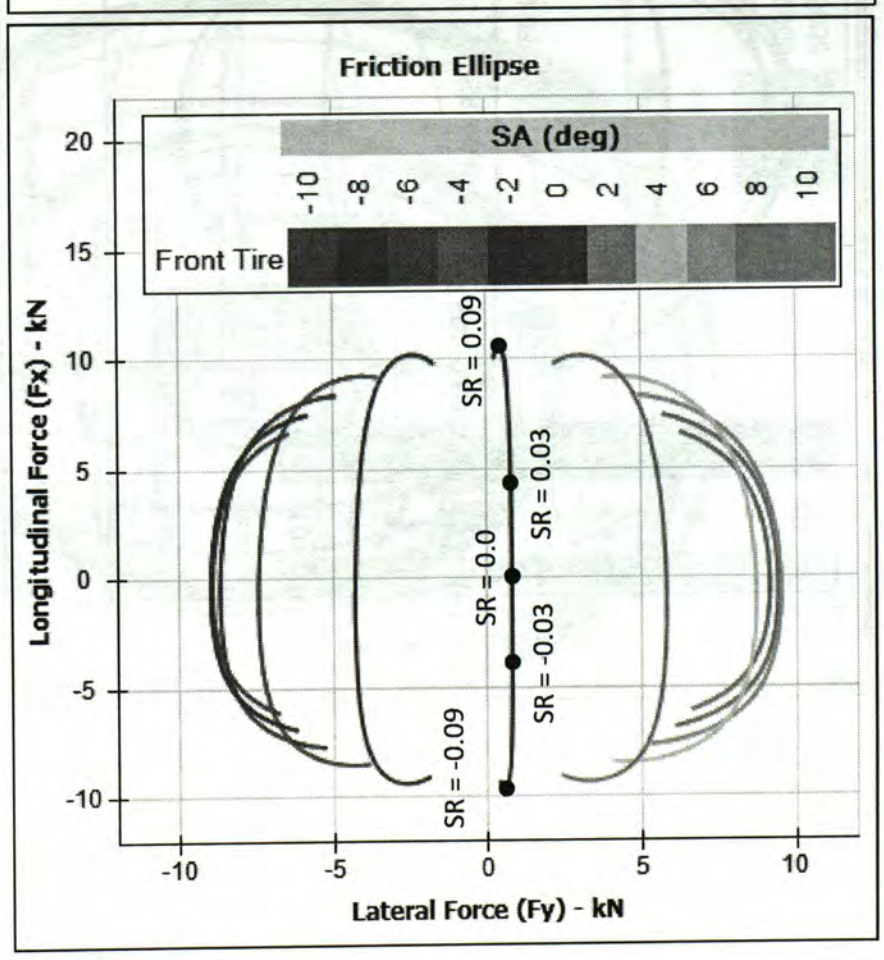
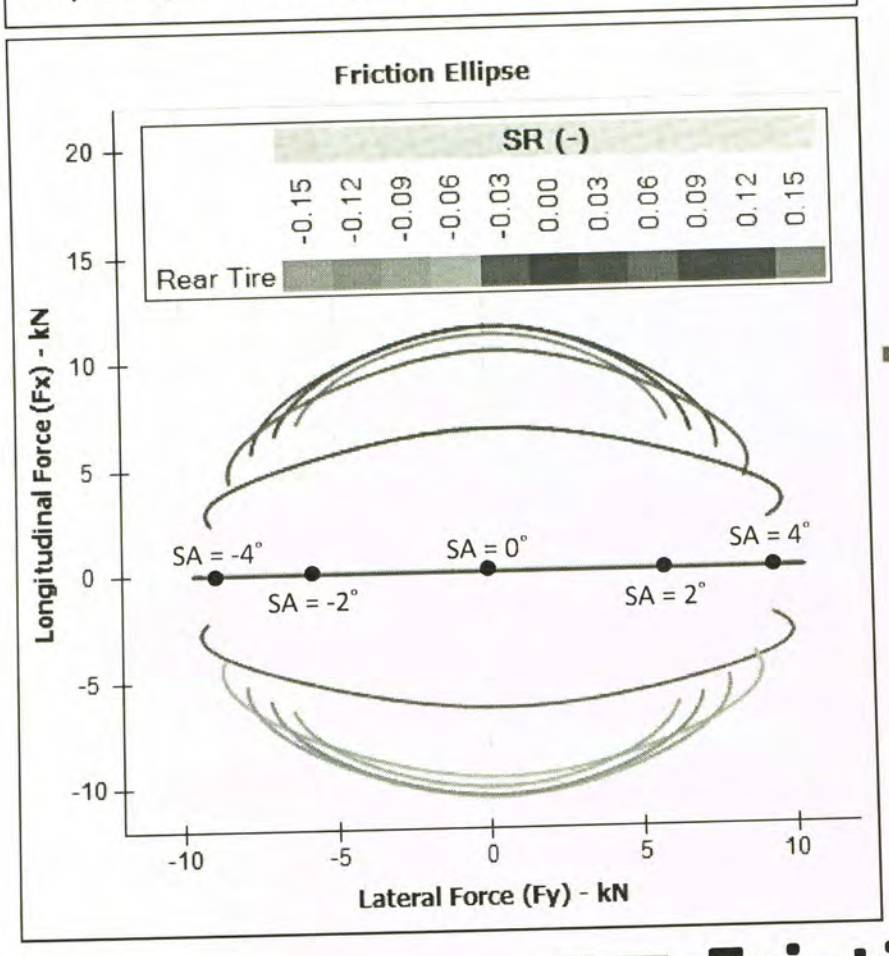


= **Friction Ellipse**

Tire Forces and Moments

Slip Ratio is constant for each line

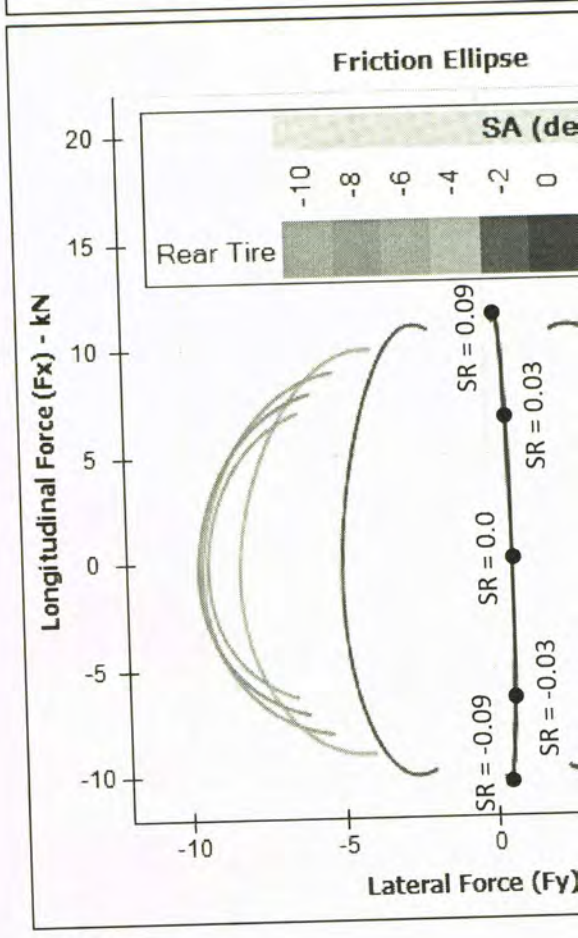
Slip Angle is swept from -10 to 10 deg



+

Slip Angle is constant for eac

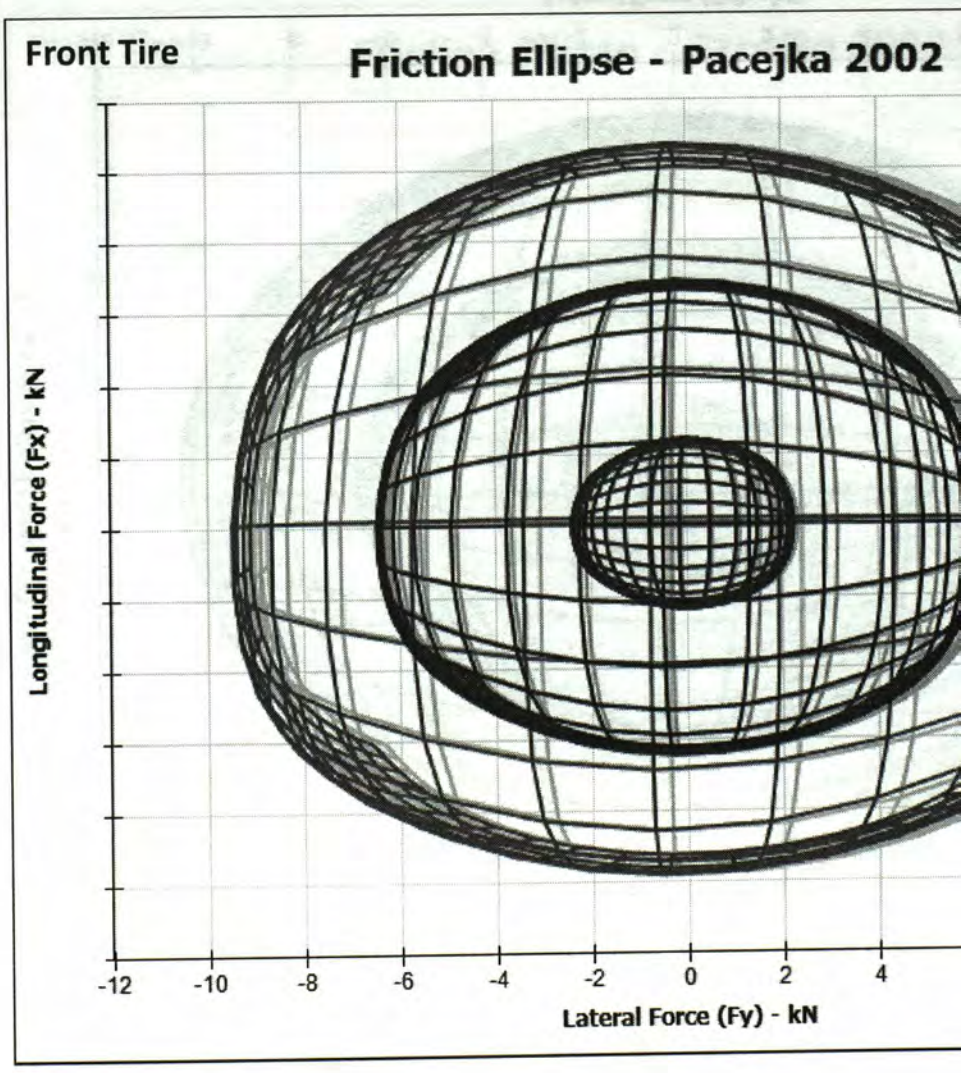
Slip Ratio is swept from -0.15



= **Friction Ellipse**

Tire Forces and Moments

- Lateral and longitudinal force as a function of SA and SR
- Lines represent constant values of SA and SR
- Multiple loads plotted



Model not representative of actual data

Tire Forces and Moments

- Lateral and longitudinal force as a function of SA and SR
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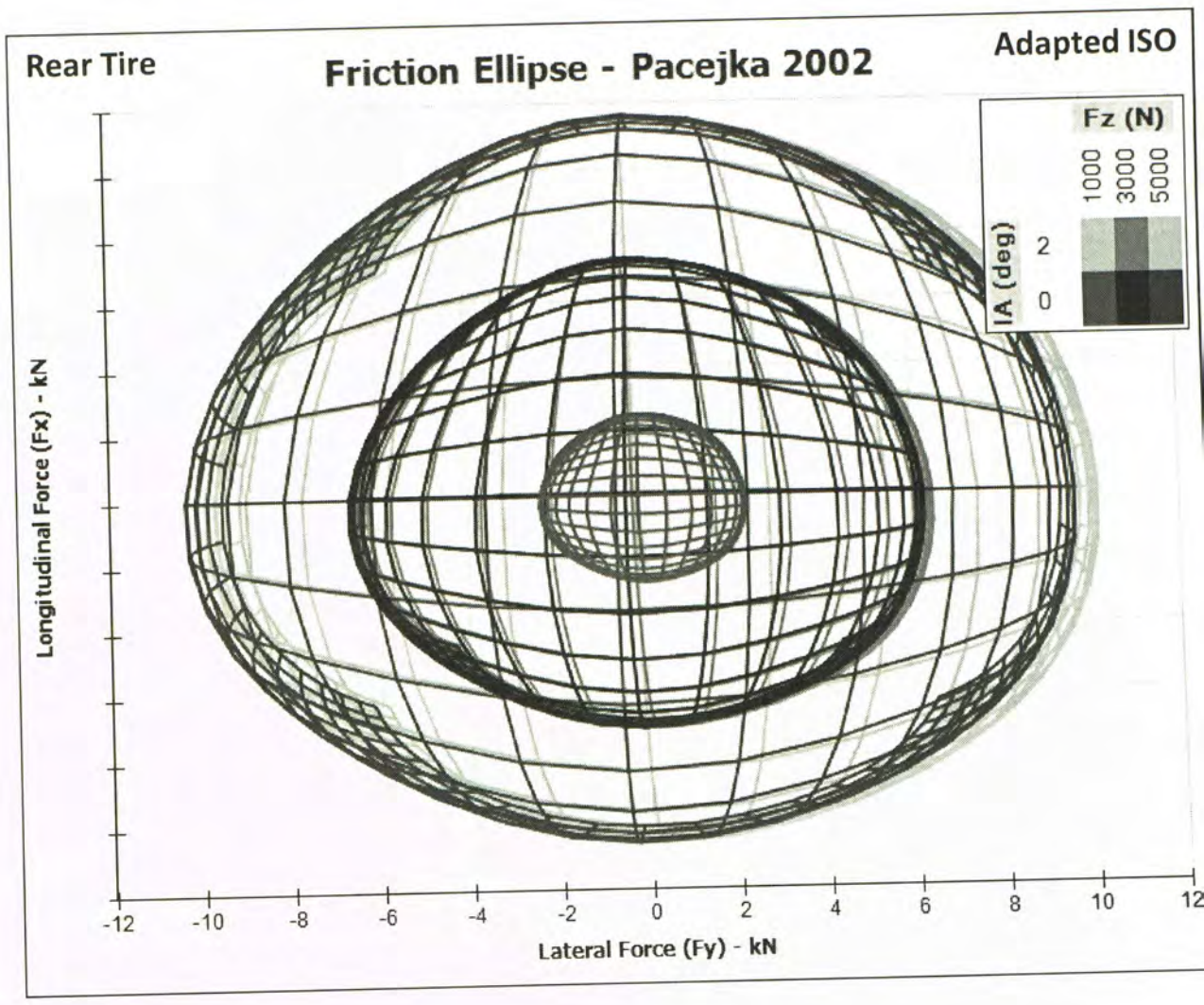
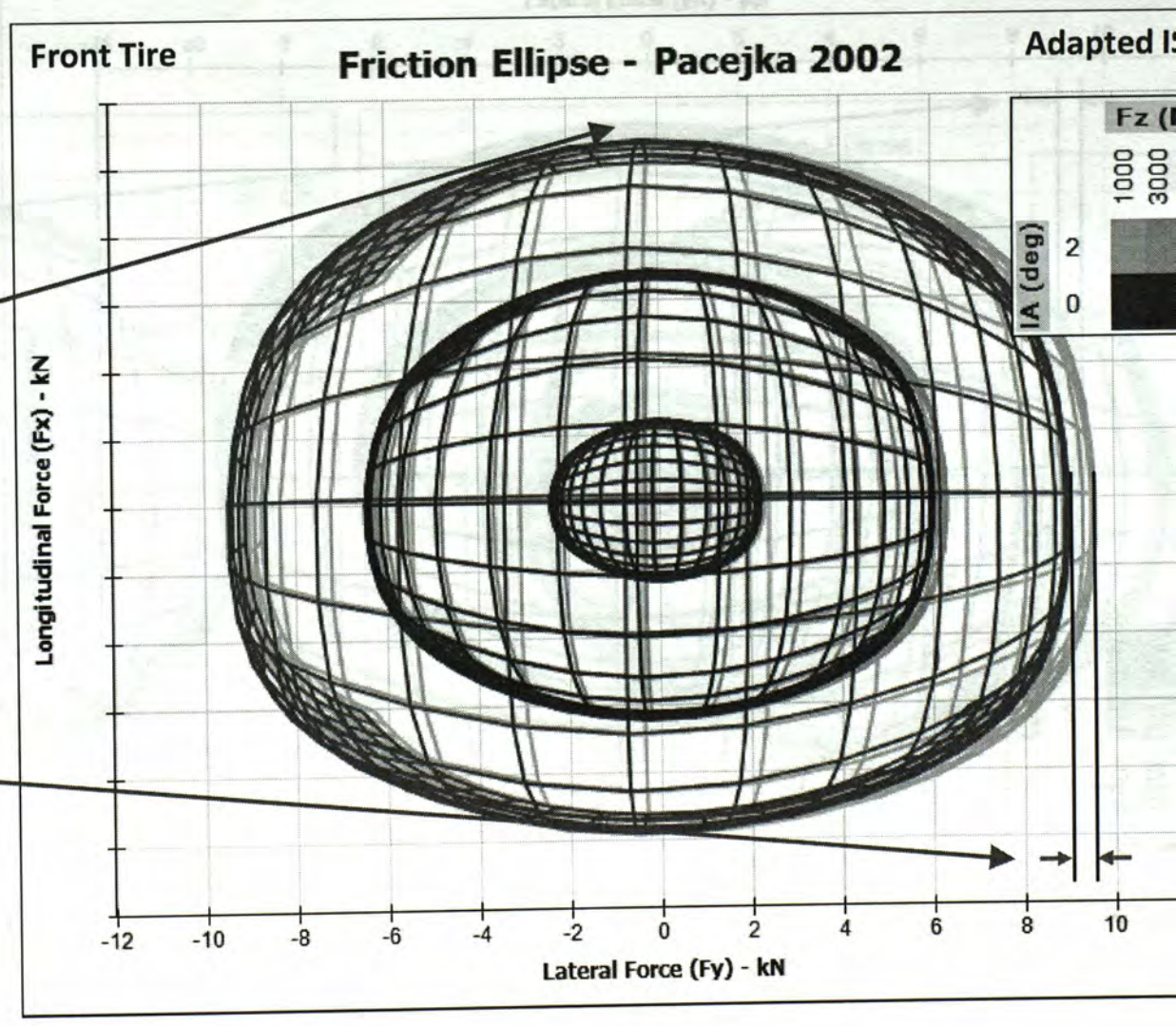


Model not representative of actual data

Tire Forces and Moments

Friction Ellipse vs. Load and Camber

- No camber effect on tire longitudinal force *in Pacejka 2002 model*
- Camber effect on tire lateral force



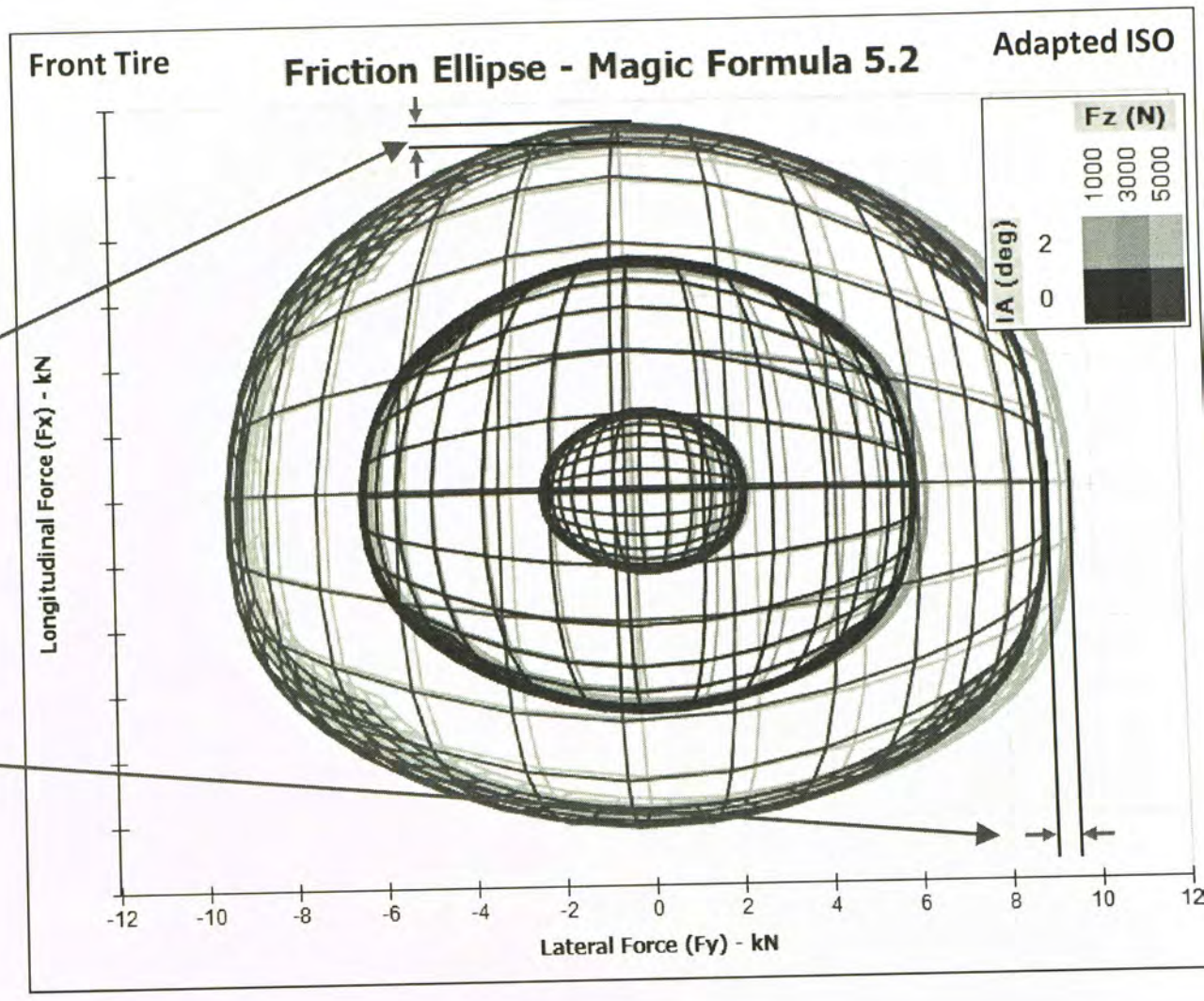
Model not representative of actual data

Tire Forces and Moments

riction Ellipse vs. Load and Camber

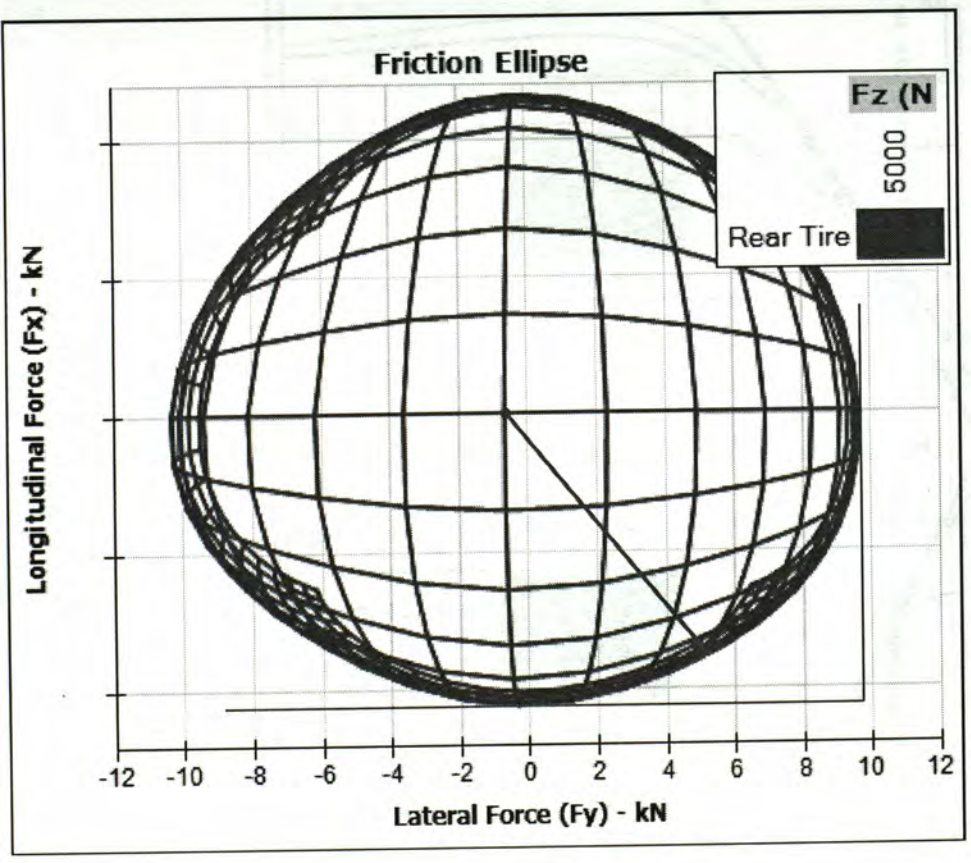
• Camber effect on tire longitudinal force

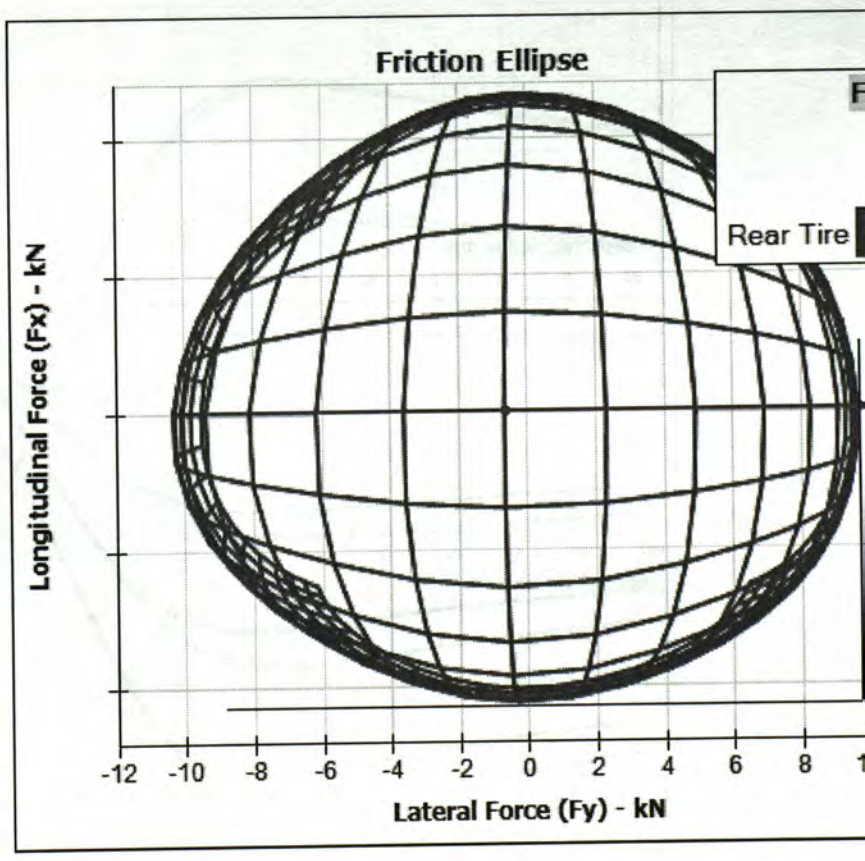
• Camber effect on tire lateral force



Model not representative of actual data

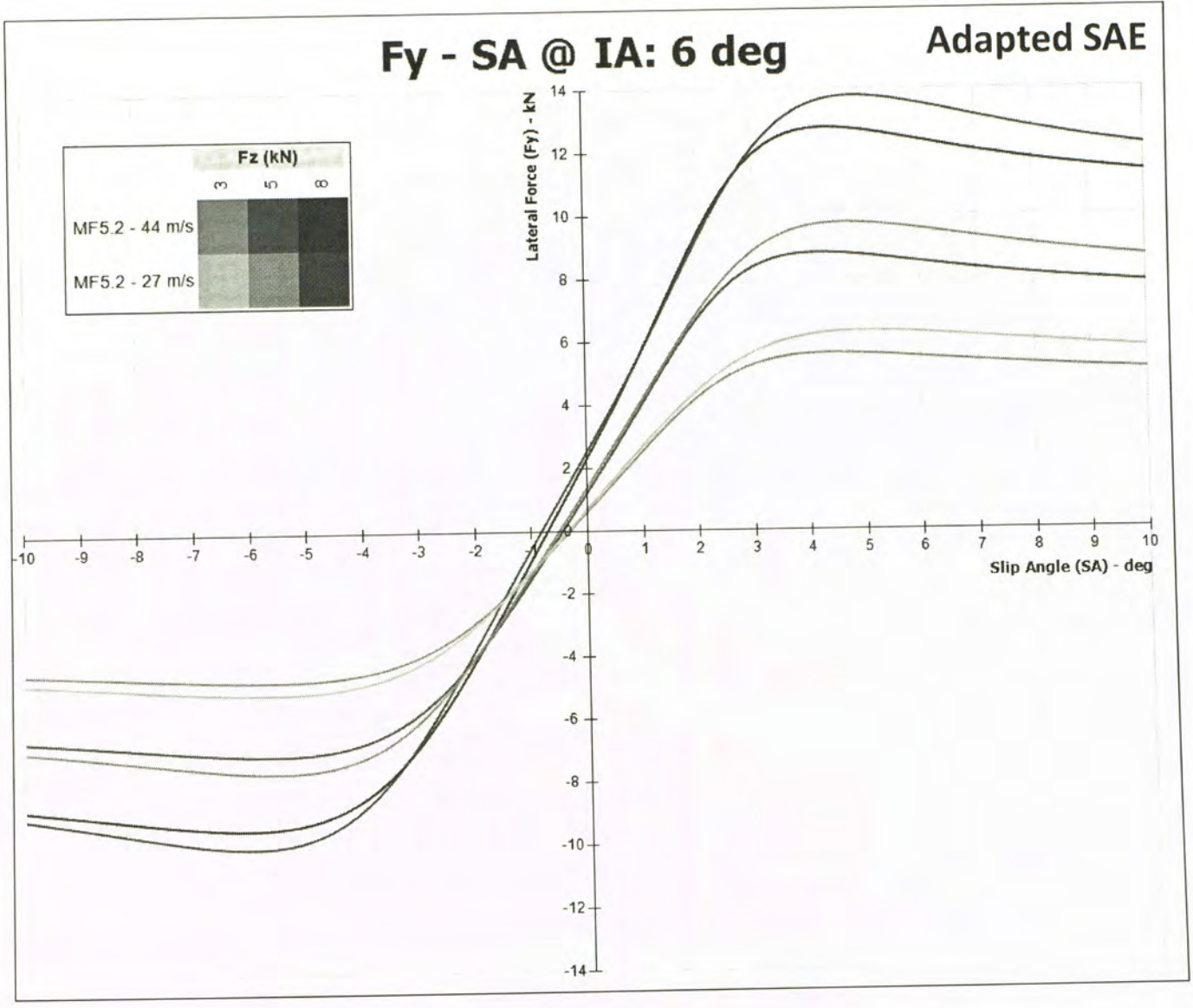
Tire Usage Efficiency



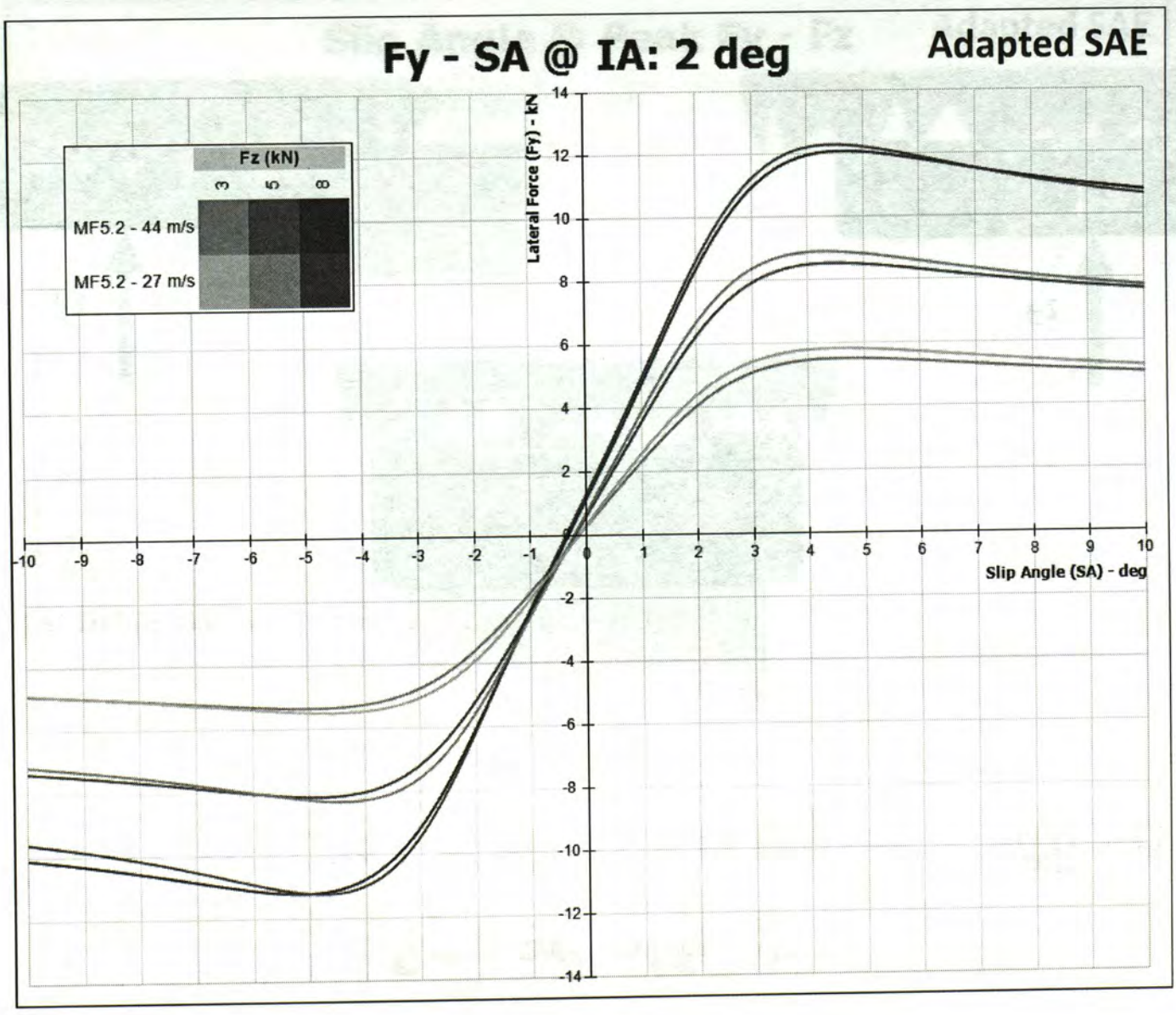


Lateral use: 53%
Longitudinal use: 76%

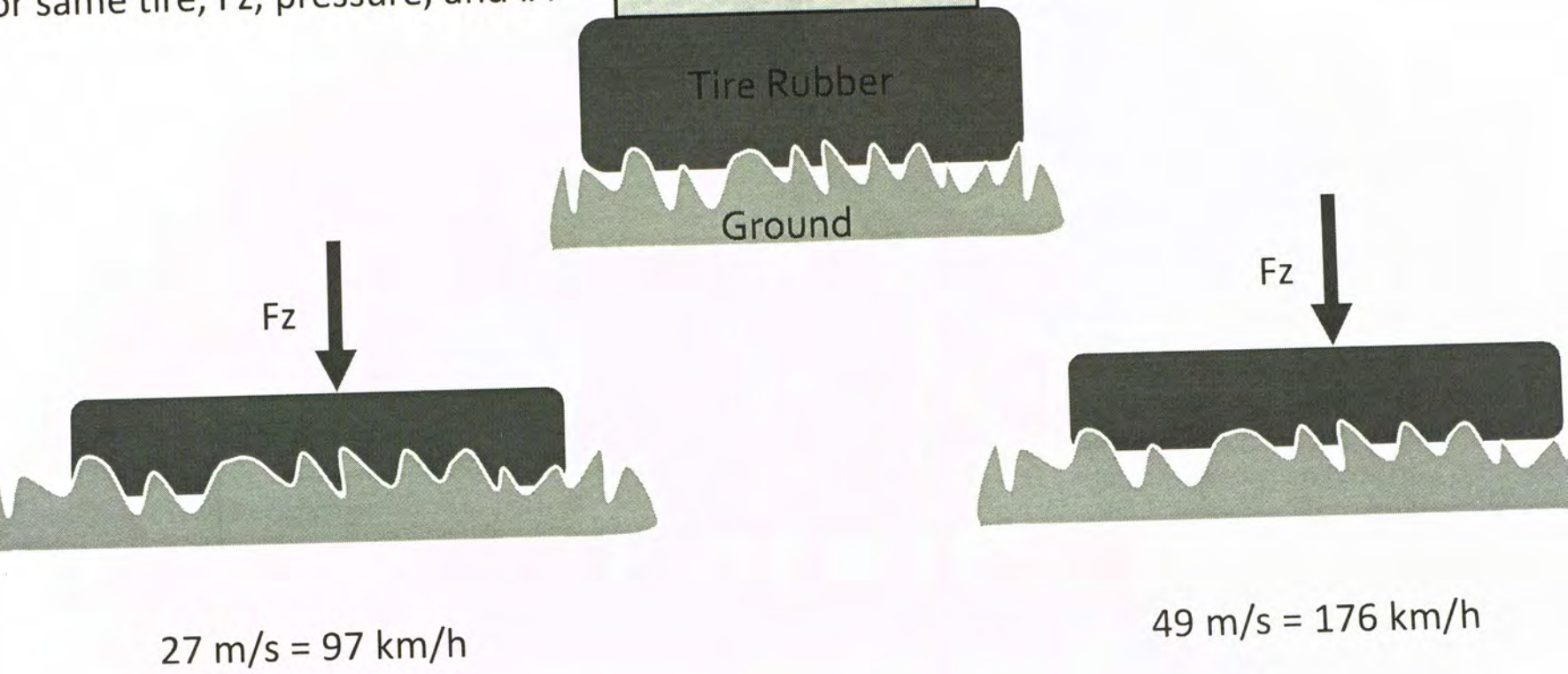
Lateral use: 100%
Longitudinal use: 0%



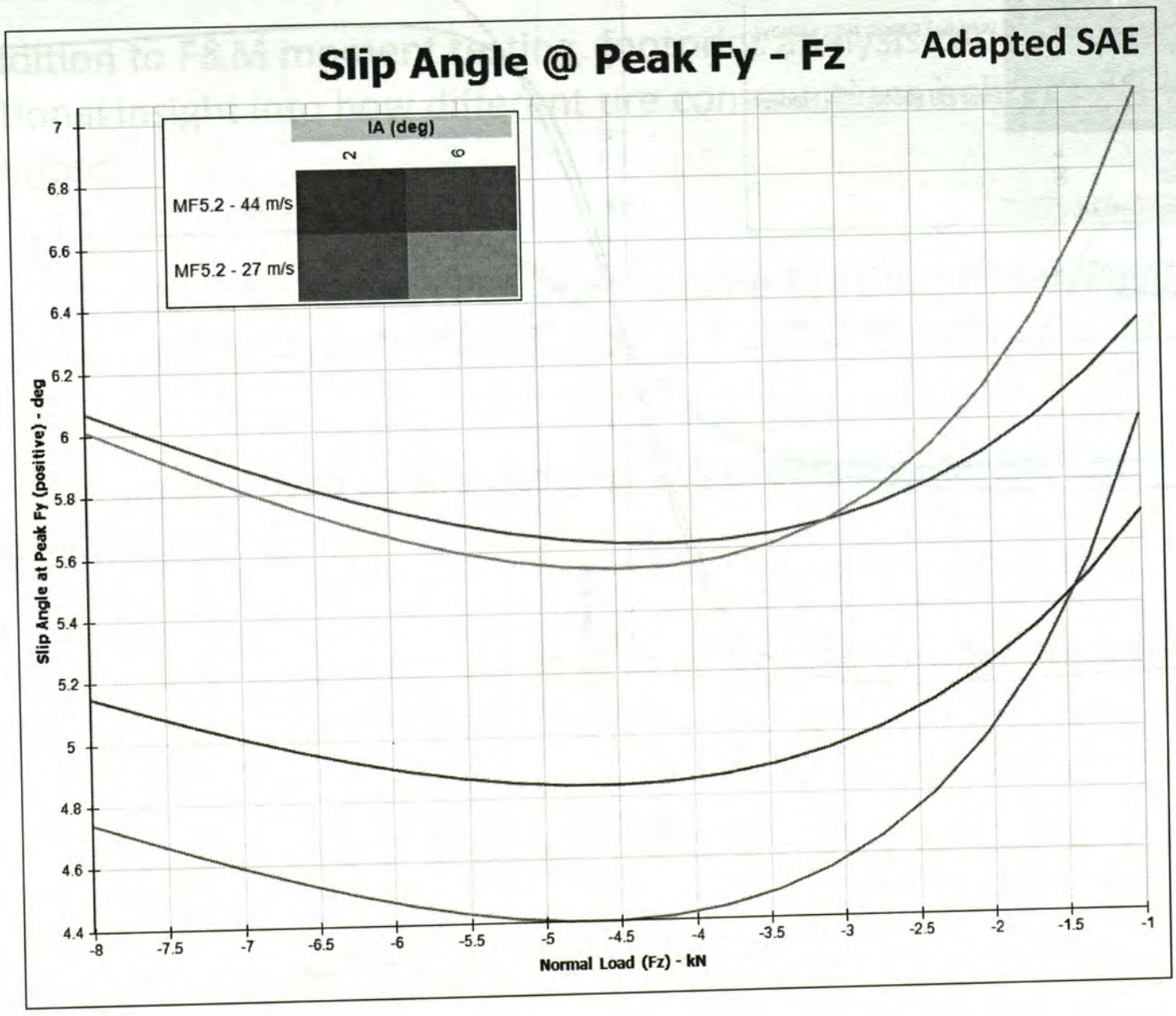
Speed effect on F_Y

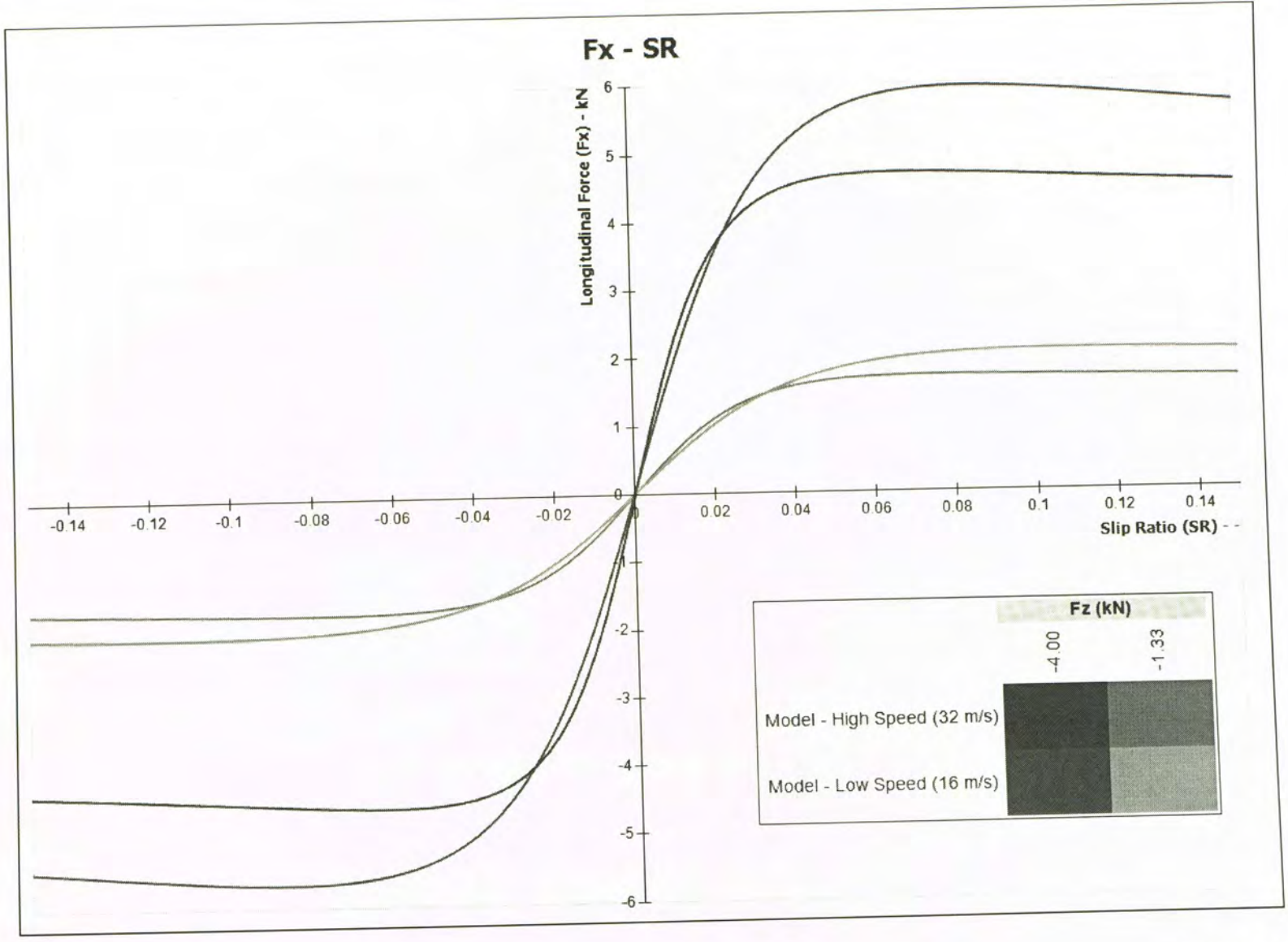


or same tire, Fz, pressure, and ...



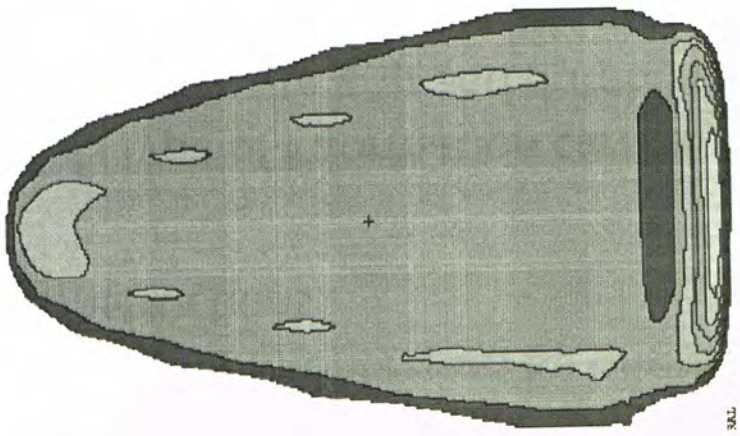
Speed effect on F_Y

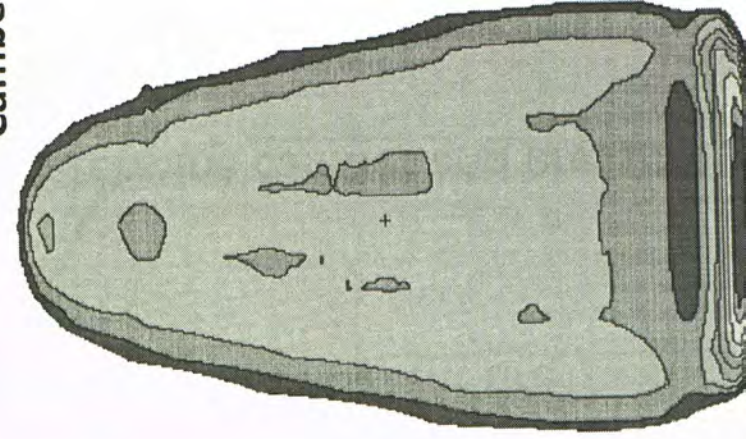




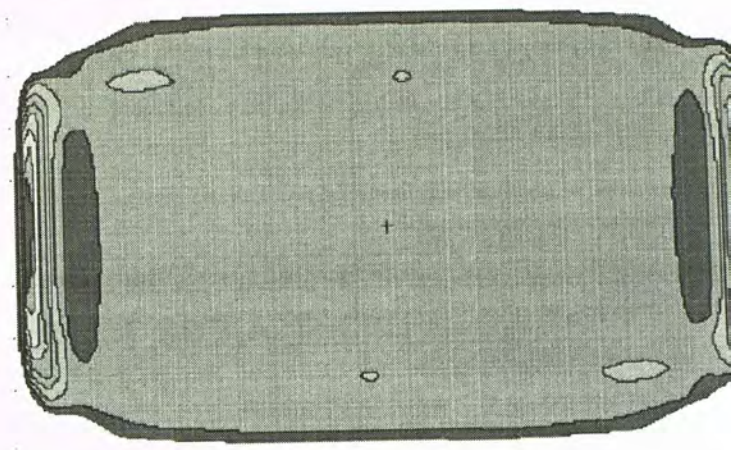
Tire Contact Patch Analysis

- In addition to F&M moment testing, footprint analysis can provide additional insight into how different tire constructions behave at different conditions.
- This can be done using both FEA and Pressure Sensitive Paper/Paint or pressure mat sensors.
- This is commonly used by tire designers to evaluate different tire constructions.
- This type of information can be used to determine camber and pressure ranges for tire grip and durability.
  - Larger contact patch area increases grip
  - High local contact pressures accelerate wear and increases risk of tire failure





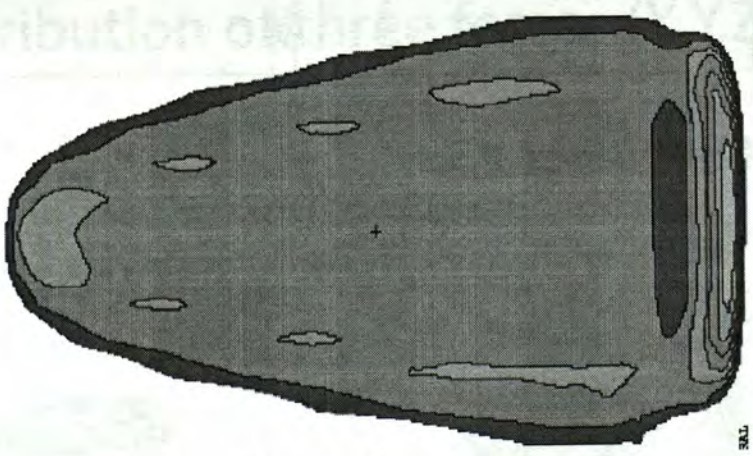
Vertical Load



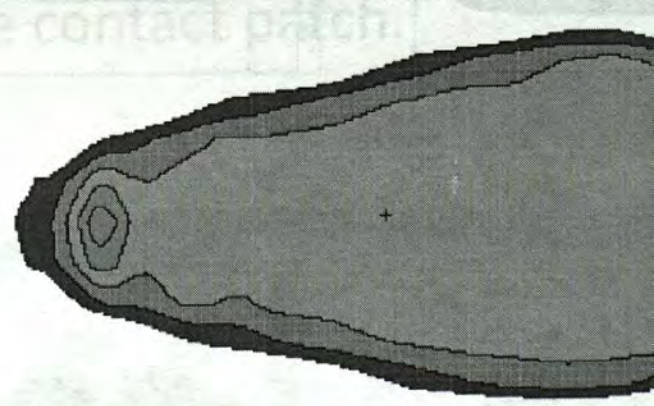
All of these conditions are at low pressure:

- High pressure areas close to shoulder
- Contact patch area reduced with camber
- At high load, reducing camber still results in high pressure areas at shoulder

Tire Footprint & Pressure



Low Pressure



High Pressure

Increasing pressure results in:

- Lower peak contact pressure
- Smaller contact patch size

→ Less grip, more durability

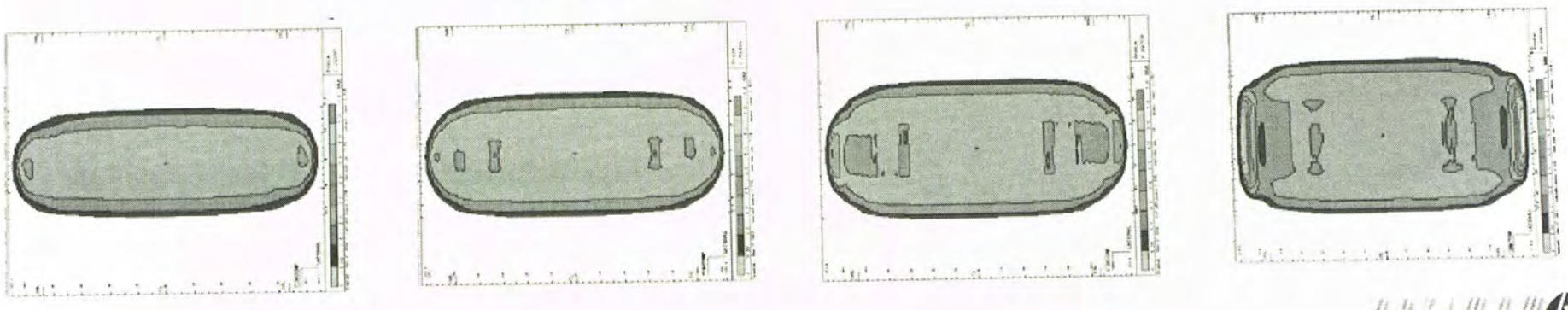
Tire FEA & Correlation

Despite advances in tire FEA, results must be validated on pressure mats or equivalent.

2.0 bar – 3000N – 4500N – 6000N – 8000N

Pressure Mat Footprints

FEA Footprints



Force Matrix Sensor

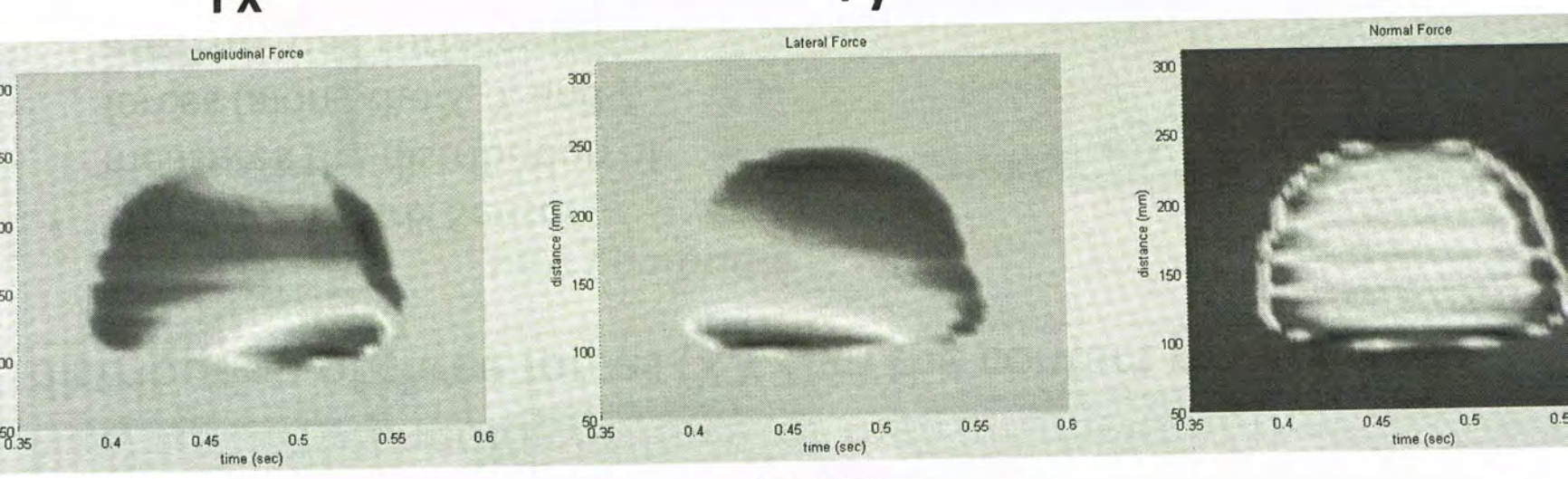
New sensor technology allows for measurement of the distribution of three forces (X,Y,Z) in the contact patch.

- The Force Matrix Sensor measures the distribution of forces (along the X, Y, and Z axes) on road surfaces using contact lines or areas made up of a combination of small sensors
- It is possible to measure the forces that running tires apply to the road at high speed

This type of sensor can help increase understanding on how the tire operates during driving, braking and cornering situations.

Measuring an actual tire has the benefit that it can capture complex frictional behaviors, which currently is difficult to model in FEA.

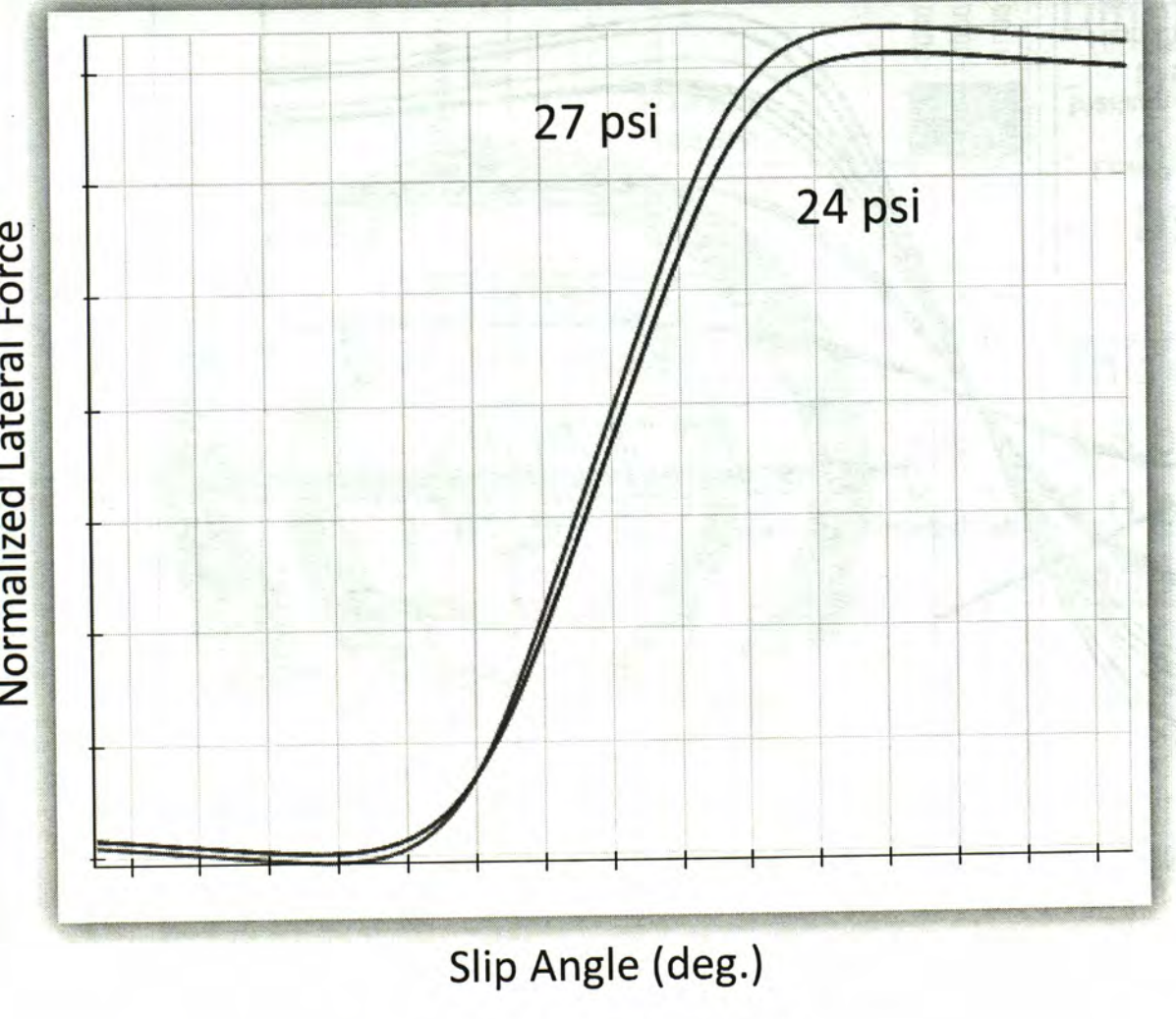
Fx **Fy** **Fz**



Front of Tire

Inside of Tire

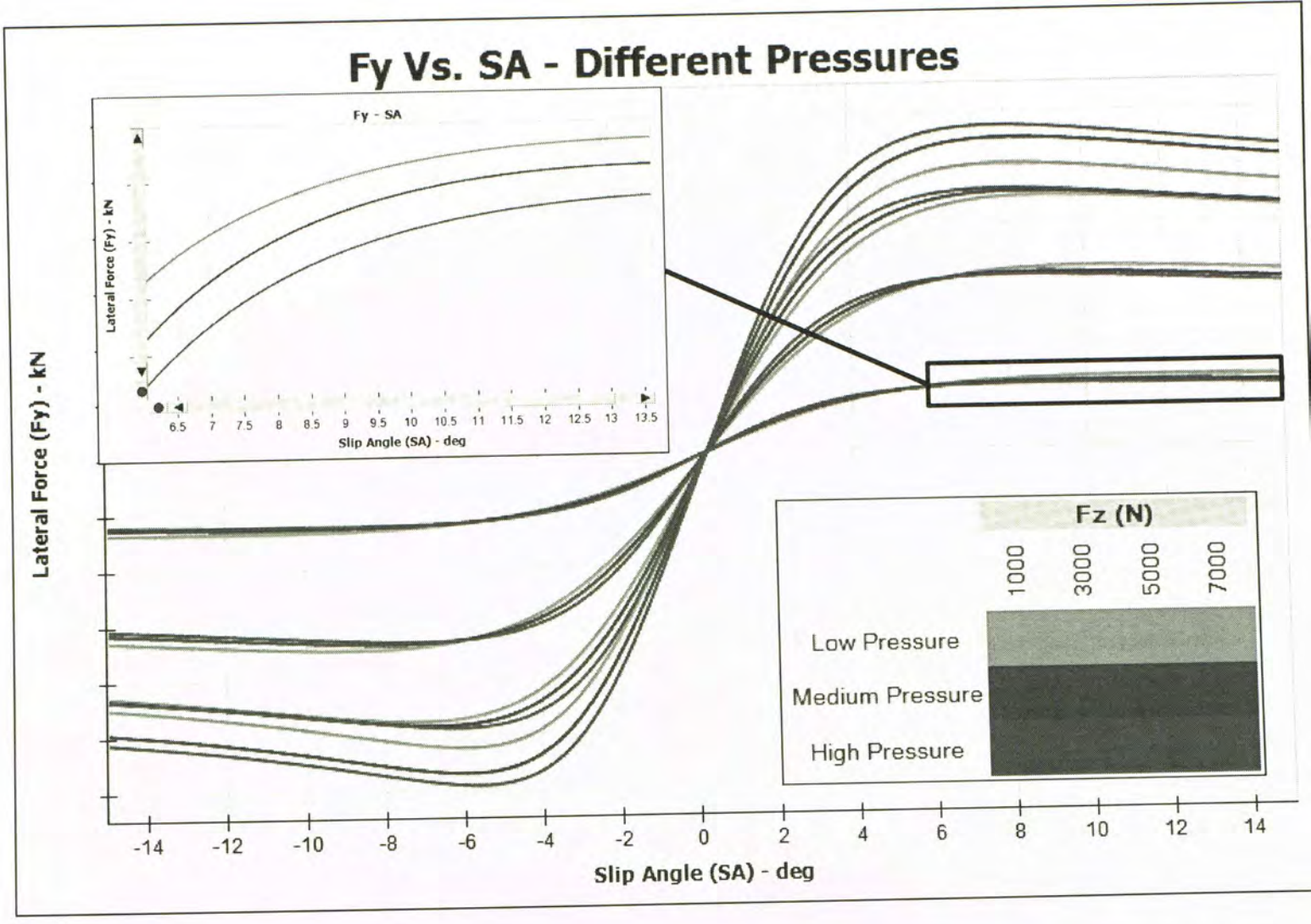
f) Pressure



Pressure changes:
- Cornering stiffn
- Peak force
- And much mor

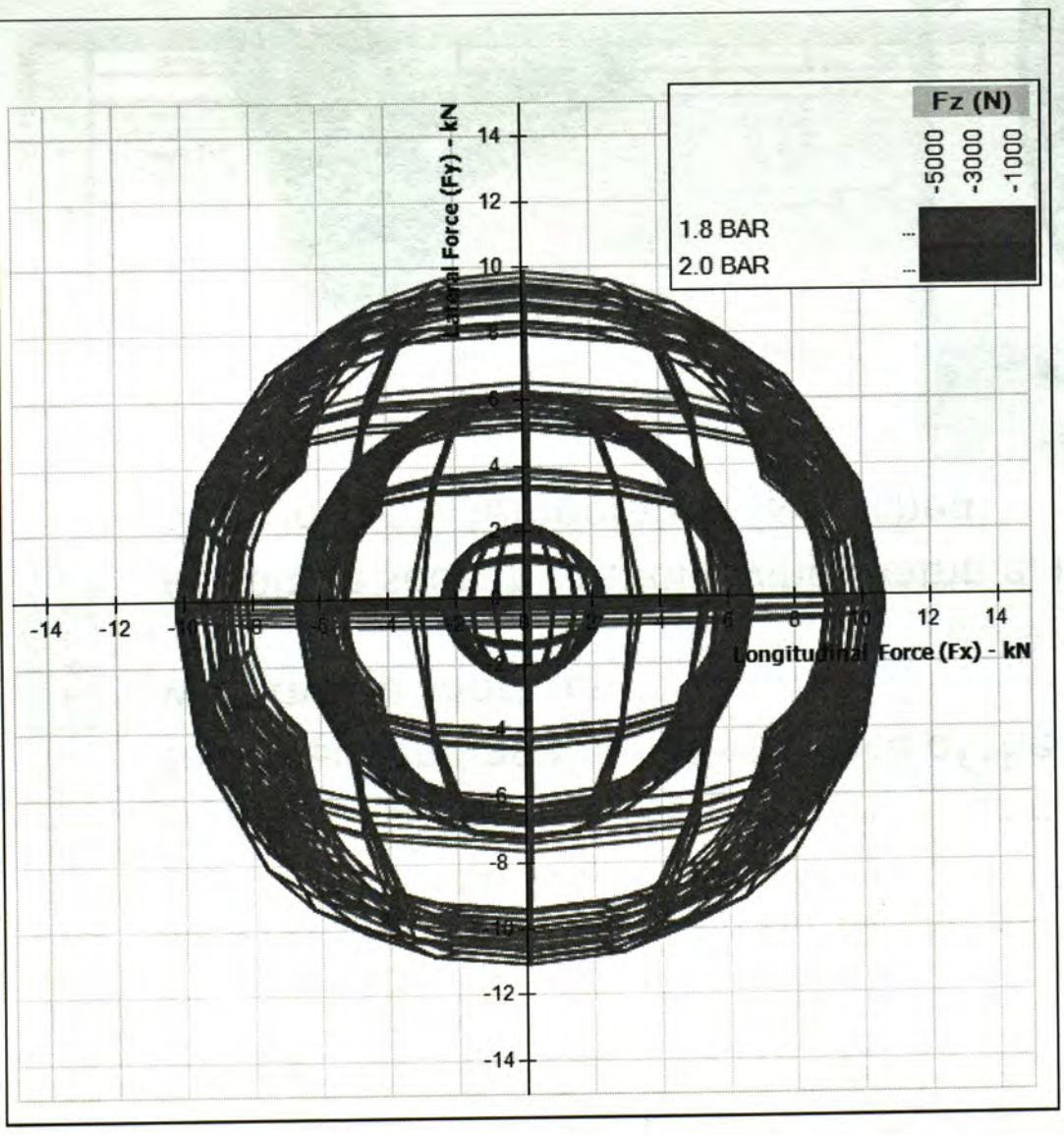
It also changes tir
vertical stiffness:
27 psi → 285 N/m
24 psi → 269 N/m
5% change

What has the bigg impact?



With this tire low pressure is better at low load and high pressure is better at high

Tire pressure

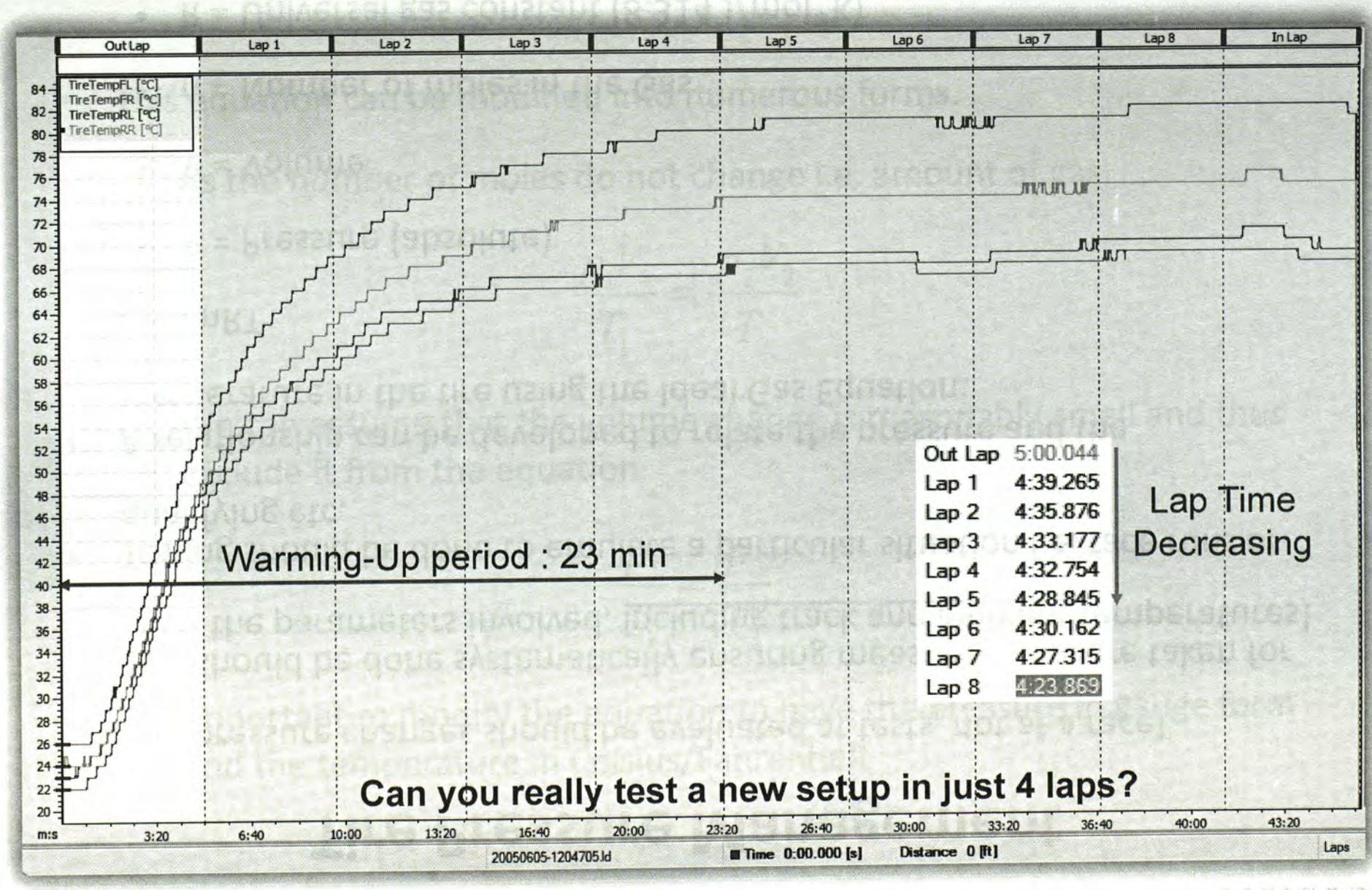


Different pressure
- Different cornering st
- Different traction ellip
- Different vertical stiff

This changes the vertical
1.8 bar → 269 N/mm
2.0 bar → 292 N/mm
<u>Modification 5%</u>

- Tire pressure sensors are mounted to the inside of the rim
- Monitoring the tire pressure during the course of a run can be important to understanding the performance of the car over a long run compared with over a short run
- As can be seen in the following example, it can take several laps until the desired "hot" temperature is reached

Tire Temperature – Le Mans Example



Tire Pressure Management

- Tire pressure changes should be evaluated at tests, not at a race!
- Test should be done systematically ensuring measurements are taken for all of the parameters involved, including track and ambient temperatures!
- Testing should be done to emulate a particular situation i.e. race runs or qualifying etc.
- A relationship can be developed to relate the pressure and the temperature in the tire using the Ideal Gas Equation:
- PV = nRT
  - P = Pressure (absolute)
  - V = Volume
  - n = Number of moles in the Gas
  - R = Universal gas constant (8.314 J/mol*K)
  - T = Temperature (absolute – Kelvin)

Tire Pressure Management

- This equation can be modified into numerous forms.
  - As the number of moles do not change i.e. amount of gas.

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

  - We can assume that the volume change is reasonably small and thus exclude it from the equation

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

  - Important to modify the equation to have the pressure in gauge form and the temperature in Celsius/Fahrenheit

Tire Pressure Management

$$\frac{P_{nom} + P_{atm}}{T_{nom} + T_{tp}} = \frac{P_{set} + P_{atm}}{T_{set} + T_{tp}}$$

Solving for P_{set}:

$$P_{set} = \left(P_{nom} + P_{atm}\right) \cdot \left(\frac{T_{set} + T_{tp}}{T_{nom} + T_{tp}}\right) - P_{atm}$$

P_{set} = Tire pressure set point (gauge) [bar ,psi] = "cold pressure"

P_{nom} = Nominal tire pressure (gauge) [bar ,psi] = "hot pressure"

P_{atm} = Atmospheric pressure [bar ,psi]

T_{set} = Tire N_2 temperature when pressure set [°C, °F] = "cold tire N_2 temperature"

T_{nom} = Nominal N_2 temperature of the tire [°C, °F] = "hot tire N_2 temperature"

T_{tp} = Triple point temperature [273.15 K and 459.7 R]

Tire Pressure Management

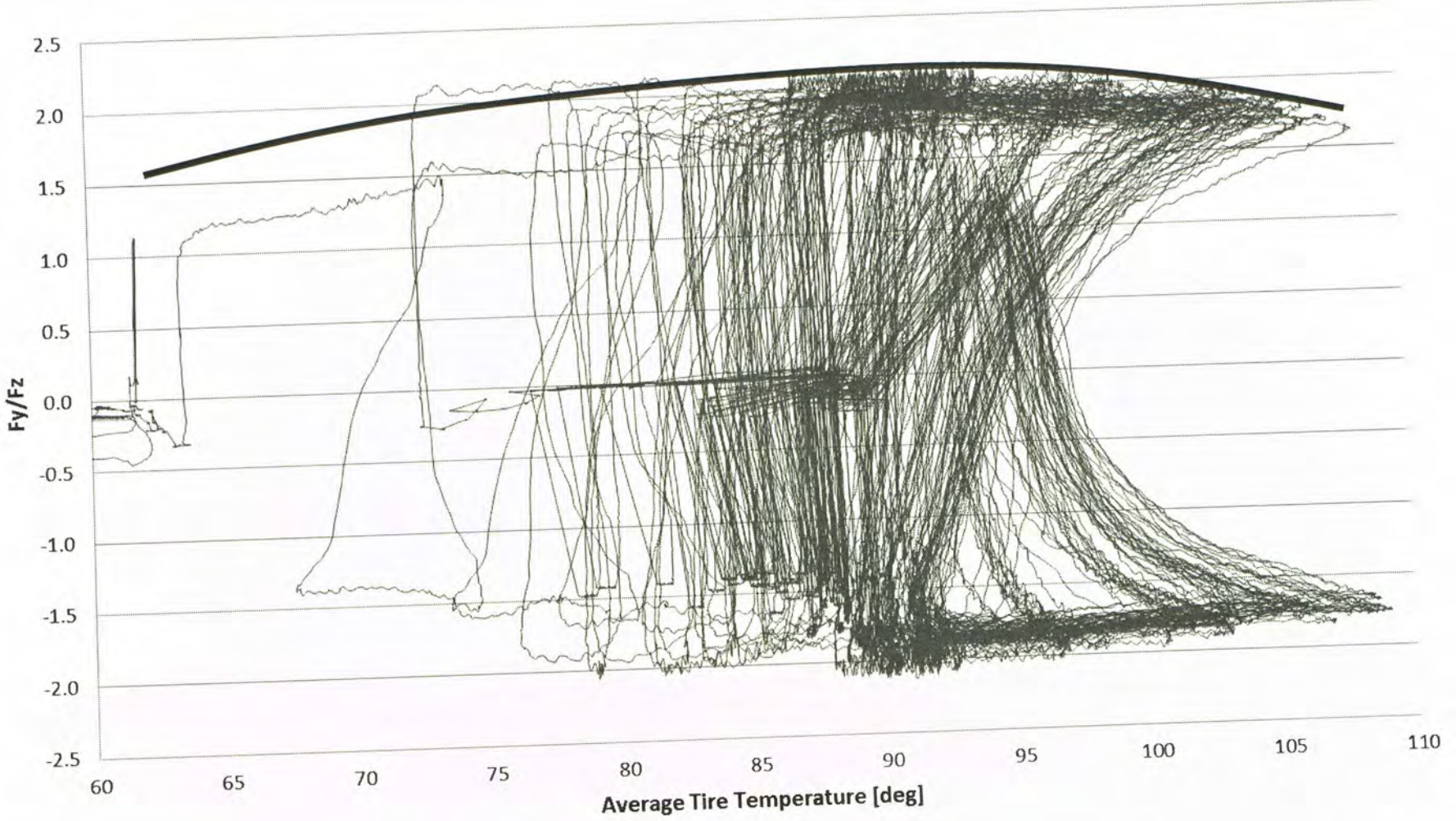
Variables					
T_{tp}	273 K	P_{atm}	101 kPa	T_{nom}	107 °C

	P_{set} (kPa)															
	T_{set} (°C)															
P_{nom} (kPa)	5	7.5	10	12.5	15	17.5	20	22.5	25	27.5	30	32.5	35	37.5	40	42.5
160	89.9	91.6	93.3	95.0	96.8	98.5	100.2	101.9	103.6	105.3	107.1	108.8	110.5	112.2	113.9	115.7
165	93.5	95.3	97.0	98.8	100.5	102.3	104.0	105.8	107.6	109.3	111.1	112.8	114.6	116.3	118.1	119.8
170	97.2	99.0	100.8	102.6	104.3	106.1	107.9	109.7	111.5	113.3	115.0	116.8	118.6	120.4	122.2	124.0
175	100.9	102.7	104.5	106.3	108.1	109.9	111.8	113.6	115.4	117.2	119.0	120.8	122.7	124.5	126.3	128.1
180	104.5	106.4	108.2	110.1	111.9	113.8	115.6	117.5	119.3	121.2	123.0	124.9	126.7	128.6	130.4	132.3
185	108.2	110.1	111.9	113.8	115.7	117.6	119.5	121.4	123.2	125.1	127.0	128.9	130.8	132.7	134.5	136.4
190	111.8	113.7	115.7	117.6	119.5	121.4	123.3	125.2	127.2	129.1	131.0	132.9	134.8	136.7	138.7	140.6
195	115.5	117.4	119.4	121.3	123.3	125.2	127.2	129.1	131.1	133.0	135.0	136.9	138.9	140.8	142.8	144.7
200	119.1	121.1	123.1	125.1	127.1	129.1	131.0	133.0	135.0	137.0	139.0	140.9	142.9	144.9	146.9	148.9
205	122.8	124.8	126.8	128.9	130.9	132.9	134.9	136.9	138.9	140.9	143.0	145.0	147.0	149.0	151.0	153.0

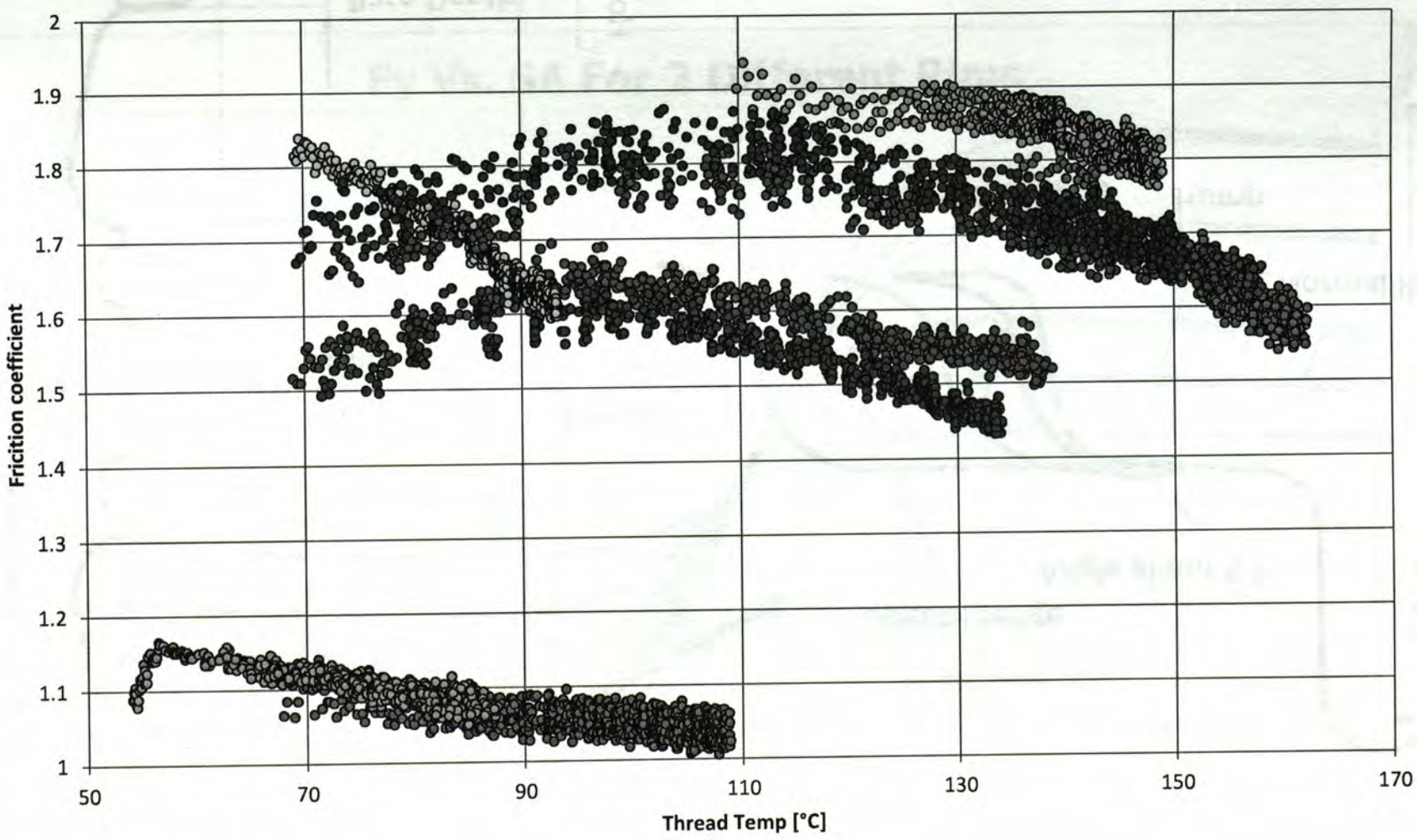
How to calculate the starting "cold" tire pressure (P_{set}) knowing:

- The targeted "hot" temperature → T_{nom}
- The ambient temperature → T_{set}
- The atmospheric pressure → P_{atm}
- The targeted "hot" pressure → P_{nom}

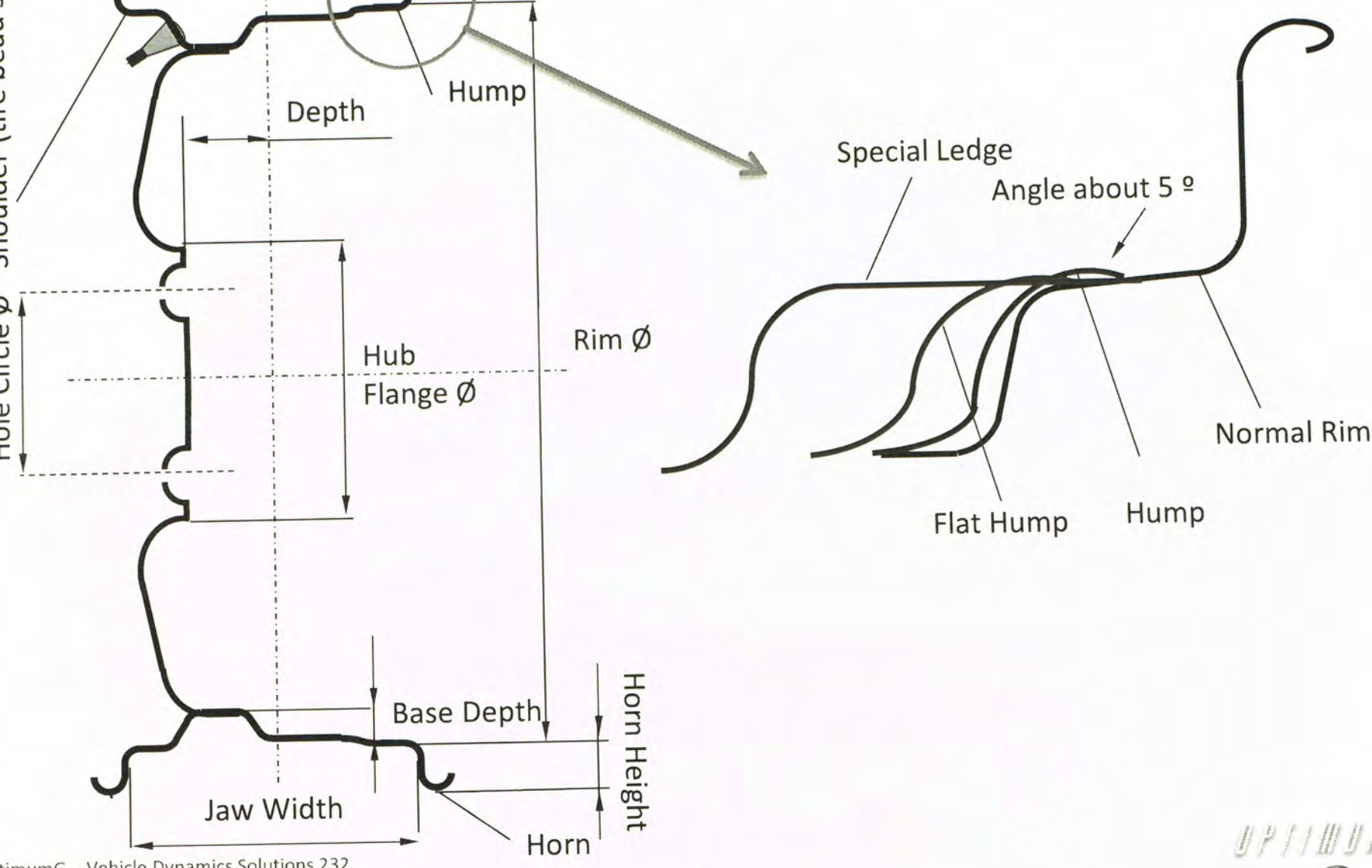
This calculation is only valid for "perfect gas".
Air, especially humid air, is far away from the "perfect gas" definition



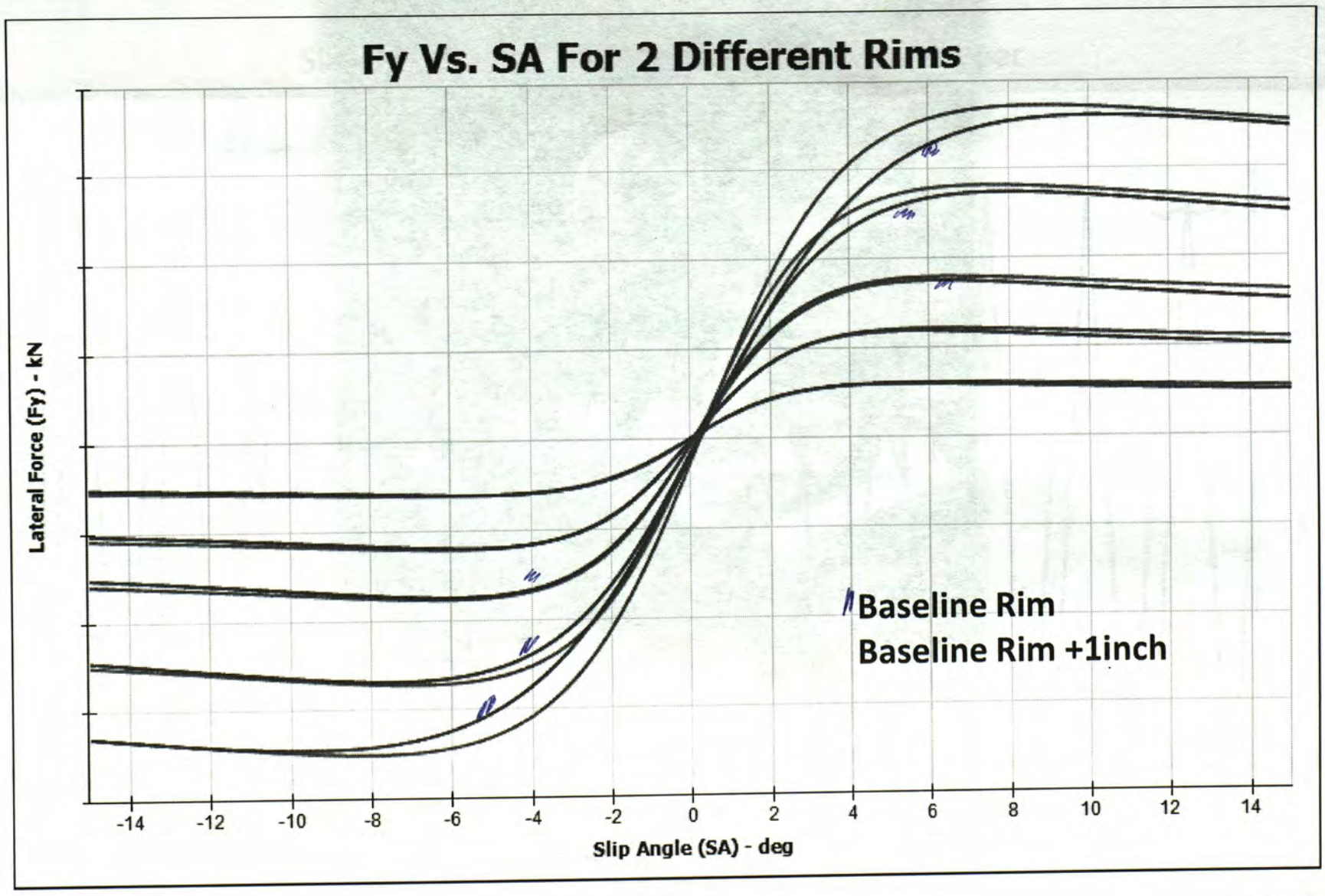
Thermal Effects

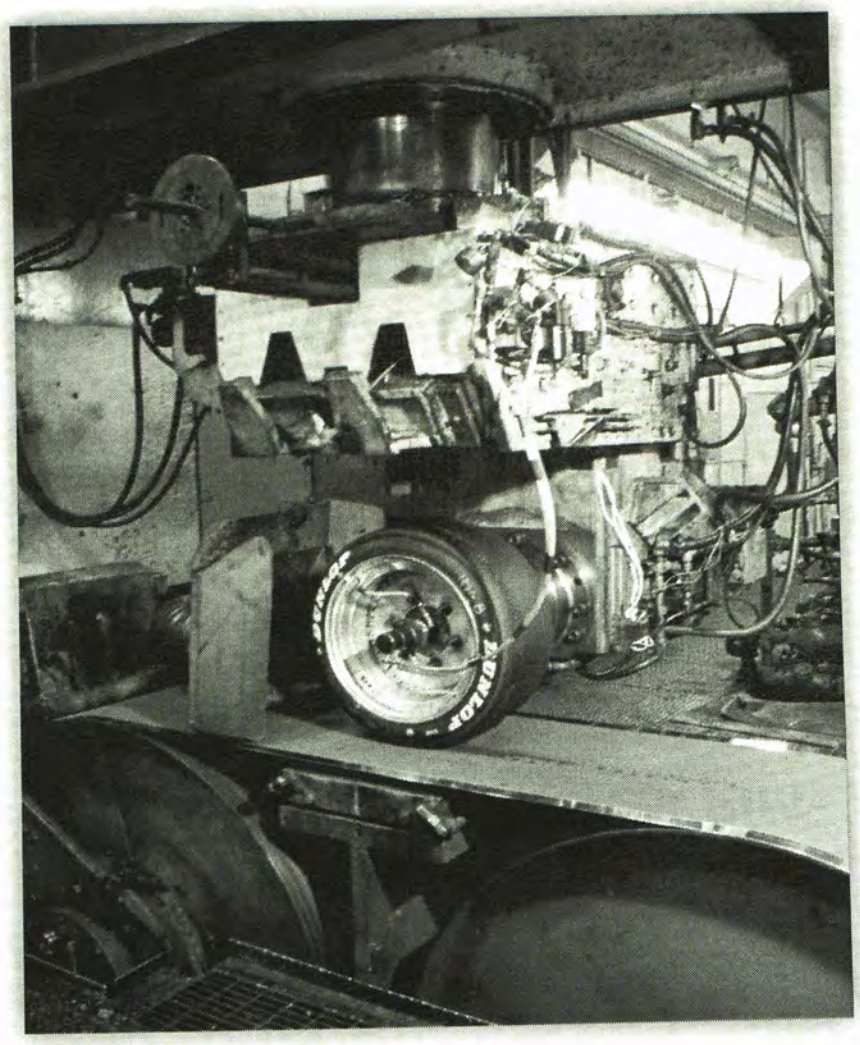


- Each color represent a specific tire
- Thermal effect on grip on different tires/compounds


Hump
Depth
Hole Circle Ø
Hub
Flange Ø
Rim Ø
Base Depth
Horn Height
Jaw Width
Horn
Special Ledge
Angle about 5 º
Normal Rim
Flat Hump
Hump

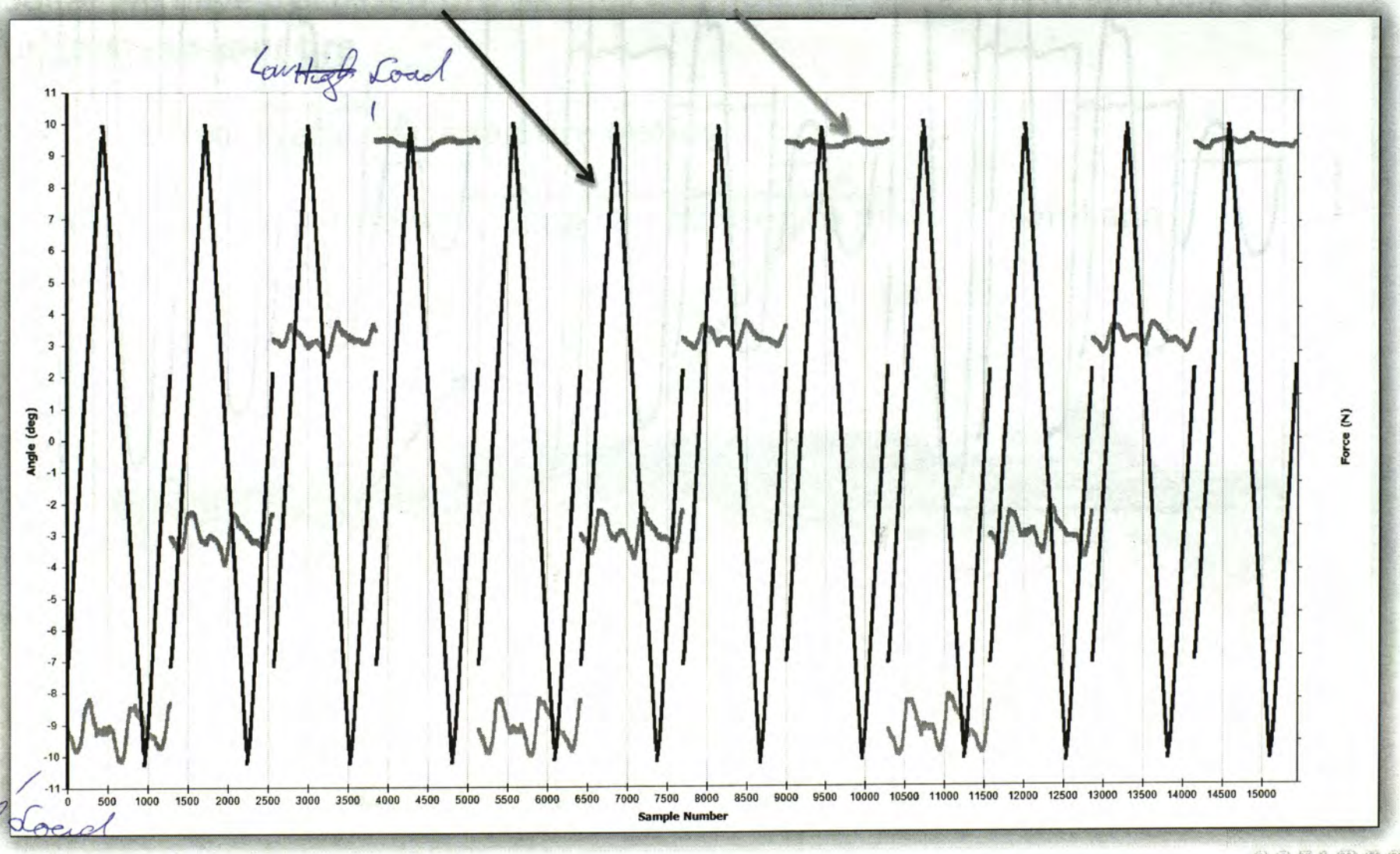
Rim Design


Fy Vs. SA For 2 Different Rims
Lateral Force (Fy) - kN
Slip Angle (SA) - deg
-14
-12
-10
-8
-6
-4
-2
0
2
4
6
8
10
12
14
Baseline Rim
Baseline Rim +1inch

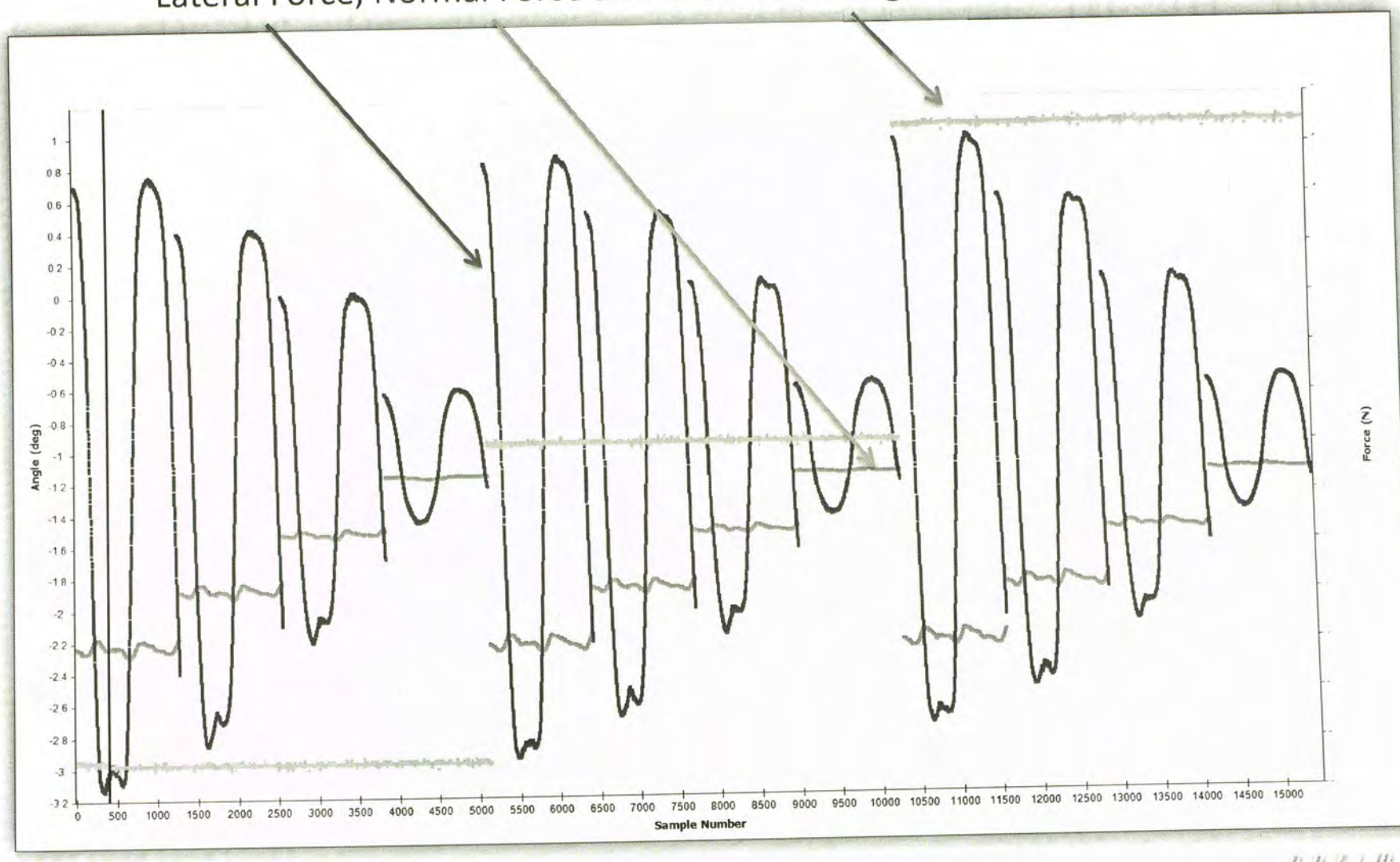


Example Run Tire Data

Slip Angle and Normal Load vs. Sample Number



Lateral Force, Normal Force and Inclination Angle vs. Sample Number



How to Obtain a Tire Model

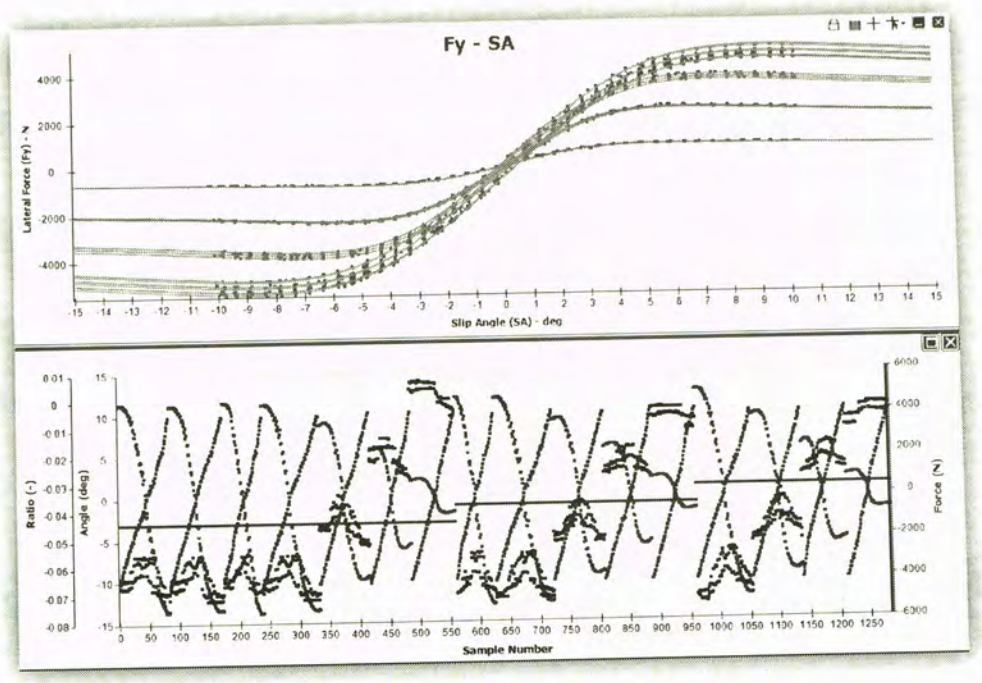
- So far we have discussed tire models and how the use of them can help to understand your tire
- We have also briefly discussed tire testing
- Now we will discuss how tire data can be used to make a useful and accurate tire model

How to Obtain a Tire Model

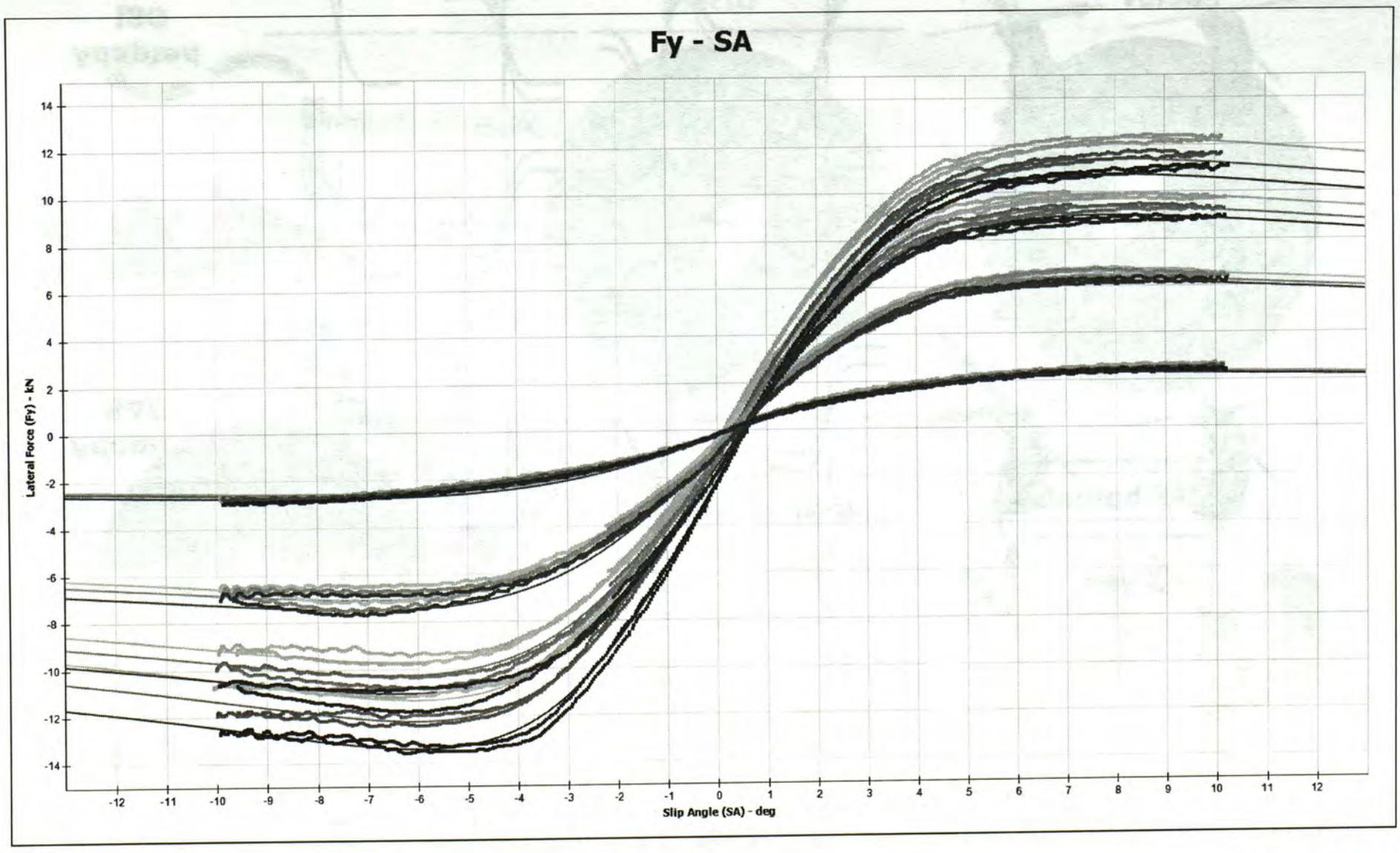
In order to fit a model to recorded tire data numerical fitting methods must be used in conjunction with data obtained from tire testing

OptimumG has developed OptimumT which enables users to quickly and easily fit a variety of models to their tire data

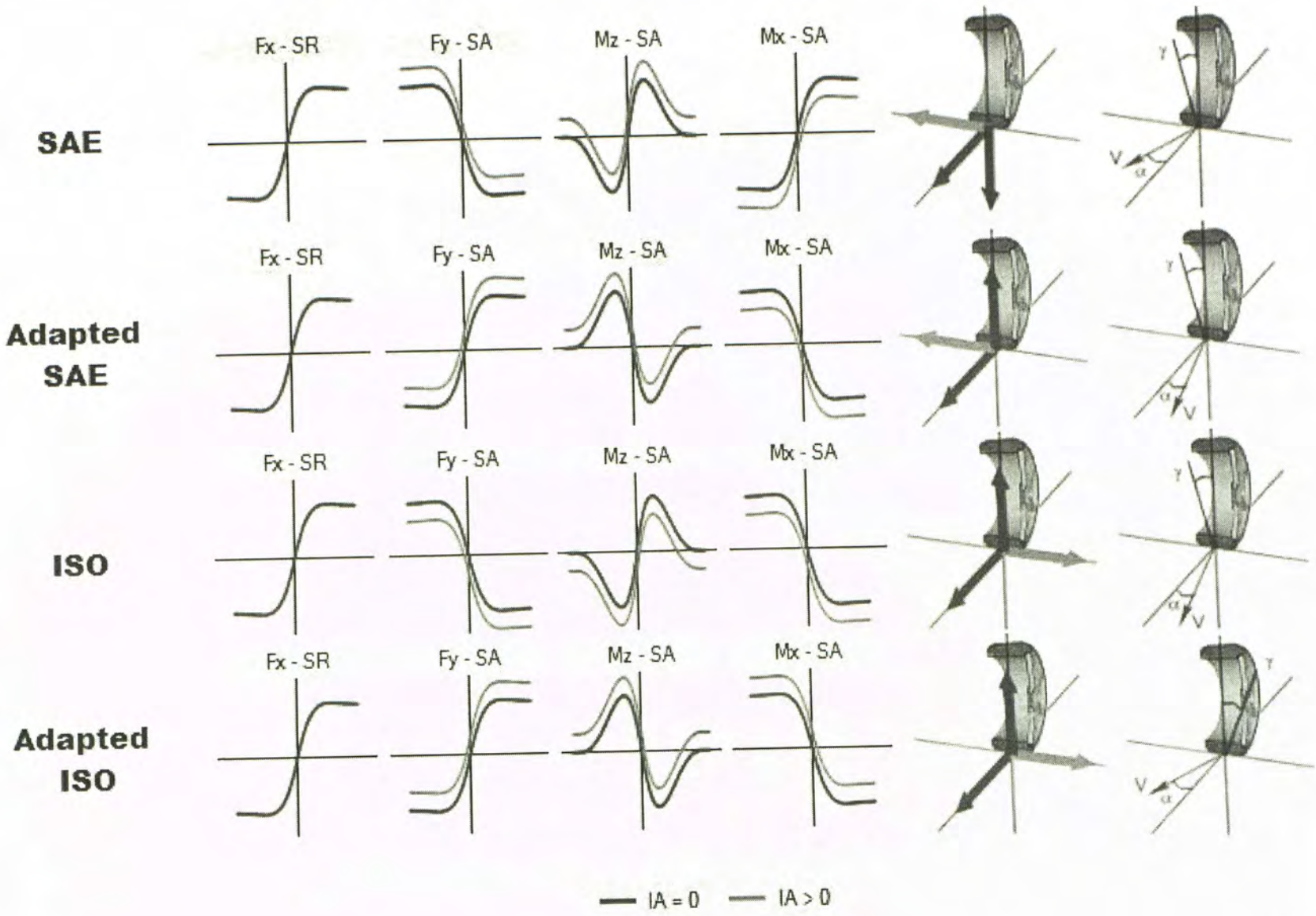
Other software packages are available from other manufacturers such as FTire



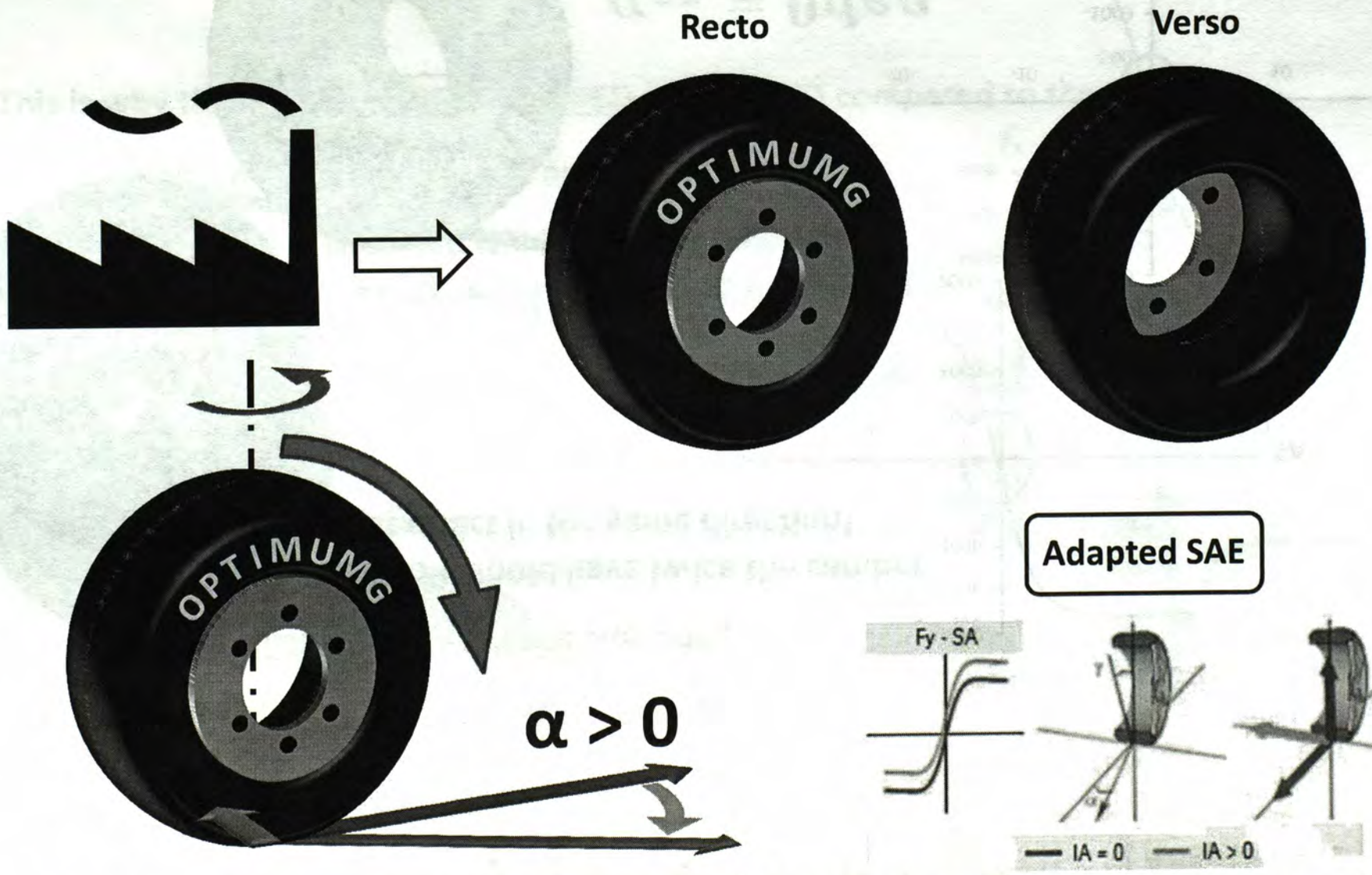
OptimumT Model Fitting



Sign Convention Summary



Tire Sign Convention

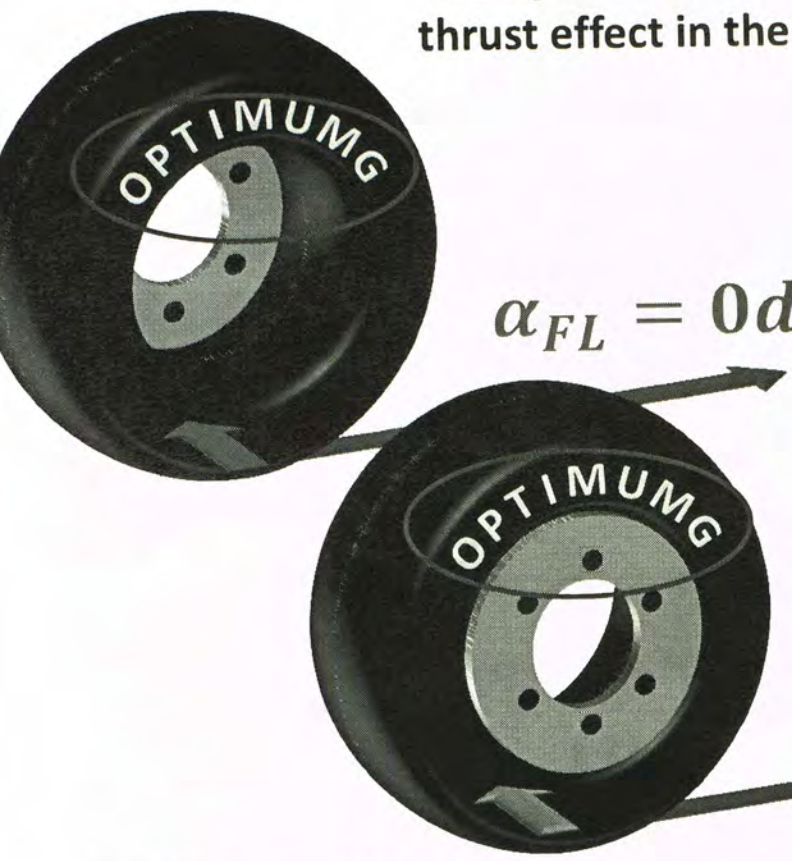


If the tire slide toward the rim side then α > 0, and Fy > 0 is toward the opposite side of the rim.

If the FL Tire was the TRANSLATION of the FR Tire.

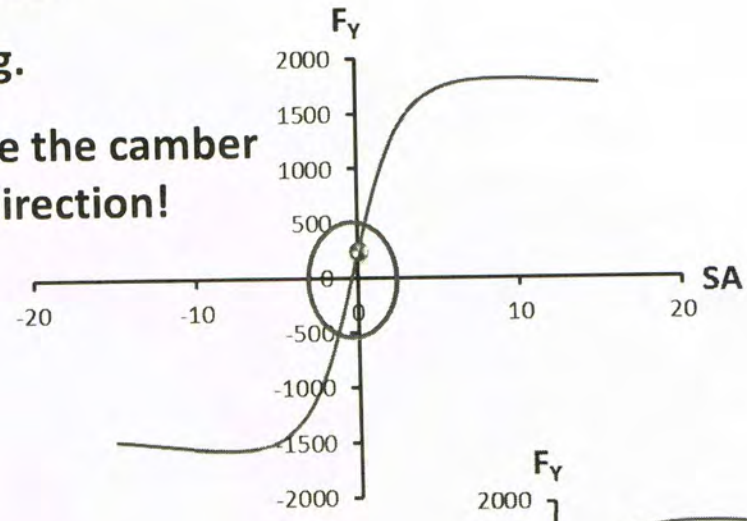
And the slip angle at each tire was 0deg.

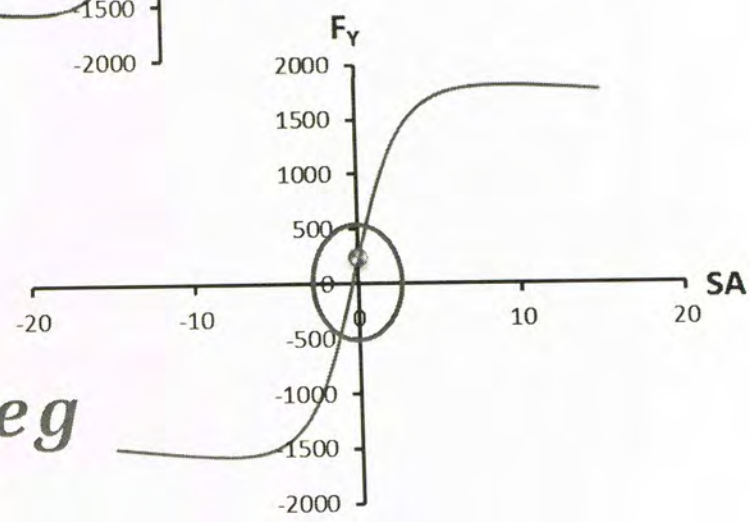
Then you would have twice the camber thrust effect in the same direction!



$\alpha_{FL} = 0deg$

$\alpha_{FR} = 0deg$





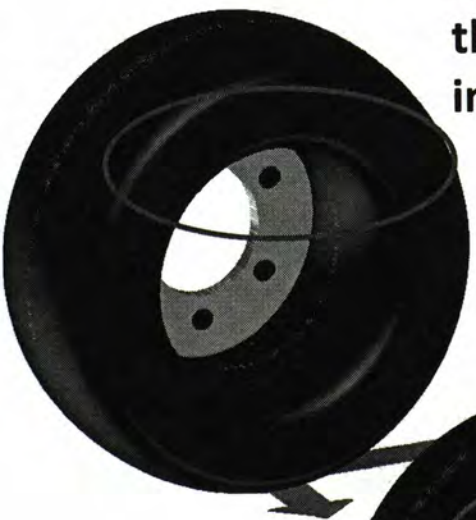
Tire Sign Convention

Adapted SAE

This is why the FL Tire has to be FLIPPED/MIRRORED compared to the FR Tire.

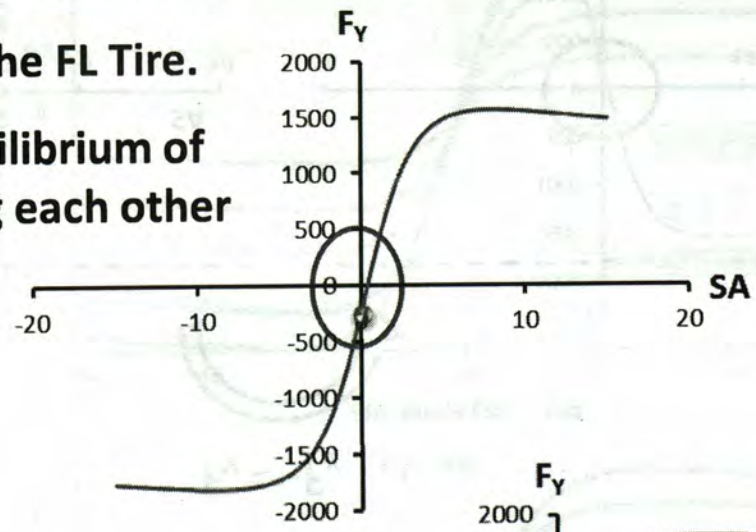
Which result in having -SA and -F_Y for the FL Tire.

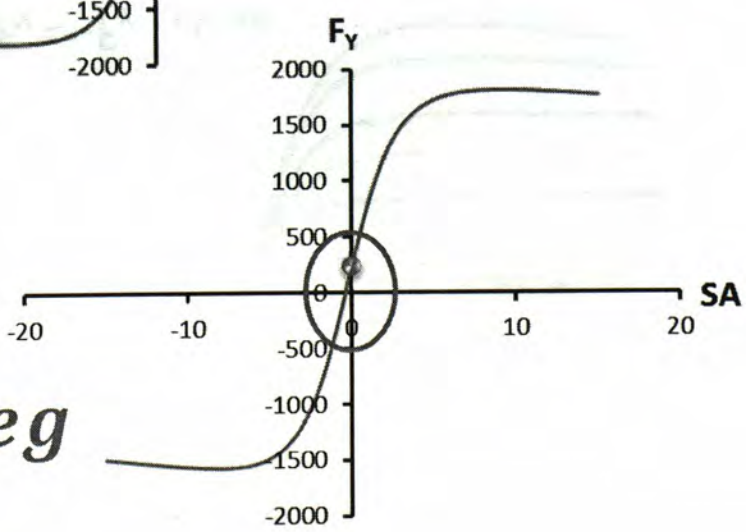
Then you will have an equilibrium of the camber thrust pushing each other in the opposite direction.



$\alpha_{FL} = 0deg$

$\alpha_{FR} = 0deg$

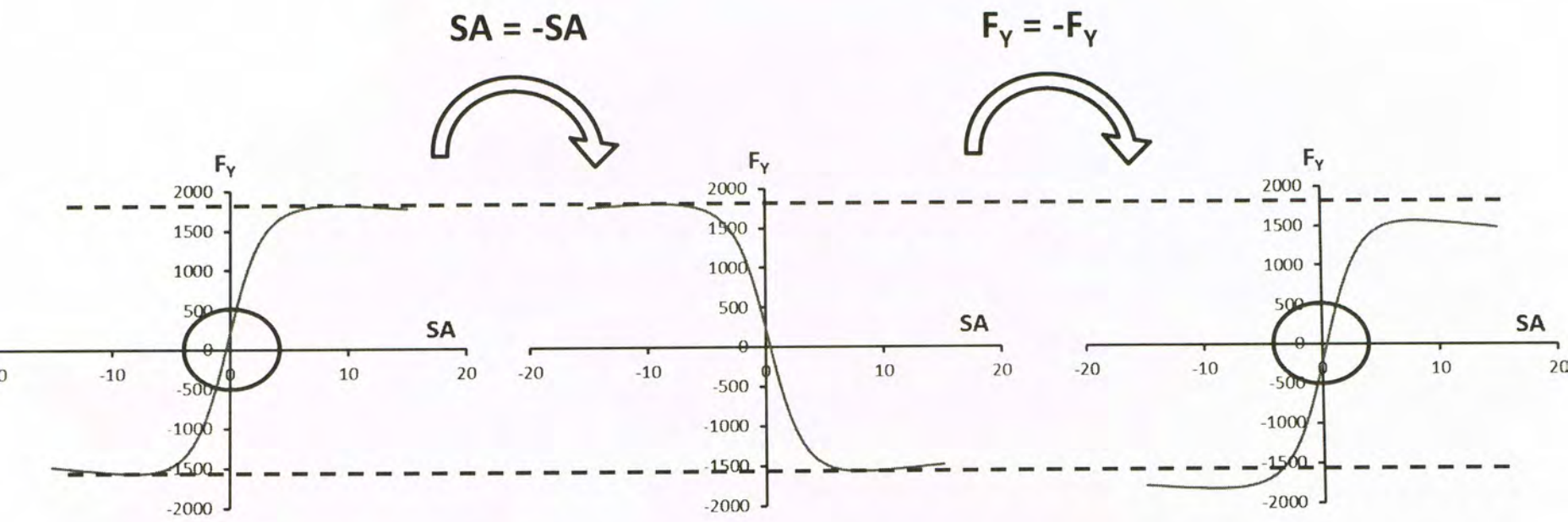




Tire Sign Convention - Flipping a tire

To mathematically flip a tire, you need:

- **To reverse the slip angle sign.**
- **To reverse the force sign.**

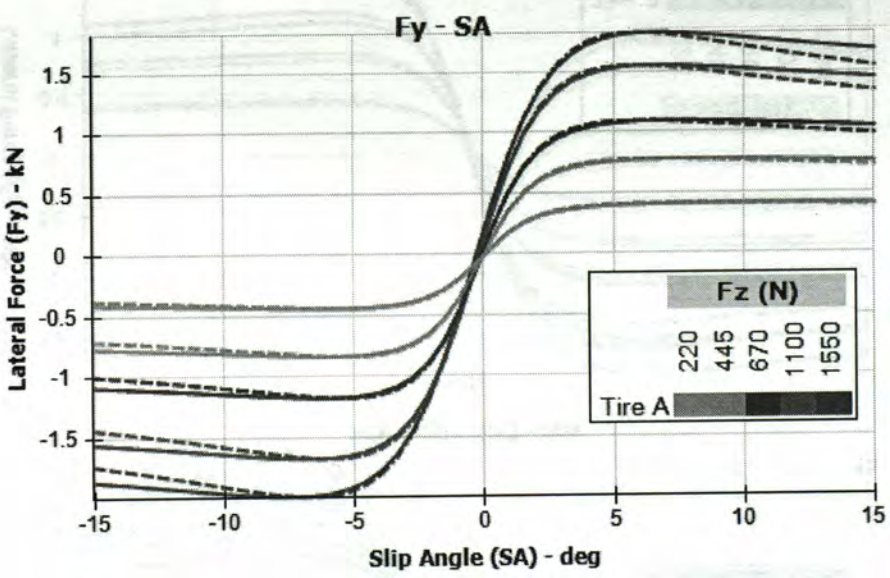


Pacejka Coefficients

Shape factor

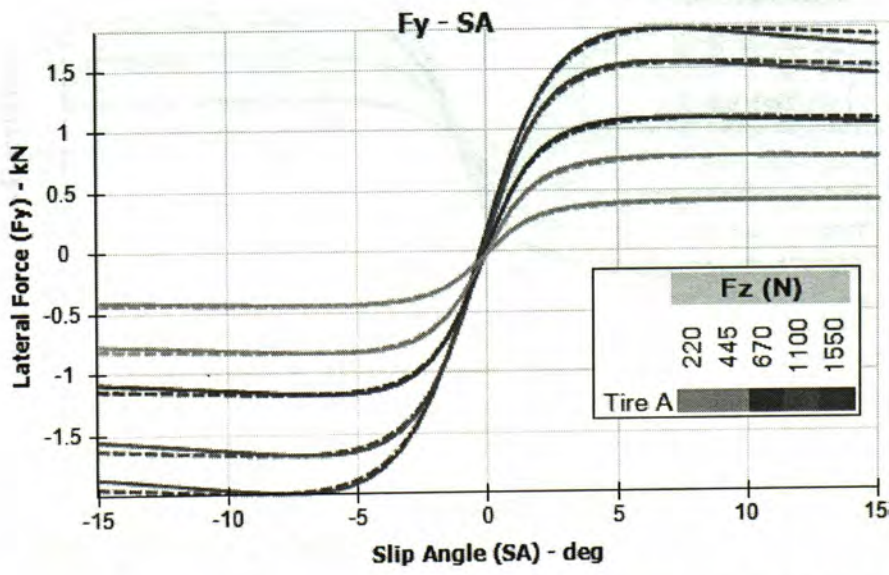
pCy1 = 1.474

pCy1 = 1.621 (+10%)



pCy1 = 1.474

pCy1 = 1.340 (-10%)

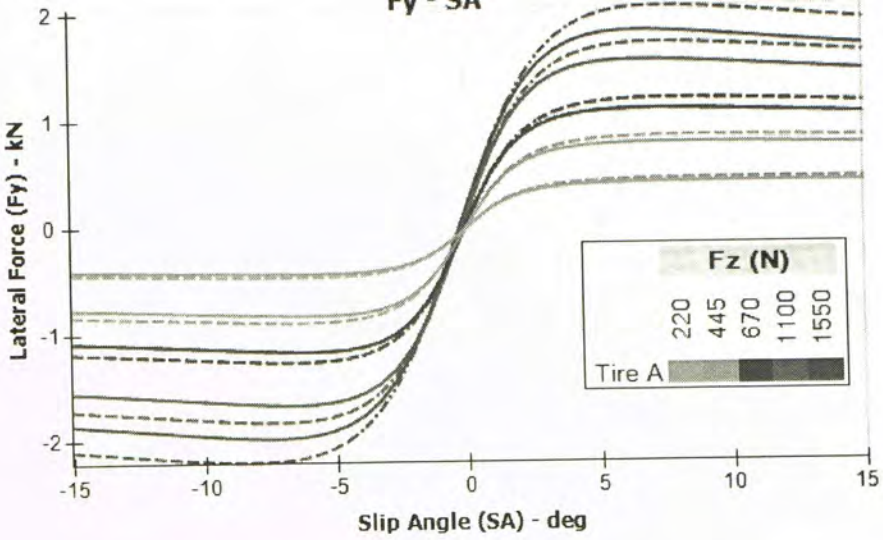


Modify the curvature (the instant cornering stiffness after the linear range) for each load.

Lateral coefficient of friction at Fz0

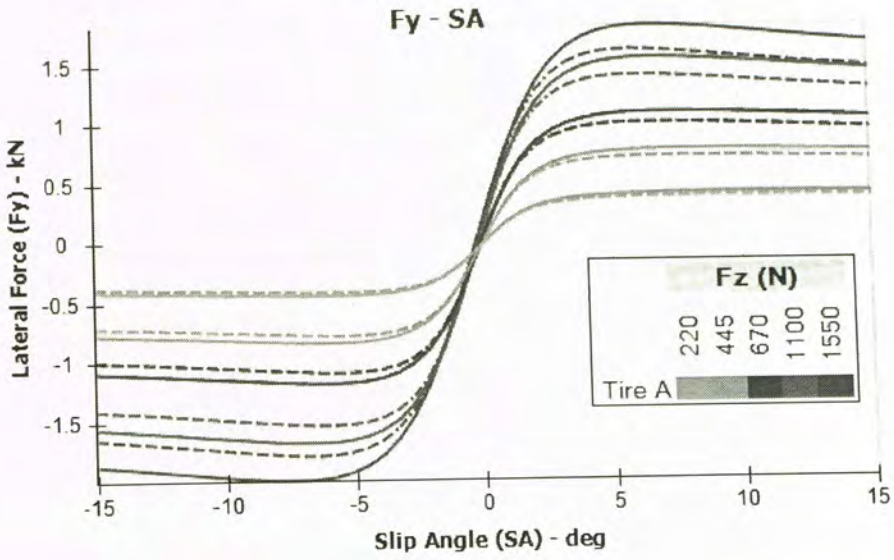
pDy1 = 1.907

pDy1 = 2.097 (+10%)



pDy1 = 1.907

pDy1 = 1.733 (-10%)



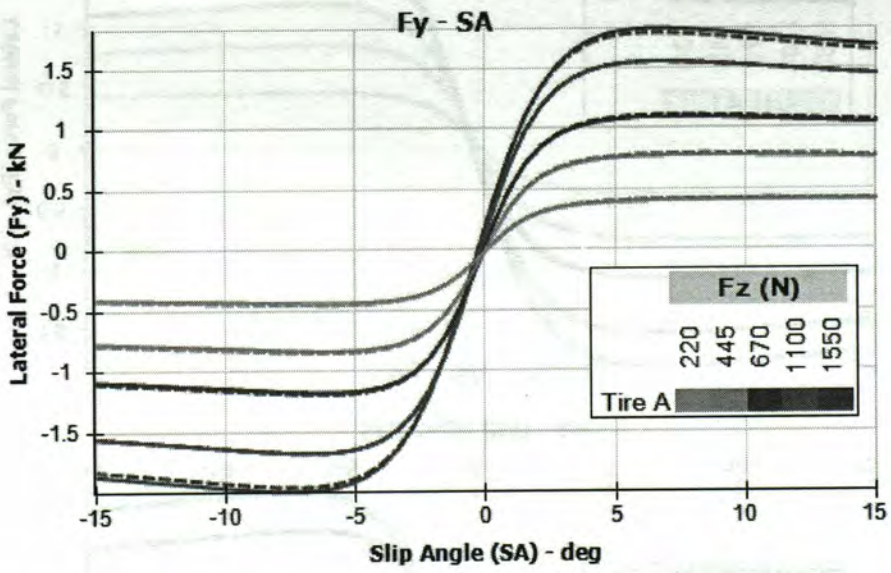
Modify the grip level for each load.

Pacejka Coefficients

Variation of friction with load

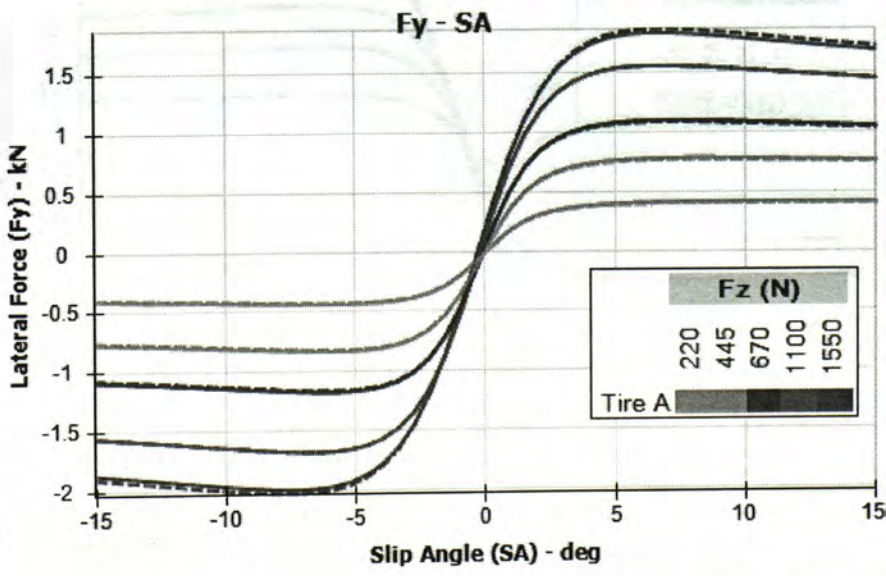
pDy2 = -0.751

pDy2 = -0.826 (+10%)



pDy2 = -0.751

pDy2 = -0.683 (-10%)

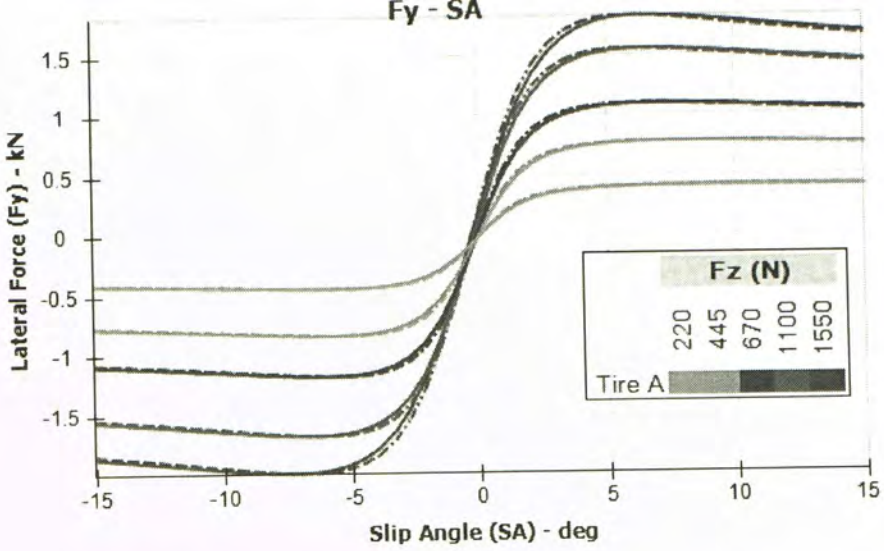


Modify the grip level for each load but the nominal one (1100N).

Maximum cornering stiffness

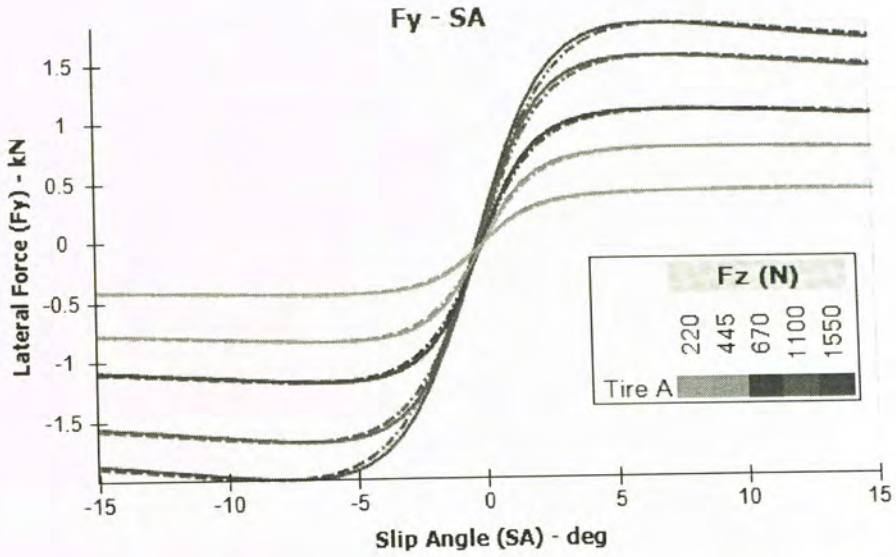
pKy1 = 37.391

pKy1 = 41.130(+10%)



pKy1 = 37.391

pKy1 = 33.992 (-10%)



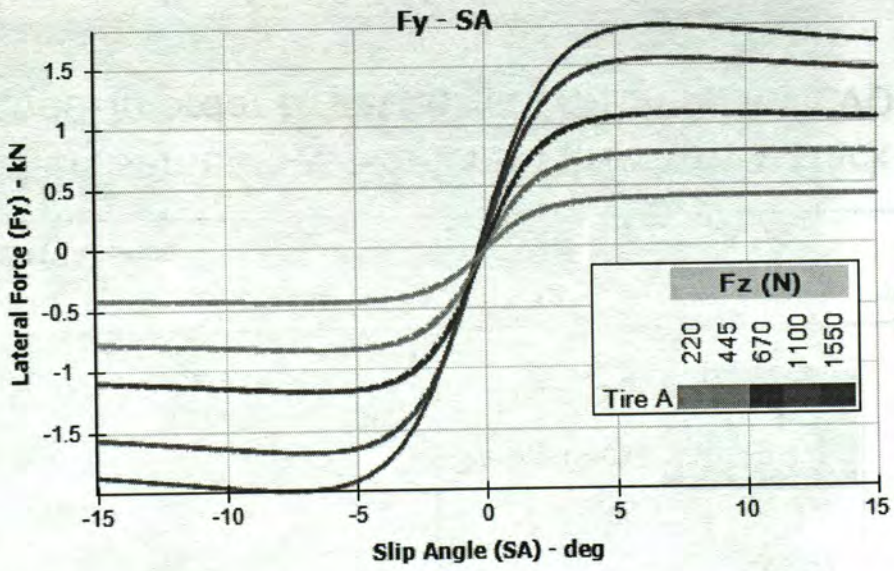
Modify the cornering stiffness for each load.

Pacejka Coefficients

Load at which maximum stiffness occurs

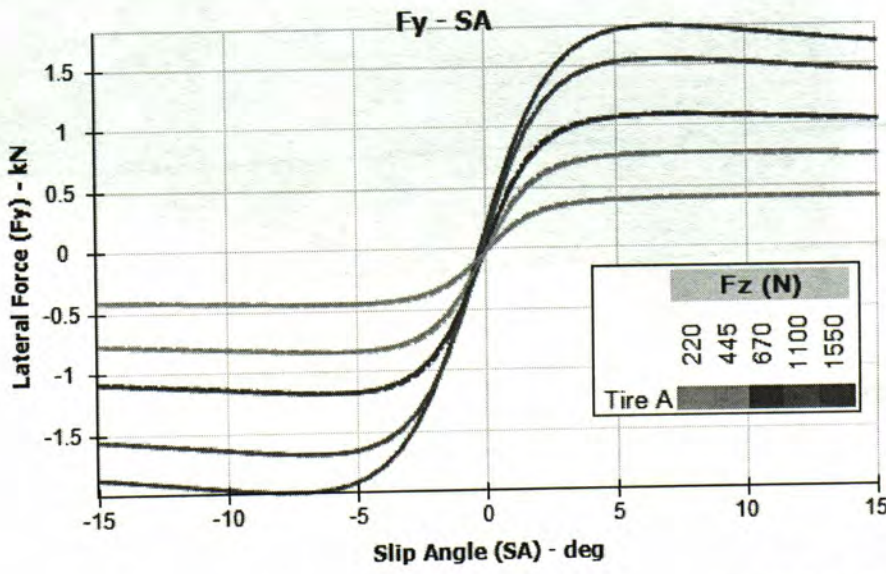
pKy2 = 1.288

pKy2 = 1.417(+10%)

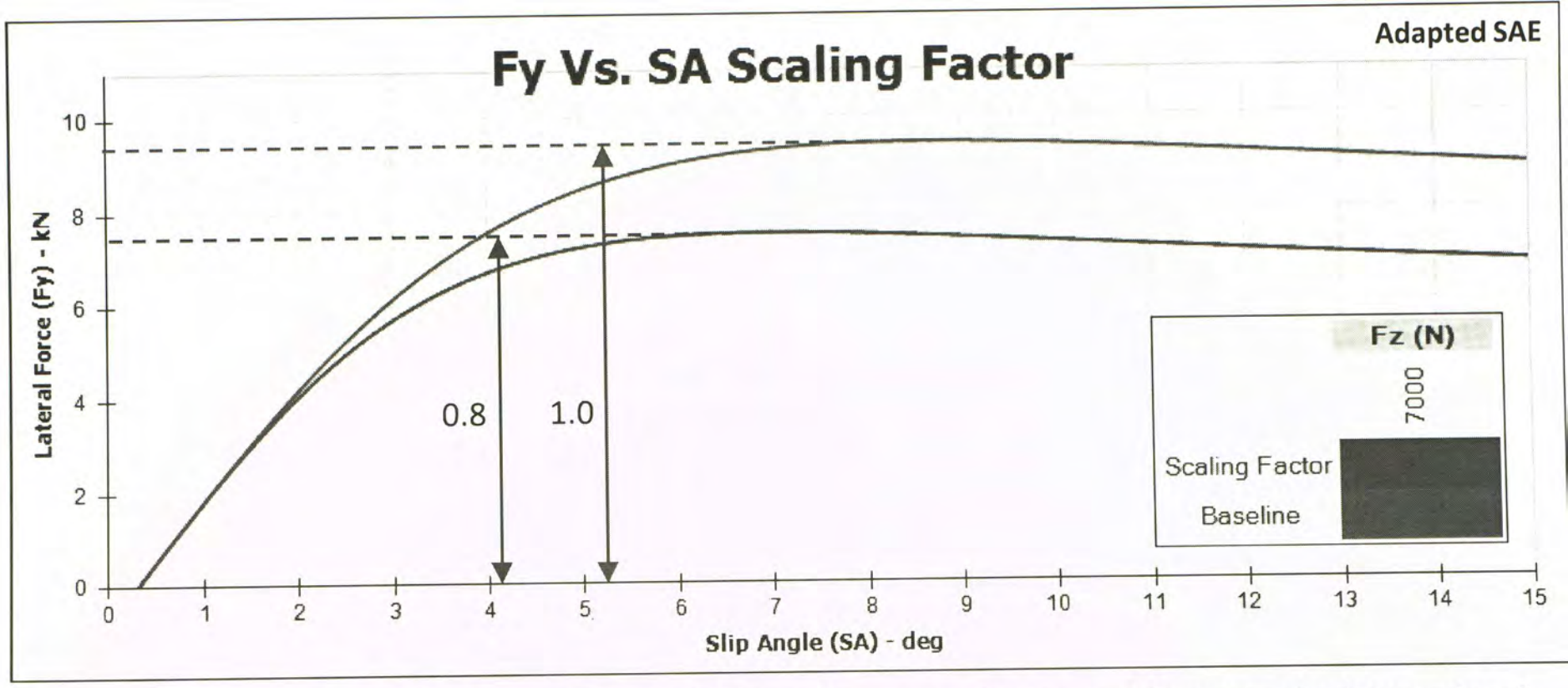


pKy2 = 1.288

pKy2 = 1.171 (-10%)



Modify the cornering stiffness for each load according to the nominal one .

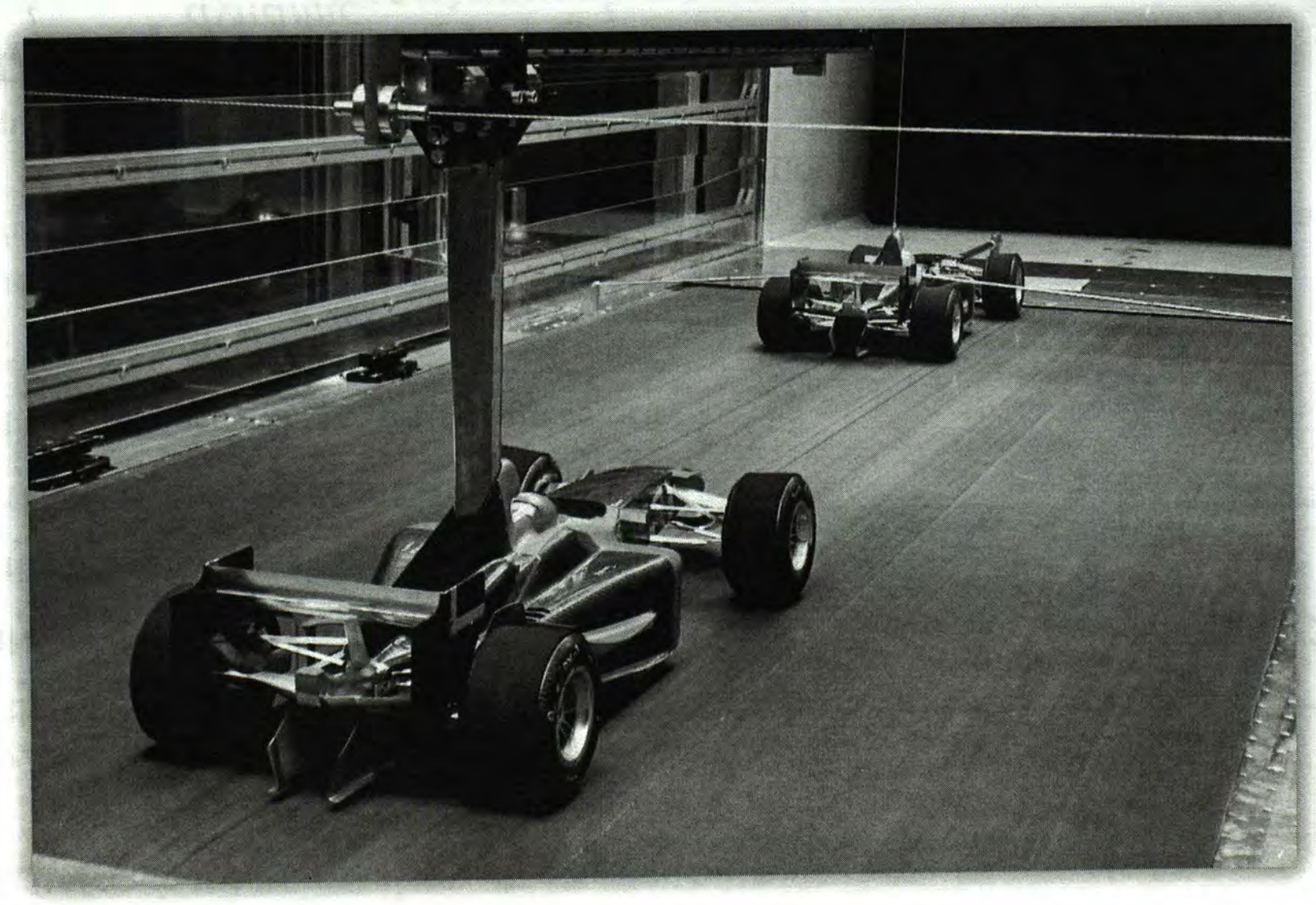


Since the rolling belt of tire testing machines offers sometimes a different grip from a track asphalt it may be necessary to scale the tire model in order to get correct result in any CVD simulation tool.

OptimumT offers this possibility just by affecting a scaling factor (which contains the λ factors) to a tire model.

In this case, we used a scaling factor "$\lambda\mu_Y$" of 0.8.

3. Aerodynamics



Aerodynamic Fundamentals

Aerodynamic Force – General Equation

F_i is the aerodynamic Force

r is the air density (Kg/m^3)

A is the reference Area (m^2)

C_i is the dimensionless Coefficient

V is the car Velocity or the free stream velocity (m/s)

$$F_i = \frac{1}{2} * \rho * A * C_i * V^2$$

$$F_i = P_{dynamic} * A * C_i$$

Subscript:

i = **drag** (into **X** direction)

i = **side** (into **Y** direction)

i = **lift** (into **Z** direction)

Aerodynamic Fundamentals

Aerodynamic Force – A and Ci

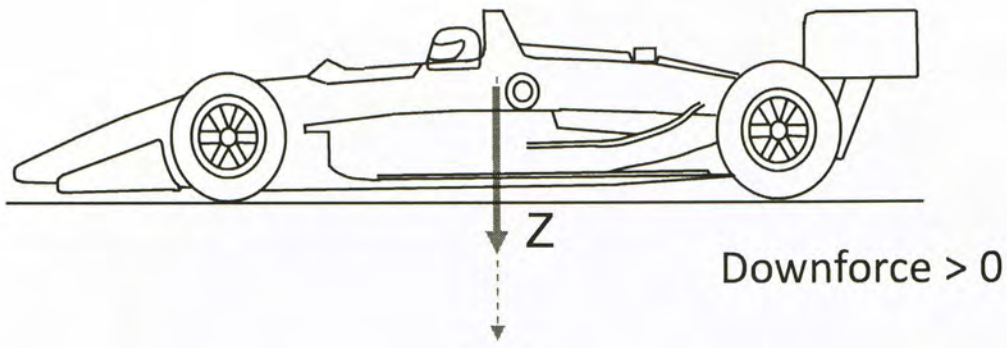
i = drag, lift or side

- **A** is the reference surface. For external flow in an automobile the reference surface is the **frontal surface.**
- C_i is a dimensionless parameter, therefore according to the last equation, we can compare the C_i on vehicles that don't have the same:
  - frontal area A
  - Velocity V
  - fluid density r
  - Aerodynamic force F applied on the body
- $A*C_i$ or simply noted AC_i, is the aerodynamic characteristic i of a given vehicle.

"A" refers to the frontal area. "C_i" is an dimensionless coefficient to compare between vehicles. AC_i is the aerodynamic characteristic "i" of a given vehicle.

Aerodynamic Fundamentals

Aerodynamic Force - Downforce



Definition: Downforce is the opposite of lift and acts in the Z direction toward the ground.

Note: C_Z can be:

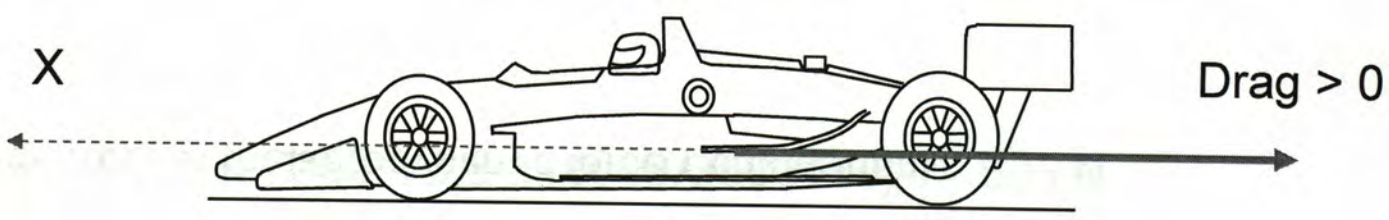
- positive: we talk about Lift
- negative: we talk about Downforce

$$Downforce = \frac{1}{2} * \rho * A * C_Z * V^2$$

Aerodynamic Fundamentals

Aerodynamic Force – Drag

X axis: Drag

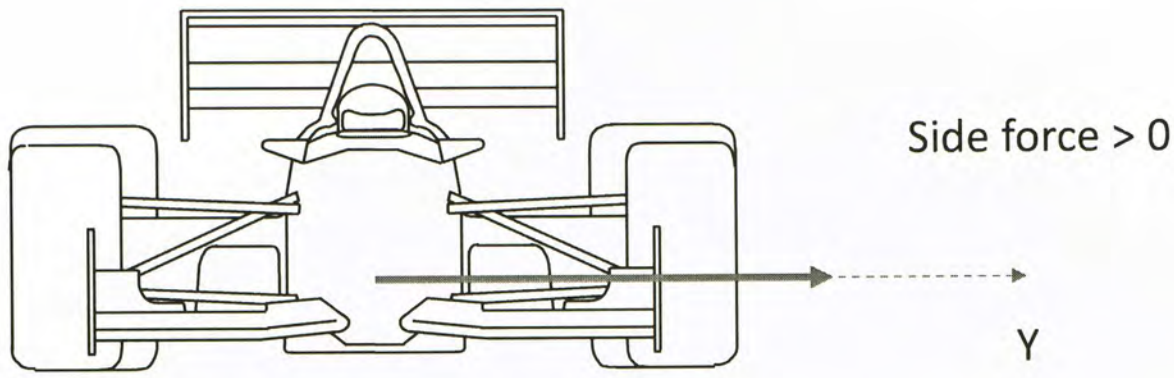


Definition: Drag is the aerodynamic resistance to motion.
i-e the aerodynamic force following x axis

$$Drag = \frac{1}{2} * \rho * A * C_D * V^2$$

Aerodynamic Fundamentals

Aerodynamic Force – Side Force



Definition: Side force is the aerodynamic force Y component.

$$Side = \frac{1}{2} * \rho * A * C_Y * V^2$$

Side force Varies considerably with yaw angle and has a critical effect on stability as we will see later.

Automotive Aerodynamics

Downforce Example – F1

$$F = 0.5 \times \rho \times A \times C_L \times V^2$$

At 50 km/h

F = 0.5 * 1.2255 *1.60 * 3.2 * $(1000/3600*50)^2$ = 605 N

At 50 km/h ----------> 605 N

At 100 km/h ----------> 2421 N

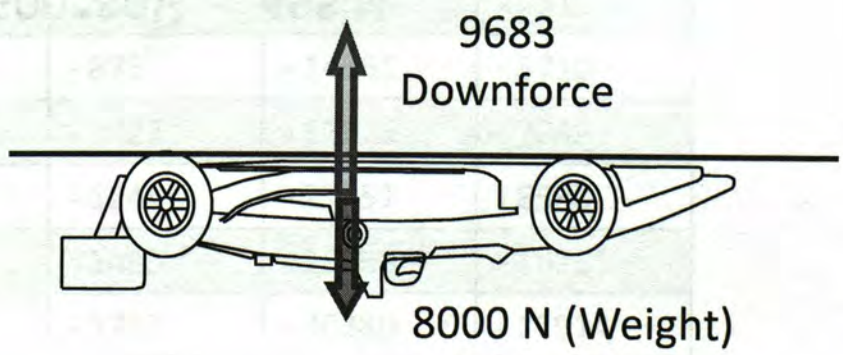
At 141.2 km/h ----------> 4842 N

At 200 km/h ----------> 9683 N

At 250 km/h ----------> 15130 N

At 300 km/h ----------> 21787 N

At 350 km/h ----------> 29654 N



Automotive Aerodynamics

Downforce Example – LMP1

$$F = 0.5 \times \rho \times A \times C_L \times V^2$$

At 50 km/h

F = 0.5 * 1.2255 *1.75 * 2.2 * $(1000/3600*50)^2$ = 455 N

At 50 km/h ----------> 455 N

At 100 km/h ----------> 1820 N

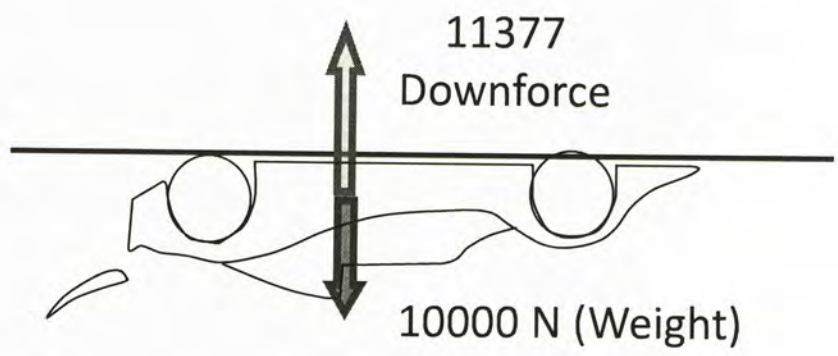
At 141.2 km/h ----------> 3640 N

At 200 km/h ----------> 7281 N

At 250 km/h ----------> 11377 N

At 300 km/h ----------> 16383 N

At 350 km/h ----------> 22298 N



	Sedan	Prototype	NASCAR Short Oval	NASCAR Super Speedway	Indy Car Road Course	Indy Car Super Speedway
Area (m^2)	1.6	1.7	1.8	1.75	1.7	1.5
CL	0.28	- 1.8	- 0.35	- 0.2	- 3.6	- 1.6

km/h	Downforce [N]					
50	53	- 360	- 74	- 41	- 719	- 282
75	118	- 508	- 167	- 93	- 1618	- 634
100	211	- 1438	- 296	- 164	- 2876	- 1128
125	329	- 2247	- 463	- 257	- 4494	- 1762
150	474	- 3226	- 666	- 370	- 6471	- 2538
175	645	- 4404	- 907	- 504	- 8808	- 3454
200	842	- 5752	- 1184	- 658	- 11505	- 4512
225	1066	- 7280	- 1499	- 833	- 14561	- 5710
250	1316	- 8988	- 1850	- 1028	- 17976	- 7049
275	1592	- 10876	- 2239	- 1244	- 21751	- 8530
300	1895	- 12943	- 2665	- 1480	- 25886	- 10151
325	2224	- 15191	- 3127	- 1737	- 30380	- 11914
350	2579	- 17617	- 3627	- 2015	- 35233	- 13817
375	2961	- 20223	- 4164	- 2313	- 40446	- 15861
400	3369	- 23009	- 4737	- 2632	- 46019	- 18047

Automotive Aerodynamics

Influence of Density on Downforce

$$F = 0.5 \times \rho \times A \times C_L \times V^2$$

Speedway at 322 km/h (200 mph)

F = 0.5 * 1.2255 * 1.05 * 1.6 * $(1000/3600*322)^2$ = 8236 N

Air density = 1.2255 at 1 bar and 15 deg C

$$= 1.2255 * \frac{(15 + 273)}{(t + 273)} * \frac{\rho}{1.000}$$

If temperature raises 5 C and pressure goes to 0.985 bar

$$= 1.2255 * \frac{(15 + 273)}{(20 + 273)} * \frac{0.985}{1.000} = 1.1865$$

Downforce = 7974 N 3.18 % less downforce !!

Automotive Aerodynamics

Influence of Density on Drag

$$F = 0.5 \times \rho \times A \times C_D \times V^2$$

Speedway at 322 kph (200 mph)

F = 0.5 * 1.2255 *1.05 * 0.68 * $(1000/3600*322)^2$ = 3500 N

Drag is Fn Speed^2

Power is Fn Speed^3

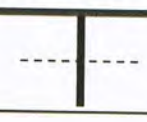
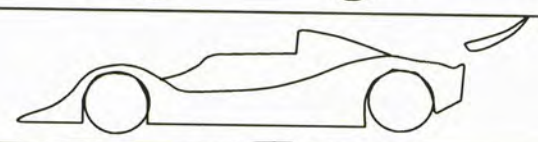
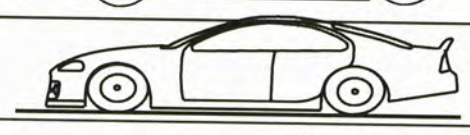
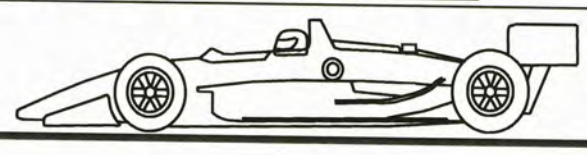
To get 322 km/h (200 mph) with a Cd of 0.68 we need 425 Hp

To get 354 km/h (220 mph) with a Cd of 0.68 we need 565 Hp

To get 355 km/h (220.6 mph) with a Cd of 0.68 we need 570 Hp

Automotive Aerodynamics

Some Drag Numbers

Body	Flow situation	C_d
Circular plate		1.17
Low drag body of revolution		0.04
Low drag vehicle near the ground		0.15
Generic sedan		0.30
Prototype car (Le Mans)		0.38
NASCAR short oval		0.38
Indy Car super speedway		0.62

	Sedan		Prototype		NASCAR Short Oval		NASCAR Super Speedway		Indy Car Road Course		Indy Car Super Speedway	
Area (m^2)	1.6		1.7		1.8		1.75		1.7		1.5	
CD	0.32		0.29		0.38		0.30		0.85		0.62	
Km/h	Drag [N] / Power [HP]											
50	60	1	58	1	80	2	62	1	170	3	109	2
75	135	4	130	4	181	5	139	4	382	11	246	7
100	241	9	232	9	321	12	247	9	679	26	437	17
125	376	18	362	17	502	24	386	19	1061	51	683	33
150	541	31	521	30	723	42	555	32	1528	88	983	57
175	737	50	710	48	984	66	756	51	2080	140	1339	90
200	962	74	927	71	1286	99	987	76	2716	209	1748	134
225	1218	105	1173	101	1627	141	1249	108	3438	297	2213	191
250	1504	145	1448	139	2009	193	1542	148	4244	408	2732	263
275	1820	192	1752	185	2431	257	1866	197	5136	543	3305	349
300	2166	250	2085	241	2893	334	2221	256	6112	705	3934	454
325	2542	318	2447	306	3395	424	2606	326	7173	896	4617	577
350	2948	397	2838	382	3938	530	3022	407	8319	1119	5354	720
375	3384	488	3258	470	4520	652	3470	500	9550	1377	6146	886
400	3850	592	3707	570	5143	791	3948	607	10866	1671	6993	1075

Automotive Aerodynamics

Aero Mapping: Single Seater Example 1

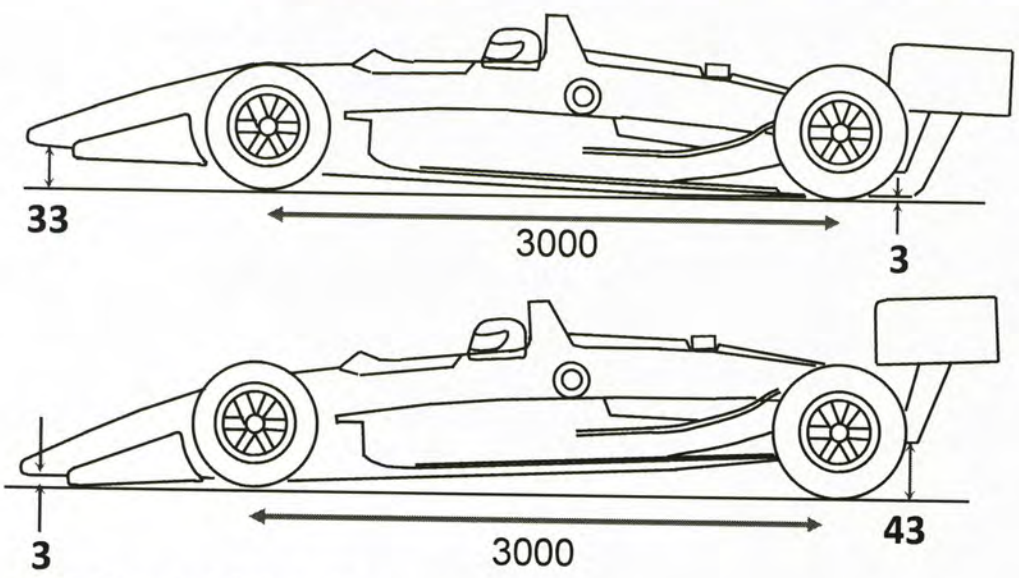
At 240 Km/h

1 bar and 15ºC

Downforce(N) Vs Front and Rear Ride Height (mm)

Rear Ride Height (mm)

Front Ride Height (mm)	3.0	13.0	23.0	33.0	43.0	53.0
33.0	6990	8450	9040	9270	9160	8910
23.0	7310	8640	9360	9620	9550	9410
13.0	7740	9080	9770	10020	9980	9850
3.0	8210	9570	10230	10550	10560	10430



$$\text{Arctan}\ \frac{3 - 33}{3000} = -0.57º$$

$$\text{Arctan}\ \frac{43 - 3}{3000} = 0.76º$$

Pitch variation = 1.33º

Automotive Aerodynamics

Aero Mapping: Single Seater Example 1

Drag (N) Vs Front and Rear Ride Height (mm)

Rear Ride Height (mm)

Front Ride Height (mm)	3.0	13.0	23.0	33.0	43.0	53.0
33.0	4310	4380	4440	4490	4520	4550
23.0	4340	4410	4490	4530	4560	4580
13.0	4380	4440	4510	4550	4590	4610
3.0	4390	4450	4520	4570	4590	4630

Lift over Drag (efficiency) Vs Front and Rear Ride Height (mm)

Rear Ride Height (mm)

Front Ride Height (mm)	3.0	13.0	23.0	33.0	43.0	53.0
33.0	1.62	1.93	2.03	2.06	2.02	1.96
23.0	1.68	1.96	2.08	2.12	2.09	2.05
13.0	1.76	2.01	2.16	2.18	2.17	2.13
3.0	1.87	2.15	2.26	2.30	2.30	2.25

Automotive Aerodynamics

Aero Mapping: Single Seater Example 1

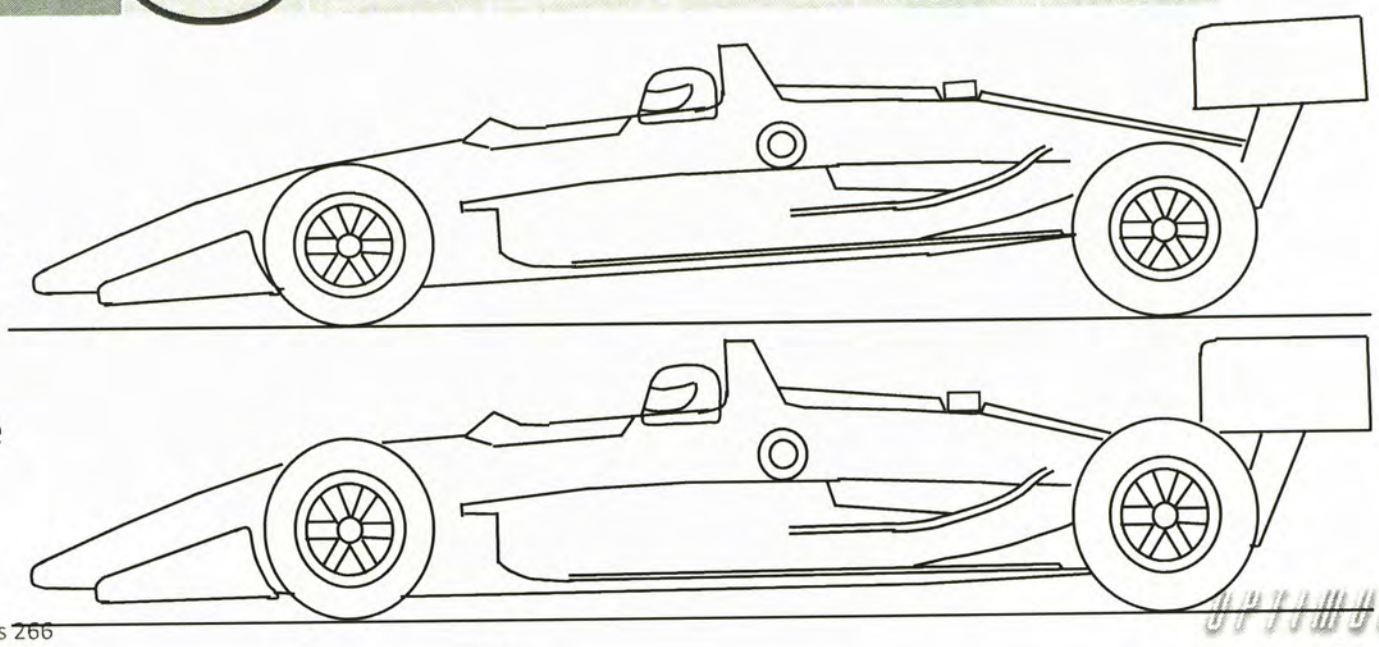
Aerobalance (% front downforce / total downforce)

Rear Ride Height (mm)

Front Ride Height (mm)	3.0	13.0	23.0	33.0	43.0	53.0
33.0	34.07	32.98	33.01	33.48	35.14	36.54
23.0	36.05	35.28	35.27	36.20	37.53	39.08
13.0	38.80	37.87	37.83	38.33	39.58	41.21
3.0	41.26	40.66	40.92	41.67	43.37	45.30

<u>Rule of thumb</u>

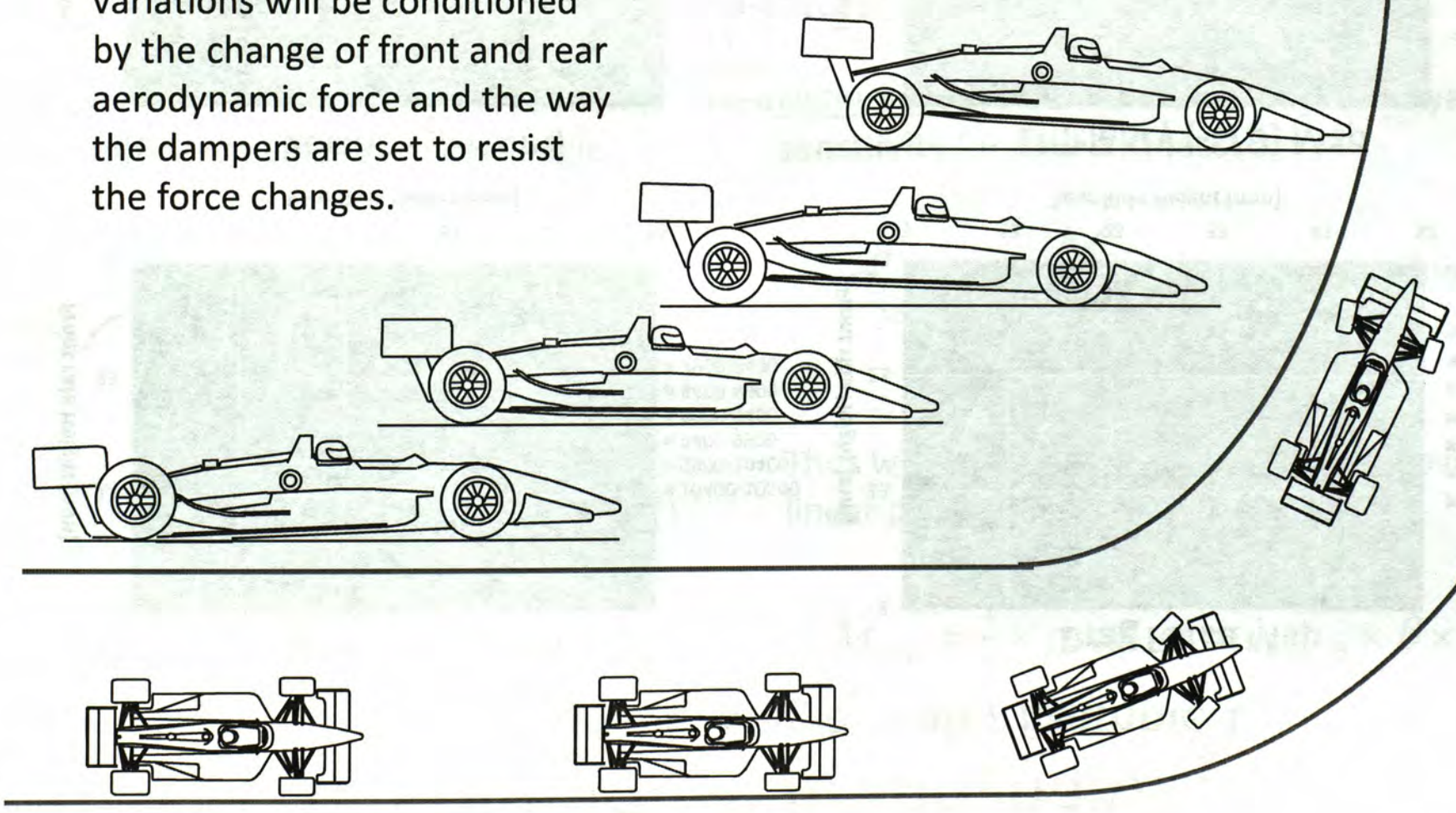
For one step of front ride height variations we need 3 or 5 steps of rear ride height variations to keep the same aerodynamic balance.



Automotive Aerodynamics

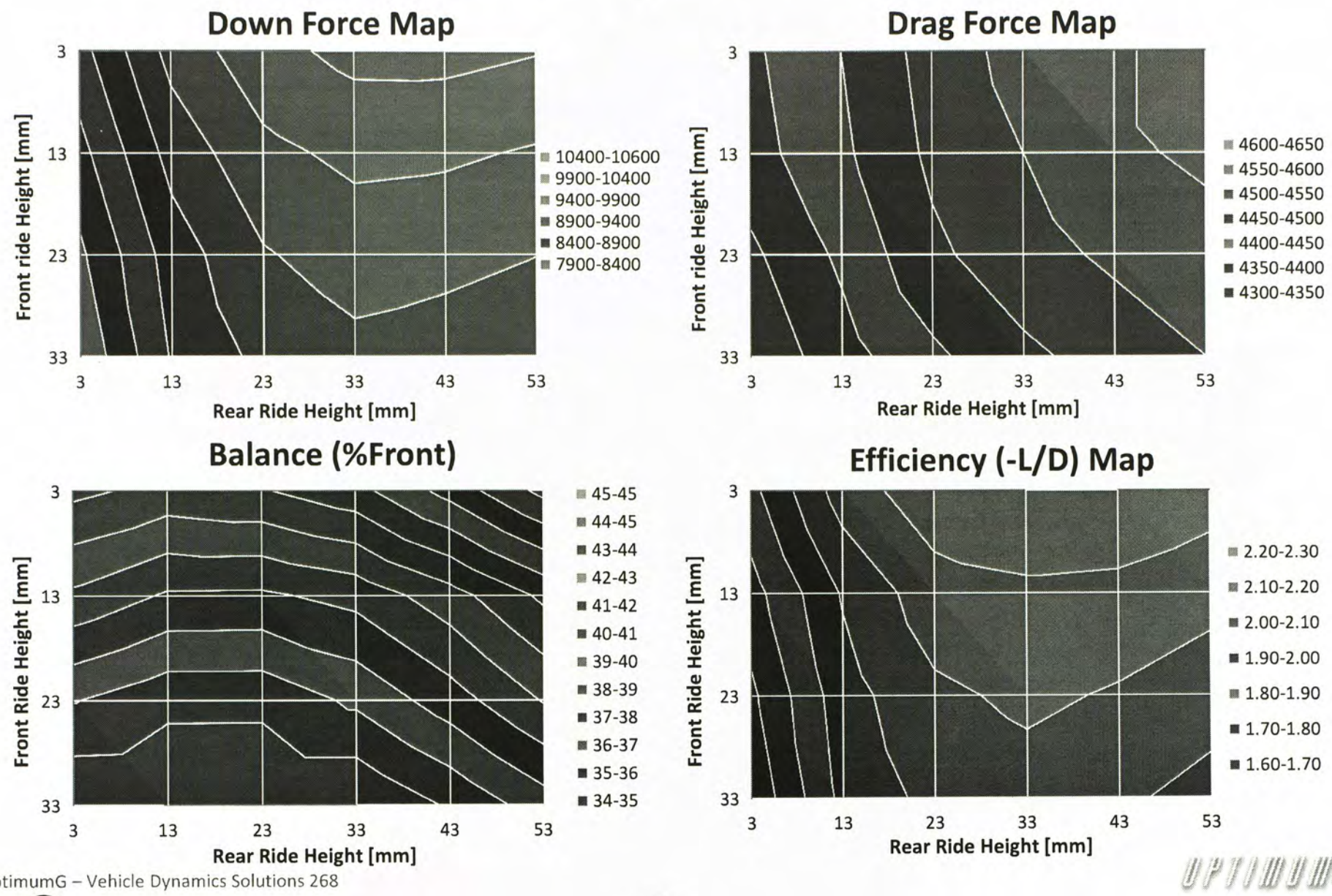
Ride Height Variations

The pitch speed and ride height variations will be conditioned by the change of front and rear aerodynamic force and the way the dampers are set to resist the force changes.



Automotive Aerodynamics

Aero Mapping: Single Seater Example 1



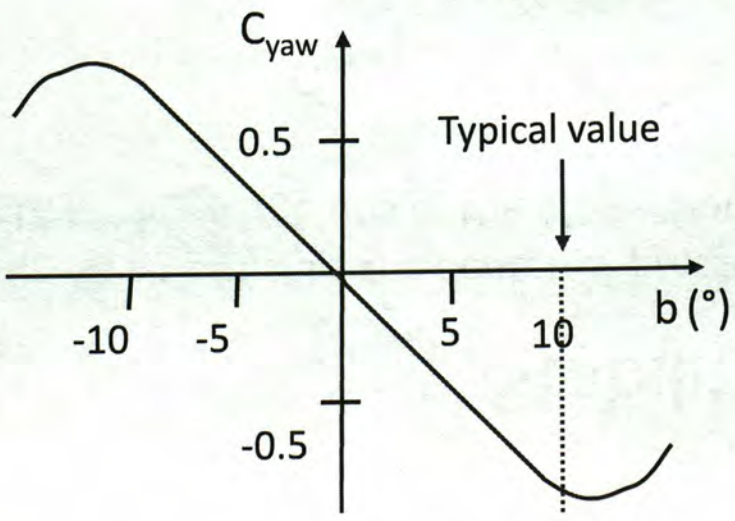
Automotive Aerodynamics

Aerodynamics and Vehicle Performance - Stability

Stability Enhancement

Quantify the stability: indicators – Aerodynamic sensitivity

Example of variation of aerodynamic coefficients versus the yaw angle



Definition: We define the **aerodynamic yaw sensitivity** (or **yaw stiffness**) by:

$$C_{yaw_\beta} = \frac{\Delta C_{yaw}}{\Delta \beta}$$ Over the linear range

Thus we can deduce the yaw moment over the linear part by:

$$M_{yaw} = \frac{1}{2} \times \rho \times A \times l \times C_{yaw_\beta} \times \beta \times V^2$$

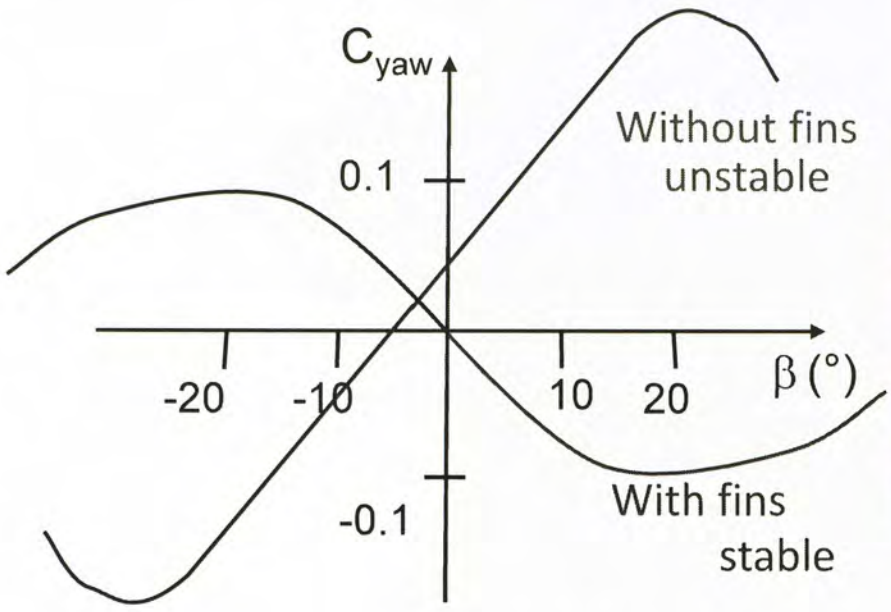
Physical sense: C_{yaw_b} gives the ratio of yaw moment generated by the yaw angle.

Automotive Aerodynamics

Aerodynamics and Vehicle Performance - Stability

Stability Enhancement

Tip: You can shift the center of pressure backward when cornering by adding large fins at the aft-section or end-plates at the rear wings.



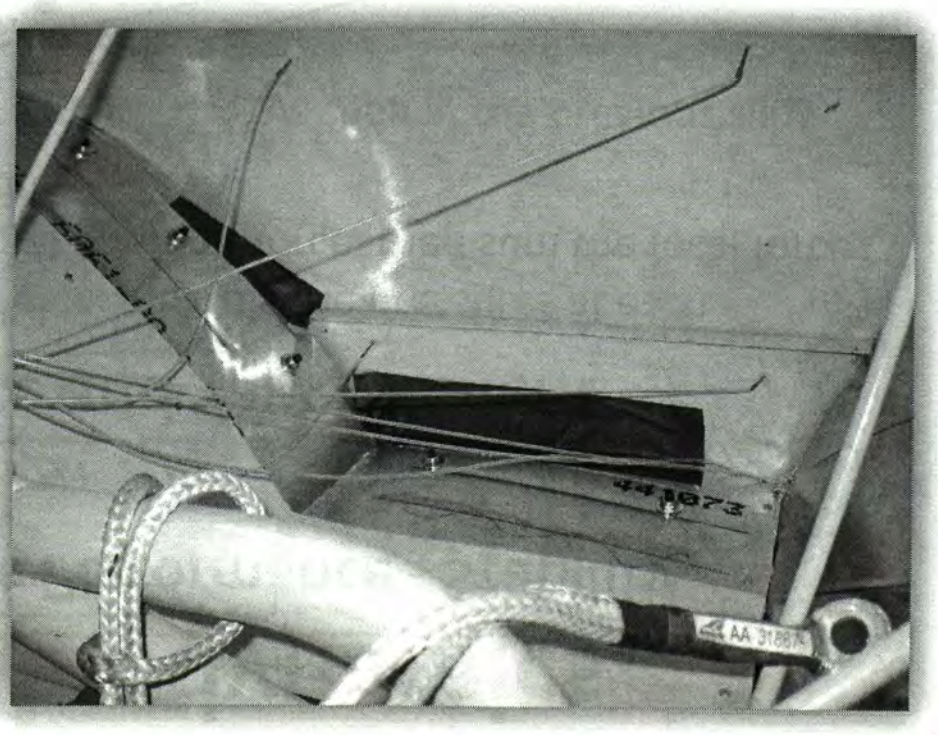
Large fins contribute to increase the rear side force coefficient ➔ static margin ↑ ➔ stability ↑ .

Automotive Aerodynamics

Aerodynamics and Vehicle Performance - Stability

Calculating Side Force and Yaw Moment

The aerodynamic side force and yaw moment can be found using a balance in a windtunnel, or estimated from pressure tap data in straight line testing. The NASCAR body below is equipped with pressure taps for this purpose. It is important to use as many pressure taps as possible, as the accuracy is increased with decreased distance between the pressure taps.



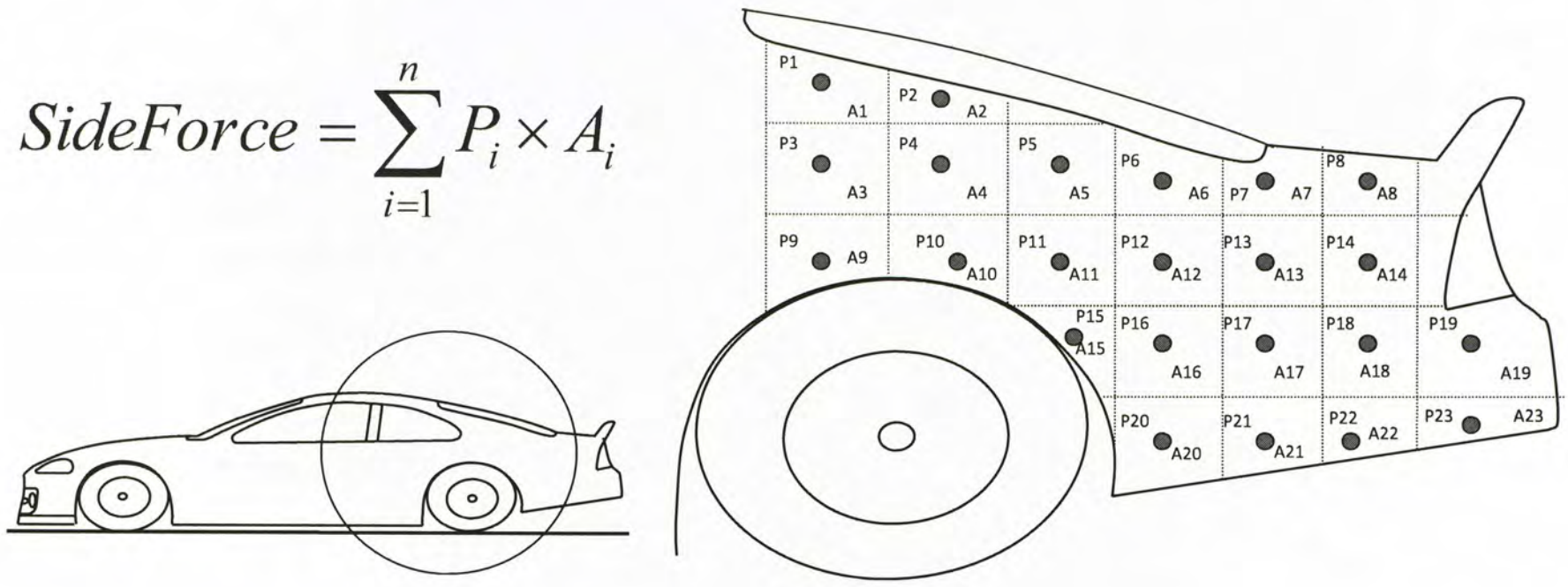
Automotive Aerodynamics

Aerodynamics and Vehicle Performance - Stability

Calculating Side Force and Yaw Moment

Side Force – To calculate approximate side force, multiply the pressures at each pressure tap by the area around the tap to get the local force, then sum the local forces across the side of the car to get total side force.

$$SideForce = \sum_{i=1}^{n} P_i \times A_i$$



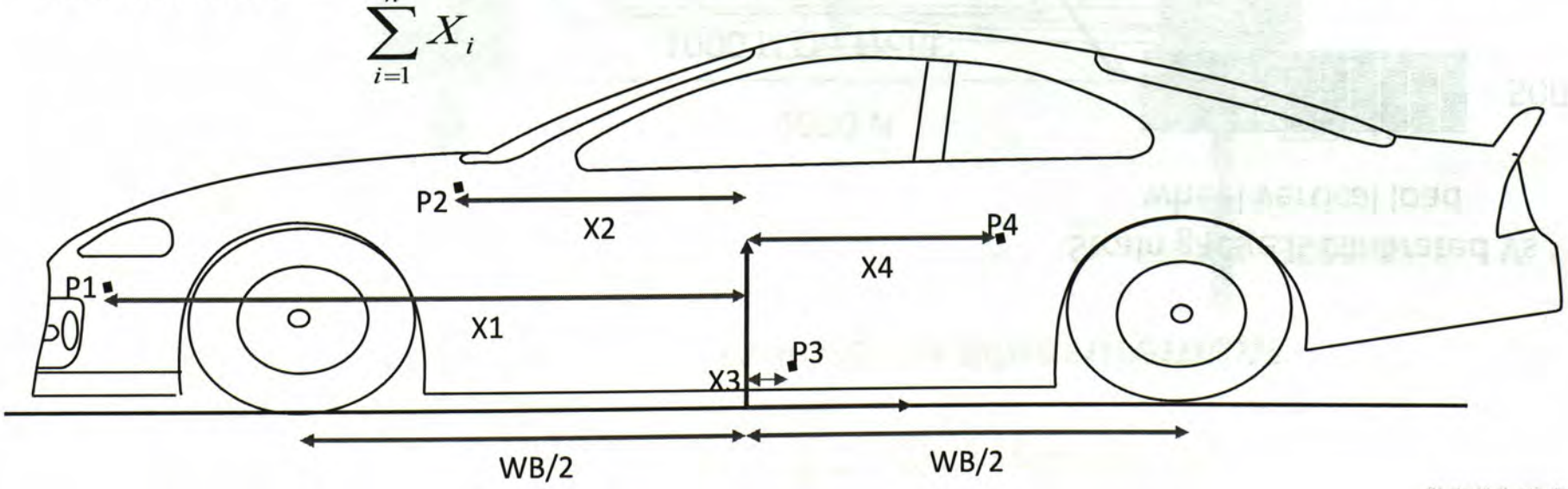
Automotive Aerodynamics

Aerodynamics and Vehicle Performance - Stability

Calculating Side Force and Yaw Moment

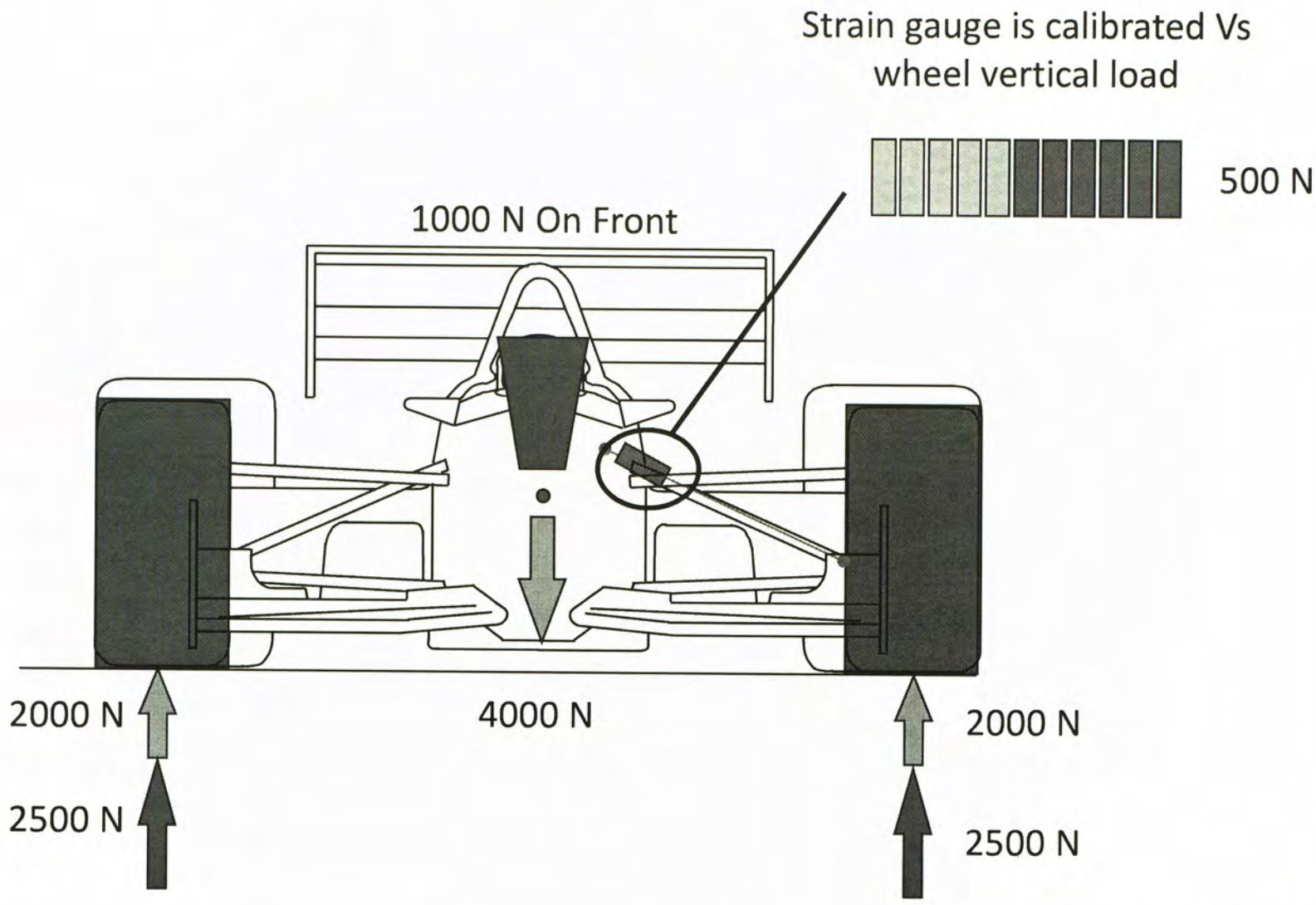
Yaw Moment – This is slightly more complex than the side force calculation, as a yaw axis must be defined. For this example we will take the moment about the vertical axis through the center of the wheelbase as the yaw axis. The yaw moment is then found using the equations below.

$$YawMoment = \frac{\sum_{i=1}^{n} F_i X_i}{\sum_{i=1}^{n} X_i}$$ where $F_i = P_i * A_i$ As seen in the previous slide



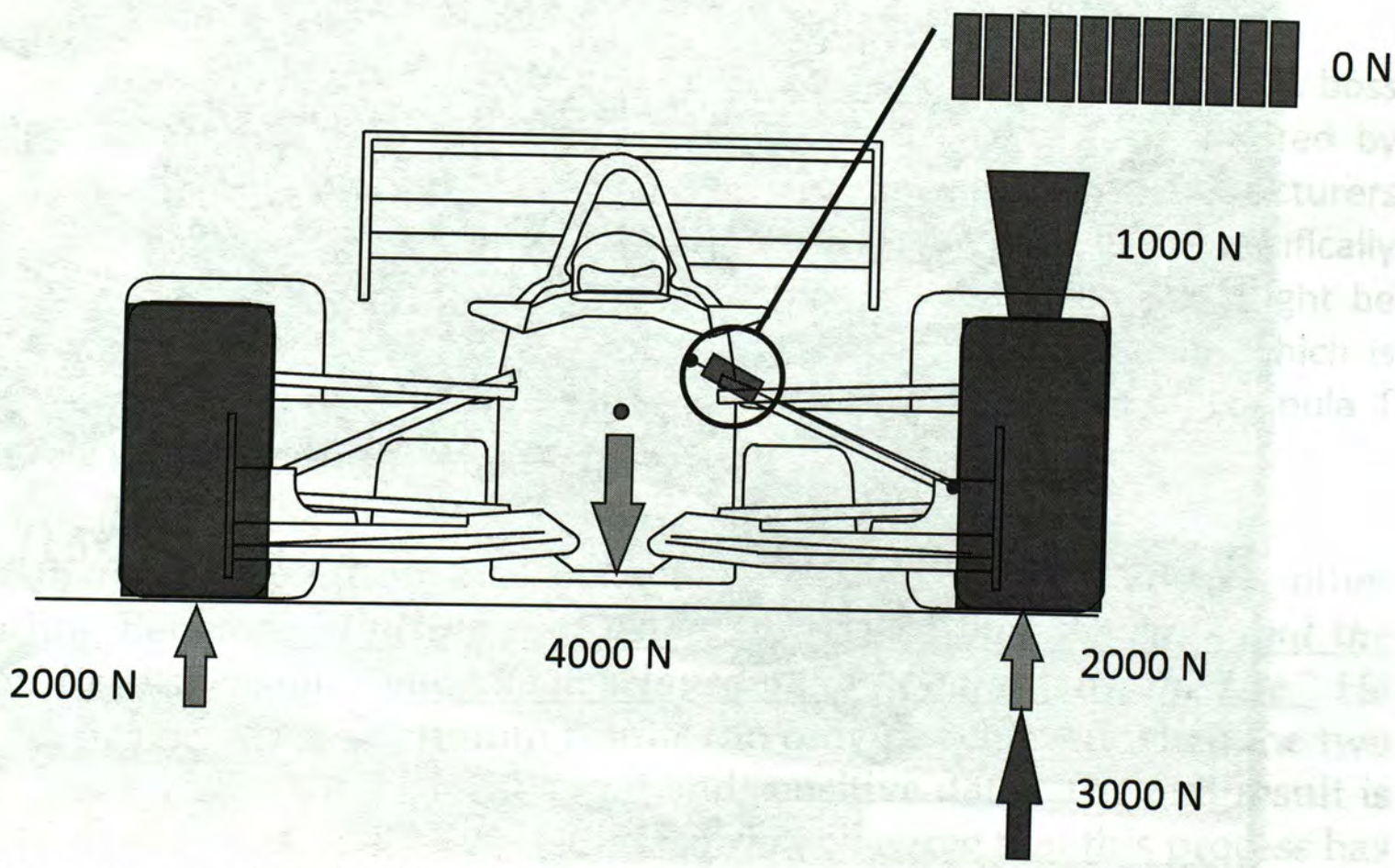
Aerodynamic Measurements

Road Testing – Downforce Measurement



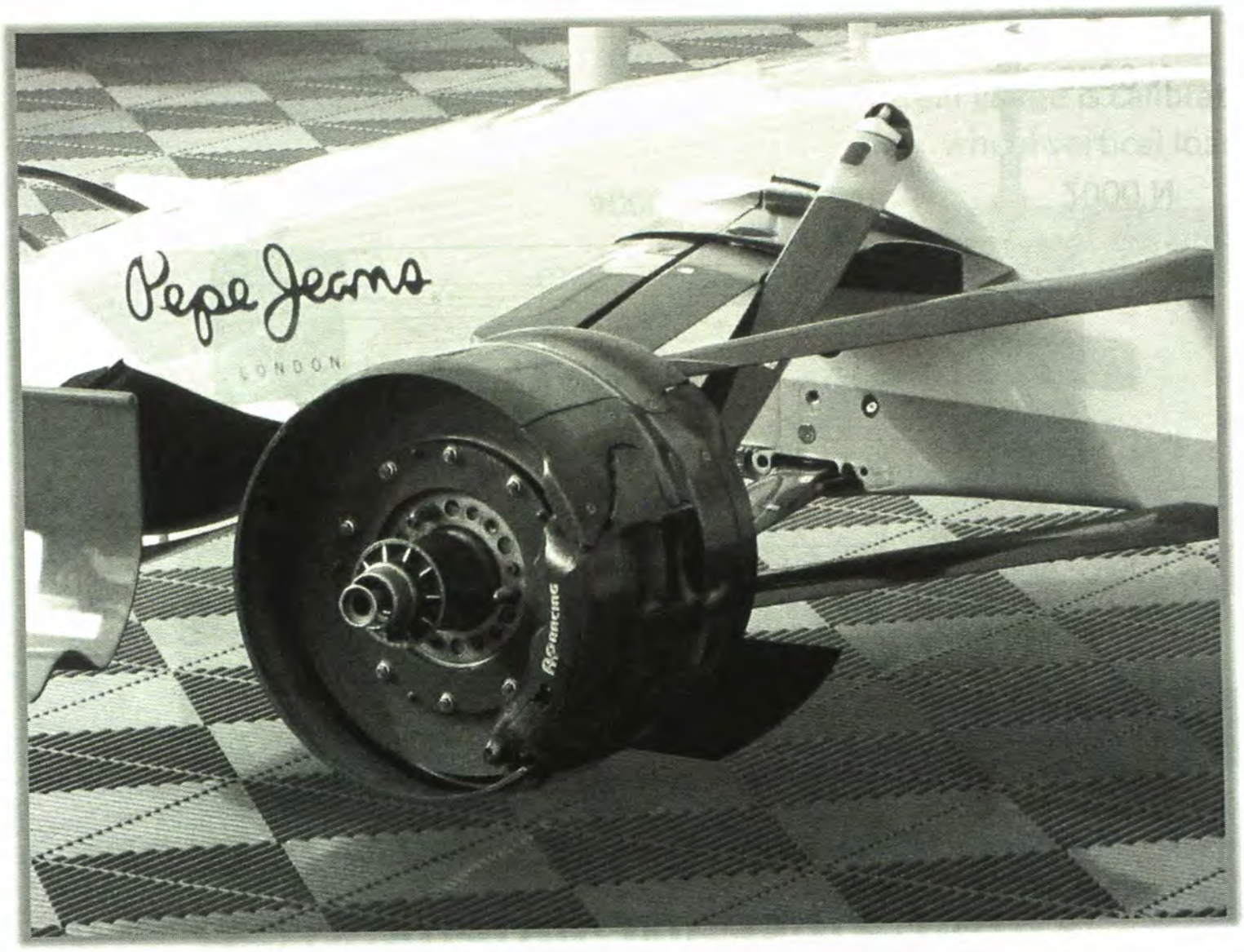
Aerodynamic Measurements

Road Testing – Downforce Measurement



A strain gauge on a pushrod will only measure forces acting on the suspended mass and subsequently do not measure the Aerodynamic lift from the wheels.

4. Kinematics



A Winning Team Opinion

McLaren boss denies Bridgestone favoritism

With the San Marino Grand Prix set to turn into a tire-and-tactics battle, McLaren boss Ron Dennis has been quick to dismiss speculation that his team is being favored by Bridgestone. Rumors have suggested that McLaren are the Japanese tire manufacturers contracted number one team - but Dennis has denied the tires are being 'specifically tailored' for his outfit. "When you say "specifically tailored" to our car, that might be interpreted as being a development that could be prejudicial to a rival team which is being supplied by Bridgestone," explained Dennis. "in fact, the needs of all Formula 1 cars are invariably served by the same technical exercise.

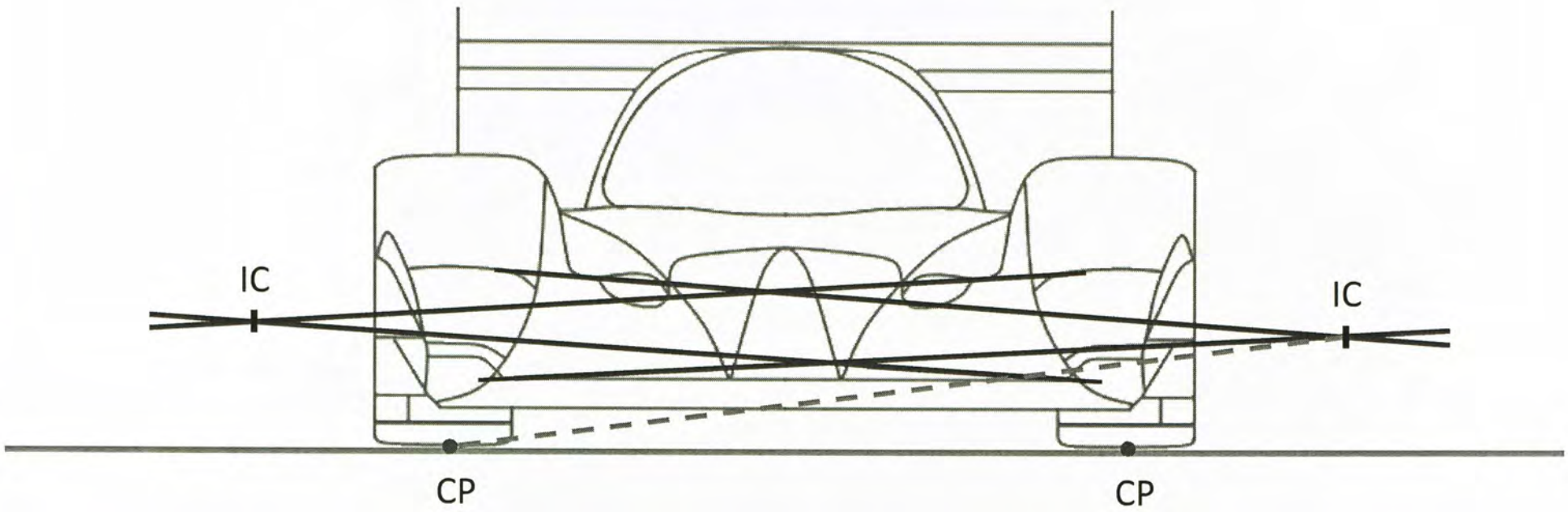
"I believe that our test program has benefited both Bridgestone and its other teams, including Benetton. ***Putting that aside, the fact is that the car - and the kinematics of its suspension - must be developed hand in hand with the tire."*** He added: "As I indicated earlier, optimum results can only be achieved when the two sides are prepared to share their most secret and sensitive data. The end result is optimized performance and I think David Richards will agree that this process has not damaged the progress of Benetton."

26/04/98

Front View Instantaneous Center

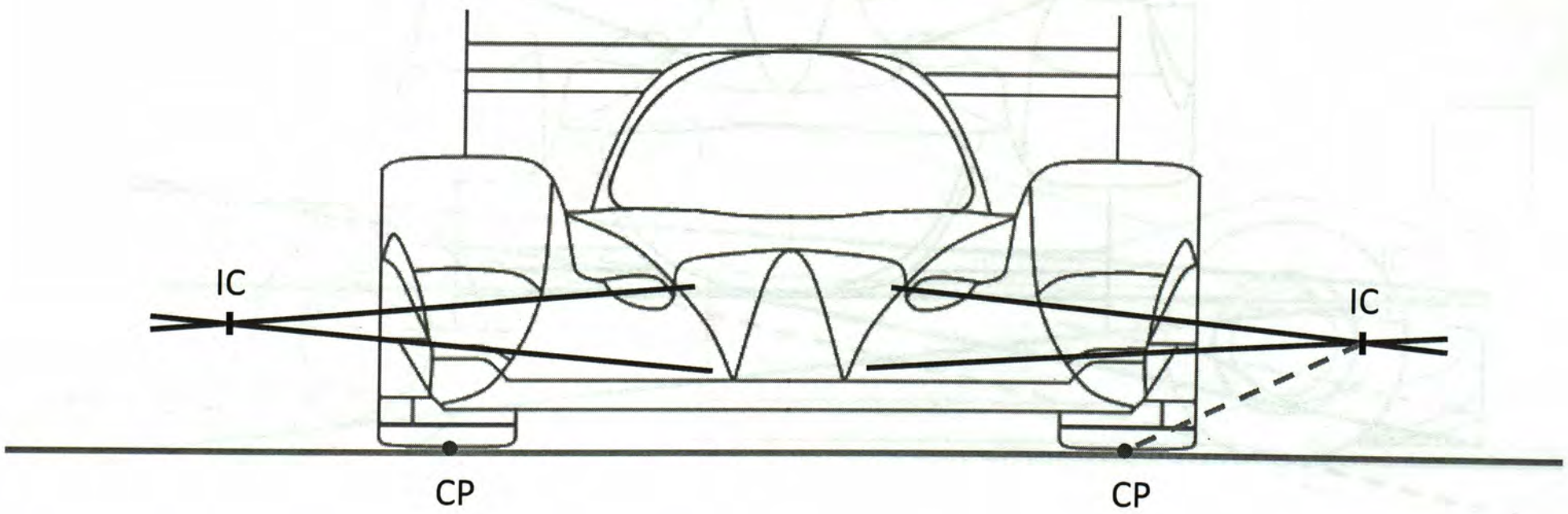
Instantaneous center of rotation of the wheel about the ground.

Instantaneous center of rotation of the wheel about the chassis.



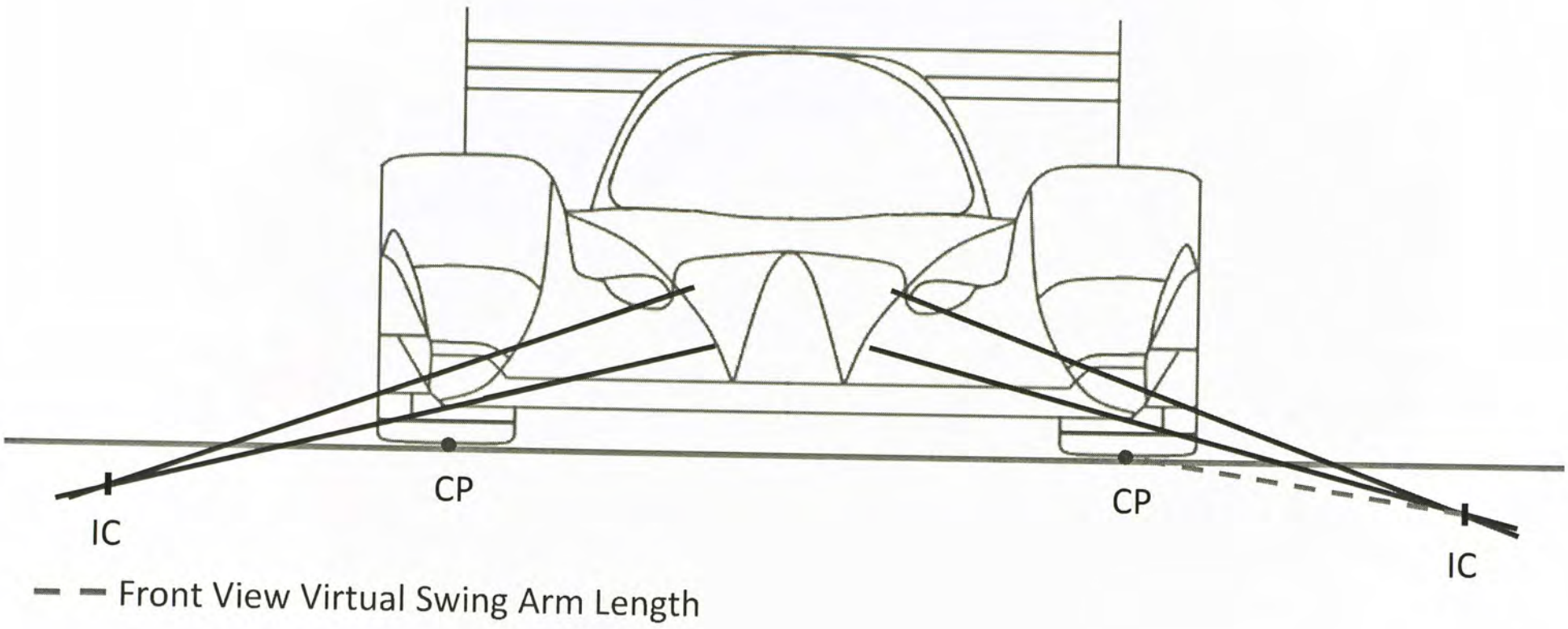
– – Front View Virtual Swing Arm Length

Front View Instantaneous Center



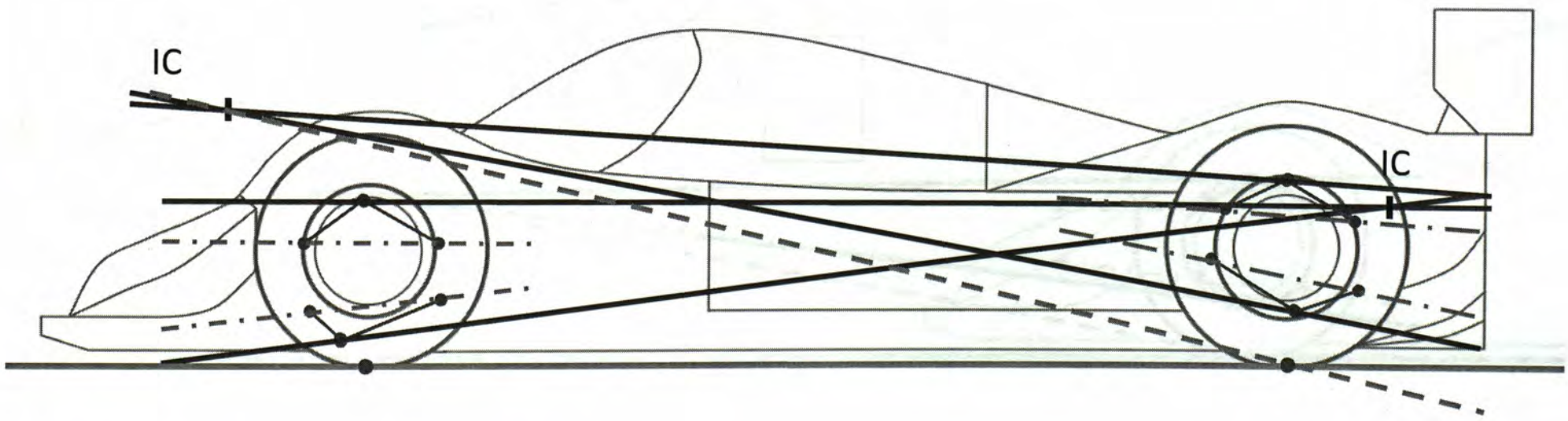
– – Front View Virtual Swing Arm Length

Front View Instantaneous Center



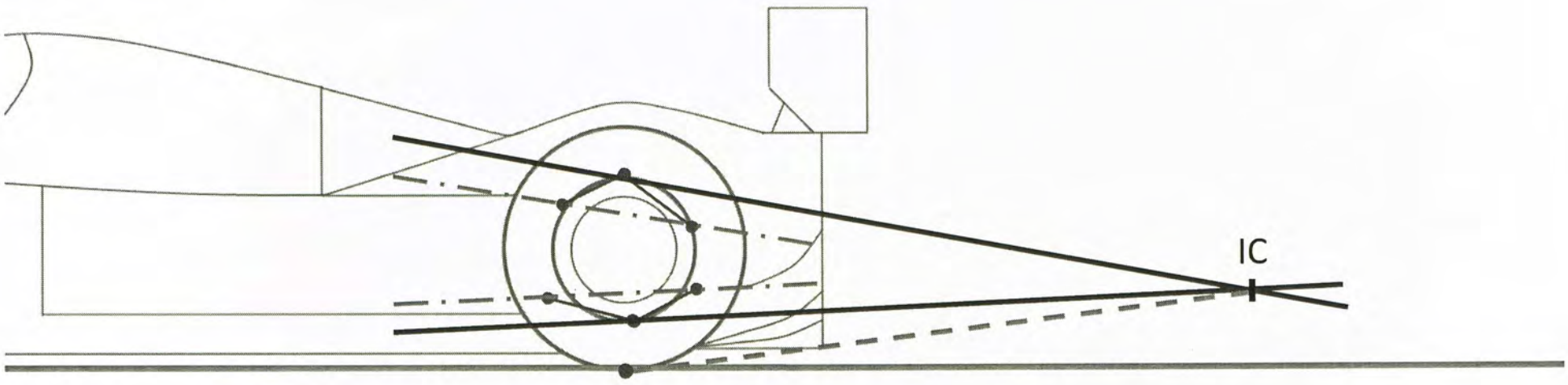
Side View Instantaneous Center

Instantaneous center of rotation of the rear wheel about the chassis.



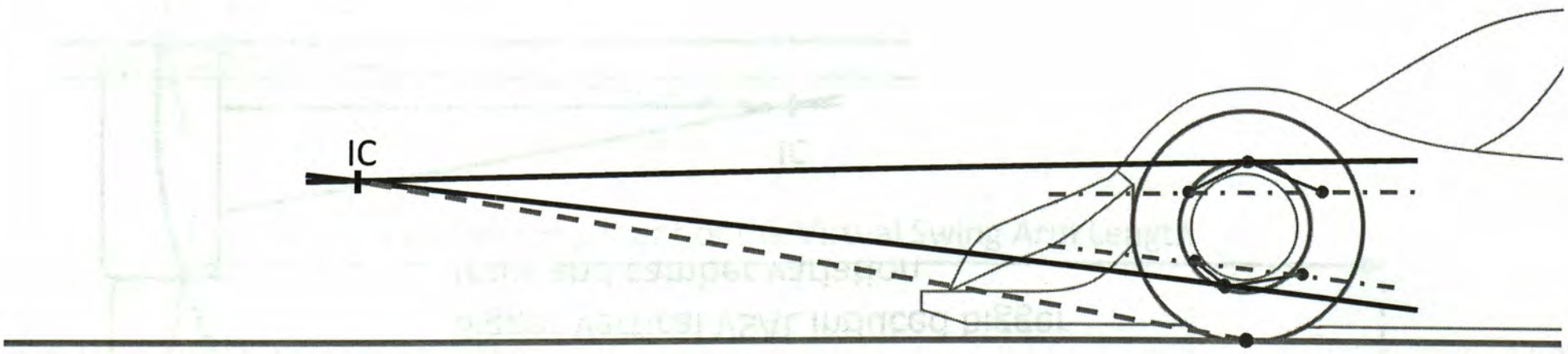
Side View Virtual Swing Arm Length

Side View Instantaneous Center



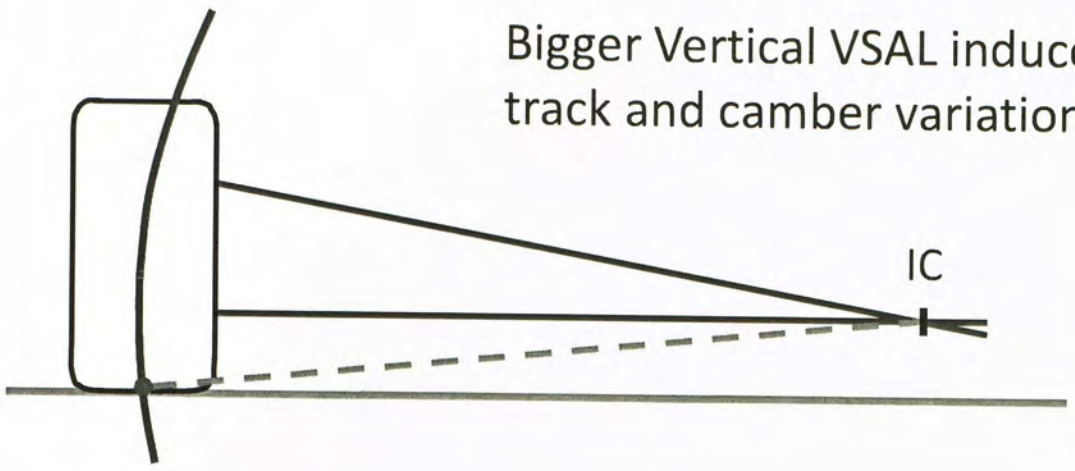
– – Side View Virtual Swing Arm Length

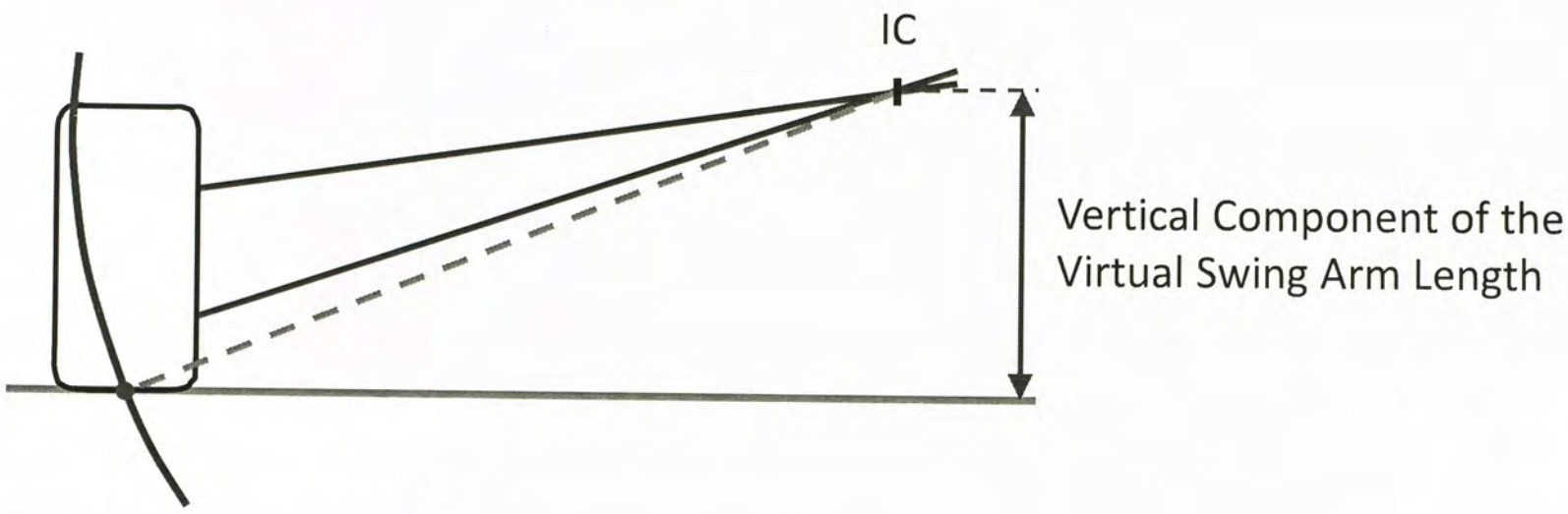
Side View Instantaneous Center



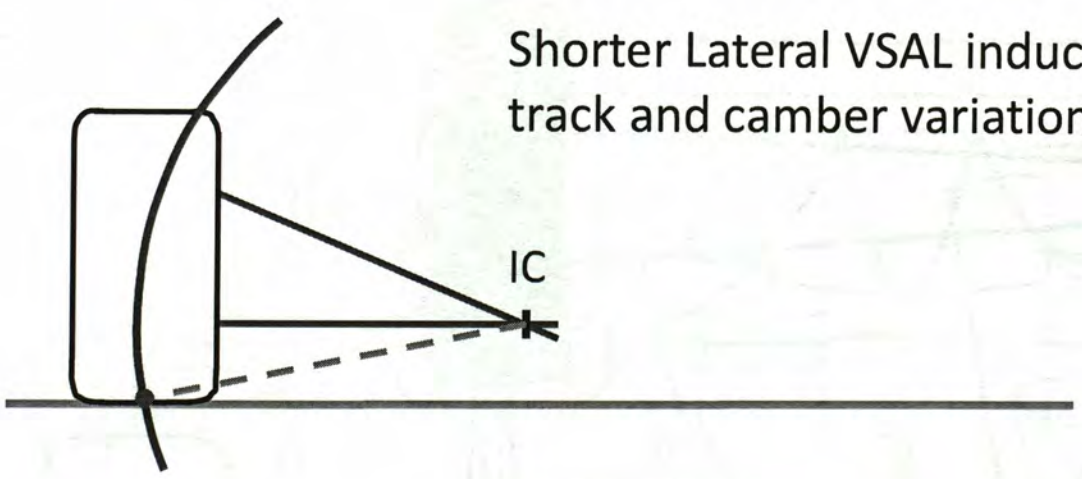
– – Side View Virtual Swing Arm Length

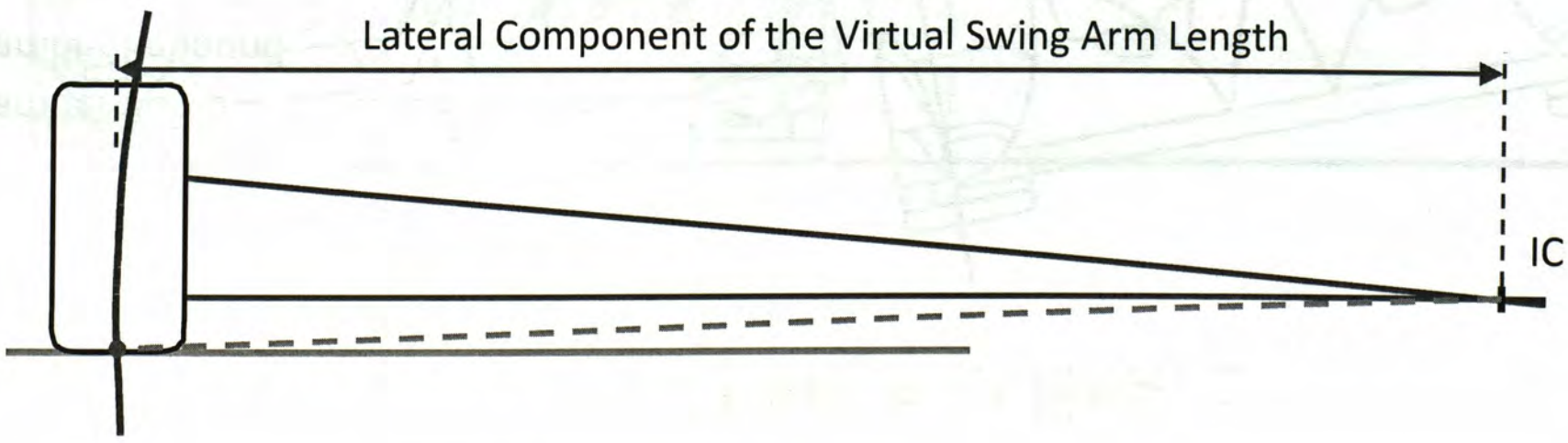
Front View IC Vertical location



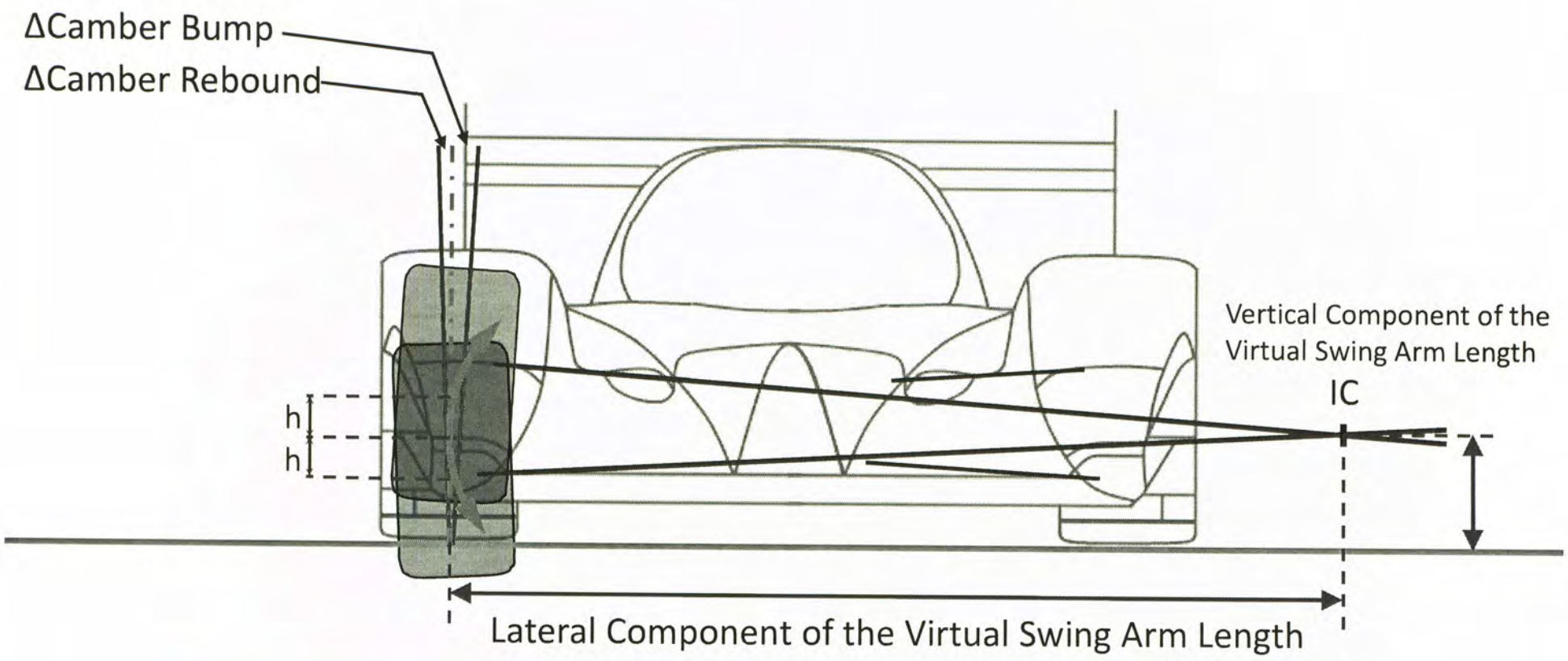


Front View IC Lateral location



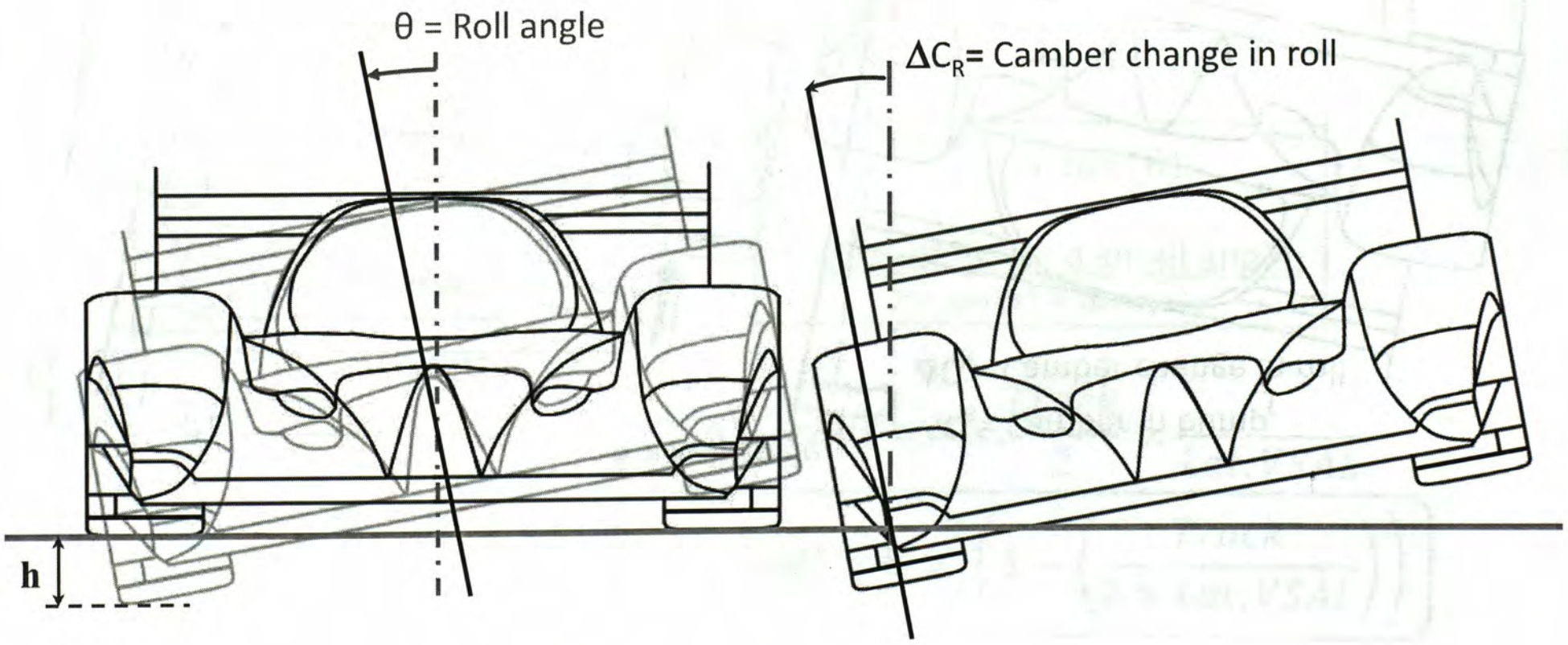


Camber Change in Heave



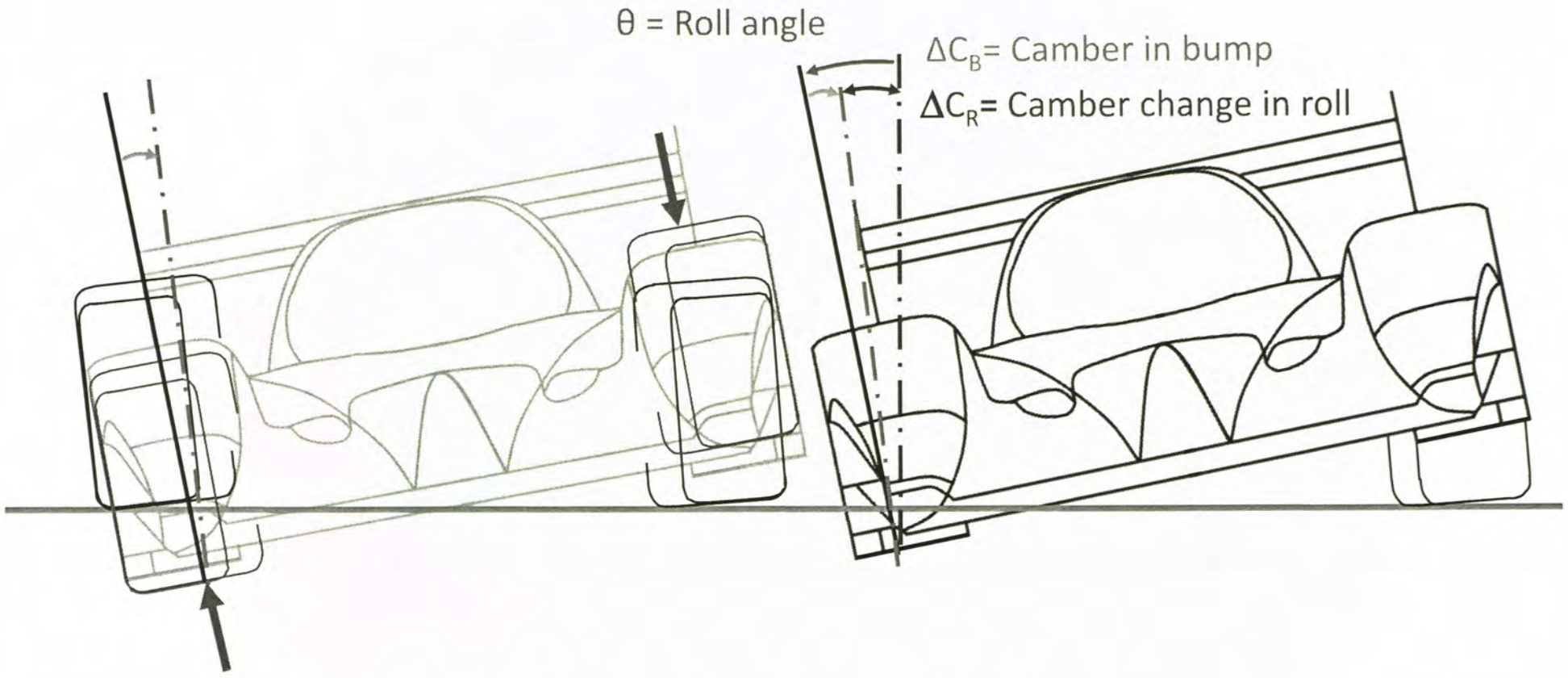
$$Camber\ change\ in\ bump/rebound\ \ \Delta C_B[rad] = Atan\left(\frac{h}{Lateral\ VSAL}\right)$$

Camber Change in Roll



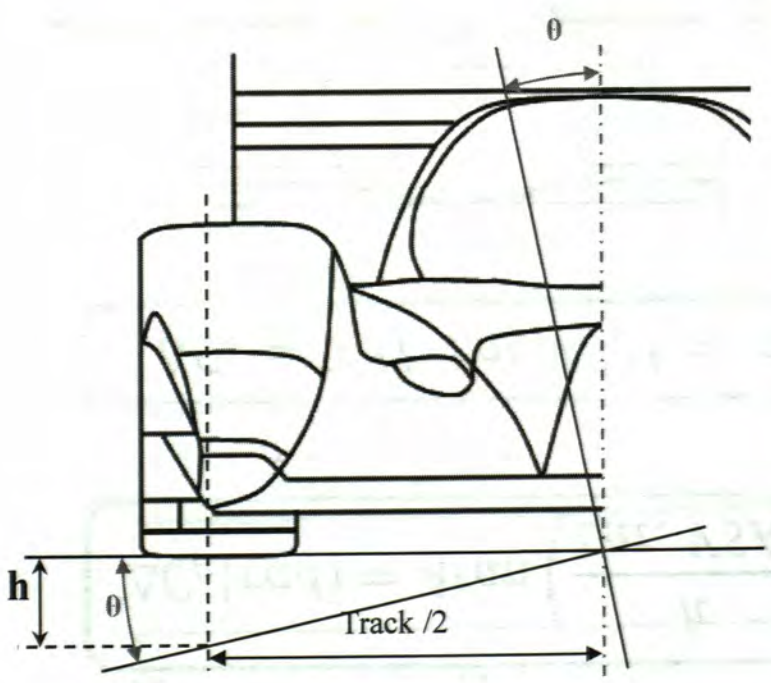
$$\Delta C_R[rad] = \theta[rad]$$

Camber Change in Roll



$$\Delta C_R[rad] = \theta[rad] - \Delta C_B[rad]$$

Camber Change in Roll



$$\Delta C_R[rad] = \theta[rad] - \Delta C_B[rad]$$

$$\Delta C_B[rad] = \left(\frac{h}{Lat.VSAL}\right)$$

- $h = \frac{Track}{2} \times tan(\theta)$
- Assuming for a small angle:
  $\propto \approx tan(\propto) \approx Atan(\propto)$

$$\Delta C_R[rad] = \theta \times \frac{Track}{2} \times \frac{1}{Lat.VSAL}$$

$$\Delta C_R[rad] = \theta \times \left(1 - \left(\frac{Track}{2 \times Lat.VSAL}\right)\right)$$

Camber Change: Comparison Heave/Roll

- Camber change in bump

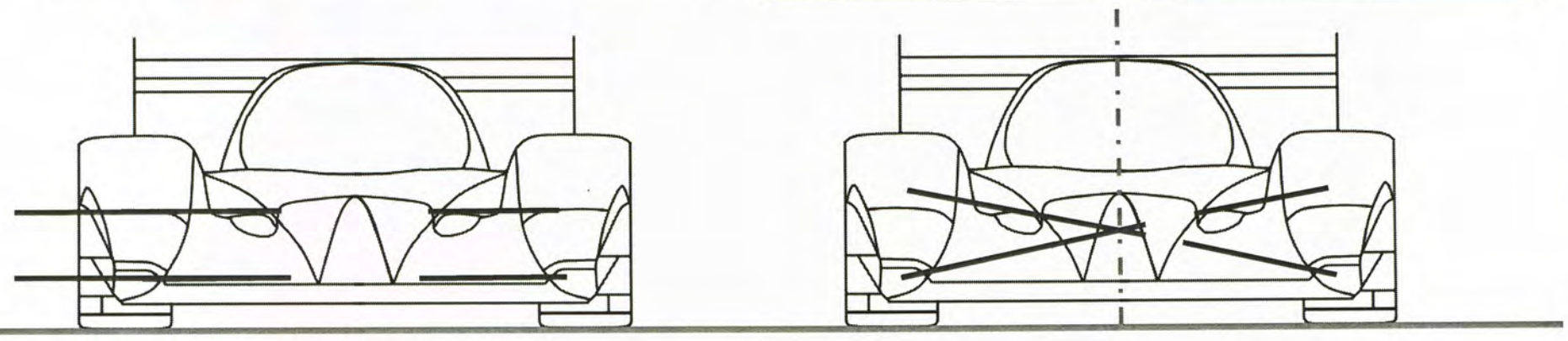
$$\Delta C_B[rad] = Atan\left(\frac{h}{Lat.\ VSAL}\right)$$

$$\Delta C_B = 0 \ \ if \ \ Lat.VSAL = \infty$$

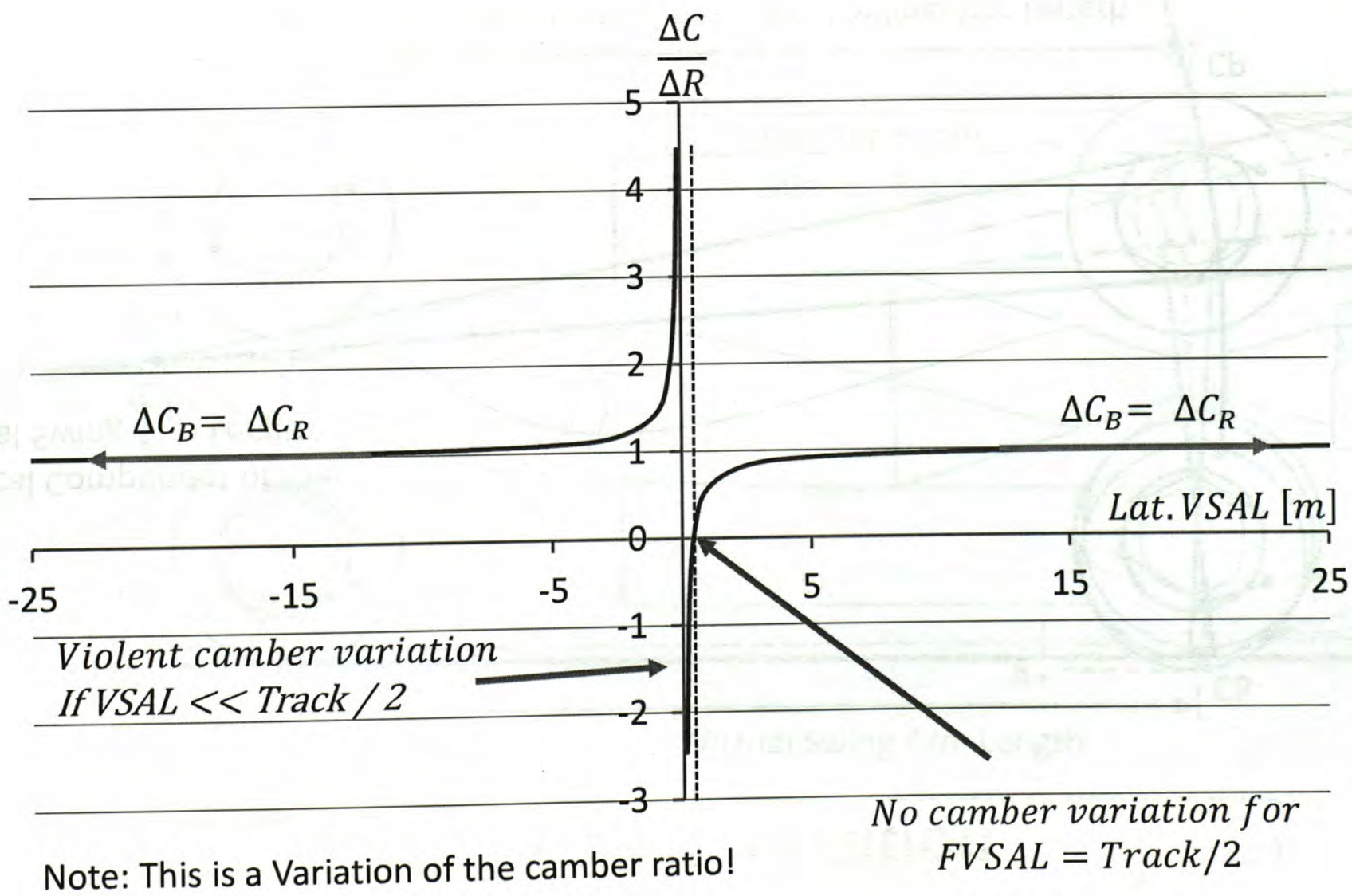
- Camber change in roll

$$\Delta C_R[rad] = \theta \times \left(1 - \left(\frac{Track}{2 \times Lat.\ VSAL}\right)\right)$$

$$\Delta C_R = 0 \ \ if \ \ Lat.VSAL = \frac{Track}{2}$$

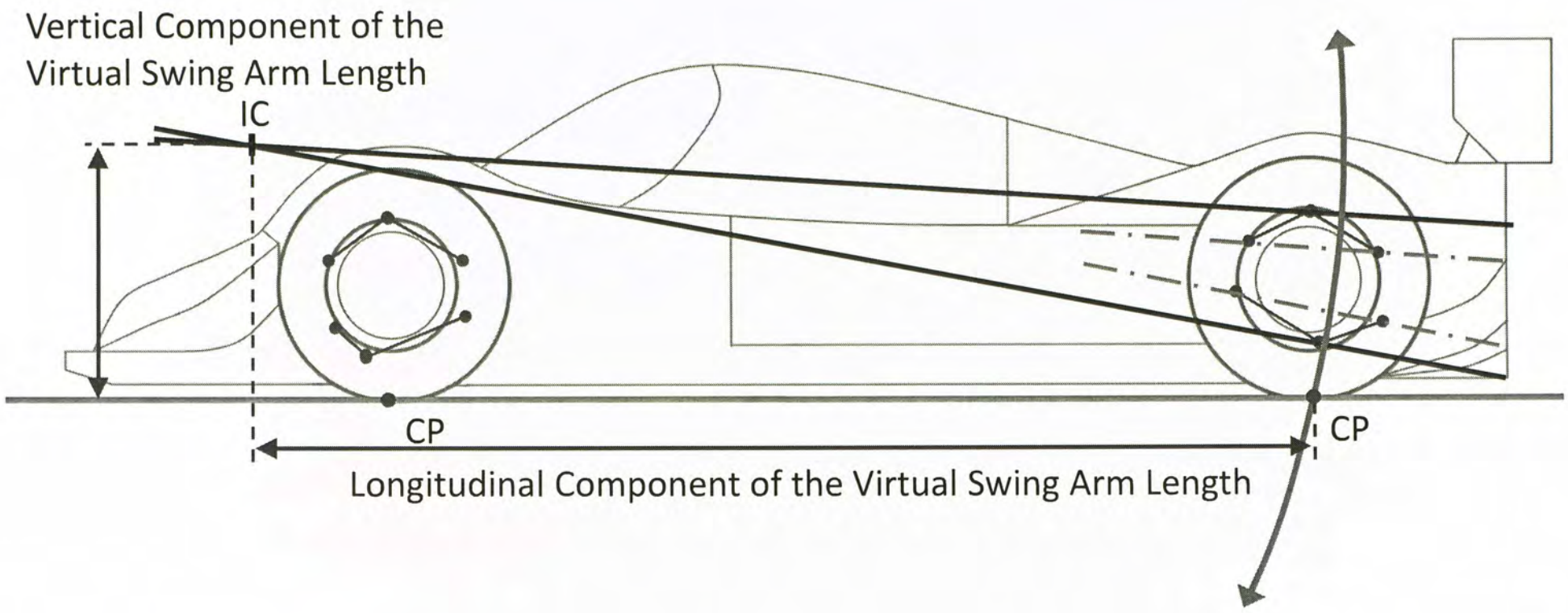


Camber Change in Roll – Camber Ratio

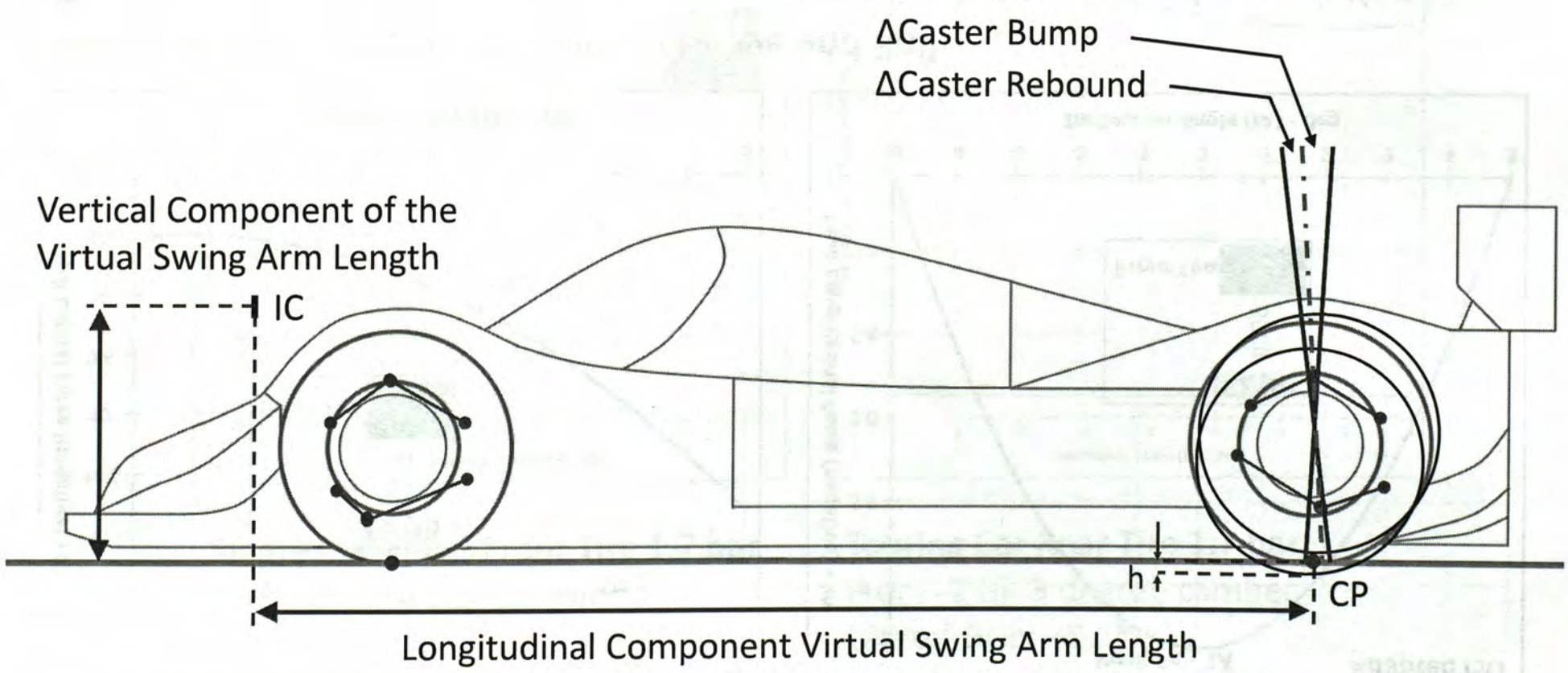


Note: This is a Variation of the camber ratio!

Side View IC Location

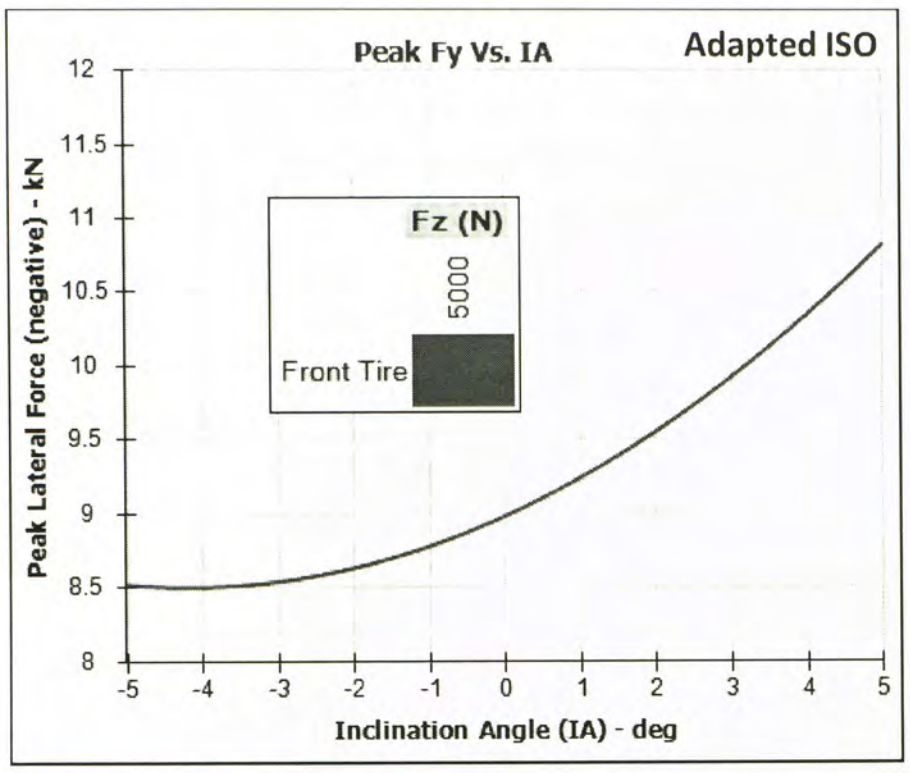


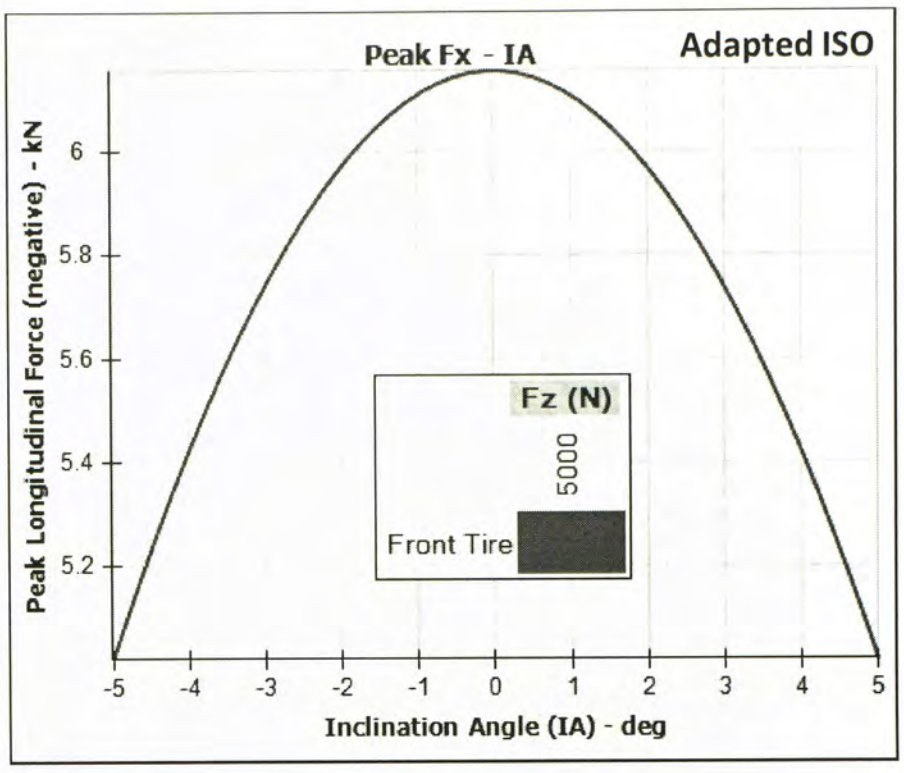
Caster change in Heave



$$Caster\ change\ in\ bump/rebound\ \ \Delta C_C[rad] = Atan\left(\frac{h}{Longitudinal\ VSAL}\right)$$

Camber Change: Heave/Roll



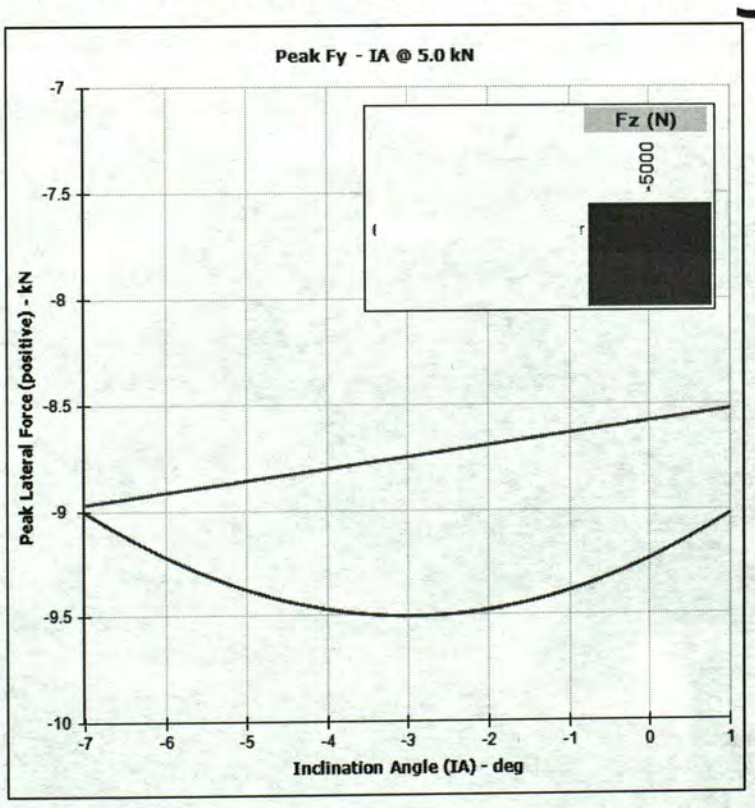


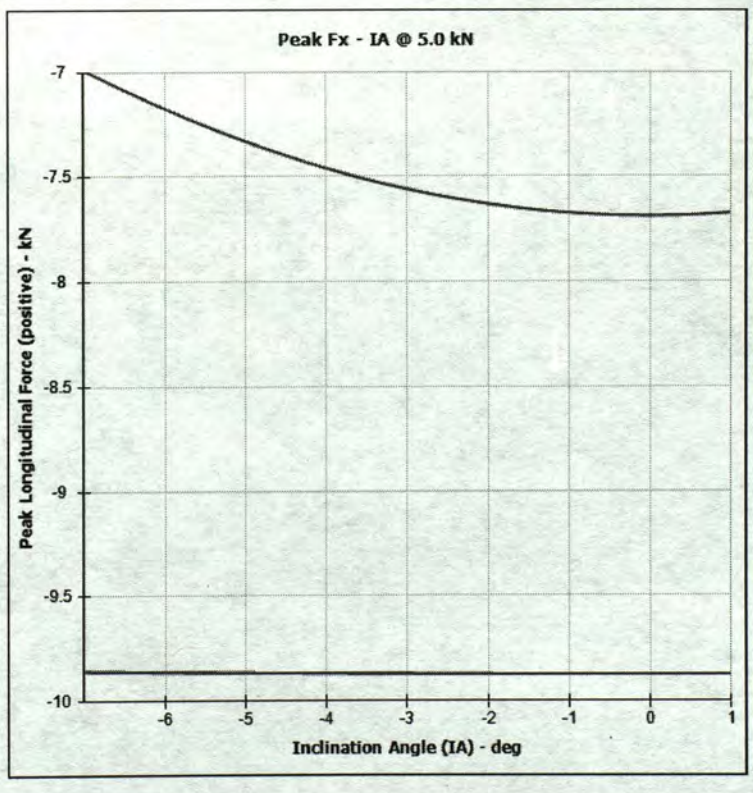
Find the ideal camber variations in Heave and Roll.

Tire data will help you how to make a compromise between camber variation in heave and roll.

Note that tire model have their flaws. It uses a second degree polynomial and no horizontal shift (in longitudinal). So the optimum inclination angle is always 0degree (whatever what the raw data are) and has symmetrical behavior in positive and negative inclination angle.

Camber Change: Heave/Roll



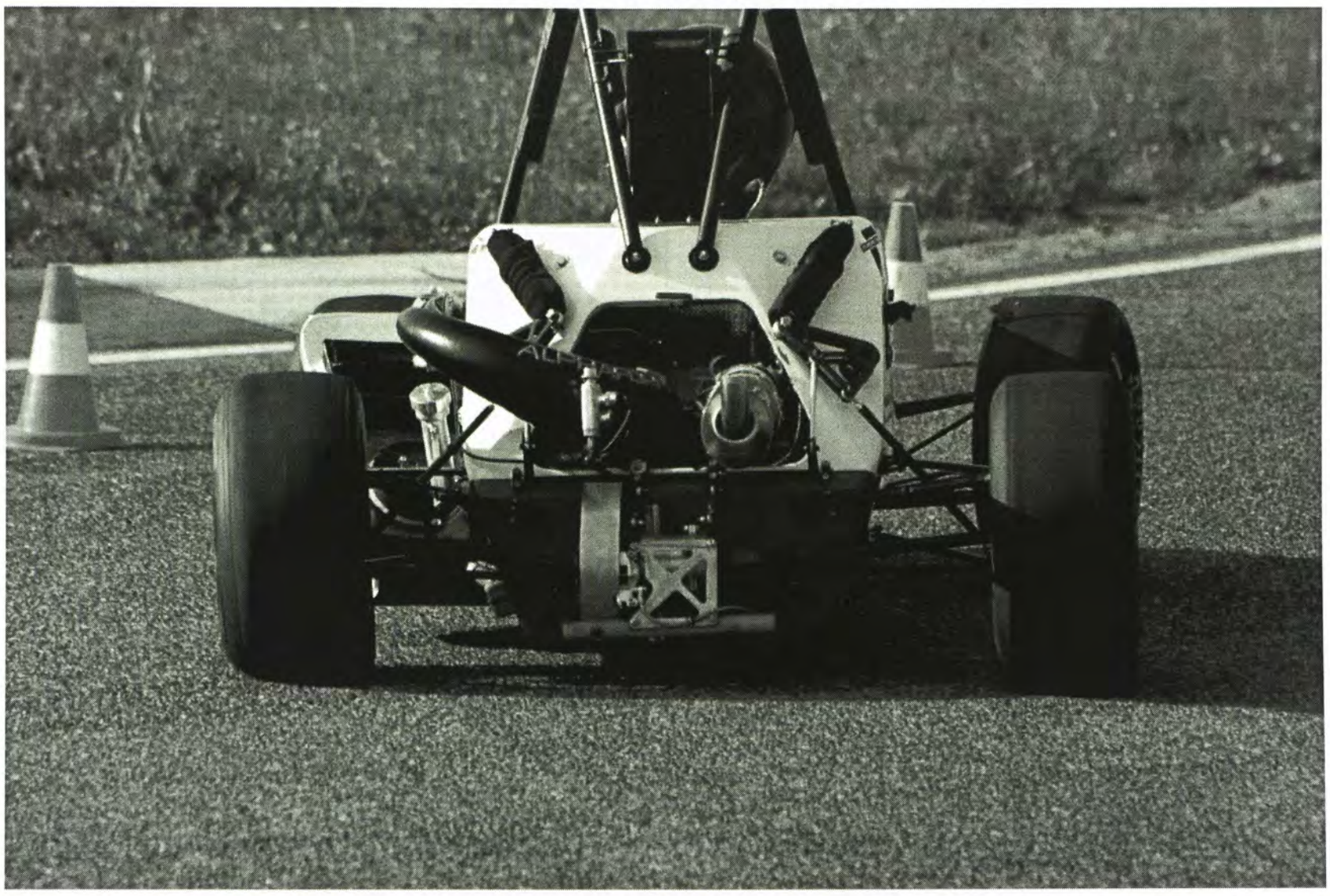


Single seater car Front Tire 1.7 bar
From -3 to -4 degree camber
Lateral Grip: +0.7%
Longitudinal Grip: -1.3%

Touring Car Rear Tire 1.7 bar
From -2 to -3 degree camber
Lateral Grip: +0.1%
Longitudinal Grip: +/- 0.0%

Camber Compliance

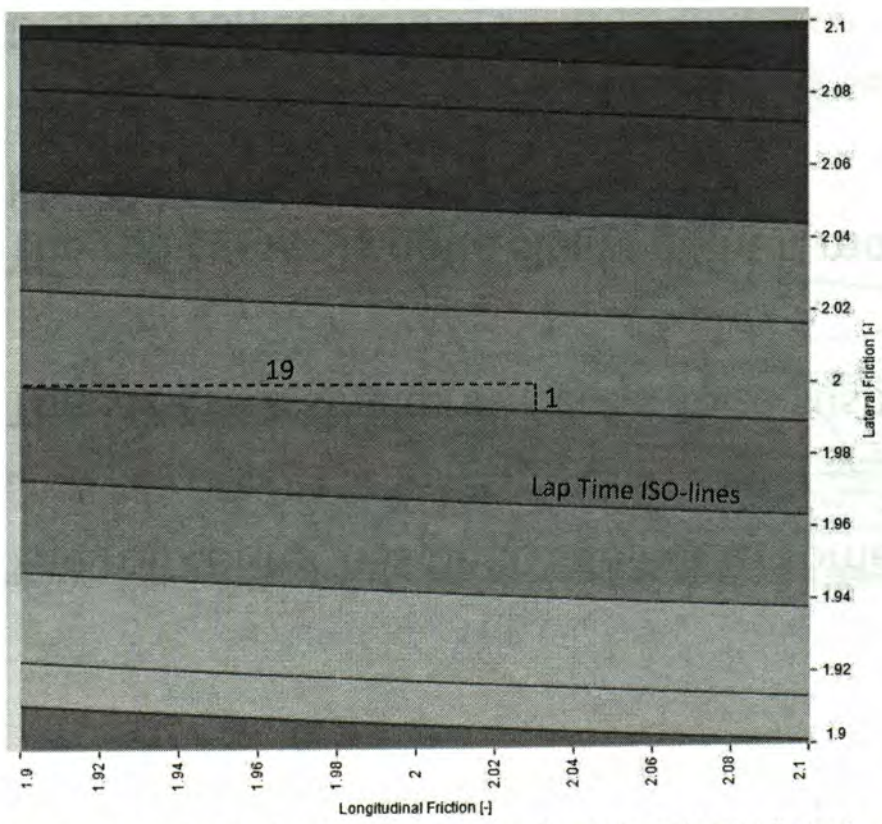
Is your camber what you think it is?



Longitudinal or Lateral Grip?

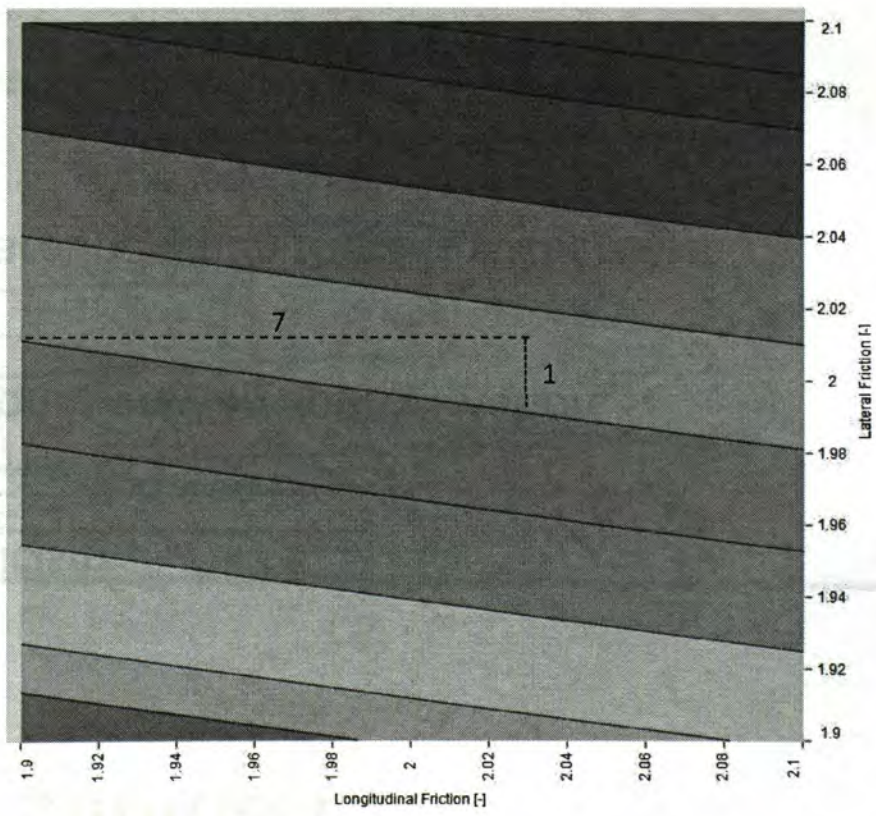
- The trade-off between longitudinal and lateral grip can be investigated using simulation.
- When the ratio is smaller than 4:1 the track is considered "longitudinal"

LMP2 @ Lime Rock



1% gain is lateral grip reduces lap time 19x more than 1% gain longitudinal grip

LMP2 @ Circuit of The Americas



Roll Center Definition

The roll center has been subject to some criticism.

The idea of a single roll center for a suspension is only an approximation.

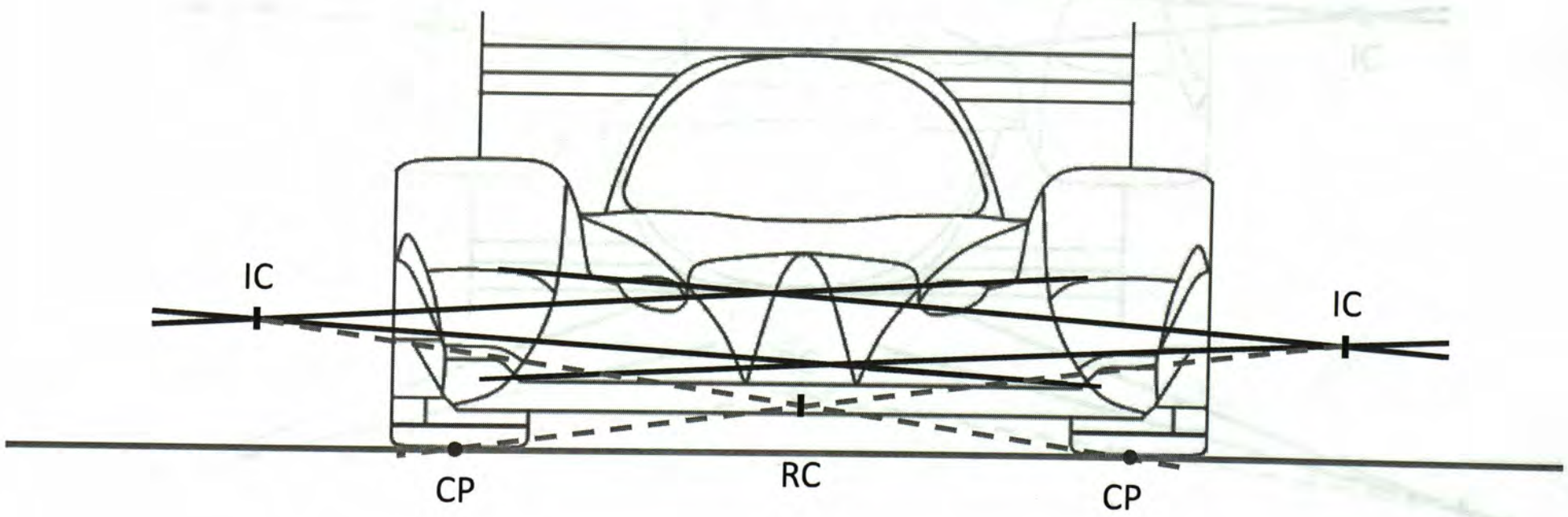
The roll center is not a single point it moves around when there is suspension bump or roll.

Distinct roll centers:

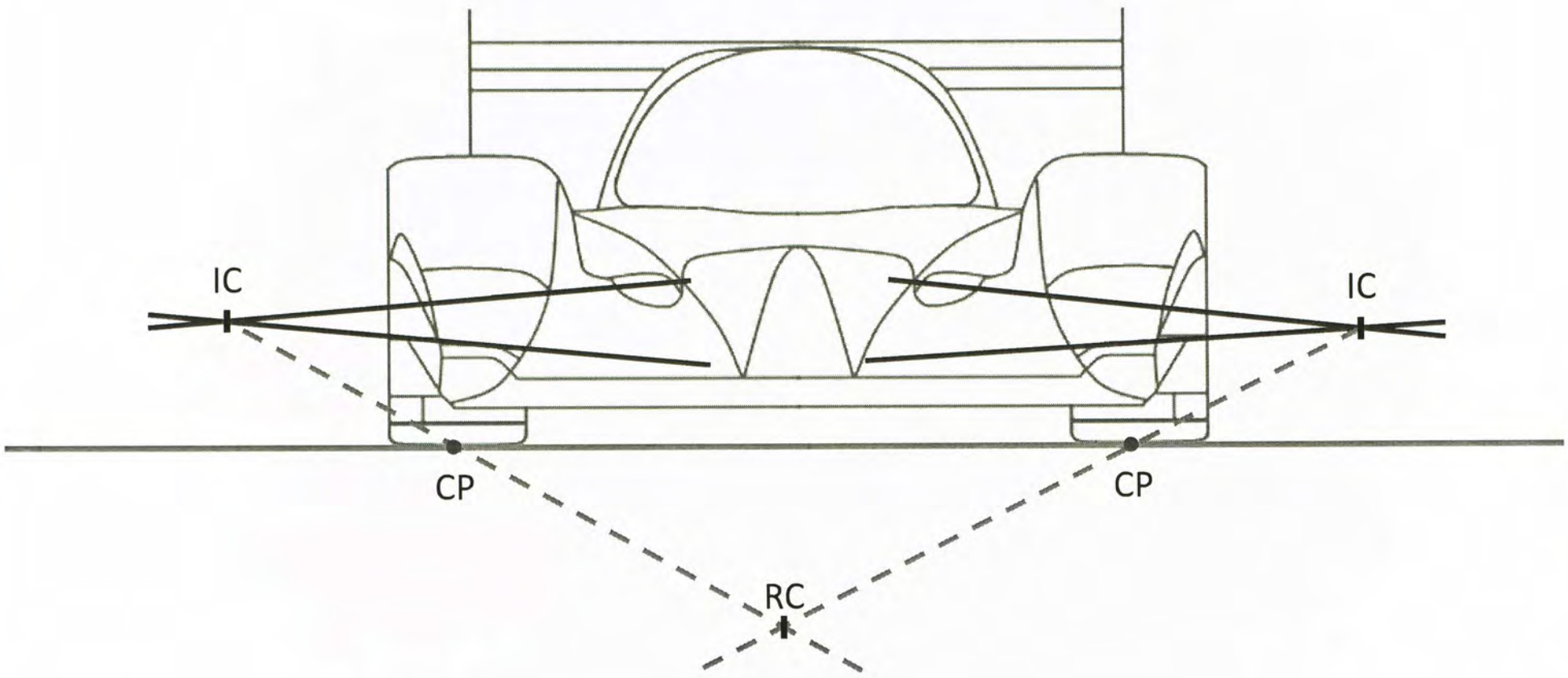
- The kinematic roll center
- The force-based roll center
- Force-Motion Roll Center

Kinematics Roll Center

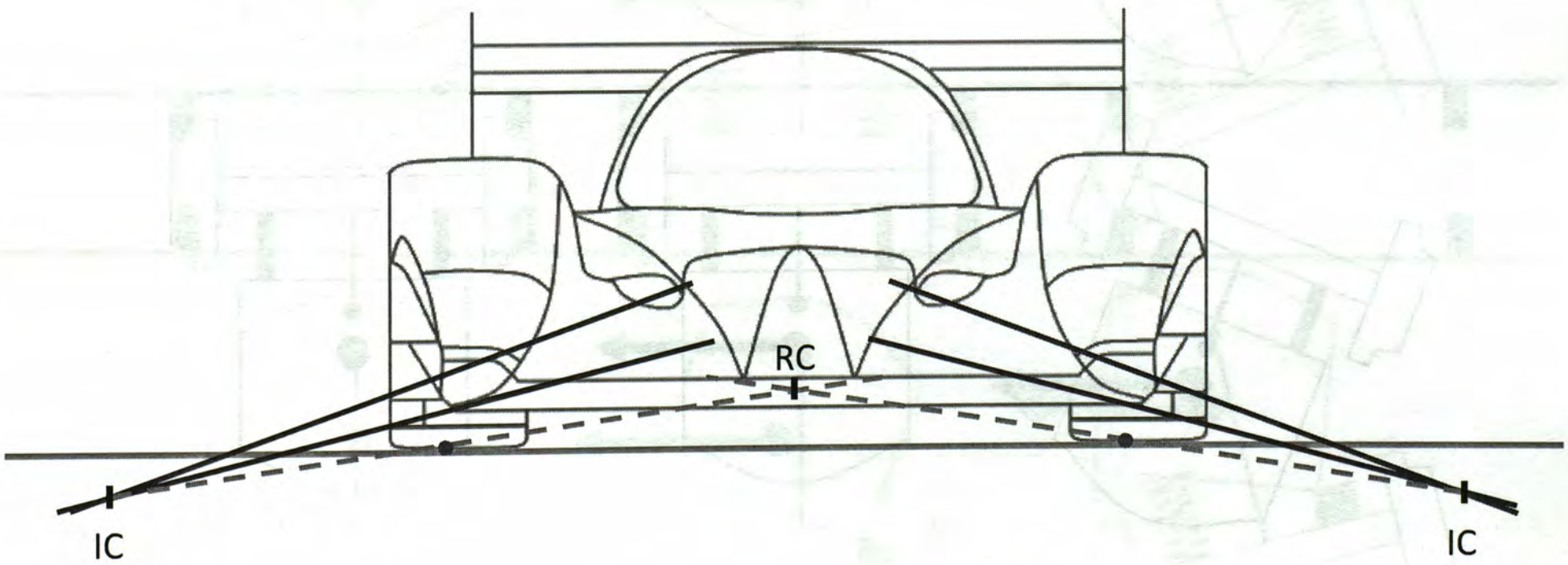
A property of the suspension, a point found by specified geometrics methods.



Kinematics Roll Center

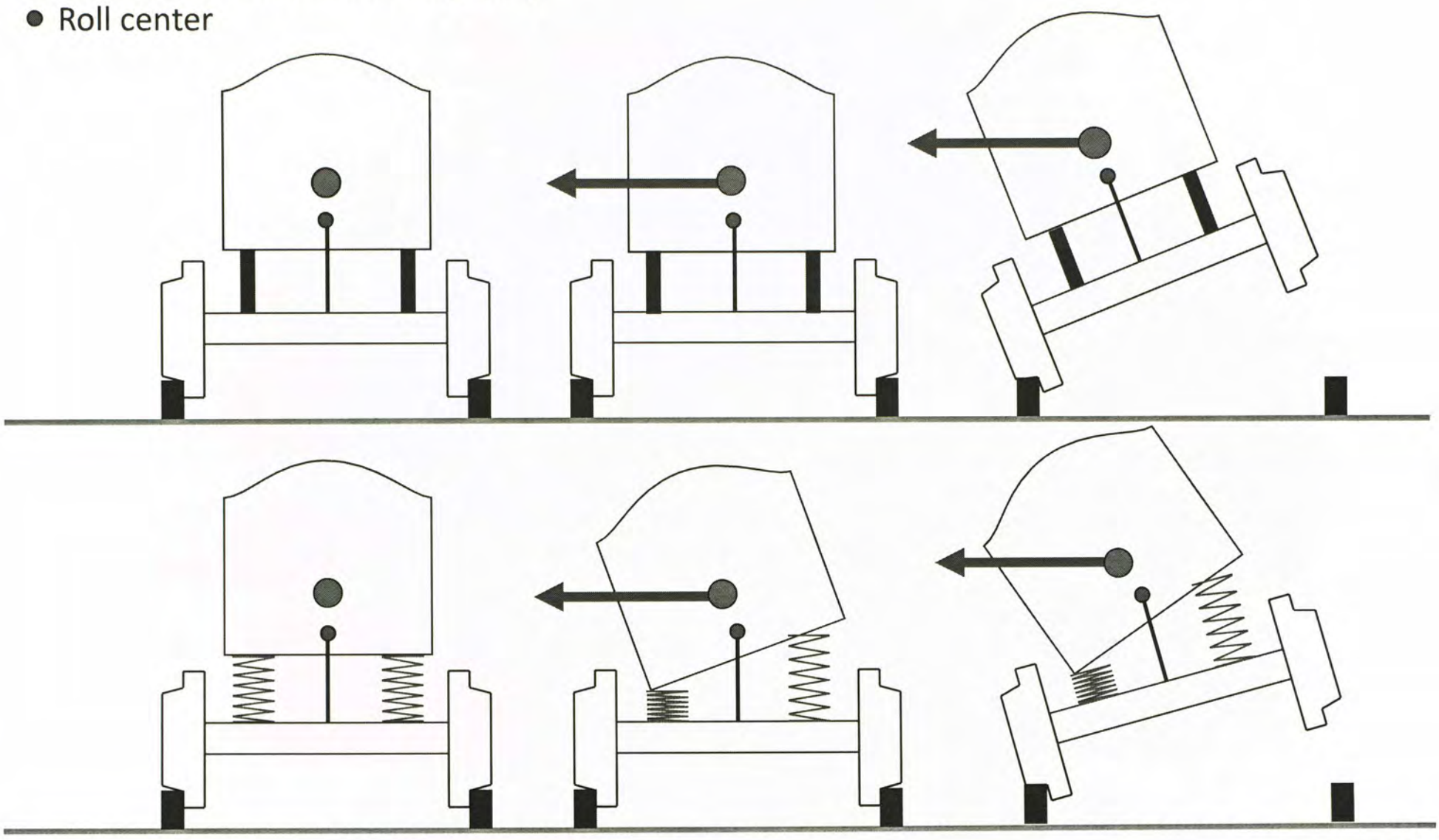


Front View Instantaneous Center



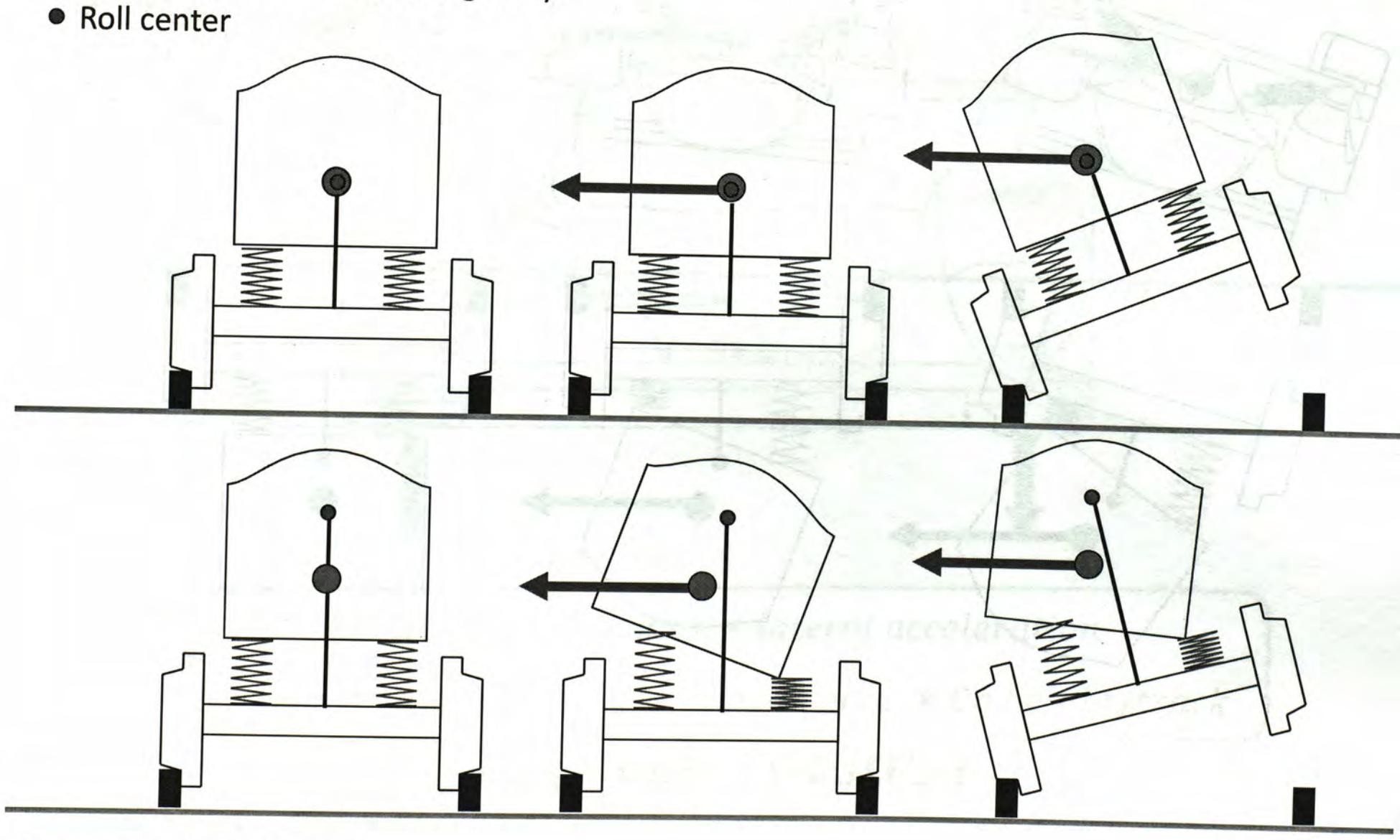
Roll Center Location

- Suspended mass center of gravity
- Roll center



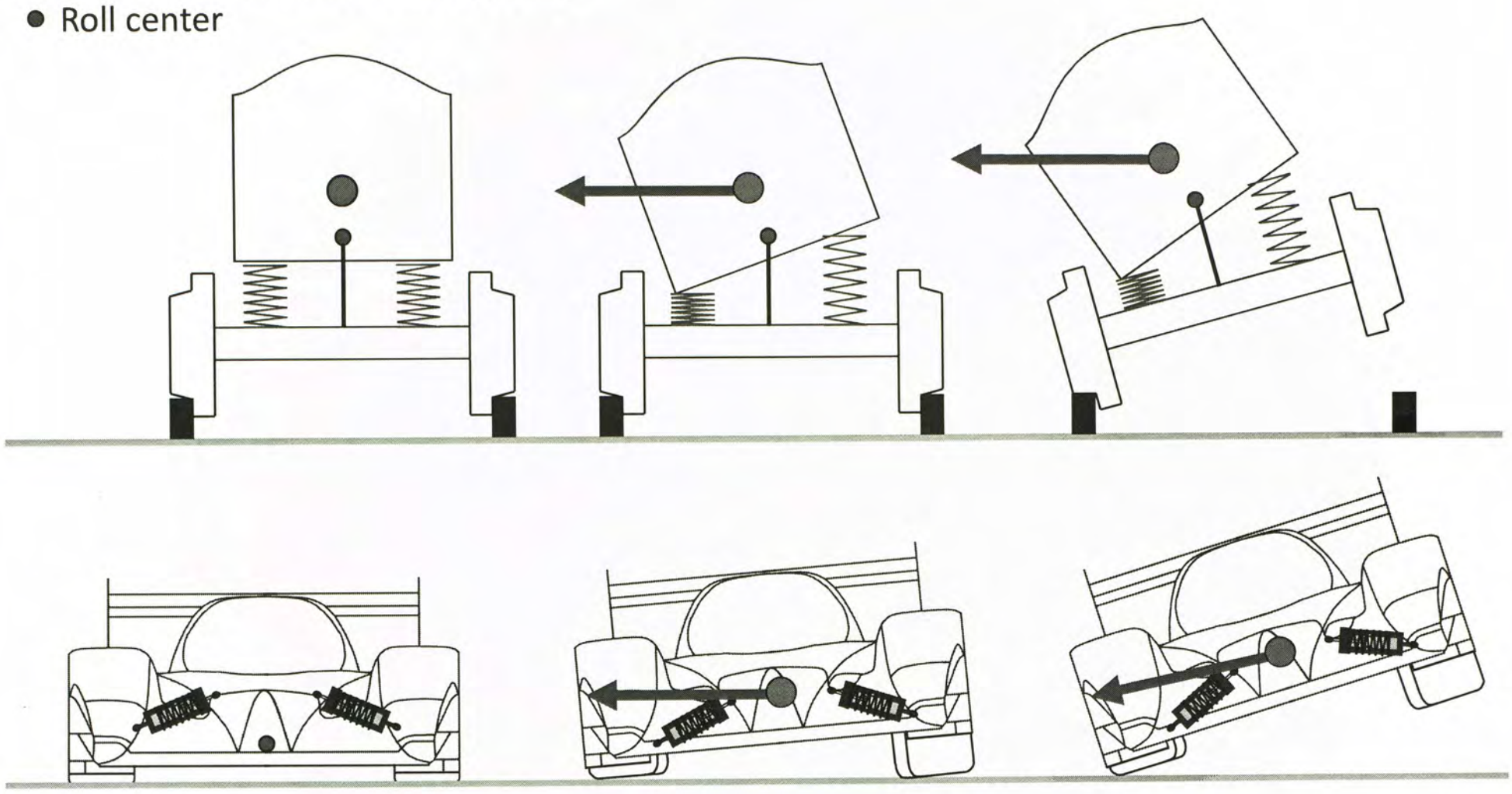
Roll Center Location

- Suspended mass center of gravity
- Roll center

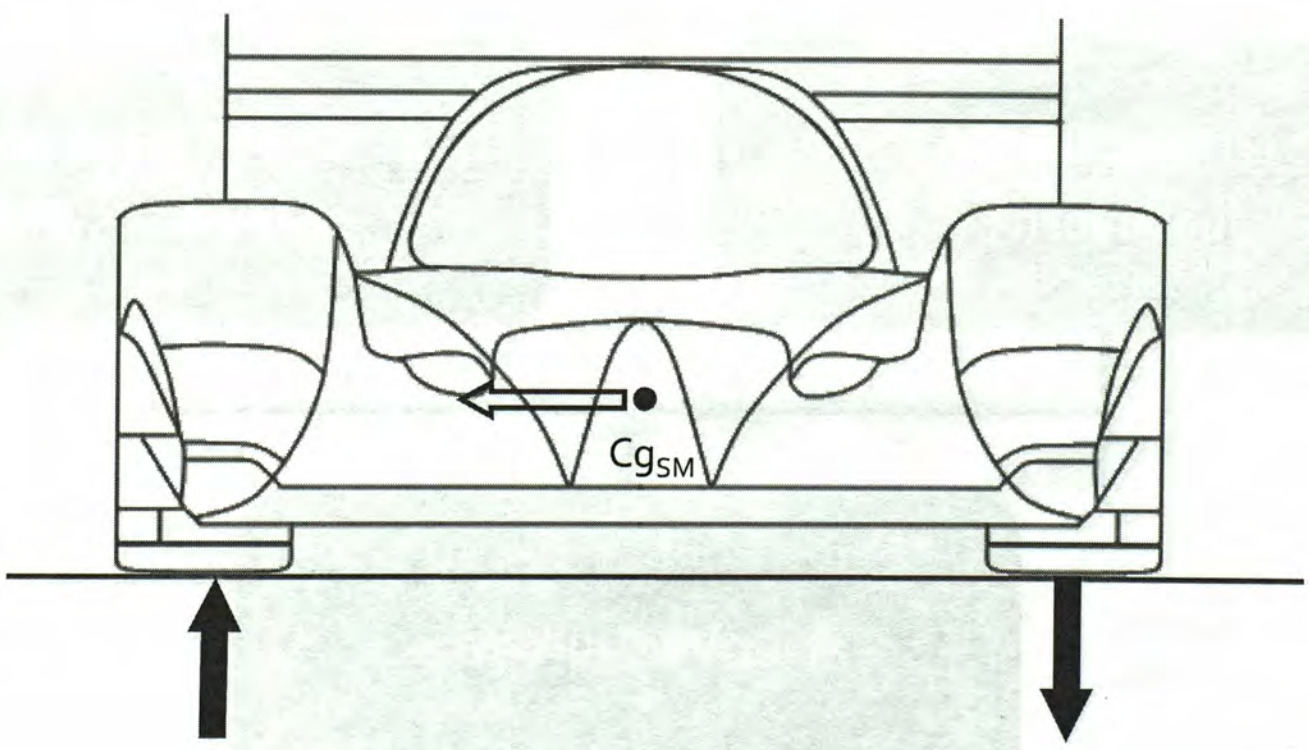


Roll Center Location

- Suspended mass center of gravity
- Roll center



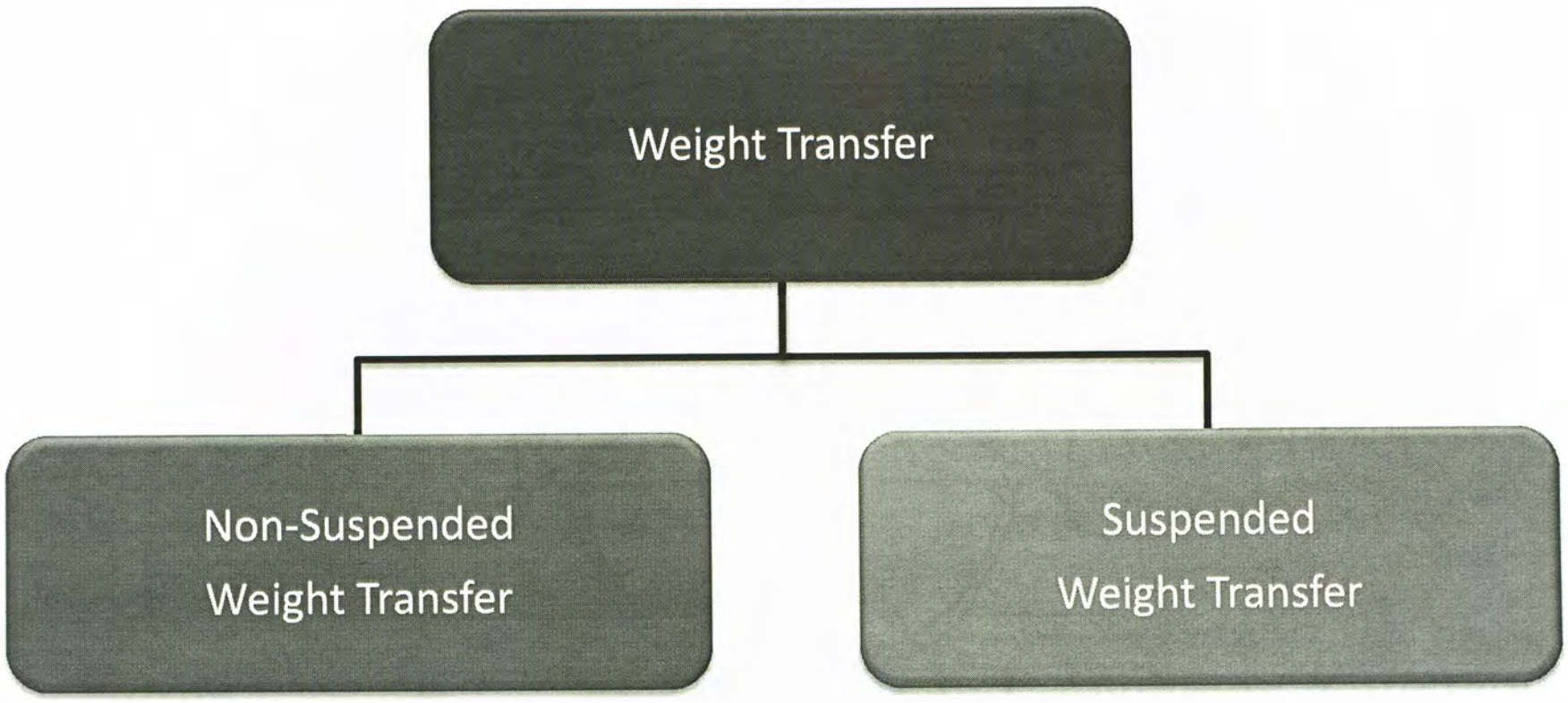
Lateral Weight Transfer Basics



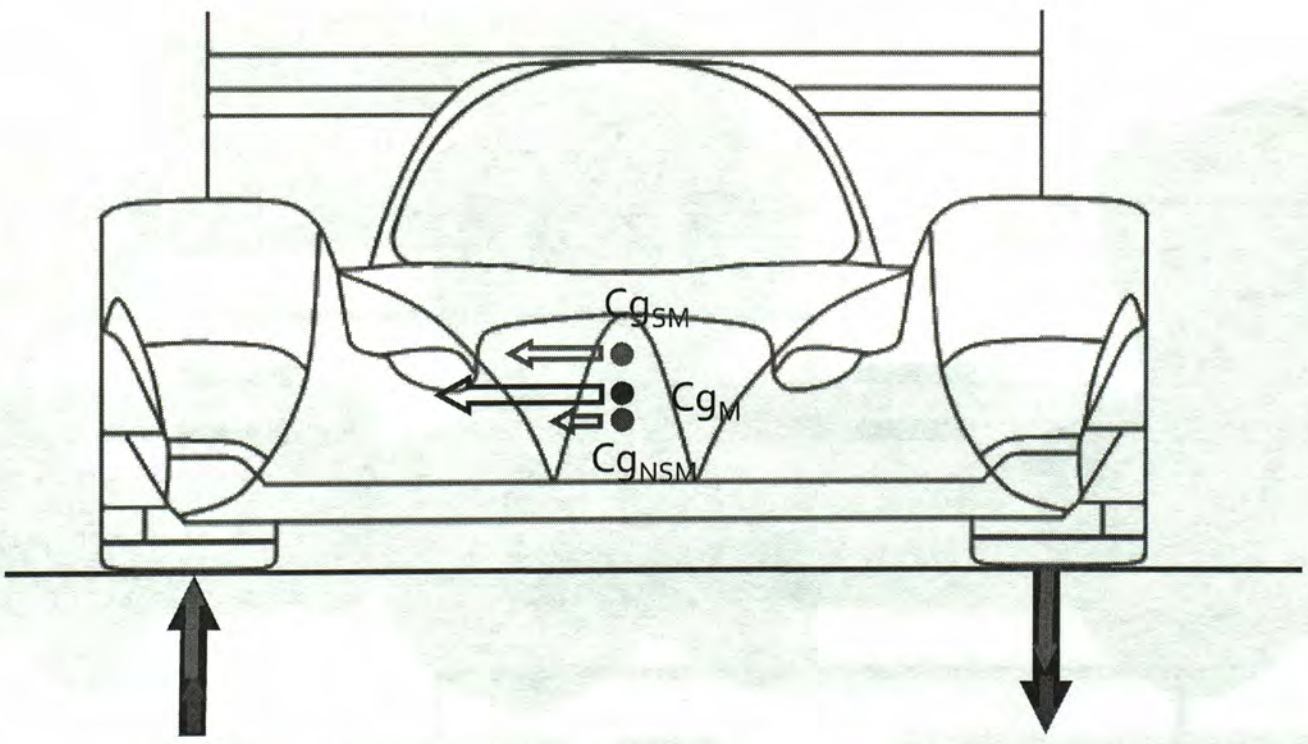
$$Lateral\ force = mass\ \times lateral\ acceleration$$

$$Lateral\ weight\ transfer = lateral\ force\ \times Cg\ height/track$$

Lateral Weight Transfer Basics



Lateral Weight Transfer Basics

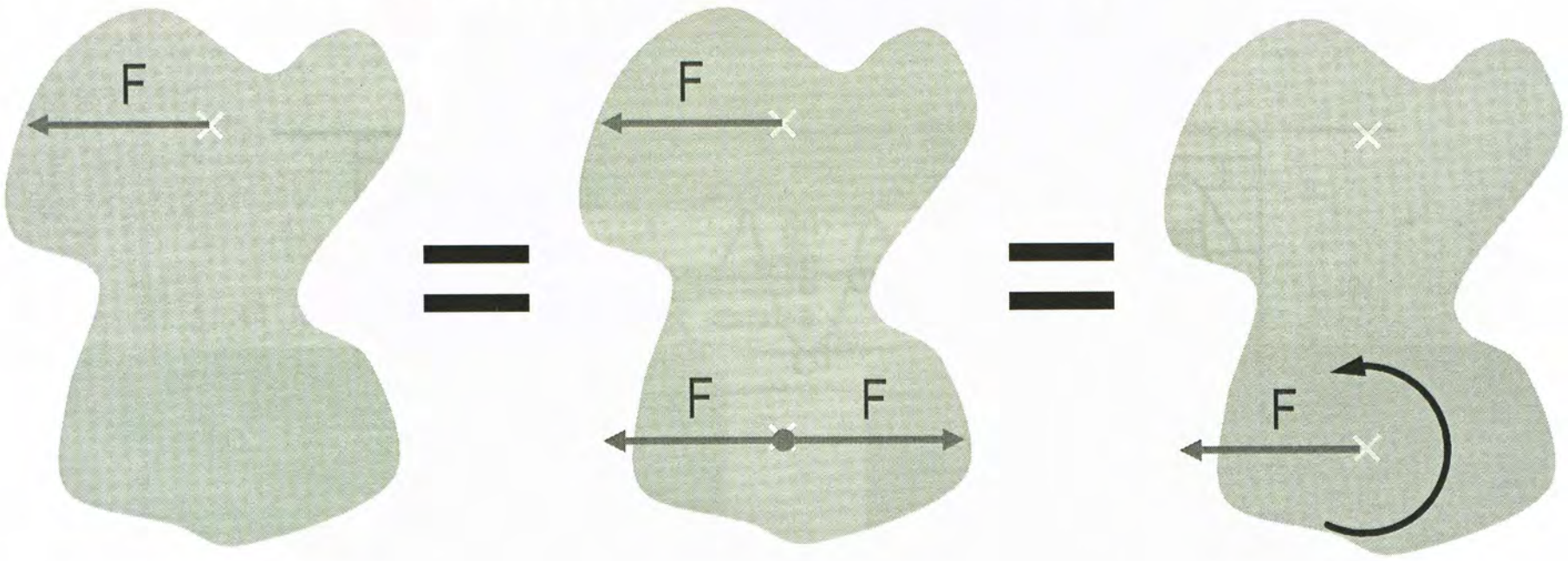


$$Lateral\ S\ WT = SM\ \times lat.acc. \times Cg_{SM\ height}/track$$

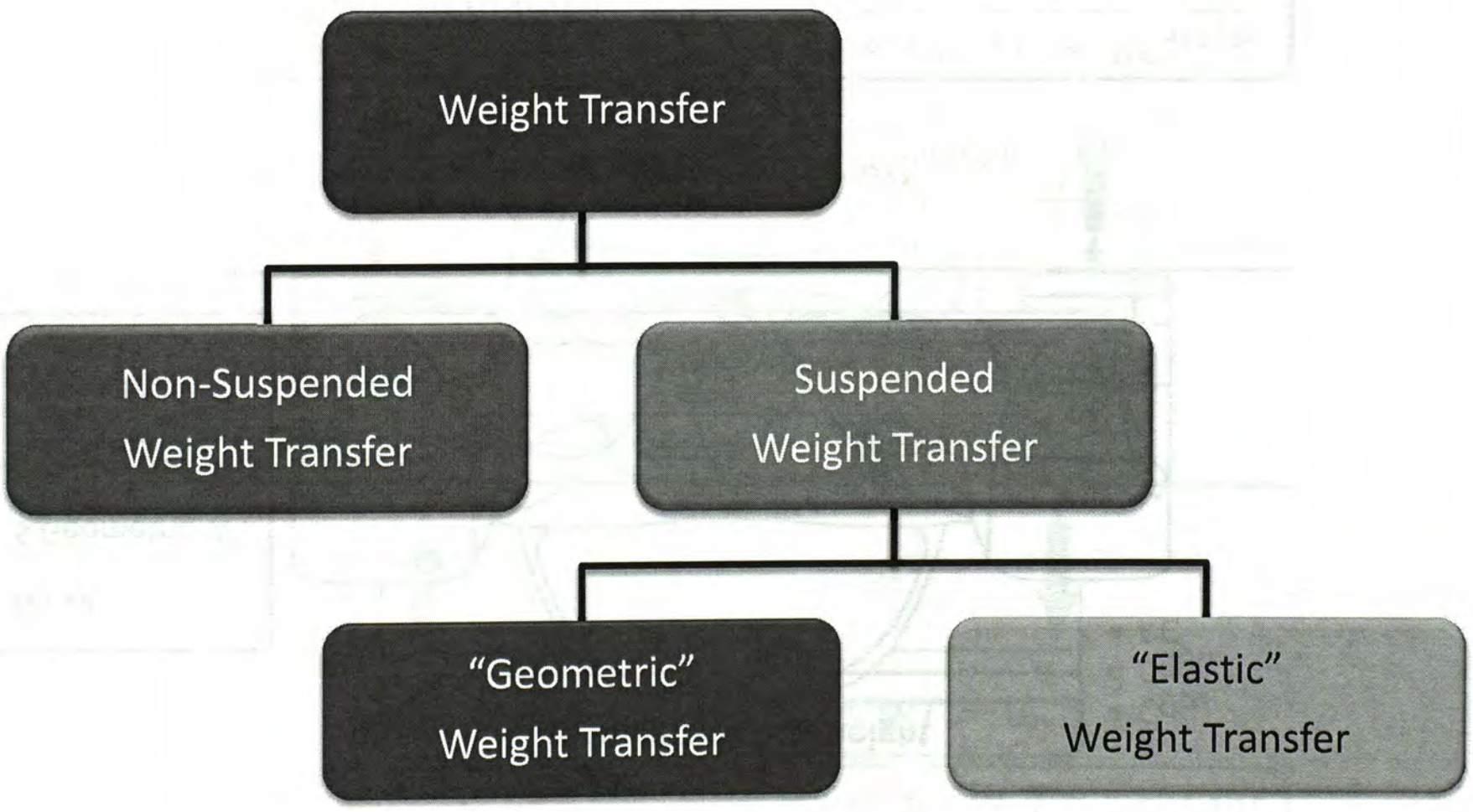
$$Lateral\ NS\ WT = NSM\ \times lat.acc. \times Cg_{NSM\ height}/track$$

Force and Moment

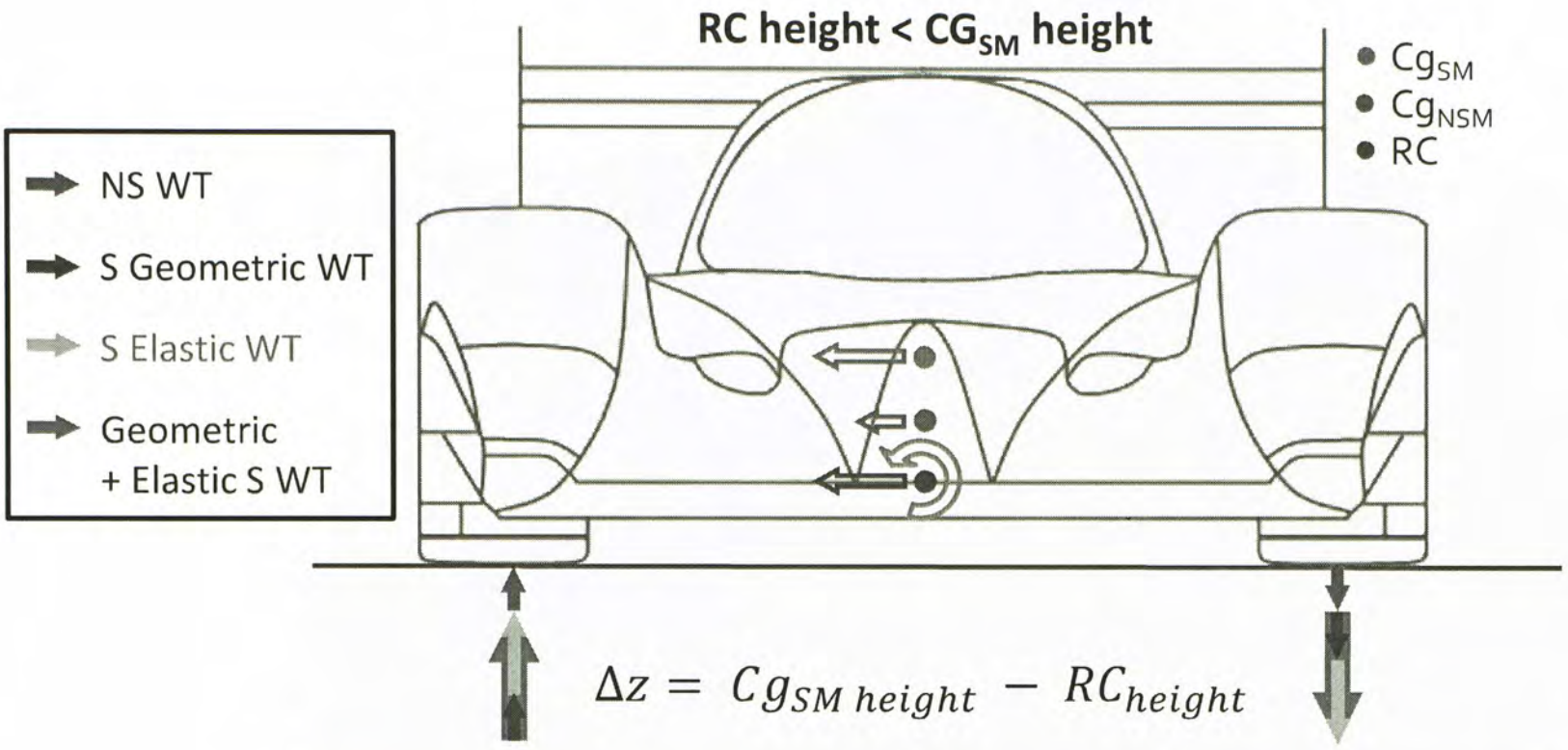
Force applied on point 1 = Force applied on point 2 + Torque



Lateral Weight Transfer Basics



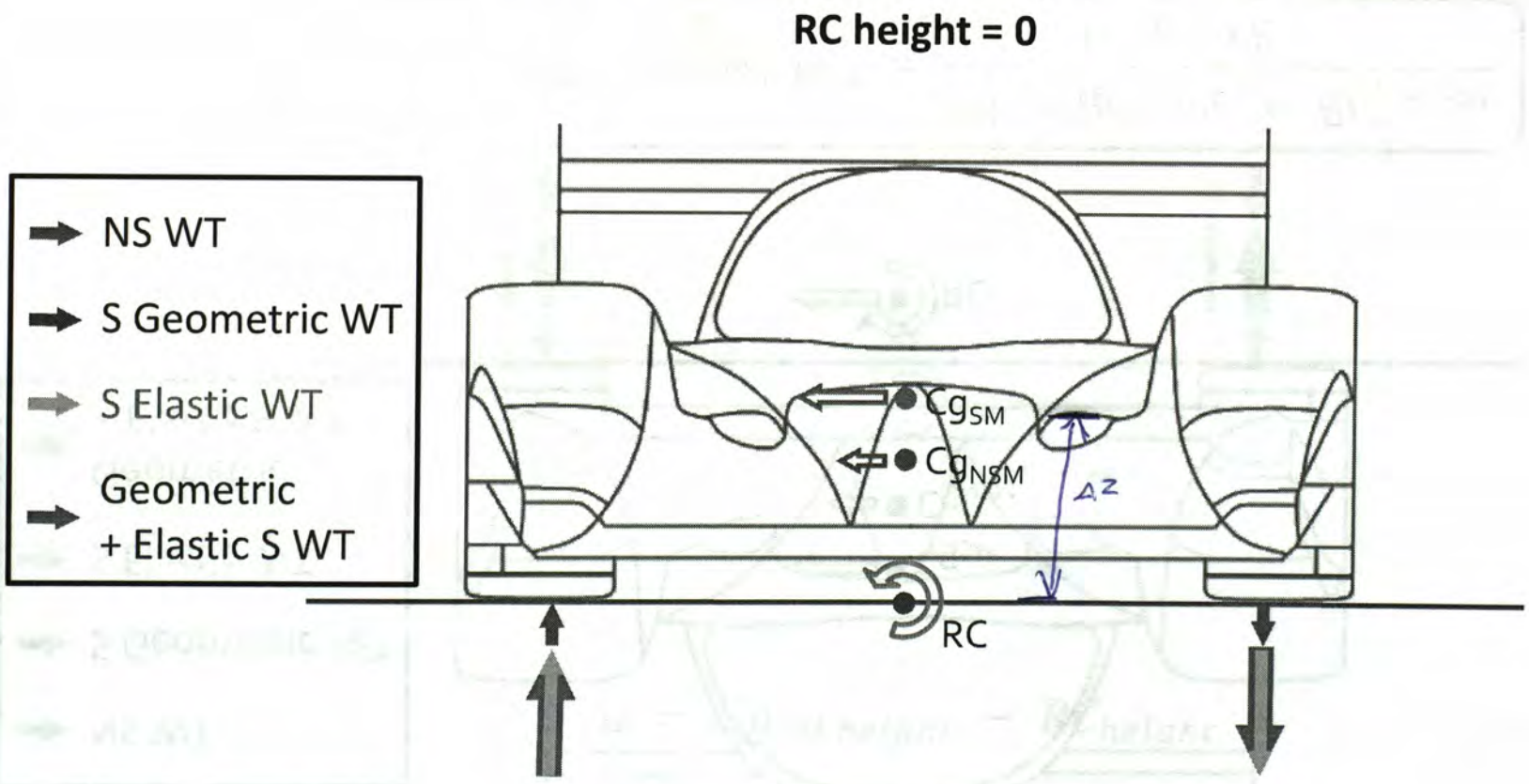
Lateral Weight Transfer



$$Lateral\ S\ "Geometric"\ WT = \frac{SM\ \times lat.acc. \times\ RC_{height}}{track}$$

$$Lateral\ S\ "Elastic"\ WT = \frac{SM\ \times lat.acc. \times\ \Delta z}{track}$$

Lateral Weight Transfer

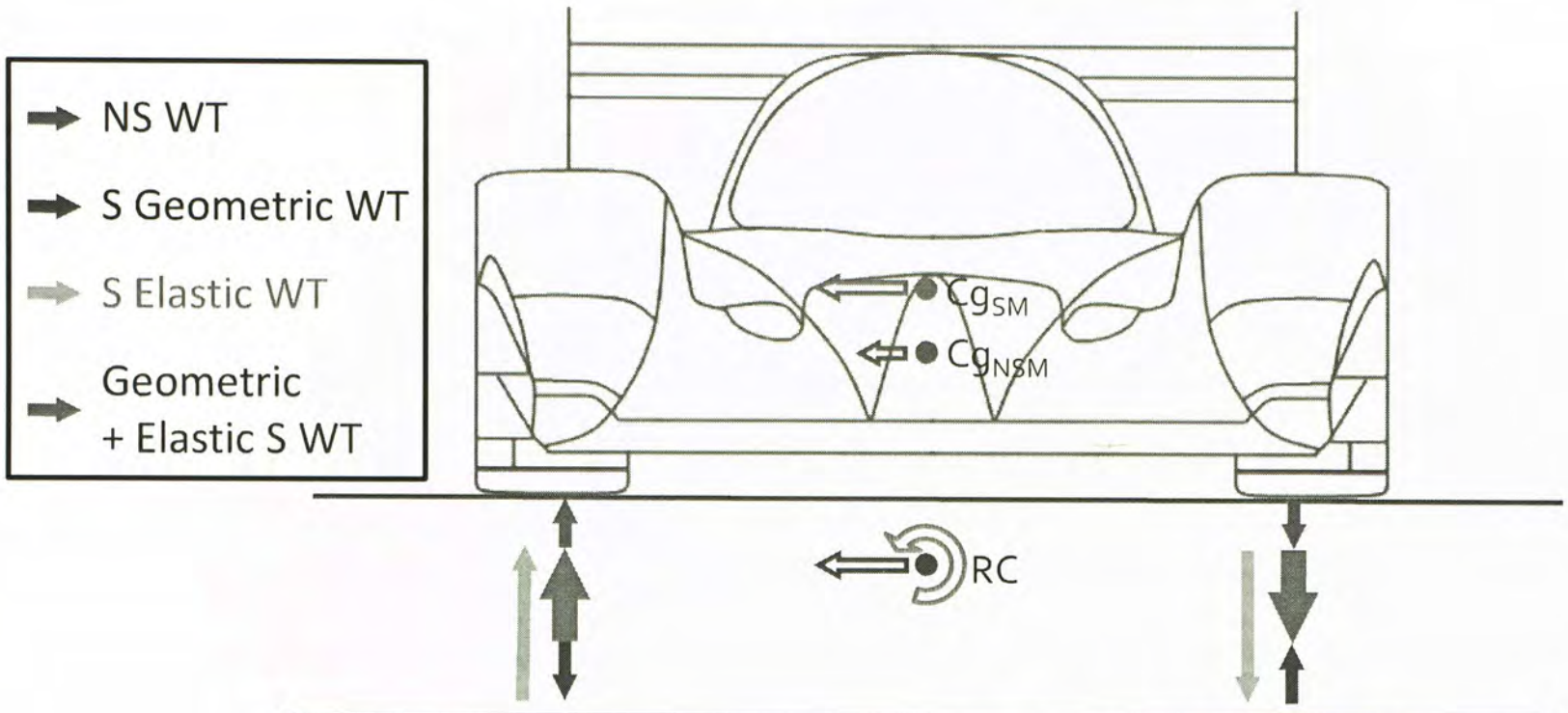


$$Lateral\ S\ "Geometric"\ WT = 0$$

$$Lateral\ S\ "Elastic"\ WT = \frac{SM\ \times lat.acc. \times\ Cg_{SM} height}{track}$$

Lateral Weight Transfer

RC height under the ground

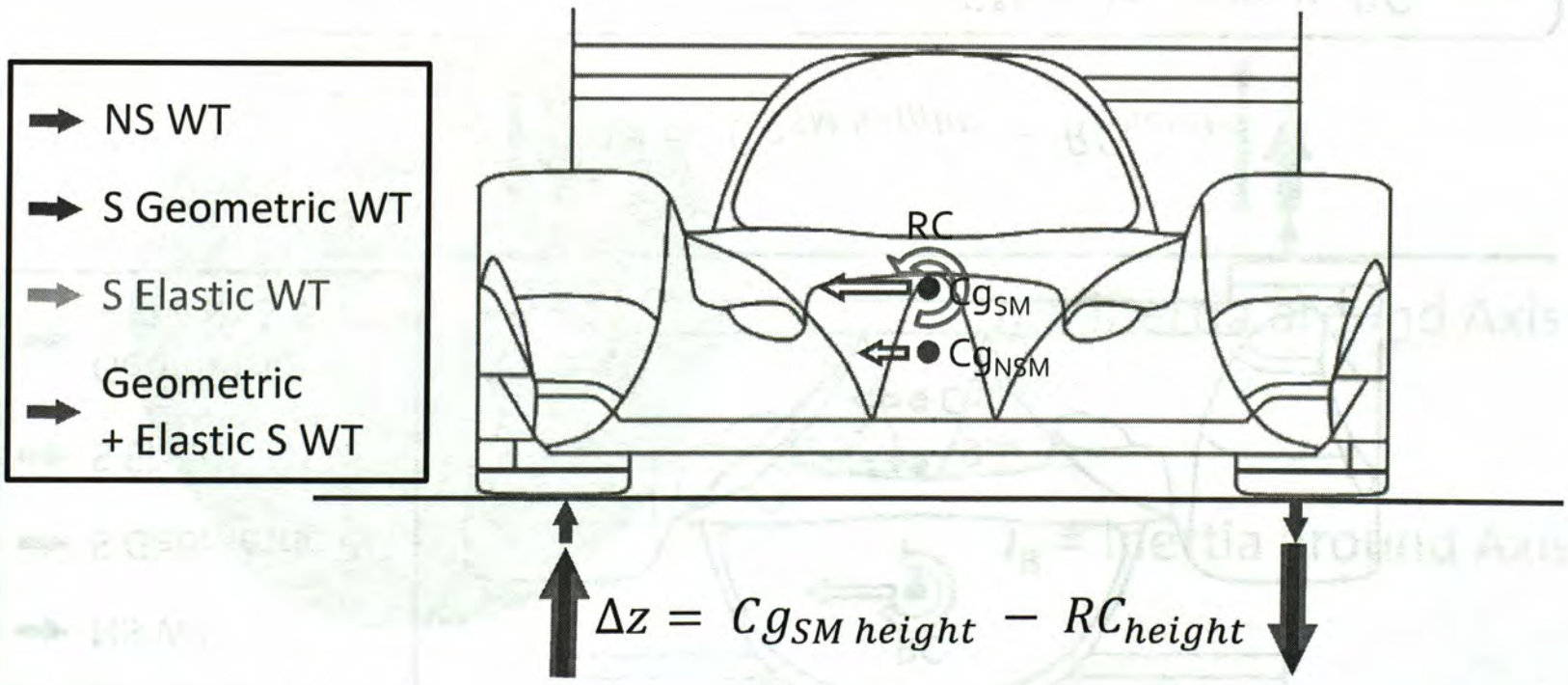


$$Lateral\ S\ "Geometric"\ WT = \frac{SM\ \times lat.acc. \times\ RC_{height}}{track}$$

$$Lateral\ S\ "Elastic"\ WT = \frac{SM\ \times lat.acc. \times\ \Delta z}{track}$$

Lateral Weight Transfer

RC height = CG_{SM} height

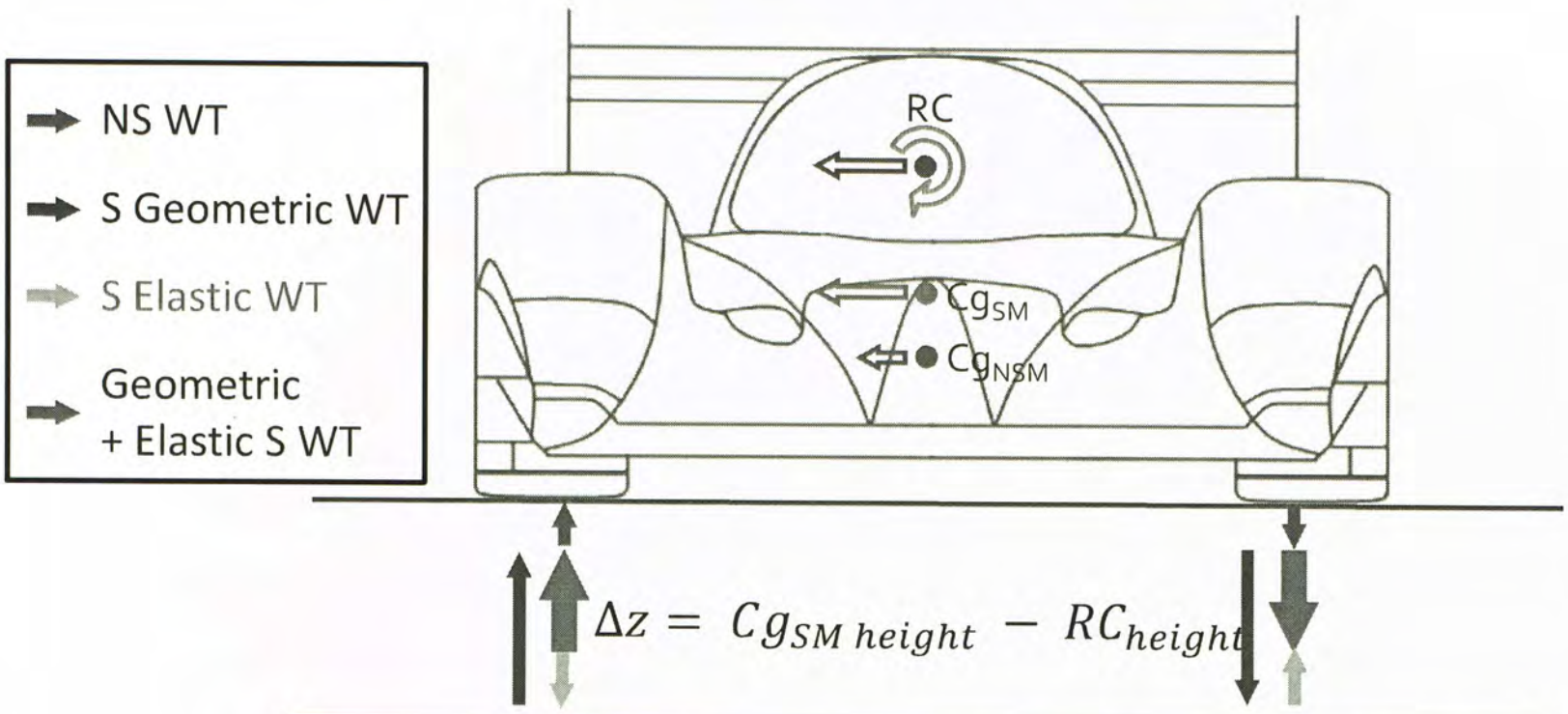


$$Lateral\ S\ "Geometric"\ WT = \frac{SM\ \times lat.acc. \times\ RC_{height}}{track}$$

$$Lateral\ S\ "Elastic"\ WT = 0$$

Lateral Weight Transfer

RC height > CG_{SW} height

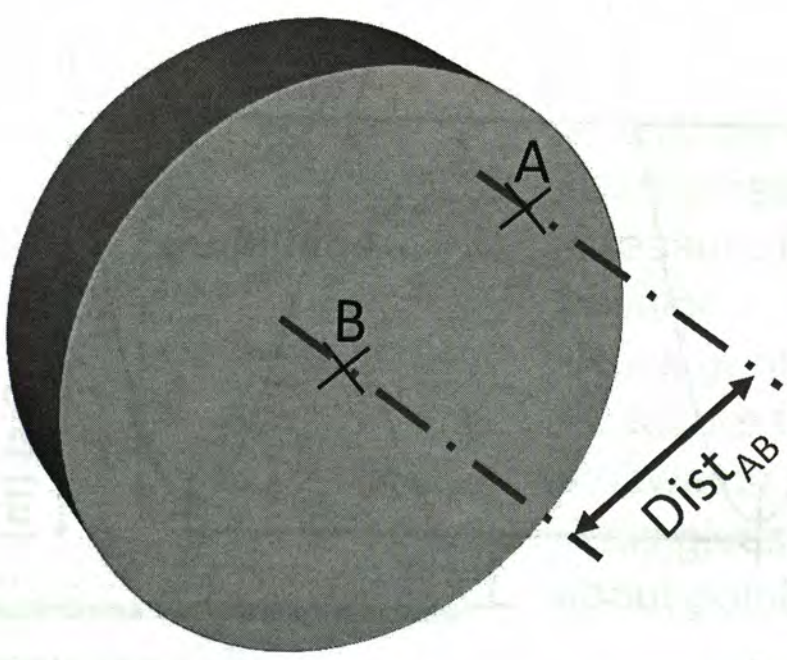


$$Lateral\ S\ "Geometric"\ WT = \frac{SM \times lat.acc. \times RC_{height}}{track}$$ → Wishbone

$$Lateral\ S\ "Elastic"\ WT = \frac{SM \times lat.acc. \times \Delta z}{track}$$ → Damper / Spring

Inertia Measurement

Huygens (or Parallel Axis) Theorem



J_A = Inertia around Axis A

J_B = Inertia around Axis B

$$J_A = J_B + \left(m \times dist_{AB}^{\ 2}\right)$$

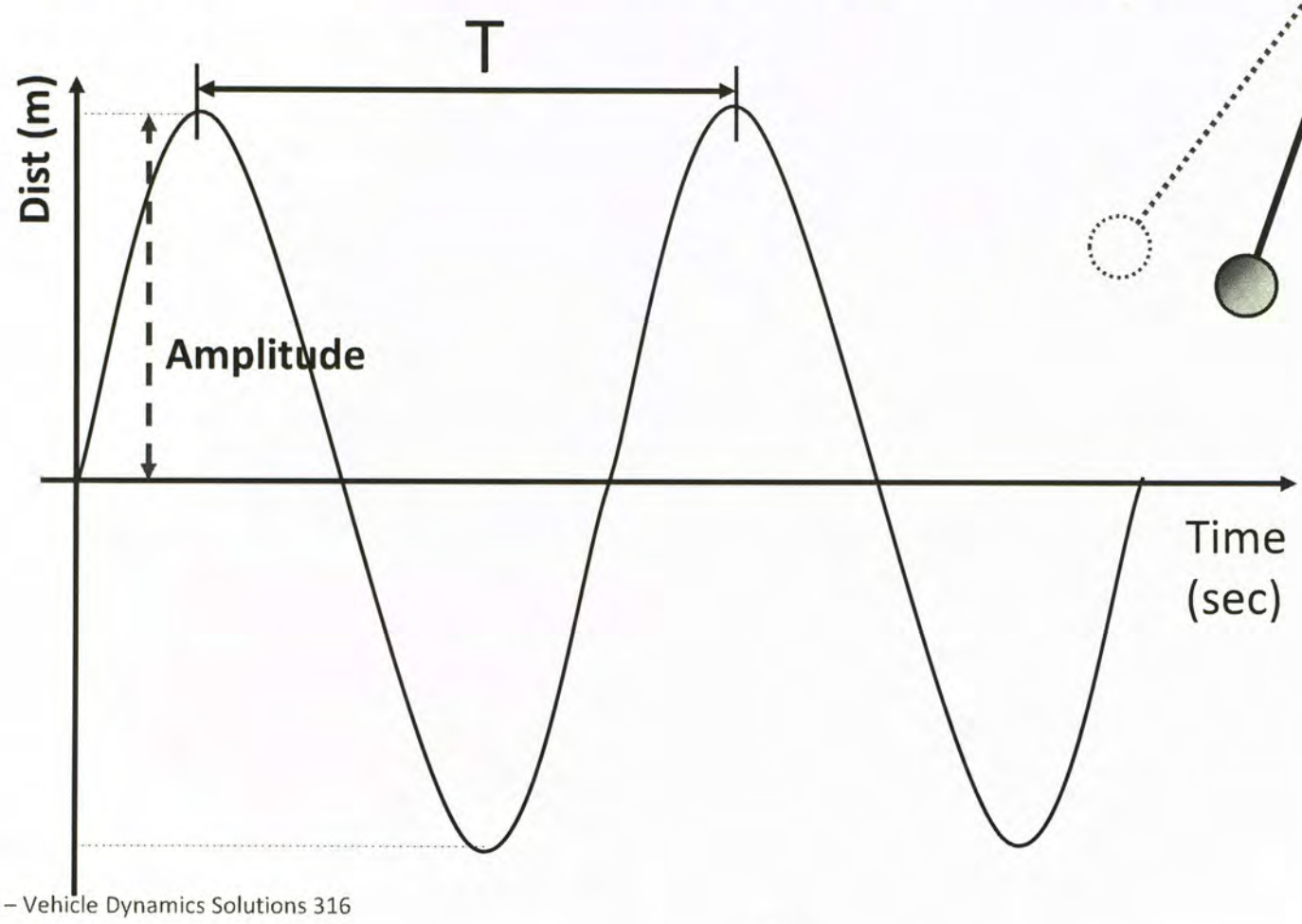
Inertia Measurement

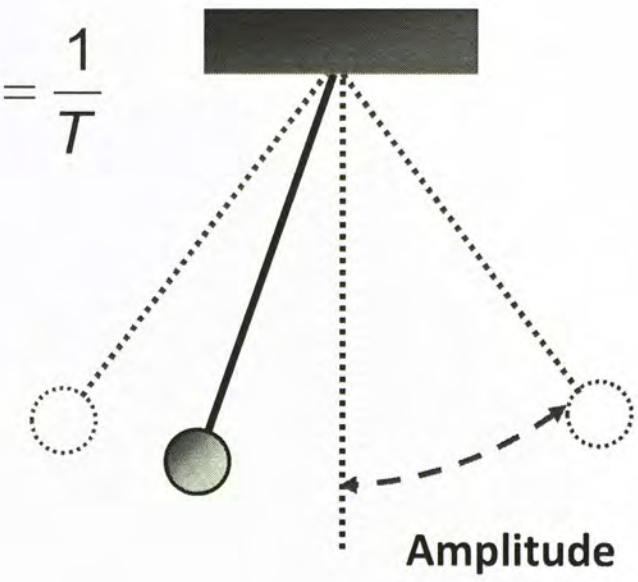
Frequency and Period

- **Frequency: f (Hz)**
- **Period:T (sec)**

$$frequency = \frac{1}{Period}$$

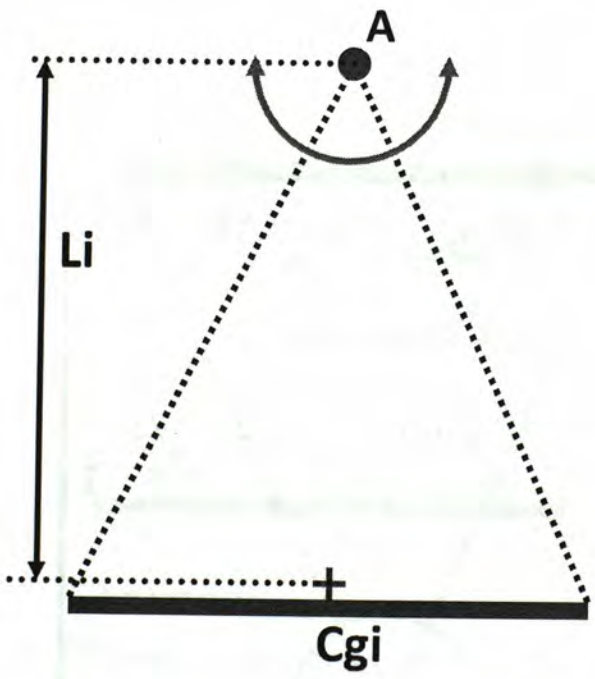
$$f = \frac{1}{T}$$





Inertia Measurement

How to Measure Inertia: 1st way



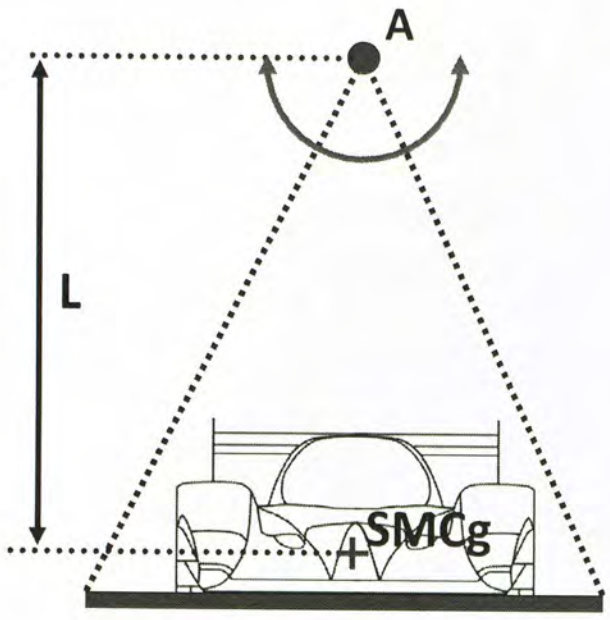
- Install a platform rotating around an axle.The platform should be fixed using inelastic cable, or chain.
- Use a bearing at point A in order to reduce friction.
- Measure the mass m_i of your installation, and its Cgi position.
- Rock gently your installation.
- Measure the period T_i of the movement with a stopwatch.
- This gives us the moment of inertia of the platform about point A.

<u>J_i: Inertia of the installation around the axis passing through the center of gravity Cgi</u>

$$J_i = m_i.g.L_i.\left(\frac{T_i^2}{(2.\pi)^2} - \frac{L_i}{g}\right)$$

Inertia Measurement

How to Measure Inertia: 1st way



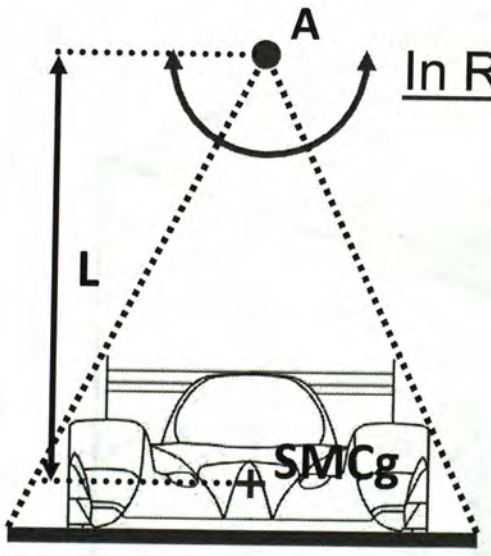
- Install the suspended mass on your platform.
- Measure the distance L between the Axis A and the center of gravity of your car.
- Induce a little movement of rotation to your installation.
- Measure the period T of the movement.
- From this we can compute the moment of inertia of the suspended mass CG about the roll axis.

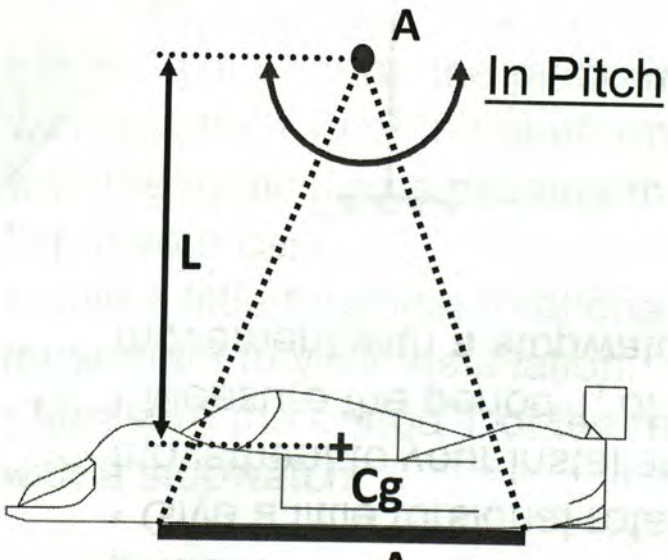
J_{Car}: Inertia of the Car around an axis passing through its center of gravity

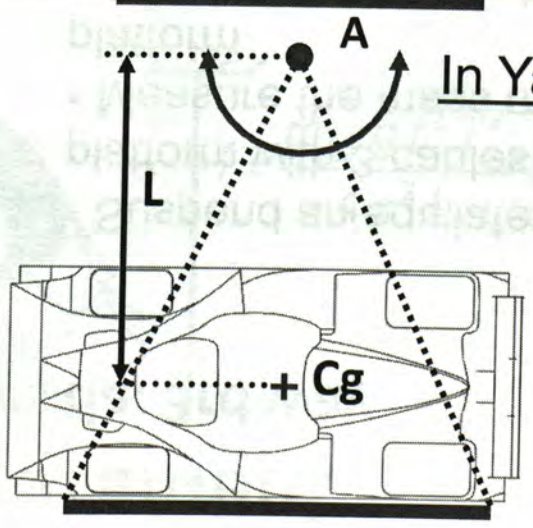
$$J_{Car} = m_{car}.g.L.\left(\frac{T^2}{4.\pi^2} - \frac{L}{g}\right) + \frac{m_i.g.L_i}{4.\pi^2}\left(T^2 - T_i^2\right)$$

Inertia Measurement

How to Measure Inertia: 1st way



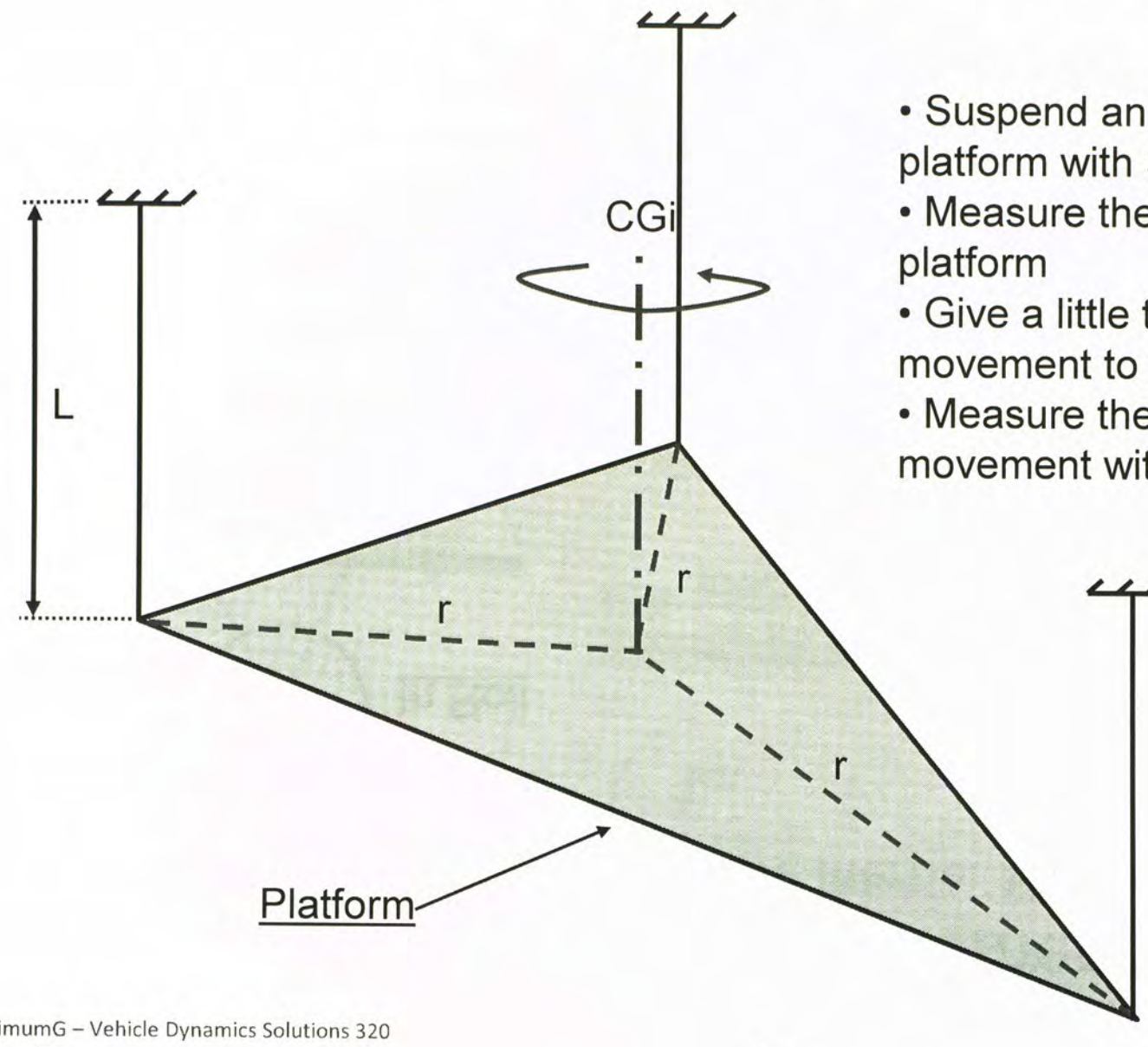




Comment:
The whole mass of the car is only taken into consideration for the computation of the yaw Inertia.
For roll and pitch inertia, the suspended mass is only taken into account.

Inertia Measurement

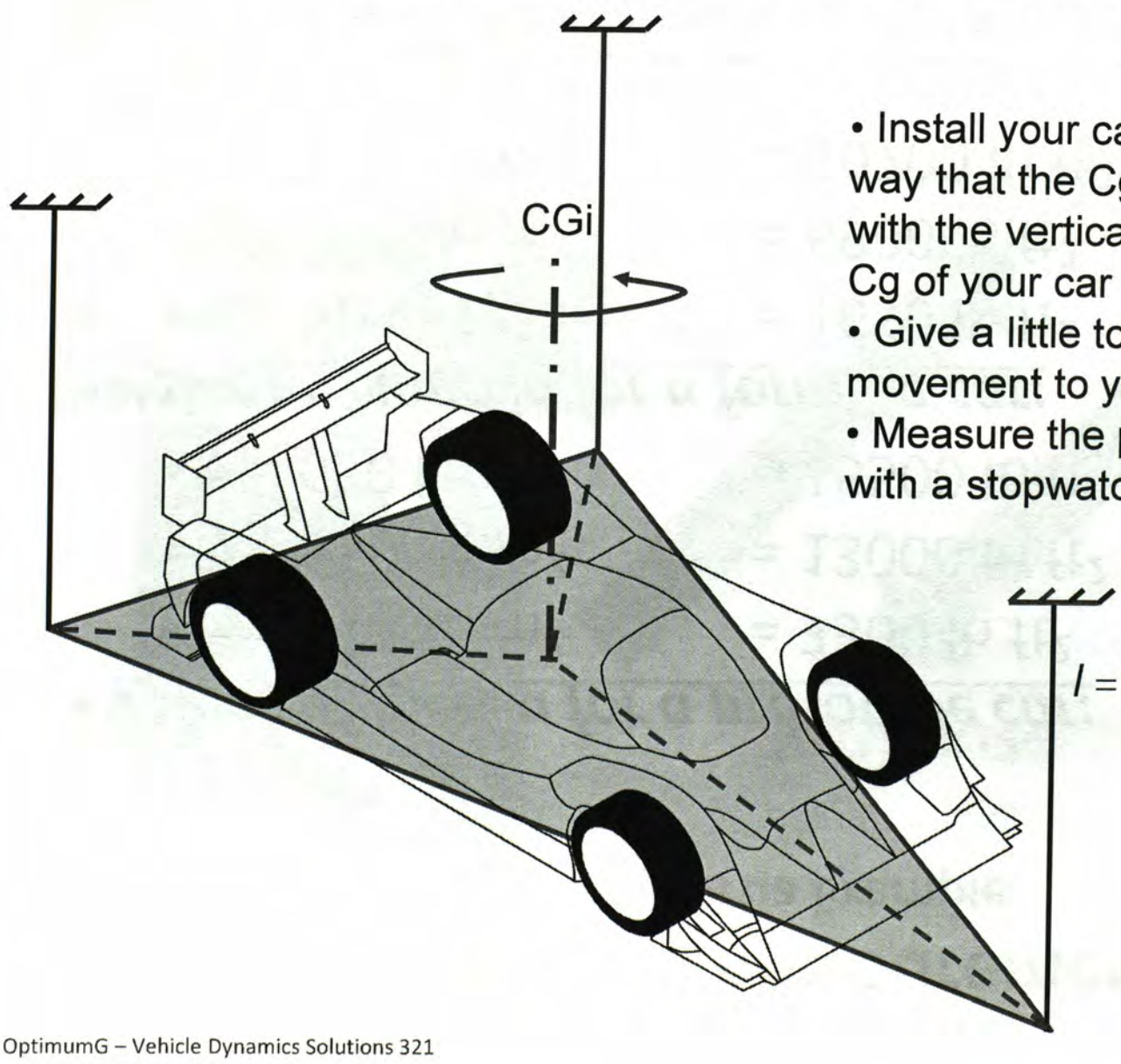
How to Measure Inertia: 3rd way



- Suspend an equilateral triangular platform with 3 cables,
- Measure the mass mi of your platform
- Give a little torsional rotational movement to your installation.
- Measure the period T_i of the movement with a stopwatch.

Inertia Measurement

How to Measure Inertia: 3rd way



- Install your car on the platform in such a way that the Cgi of the platform coincides with the vertical axis passing through the Cg of your car
- Give a little torsional rotational movement to your installation.
- Measure the period T of the movement with a stopwatch.

$$I = \frac{m_{car}.g.r^2.T^2}{4.\pi^2.L} + \frac{m_i.g.r^2}{4.\pi^2.L}.\left(T^2 - T_i^2\right)$$

Inertia Measurement

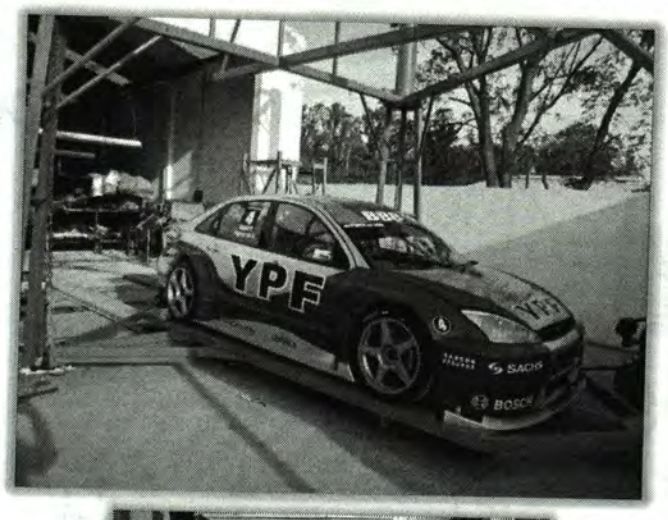
Inertia Example

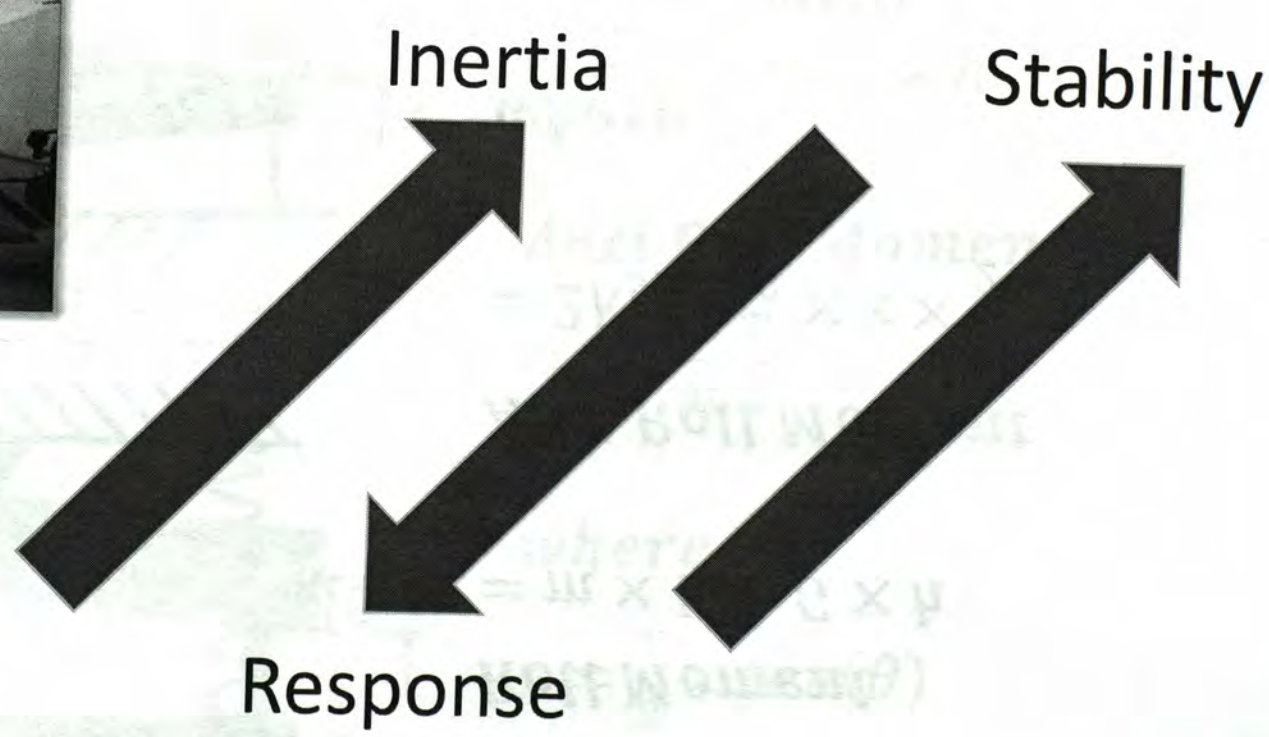
- ***Values of Inertia for a prototype car:***
  - Ix = 140 kg.m^2 = 3300 lb.ft^2
  - Iy = 550 kg.m^2 = 13000 lb.ft^2
  - Iz = 650 kg.m^2 = 15500 lb.ft^2
- ***Values of Inertia for a formula car:***
  - Ix = 70 kg.m^2 = 1650 lb.ft^2
  - Iy = 290 kg.m^2 = 6900 lb.ft^2
  - Iz = 450 kg.m^2 = 10700 lb.ft^2

Check out the NHTSA (National Highway Traffic Safety Administration) website for more examples:
http://www-nrd.nhtsa.dot.gov/vrtc/ca/nhtsa_inertia_database_metric.pdf

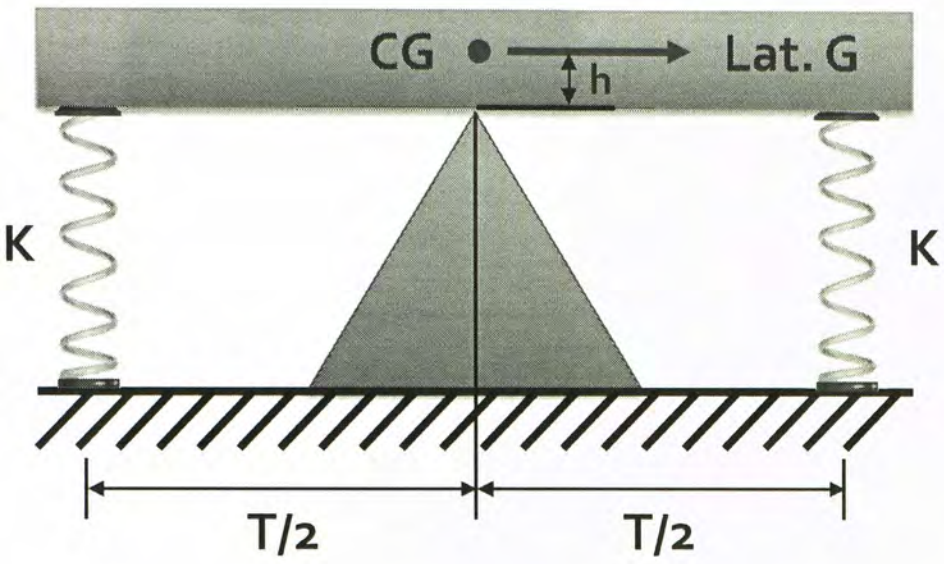
Influence of Inertia and Wheelbase

Response and Stability vs Inertia





Transient Lateral Weight Transfer - Springs

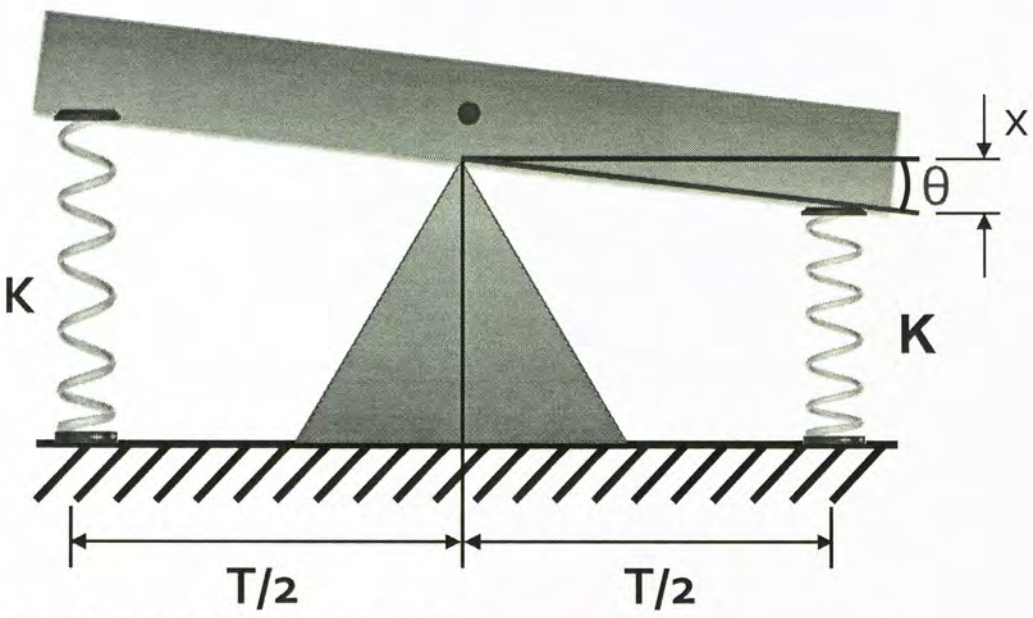


Roll Moment

$$= m \times Lat.G \times h$$

Anti Roll Moment

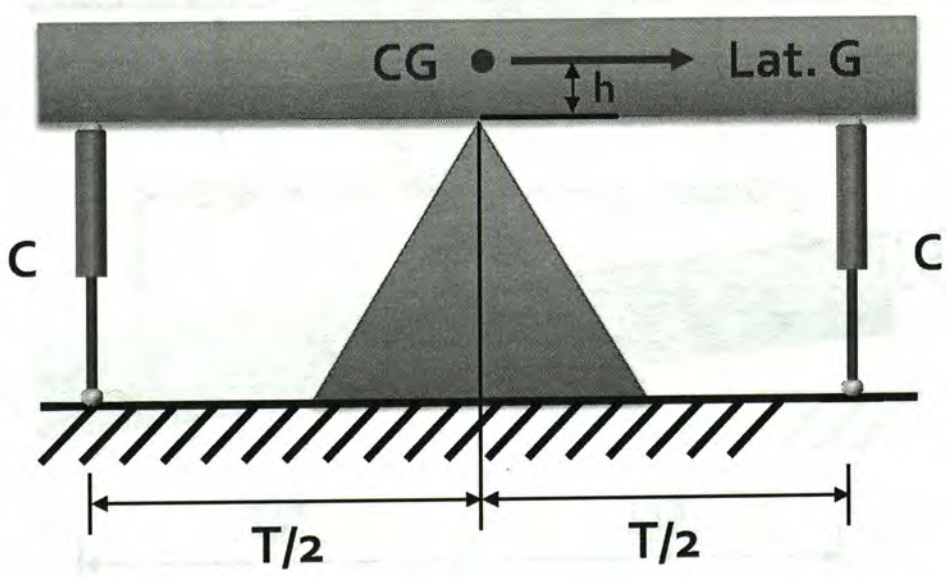
$$= 2K_{Spring} \times x \times \frac{T}{2}$$



where

$$x = \frac{T}{2} \times tan(\theta)$$

Transient Lateral Weight Transfer - Dampers

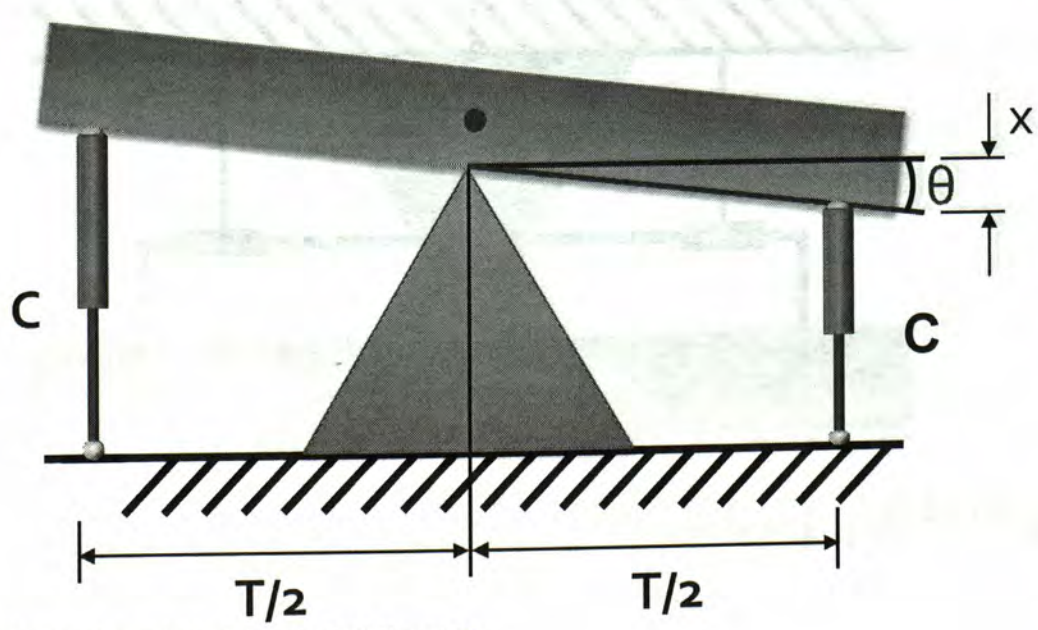


Roll Moment

$$= m \times Lat.G \times h$$

Anti Roll Moment

$$= 2C\frac{dx}{dt}\frac{T}{2}$$

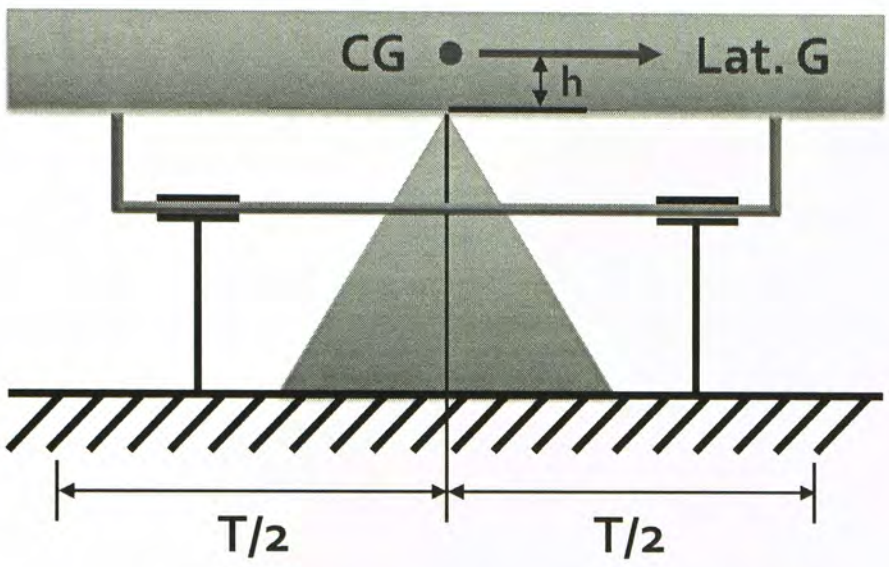


where

$$x = \frac{T}{2} \times tan(\theta)$$

$$\frac{dx}{dt} = \frac{T}{2} \times \dot{\theta}$$

Transient Lateral Weight Transfer - ARB

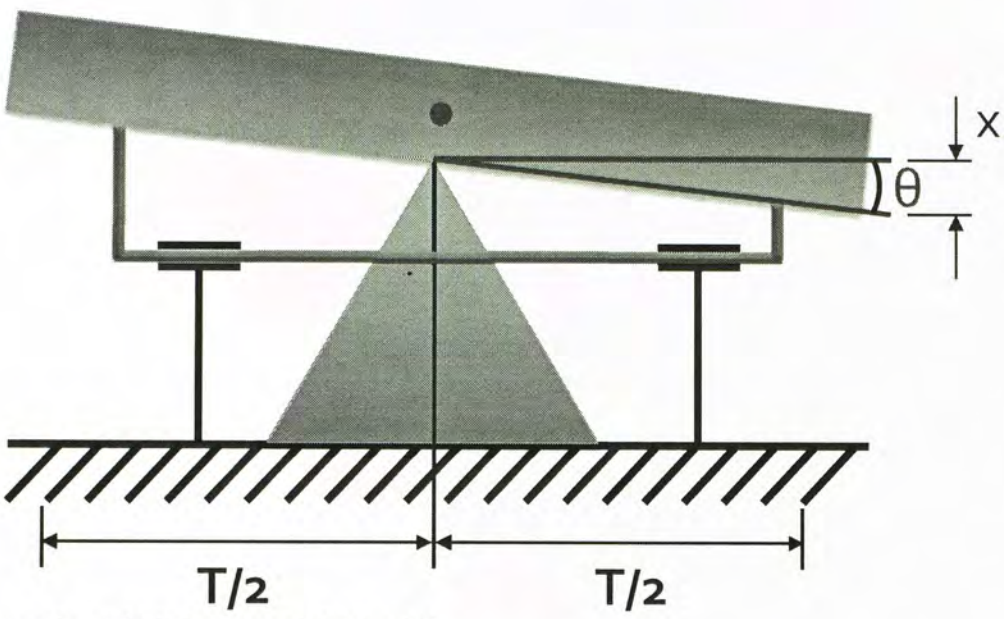


Roll Moment

$$= m \times Lat.G \times h$$

Anti Roll Moment

$$= 2 \times K_{ARB}{}^{*} \times x \times \frac{T}{2}$$

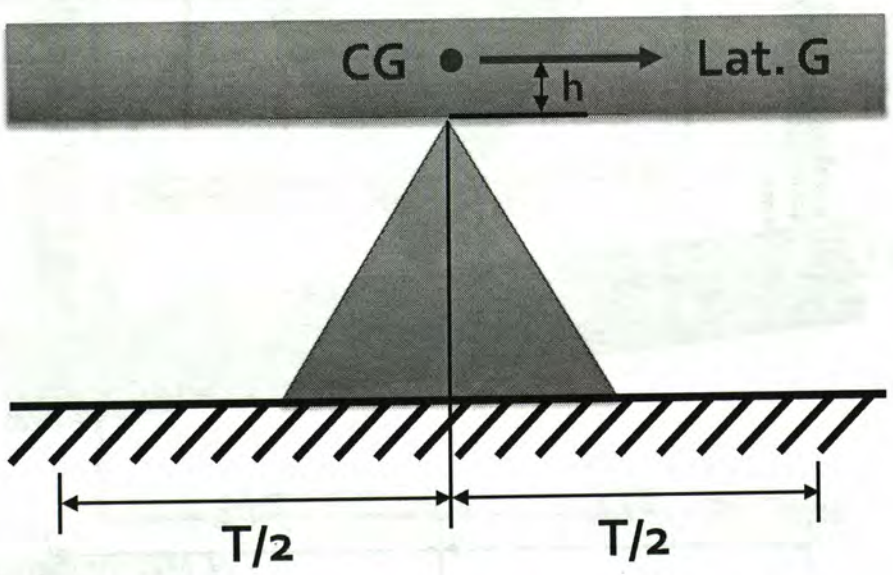


where

$$x = \frac{T}{2} \times tan(\theta)$$

* : *[N/mm]*

Transient Lateral Weight Transfer - Inertia



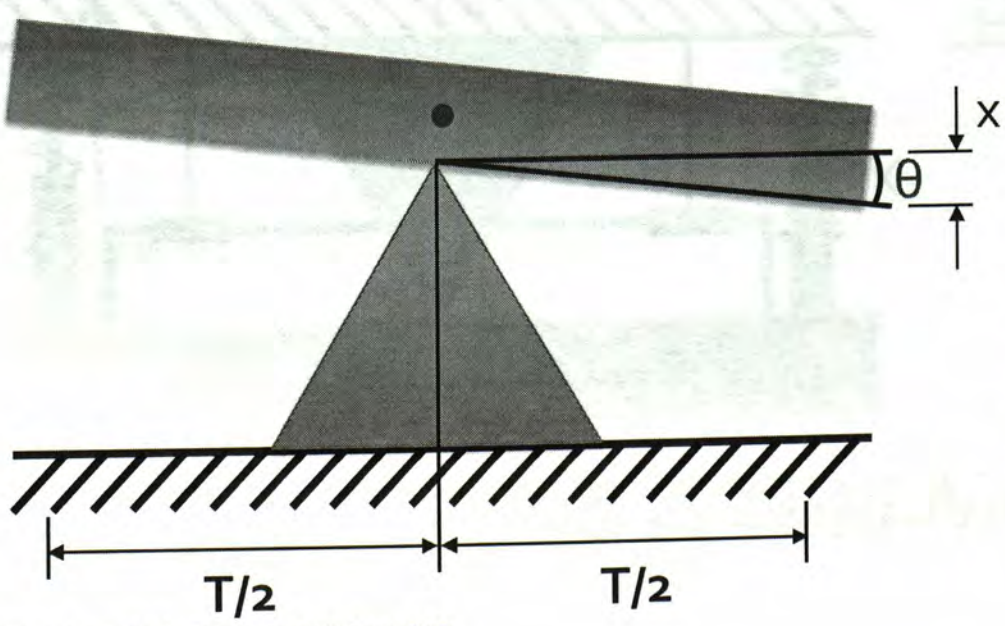
Roll Moment

$$= m \times Lat.G \times h$$

Anti Roll Moment

$$= I\ddot{\theta}$$

If I not *constant then* $\frac{d(I\dot{\theta})}{dt} = \dot{I}\dot{\theta} + I\ddot{\theta}$

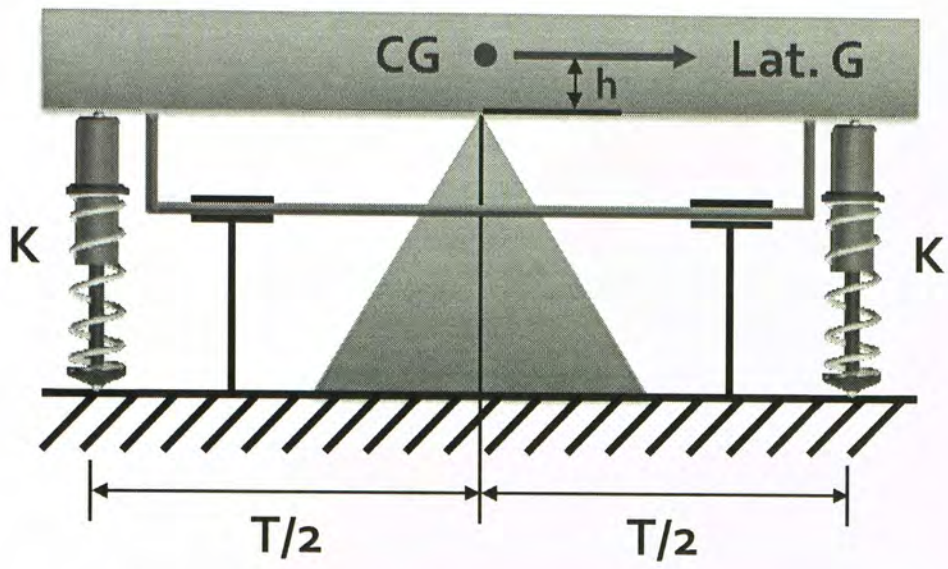


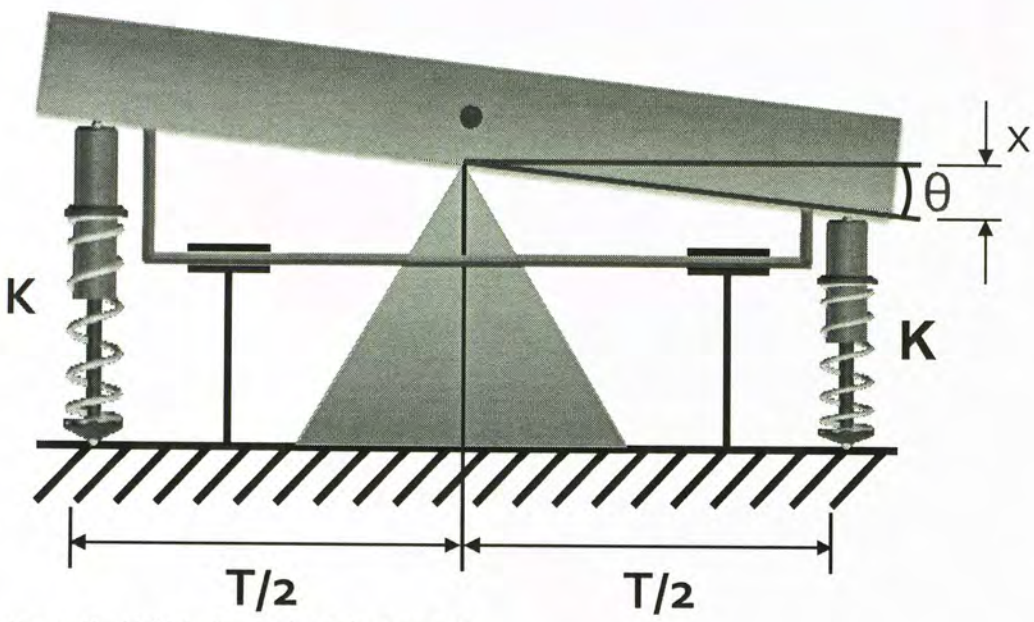
where

$$x = \frac{T}{2} \times tan(\theta)$$

$$\frac{dx}{dt} = \frac{T}{2} \times \dot{\theta}$$

Transient Lateral Weight Transfer





Roll Moment

$$= m \times Lat.G \times h$$

Anti Roll Moment

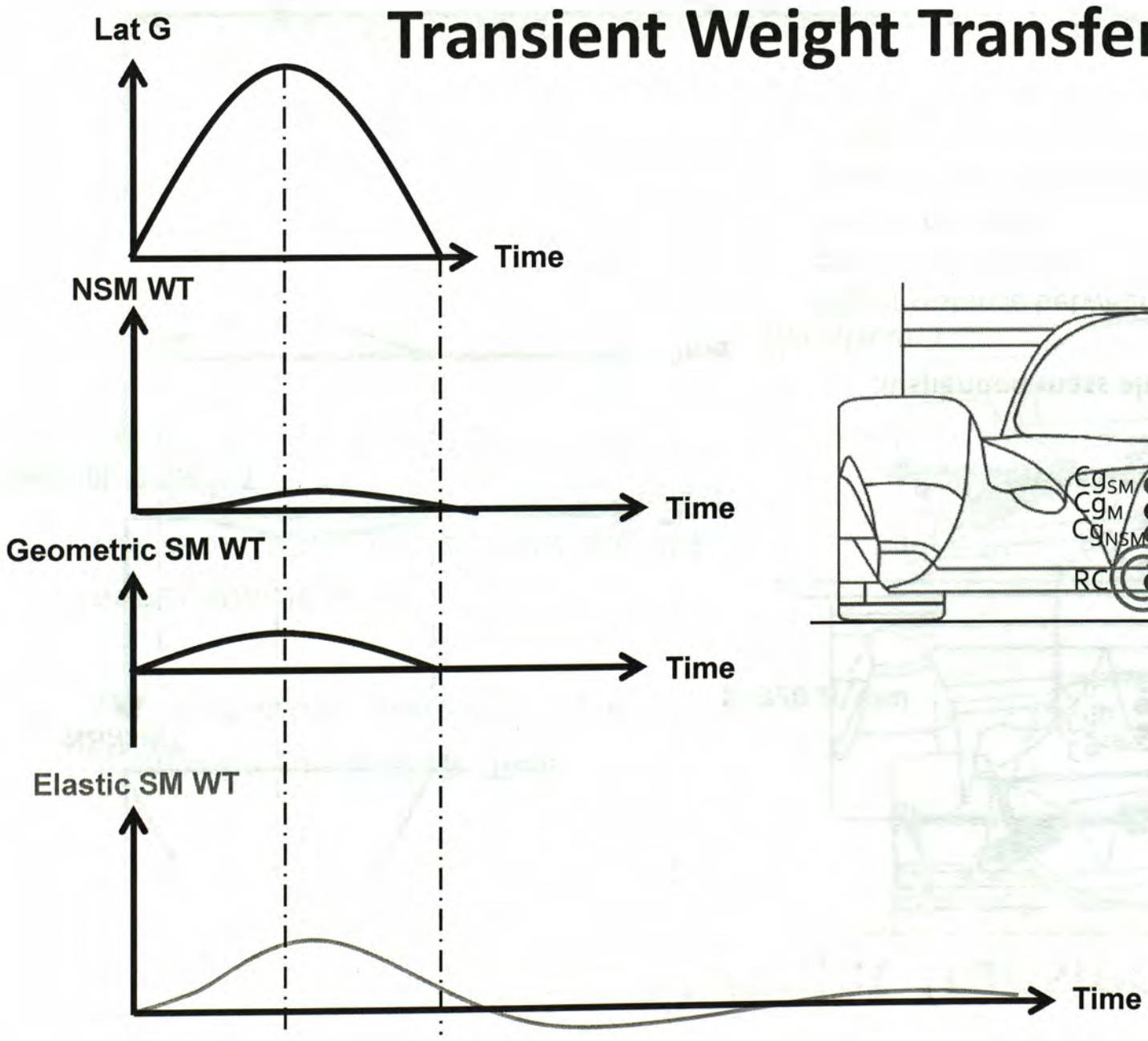
$$= 2(K_{Spring} + K_{ARB}) \times x \times \frac{T}{2} + 2C\frac{dx}{dt}\frac{T}{2} + I\ddot{\theta}$$

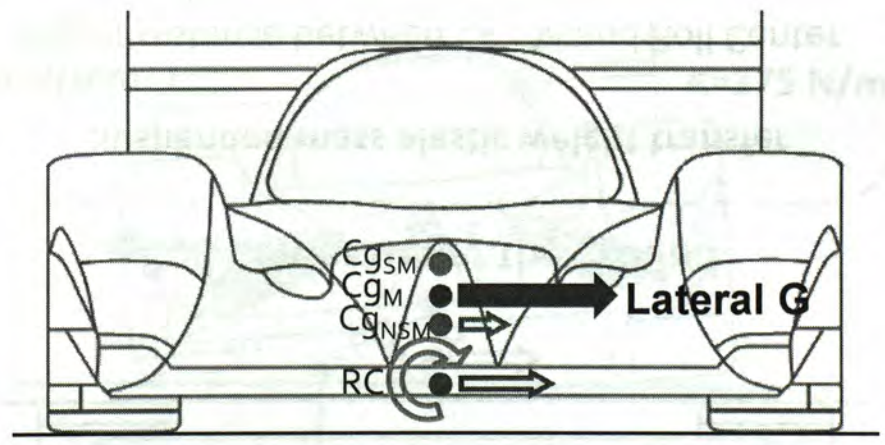
where

$$x = \frac{T}{2} \times tan(\theta)$$

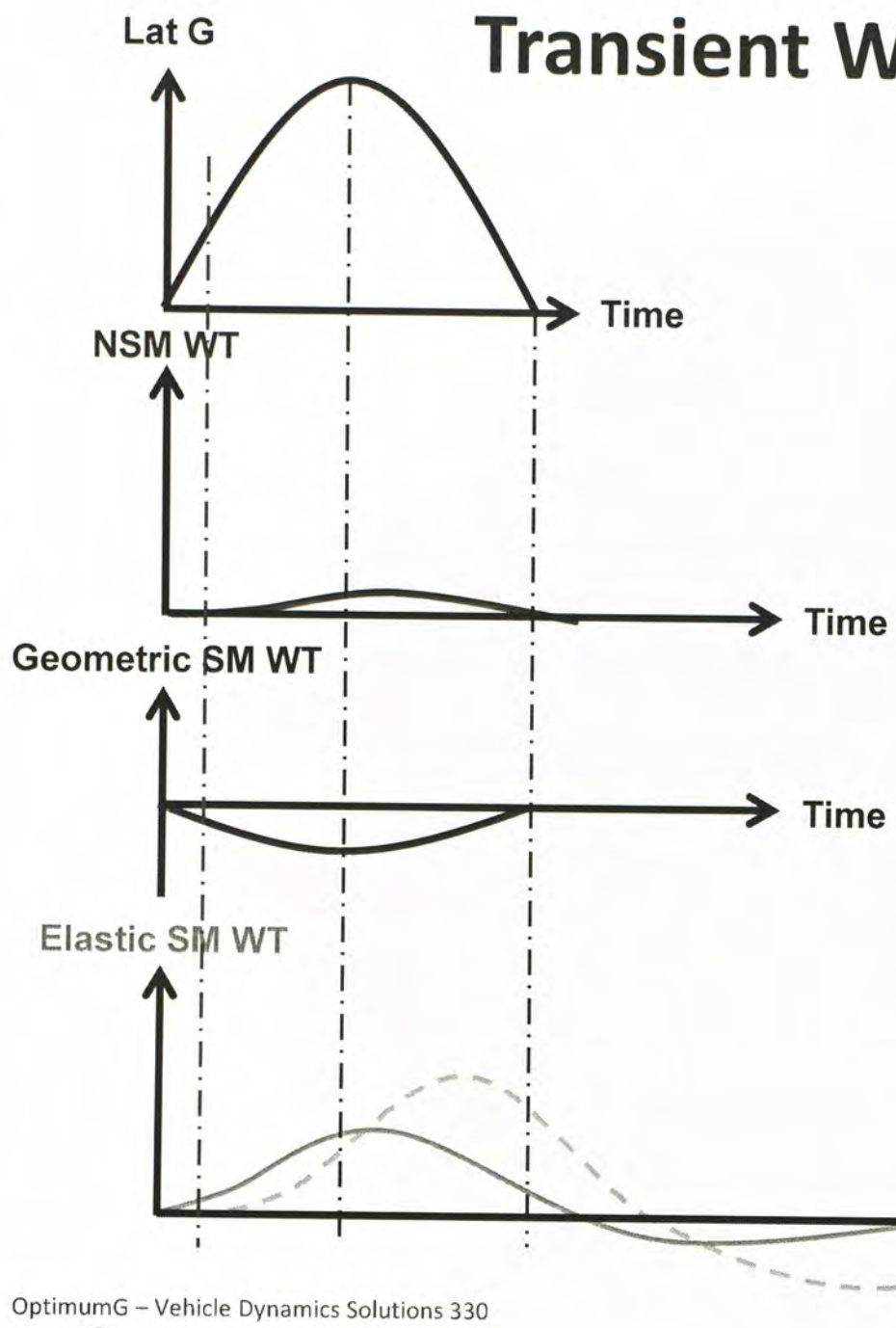
$$\frac{dx}{dt} = \frac{T}{2} \times \dot{\theta}$$

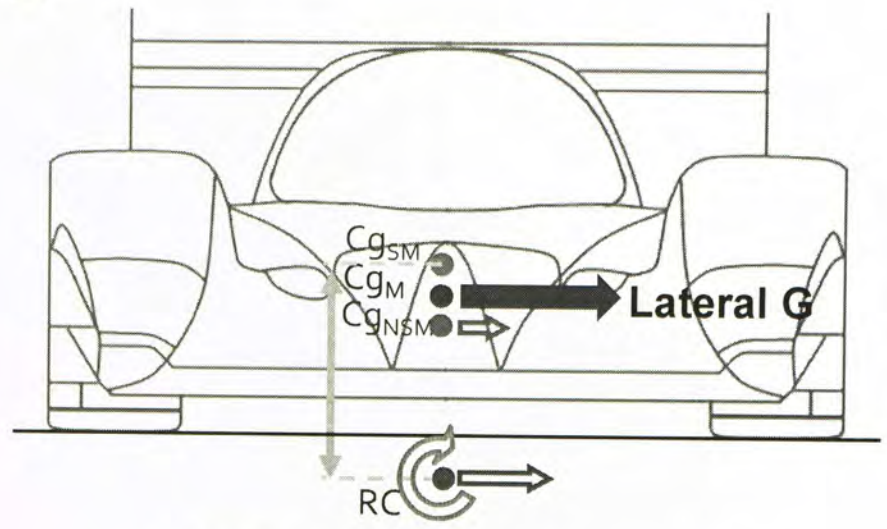
Transient Weight Transfer





Transient Weight Transfer





Roll Center under the ground

Suspended mass elastic weight transfer

Bigger distance between Cg SM and Roll Center
Bigger roll moment
Bigger roll angle
Bigger suspended mass roll inertia around RC
Longer delays
Smaller roll frequencies

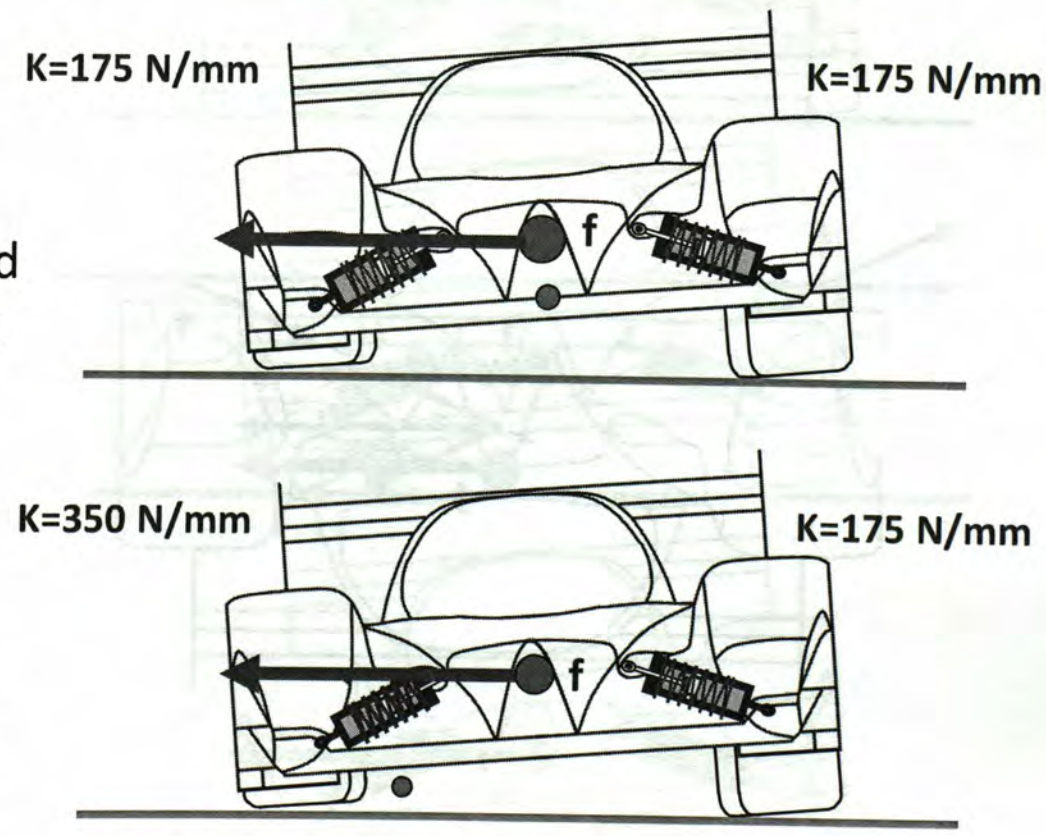
Kinematics Roll Center Limitations

Asymmetric setup. For example different spring stiffness left and right.

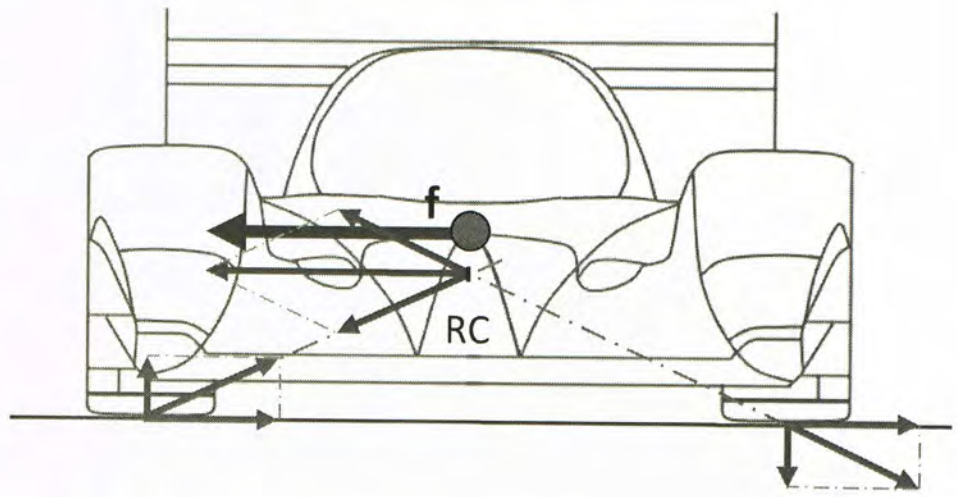
Force balance stayed the same. Force acting on the spring remained the same but displacement are not equal anymore.

The suspended mass will have combined roll and heave.

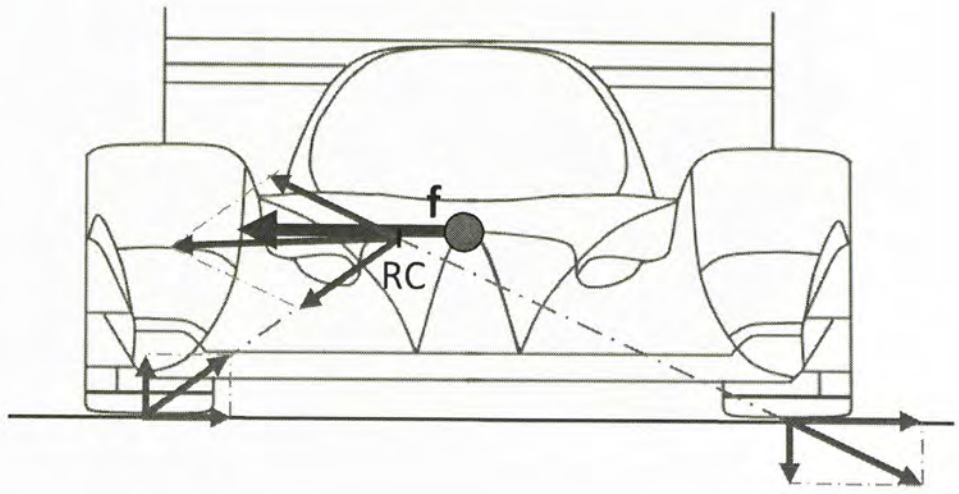
The suspended mass seems to roll around a point closer to the stiffer spring.



Kinematics Roll Center Limitations



The roll center tends to move to the side where the grip is lower.

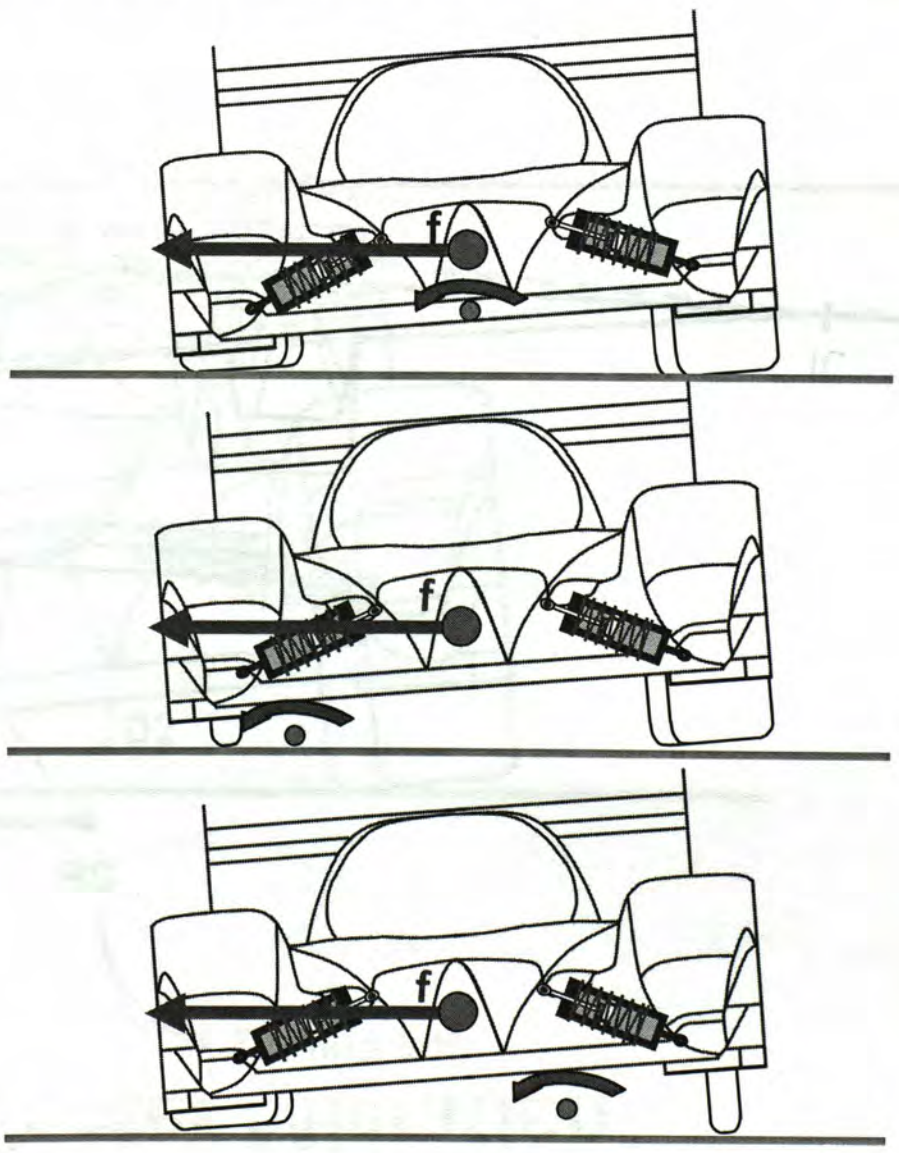


Kinematics Roll Center Limitations

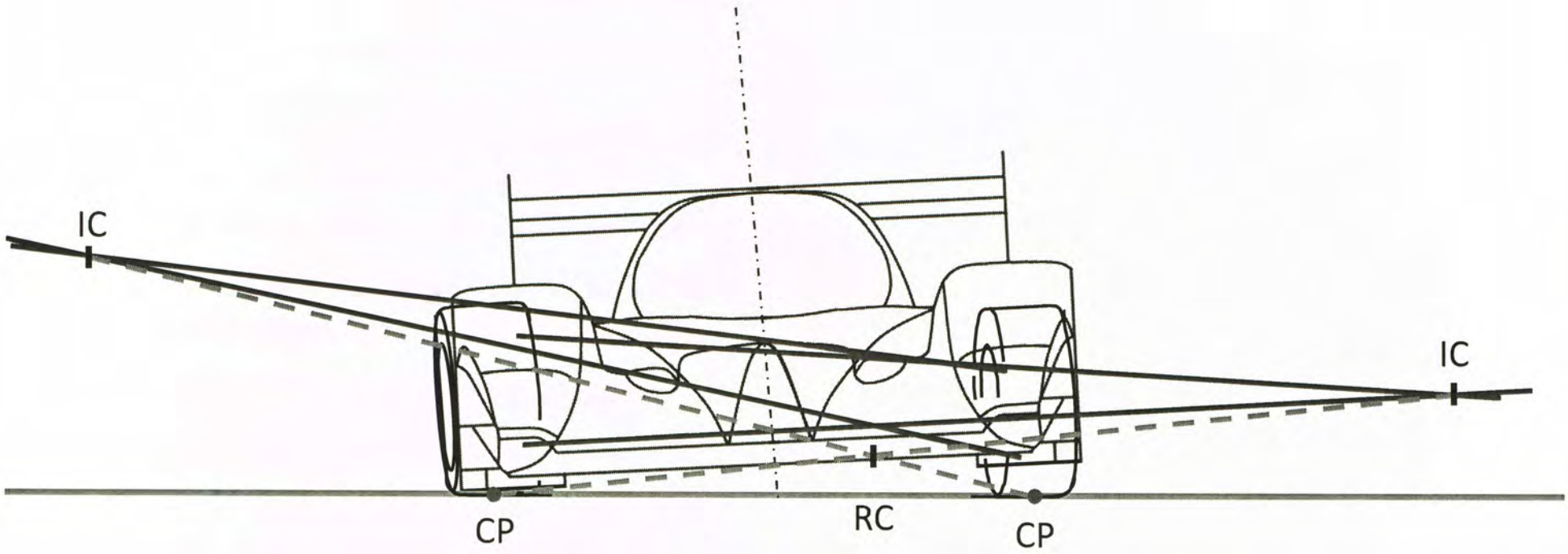
Tire are not infinitely stiff.

If tires are infinitely stiff the suspended mass rolls in the suspension spring as predicted by the kinematic roll center theory.

If the spring are infinitely stiff the suspended mass rolls in the tire.

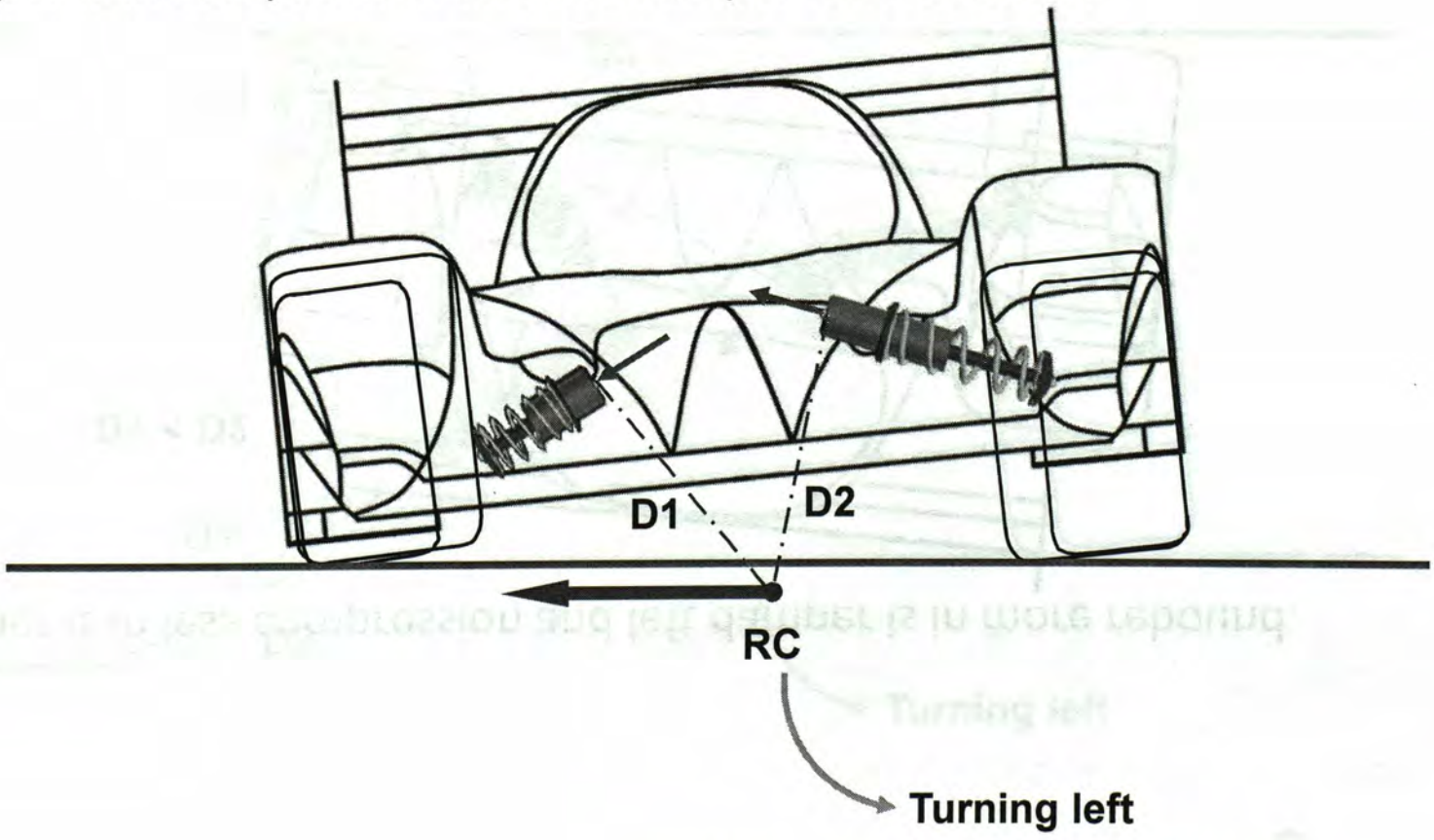


Kinematics Roll Center Migration



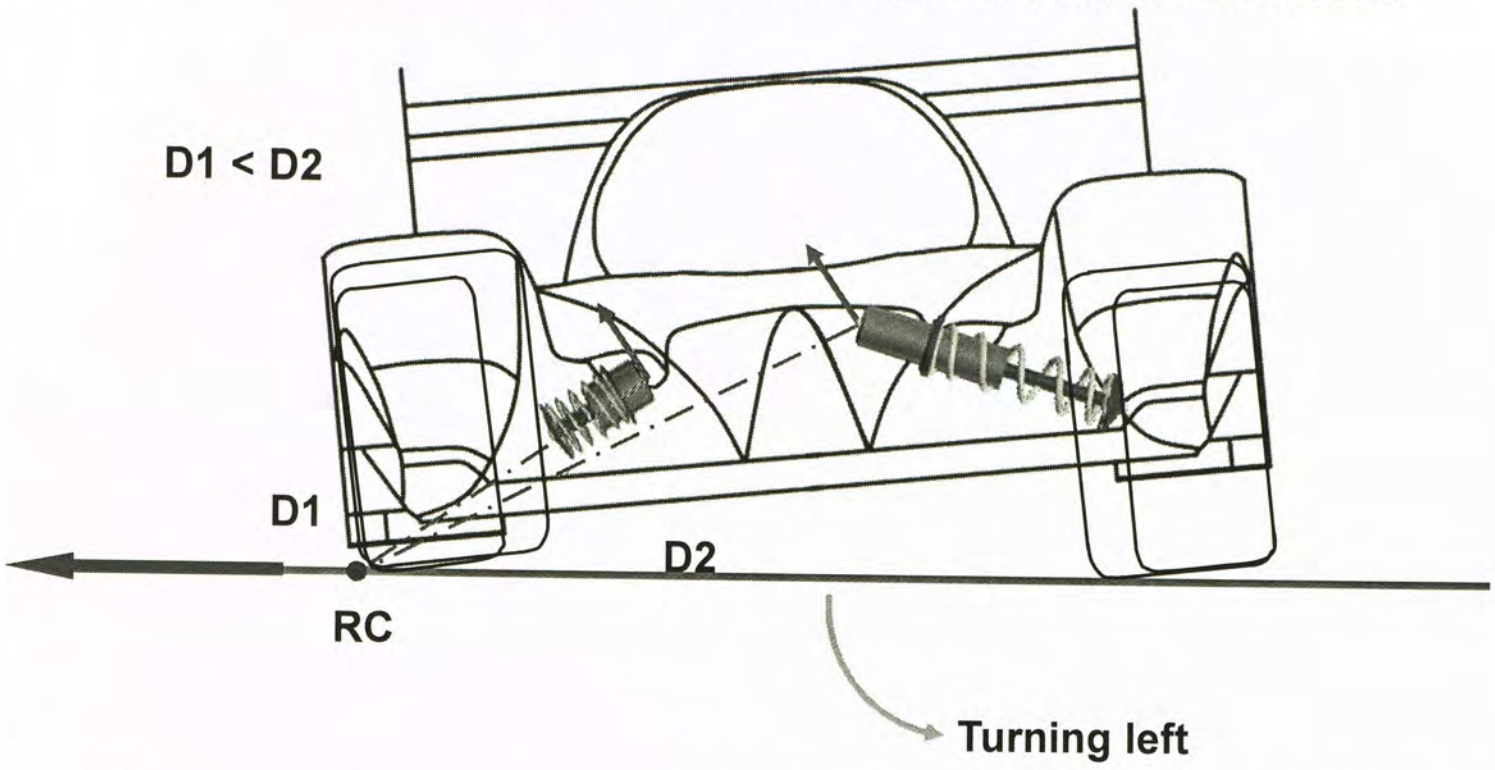
Roll Center Location Under Rolling

Right damper is in compression and left damper is in rebound.



Roll Center Location Under Rolling

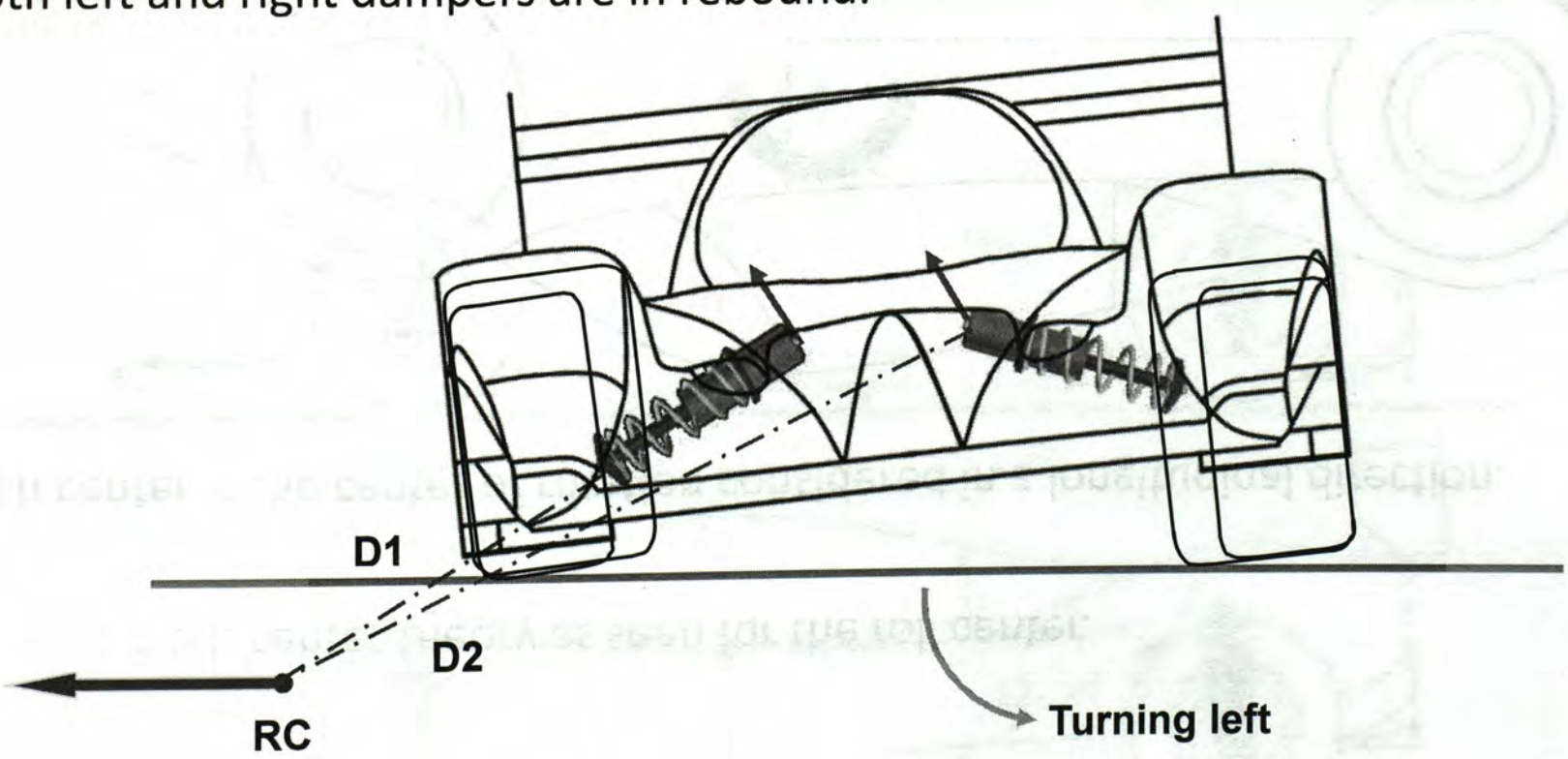
Right damper is in less compression and left damper is in more rebound.



Having the roll center closer to one corner of the car and inside the track, is like having stiffer springs in that corner. This is because the kinematics is limiting the spring movement.

Roll Center Location Under Rolling

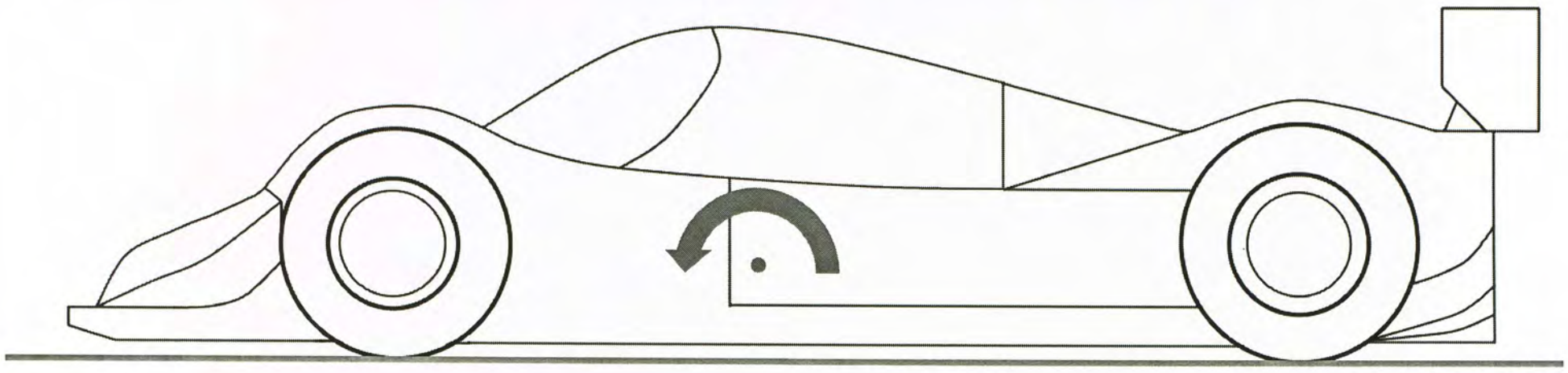
Both left and right dampers are in rebound.



Pitch Center Definition

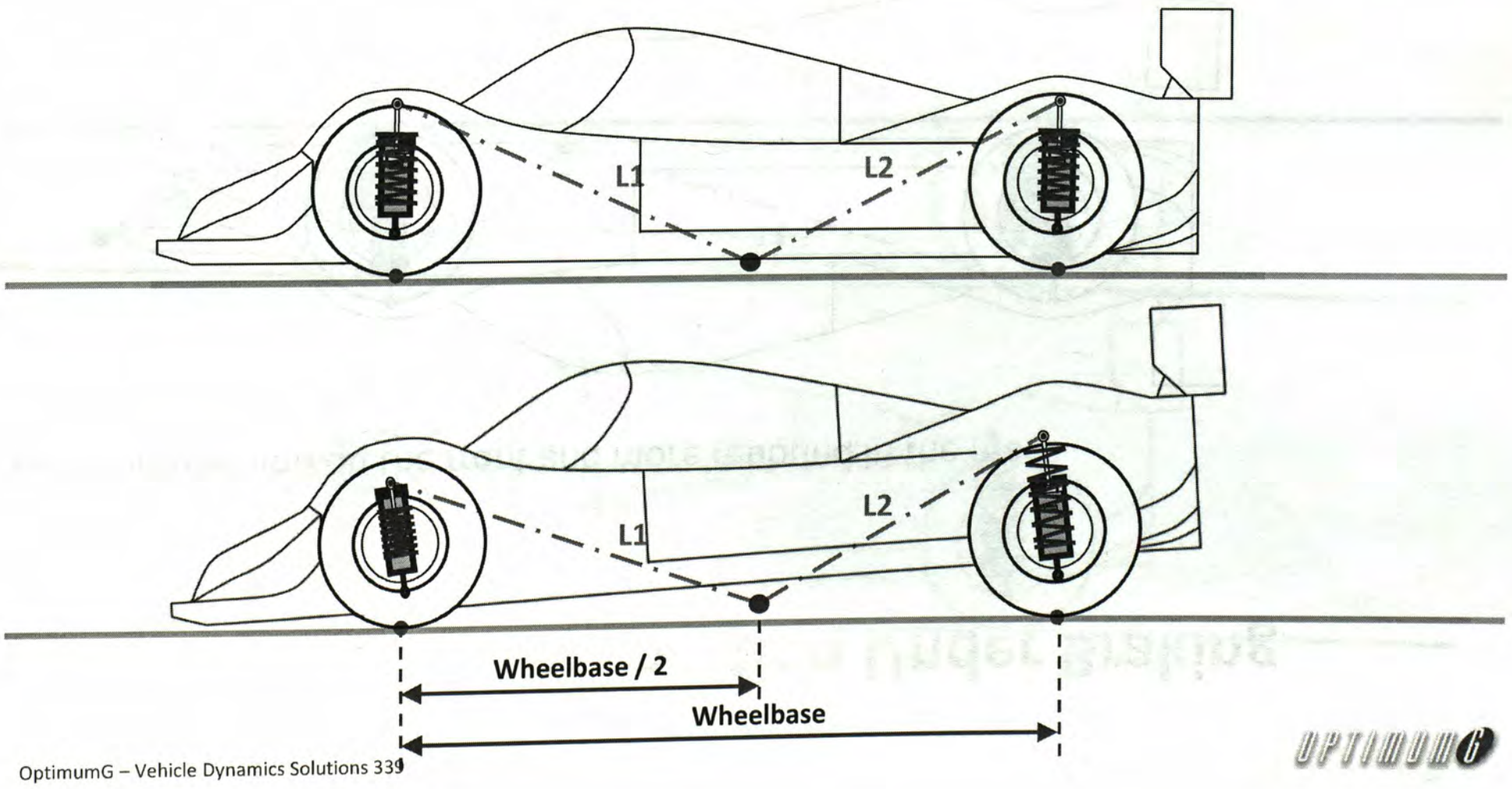
Multiple pitch center theory as seen for the roll center.

Pitch center is the center of rotation considered in a longitudinal direction.



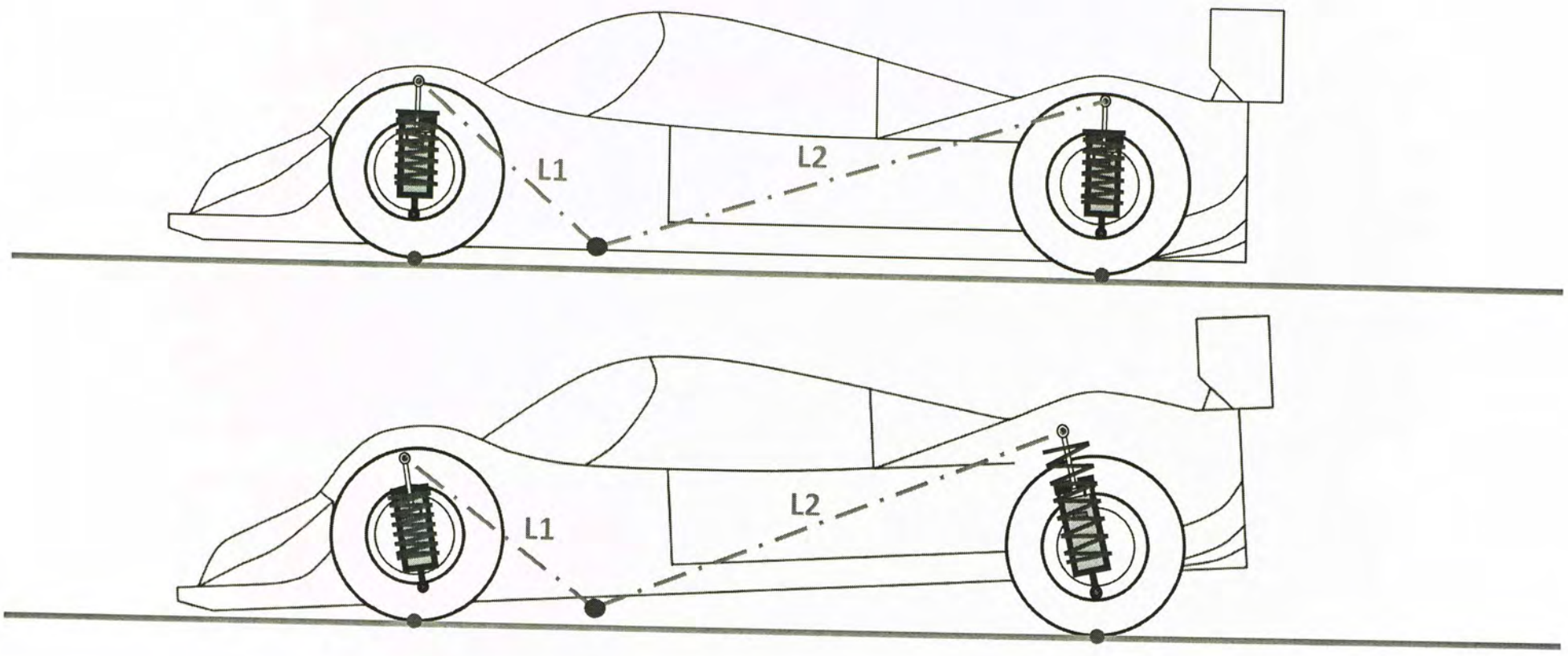
Pitch Center Location Under Braking

Front bump compression and rear rebound extension have the same value.



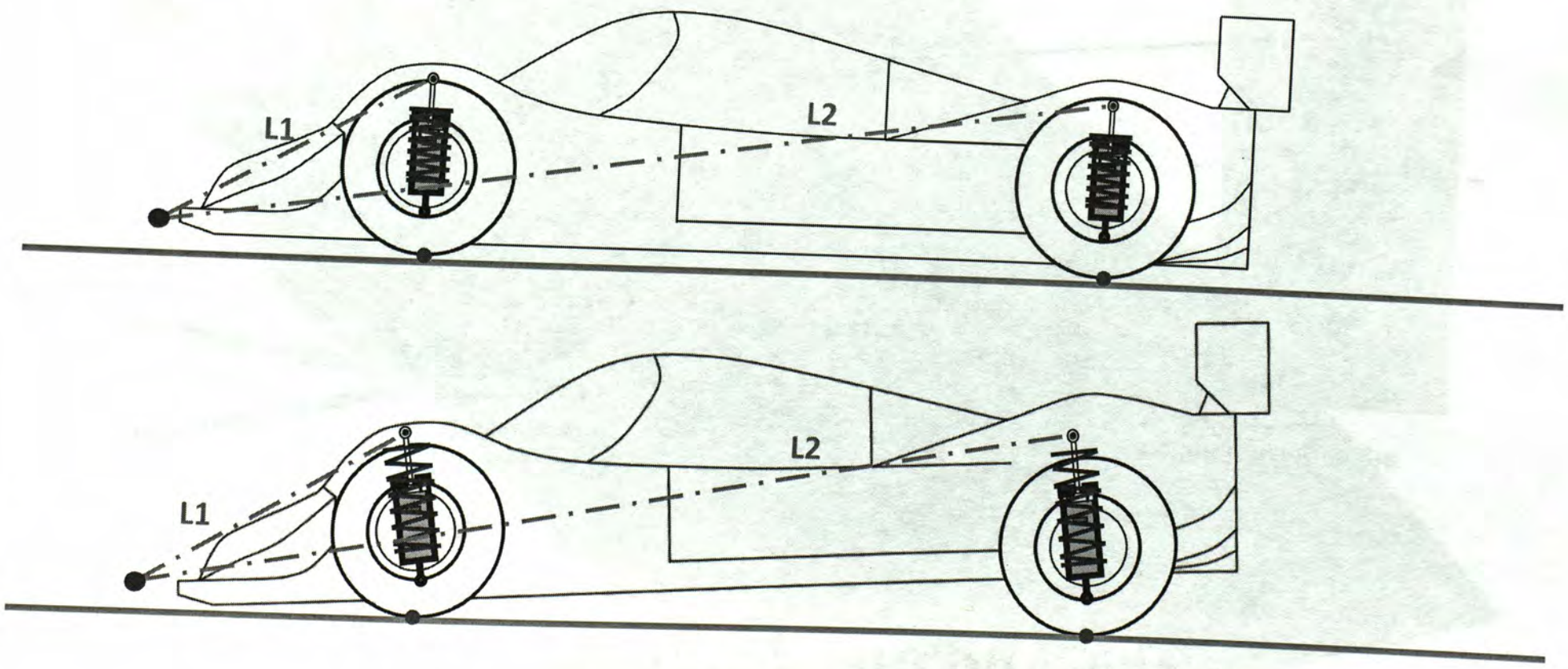
Pitch Center Location Under Braking

Less compression on the front and more rebound in the rear.

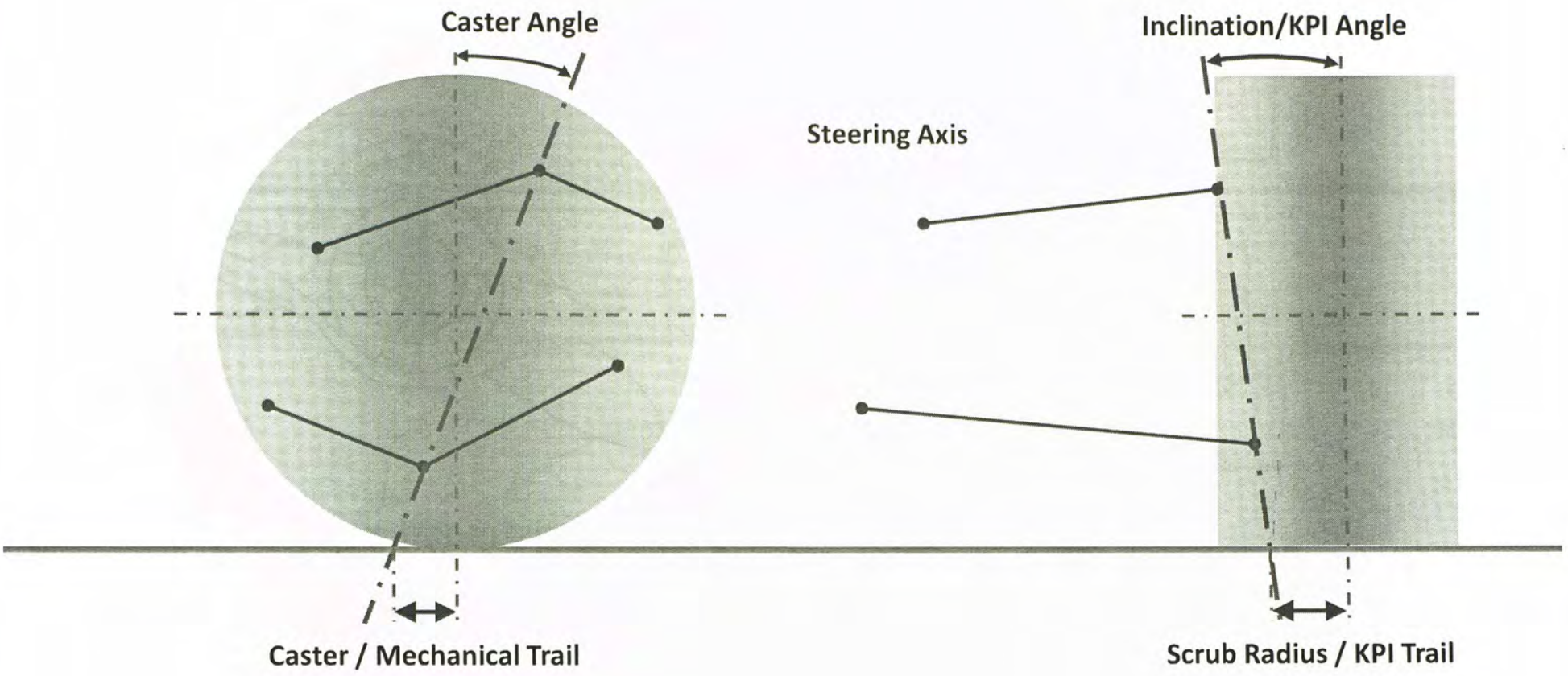


Pitch Center Location Under Braking

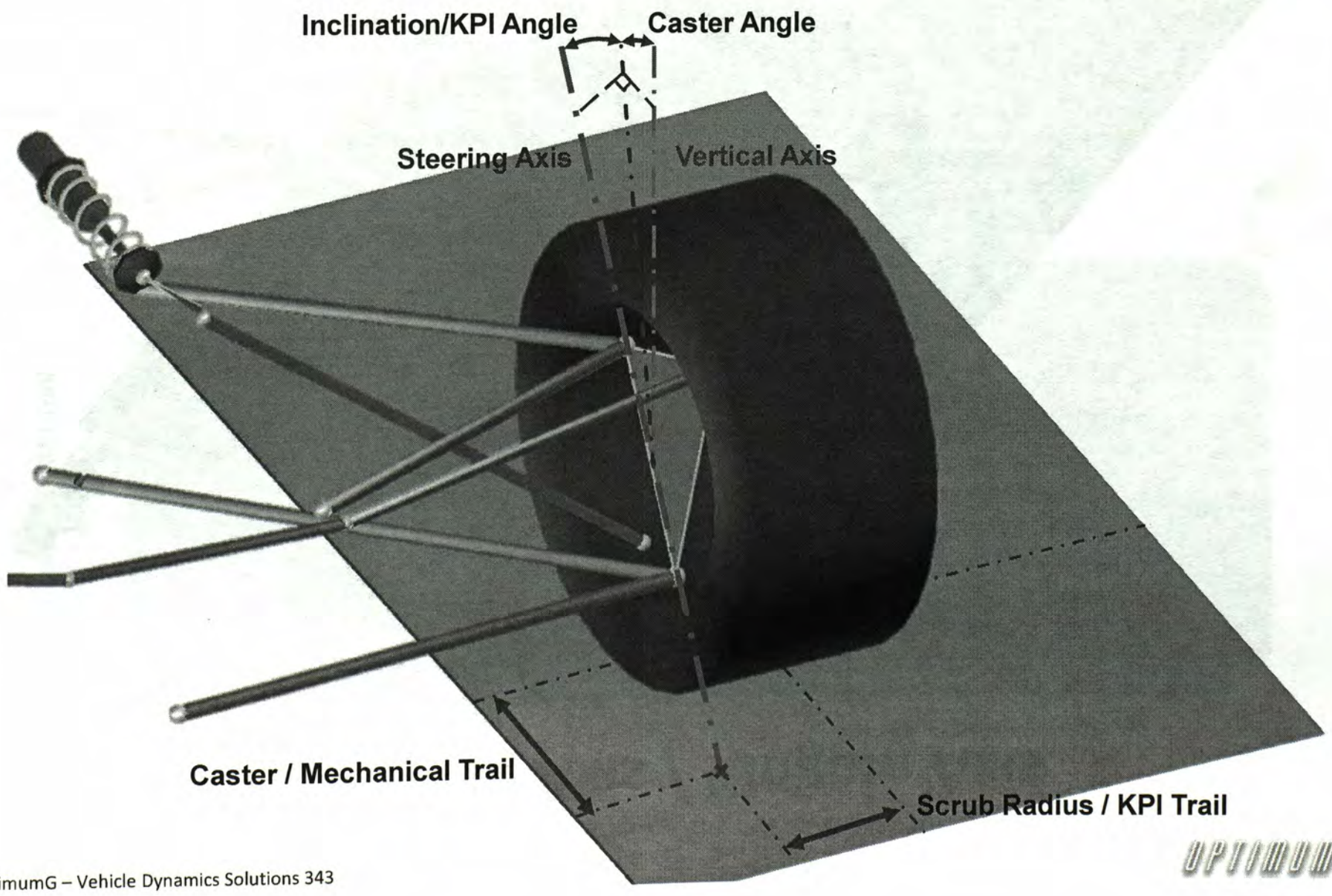
Both spring and damper are in rebound.



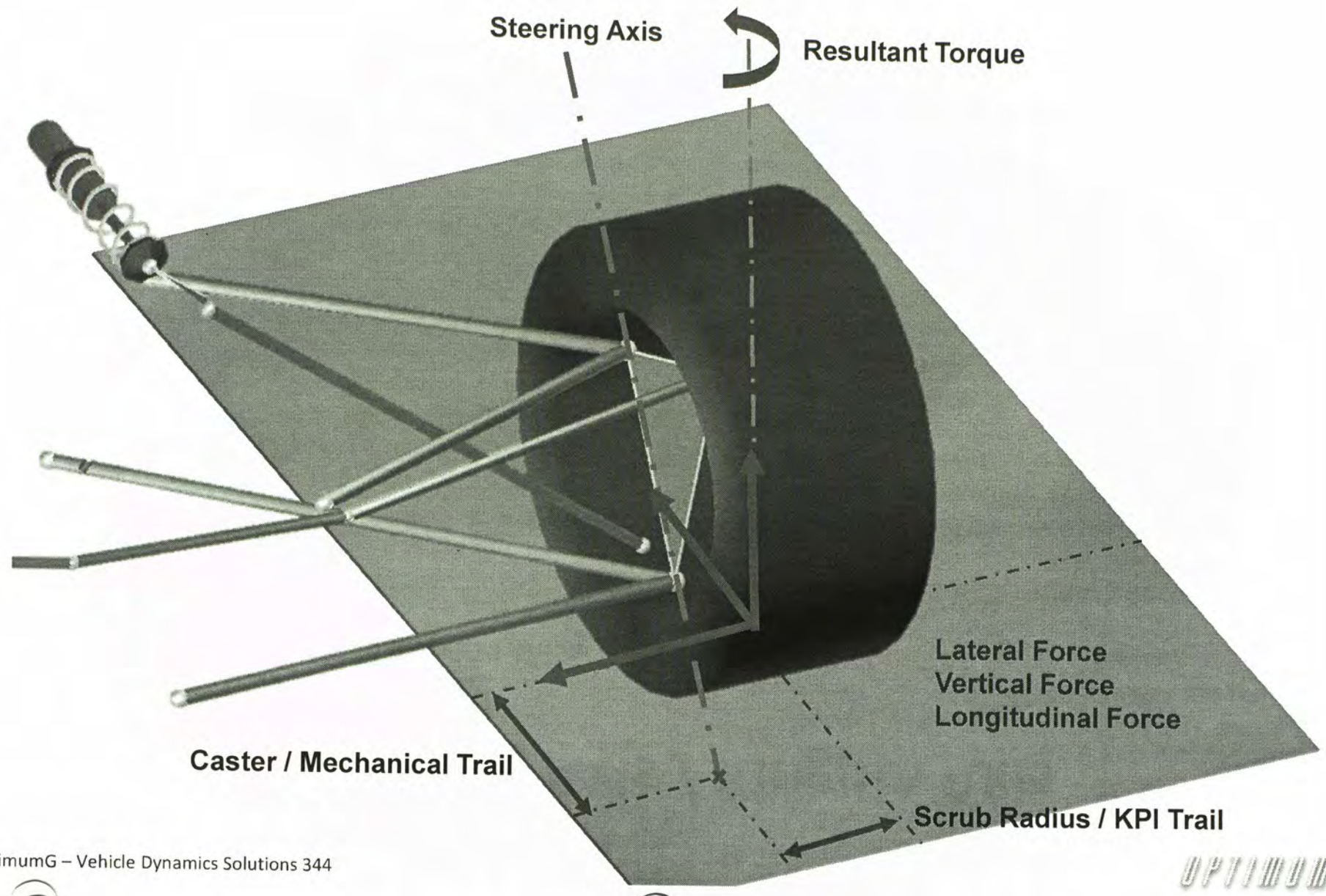
Steering Axis / Kingpin Axis



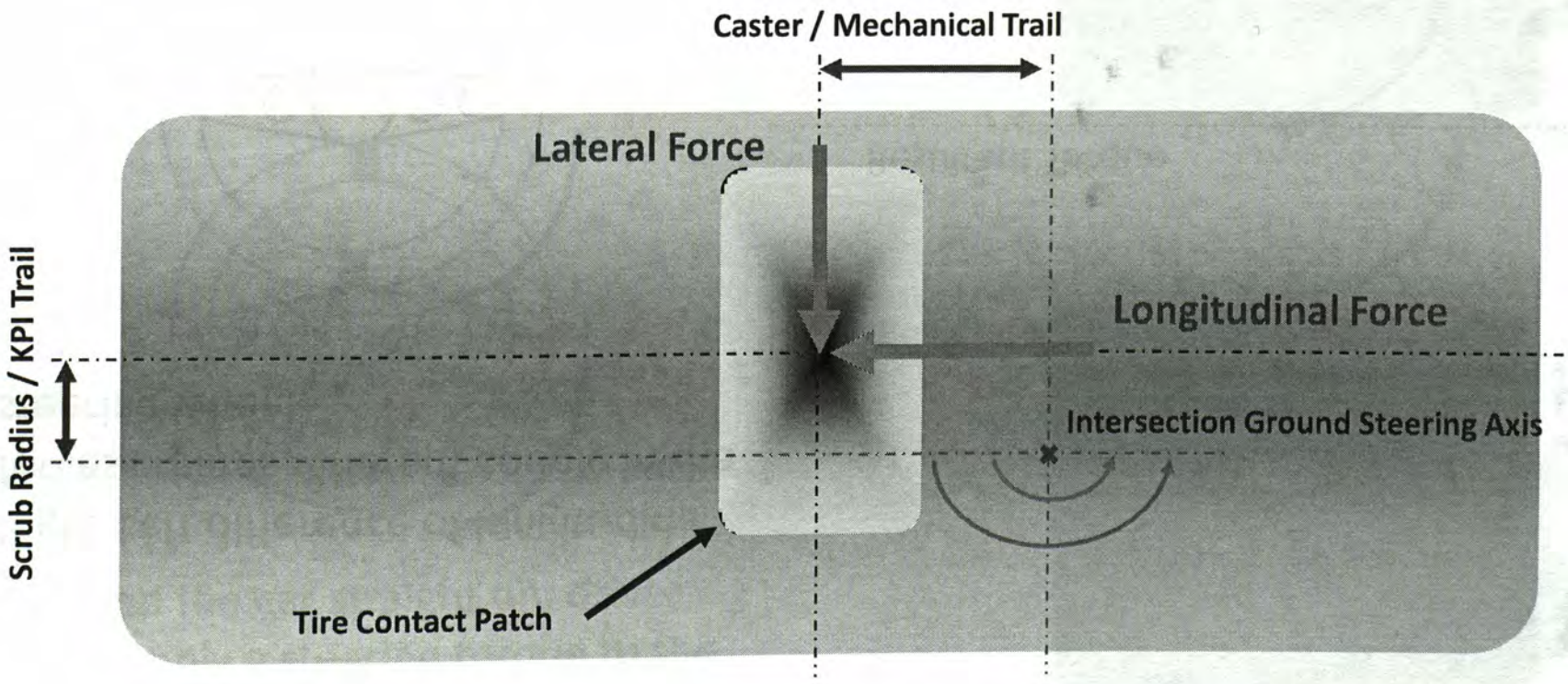
Steering Axis / Kingpin Axis



Steering Axis / Kingpin Axis

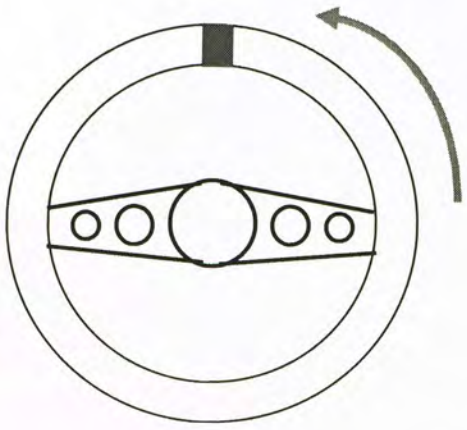


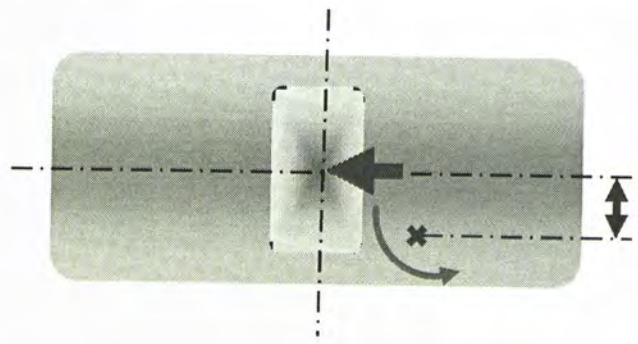
Steering Axis / Kingpin Axis



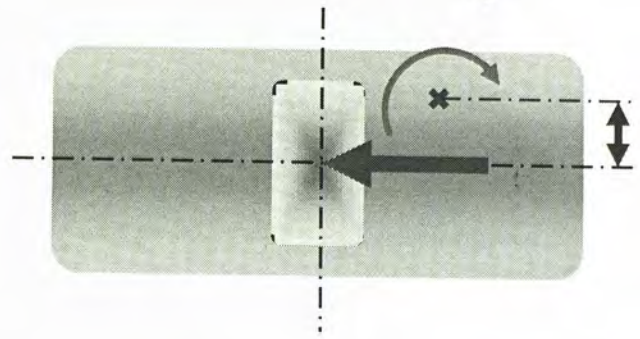
Steering Axis / Kingpin Axis

Right-Left difference of longitudinal forces creates different couple in the steering wheel.





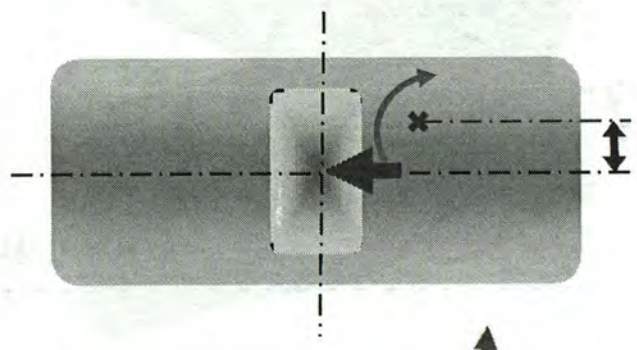
Resultant Torque

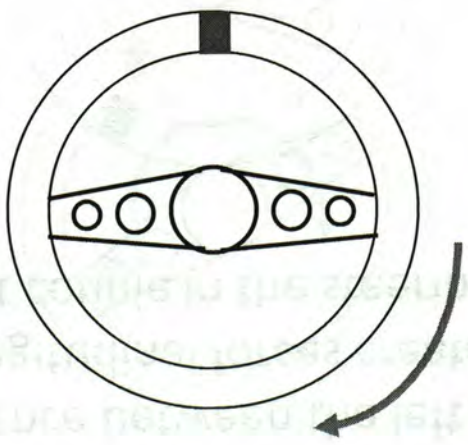


To keep the car straight on, driver must apply a steering torque to the left.

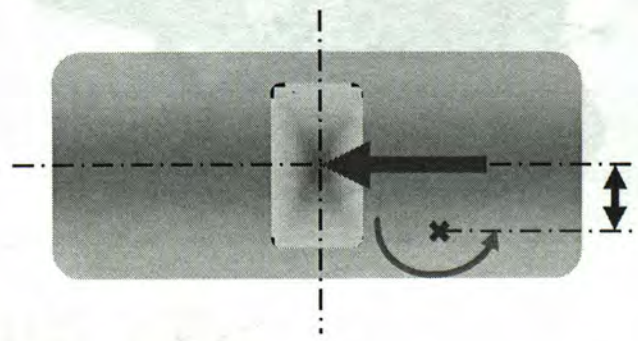
Steering Axis / Kingpin Axis

Right-Left difference of longitudinal forces creates different couple in the steering wheel.





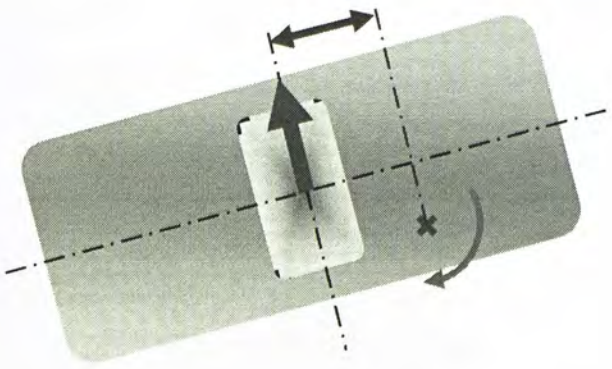
Resultant Torque

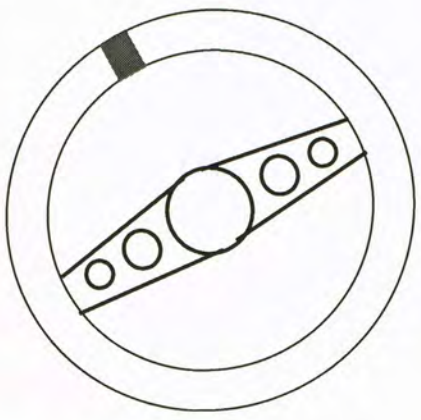


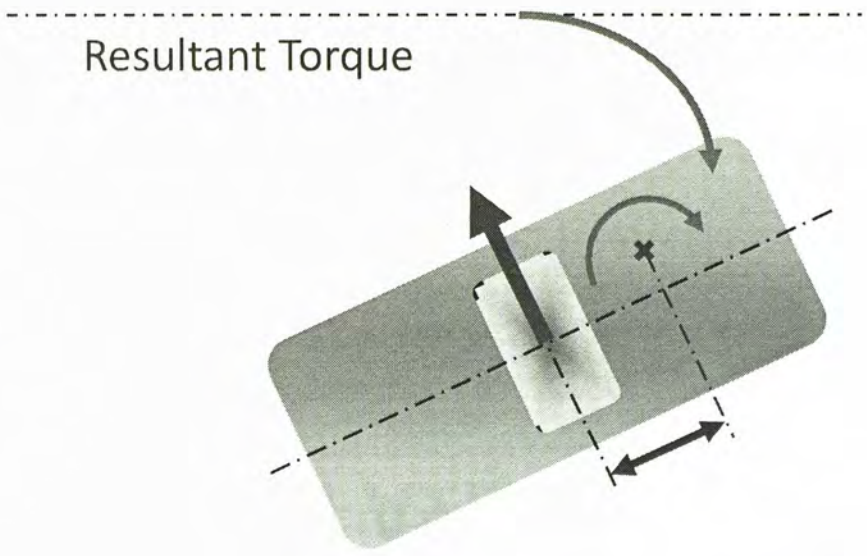
To keep the car straight on, driver must apply a steering torque to the right.

Steering Axis / Kingpin Axis

A difference between the left and right longitudinal forces creates a different couple in the steering wheel.



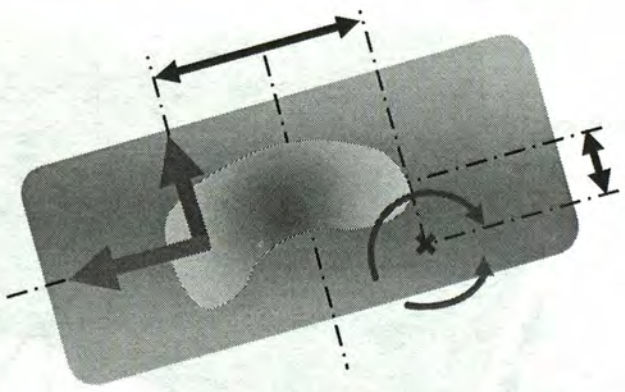


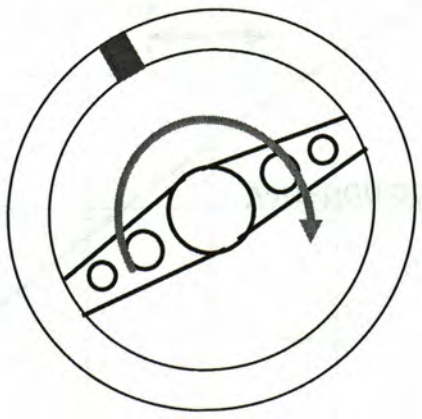


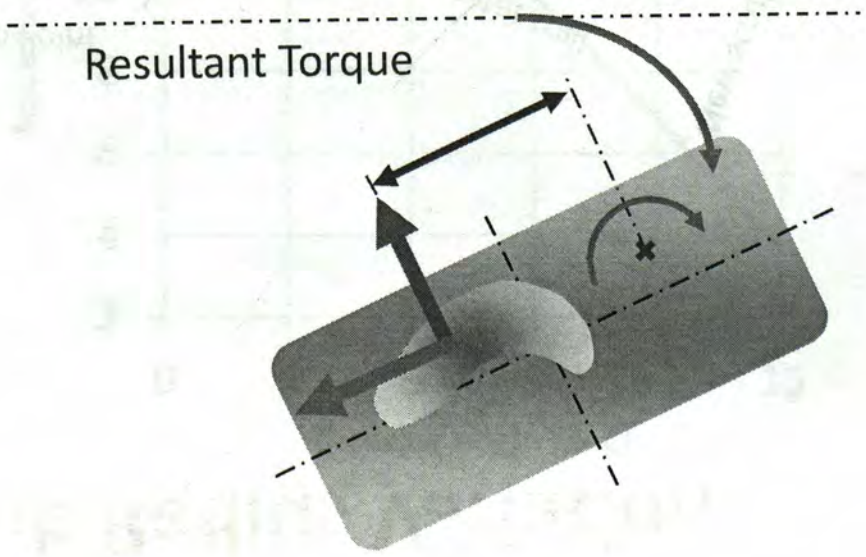
To keep the car in corner, driver must resist this torque.

Steering Axis / Kingpin Axis

When taking into account the tire longitudinal and lateral deformations, the aligning torque increases a lot.

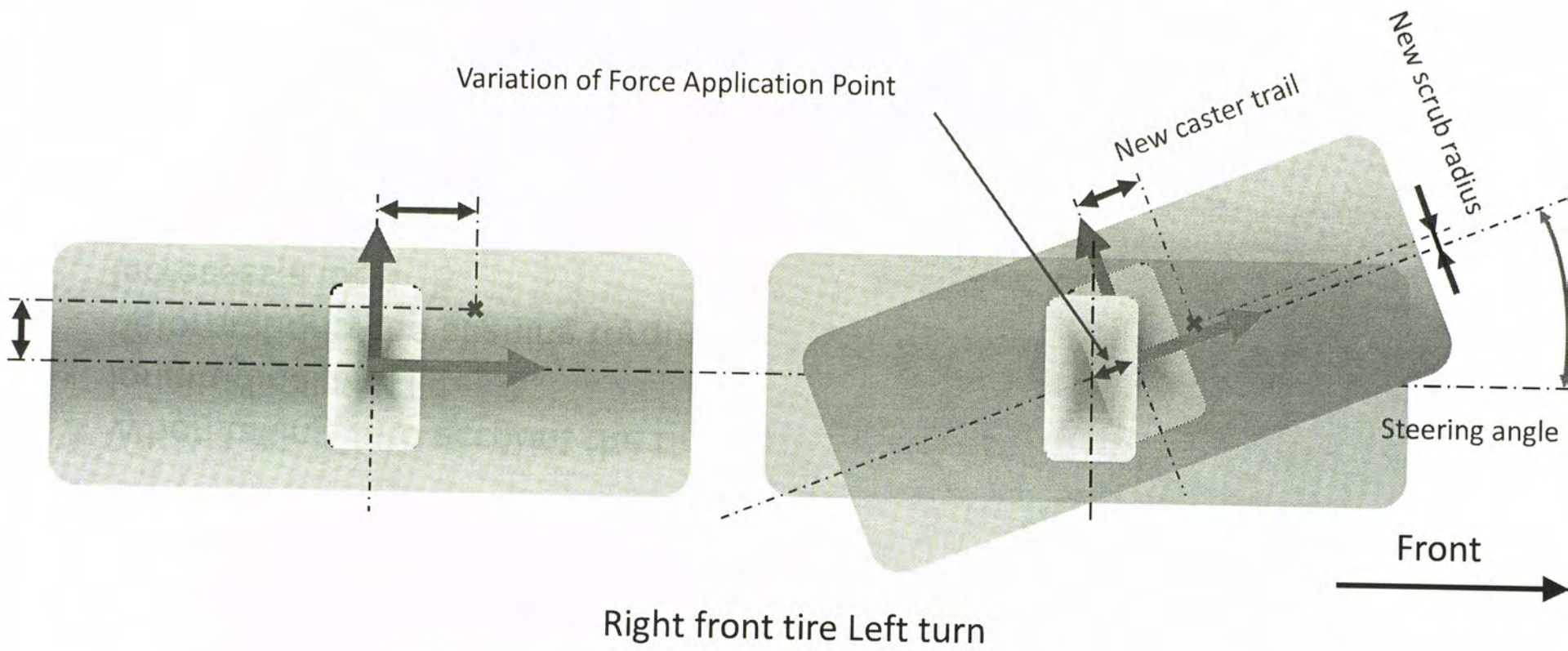




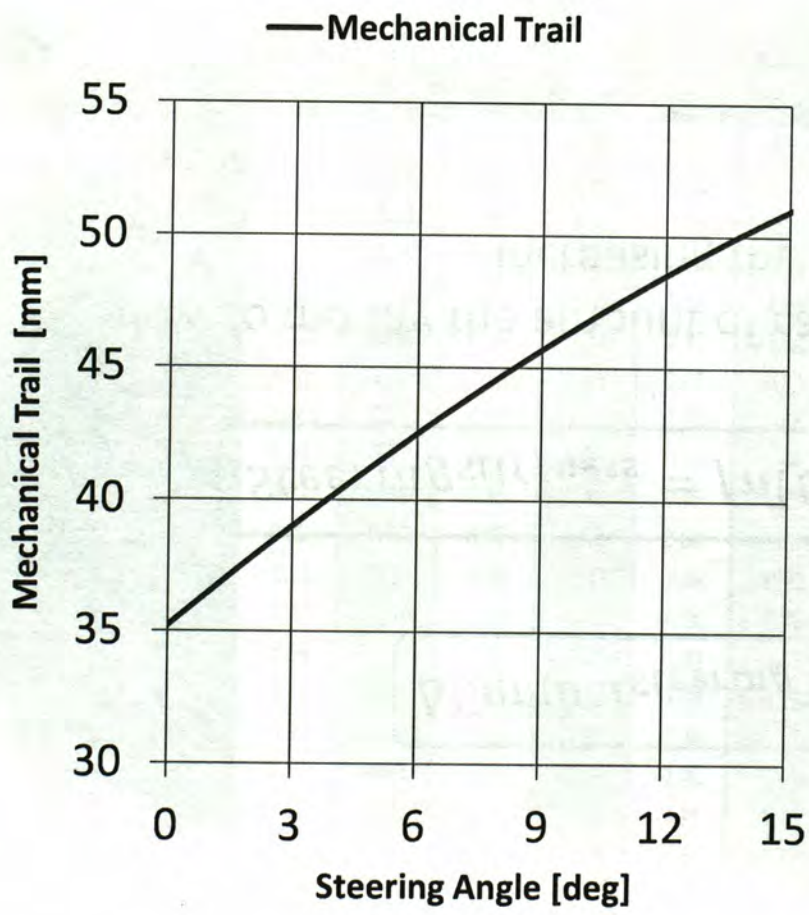


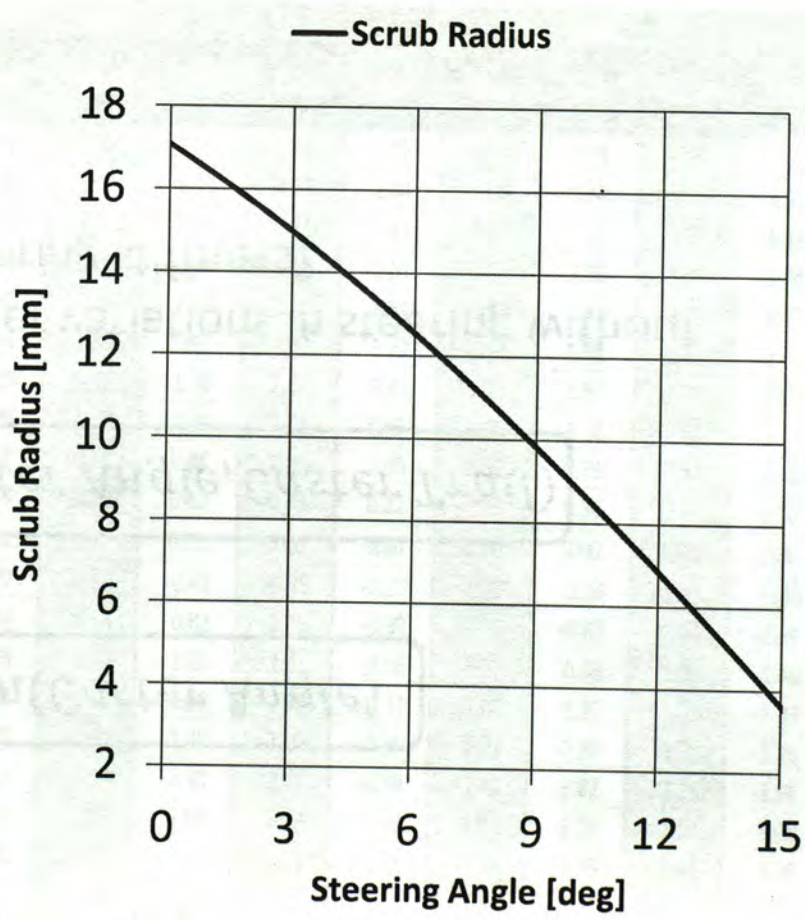
To keep the car in corner, driver must resist this torque

Caster Trail and Scrub Radius Variation



Caster Trail and Scrub Radius Variation





Steering Axis / Kingpin Axis

$$\Delta Camber_{Steering} = fn(Caster\ Angle)$$

$$Steering_{Stiffness} = fn(Caster\ Angle, Caster\ Trail)$$

How to modify the amount of camber variations in steering without increasing the steering stiffness?

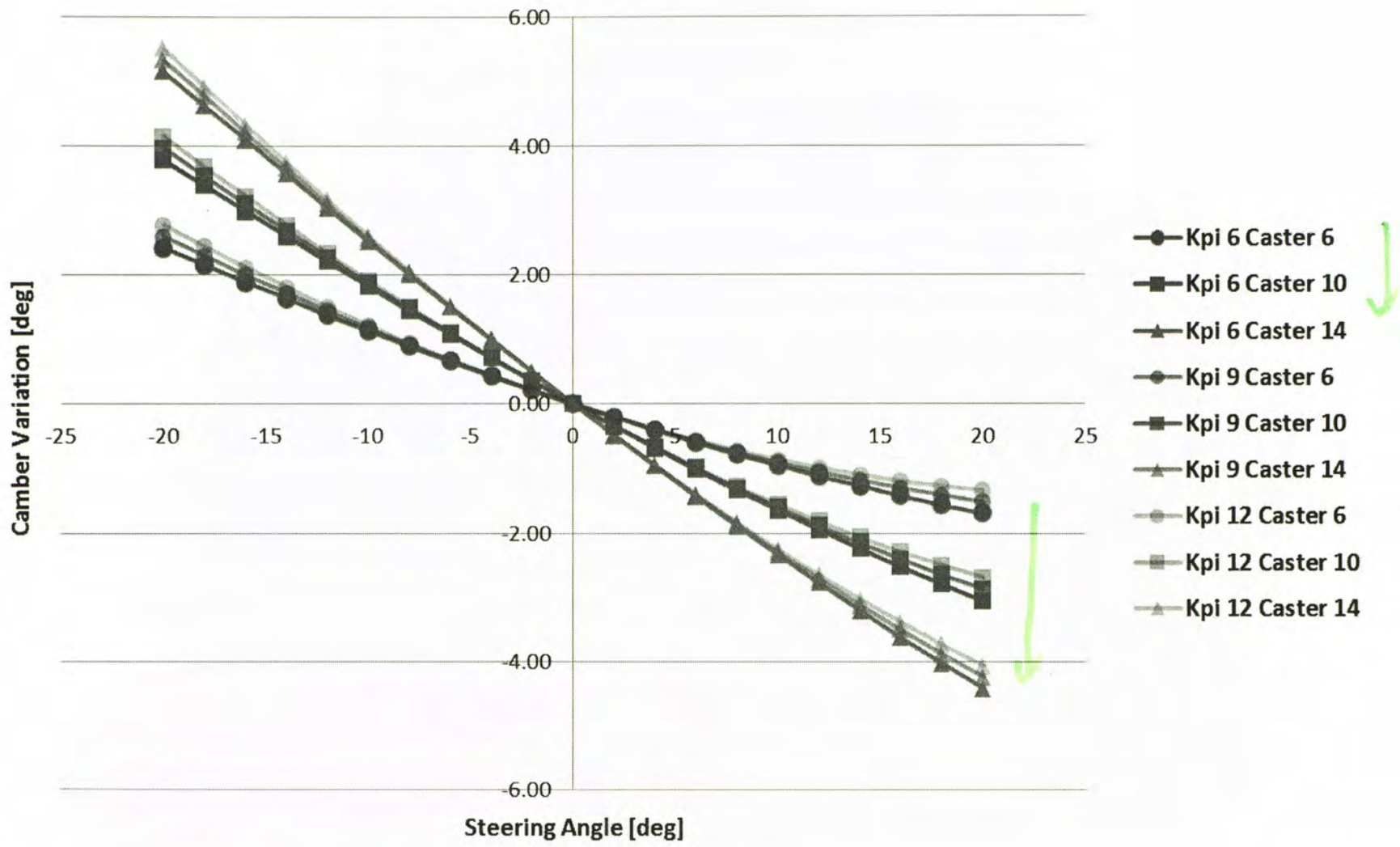
Camber Variation [deg] in Steering

Camber var. in steering = KPI Angle x (1-cos(Steering angle))- Caster angle x Sin (steering angle)

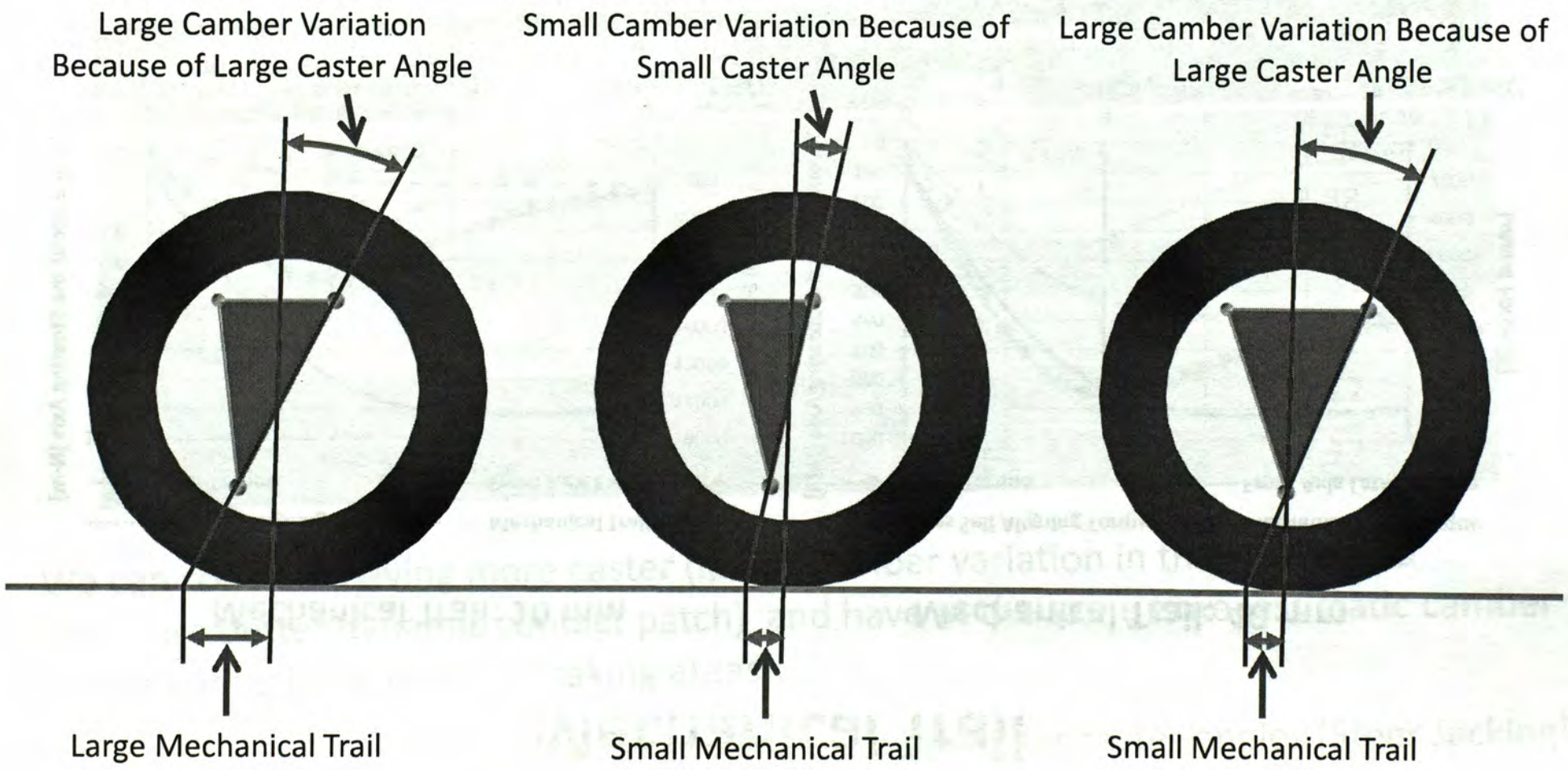
Positive Steering Angle

Inclination Angle [deg]		6.00					9.00					12.00				
Caster Angle [deg] / Steering Angle [deg]		6.00	8.00	10.00	12.00	14.00	6.00	8.00	10.00	12.00	14.00	6.00	8.00	10.00	12.00	14.00
Inside	-20.00	2.41	3.10	3.78	4.47	5.15	2.59	3.28	3.96	4.65	5.33	2.78	3.46	4.14	4.83	5.51
	-18.00	2.15	2.77	3.38	4.00	4.62	2.29	2.91	3.53	4.15	4.77	2.44	3.06	3.68	4.30	4.91
	-16.00	1.89	2.44	2.99	3.54	4.09	2.00	2.55	3.11	3.66	4.21	2.12	2.67	3.22	3.77	4.32
	-14.00	1.63	2.11	2.60	3.08	3.57	1.72	2.20	2.69	3.17	3.65	1.81	2.29	2.78	3.26	3.74
	-12.00	1.38	1.79	2.21	2.63	3.04	1.44	1.86	2.28	2.69	3.11	1.51	1.93	2.34	2.76	3.17
	-10.00	1.13	1.48	1.83	2.17	2.52	1.18	1.53	1.87	2.22	2.57	1.22	1.57	1.92	2.27	2.61
	-8.00	0.89	1.17	1.45	1.73	2.01	0.92	1.20	1.48	1.76	2.04	0.95	1.23	1.51	1.79	2.07
	-6.00	0.66	0.87	1.08	1.29	1.50	0.68	0.89	1.09	1.30	1.51	0.69	0.90	1.11	1.32	1.53
	-4.00	0.43	0.57	0.71	0.85	0.99	0.44	0.58	0.72	0.86	1.00	0.45	0.59	0.73	0.87	1.01
	-2.00	0.21	0.28	0.35	0.42	0.49	0.21	0.28	0.35	0.42	0.49	0.22	0.29	0.36	0.43	0.50
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Outside	2.00	-0.21	-0.28	-0.35	-0.42	-0.48	-0.20	-0.27	-0.34	-0.41	-0.48	-0.20	-0.27	-0.34	-0.41	-0.48
	4.00	-0.40	-0.54	-0.68	-0.82	-0.96	-0.40	-0.54	-0.68	-0.82	-0.95	-0.39	-0.53	-0.67	-0.81	-0.95
	6.00	-0.59	-0.80	-1.01	-1.22	-1.43	-0.58	-0.79	-1.00	-1.21	-1.41	-0.56	-0.77	-0.98	-1.19	-1.40
	8.00	-0.78	-1.05	-1.33	-1.61	-1.89	-0.75	-1.03	-1.30	-1.58	-1.86	-0.72	-1.00	-1.27	-1.55	-1.83
	10.00	-0.95	-1.30	-1.65	-1.99	-2.34	-0.91	-1.25	-1.60	-1.95	-2.29	-0.86	-1.21	-1.55	-1.90	-2.25
	12.00	-1.12	-1.53	-1.95	-2.36	-2.78	-1.05	-1.47	-1.88	-2.30	-2.71	-0.99	-1.40	-1.82	-2.23	-2.65
	14.00	-1.27	-1.76	-2.24	-2.72	-3.21	-1.18	-1.67	-2.15	-2.64	-3.12	-1.10	-1.58	-2.06	-2.55	-3.03
	16.00	-1.42	-1.97	-2.52	-3.08	-3.63	-1.31	-1.86	-2.41	-2.96	-3.51	-1.19	-1.74	-2.29	-2.84	-3.39
	18.00	-1.56	-2.18	-2.80	-3.41	-4.03	-1.41	-2.03	-2.65	-3.27	-3.89	-1.27	-1.88	-2.50	-3.12	-3.74
	20.00	-1.69	-2.37	-3.06	-3.74	-4.43	-1.51	-2.19	-2.88	-3.56	-4.25	-1.33	-2.01	-2.70	-3.38	-4.06

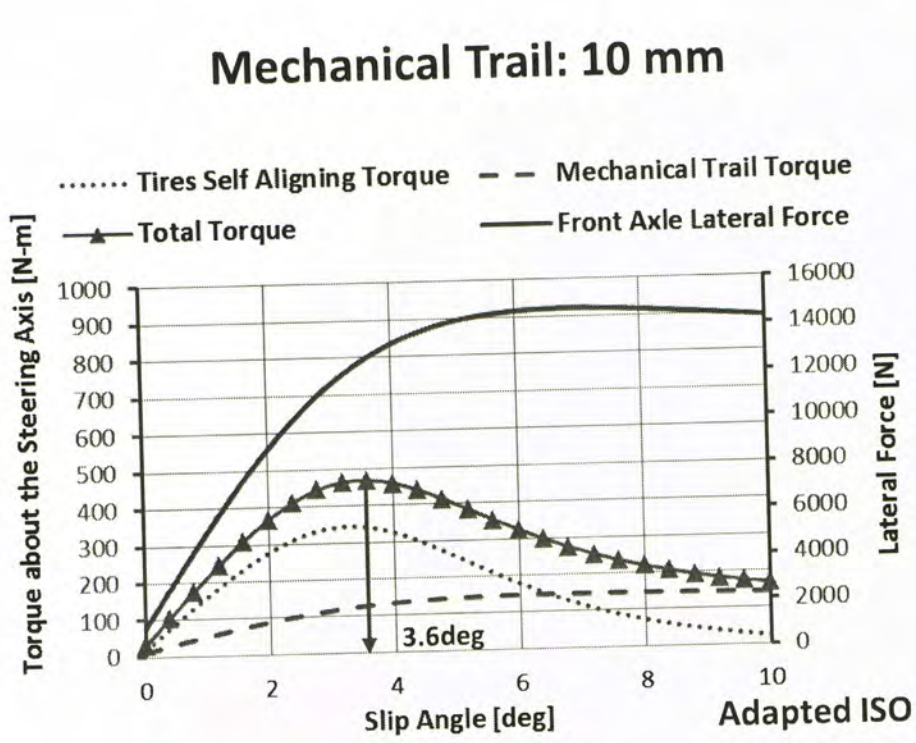
Suspension Design – Camber/Caster/Kpi

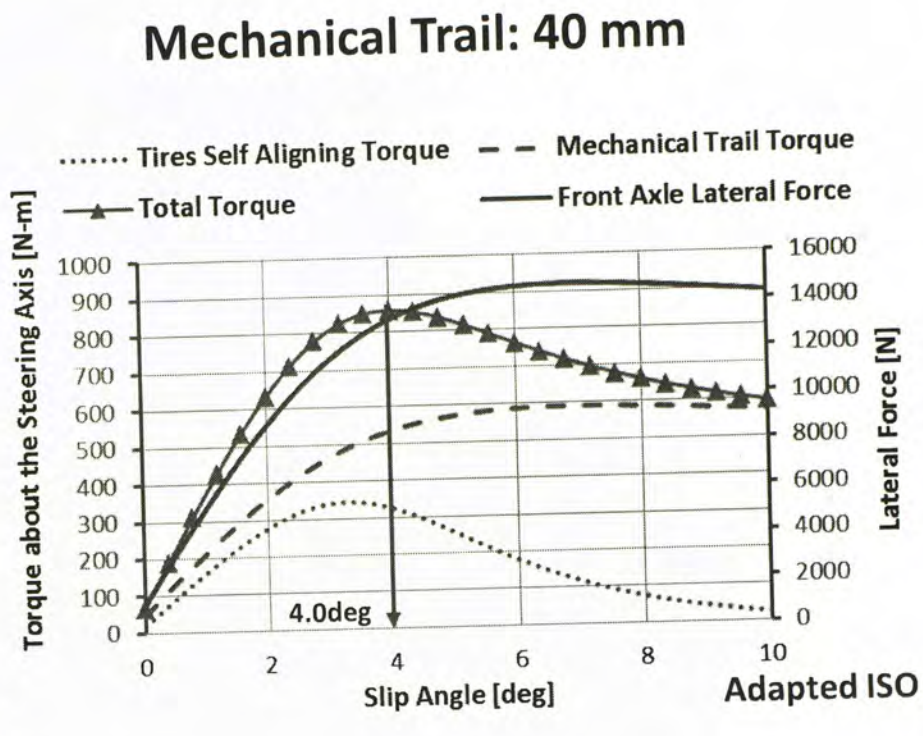


Caster Angle and Mechanical Trail



Brief Discussion on Pneumatic and Mechanical Trail





Does not take into account the static torque due to Right/Left Jacking effect.
Assume Fz_{FR} = 5600N and Fz_{FL} = 2000N continuously.

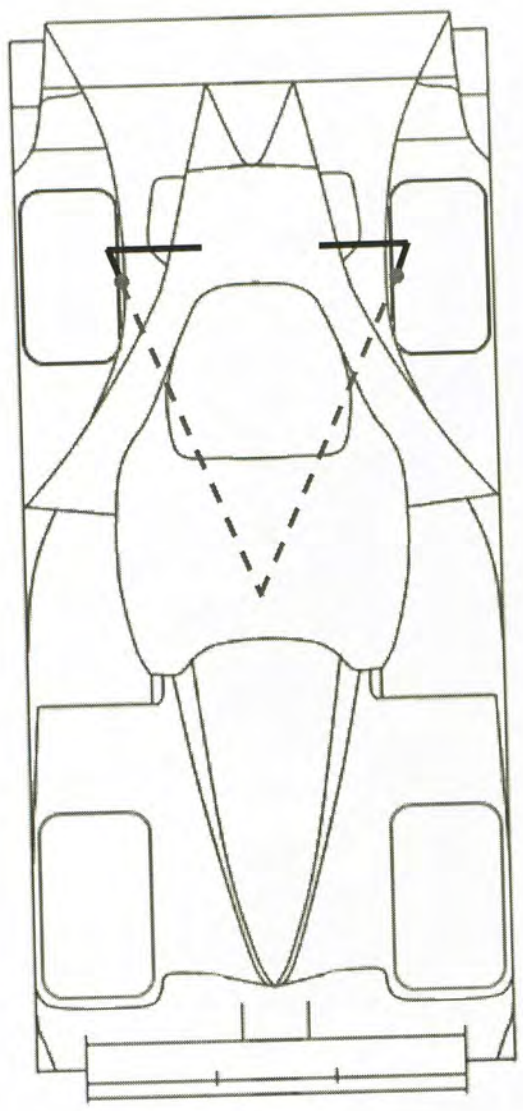
Camber Variation in Steering

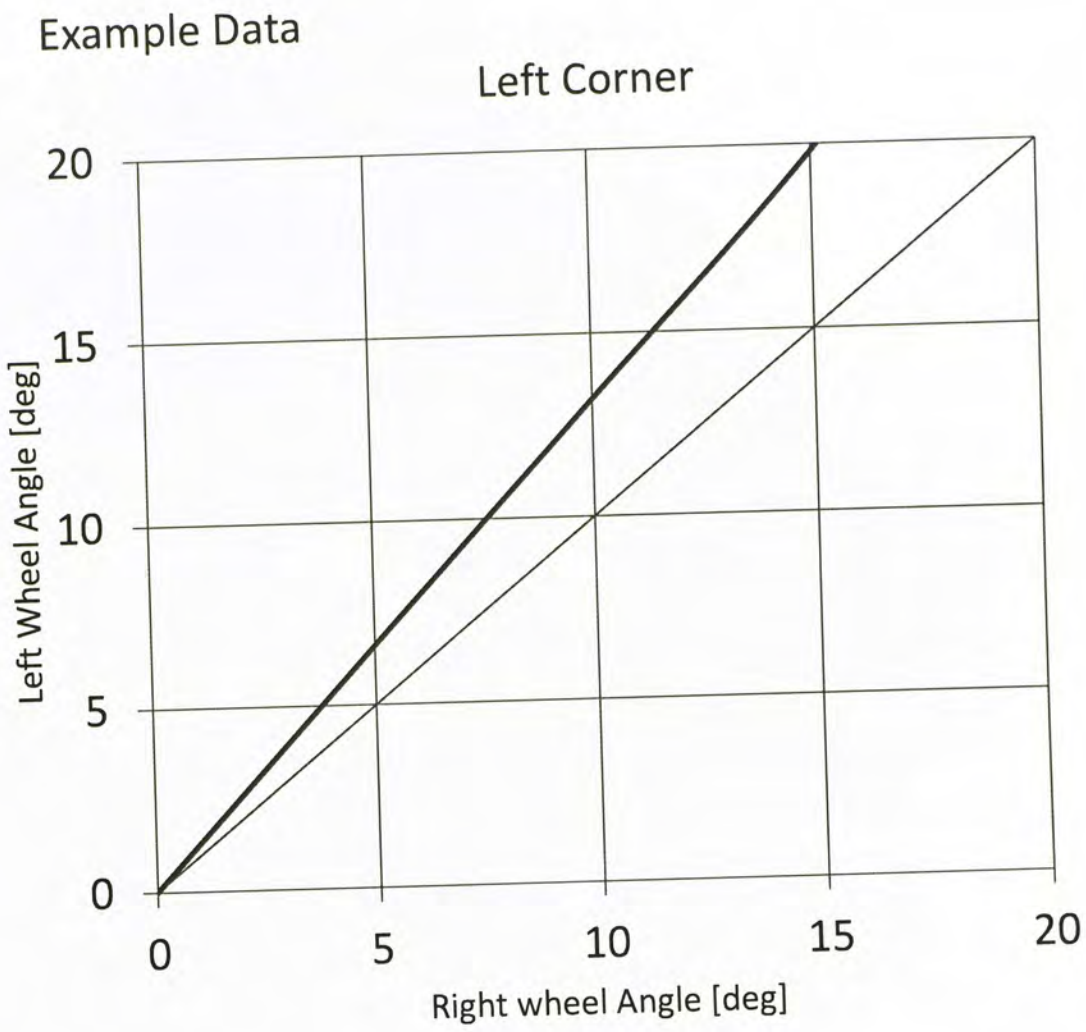
Caster Angle [deg]	Steering Angle (Left) [deg]	Left Font Camber [deg]	Right Front Camber [deg]
6.0	0.0	-1.50	-1.50
6.0	15.0	+0.25	-2.88
ΔCamber		+1.75	-1.38
9.0	0.0	-1.50	-1.50
9.0	15.0	+1.03	-3.72
ΔCamber		+2.53	-2.22

We can consider having more caster (larger camber variation in the corner and therefore better dynamic contact patch) and have at the same time less static camber (larger contact patch in the braking areas).

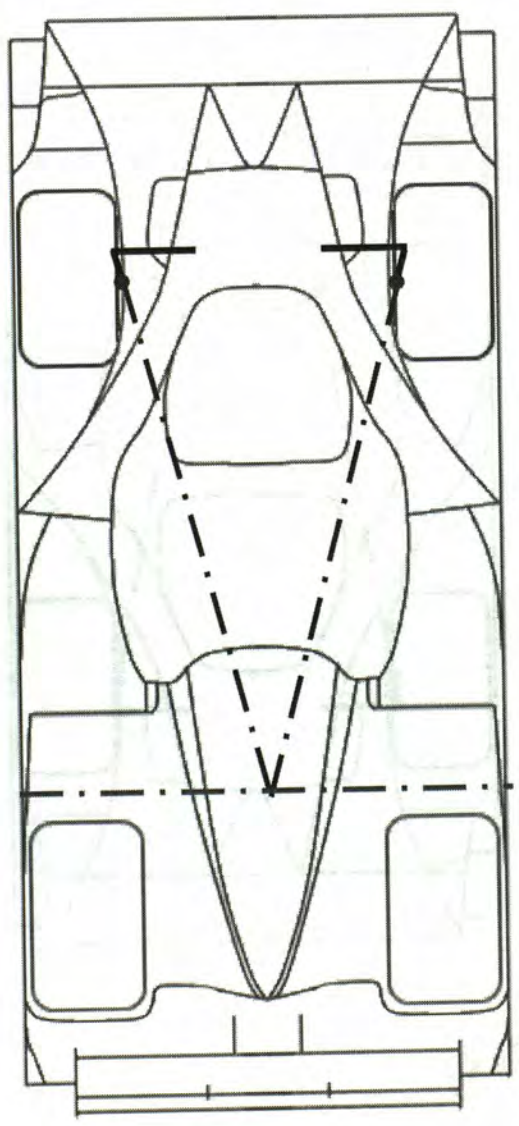
Do not forget: bigger front ride height changes with higher caster angles [Steer Jacking].

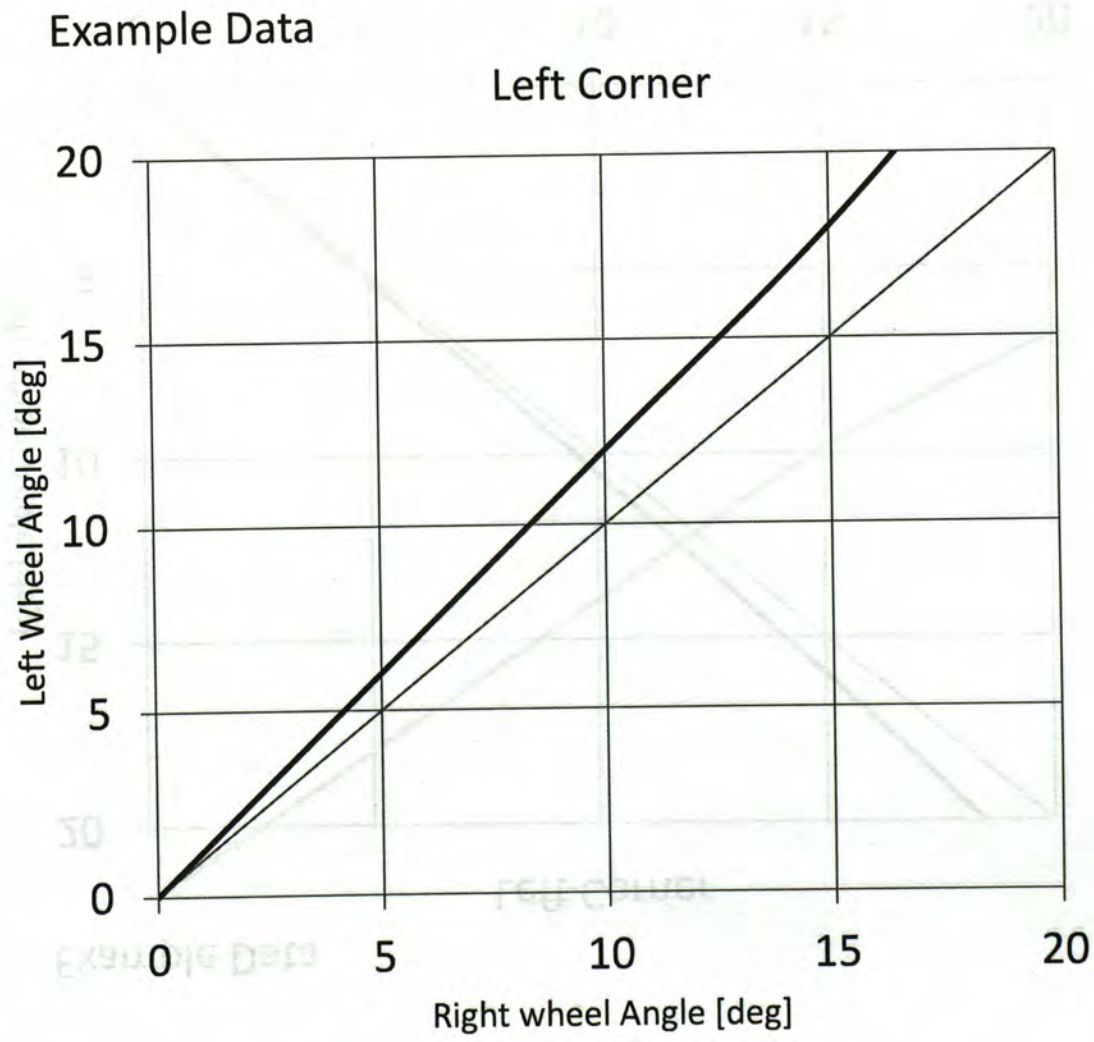
Ackermann Geometry



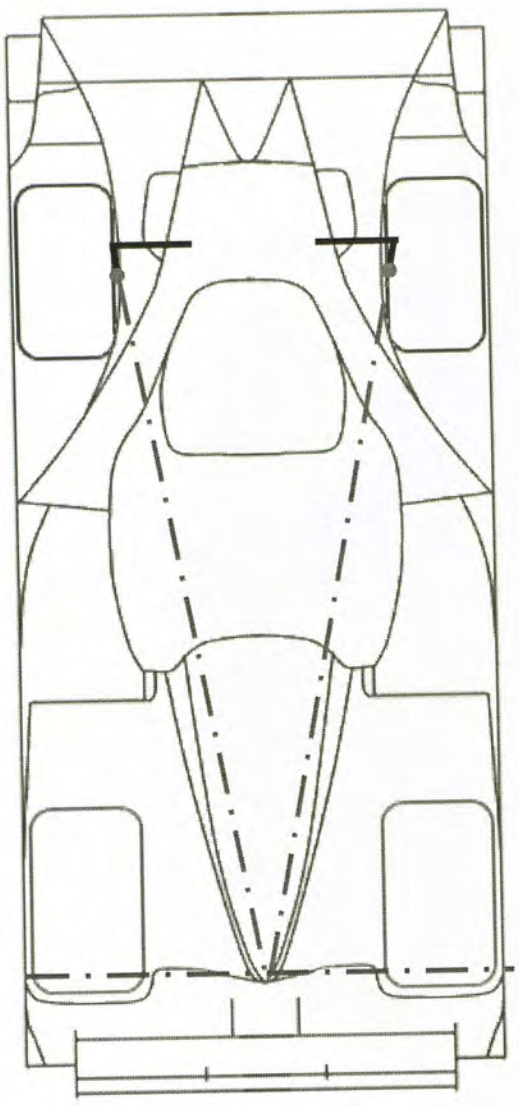


Ackermann Geometry

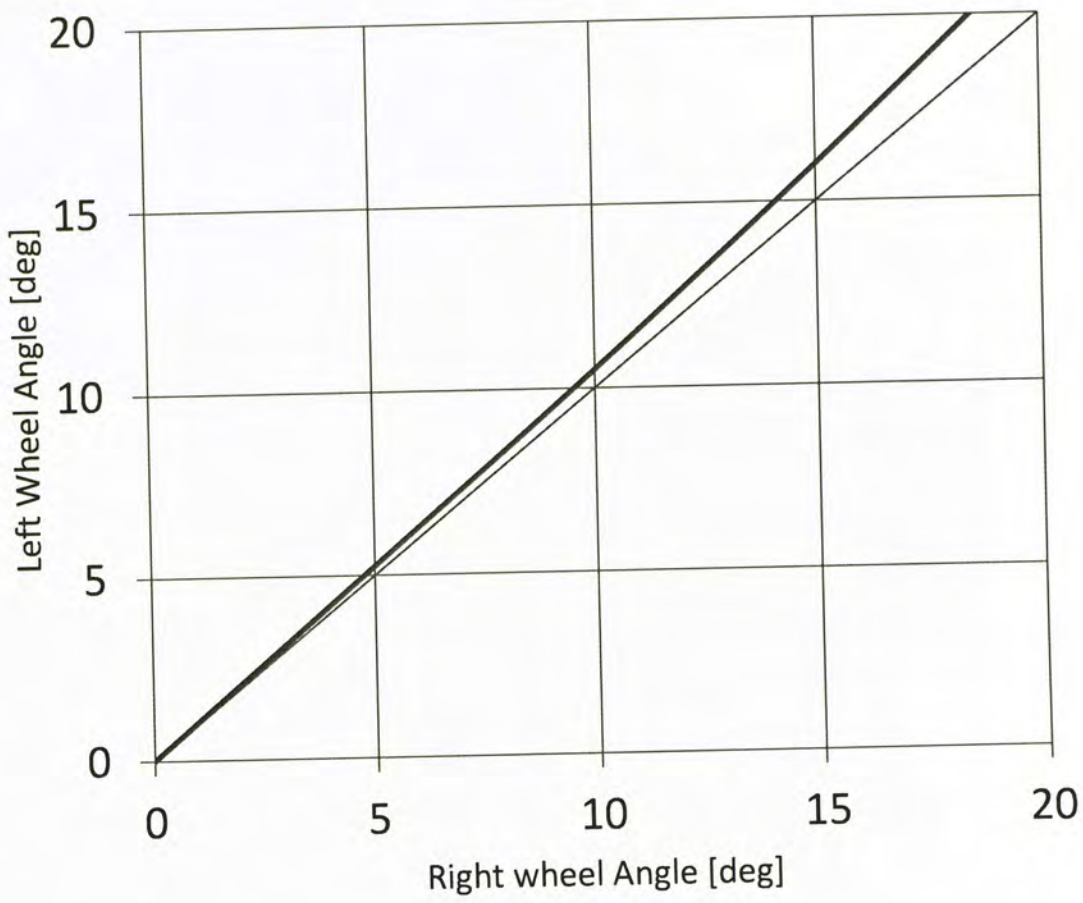




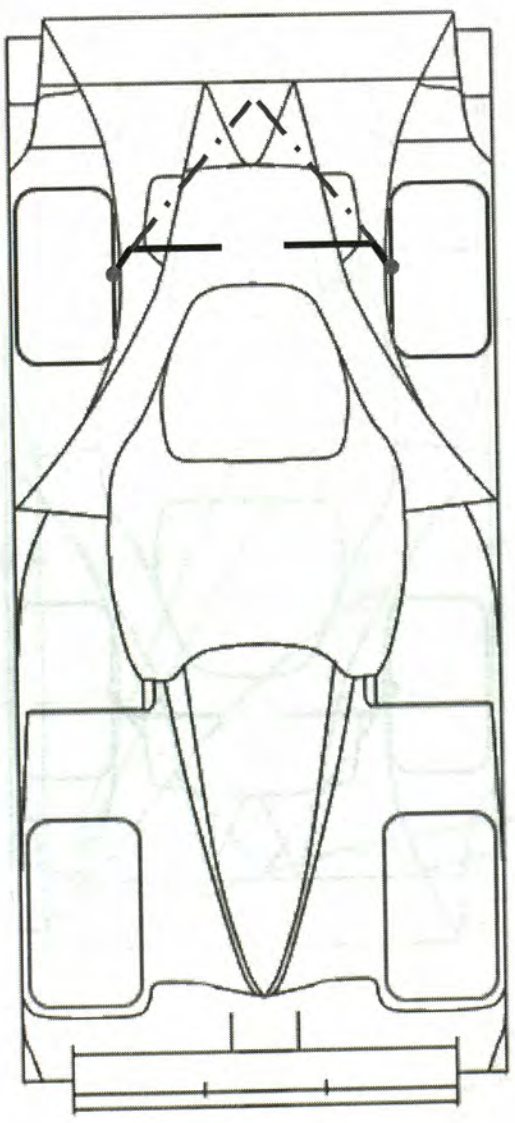
Ackermann Geometry



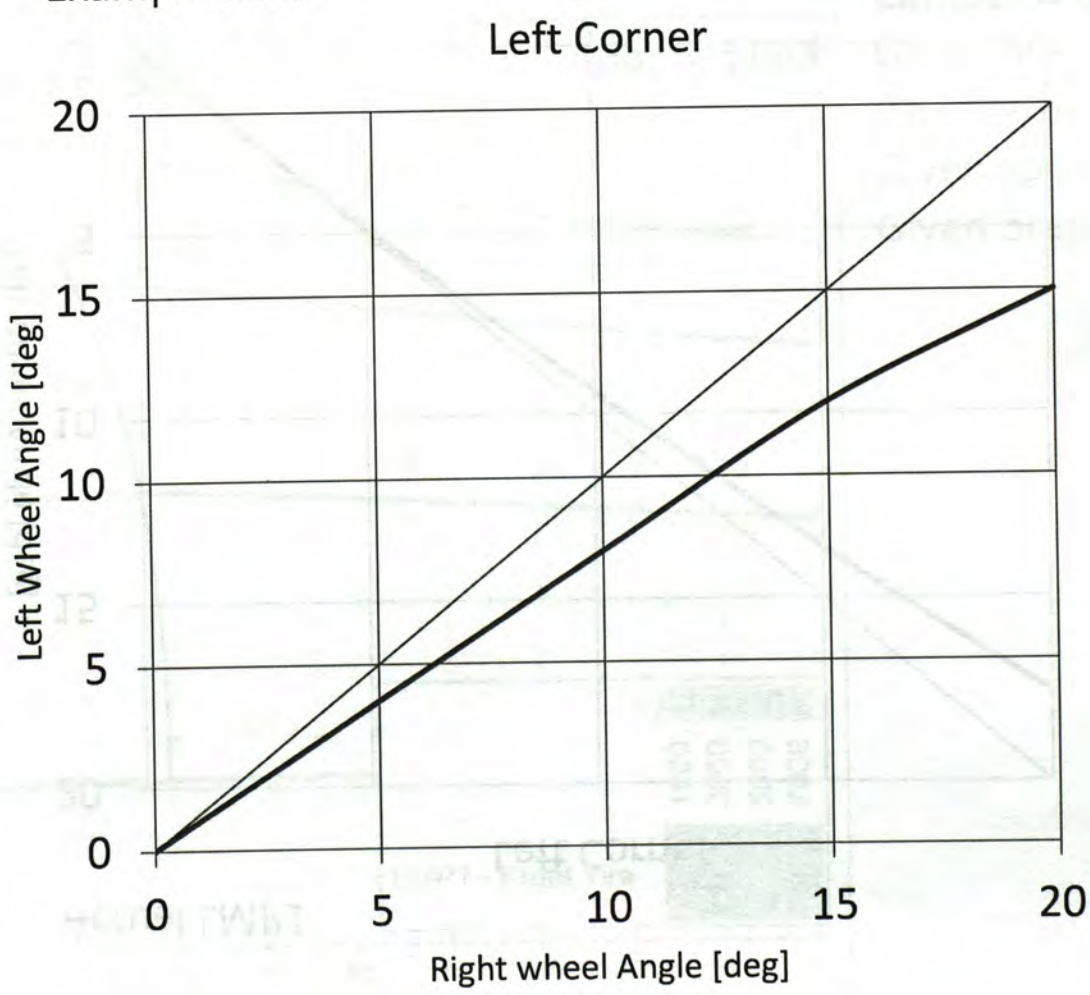
Example Data



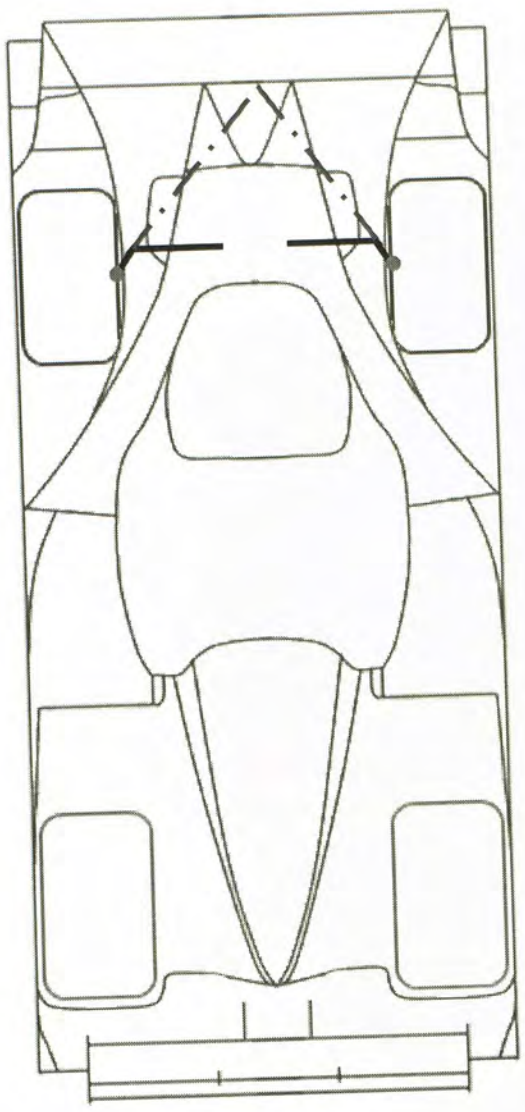
Ackermann Geometry

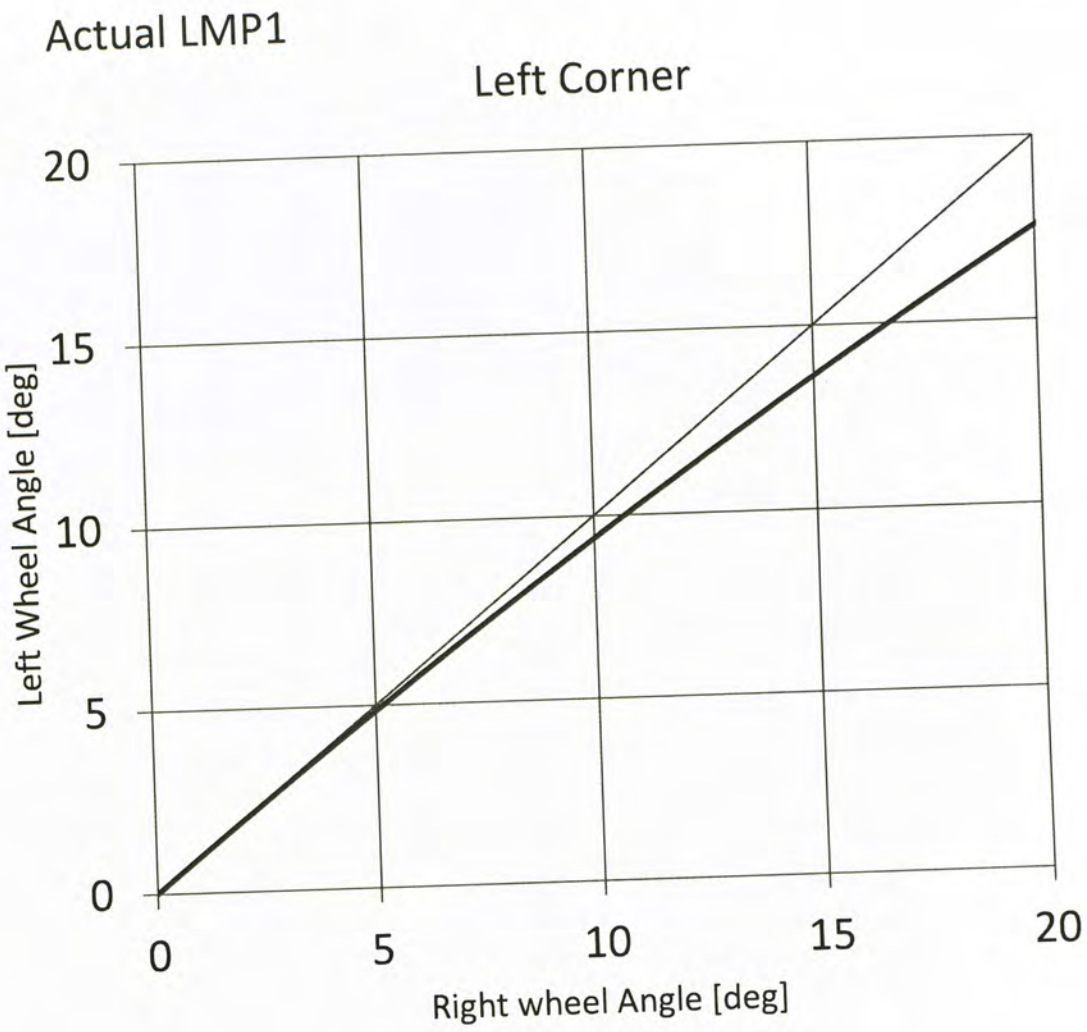


Example Data

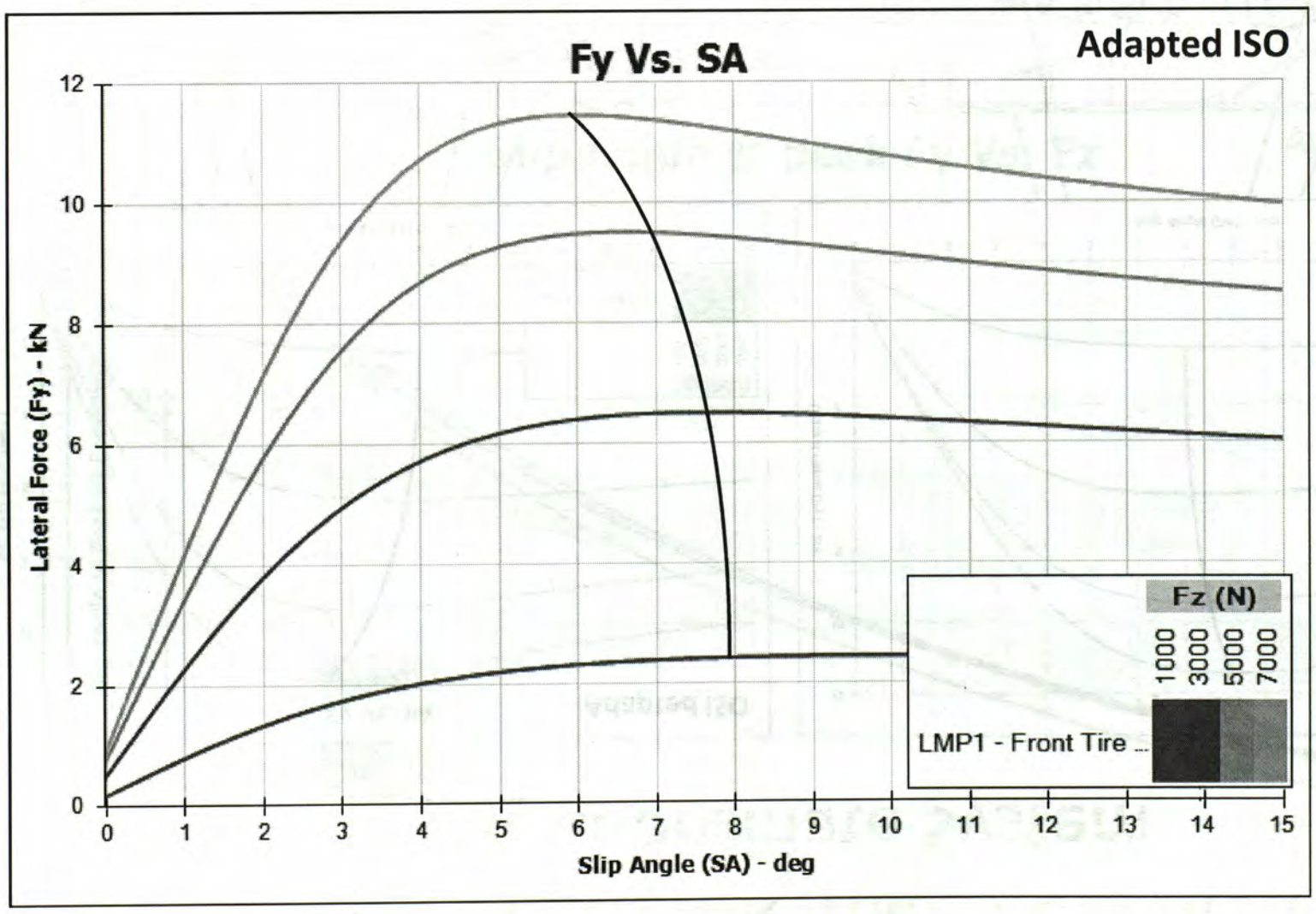


Ackermann Geometry



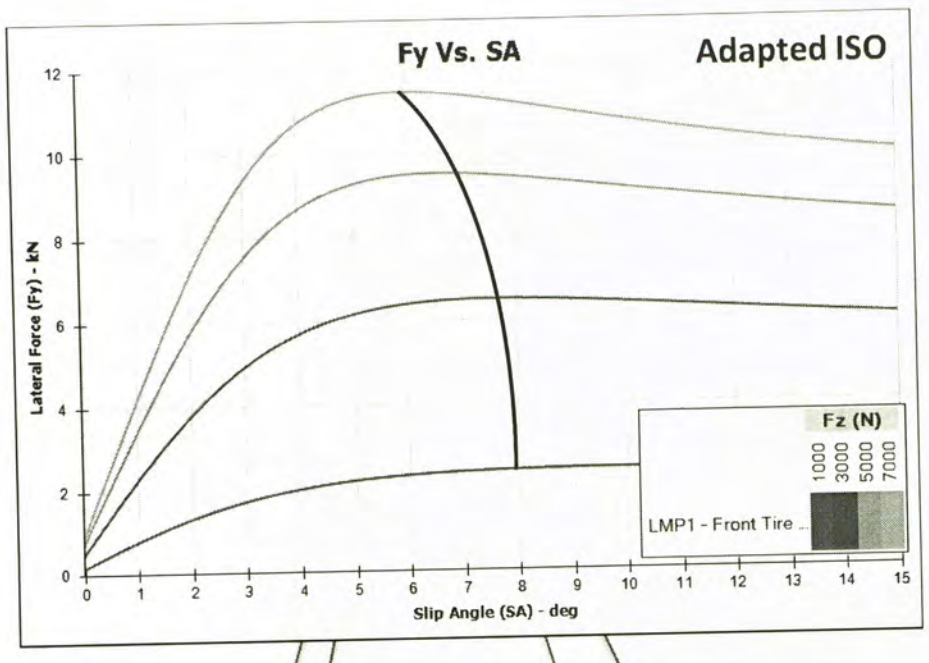


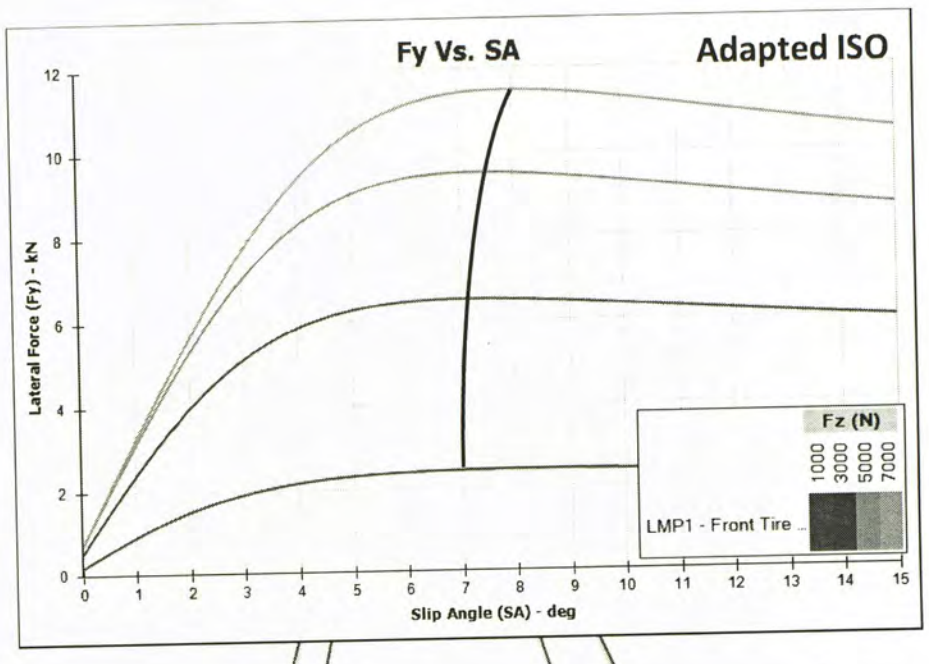
Tire Lateral Force vs. Slip Angle

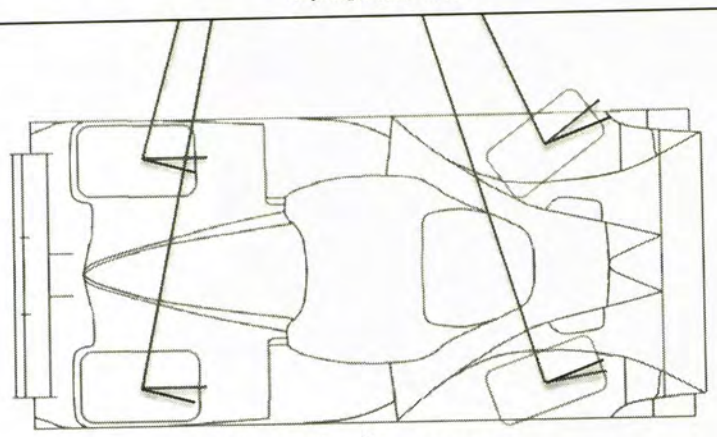


Camber 0 deg
Given tire
Given ground
Given speed
Given pressure

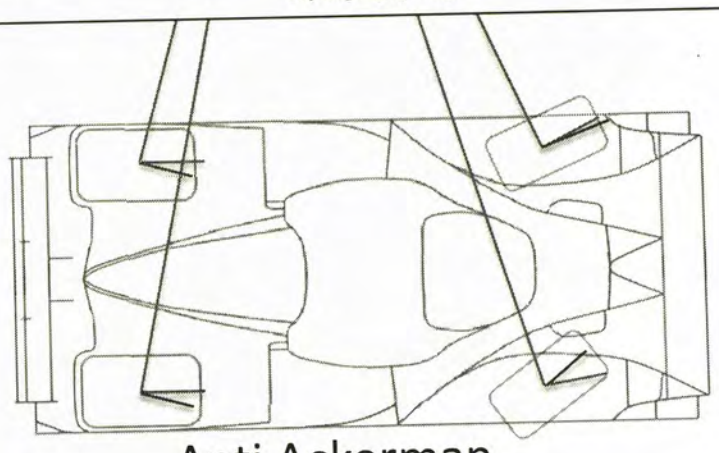
Choice of an Ackermann Geometry
Car Coordinate System







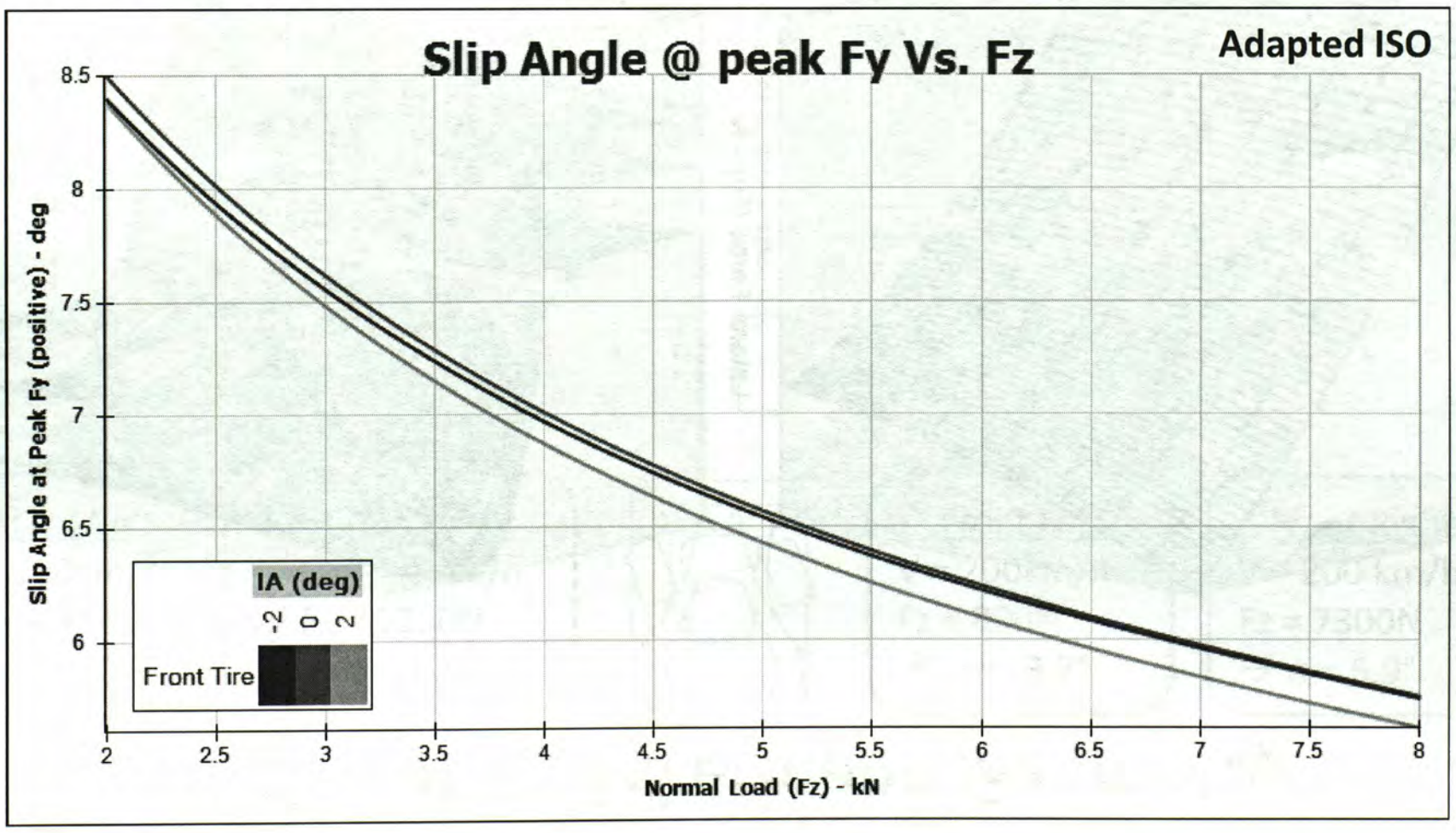
Pro Ackerman



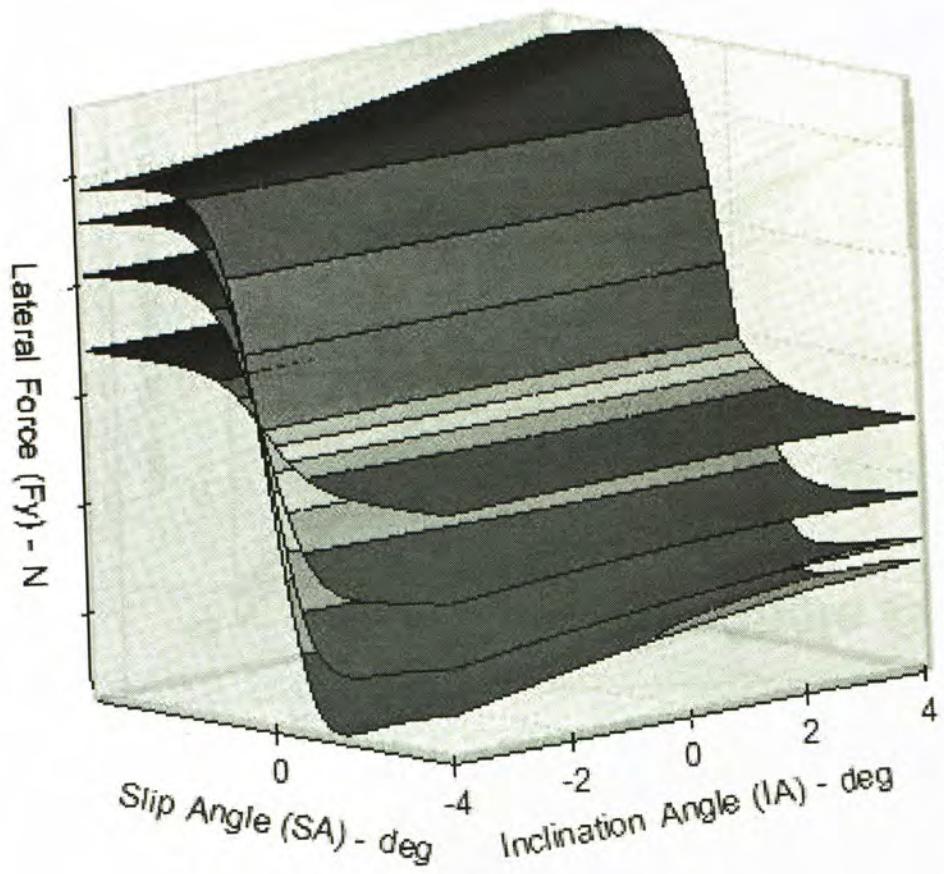
Anti Ackerman

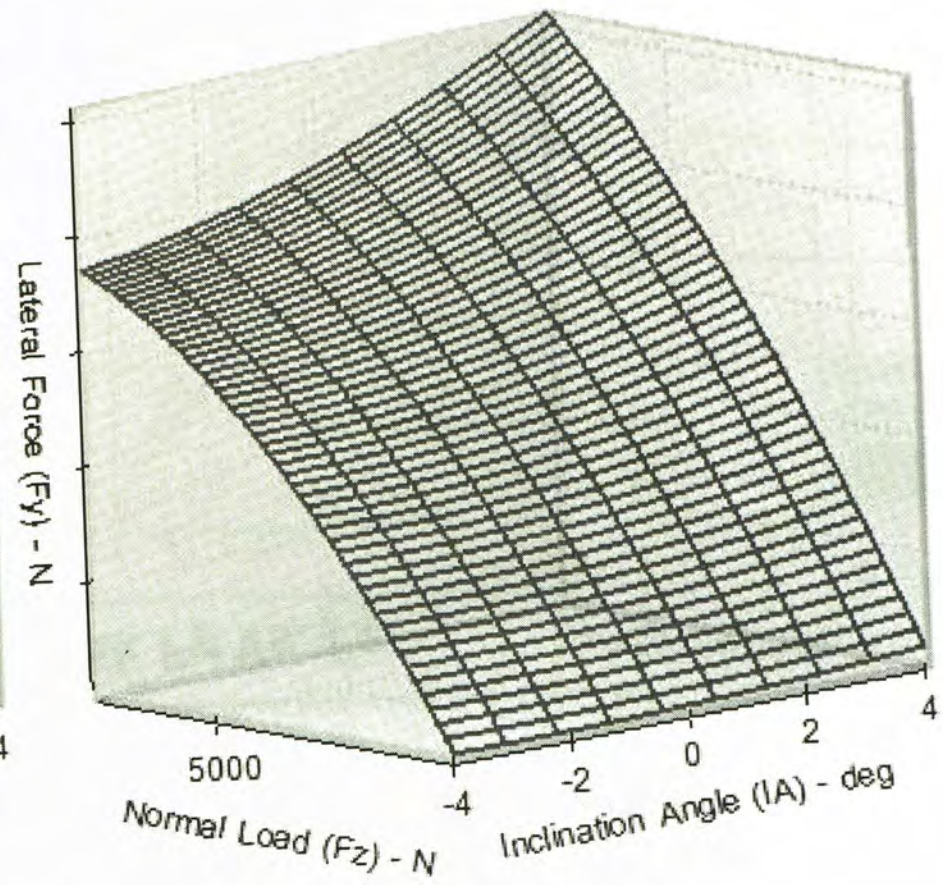
Choice of an Ackermann Geometry

Peak slip angle (slip angle at which the maximum lateral grip occurs) per camber angle for different vertical load.

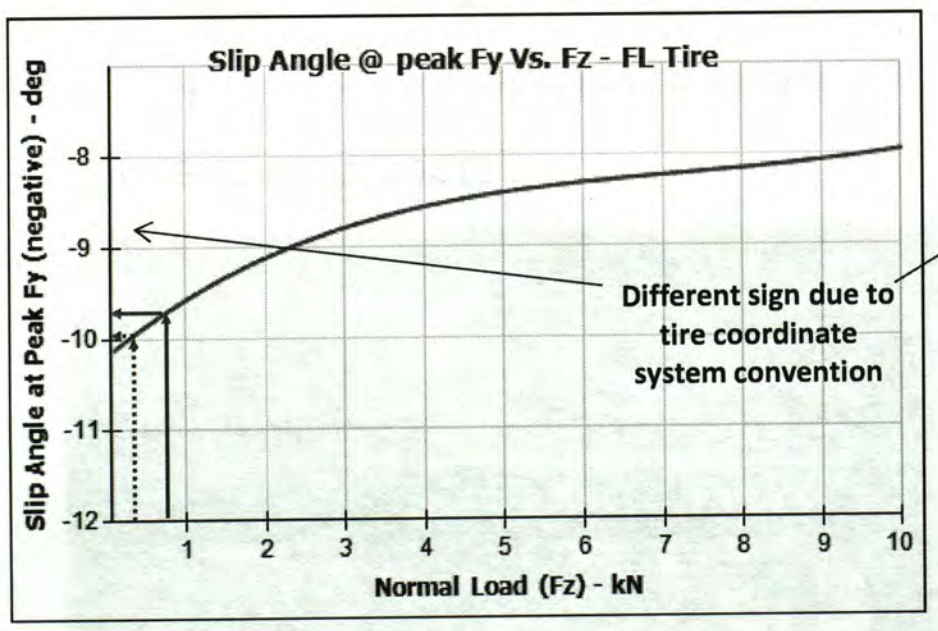


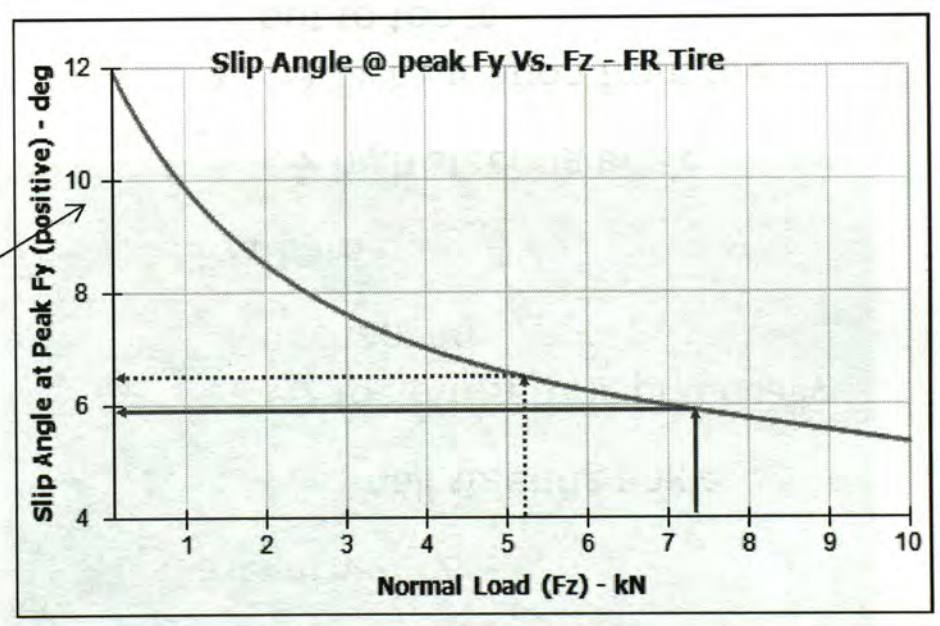
Choice of an Ackermann Geometry





Choice of an Ackermann Geometry
Tire Coordinate System

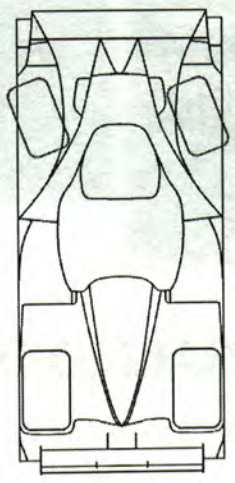




Front Left
V = 100km/h
Fz = 360N
→ α = -10.0°

Front Right
V = 100 km/h
Fz = 5150N
→ α = 6.5°

→ ABS(-10) - 6.5 = 3.5° difference



Front Left
V = 200km/h
Fz = 800N
→ α = -9.7°

Front Right
V = 200 km/h
Fz = 7300N
→ α = 5.9°

→ ABS(-9.7) - 5.9 = 3.8° difference

Choice of an Ackermann Geometry

How to decide pro or reverse Ackermann with no tire model… but a very sensitive driver?

	Slow Corner	Fast Corner
Static Toe In	+++	-
Static Toe Out	-	+++

Driver's comments :

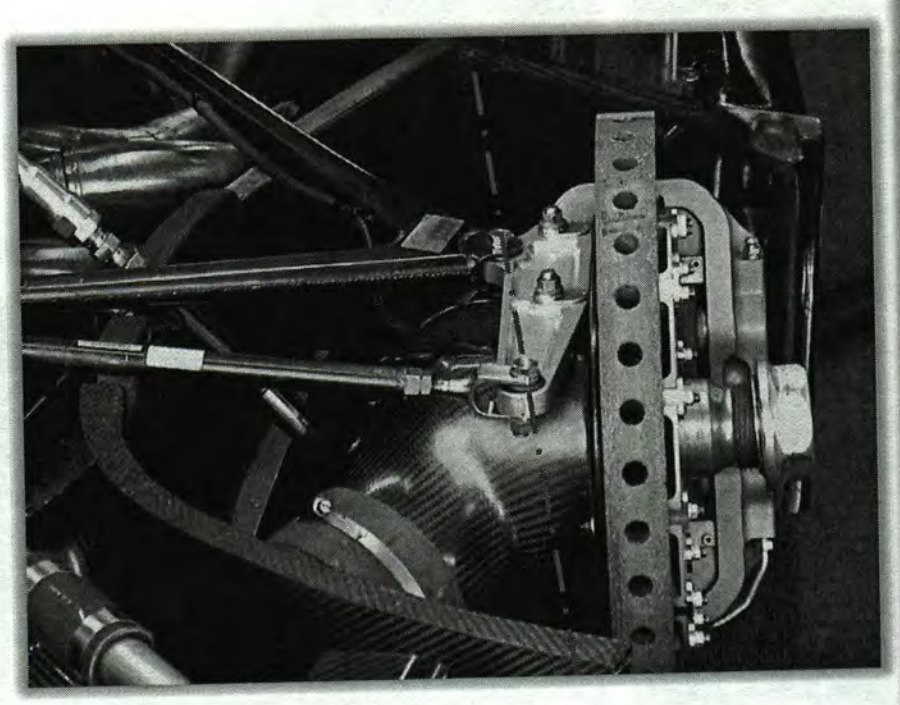
- Toe-out is better in fast corner.
- Toe-in is better in slow corner.

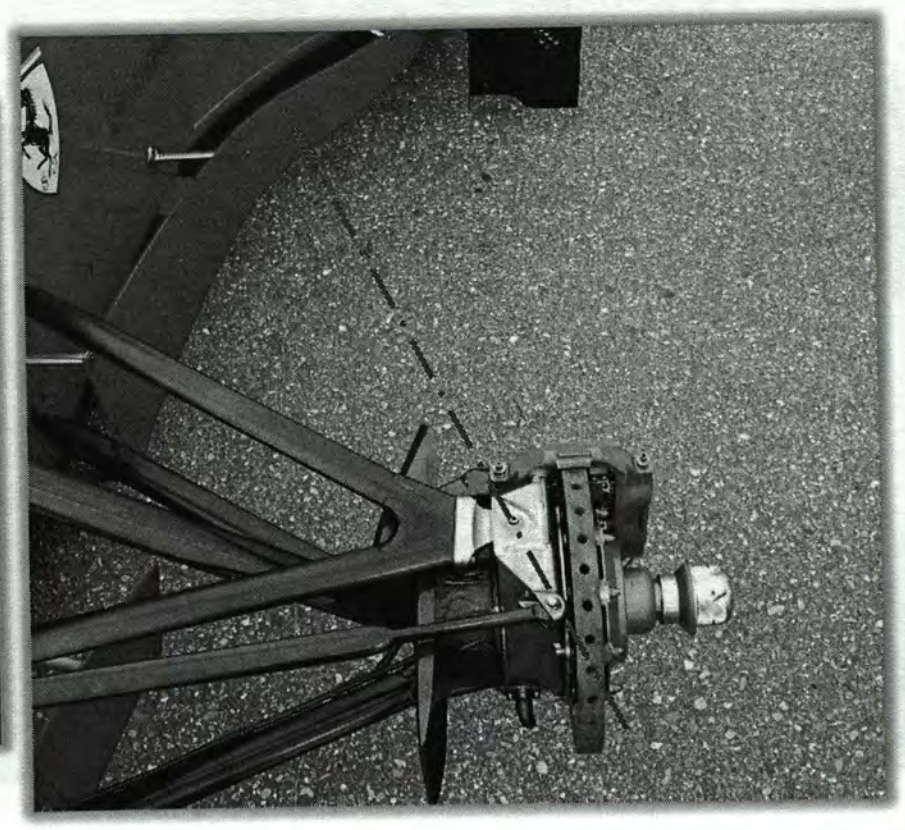
Solution is Static Toe Out with Reverse Ackermann Geometry because :

- Fast corner
  - ↔ small steering angle
  - ↔ toe out setting practically unchanged
- Slow corner
  - ↔ high steering angle
  - ↔ fast variation from toe out to toe in

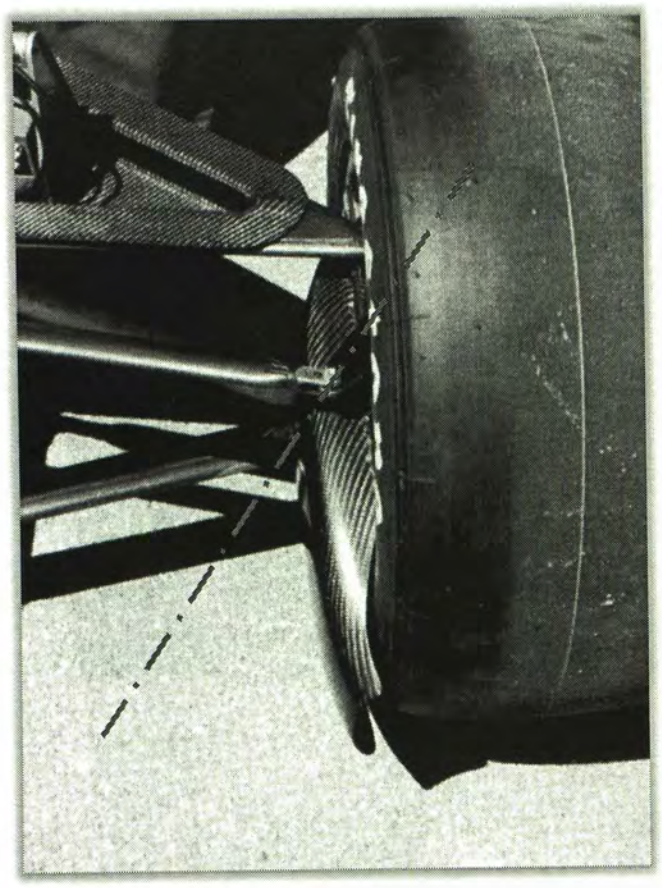
This is an example, not a recipe!

Ackermann Geometry



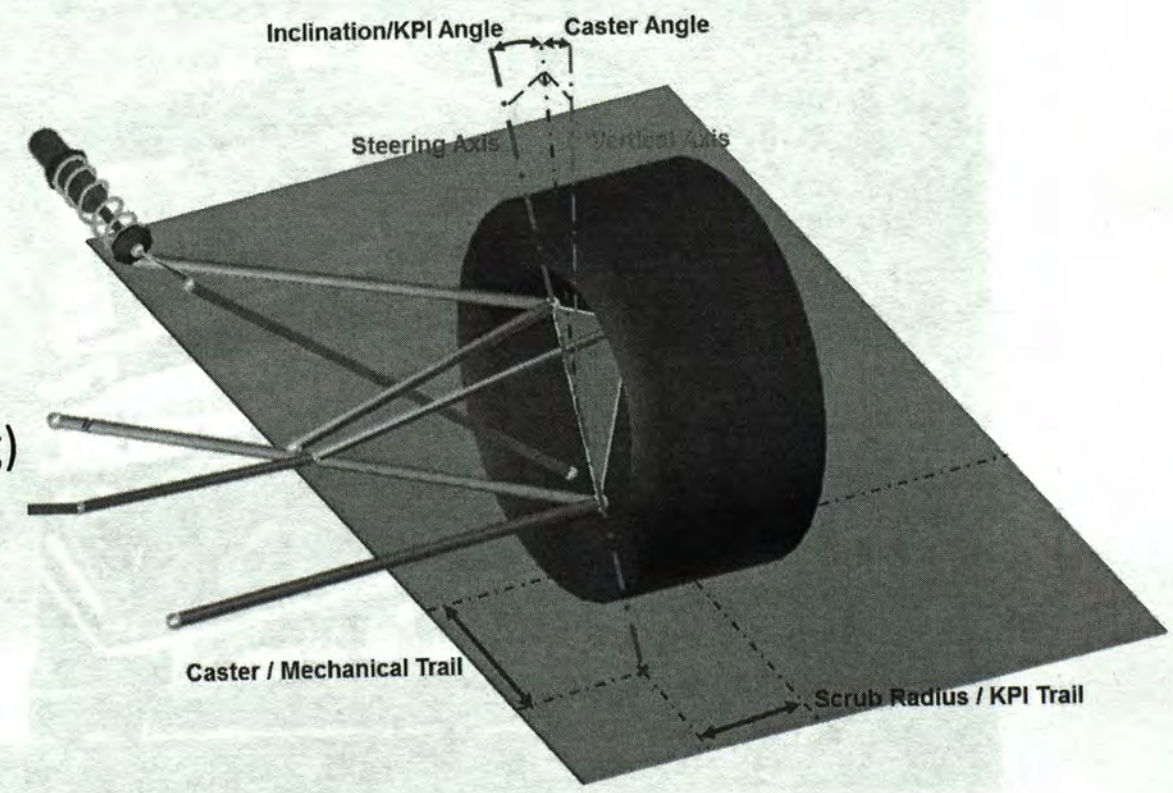


Reverse Ackermann Geometry

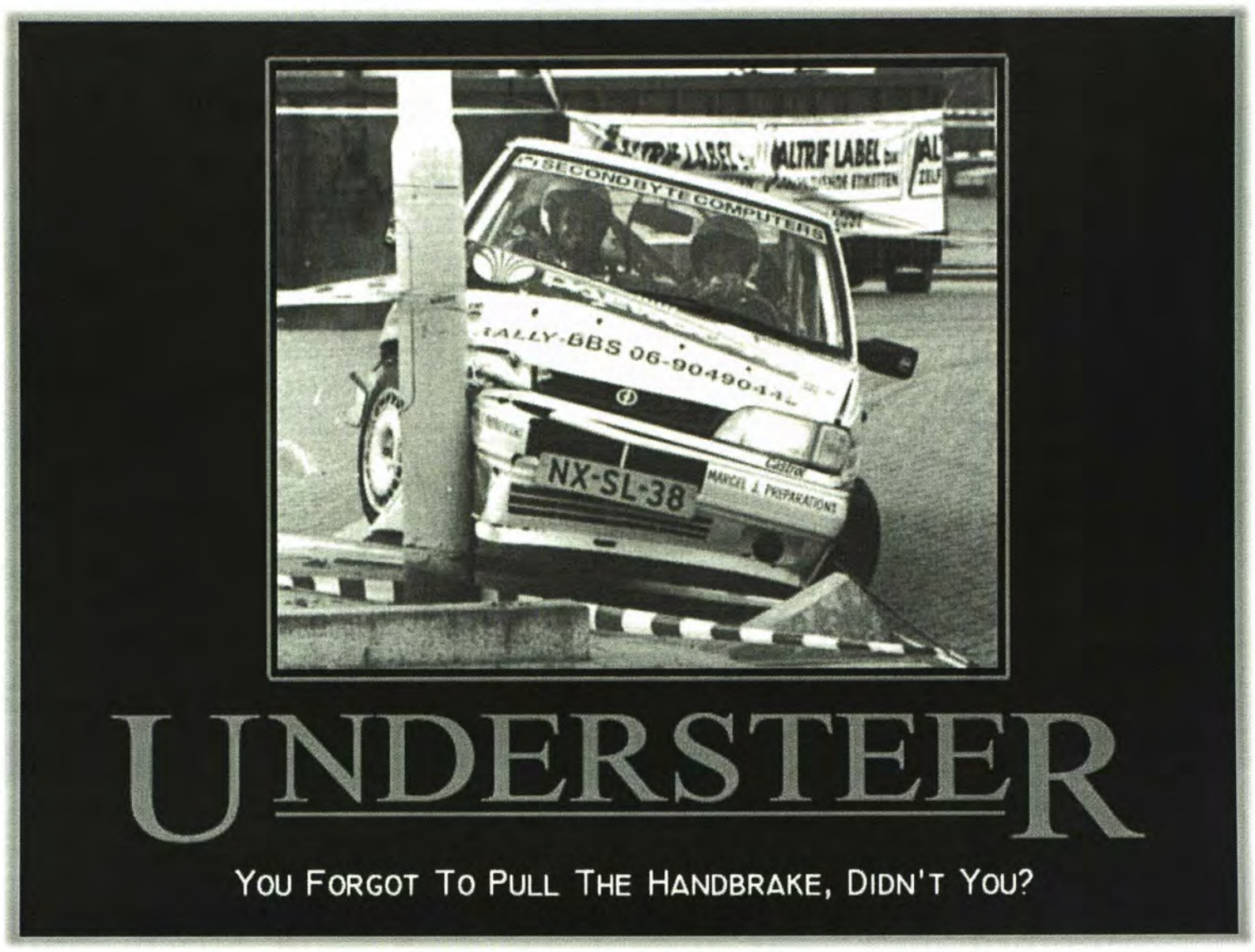


Steering Variations

- Tire KPI trail
- Tire pneumatic trail
- Camber
- Ride heights (steer jacking)
- Roll angle (steer jacking)
- Pitch (steer jacking)
- Corner weights (steer jacking)
- Steering stiffness
- Front track
- Wheelbases
- ...



5. Steady State - Basics

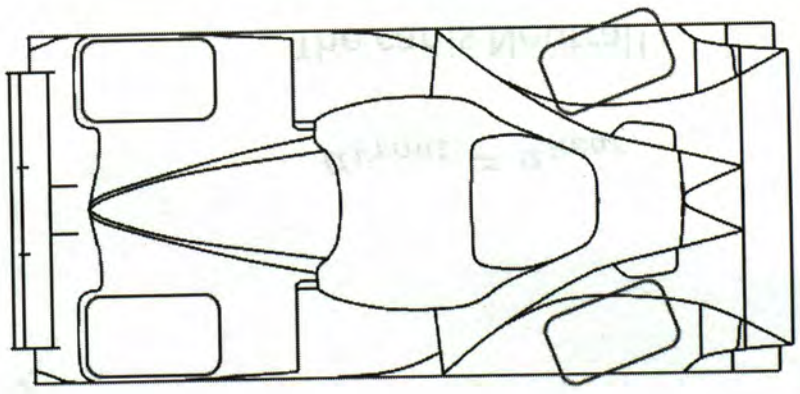


Steady State

In this Chapter we are going to define the understeering and oversteering behavior.

First with a bicycle model

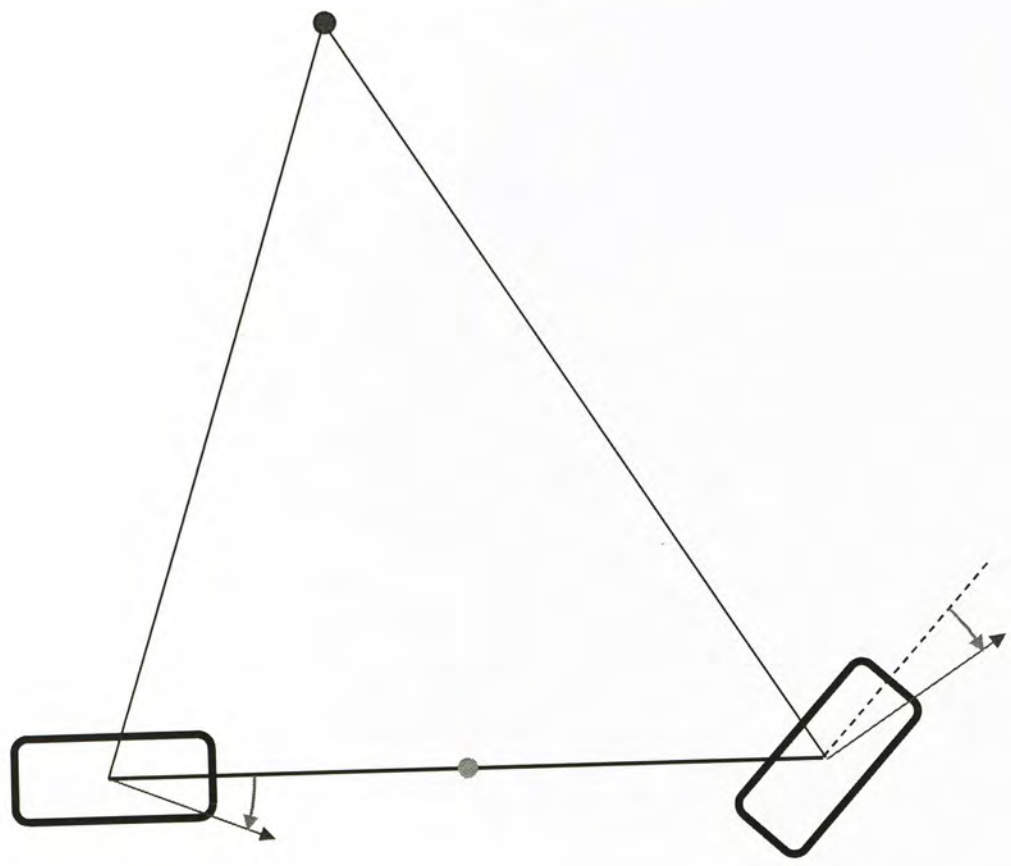
Then with a four wheel model



First definition

(incomplete and inaccurate) based only on front and rear chassis slip angle

Neutral Car



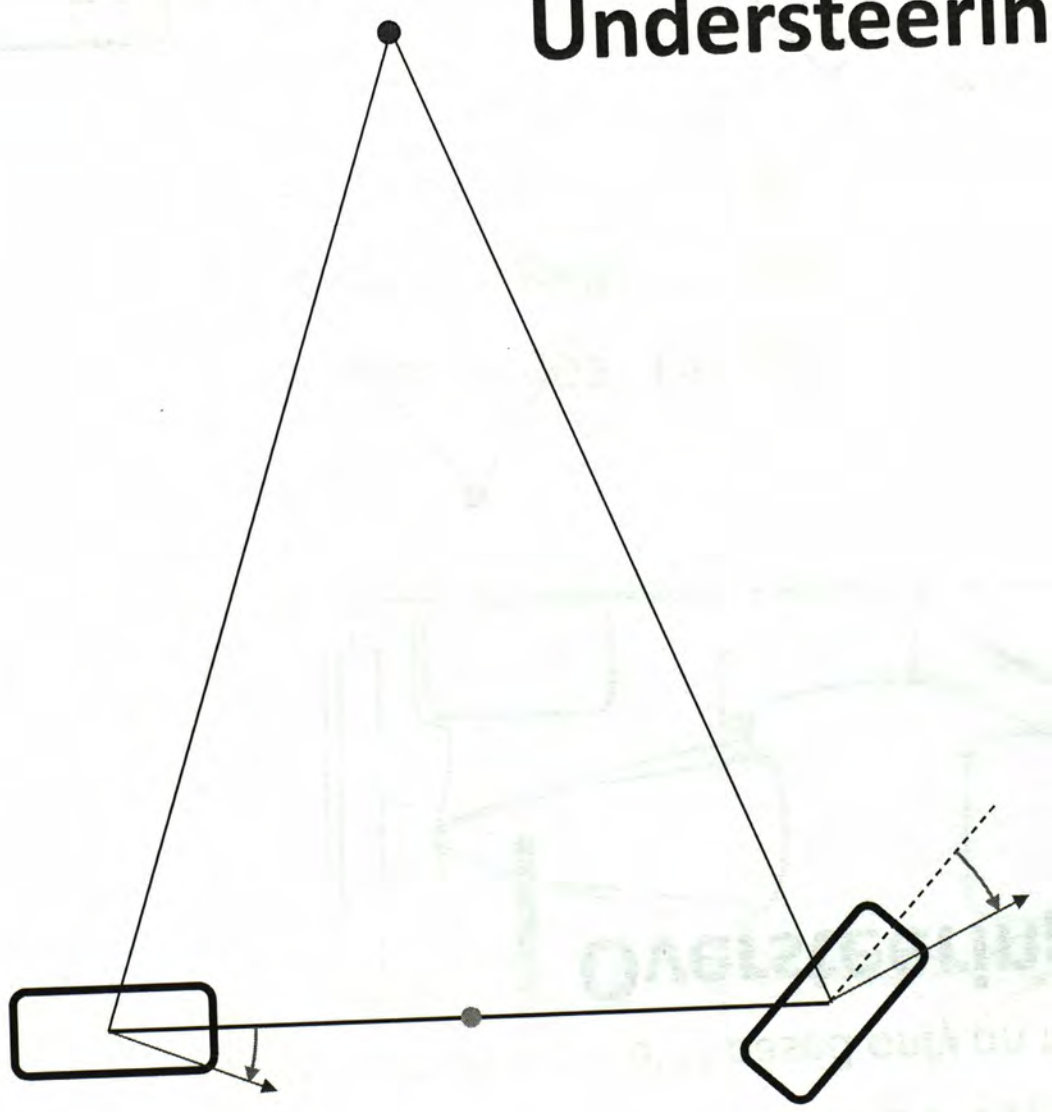
$\alpha_{Front} = \alpha_{Rear}$

The car is Neutral!

First definition

(incomplete and inaccurate) based only on front and rear chassis slip angle

Understeering Car



$\alpha_{Front} > \alpha_{Rear}$

The car understeers!

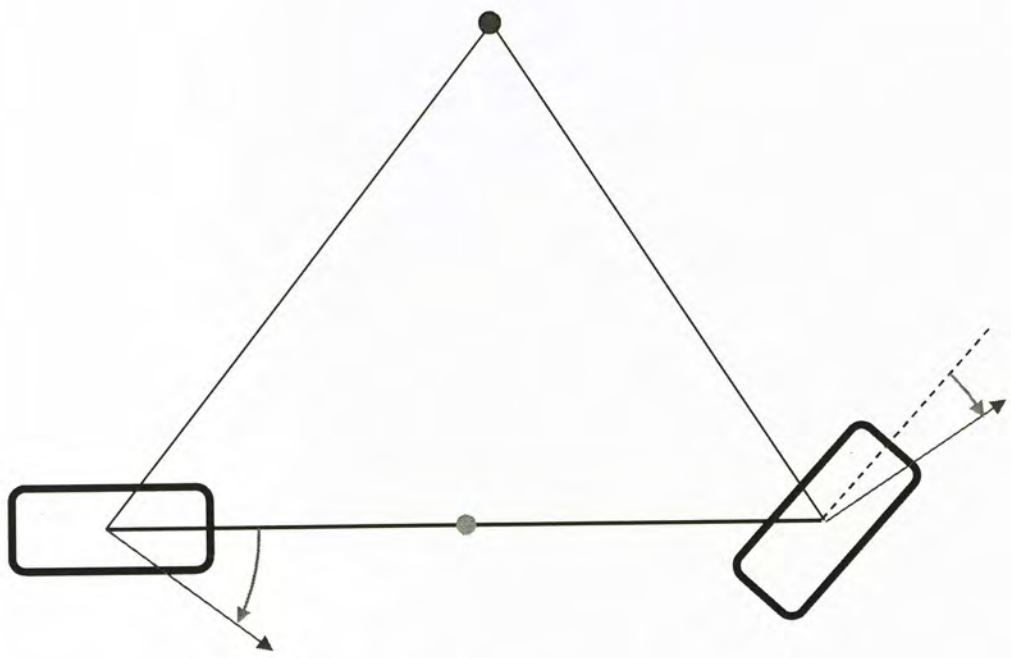
First definition

(incomplete and inaccurate) based only on front and rear chassis slip angle

Oversteering Car

$\alpha_{Front} < \alpha_{Rear}$

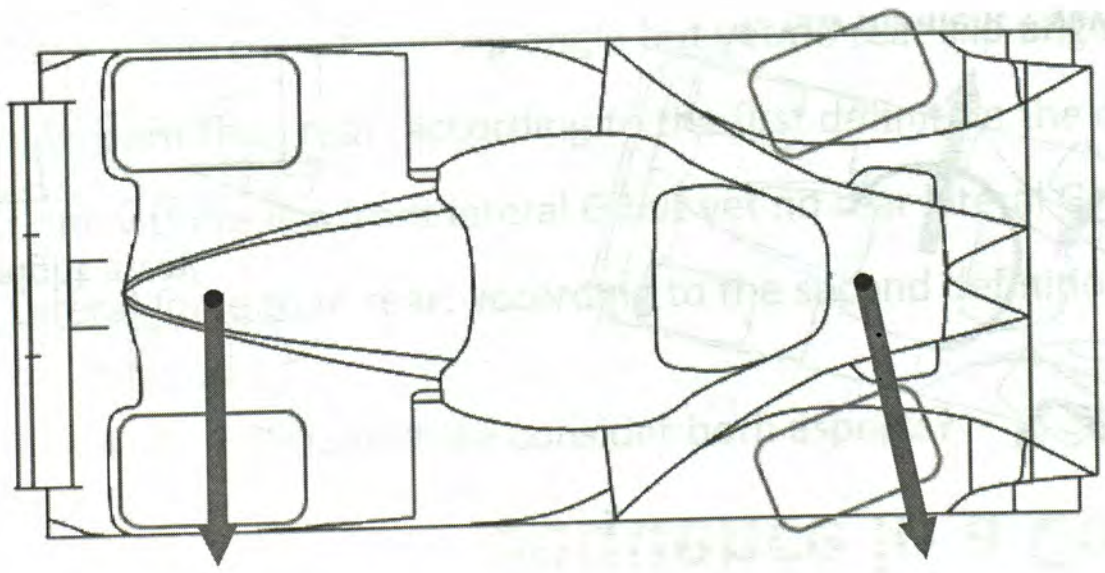
The car oversteers!



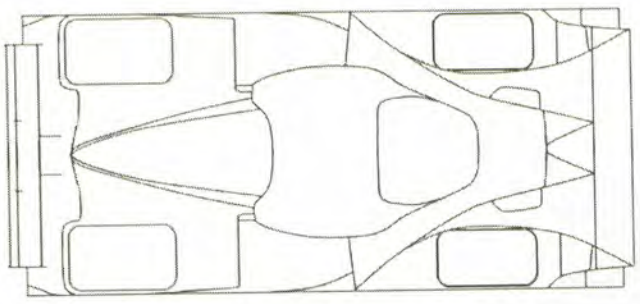
Second definition

(incomplete and inaccurate) based only on front and rear chassis Lateral G

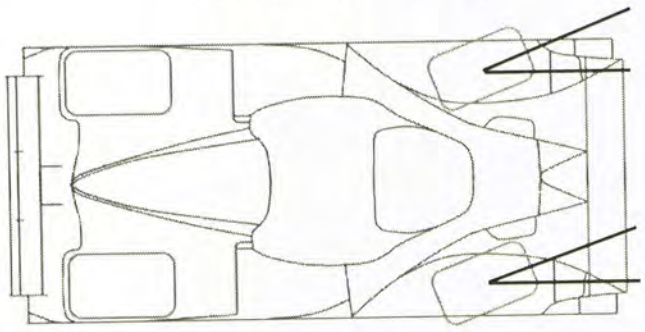
If Front Lat. Acc. ≈ Rear Lat. Acc.	:	Neutral
If Front Lat. Acc. < Rear Lat. Acc.	:	U/S
If Front Lat. Acc. > Rear Lat. Acc.	:	O/S



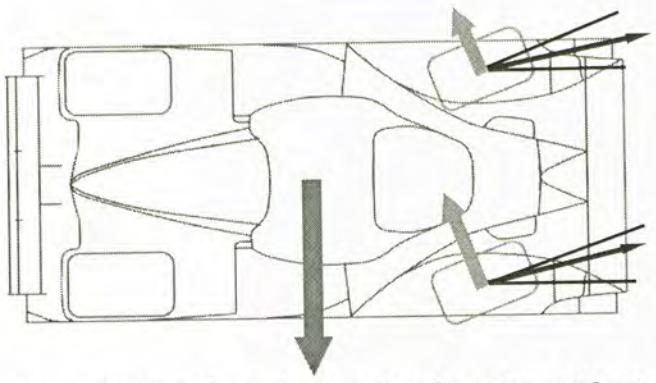
Sequence in a Corner



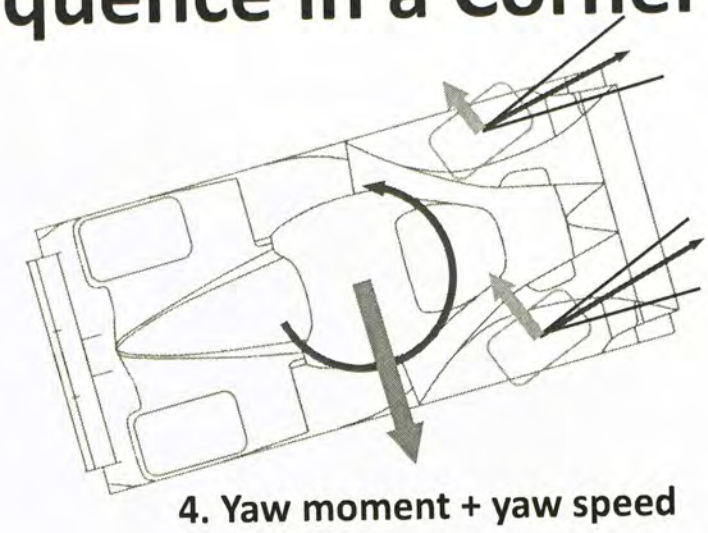
1. Straight away



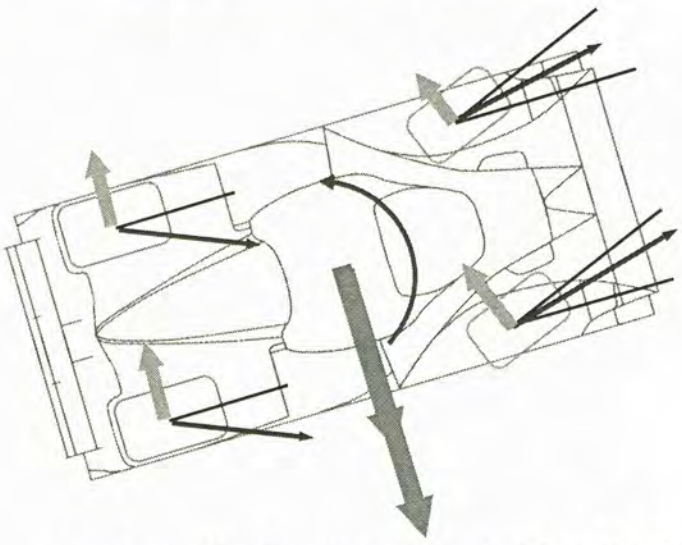
2. Front Wheel Steering



3. Front Wheel Slip Angle, Front Lateral Grip, Lateral Acceleration



4. Yaw moment + yaw speed



5. Rear Slip Angles, Rear Lateral Grip, more Lateral Acceleration, more yaw speed but less yaw moment

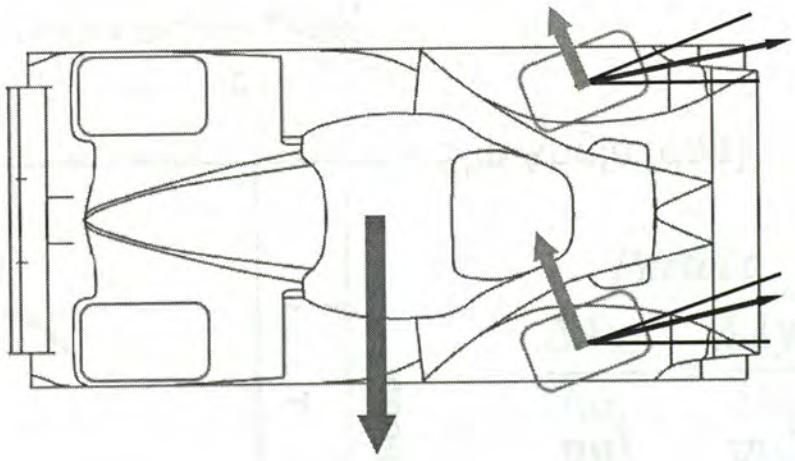
Under and Oversteer Definition

So is understeer and oversteer defined....

With front and rear slip angle?

Or front and rear chassis lateral G?

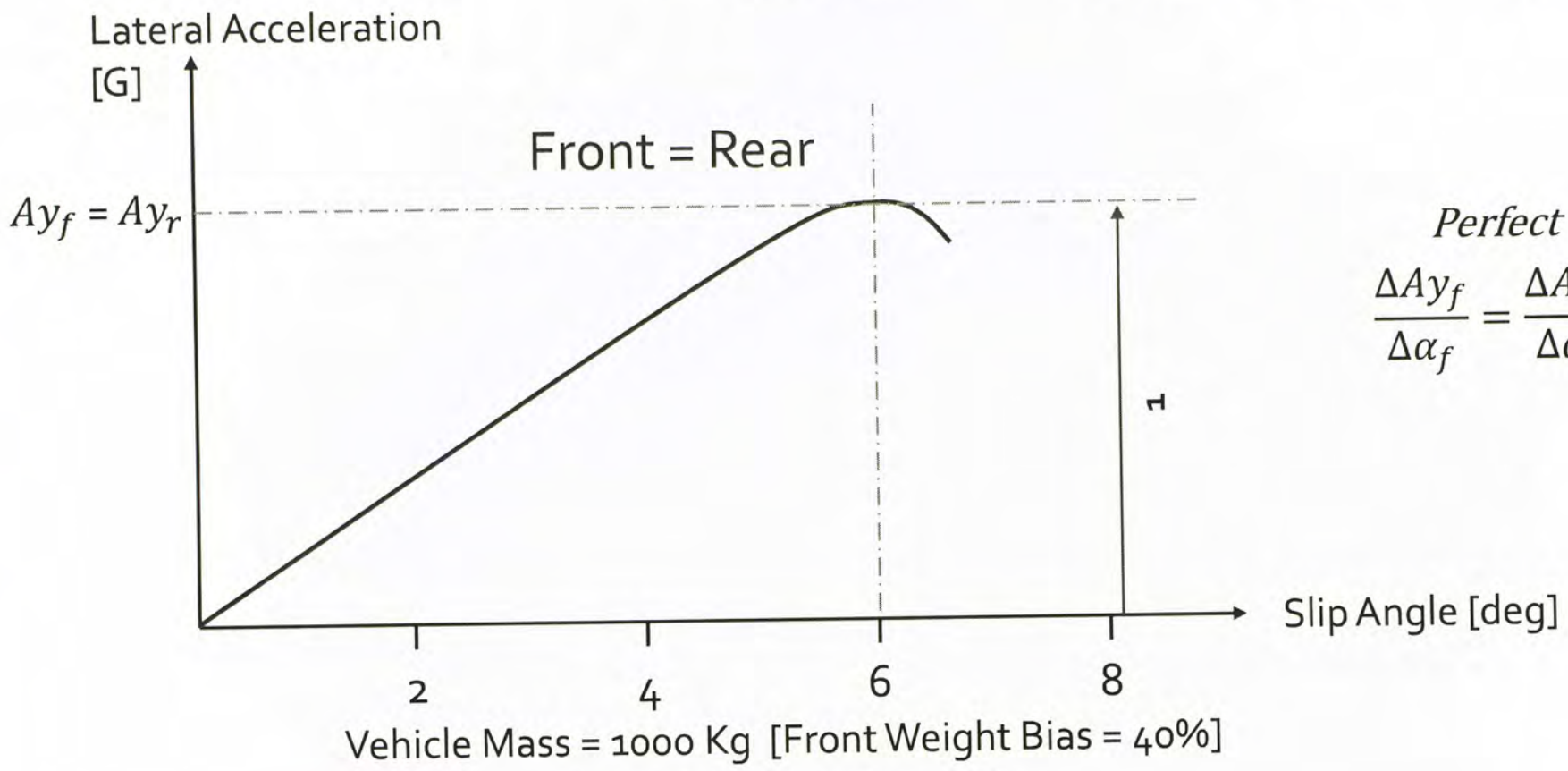
Or both?



- At the corner entry there is a front slip angle but yet no rear slip angle.
- Bigger slip angle front than rear: according to the first definition the car understeers.
- At the corner entry there is a front lateral G but yet no rear lateral G
- Bigger front Lateral force than rear: according to the second definition the car oversteers.
- So .... Which is which...? Shouldn't we consider both aspects?

Choice between Balance and Grip

Suppose that $Fy_f/Fy_r = Fz_f/Fz_r \rightarrow Ay_f = Ay_r$



Perfect

$$\frac{\Delta Ay_f}{\Delta \alpha_f} = \frac{\Delta Ay_r}{\Delta \alpha_r}$$

Vehicle Mass = 1000 Kg [Front Weight Bias = 40%]

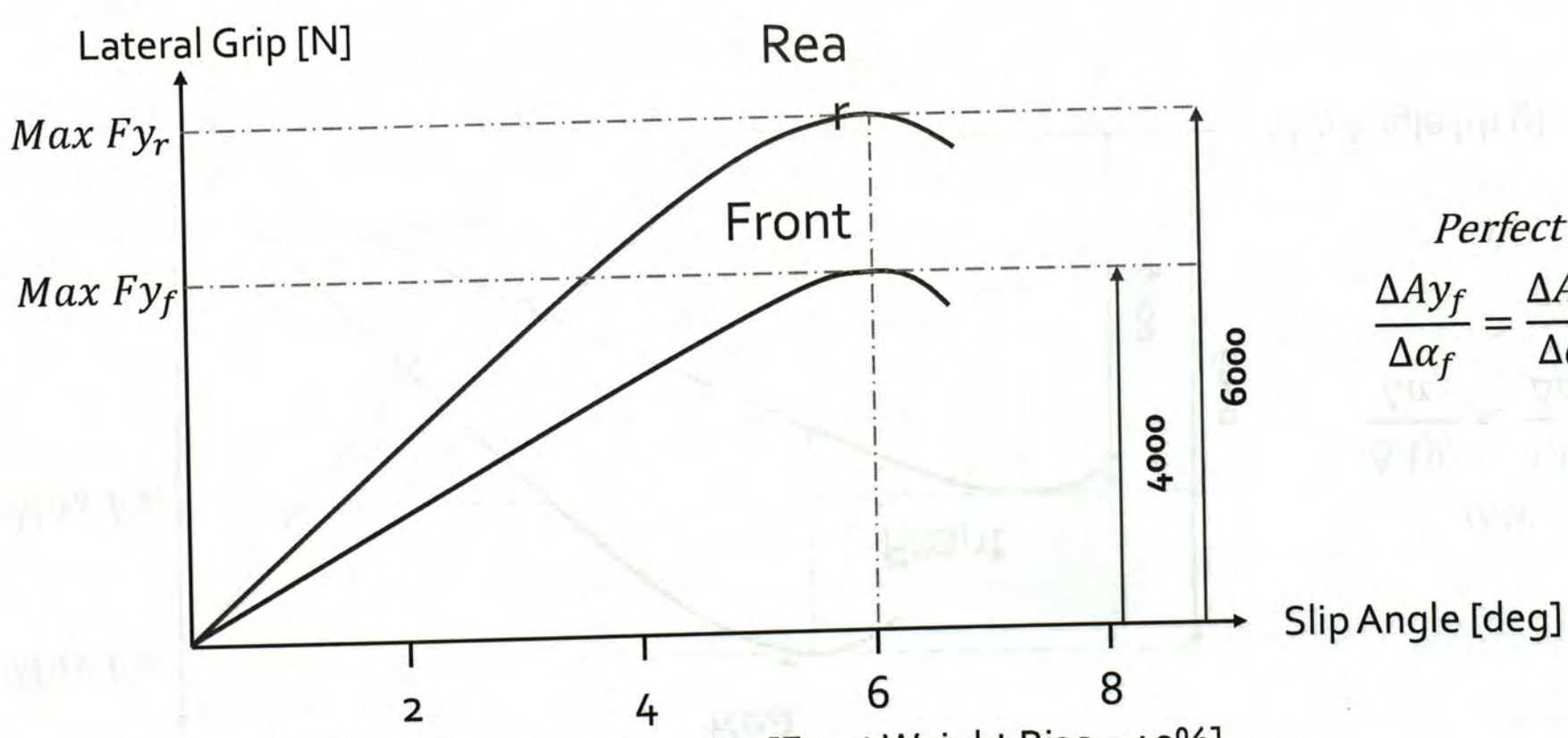
Lateral Acceleration = 1G

Vehicle is simplified to 2 wheels

Same lat. acc. front and rear, same slip angle front and rear ⇒ Neutral

Choice between Balance and Grip

Suppose that $Fy_f/Fy_r = Fz_f/Fz_r \rightarrow Ay_f = Ay_r$



Perfect

$$\frac{\Delta Ay_f}{\Delta \alpha_f} = \frac{\Delta Ay_r}{\Delta \alpha_r}$$

Vehicle Mass = 1000 Kg [Front Weight Bias = 40%]

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Vehicle is simplified to 2 wheels

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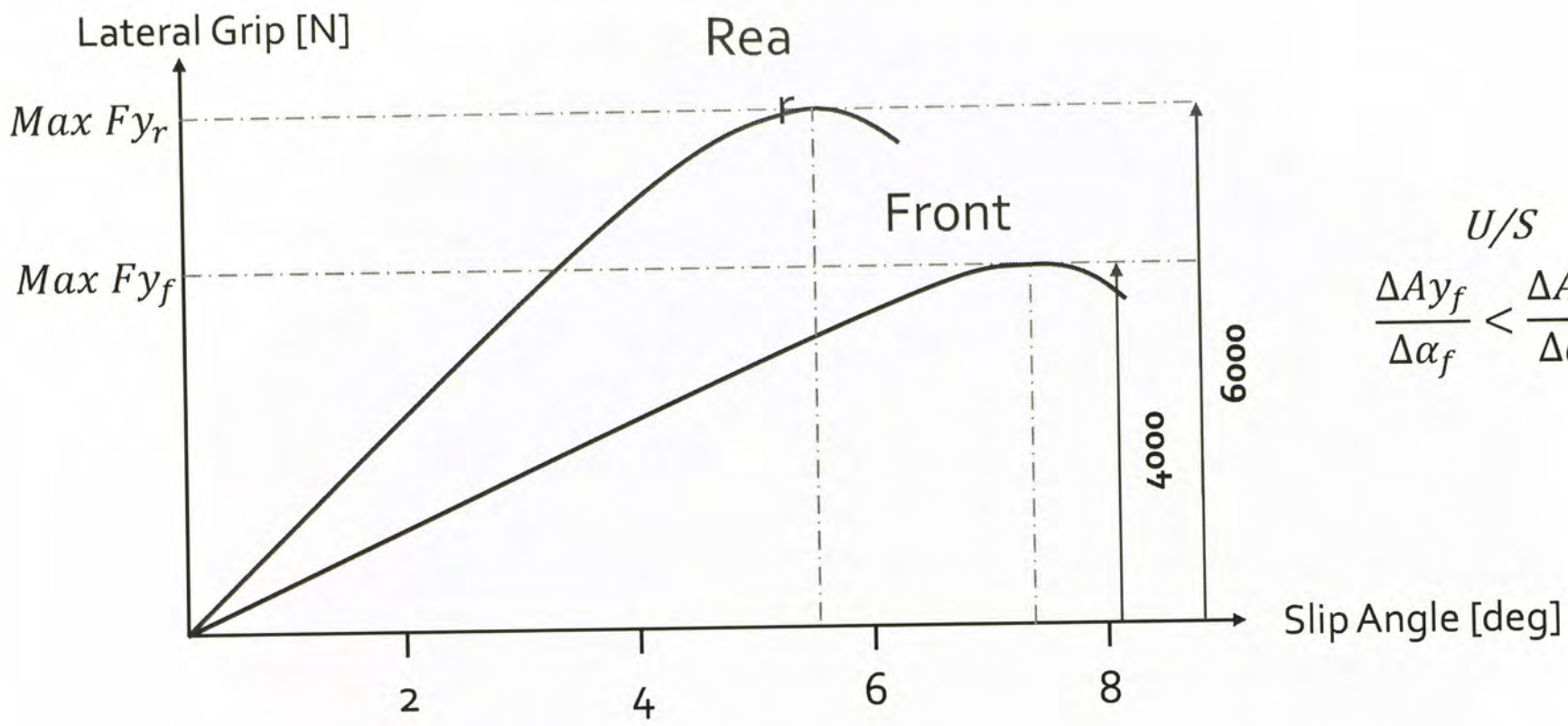
Choice between Balance and Grip



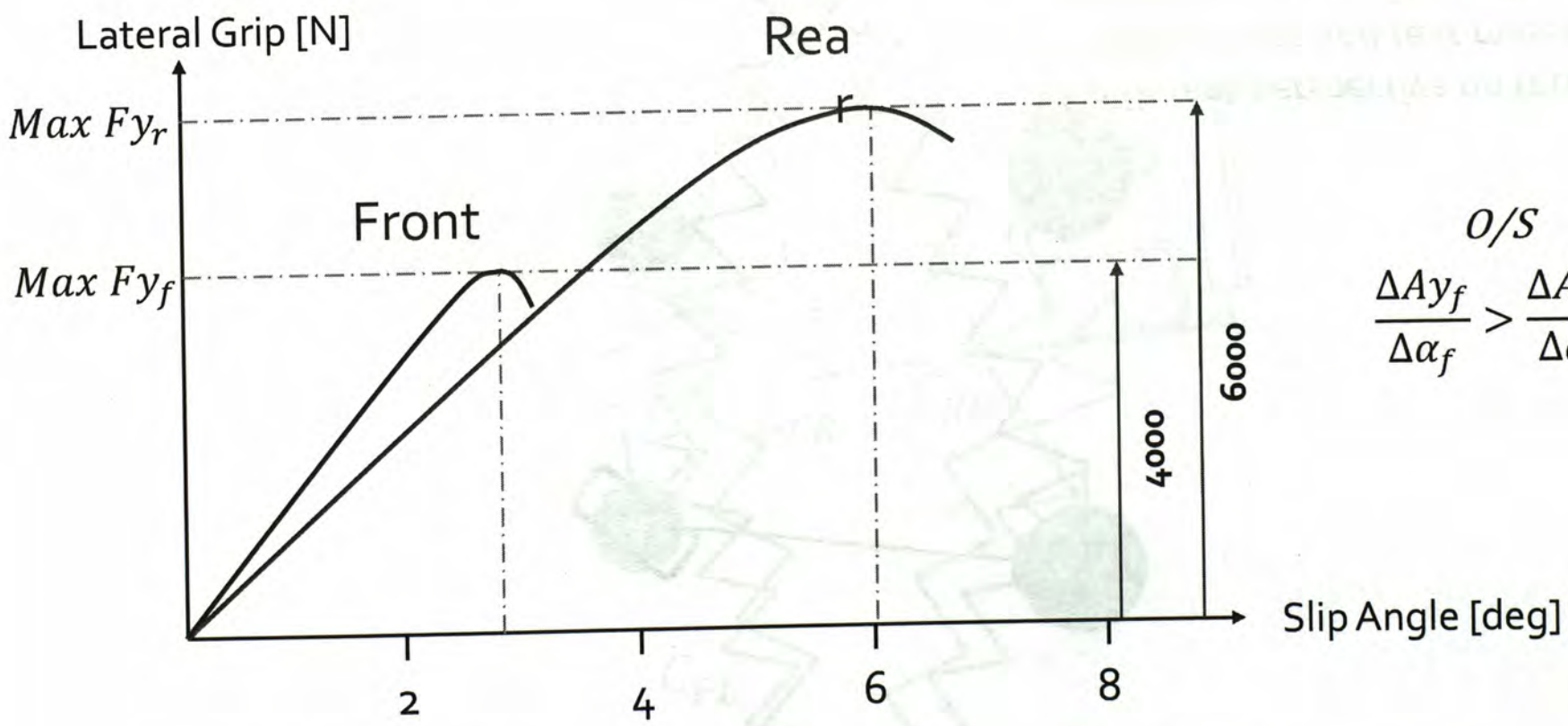
$$U/S \quad \frac{\Delta Ay_f}{\Delta \alpha_f} < \frac{\Delta Ay_r}{\Delta \alpha_r}$$

Same grip, but front slip angle is bigger ⇒ UNDERSTEER

Maximum front and rear grip for 2 different slip angles ⇒

Impossible to have a neutral car with the maximum grip on each end

Choice between Balance and Grip



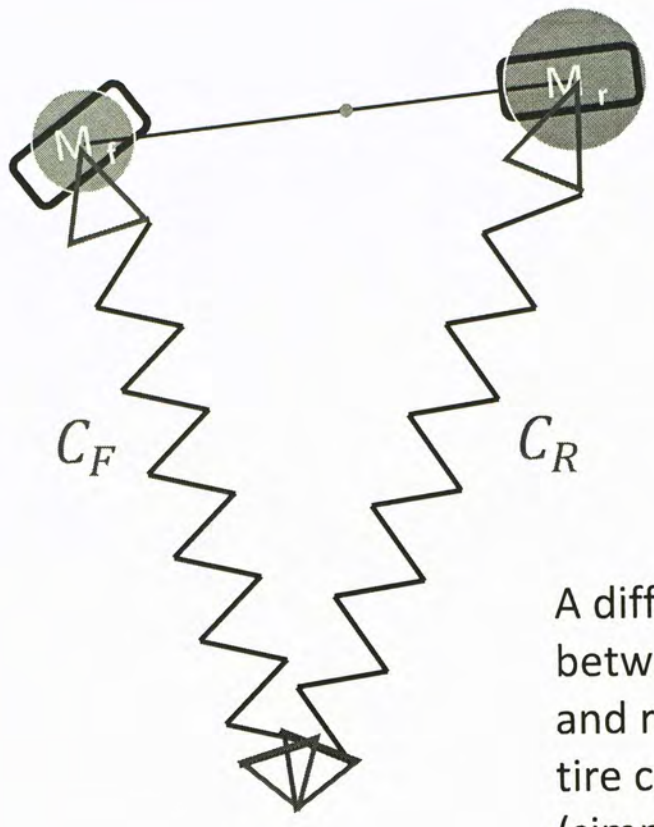
$$O/S \quad \frac{\Delta Ay_f}{\Delta \alpha_f} > \frac{\Delta Ay_r}{\Delta \alpha_r}$$

Same grip, but rear slip angle is bigger ⇒ OVERSTEER

Maximum front and rear grip for 2 different slip angles ⇒

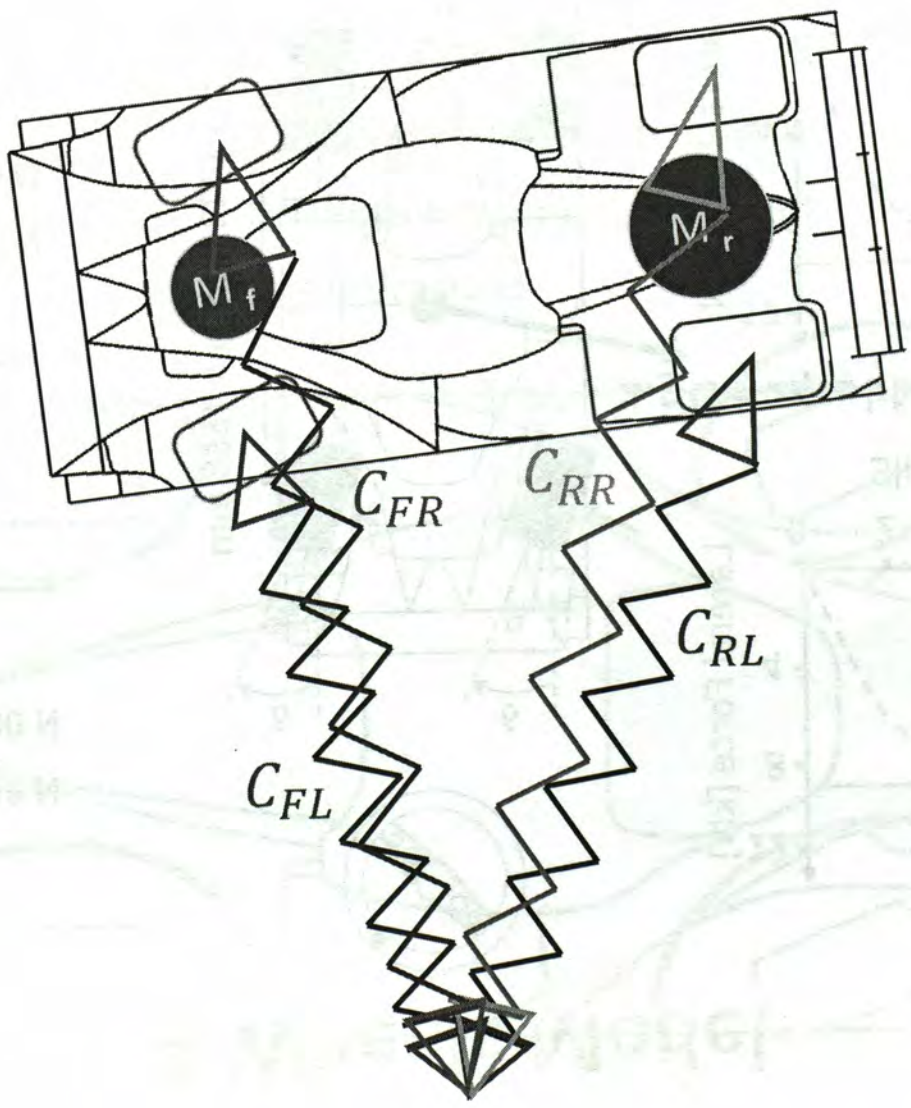
Impossible to have a neutral car with the maximum grip on each end

Choice between Balance and Grip

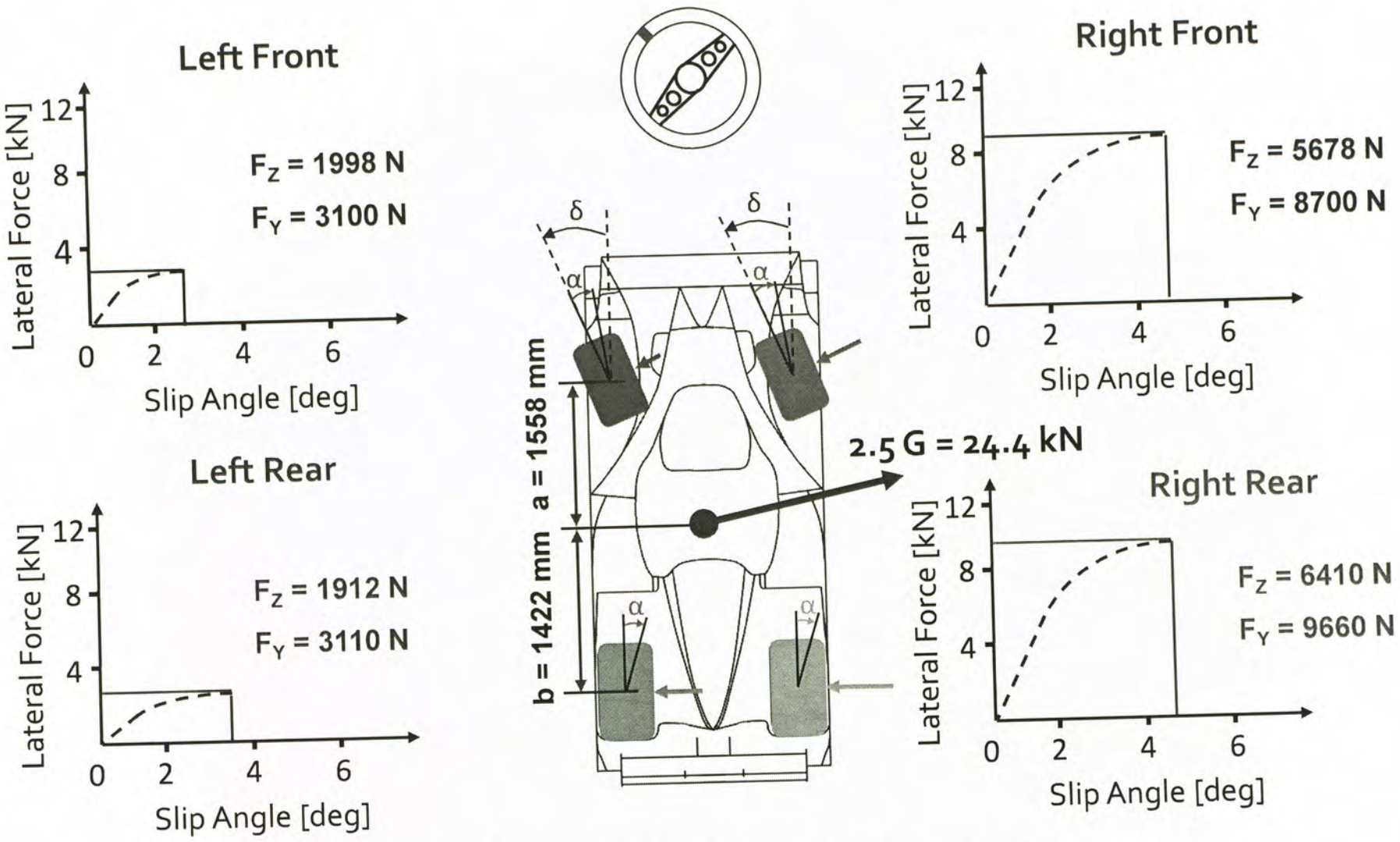


A different *perspective* on ratio between front and rear masses and ratio between front and rear tire cornering stiffness .
(simplified 2 wheels example)

Choice between Balance and Grip



4 Wheels Model



$$M_{yaw} = [(Fy_{fl} + Fy_{fr}) \times a] \times cos(\delta) - (Fy_{rl} + Fy_{rr}) \times b$$

$$M_{yaw} = [(3100 + 8700)] \times cos(5) \times 1.558 - (3110 + 9660) \times 1.422 = 155.5\ Nm$$

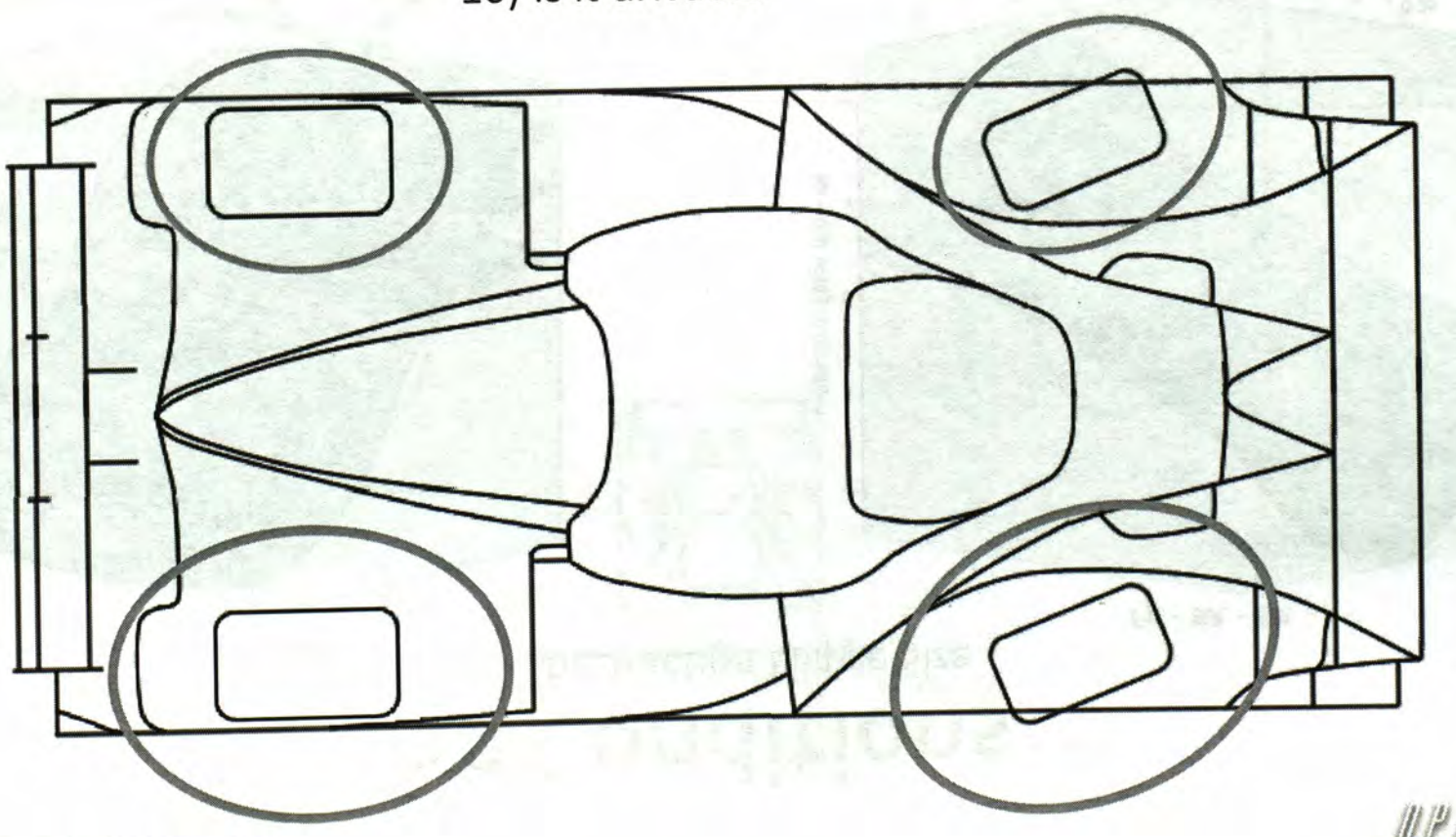
10 Conditions

1-4) The traction ellipse size

5-8) Traction ellipse exploitation

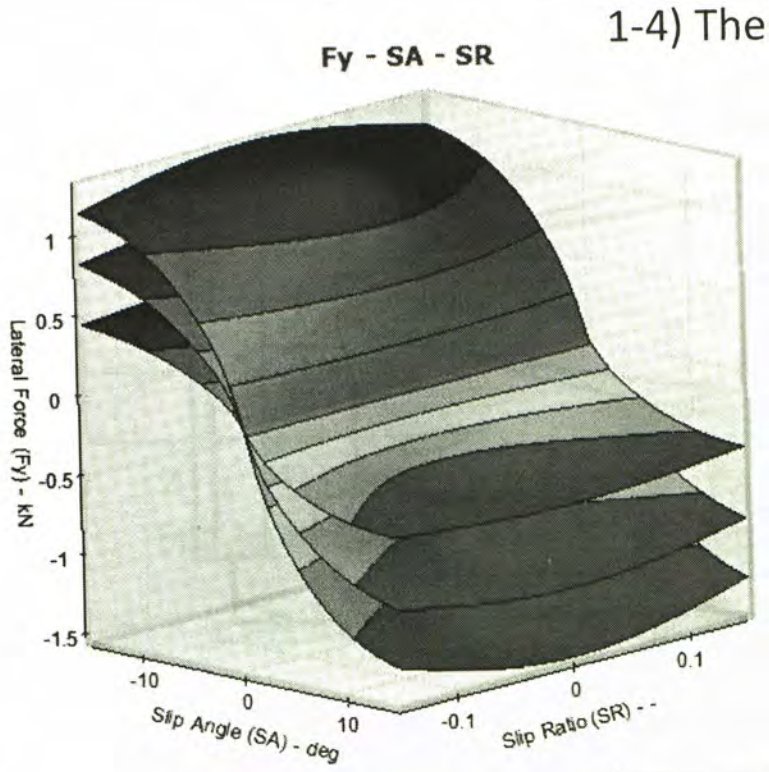
9) The yaw moment you want

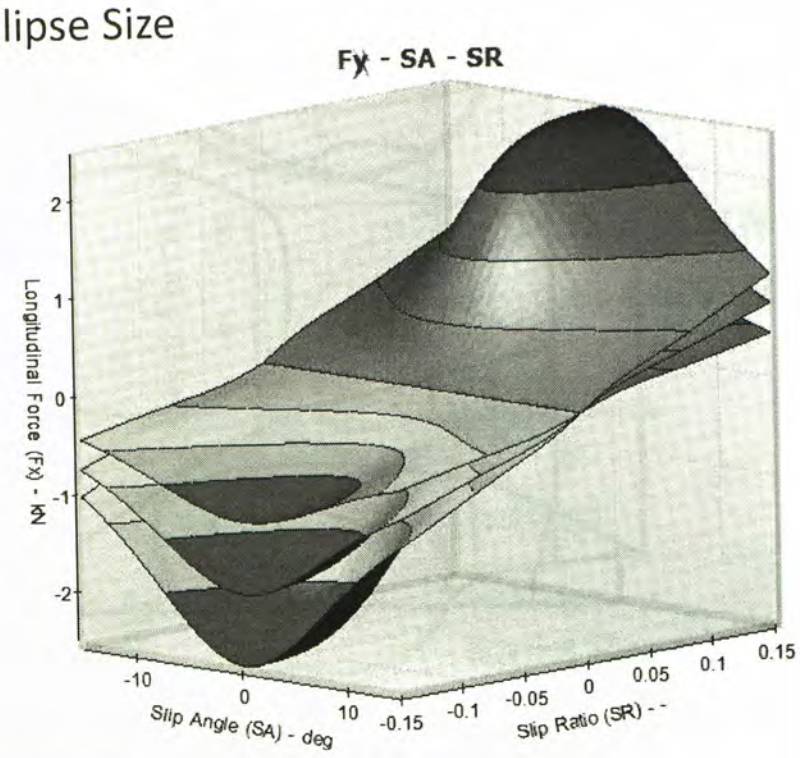
10) Is it drivable



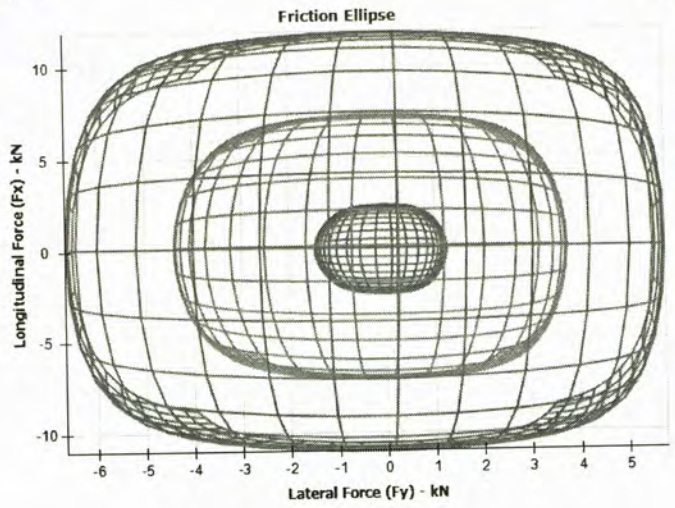
10 Conditions

1-4) The Traction Ellipse Size



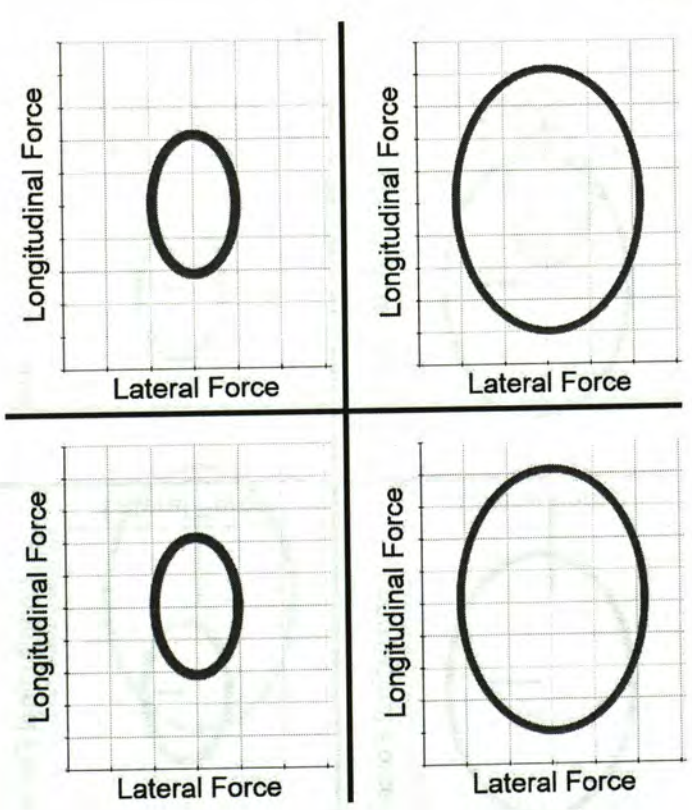


The bigger the combined forces, the better

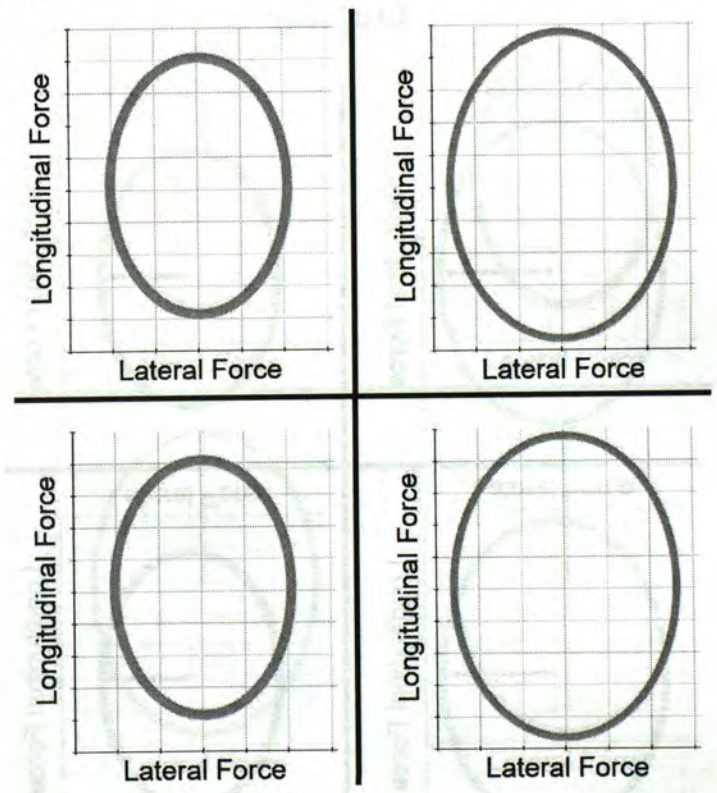


10 Conditions

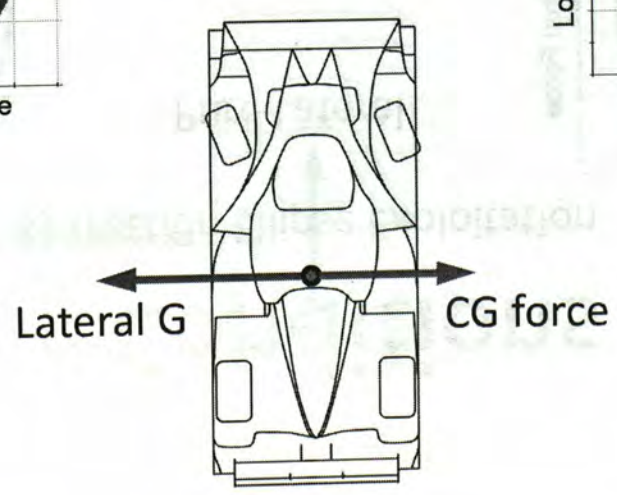
1-4) The Traction Ellipse Size



Bad



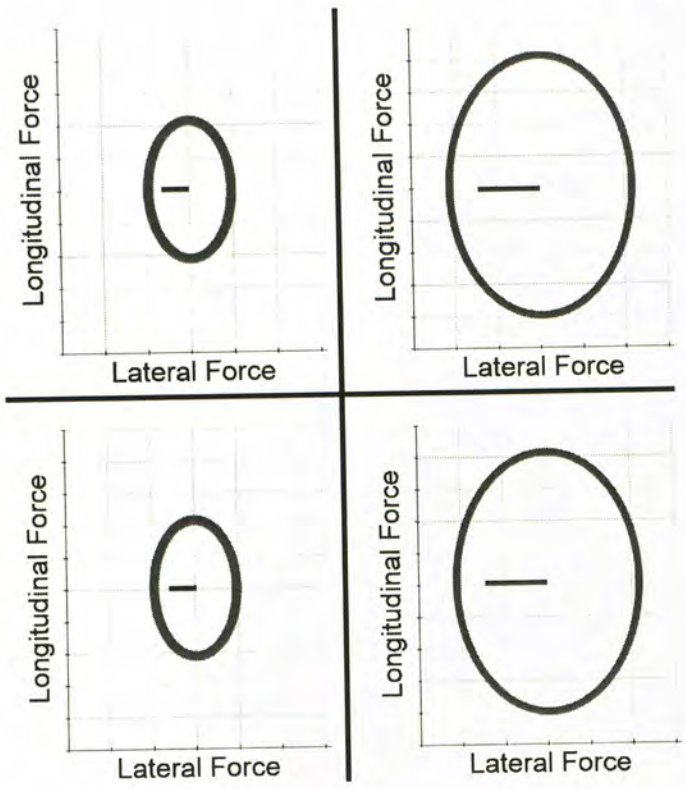
Better



10 Conditions

5-8) Traction Ellipse Exploitation

Pure Lateral



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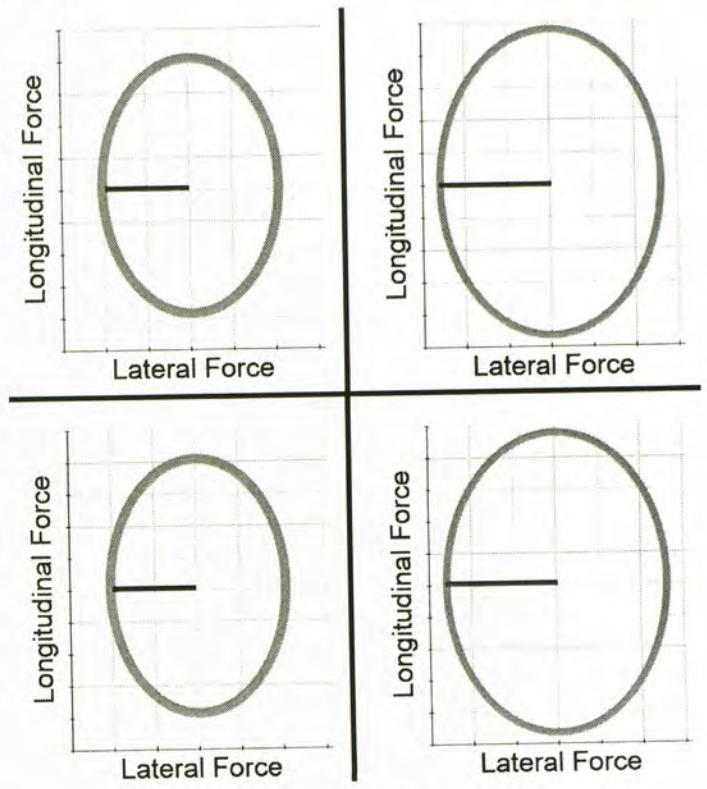
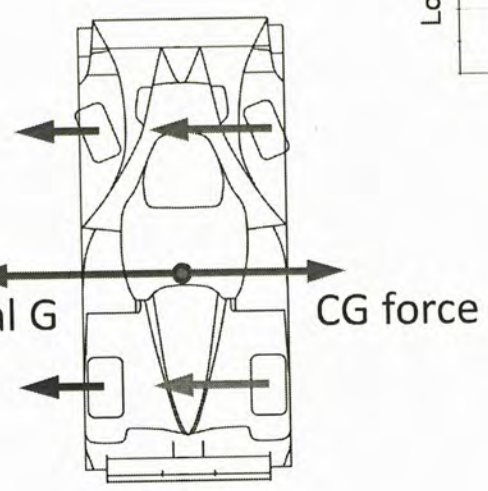


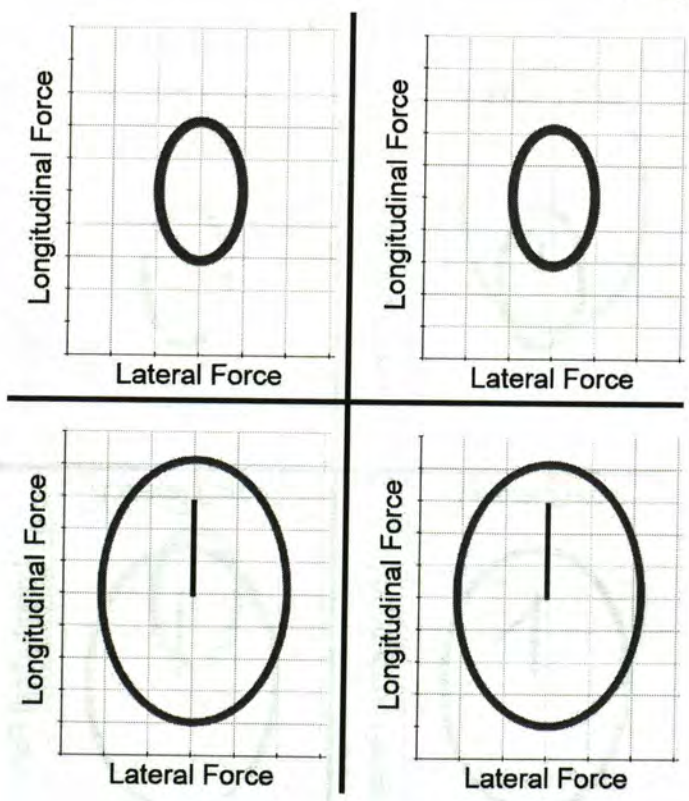
Perfect



10 Conditions

5-8) Traction Ellipse Exploitation

Pure Longitudinal Accelerating



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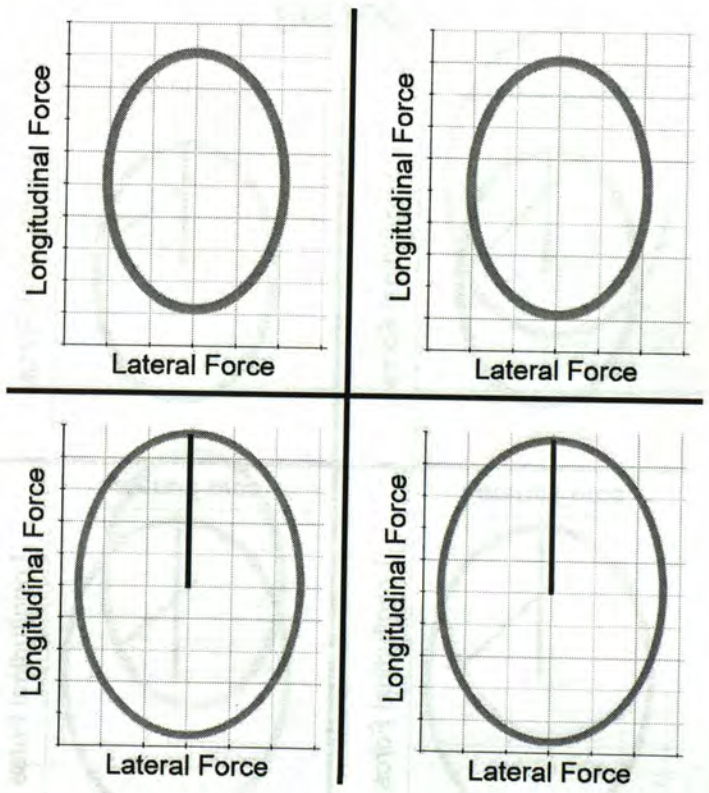
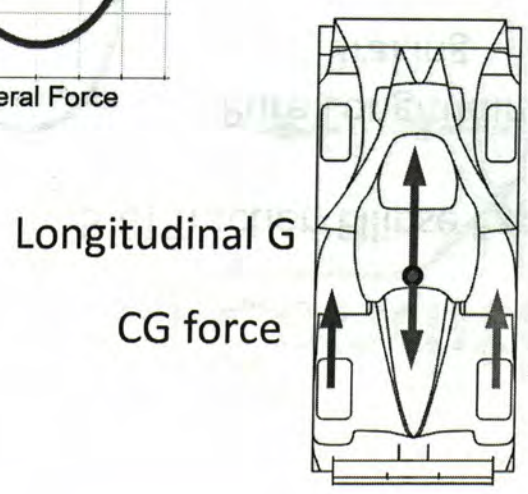
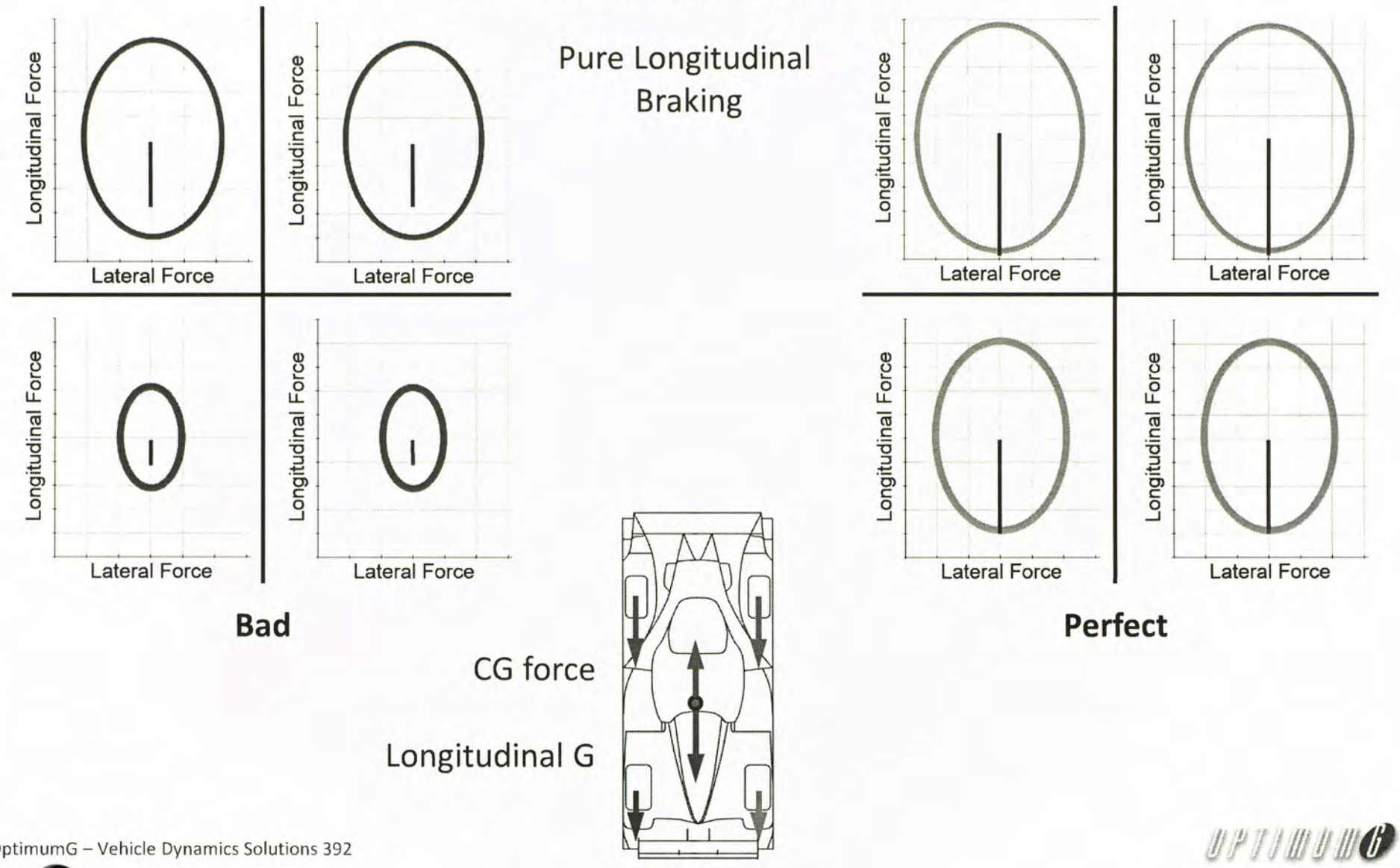


Perfect



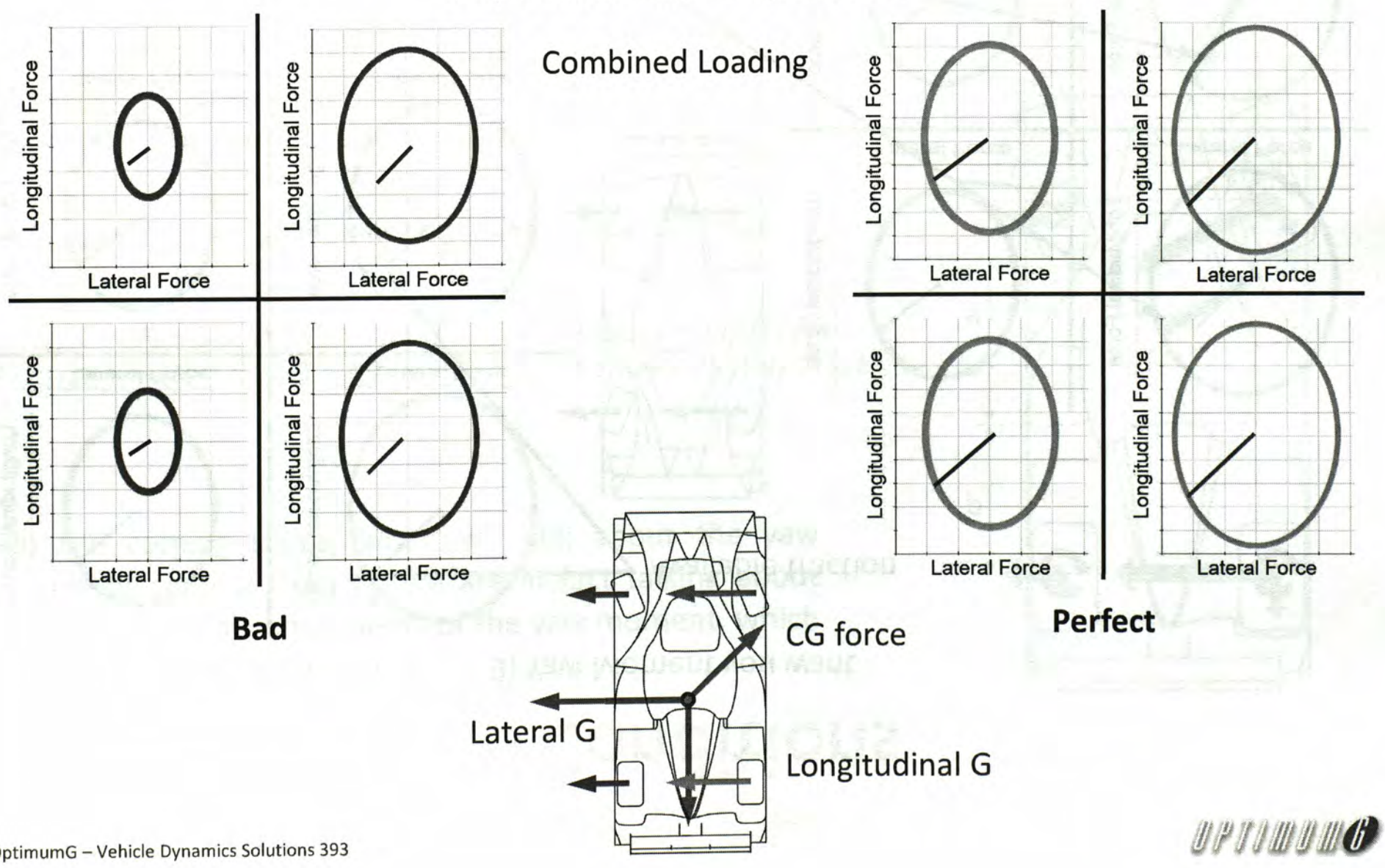
10 Conditions

5-8) Traction Ellipse Exploitation



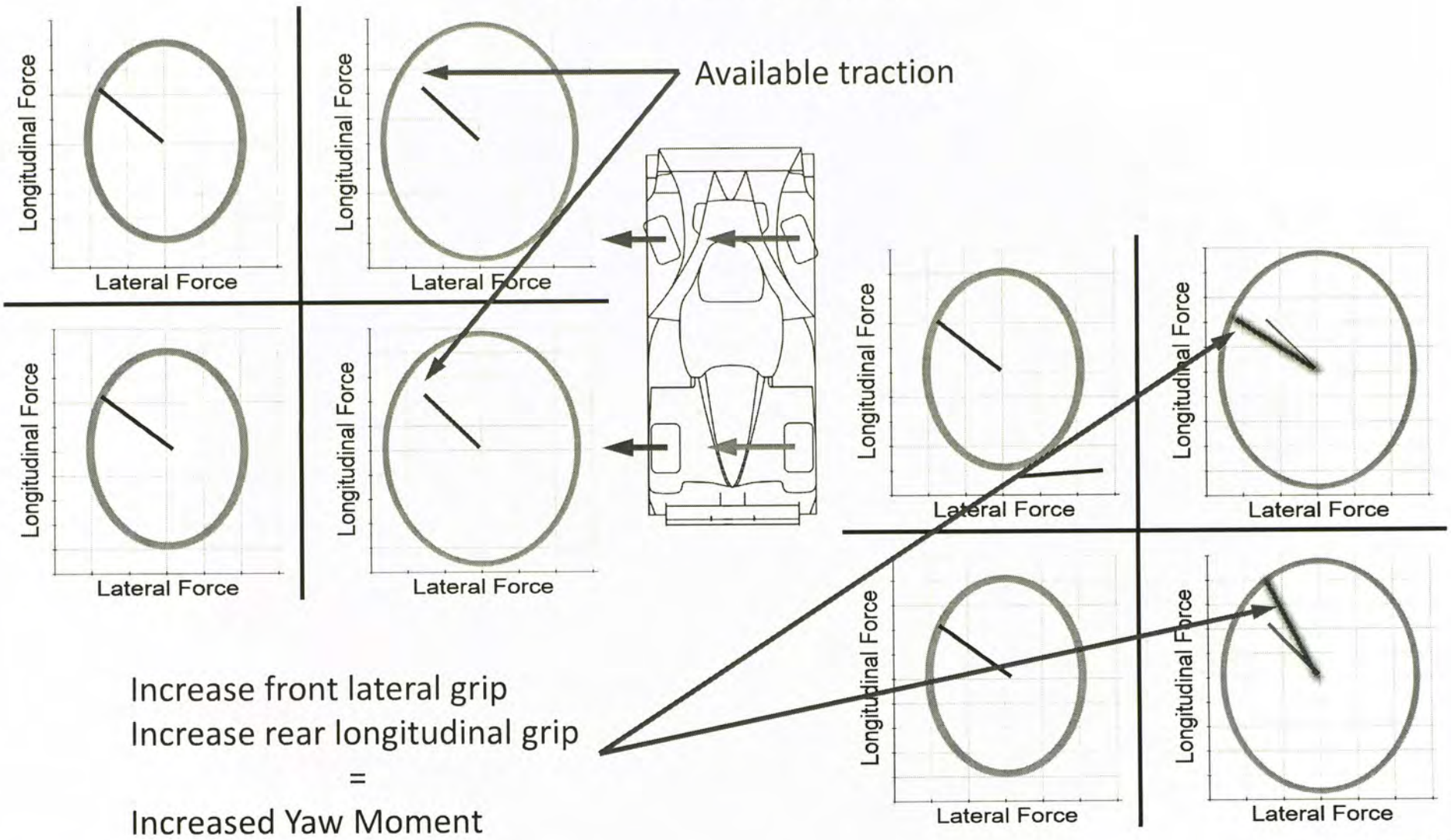
10 Conditions

5-8) Traction Ellipse Exploitation



10 Conditions

9) Yaw Moment you want



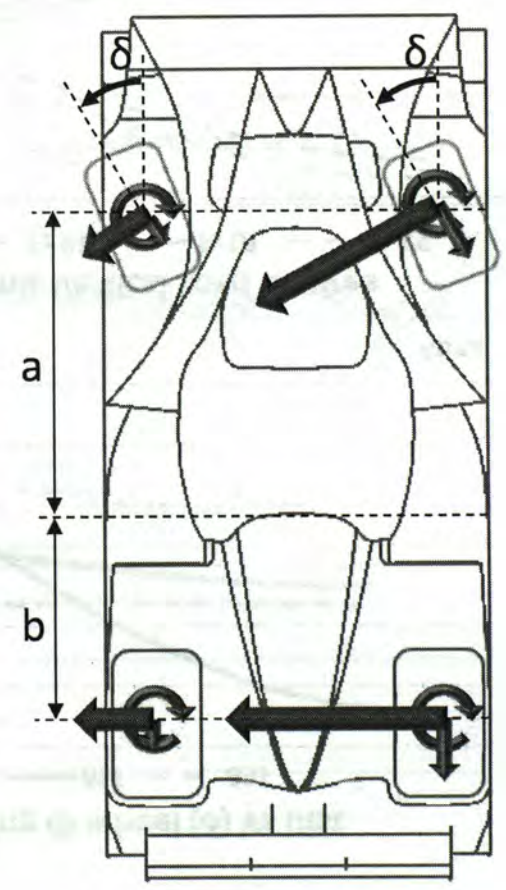
Increase front lateral grip
Increase rear longitudinal grip
=
Increased Yaw Moment

Dynamic N - Calculations

$F_Y = Coming\ from\ the\ tyre\ model$
$F_X = F_Z \times Rolling\ Resistance$
$M_Z = Coming\ from\ the\ tyre\ model$

$$N_{F_{Y-Front}} = ((F_{Y_{LF}} + F_{Y_{RF}}) \times \mathrm{Cos}(\delta) - (F_{X_{LF}} + F_{Y_{RF}}) \times Sin(\delta)) \times a$$
$$N_{F_{Y-Rear}} = -F_Y \times b$$

$$N_{F_{X-Left}} = (F_X \times \mathrm{Cos}(\delta) + F_Y \times Sin(\delta) + F_Z \times Roll.Res.) \times Half\ Track$$
$$N_{F_{X-Right}} = -(F_X \times \mathrm{Cos}(\delta) + F_Y \times Sin(\delta) + F_Z \times Roll.Res.) \times Half\ Track$$

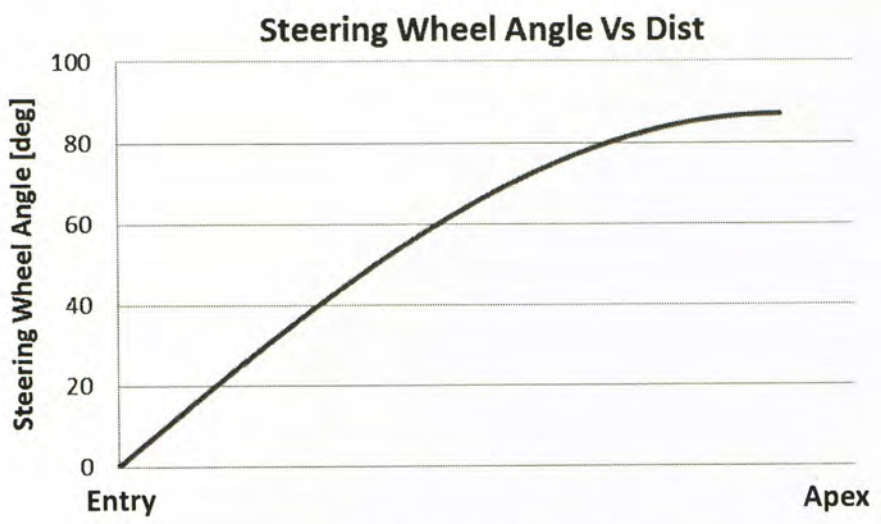


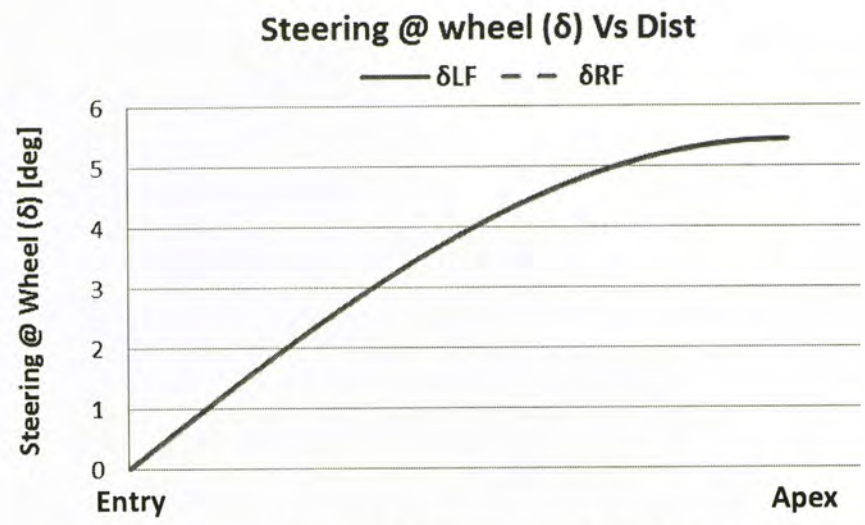
In the coming slides, when we talk about the yaw moment coming from F_X, we are in fact talking about the longitudinal components of the yaw moment, which for a part are coming from F_Y.

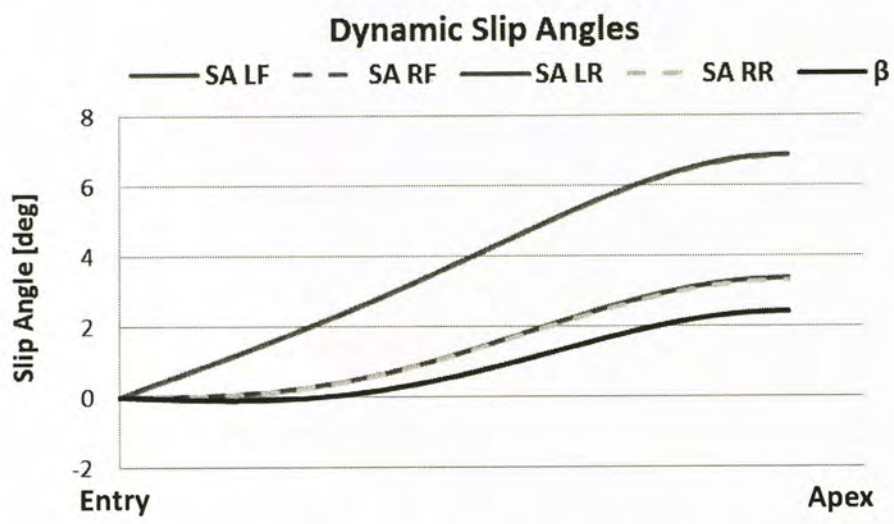
Skipped with clutch pushed

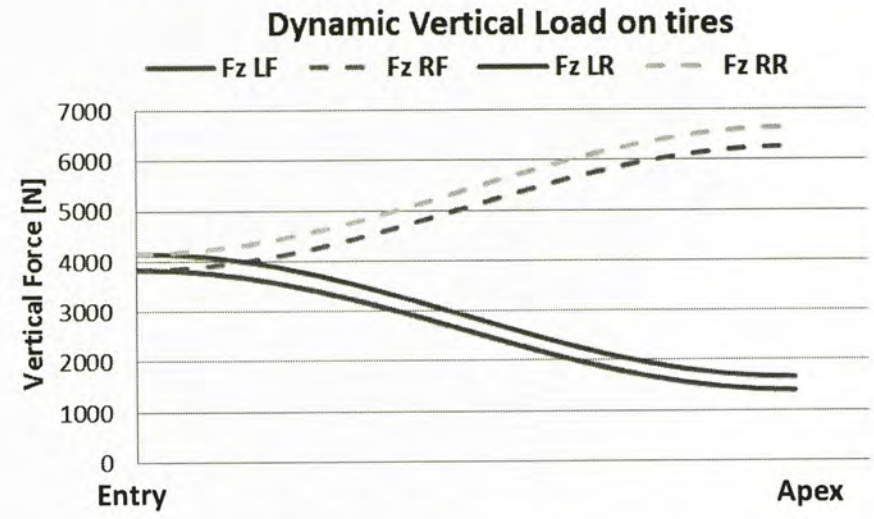
Assumes there is no Ackermann or Toe.

Dynamic Corner Entry to Apex - Inputs



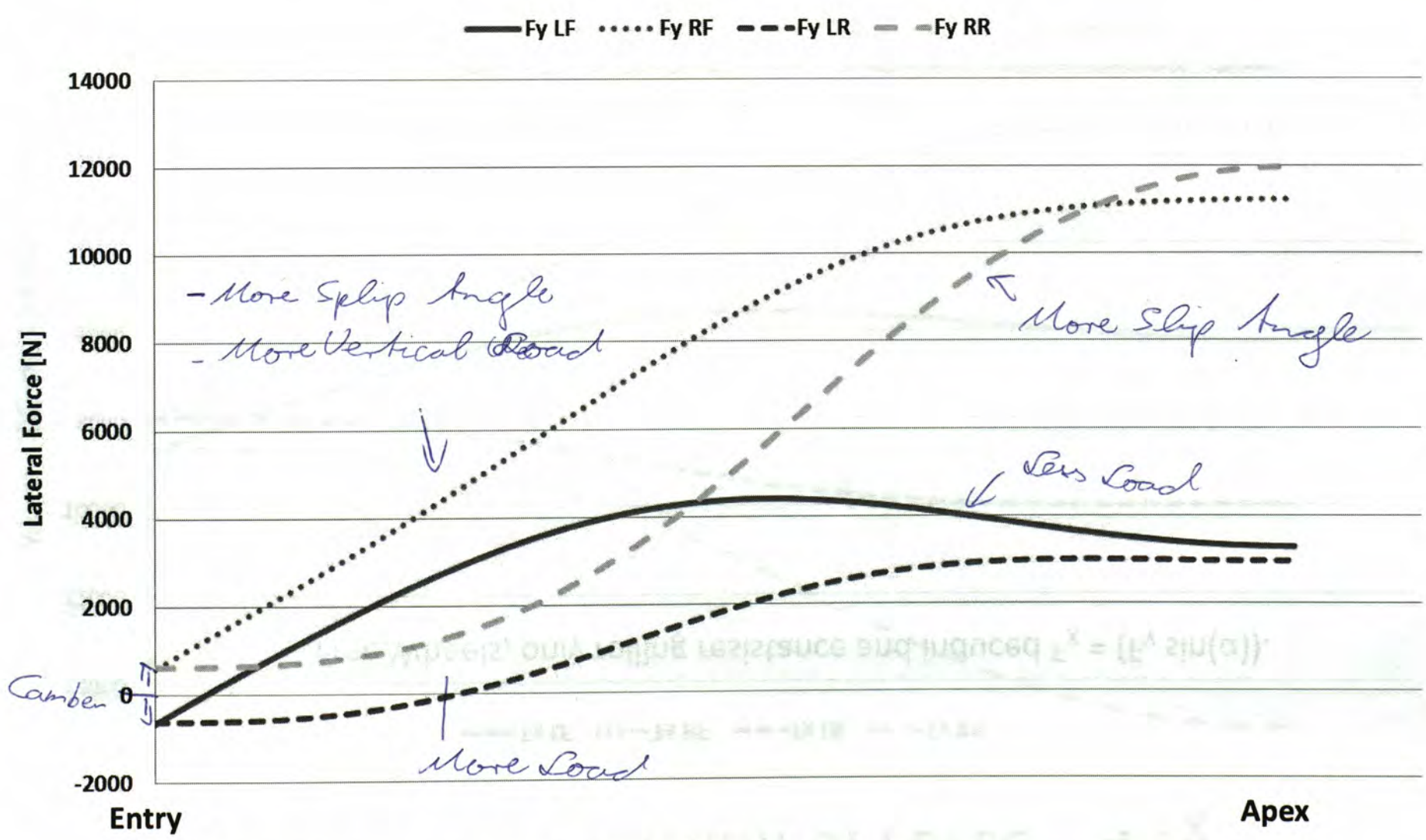






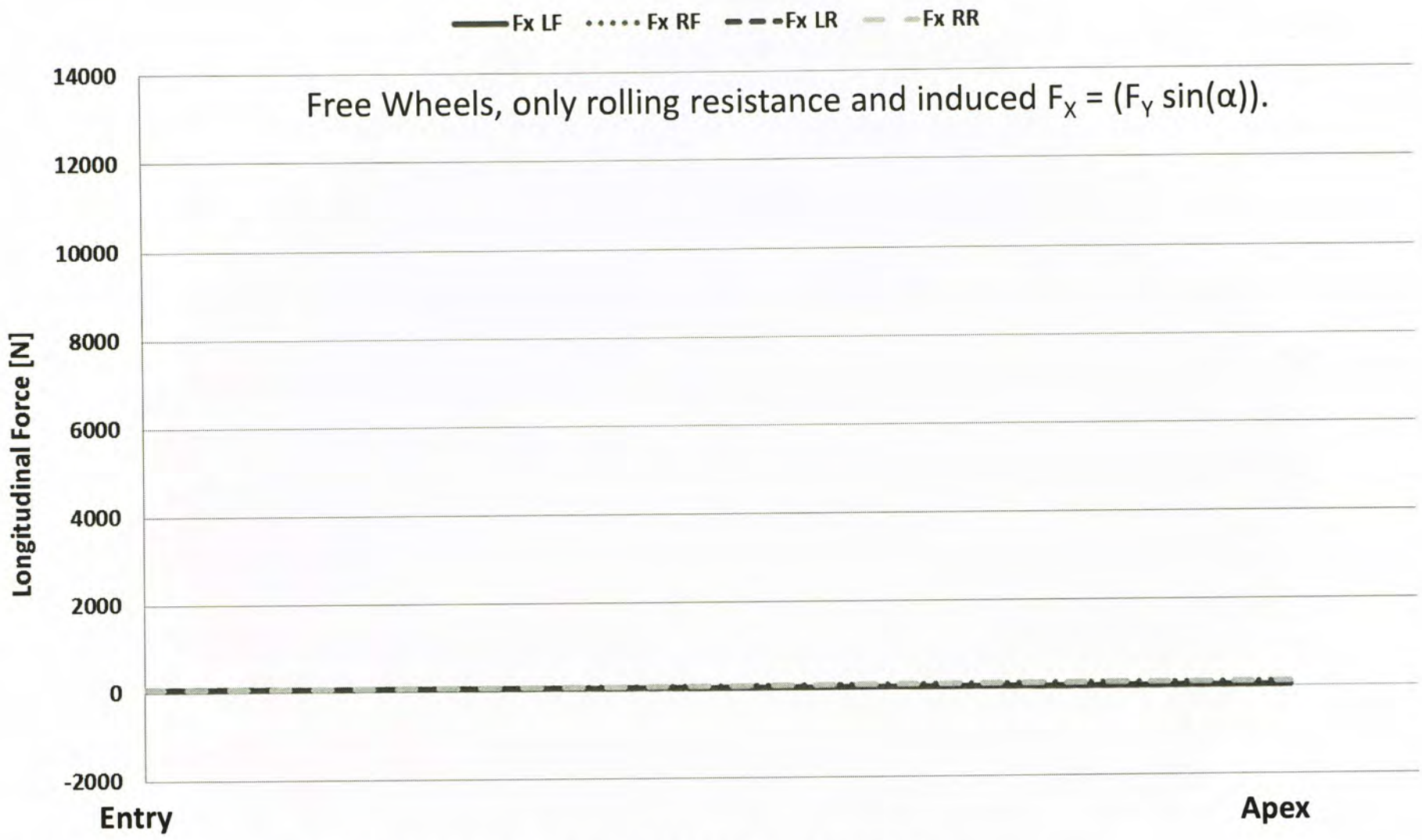
Note: Radius goes from INF to 89m so the slip angles are almost the same for the right and left sides of an axle.

Dynamic Lateral Force – 4 F_Y



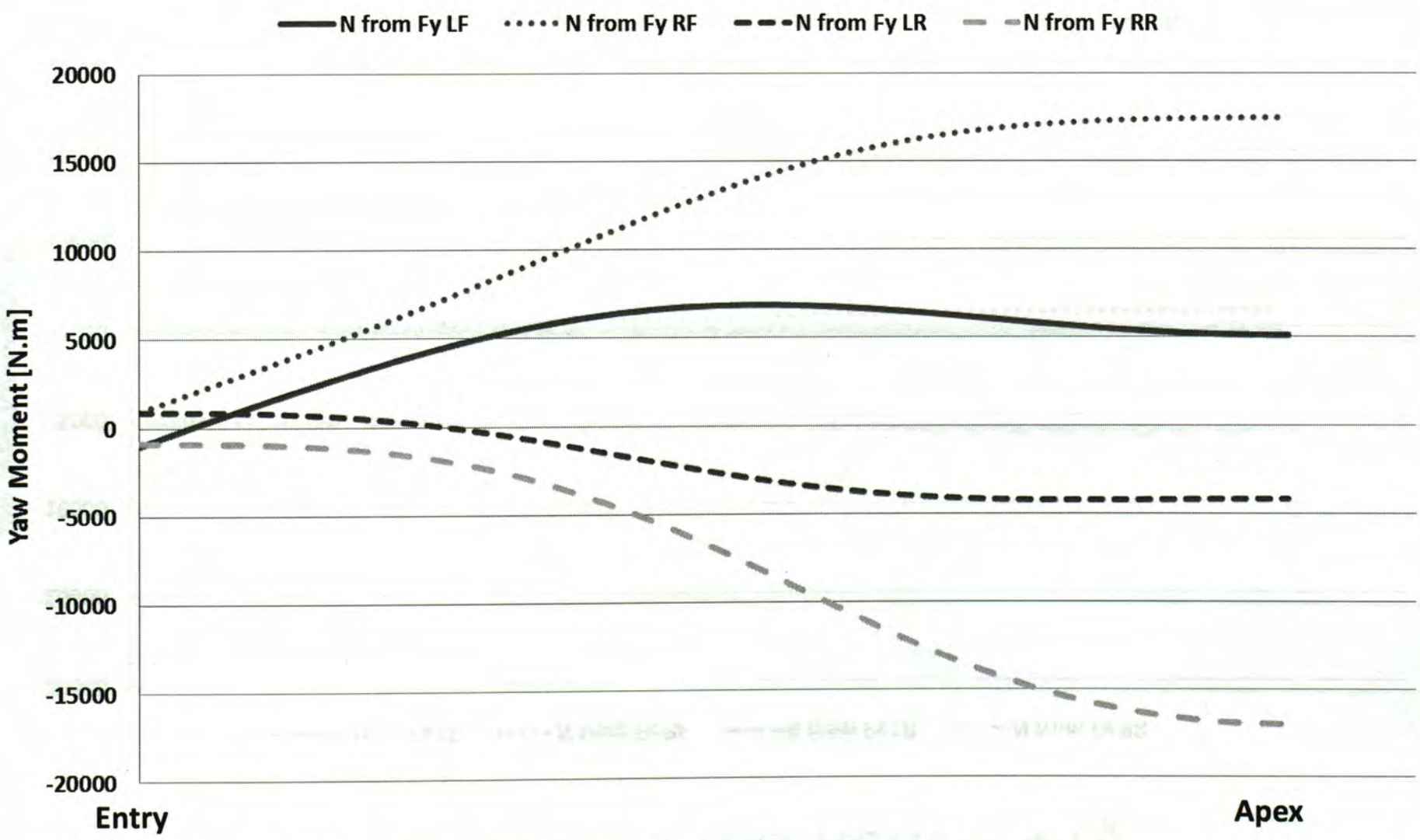
Note : Lateral Force from corner entry to apex (0N.m) – Left Turn.

Dynamic Longitudinal Force – 4 F_X



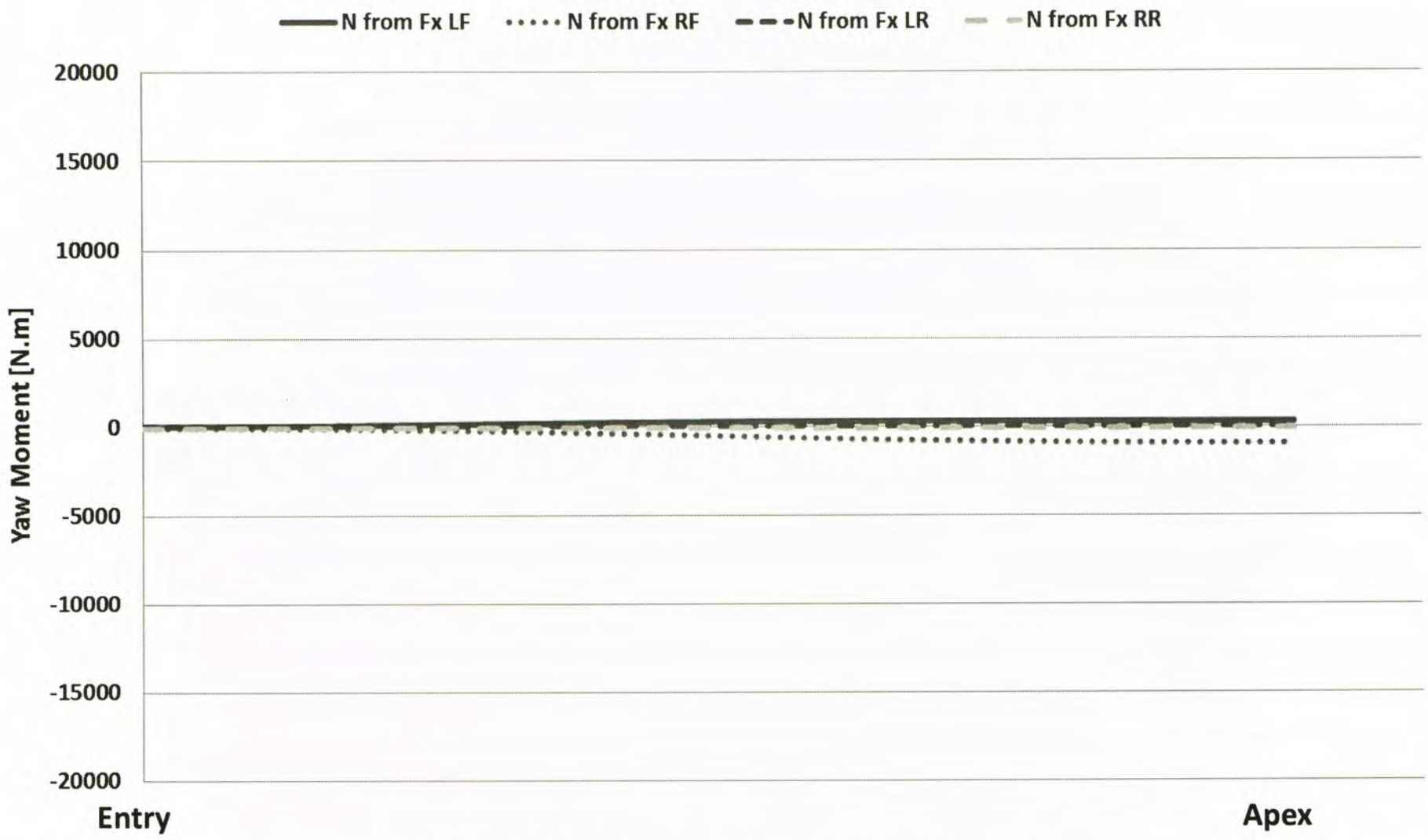
Note : Longitudinal Force from corner entry to apex (0N.m) – Left Turn.

Dynamic Yaw Moment – 4 F_Y



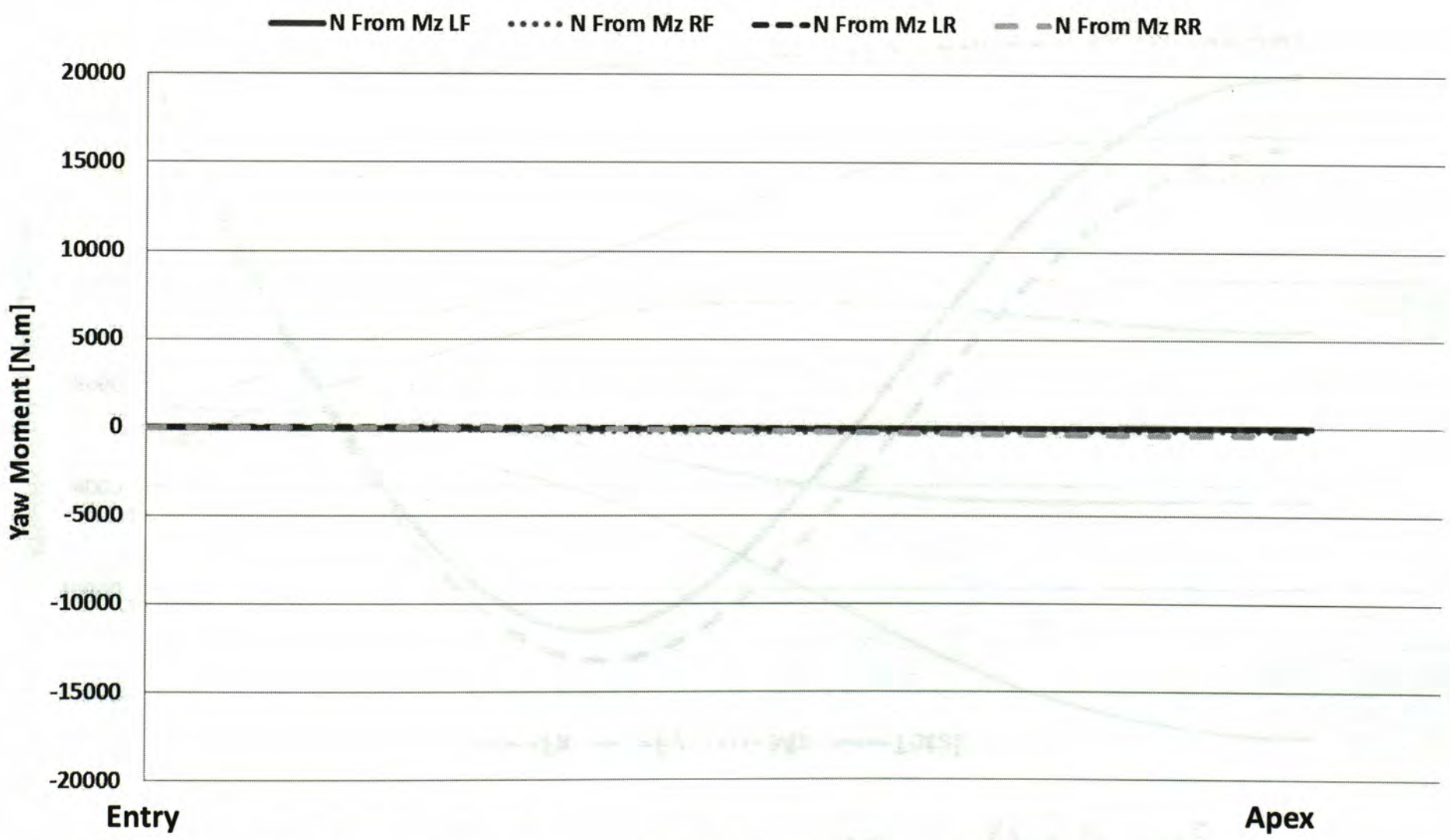
Note : Dynamic yaw moment from corner entry to apex (0N.m) – Left Turn.

Dynamic Yaw Moment – 4 F_X



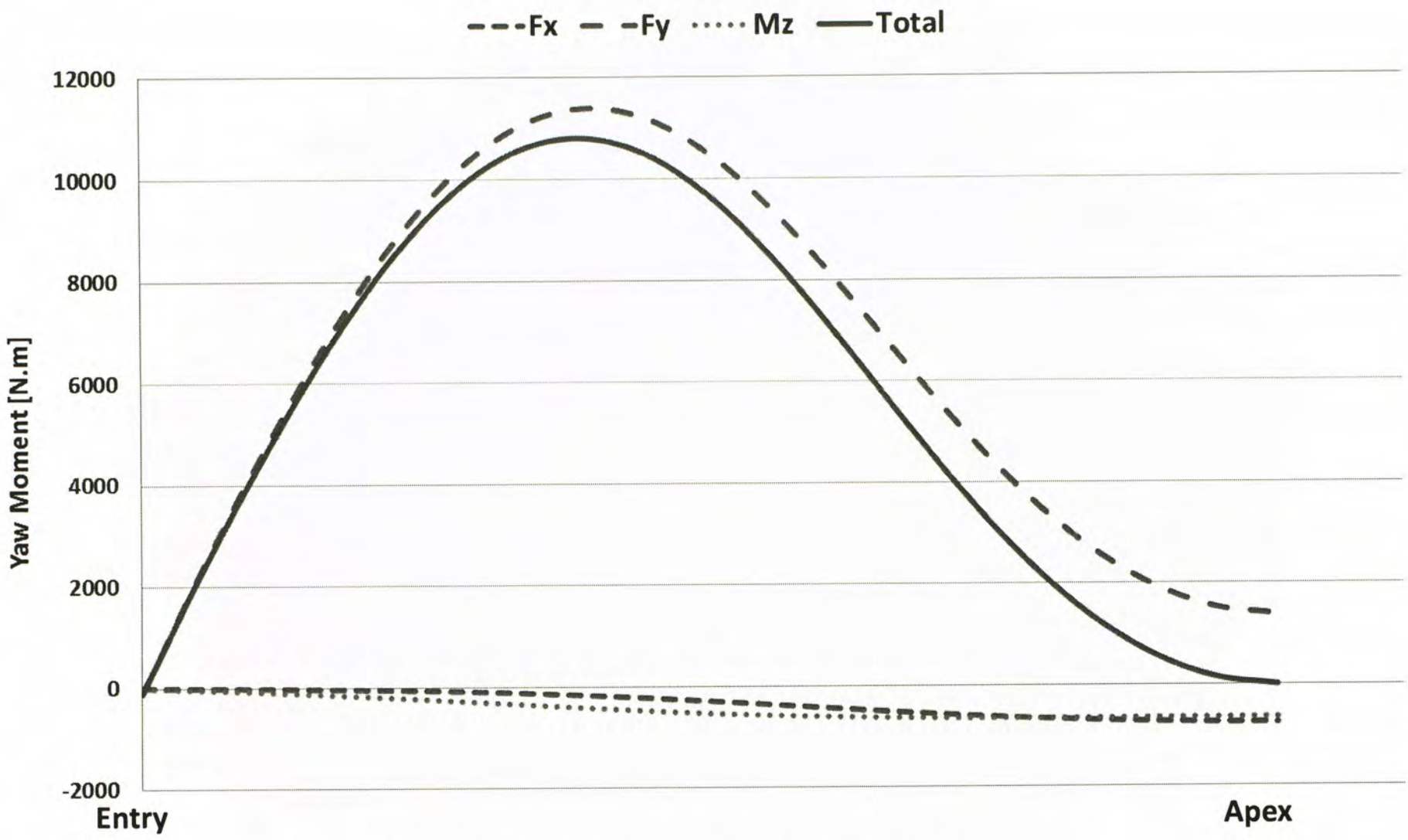
Note : Dynamic yaw moment from corner entry to apex (0N.m) – Left Turn.

Dynamic Yaw Moment – 4 M_Z



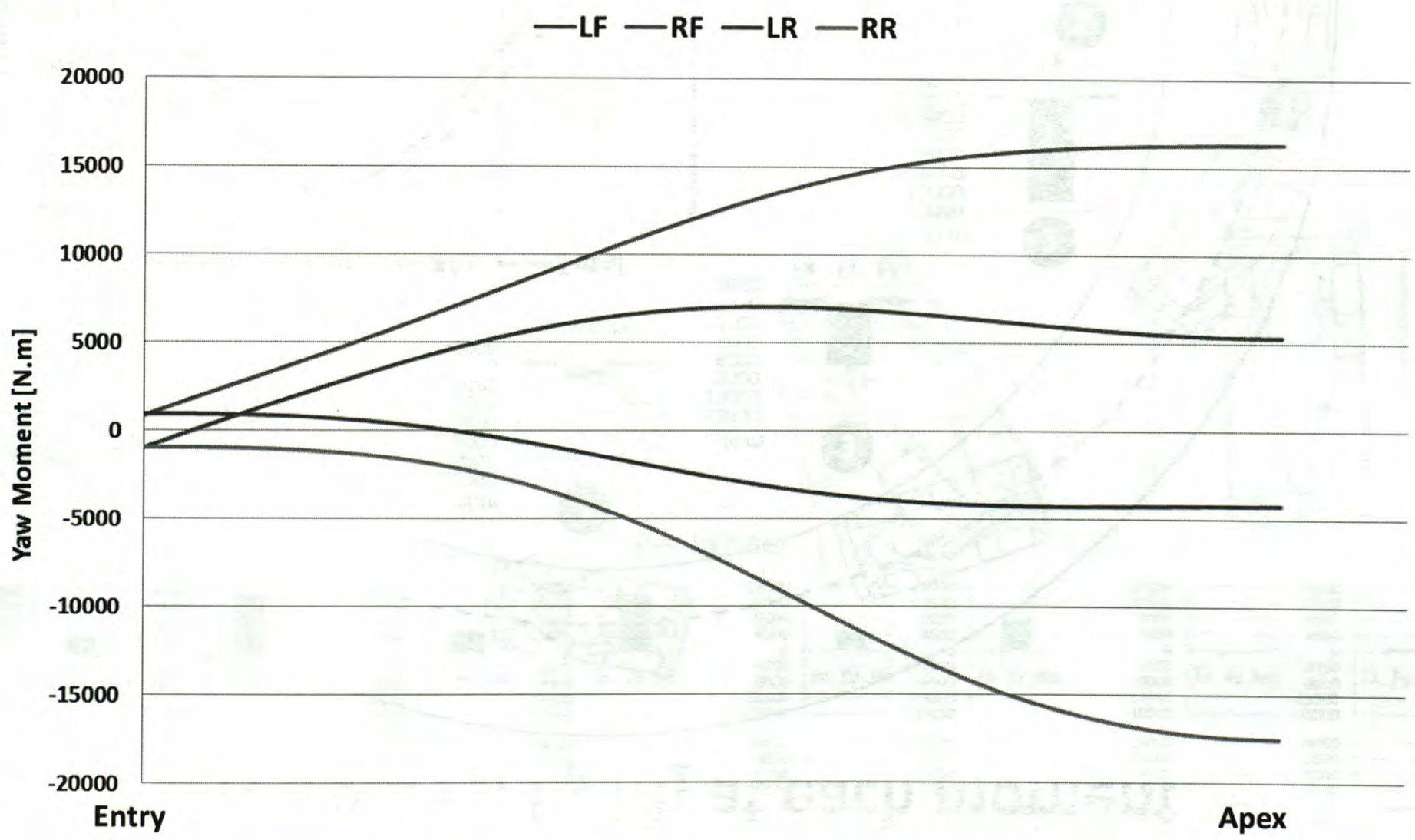
Note : Dynamic yaw moment from corner entry to apex (0N.m) – Left Turn.

Dynamic Yaw Moment – F_X, F_Y, M_Z



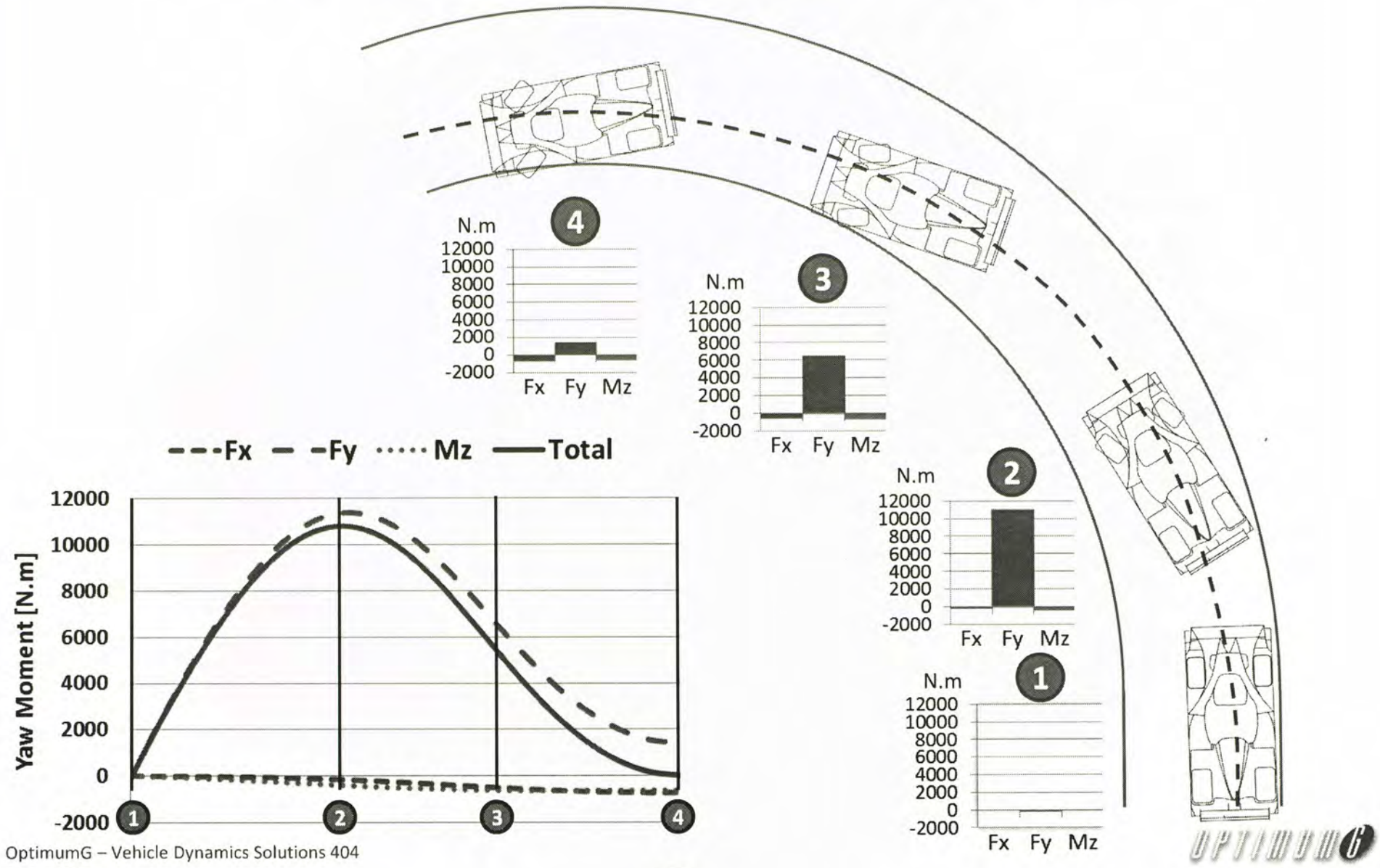
Note : Dynamic yaw moment from corner entry to apex (0N.m) – Left Turn.

Dynamic Yaw Moment – Each Wheel

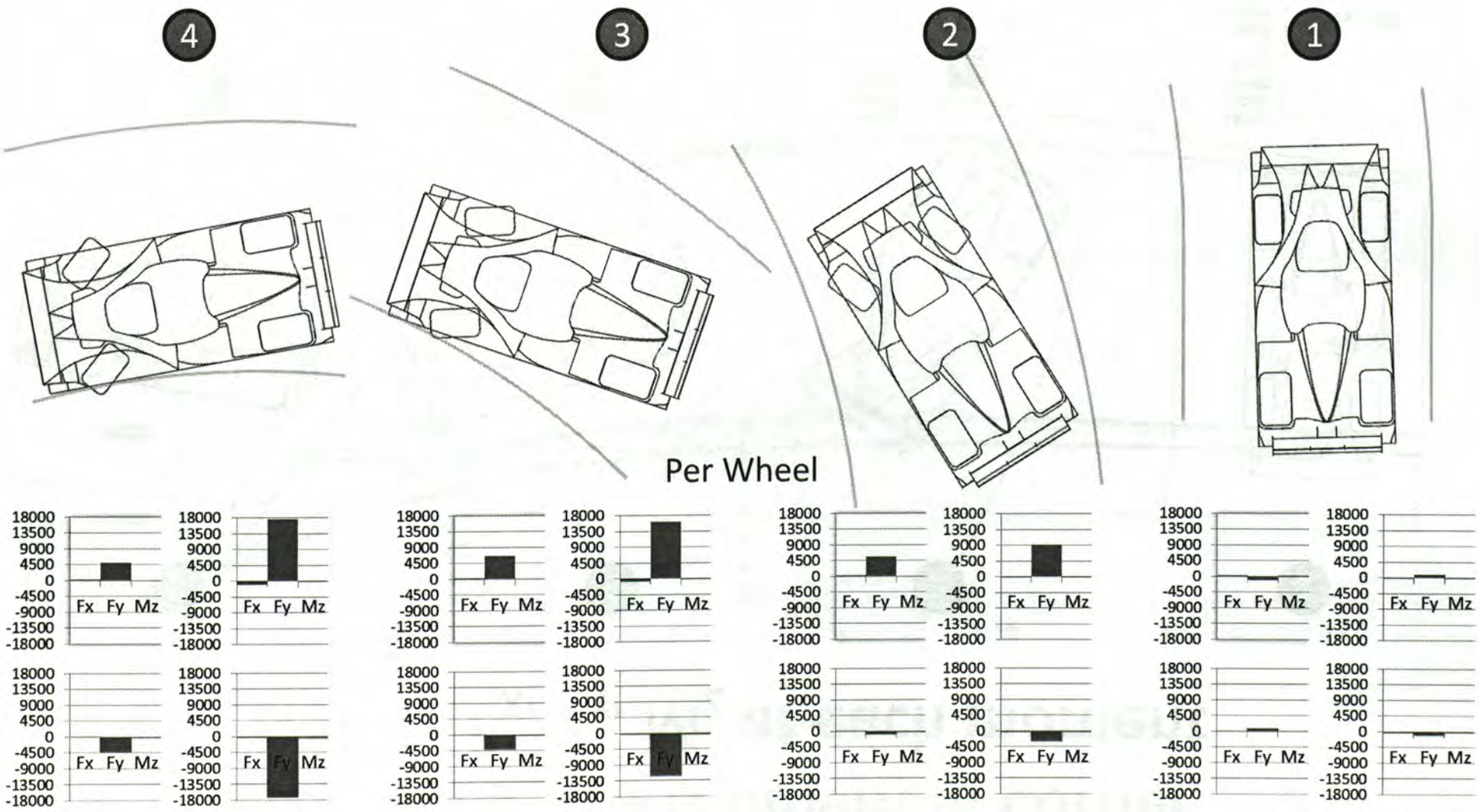


Note : Dynamic yaw moment from corner entry to apex (0N.m) – Left Turn.

Dynamic Yaw Moment in Corner
N from F_X, F_Y, M_Z at each moment

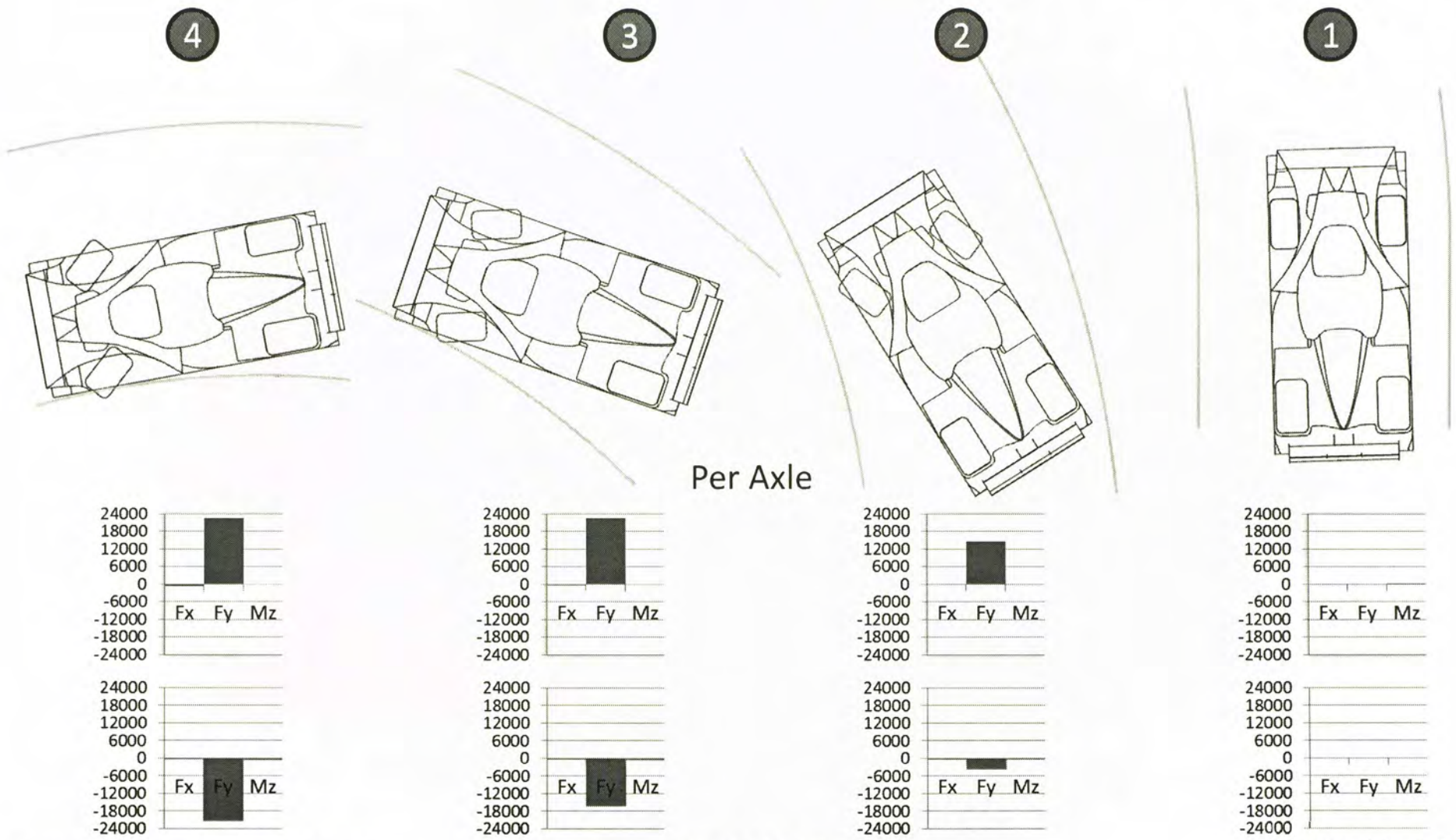


Dynamic Yaw Moment in Corner
N from F_X, F_Y, M_Z at each moment

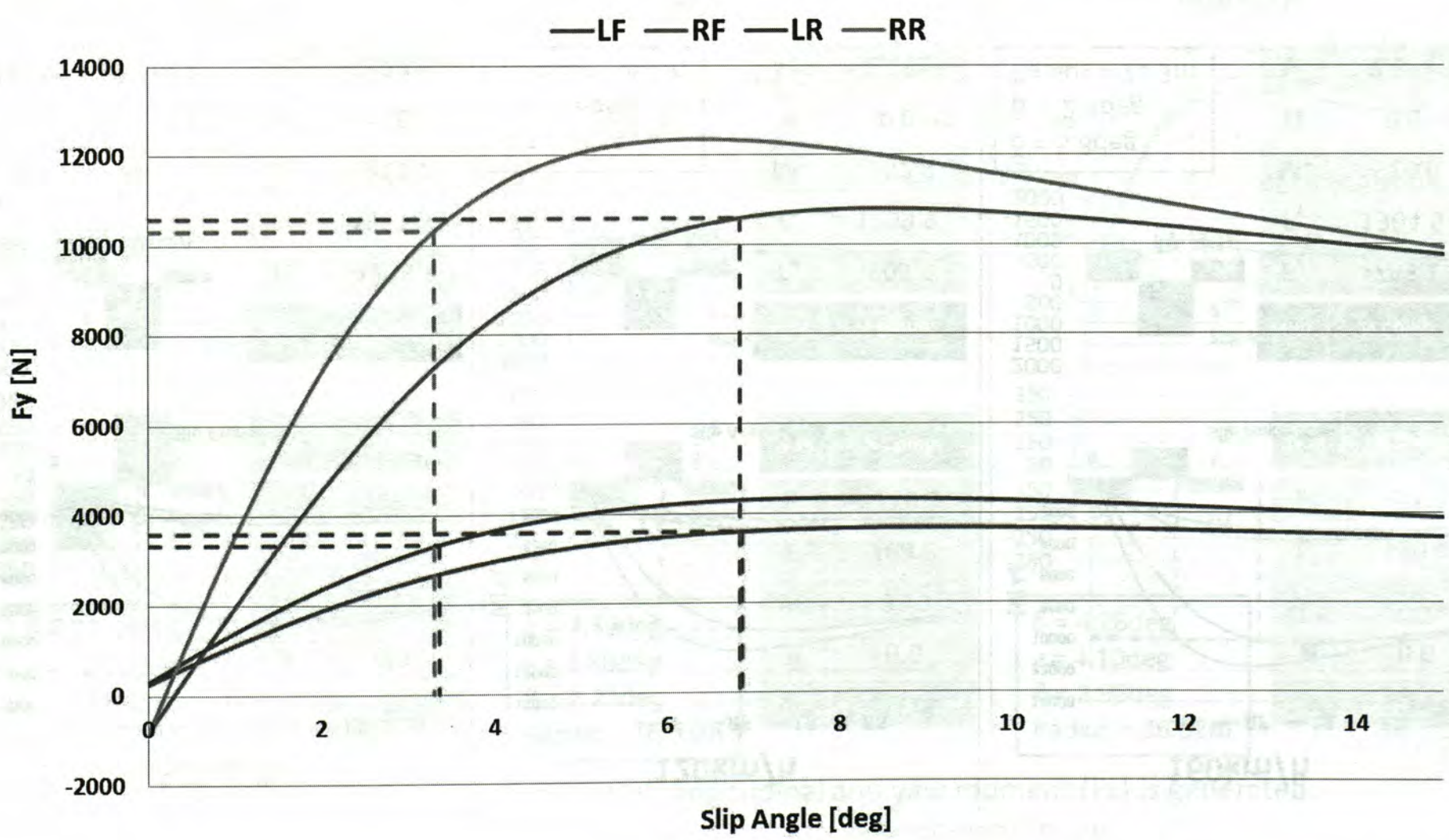


Units : [N.m]

Dynamic Yaw Moment in Corner
N from F_X, F_Y, M_Z at each moment

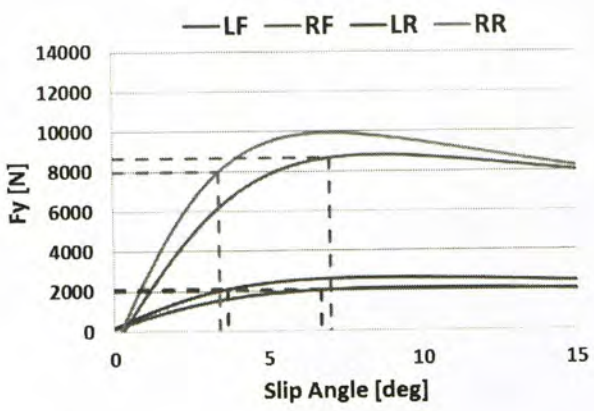


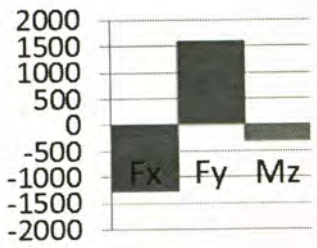
Tire Use @ apex (equilibrium N=0)



Yaw Moment and Speed – LMP1 Car

80km/h

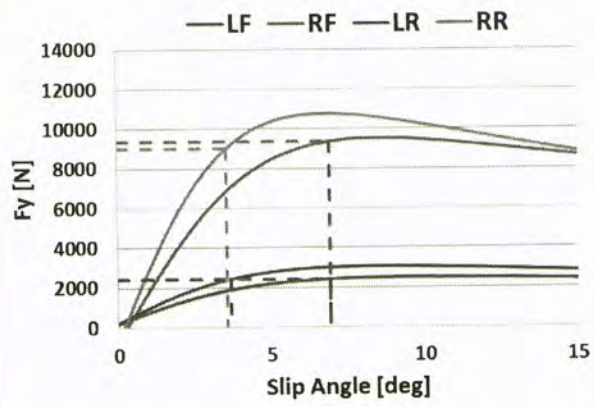


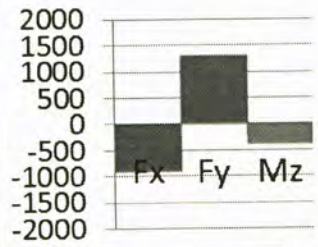


N.m	N Comp.
F_X	-1273.3
F_Y	1587.5
M_Z	-313.7
N	0.0
Ac.	2.27g

δ = 1.1deg
β = -0.1deg
Radius = 22.1m

120km/h

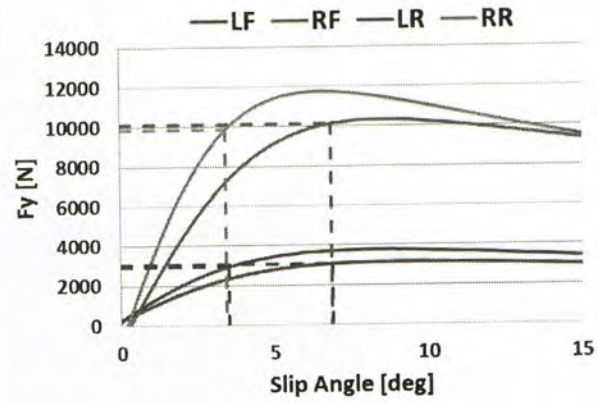


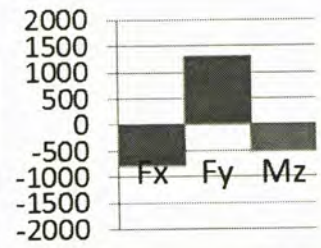


N.m	N Comp.
F_X	-906.4
F_Y	1299.9
M_Z	-392.8
N	0.0
Ac.	2.53g

δ = 7.1deg
β = 1.8deg
Radius = 44.7m

160km/h





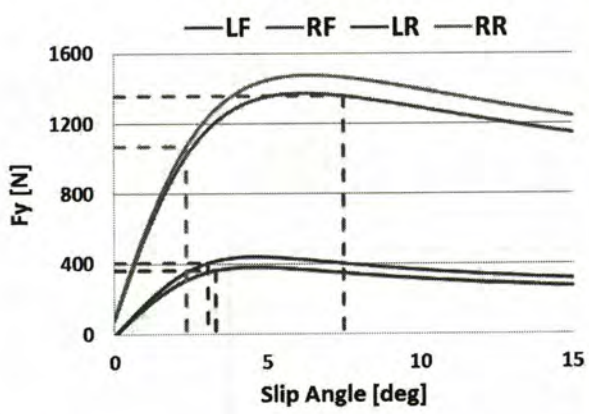
N.m	N Comp.
F_X	-785.1
F_Y	1304.5
M_Z	-520.0
N	0.0
Ac.	2.88g

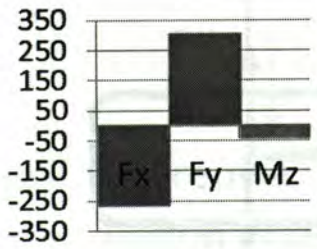
δ = 5.8deg
β = 2.3deg
Radius = 71.3m

At low speed steering angle δ is high and a lot of longitudinal anti yaw moment (Fx) is generated.
At high speed steering angle δ is low and the longitudinal anti yaw moment is low.
At low speed (High steering angle), a large amount of the tires lateral force is used to fight the longitudinal yaw moment. This is as much lateral force not used to achieve lateral acceleration.

Yaw Moment and Speed – FSAE Car

30km/h

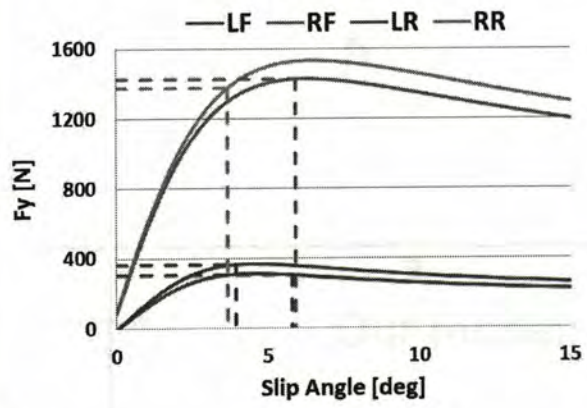


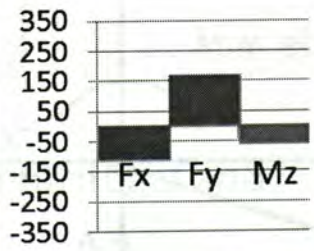


N.m	N Comp.
F_X	-263.7
F_Y	304.7
M_Z	-40.9
N	0.0
Ac.	1.53g

δ = 21.64deg
γ = 2.66deg
β = -6.39deg
Radius = 4.63m

60km/h

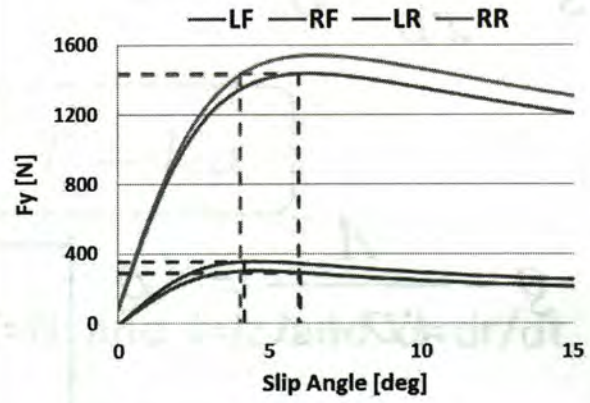


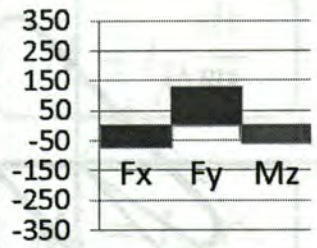


N.m	N Comp.
F_X	-110.7
F_Y	169.9
M_Z	-59.1
N	0.0
Ac.	1.71g

δ = 7.36deg
γ = 3.80deg
β = 2.23deg
Radius = 16.53m

90km/h



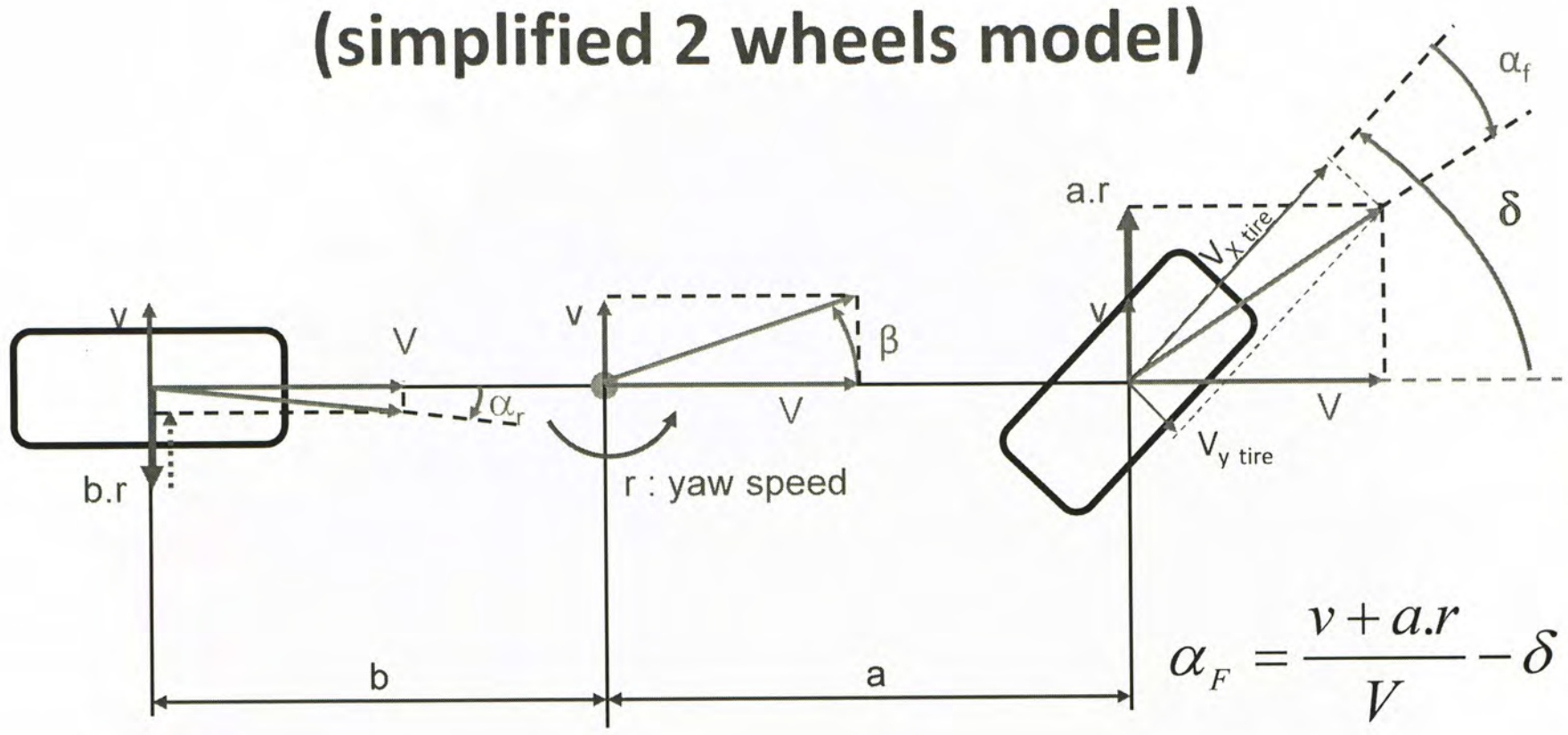


N.m	N Comp.
F_X	-71.5
F_Y	130.9
M_Z	-59.5
N	0.0
Ac.	1.74g

δ = 4.28deg
γ = 4.10deg
β = 2.96deg
Radius = 36.62m

At low speed steering angle δ is high and a lot of longitudinal anti yaw moment (Fx) is generated.
At high speed steering angle δ is low and the longitudinal anti yaw moment is low.
At low speed (High steering angle), a large amount of the tires lateral force is used to fight the longitudinal yaw moment. This is as much lateral force not used to achieve lateral acceleration.

Tire slip angle calculation (simplified 2 wheels model)



$$\alpha_F = \frac{v + a.r}{V} - \delta$$

$$\alpha_R = \frac{v - b.r}{V}$$

$$\beta = \frac{v}{V}$$

$$\alpha_F = \beta + \frac{a.r}{V} - \delta$$

$$\alpha_R = \beta - \frac{b.r}{V}$$

Equations of Motion

$$F = m.a$$

Our model: F=Y and $a=a_y$

$$Y = m.a_y$$

Moreover a_y = the sum of centrifugal and lateral acceleration.

$$a_y = \frac{V^2}{R} + \dot{v} \quad \text{Or} \quad a_y = V.r + V.\dot{\beta}$$

since r=V/R and $\dot{v} = V.\dot{\beta}$

$$\rightarrow Y = m.V.(r + \dot{\beta})$$

$$T = I.\alpha$$

Our model: T=N and I=Iz and α=dr/dt

$$N = I_z.\frac{dr}{dt}$$

$$N = I_z.\dot{r}$$

Equations of Motion

Total tire lateral force Y

$$Y = Y_f + Y_r$$

$$Y = C_f.\alpha_f + C_r.\alpha_r$$

$$Y = C_f.\left(\beta + \frac{a.r}{V} - \delta\right) + C_r.\left(\beta - \frac{b.r}{V}\right)$$

$$Y = \underbrace{(C_f + C_r)}_{Y_\beta}.\beta + \underbrace{\frac{1}{V}.(a.C_f - b.C_r)}_{Y_r}.r - \underbrace{C_f}_{Y_\delta}.\delta$$

$$\boxed{Y = Y_\beta.\beta + Y_r.r + Y_\delta.\delta}$$

Total tire yawing moment about the z axis thought the CG

$$N = N_f + N_r$$

$$N = a.Y_f - b.Y_r$$

Yaw Damping

$$N = \underbrace{(a.C_f - b.C_r)}_{N_\beta}.\beta + \underbrace{\frac{1}{V}.(a^2.C_f + b^2.C_r)}_{N_r}.r - \underbrace{a.C_f}_{N_\delta}.\delta$$

$$\boxed{N = N_\beta.\beta + N_r.r + N_\delta.\delta}$$

Equations of Motion

Total tire lateral force Y

Lateral Force variation for a $\Delta\beta$:

$$\boxed{Y_\beta = \frac{\partial Y}{\partial \beta} = (C_f + C_r)}$$

Lateral Force variation for a Δr :

$$\boxed{Y_r = \frac{\partial Y}{\partial r} = \frac{1}{V}(a.C_f - b.C_r)}$$

Lateral Force variation for a $\Delta\delta$:

$$\boxed{Y_\delta = \frac{\partial Y}{\partial \delta} = -C_f}$$

$$\boxed{Y = Y_\beta.\beta + Y_r.r + Y_\delta.\delta}$$

Total tire yawing moment about the z axis thought the CG

Lateral Force variation for a $\Delta\beta$:

$$\boxed{N_\beta = \frac{\partial N}{\partial \beta} = (a.C_f - b.C_r)}$$

Lateral Force variation for a Δr :

$$\boxed{N_r = \frac{\partial N}{\partial r} = \frac{1}{V}(a^2.C_f + b^2.C_r)}$$

Lateral Force variation for a $\Delta\delta$:

$$\boxed{N_\delta = \frac{\partial N}{\partial \delta} = -a.C_f}$$

$$\boxed{N = N_\beta.\beta + N_r.r + N_\delta.\delta}$$

Definition of Understeer and Oversteer

- Race Car Performance is about maximum acceleration
- Lateral and longitudinal acceleration are functions of
  - The lateral and longitudinal tire properties as well as moments
  - The suspension design and tuning to exploit them
  - and the ability of your driver to exploit them.
- **It is also and mainly about yaw acceleration**.
- The yaw acceleration is a function of the cars yaw inertia and your cars yaw moment
- The yaw moment is a function of the longitudinal and lateral tire forces, the wheelbase and the front and rear tracks.
- The ideal yaw acceleration is 0 on a skid pad (steady state)
- The ideal yaw acceleration you need in transient depends on the speed and the radius of the corner.
- **Because the speed, the radius, the lateral acceleration and the CG slip angle never stop changing, the ideal yaw acceleration (and therefore the ideal yaw moment) never stops changing**.

$$\text{Yaw Acceleration} = \dot{r} = \frac{d\,\dfrac{\text{Lateral G}}{V_x}}{dt} + \ddot{\beta}$$

$$\text{Yaw Moment} = \text{Inertia}_{Yaw} \times \dot{r}$$

Oversteer/Understeer Definition

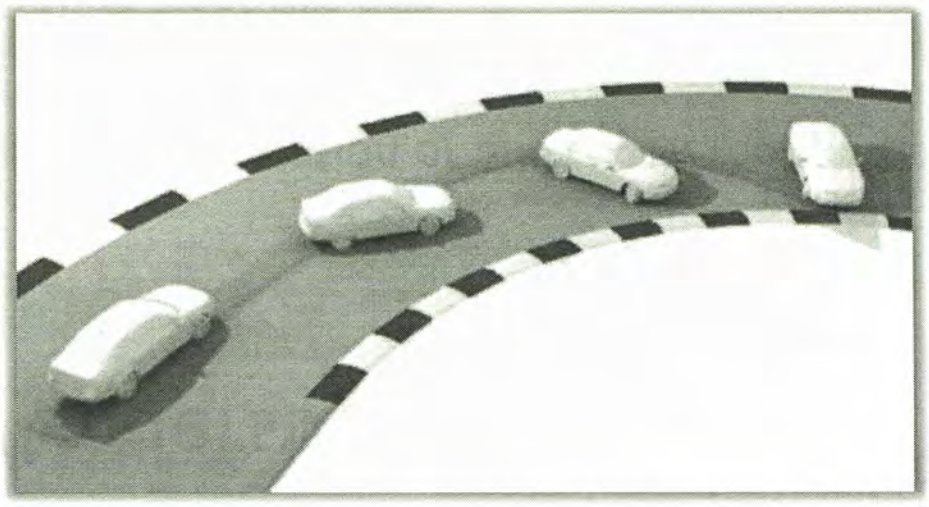
You can get understeer (under yaw acceleration or under yaw moment) because:

1. You do not have enough front tire grip
2. You have too much rear tire grip
3. You do not have the ideal weight distribution (distance a and b)
4. You have too much yaw inertia
5. All of the above

Oversteer/Understeer Definition

You can get oversteer (over yaw acceleration or over yaw moment) because:

1. You do not have enough rear tire grip
2. You have too much front tire grip
3. You do not have the ideal weight distribution (distance a and b)
4. You have too little yaw inertia
5. All of the above

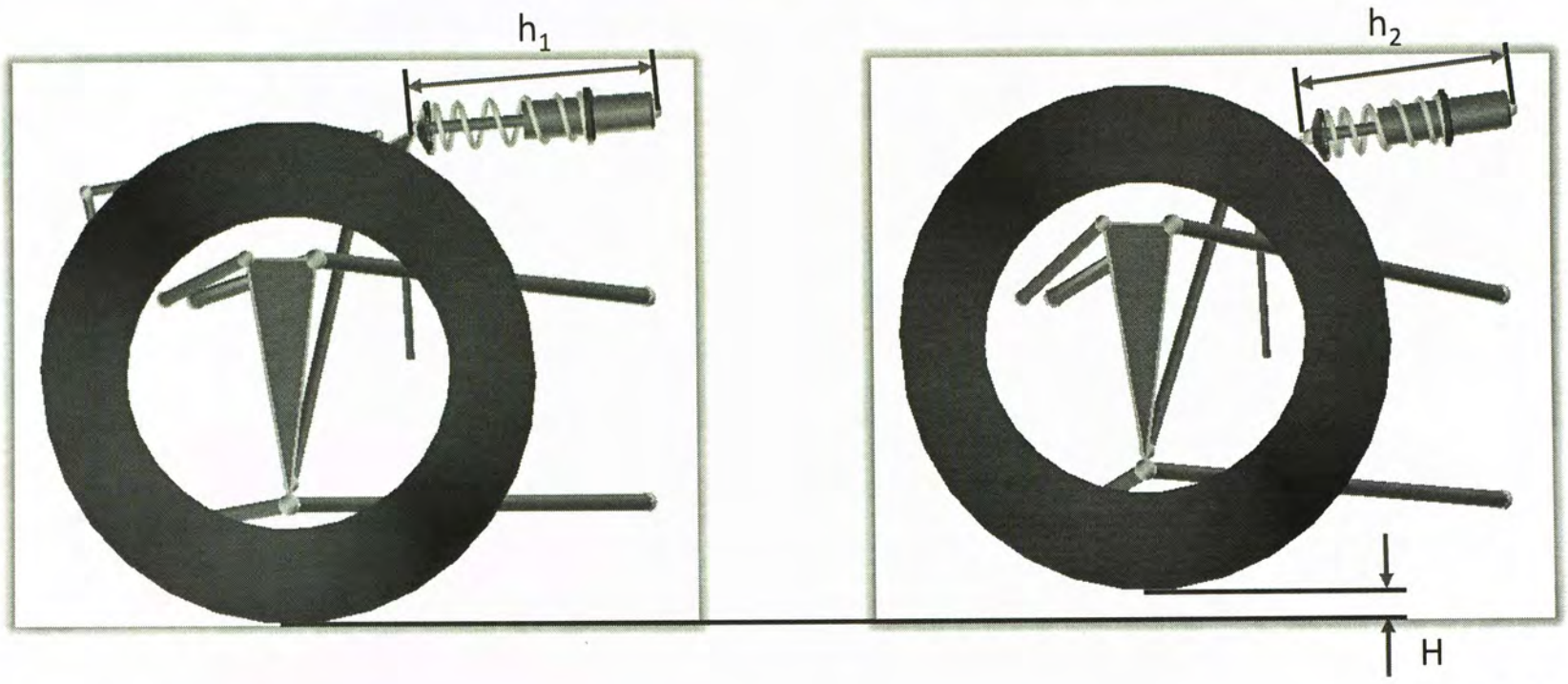


6. Steady State - Weight Transfer

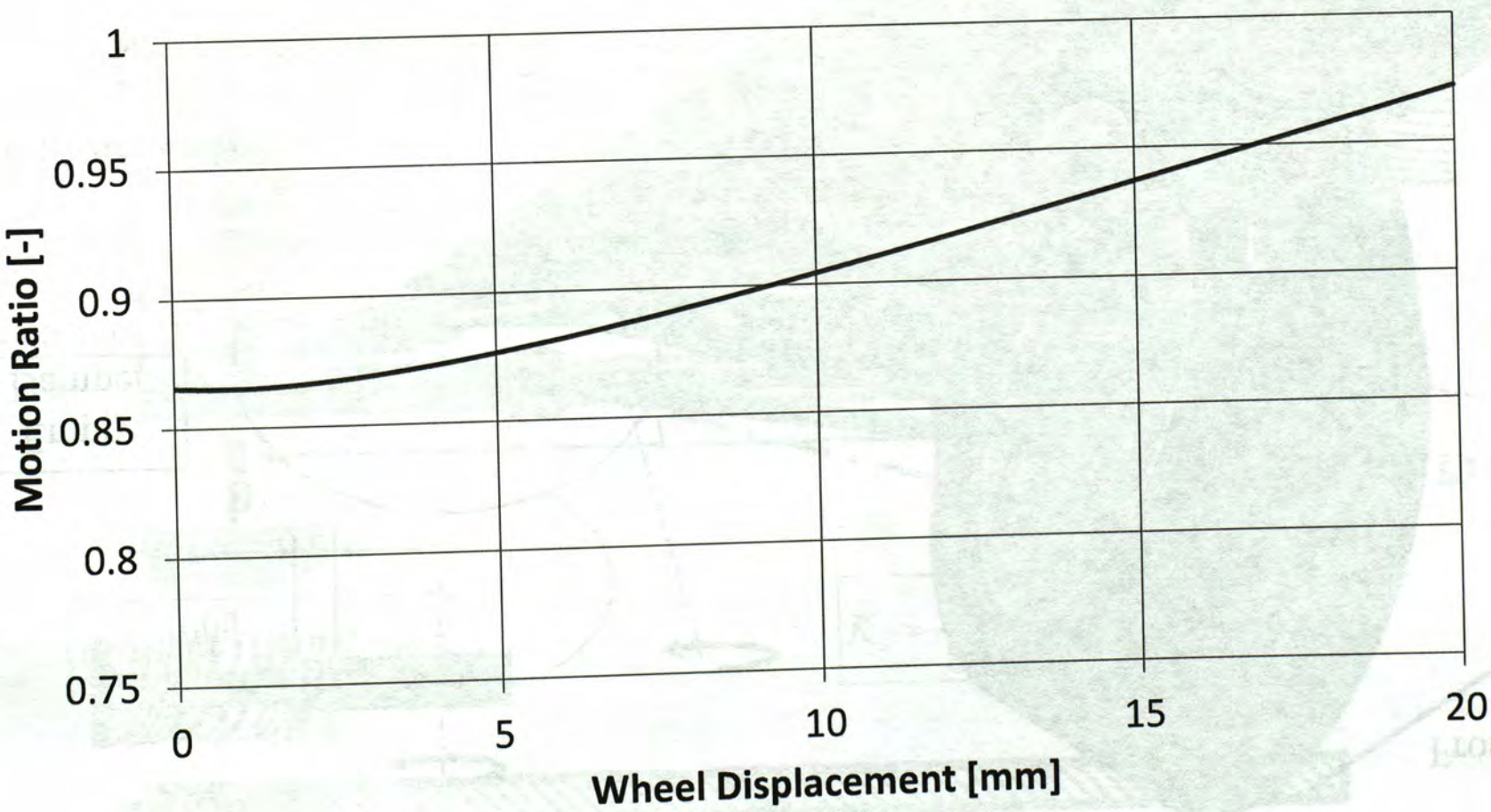
Motion Ratio

The **motion ratio** of a mechanism is the ratio of the displacement between two elements of the system.

$$MR_{wheel/spring} = \frac{Wheel\ displacement}{Spring\ Displacement} = \frac{H}{h} = \frac{H}{(h_2 - h_1)}$$

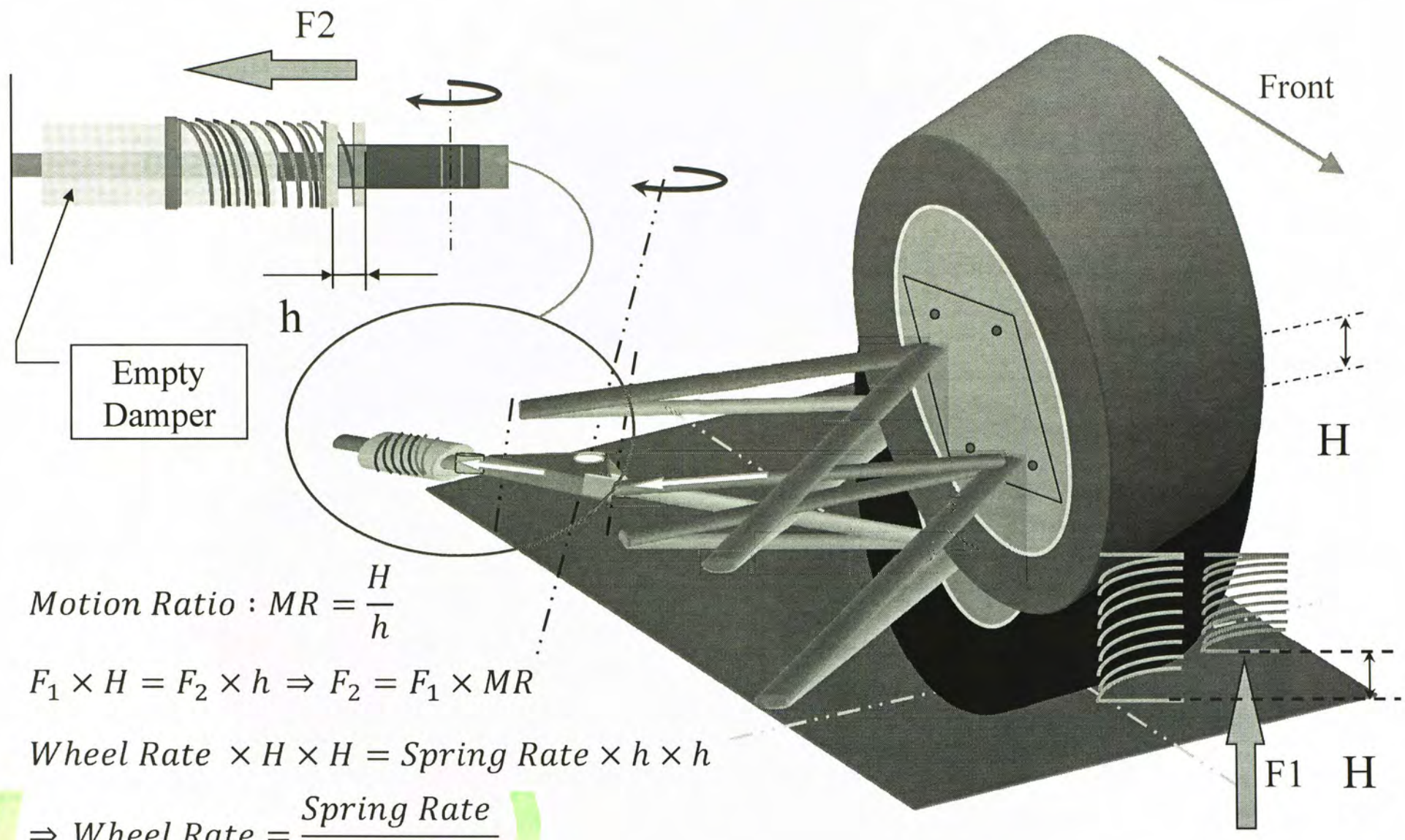


Motion Ratio



Warning: Motion ratios are NOT always perfectly linear!

Motion Ratio and Forces



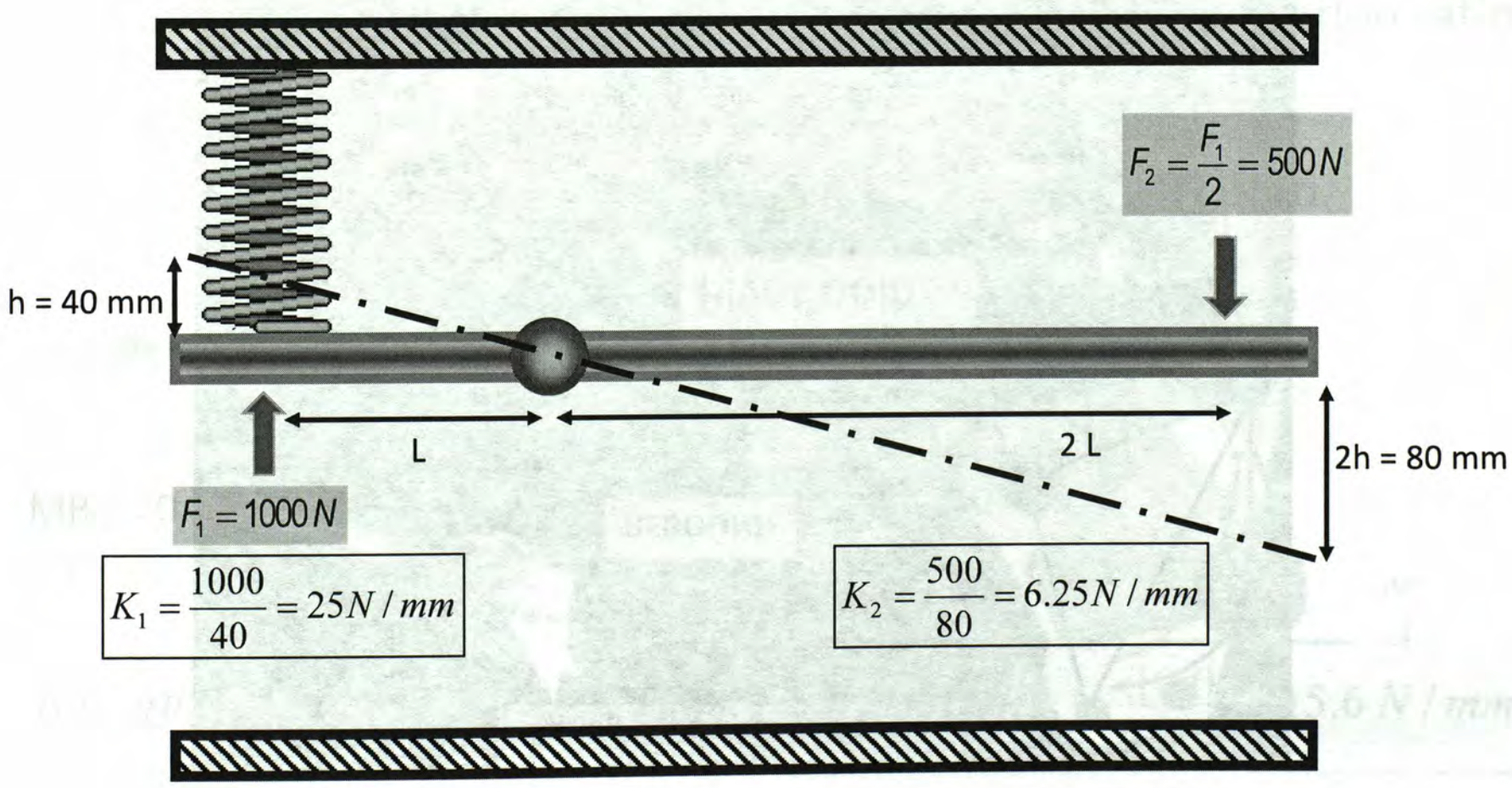
$$Motion\ Ratio : MR = \frac{H}{h}$$

$$F_1 \times H = F_2 \times h \Rightarrow F_2 = F_1 \times MR$$

$$Wheel\ Rate \times H \times H = Spring\ Rate \times h \times h$$

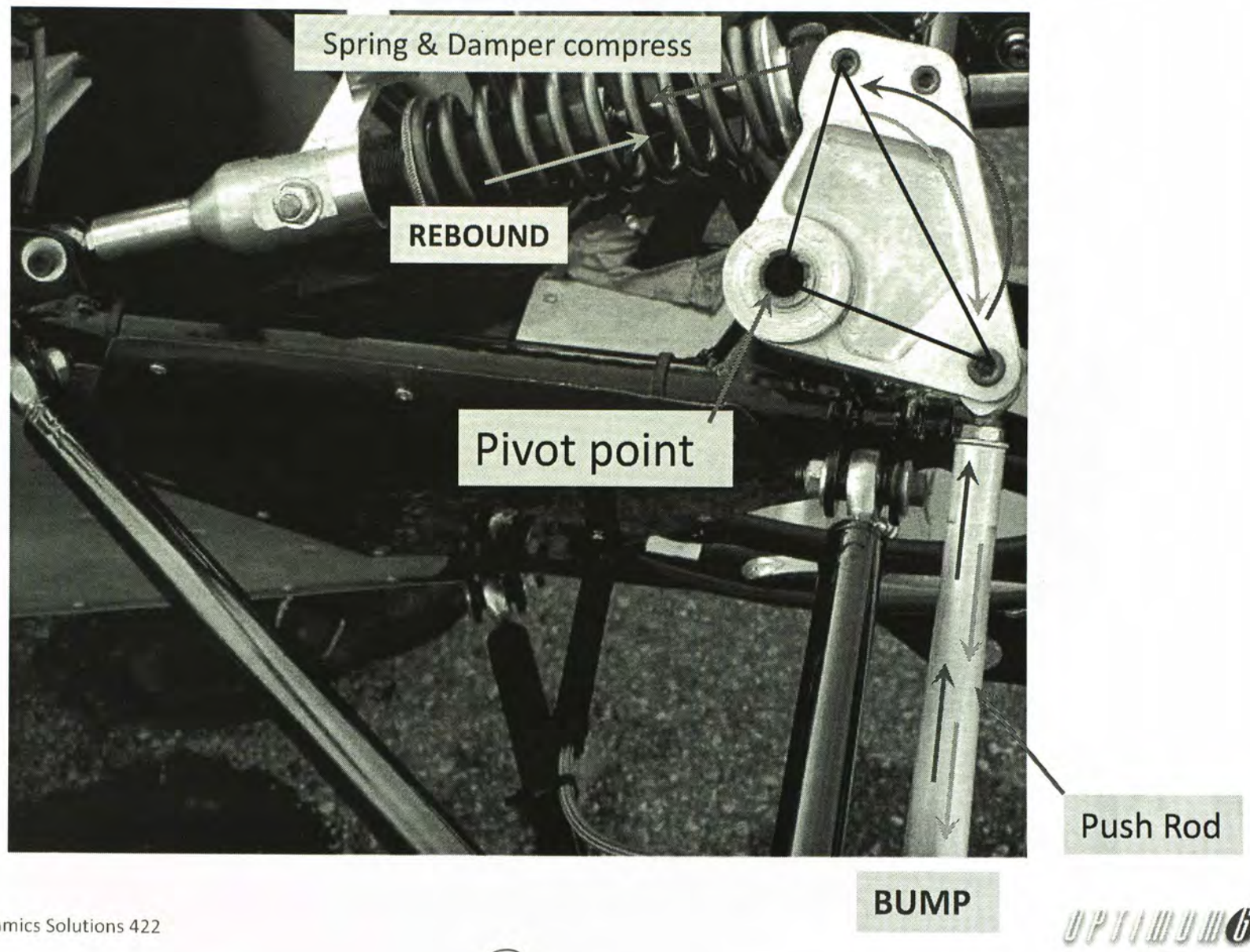
$$\Rightarrow Wheel\ Rate = \frac{Spring\ Rate}{MR^2}$$

Motion Ratio and Forces



$$K_2 = \frac{K_1}{MR^2} \qquad K_2 = \frac{25}{2^2} = 6.25N/mm$$

Motion Ratio and Forces

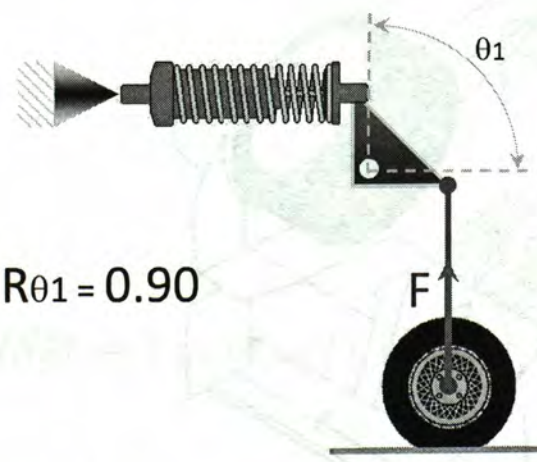


Motion Ratio and Forces

Variation of Wheel Rate with Stiffness Springs and with Motion ratio

2 Different spring stiffness are tested

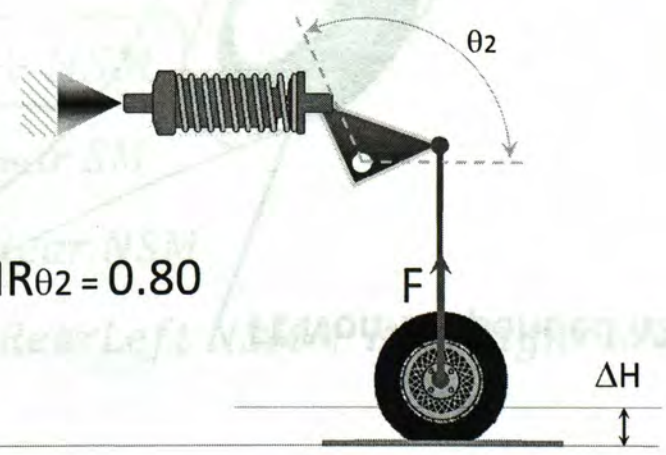
11.5 N/mm



$MR_{\theta 1} = 0.90$

$$WheelRate_{\theta 1} = \frac{11.5}{0.90^2} = 14.2\ N/mm$$

10.0 N/mm



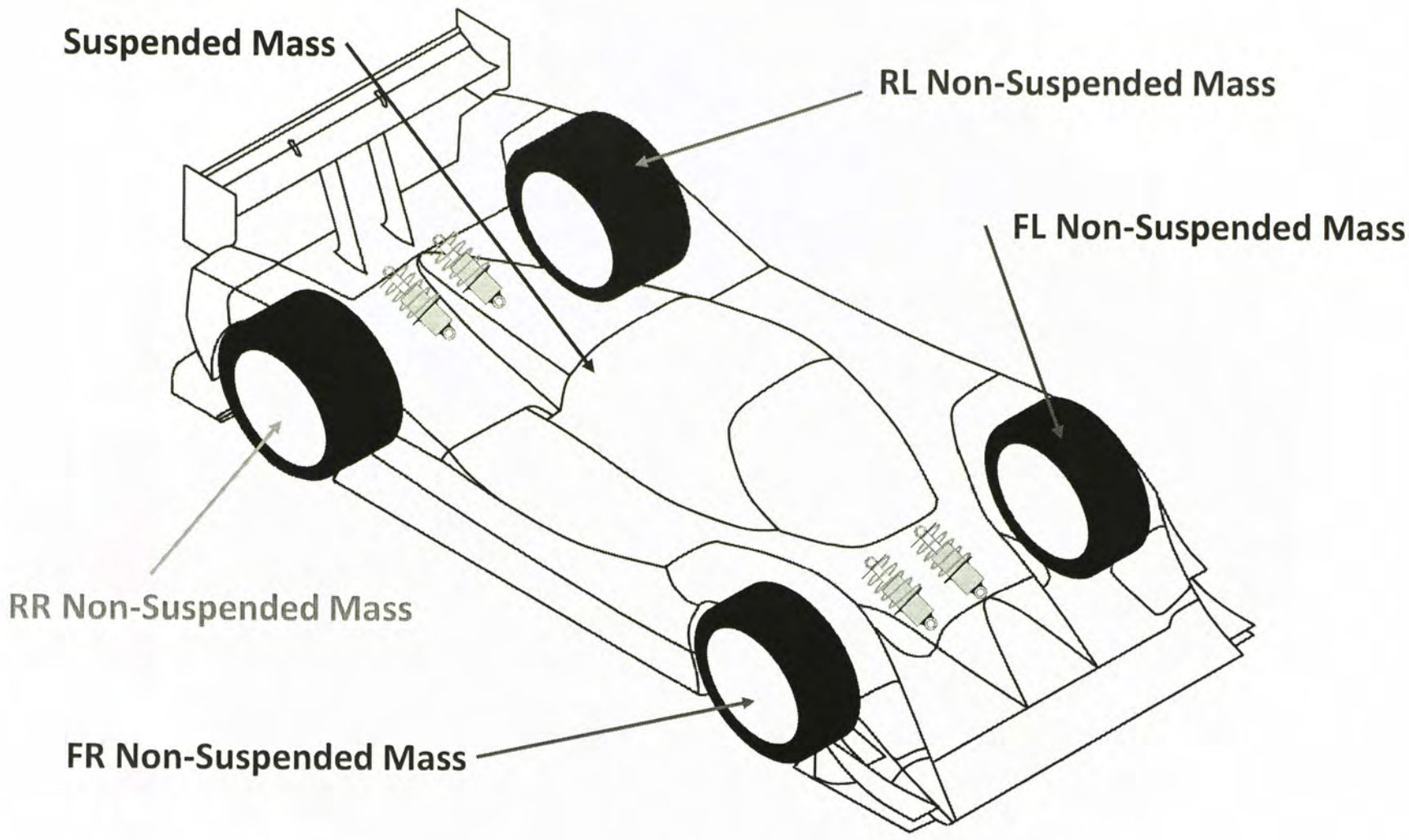
$MR_{\theta 2} = 0.80$

$$WheelRate_{\theta 2} = \frac{10.0}{0.80^2} = 15.6\ N/mm$$

So by decreasing the stiffness of the spring you have increased the Wheel Rate.

KPI ⟷ MR

Suspended and Non-Suspended Masses



Suspended and Non-Suspended Masses

SM: Suspended Mass

NSM: Non-Suspended Mass

$$\boxed{Total\ Mass = SM + NSM}$$

$$SM = Front\ SM + Rear\ SM$$

$$NSM = Front\ NSM + Rear\ NSM$$

$$NSM = FrontLeft\ NSM + FrontRight\ NSM + RearLeft\ NSM + RearRight\ NSM$$

Same equations can be re-written with the total, suspended and non-suspended weight.

Non-Suspended Mass Determination

Three methods:

- English method
  - Measure weight of all suspension components.
  - Assume 1/3 of suspension mass is suspended, 2/3 non-suspended.
- Scale method
  - Set chassis on blocks, disconnect pushrods, measure weight of suspension on scale under tire.
- Analytical method
  - Disassemble suspension, measure mass and Cg of every component.
  - Reassemble, move suspension- if control arm Cg moves 40mm vertically for 100mm of vertical tire movement, control arm mass is 40% non-suspended.

half Drive Shaft unsuspended

Weight Measurement

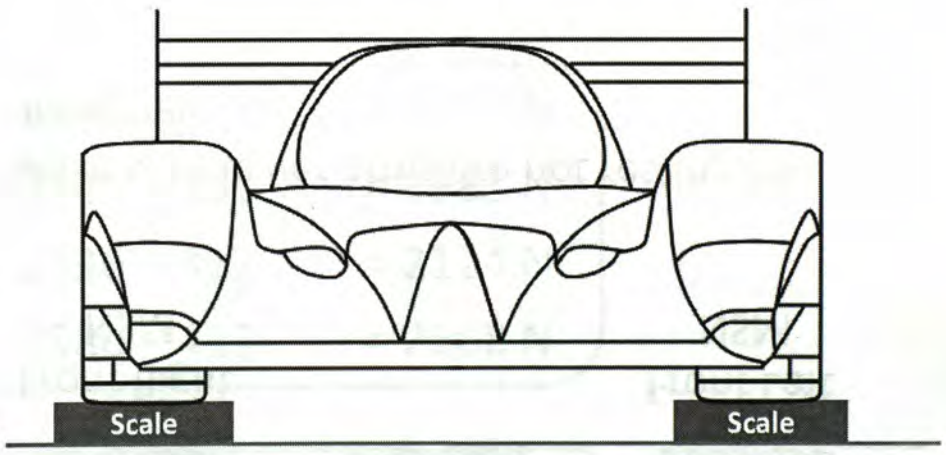
Measured corner weight:

CW_{fl} = 2300.0 N

CW_{fr} = 2344.0 N

CW_{rl} = 2536.0 N

CW_{rr} = 2580.0 N



Front Right Corner Weight

Front Left Corner Weight

The planarity of the scales and the friction in suspension ball joints and damper affects the measurement.

Shake the car before taking the measure

Weight Measurement

Disconnect the suspension at the rocker arm.

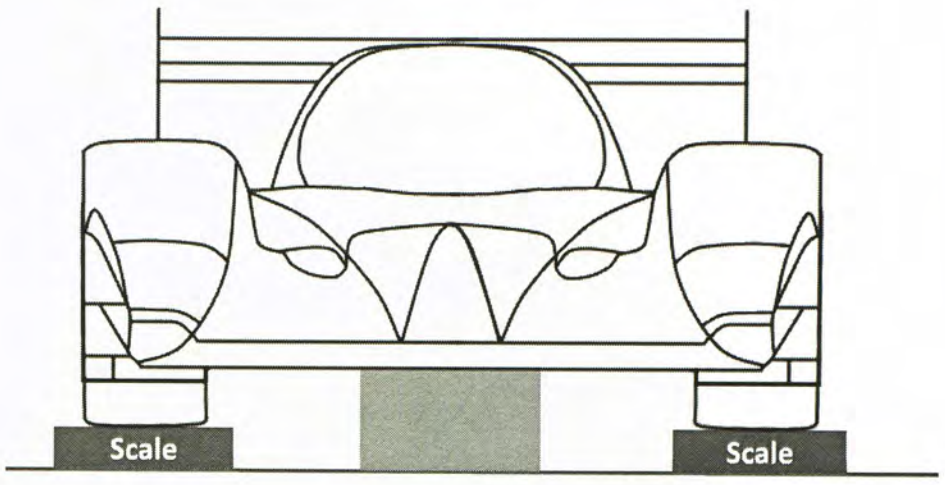
Measured non-suspended weight:

NSW_{fl} = 363.0 N

NSW_{fr} = 363.0 N

NSW_{rl} = 412.0 N

NSW_{rr} = 412.0 N



Front Right NSM

Front Left NSM

Make sure lower arms are not resting on undertray.

SW and NSW Calculations

$SW_{fl} = CW_{fl} - NSW_{fl}$ = 2300 – 363 = 1937 N

$SW_{fr} = CW_{fr} - NSW_{fr}$ = 2344 – 363 = 1981 N

$SW_{rl} = CW_{rl} - NSW_{rl}$ = 2536 – 412 = 2124 N

$SW_{rr} = CW_{rr} - NSW_{rr}$ = 2580 – 412 = 2168 N

$SW_{front} = SW_{fl} + SW_{fr}$ = 3918 N

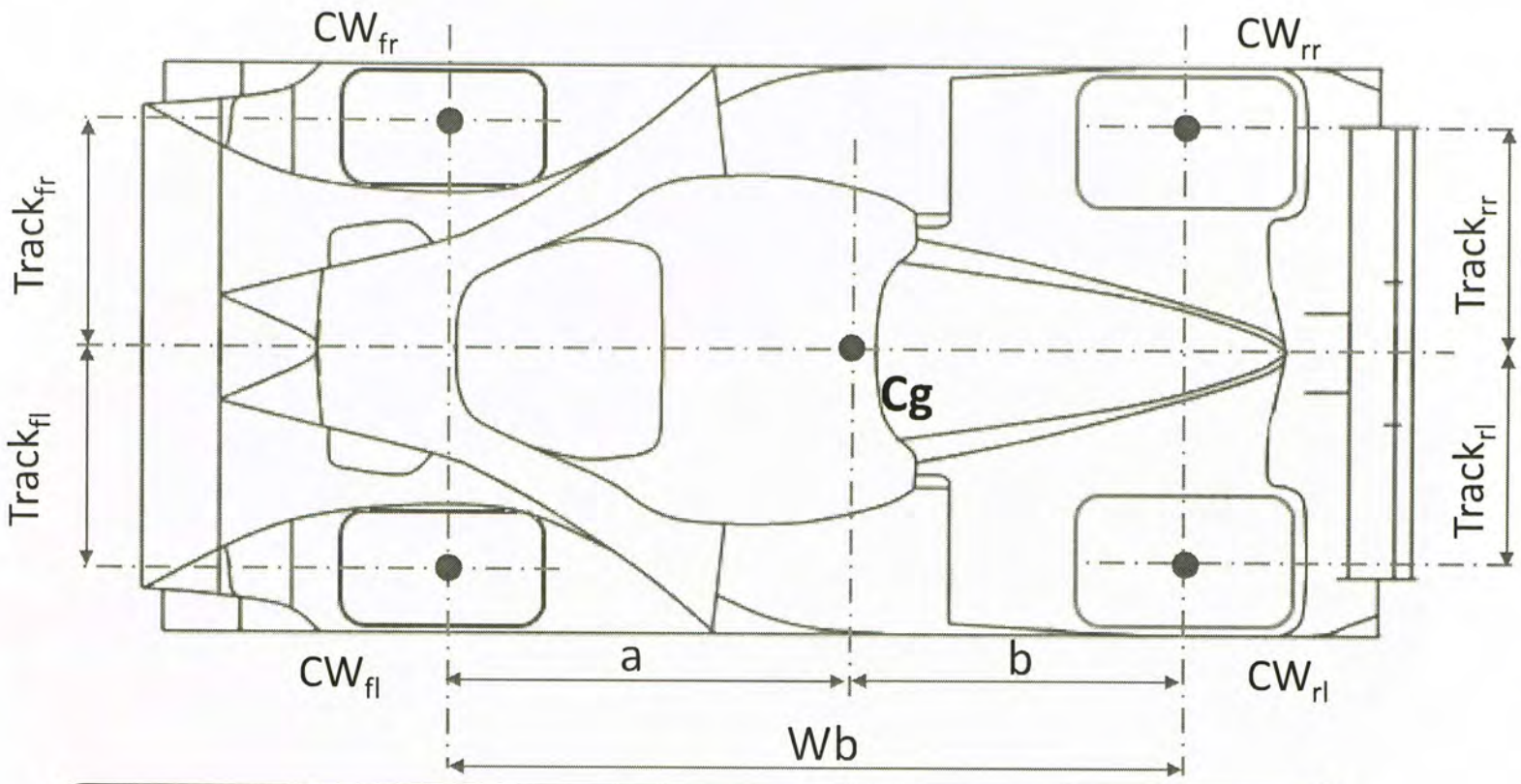
$SW_{rear} = SW_{rl} + SW_{rr}$ = 4292 N

$SW_{total} = SW_{front} + SW_{rear}$ = 8210 N

$W_{total} = CW_{fl} + CW_{fl} + CW_{fl} + CW_{fl}$ = 9760 N

$M_{total} = W_{total}/9.81$ = 995 Kg

Center of Gravity Location

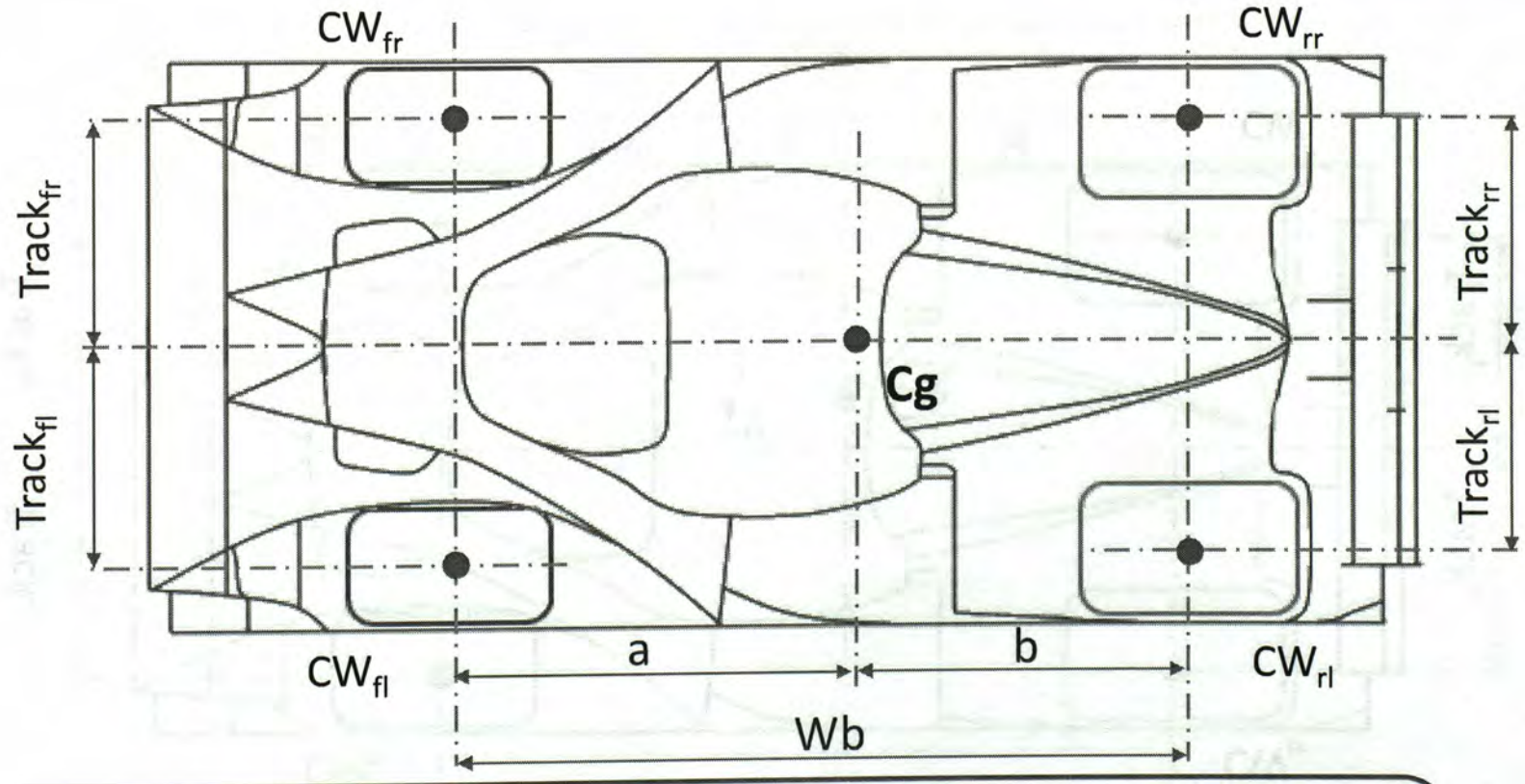


$$Wb = a + b$$

$$a = W_{rear}/W_{total} \times Wb = (CW_{rl} + CW_{rr})/Wtotal \times Wb$$

$$b = W_{front}/W_{total} \times Wb = (CW_{fl} + CW_{fr})/Wtotal \times Wb$$

Center of Gravity Location

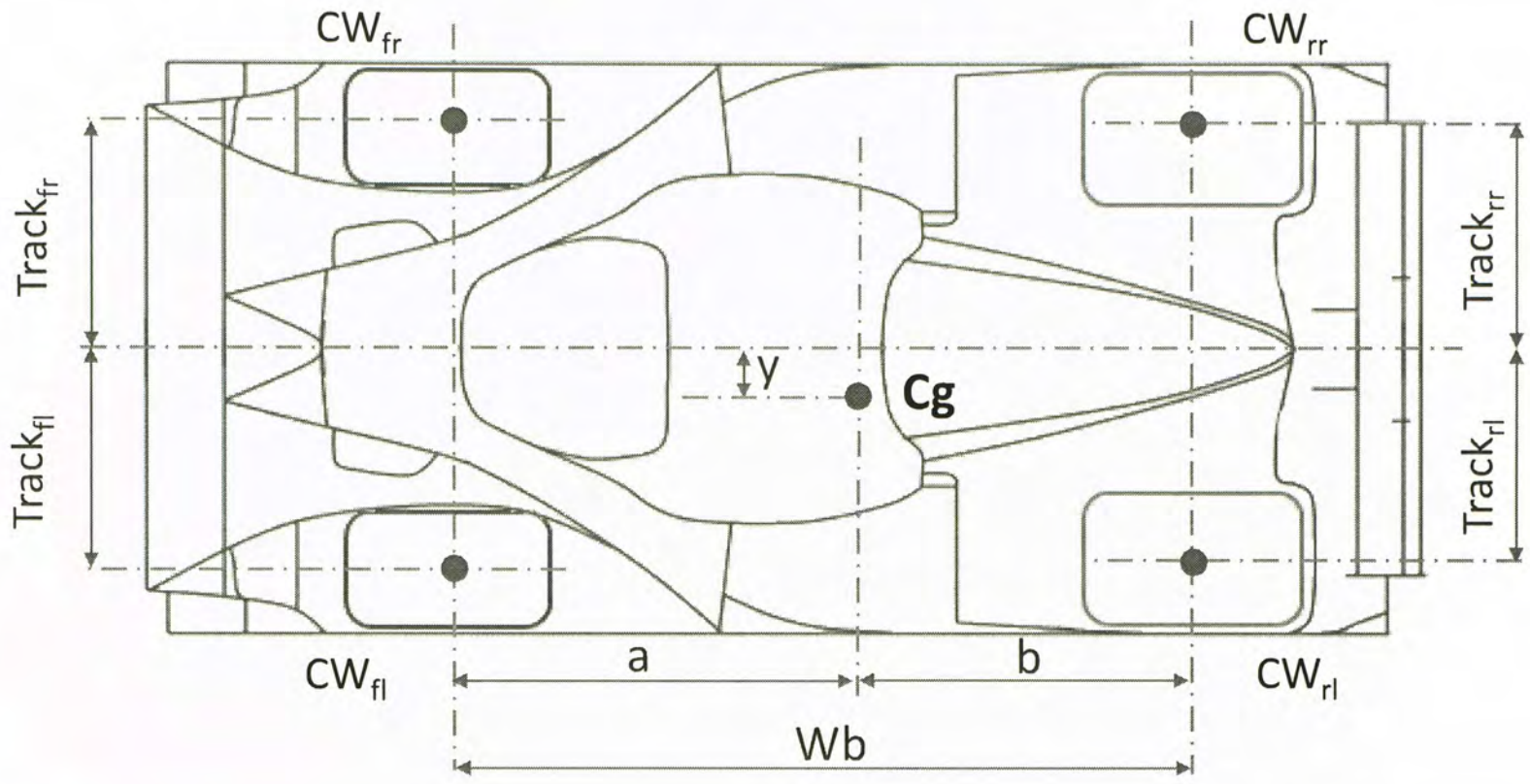


$$Wb = a + b$$

$$a = 5116/9760 \times 2980 = (2536 + 2580)/9760 \times 2980 = 1562\,mm$$

$$b = 4644/9760 \times 2980 = (2300 + 2344)/9760 \times 2980 = 1418\,mm$$

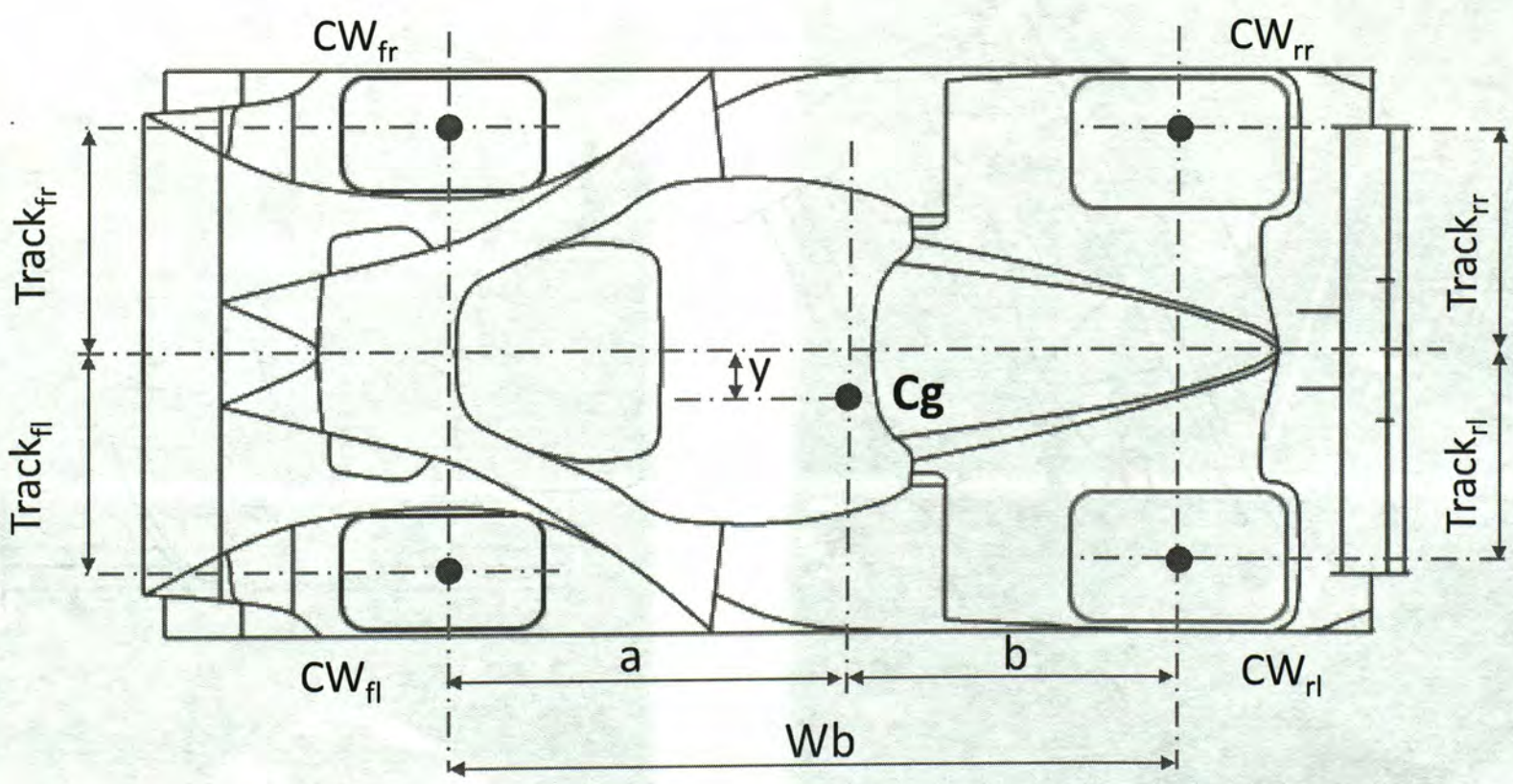
Center of Gravity Location



$$W_{total} \times (T_{rl} - y) + CW_{fl} \times (T_{fl} - T_{rl}) = CW_{fr} \times (T_{fr} + T_{rl}) + CW_{rr} \times (T_{rr} + T_{rl})$$

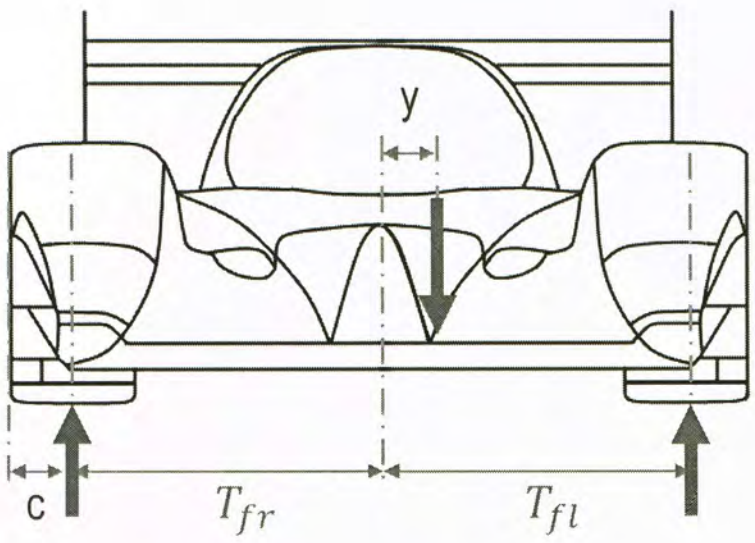
$$y = (CW_{fl} \times (T_{fl} - T_{rl}) - CW_{rr} \times (T_{rr} + T_{rl}) - CW_{fr} \times (T_{fr} + T_{rl}))/W_{total} + T_{rl}$$

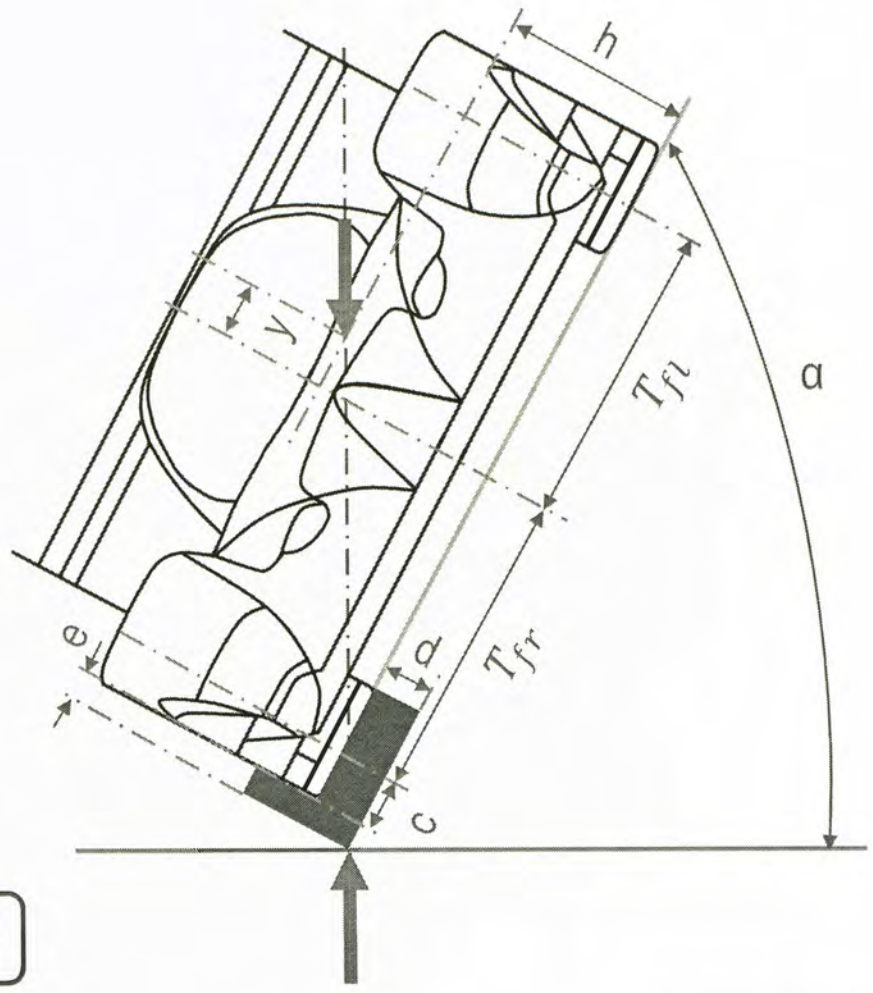
Center of Gravity Location



$$y = 796 + (2300 \times (820 - 796) - 2344 \times (820 + 796) - 2580 \times (796 + 796))/9760 = -7.3\ mm$$

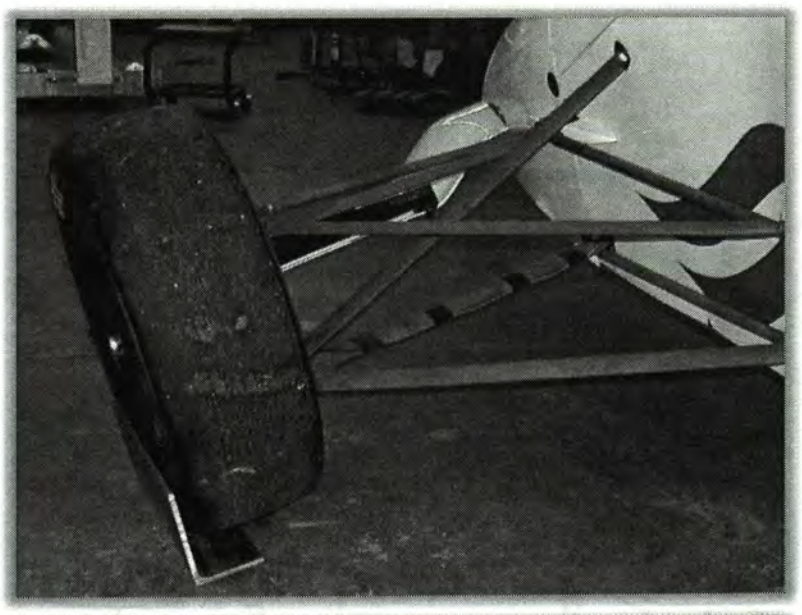
Center of Gravity Location

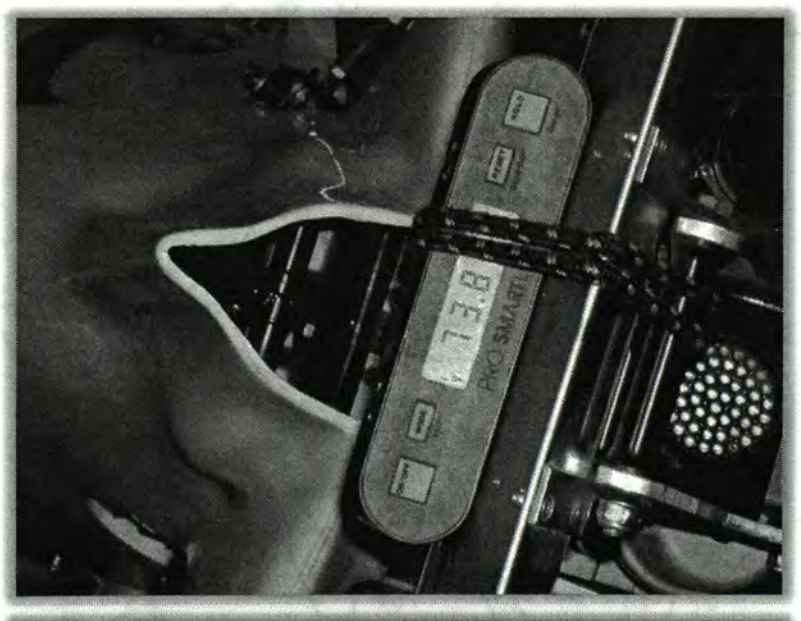


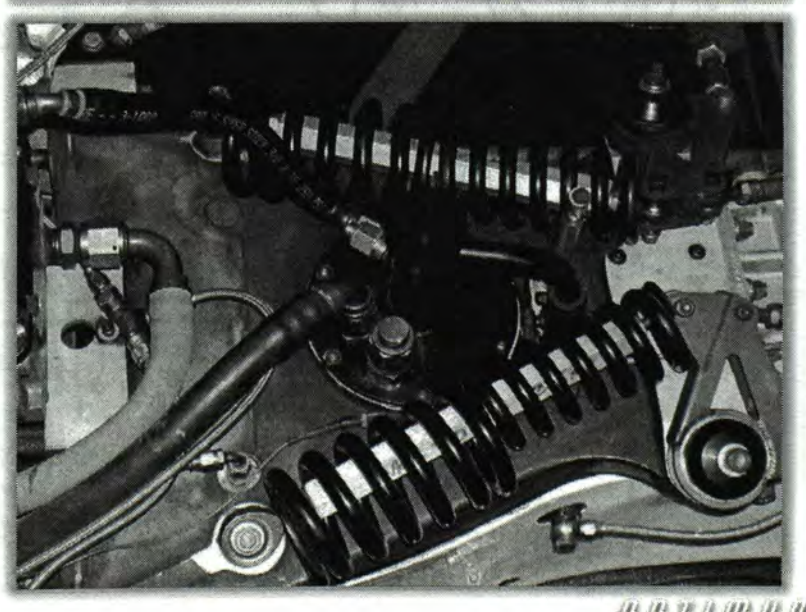


$$h = tan(90° - \alpha) \times (T_{fr} + y + c + e) - d$$

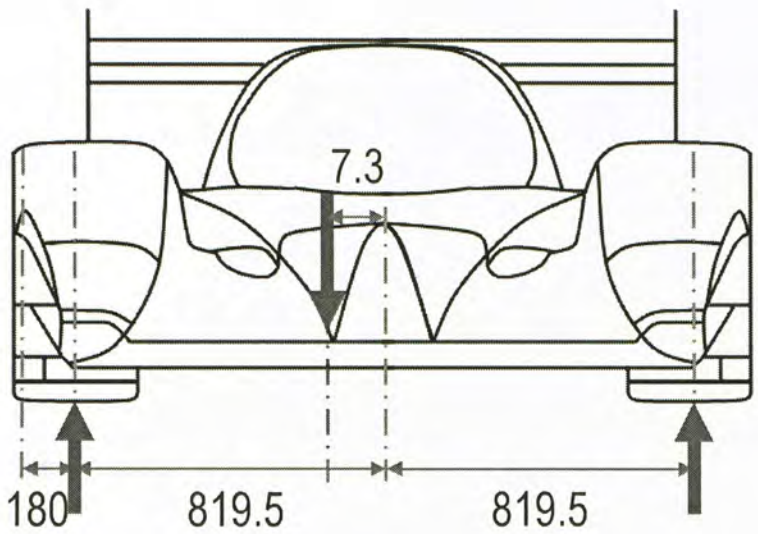
Center of Gravity Measurement

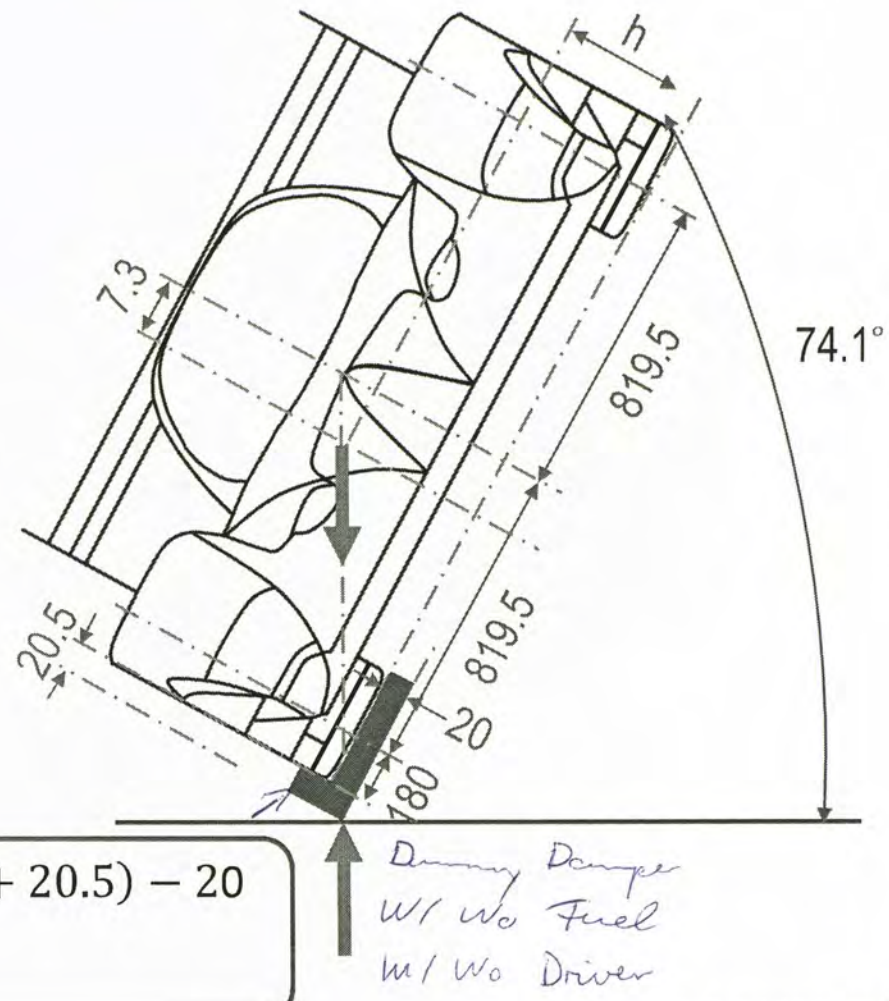






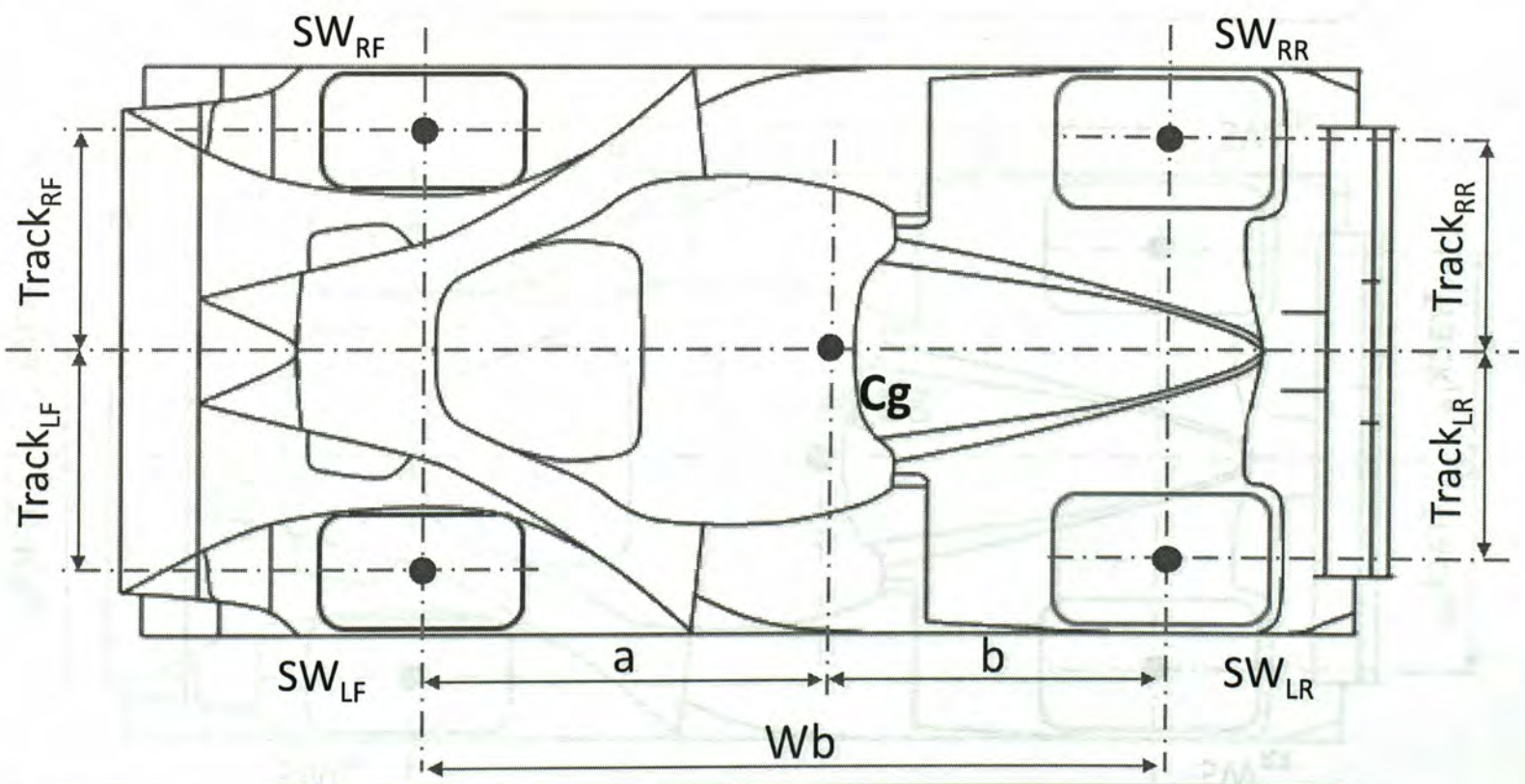
Center of Gravity Location





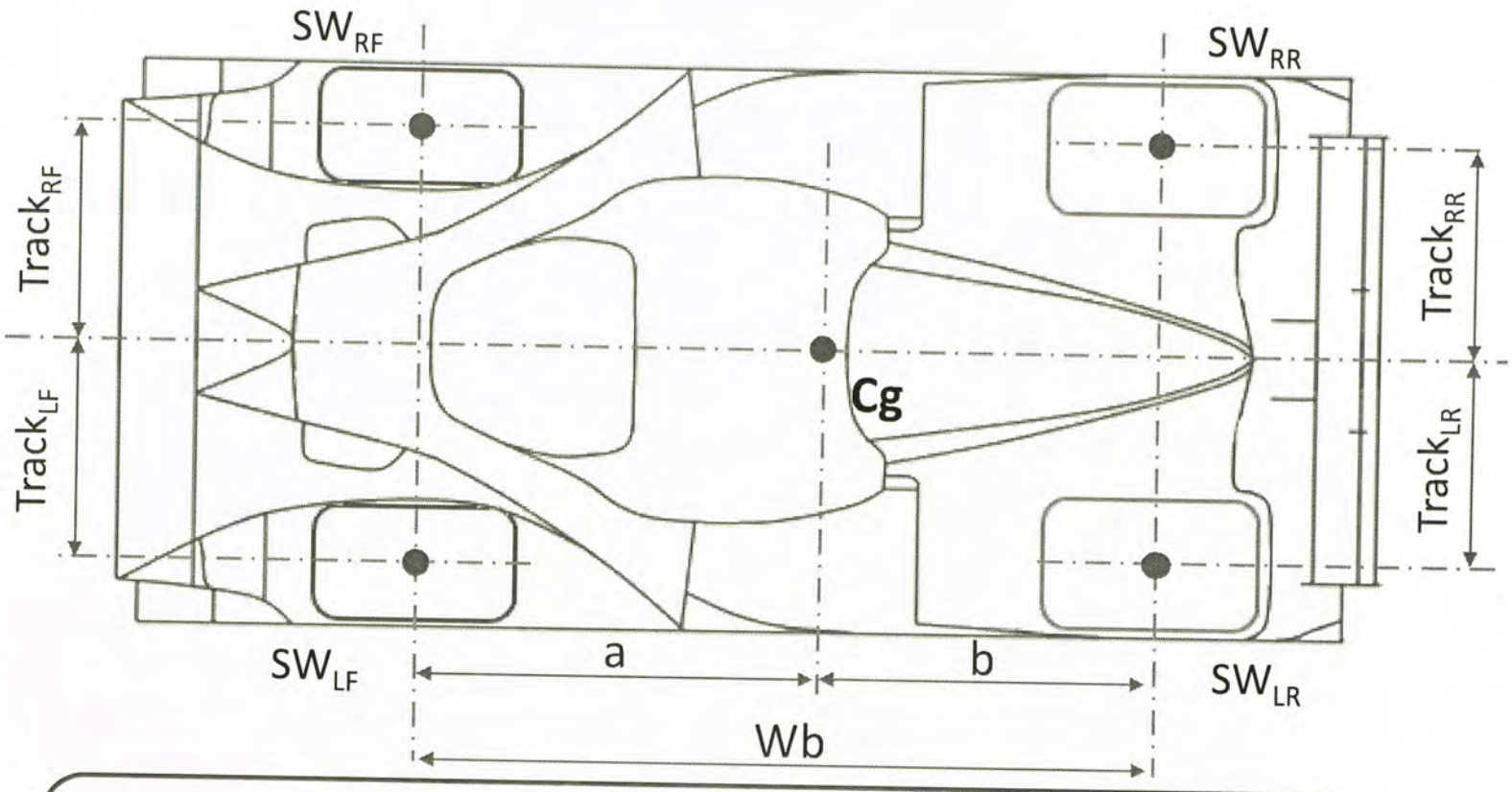
$$h = tan(90° - 74.1°) \times (819.5 - 7.3 + 180 + 20.5) - 20$$
$$= 270.4\ mm$$

Suspended Weight Cg Location



$$Wb = a_{SW} + b_{SW}$$
$$a_{SW} = SW_{Rear}/SW_{Total} \times Wb = (SW_{LR} + SW_{RR})/SW_{Total} \times Wb$$
$$b_{SW} = SW_{Front}/SW_{Total} \times Wb = (SW_{LF} + SW_{RF})/SW_{Total} \times Wb$$

Suspended Weight Cg Location

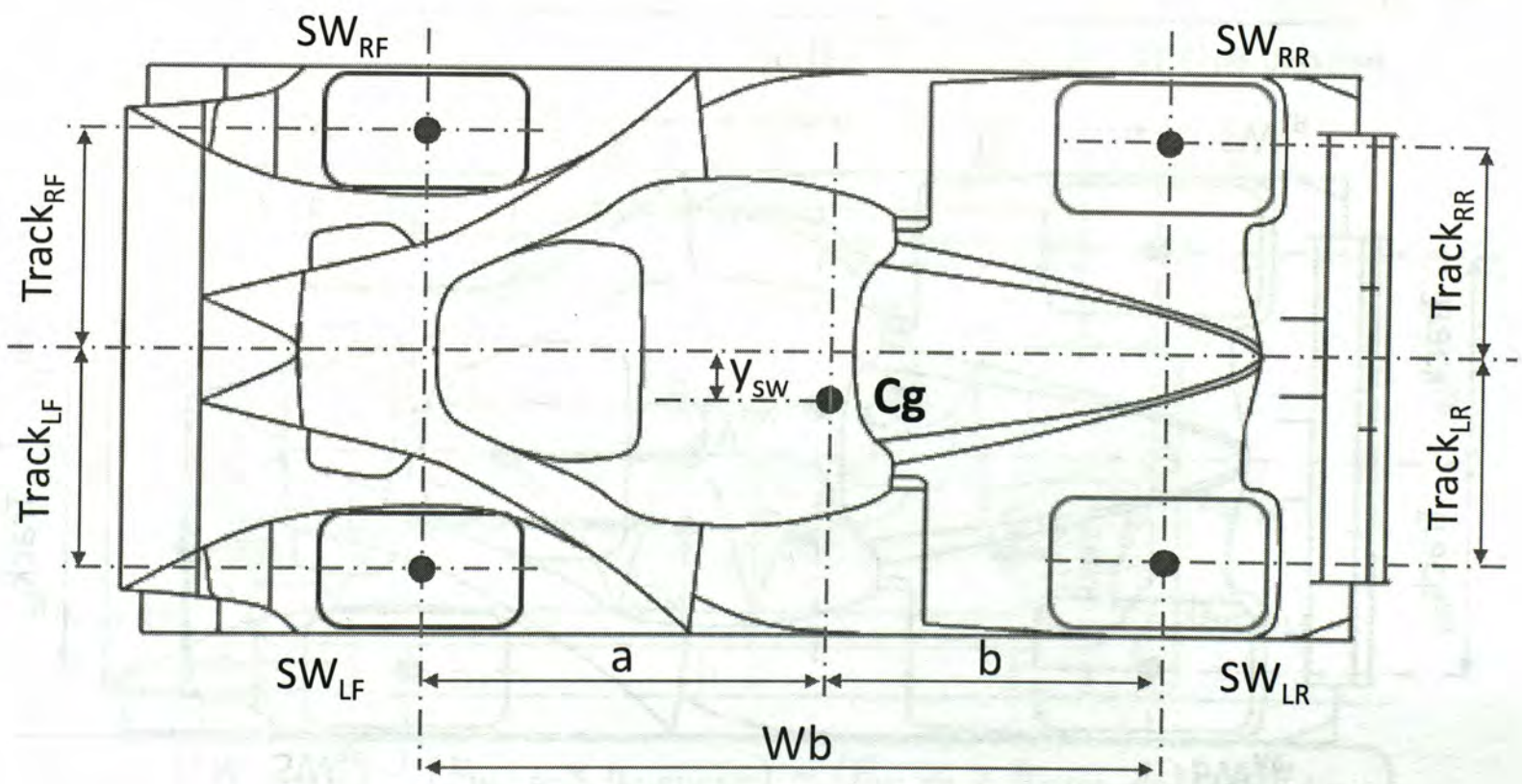


$$Wb_{SW} = a_{SW} + b_{SW}$$

$$a_{SW} = (2124 + 2168)/8210 \times 2980 = 1558\,mm$$

$$b_{SW} = (1937 + 1981)/8210 \times 2980 = 1422\,mm$$

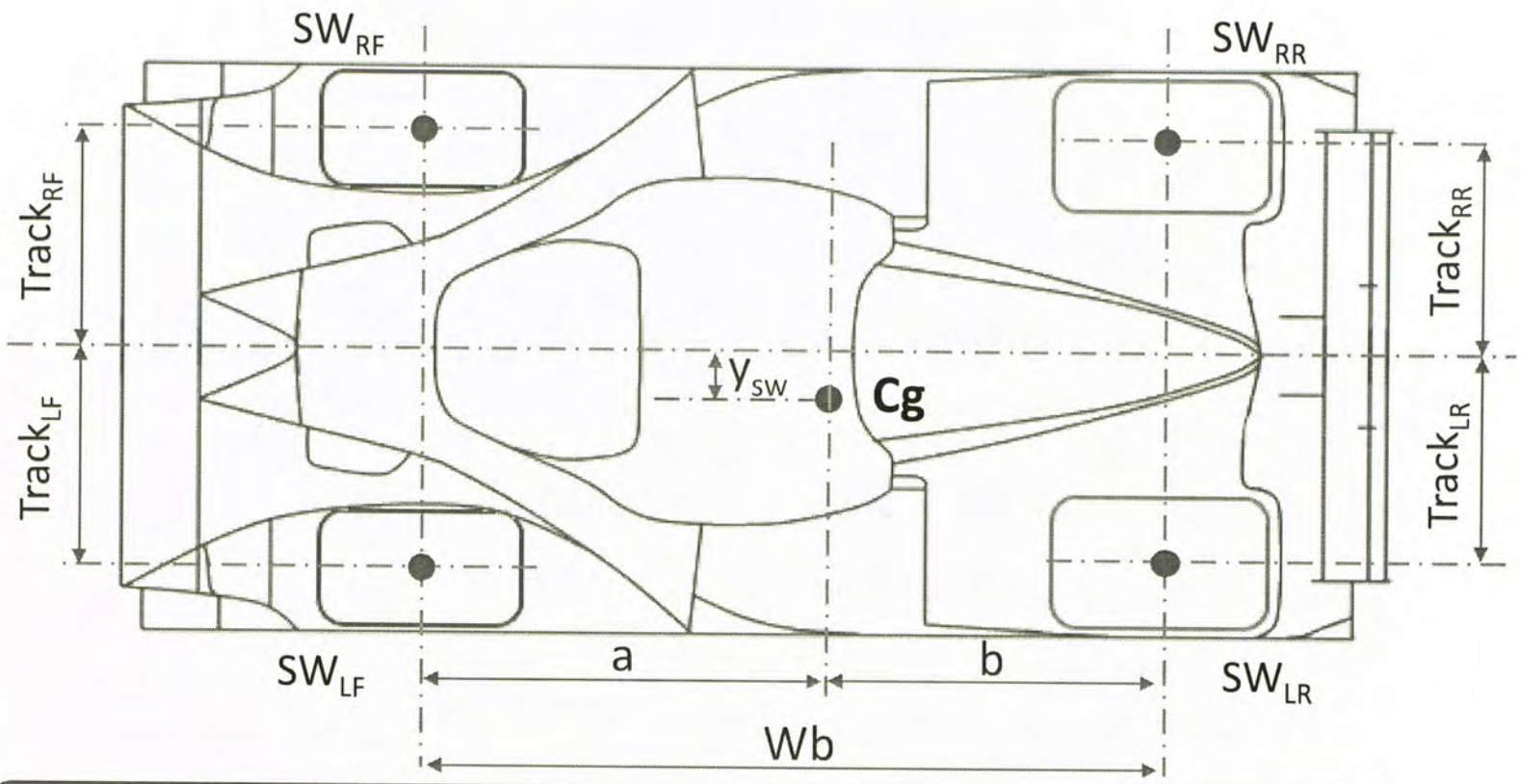
Suspended Weight Cg Location



$$SW_{total} \times (T_{LR} - y_{SW}) + SW_{LF} \times (T_{LF} - T_{LR}) = SW_{RF} \times (T_{RF} + T_{LR}) + SW_{RR} \times (T_{RR} + T_{LR})$$

$$y_{SW} = (SW_{LF} \times (T_{LF} - T_{LR}) - SW_{RF} \times (T_{RF} + T_{LR}) - SW_{RR} \times (T_{RR} + T_{LR}))/SW_{Total} + T_{LR}$$

Suspended Weight Cg Location



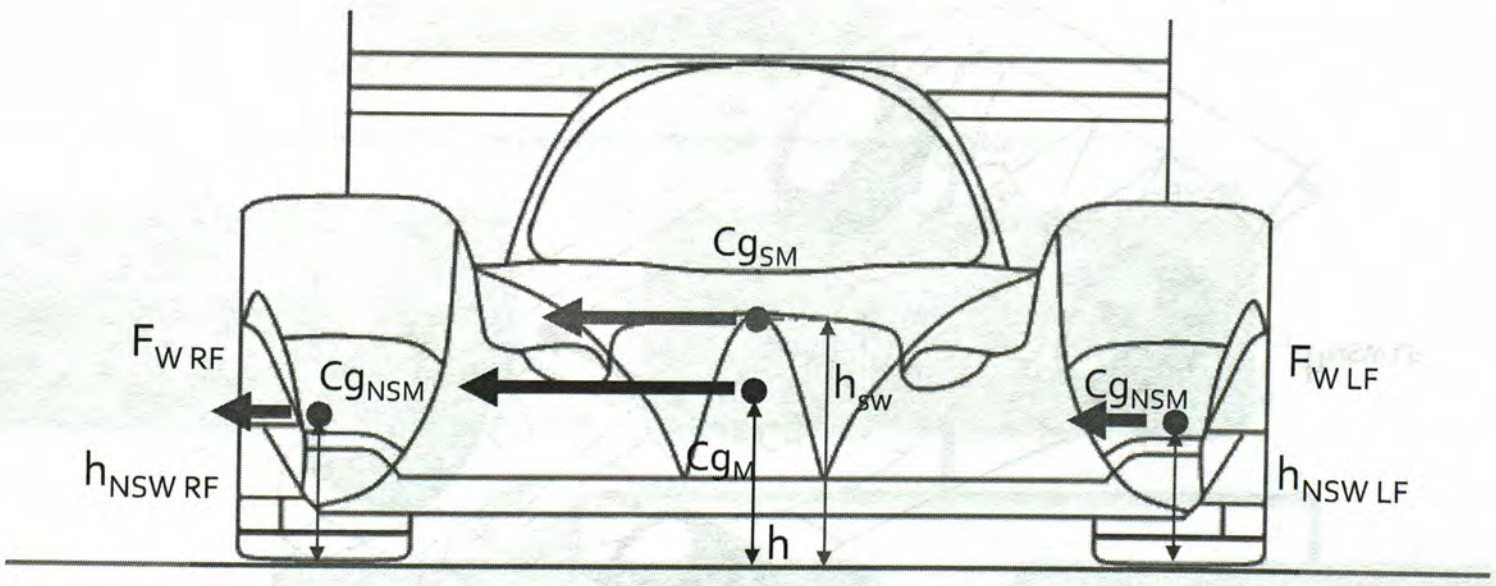
$$y_{SW} = 796 + (1937 \times (820 - 796) - 1981 \times (820 + 796) - 2168 \times (796 + 796))/8210 = -8.5\ mm$$

Suspended Weight Cg Location

Applying a lateral acceleration a_y= 1G

F = m x a = W

Assumption: the center of gravity of the NSW is located at the wheel center.



$$h_{sw} = [(W_f \times h) - (F_{W\,LF} \times h_{NSW\,LF}) - (F_{W\,RF} \times h_{NSW\,RF})]/SW_f$$

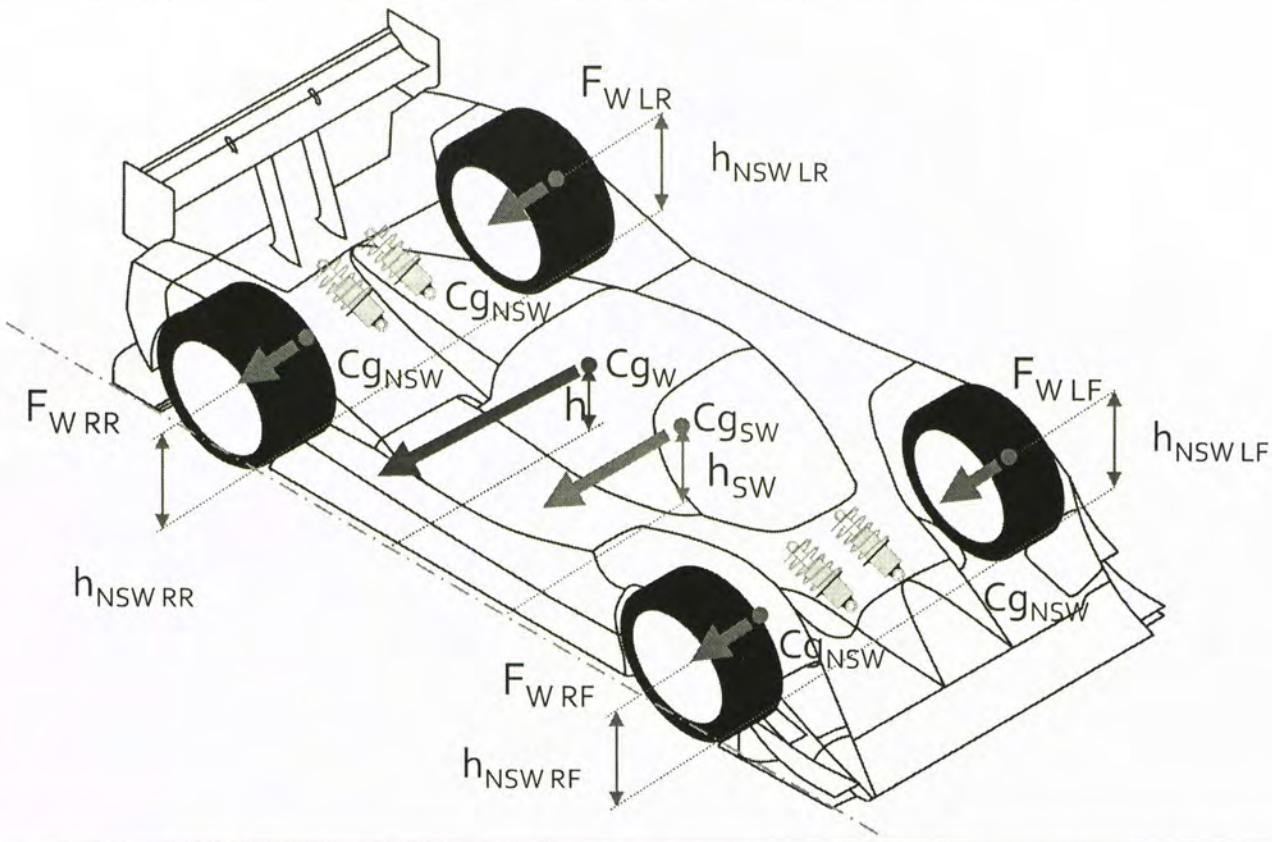
$$W_f \times h = (F_{W\,LF} \times h_{NSW\,LF}) + (F_{W\,RF} \times h_{NSW\,RF}) + (SW_f \times h_{SW})$$

$$h_{FSW} = [(4644 \times 270) - (363 \times 291) - (363 \times 291)]/3918$$

$$h_{FSW} = 266.1\ mm$$

Note: 2D Calculation (Front)

Suspended Weight Cg Location



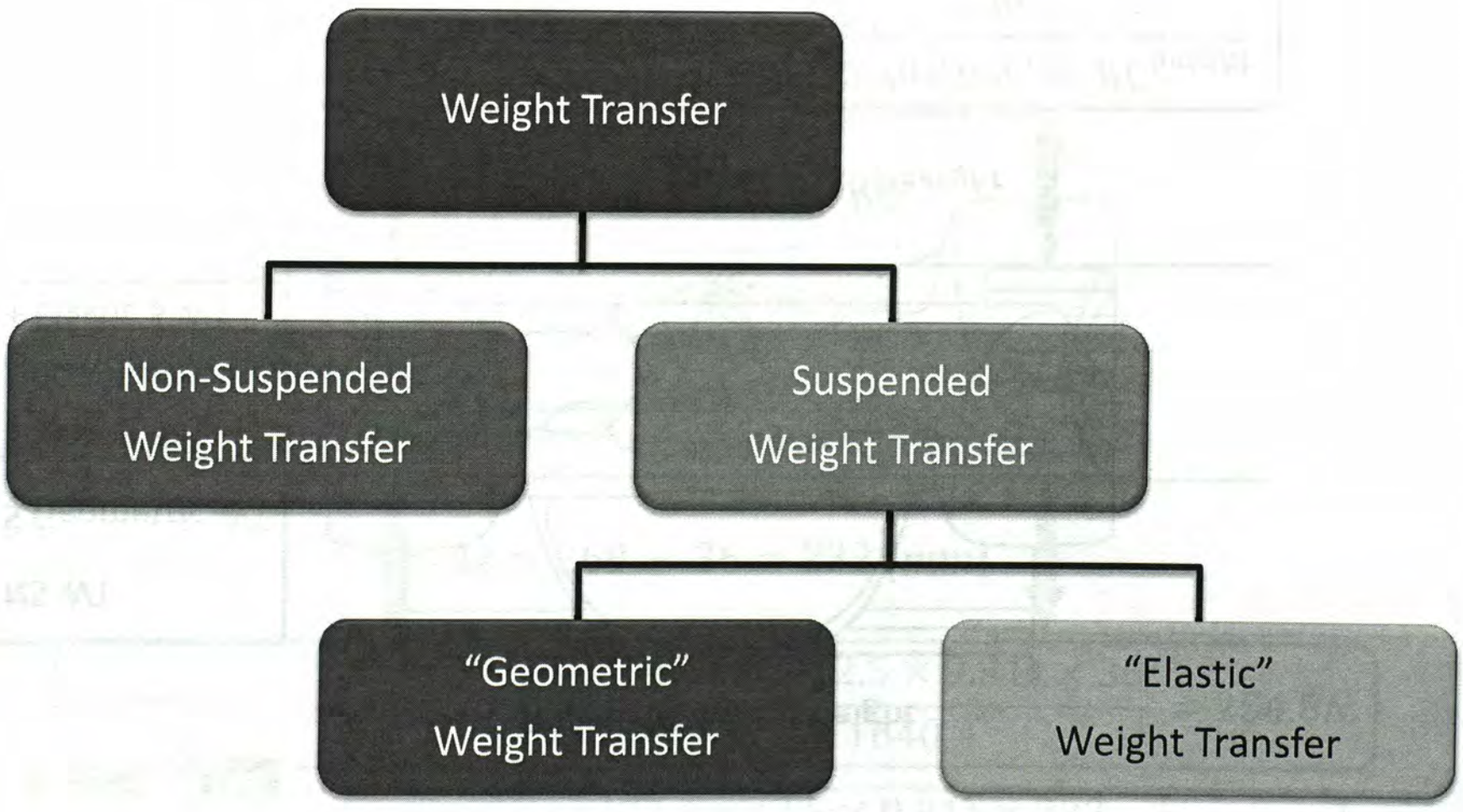
$$W \times h = (F_{W\,LF} \times h_{NSW\,LF}) + (F_{W\,RF} \times h_{NSW\,RF}) + (F_{W\,LR} \times h_{NSW\,LR}) + (F_{W\,RR} \times h_{NSW\,RR}) + (SW \times h_{SW})$$

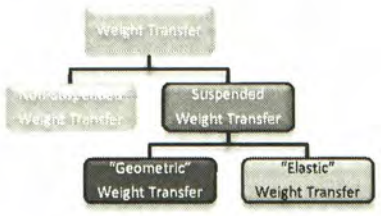
$$h_{sw} = [(W \times h) - (F_{W\,LF} \times h_{NSW\,LF}) - (F_{W\,RF} \times h_{NSW\,RF}) - (F_{W\,LR} \times h_{NSW\,LR}) - (F_{W\,rr} \times h_{NSW\,RR})]/SW$$

$$h_{sw} = [(9760 \times 270) - (363 \times 280) - (363 \times 280) - (412 \times 280) - (412 \times 280)]/8210$$

$$h_{sw} = 268.0\ mm$$

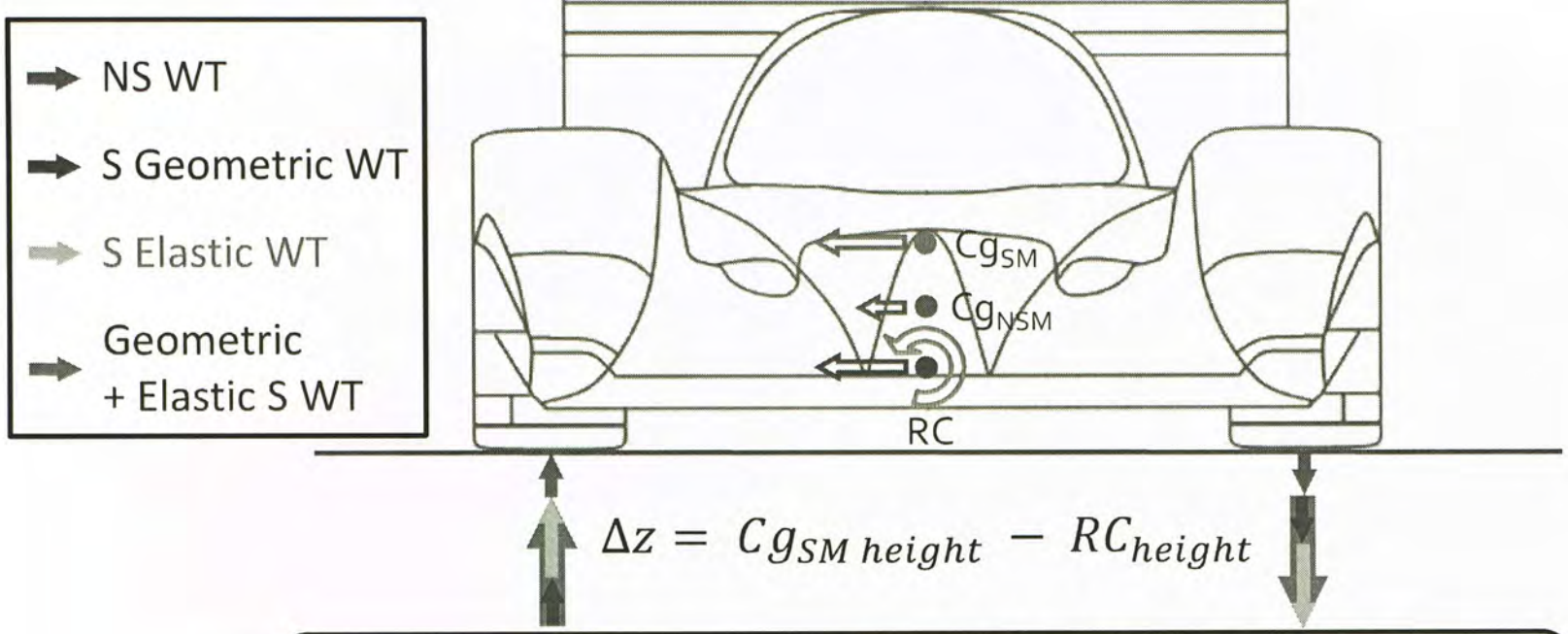
Lateral Weight Transfer Basics





Lateral Weight Transfer

RC height < CG_{SM} height

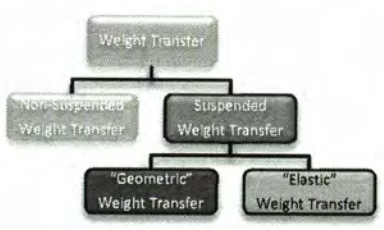


$$\Delta z = Cg_{SM\ height} - RC_{height}$$

$$Lateral\ S\ "Geometric"\ WT = \frac{SM \times lat.acc. \times RC_{height}}{track}$$

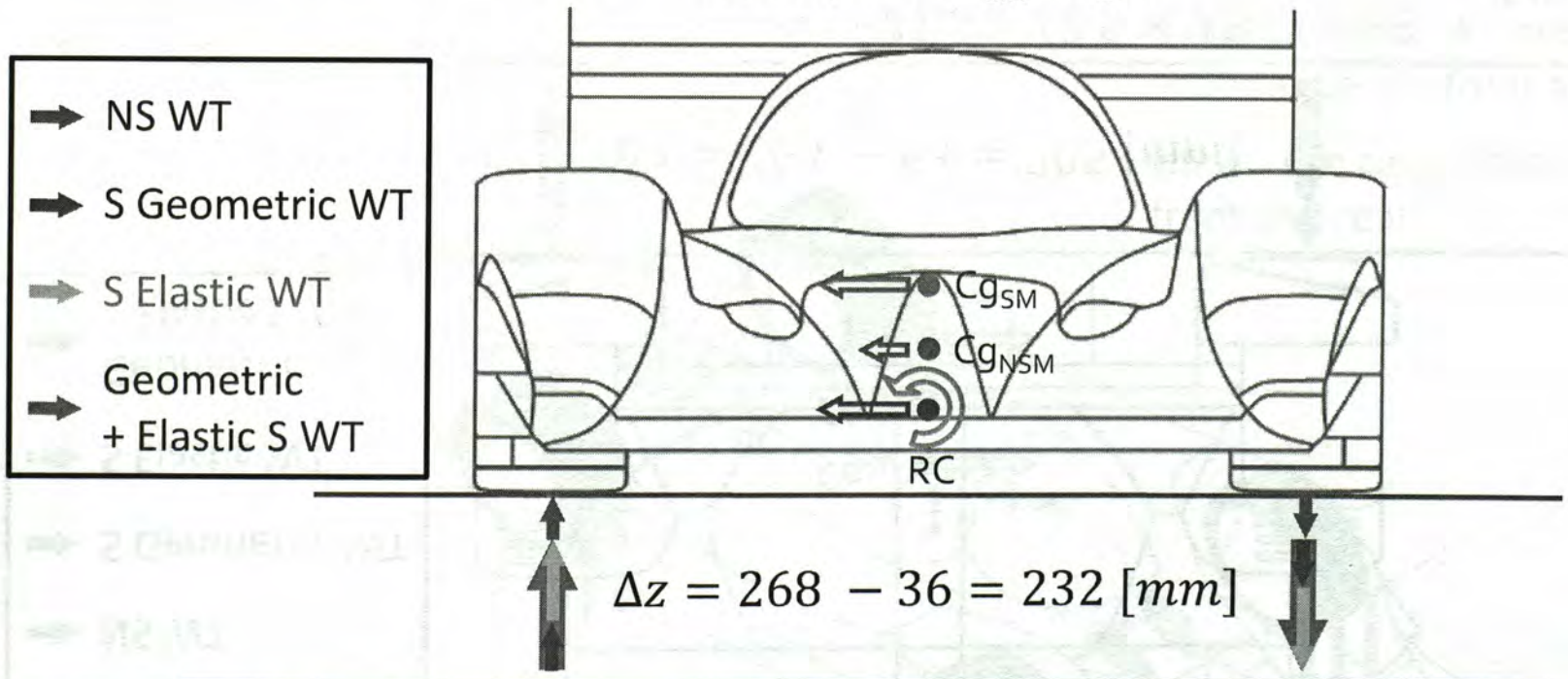
$$Lateral\ S\ "Elastic"\ WT = \frac{SM \times lat.acc. \times \Delta z}{track}$$

Note: 2D Calculation (Front)



Lateral Weight Transfer

RC height < CG_{SM} height

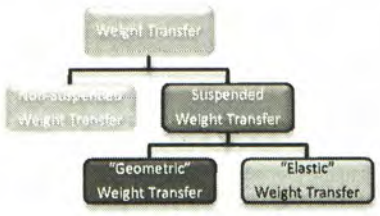


$$\Delta z = 268 - 36 = 232\ [mm]$$

$$Lateral\ S\ "Geometric"\ WT = \frac{473.4 \times (2.5 \times 9.81) \times 36}{1640} = 254.8N$$

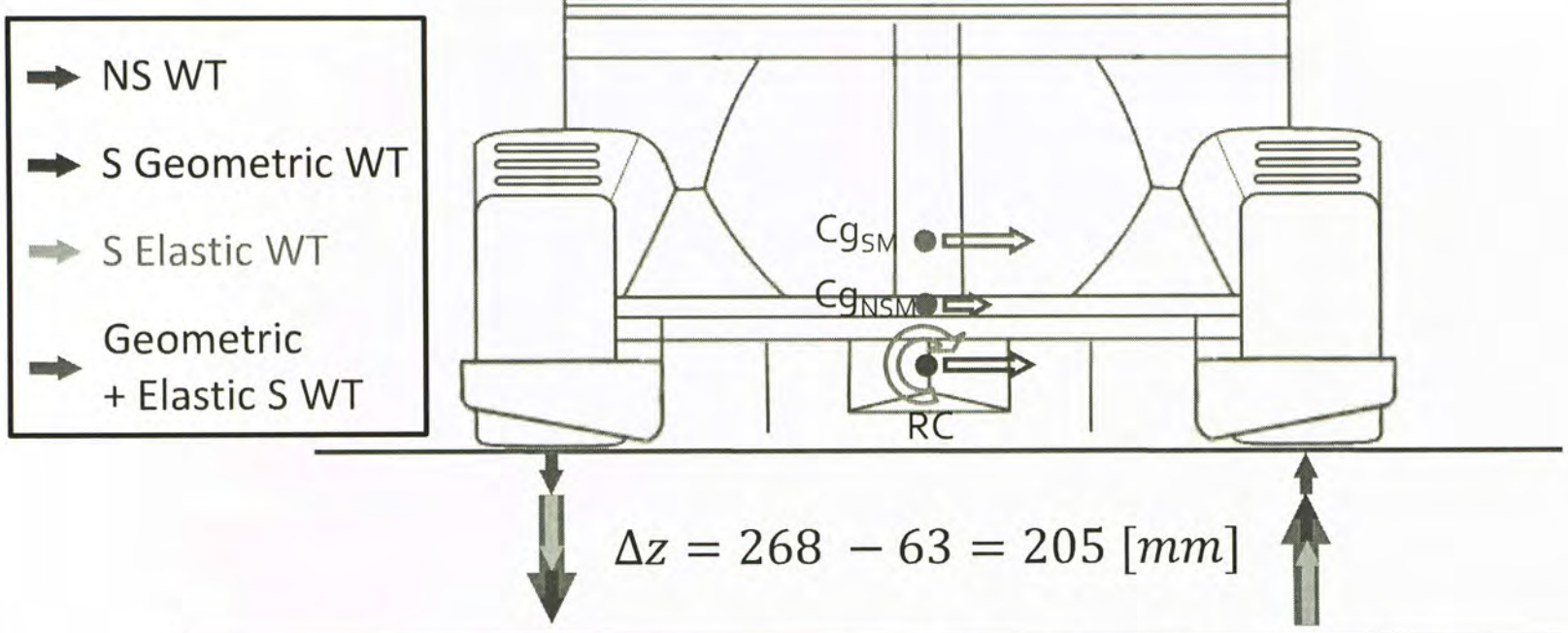
$$Lateral\ S\ "Elastic"\ WT = \frac{473.4 \times (2.5 \times 9.81) \times 232}{1640} = 1642.4N$$

Note: 2D Calculation (Front)



Lateral Weight Transfer

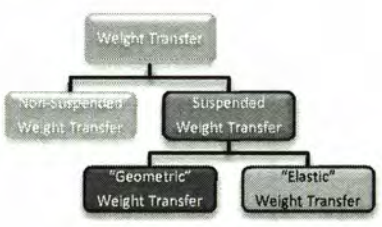
RC height < CG_{SM} height



$$Lateral\ S\ "Geometric"\ WT = \frac{521.5 \times (2.5 \times 9.81) \times 63}{1592} = 506.1N$$

$$Lateral\ S\ "Elastic"\ WT = \frac{521.5 \times (2.5 \times 9.81) \times 203}{1592} = 1630.8N$$

Note: 2D Calculation (Rear)

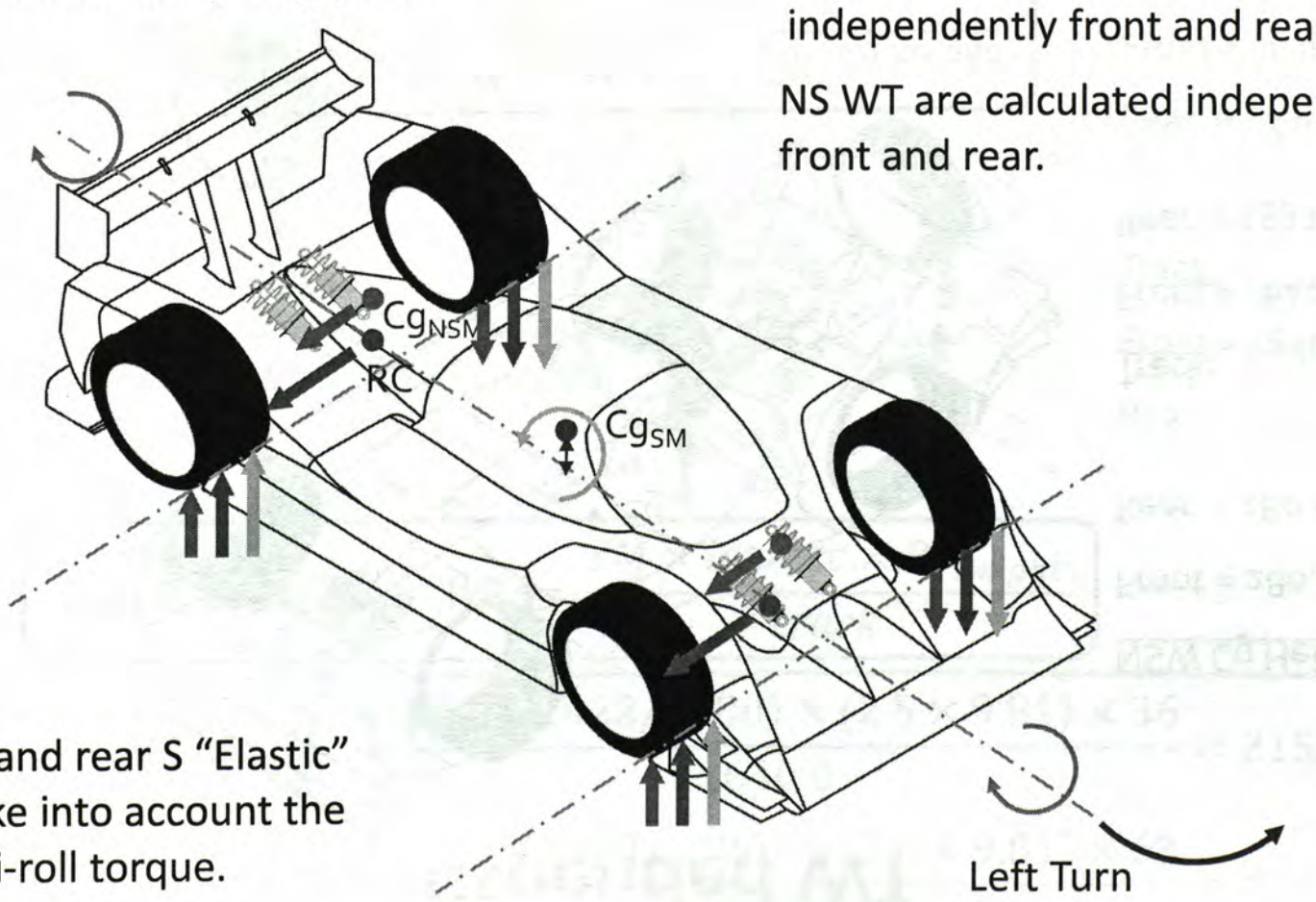


Lateral Weight Transfer

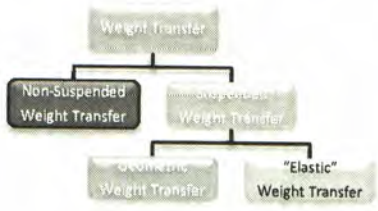
Anti-Roll torque due to springs, ARB

S "Geometric" WT are calculated independently front and rear.

NS WT are calculated independently front and rear.



To calculate front and rear S "Elastic" WT we have to take into account the front and rear anti-roll torque.



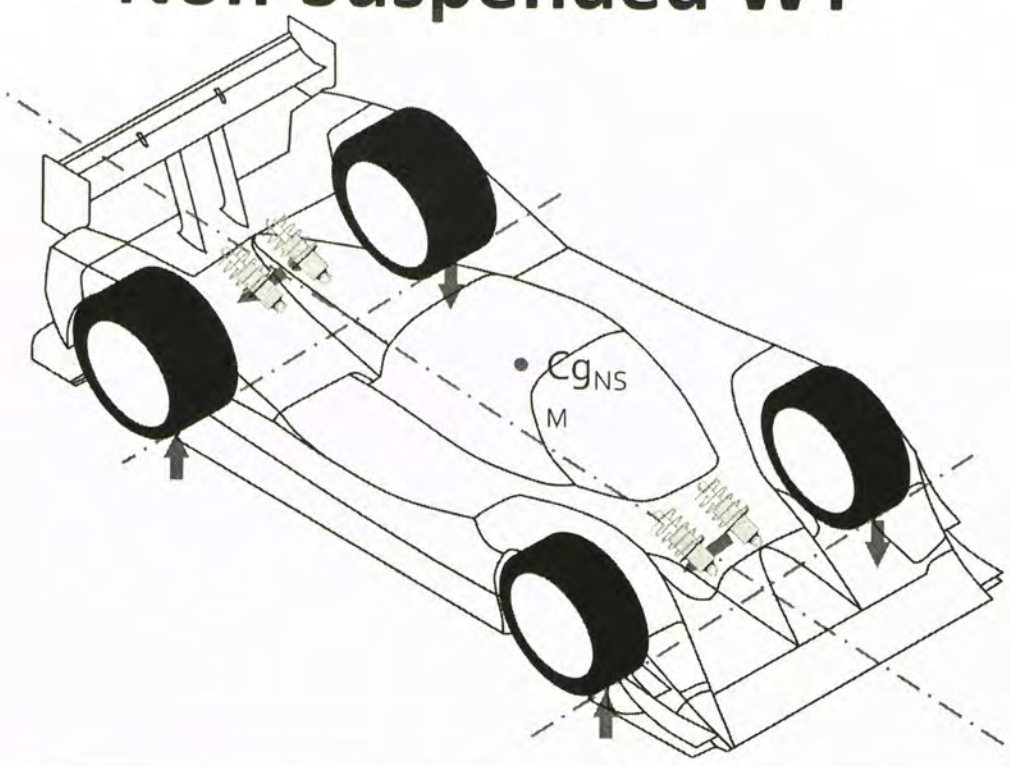
Non-Suspended WT

NSM_f = 74 kg

NSM_r = 84 kg

$Lat.acc_f$ = 2.5 g

$Lat.acc_r$ = 2.5 g



NSW Cg Height:

Front = 280.0 mm

Rear = 280.0 mm

Track:

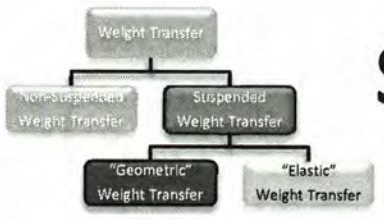
Front = 1640.0 mm

Rear = 1592.0 mm

$$Lat.NS\ WT = \frac{NSM \times lat.acc. \times Cg_{NSM\ height}}{track}$$

$$Lat.NS\ WT_{front} = \frac{74 \times (9.81 \times 2.5) \times 280}{1640.0} = 309.9\ N$$

$$Lat.NS\ WT_{rear} = \frac{84 \times (9.81 \times 2.5) \times 280}{1592.0} = 362.3\ N$$



Suspended "Geometric" WT

Wb = 2980 mm

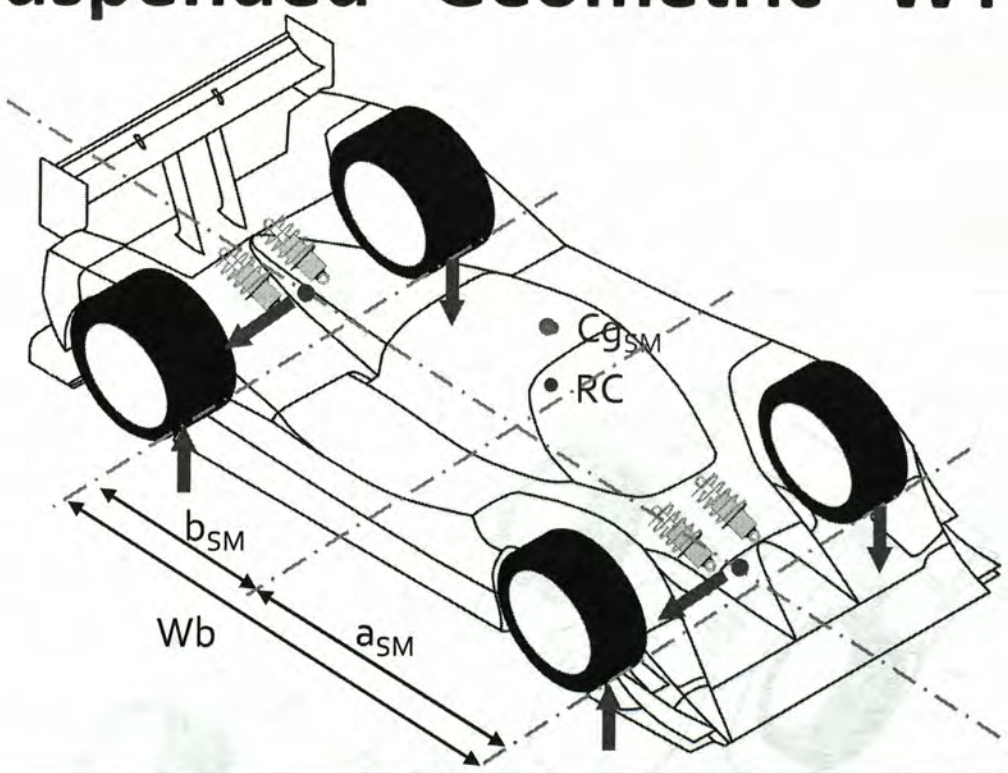
a = 1562 mm

b = 1418 mm

a_{SM} = 1558 mm

b_{SM} = 1422 mm

SM = 836.9 Kg



Roll Center Height:

Front = 36 mm

Rear = 63 mm

Track:

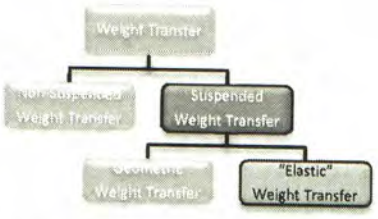
Front = 1640.0 mm

Rear = 1592.0 mm

$$Lat.S\ "Geometric"\ WT = \frac{SM \times lat.acc. \times RC_{height}}{track}$$

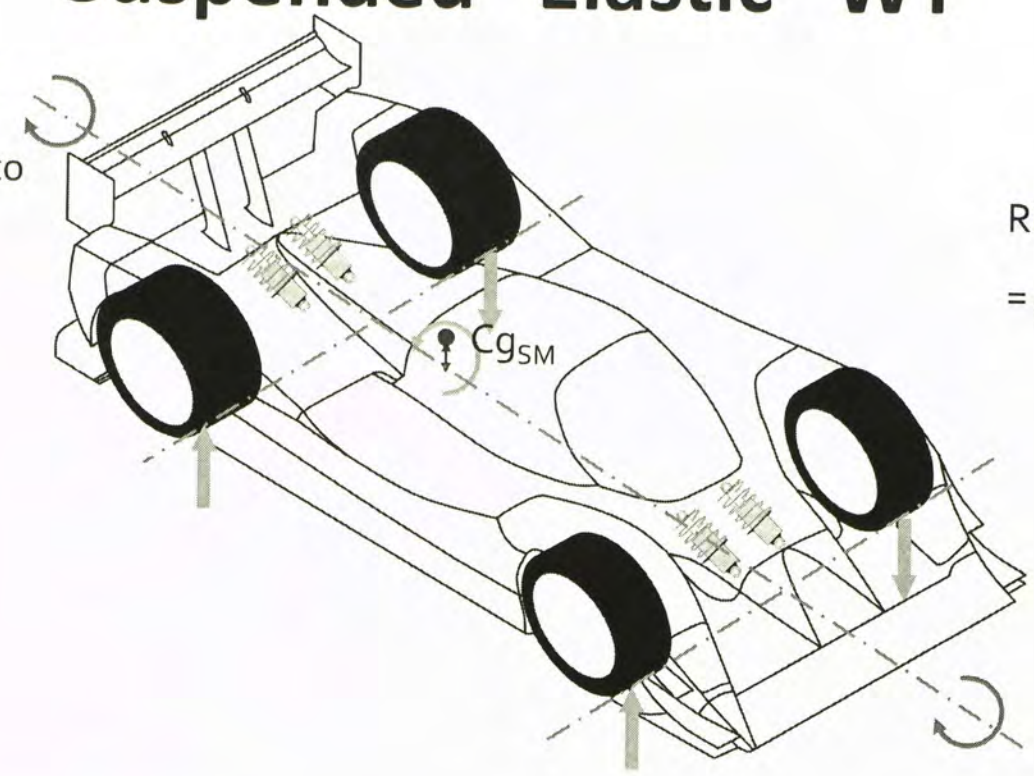
$$Lat.S\ "Geometric"\ WT_{front} = \frac{836.9 \times (1422/2980) \times (2.5 \times 9.81) \times 36}{1640.0} = 215.0\ N$$

$$Lat.S\ "Geometric"\ WT_{rear} = \frac{836.9 \times (1558/2980) \times (2.5 \times 9.81) \times 63}{1592.0} = 424.6N$$



Suspended "Elastic" WT

Anti-Roll torque due to springs, ARB



RM: Resisting Moment

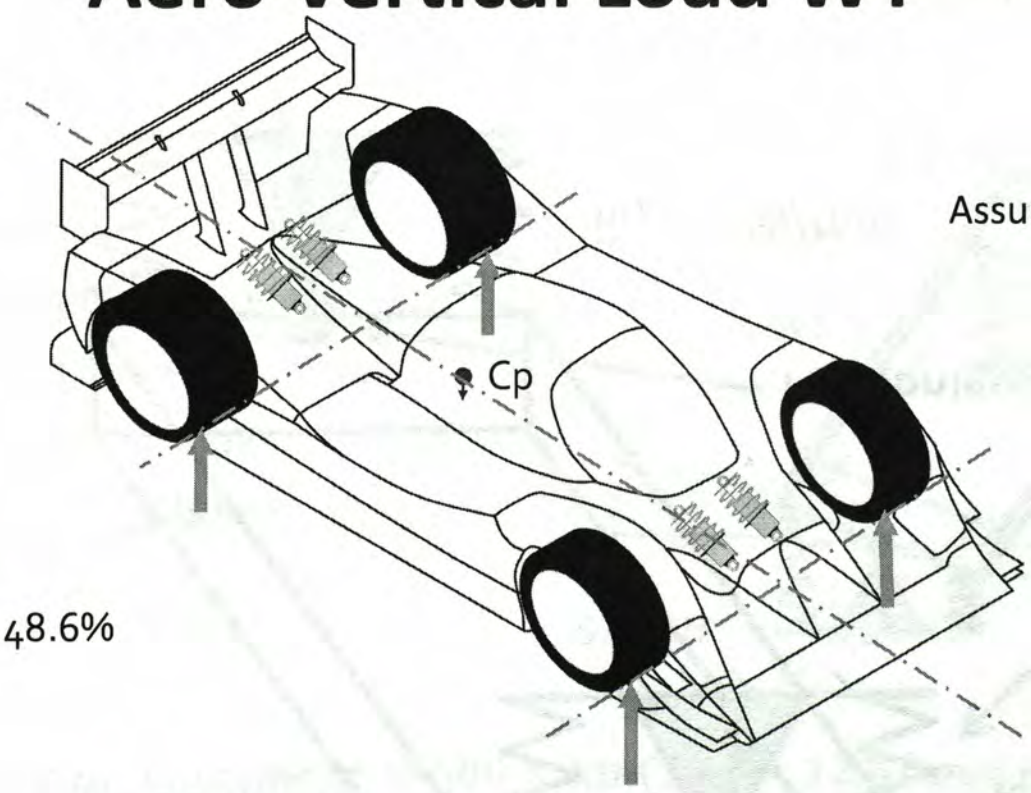
= Anti-Roll Torque

$$S\ "Elastic"\ WT_{front} = \frac{SM \times lat.acc. \times \Delta z(Cg_{SW} - Roll\ Axis) \times (RM_{front}/RM_{total})}{track_{front}}$$

$$S\ "Elastic"\ WT_{rear} = \frac{SM \times lat.acc. \times \Delta z(Cg_{SW} - Roll\ Axis) \times (RM_{rear}/RM_{total})}{track_{rear}}$$

Aero Vertical Load WT

Vertical Load due to Aerodynamic load



Assuming a Corner Radius of 110m at 2.5g.

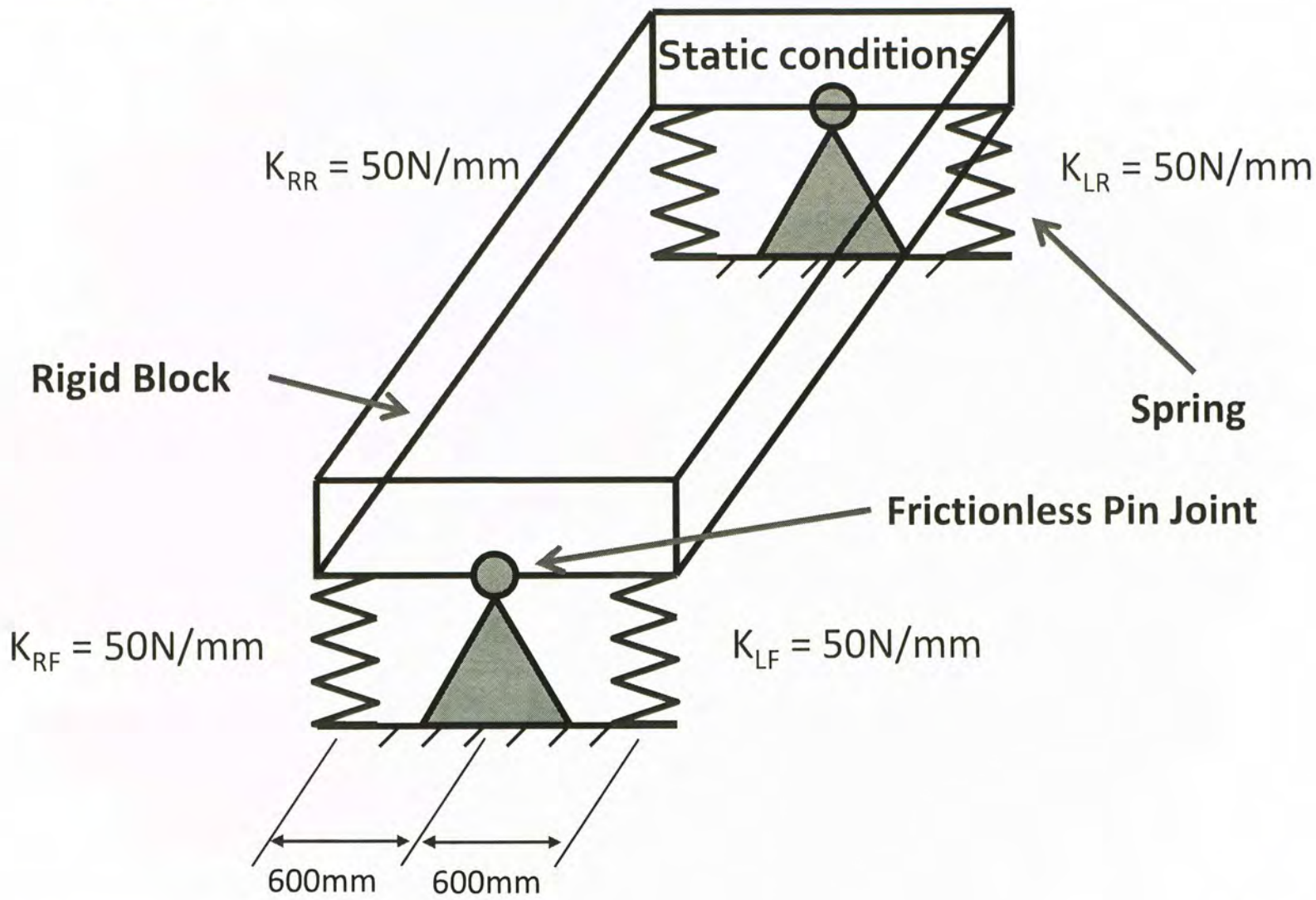
AeroBalance = 48.6%

$$V = \sqrt{LatG \times R \times g} = 51.4\ m/s$$

$$AeroLoad_{Front} = 0.5 \times \rho \times A \times CL \times V^2 \times CL_{Dist} = 0.5 \times 1.2255 \times 1.75 \times 2.2 \times 51.42 \times 0.486 = 3032.9\ N$$

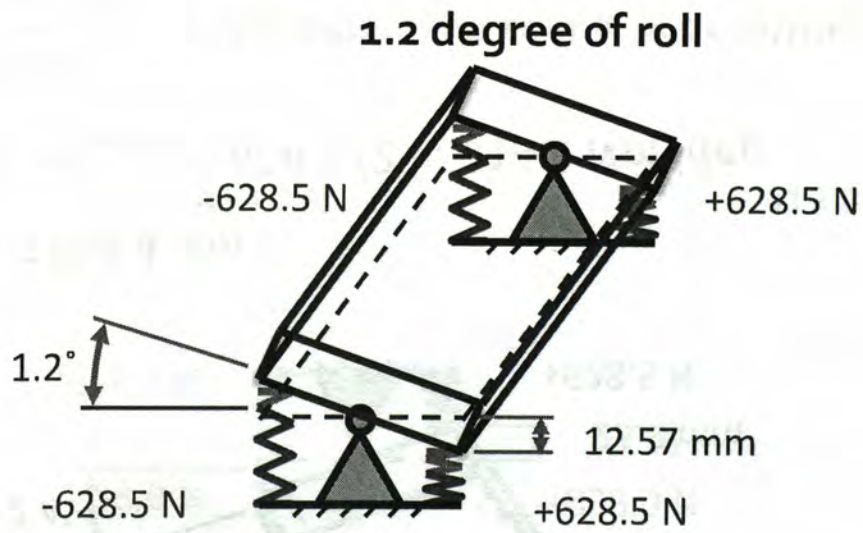
$$AeroLoad_{Rear} = 0.5 \times \rho \times A \times CL \times V^2 \times (1 - CL_{Dist}) = 0.5 \times 1.2255 \times 1.75 \times 2.2 \times 51.42 \times (1 - 0.486) = 3206.9\ N$$

Calculating Elastic Anti-Roll Stiffness



Calculating Elastic Anti-Roll Stiffness

1. Impose Roll Angle
2. Calculate Spring Deflection
3. Calculate Spring Force Variation
4. Calculate Anti-Roll Torques
5. Calculate Anti-Roll Stiffness



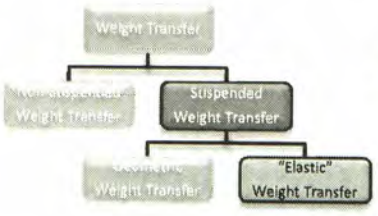
$$Spring\ movement = 600 \times tan(1.2) = 12.57\ mm$$

$$Spring\ force = 50 \times 12.57 = 628.5\ N$$

$$AntiRoll\ Torque_{front} = AntiRoll\ Torque_{rear} = 2 \times 628.5 \times 0.6 = 754.2\ Nm$$

$$AntiRoll\ Stiffness_{front} = AntiRoll\ Stiffness_{rear} = 754.2/1.2 = 628.5\ Nm/deg$$

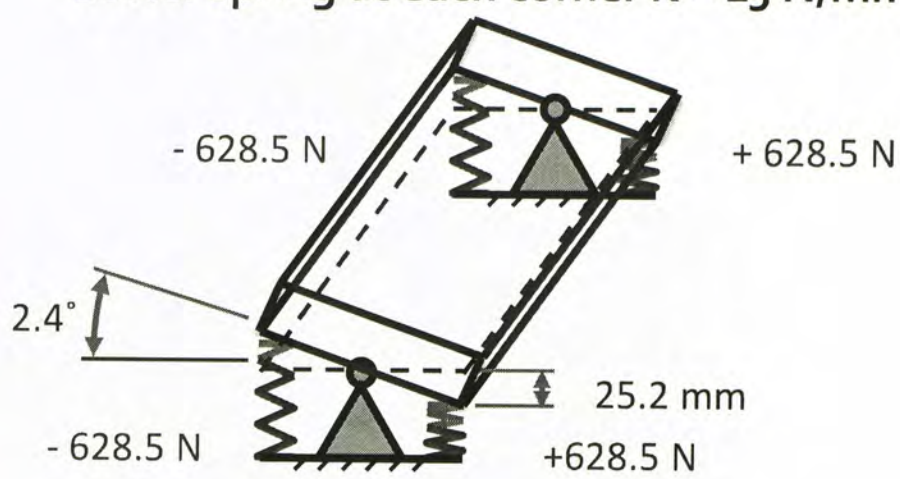
$$\boxed{AntiRoll\ Stiffness_{total} = 1508.4/1.2 = 1257.0\ Nm/deg}$$



Calculating Elastic Anti-Roll Stiffness

1. Impose Same Roll Torque
2. Calculate Anti-Roll Torques
3. Calculate Roll Angle
4. Calculate Spring Movement
5. Weight Transfer Front and Rear

Softer spring at each corner K = 25 N/mm



$$Roll\ Torque = 1508.4\ Nm$$

$$AntiRoll\ Stifness_{front} = AntiRoll\ Stifness_{rear} = 628.5/2 = 314.2\ Nm/deg$$

$$Roll\ Angle = \frac{1508.4}{314.2 + 314.2} = 2.4\ deg$$

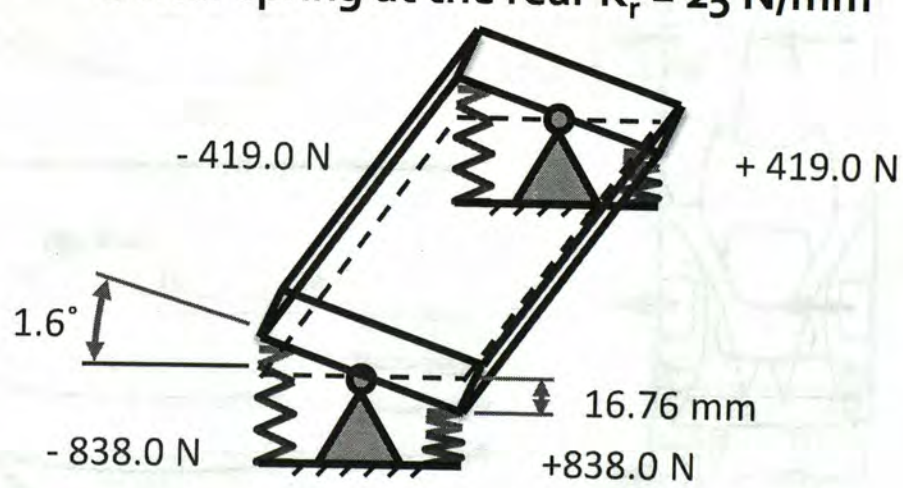
$$Spring\ movement = 600 \times tan(2.4) = 25.15\ mm$$

$$\boxed{Weight\ transfer_{front} = Weight\ transfer_{rear} = 628.4\ N}$$

Calculating Elastic Anti-Roll Stiffness

1. Impose Same Roll Torque
2. Calculate Anti-Roll Torques
3. Calculate Roll Angle
4. Calculate Spring Movement
5. Weight Transfer Front and Rear

Softer spring at the rear K_r = 25 N/mm



$$Roll\ Torque = 1508.4\ Nm$$

$$AntiRoll\ Stifness_{front} = 2 \times AntiRoll\ Stifness_{rear} = 628.4\ Nm/deg$$

$$Roll\ Angle = \frac{1508.4}{628.4 + 314.2} = 1.6\ deg$$

$$Spring\ movement = 600 \times tan(1.6) = 16.76\ mm$$

$$\boxed{Weight\ transfer_{front} = 2 \times Weight\ transfer_{rear} = 50 \times 16.76 = 838.0\ N}$$

WT Summary

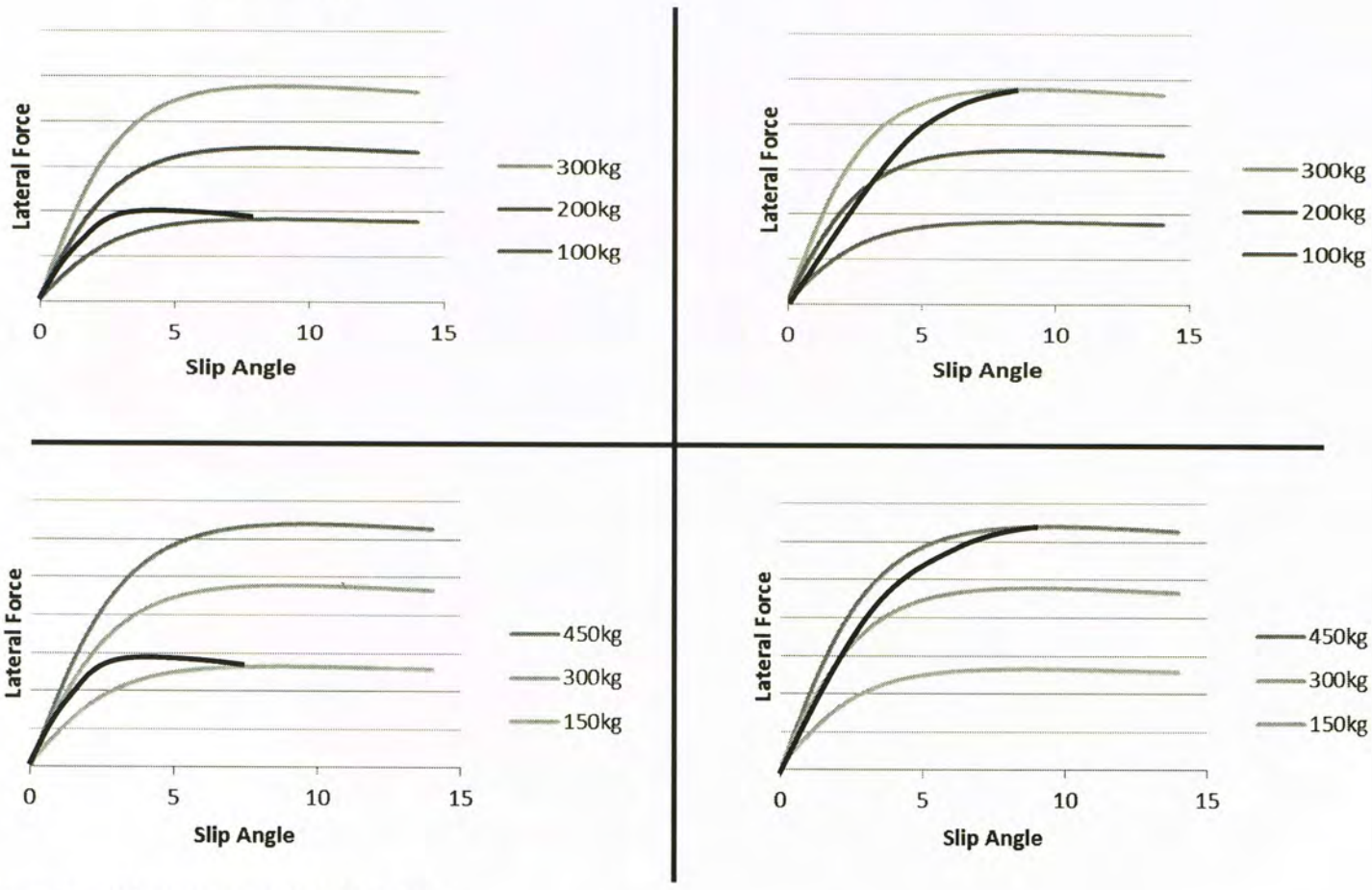
Static corner weights

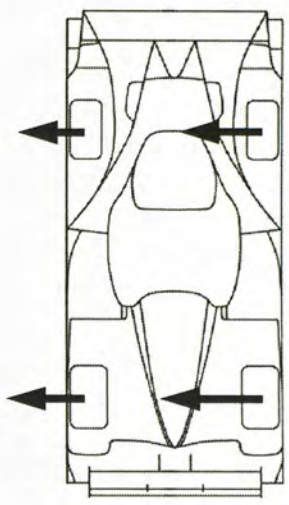
FL	FR
200	200
300	300
RL	RR

Weight transfer

Dynamic corner weights

100	300
150	450





WT Summary

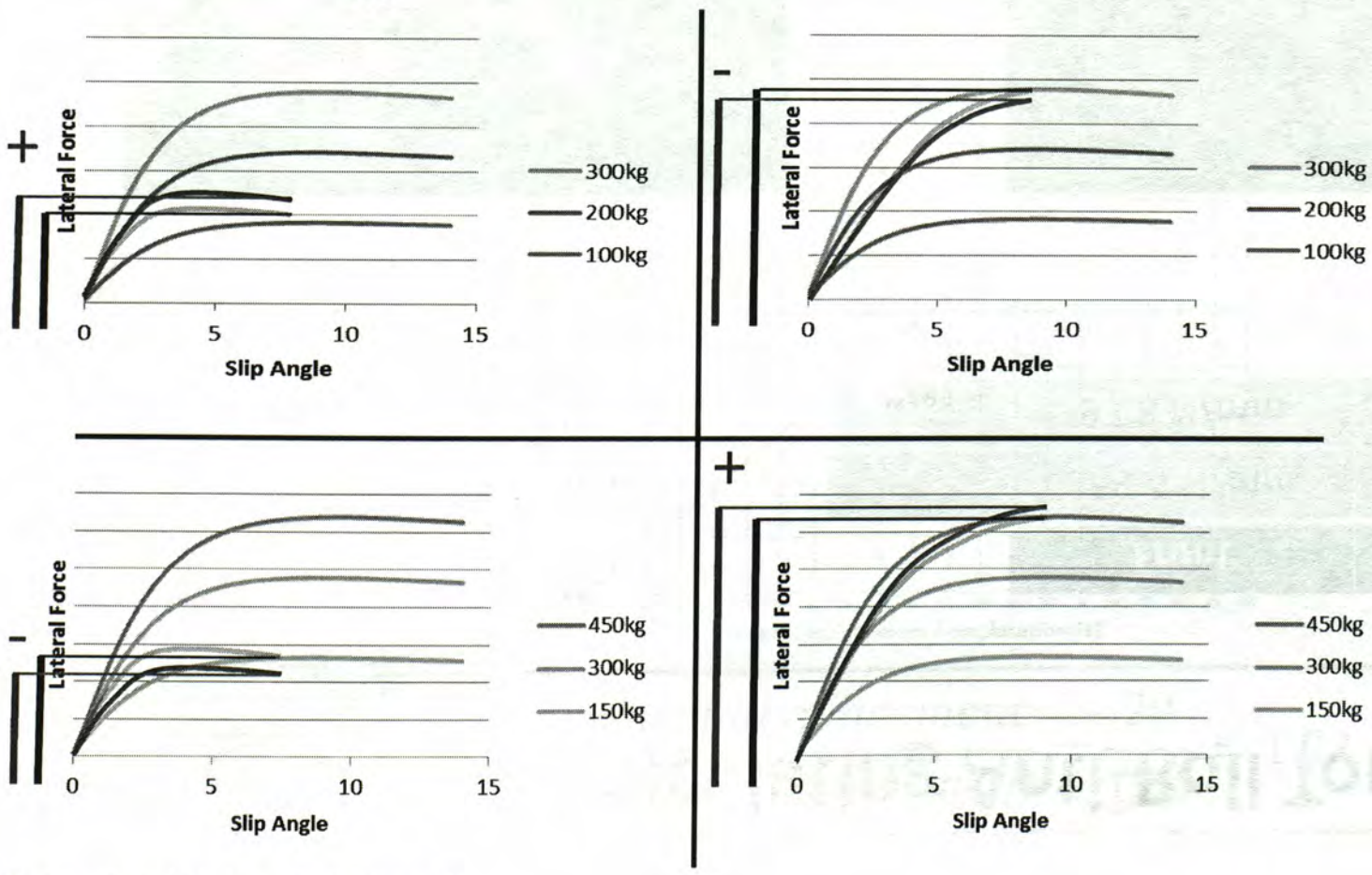
Static corner weights

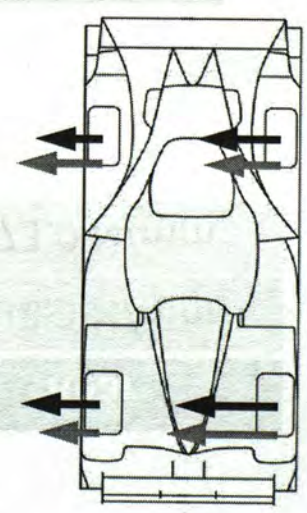
FL	FR
200	200
300	300
RL	RR

Weight transfer
with softer front ARB

Dynamic corner weights

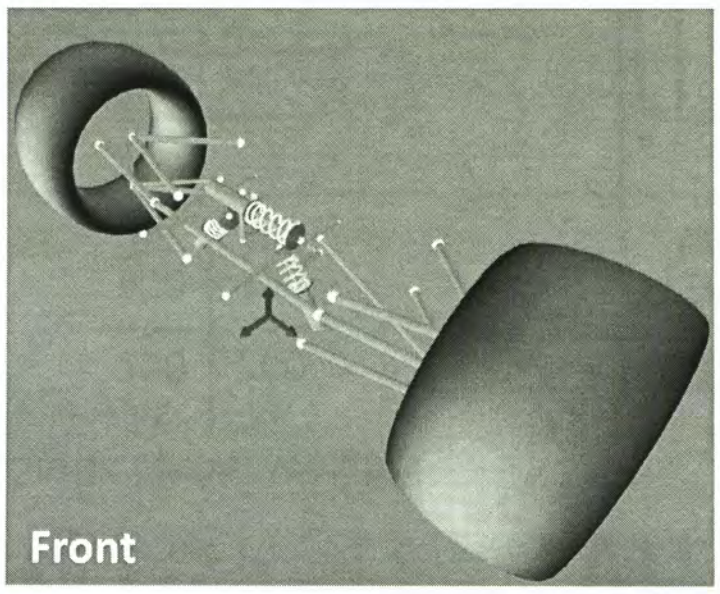
120	280
130	470

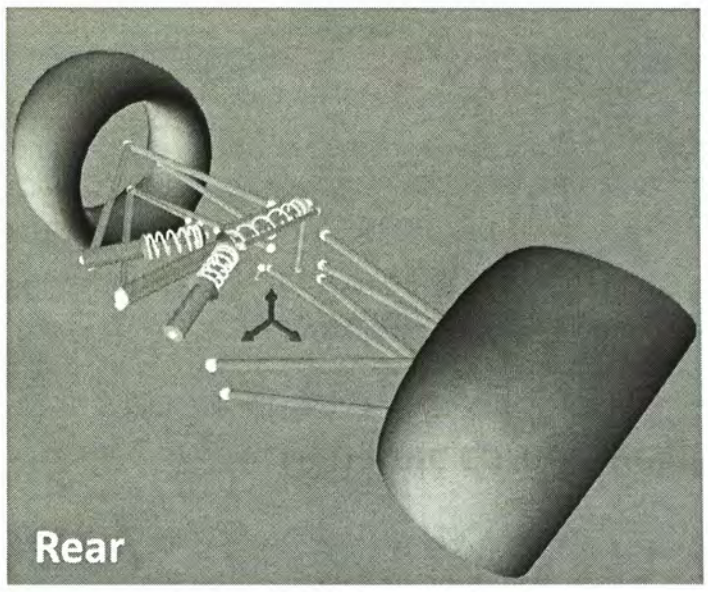




Calculating Anti-Roll Torque

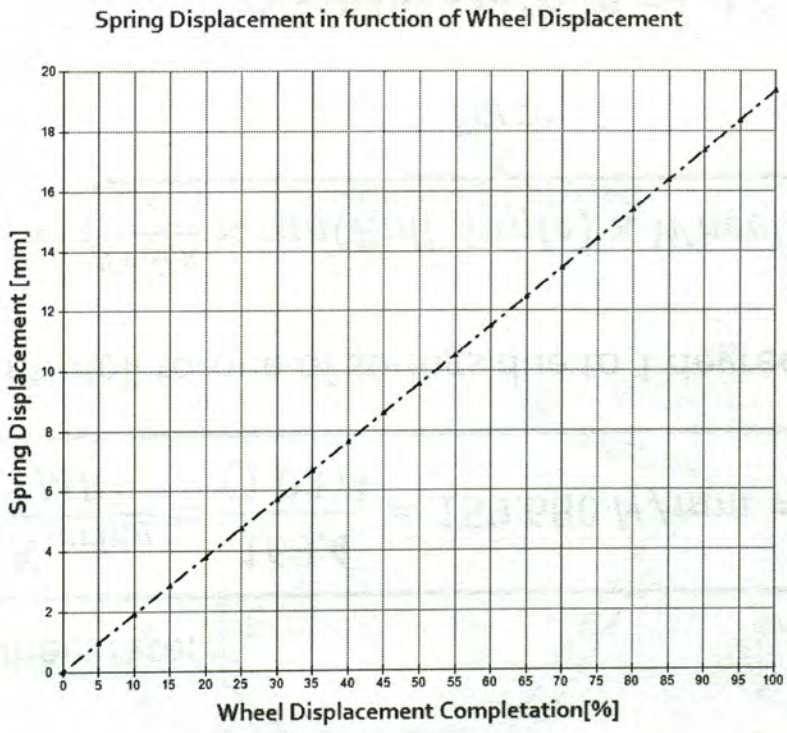
	Front	Rear
k_s	169.4 N/mm	138.3 N/mm
K_{ARB}	97.8 N/mm	173.0 N/mm



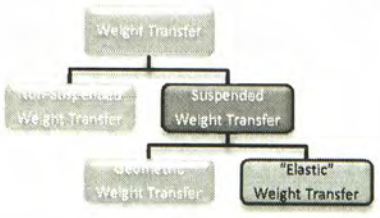


Calculating Anti-Roll torque due to Front Springs

1. Calculate the spring motion ratio:



$$MR = \frac{wheel\ displacement}{spring\ displacement} = \frac{20}{19.4} = 1.03$$



Calculating Anti-Roll torque due to Front Springs

2. Calculate the wheel rate:

$$Wheel\ Rate = \frac{K_{spring}}{MR^2} = \frac{169.4}{(1.03)^2} = 159.680\ N/mm = 159680\ \ N/m$$

3. Calculate the Anti-Roll torque of springs due to 1 degree of roll angle:

$$AntiRoll\ Torque_{spring} = \underbrace{\left(\frac{Track}{2} \times tan(Roll\ Angle) \times Wheel\ Rate\right)}_{Force} \times \underbrace{\frac{Track}{2}}_{Distance} \times 2$$

2 sides

$$AntiRoll\ Torque_{spring} = \frac{Track^2 \times tan(Roll\ Angle) \times Wheel\ Rate}{2}$$

$$AntiRoll\ Torque_{spring} = \frac{1.640^2 \times tan(1 \times \pi/180) \times 159680}{2} = 3748.2\ Nm$$

$$AntiRoll\ Torque\ due\ to\ front\ springs = 3748.2\ Nm/deg$$

Measuring ARB Stiffness

$$ARB\ Stiffness = \frac{F}{d}$$

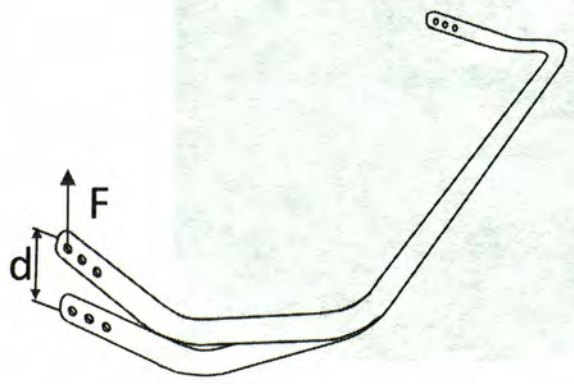
Expressed in N/mm

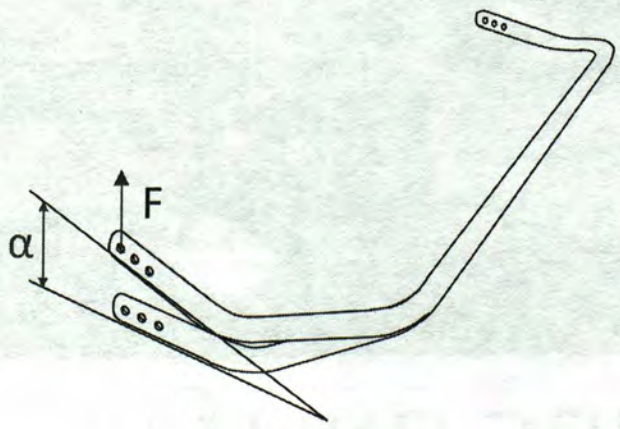
$$ARB\ Stiffness = \frac{F}{\alpha}$$

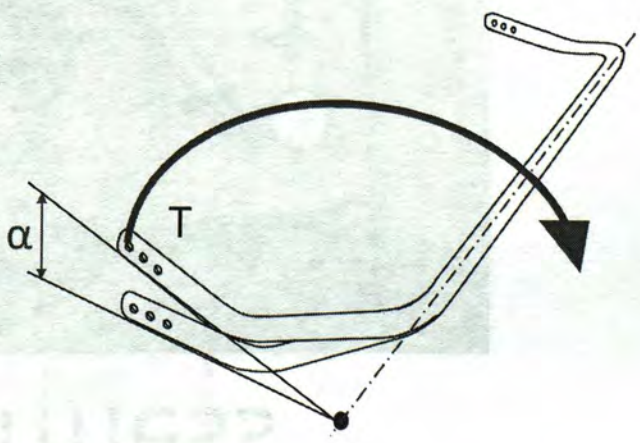
Expressed in N/deg

$$ARB\ Stiffness = \frac{T}{\alpha}$$

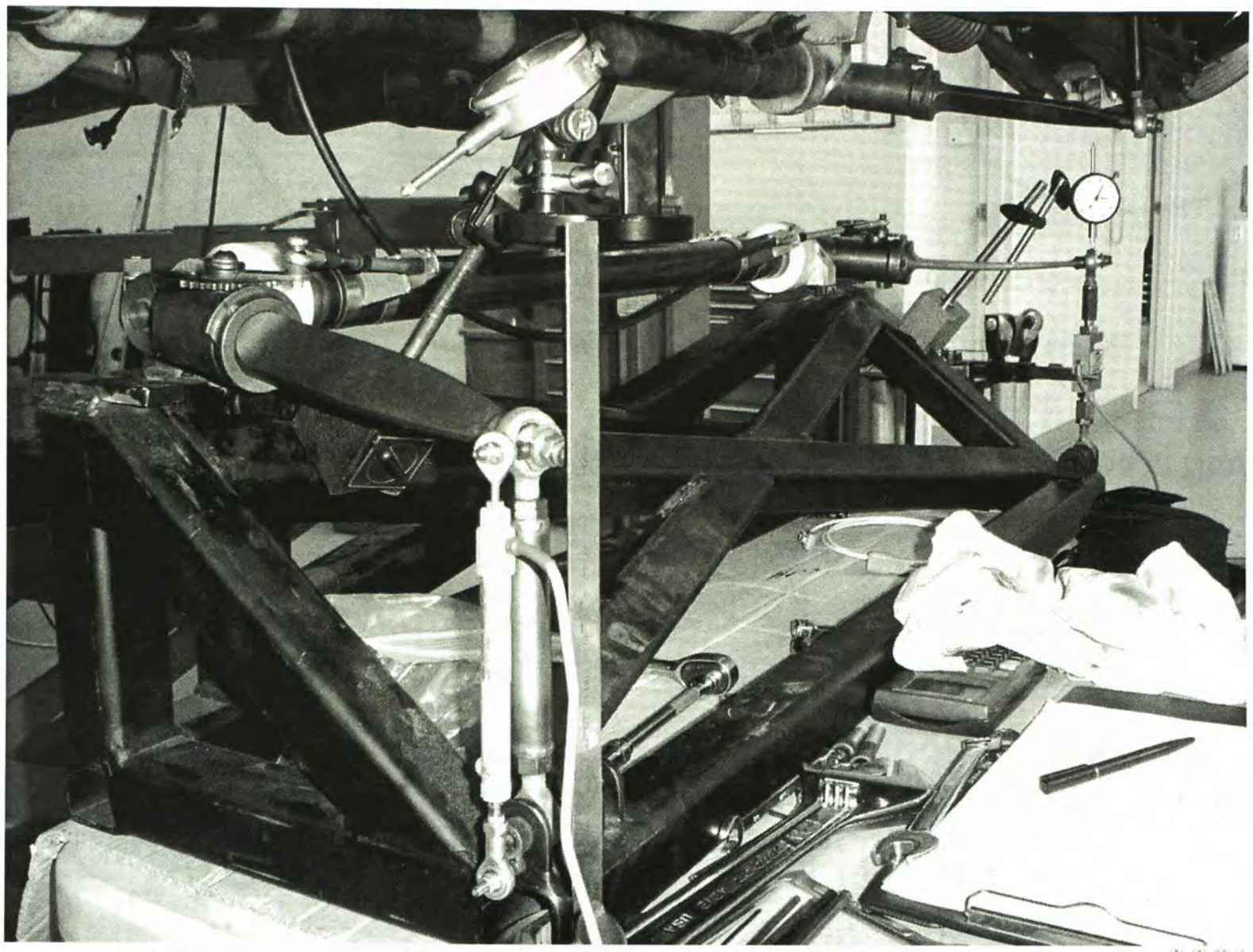
Expressed in Nm/deg

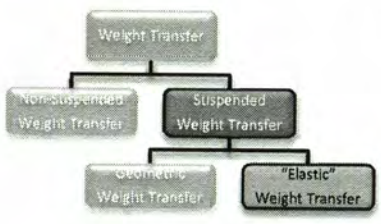






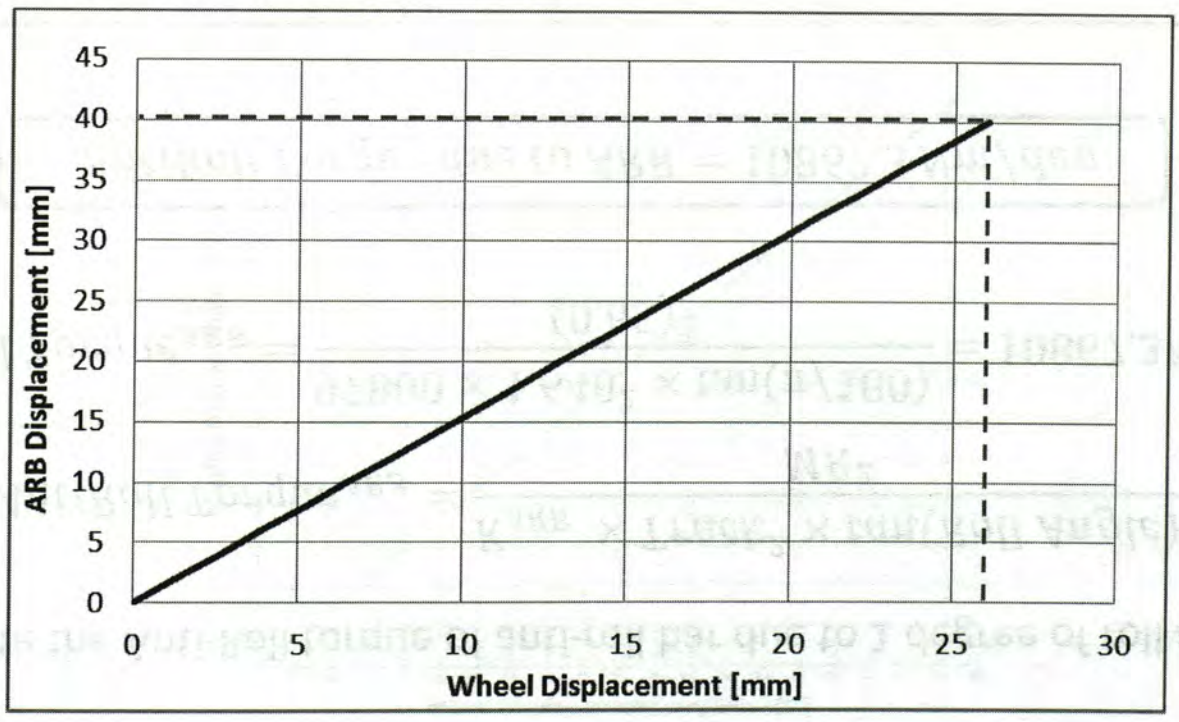
Measuring ARB Stiffness





Calculating Anti-Roll torque due to Front ARB

1. Calculate the anti-roll bar motion ratio:



$$MR = \frac{Wheel\ Displacement}{ARB\ Displacement} = \frac{26}{40} = 0.65$$

Calculating Anti-Roll torque due to Front ARB

2. Calculate the Anti-Roll torque of anti-roll bar due to 1 degree of roll angle:

$$AntiRoll\ Torque_{ARB} = \frac{K_{ARB} \times Track^2 \times tan(Roll\ Angle)}{MR^2}$$

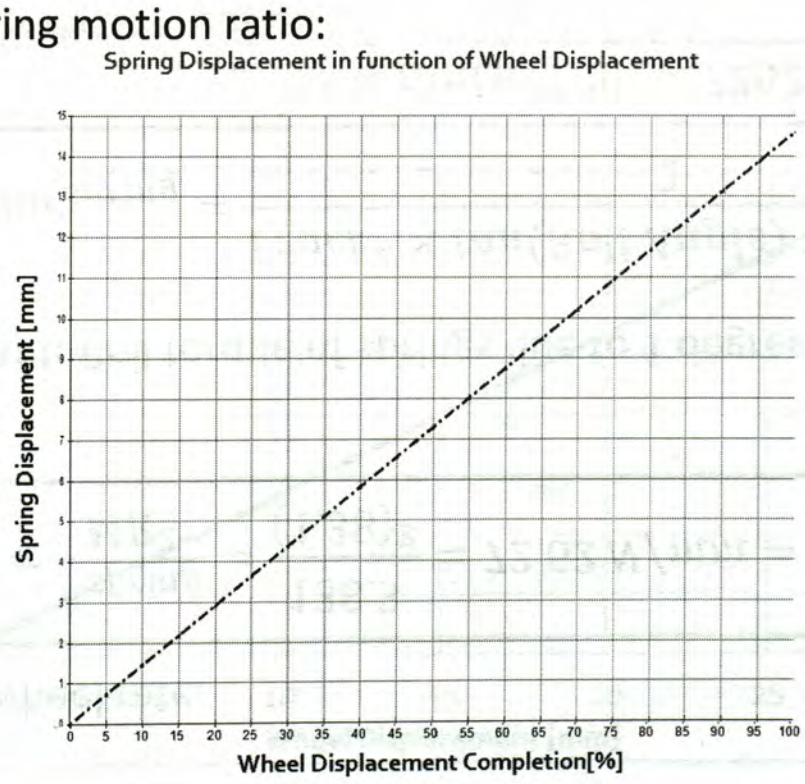
$$AntiRoll\ Torque_{ARB} = \frac{97800 \times 1.640^2 \times \tan(\pi/180)}{(0.65)^2} = 10867.3 Nm/deg$$

$$\boxed{AntiRoll\ Torque\ due\ to\ ARB = 10867.3\ Nm/deg}$$

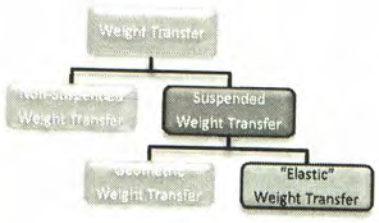
$$\boxed{AntiRoll\ Torque_{\ Front\ axle} = 3748.2 + 10867.3 = 14615.5\ Nm/deg}$$

Calculating Anti-Roll torque due to Rear Springs

1. Calculate the spring motion ratio:



$$\boxed{MR = \frac{Wheel\ Displacement}{Spring\ Displacement} = \frac{20}{14.5} = 1.38}$$



Calculating Anti-Roll torque due to Rear Springs

2. Calculate the wheel rate:

$$Wheel\ Rate = \frac{K_{spring}}{MR^2} = \frac{138.3}{(1.38)^2} = 72.62\ N/mm = 72620\ \ N/m$$

3. Calculate the Anti-Roll torque of springs due to 1 degree of roll angle:

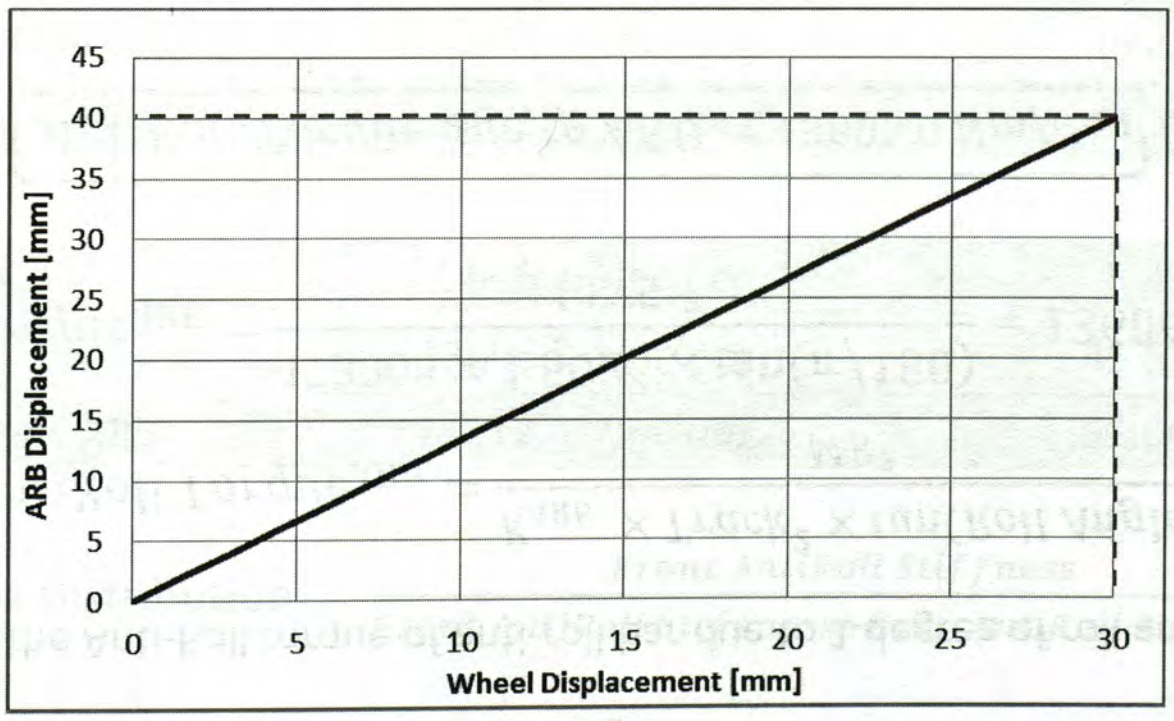
$$AntiRoll\ Torque_{spring} = \frac{Track^2 \times tan(Roll\ Angle) \times Wheel\ Rate}{2}$$

$$AntiRoll\ Torque_{spring} = \frac{1.592^2 \times tan(\pi/180) \times 72620}{2} = 1606.4\ \ Nm$$

$$AntiRoll\ Torque\ due\ to\ rear\ springs = 1606.4\ \ Nm/deg$$

Calculating Anti-Roll torque due to Rear ARB

1. Calculate the anti-roll bar motion ratio:



$$MR = \frac{Wheel\ Displacement}{ARB\ Displacement} = \frac{30}{40} = 0.75$$

Calculating Anti-Roll torque due to Rear ARB

2. Calculate the Anti-Roll torque of anti-roll bar due to 1 degree of roll angle:

$$AntiRoll\ Torque_{ARB} = \frac{K_{ARB} \times Track^2 \times tan(Roll\ Angle)}{MR^2}$$

$$AntiRoll\ Torque_{ARB} = \frac{173000 \times 1.592^2 \times \tan(\pi/180)}{(0.75)^2} = 13606.0\ Nm/deg$$

$$\boxed{AntiRoll\ Torque\ due\ to\ ARB = 13606.0\ Nm/deg}$$

$$\boxed{AntiRoll\ Torque_{Rear\ Axle} = 1606.4 + 13606.0 = 15212.4\ Nm/deg}$$

Anti-Roll Stiffness Summary

	Anti-Roll Torque	Percentage of total
Front Springs	3748.2 Nm/deg	12.57 %
Front ARB	10867.3 Nm/deg	36.43 %
Front (Springs + ARB)	**14615.5 Nm/deg**	**49.00 %**
Rear Springs	1606.4 Nm/deg	5.39 %
Rear ARB	13606.0 Nm/deg	45.61 %
Rear (Springs + ARB)	**15212.4 Nm/deg**	**51.00 %**

$$\text{Anti-Roll Stiffness Distribution} = \frac{Front\ AntiRoll\ Stiffness}{Front\ AntiRoll\ Stiffness + Rear\ AntiRoll\ Stiffness} \times 100$$

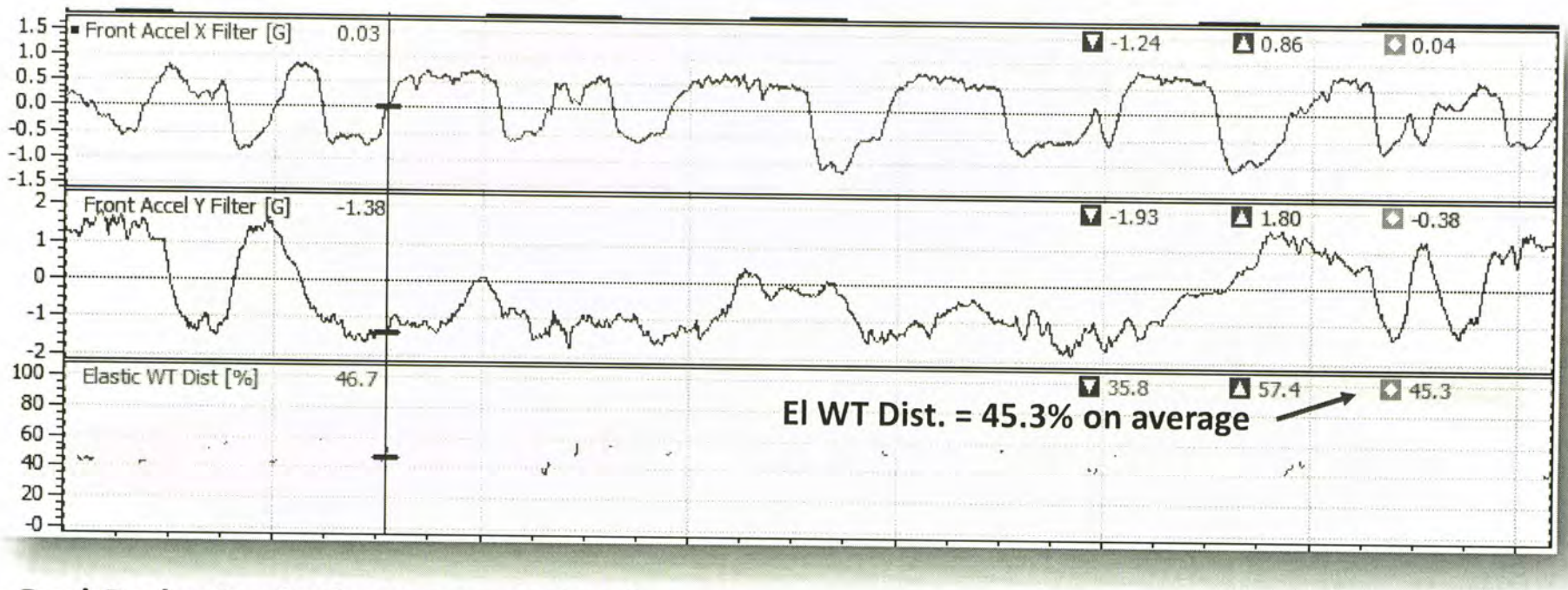
$$= \frac{14615.5}{14615.5 + 15212.4} \times 100$$

= 49.00 % → **MAGIC NUMBER!**

→ Suspended Mass Distri

Car Analysis

Elastic Weight Transfer Distribution



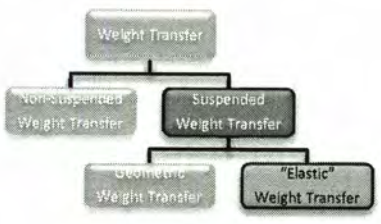
Sg_PushRod = Strain Gauge on pushrod.

Condition function "if"

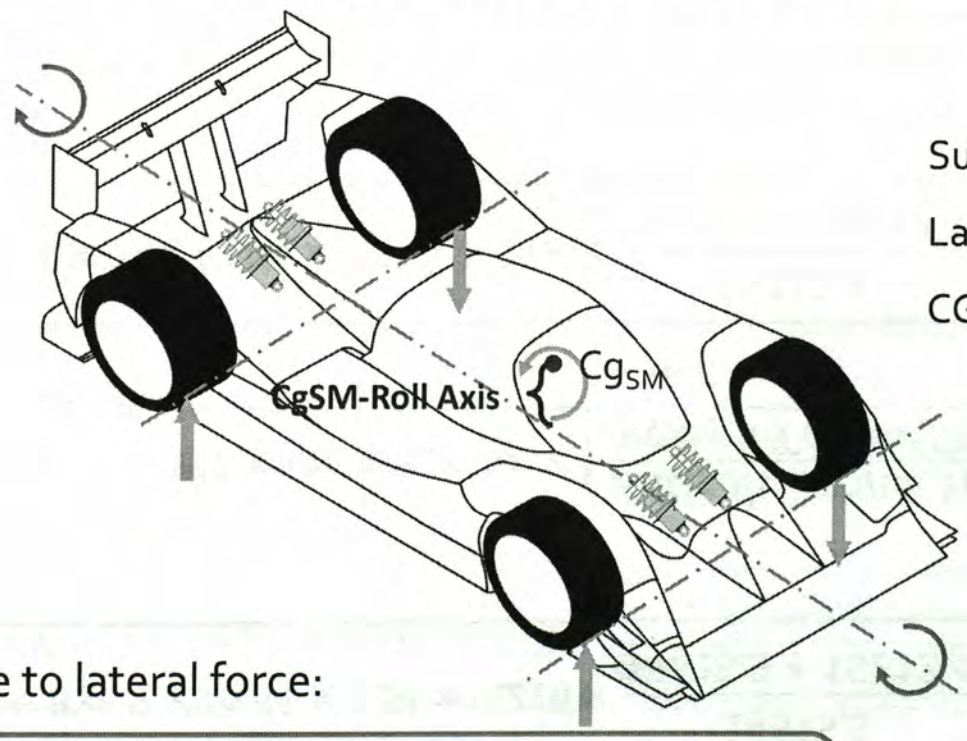
Elstic Weight Transfert Distribution [%] = choose(abs(**'Long Accel'** [G]) < 0.1 AND abs(**'Lat Accel'** [G]) > 0.5, (**'Sg_PushRod_Filter_FR'** [N] - **'Sg_PushRod_Filter_FL'** [N]) / (**'Sg_PushRod_Filter_FR'** [N] - **'Sg_PushRod_Filter_FL'** [N] +**'Sg_PushRod_Filter_RR'** [N] - **'Sg_PushRod_Filter_RL'** [N]), 1/0)

Return nothing if condition false

The actual vertical load is obtained by the product of the strain gauge and a coefficient relative to the pushrod inclination angle.



Suspended "Elastic" WT



Suspended Mass = 836.9 kg

Lat. acc. = 2.5 G

CG_{SM} - Roll Axis = 218 mm

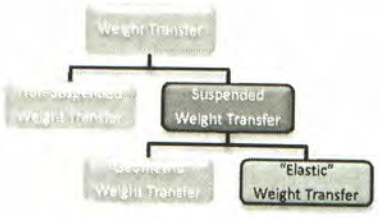
Moment due to lateral force:

$$M = SM \times lat.acc. \times \Delta z$$

$$M = 836.9 \times (9.81 \times 2.5) \times 0.218 = 4474.4\ Nm$$

$$Roll\ Angle = 4474.4/(14983.2 + 15389.4) = 0.15\ deg$$

Note - Roll Rate = 0.06 deg/g



Suspended "Elastic" WT

$$S\ "Elastic"\ WT_{front} = \frac{SM \times lat.acc. \times \Delta z \times \left(\frac{Antiroll\ Torque\ Stiffness_{front}}{Antiroll\ Torque\ Stiffness_{Total}}\right)}{track_{front}}$$

$$S\ "Elastic"\ WT_{front} = 836.9 \times (9.81 \times 2.5) \times 0.218 \times \frac{14615.5}{14615.5 + 15212.4} / 1.640 = 1336.8\ N$$

$$S\ "Elastic"\ WT_{rear} = \frac{SM \times lat.acc. \times \Delta z \times \left(\frac{Antiroll\ Torque\ Stiffness_{rear}}{Antiroll\ Torque\ Stiffness_{Total}}\right)}{track_{rear}}$$

$$S\ "Elastic"\ WT_{rear} = 836.9 \times (9.81 \times 2.5) \times 0.218 \times \frac{15212.4}{14615.5 + 15212.4} / 1.592 = 1433.4\ N$$

$$Suspended\ Elastic\ WT\ distribution\ [\%\ Front] = \frac{1336.8}{1336.8 + 1433.4} = 48.26\ \%$$

↑ to 49% because Track difference

Suspended "Elastic" WT

If we change the front spring or the ARB (so they are softer) so that the Front Anti-Roll torque goes down to 14000.0 Nm/deg.

$$S\ "Elastic"\ WT_{front} = 836.9 \times (9.81 \times 2.5) \times 0.218 \times \frac{\mathbf{14000.0}}{\mathbf{14000.0} + 15212.4} / 1.640 = 1307.5\ N$$

$$S\ "Elastic"\ WT_{rear} = 836.9 \times (9.81 \times 2.5) \times 0.218 \times \frac{15212.4}{\mathbf{14000.0} + 15212.4} / 1.592 = 14\cancel{20}.8\ N$$

63

$$Suspended\ Elastic\ WT\ distribution\ [\%\ Front] = \frac{1307.7}{1307.7 + 1420.8} = 47.18\ \%$$

WT Summary

Static Corner Load	2300.0 N	FL	2344.0 N	FR
NS WT	- 309.9 N		+ 309.9 N	
S "Geometric" WT	- 215.0 N		+ 215.0 N	
S "Elastic" WT	- 1336.8 N	- 1307.5 N	+ 1336.8 N	+ 1307.5 N
Aero Load	+ 1516.4 N		+ 1516.4 N	
Dynamic Corner Load	1954.7 N	1984.0 N	5722.1 N	5692.8 N
Static Corner Load	2536.0 N	RL	2580.0 N	RR
NS WT	- 376.2 N		+ 376.2 N	
S "Geometric" WT	- 424.6 N		+ 424.6 N	
S "Elastic" WT	- 1433.4 N	- 1463.6 N	+ 1433.4 N	+ 1463.6 N
Aero Load	+ 1603.4 N		+ 1603.4 N	
Dynamic Corner Loa	1905.2 N	1875.0 N	6417.6 N	6447.8 N

With softer ARB at the front

Steady-State Magic Numbers

1. Elastic Weight Transfer distribution on Front axle (%) **OR** anti-roll stiffness distribution on Front axle (%). Chassis and tire stiffness are neglected

$$= 100 \times Elastic\ WT\ Front/(Elastic\ WT\ Front + Elastic\ WT\ Rear)$$ [N/N]

$$= 100 \times Front\ Anti\ Roll\ Stiffness/(Front\ Anti\ Roll\ Stiffness + Rear\ Anti\ Roll\ Stiffness)$$ [(Nm/deg)/(Nm/deg)]

2. Total (Elastic, Geometric, and Non-suspended) Weight transfer distribution on Front axle (%). Chassis and tire stiffness are neglected.

$$= 100 \times Total\ WT\ Front/(Total\ WT\ Front + Total\ WT\ Rear)$$ [N/N]

3. Total Weight transfer distribution on Front axle (%) **OR** anti-roll stiffness distribution on Front axle (%). Tire stiffness is included in the Weight transfer calculation.

$$= 100 \times Total\ WT\ Front/(Total\ WT\ Front + Total\ WT\ Rear)$$ [N/N]

$$= 100 \times Front\ Anti\ Roll\ Stiffness/(Front\ Anti\ Roll\ Stiffness + Rear\ Anti\ Roll\ Stiffness)$$ [(Nm/deg)/(Nm/deg)]

Steady-State Magic Numbers

1st Magic Number (both anti-roll stiffness and elastic WT distribution):

$$Anti\ Roll\ Stiffness\ Distribution\ [\%\ Front] = \frac{14615.5}{14615.5 + 15212.4} \times 100 = \mathbf{49.00}\ \%$$

$$Suspended\ Elastic\ WT\ Distribution\ [\%\ Front] = \frac{1336.8}{1336.8 + 1433.4} \times 100 = \mathbf{48.26}\ \%$$

2nd Magic Number:

$Total\ WTfront = Lat.NS\ WT_{front} + Lat.S\ "Geometric"\ WT_{front} + S\ "Elastic"\ WT_{front} = 1861.7\ N$

$Total\ WTrear = Lat.NS\ WT_{rear} + Lat.S\ "Geometric"\ WT_{rear} + S\ "Elastic"\ WT_{rear} = 2220.4\ N$

$$Total\ WT\ distribution[\%\ Front] = \frac{1861.7}{1861.7 + 2220.4} \times 100 = \mathbf{45.61}\%$$

Steady-State Magic Numbers

3rd Magic Number :

Front:

Calculate the Anti-Roll torque of tires due to 1 degree of roll angle ($k_{tire\text{-}F}$ = 380 N/mm):

$$AntiRoll\ Torque_{tire-F} = \frac{Track^2 \times tan(Roll\ Angle) \times k_{tire-F}}{2}$$

$$AntiRoll\ Torque_{tire-F} = \frac{1.640^2 \times tan(\pi/180) \times 380000}{2} = 8920.0\ Nm$$

Calculate the Total front Anti-Roll torque due to 1 degree of roll angle:

$$AntiRoll\ Torque_{Total-F} = \frac{AntiRoll\ Torque_{SM-F} \times AntiRoll\ Torque_{tire-F}}{AntiRoll\ Torque_{SM-F} + AntiRoll\ Torque_{tire-F}}$$

$$AntiRoll\ Torque_{Total-F} = \frac{14615.5 \times 8920.0}{14615.5 + 8920.0} = 5539.3\ Nm$$

$$Front\ AntiRoll\ Stiffness = 5539.3\ Nm/deg$$

Note: $AntiRollStiffness_{SM\text{-}F} = AntiRollStiffness_{spring\text{-}F} + AntiRollStiffness_{ARB\text{-}F}$

Steady-State Magic Numbers

3rd Magic Number :

Rear:

Calculate the Anti-Roll torque of tires due to 1 degree of roll angle ($k_{tire\text{-}R}$ = 410 N/mm):

$$AntiRoll\ Torque_{tire-R} = \frac{Track^2 \times tan(Roll\ Angle) \times k_{tire-R}}{2}$$

$$AntiRoll\ Torque_{tire-R} = \frac{1.592^2 \times tan(\pi/180) \times 410000}{2} = 9069.0\ Nm$$

Calculate the Total rear Anti-Roll torque due to 1 degree of roll angle:

$$AntiRoll\ Torque_{Total-R} = \frac{AntiRoll\ Torque_{SM-R} \times AntiRoll\ Torque_{tire-R}}{AntiRoll\ Torque_{SM-R} + AntiRoll\ Torque_{tire-R}}$$

$$AntiRoll\ Torque_{Total-R} = \frac{15212.4 \times 9069.0}{15212.4 + 9069.0} = 5681.8\ Nm$$

$$Rear\ AntiRoll\ Stiffness = 5681.8\ Nm/deg$$

Note: $AntiRollStiffness_{M\text{-}R} = AntiRollStiffness_{spring\text{-}R} + AntiRollStiffness_{ARB\text{-}R}$

Steady-State Magic Numbers

3rd Magic Number :

	Anti-Roll Stiffness
Front Springs	3748.2 Nm/deg
Front ARB	10867.3 Nm/deg
Front Tire	8920.0 Nm/deg
Rear Springs	1606.4 Nm/deg
Rear ARB	13606.0 Nm/deg
Rear Tire	9069.0 Nm/deg

	Anti-Roll Stiffness	Percentage of total
Front	5539.3 Nm/deg	49.37 %
Rear	5681.8 Nm/deg	50.63 %

Steady-State Magic Numbers

3rd Magic Number :

Calculate the Elastic WT:

$$S\ "Elastic"\ WT_{front} = \frac{SM \times lat.acc. \times \Delta z \times \left(\frac{Antiroll\ Torque\ Stiffness_{front}}{Antiroll\ Torque\ Stiffness_{Total}}\right)}{track_{front}}$$

$$S\ "Elastic"\ WT_{front} = 836.9 \times (2.5 \times 9.81) \times 0.218 \times \frac{5539.3}{5539.3 + 5681.8} / 1.640 = 1346.8N$$

$$S\ "Elastic"\ WT_{rear} = \frac{SM \times lat.acc. \times \Delta z \times \left(\frac{Antiroll\ Torque\ Stiffness_{rear}}{Antiroll\ Torque\ Stiffness_{Total}}\right)}{track_{rear}}$$

$$S\ "Elastic"\ WT_{rear} = 836.9 \times (2.5 \times 9.81) \times 0.218 \times \frac{5681.8}{5539.3 + 5681.8} / 1.592 = 1423.1N$$

Steady-State Magic Numbers

3rd Magic Number :

$$Anti\ Roll\ Stiffness\ Distribution\ [\%\ Front] = \frac{5539.3}{5539.3 + 5681.8} \times 100 = \mathbf{49.37}\ \%$$

Calculate the Total WT:

$$Total\ WTfront = Lat.NS\ WT_{front} + Lat.S\ "Geometric"\ WT_{front} + S\ "Elastic"\ WT_{front} = 1871.7\ N$$

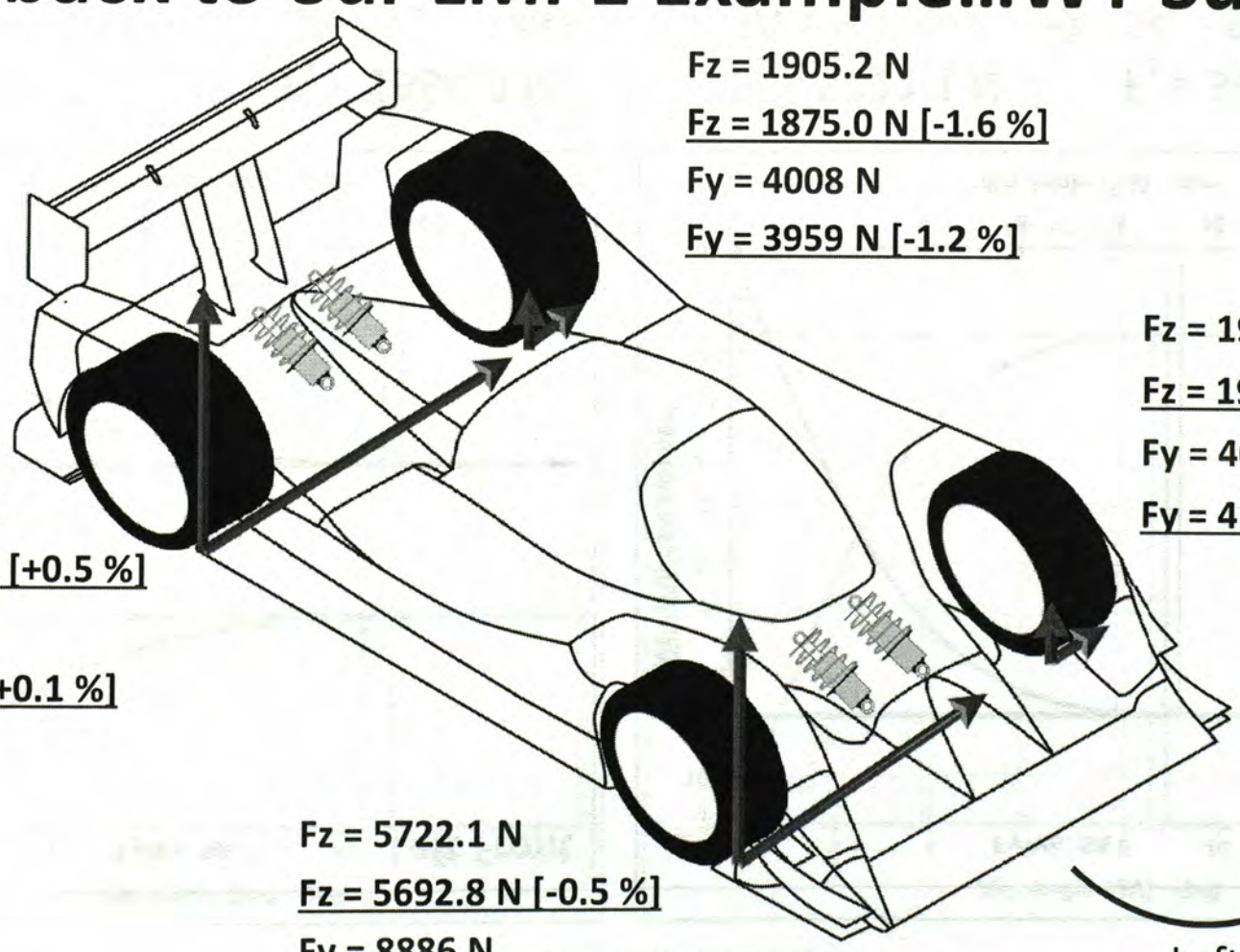
$$Total\ WTrear = Lat.NS\ WT_{rear} + Lat.S\ "Geometric"\ WT_{rear} + S\ "Elastic"\ WT_{rear} = 2210.1\ N$$

$$Total\ WT\ distribution\ [\%\ Front] = \frac{1871.7}{1871.7 + 2210.1} \times 100 = \mathbf{45.85}\ \%$$

Steady-State Magic Numbers Summary

	Anti-Roll Stiffness Distribution [%]	Elastic WT Distribution [%]	Total WT Distribution [%]
1st Magic Number	49.00%	48.26 %	-
2nd Magic Number	-	-	45.61 %
3rd Magic Number	49.37 %	-	45.85 %

Coming back to our LMP1 Example…WT Summary

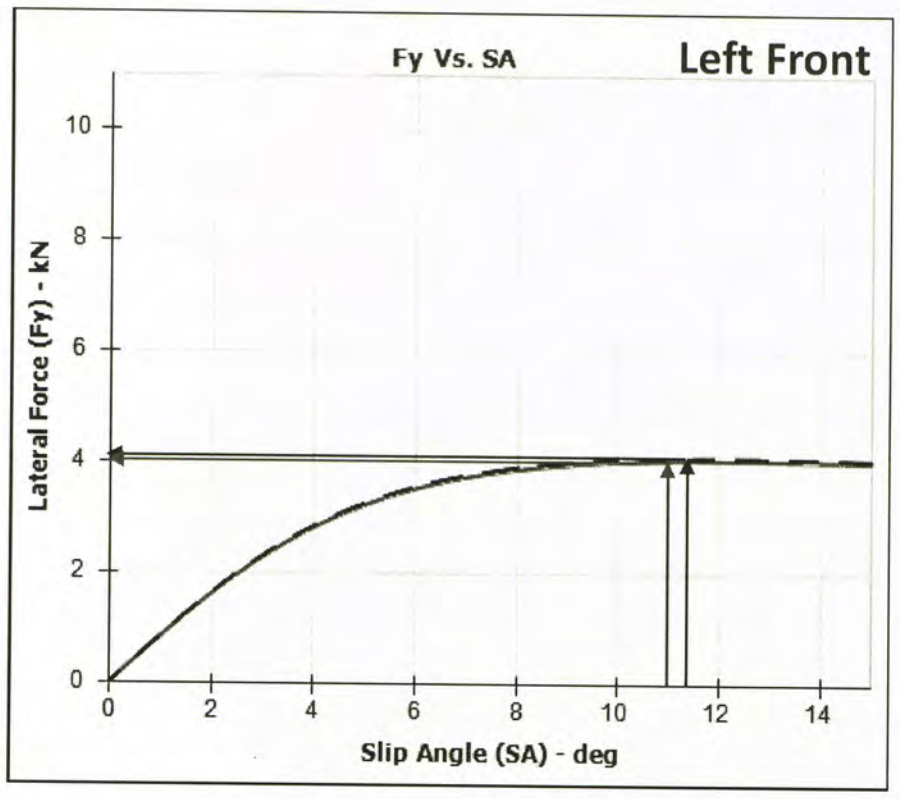


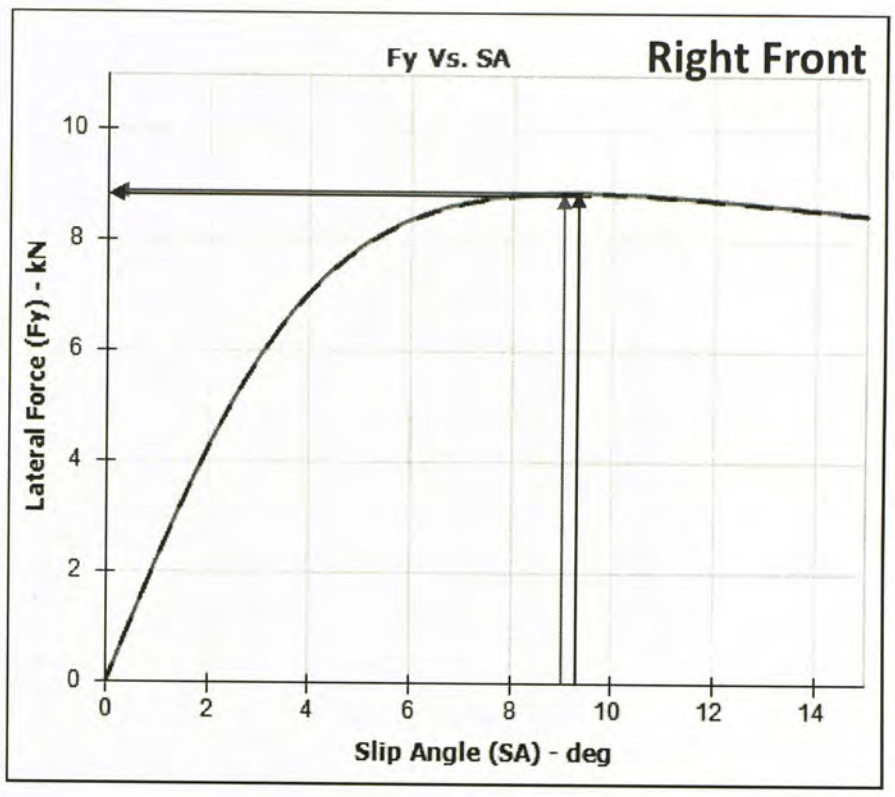
Initial Front ARB

With softer ARB at the front

Lateral Force Front

Calculation made with CVD using a "tricked" tire model!





F_z = 1954.7 N
SA = 10.85 deg →
F_y = 4055 N

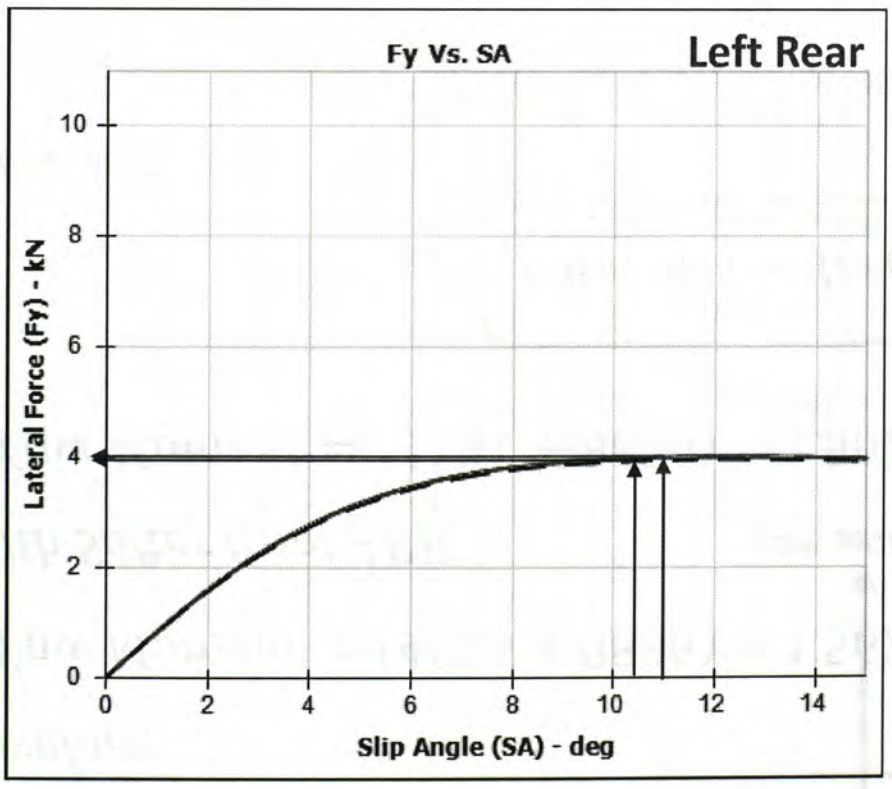
F_z = 1984.0 N
SA = 11.35 deg
F_y = 4101.0 N

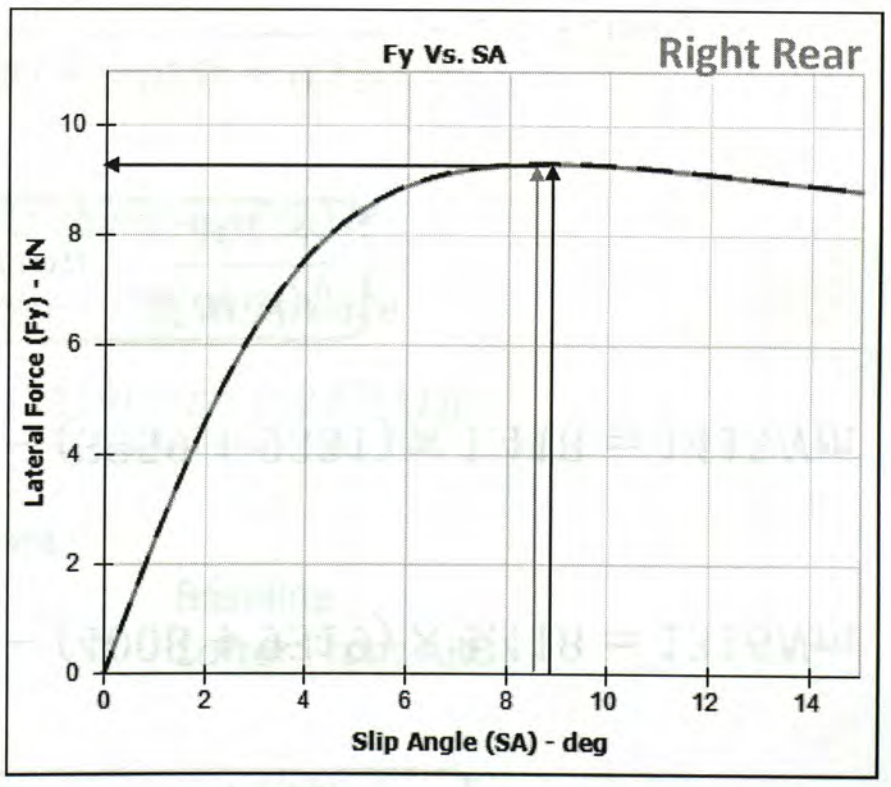
F_z = 5722.1 N
SA = 9.1 deg →
F_y = 8886.0 N

F_z = 5692.8 N
SA = 9.2 deg
F_y = 8870.0 N

Lateral Force Rear

Calculation made with CVD using a "tricked" tire model!





F_z = 1905.2 N
SA = 11.28 deg →
F_y = 4008.0 N

F_z = 1875.0 N
SA = 10.53 deg
F_y = 3959.0 N

F_z = 6417.6 N
SA = 8.40 deg →
F_y = 9319.0 N

F_z = 6447.8 N
SA = 8.95 deg
F_y = 9331.0 N

WT Summary

$$\boxed{Yaw\ Moment = F_{Y\ Front} \times a - F_{Y\ Rear} \times b}$$

Baseline:

$$Yaw\ Moment = (4055 + 8886) \times 1.562 - (4008 + 9319) \times 1.418 = 1316Nm$$

With Softer front ARB:

$$Yaw\ Moment = (4101 + 8870) \times 1.562 - (3959 + 9331) \times 1.418 = 1415Nm$$

$$\boxed{Lateral\ Acceleration = \frac{Total_{F_Y}}{Mass}}$$

Baseline:

$$Lateral\ Acceleration = \frac{4055 + 8886 + 4008 + 9319}{994} = 26.426m/s^2$$

With Softer front ARB:

$$Lateral\ Acceleration = \frac{4101 + 8870 + 3959 + 9331}{994} = 26.419m/s^2$$

WT Summary

Baseline

$$Yaw\ moment = 1316Nm$$

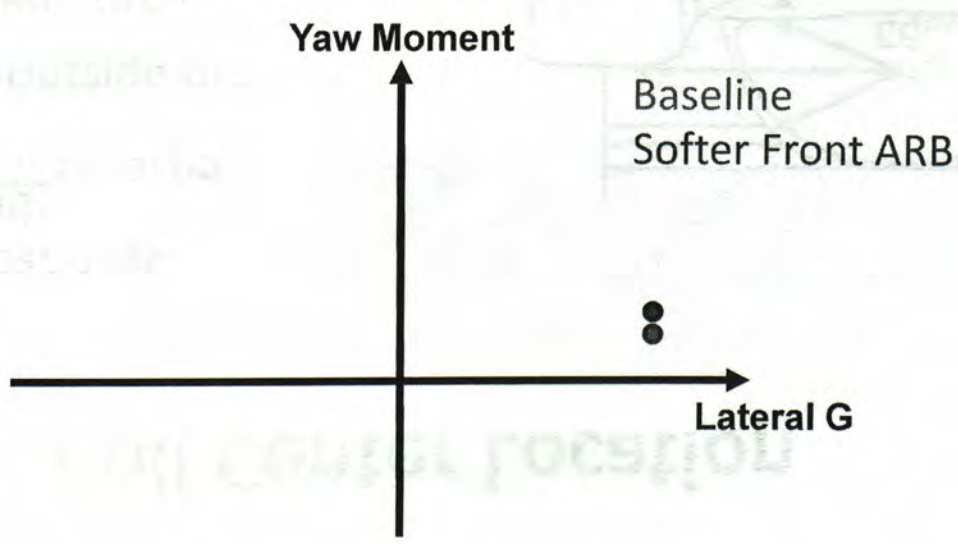
$$Lateral\ Acceleration = 26.426m/s^2 = 2.6938g$$

With Softer ARB

Dynamic corner weight changes of 30 N

$$Yaw\ moment = 1415Nm$$ ***7% difference***

$$Lateral\ Acceleration = 26.419m/s^2 = 2.6931g$$



Roll Center Location

RC above the ground:

Less roll

More jacking

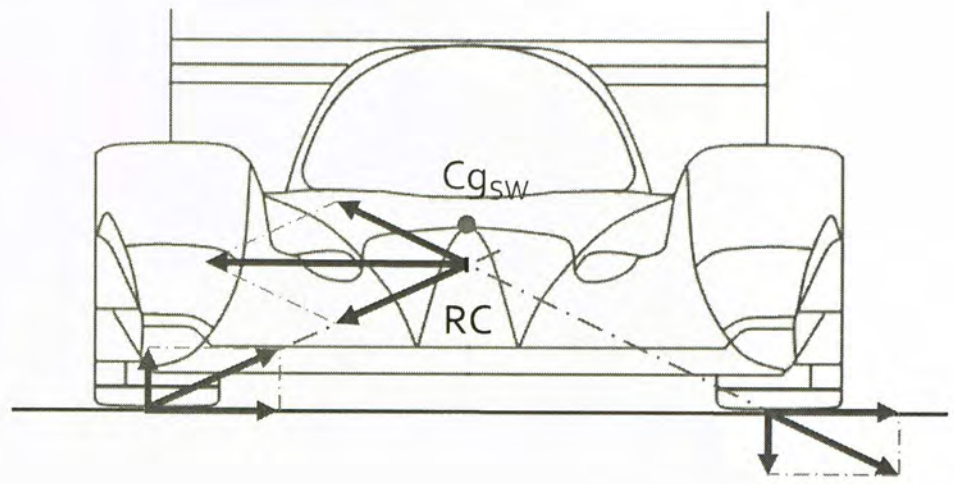
Ride height increases

Instant geometric transfer

- loads the outside tire
- unloads the inside tire

Less suspended mass inertia

Increases turn in response



Rear Higher Than Front

(Roll Center Lukianov)

Roll Center Location

RC under the ground:

More roll

Anti jacking

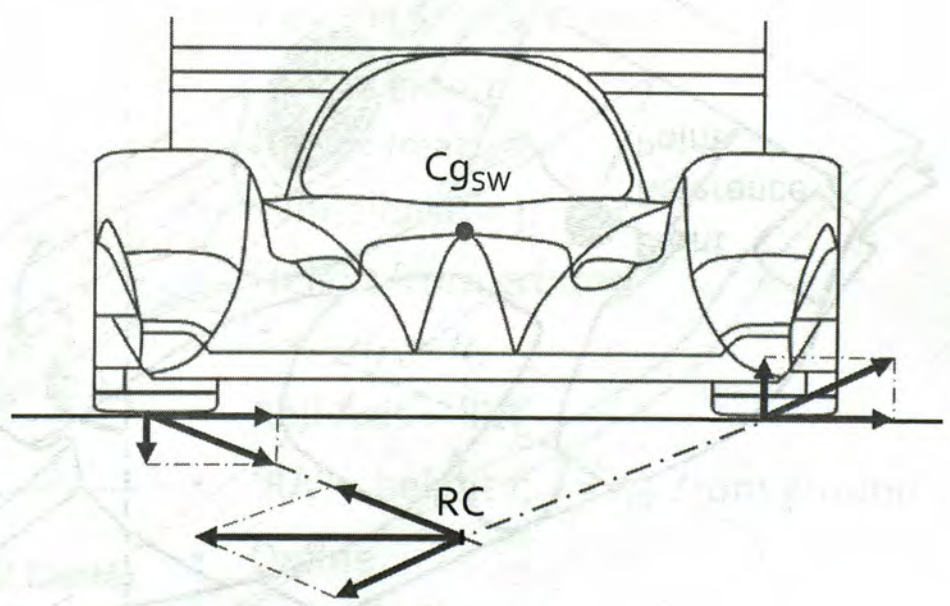
Ride height decreases

Instant geometric transfer

- loads the inside tire
- unloads the outside tire

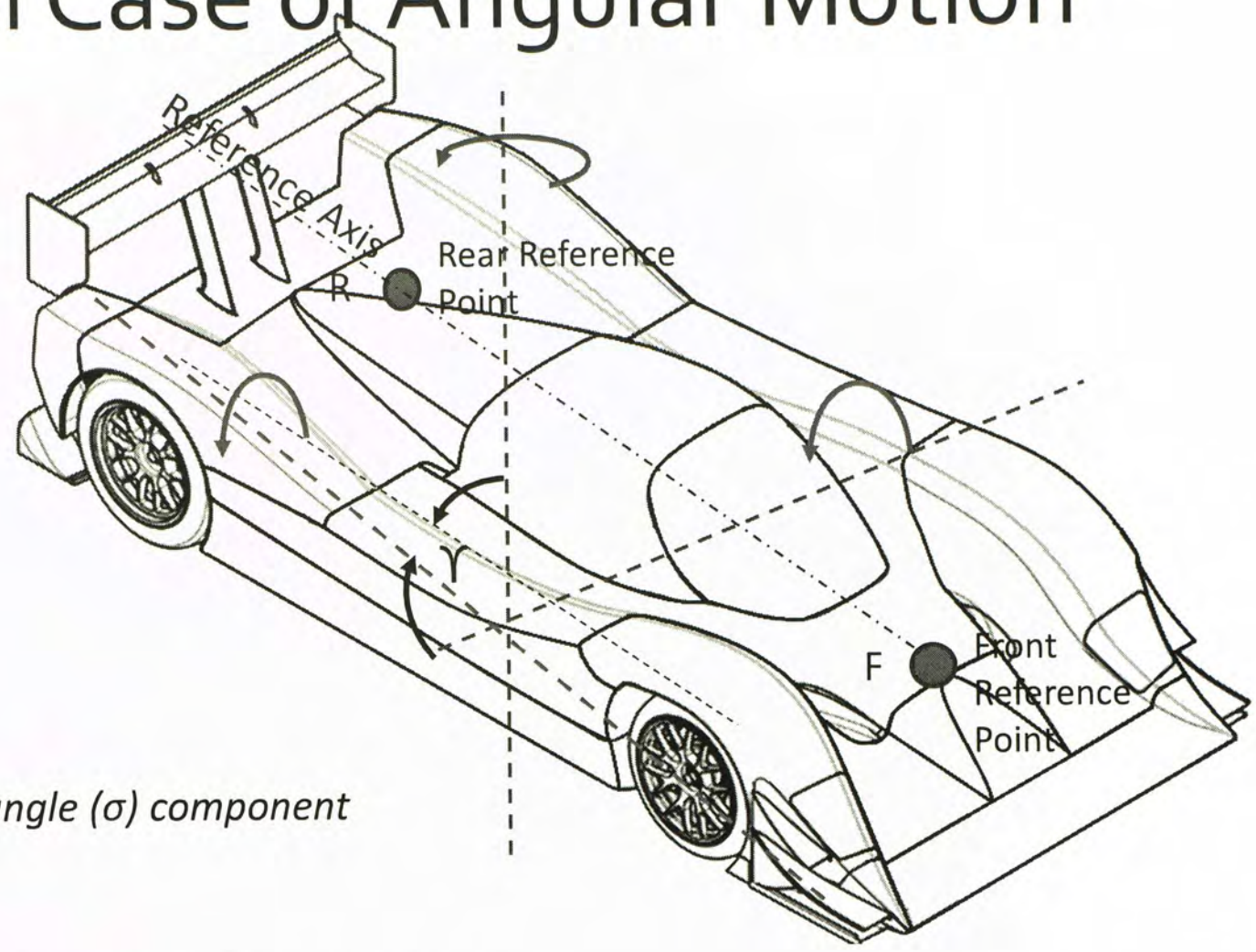
More suspended mass inertia

"Sluggish" turn in response

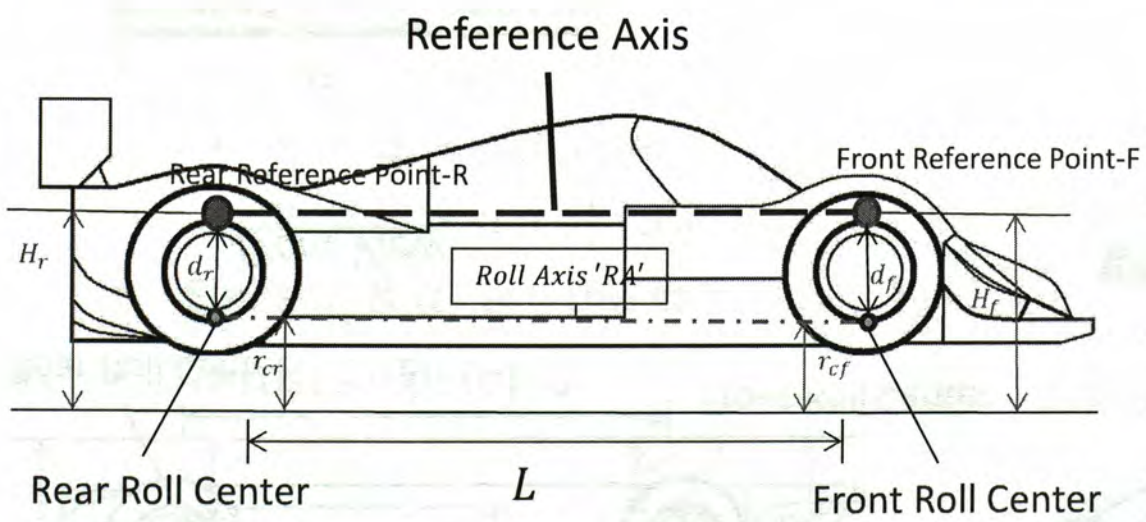


General Case of Angular Motion

- Pitch (θ)
- Roll (ϕ)
- Yaw (ψ)
  - *Has a thrust angle (σ) component*
- Roll Angle Inclination (ϒ)

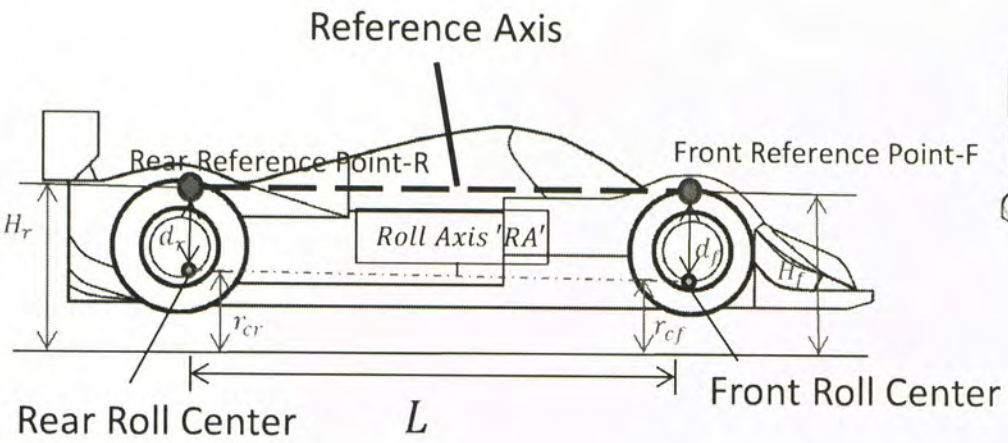


Simple Illustration

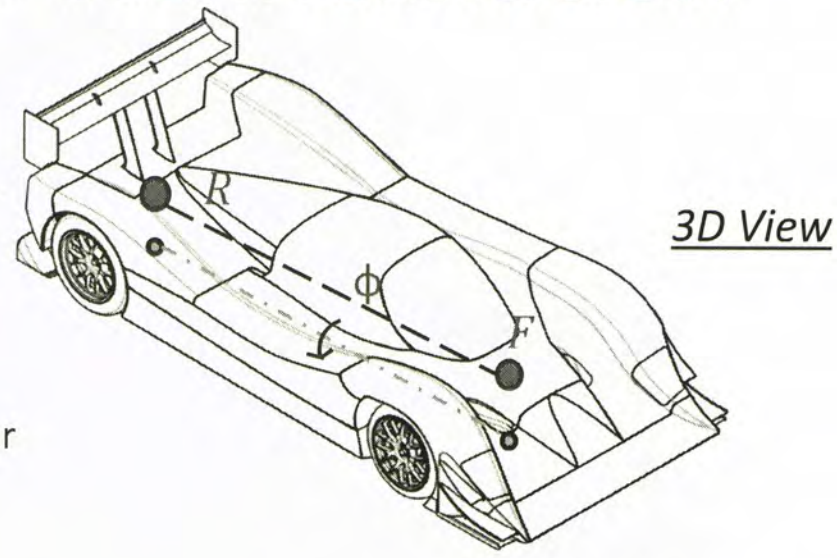


- Pick Reference Points
  - F (front)
  - R (rear)
- Wheelbase = L
- Height from ground
  - $H_f = H_r$
- Roll Axis – 'RA'
- 'RA' - height $r_{cf} = r_{cr}$ from ground
- Define
  - $d_f = H_f - r_{cf}$
  - $d_r = H_r - r_{cr}$
- **<u>NO Roll Axis Movement</u>**

Case 1 – No Roll Axis Inclination

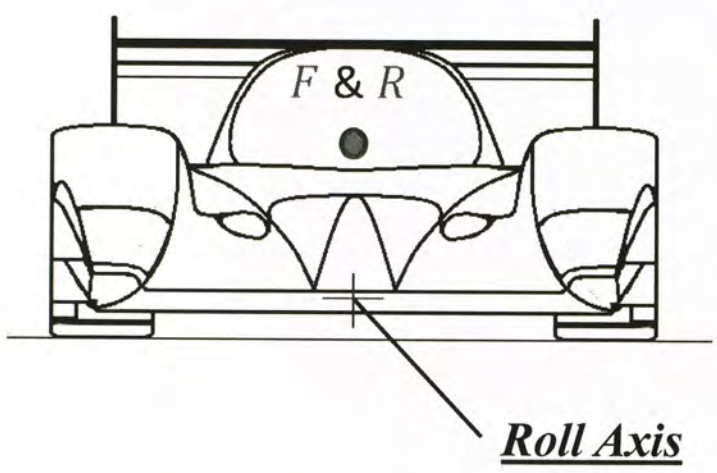


Front View



3D View

Front View



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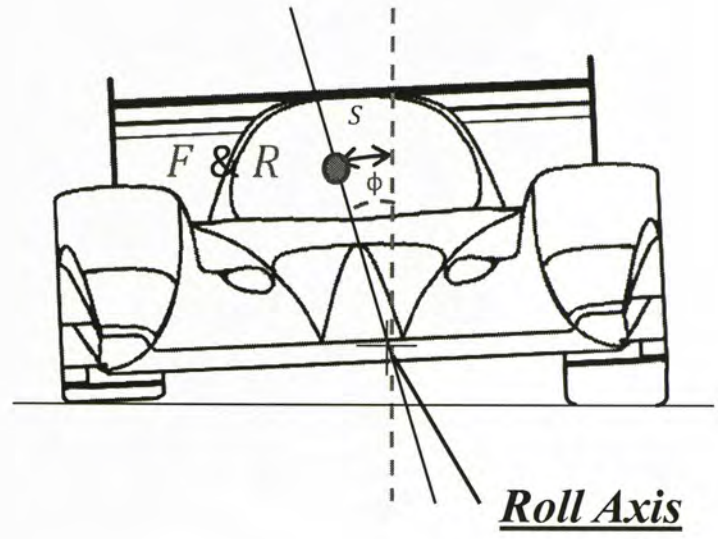
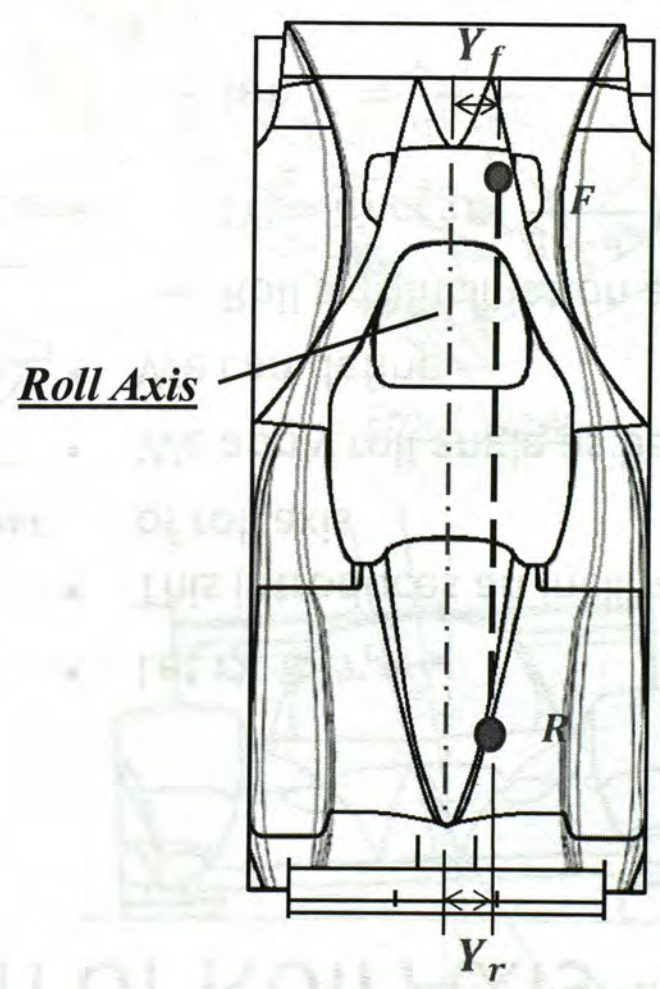


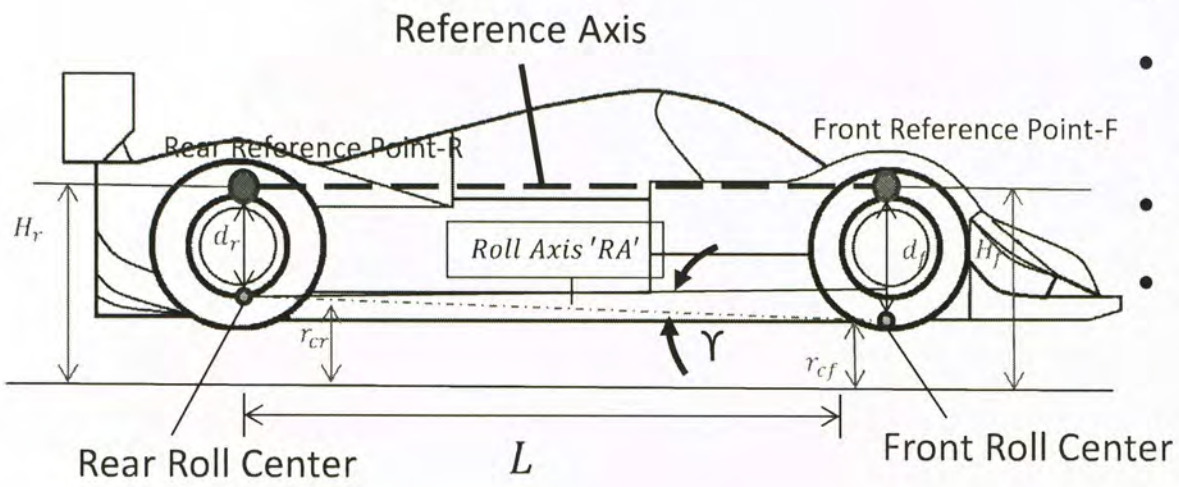
Case 1 - What Happened?

- Both points F & R travel the same arc distance 'S' .
- So lateral distance -
  - $Y_f = d_f * \sin(\phi)$ for F
  - $Y_r = d_r * \sin(\phi)$ for R
- Since d_f & d_r are the same, $Y_f = Y_r$
- Thrust Angle(σ)=0

Top View

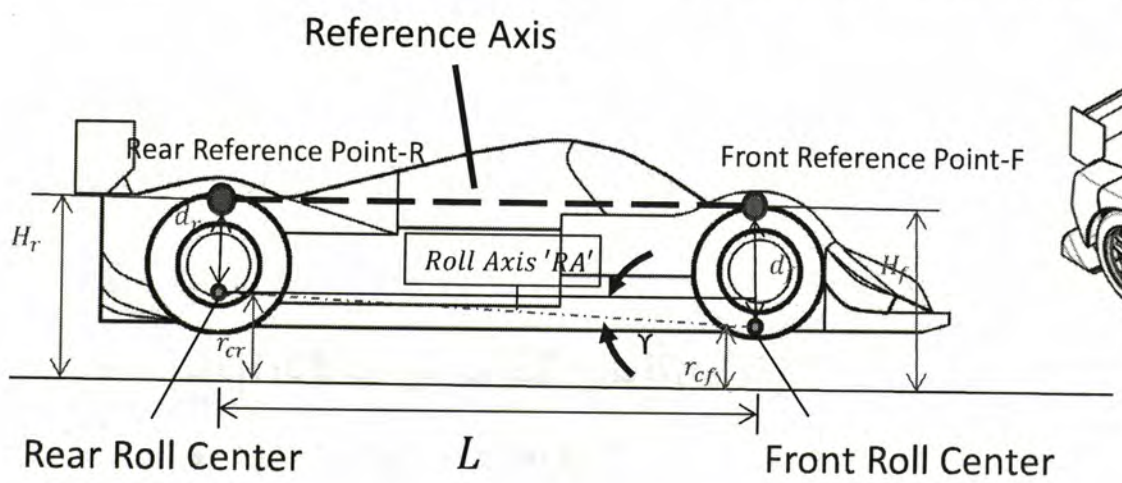


Case 2 - Inclination of Roll Axis

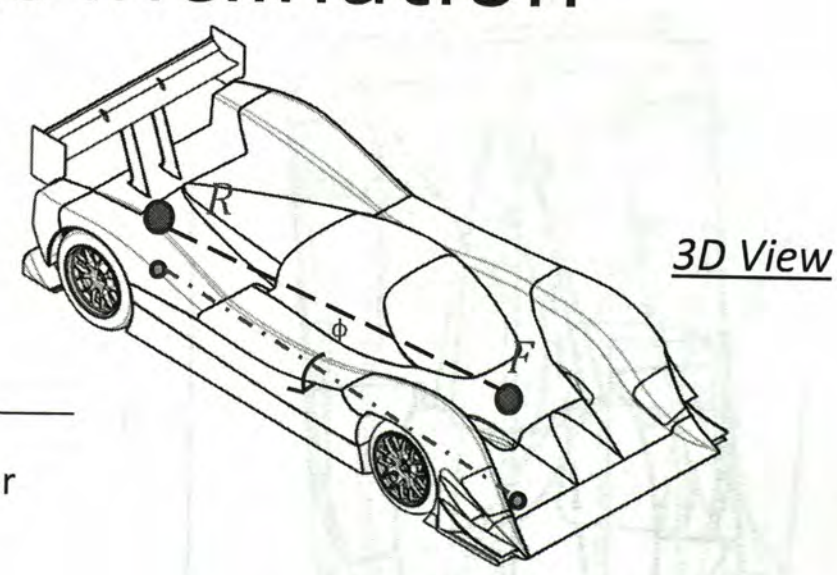


- Let $r_{cr} > r_{cf}$
- This introduces an inclination of roll axis
- We apply roll angle as before
- We can define
  - Roll axis inclination angle (Υ)= $arctan(\frac{d_f - d_r}{L})$
  - $\tan \Upsilon = \frac{d_f - d_r}{L}$

Case 2 – Roll Axis Inclination

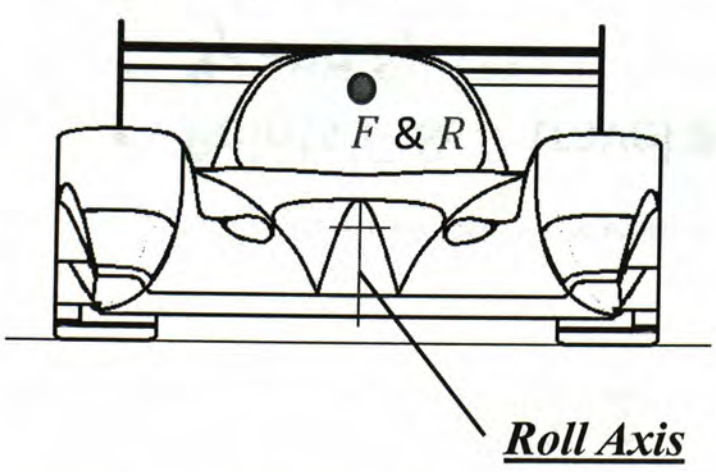


Front View



3D View

Front View



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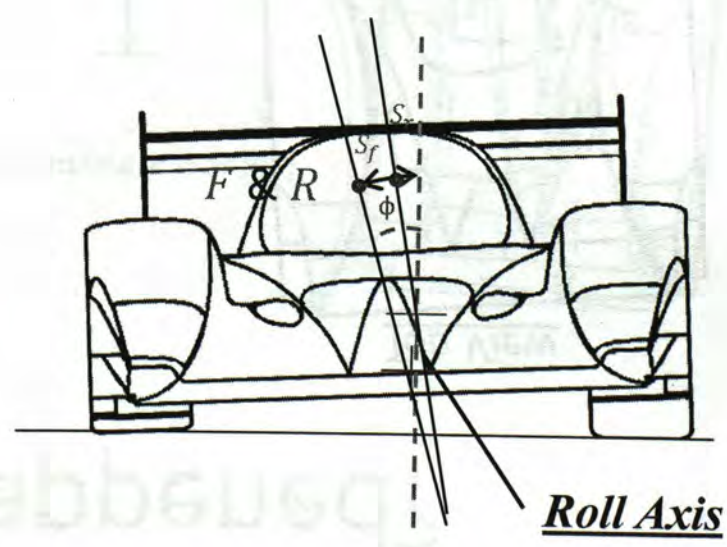
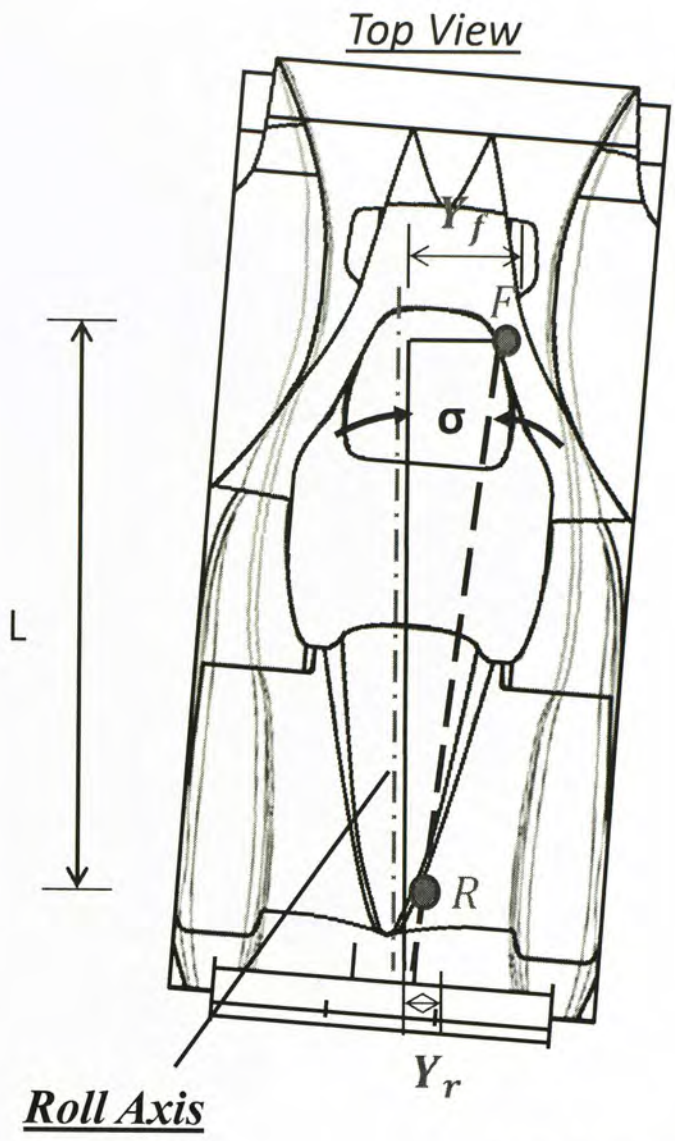


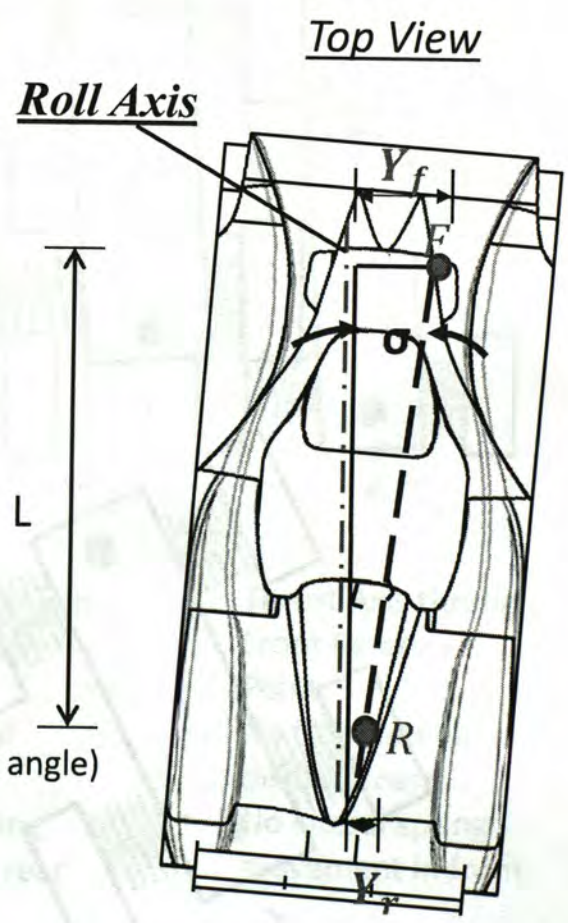
Case 2 - What Happened?

- Points F & R travel arcs S_f and S_r.
- Thus lateral distance travelled
  - $Y_f = d_f * \sin(\phi)$
  - $Y_r = d_r * \sin(\phi)$
- Induced Thrust Angle(σ)

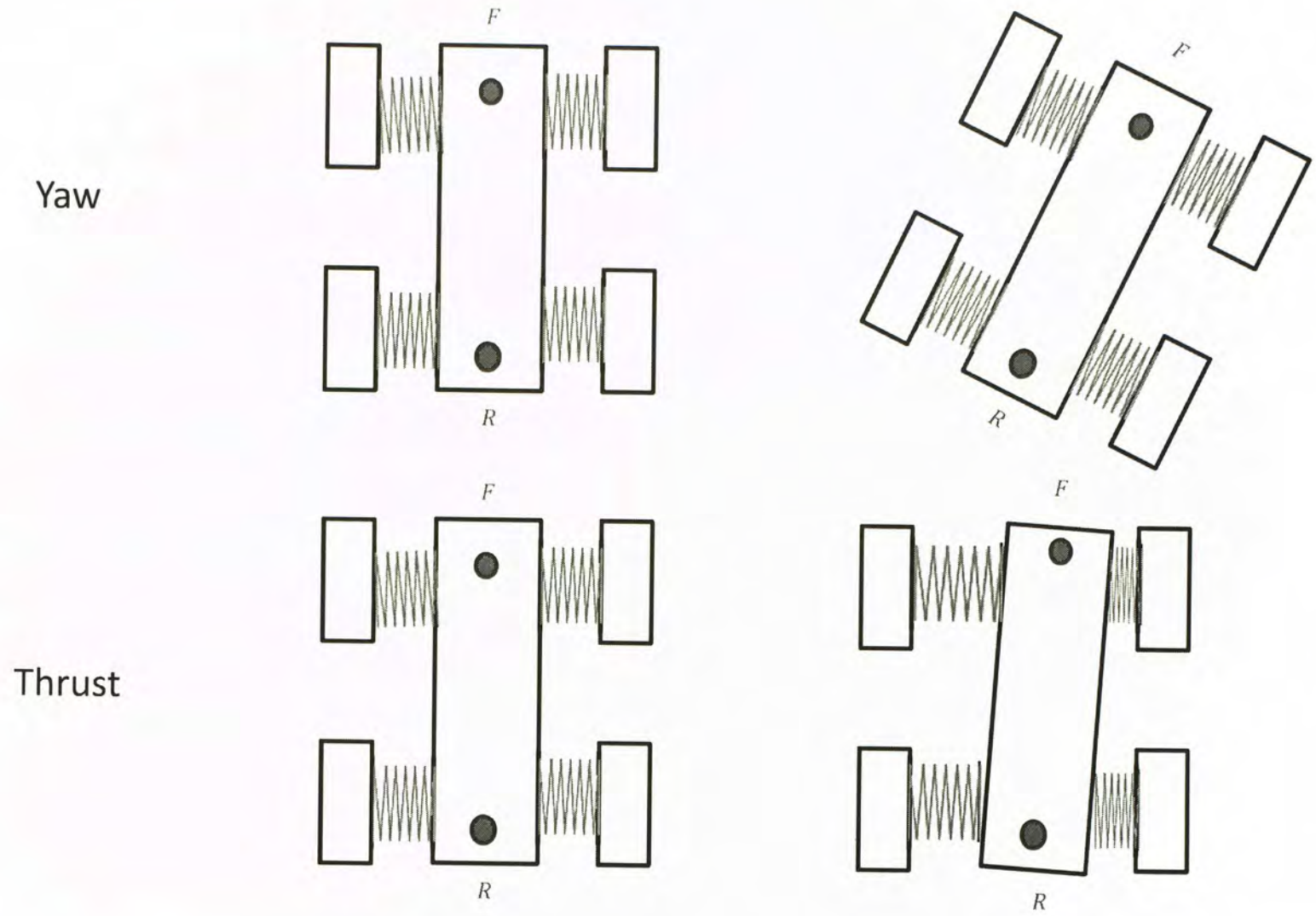


Calculate Thrust Angle (σ)

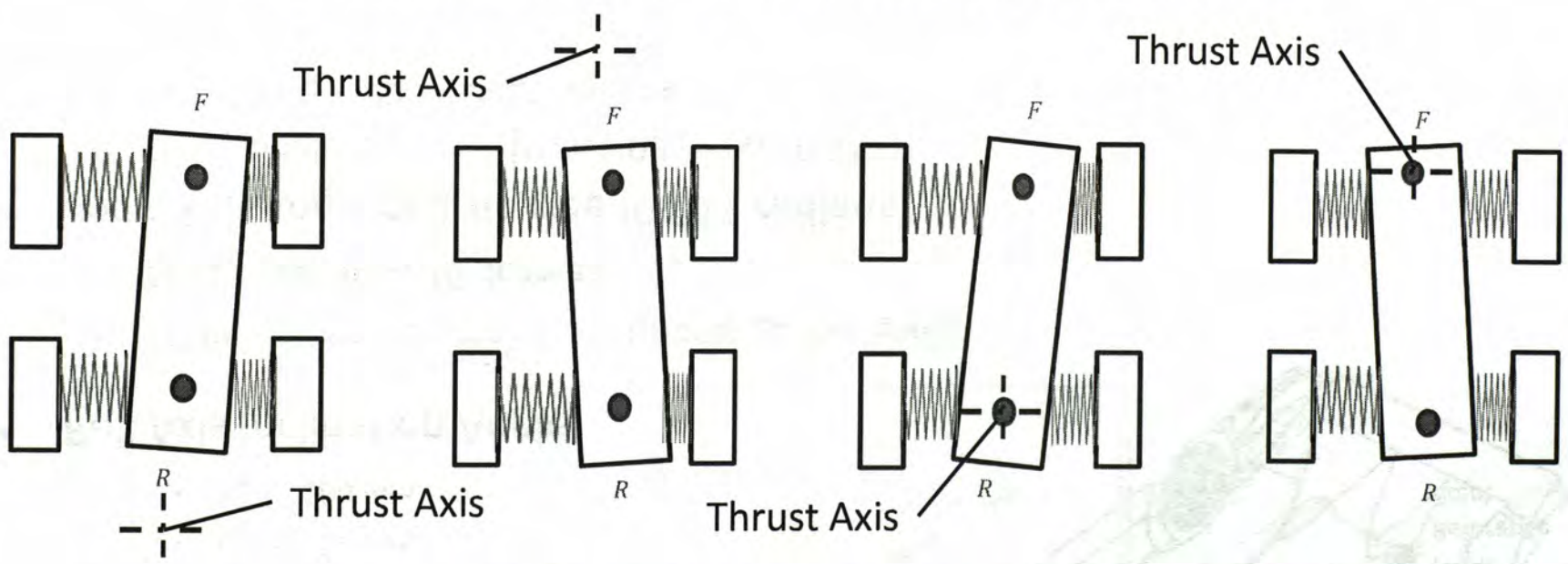
- From the figure
  - $\tan\sigma = \frac{Y_f - Y_r}{L}$
  - $\tan\sigma = \frac{(d_f - d_r) * \sin\phi}{L}$
- Thus Thrust Angle is
  - $\sigma = arctan(\frac{(d_f - d_r) * \sin\phi}{L})$
- Alternatively,
  - $\sigma = arctan(\tan\Upsilon * \sin\phi)$ (where Υ is the roll axis inclination angle)
  - *or*
  - $\sigma \sim \Upsilon * \phi$ (in radians using small angle approximation)



Thrust Angle is NOT Yaw Angle



Influence of Thrust Axis



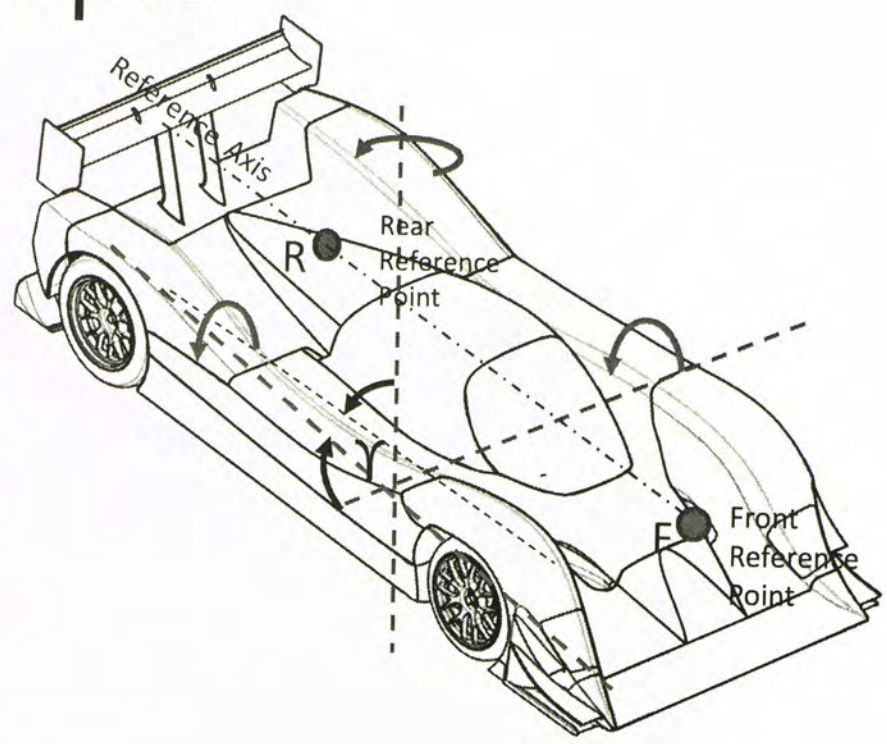
- Thrust Axis behind Rear Reference Point
- Lateral displacement
  - Front>Rear
- Extension/Compression of lateral springs
  - Front>Rear

- Thrust Axis ahead of Front Reference Point
- Lateral displacement
  - Rear>Front
- Extension/Compression of lateral springs
  - Rear>Front

- Thrust Axis through Rear Reference Point
- No rear lateral displacement
- No lateral spring movement in rear

- Thrust Axis through Front Reference Point
- No front lateral displacement
- No lateral spring movement in front

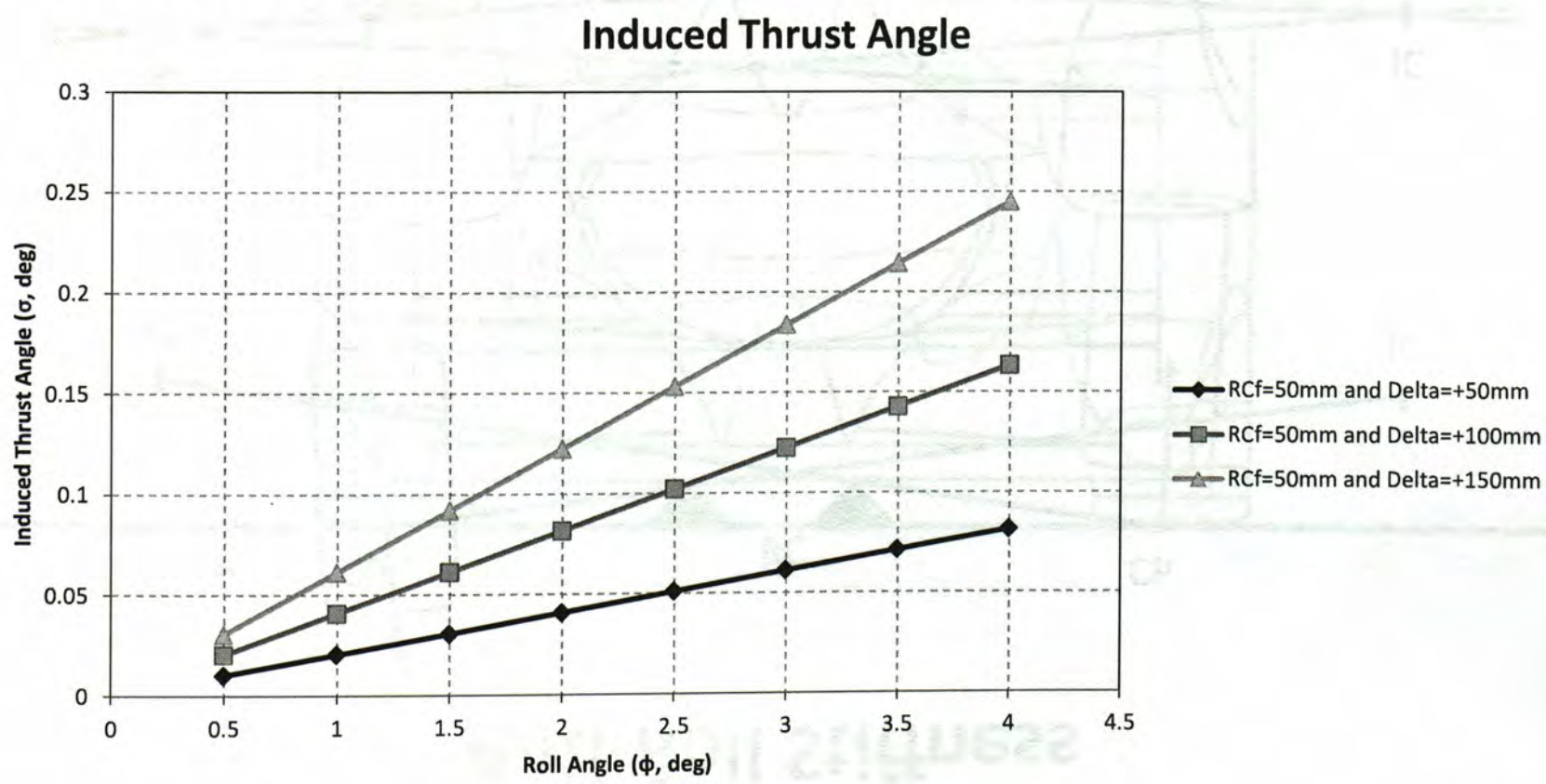
Example



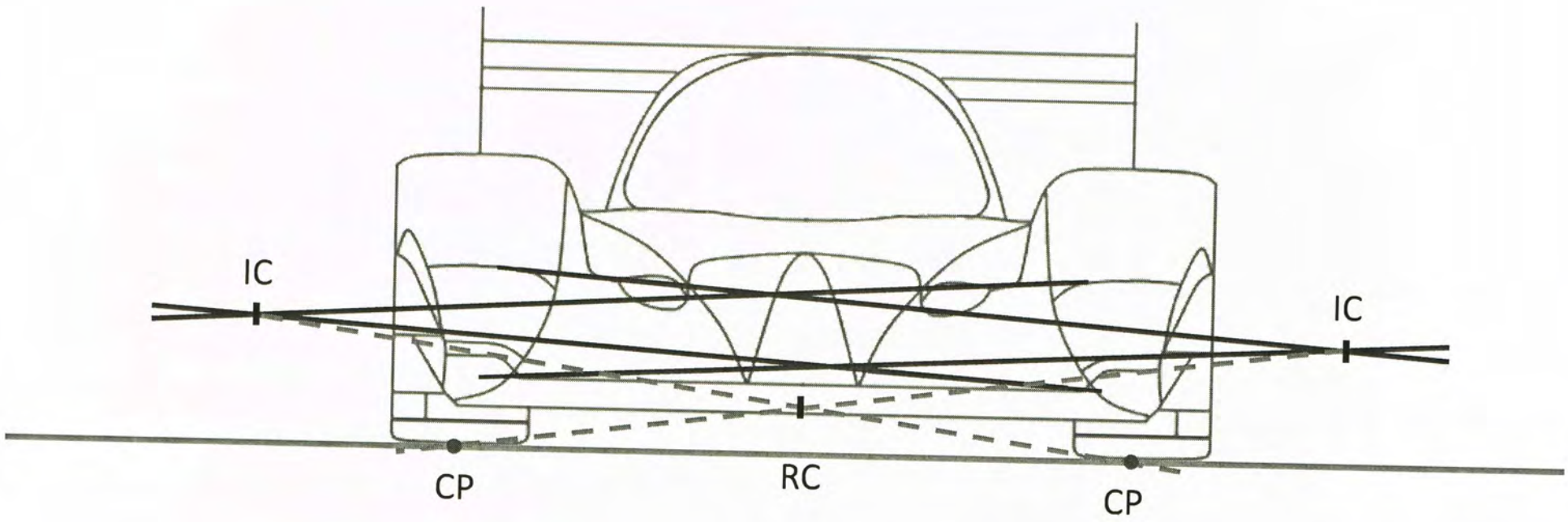
- Let's take an example of a WRC Car
  - Parameters
    - H_f = 430 mm
    - H_r = 430 mm
    - L = 2450 mm
    - r_{cf} = 50 mm
    - r_{cr} = 100 mm
- Roll Axis Inclination Angle
  - $\Upsilon = \tan^{-1}\frac{(H_f - r_{cf}) - (H_r - r_{cr})}{L}$
  - **Υ=0.02 radians=1.16 degrees**
- For a Roll Angle of **1 degree** (0.017 radians)
  - Thrust Angle, $\sigma = \tan^{-1}[(\tan 0.02) * (\sin 0.017)]$
  - **σ= 0.00035 radians=0.02 degrees**

Calculation

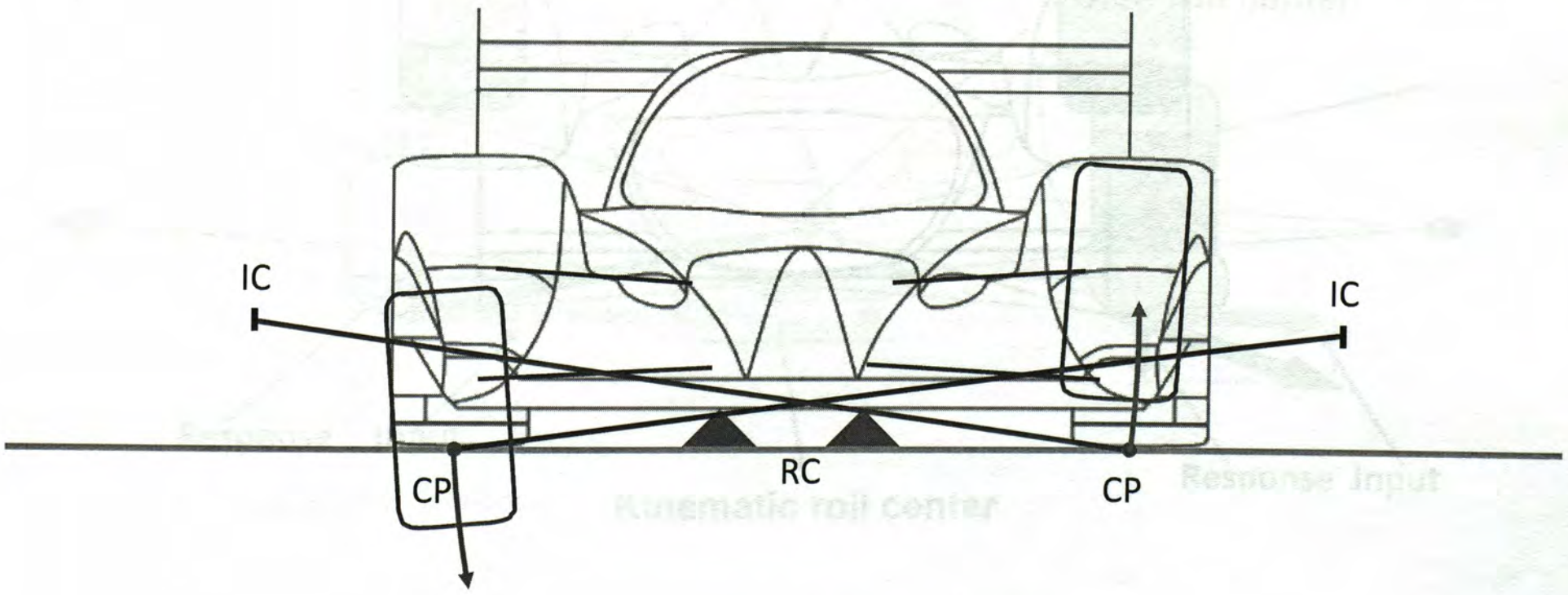
- For a sweep of Roll Angle
- Let Delta = r_{cr} - r_{cf}



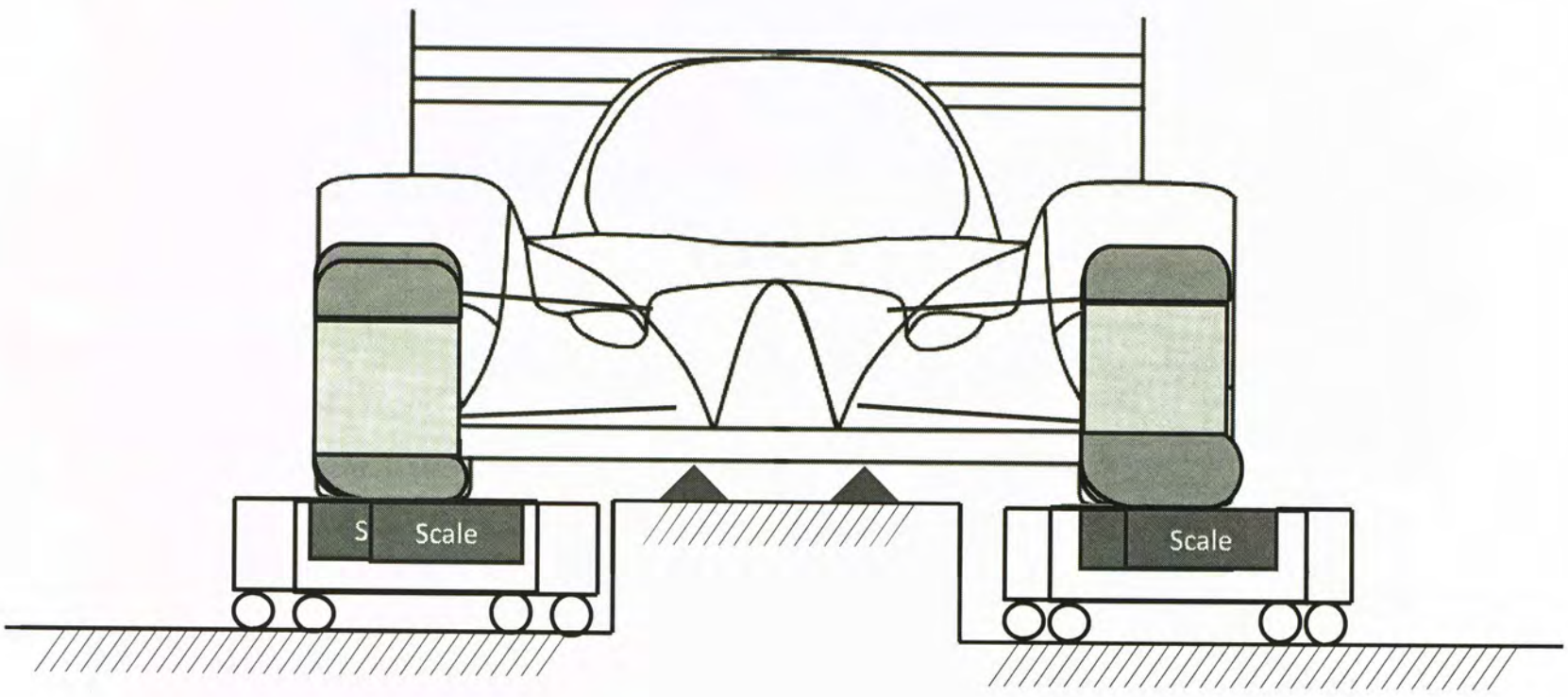
Anti-Roll Stiffness



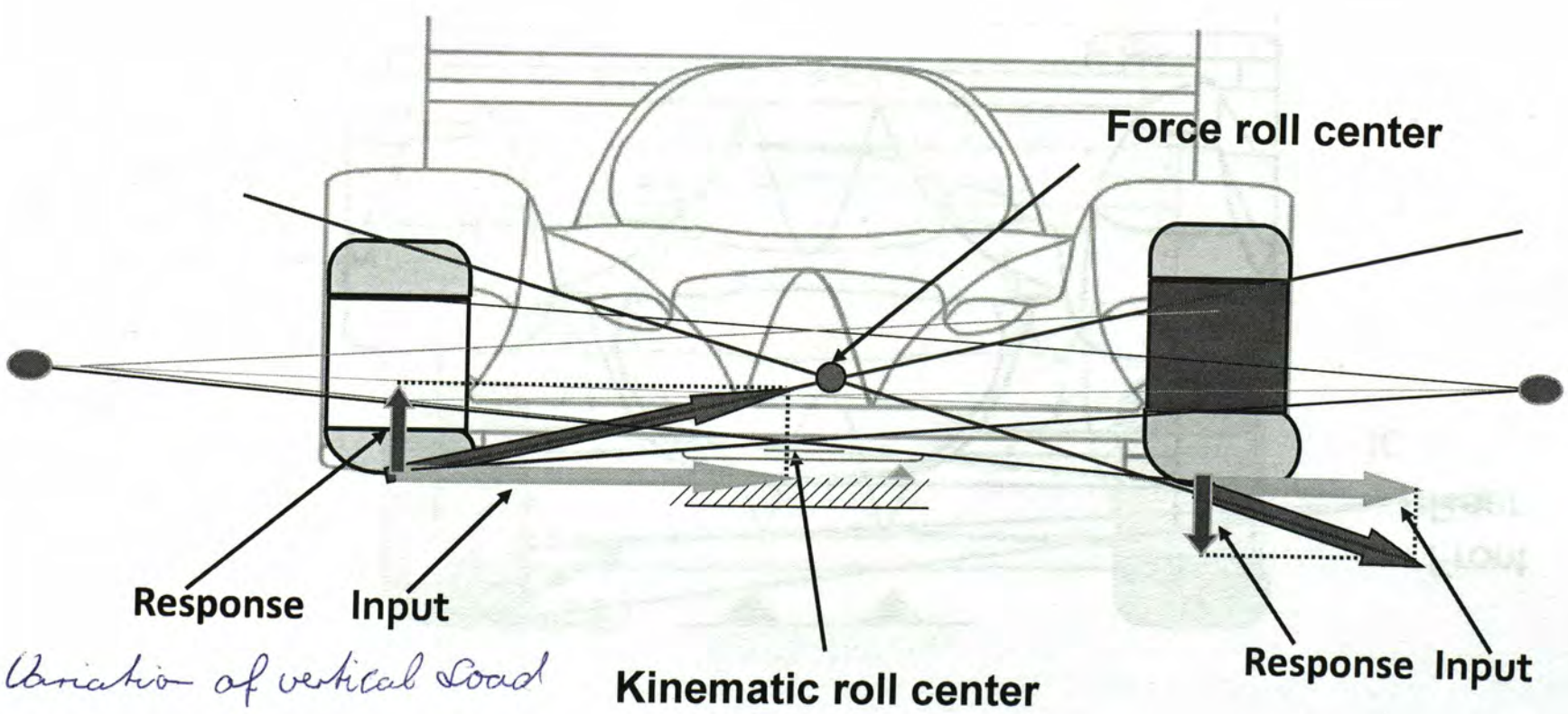
Anti-Roll Stiffness



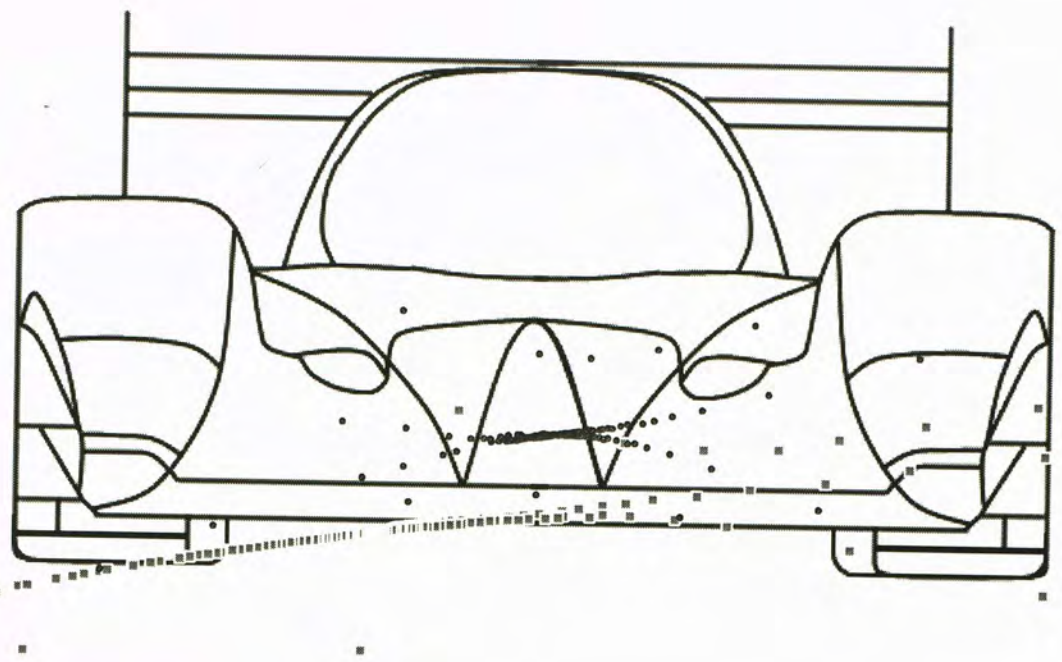
Anti-Roll Stiffness



Kinematics and Force Roll Centers



Kinematics and Force Roll Centers

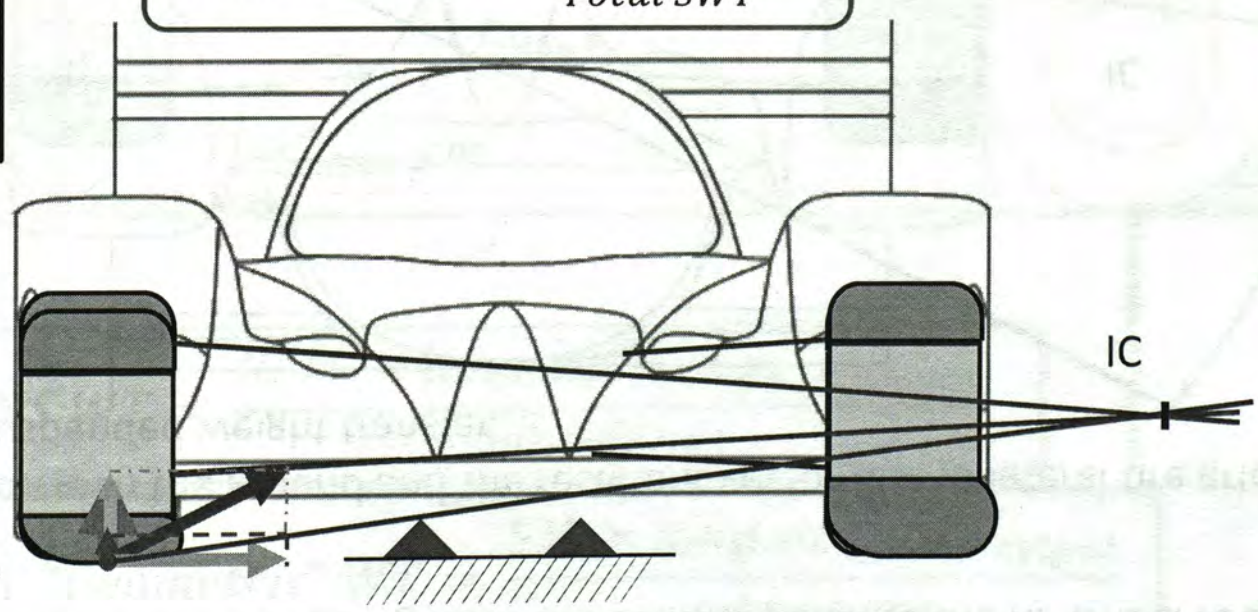


- Front
- Rear

Anti-Roll Stiffness

- S Geometric WT
- S Elastic WT
- Geometric + Elastic S WT

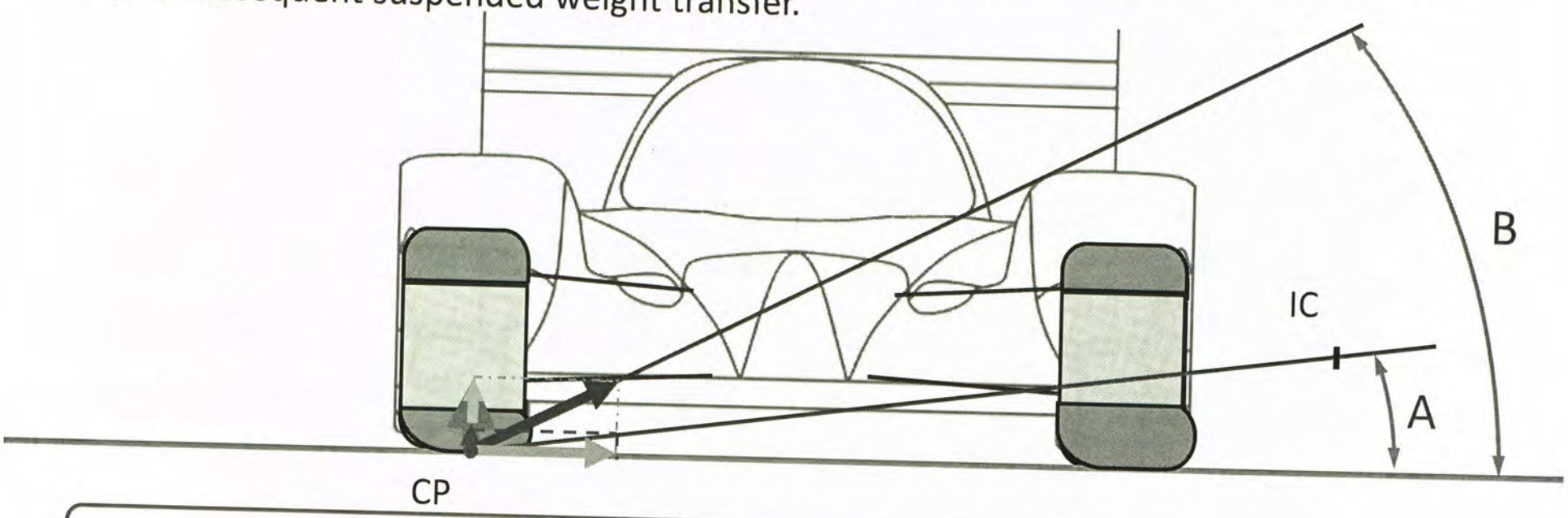
$$\% \, Antiroll = \frac{\text{"Geometric" } SWT}{Total\ SWT}$$



Anti-Roll Stiffness

Angle A = Angle between the axis going from the contact patch to the RF wheel IC and the ground.

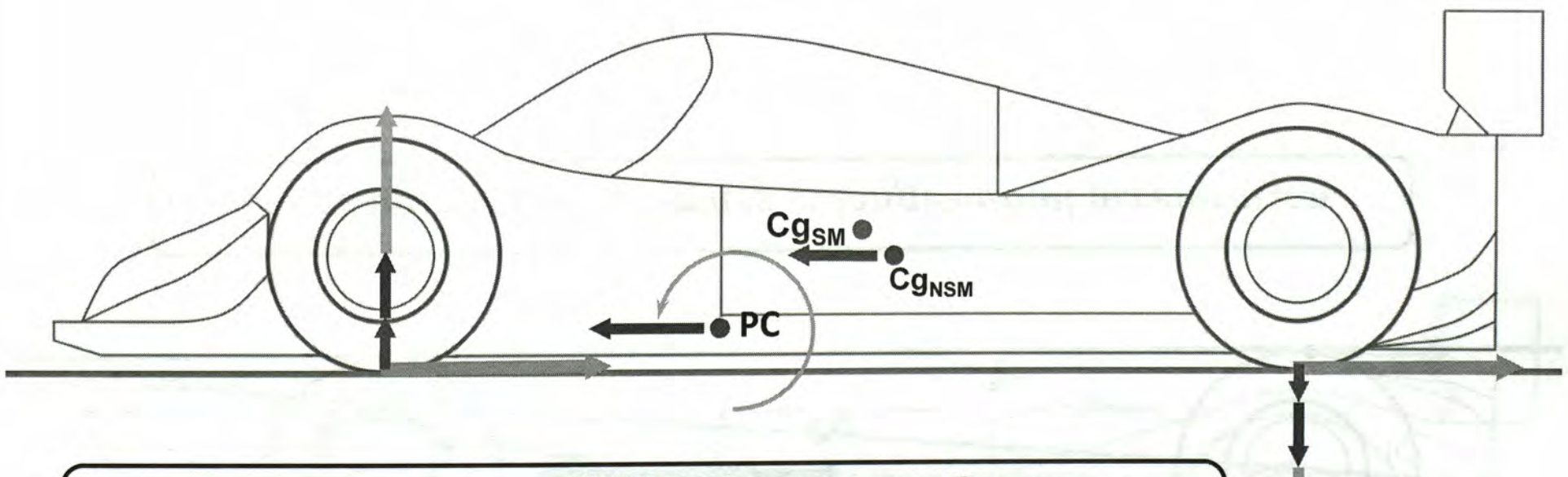
Angle B = Angle between the ground and the resultant vector due to lateral tire grip and subsequent suspended weight transfer.



$$AntiRoll\ from\ right\ wheel\ [\%] = 100 \times tan(A)/\tan(B)$$

If Tan A = Tan B then there is 100% anti-roll from right wheel suspension

Longitudinal Weight Transfer

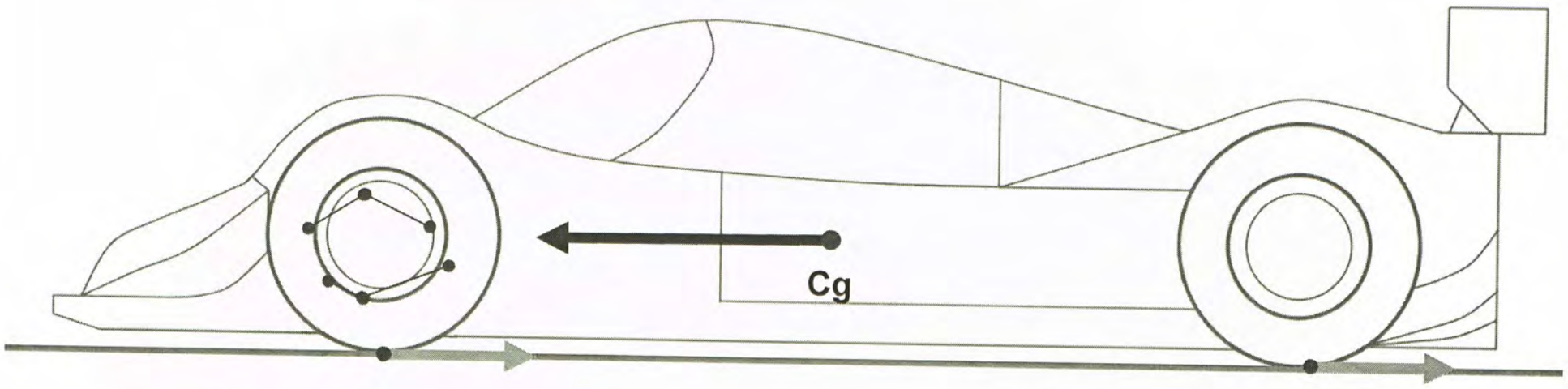


$$Longitudinal\ NS\ WT = \frac{NSM\ \times long.acc. \times\ Cg_{NSM\ height}}{wheelbase}$$

$$Longitudinal\ S\ "Geometric"\ WT = \frac{SM\ \times long.acc. \times\ PC_{height}}{wheelbase}$$

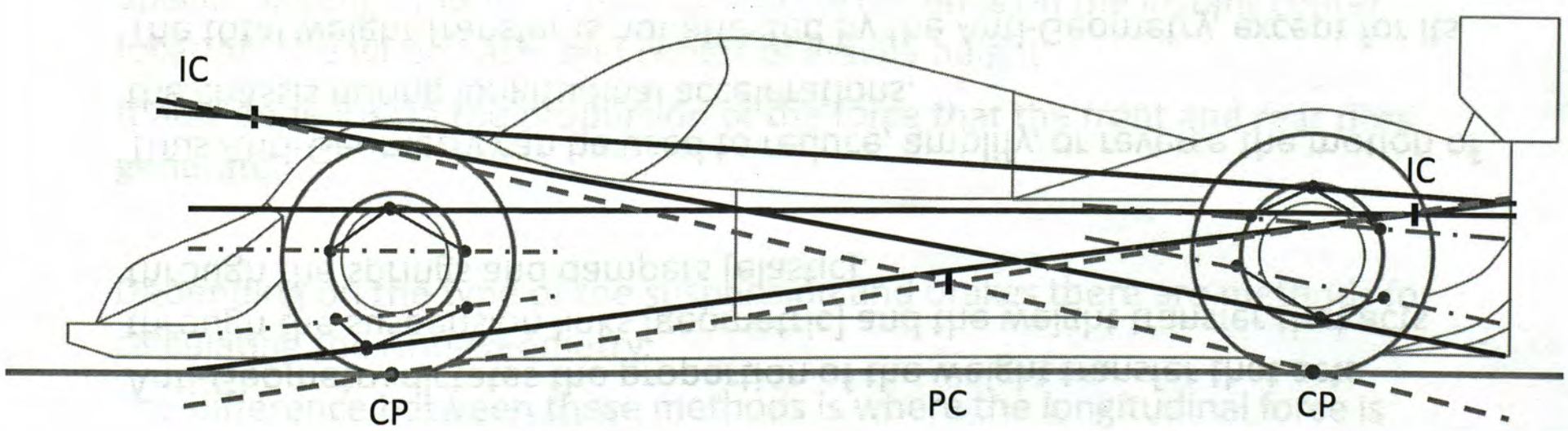
$$Longitudinal\ S\ "Elastic"\ WT = \frac{NSM\ \times long.acc. \times\ \Delta z(Cg_{SM} - PC)}{wheelbase}$$

Anti-Dive / Anti-Lift / Anti-Squat



$$Longitudinal\ force = mass\ \times longitudinal\ acceleration$$

Kinematics Pitch Center Definition

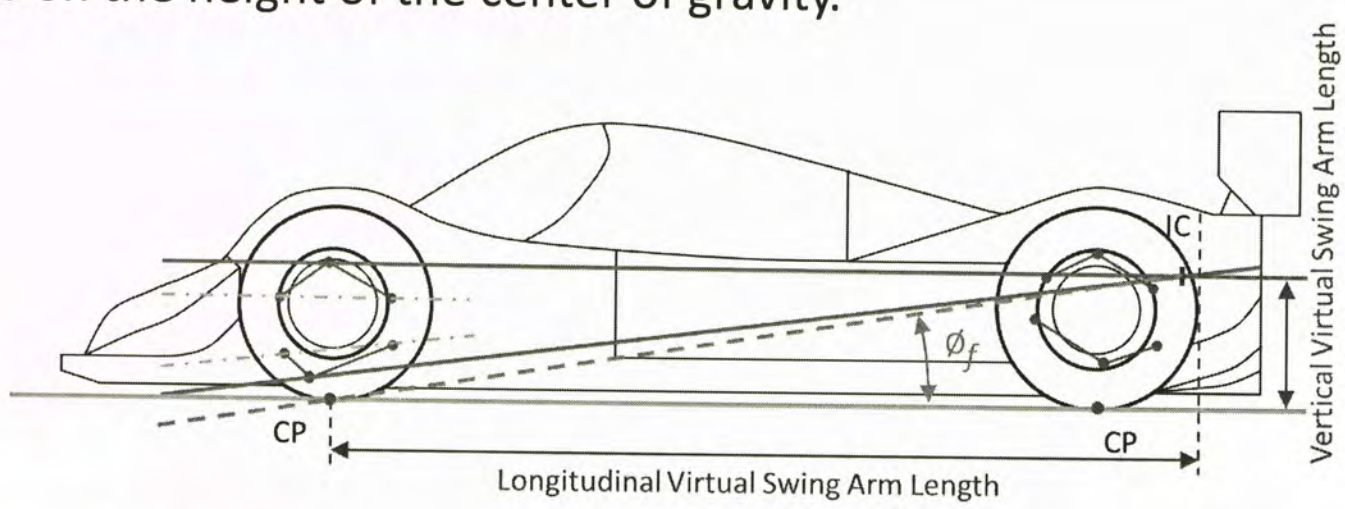


Longitudinal Anti-Geometry

Anti-Geometry dictates the proportion of the weight transfer that acts through the suspension links [geometric] and the weight transfer that acts through the springs and dampers [elastic].

Thus Anti-Geometry can be used to reduce, amplify, or reverse the motion of the chassis during longitudinal accelerations.

The total weight transfer is not affected by the Anti-Geometry, except for its effects on the height of the center of gravity.



Longitudinal Anti-Geometry

The Anti-Geometry can be determined from knowledge of the suspension kinematics and other basic properties.

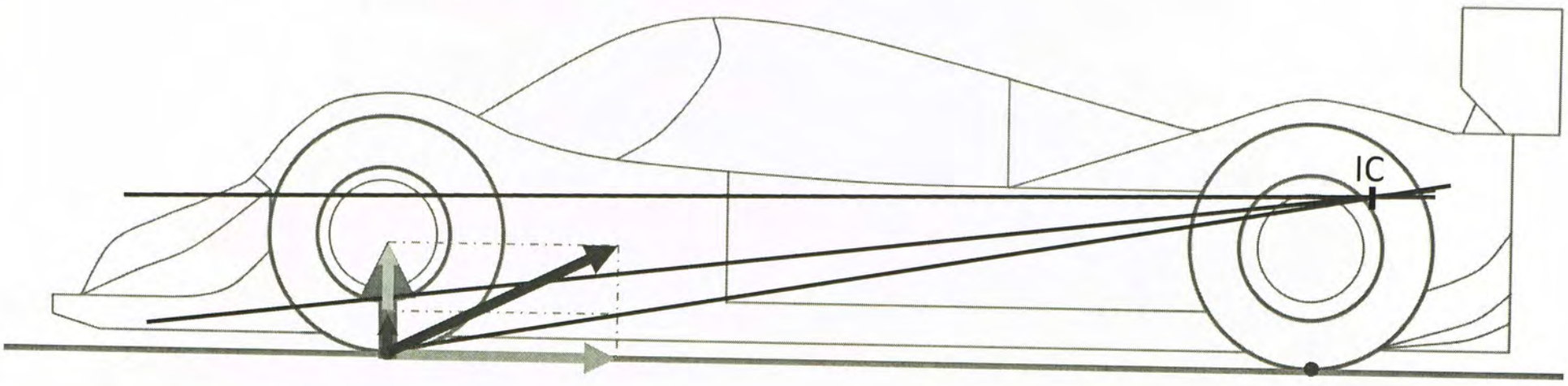
Specifically the amount of Anti-Geometry depends on the instant center location, the wheelbase, and center of gravity height.

It also depends on the proportion of the force that the front and rear tires generate.

Depending on the type of the suspension and brakes there are methods for calculating the Anti-Geometry.

The difference between these methods is where the longitudinal force is applied to the suspension links.

Anti-Dive / Anti-Lift / Anti-Squat

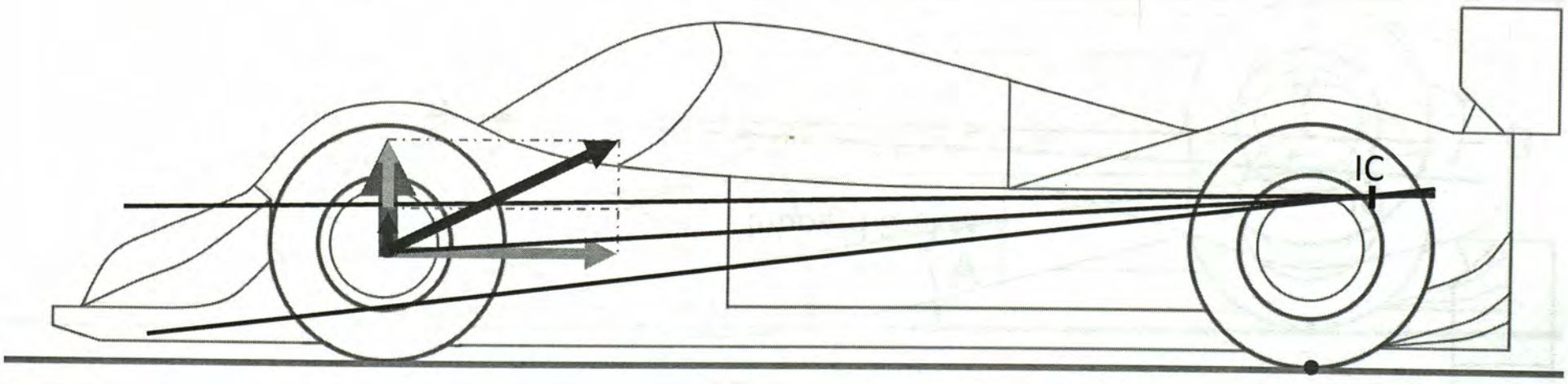


Inline Weight Transfer

Forces are applied at the front tire contact patch center if brakes are outboard (in the wheel/upright).

- S Geometric WT
- S Elastic WT
- Geometric + Elastic S WT

Anti-Dive / Anti-Lift / Anti-Squat



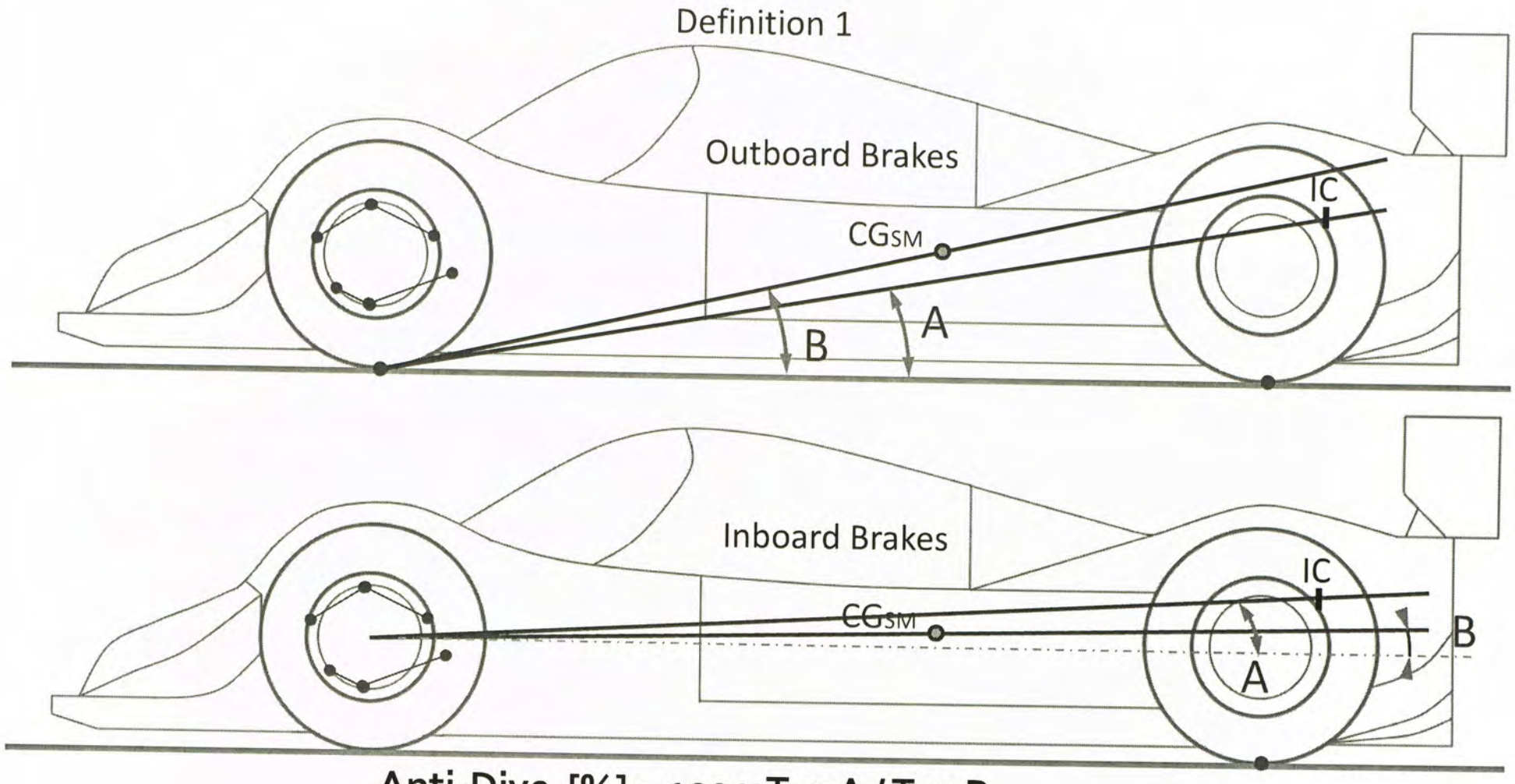
Inline Weight Transfer

Forces are applied at the front wheel center if brakes are inboard (in the chassis).

- S Geometric WT
- S Elastic WT
- Geometric + Elastic S WT

Anti-Dive

Definition 1



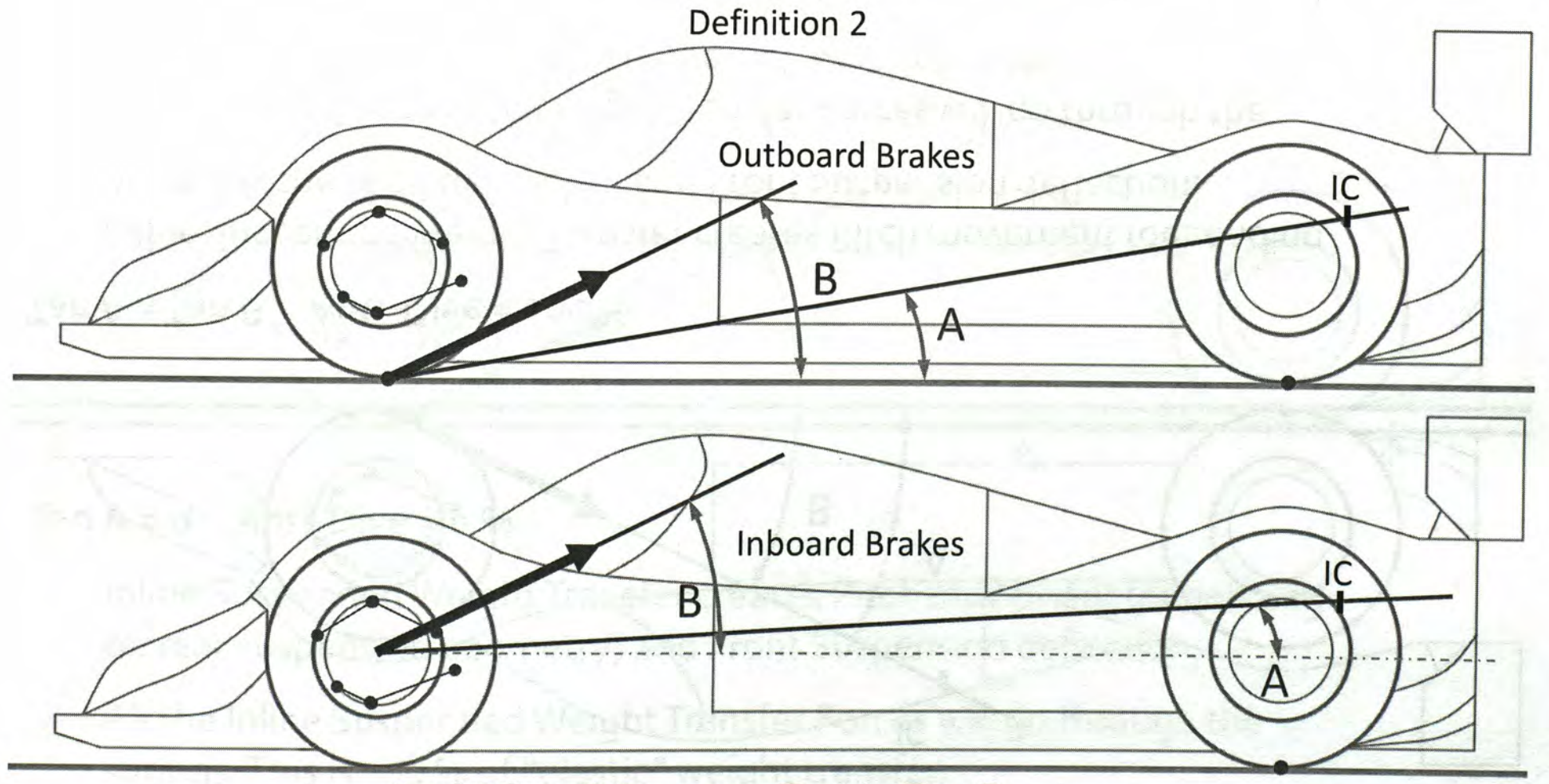
Anti-Dive [%] = 100 x Tan A / Tan B

Angle A = Fn (Instant suspension geometry)

Angle B = Fn (Sprung mass CG position)

Anti-Dive

Definition 2



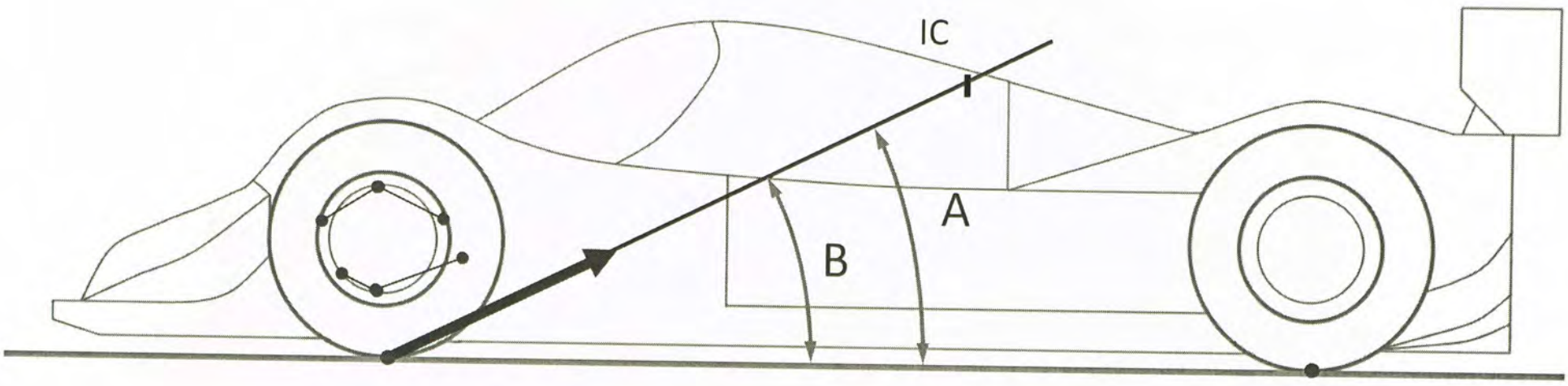
Anti-Dive [%] = 100 x Tan A / Tan B

Angle A = Fn (Instant suspension geometry)

Angle B = Fn (Inline Weight Transfer, Brake balance)

This definition will be use for the following examples

Anti-Dive



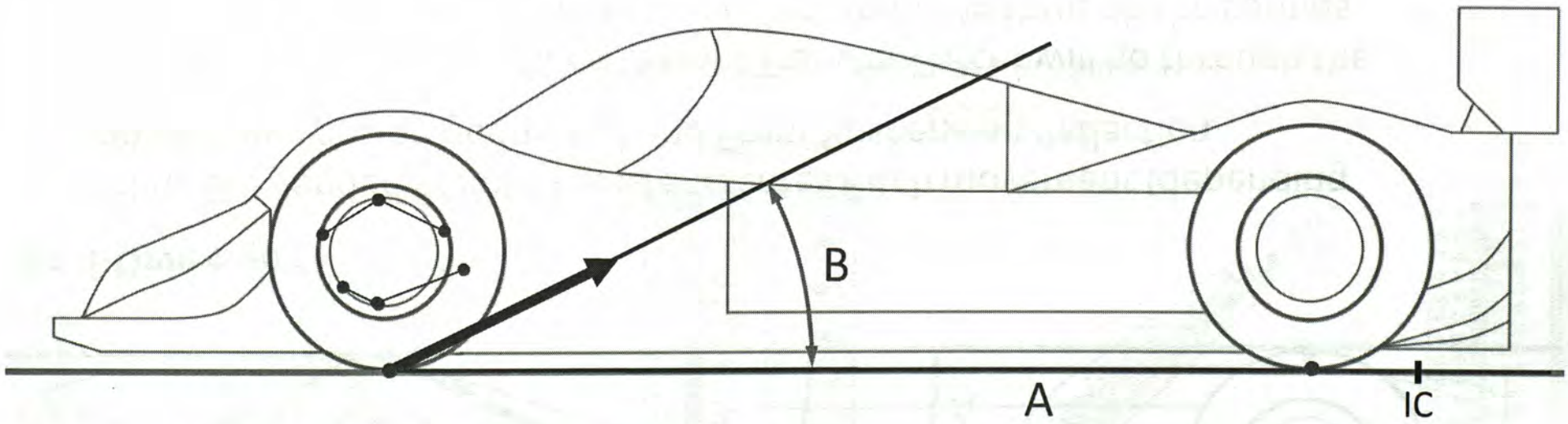
Tan A = Tan B Anti-Dive = 100 %

Inline Suspended Weight Transfer creates Pitch movement (depending on rear suspension geometry) and Front Suspension deflection.

All the Inline Suspended Weight Transfer Forces will go through the wishbones. This is 100 % of "geometric" weight transfer.

No forces are measured on the Front pushrod strain gauges.

Anti-Dive



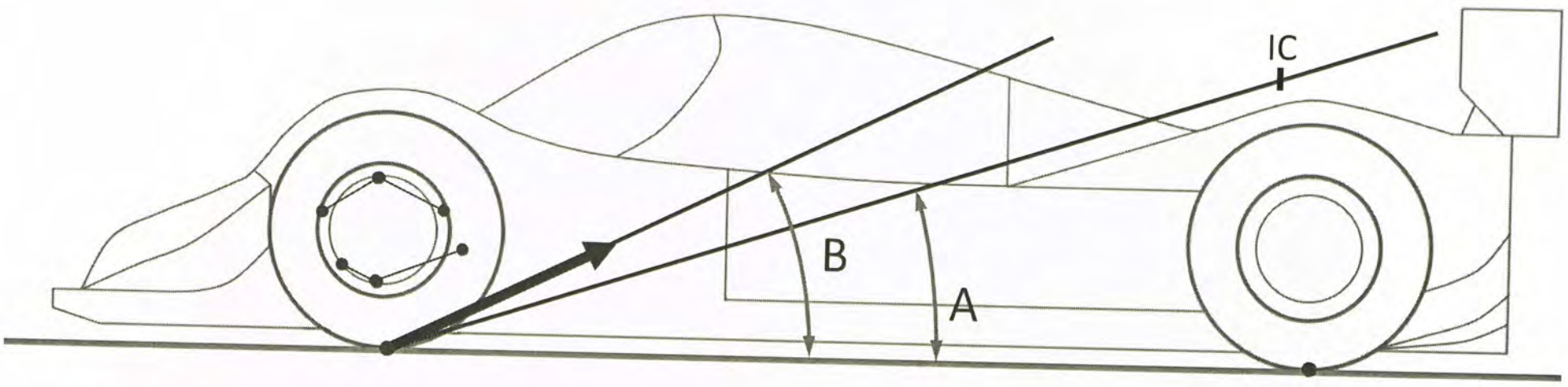
Tan A = 0 Anti-Dive = 0 %

Inline Suspended Weight Transfer creates Pitch movement (depending on rear suspension geometry) and Front Suspension deflection.

All the Inline Suspended Weight Transfer Forces will go through the springs. This is 100 % of "elastic" weight transfer.

All forces are measured on the Front pushrod strain gauges.

Anti-Dive



Anti-Dive = 30 %

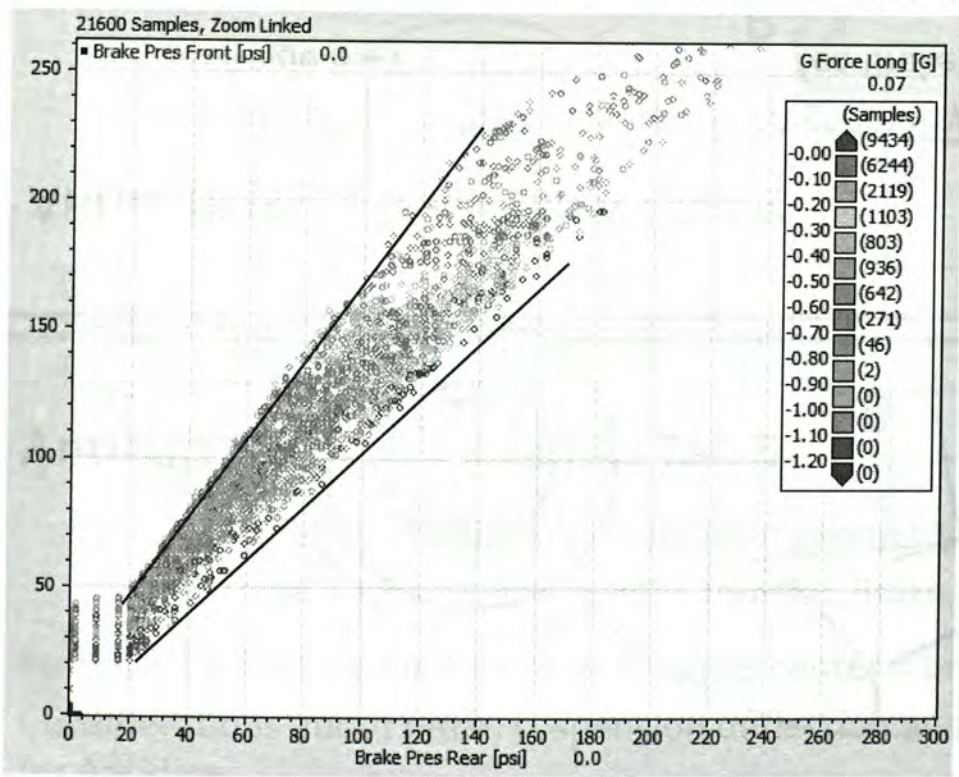
Inline Suspended Weight Transfer creates Pitch movement (depending on rear suspension geometry) and Front Suspension deflection.

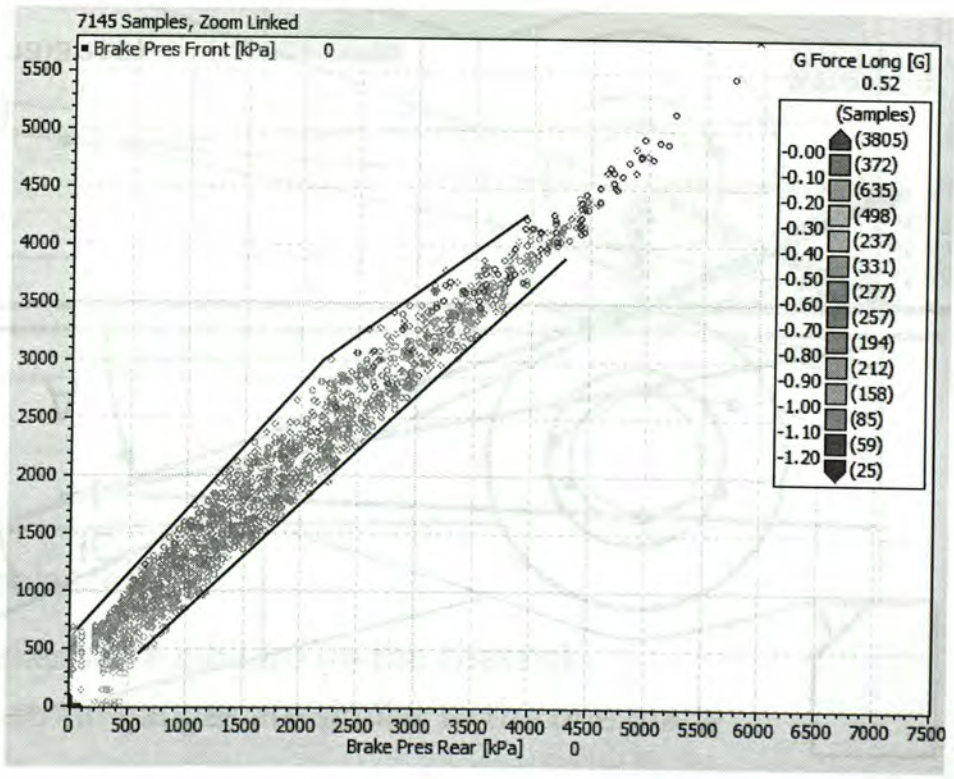
30 % of the Inline Suspended Weight Transfer Forces will go through the wishbones ("geometric" weight transfer) and 70 % through the springs ("elastic "weight transfer). Only 70 % of these forces will be measured on the Front pushrod strain gauges.

Brake Balance

Is your brake balance what you think it is?

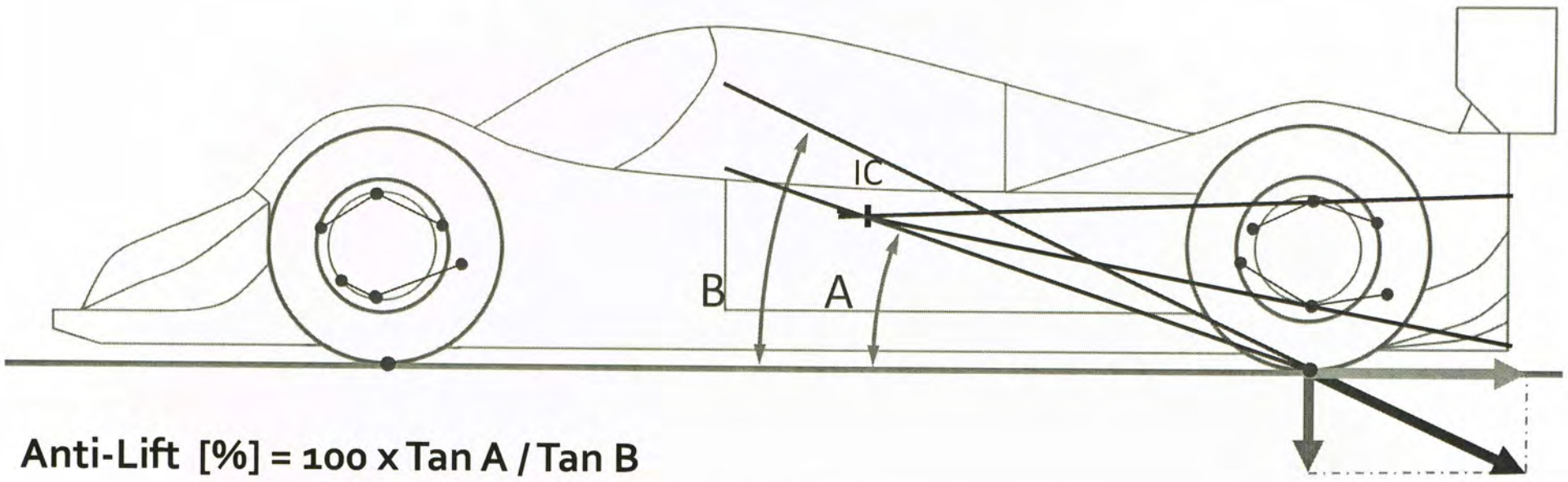
Front Brake Pressure vs. Rear Brake Pressure





Compliance in the brake assembly can create variance in brake balance.

Anti-Lift



Anti-Lift [%] = 100 x Tan A / Tan B

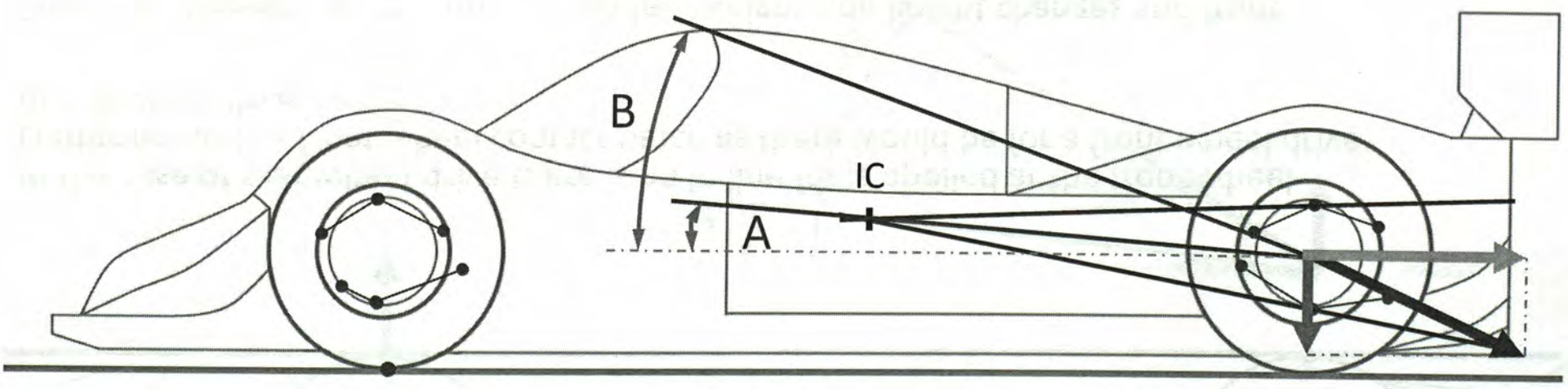
Angle A = Fn. (Instant suspension geometry)

Angle B = Fn. (Inline Weight Transfer, Brake balance)

Forces are applied at the rear tire contact patch center if brakes are outboard (in the wheel).

Considerations about pitch, suspension deflection and strain gauges are similar to the one made for Antidive.

Anti-Lift



Anti-Lift [%] = 100 x Tan A / Tan B

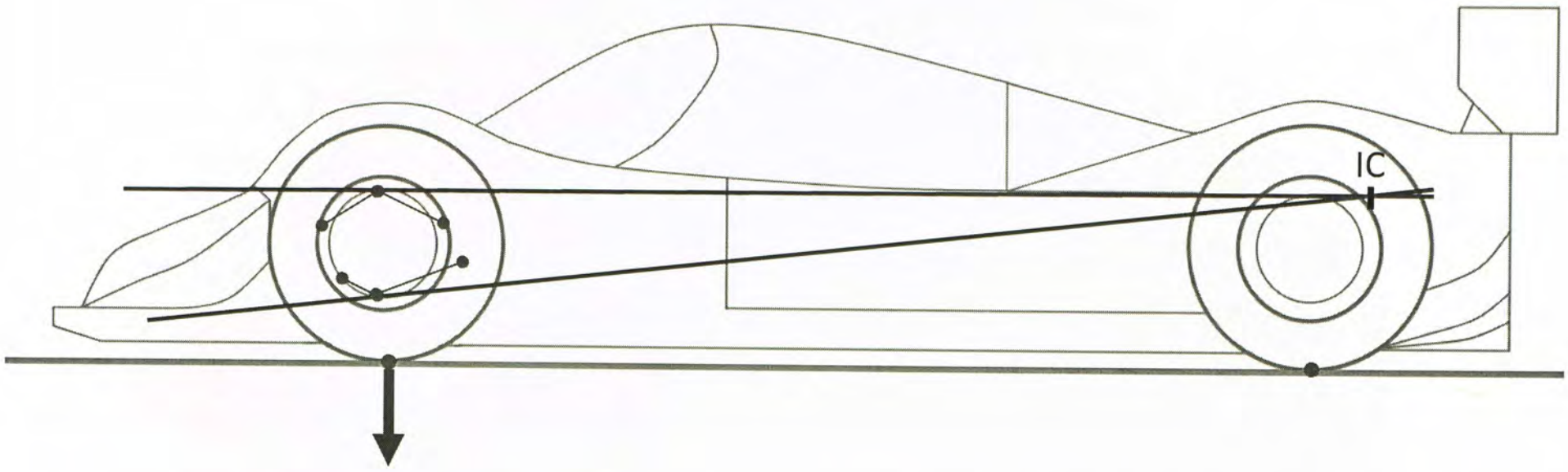
Angle A = Fn. (Instant suspension geometry)

Angle B = Fn. (Inline Weight Transfer, Brake balance)

Forces would are applied at the rear wheel center if brakes were inboard (in the chassis).

Considerations about pitch, suspension deflection and strain gauges are similar to the one made for Antidive.

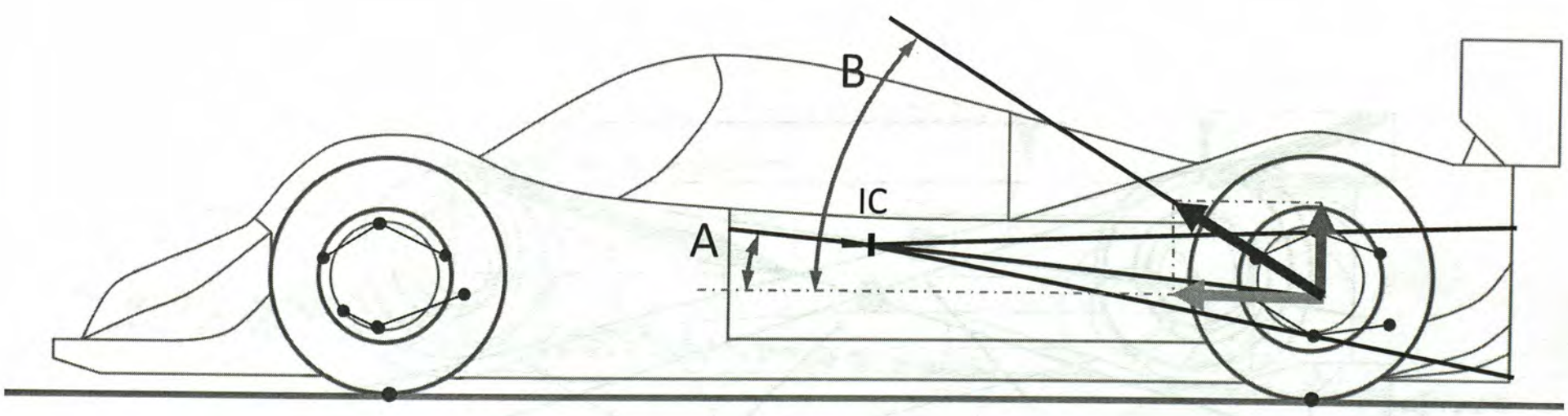
Anti-Lift [FWD & 4WD]



In the case of rear wheel drive there is no in line force applied at the front wheel center nor at the front wheel contact patch as there would be for a front wheel drive or a 4 wheel drive car.

Suspended weight pitch, front suspended weight ride height changes and front suspension deflection will be 100 % function of the inline weight transfer.

Anti-Squat



Anti-Squat [%]= 100 x Tan A / Tan B

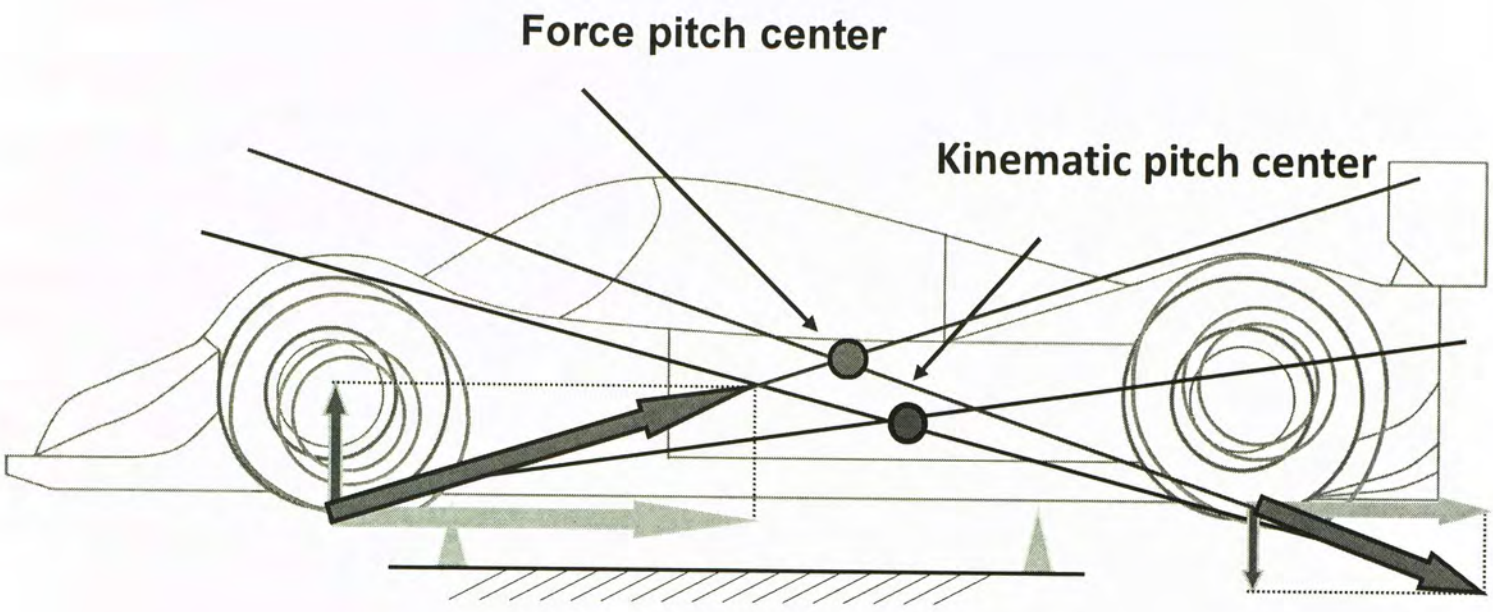
Angle A = Fn. (Instant suspension geometry)
Angle B = Fn. (Inline Weight Transfer, Brake balance)

Forces are applied at the wheel center in the case of an independent suspension.

Forces would be applied at the tire contact patch center in case of a solid axle.

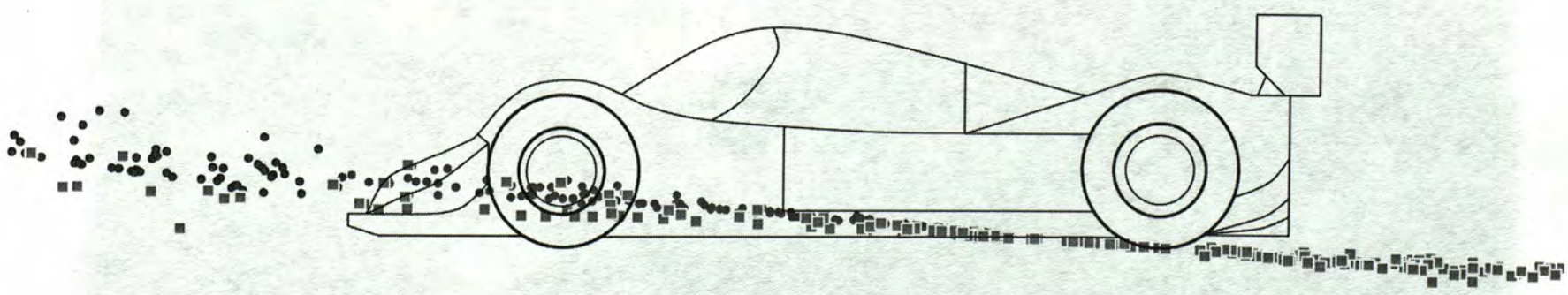
Considerations about pitch, suspension deflection and strain gauges are similar to the one made for Anti-Lift.

Kinematic and Force Pitch Centers



Kinematic and force pitch centers

In this example the car has already 1 deg of roll.

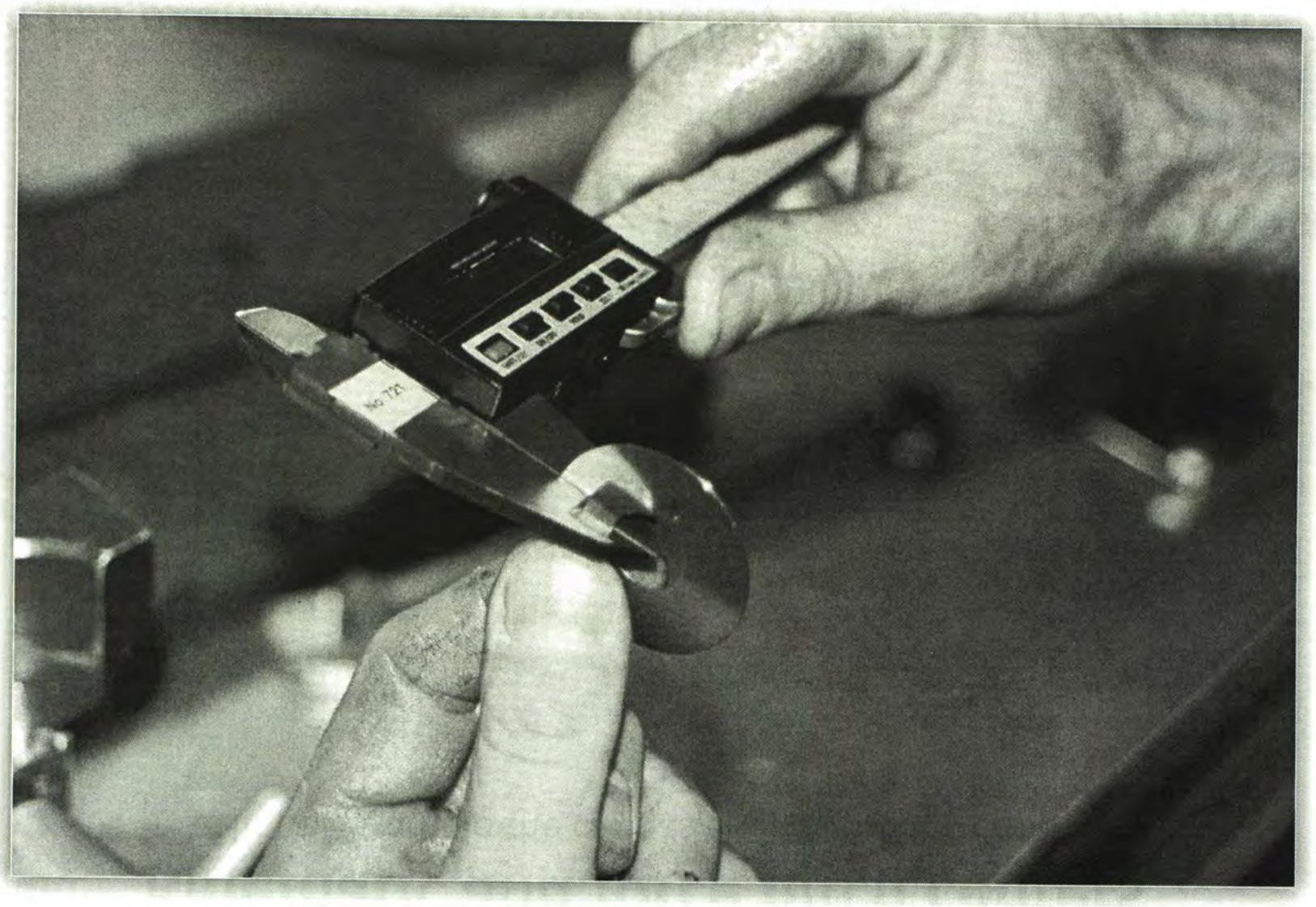


• Right Side

▪ Left Side

In this example the car has already1 degree of roll input on the K&C machine.

7. Damping



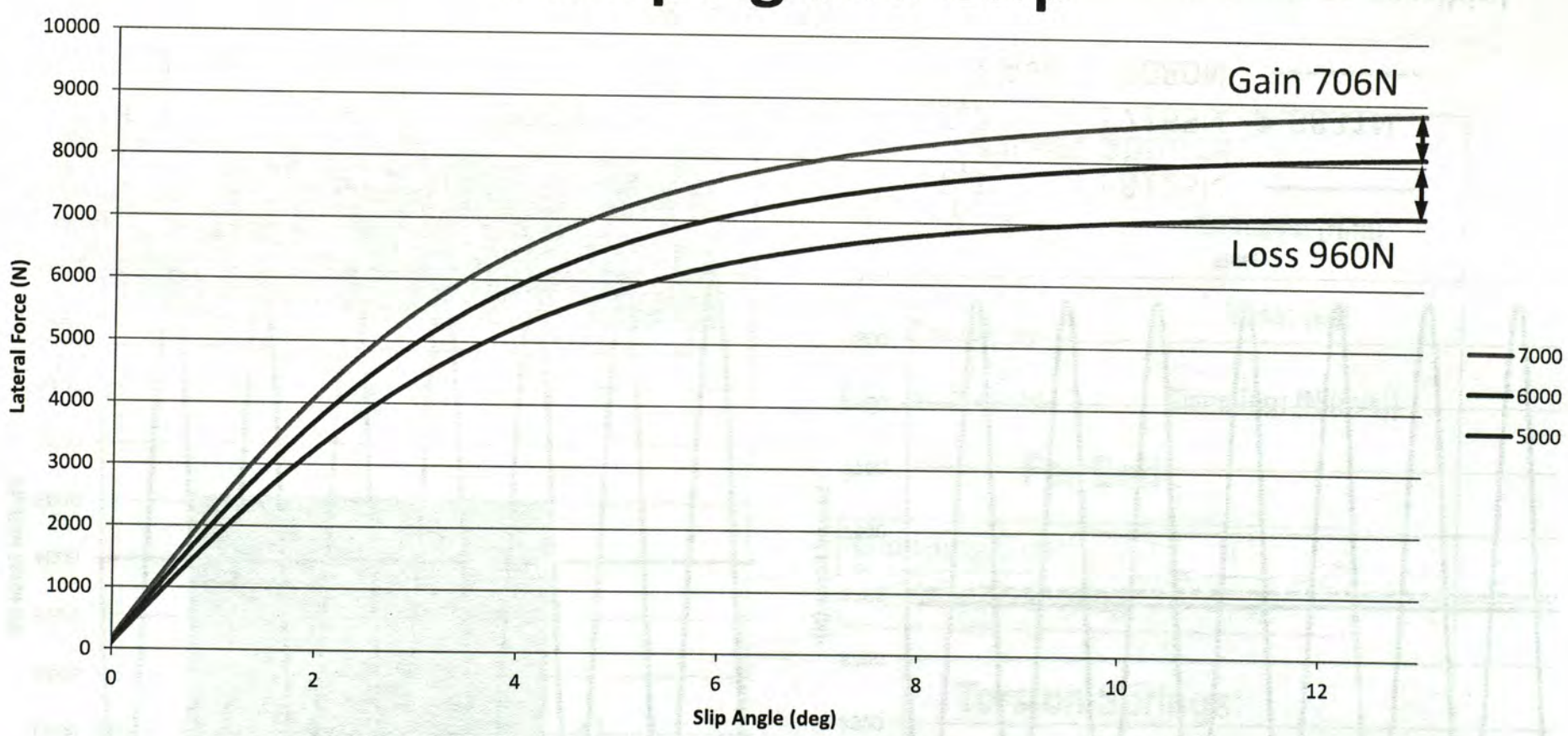
Purpose of Dampers

- Control transient balance
  - Dictates rate of weight transfer, and consequently weight transfer distribution, during transient maneuvers 2-10Hz
- Stabilize Aerodynamic platform
  - Reduce speed of chassis motions to reduce rate of change of aerodynamic forces
  - Force chassis to stay at lower ride heights 10-50
- Control tire contact patch consistency >100Hz
  - Change tire load variation to increase grip or tire temperature
  - Reduce relaxation length effect
- Control unsprung mass oscillations 10-50Hz
  - Eliminate resonant vibrations (i.e. wheel hop, shimmy, tramp)

Purpose of Dampers

- Weight transfer and chassis motion
  - Low frequency (< 5 Hz)
  - Primarily low damper speeds (< 25 – 50 mm/sec)
  - Stiffer dampers increase the rate of weight transfer and decrease the speed of the body motions
- Unsprung oscillations
  - Medium frequency (5 – 20 Hz)
  - Excitement of unsprung mass natural frequencies
- Tire contact patch consistency
  - High frequency (> 20 Hz)
  - Primarily high damper speeds (> 100 – 150 mm/sec)
  - Reduce tire relaxation length effect

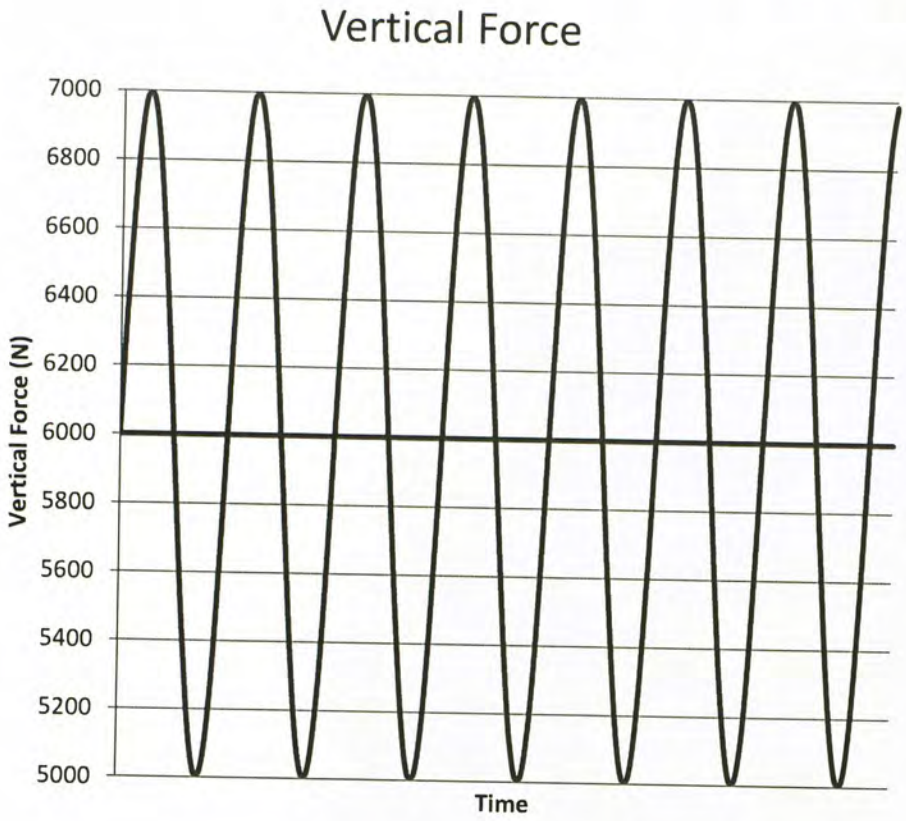
Damping and Grip

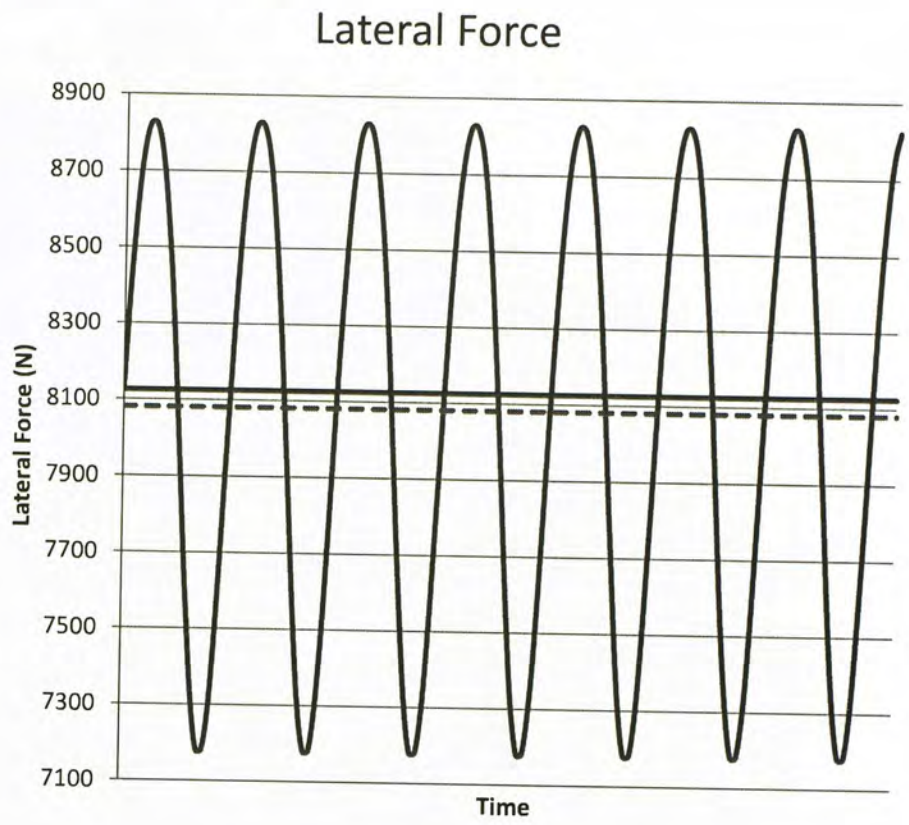


No vertical force variation	F_Z1	: 6000N	F_Y1	: 8125N
With vertical force variation	F_Z2	: 6000N +/- 1000N	F_Y2	: 7165N ➔ 8831N
	F_Z2 Avg	: 6000N	F_Y2 Avg	: 8080N

We suppose slip angle remains constant
In reality the slip angle is changing, creating even more variation

Damping and Grip





F_Z1 : 6000N

F_Z2 : 6000N +/- 1000N

F_Z2 Avg : 6000N

F_Y1 : 8125N ————

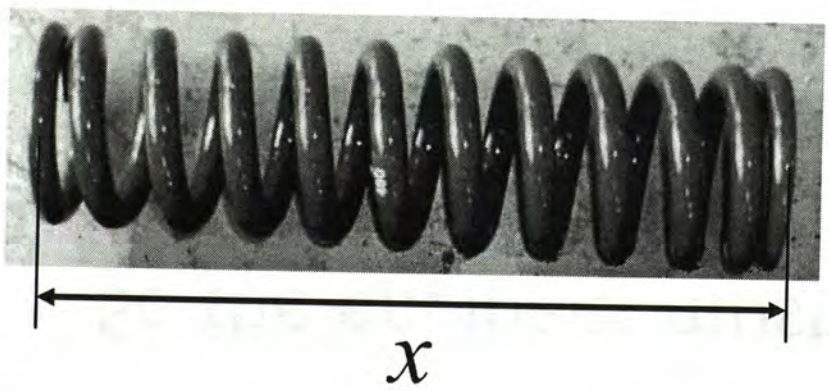
F_Y2 : 7165N ➔ 8831N

F_Y2 Avg : 8080N ---------

This is why we need a damping that minimize the tire load variation as much as possible!

Modes of Vibration

Basic Modes



Linear Springs:

$$F = k \cdot x$$ **Stiffness: [N/m]**

$$f = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}}$$ **Mass: [kg]**

$$C_{Crit} = 2.\sqrt{k.m}$$ **Damping: [N/(m/s)]**

For Both:

$$\frac{\text{Damping Coefficient}}{\text{Critical Damping}} = \frac{C}{C_{Crit}} = \zeta$$

Torsion Springs:

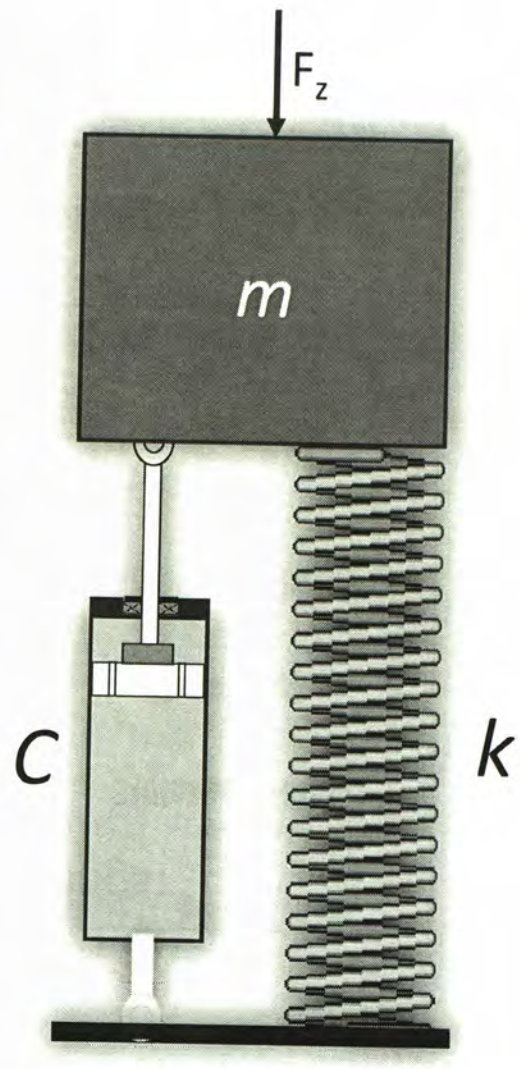
$$M = k_t \cdot \theta$$ **Stiffness: [N.m/deg]**

$$f = \frac{1}{2\pi} \times \sqrt{\frac{k_t}{I}}$$ **Inertia: [kg.m2]**

$$C_{Crit} = 2.\sqrt{k_t.I}$$ **Damping: [Nm/(deg/s)]**

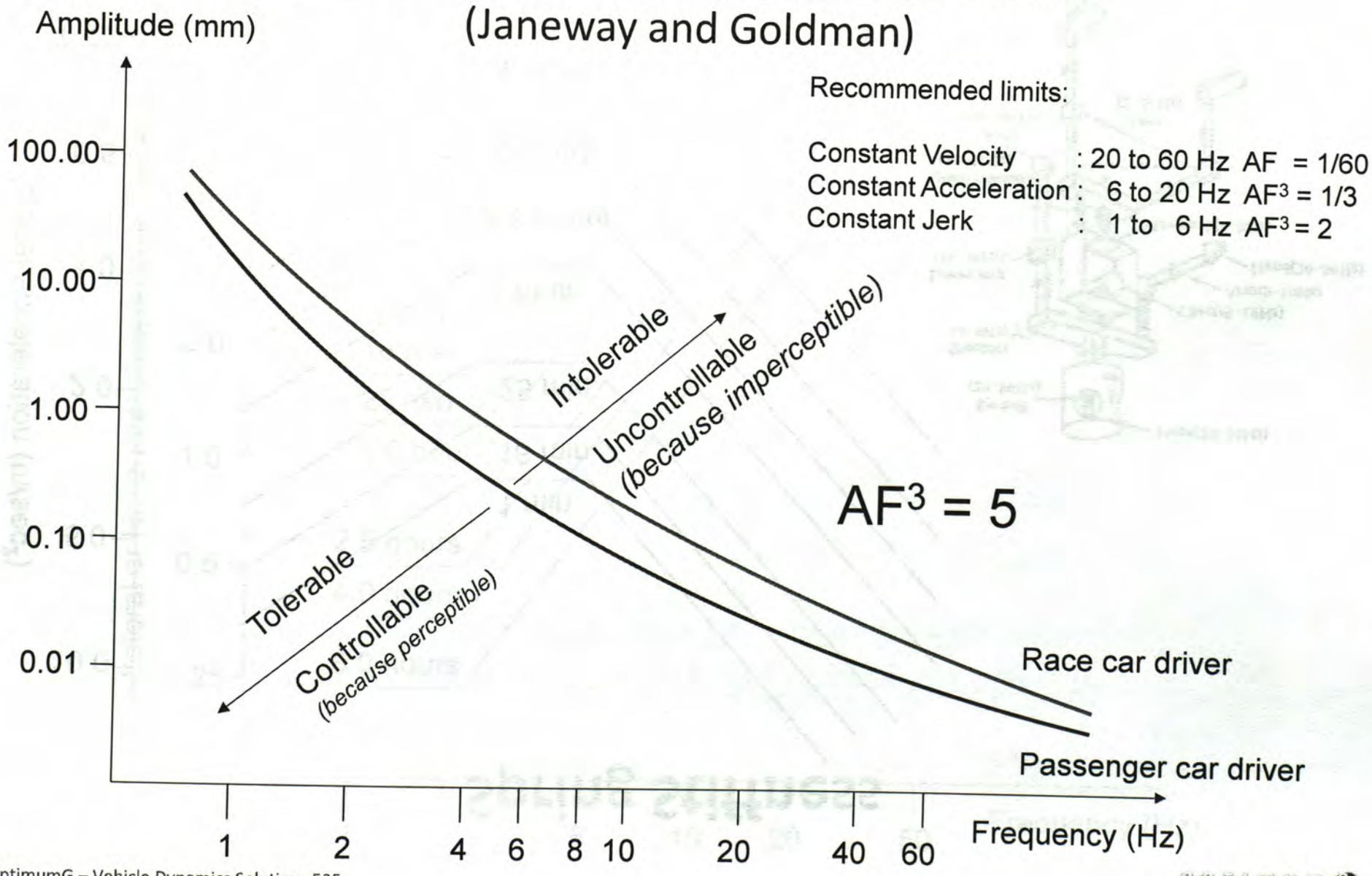
Spring and Dampers

- How to determine an initial spring and damper setup
- On race cars the mass should already be at its minimum value
- So the engineer must maximize the performance with the springs, dampers and anti-roll bars

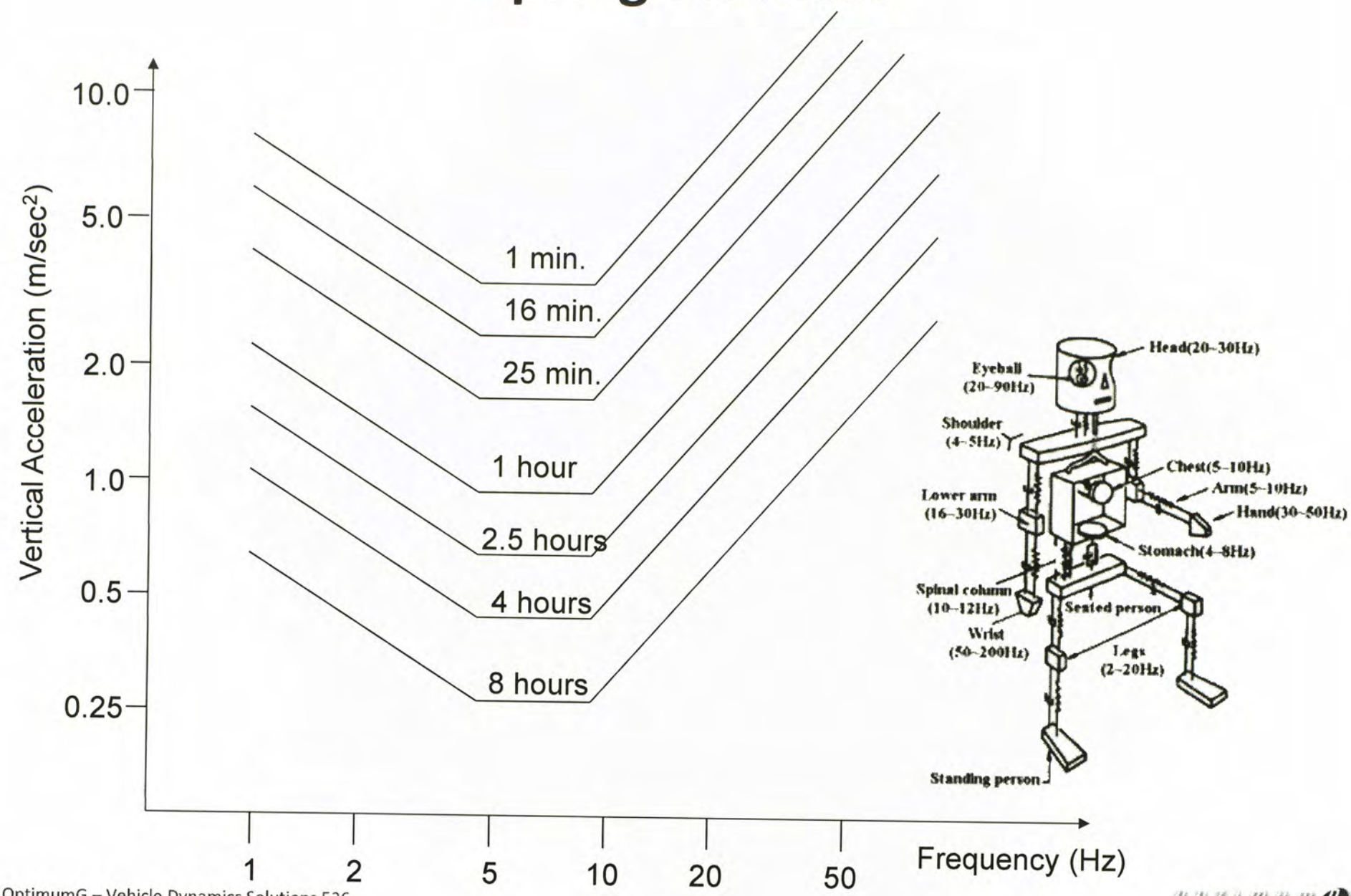


Spring Stiffness

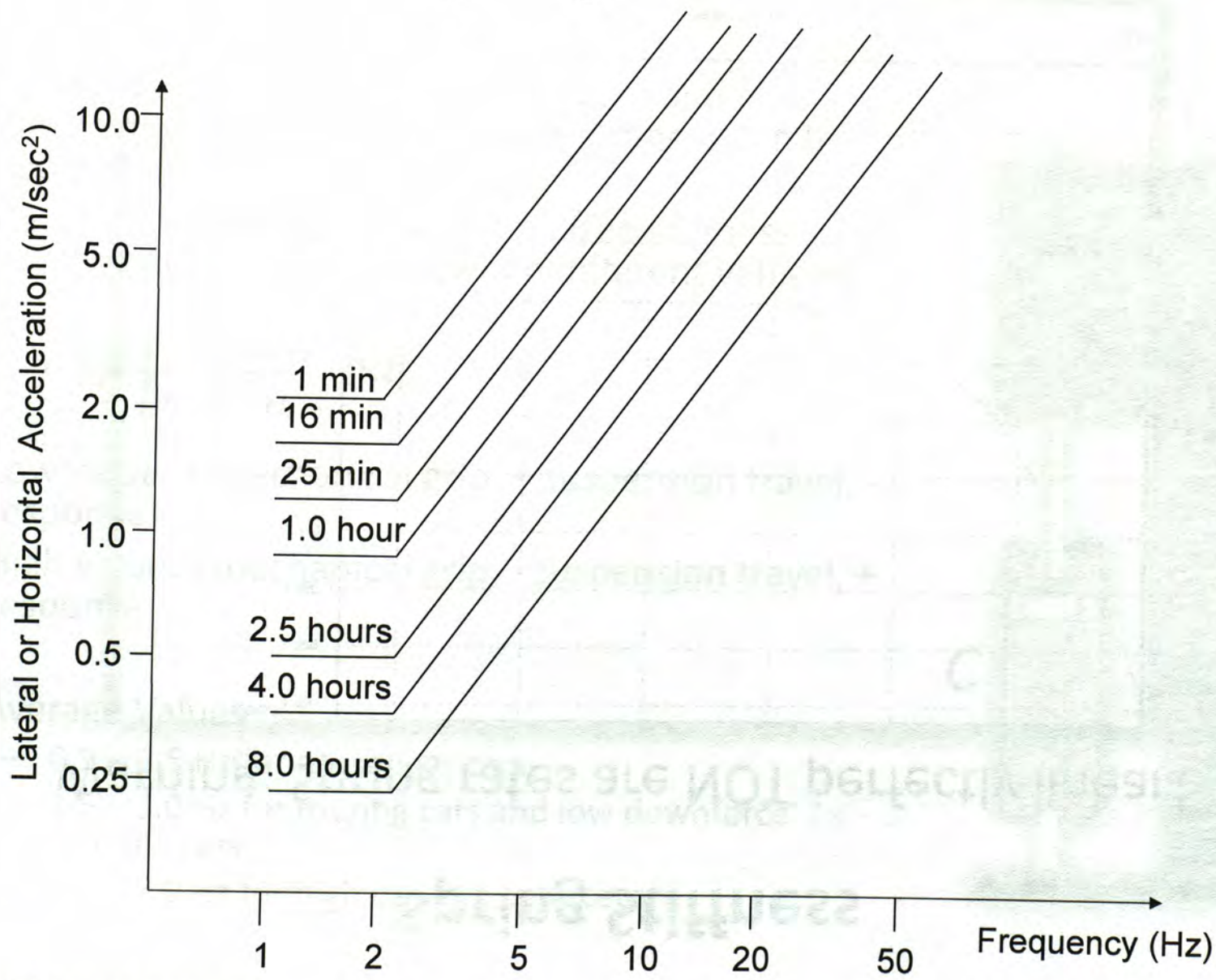
Vertical vibration limit for passenger comfort (Janeway and Goldman)



Spring Stiffness

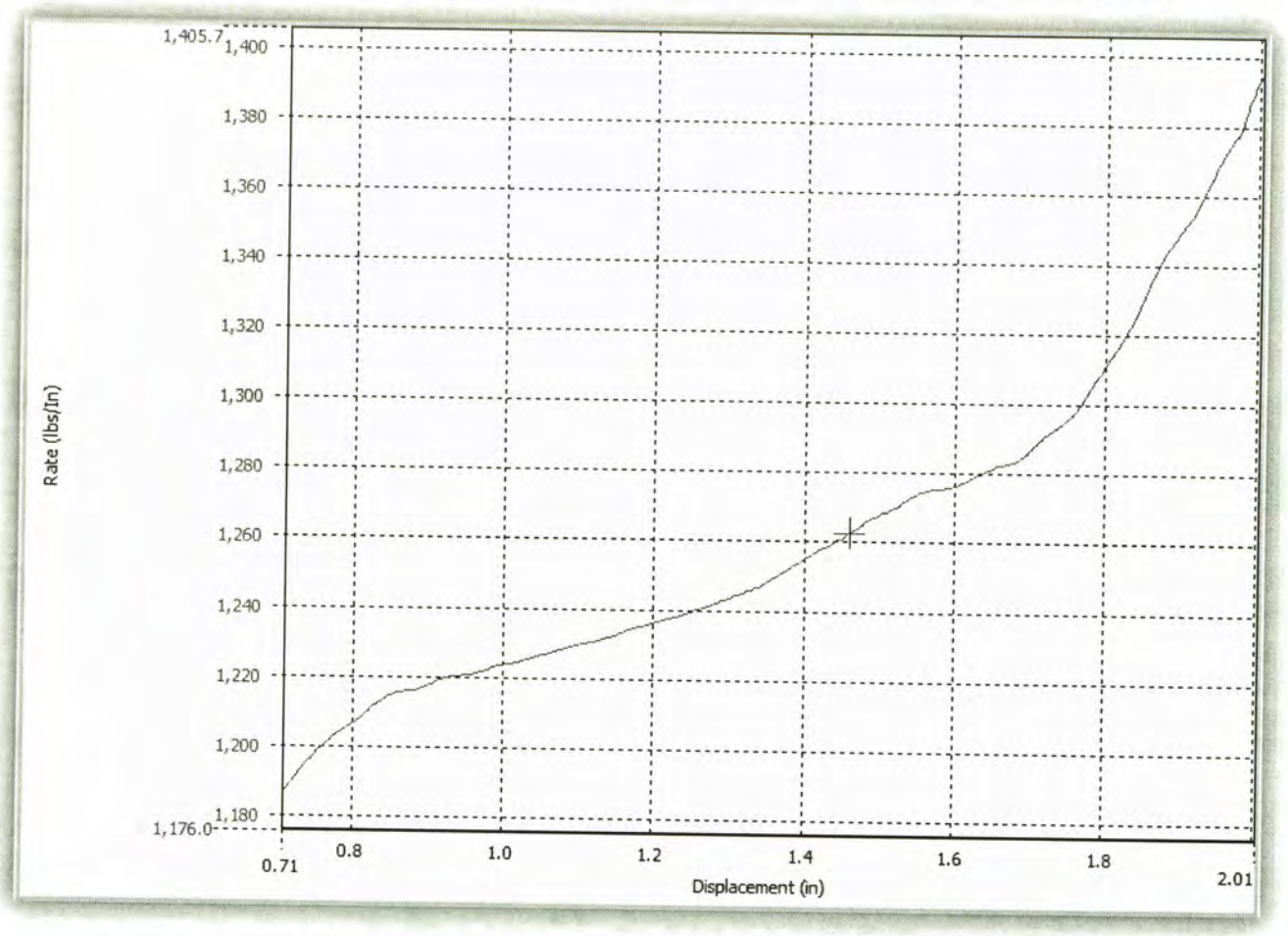


Spring Stiffness



Spring Stiffness

Warning: Spring rates are NOT perfectly linear!



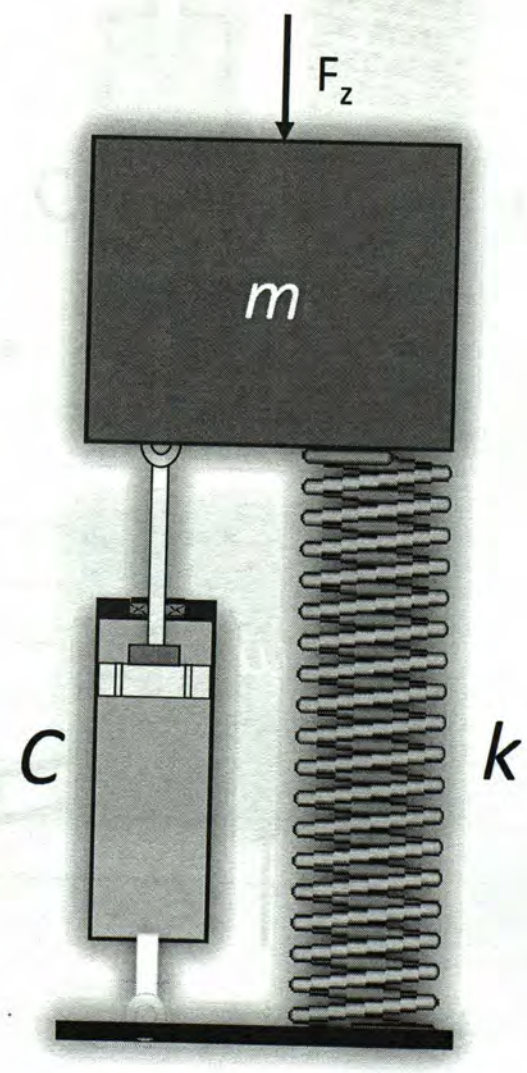
Actual rates can be determined from a spring / damper dyno

Spring Stiffness

- The first step is to decide on the front and rear natural frequencies so the spring stiffness can be determined
- Natural frequencies are a normalized stiffness so they can be compared between different vehicles

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \quad \text{[Hz]}$$

- Low value: + mechanical grip, + suspension travel, - response
- High value: - mechanical grip, - suspension travel, + response
- Average Values:
  - 0.9 – 1.5 Hz for passenger cars
  - 1.5 – 3.0 Hz for touring cars and low downforce formula cars 2,5 - 3
  - 3.0 – 7.0 Hz for high downforce racecars

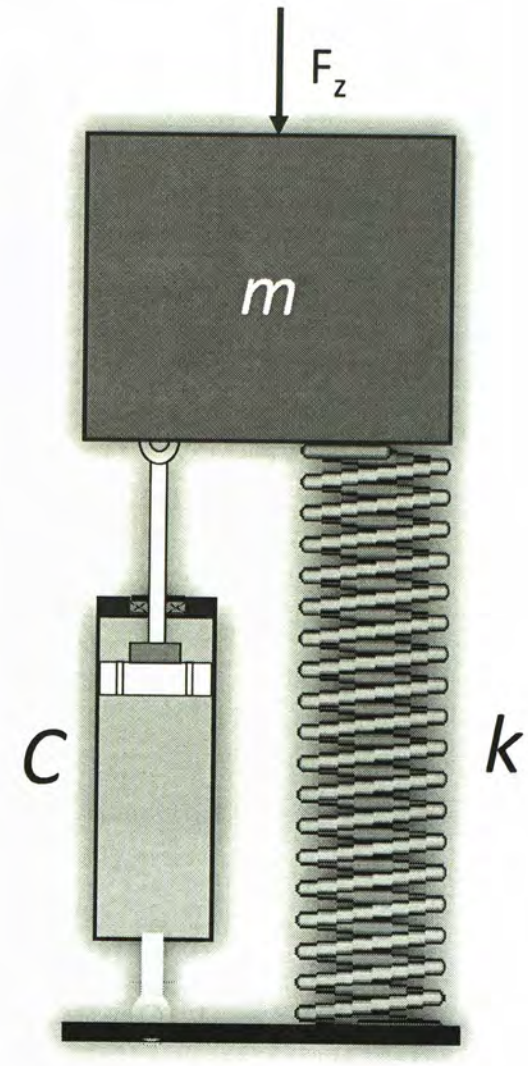


Spring Stiffness

The required frequency can be determined from the sprung mass, the motion ratio, and the desired spring rates

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \qquad k_s = K \times MR^2$$

$$k_S = 4\pi^2 f^2 m_{sm} MR^2$$



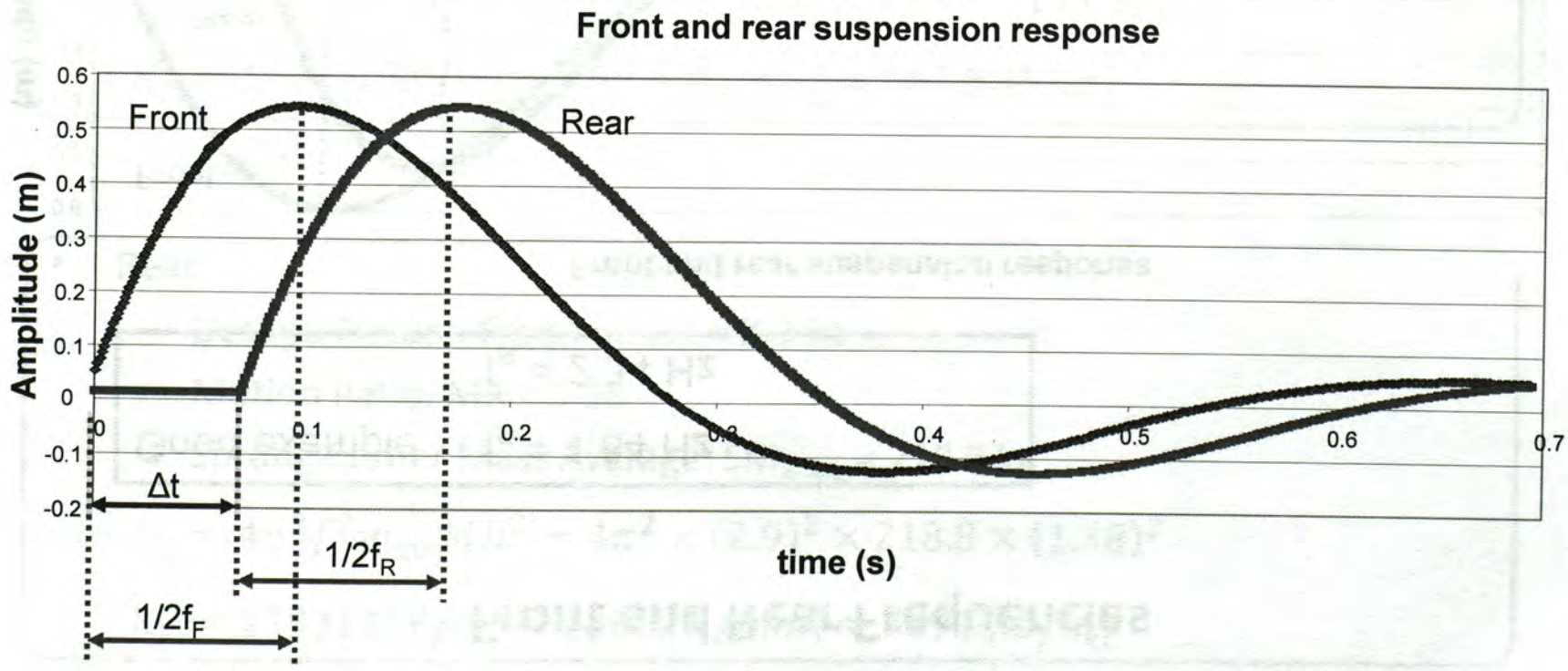
K	Wheel rate (N/m)
k_s	Spring rate (N/m)
f	Ride frequency (Hz)
m_{sm}	Sprung corner mass (kg)
MR	Motion ratio (wheel travel/spring travel)

Modes of Vibration

Front and Rear Frequencies

Bad example : $f_F = 2.05$ Hz

$f_R = 2.05$ Hz



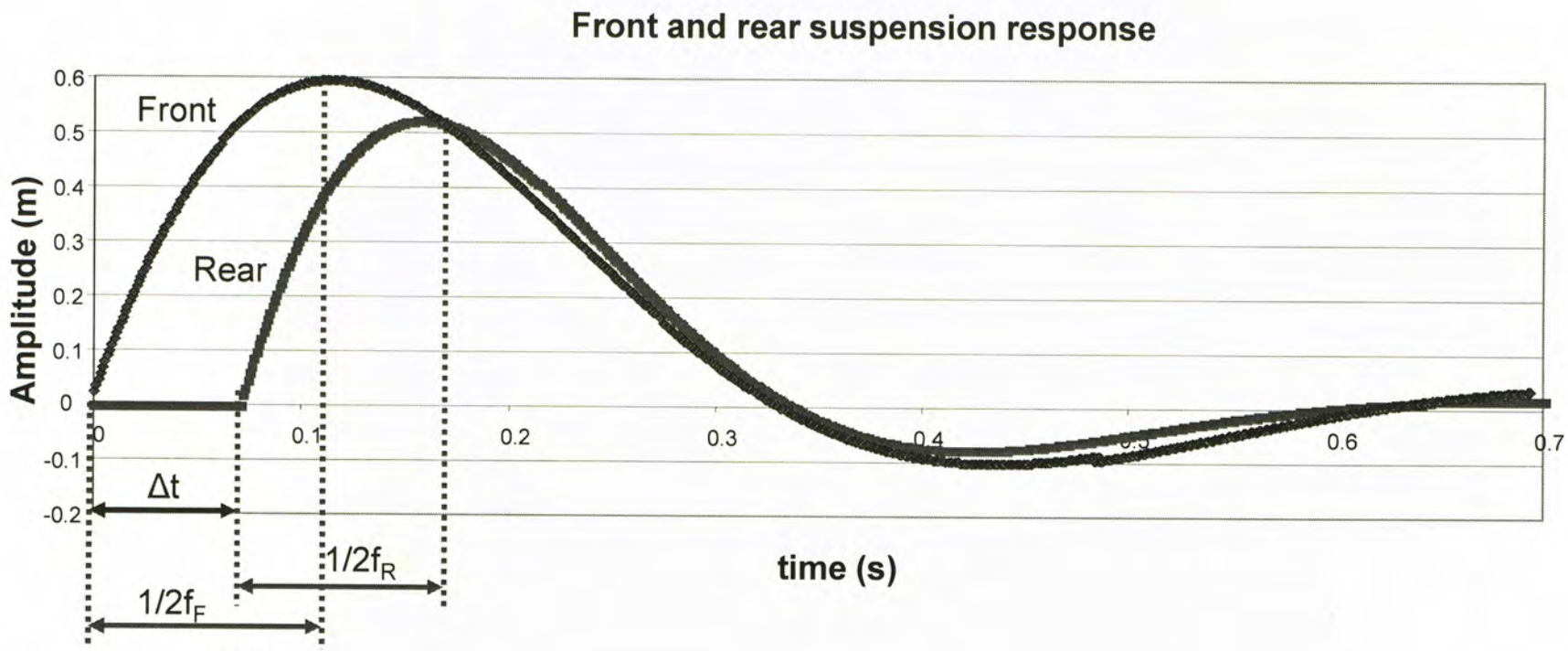
$$\Delta t = \frac{WB}{v}$$

Modes of Vibration

Front and Rear Frequencies

Good example : $f_F = 1.84$ Hz

$f_R = 2.34$ Hz



$$\Delta t = \frac{WB}{v}$$

Example

Spring Stiffness

- **Front**
  - Desired Natural Frequency: f = 4.5 Hz
  - Motion Ratio: MR = 1.03
  - Sprung Corner Mass Average: $SM_{F\text{-}Avg}$ = 199.7 kg

$$K_F = 4\pi^2 f^2 m_{sm} MR^2 = 4\pi^2 \times (4.5)^2 \times 199.7 \times (1.03)^2$$

$$K_F = 169370[N/m] \approx 169.4[N/mm] \approx 967.3[lb/in]$$

- **Rear**
  - Desired Natural Frequency: f = 2.9 Hz
  - Motion Ratio: MR = 1.38
  - Sprung Corner Mass Average: $SM_{R\text{-}Avg}$ = 218.8 kg

$$K_F = 4\pi^2 f^2 m_{sm} MR^2 = 4\pi^2 \times (2.9)^2 \times 218.8 \times (1.38)^2$$

$$K_F = 138313[N/m] \approx 138.3[N/mm] \approx 789.8[lb/in]$$

Spring Stiffness

Example A

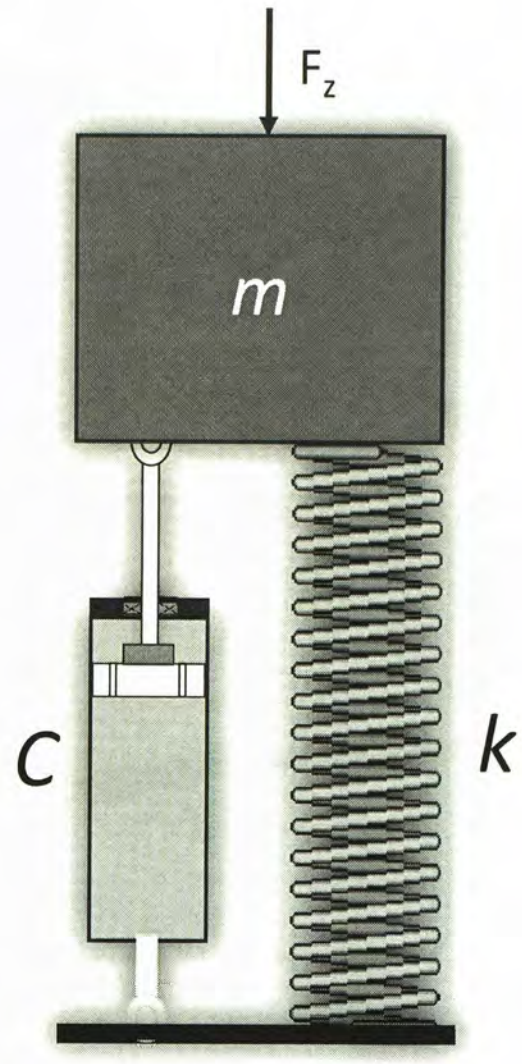
The required frequencies can be determined from the sprung mass, the motion ratio, and the desired spring rates

$$f_{Front} = \frac{1}{2\pi} \times \sqrt{\frac{1}{SM} \times \frac{K_s}{MR^2}}$$

$$\frac{1}{2\pi} \times \sqrt{\frac{1}{199.7} \times \frac{169370}{1.03^2}} = 4.5Hz$$

$$f_{Front} = 4.5Hz$$

k_s Spring rate (N/m)
f Ride frequency (Hz)
SM Average sprung corner mass (kg)
MR Motion ratio (wheel travel/spring travel)



Spring Stiffness

Example A

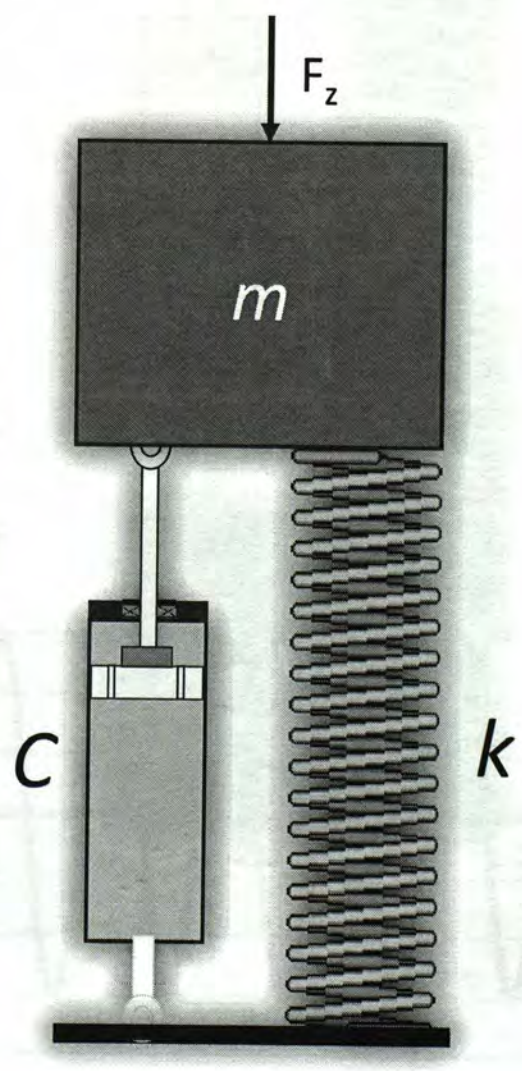
The required frequencies can be determined from the sprung mass, the motion ratio, and the desired spring rates

$$f_{Rear} = \frac{1}{2\pi} \times \sqrt{\frac{1}{SM} \times \frac{K_s}{MR^2}}$$

$$\frac{1}{2\pi} \times \sqrt{\frac{1}{218.8} \times \frac{138313}{1.38^2}} = 2.9Hz$$

$$f_{Rear} = 2.9Hz$$

k_s Spring rate (N/m)
f Ride frequency (Hz)
SM Average sprung corner mass (kg)
MR Motion ratio (wheel travel/spring travel)



Damping

- Critical Damping:

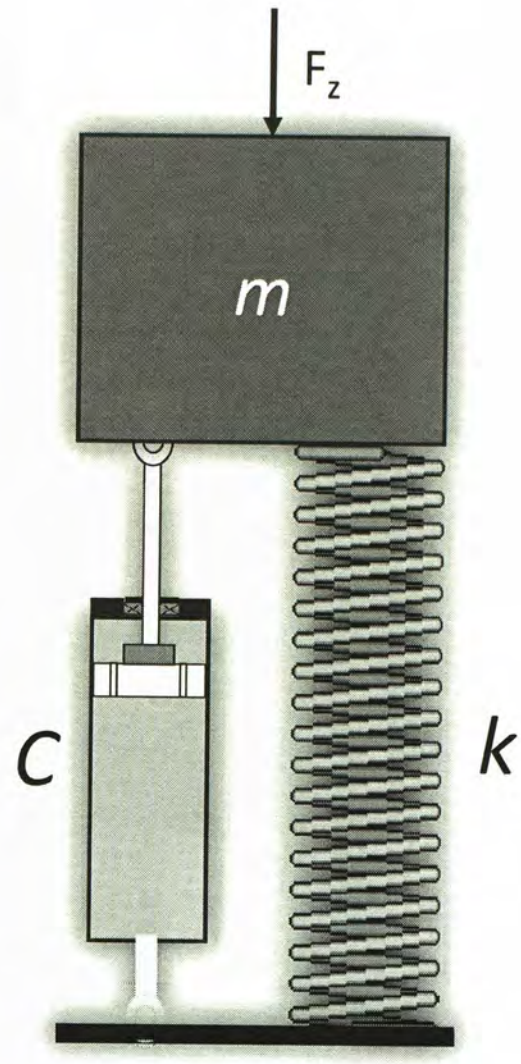
$$C_{crit} = 2\sqrt{Km}$$

- Damping Ratio:

$$\zeta = \frac{C}{C_{crit}} = \frac{C}{2\sqrt{Km}}$$

Motion Ratio

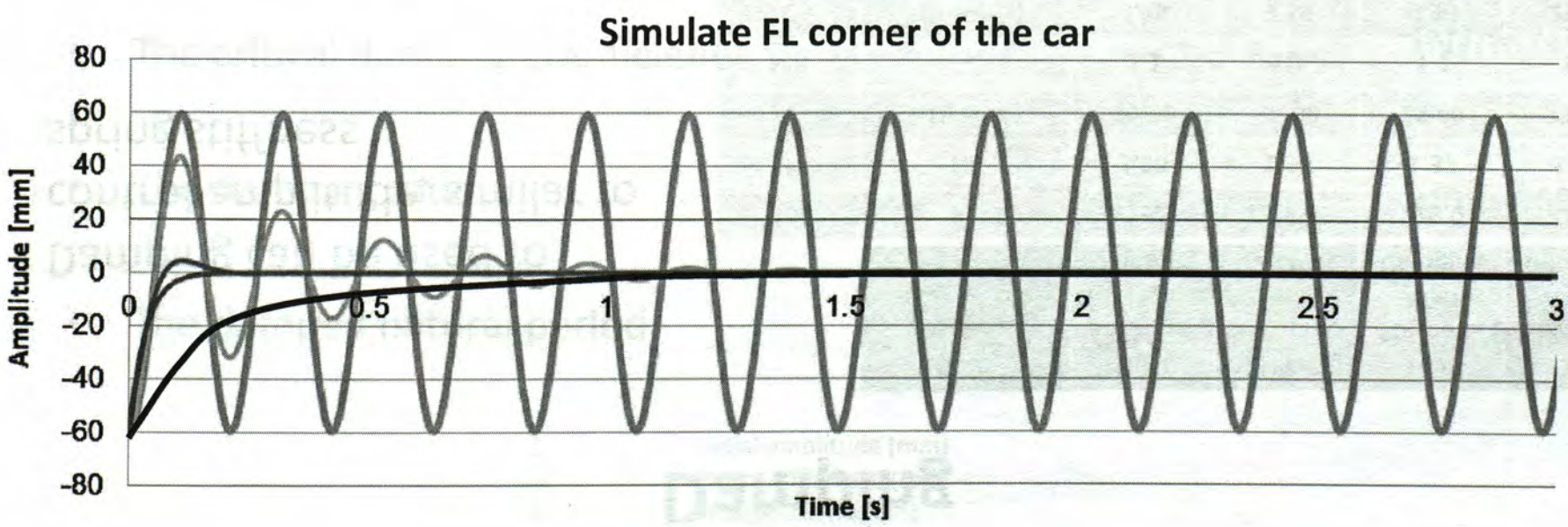
- Average Values:
  - 0.25 - 0.3 – Passenger cars
  - 0.65 - 0.7 – Most race cars
  - > 1 – Some high downforce race cars



Damping

	Time to equilibrium (10mm window)
• No damping	Infinite
• ζ = 0.1 Underdamped	0.768s
• ζ = 0.7 Underdamped	0.060s
• ζ = 1.0 Critically damped	0.084s
• ζ = 1.8 Overdamped	1.532s

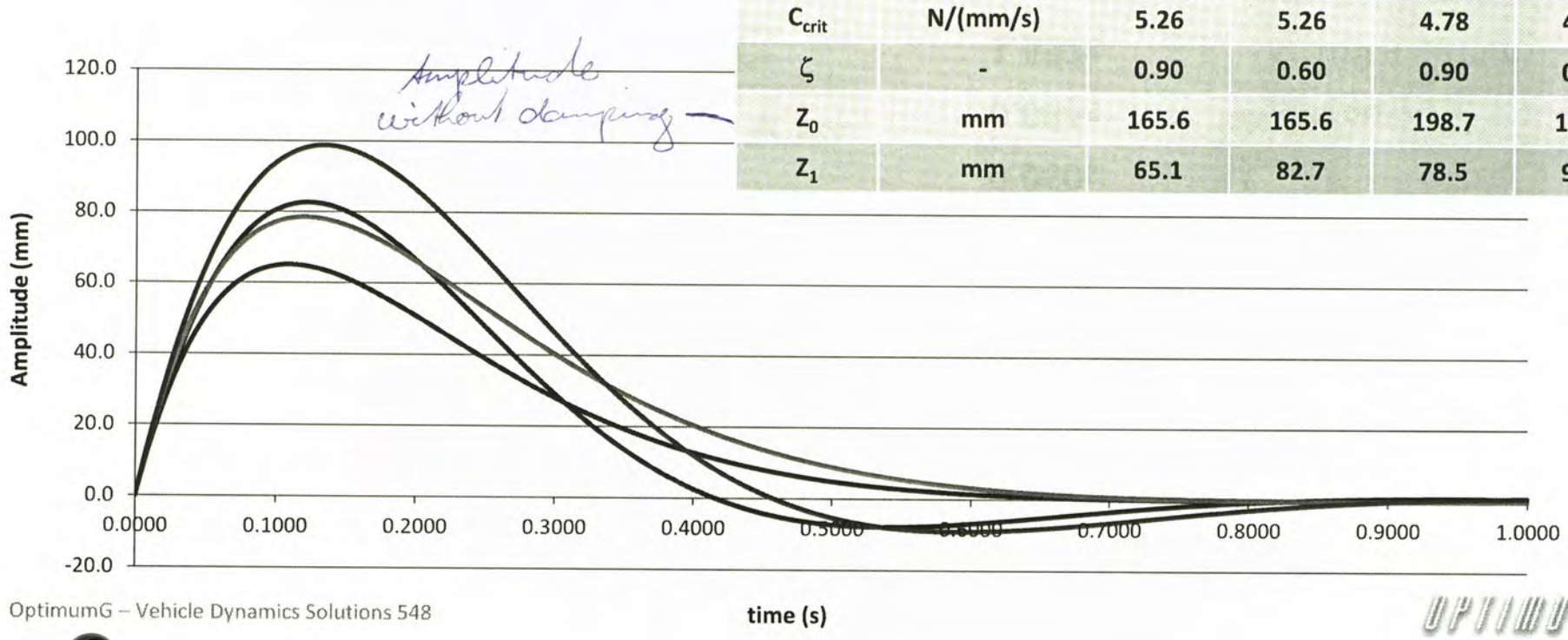
m: 199.7 kg
K: 169.4 N/mm



Damping

Damping can be used to control amplitude, similar to spring stiffness

	Units	High Stiffness, High Damping	High Stiffness, Low Damping	Low Stiffness, High Damping	Low Stiffness, Low Damping
m	kg	279.0	279.0	279.0	279.0
f	Hz	1.50	1.50	1.37	1.37
k_s	N/mm	30.00	30.00	25.00	25.00
MR	-	1.1	1.1	1.1	1.1
C	N/(mm/s)	4.75	3.15	4.30	2900
C_{crit}	N/(mm/s)	5.26	5.26	4.78	4.78
ζ	-	0.90	0.60	0.90	0.60
Z_0	mm	165.6	165.6	198.7	198.7
Z_1	mm	65.1	82.7	78.5	98.8



Front and Rear Damping

The suspensions response can be represented as follows for $0 < \zeta < 1$:

$$z = Z_0 e^{-\zeta\omega n t}\sin(\omega_D t + \varphi)$$

with:

- The undamped (C=0) natural period $\omega_N = \sqrt{\frac{K}{m}}$ [rad/s]
- The critical damping coefficient $C_{crit} = 2\sqrt{Km}$ [N/(m/s)]
- The damping ratio $\zeta = \frac{C}{C_{crit}} = \frac{C}{2\sqrt{Km}}$ [unit less]
- The damped natural period $\omega_D = \omega_N\sqrt{1-\zeta^2}$ [rad/s]

Z_0	Initial amplitude (mm)
φ	Phase difference (rad)
K	Wheel rate (N/mm)
m	Mass (kg)
C	Damping coefficient (N/(m/s))

Example

Front and Rear Damping

- **Front**
  - Desired Damping Ratio: $\zeta = 0.7$ (@ the wheel)

$$\zeta = \frac{C}{C_{Crit}} = \frac{C}{2\sqrt{Km}} = \frac{C}{2\sqrt{Wheel\ Rate \times SM_{F-Avg}}}$$

$$C = \zeta \times C_{Crit} = 0.7 \times 2\sqrt{159676 \times 199.7} = 7905\ [N/m/s]\ (@\ the\ wheel)$$

$$C_{Damper} = C_{Wheel} \times MR = 7905 \times 1.03 = 8142\ [N/m/s]$$

- **Rear**
  - Desired Damping Ratio: $\zeta = 0.7$ (@ the wheel)

$$\zeta = \frac{C}{C_{Crit}} = \frac{C}{2\sqrt{Km}} = \frac{C}{2\sqrt{Wheel\ Rate \times SM_{R-Avg}}}$$

$$C = \zeta \times C_{Crit} = 0.7 \times 2\sqrt{72621 \times 218.8} = 5580\ [N/m/s]\ (@\ the\ wheel)$$

$$C_{Damper} = C_{Wheel} \times MR = 5580 \times 1.38 = 7700\ [N/m/s]$$

Front and Rear Damping

Way to determinate the initial amplitudes (Z_0) of the front and rear rides assuming a vertical acceleration of 1.5G:

$$Z_0 = \frac{SM \times VerticalG \times 9.81}{K}$$

On the front (assuming the car is symmetric):

$$Z_{0\,F} = \frac{SM_{F-Avg} \times VerticalG \times 9.81}{K_F} = \frac{199.7 \times 1.5 \times 9.81}{\frac{169.4}{1.03^2}} = 17.84mm$$

On the rear (assuming the car is symmetric):

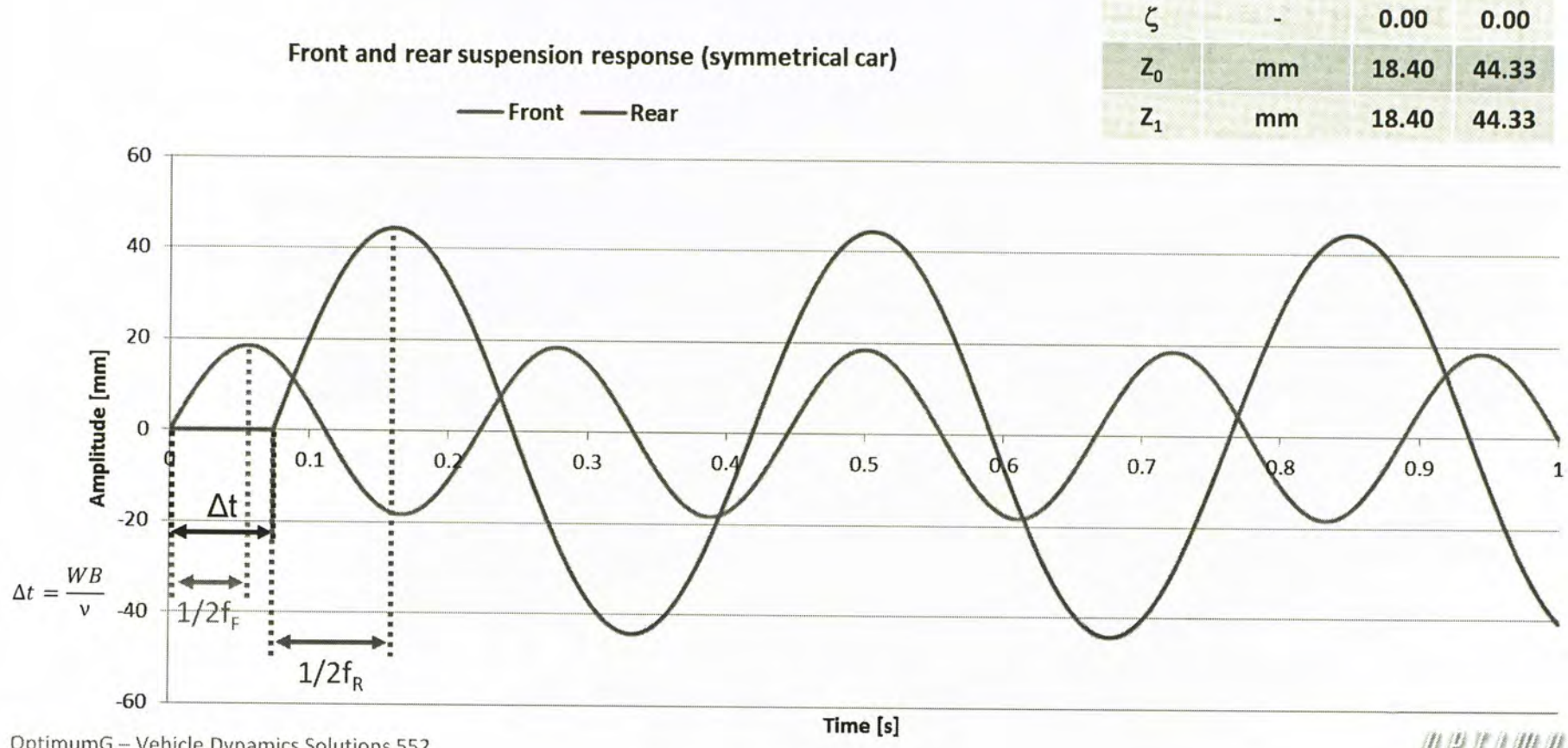
$$Z_{0\,R} = \frac{SM_{R-Avg} \times VerticalG \times 9.81}{K_R} = \frac{218.8 \times 1.5 \times 9.81}{\frac{138.3}{1.38^2}} = 44.3mm$$

Front and Rear Damping

Example

No damping

	Units	Front	Rear
f	Hz	4.50	2.90
k_s	N/mm	169.4	138.3
MR	-	1.03	1.38
C	N/(mm/s)	0.00	0.00
C_{crit}	N/(mm/s)	11.29	7.97
ζ	-	0.00	0.00
Z_0	mm	18.40	44.33
Z_1	mm	18.40	44.33

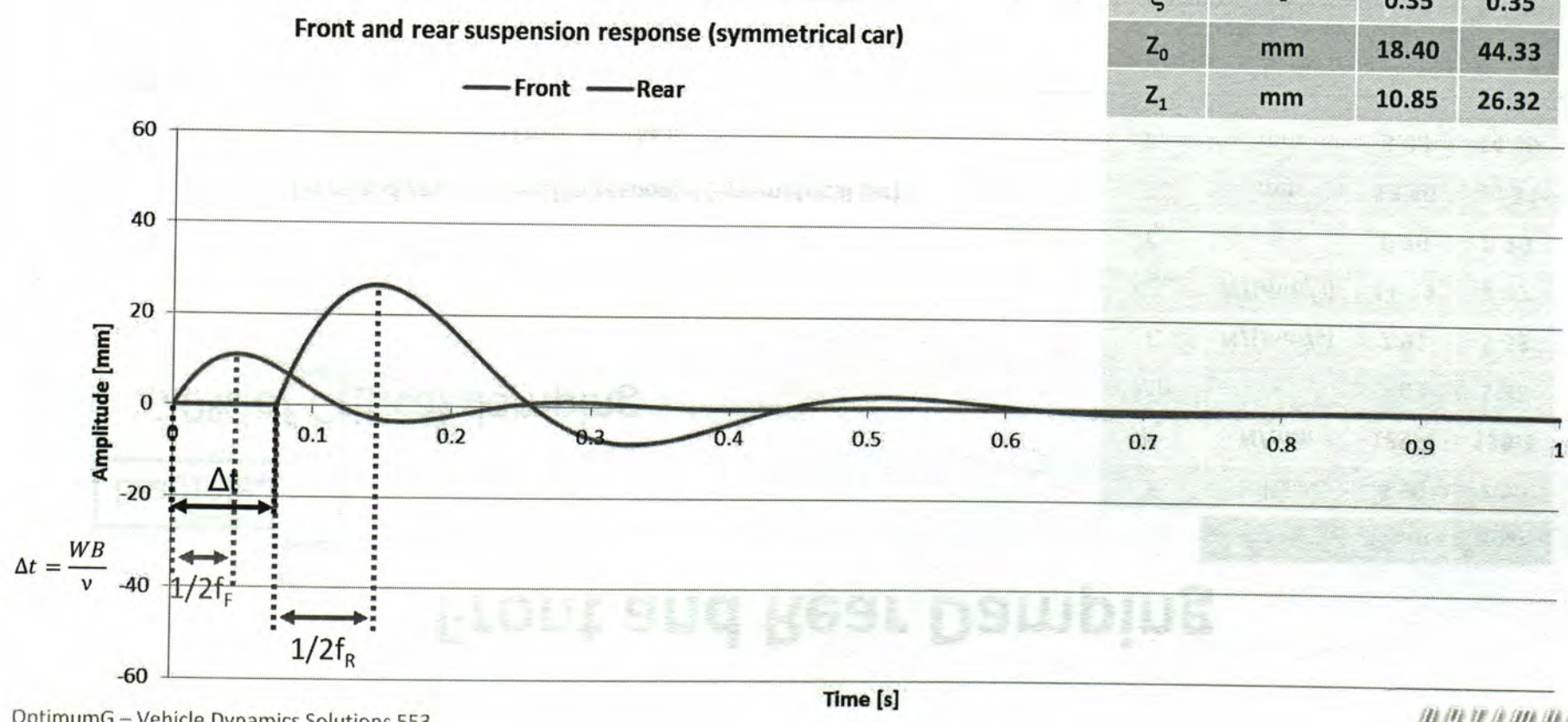


Front and Rear Damping

Example

Low damping

	Units	Front	Rear
f	Hz	4.50	2.90
k_s	N/mm	169.4	138.3
MR	-	1.03	1.38
C	N/(mm/s)	3.95	2.79
C_{crit}	N/(mm/s)	11.29	7.97
ζ	-	0.35	0.35
Z_0	mm	18.40	44.33
Z_1	mm	10.85	26.32

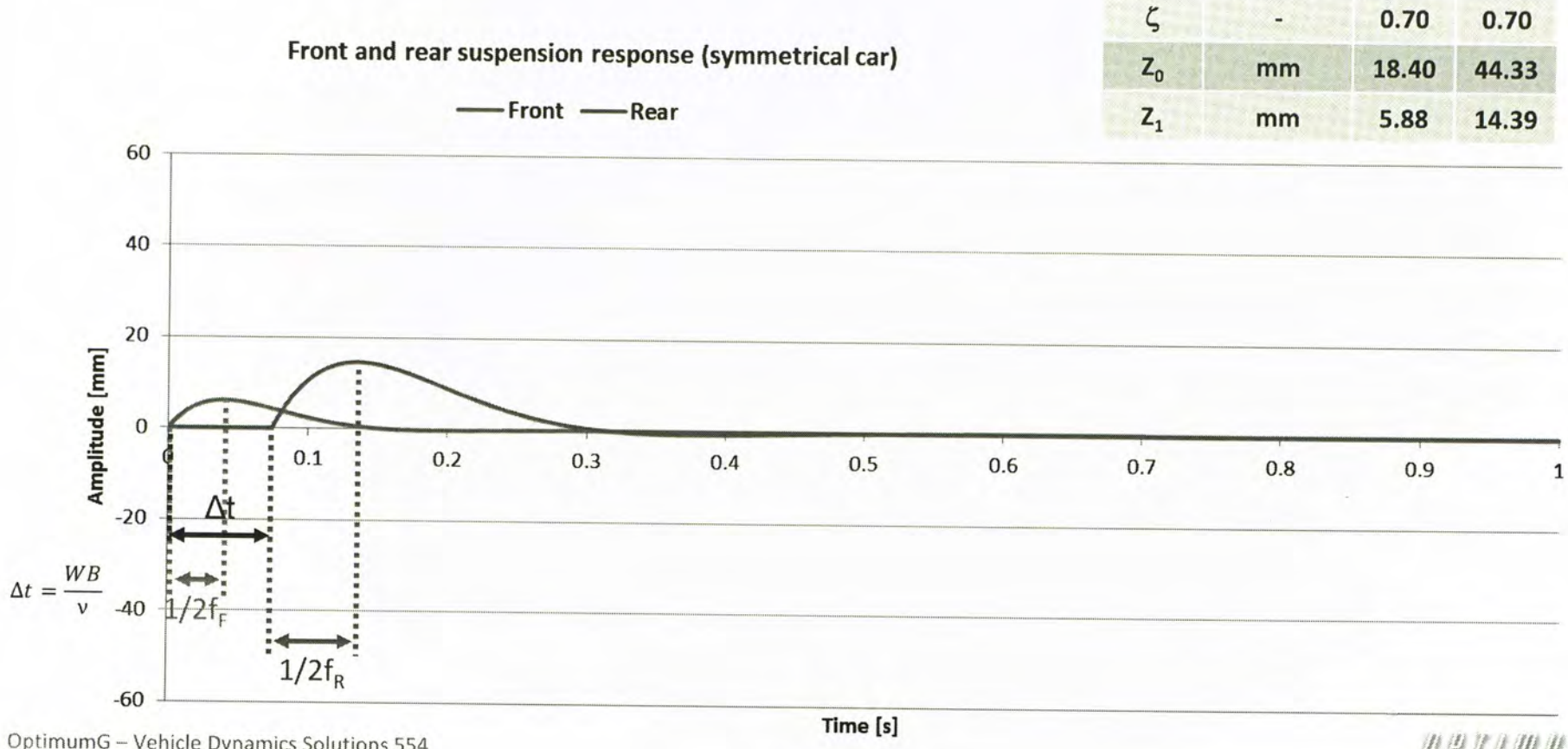


Front and Rear Damping

Example

70% of Critical damping

	Units	Front	Rear
f	Hz	4.50	2.90
k_s	N/mm	169.4	138.3
MR	-	1.03	1.38
C	N/(mm/s)	7.91	5.58
C_{crit}	N/(mm/s)	11.29	7.97
ζ	-	0.70	0.70
Z_0	mm	18.40	44.33
Z_1	mm	5.88	14.39

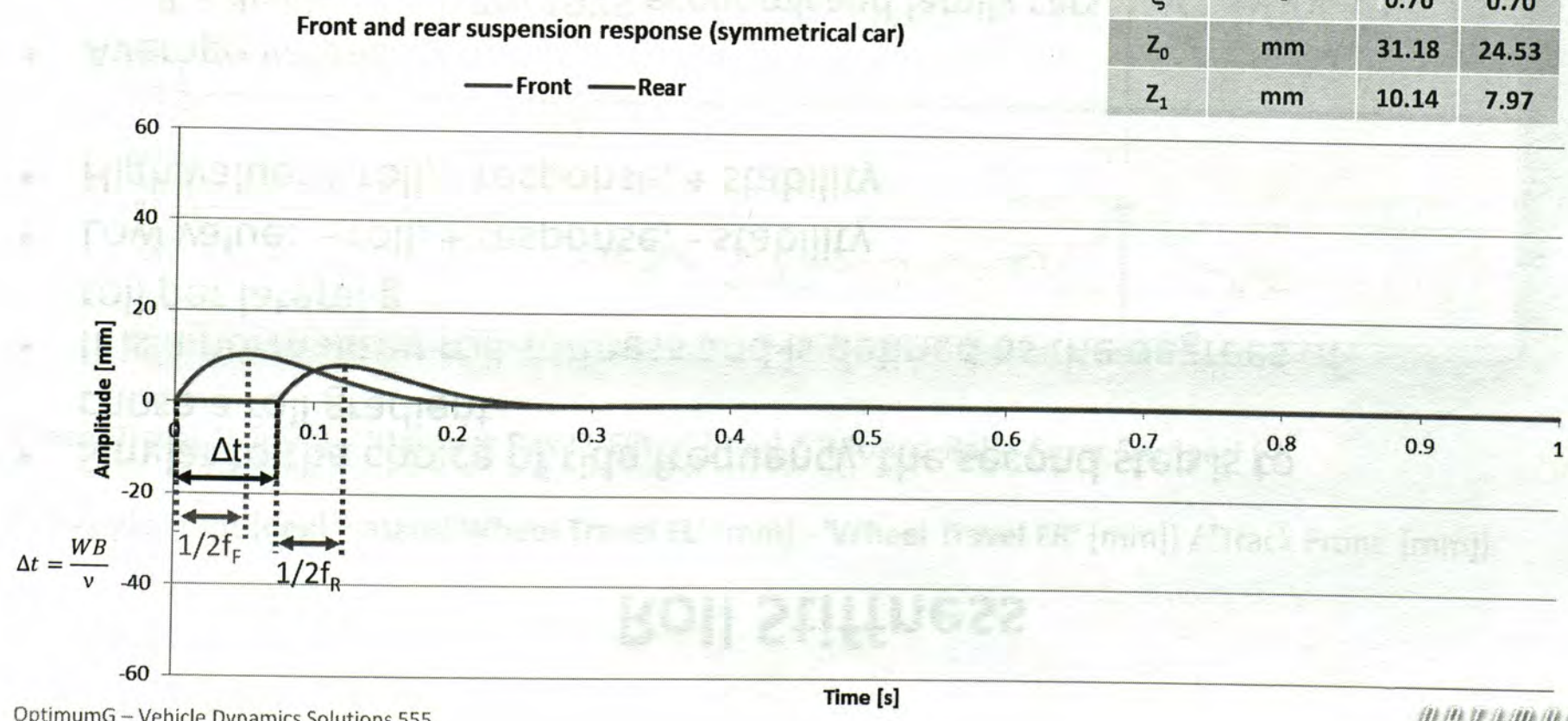


Front and Rear Damping

Example

70% of Critical damping
Different frequency
Better example

	Units	Front	Rear
f	Hz	3.46	3.91
k_s	N/mm	100	250
MR	-	1.03	1.38
C	N/(mm/s)	6.06	7.49
C_{crit}	N/(mm/s)	8.68	10.70
ζ	-	0.70	0.70
Z_0	mm	31.18	24.53
Z_1	mm	10.14	7.97



Roll Stiffness

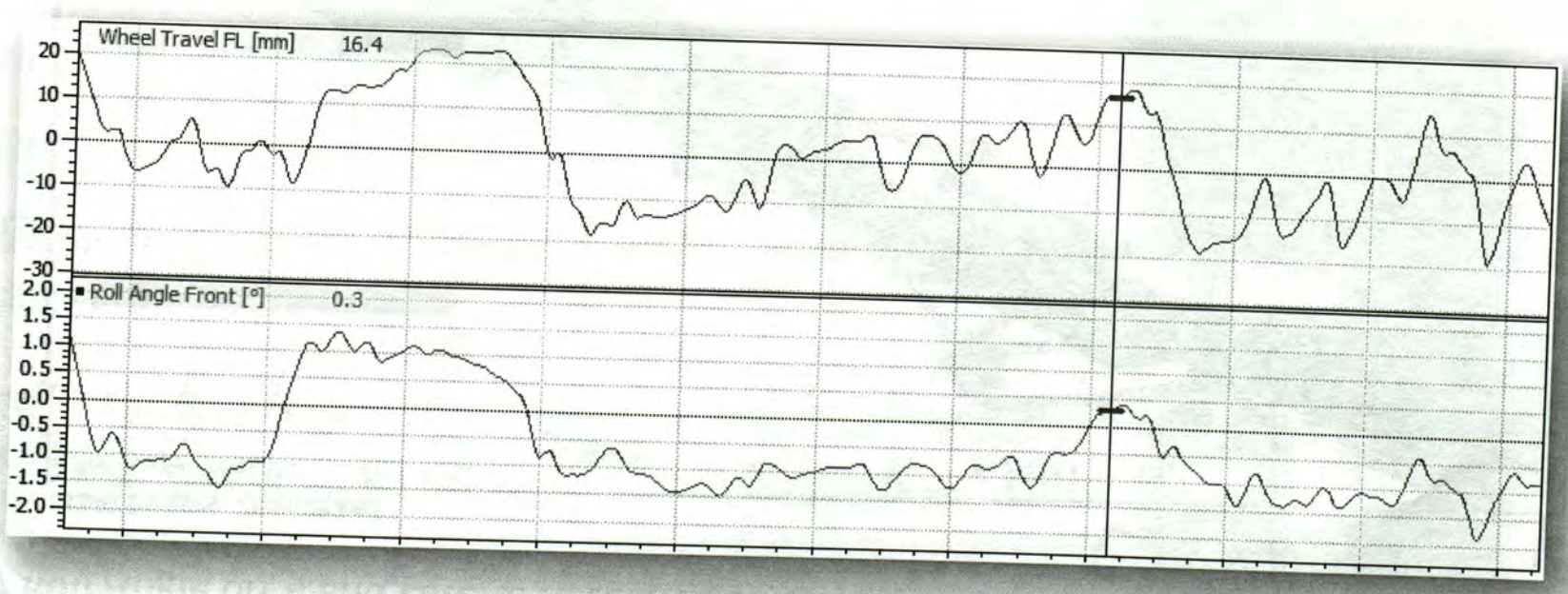
- Similar to the choice of ride frequency, the second step is to chose a roll gradient
- It is a normalized roll stiffness and is defined as the degrees of roll per lateral g
- Low value: - roll, + response, - stability
- High value: + roll, - response, + stability

- Average values:
  - 8.5 deg/g Pre 1975 economy and family cars
  - 7.5 deg/g Basic family cars
  - 5.0 deg/g Sports sedans
  - 3.0 - 4.0 deg/g High performance sports cars
  - 1.0 – 1.8 deg/g Low downforce racecars
  - 0.2 – 0.7 deg/g High downforce racecars

Car Analysis

Roll Gradient*

(*) Roll Gradient: Stiffness of the car in roll [deg/g]



Wheel Travel FL [mm] = **'Damper Pos FL Filter'** [mm] * 'Motion Ratio Front Damper' [-]

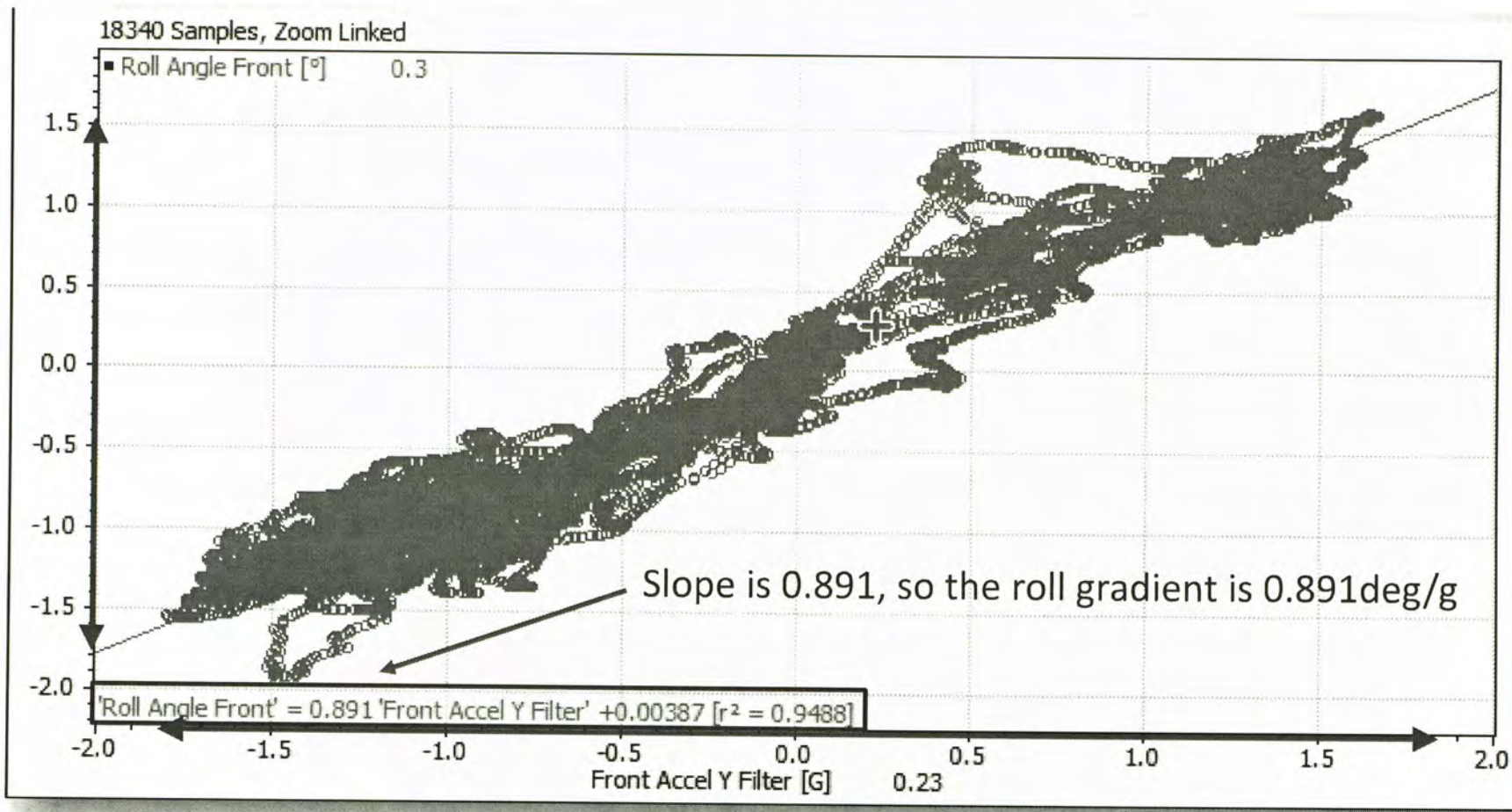
Roll Angle Front [deg] = atan(('**Wheel Travel FL**' [mm] - **'Wheel Travel FR'** [mm]) / 'Track Front' [mm])

Car Analysis

Roll Gradient*

(*) Roll Gradient: Stiffness of the car in roll [deg/g]

Display **Roll Angle** on Y, and **Lateral Acceleration** on X axis of a scatter plot.



Roll Stiffness

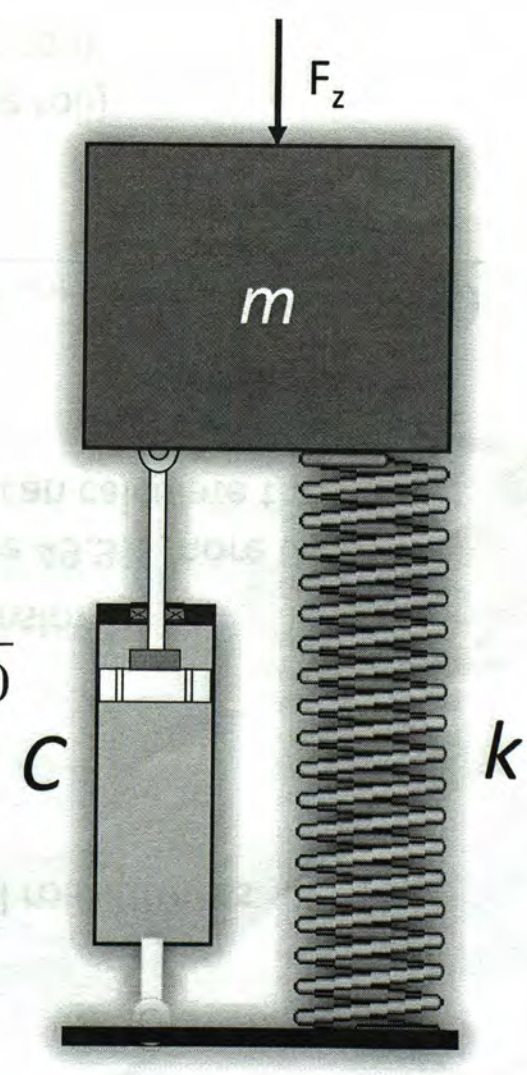
- The roll gradient depends on both the springs and anti-roll bars

$$\frac{\phi}{A_y} = \frac{mg \times H_{cg-rc}}{K_{\phi F} + K_{\phi R}}$$

- The front and rear roll stiffness due to the suspension springs are:

$$K_{\phi S-F} = \frac{t_F^{\,2} K_{LF} K_{RF}}{(K_{LF} + K_{RF})} \frac{\pi}{180} \qquad K_{\phi S-R} = \frac{t_R^{\,2} K_{LR} K_{RR}}{(K_{LR} + K_{RR})} \frac{\pi}{180}$$

$K_{\phi S-F}$ Front roll stiffness from springs (Nm/ deg roll)
t_F Front track width (m)
K_{LF} Front left wheel rate (N/m)
K_{RF} Front right wheel rate (N/m)



Roll Stiffness

- From a chosen roll gradient we can calculate the desired total roll stiffness

$$K_{\phi DES} = \frac{m_s g \times H_{scg-rc}}{\phi / A_y}$$

- This stiffness is distributed between the front and rear suspension.
- As a baseline the front roll gradient (magic number) should be 49.3% more than the static suspended weight distribution. From this value we can calculate the roll stiffness needed at the front and rear.

$$K_{\phi DES-F} = \frac{K_{\phi DES} \times N_{magic}}{100} \qquad K_{\phi DES-R} = \frac{K_{\phi DES} \times (100 - N_{magic})}{100}$$

$K_{\phi DES-F}$ Desired front roll stiffness (Nm/deg roll)
$K_{\phi DES-R}$ Desired rear roll stiffness (Nm/deg roll)
N_{magic} Magic Number (%)

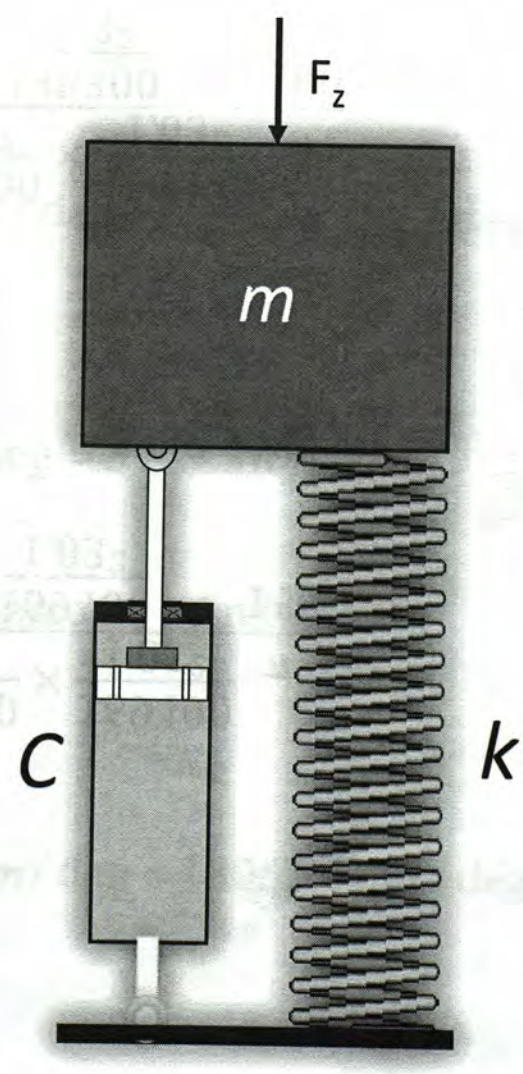
Roll Stiffness

- Now from the desired front and rear roll stiffness we can calculate the required anti-roll bar stiffness. This must be done for both the front and rear of the car.
- First, we must determine the total roll stiffness provided by the tires

$$K_{\phi T} = \left(K_{TF} \frac{t_F^{\,2}}{2} + K_{TR} \frac{t_R^{\,2}}{2} \right) \frac{\pi}{180}$$

- We know that the springs and ARB act in parallel with each other and in series with the tire stiffness
- From this can determine the additional roll stiffness required from anti-roll bars

$$K_{\phi ARB} = \left[\frac{K_{\phi DES}(K_{\phi T} + K_{\phi S}) - K_{\phi T} K_{\phi S}}{K_{\phi T} - K_{\phi DES}} \right]$$



Roll Stiffness

- Roll stiffness from front and rear springs
  - Front: t_F = 1.640 m, k_F = 169400 N/m, MR_F = 1.03

$$K_{\emptyset S-F} = \frac{{t_F}^2 \times K_{LF} \times K_{RF}}{K_{LF+}K_{RF}} \frac{\pi}{180} = \frac{1.64^2 \times \frac{169400}{1.03^2} \times \frac{169400}{1.03^2}}{\frac{169400}{1.03^2} + \frac{169400}{1.03^2}} \frac{\pi}{180}$$

$$K_{\emptyset S-F} = 3748.2\ Nm/deg$$

  - Rear: t_R = 1.592 m, k_R = 138300 N/m, MR_R = 1.38

$$K_{\emptyset S-R} = \frac{{t_R}^2 \times K_{LR} \times K_{RR}}{K_{LR+}K_{RR}} \frac{\pi}{180} = \frac{1.592^2 \times \frac{138300}{1.03^2} \times \frac{138300}{1.03^2}}{\frac{138300}{1.03^2} + \frac{138300}{1.03^2}} \frac{\pi}{180}$$

$$K_{\emptyset S-R} = 1606.4\ Nm/deg$$

Roll Stiffness

- Desired Roll Gradient: 0.06 deg/G = ϕ ?

$$K_{\phi DES} = \frac{m_s g \times H_{scg-rc}}{\phi / A_y = g?} = \frac{836.9 \times 9.81 \times 0.218}{0.06} = 29829.6\ Nm/\deg = 22001.2\ lbft/\deg$$

- Total roll stiffness from springs

$$K_{\phi S} = K_{\phi S-F} + K_{\phi S-R} = 3748.2 + 1606.4 = 5354.5\ Nm/\deg = 3949.3\ lbft/\deg$$

- Total roll stiffness from tires not taken into consideration (assume infinitely stiff)

- Roll stiffness total needed from anti-roll bars

$$K_{\phi ARB@Wheel} = K_{\phi DES} - K_{\phi S} = 29829.6 - 5354.5 = 24475.2\ Nm/\deg = 18052.0\ lbft/\deg$$

Roll Stiffness

- Desired Magic Number: N_{magic} = 49%
- ARB Motion ratio = Chassis roll angle / ARB twist angle
- Stiffness from front anti-roll bar [MR = 0.65]

$$K_{\emptyset DES-F\ @\ Wheel} = K_{\emptyset DES} \times N_{Magic} = 29829.6 \times 0.49 = 14616.5$$

$$K_{\emptyset ARB-F\ @\ Wheel} = K_{\emptyset DES-F} - K_{\emptyset S-F}$$

$$K_{\emptyset ARB-F\ @\ Wheel} = 14616.5 - 3748.2 = 10868.3\ Nm/deg$$

$$K_{\emptyset ARB-F} = \frac{(K_{\emptyset ARB-F\ @\ Wheel}) \times MR^2}{{Track_F}^2 \times Tan\ (\pi/180)} = \frac{(10868.3) \times 0.65^2}{1.640^2 \times Tan\ (\pi/180)} = 97809\ N/m$$

- Stiffness from rear anti-roll bar [MR = 0.75]

$$K_{\emptyset DES-R\ @\ Wheel} = K_{\emptyset DES} \times (1 - N_{Magic}) = 29829.6 \times (1 - 0.49) = 15213.1$$

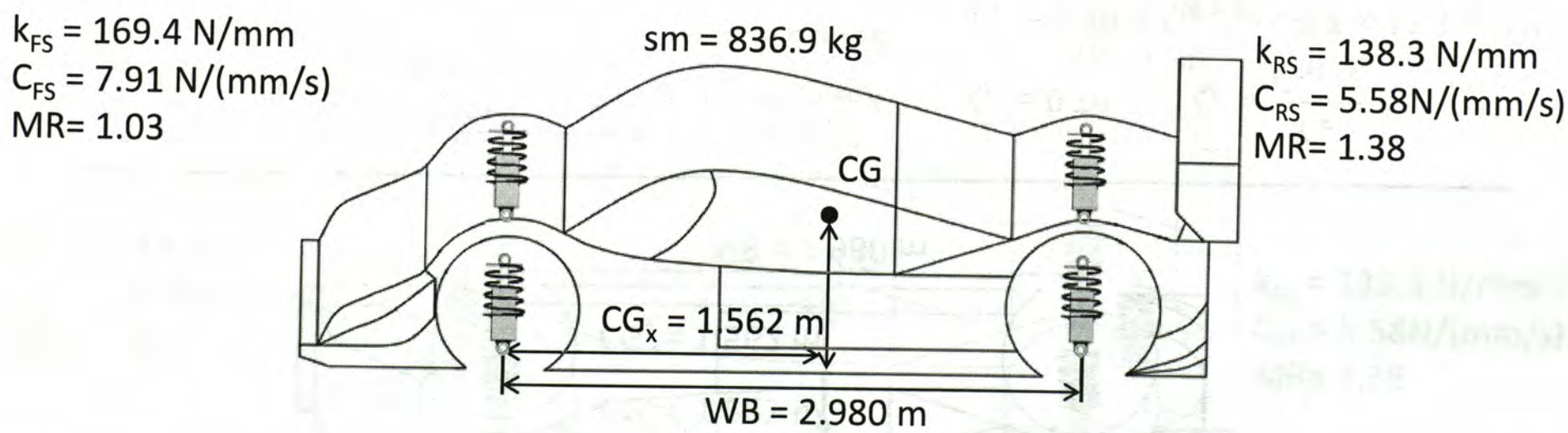
$$K_{\emptyset ARB-R\ @\ Wheel} = K_{\emptyset DES-R} - K_{\emptyset S-F}$$

$$K_{\emptyset ARB-R\ @\ Wheel} = 15213.1 - 1606.4 = 13606.7\ Nm/deg$$

$$K_{\emptyset ARB-R} = \frac{(K_{\emptyset ARB-R\ @\ Wheel}) \times MR^2}{{Track_R}^2 \times Tan\ (\pi/180)} = \frac{(13606.7) \times 0.75^2}{1.592^2 \times Tan\ (\pi/180)} = 173009/m$$

Modes of Vibration

Heave



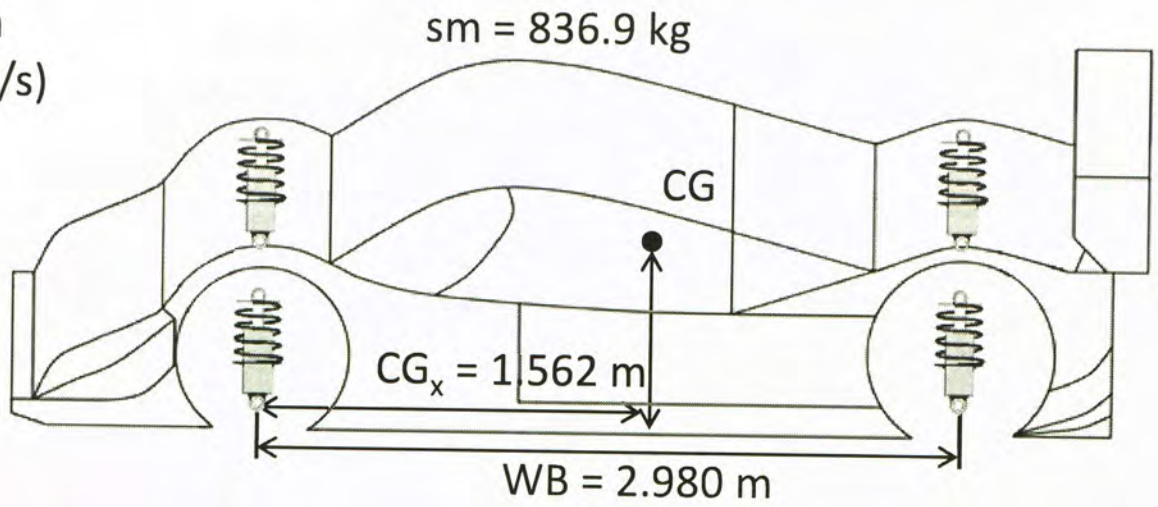
$$sm_{F-Avg} = 199.7kg \qquad f_F = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \times \sqrt{\frac{1}{199.7} \times \frac{169400}{1.03^2}} = 4.5Hz$$

$$sm_{R-Avg} = 218.8kg \qquad f_R = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \times \sqrt{\frac{1}{218.8} \times \frac{138300}{1.38^2}} = 2.9Hz$$

Modes of Vibration

Heave

k_{FS} = 169.4 N/mm
C_{FS} = 7.91 N/(mm/s)
MR= 1.03



k_{RS} = 138.3 N/mm
C_{RS} = 5.58N/(mm/s)
MR= 1.38

$$C_{Crit-F} = 2\sqrt{km} = 2\sqrt{\frac{169400}{1.03^2} \times 199.7} = 11.3N/mm/s \qquad \zeta_F = 0.70 \qquad \zeta_F = \frac{C_F}{C_{Crit\ F}}$$

$$C_F = 0.70 \times C_{Crit\ F} = 0.7 \times 11.3 = 7.91$$

$$C_{Crit-R} = 2\sqrt{km} = 2\sqrt{\frac{137800}{1.38^2} \times 218.8} = 8.0N/mm/s \qquad \zeta_R = 0.70 \qquad \zeta_R = \frac{C_R}{C_{Crit\ R}}$$

$$C_R = 0.70 \times C_{Crit\ R} = 0.7 \times 8.0 = 5.58$$

Modes of Vibration

Heave

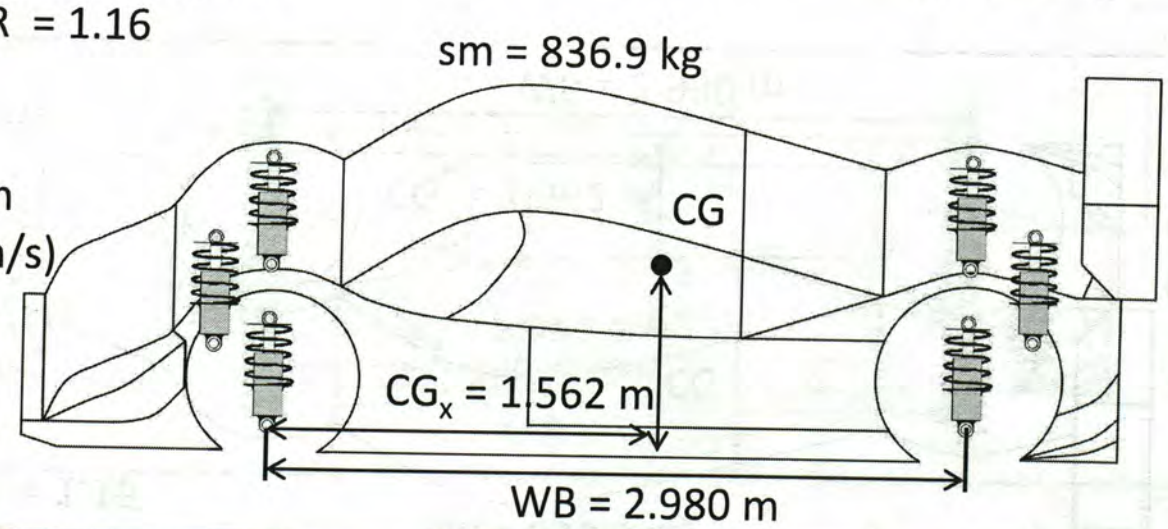
For aero reasons we need to increase the heave stiffness by adding a 3rd spring.

+ 3rd springs

k_{F3rd} = 350 N/mm
MR = 1.16

k_{R3rd} = 280 N/mm
MR= 1.36

k_{FS} = 169.4 N/mm
C_{FS} = 7.91 N/(mm/s)
MR= 1.03



k_{RS} = 138.3 N/mm
C_{RS} = 5.58N/(mm/s)
MR= 1.38

$$m_F = 399.4kg \qquad f_F = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \times \sqrt{\frac{1}{399.4} \times \left[\left(2 \times \frac{169400}{1.03^2}\right) + \left(\frac{350000}{1.16^2}\right)\right]} = 6.1Hz$$

$$m_R = 437.6kg \qquad f_R = \frac{1}{2\pi} \times \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \times \sqrt{\frac{1}{437.6} \times \left[\left(2 \times \frac{138300}{1.38^2}\right) + \left(\frac{280000}{1.36^2}\right)\right]} = 4.1Hz$$

Modes of Vibration

Heave

So the damping needs to be increased if we want to keep a damping ratio which is 70% of the critical damping.

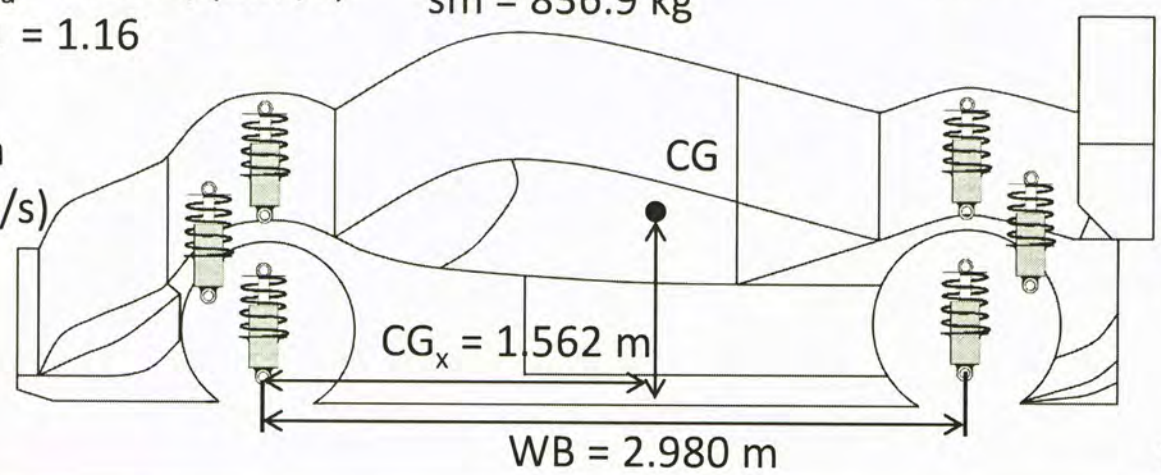
+ 3rd springs

k_{F3rd} = 350 N/mm
C_{F3rd} = 5.48 N/(mm/s)
MR = 1.16

k_{R3rd} = 280 N/mm
C_{R3rd} = 4.79 N/(mm/s)
MR= 1.36

sm = 836.9 kg

k_{FS} = 169.4 N/mm
C_{FS} = 7.91 N/(mm/s)
MR= 1.03



k_{RS} = 138.3 N/mm
C_{RS} = 5.58N/(mm/s)
MR= 1.38

$m_F = 399.4kg$ $\qquad$ $m_R = 437.6kg$

$$C_{Crit-F} = 2\sqrt{km} = 2\sqrt{\left(2 \times \frac{169400}{1.03^2} + \frac{350000}{1.16^2}\right) \times 399.4} = 30.4N/mm/s$$

$$\zeta_F = 0.70 \qquad \zeta_F = \frac{C_F}{C_{Crit\,F}}$$

$$C_F = 0.70 \times C_F = 0.7 \times 30.4 = 21.30$$

$$C_{3rd-F} = C - 2 \times C_{fs} = 21.30 - 2 \times 7.91 = 5.48$$

$$C_{Crit-R} = 2\sqrt{km} = 2\sqrt{\left(2 \times \frac{138300}{1.38^2} + \frac{\frac{280000}{2}}{1.36^2}\right) \times 437.6} = 22.8N/mm/s$$

$$\zeta_R = 0.70 \qquad \zeta_R = \frac{C_R}{C_{Crit\,R}}$$

$$C_R = 0.70 \times C_R = 0.7 \times 22.8 = 15.95$$

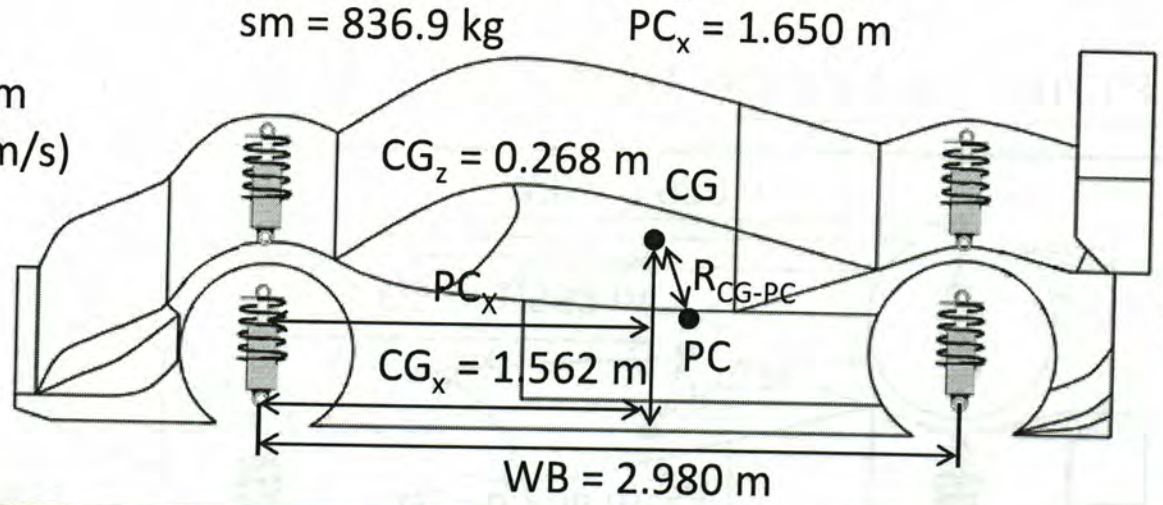
$$C_{3rd-R} = C - 2 \times C_{rs} = 15.95 - 2 \times 5.58 = 4.79$$

Modes of Vibration

Pitch

I_{pitch} = 1250 kg m^2 $\qquad$ PC_z = 0.052 m
sm = 836.9 kg $\qquad$ PC_x = 1.650 m

k_{FS} = 169.4 N/mm
C_{FS} = 7.91 N/(mm/s)
MR= 1.03



k_{RS} = 138.3 N/mm
C_{RS} = 5.58 N/(mm/s)
MR = 1.38

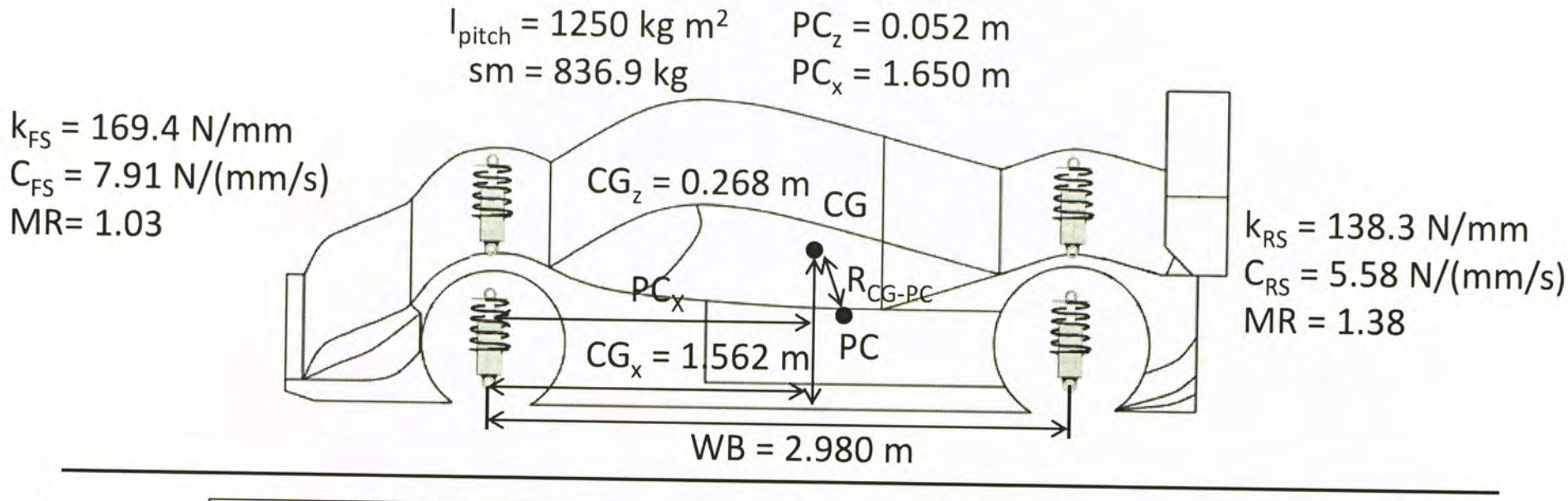
$$K_{pitch} = \left[PC_x^2 \left(\frac{k_{FL}}{MR_{FL}^2} + \frac{k_{FR}}{MR_{FR}^2} \right) + (WB - PC_x)^2 \left(\frac{k_{RL}}{MR_{RL}^2} + \frac{k_{RR}}{MR_{RR}^2} \right) \right] \frac{\pi}{180}$$

$$= \left(1.650^2 \left(\frac{169300}{1.03^2} \times 2 \right) + (2.980 - 1.650)^2 \left(\frac{138300}{1.38^2} \times 2 \right) \right) \frac{\pi}{180} = 16966\, Nm/\deg$$

$$\frac{\theta}{A_y} = \frac{mg \times (CG_z - PC_z)}{K_{Pitch}} = \frac{836.9 \times 9.81 \times (0.268 - 0.052)}{(16966)} = 0.105 \deg/g$$

Modes of Vibration

Pitch



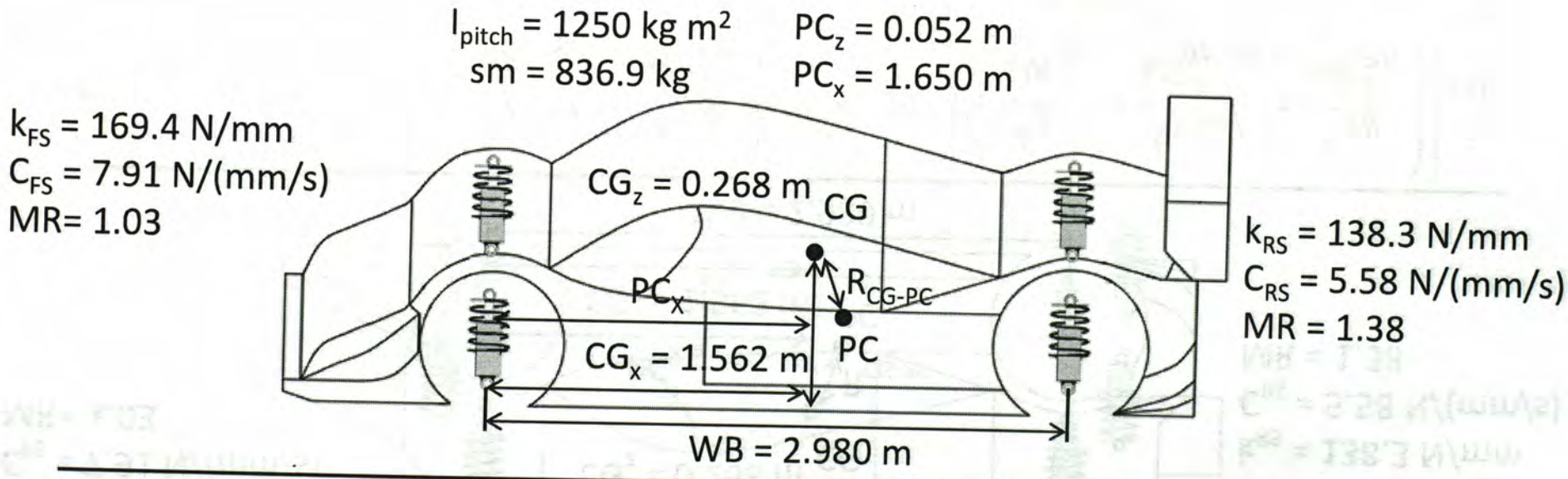
$$R_{CG-PC} = \sqrt{(CG_x - PC_x)^2 + (CG_z - PC_z)^2} = \sqrt{(1.562-1.650)^2 + (0.268-0.052)^2} = 0.233\,m$$

Parallel Axis Theorem: $I_{PC} = I_{pitch} + \left(m \times R_{CG-PC}{}^2\right) = 1250 + \left(836.9 \times (0.233)^2\right) = 1295.4\,kg \cdot m^2$

$$f_{Pitch} = \frac{1}{2\pi} \times \sqrt{\frac{K_{pitch}}{I_{PC}}} = \frac{1}{2\pi} \times \sqrt{\frac{16966 \times \frac{180}{\pi}}{1295.4}} = 4.36\,Hz$$

Modes of Vibration

Pitch



$$C_{pitch} = \left[PC_x{}^2 \times (C_{FL} + C_{FR}) + (WB - PC_x)^2 \times (C_{RL} + C_{RR})\right] \times \frac{\pi}{180}$$

$$= \left(1.65^2 \times (7910 + 7910) + (2.980 - 1.650)^2 \times (5580 + 5580)\right) \times \frac{\pi}{180} = 1096\,Nm/(\deg/s)$$

$$C_{Crit-Pitch} = 2\sqrt{K_{pitch} I_{PC}} = 2\sqrt{16966 \times \frac{180}{\pi} \times 1295.4} \times \left(\frac{\pi}{180}\right) = 1239\,Nm/(\deg/s)$$

$$\zeta_{Pitch} = \frac{C_{Pitch}}{C_{Crit-Pitch}} = \frac{1096}{1239} = 0.89$$

Modes of Vibration

Pitch

+ 3rd springs

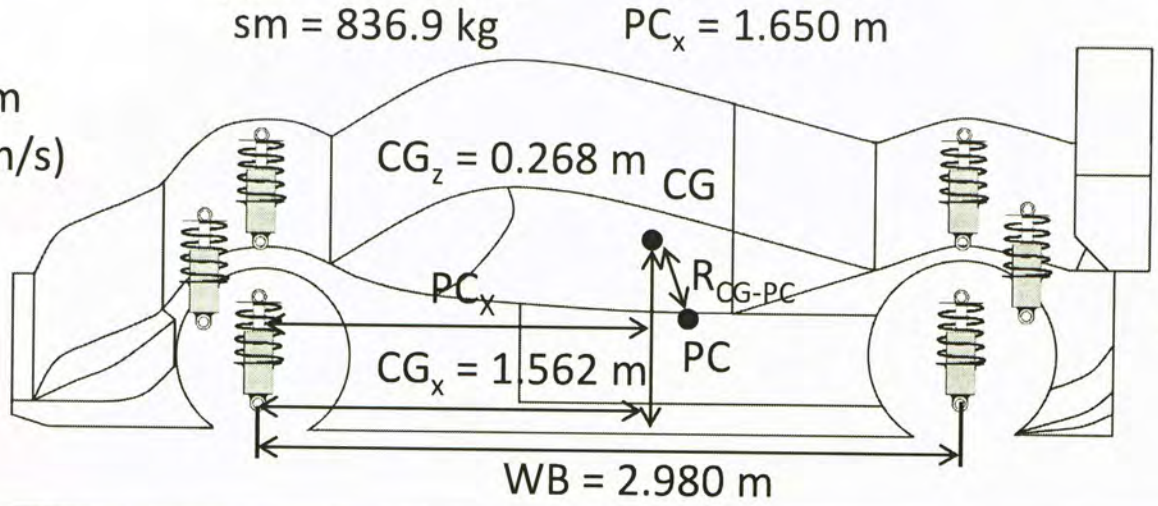
k_{F3rd} = 350 N/mm
MR = 1.16

k_{R3rd} = 280 N/mm
MR = 1.36

I_{pitch} = 1250 kg m^2
sm = 836.9 kg

PC_z = 0.052 m
PC_x = 1.650 m

k_{FS} = 169.4 N/mm
C_{FS} = 7.91 N/(mm/s)
MR= 1.03



k_{RS} = 138.3 N/mm
C_{RS} = 5.58 N/(mm/s)
MR = 1.38

$$K_{pitch} = \left[PC_x^2 \left(\frac{k_{FL}}{MR_{FL}^2} + \frac{k_{F3rd}}{MR_{F3rd}^2} + \frac{k_{FR}}{MR_{FR}^2} \right) + (WB - PC_x)^2 \left(\frac{k_{RL}}{MR_{RL}^2} + \frac{k_{R3rd}}{MR_{R3rd}^2} + \frac{k_{RR}}{MR_{RR}^2} \right) \right] \frac{\pi}{180}$$

$$= \left(1.650^2 \left(\frac{169300}{1.03^2} \times 2 + \frac{350000}{1.16^2} \right) + (2.980 - 1.650)^2 \left(\frac{138300}{1.38^2} \times 2 + \frac{280000}{1.36^2} \right) \right) \frac{\pi}{180} = 36241\, Nm/\deg$$

$$\frac{\theta}{A_y} = \frac{mg \times (CG_z - PC_z)}{K_{Pitch}} = \frac{836.9 \times 9.81 \times (0.268 - 0.052)}{(36241)} = 0.049\, \deg/g$$

Modes of Vibration

Pitch

+ 3rd springs

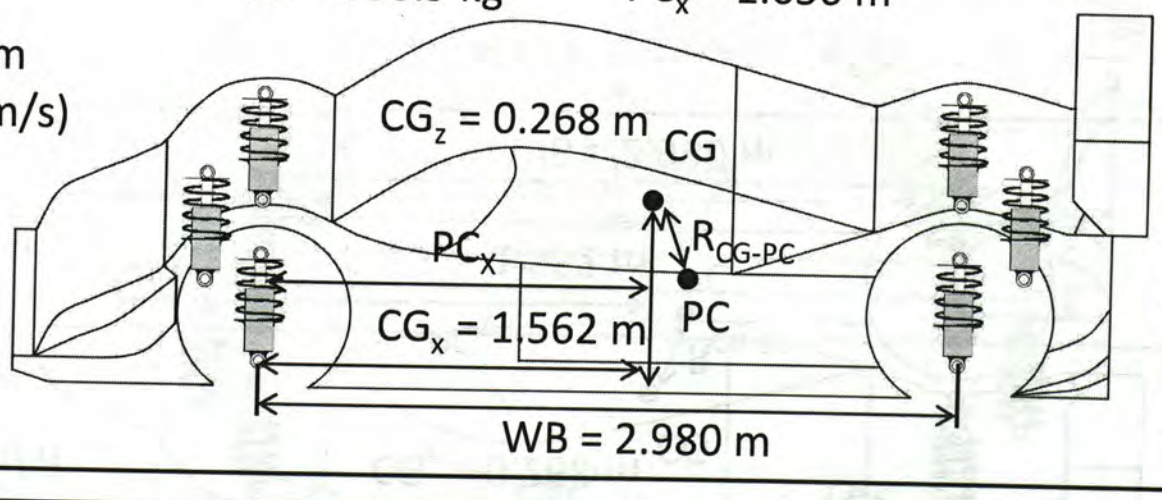
k_{F3rd} = 350 N/mm
C_{F3rd} = 5.48 N/(mm/s)
MR = 1.16

k_{R3rd} = 280 N/mm
C_{R3rd} = 4.79 N/(mm/s)
MR = 1.36

I_{pitch} = 1250 kg m^2
sm = 836.9 kg

PC_z = 0.052 m
PC_x = 1.650 m

k_{FS} = 169.4 N/mm
C_{FS} = 7.91 N/(mm/s)
MR= 1.03



k_{RS} = 138.3 N/mm
C_{RS} = 5.58 N/(mm/s)
MR = 1.38

$$R_{CG-PC} = \sqrt{(CG_x - PC_x)^2 + (CG_z - PC_z)^2} = \sqrt{(1.562 - 1.650)^2 + (0.268 - 0.052)^2} = 0.233\, m$$

Parallel Axis Theorem: $I_{PC} = I_{pitch} + \left(m \times R_{CG-PC}{}^2 \right) = 1250 + \left(836.9 \times (0.233)^2 \right) = 1295.4\, kg \cdot m^2$

$$f_{Pitch} = \frac{1}{2\pi} \times \sqrt{\frac{K_{pitch}}{I_{PC}}} = \frac{1}{2\pi} \times \sqrt{\frac{36241 \times \frac{180}{\pi}}{1295.4}} = 6.37\, Hz$$

Modes of Vibration

Pitch

+ 3rd springs

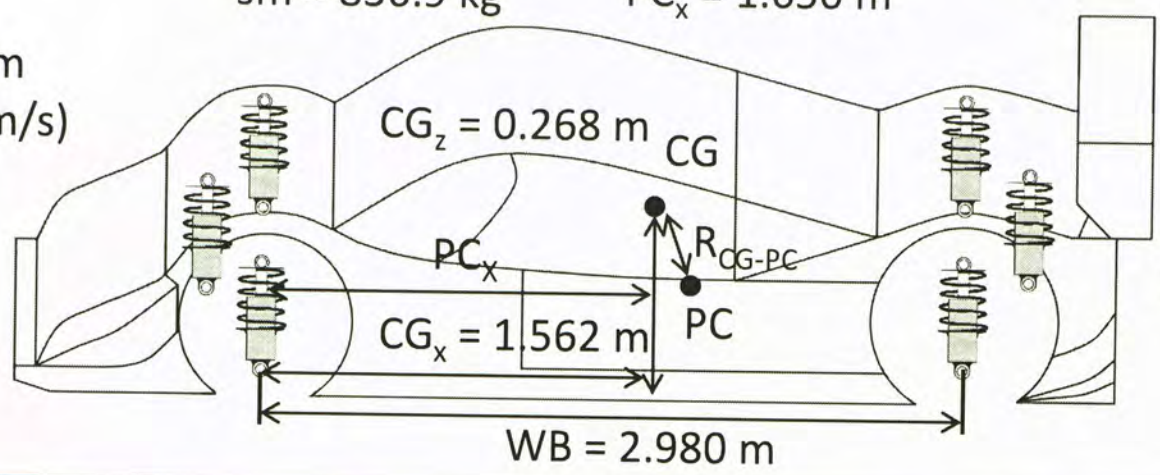
k_{F3rd} = 350 N/mm
C_{F3rd} = 5.48 N/(mm/s)
MR = 1.16

k_{FS} = 169.4 N/mm
C_{FS} = 7.91 N/(mm/s)
MR= 1.03

I_{pitch} = 1250 kg m²
sm = 836.9 kg

PC_z = 0.052 m
PC_x = 1.650 m

k_{R3rd} = 280 N/mm
C_{R3rd} = 4.79 N/(mm/s)
MR = 1.36



k_{RS} = 138.3 N/mm
C_{RS} = 5.58 N/(mm/s)
MR = 1.38

$$C_{pitch} = \left[PC_x^2 \times (C_{FL} + C_{F3rd} + C_{FR}) + (WB - PC_x)^2 \times (C_{RL} + C_{R3rd} + C_{RR})\right] \times \frac{\pi}{180}$$

$$= \left(1.65^2 \times (7910 + 5480 + 7910) + (2.980 - 1.650)^2 \times (5580 + 4790 + 5580)\right) \times \frac{\pi}{180} = 1504\, Nm/(\deg/s)$$

$$C_{Crit-Pitch} = 2\sqrt{K_{pitch} I_{PC}} = 2\sqrt{36241 \times \frac{180}{\pi} \times 1295.4} \times \left(\frac{\pi}{180}\right) = 1810\, Nm/(\deg/s)$$

$$\zeta_{Pitch} = \frac{C_{Pitch}}{C_{Crit-Pitch}} = \frac{1504}{1810} = 0.83$$

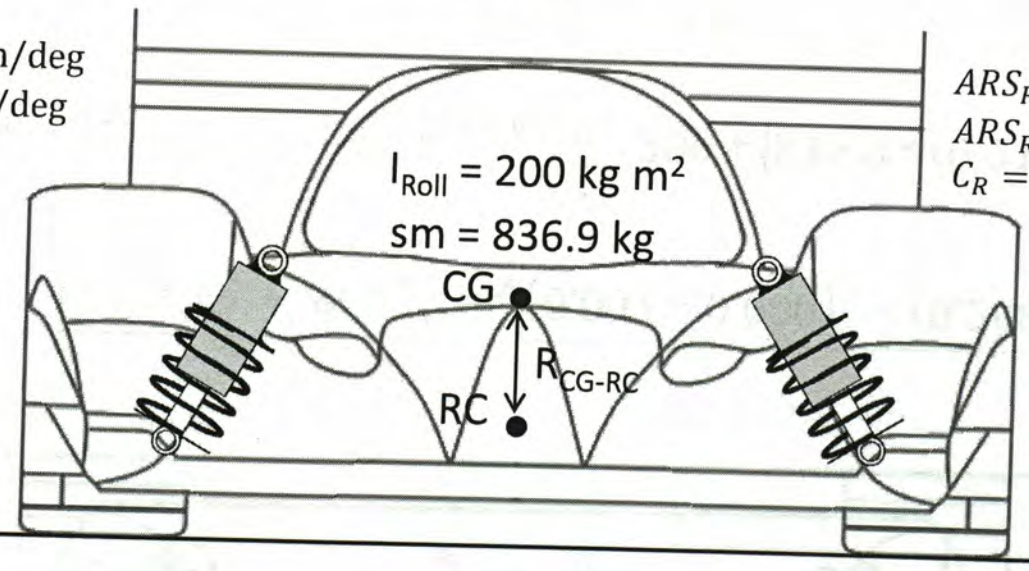
Modes of Vibration

Roll

Front View

$ARS_{F-Spring}$ = 3748.2 Nm/deg
ARS_{F-ARB} = 10868.3 Nm/deg
C_F = 7.91 N/(mm/s)

CG_z = 0.268 m
CG_y = 0.007 m



$ARS_{R-Spring}$ = 1606.4 Nm/deg
ARS_{R-ARB} = 10868.3 Nm/deg
C_R = 5.58 N/(mm/s)

RC_z = 0.050 m
RC_y = 0.000 m

$$K_{Roll} = ARS_{F-Spring} + ARS_{F-ARB} + ARS_{R-Spring} + ARS_{R-ARB}$$
$$= 3748.2 + 10868.3 + 1606.4 + 10868.3 = 29828.8 \text{ Nm/deg}$$

$$\frac{\phi}{A_y} = \frac{mg \times (CG_z - RC_z)}{K_{roll}} = \frac{836.9 \times 9.81 \times (0.268 - 0.050)}{29828.8} = 0.06 \deg/g$$

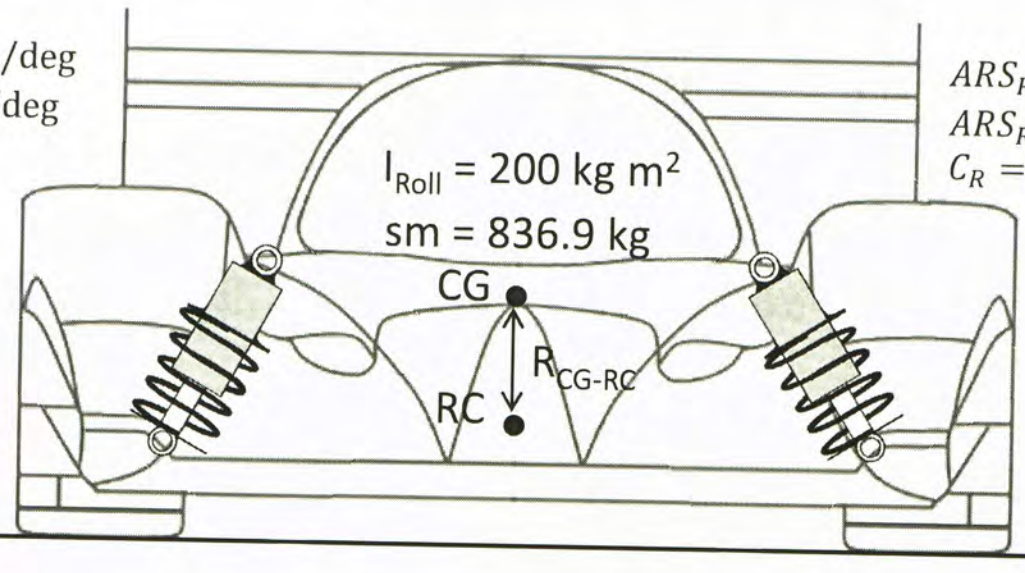
Modes of Vibration

Front View

Roll

$ARS_{F-Spring} = 3748.2$ Nm/deg
$ARS_{F-ARB} = 10868.3$ Nm/deg
$C_F = 7.91$ *N/(mm/s)*

$ARS_{R-Spring} = 1606.4$ Nm/deg
$ARS_{R-ARB} = 10868.3$ Nm/deg
$C_R = 5.58$ *N/(mm/s)*



$CG_z = 0.268$ m
$CG_y = 0.007$ m

$RC_z = 0.050$ m
$RC_y = 0.000$ m

$$R_{CG-RC} = \sqrt{(CG_y - RC_y)^2 + (CG_z - RC_z)^2} = \sqrt{(0.007-0.000)^2 + (0.268-0.050)^2} = 0.218\,m$$

Parallel Axis theorem: $I_{RC} = I_{roll} + \left(m \times R_{CG-RC}{}^2\right) = 200 + \left(836.9 \times (0.218)^2\right) = 239.8\,kgm^2$

$$f_{Roll} = \frac{1}{2\pi} \times \sqrt{\frac{K_{Roll}}{I_{RC}}} = \frac{1}{2\pi} \times \sqrt{\frac{29828.8 \times \frac{180}{\pi}}{239.8}} = 13.44\,Hz$$

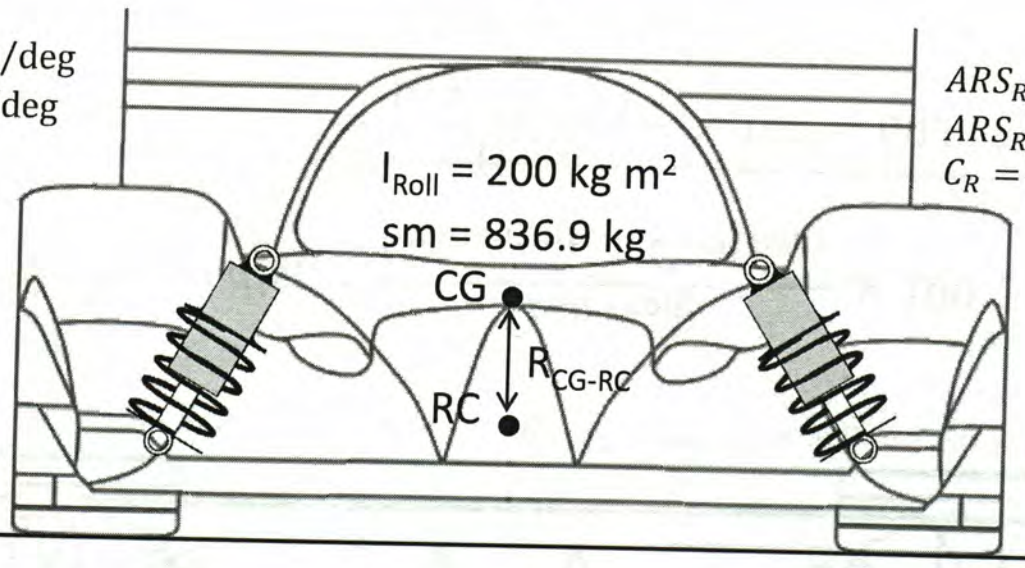
Modes of Vibration

Front View

Roll

$ARS_{F-Spring} = 3748.2$ Nm/deg
$ARS_{F-ARB} = 10868.3$ Nm/deg
$C_F = 7.91$ *N/(mm/s)*

$ARS_{R-Spring} = 1606.4$ Nm/deg
$ARS_{R-ARB} = 10868.3$ Nm/deg
$C_R = 5.58$ *N/(mm/s)*



$CG_z = 0.268$ m
$CG_y = 0.007$ m

$T_R = 1.592$ m

$RC_z = 0.050$ m
$RC_y = 0.000$ m

$T_F = 1.640$ m

$$C_{Roll} = C_{Roll_F} + C_{Roll_R} = \frac{t_F^2 C_{LF} C_{RF}}{C_{LF} + C_{RF}} \frac{\pi}{180} + \frac{t_R^2 C_{LR} C_{RR}}{C_{LR} + C_{RR}} \frac{\pi}{180} =$$

$$= \frac{1.640^2 \times 7910 \times 7910}{7910 + 7910} \frac{\pi}{180} + \frac{1.592^2 \times 5580 \times 5580}{5580 + 5580} \frac{\pi}{180} = 185.6 + 123.4 = 309\,Nm/(\deg/s)$$

$$C_{Crit-Roll} = 2\sqrt{K_{Roll} I_{RC}} = 2\sqrt{29828.8 \times \frac{180}{\pi} \times 239.8}\left(\frac{\pi}{180}\right) = 707\,Nm/(\deg/s)$$

$$\zeta_{Roll} = \frac{C_{Roll}}{C_{Crit-Roll}} = \frac{309}{707} = 0.44$$

Modes of Vibration

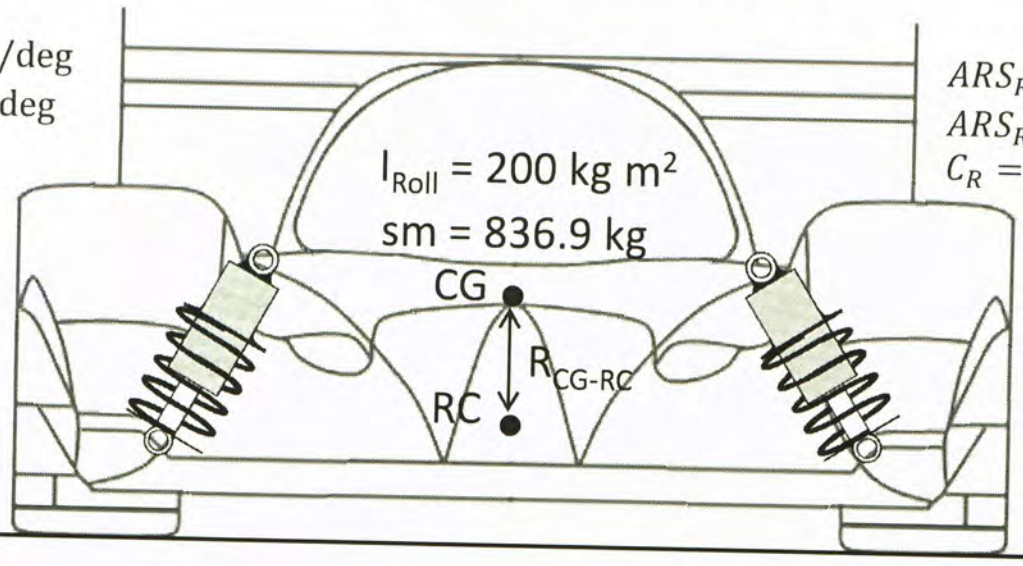
Roll Damping Distribution

Front View

$ARS_{F-Spring} = 3748.2$ Nm/deg
$ARS_{F-ARB} = 10868.3$ Nm/deg
$C_F = 7.91$ *N/(mm/s)*

$ARS_{R-Spring} = 1606.4$ Nm/deg
$ARS_{R-ARB} = 10868.3$ Nm/deg
$C_R = 5.58$ *N/(mm/s)*

$CG_z = 0.268$ m
$CG_y = 0.007$ m
$T_R = 1.592$ m



$RC_z = 0.050$ m
$RC_y = 0.000$ m
$T_F = 1.640$ m

$$Roll\ Damping\ dist. = \frac{C_{Roll\ Front}}{C_{Roll\ Front} + C_{Roll\ Rear}} \times 100$$

$$= \frac{185.6}{185.6 + 123.4} \times 100 = 60.07\%\ Front$$

60.07% : Transient Magic Number!

Modes of Vibration

Heave

Force Vs. Velocity curve

Low speed adjustments

$\zeta = 0.7$ **at the wheels :**

$C_{W-FL} = 0.7 \times C_{Crit-Fl} = 0.7 \times 11.3 = 7.91\ N/mm/s$ $\quad C_{W-RL} = 0.7 \times C_{Crit-RL} = 0.7 \times 8.00 = 5.60\ N/mm/s$

$\zeta = 0.7$ **at the damper:**

$C_{D-FL} = C_{W-FL} \times MR = 7.91 \times 1.03 = 8.14\ N/mm/s$ $\quad C_{D-RL} = C_{W-RL} \times MR = 5.60 \times 1.38 = 7.73\ N/mm/s$

$\zeta = 0.8$ **at the wheels :**

$C_{W-FL} = 0.8 \times C_{Crit-Fl} = 0.8 \times 11.3 = 9.04\ N/mm/s$ $\quad C_{W-RL} = 0.8 \times C_{Crit-RL} = 0.8 \times 8.00 = 6.40\ N/mm/s$

$\zeta = 0.8$ **at the damper:**

$C_{D-FL} = C_{W-FL} \times MR = 9.04 \times 1.03 = 9.31\ N/mm/s$ $\quad C_{D-RL} = C_{W-RL} \times MR = 6.40 \times 1.38 = 8.83\ N/mm/s$

$\zeta = 0.6$ **at the wheels :**

$C_{W-FL} = 0.6 \times C_{Crit-FL} = 0.6 \times 11.3 = 6.78\ N/mm/s$ $\quad C_{W-RL} = 0.6 \times C_{Crit-RL} = 0.6 \times 8.00 = 4.80\ N/mm/s$

$\zeta = 0.6$ **at the damper:**

$C_{D-FL} = C_{W-FL} \times MR = 6.78 \times 1.03 = 6.98\ N/mm/s$ $\quad C_{D-RL} = C_{W-RL} \times MR = 4.80 \times 1.38 = 6.62\ N/mm/s$

Note: symmetric in bump and rebound

Modes of Vibration

Heave

Force Vs. Velocity curve

High speed adjustments

$\zeta = 0.35$ **at the wheels** :

$C_{W-FL} = 0.35 \times C_{Crit-Fl} = 0.35 \times 11.3 = 3.96\ N/mm/s$ $\quad C_{W-RL} = 0.7 \times C_{Crit-RL} = 0.35 \times 8.00 = 2.80\ N/mm/s$

$\zeta = 0.35$ **at the damper:**

$C_{D-FL} = C_{W-FL} \times MR = 3.96 \times 1.03 = 4.07\ N/mm/s$ $\quad C_{D-RL} = C_{W-RL} \times MR = 2.80 \times 1.38 = 3.86\ N/mm/s$

$\zeta = 0.4$ **at the wheels** :

$C_{W-FL} = 0.4 \times C_{Crit-Fl} = 0.4 \times 11.3 = 4.52\ N/mm/s$ $\quad C_{W-RL} = 0.4 \times C_{Crit-RL} = 0.4 \times 8.00 = 3.20\ N/mm/s$

$\zeta = 0.4$ **at the damper:**

$C_{D-FL} = C_{W-FL} \times MR = 4.52 \times 1.03 = 4.66\ N/mm/s$ $\quad C_{D-RL} = C_{W-RL} \times MR = 3.20 \times 1.38 = 4.42\ N/mm/s$

$\zeta = 0.3$ **at the wheels** :

$C_{W-FL} = 0.3 \times C_{Crit-FL} = 0.3 \times 11.3 = 3.39\ N/mm/s$ $\quad C_{W-RL} = 0.3 \times C_{Crit-RL} = 0.3 \times 8.00 = 2.40\ N/mm/s$

$\zeta = 0.3$ **at the damper:**

$C_{D-FL} = C_{W-FL} \times MR = 3.39 \times 1.03 = 3.49\ N/mm/s$ $\quad C_{D-RL} = C_{W-RL} \times MR = 2.40 \times 1.38 = 3.31\ N/mm/s$

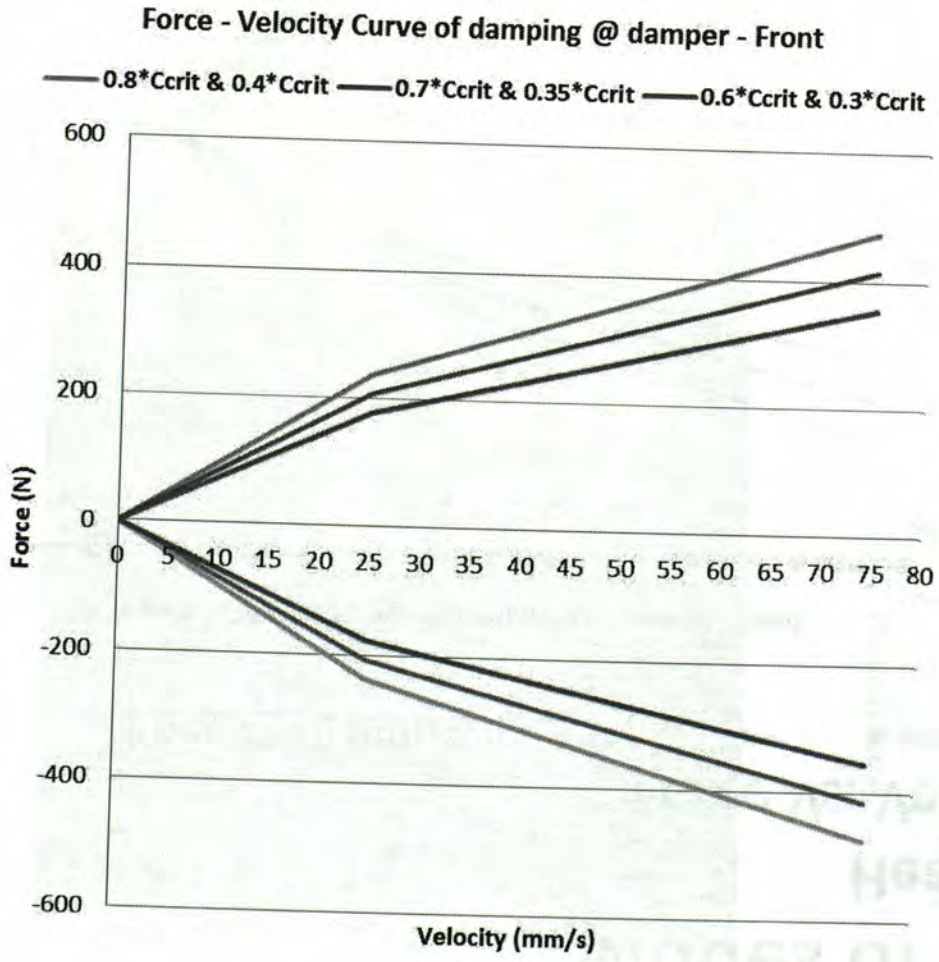
Note: symmetric in bump and rebound

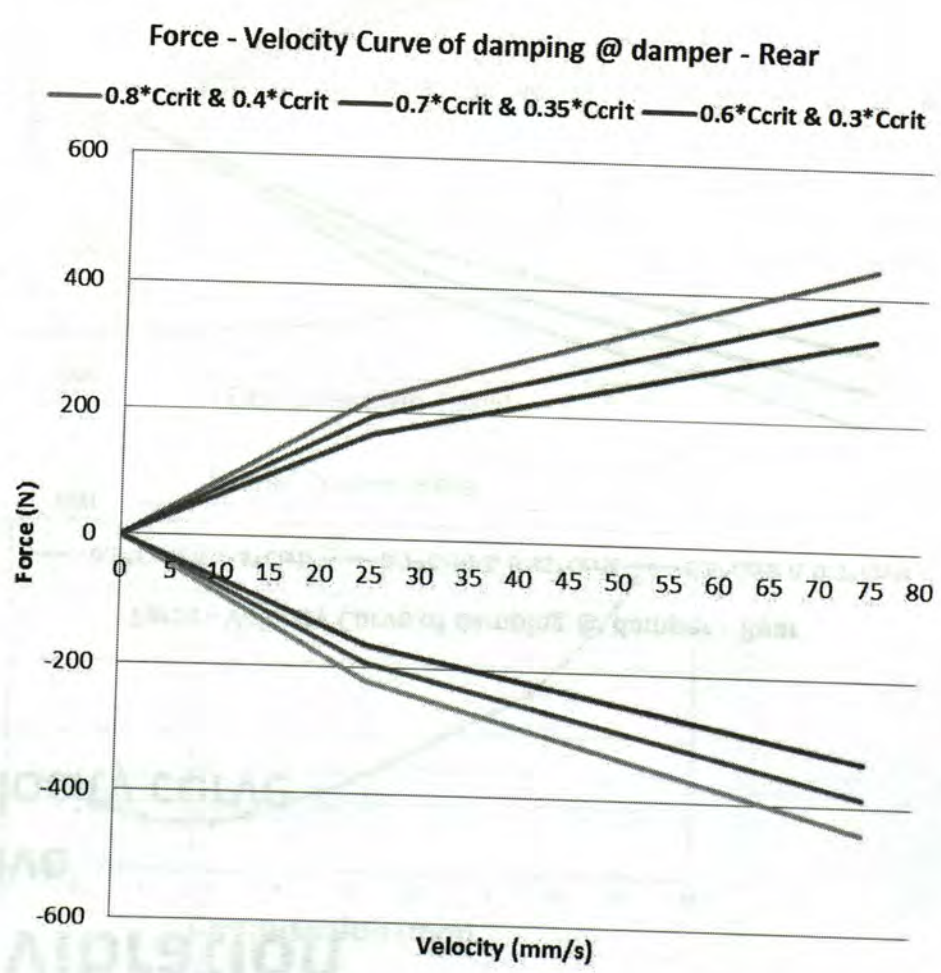
Modes of Vibration

Heave

Force Vs. Velocity curve

Knee at 25 mm/s:





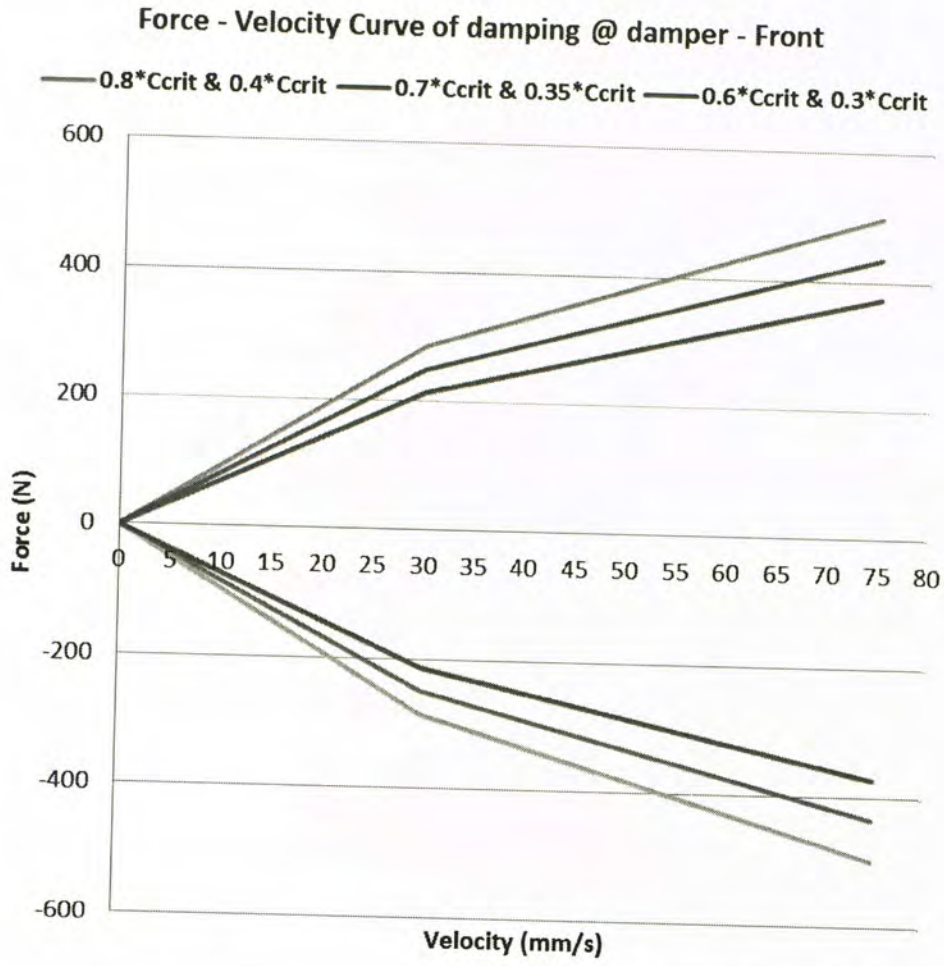
Note: symmetric in bump and rebound

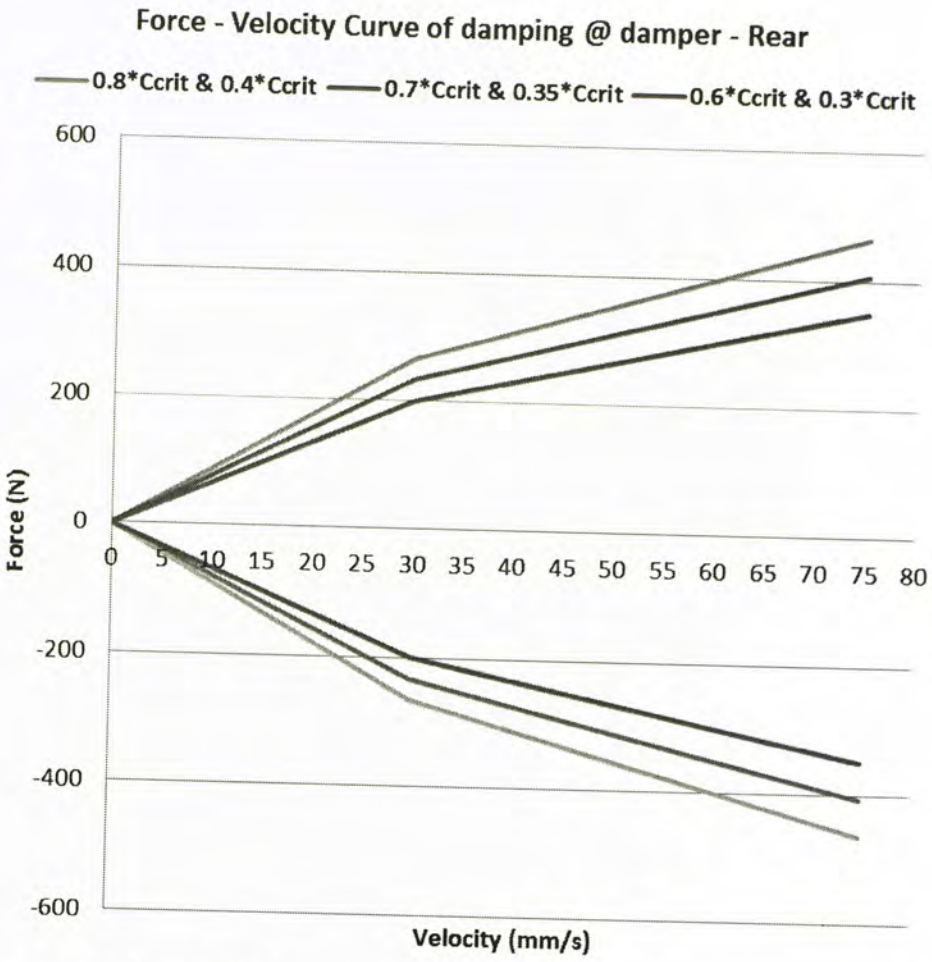
Modes of Vibration

Heave

Force Vs. Velocity curve

Knee at 30 mm/s:



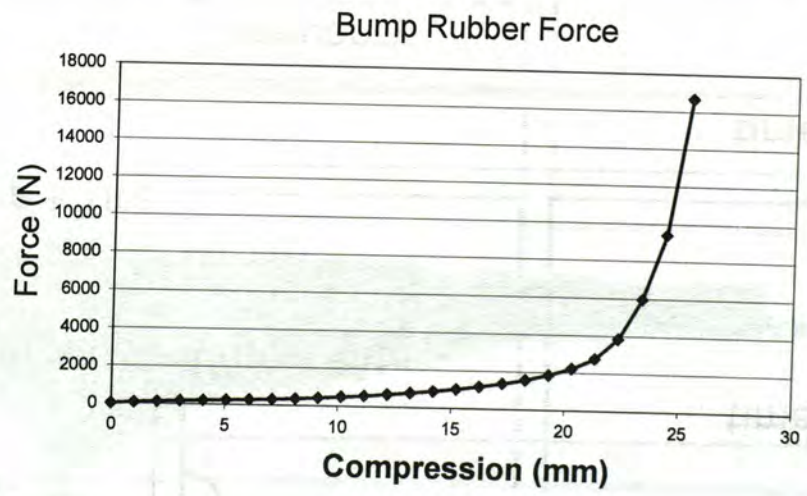


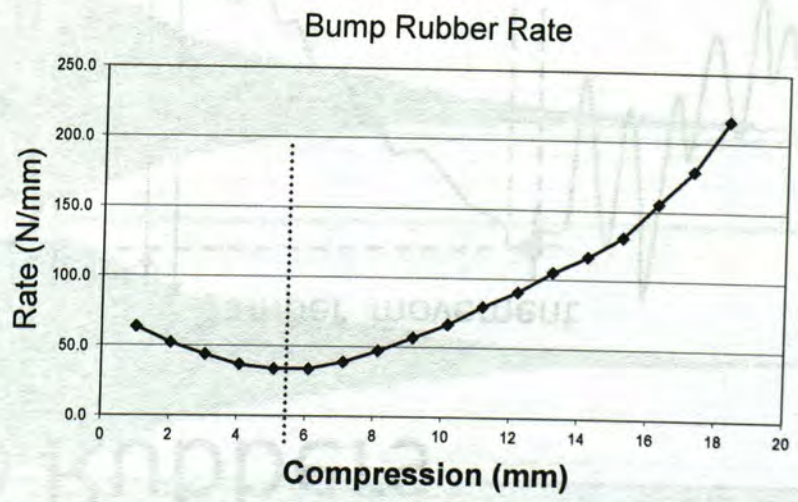
Note: symmetric in bump and rebound

Bump Rubbers

Very nonlinear and difficult to determine amount of damping

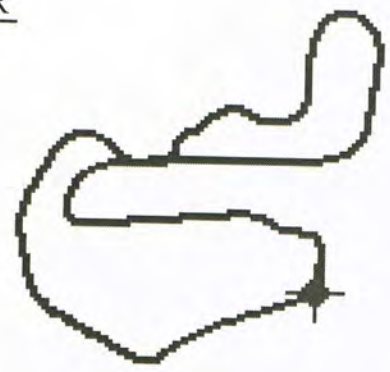
Compression mm	Force N		Rate (N/mm)
0.00	0.0		
1.02	64.5		63.5
2.03	117.4		52.1
3.05	161.9		43.8
4.06	199.3		36.8
5.08	233.5		33.7
6.10	267.8		33.7
7.11	307.4		39.0
8.13	355.4		47.3
9.14	413.2		56.9
10.16	480.9		66.5
11.18	561.8		79.7
12.19	653.9		90.6
13.21	760.6		105.1
14.22	879.0		116.5
15.24	1012.0		130.9
16.26	1169.9		155.4
17.27	1351.8		179.1
18.29	1570.7		215.4
19.30	1851.3		276.3
20.32	2223.7		366.5
21.34	2763.2		531.1
22.35	3847.7		1067.4
23.37	5960.6		2079.6
24.38	9381.3		3366.8
25.40	16675.5		7179.3

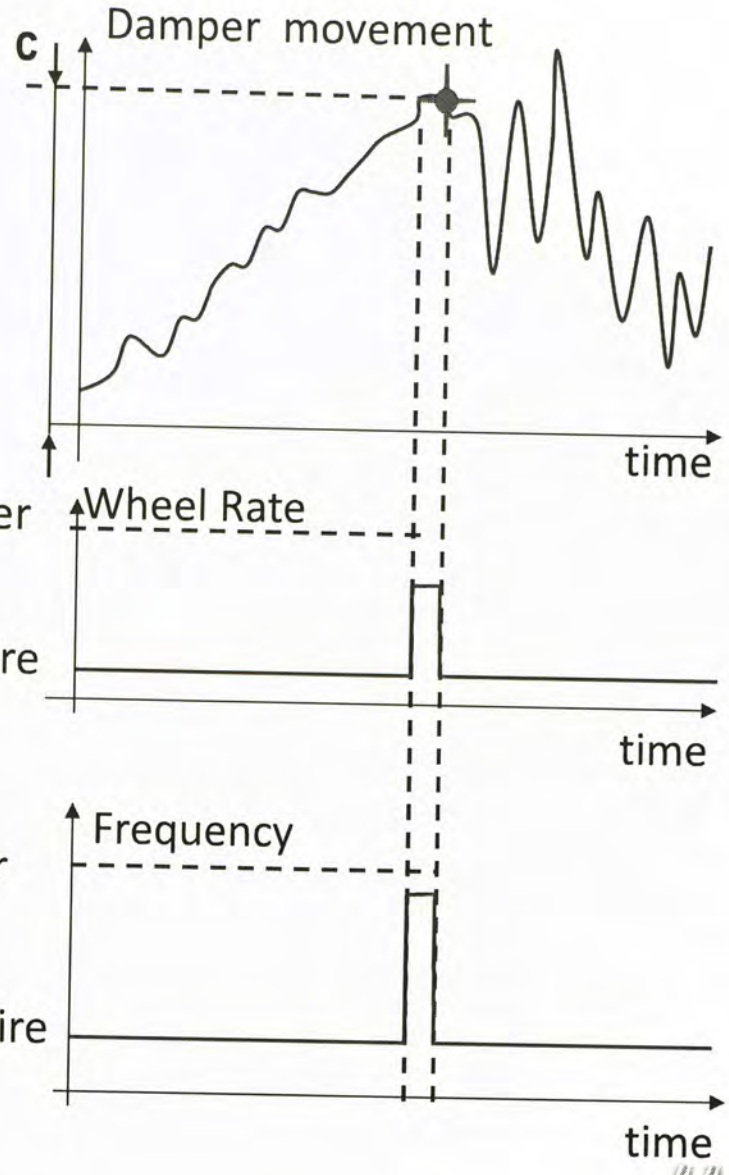


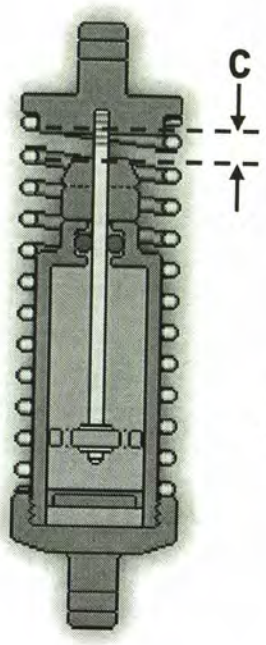


Bump Rubbers

$$f_N = \frac{1}{2\pi} \times \sqrt{\frac{K_{s+t+\mathrm{BR}}}{m_s}}$$







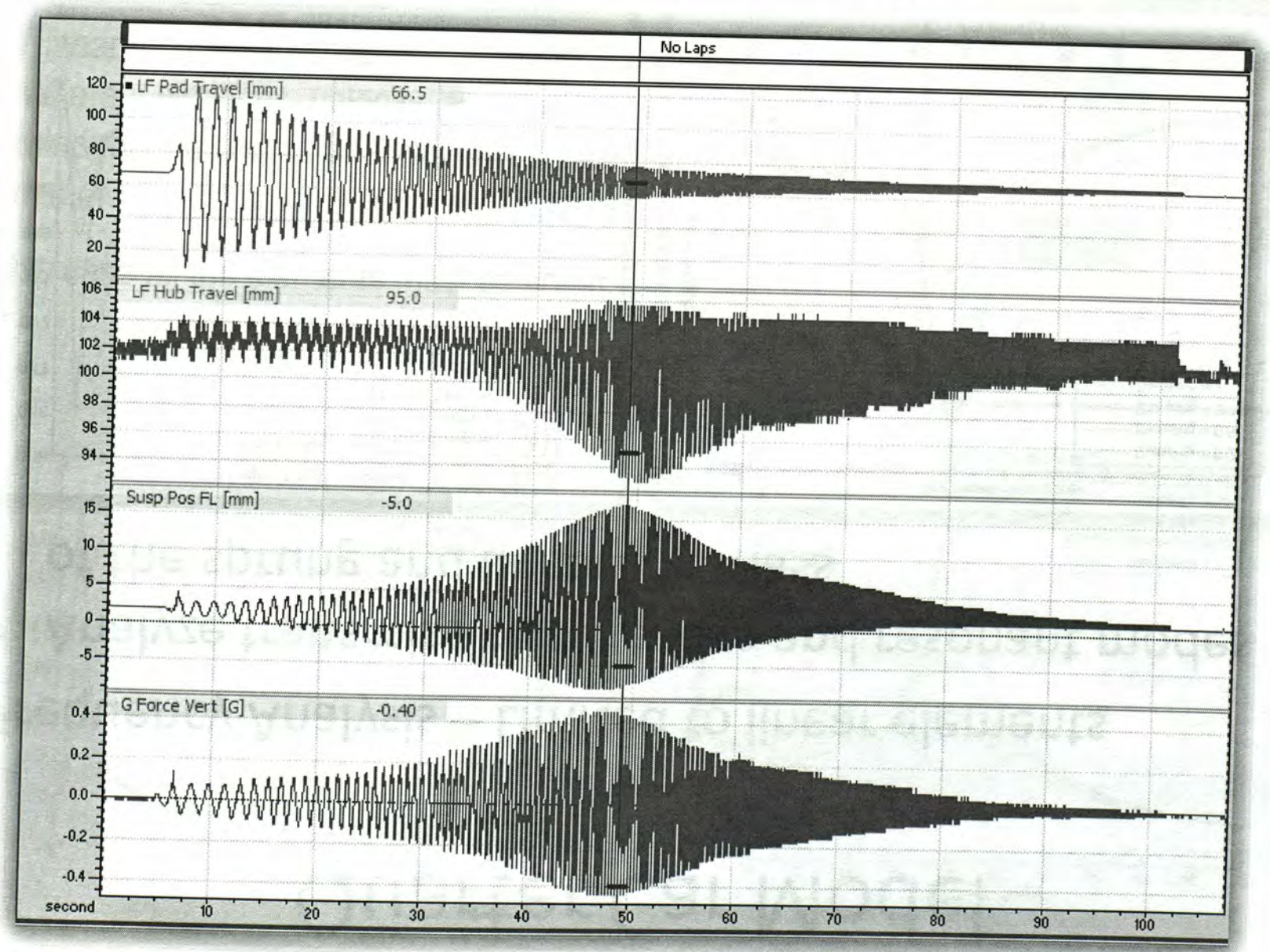
$K_{\text{Spring + Tire + Bump Rubber}}$

$K_{\text{Spring + Tire}}$

$f_{\text{Spring + Tire + Bump Rubber}}$

$f_{\text{Spring + Tire}}$

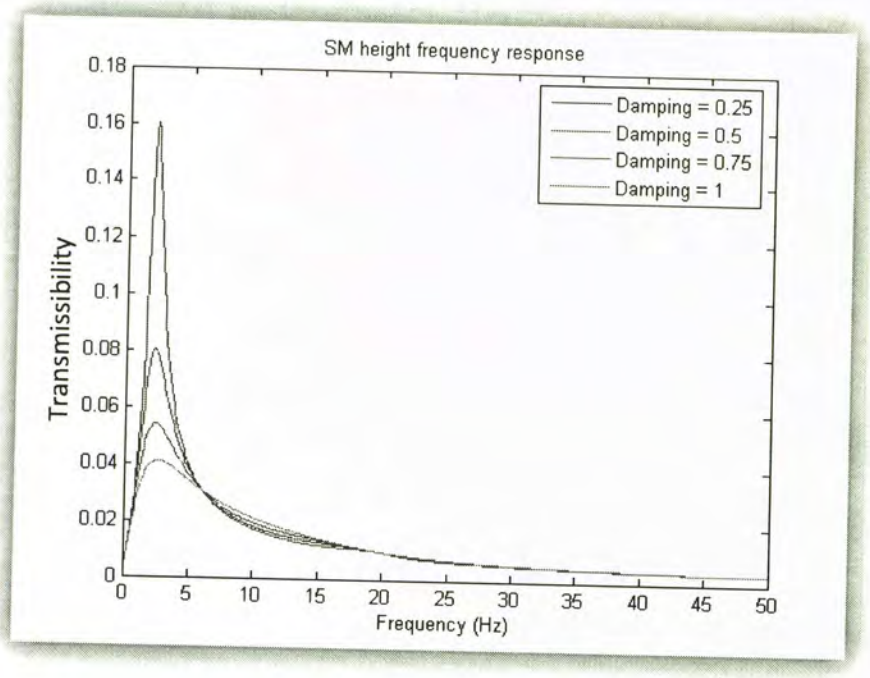
Vibration Theory

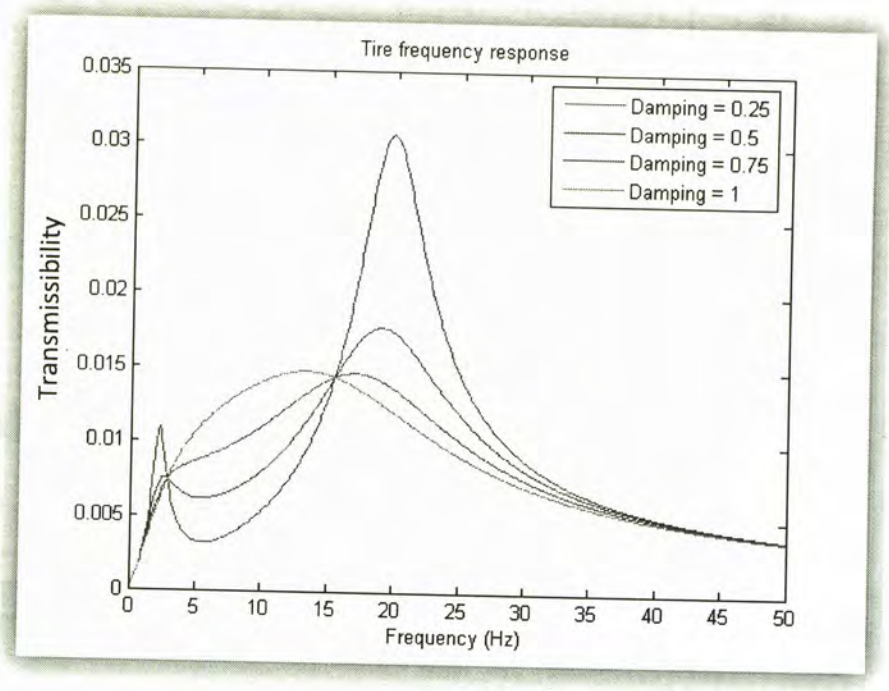


Quarter Car Model

Frequency Analysis – Limited to linear elements

- Analyze transmissibility, phase and resonant modes of the sprung and unsprung mass

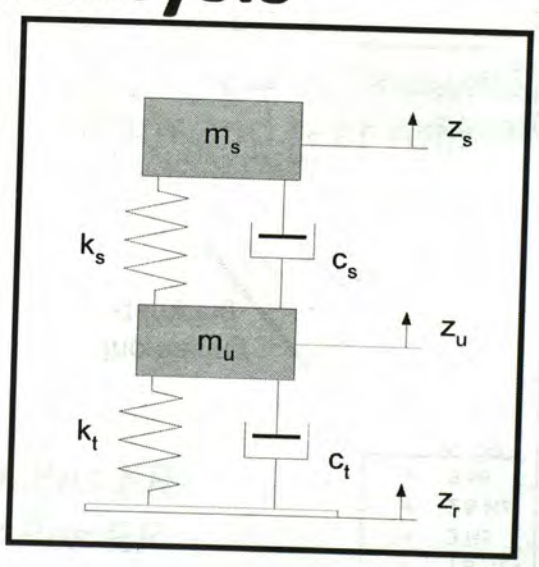


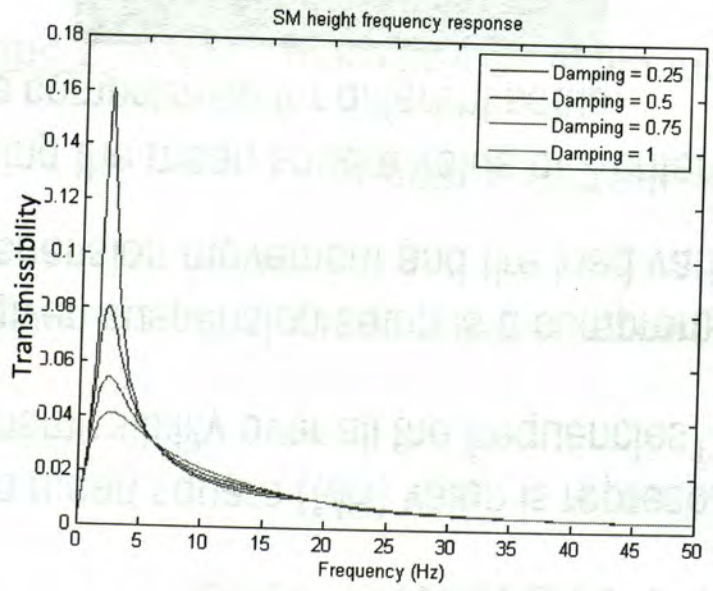


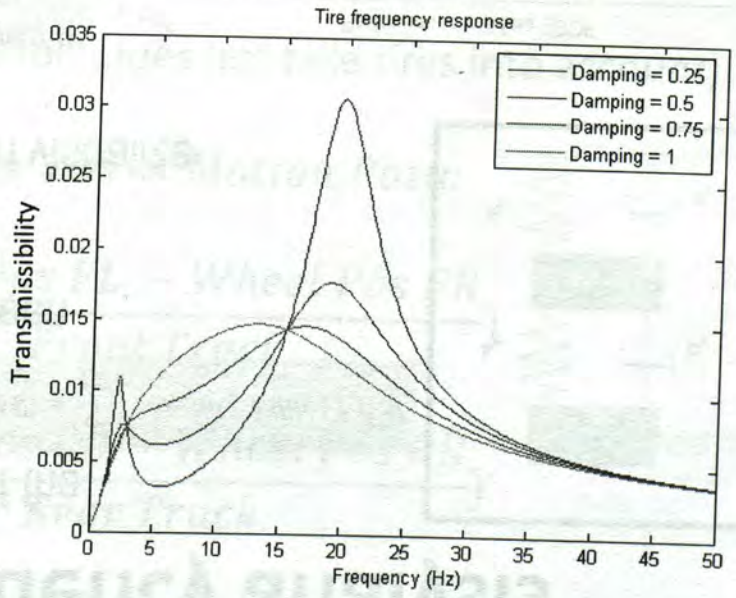
Simulation : Frequency analysis

The linear theory is used to calculate the transmissibility of the different positions, velocity, acceleration and forces to road inputs. Using this analysis we can see the resonance modes of the sprung and unsprung masses.

In this case all the elements must be linear.



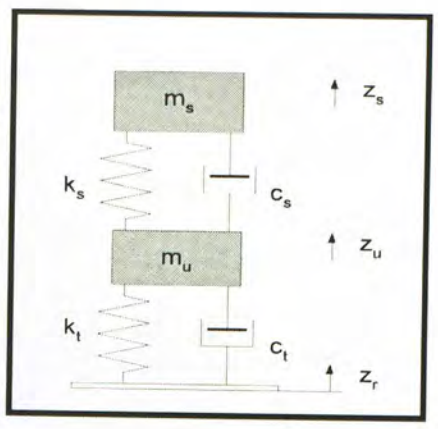




Simulation : Frequency analysis

The mean square (MS) value is representative of the transmissibility over all the frequencies.

$$MS = \sum_{f=0.1}^{f=50} Transmissibility(f)^2 df$$

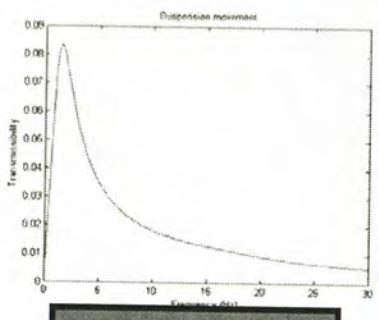


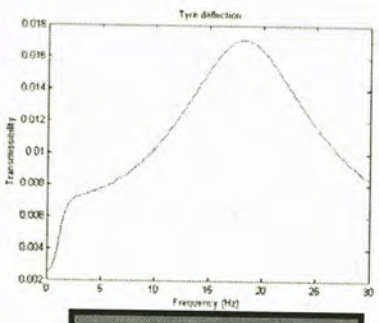
A given suspension setup is a compromise between suspension movement and tire load variation.

Using the mean square value of 2 criteria we can visualize the compromise for different setup.

Example of one setup :

f = 1.5Hz c = $0.7c_{crit}$

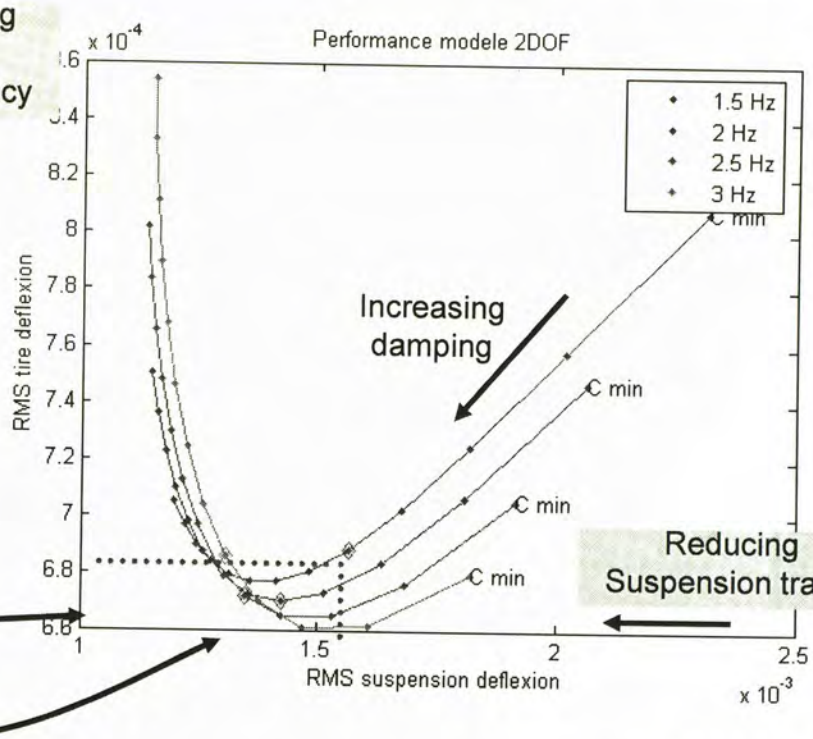




susp_rms = 0.0016

tire_rms = 6.8864e-004

Improving Grip Consistency



Lag or Lead in Roll

Stability issues could come from a lag or lead between front and rear roll.

Roll calculation

Method 1 : Gives a relative estimation of the roll

$$Front\ Roll = Damper\ Pos\ FL - Damper\ Pos\ FR$$
$$Rear\ Roll = Damper\ Pos\ RL - Damper\ Pos\ RR$$

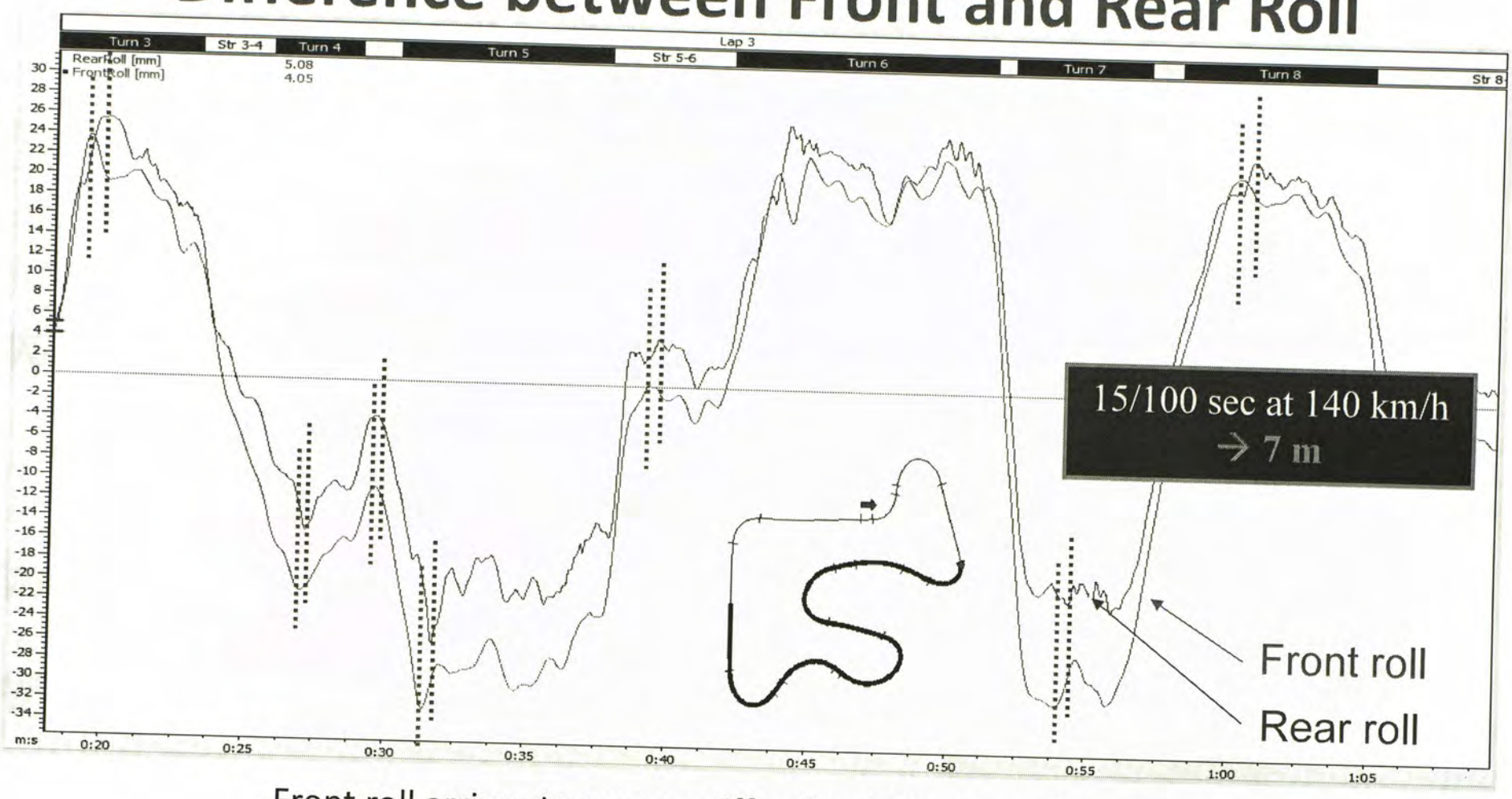
Method 2 : Gives a more precise estimation of the roll (does not take tires into account)

$$Wheel\ Position = Damper\ Pos \times Motion\ Ratio$$

$$Front\ Roll = Atan\left(\frac{Wheel\ Pos\ FL - Wheel\ Pos\ FR}{Front\ Track}\right)$$

$$Rear\ Roll = Atan\left(\frac{Wheel\ Pos\ RL - Wheel\ Pos\ RR}{Rear\ Track}\right)$$

Damper Speed Histogram
Difference between Front and Rear Roll

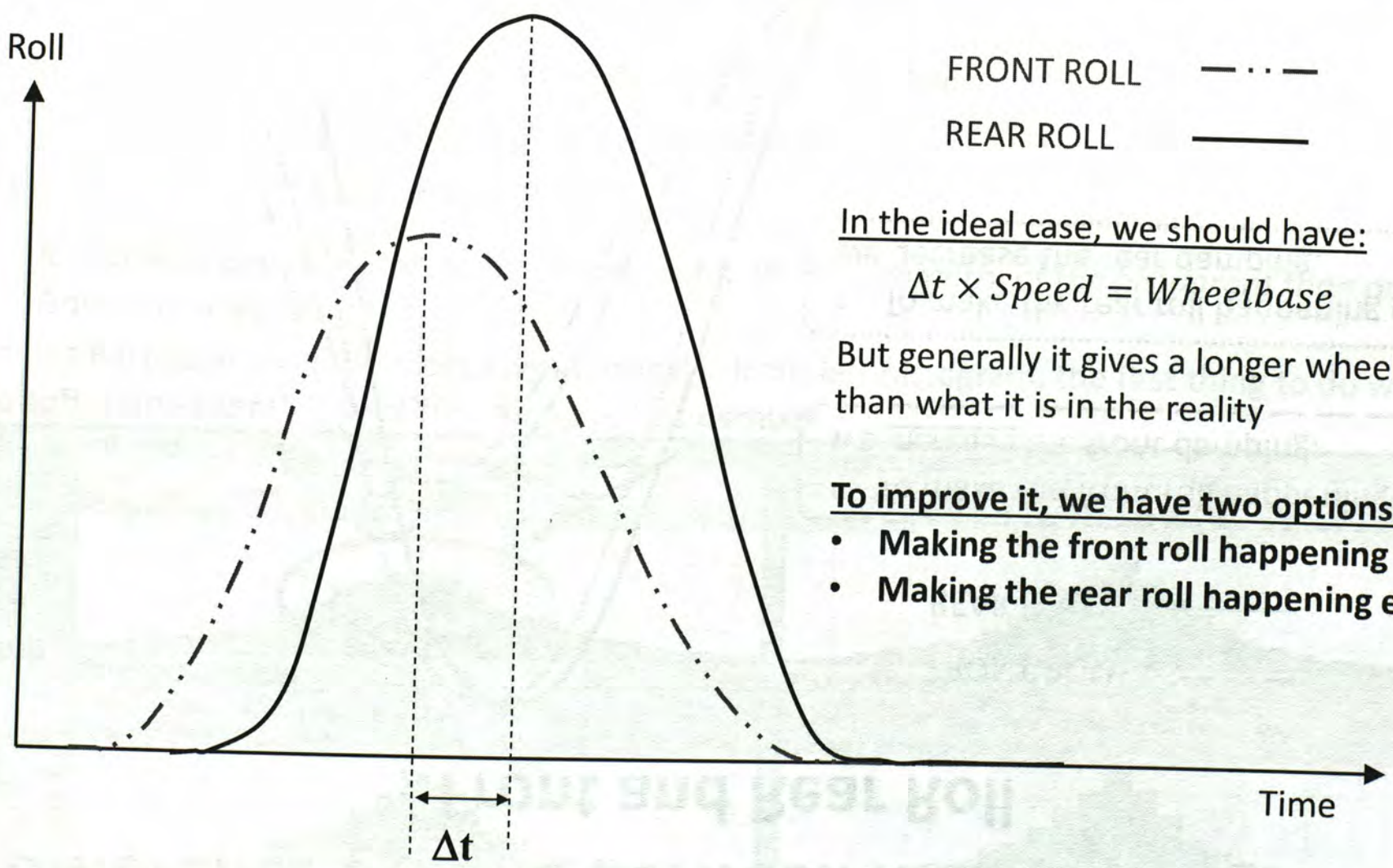


-Front roll arrives too soon: stiffen front bump and/or rebound

-Rear roll arrives too late: soften rear bump and/or rebound

Be careful! Before making the final decision, look at the damper histogram on the complete lap.

Difference of Time Between Maximum Value of Front and Rear Roll



<u>In the ideal case, we should have:</u>

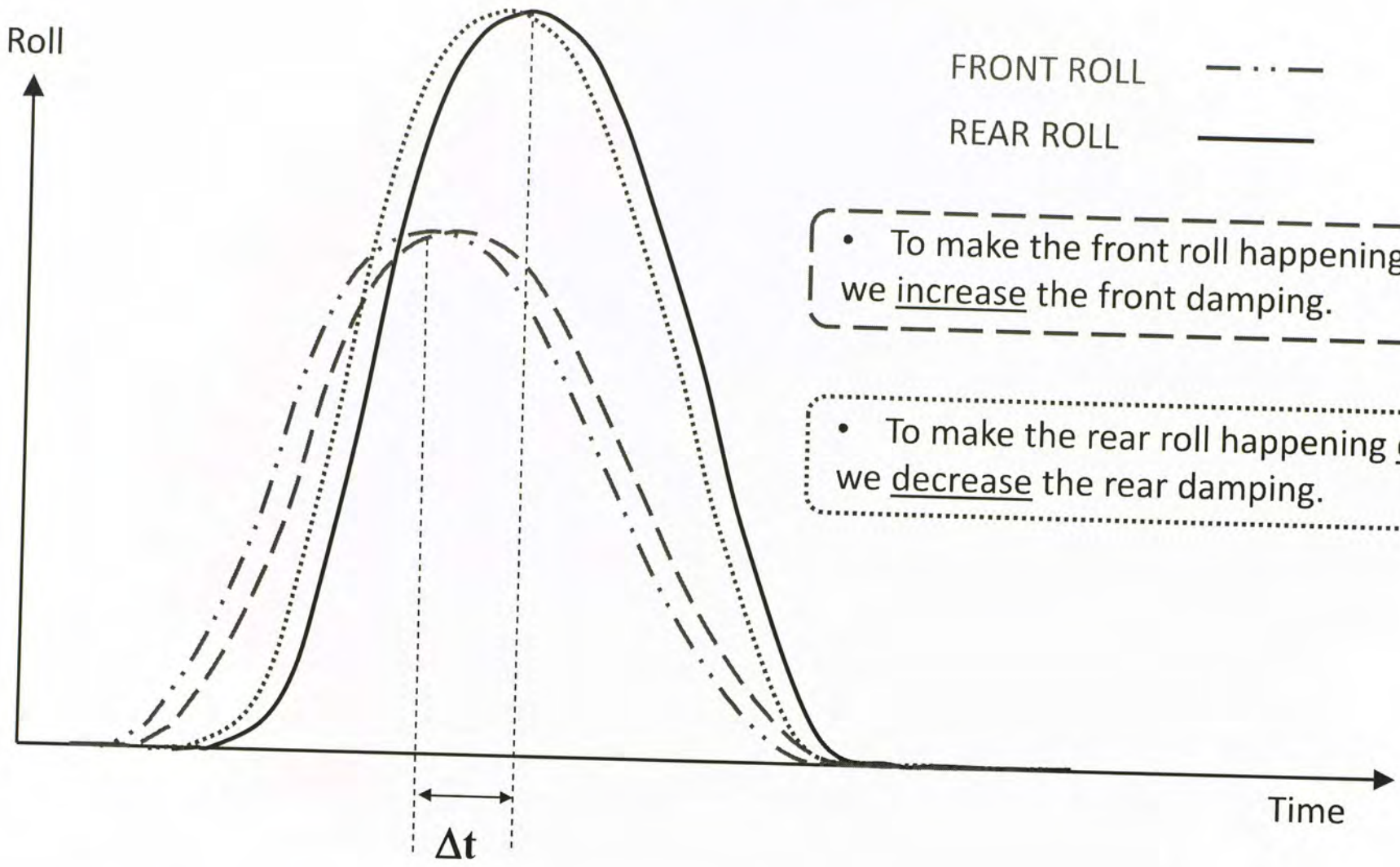
$$\Delta t \times Speed = Wheelbase$$

But generally it gives a longer wheelbase than what it is in the reality

<u>To improve it, we have two options:</u>

- **Making the front roll happening later**
- **Making the rear roll happening earlier**

Difference of Time Between Maximum Value of Front and Rear Roll



- To make the front roll happening <u>later</u> we <u>increase</u> the front damping.

- To make the rear roll happening <u>earlier</u> we <u>decrease</u> the rear damping.

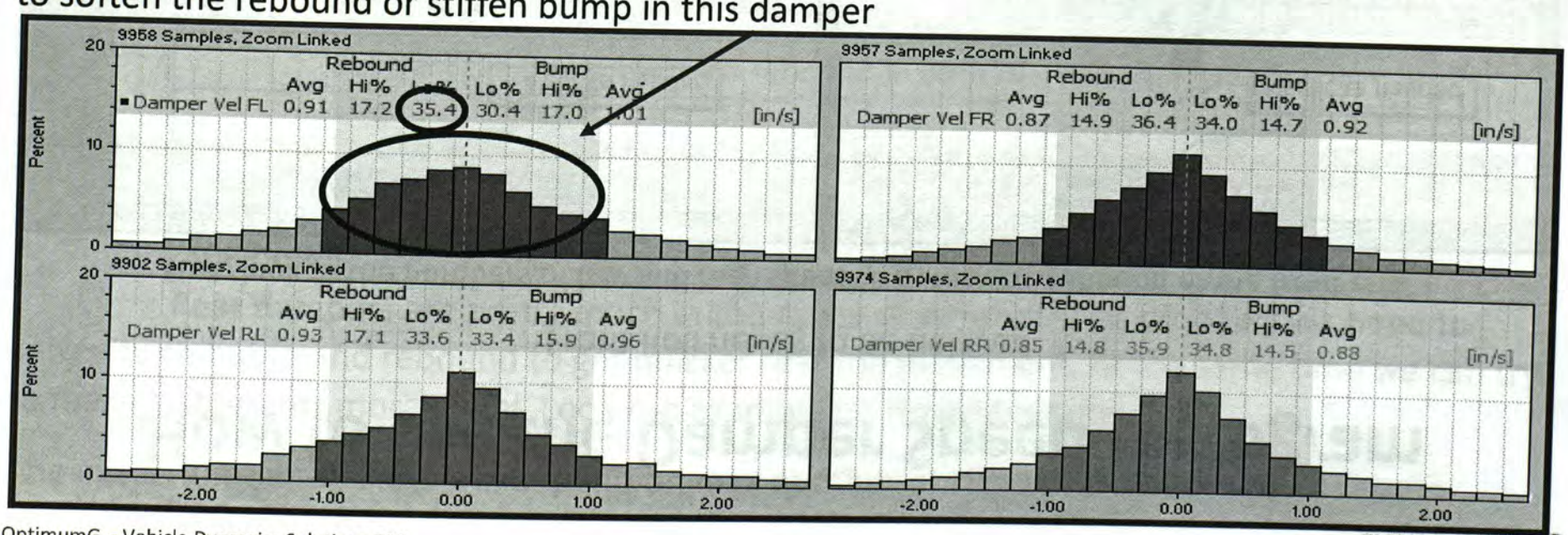
How to use the Damper Speed Histogram

<u>Steps of reasoning:</u>

1) Which car end do we have to stiffen or soften, front or rear?
2) On which side do we have to work first, right or left?
3) Which Gauss curve do I expect?

It is almost always better to first have a look on the symmetry of the histogram than on the Gauss curve expected.

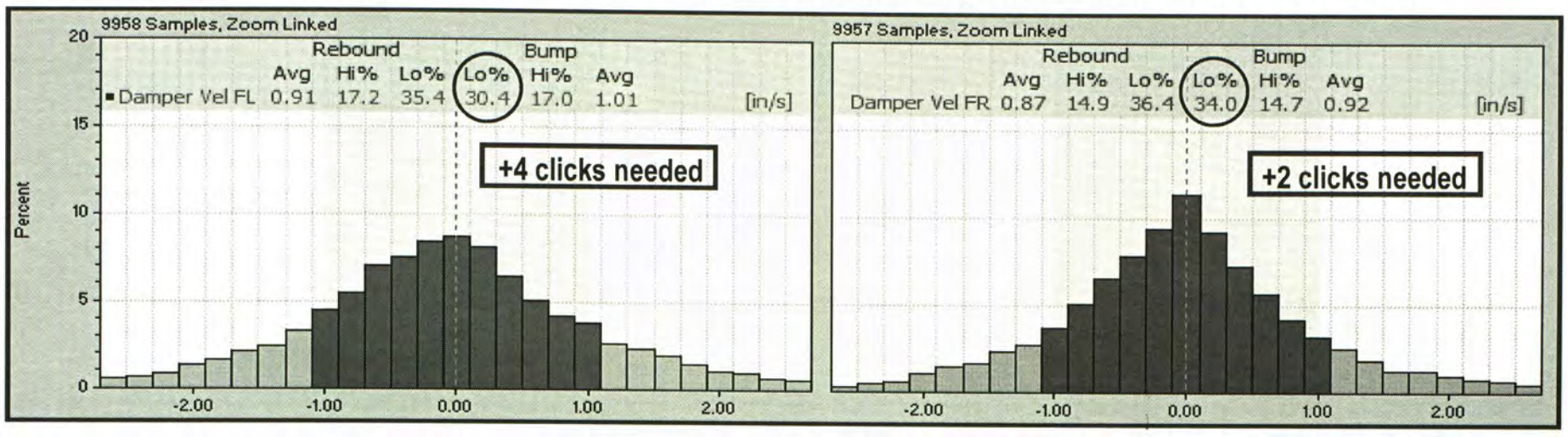
In this particular case the most asymmetrical is front left histogram, the first thing to do will be to soften the rebound or stiffen bump in this damper



How to use the Damper Speed Histogram

Front Roll needs to be delayed

First, look at the numbers of low and high speed in bump and rebound of the front end.



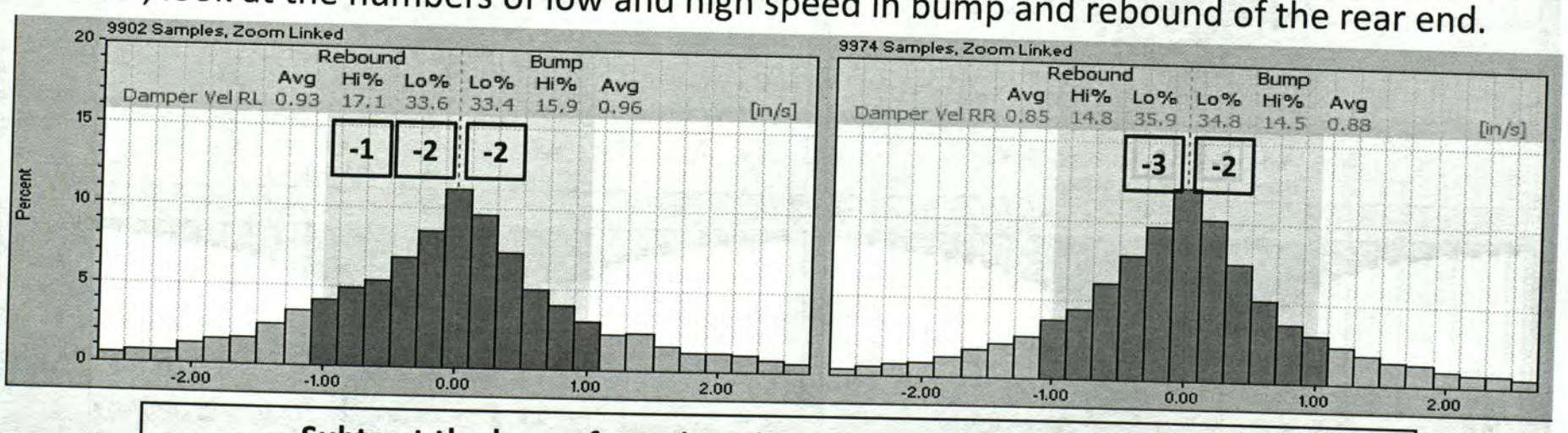
Subtract the bump from the rebound in both low and high speed		
	Left Front	Right Front
(Rebound – Bump) Low Speed	5.0	2.4
(Rebound – Bump) High Speed	0.2	0.2

In this case, it will be better to stiffen both front dampers low speed bump: for example 4 clicks on left and 2 clicks on right low speed bump. The exact number of clicks will depend on max number of clicks and damper dyno results.

How to use the Damper Speed Histogram

Rear Roll needs to be advanced

First, look at the numbers of low and high speed in bump and rebound of the rear end.



Subtract the bump from the rebound in both low and high speed		
	Left Rear	Right Rear
(Rebound – Bump) Low Speed	0.2	1.1
(Rebound – Bump) High Speed	1.2	0.3

Rear dampers are pretty much in the range of symmetry, so probably we need to soften both bump and rebound to get quicker rear roll movement. At the same time we can try to make histograms more equal. Look the example on the histograms.

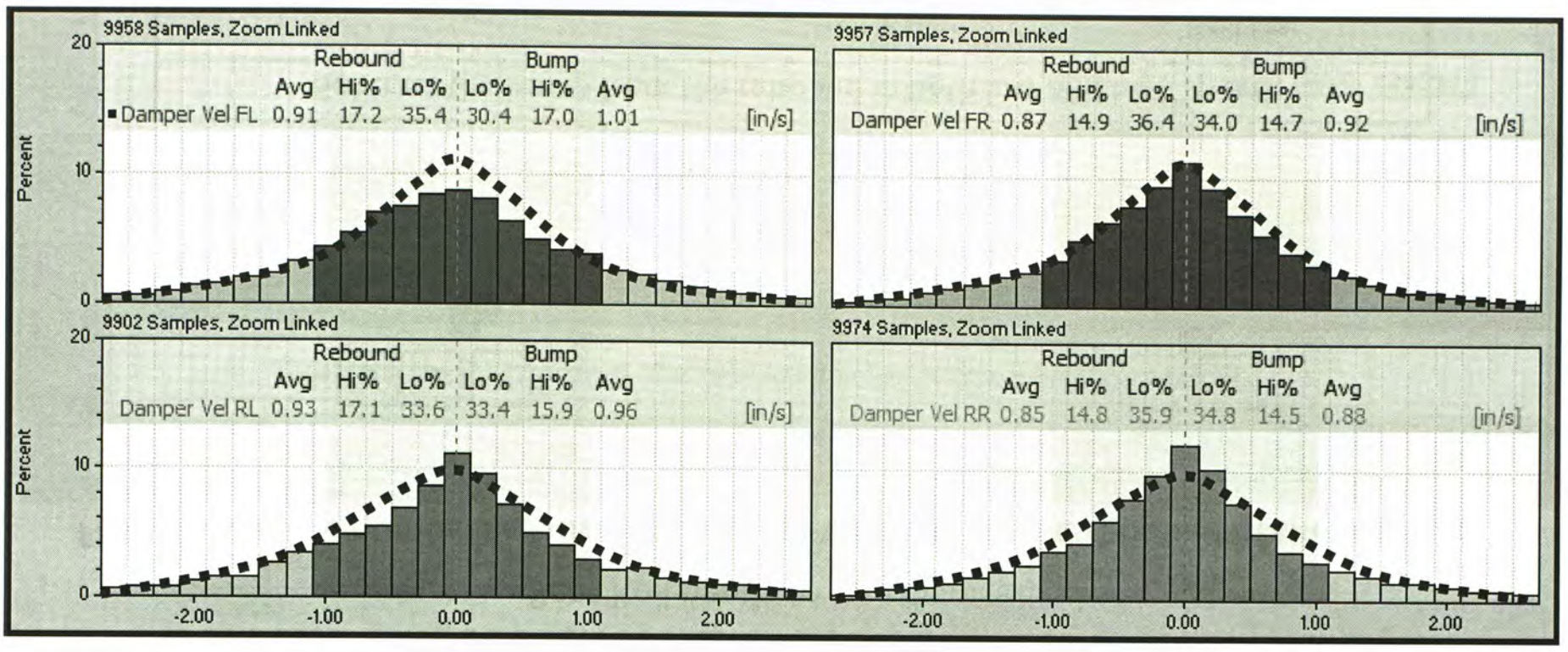
The exact number of clicks will depend on max number of clicks and damper dyno results

How to use the Damper Speed Histogram

Regarding the ideal Gauss curve…

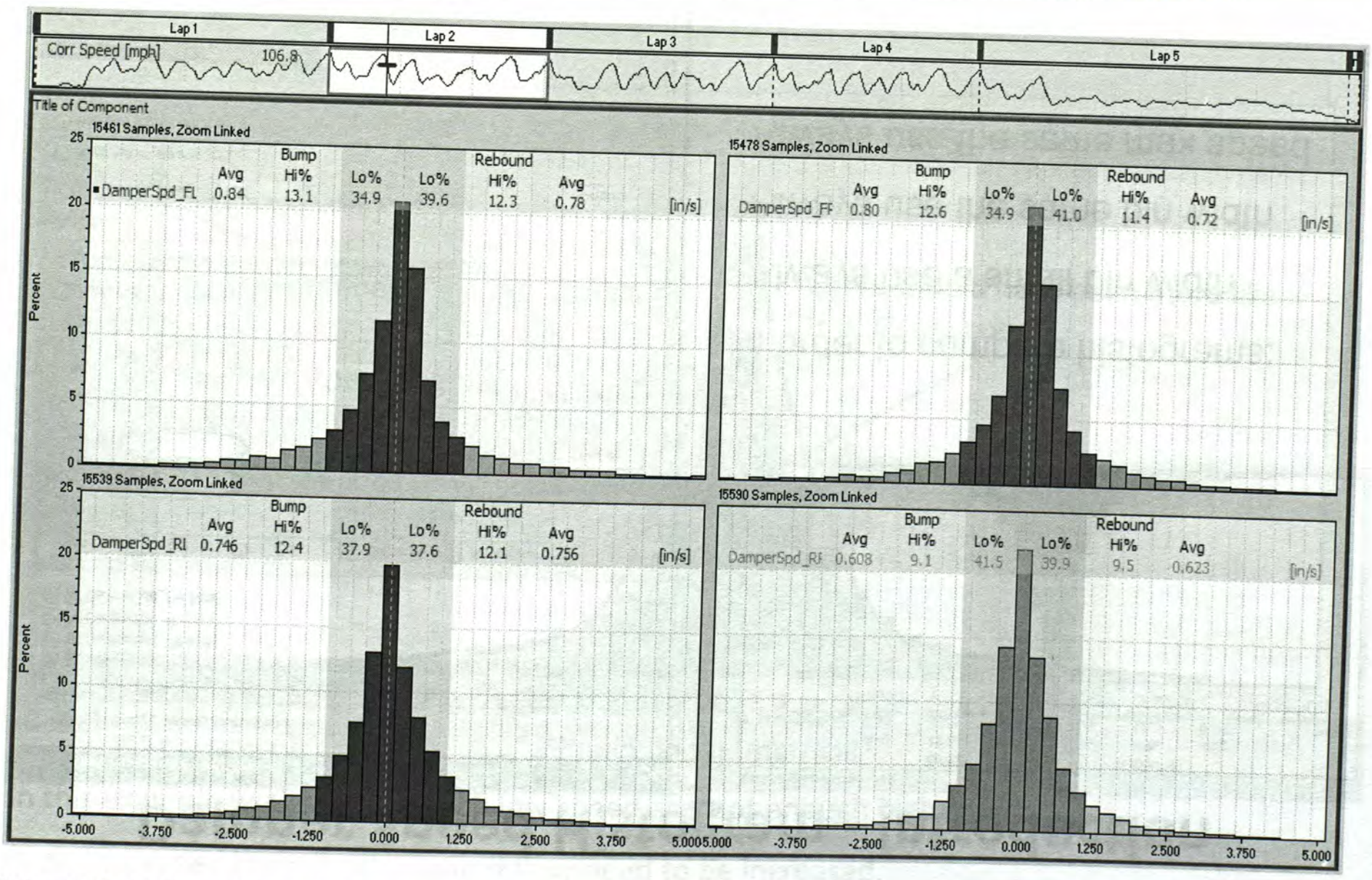
We need to stiffen front bump and/or rebound to delay front roll

By experience, we know that the ideal Gauss curve for this track is:

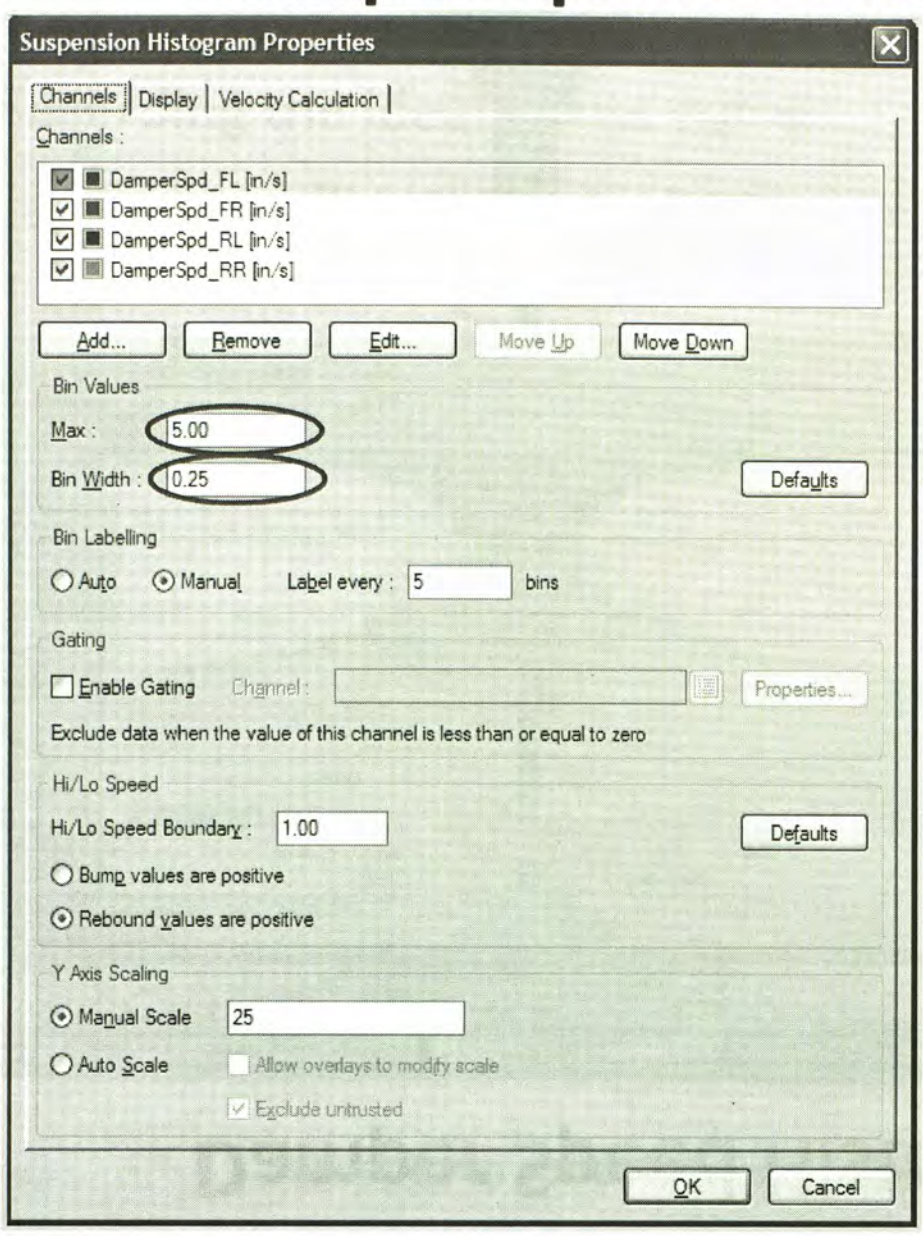


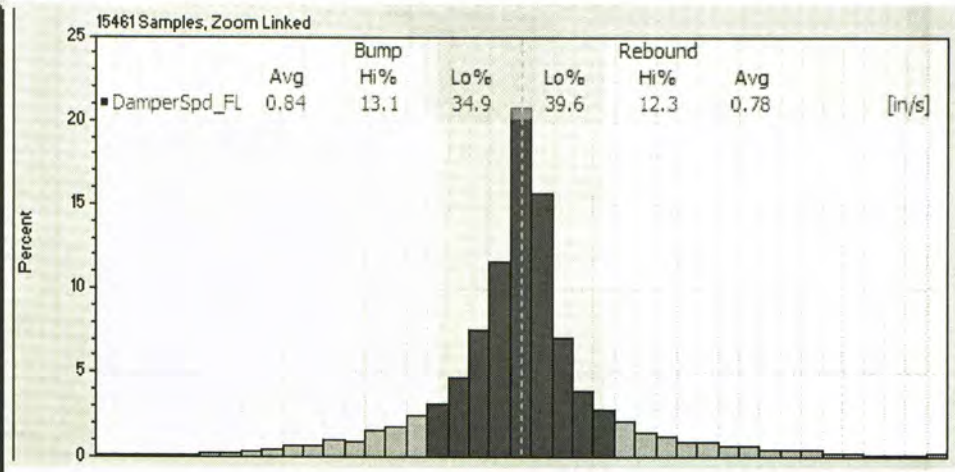
We need to softer rear bump and/or rebound to advance rear roll

Damper Speed Histogram - Introduction



Damper Speed Histogram - Introduction





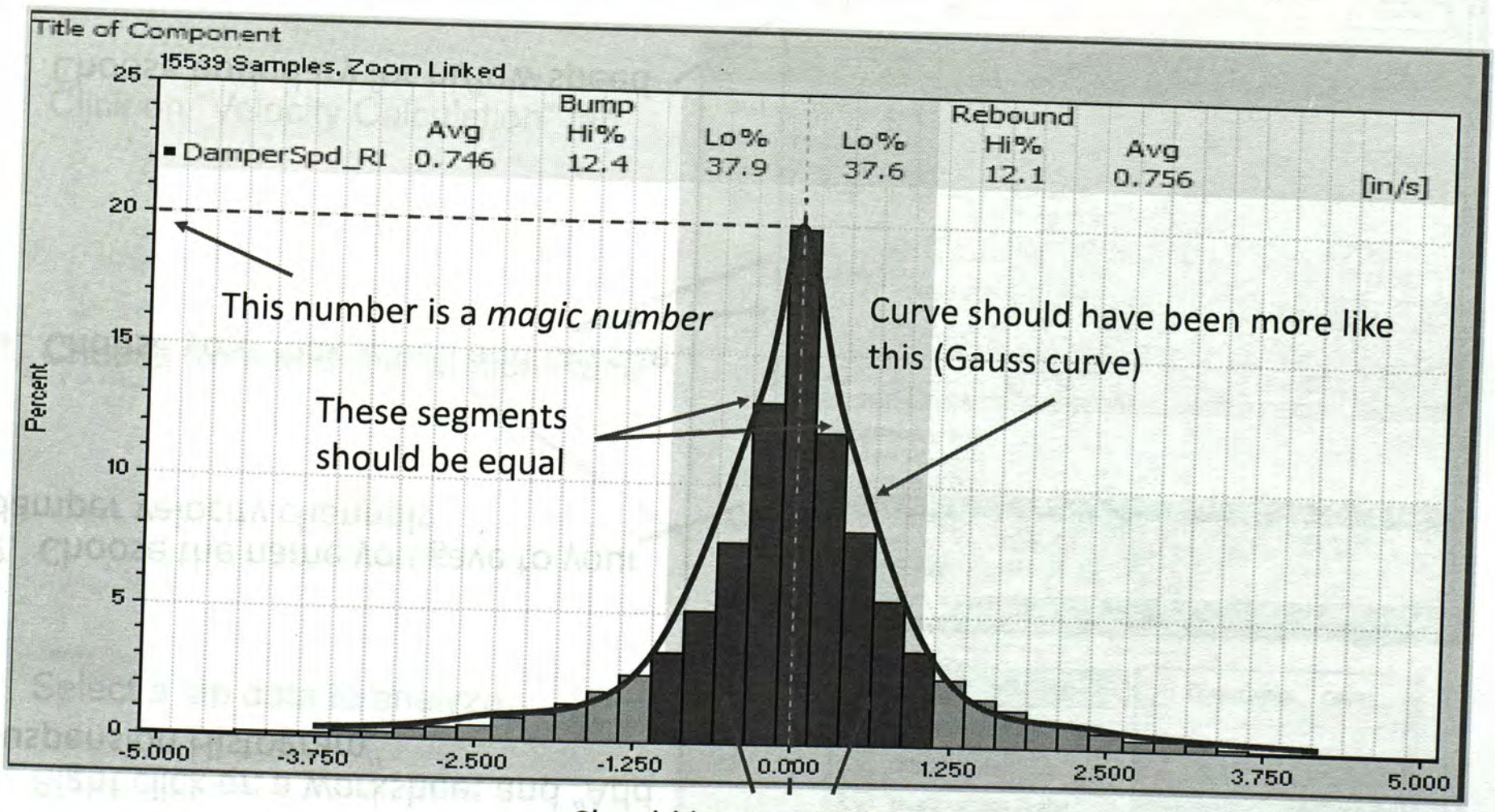
In order to compare histograms:

-Always use a small bin width

-Always use the same bin width

-Always use the same max speed

Damper Speed Histogram – Ideal histogram



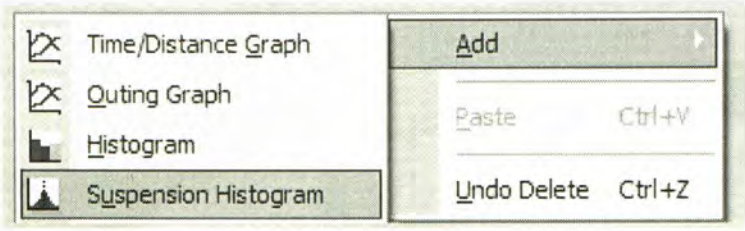
Should be symmetrical

In this example we have too much low speed and not enough high speed.

Both low speed bump and rebound flow need to be increased.

Damper Speed Histogram – How to…

1. Right click on a worksheet and "Add Suspension Histogram"

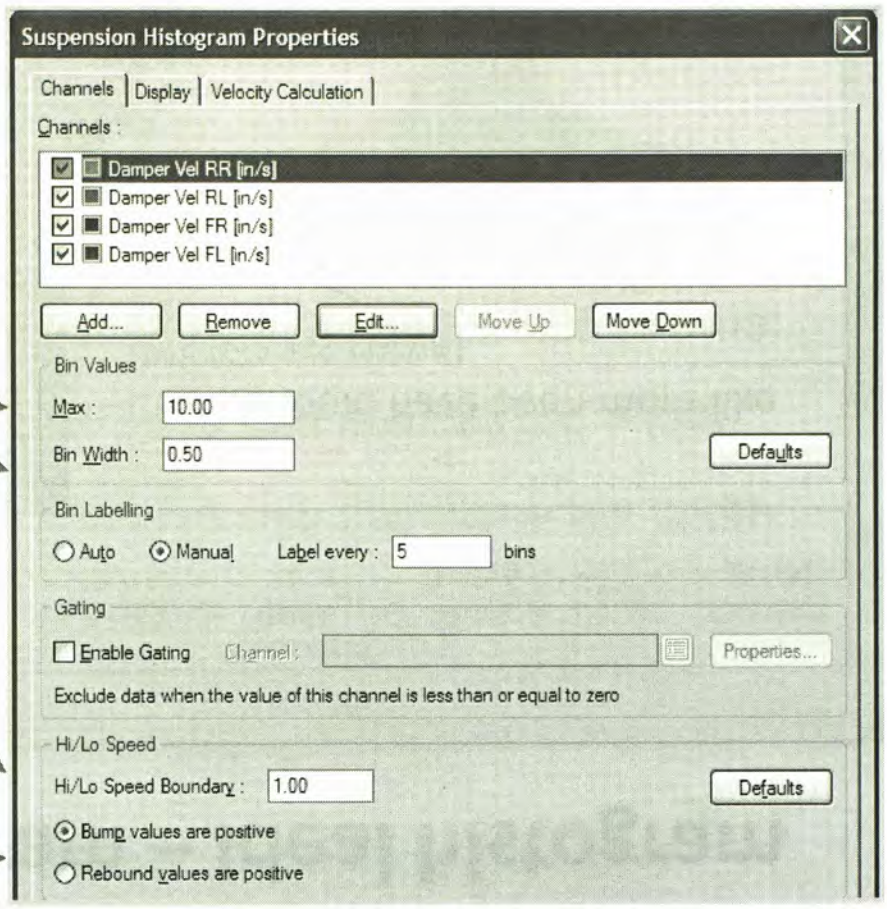


2. Choose the name you gave to your damper velocity channels

3. Choose your max speed and bin size

4. Choose boundary for hi/low speed

5. Determine if bump values are positive

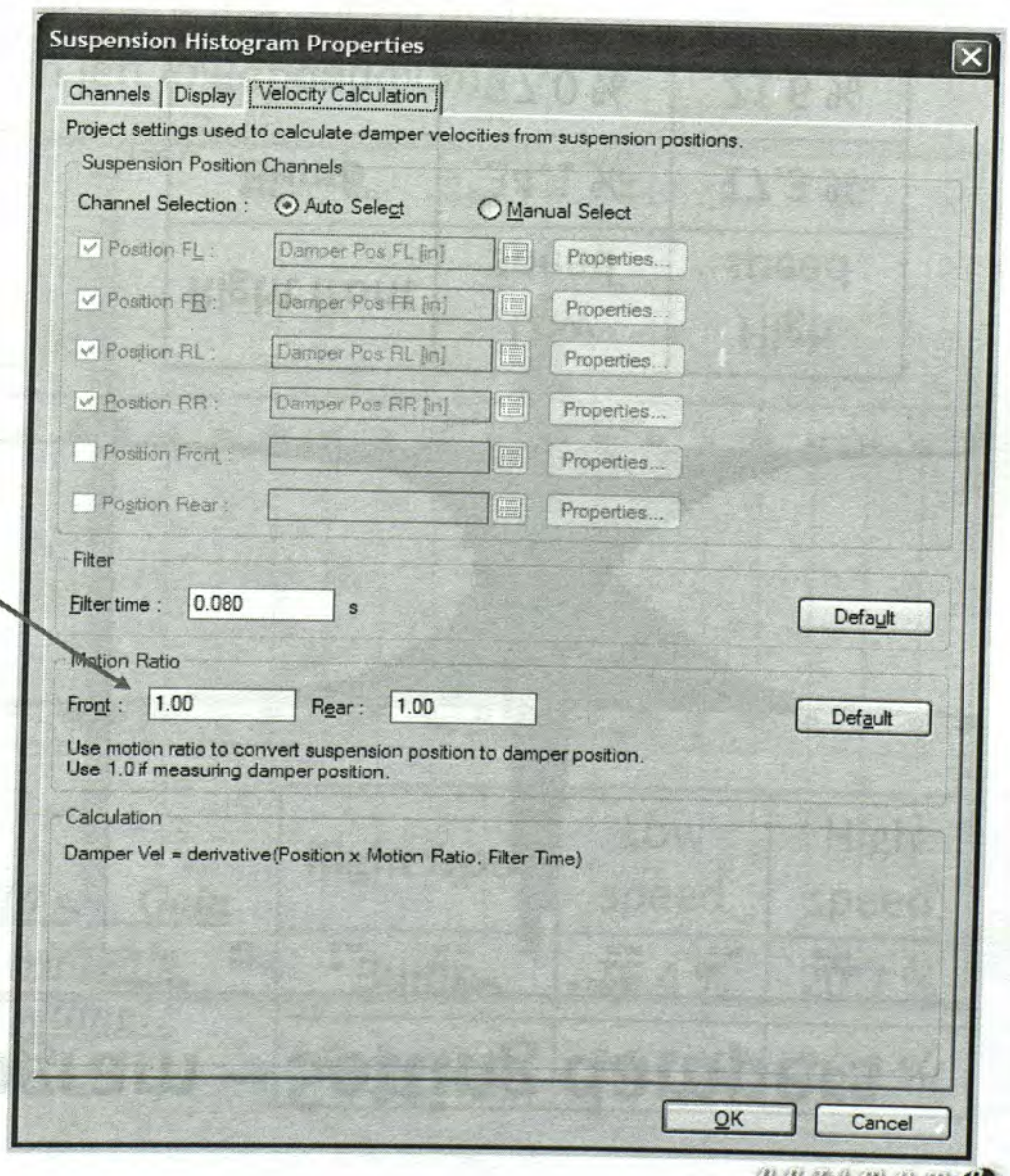


Damper Speed Histogram – How to…

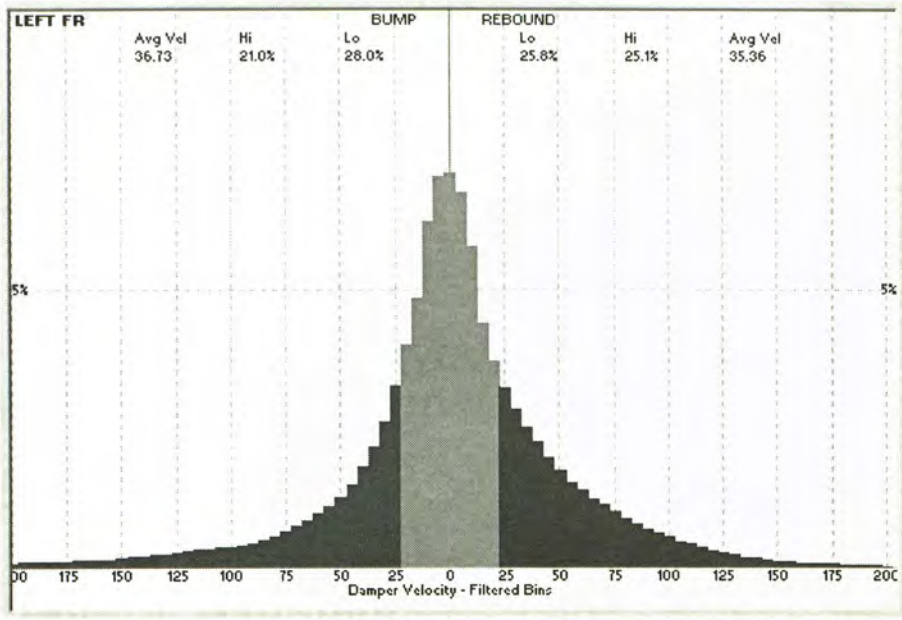
6. Click on "Velocity Calculation" tab

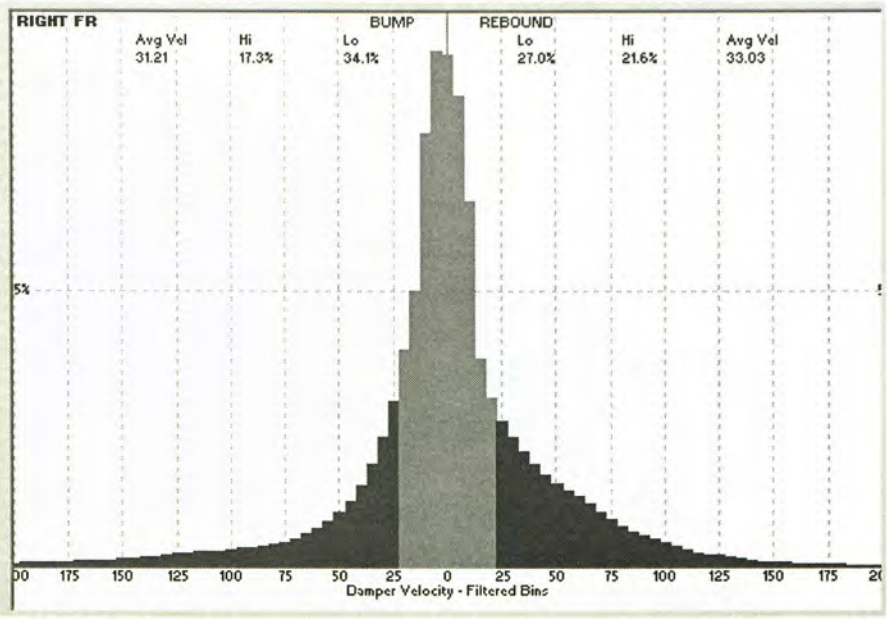
7. Choose front and rear Motion Ratio

8. Select a lap data to analyze



Damper Speed Histogram – Setting dampers





Left front	Low speed	High speed
Bump	28.0 %	21.0 %
Rebound	25.8 %	25.1 %

Right front	Low speed	High speed
Bump	34.1 %	17.3 %
Rebound	27.0 %	21.6 %

Left bump low speed is too stiff

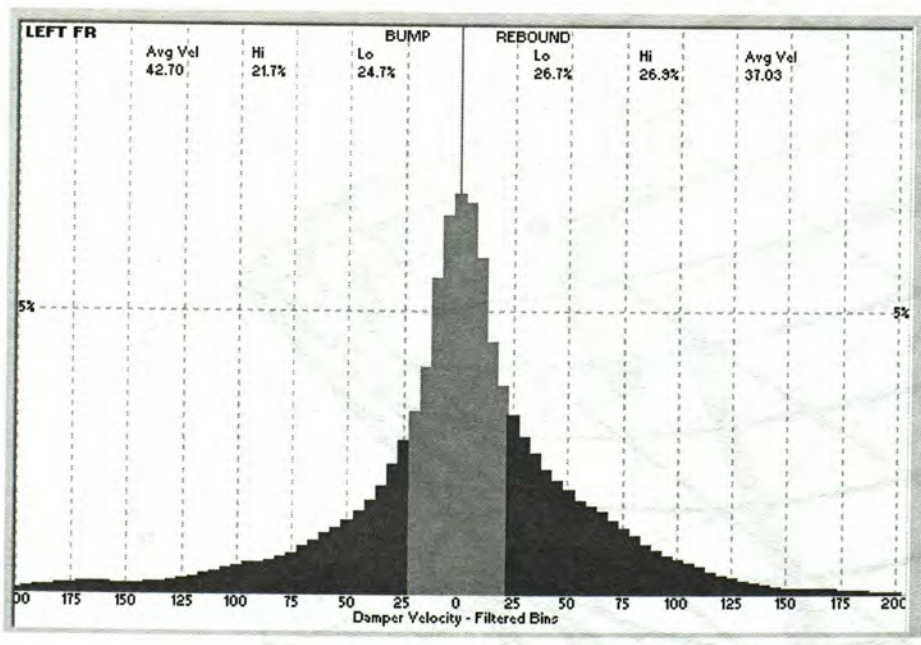
Right low speed bump is too stiff

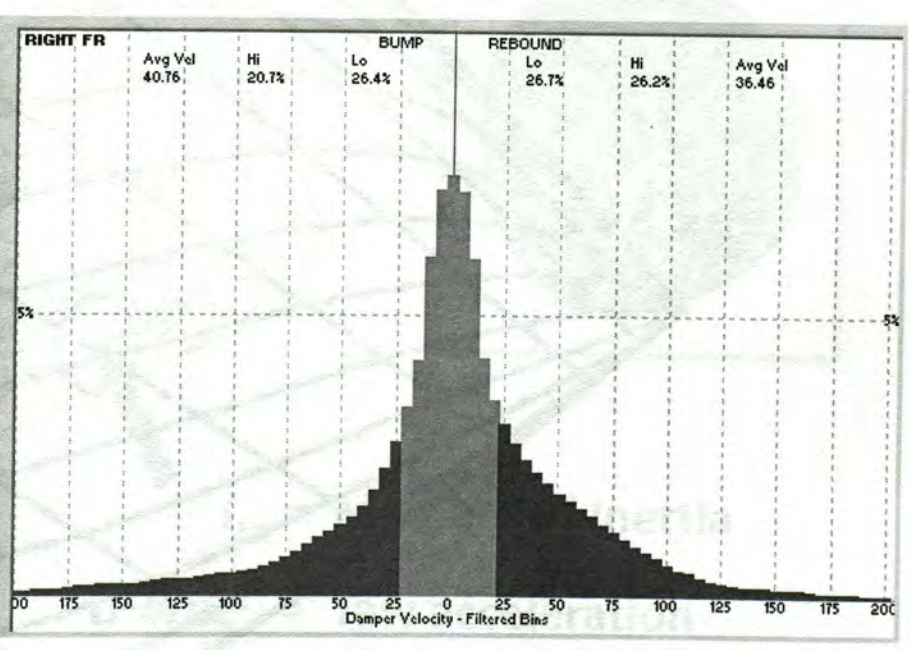
Right damper is stiffer than left damper

Damper Speed Histogram – Setting dampers

Set-up modification : 2 clicks softer in right low speed bump

1 click softer in left low speed bump





Left front	Low speed	High speed
Bump	24.7 %	21.7 %
Rebound	26.7 %	26.9 %

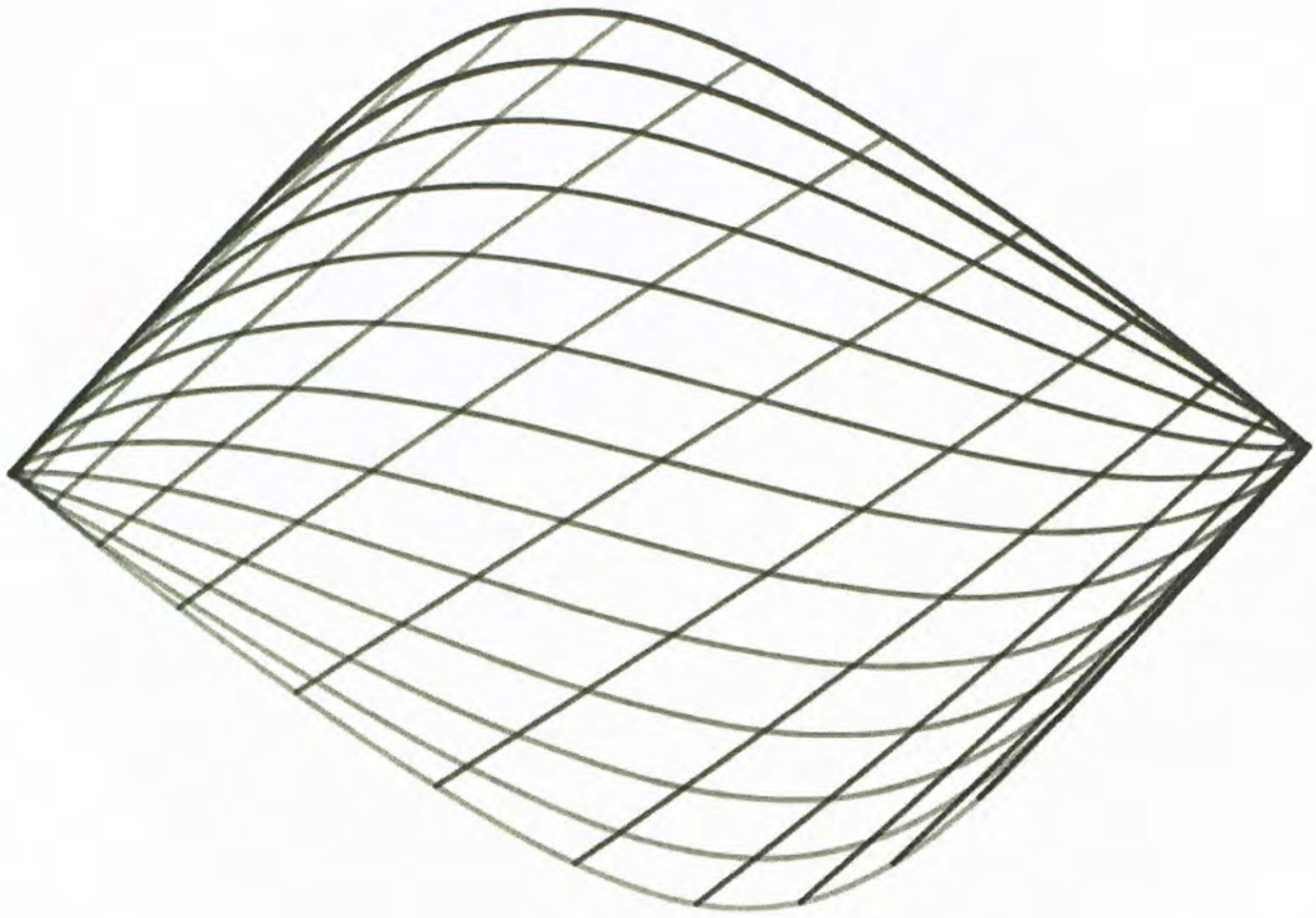
3/10 sec. Gain with this setting!!

Right front	Low speed	High speed
Bump	26.4 %	20.7 %
Rebound	26.7 %	26.2 %

8. Balance Vs. Grip

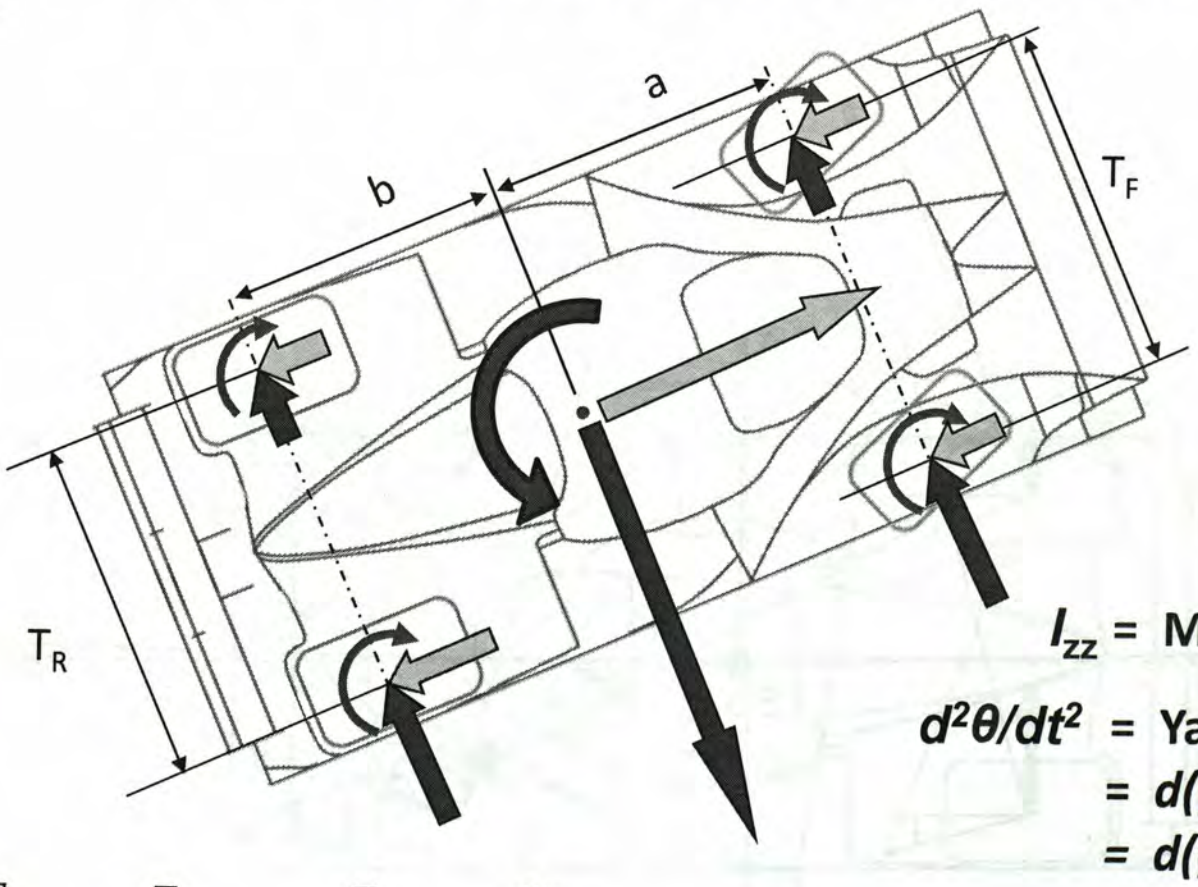
Yaw moment

Lat G



Yaw Moment Measurement

Third method: with Kistler Wheel Force Transducer



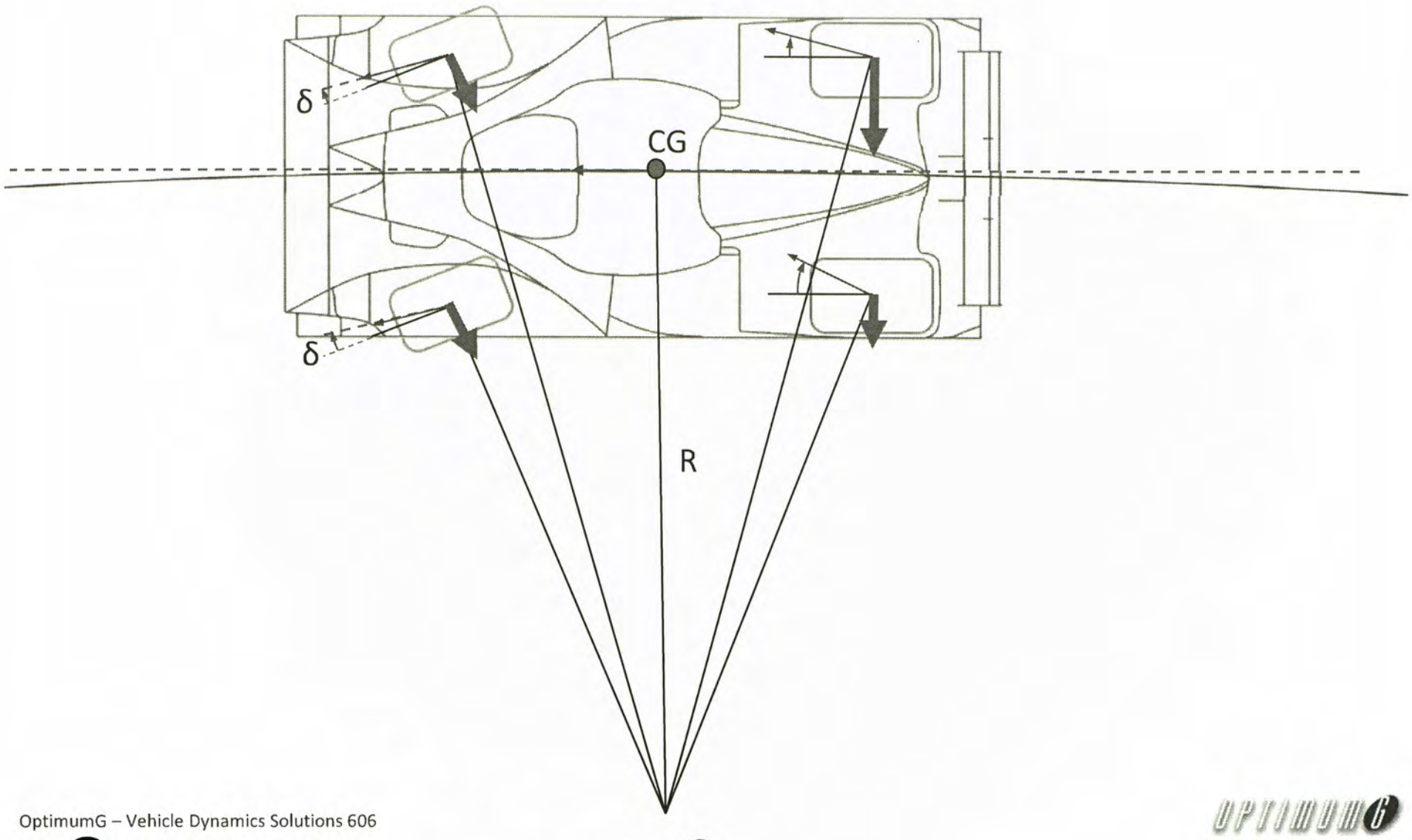
I_{zz} = Moment of Inertia

$d^2\theta/dt^2$ = Yaw acceleration
= *d(yaw rate)/dt*
= *d(gyro)/dt*

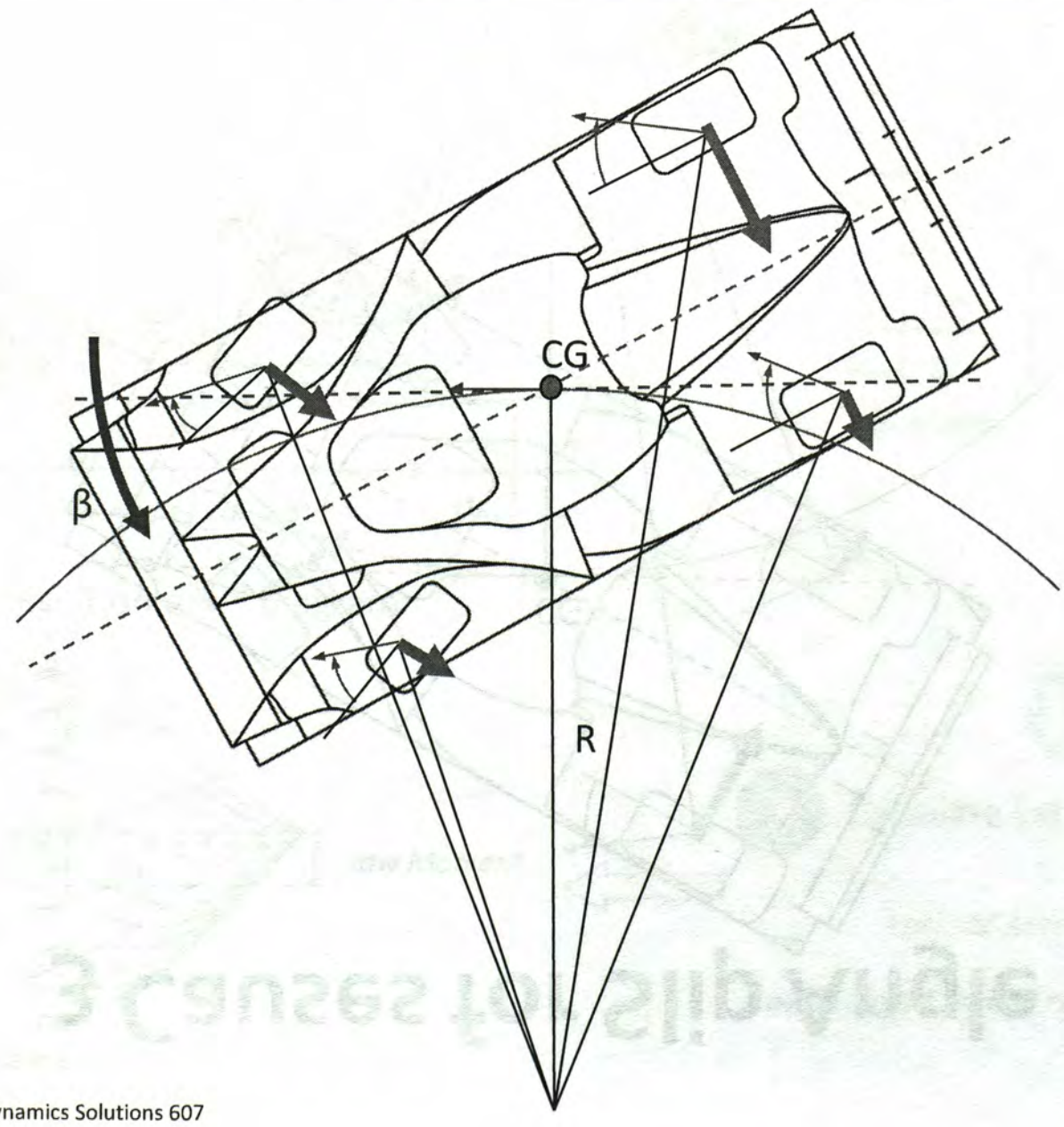
$$[(F_{XLF}\frac{T_F}{2}+F_{xLR}\frac{T_R}{2})-(F_{xRF}\frac{T_F}{2}+F_{xRR}\frac{T_R}{2})] + [(F_{yFL}+F_{yFR})\,a-(F_{yRL}+F_{yRR})\;b]-M_{zRF}-M_{zLF}-M_{zRR}-M_{zLR}$$

Allows us to determine and quantify each of the 12 causes of the yaw moment

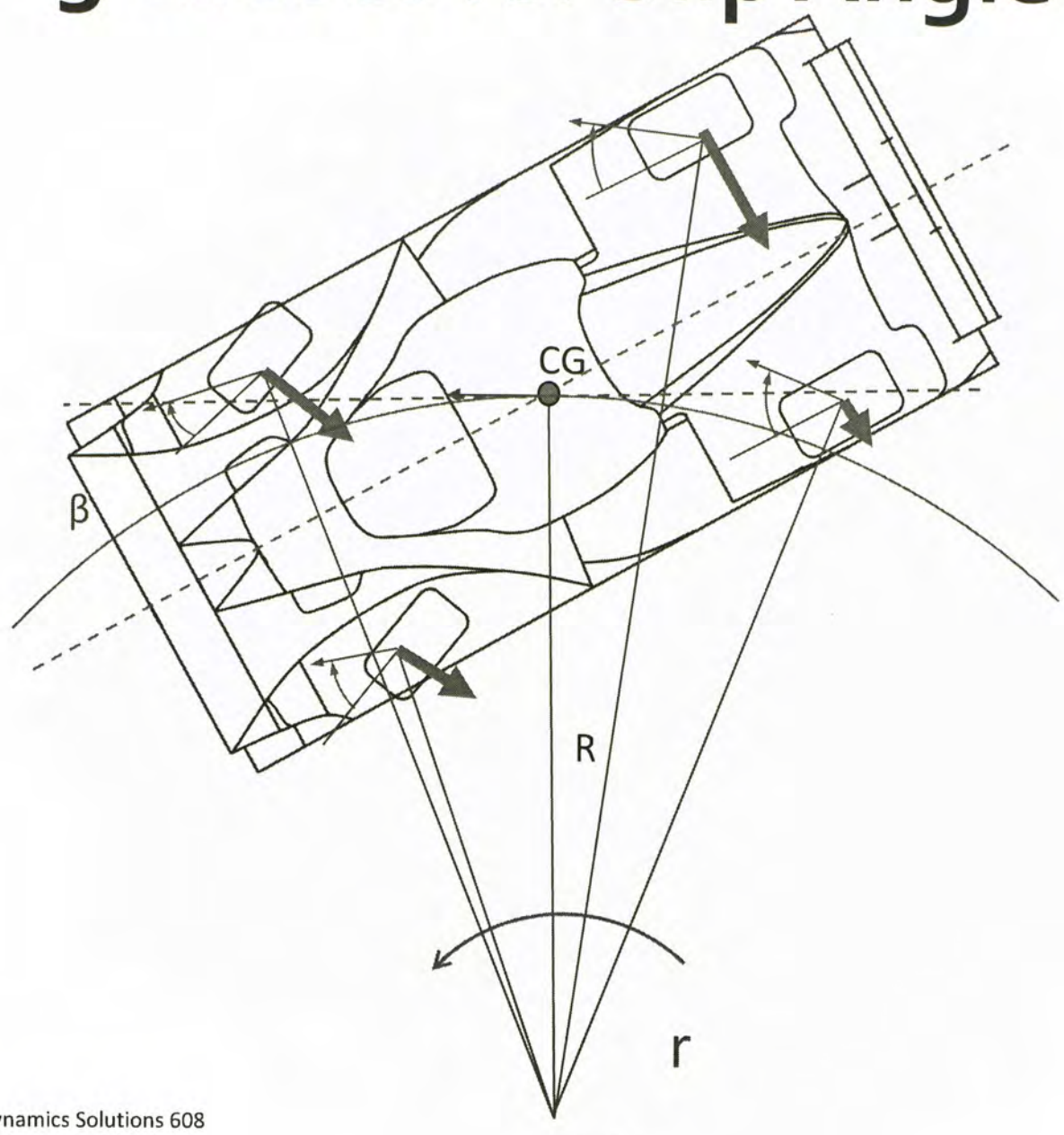
3 Causes for Slip Angle - δ



3 Causes for Slip Angle - β



3 Causes for Slip Angle - r



Yaw Moment Diagram Flow Chart

1 Impose Arbitrary Car Orientation

Imposed β & δ combinations

α	δ = -6		δ = -3		δ = 0		δ = 3		δ = 6	
β = -8	11.83	12.16	8.82	9.15	5.85	6.18	3.04	3.35	0.56	0.80
	9.91	9.73	9.92	9.74	9.89	9.71	9.75	9.51	9.39	9.01
β = -4	7.96	8.07	5.02	5.13	2.30	2.40	-0.09	0.00	-2.38	-2.35
	6.00	5.61	5.95	5.54	5.73	5.27	5.23	4.67	4.66	4.01
β = 0	4.68	4.64	2.10	2.08	0.00	0.00	-2.08	-2.10	-4.64	-4.68
	1.54	0.89	1.16	0.49	0.35	-0.35	0.49	-1.16	-0.89	-1.54
β = 4	2.55	2.38	0.00	0.01	-2.40	-2.30	-5.13	-5.02	-8.07	-7.96
	-4.01	-4.66	-4.67	-5.23	-5.27	-5.73	-5.54	-5.95	-5.61	-6.00
β = 8	-0.80	-0.56	-3.35	-3.04	-6.18	-5.85	-9.15	-8.82	-12.16	-11.83
	-9.01	-9.39	-9.51	-9.75	-9.71	-9.89	-9.74	-9.92	-9.73	-9.91

Slip Angles Calculation

α_{FL}	α_{FR}
α_{RL}	α_{RR}

2 Calculate Fy thanks to Pacejka Model

Static Fz

Fz_{FL}	Fz_{FR}
Fz_{RL}	Fz_{RR}

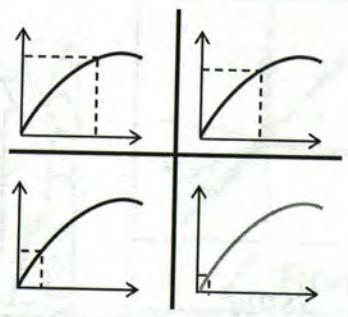
+

α_{FL}	α_{FR}
α_{RL}	α_{RR}

Slip Angles

Pacejka Model

pDy1	-2.29
pDy2	0.2782
pDy3	9.481
pEy1	-0.009
pEy2	0.0049
pEy3	-17.56
pEy4	-345.3
pKy1	31.447



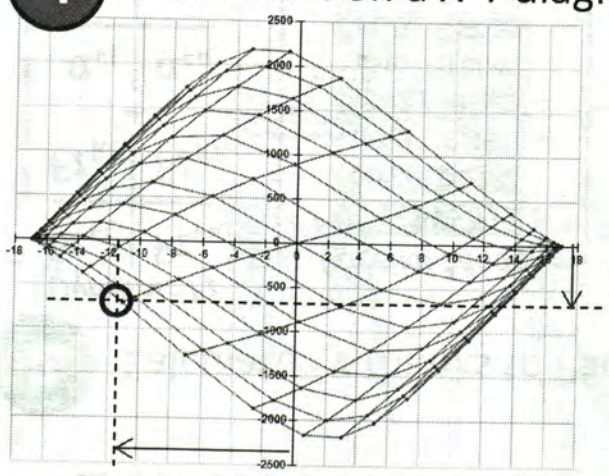
Fy on each Wheel

3 Calculate Lat. Accel. & Yaw Mom.

$$Lateral\ Acceleration = \frac{\sum Fy}{Mass}$$

$$Yaw\ Moment = FyFront \times a - FyRear \times b$$

4 Plot result on a X-Y diagram



Yaw Moment Diagram Flow Chart – "2" Details

2 Calculate Fy thanks to Pacejka Model

Dynamic Fz

Fz_{FL}	Fz_{FR}
Fz_{RL}	Fz_{RR}

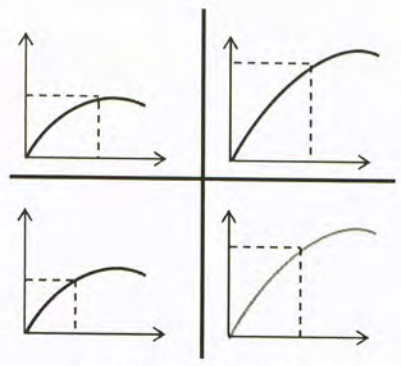
+

α_{FL}	α_{FR}
α_{RL}	α_{RR}

Slip Angles

Pacejka Model

pDy1	-2.29
pDy2	0.2782
pDy3	9.481
pEy1	-0.009
pEy2	0.0049
pEy3	-17.56
pEy4	-345.3
pKy1	31.447



Fy on each Wheel

2b Calculate Lateral Acceleration

$$Lateral\ Acceleration = \frac{\sum Fy}{Mass}$$

2c Calculate Weight Transfer

$$S\ "Elastic"\ WT_{front} = \frac{SM \times lat.acc. \times \Delta z(Cg_{SW} - Roll\ Axis) \times (RM_{front}/RM_{total})}{track_{front}}$$

$$S\ "Elastic"\ WT_{rear} = \frac{SM \times lat.acc. \times \Delta z(Cg_{SW} - Roll\ Axis) \times (RM_{rear}/RM_{total})}{track_{rear}}$$

$$Lat.S\ "Geometric"\ WT = \frac{SM \times lat.acc. \times RC_{height}}{track}$$

$$Lat.NS\ WT = \frac{NSM \times lat.acc. \times Cg_{NSM\ height}}{track}$$

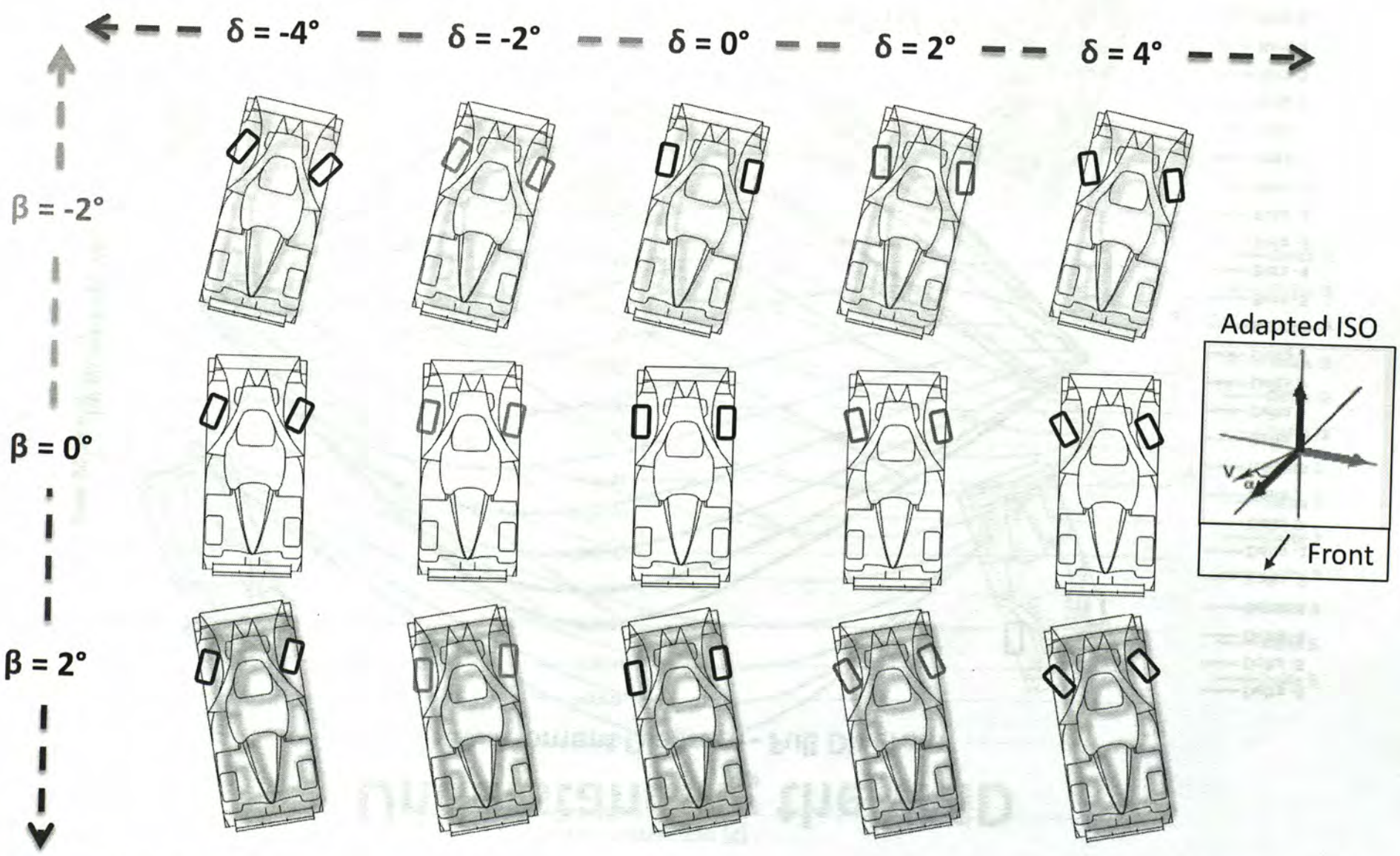
2d Calculate Weight Transfer

$$Fz\ Dynamic = Fz\ Static + Weight\ Transfer$$

Fz_{FL}	Fz_{FR}
Fz_{RL}	Fz_{RR}

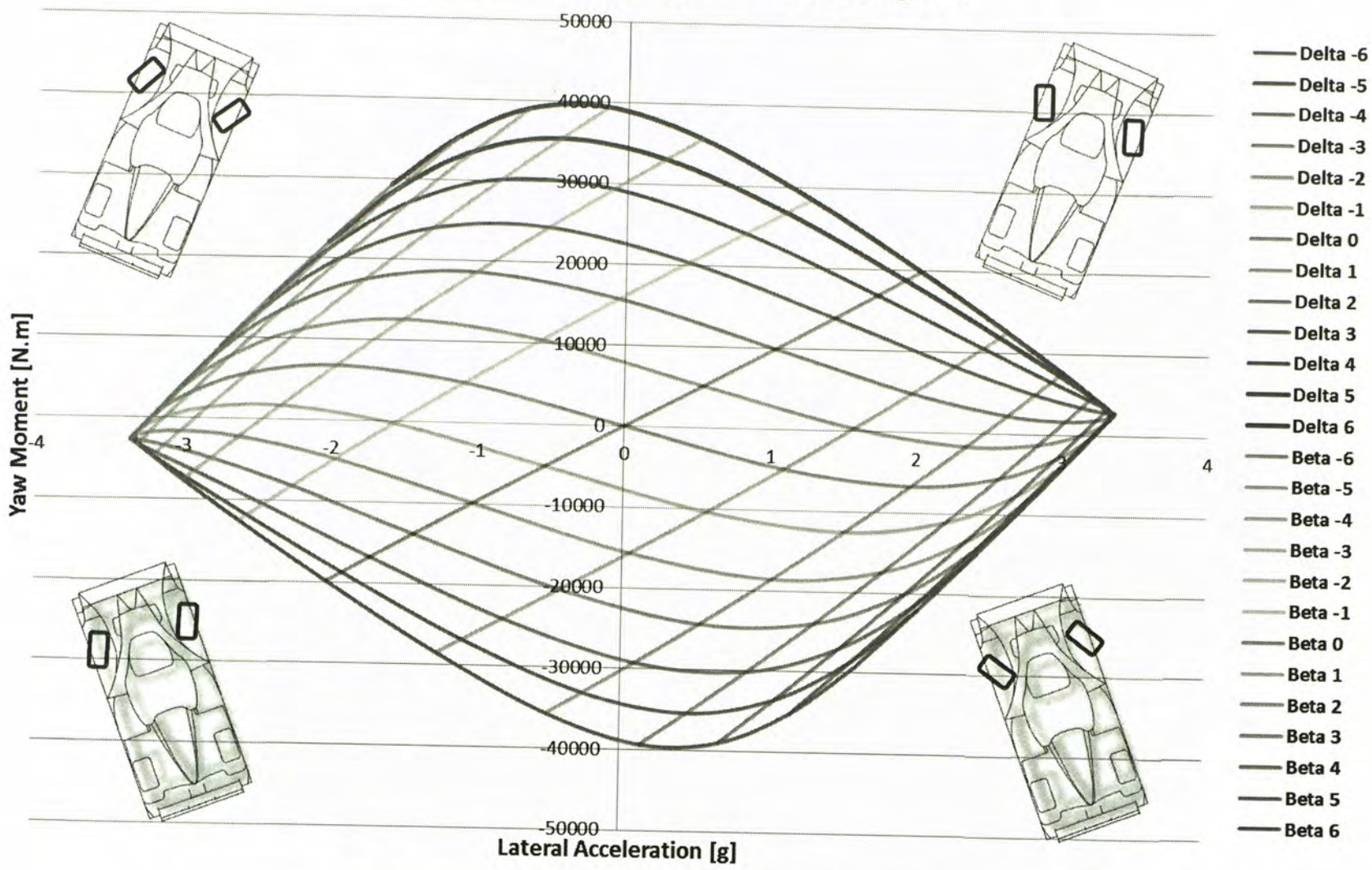
Loop done until : Lateral Acceleration (n-1) – Lateral Acceleration (n)< Tolerance

What is a Yaw Moment Diagram?



Understanding the YMD

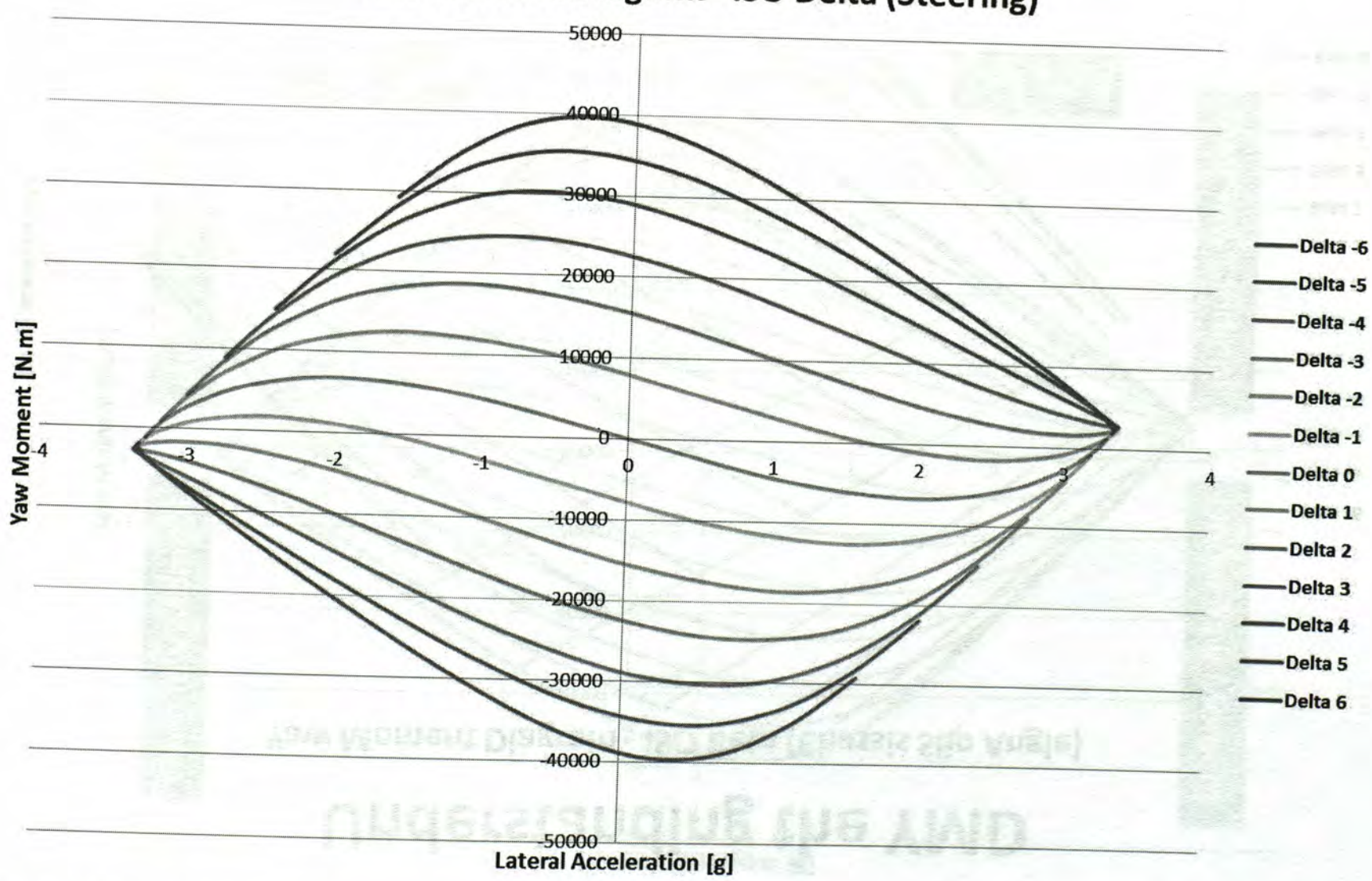
Yaw Moment Diagram - Full Diagram



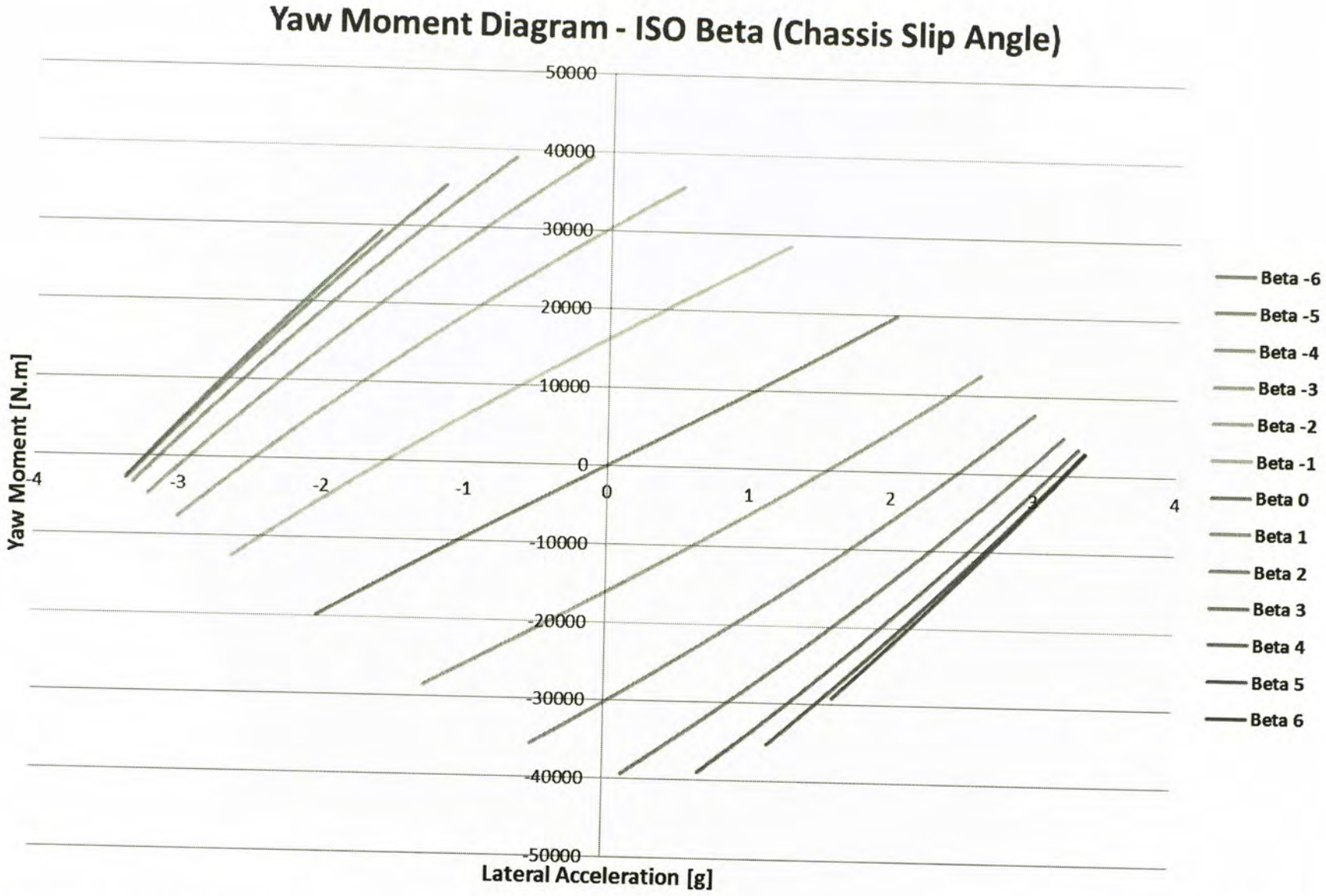
A diagram can be made for either constant speed or constant radius

Understanding the YMD

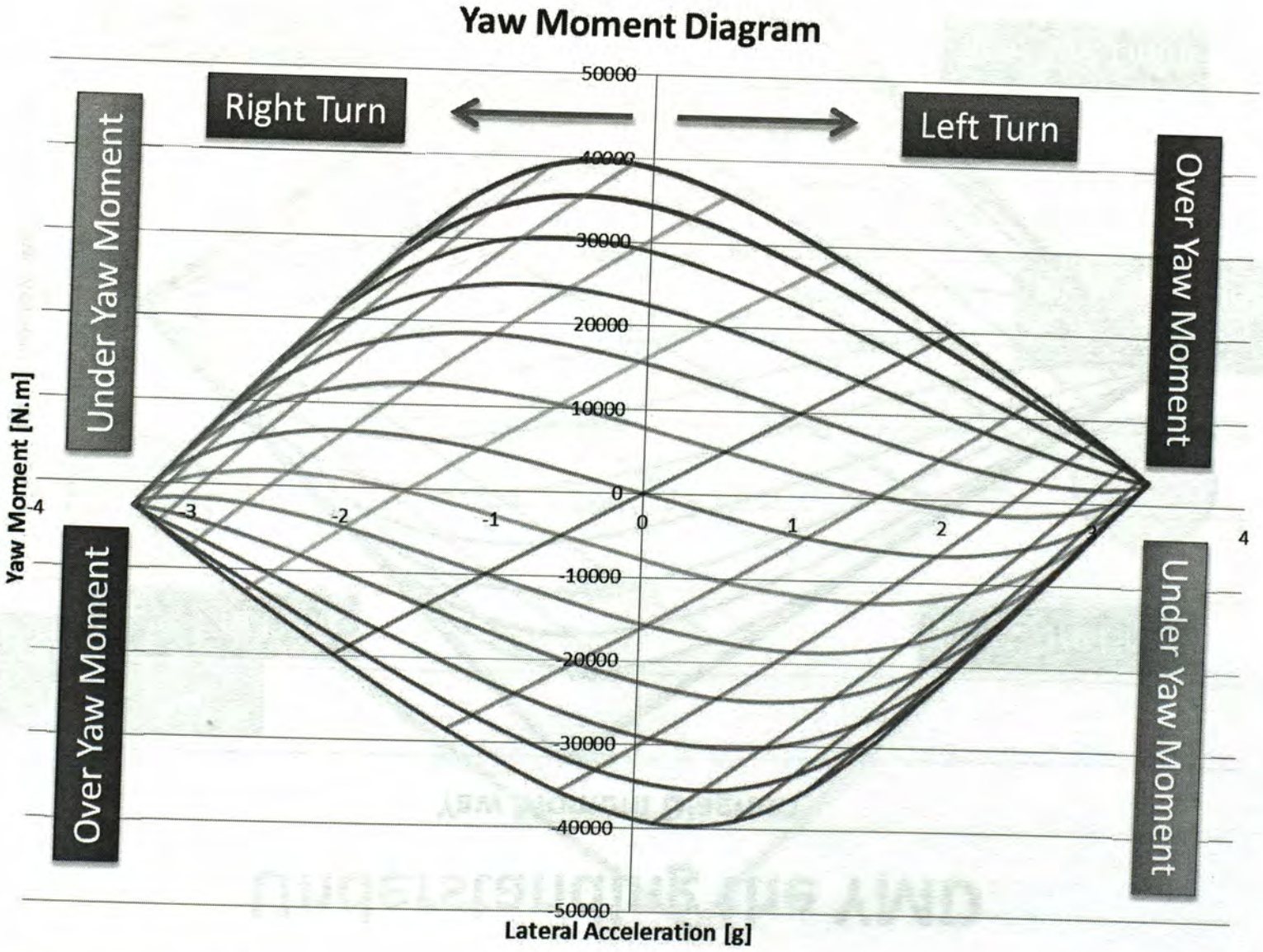
Yaw Moment Diagram - ISO Delta (Steering)



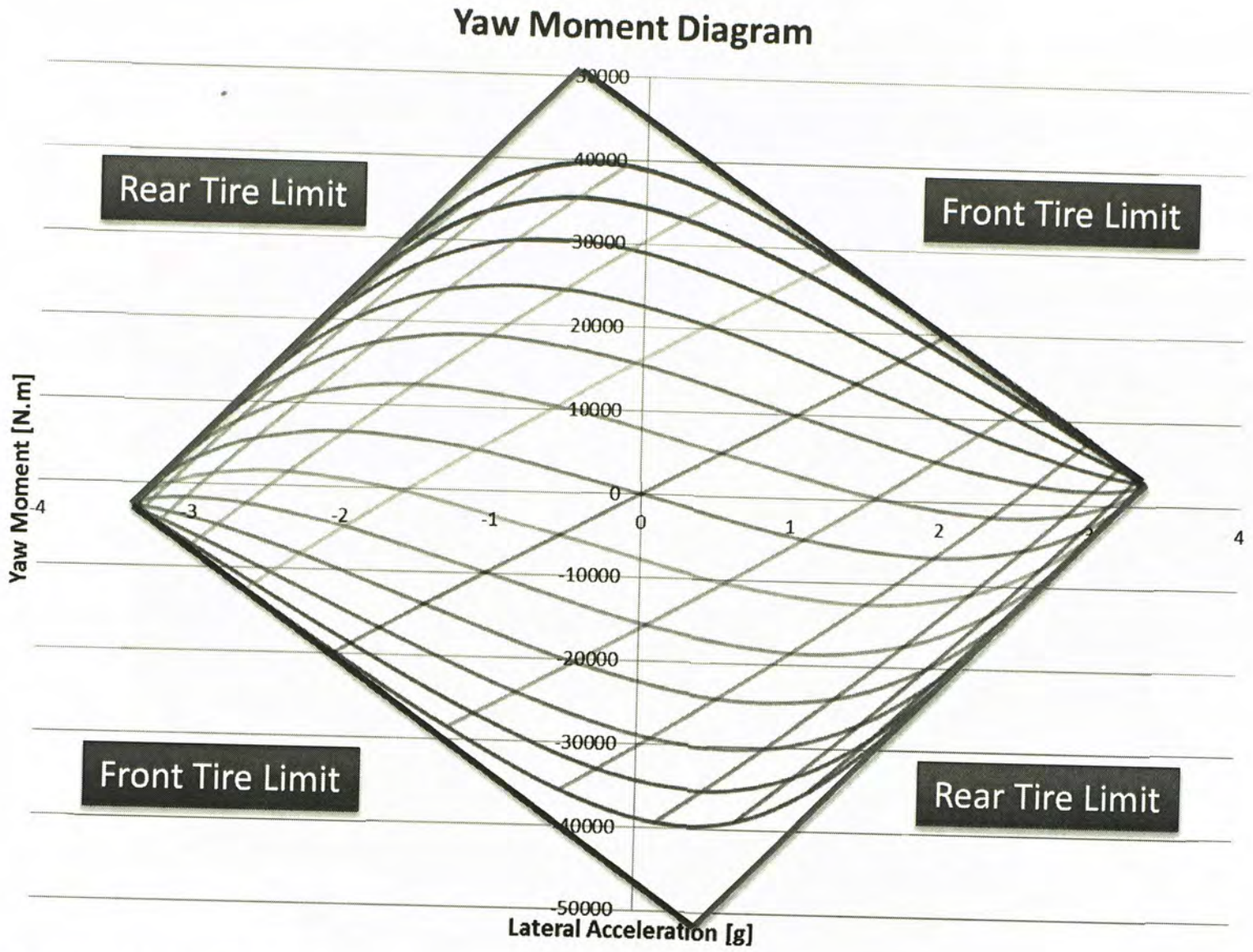
Understanding the YMD



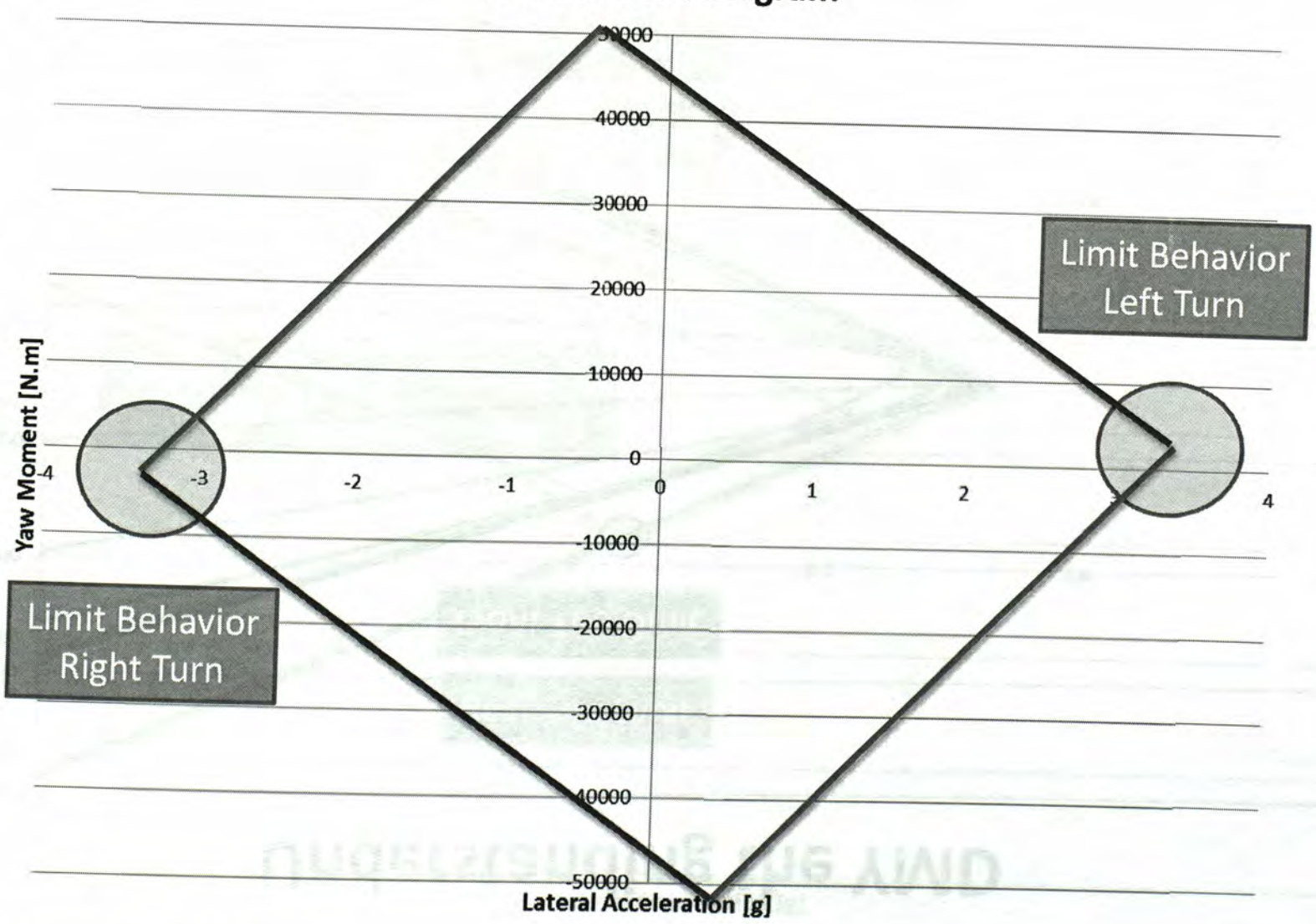
Understanding the YMD



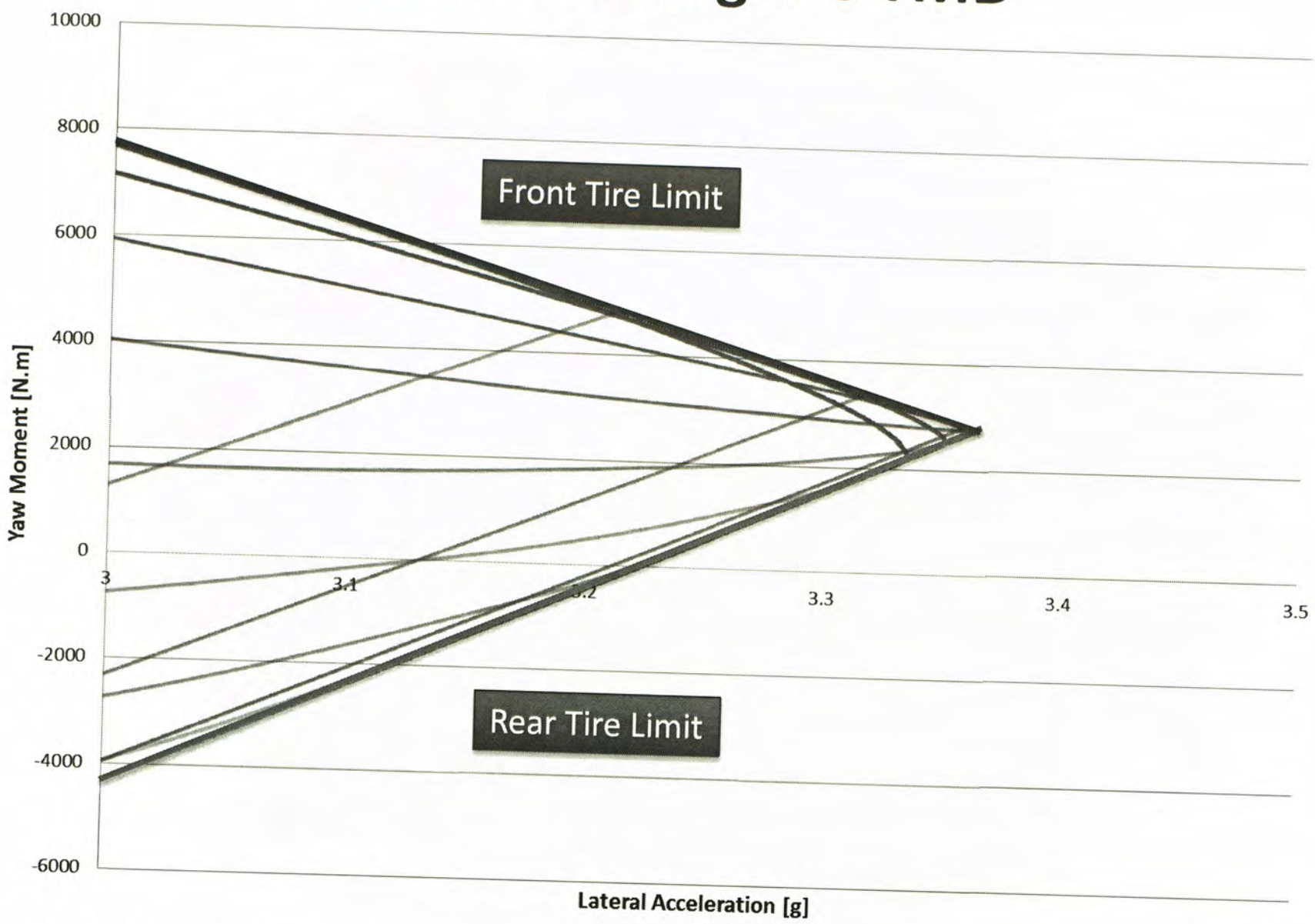
Understanding the YMD



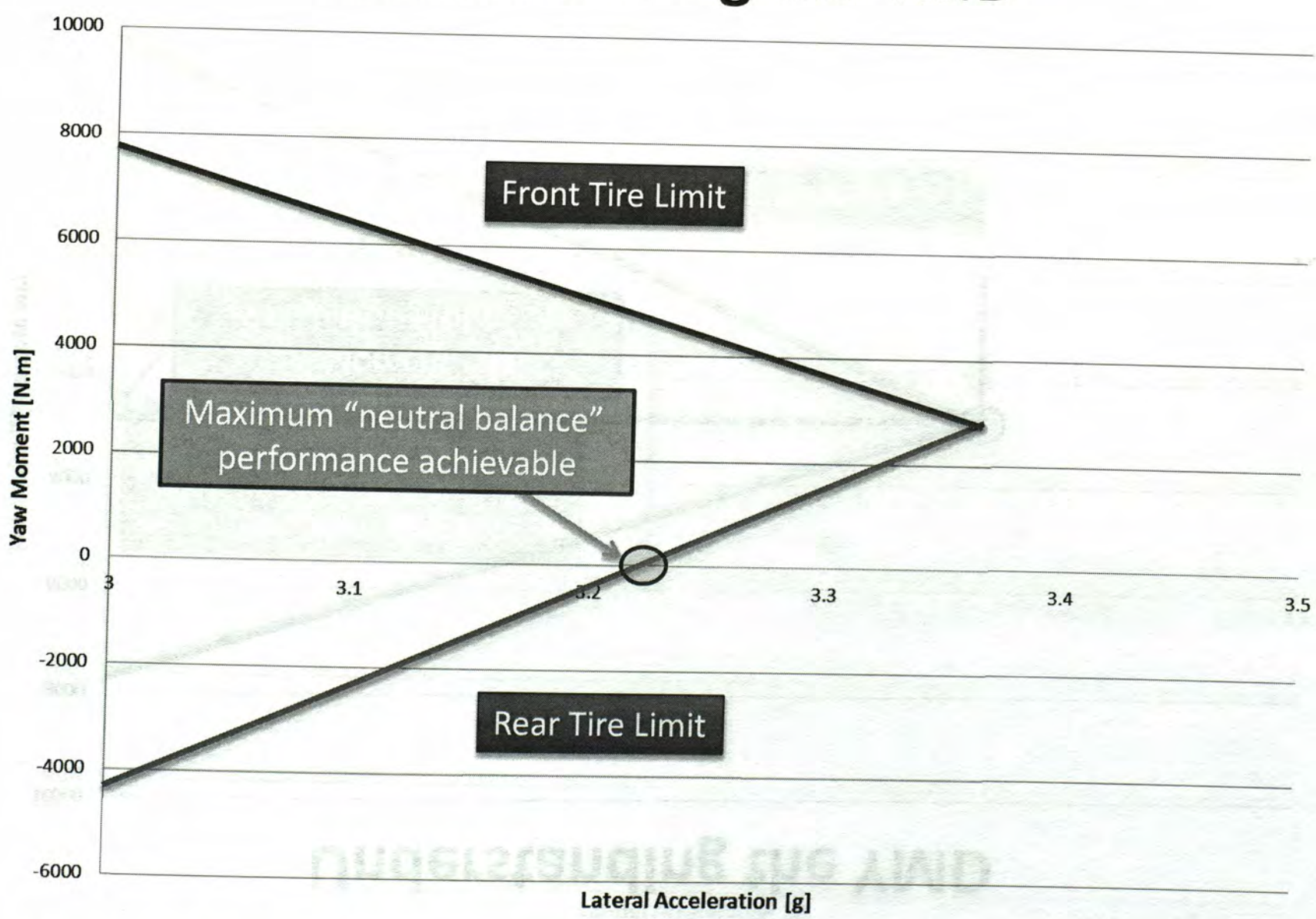
Understanding the YMD



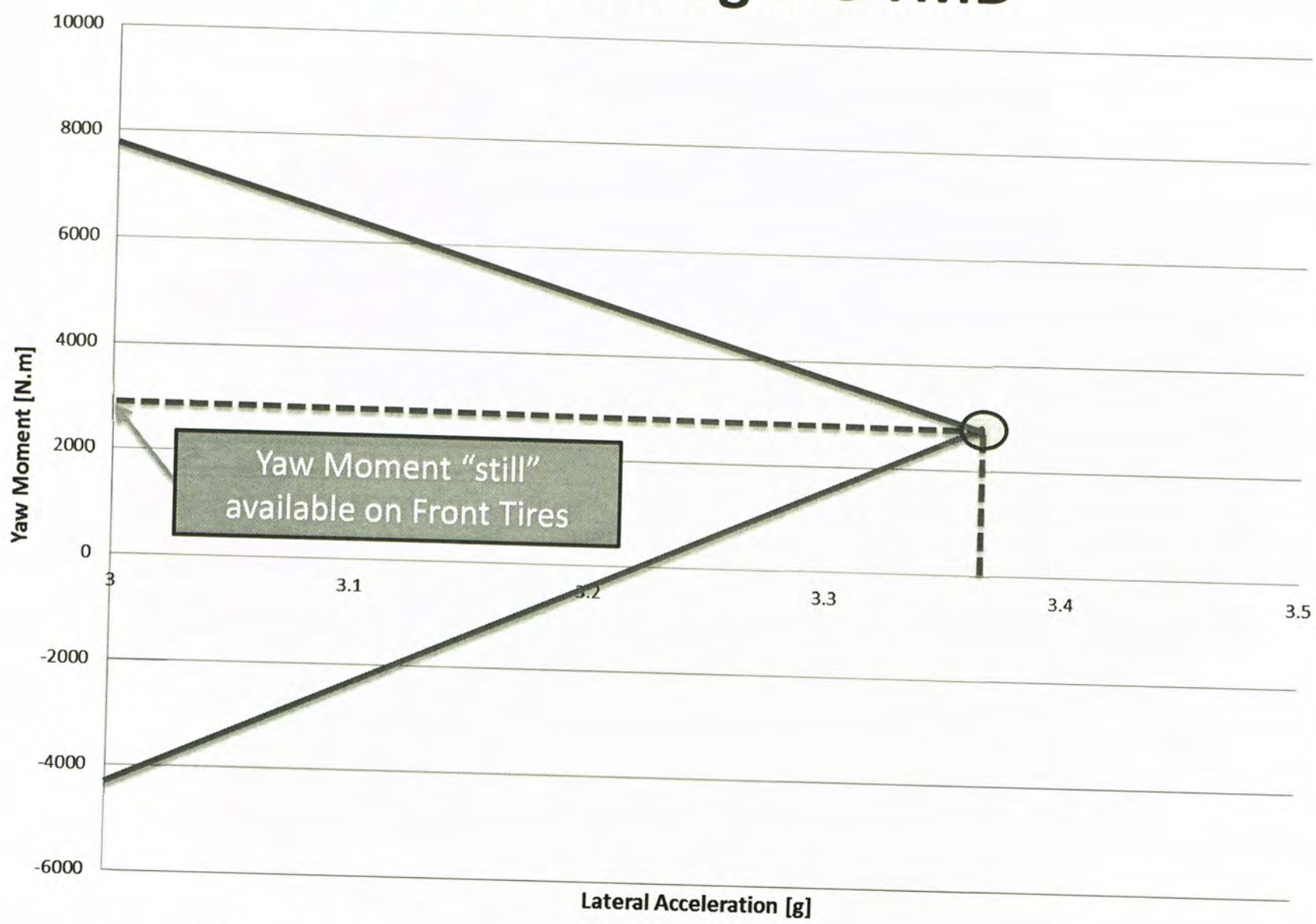
Understanding the YMD



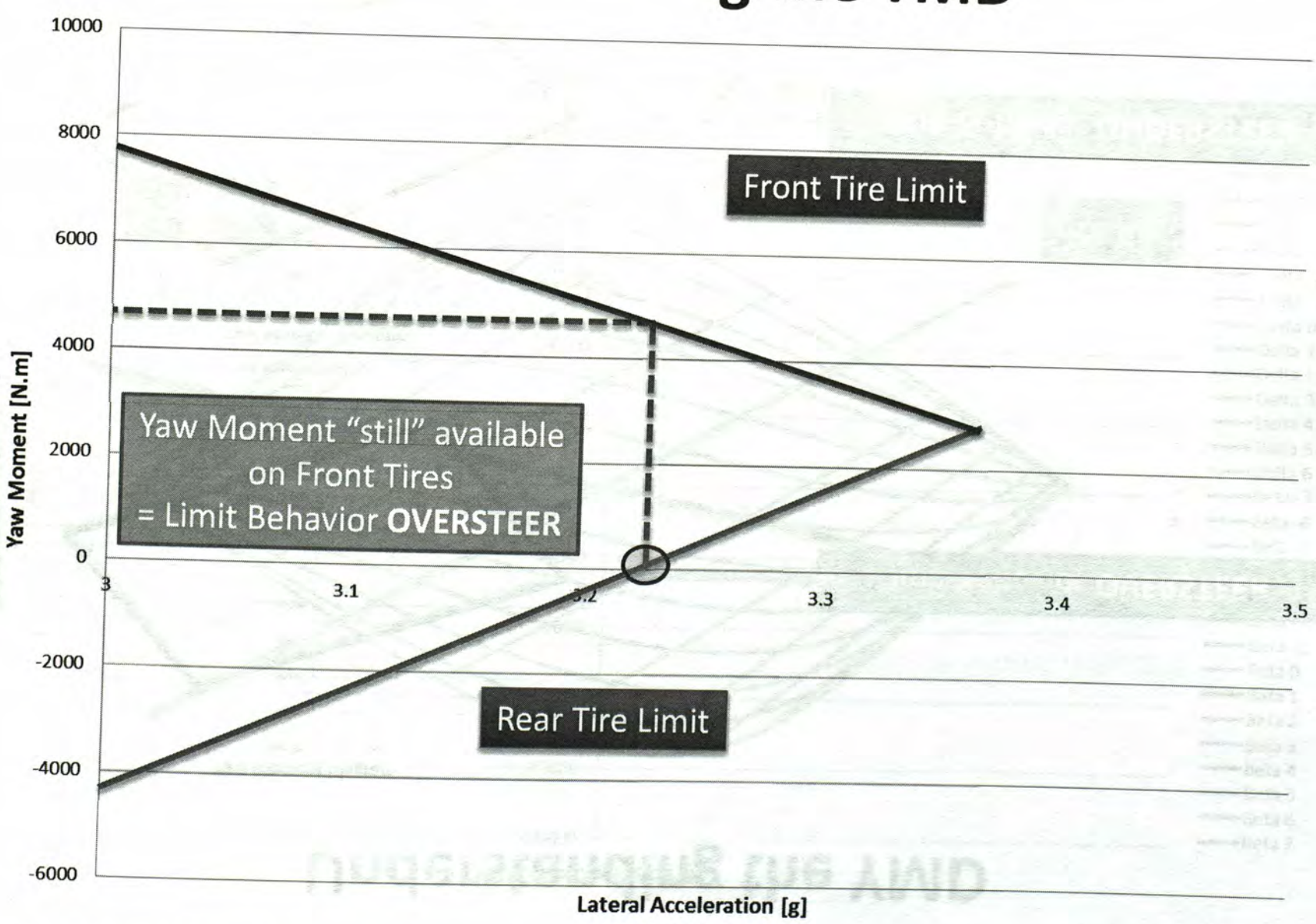
Understanding the YMD



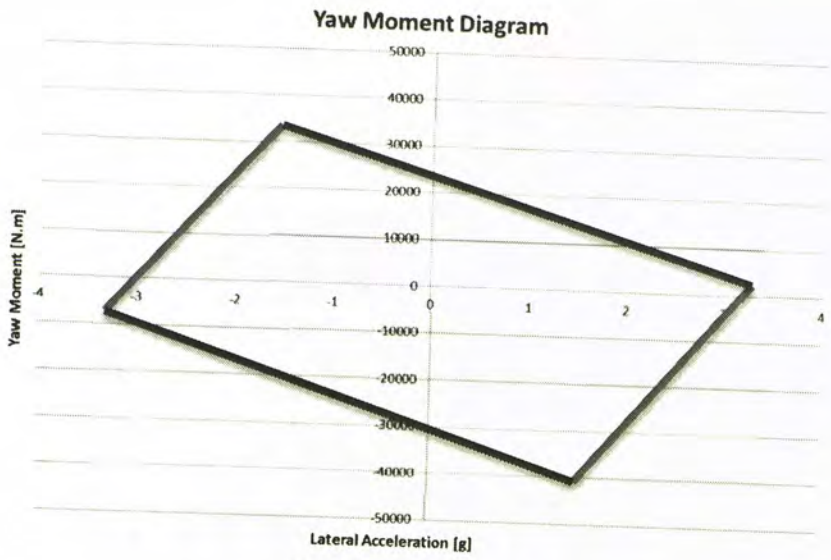
Understanding the YMD



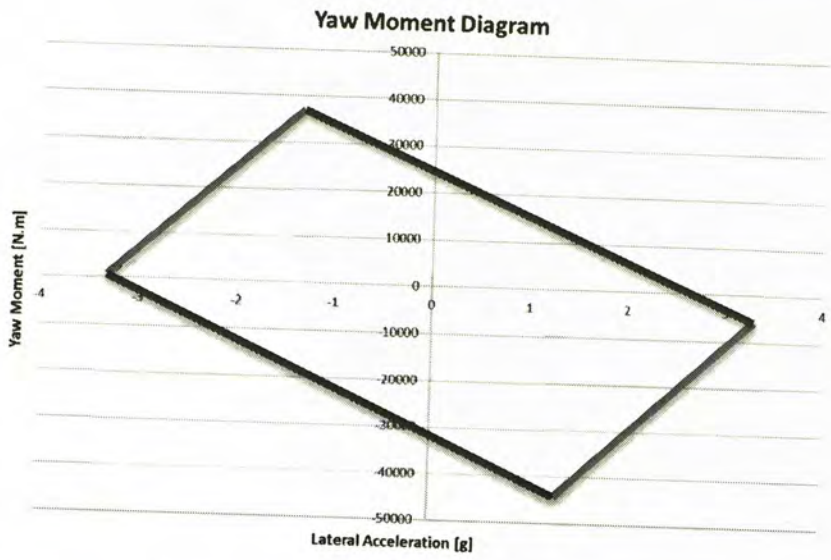
Understanding the YMD



Understanding the YMD

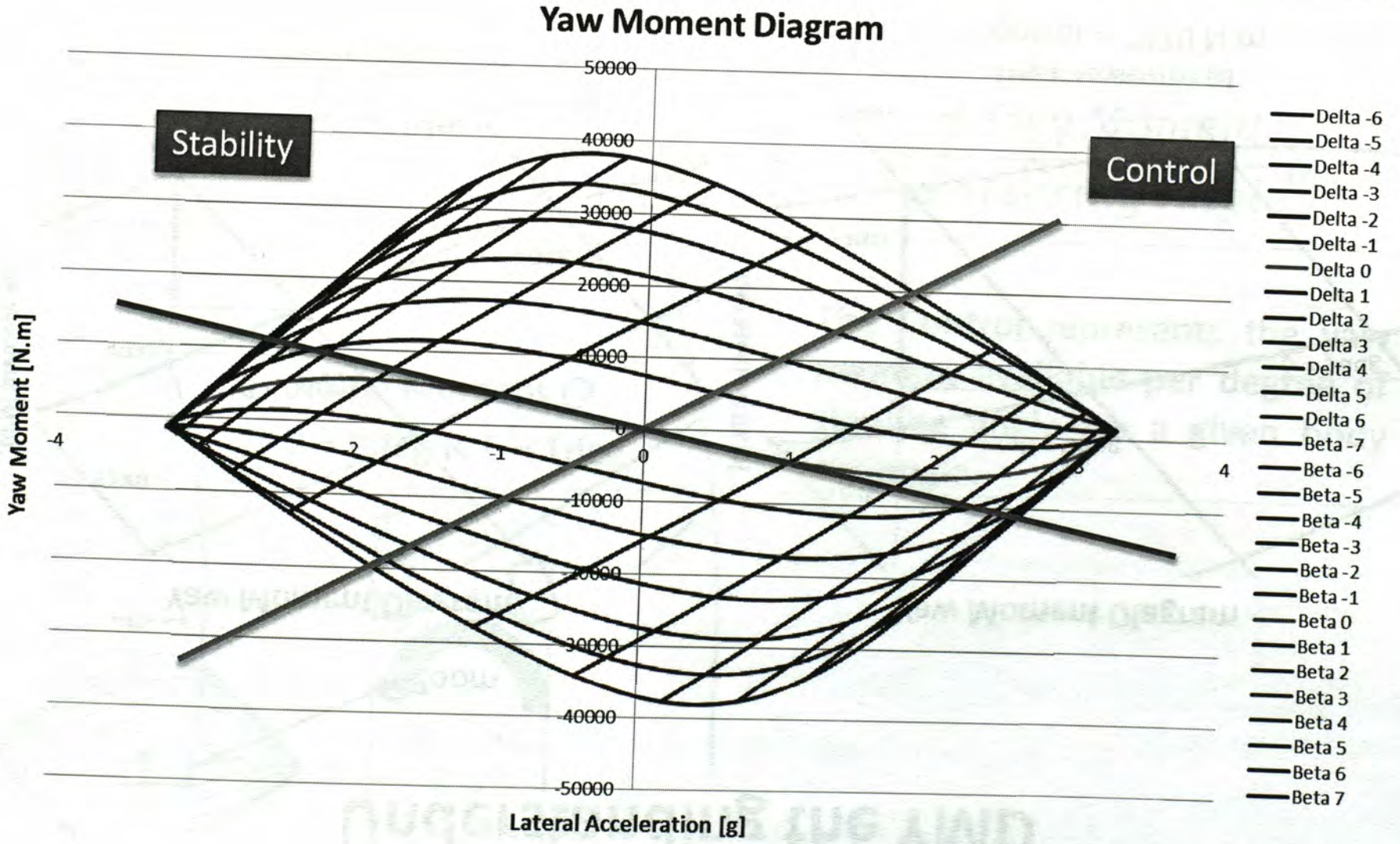


Limit Behavior **OVERSTEER**

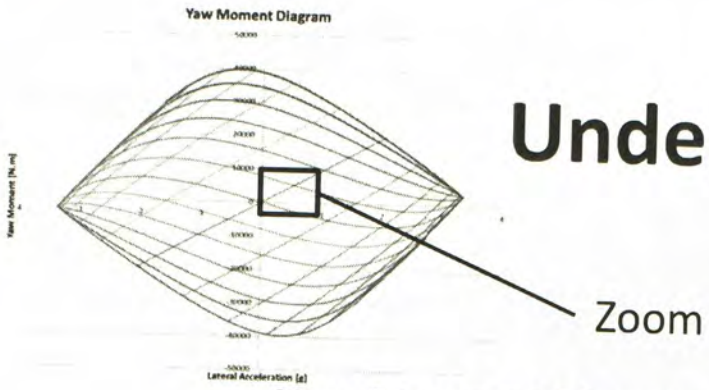


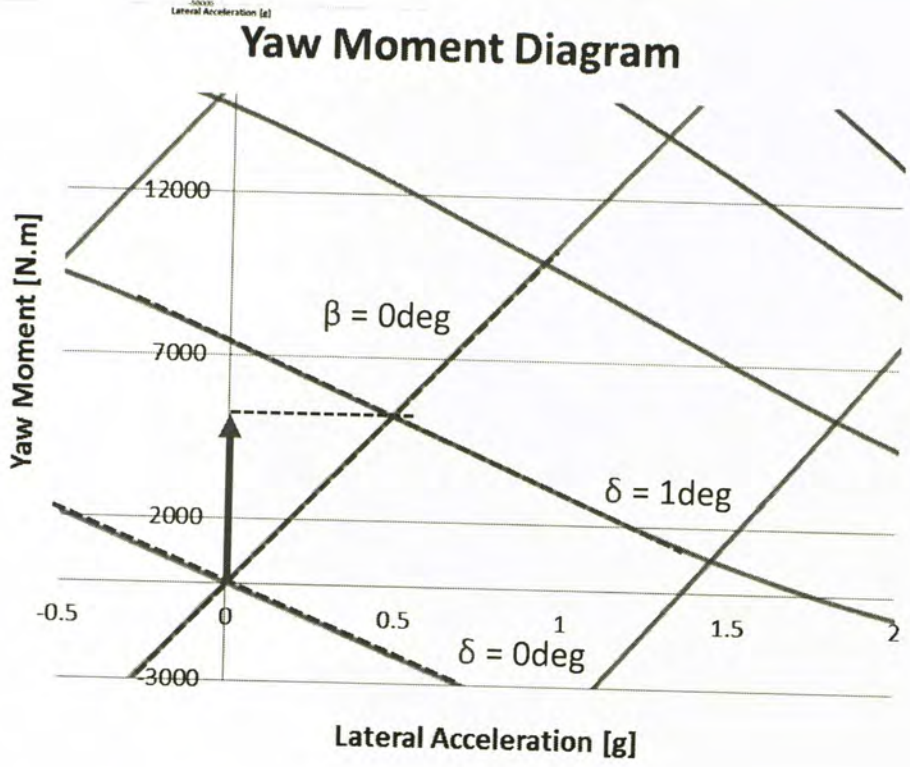
Limit Behavior **UNDERSTEER**

Understanding the YMD

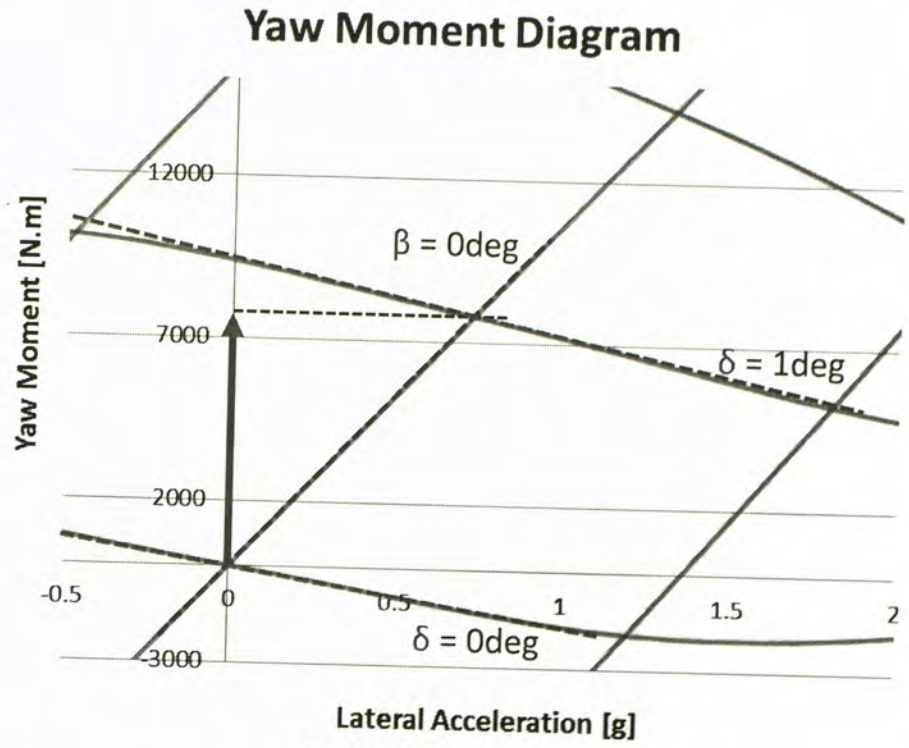


Understanding the YMD





Control = 5275 N.m



Control = 7970 N.m

Front tire with higher cornering stiffness!

Control is a function of:

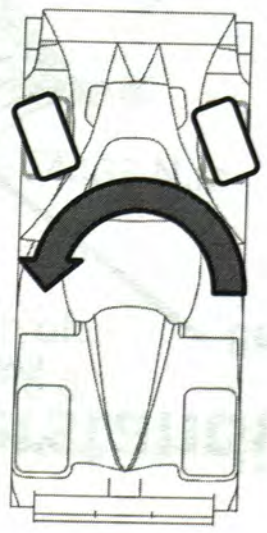
- Slope of an iso-line β
- Distance between two iso-line δ for a given β

Understanding the YMD: Control

$$\frac{d\ Yaw\ Moment}{d\ Steering\ Angle}$$

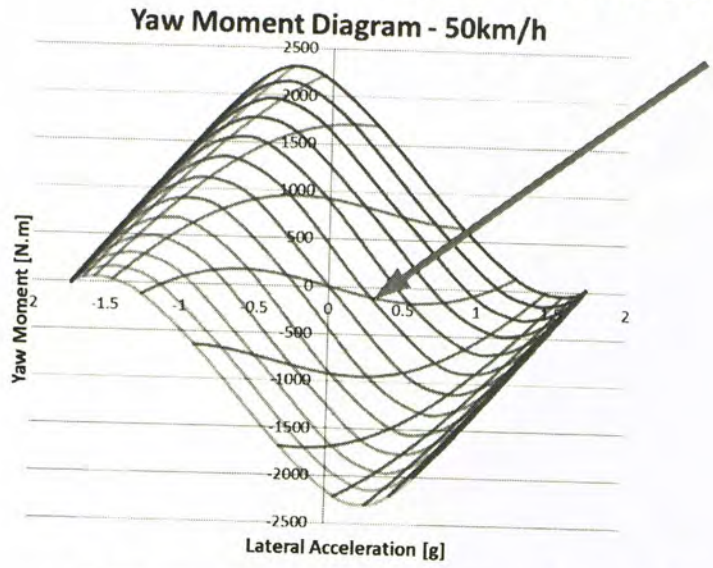
Yaw Moment @

β = 0deg & δ = 1deg



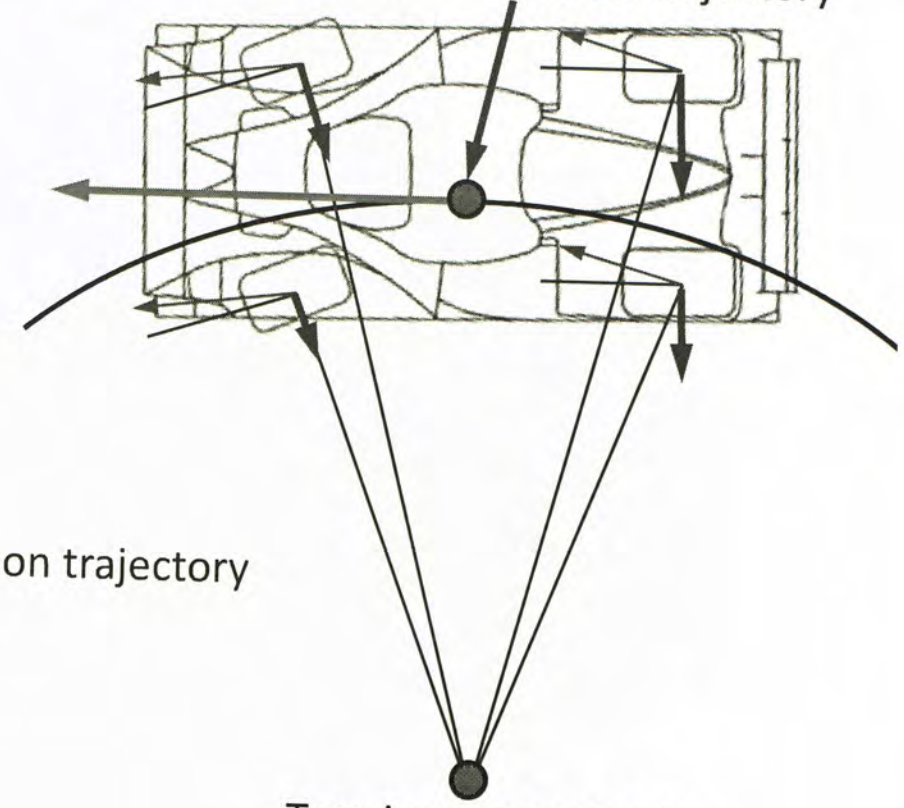
The **Control** represents the **yaw moment available per degree of steering angle for a given body slip angle.**

Understanding the YMD: Control

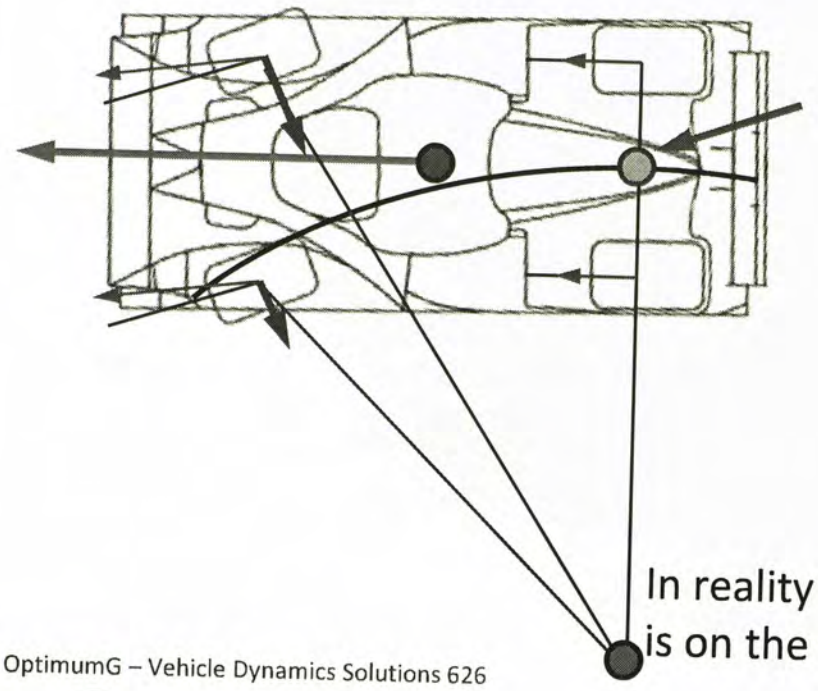


Be careful of Negative control at $\beta = 0$

CG on trajectory



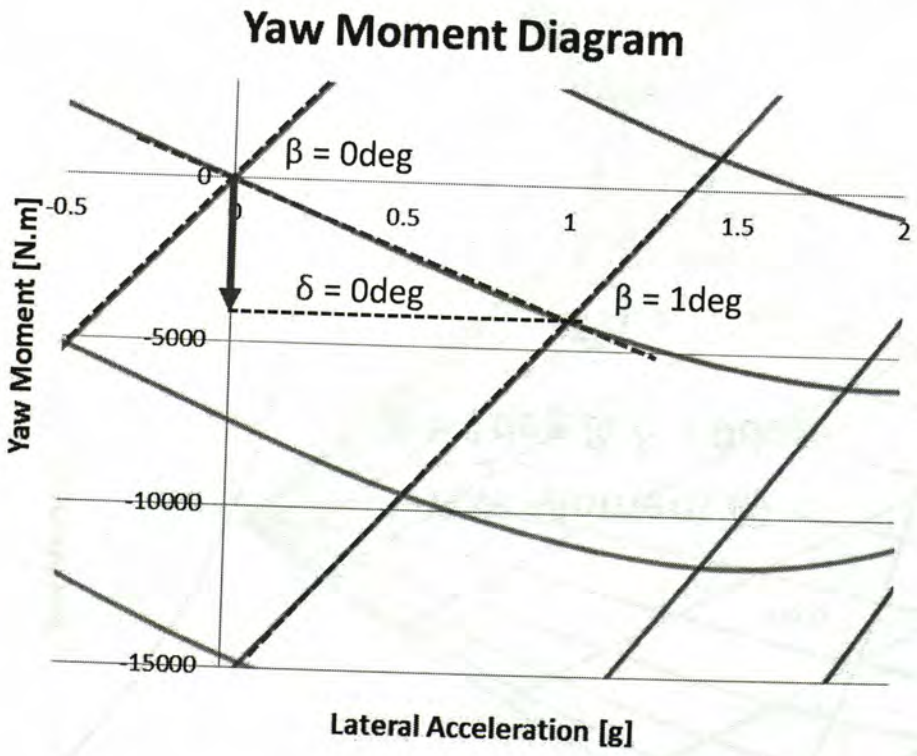
Turn instant center is calculated under the cg



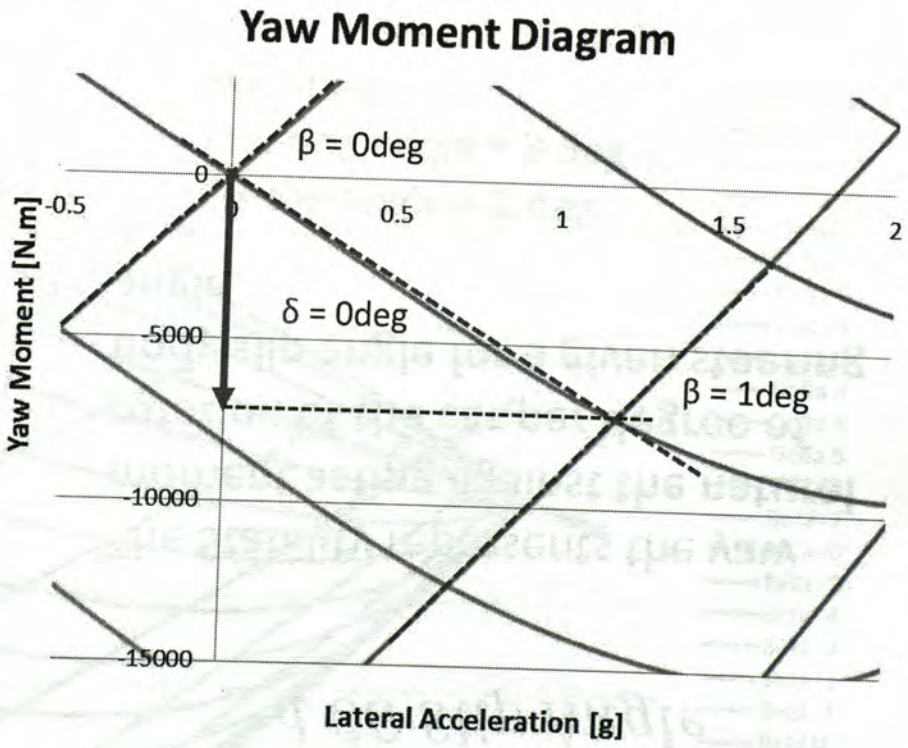
Rear axle on trajectory

In reality the turn instant center is on the rear axle axis

Understanding the YMD



Stability = -4463 N.m



Stability = -8108 N.m

Rear tire with higher cornering stiffness!

Stability is a function of:

- Slope of an iso-line δ
- Distance between two iso-line β for a given δ

Understanding the YMD: Stability

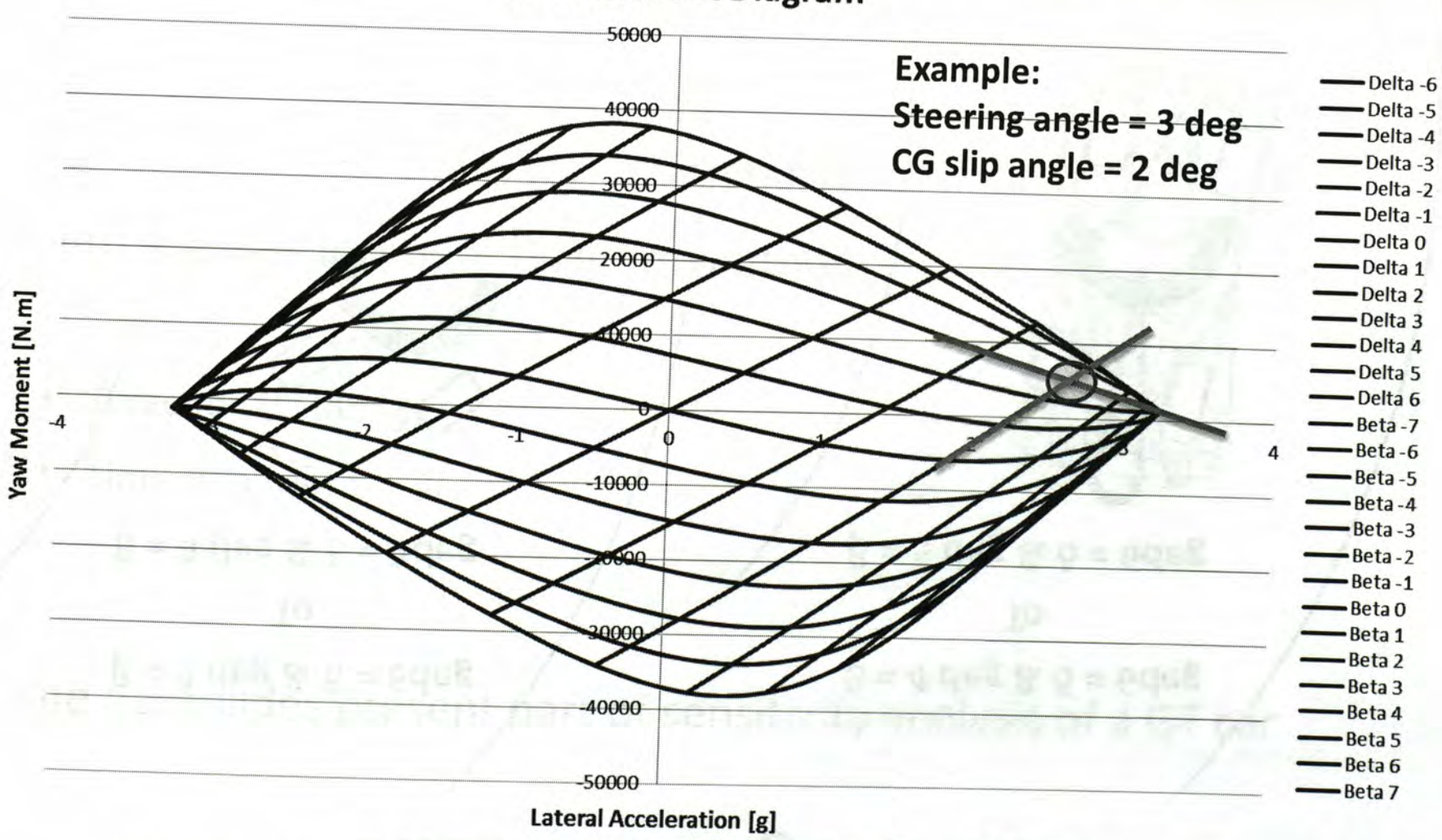
$$\frac{d\ Yaw\ Moment}{d\ CG\ Slip\ Angle}$$

Yaw Moment @
β = 1deg & δ = 0deg

The **Stability** represents the **yaw moment acting against the natural rotation of the car per degree of body slip angle for a given steering angle.**

Understanding the YMD

Yaw Moment Diagram



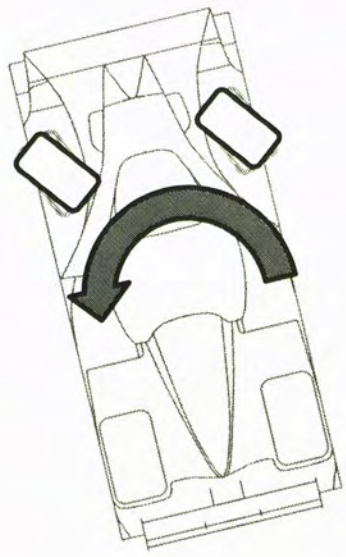
STABILITY AND **CONTROL** can be measured at every point on the YMD

Understanding the YMD

β = 4 deg & δ = 6deg

To

β = 4 deg & δ = 7deg

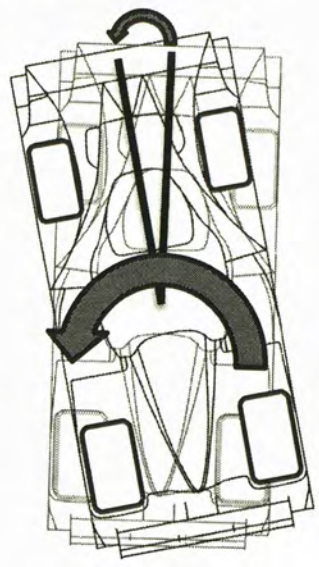


$$\frac{d\ Yaw\ Moment}{d\ Steering\ Angle}$$

β = 4 deg & δ = 6deg

To

β = 5 deg & δ = 6deg



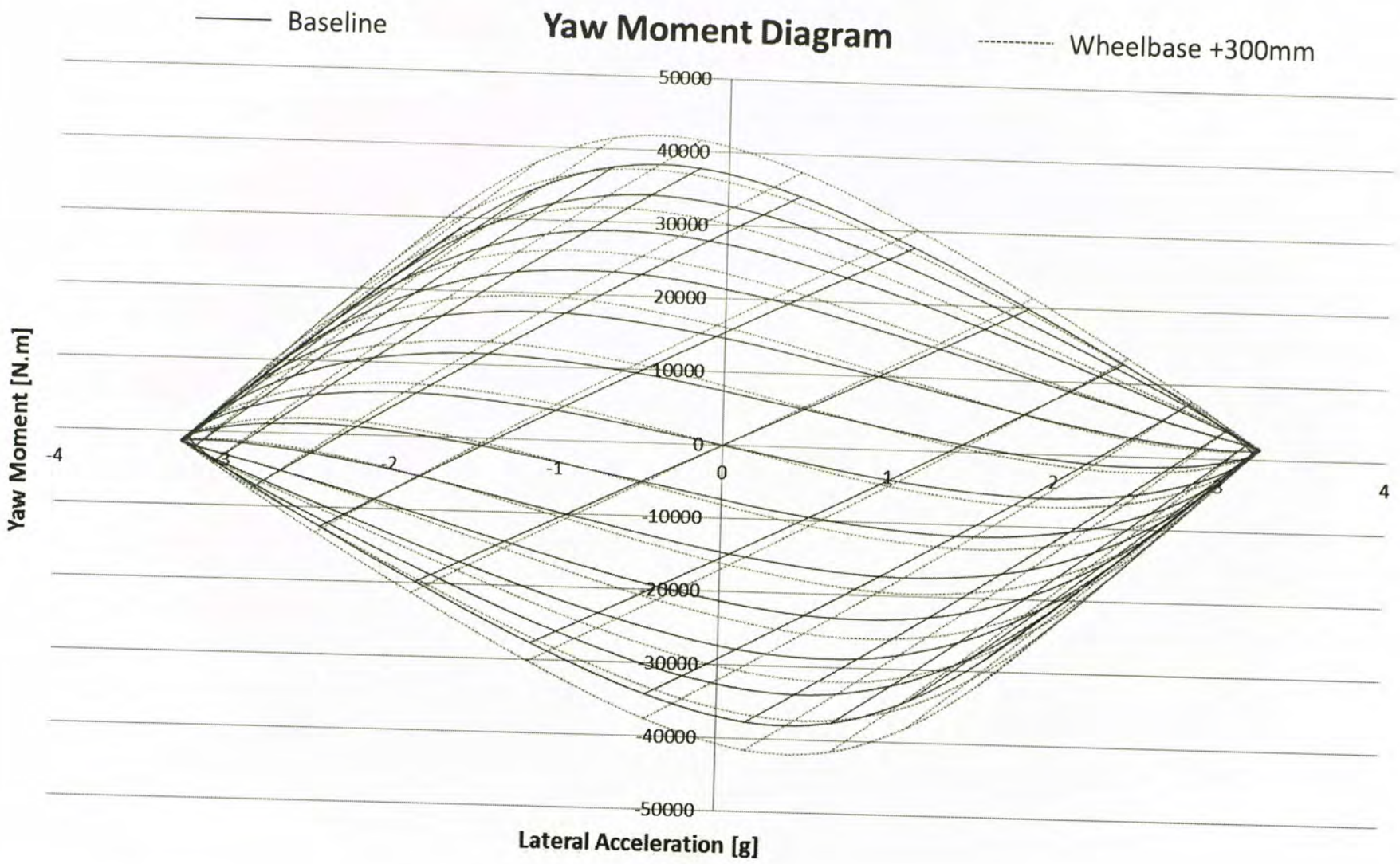
$$\frac{d\ Yaw\ Moment}{d\ CG\ Slip\ Angle}$$

Sensitivity Analysis

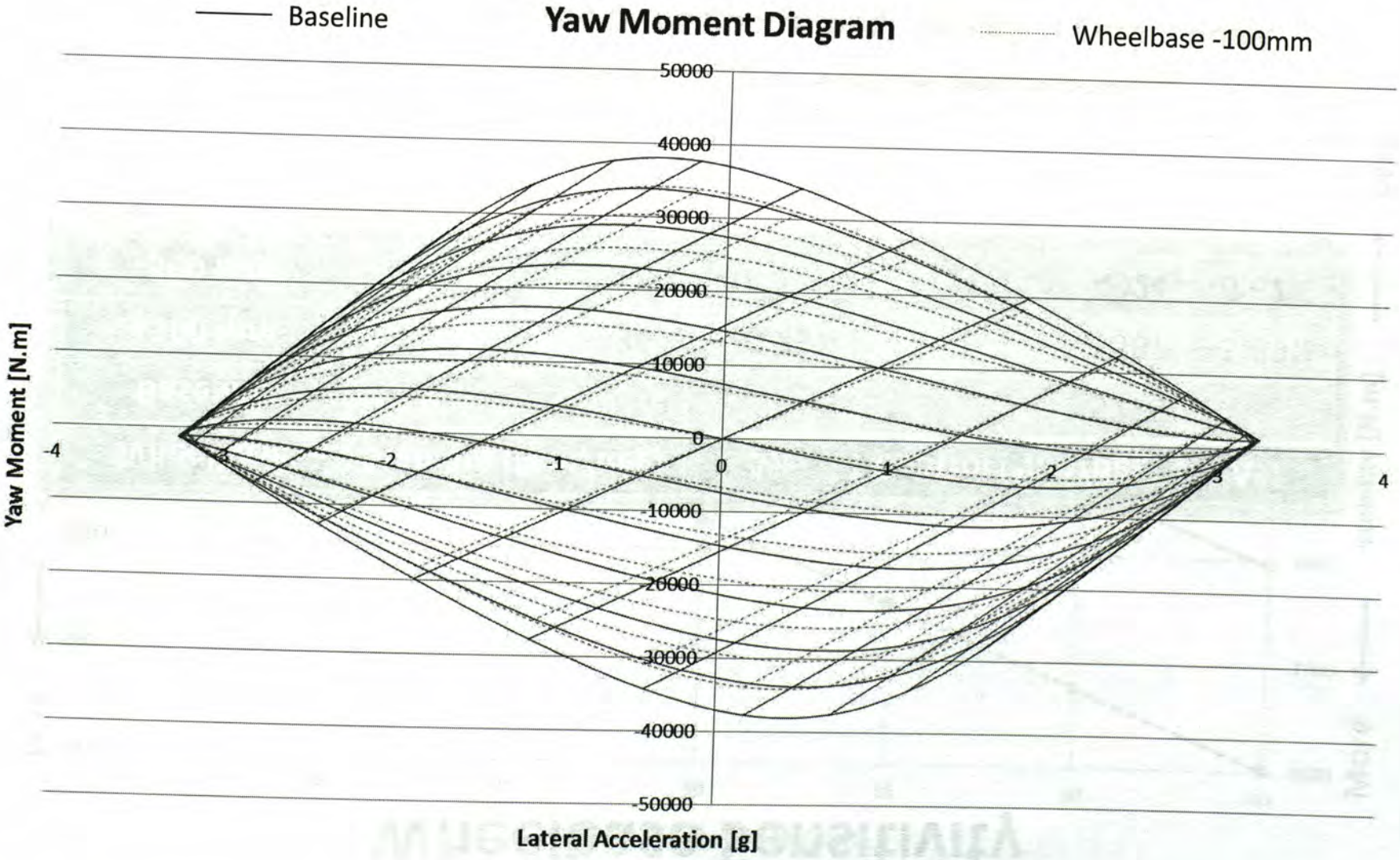
- Useful in car design and conception
- Useful in car setup and car understanding
- With the yaw moment diagram method:
  - Control
  - Directional Stability
  - Limit Behavior

- The next slides present part of sensitivity analysis of a GT car

Wheelbase Sensitivity



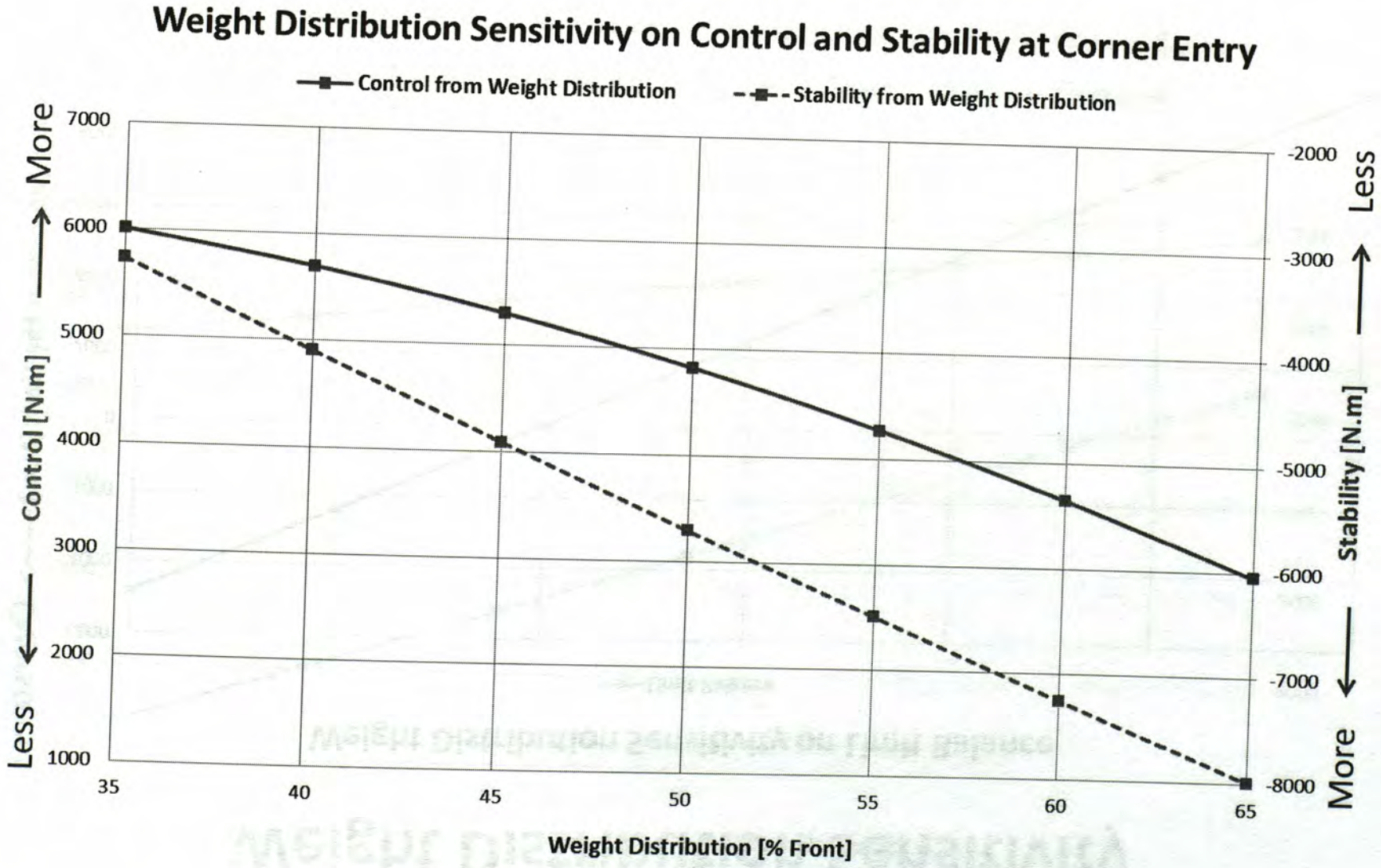
Wheelbase Sensitivity



Wheelbase Sensitivity

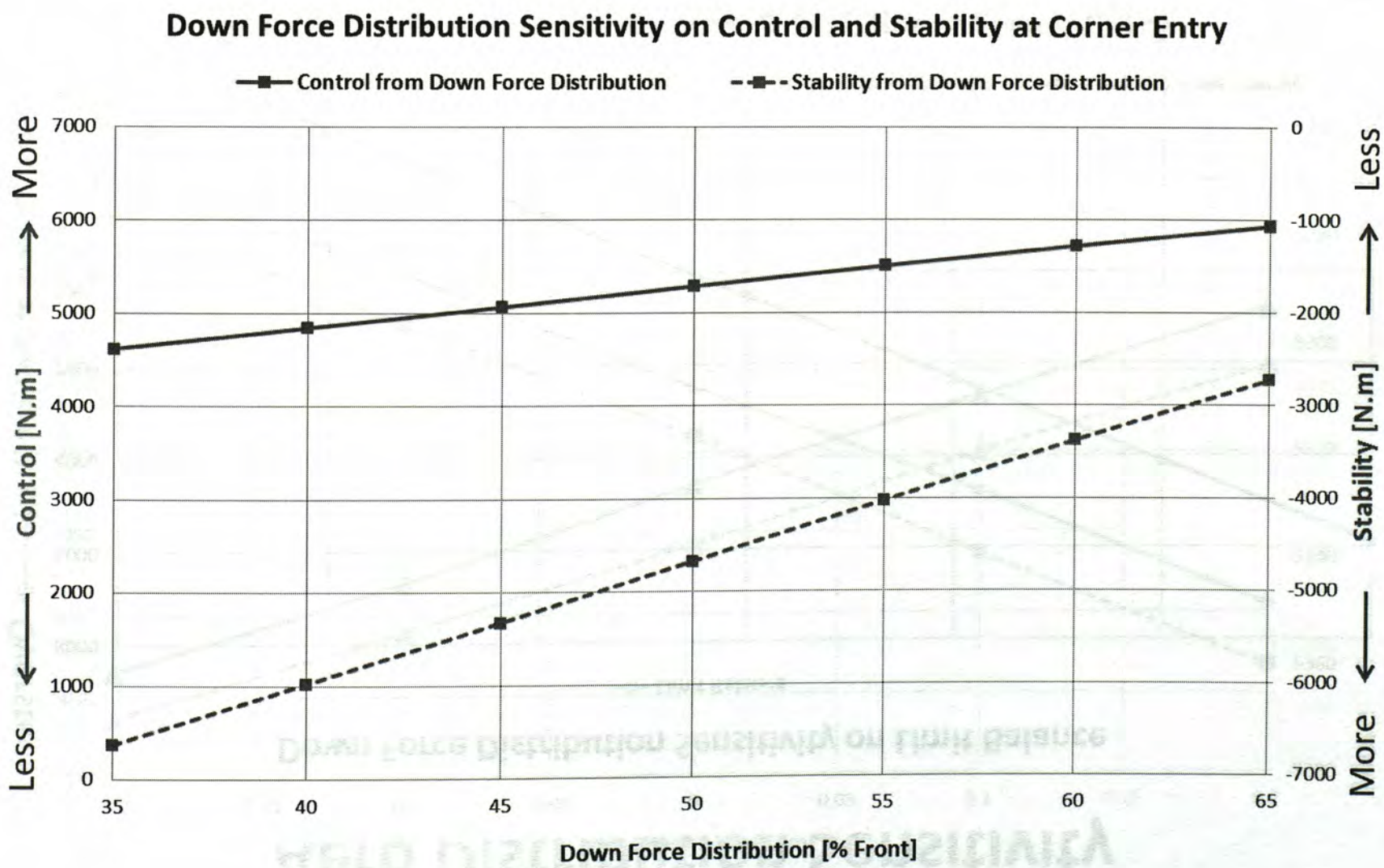
Wheelbase	Stability (N.m/deg)	Δ%	Control (N.m/deg)	Δ%
Baseline	-4463	-	5276	-
+300 mm	-5367	+20.25%	5587	+5.89%
-300 mm	-3643	-18.37%	4924	-6.67%

Weight Distribution Sensitivity



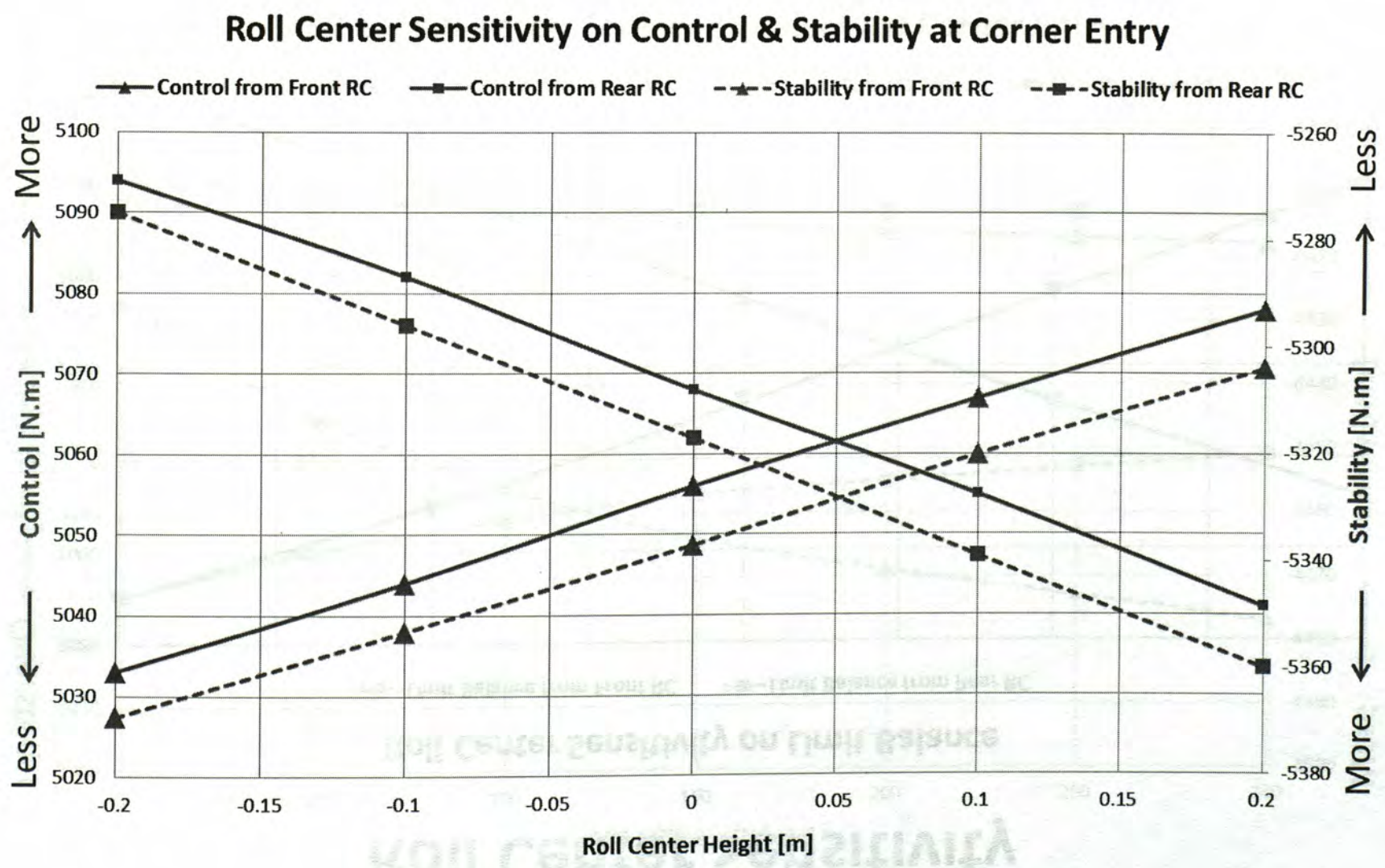
Weight Distribution Sensitivity

Aero Distribution Sensitivity

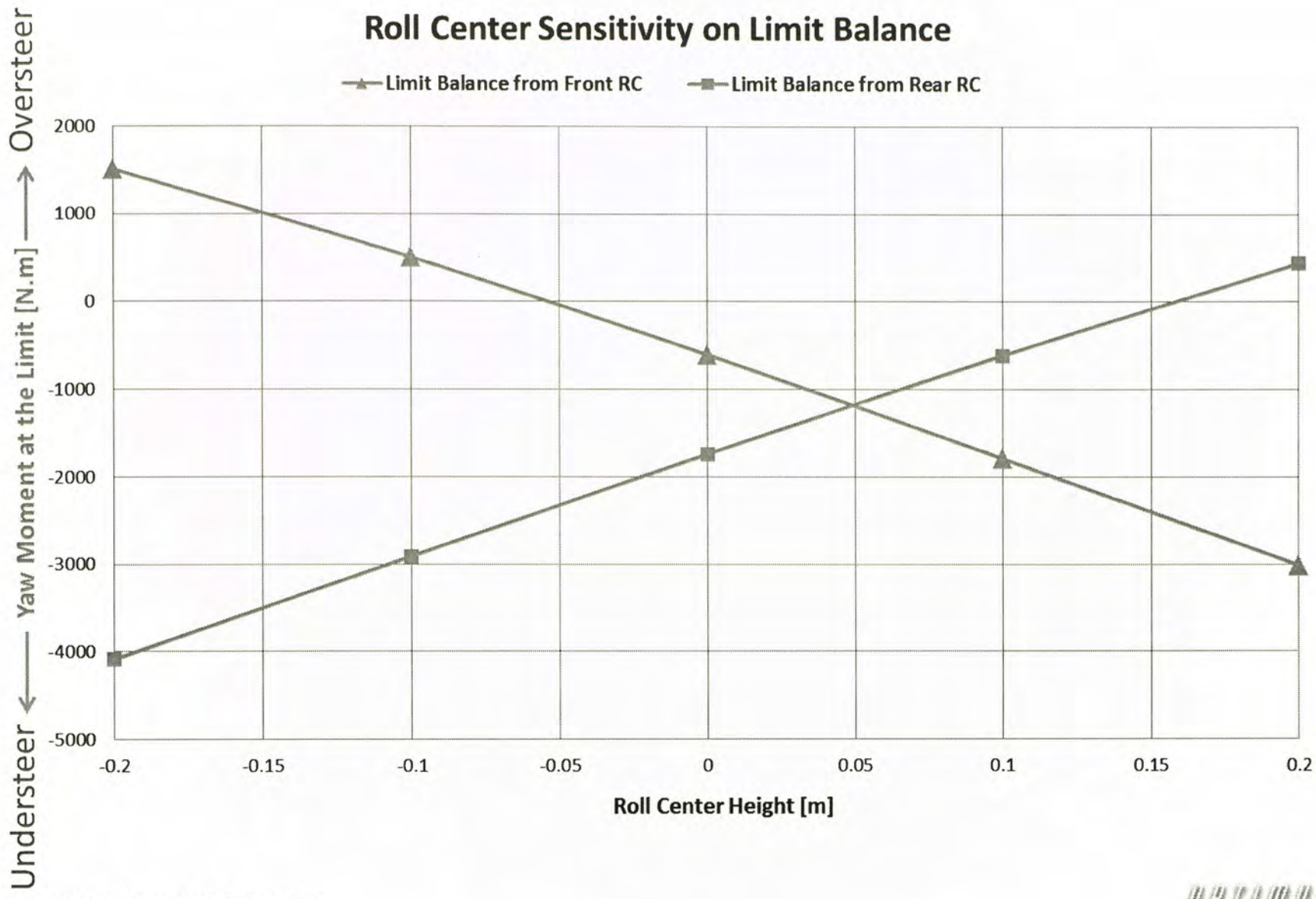


Aero Distribution Sensitivity

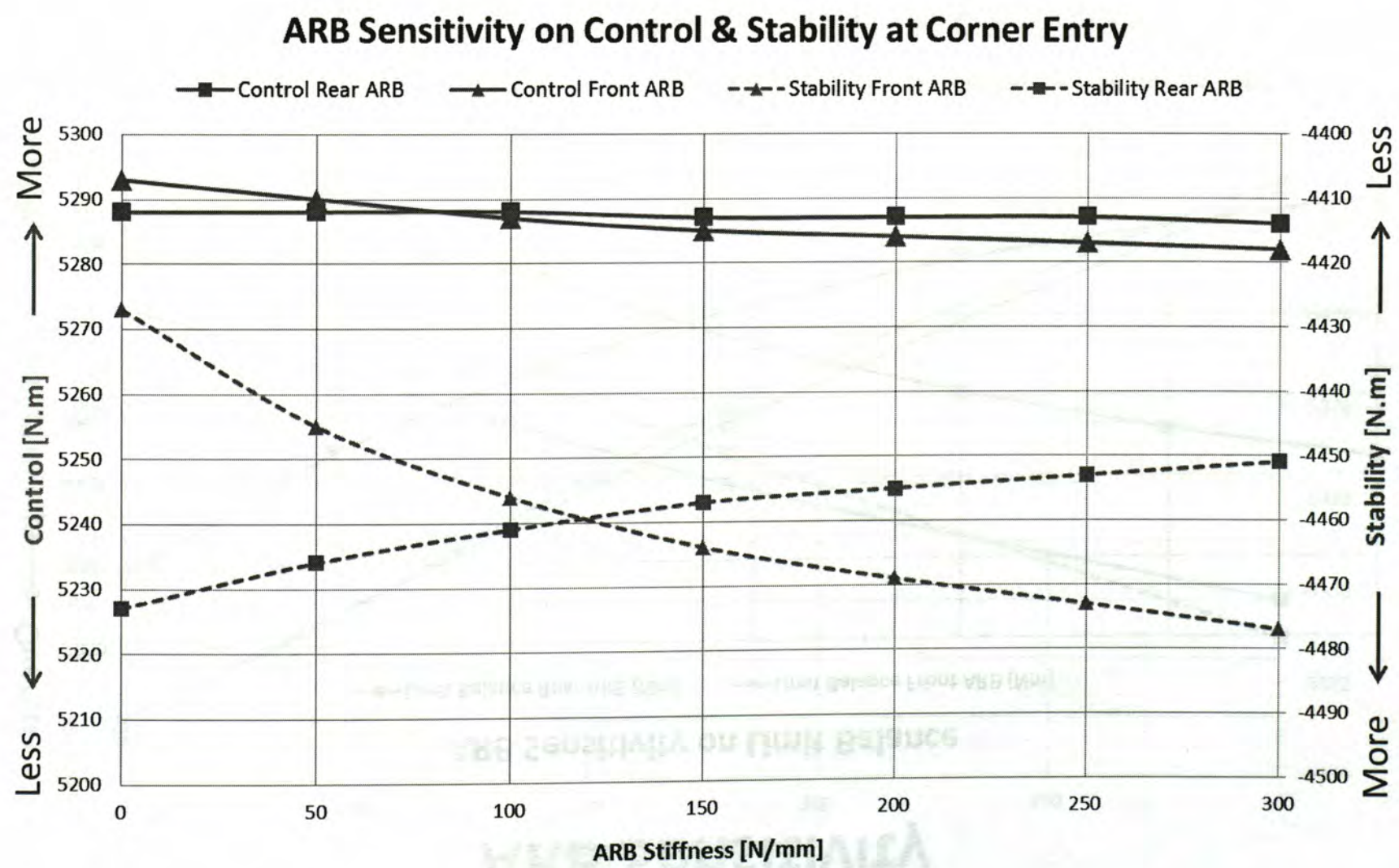
Roll Center Sensitivity



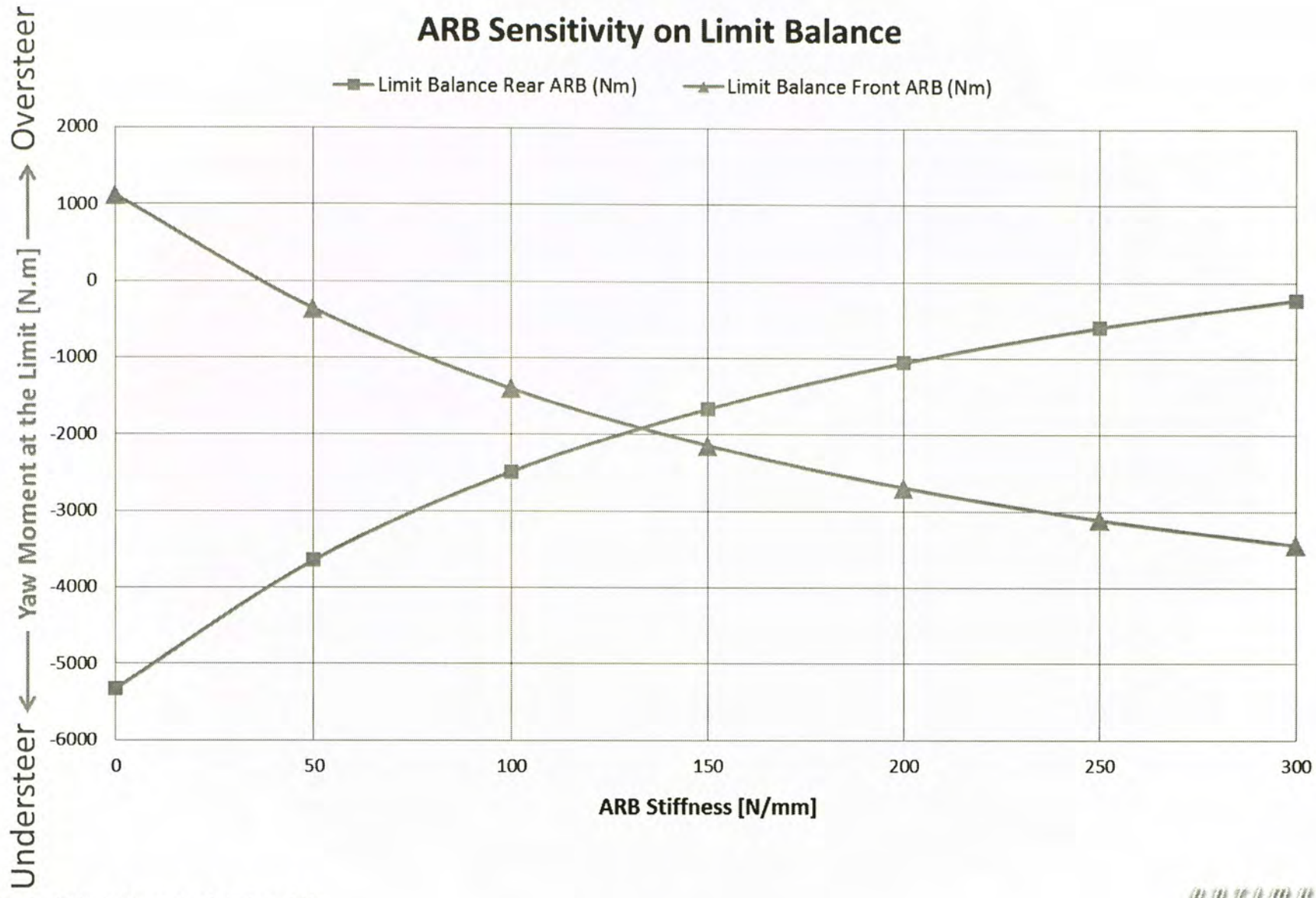
Roll Center Sensitivity



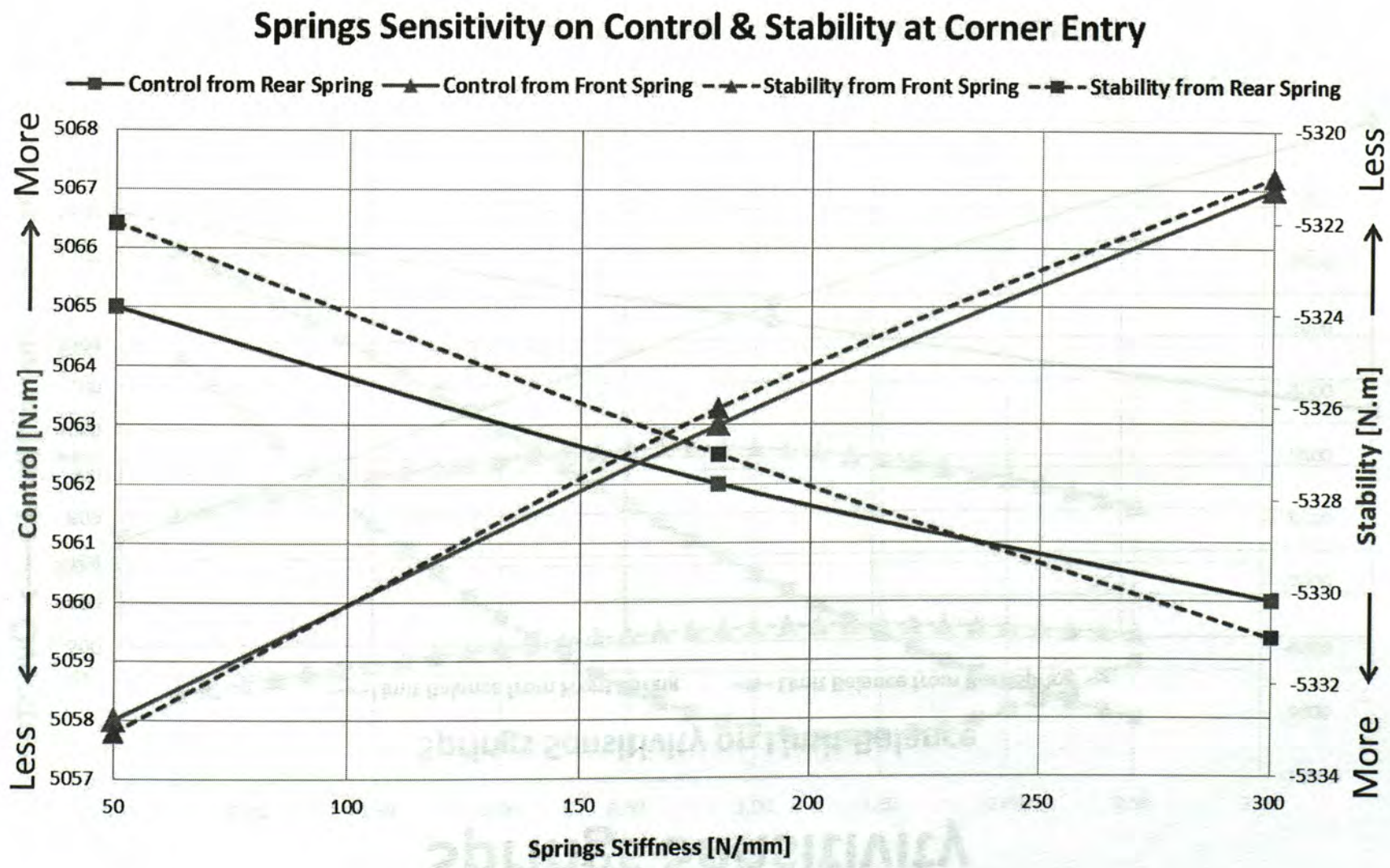
ARB Sensitivity



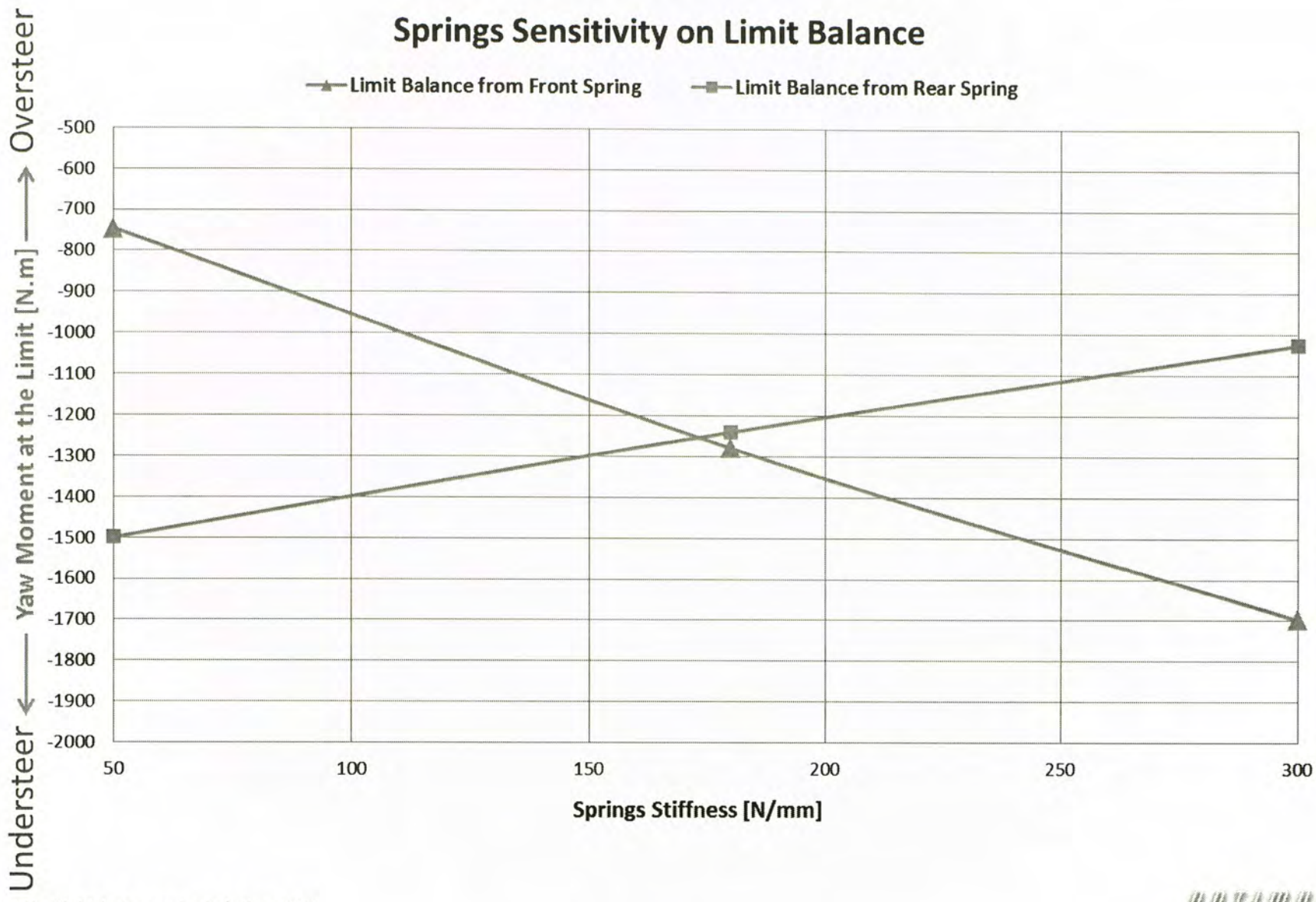
ARB Sensitivity



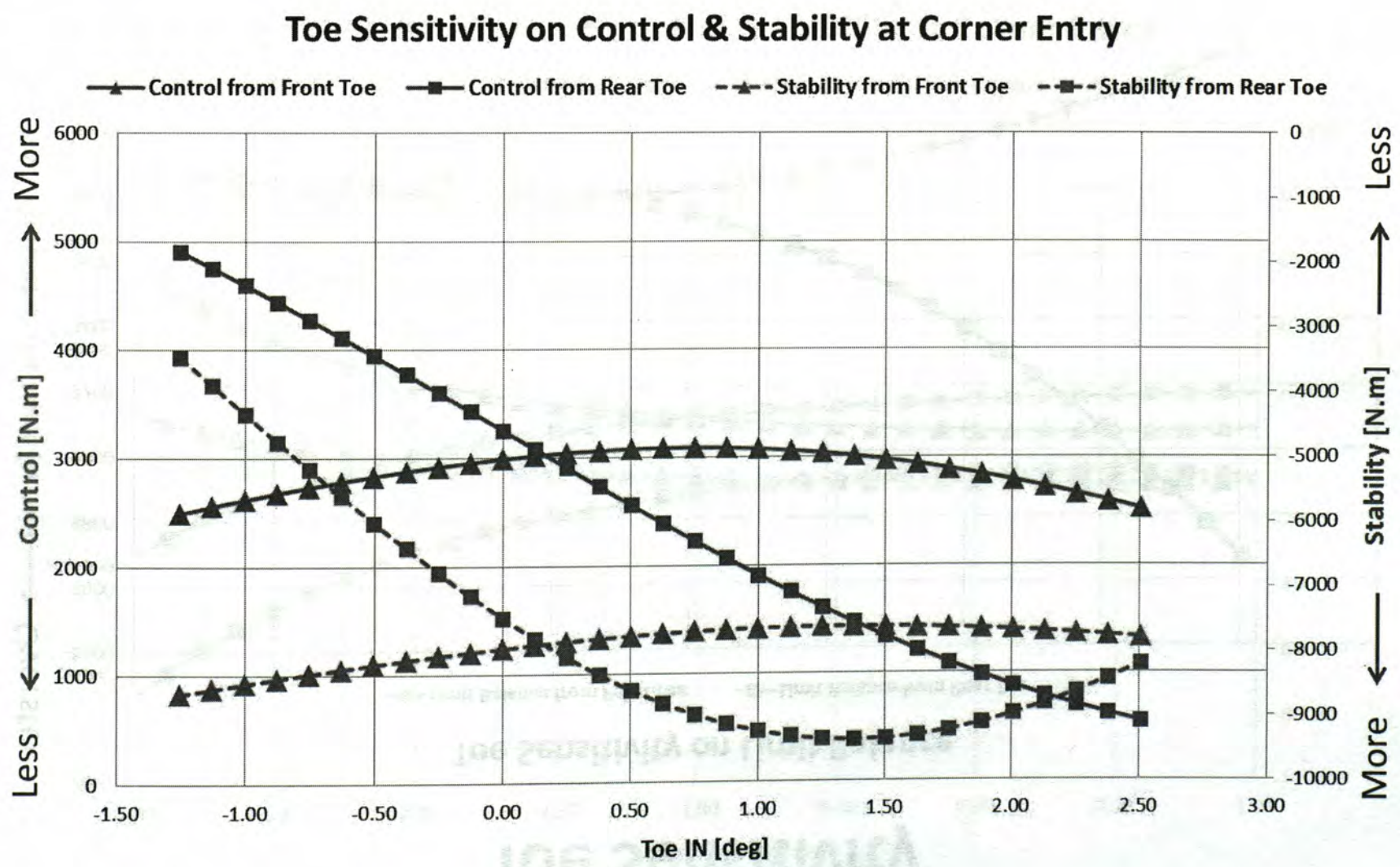
Springs Sensitivity



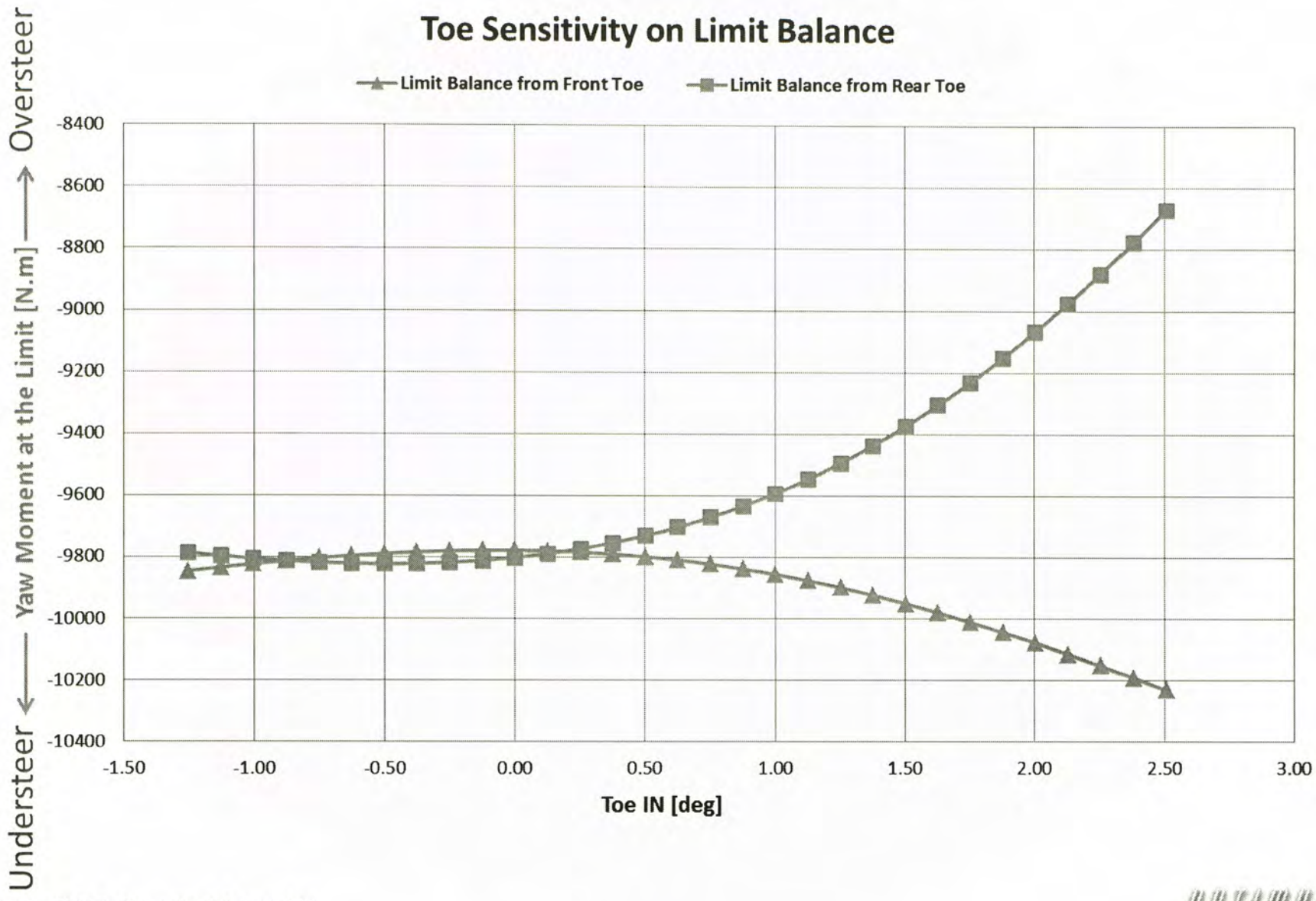
Springs Sensitivity



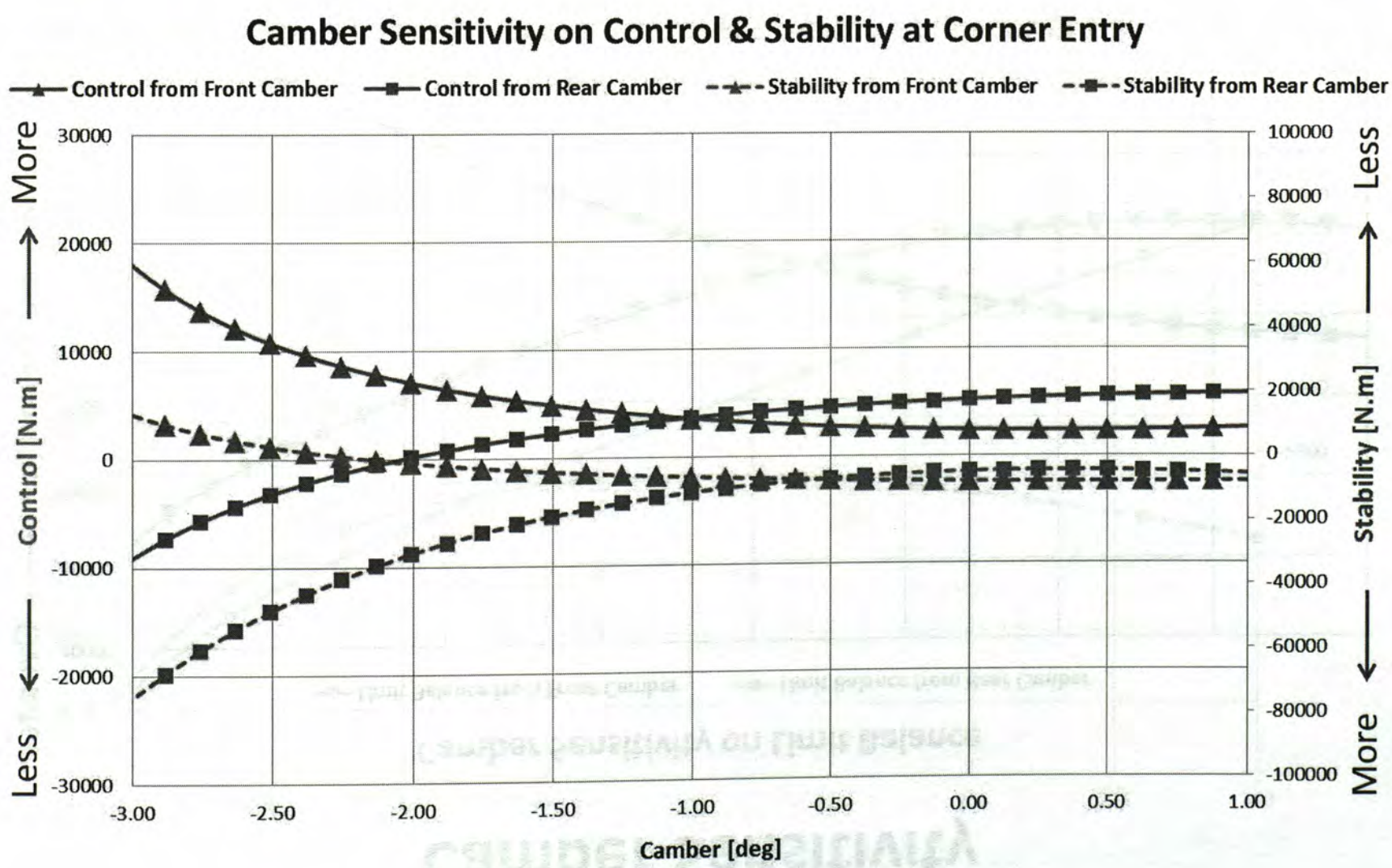
Toe Sensitivity



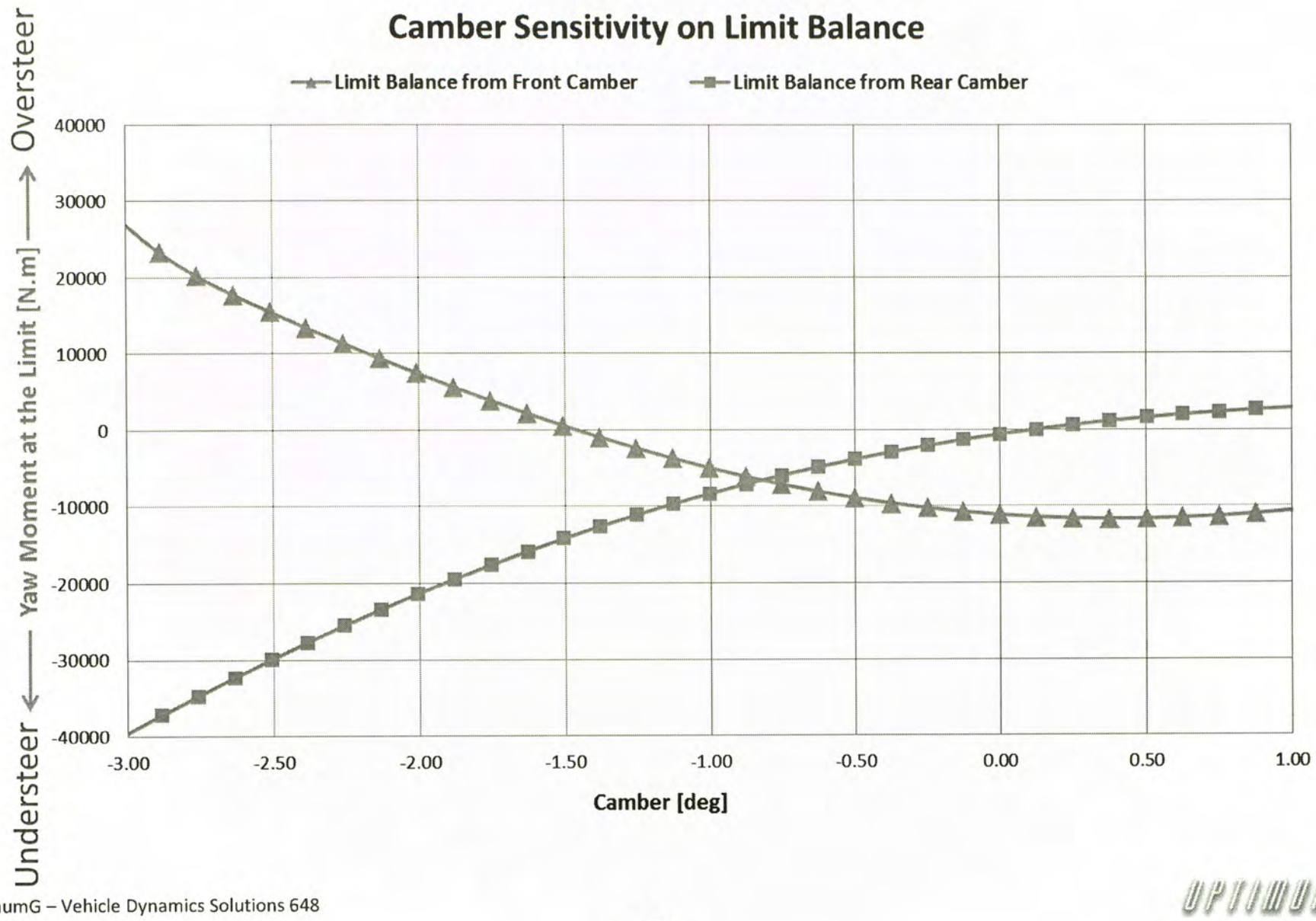
Toe Sensitivity



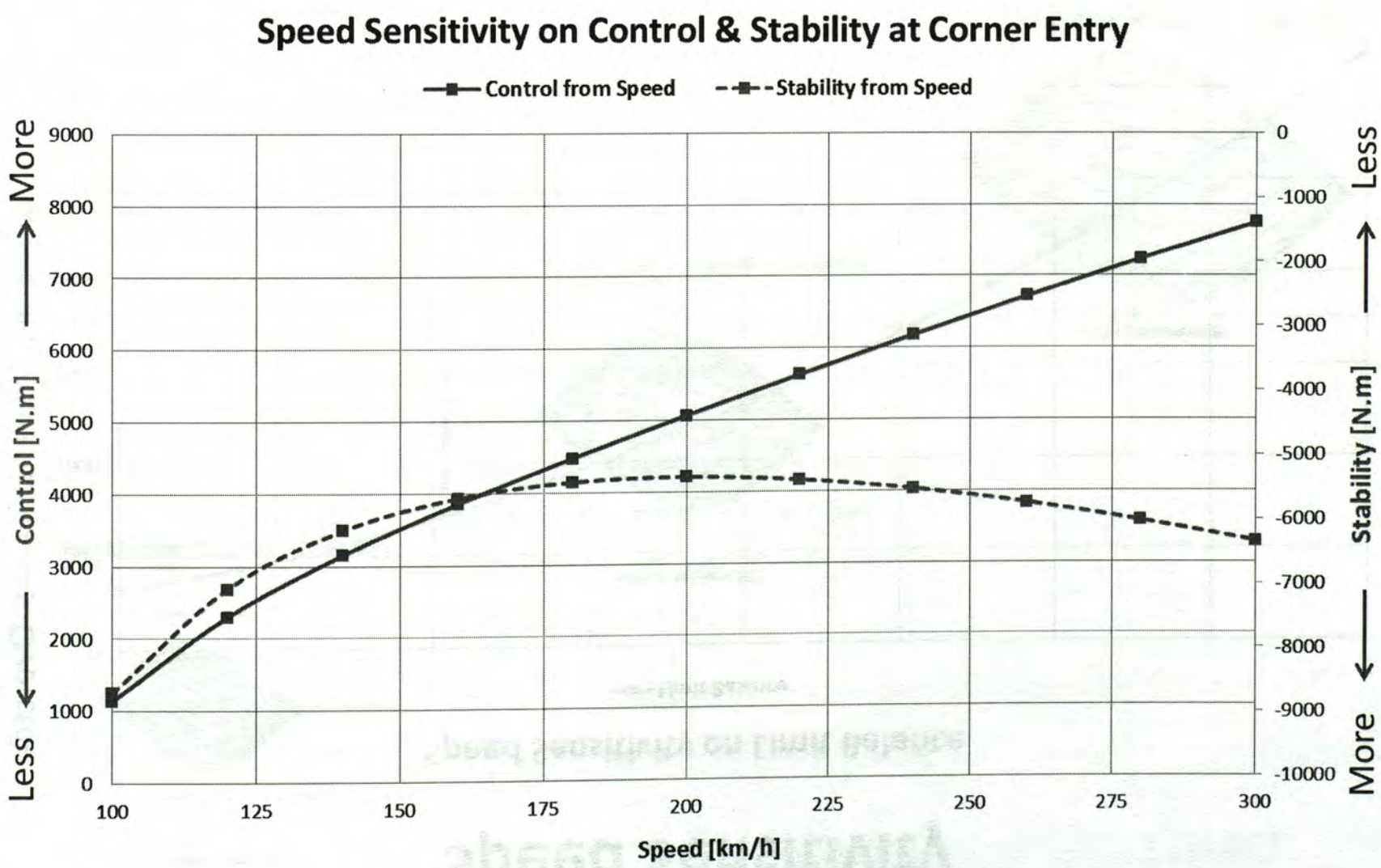
Camber Sensitivity



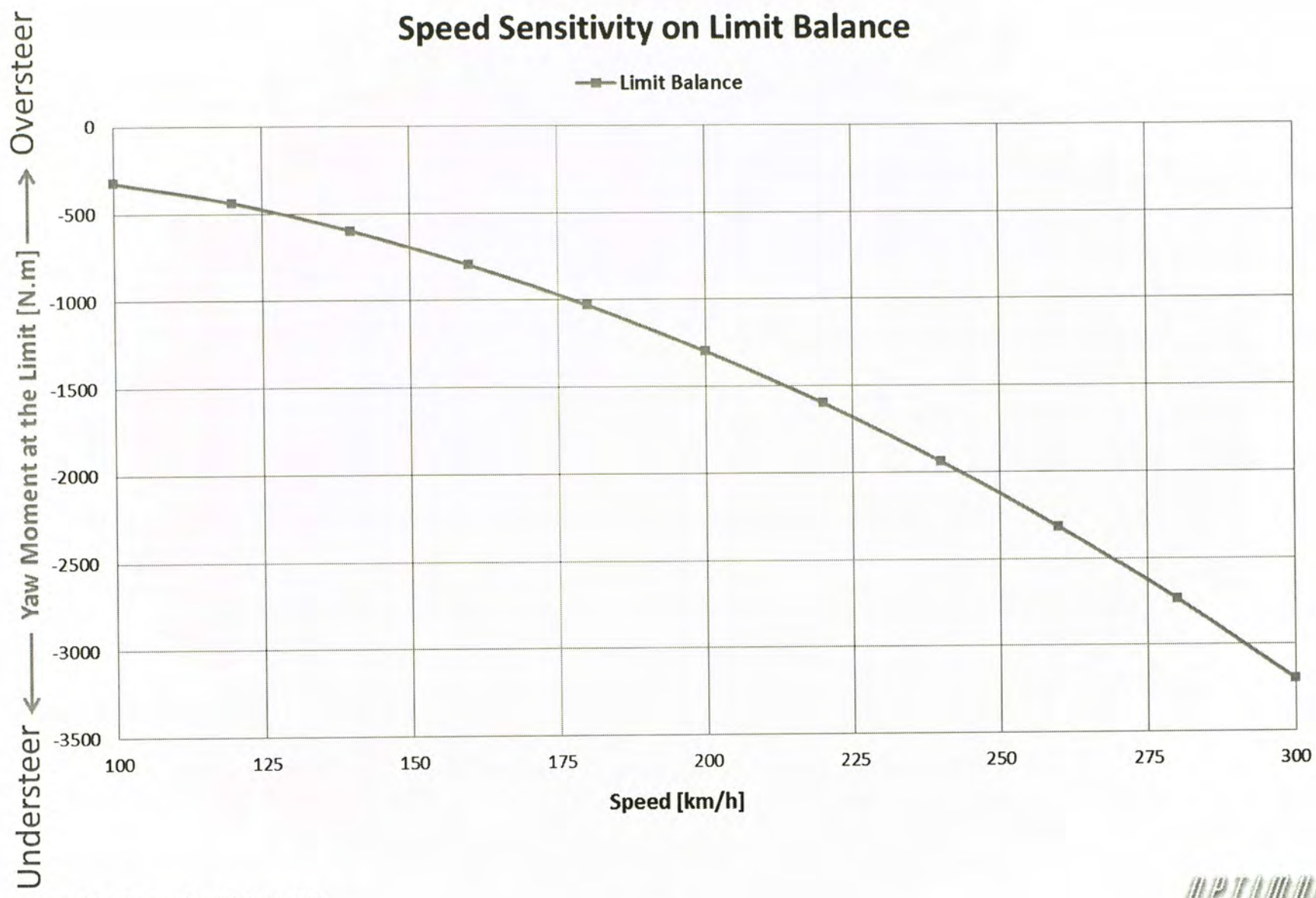
Camber Sensitivity



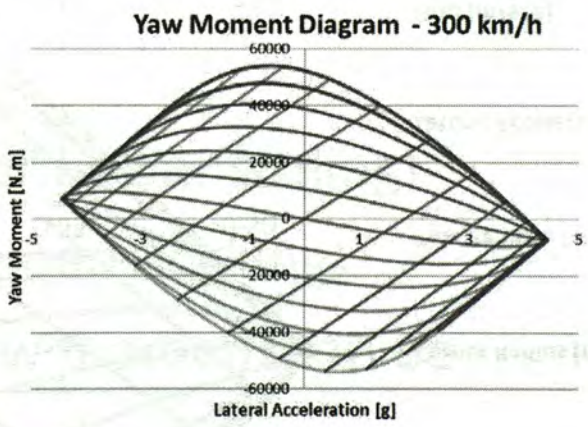
Speed Sensitivity

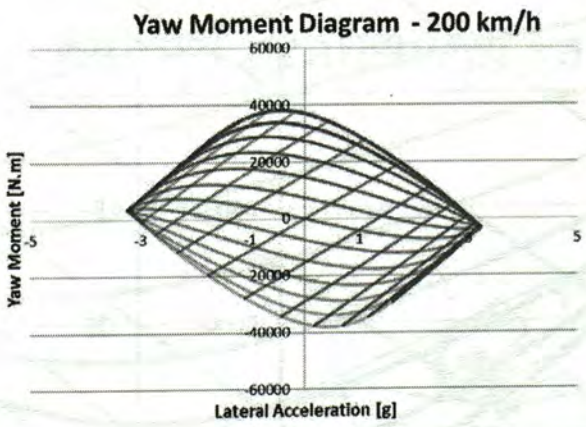


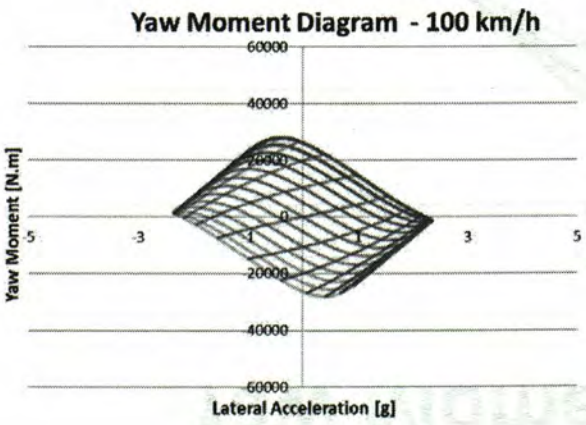
Speed Sensitivity



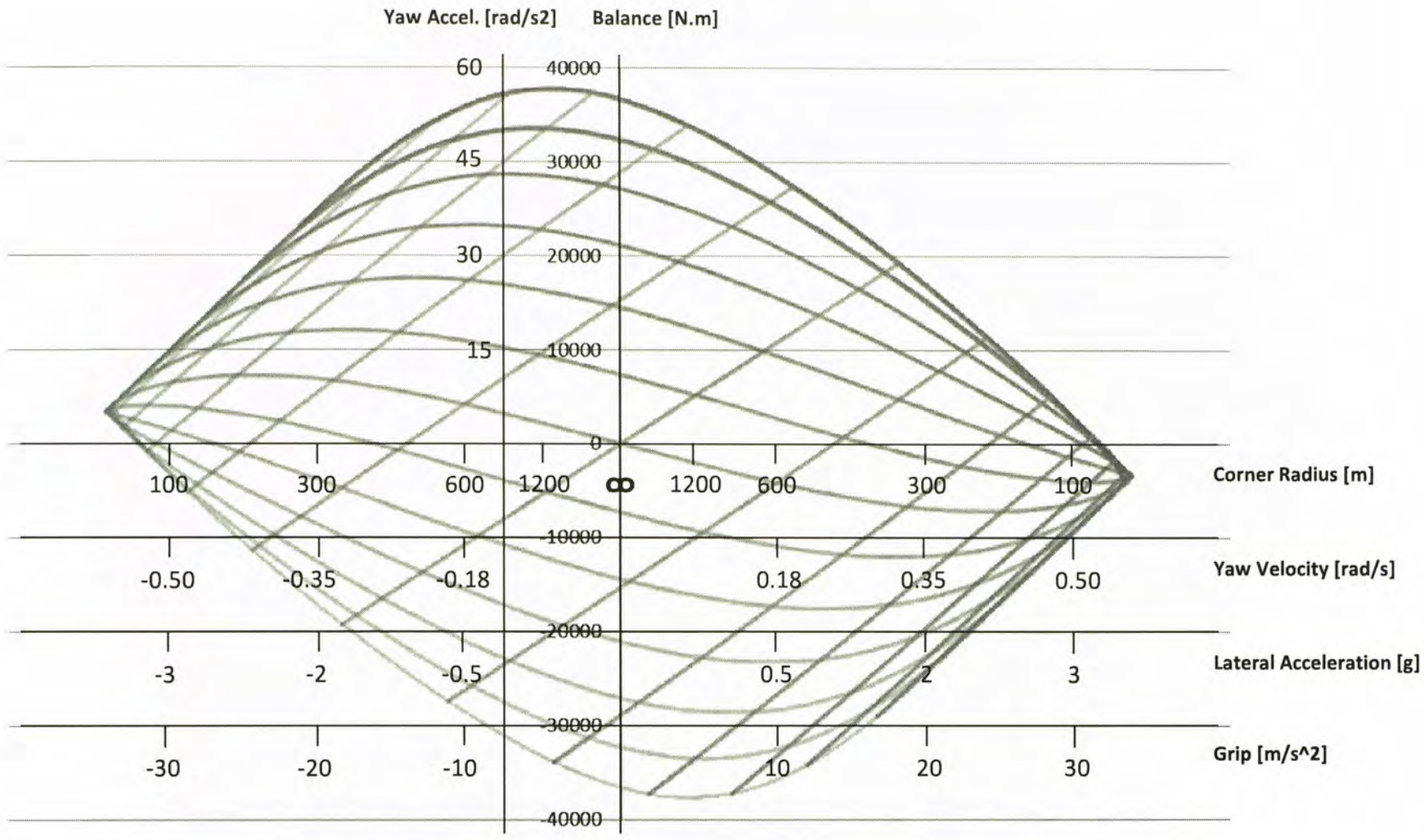
Speed Sensitivity







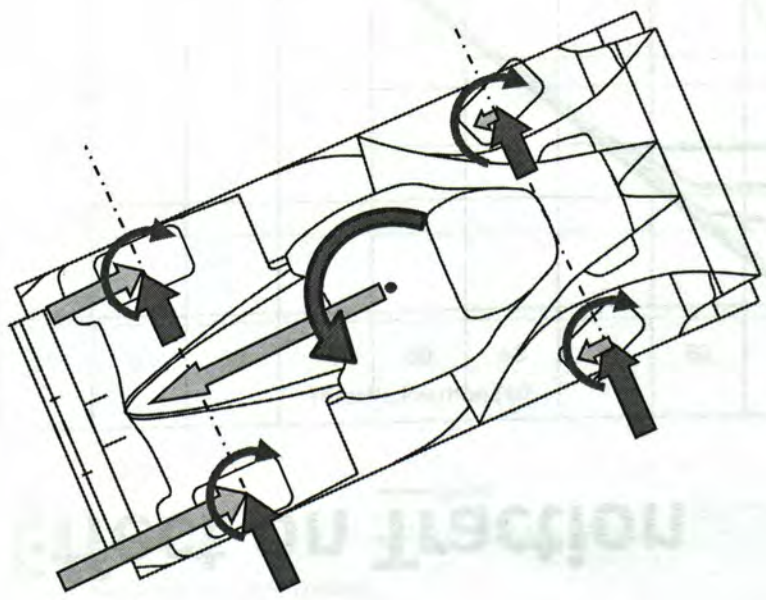
Yaw Moment Vs. Different Perspectives



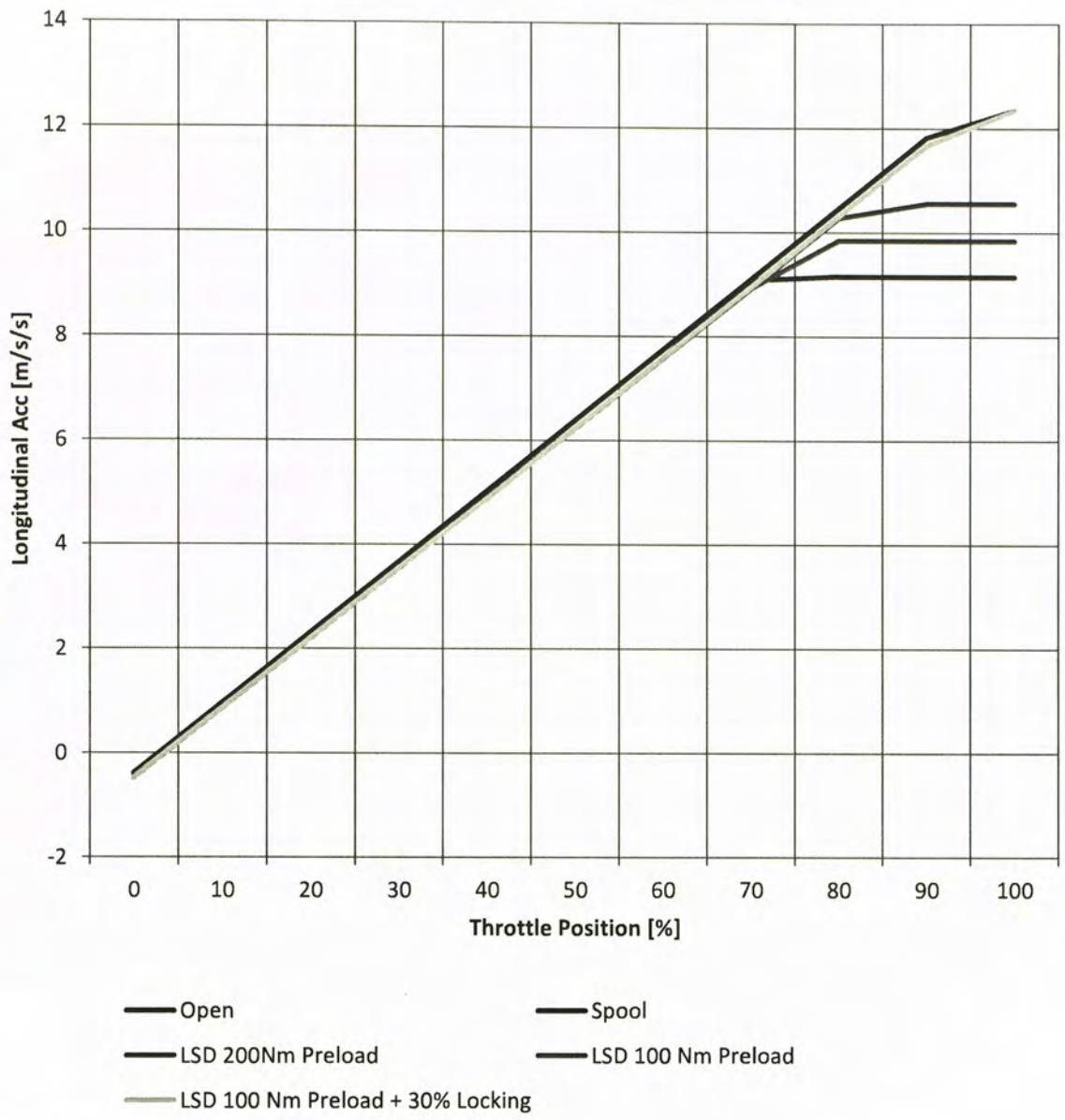
Note : 200 km/h ; I_{ZZ} = 650kg.m^2

Differential Effect on Yaw Moment

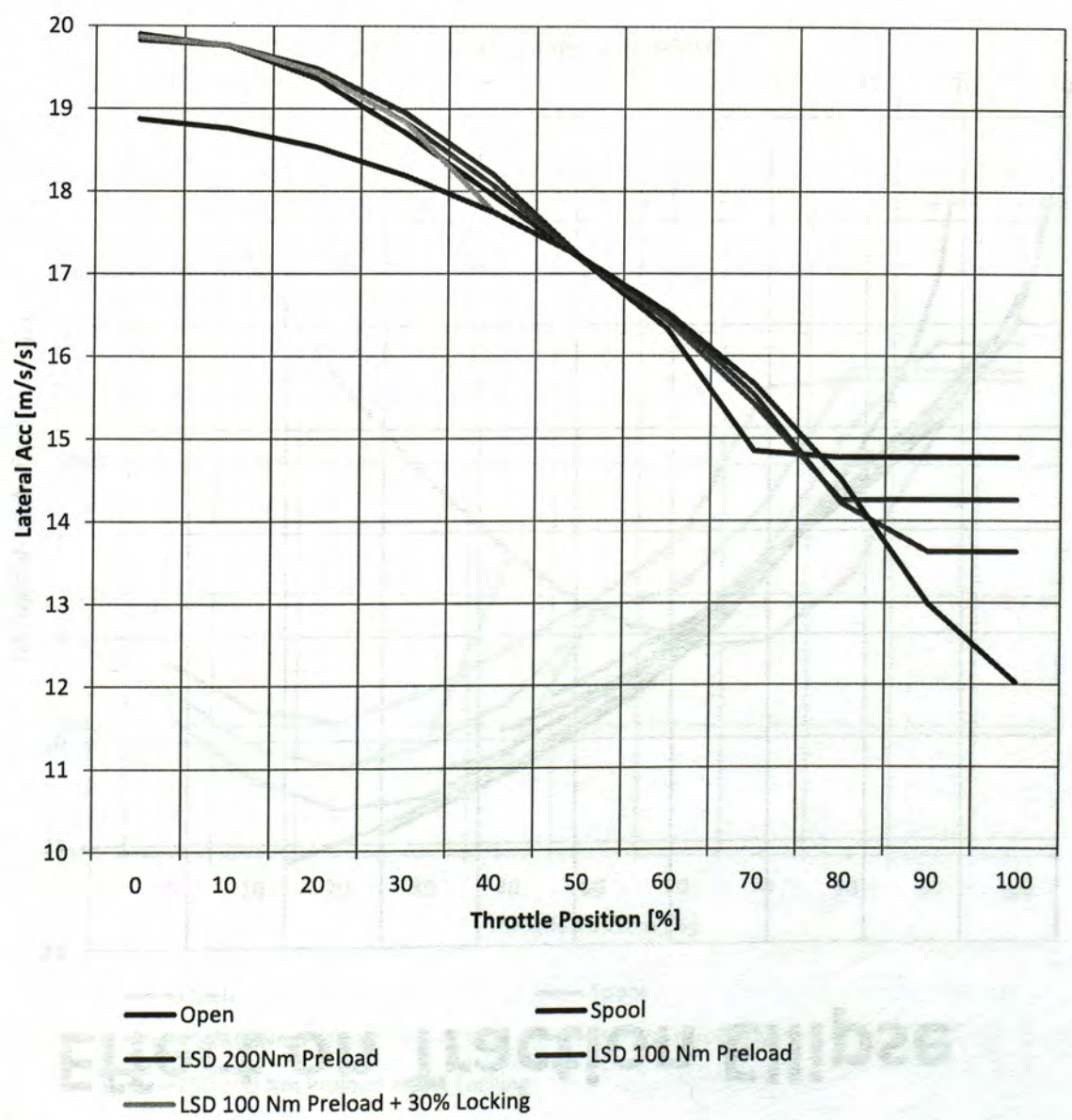
- Road Course Setup at 80 km/h
- "Throttle" swept from 0% to 100% (linear power scaling)
- Open Diff, Spool, LSD (preload), LSD (preload+locking)
- Interaction between the 12 causes of yaw moment can be investigated



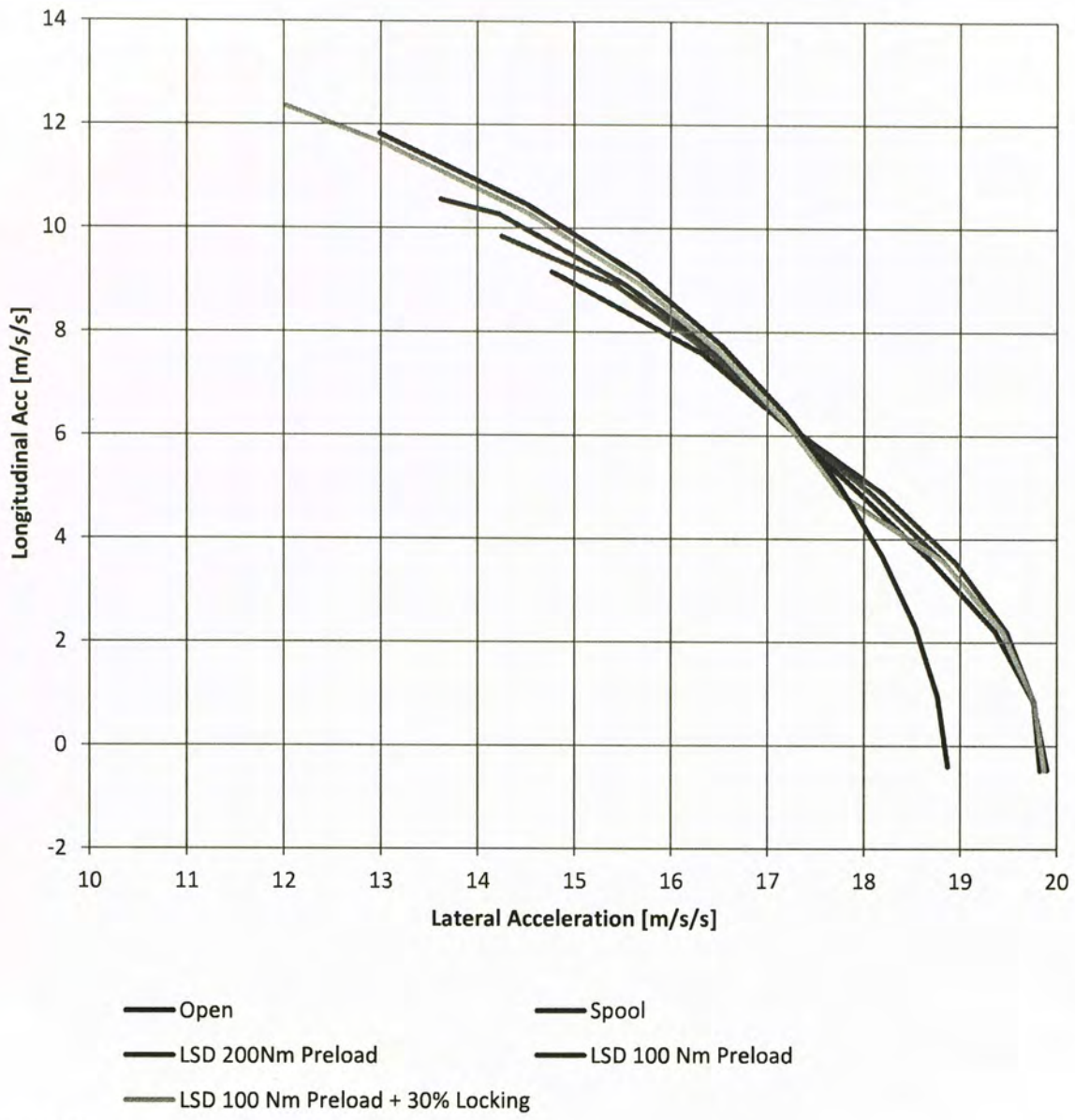
Effect on Traction



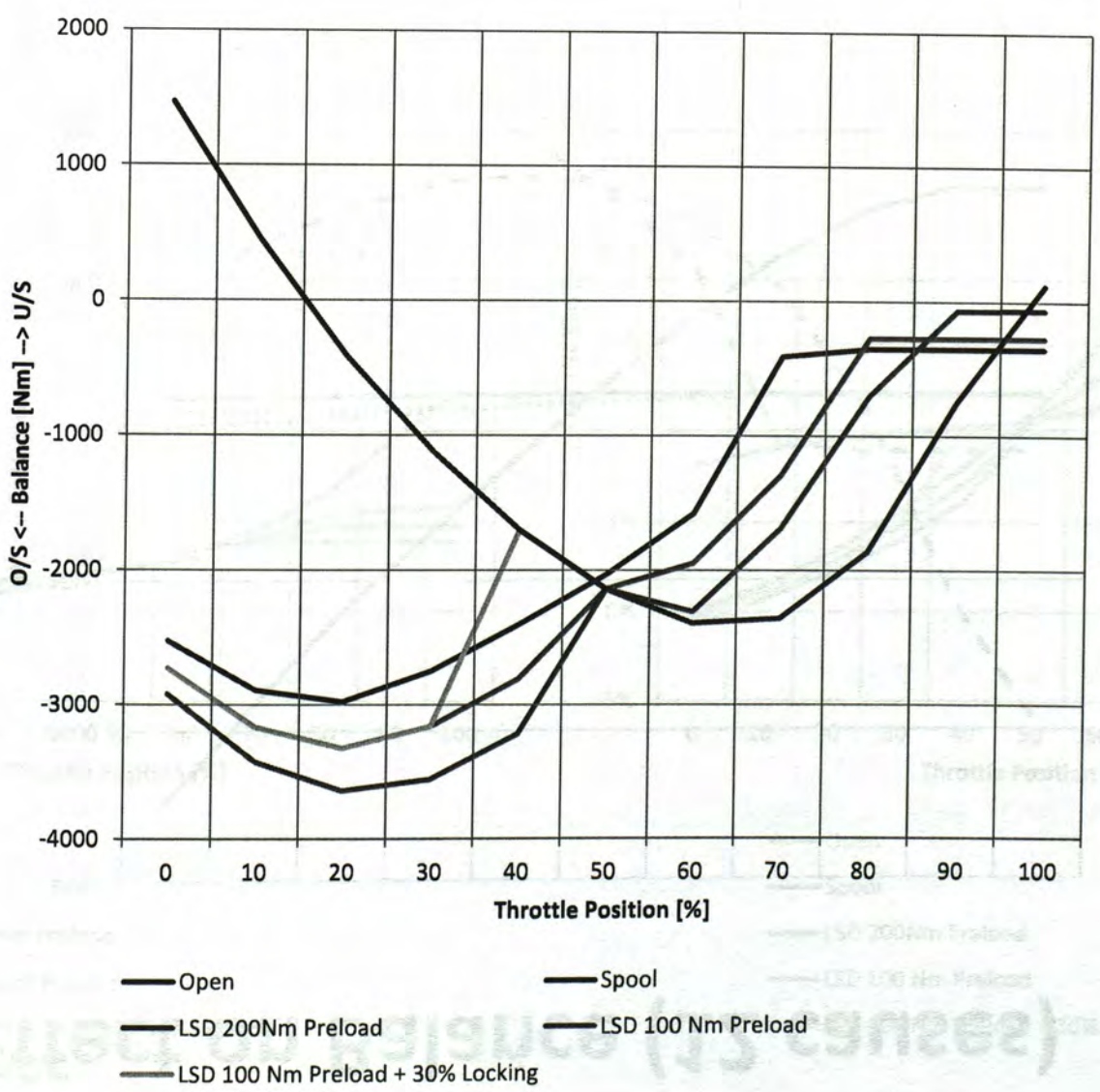
Effect on Grip



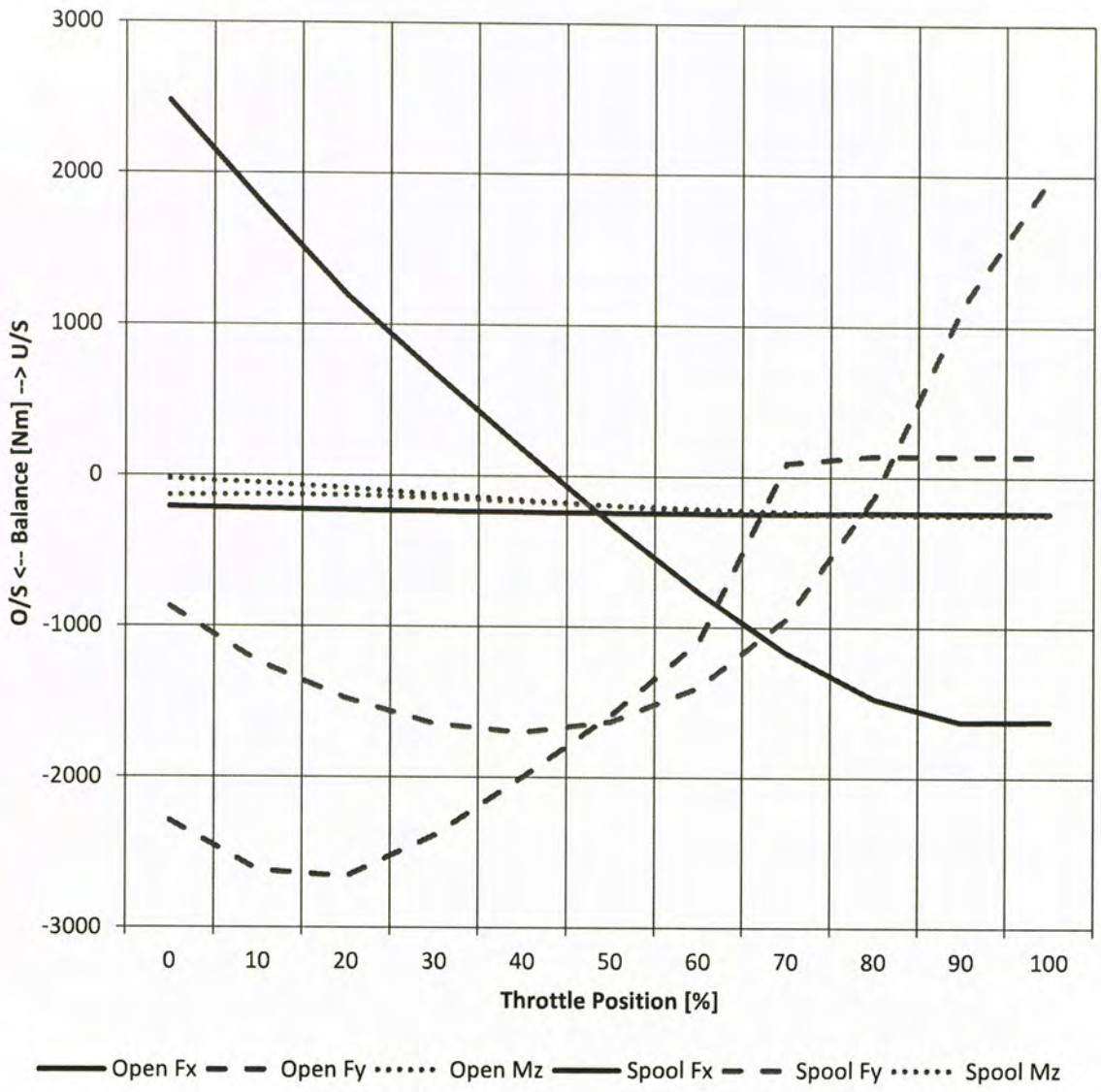
Effect on Traction Ellipse



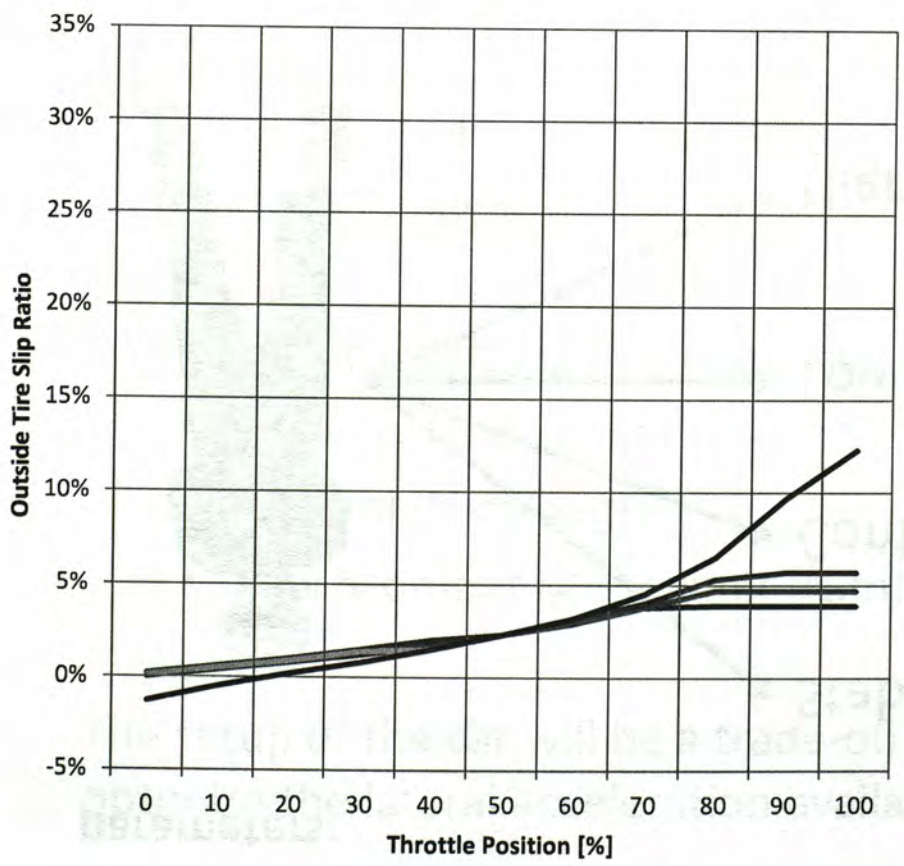
Effect on Balance

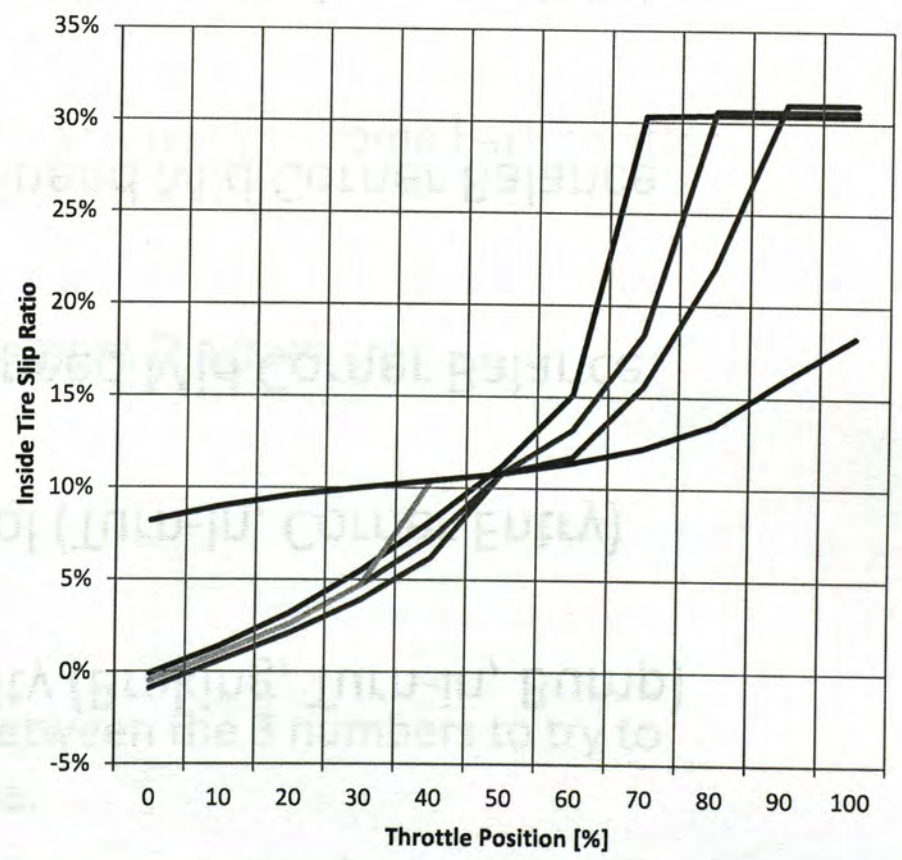


Effect on Balance (12 causes)



Effect on Tire Slip Ratio





Open
Spool
LSD 200Nm Preload
LSD 100 Nm Preload
LSD 100 Nm Preload + 30% Locking

Open
Spool
LSD 200Nm Preload
LSD 100 Nm Preload
LSD 100 Nm Preload + 30% Locking

Optimizing setup with the YMD

After each outing, you need to ask your driver to quantify the following parameters:

- Stability (Braking, Turn-in, Bump)
- Control (Turn-in, Corner Entry)
- Low Speed Mid Corner Balance
- High Speed Mid Corner Balance

For faster results, the problem of finding the correct balance needs to be taken in order.

Optimizing setup with the YMD

The goal is to find 3 "magic numbers" that you need to come back to after each setup changes.

The 3 "magic numbers" from the Yaw Moment Diagram are:

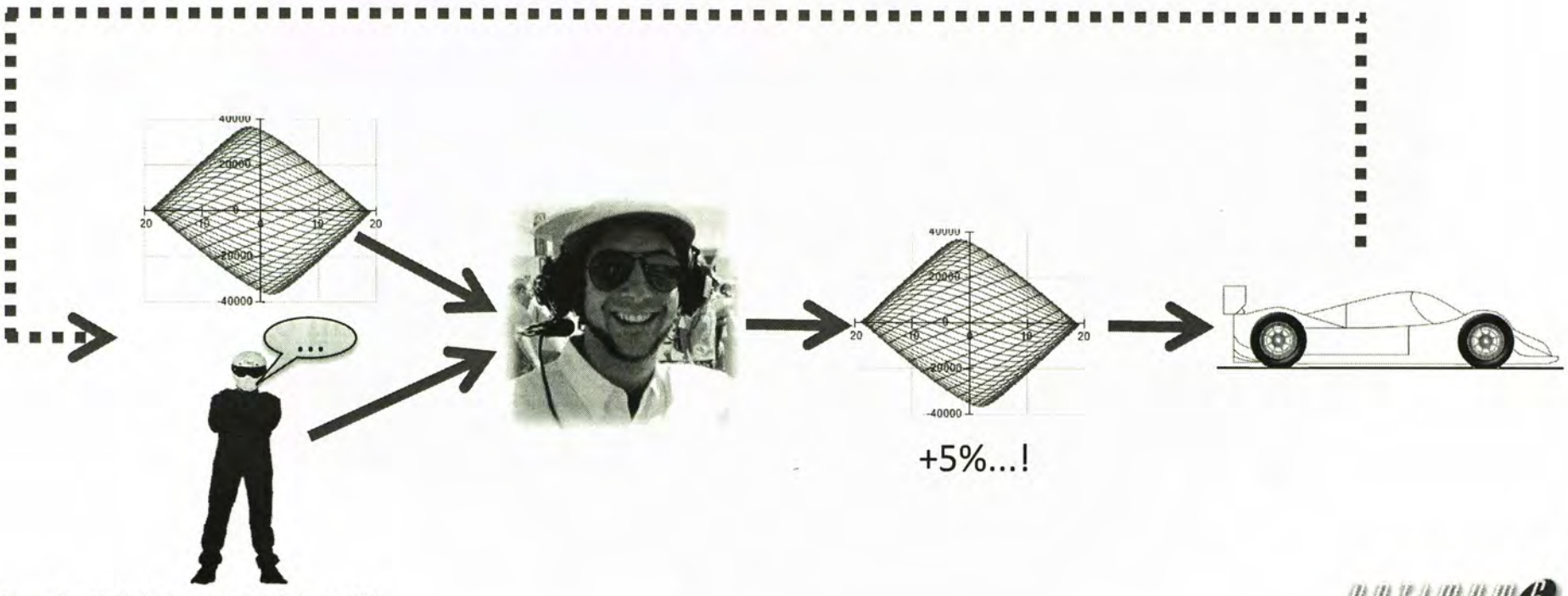
- Stability (N.m/deg)
- Control (N.m/deg)
- Yaw Moment @ the limit (N.m)

The setup of the car will be a trade-off between the 3 numbers to try to optimize the lateral acceleration available.

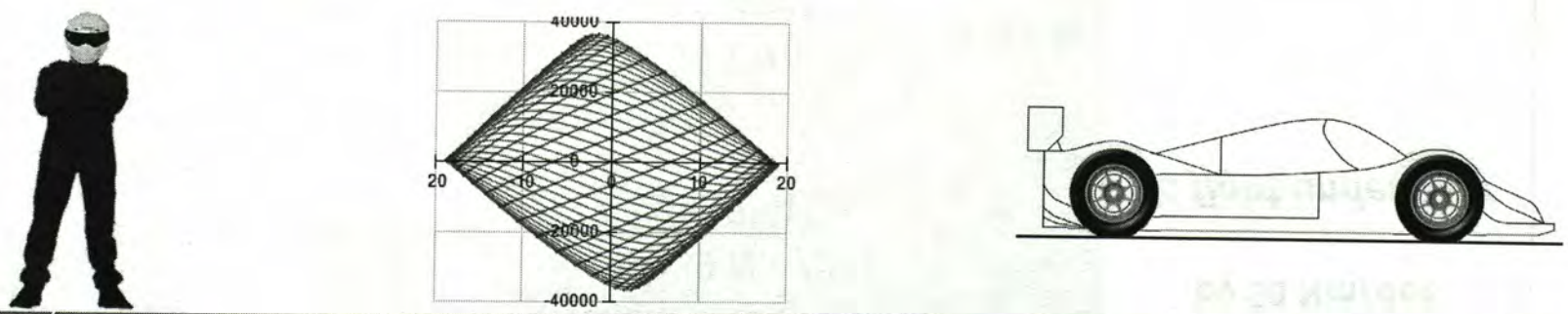
Note: The Yaw Moment @ the limit is not necessarily 0. It is driver dependent! (It also depends on your simulation accuracy)

Optimizing setup with the YMD

1. Listen to driver comments
2. Correlate with YMD predictions
3. Set goal for next outing
4. Find setup that meet goals
5. Test setup
6. Repeat

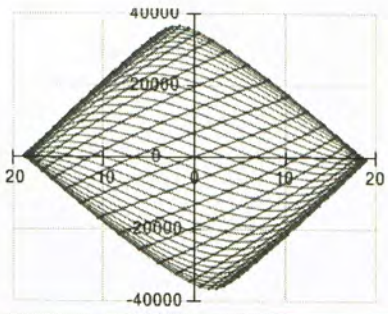


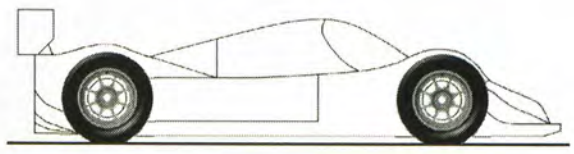
Optimizing setup with the YMD



Run 1	Driver	YMD	Goal	Setup Change
Stability	*Really bad over bumps and under braking. Car really not stable, rear moving a lot*	-8200 Nm/deg	+5%	**Rear Toe-In Increase 0.5 mm → 2 mm** ***To increase stability***
Control	*Hard to describe because of lack of stability*	11480 Nm/deg	-	
Balance	*OK, but not at the limit yet due to poor corner entry*	204 Nm	-	

Optimizing setup with the YMD

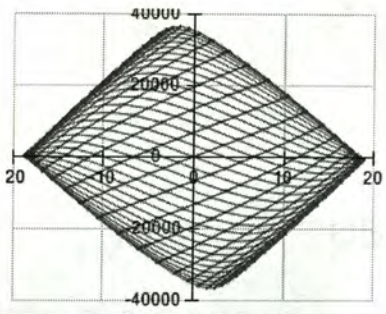


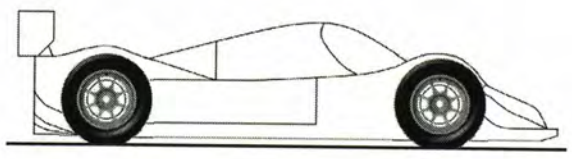


Run 2	Driver	YMD	Goal	Setup Change
Stability	*No problem, much better*	-8598 Nm/deg (+4.9%)*	-	**Front ARB decreased by 50 Nm/deg** ***To fight understeer***
Control	*Sharp turn-in, really good*	11289 Nm/deg (-1.7%)*	-	
Balance	*Big U/S everywhere, loosing a lot of time*	193 Nm (-5.4%)*	+100%	

** Compared to Run 1*

Optimizing setup with the YMD



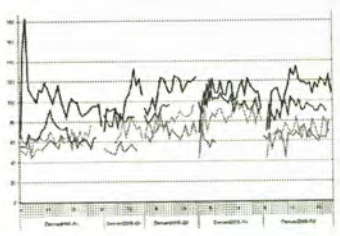


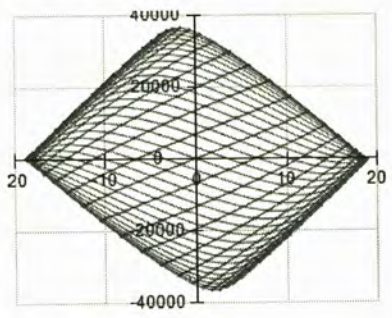
Run 3	Driver	YMD
Stability	*Still good*	-8621 Nm/deg (+0.3%)*
Control	*Still good*	11265 Nm/deg (-0.2%)*
Balance	*Good balance, car is good*	407 Nm (+110.9%)*

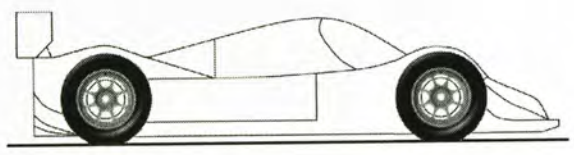
** Compared to Run 2*

Optimizing setup with the YMD

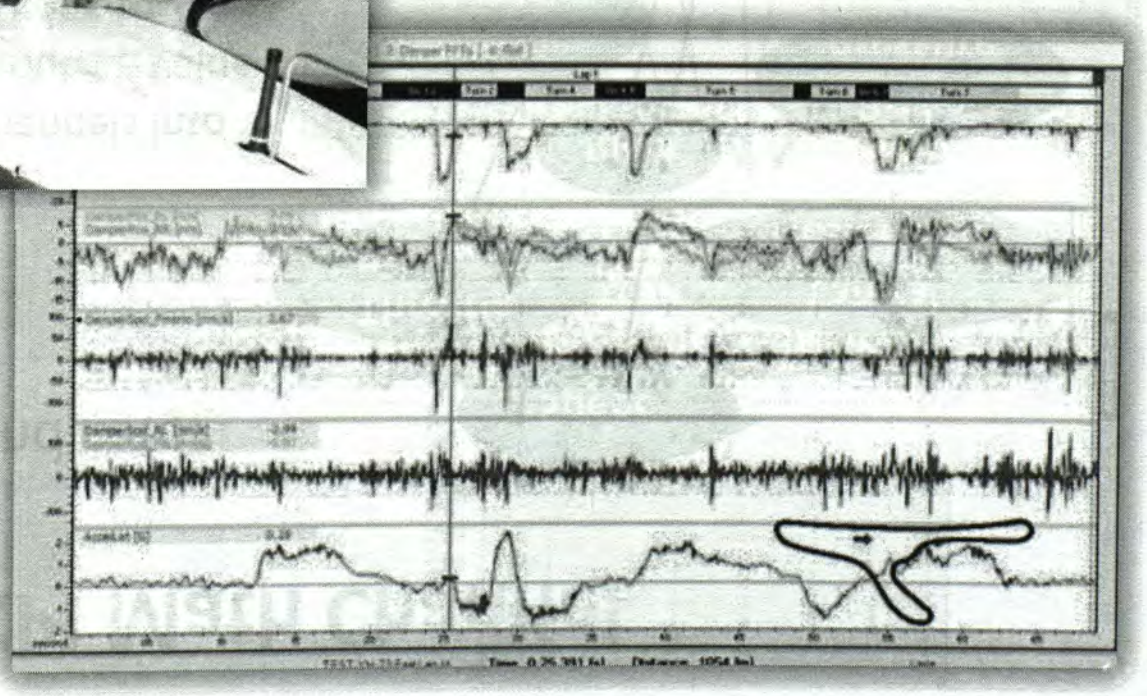
- DAQ is also an input to this process, it should correlate with driver comments
- Stability and Control are usually driver and car dependent
- Balance is more driver, car, track and condition dependent
- The goal is to find the good balance while not disturbing control and stability
  - And maximizing grip
- Sometimes more that one parameter has to be changed to maintain some parameters and change others
- The result should be kept in a database





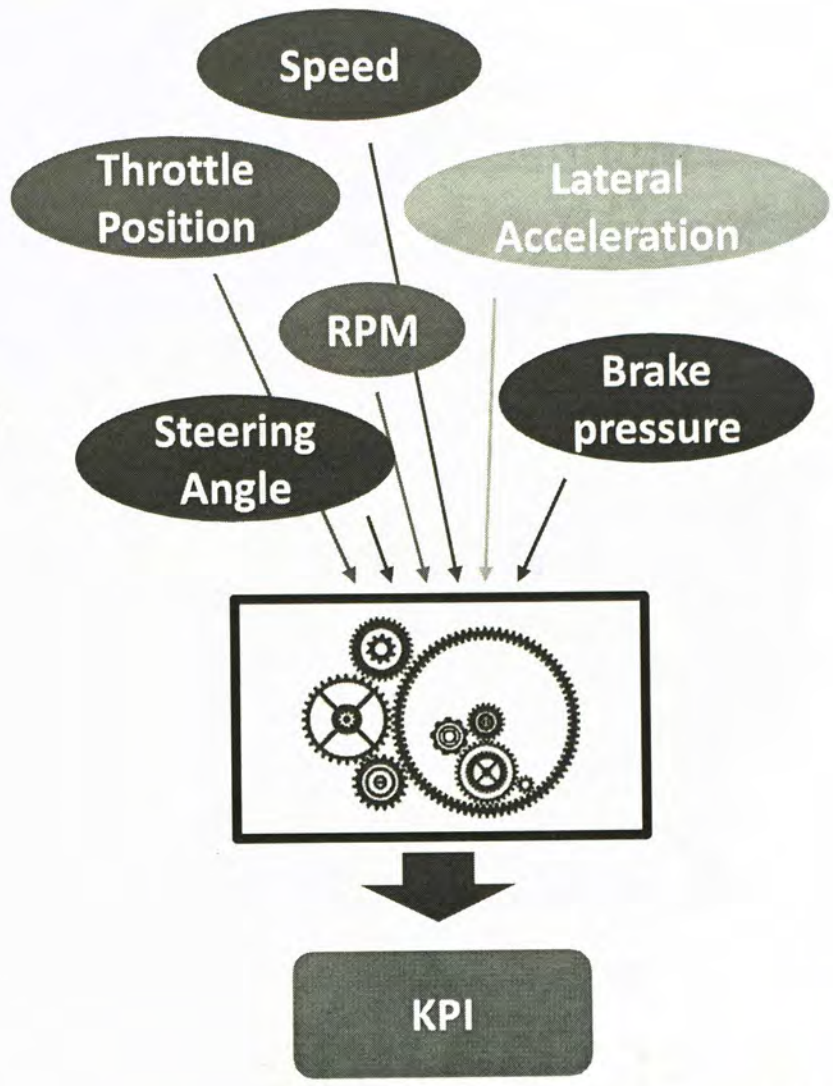


9. Simple Driver Acquired Data Analysis



KPI Math Channel

KPI: Key Performance Indicator

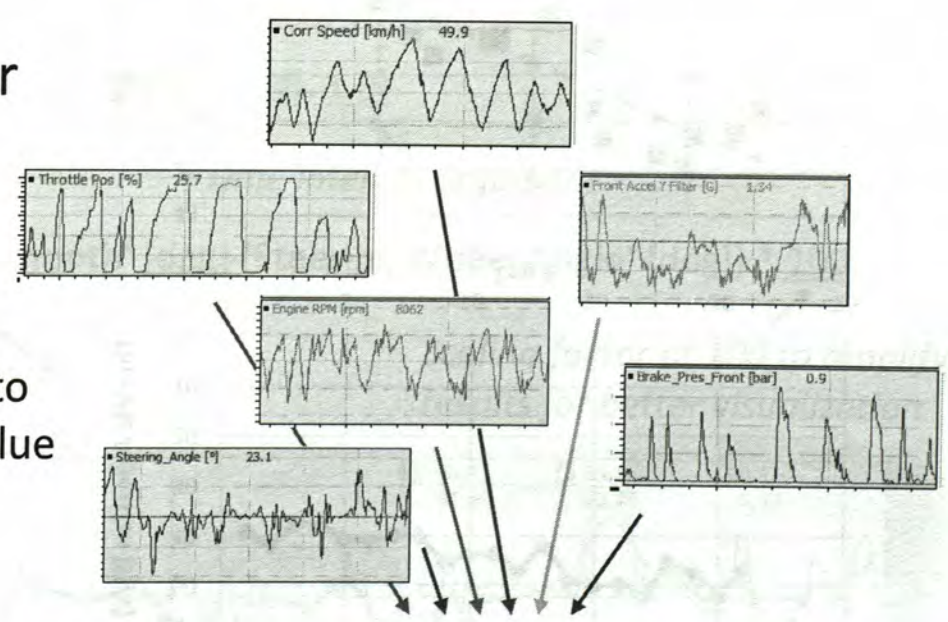


Goal: Combining several log channels into a math channel to get a single output value relevant of the state your are trying to characterize:

- Driver style
- Driver confidence
- Car Balance
- …

KPI Math Channel

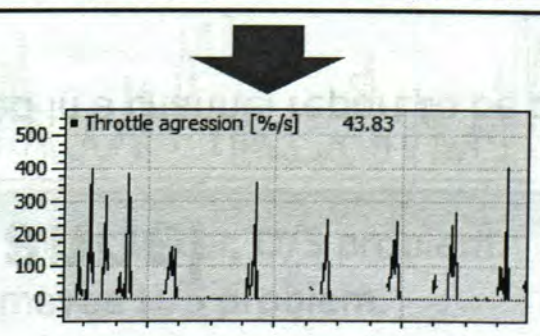
KPI: Key Performance Indicator



Goal: Combining several log channels into a math channel to get a single output value relevant of the state your are trying to characterize:

- Driver style
- Driver confidence
- Car Balance
- …

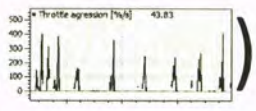
choose((derivative('**Throttle Pos**' [%],0.5) > 25), derivative ('**Throttle Pos**' [%], 0.05),1/0)*('**Lat Accel**' [g]<50)



KPI Math Channel

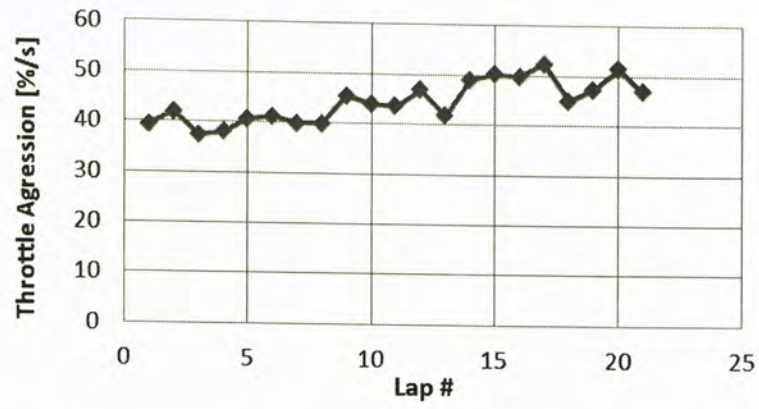
Once implemented, the math channel has to be added in a channel report to be displayed as the min, average, or max and lap by lap.

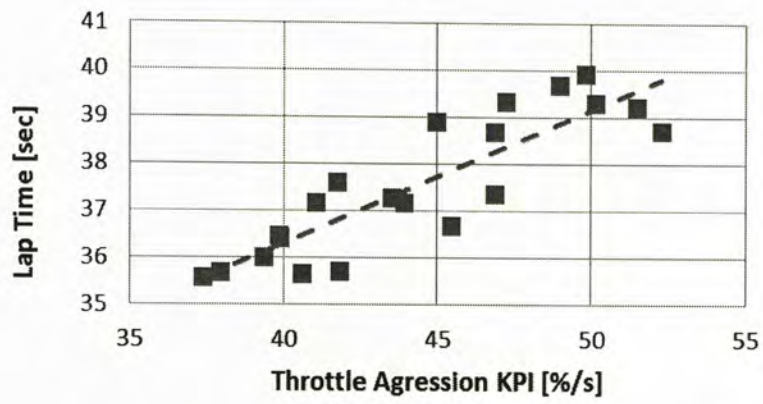
		Lap 1 [0:55.080]	Lap 2 [0:37.400]	Lap 3 [0:37.240]	Lap 4 [0:37.160]	Lap 5 [0:37.080]	Lap 6 [0:37.480]	Lap 7 [0:45.680]	Lap 8 [0:40.600]	Lap 9 [0:37.760]	Lap 10 [0:37.600]	Lap 11 [0:37.480]	Lap 12 [0:37.880]	Lap 13 [0:37.640]
Throttle agression [%/s]	Avg	61.27	59.25	60.14	54.17	68.66	65.05	26.35	28.82	49.29	41.75	44.68	40.13	66.31

AVG_{Lap1}()

Then you can display it in a scatter plot to see:

- The evolution lap by lap.
- Versus something else as lap time, tire temp…





Driver Analysis

Steering Smoothness KPI

Motec Math Channel

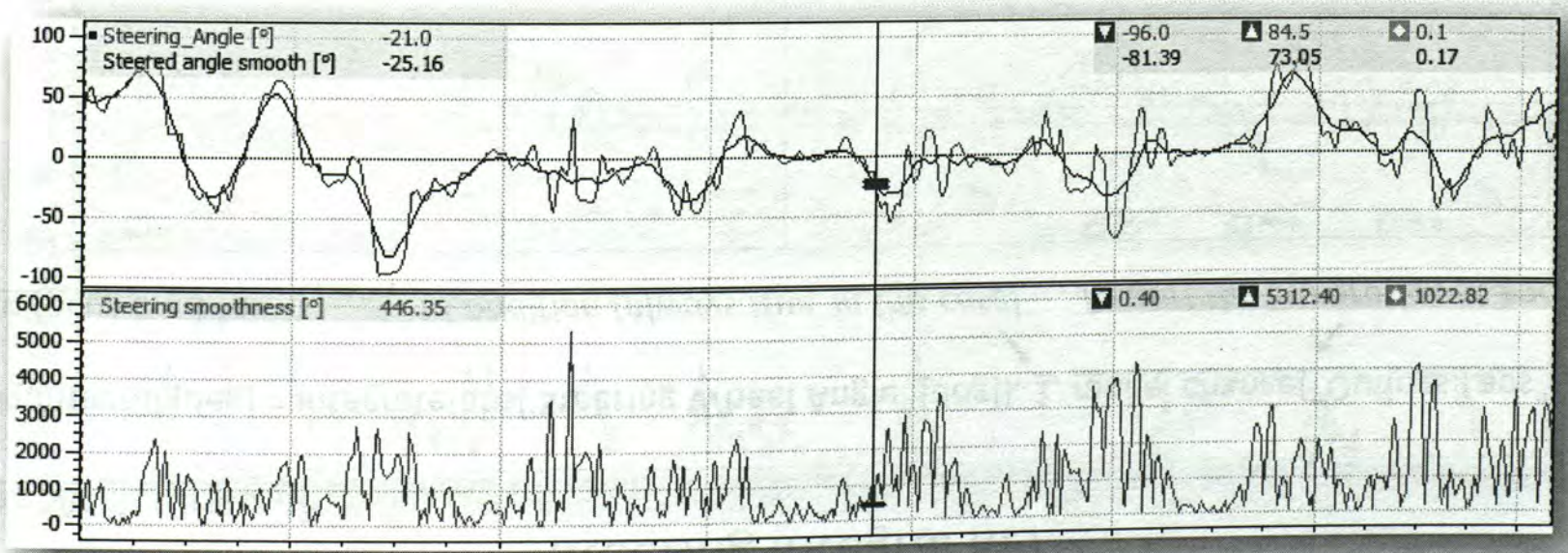
Steered angle smooth [deg]= smooth(**'Steering Wheel Angle'** [deg], 1.0)

Averaging function *Time interval argument*

Steering Smoothness [deg] = abs(**'Steered angle smooth'** [deg]-**'Steering Wheel Angle'** [deg]) * 100

Scaling factor of 100 to amplify results for better visualization

Value to focus on: Average



High Value: The driver is not smooth, he is fighting with the car, O/S problem.
Low Value: The driver is smooth, good or lazy car, maybe U/S problem.

Driver Analysis

Steering Integral KPI

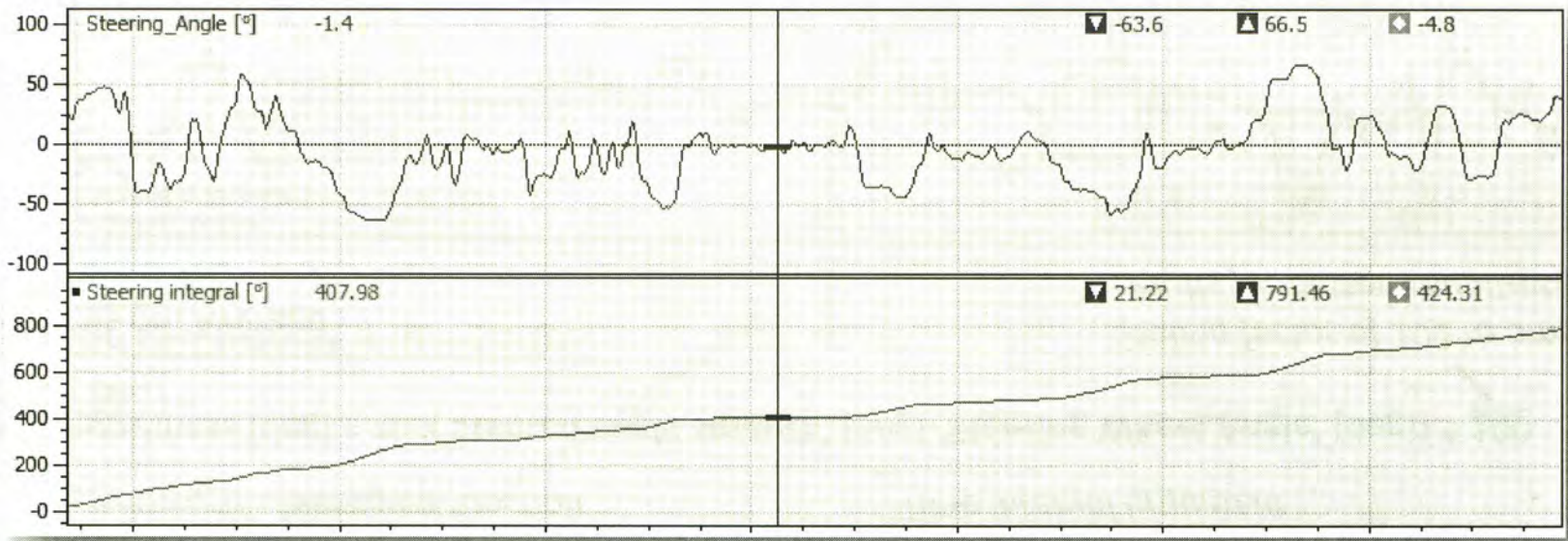
Motec Math Channel

Steering Integral [deg] = integrate(abs(**'Steering Wheel Angle'** [deg]), 1, range_change("Outings:Laps"))

Value to focus on: Max

Condition (always true in this case)

Restart the integration at each lap



High Value: The driver is steering a lot, U/S problem.
Low Value: The driver is counter-steering, O/S problem.
The trick is to do a low speed lap to get the "Nominal steering", and then compare the steering integral of this lap with the one of a normal lap. Steers more (U/S), Steers less (O/S).

Driver Analysis

Throttle Aggression KPI

Motec Math Channel

Condition function "if"

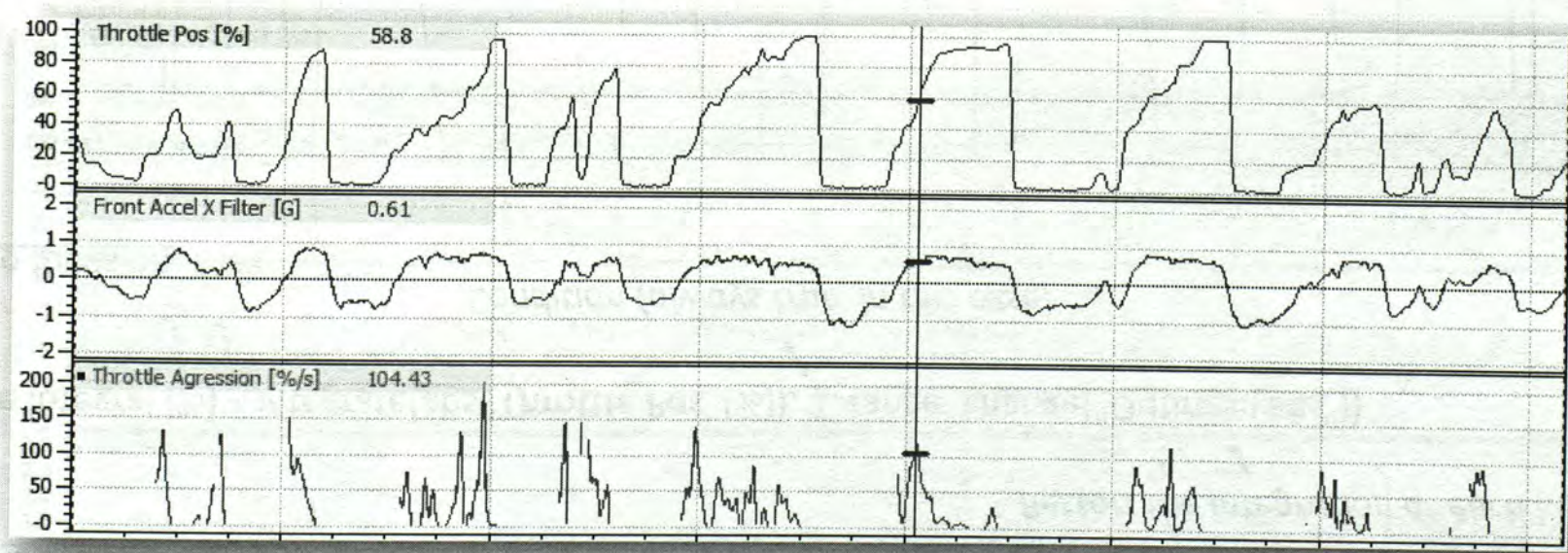
Acceleration phase

Not releasing

Throttle Aggression [%/s] = choose(**'Long Accel'** [G] > 0.2 AND derivative(**'Throttle Pos'** [%],0.1) > 0, derivative(**'Throttle Pos'** [%], 0.1), 1/0)

Value to focus on: Average

Return nothing if condition false



High Value: The driver is aggressive, confident.
Low Value: The driver is smooth, not confident, differential problem?

Driver Analysis

Throttle Integral KPI

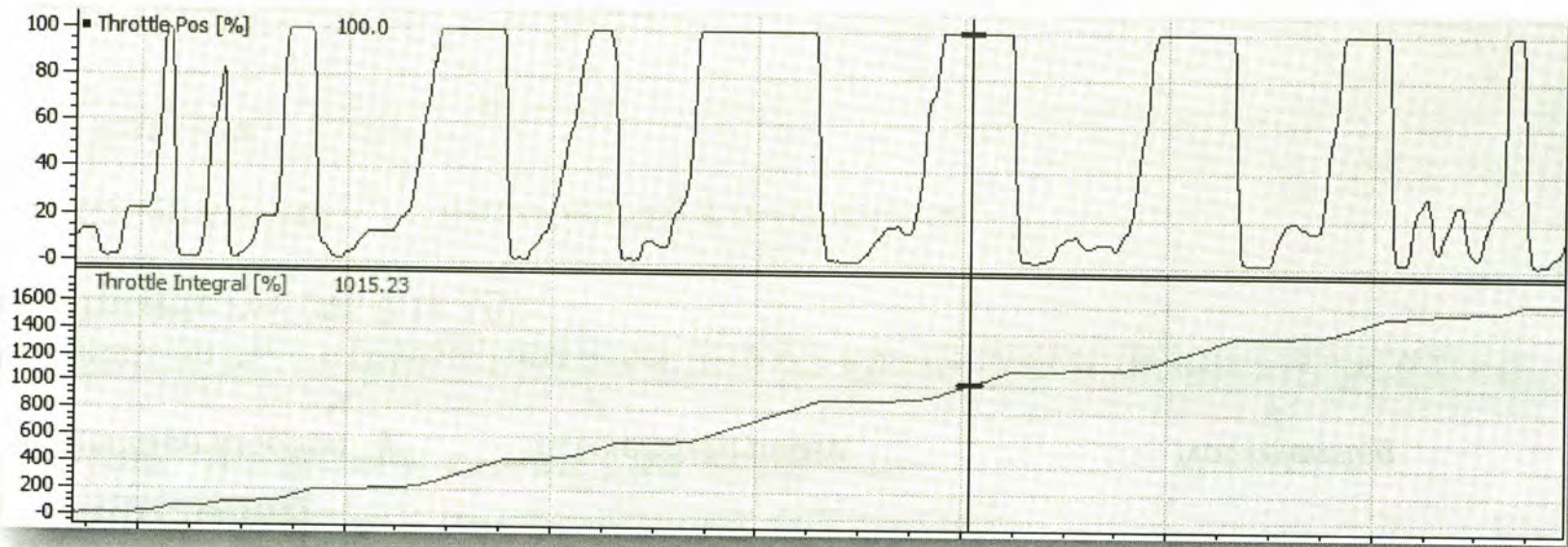
Motec Math Channel

Restart the integration at each lap

Throttle Integral [%] = integrate(abs(**'Throttle Pos'** [%]), 1, range_change("Outings:Laps"))

Condition (always true in this case)

Value to focus on: Max



High Value: The driver use a lot the throttle, confident.
Low Value: The driver does use a lot of throttle, not confident, poor grip…

Driver Analysis

Throttle Efficiency KPI

Motec Math Channel

Condition function "if"

Throttle Efficiency [g/%] = choose(**'Front Accel X Filter'** [G] > 0.2,**'Front Accel X Filter'** [G]/**'Throttle Pos'** [%], 1/0)*100

Scaling factor of 100 to amplify results for better visualization

Return nothing if condition false

Value to focus on: Average

High Value: Good traction, the driver is good on the throttle.
Low Value: Bad traction, the driver is bad on the throttle, mechanical problem?…

Driver Analysis

Brake Pressure Smoothness KPI

Motec Math Channel

Brake pressure total [bar] = **'Brake pressure Front'** [bar] + **'Brake pressure Rear'** [bar]

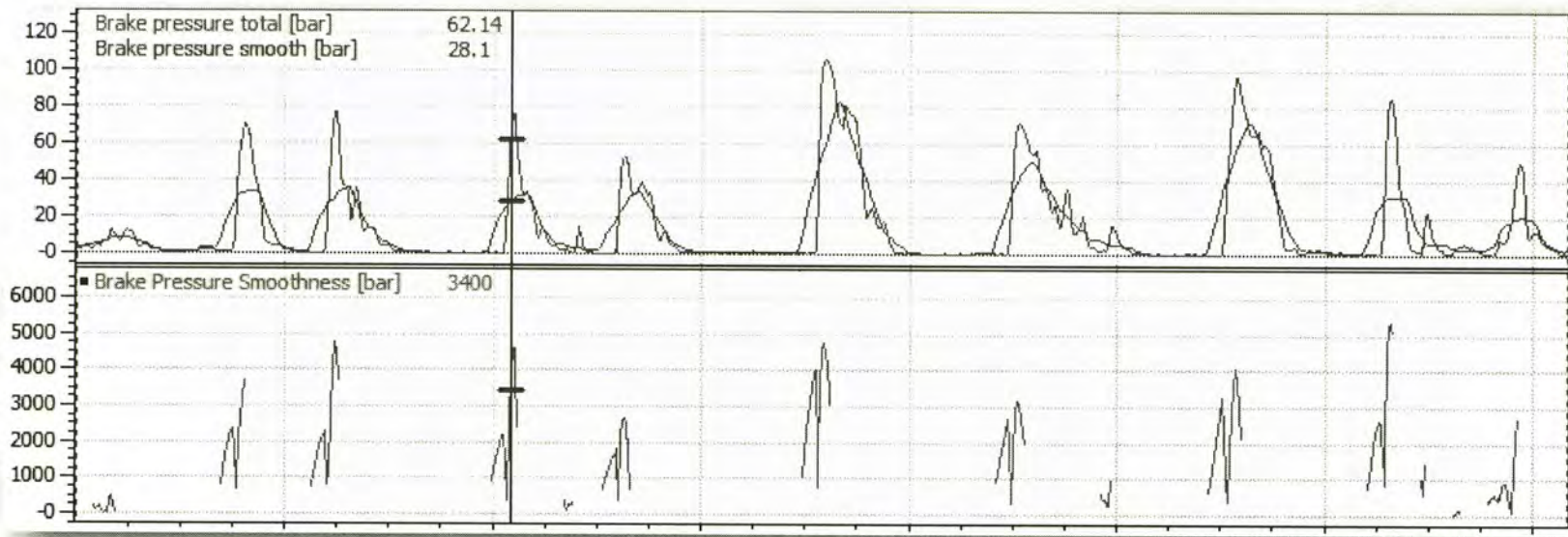
Averaging function ↘ *Time interval argument* ↘

Brake Pressure Smooth [bar]= smooth(**'Brake pressure total'** [bar],1.0)

Brake Pressure Smoothness [bar]= choose((derivative(**'Brake pressure total'** [bar],0.75) > 5),abs(**'Brake pressure total'** [bar]-**'Brake pressure smooth'** [bar]),1/0)*100

Scaling factor of 100 to amplify results for better visualization

Value to focus on: Average



High Value: The driver is aggressive, put a lot of pressure quickly, confident.
Low Value: The driver is smooth, not confident, not stable enough under braking.

Driver Analysis

Brake Release Smoothness KPI

Motec Math Channel

Brake pressure total [bar] = **'Brake pressure Front'** [bar] + **'Brake pressure Rear'** [bar]

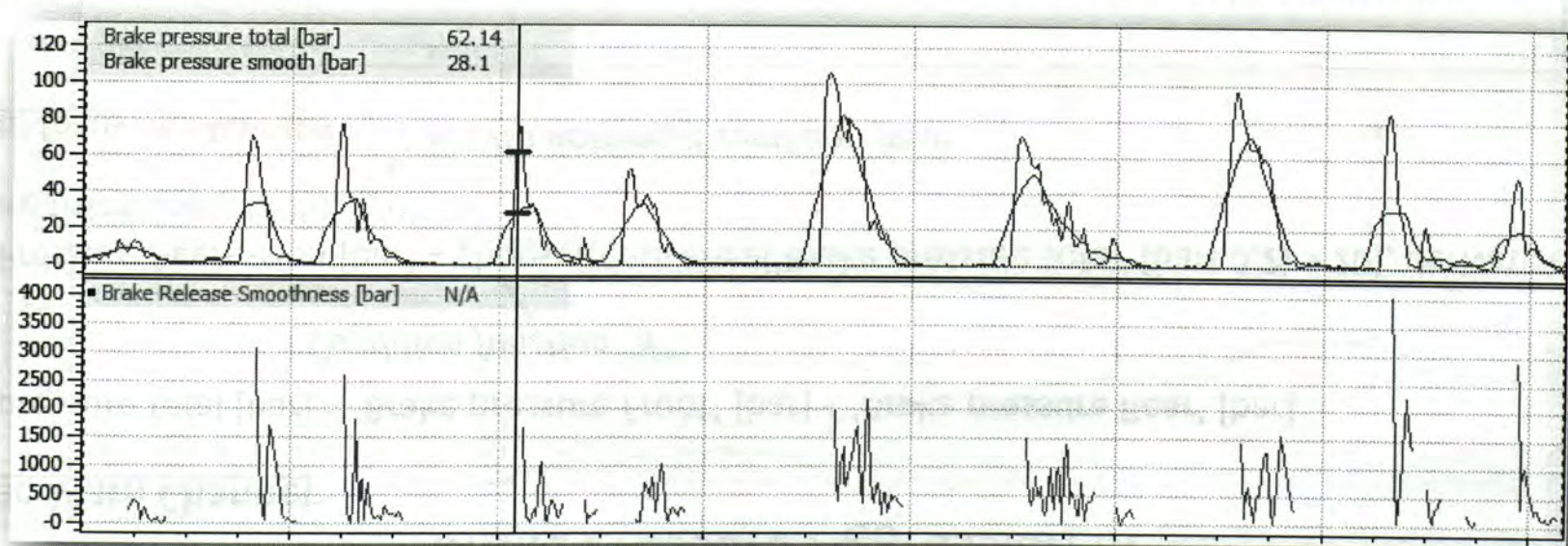
Averaging function ↘ *Time interval argument* ↘

Brake Pressure Smooth [bar]= smooth(**'Brake pressure total'** [bar],1.0)

Brake Release Smoothness [bar]= choose((derivative(**'Brake pressure total'** [bar],0.75) < -5),abs(**'Brake pressure total'** [bar]-**'Brake pressure smooth'** [bar]),1/0)*100

Scaling factor of 100 to amplify results for better visualization

Value to focus on: Average



High Value: Remove pressure too quickly, bad technique, risk of wheel locking.
Low Value: Remove pressure as it should be, good technique.

Driver Analysis

Brake Pressure Aggression KPI

Motec Math Channel

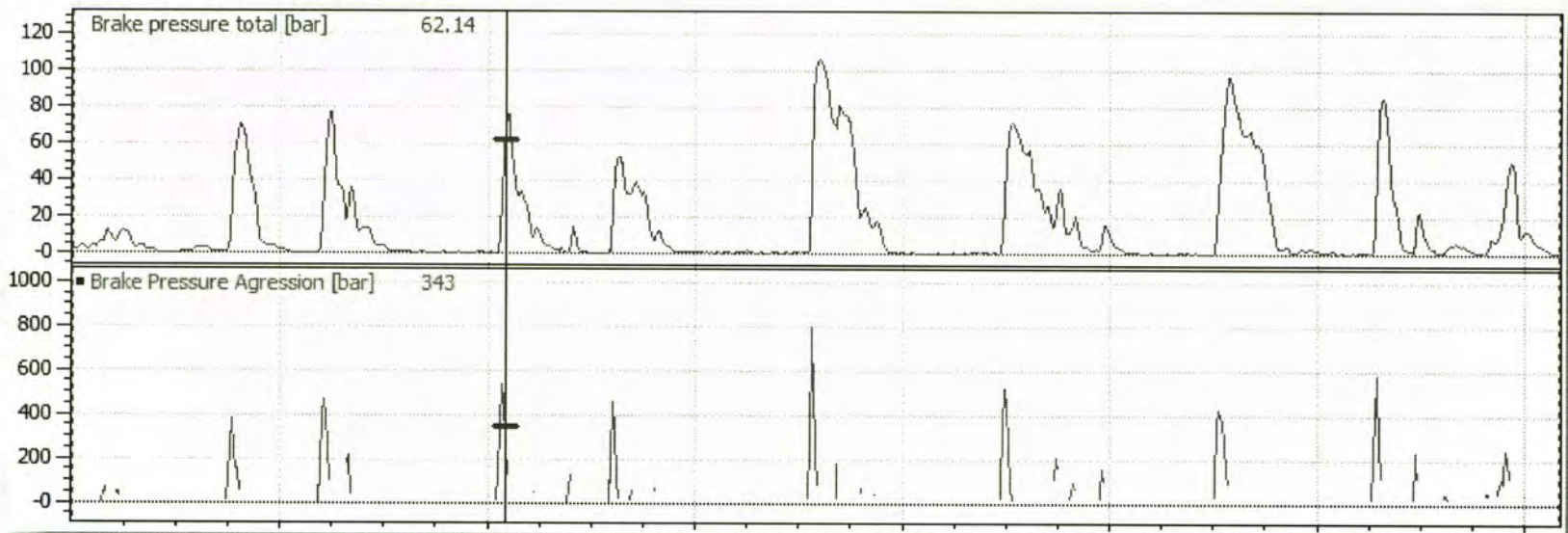
Brake pressure total [bar] = **'Brake pressure Front'** [bar] + **'Brake pressure Rear'** [bar]

Condition function "if"

Brake Pressure Aggression [bar] = choose((derivative('**Brake pressure total**' [bar],0.2) > 20), derivative('**Brake pressure total**' [bar], 0.05), 1/0)

Return nothing if condition false

Value to focus on: Average



High Value: The driver is fast on brakes, good technique, confident.
Low Value: The driver is slow on brake, bad technique not confident.

Driver Analysis

Brake Pressure Integral KPI

Motec Math Channel

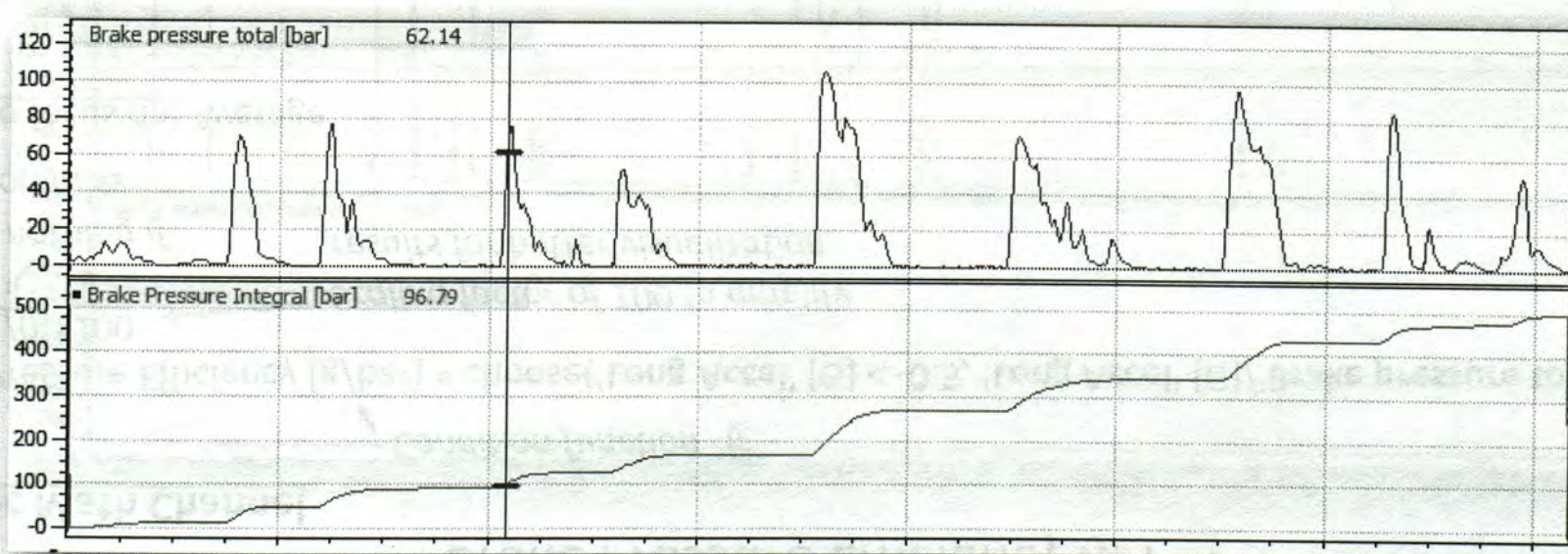
Brake pressure total [bar] = **'Brake pressure Front'** [bar] + **'Brake pressure Rear'** [bar]

Restart the integration at each lap

Brake Pressure Integral [bar] = integrate(abs('**Brake pressure total**' [bar]), 1, range_change("Outings:Laps"))

Condition (always true in this case)

Value to focus on: Max



High Value: The driver slows down a lot, not confident.
Low Value: The driver goes fast, efficient on brakes, confident.

Driver Analysis

Brake Pressure Efficiency KPI

Motec Math Channel

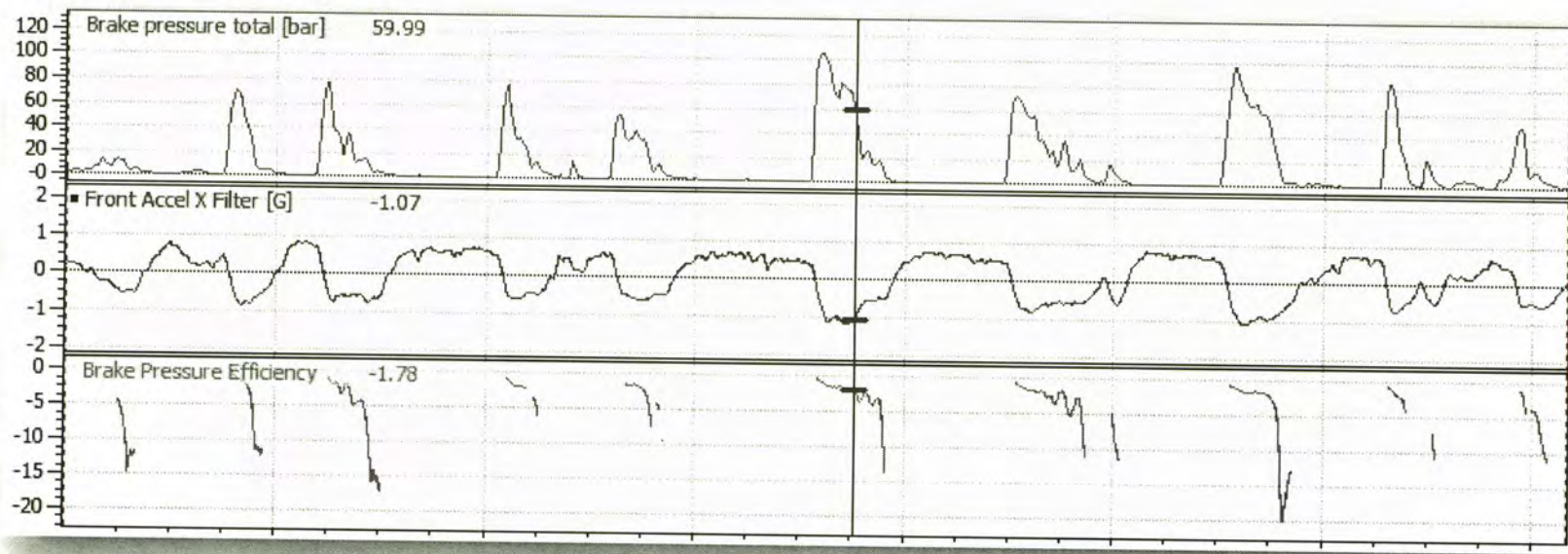
Condition function "if"

Brake Pressure Efficiency [g/bar] = choose(**'Long Accel'** [G] < -0.5, **'Long Accel'** [G]/**'Brake pressure total'** [bar], 1/0)*100

Scaling factor of 100 to amplify results for better visualization

Return nothing if condition false

Value to focus on: Average



Gives an indication on the efficiency of the braking system. Good way to detect problems.

Driver Analysis

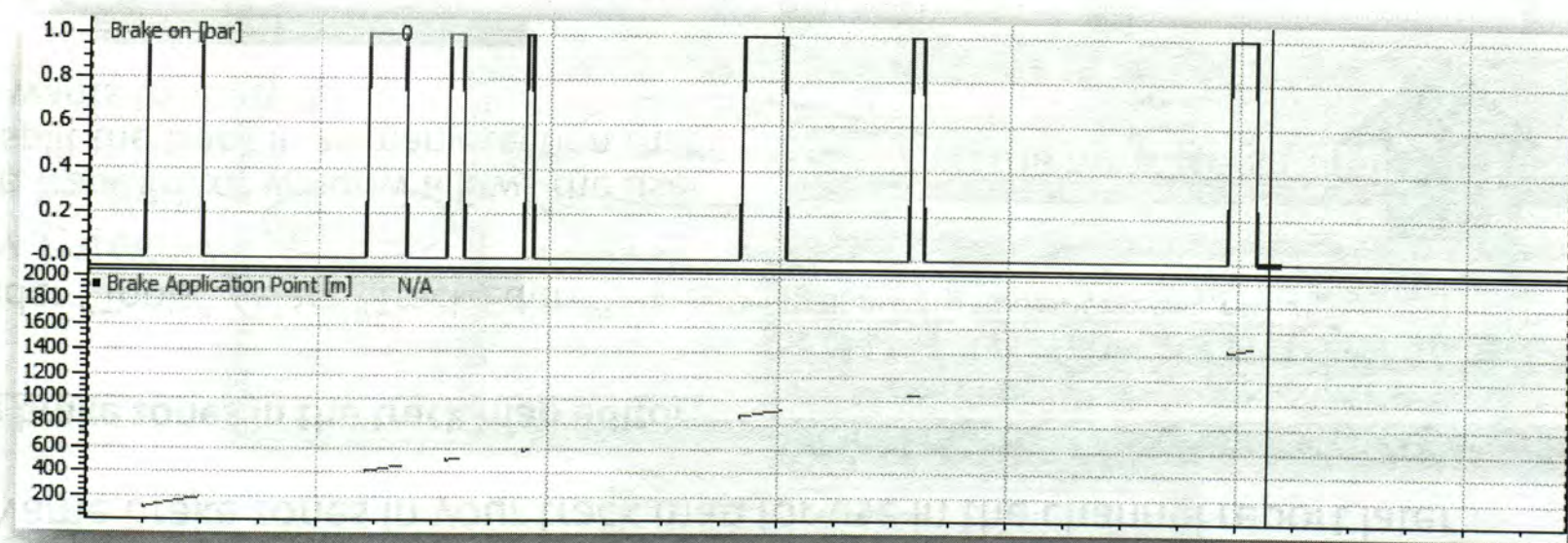
Brake Point Application KPI

Motec Math Channel

Brake on [-] = **'Brake Pedal Pos'** [%] > 'Brake pressure limit' [bar]

Brake Application Point [m] = choose(**'Brake on'** [bar] > 0,**'Lap Distance'** [m],1/0)

Value to focus on: Min



Then there is a bit of work to do on the track editor!

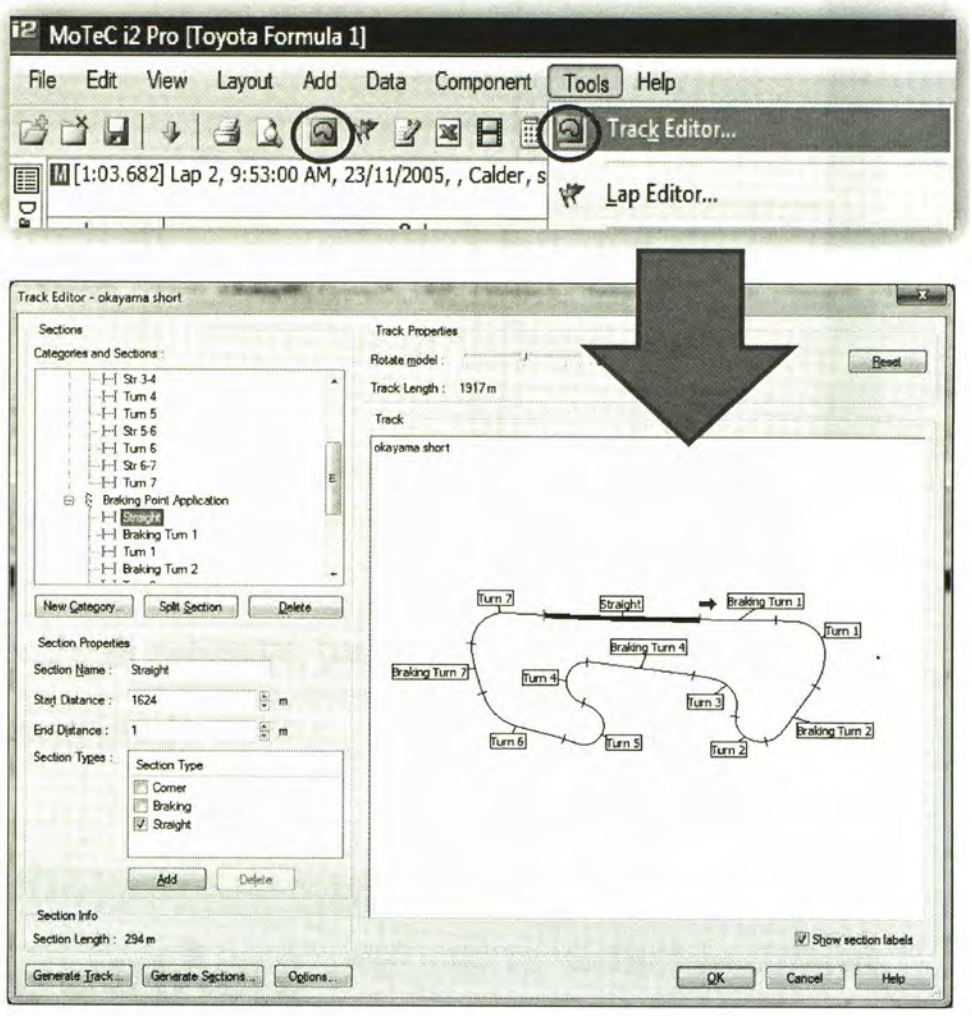
Driver Analysis

Brake Point Application KPI

Name brake zones in your track map for use in the channel report later

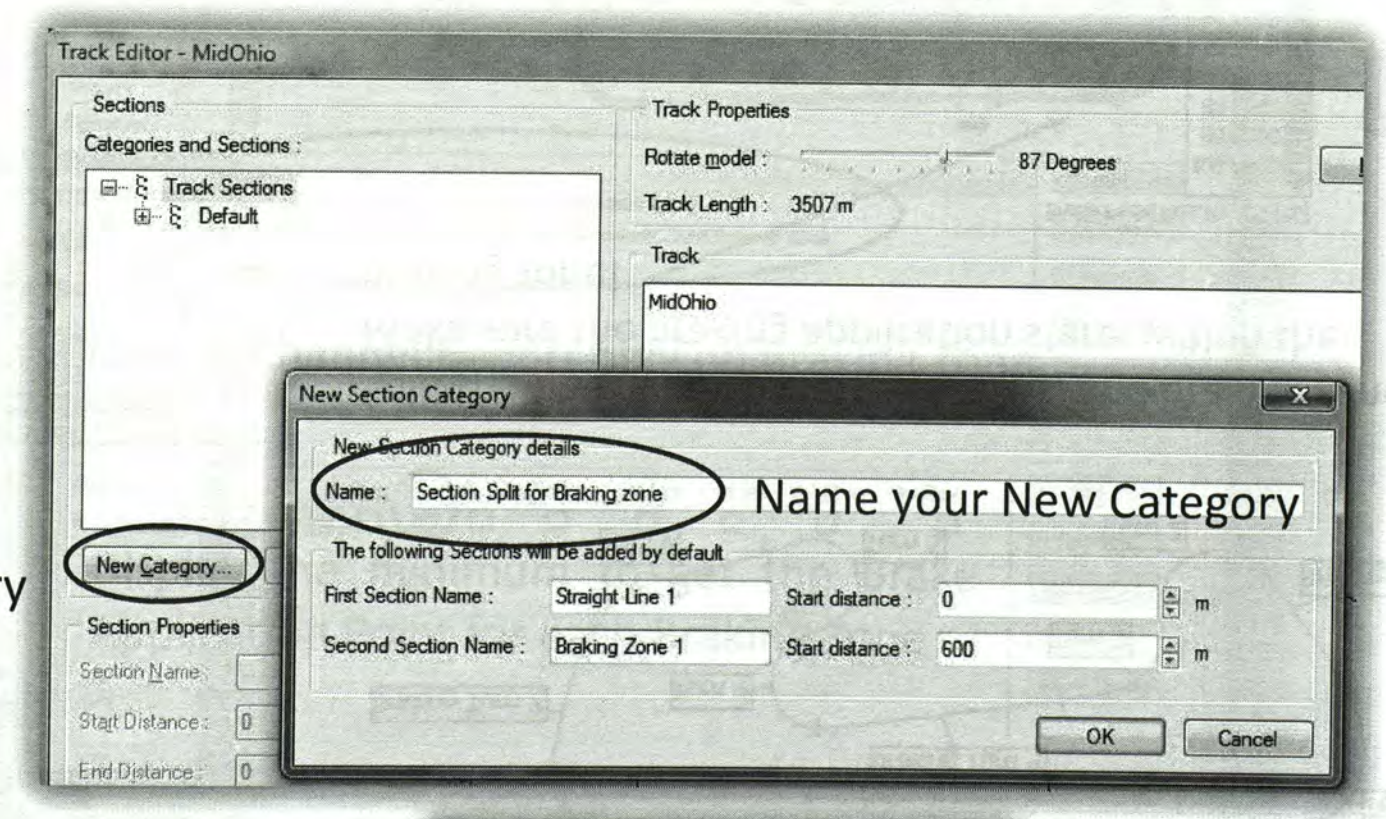
Create brake zones in the track map editor:

- Under "Tools", select "Track Editor"
- The Track Editor window allows the user to split the track in as many section that he wants to study it.
- By default, you can generate automatically the straight lines and the corners.
- Next slide explains the way to generate brake sections.

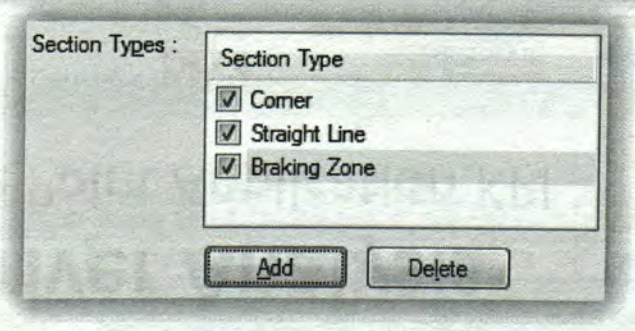


Driver Analysis

Brake Point Application KPI



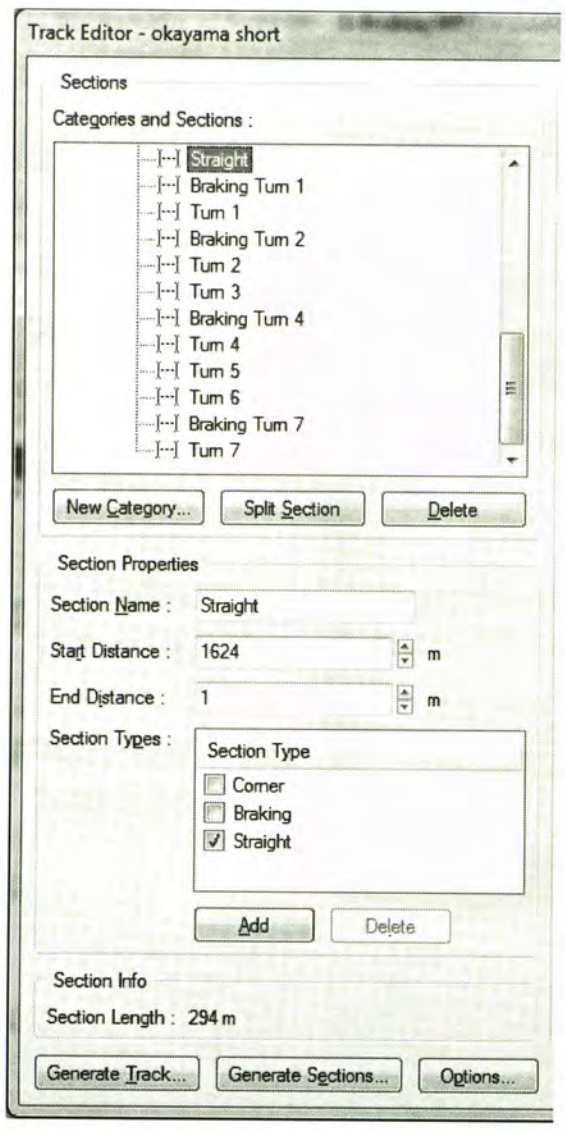
Create a New Category

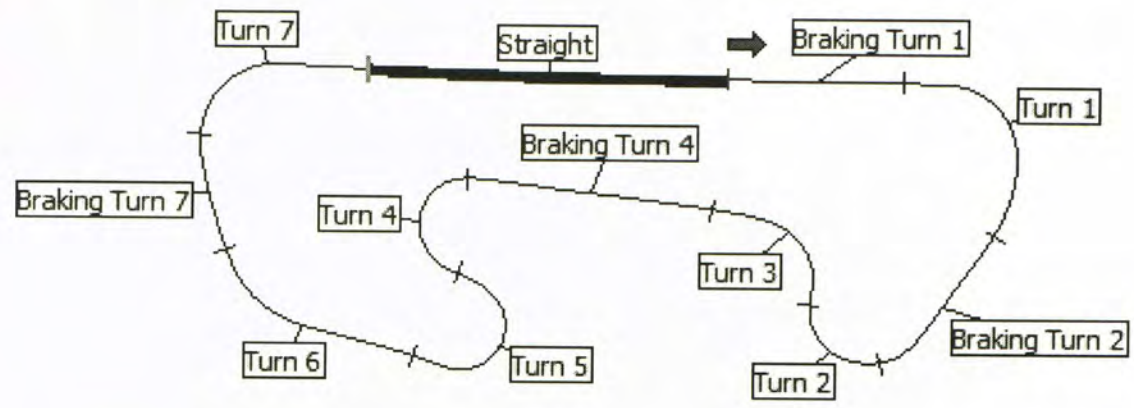


Create the Section Types that you need for a further use in a Channel report

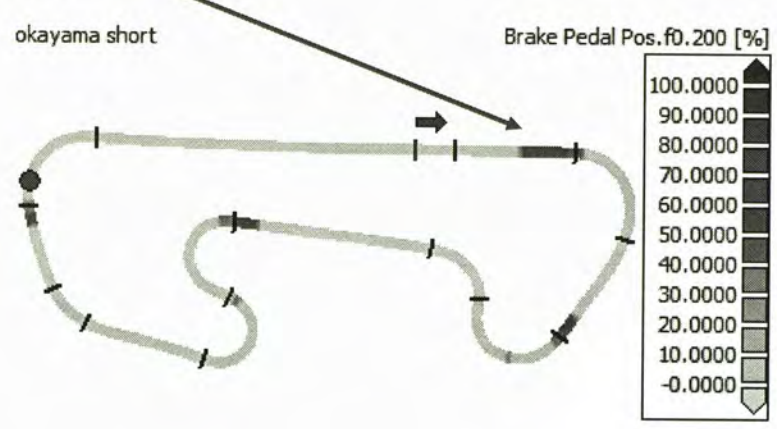
Driver Analysis

Brake Point Application KPI



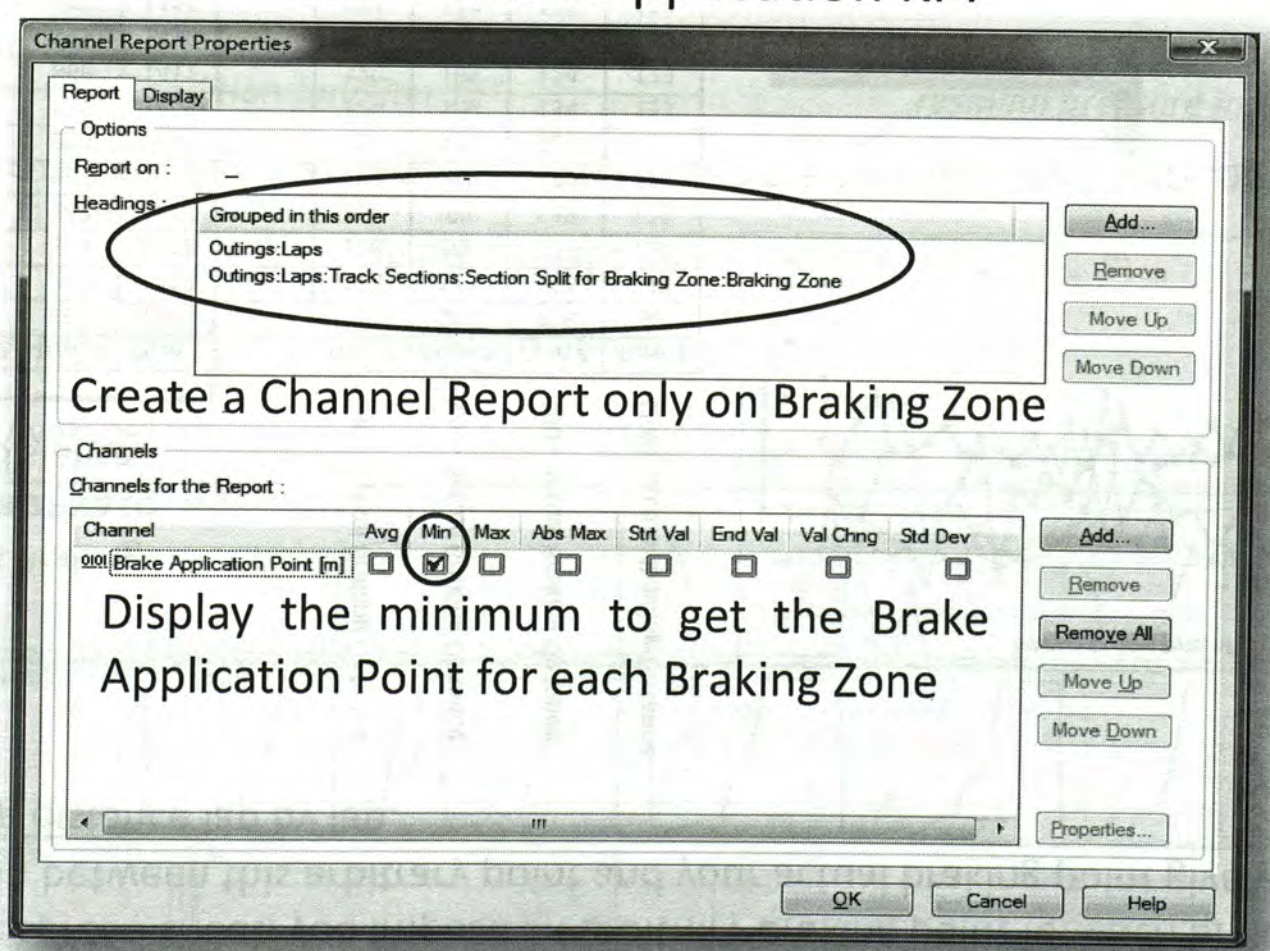


Split the track into corners, straights, and braking zone. Make sure the braking application starts within the braking zone!



Driver Analysis

Brake Point Application KPI



Create a Channel Report only on Braking Zone

Display the minimum to get the Brake Application Point for each Braking Zone

Channel Report - Laps

		Lap 13 [1:28.155]						
		Braking Zone 1	Braking Zone 2	Braking Zone 3	Braking Zone 4	Braking Zone 5	Braking Zone 6	Braking Zone 7
Brake Application Point [m]	Min	143.12	754.19	1757.88	2134.96	2456.53	2887.18	3094.43

Driver Analysis

Brake Point Application KPI

Then in a table spread sheet, you impose an arbitrary braking point for each braking zone. The comparison between this arbitrary point and your actual braking point give you a relative braking distance lap by lap.

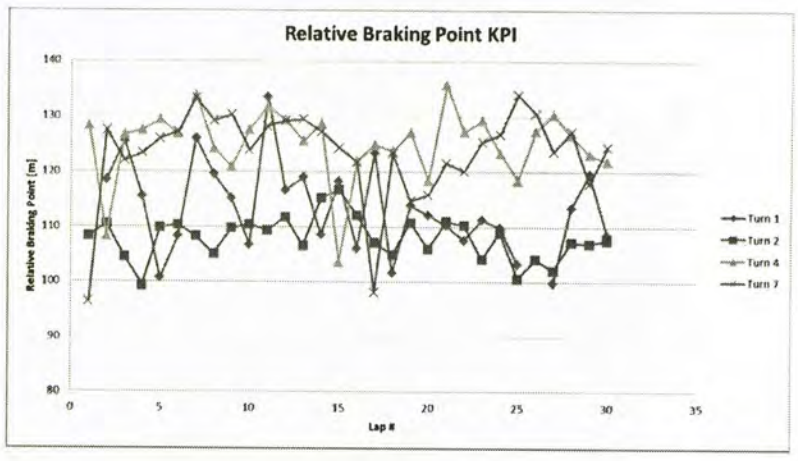
Actual Braking point relative to track distance.

230 - Actual Braking Distance
520 - Actual Braking Distance
990 - Actual Braking Distance
1540 - Actual Braking Distance

	Turn 1	Turn 2	Turn 4	Turn 7		Turn 1	Turn 2	Turn 4	Turn 7
Lap 1		412	862	1444			108	128	96
Lap 2	112	410	882	1413		118	111	108	127
Lap 3	104	415	863	1418		126	105	127	122
Lap 4	114	421	862	1417		116	99	128	123
Lap 5	129	410	861	1414		101	110	129	126
Lap 6	122	410	863	1413		108	110	127	127
Lap 7	104	412	856	1407		126	108	134	133
Lap 8	110	415	866	1411		120	105	124	129
Lap 9	115	410	869	1410		115	110	121	130
Lap 10	123	410	862	1416		107	110	128	124

Latest braking points

Arbitrary Distance - Actual Braking Distance



Driver Analysis

Coast Factor KPI

Motec Math Channel

Condition function "if"

To determine according to the residual pressure in the brakes

Coast Factor [%] = choose(**'Brake pressure total'** [bar] < 'Brake pressure limit' [bar] AND **'Throttle Pos'** [%] < 'Throttle Limit' [%], 1, 0)*100

To get a percentage in average

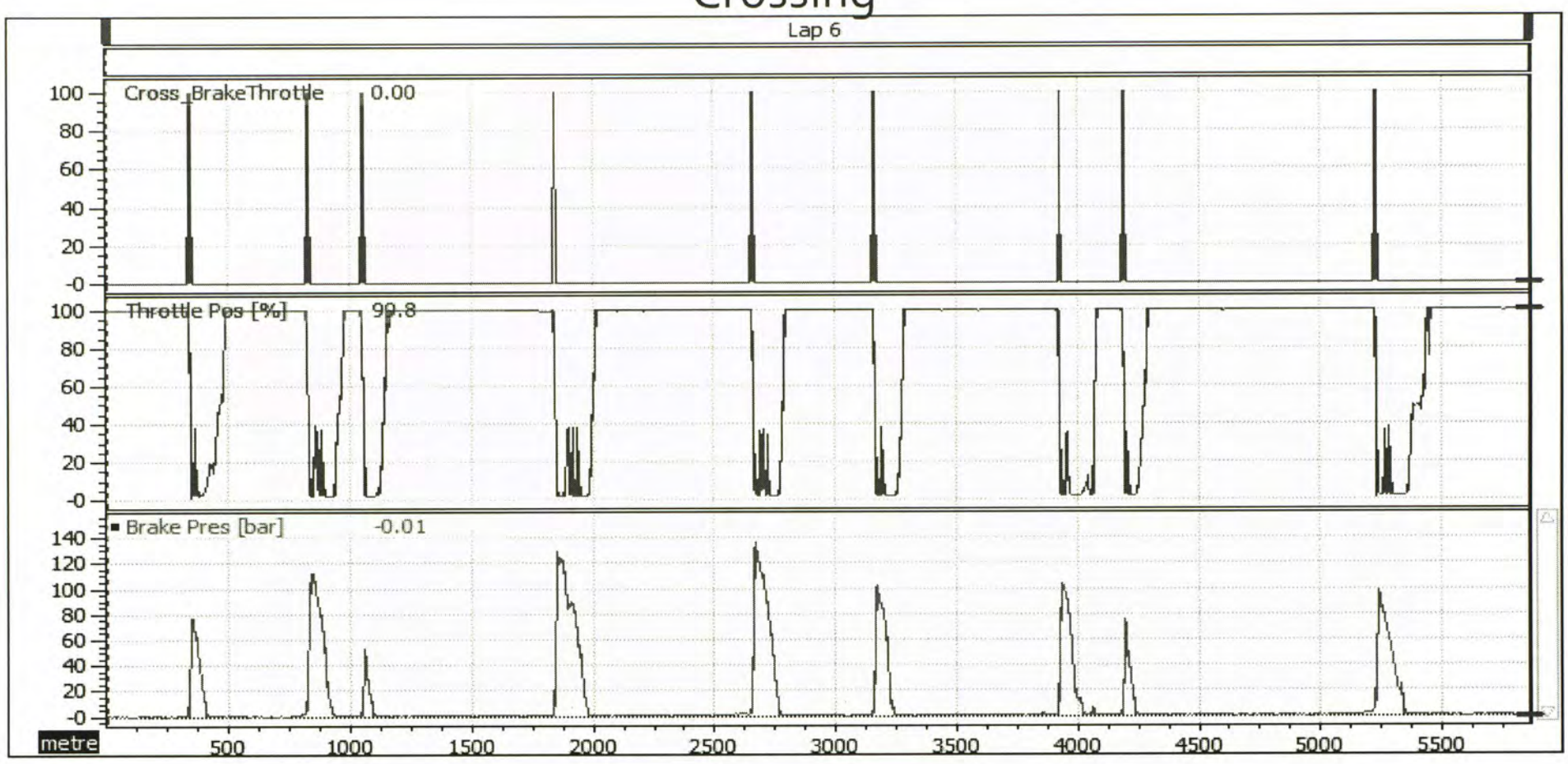
To determine according to the min throttle position

Value to focus on: Average

High Value: Poor driving technique, difficult car.
Low Value: Good driving technique, good car, confident.

Driver Analysis

Crossing

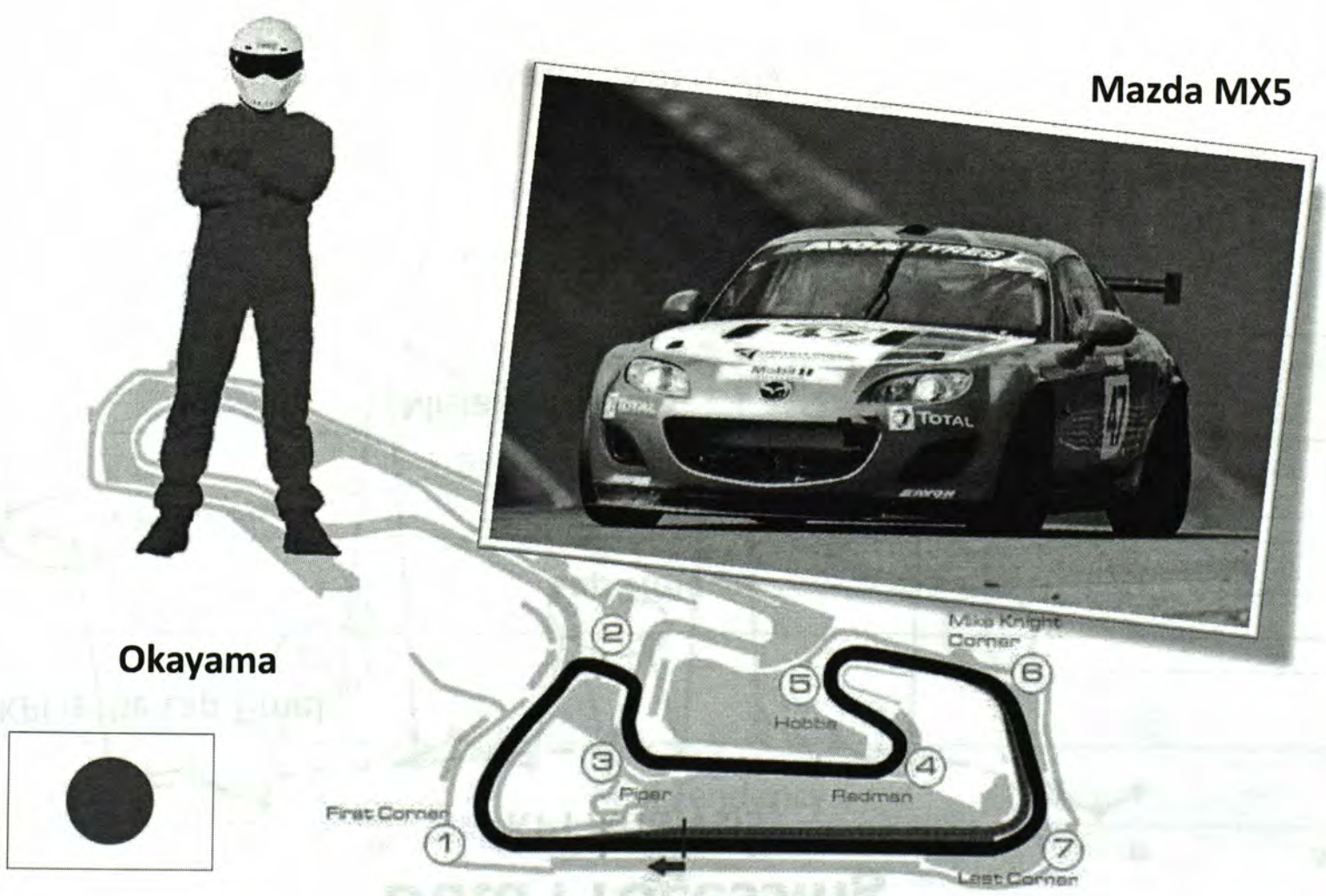


choose('**Brake Pres**' [bar]>5 AND '**Throttle Pos**' [%]>40,100,0)

Crossing is when the driver is pressing both the brake and throttle

Driver Analysis

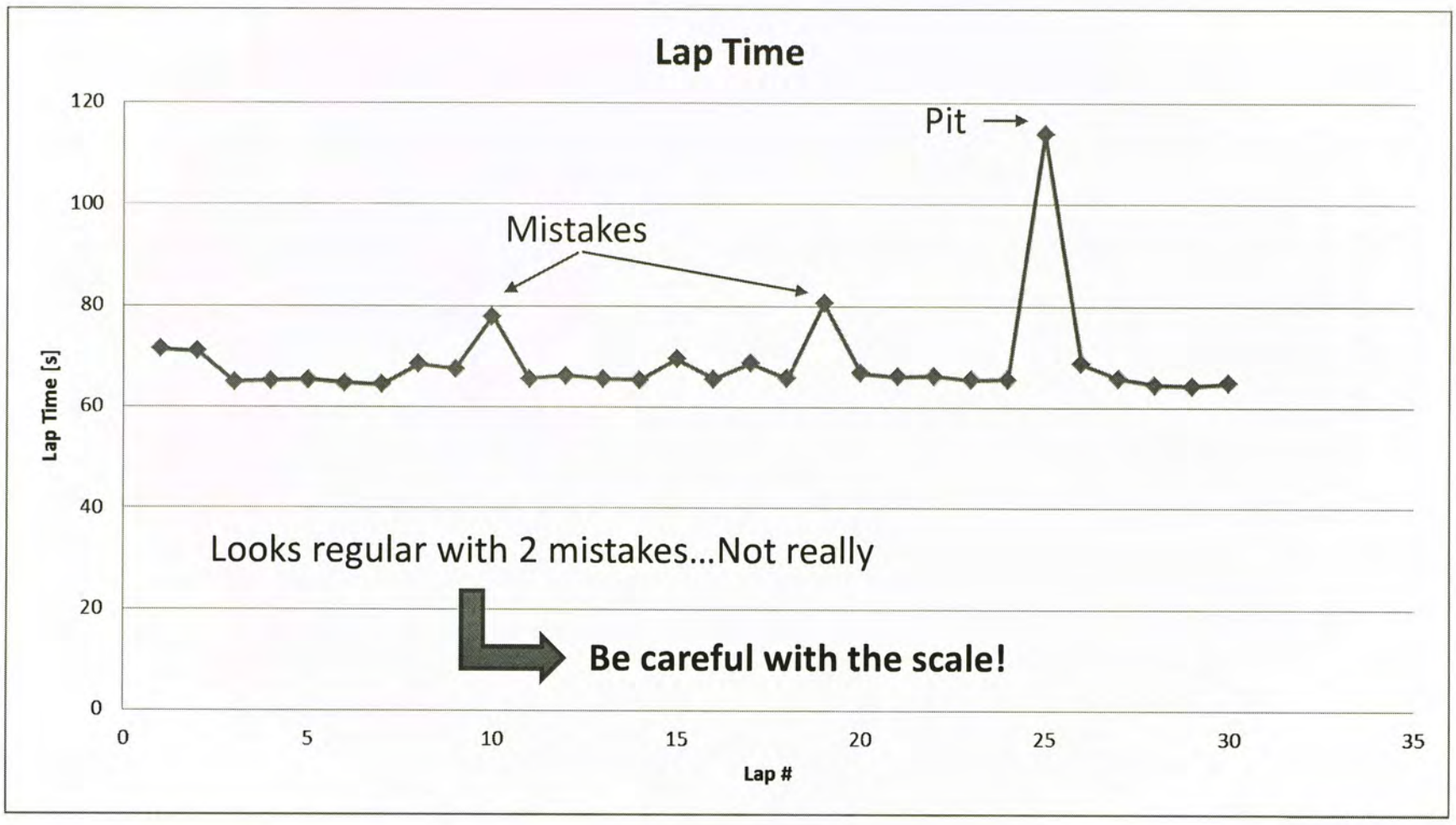
Example



Data Processing

KPI Analysis

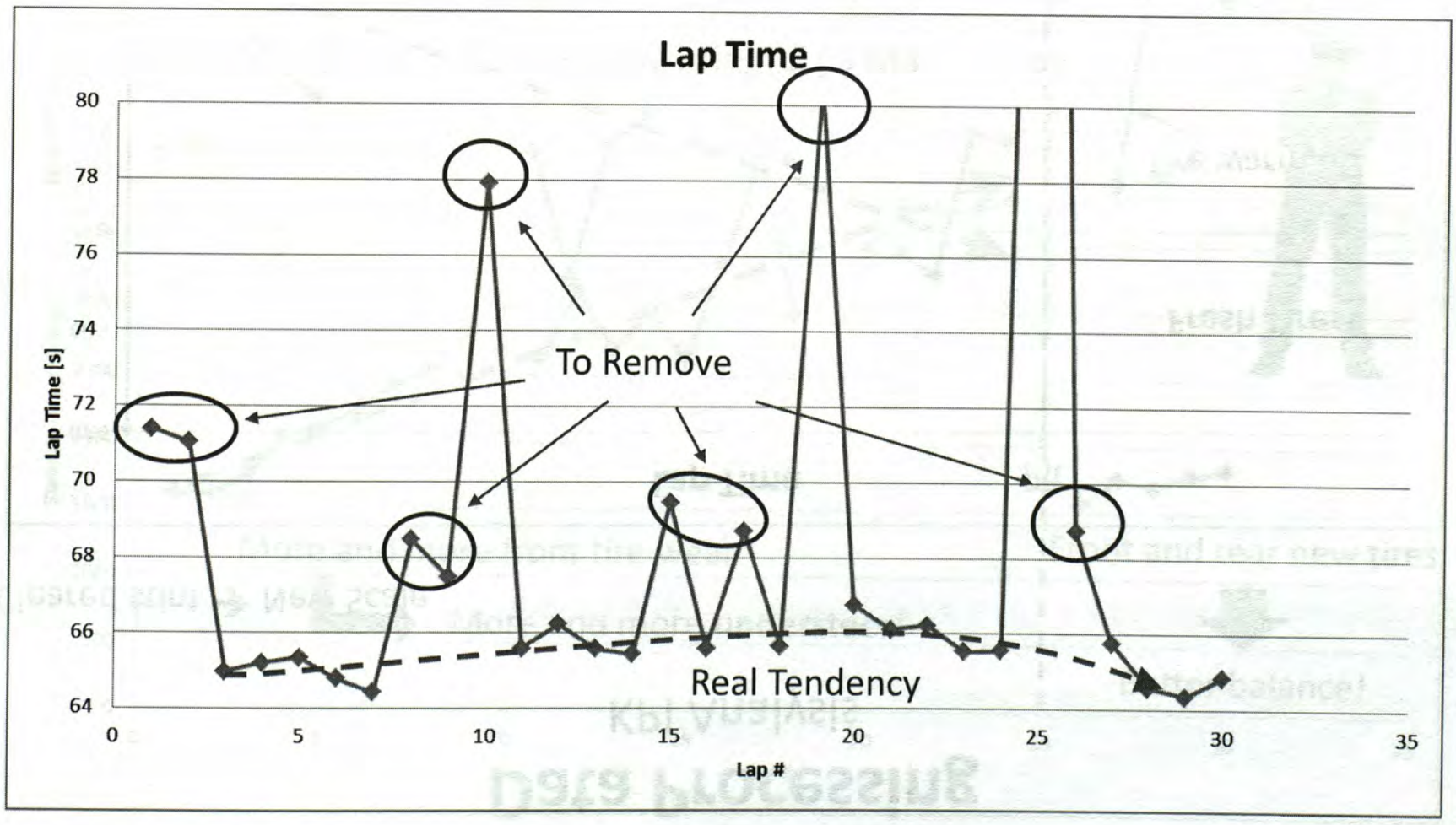
The #1 KPI is the Lap Time!



Data Processing

KPI Analysis

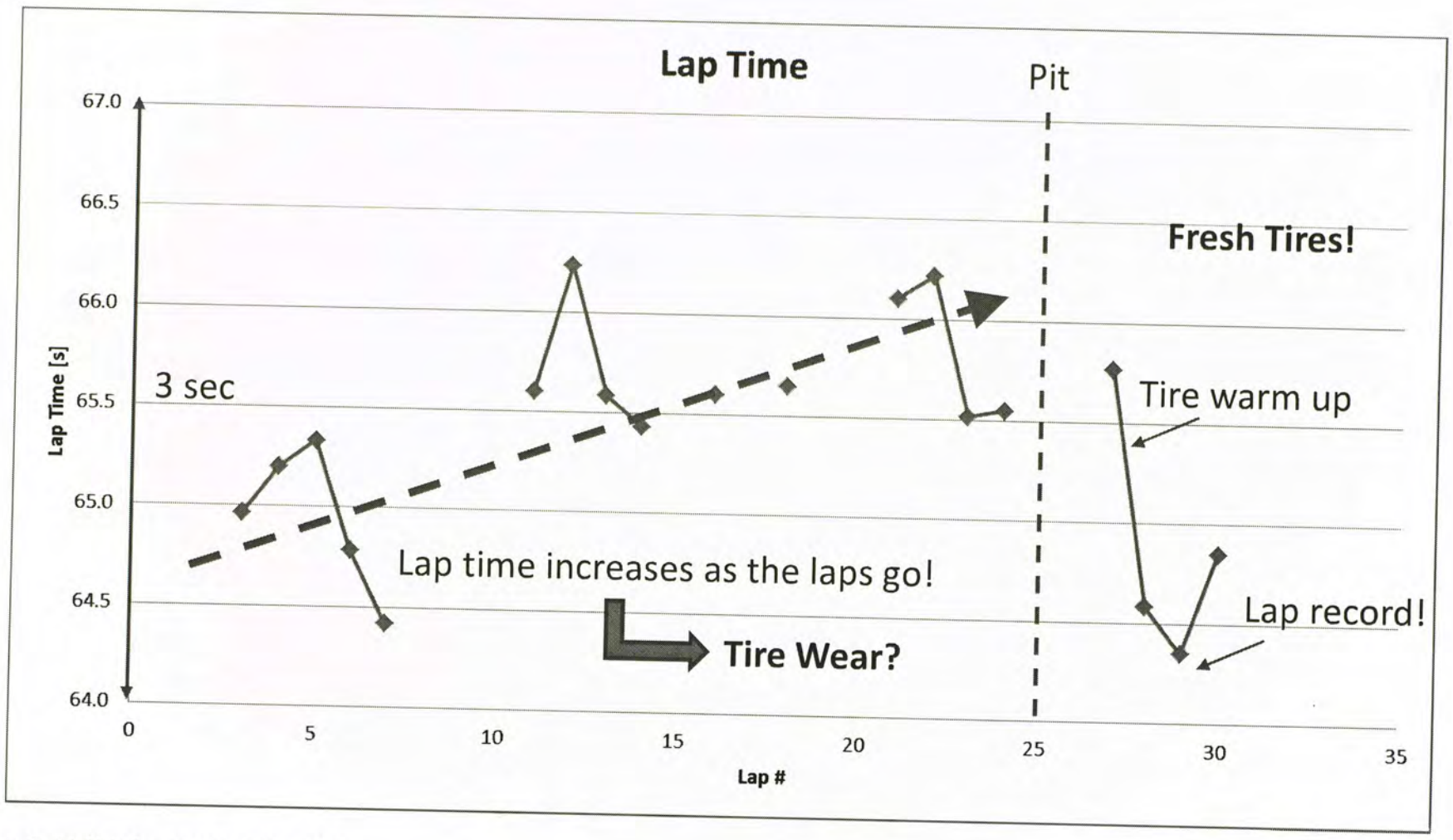
Same Stint ➔ New scale. Looks much more irregular.



Data Processing

KPI Analysis

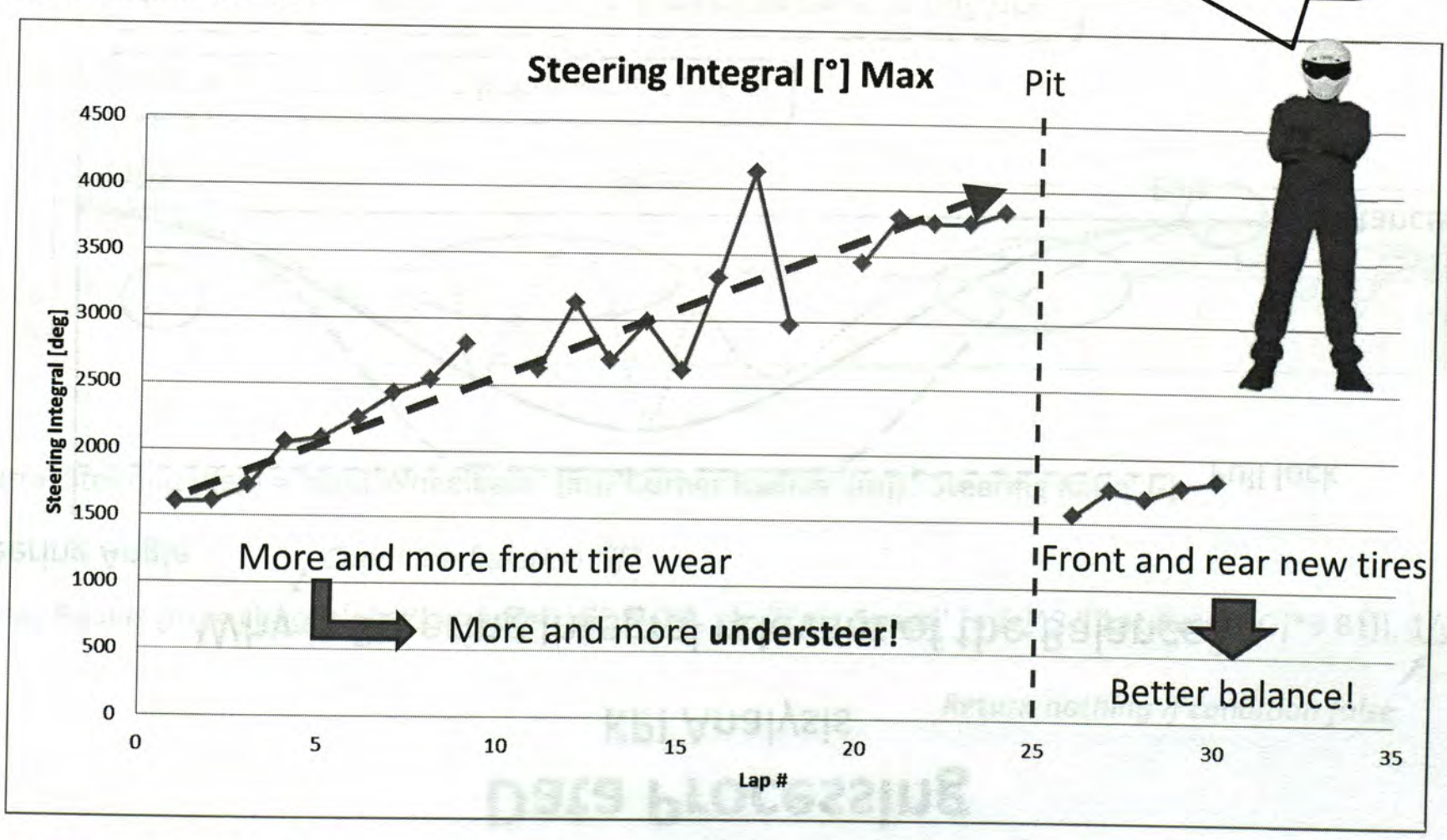
Cleared stint ➔ New Scale



Data Processing

KPI Analysis

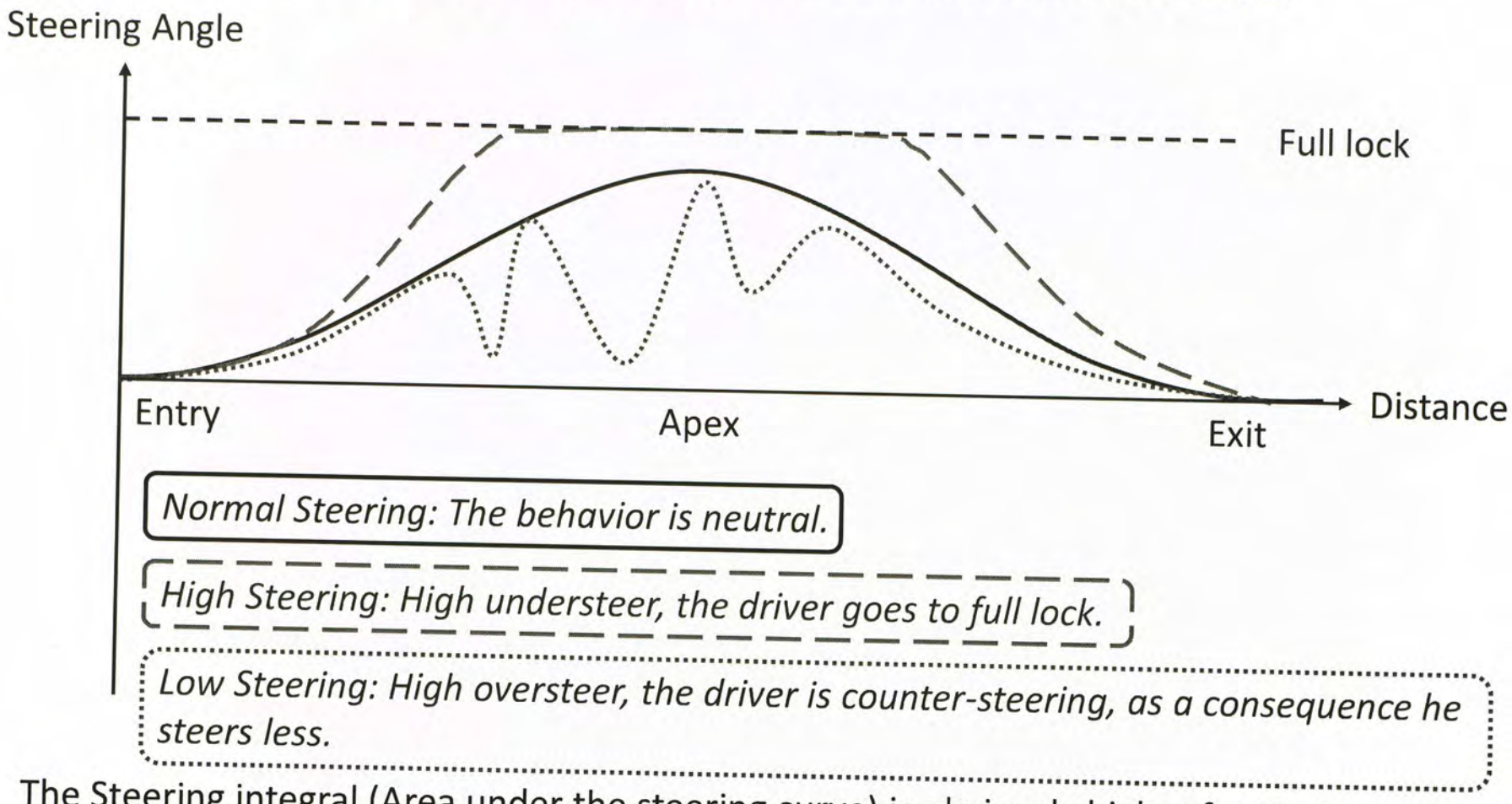
The lap time issue was due to a more and more U/S behavior of the car.



Data Processing

KPI Analysis

Why is Steering Integral relevant of the Balance?



The Steering integral (Area under the steering curve) is obviously higher for **Understeer** and lower for **Oversteer.**

Data Processing

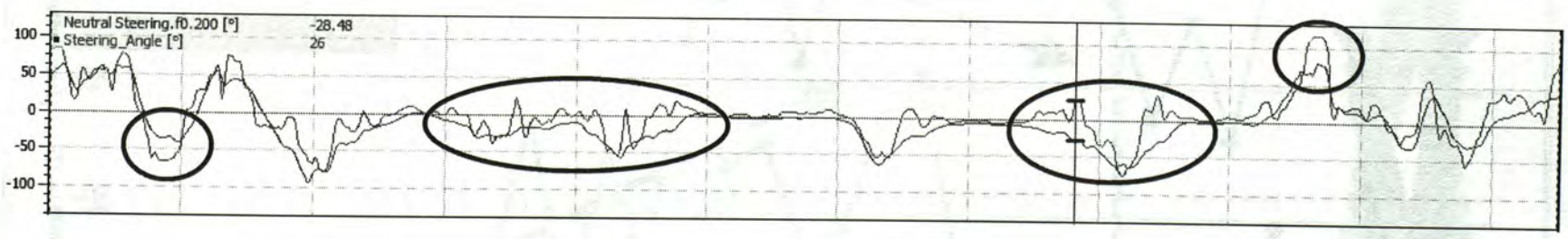
KPI Analysis

Why is Steering Integral relevant of the Balance?

Real Data

Oversteer

Understeer



Neutral Steering [deg] = atan('Wheelbase' [m]/**'Corner Radius'** [m])*'Steering Ratio' [-]

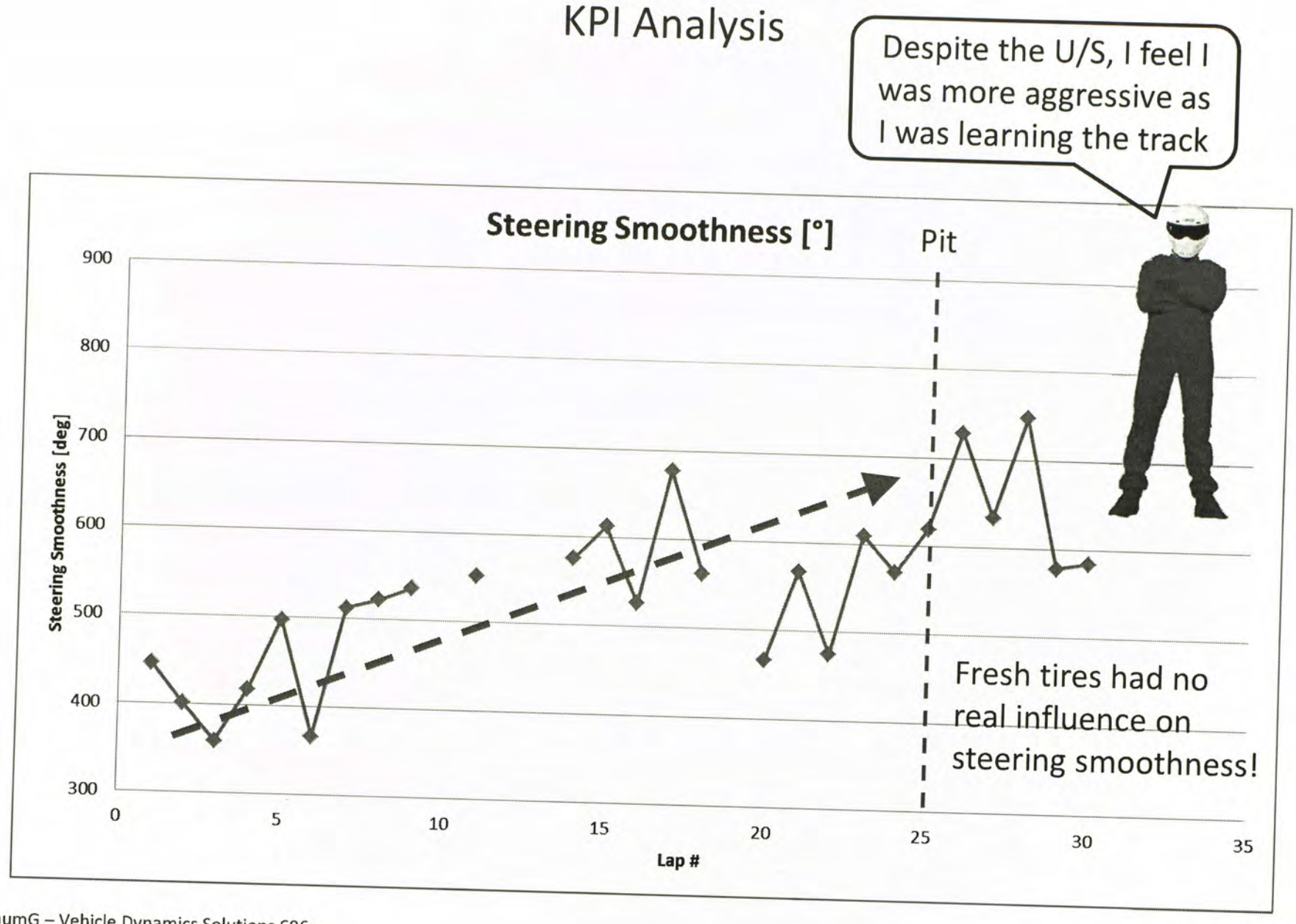
Condition function "if"

Corner Radius [m] = choose(abs(**'Lat Accel'** [G]) > 0.2, abs(**'Corr Speed'** [m/s]^2/(**'Lat Accel'** [G]*9.81)), 1/0)

Return nothing if condition false

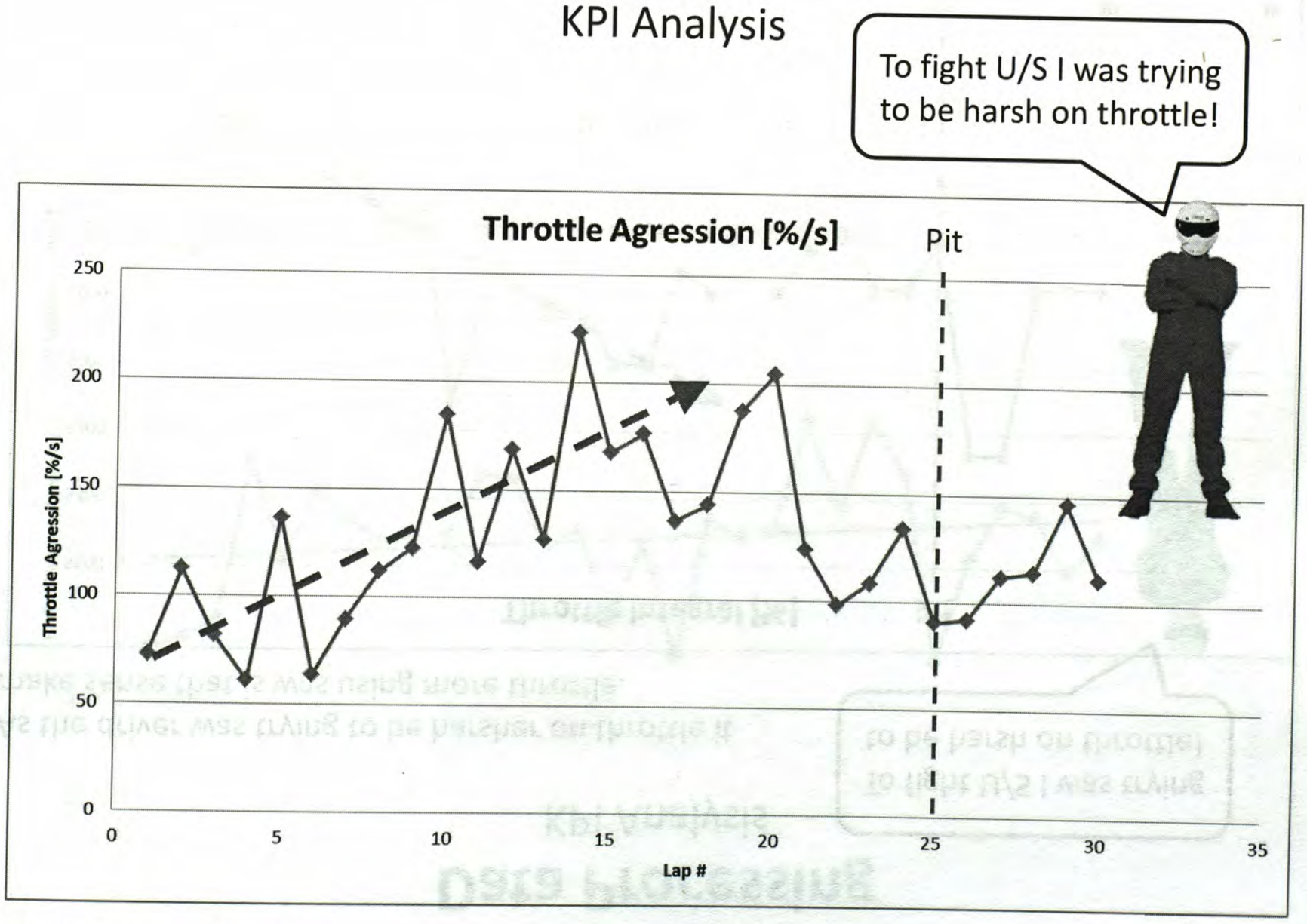
Data Processing

KPI Analysis



Data Processing

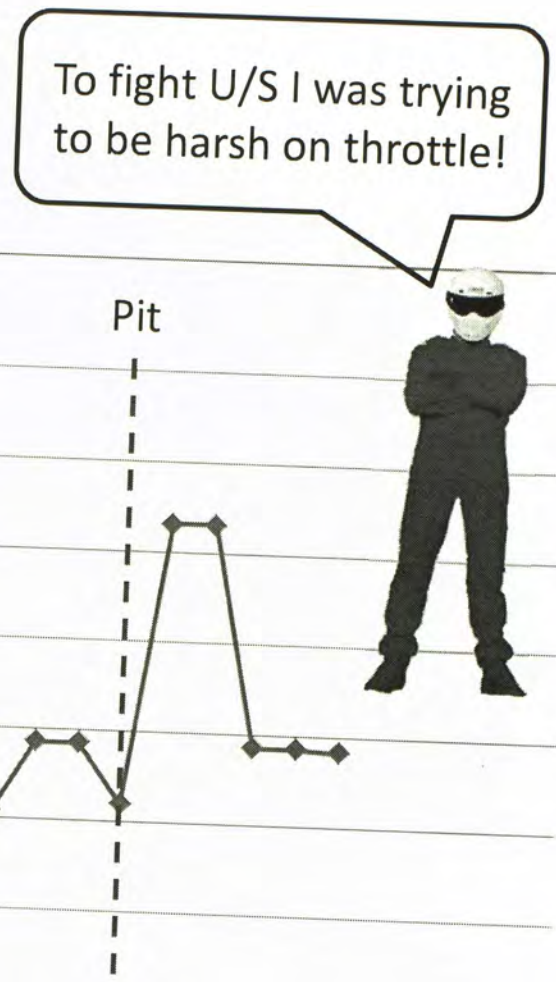
KPI Analysis

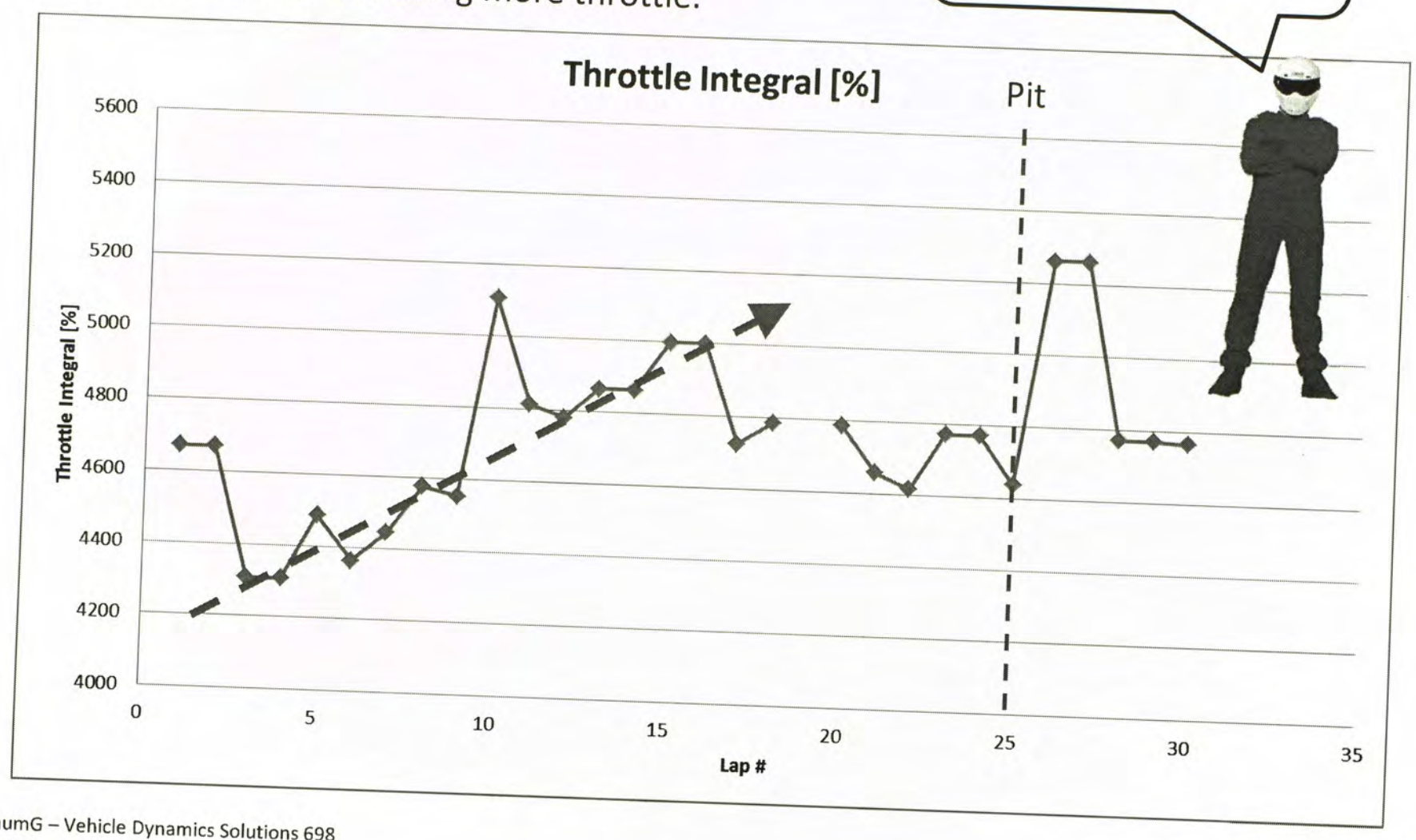


Data Processing

KPI Analysis

As the driver was trying to be harsher on throttle it make sense that is was using more throttle.

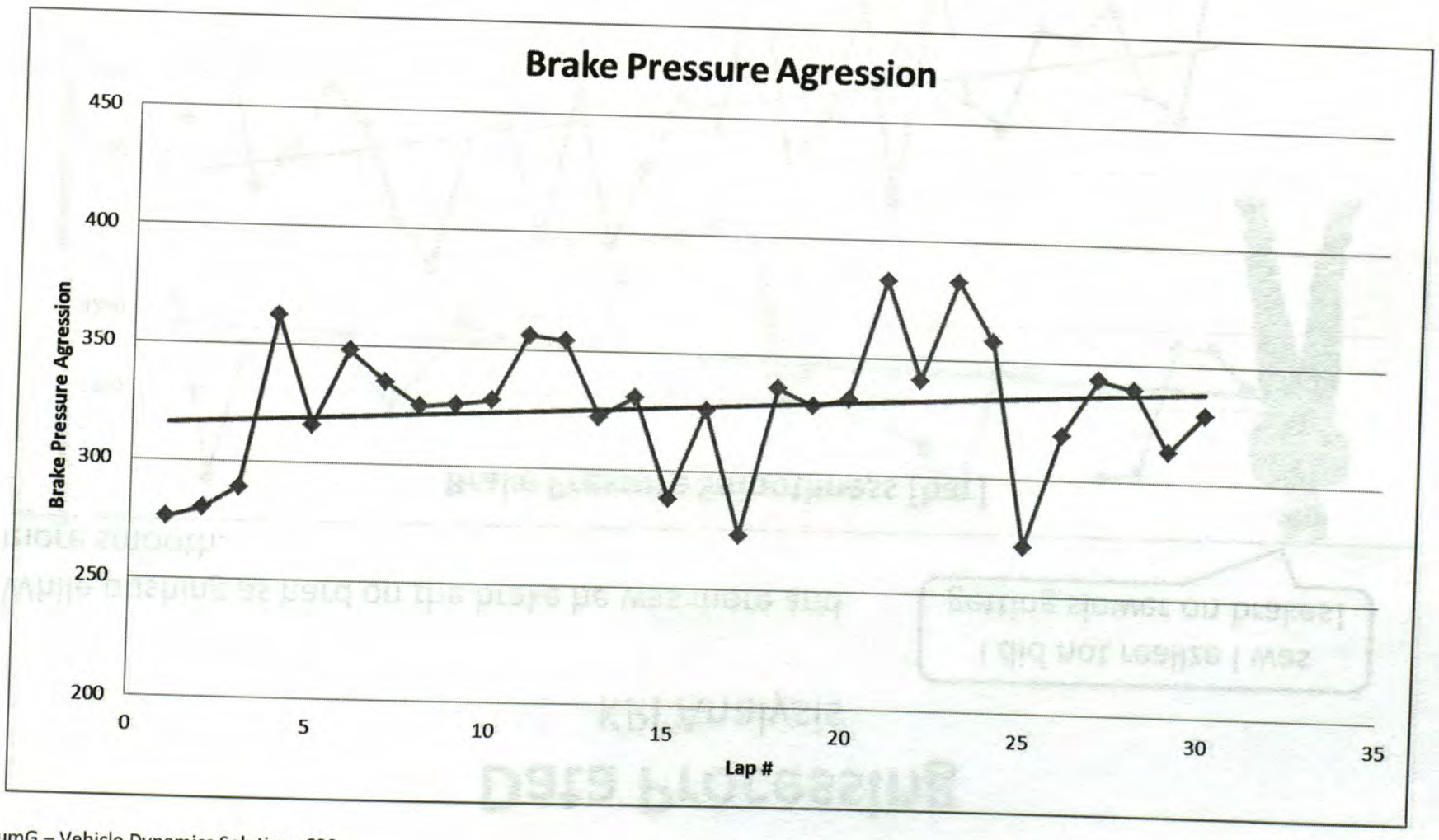




Data Processing

KPI Analysis

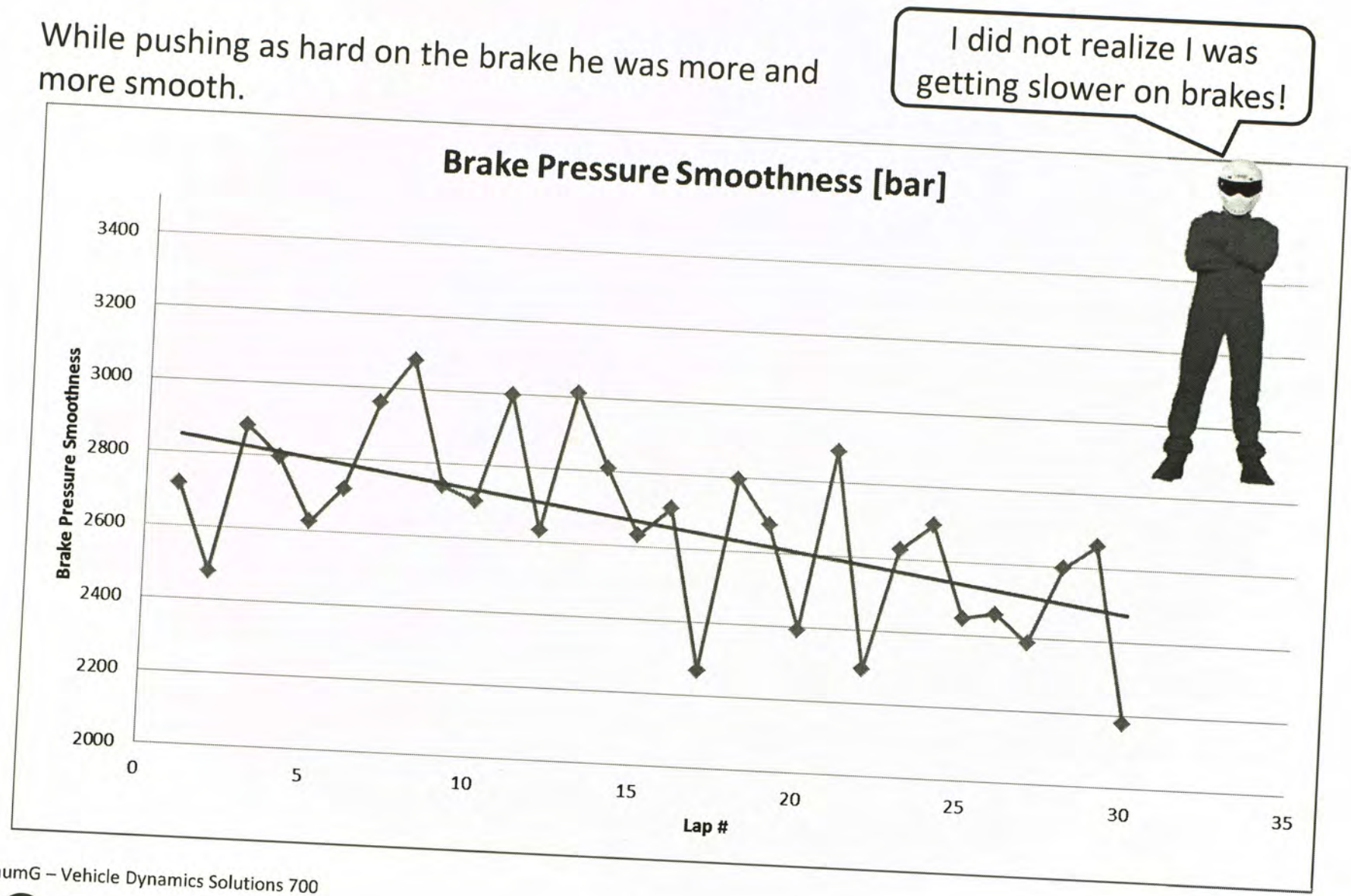
The average brake pressure remained the same during the stint.



Data Processing

KPI Analysis

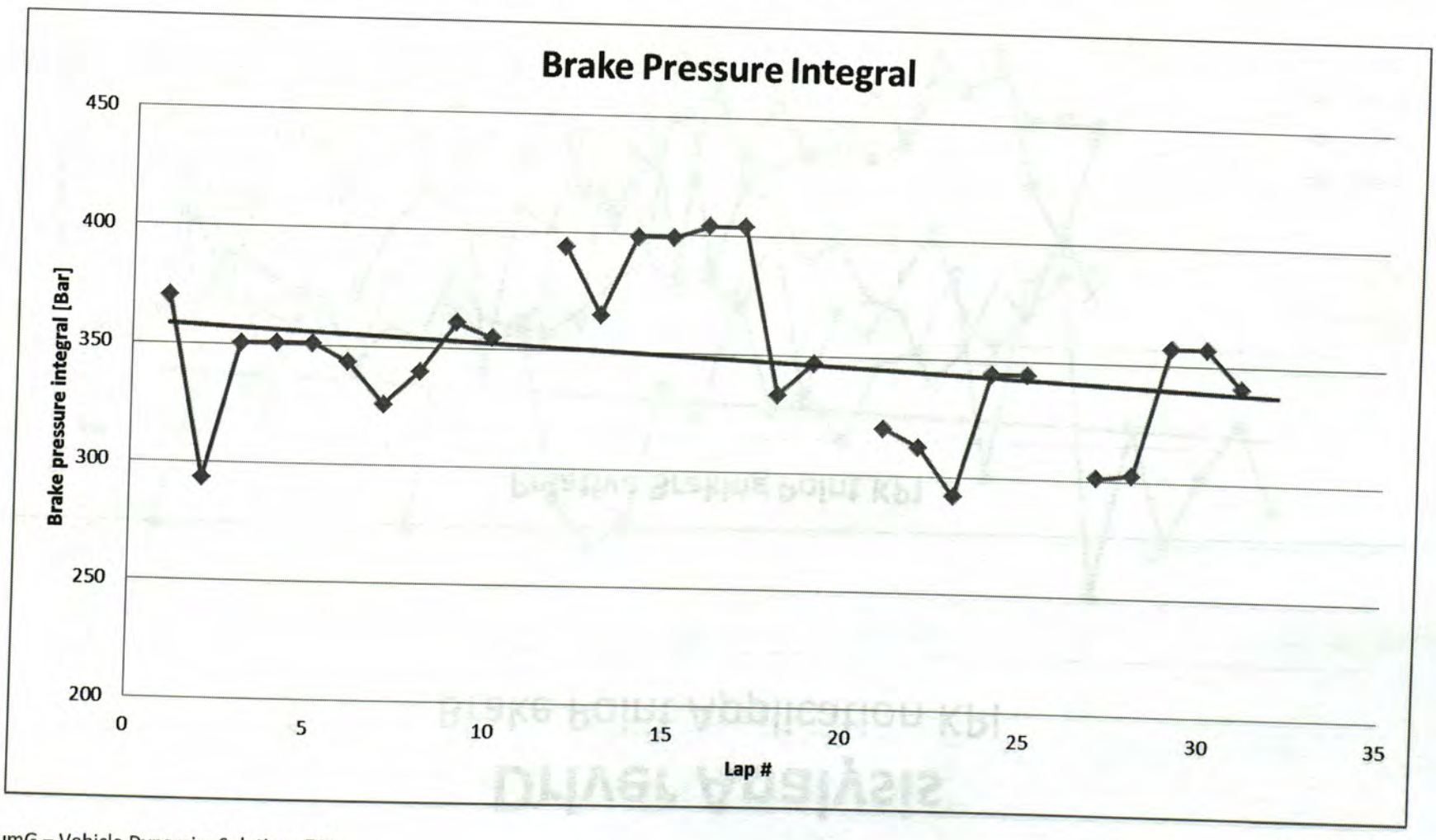
While pushing as hard on the brake he was more and more smooth.



Data Processing

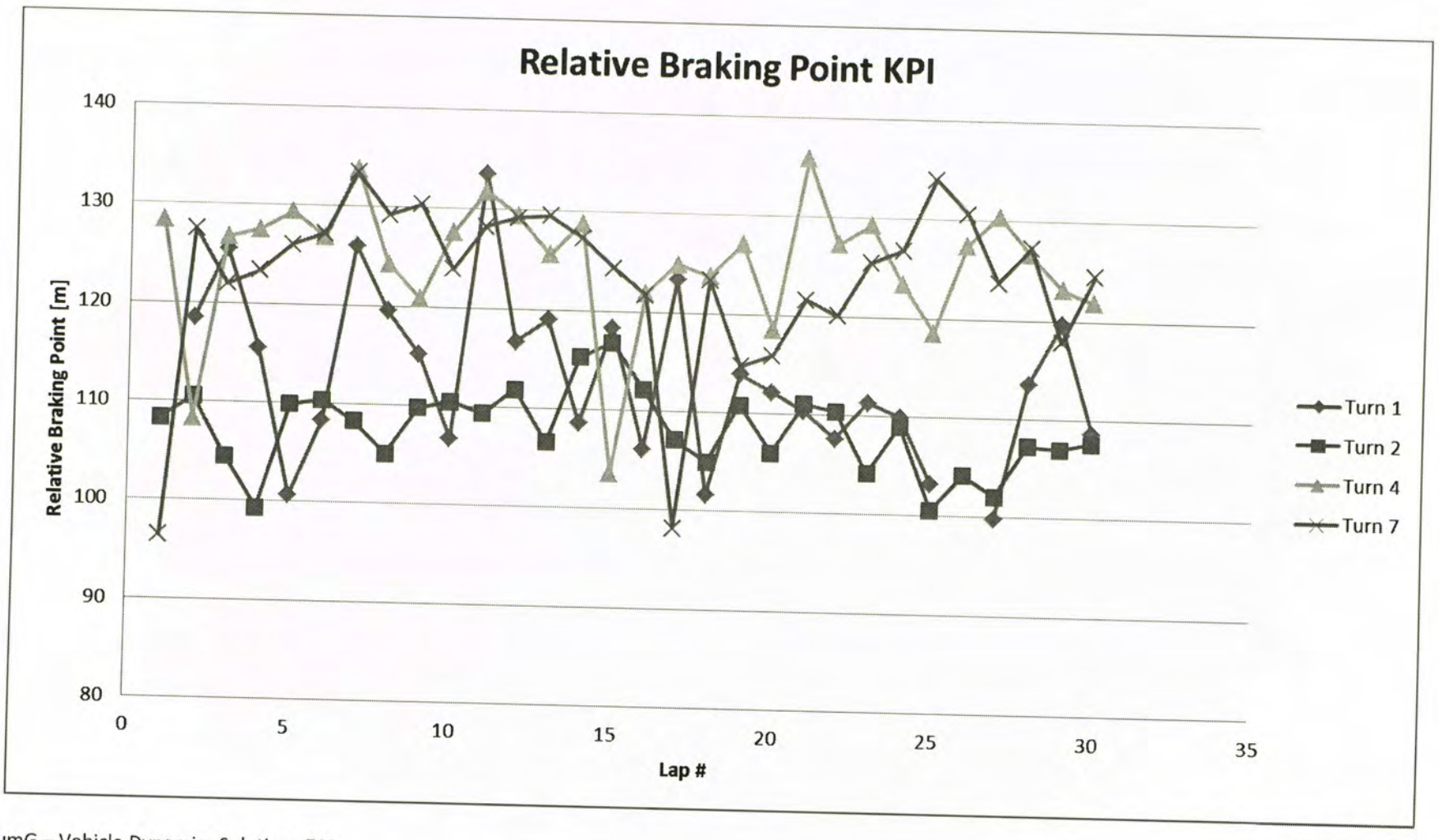
KPI Analysis

Brake Pressure Integral is quite constant during the stint.



Driver Analysis

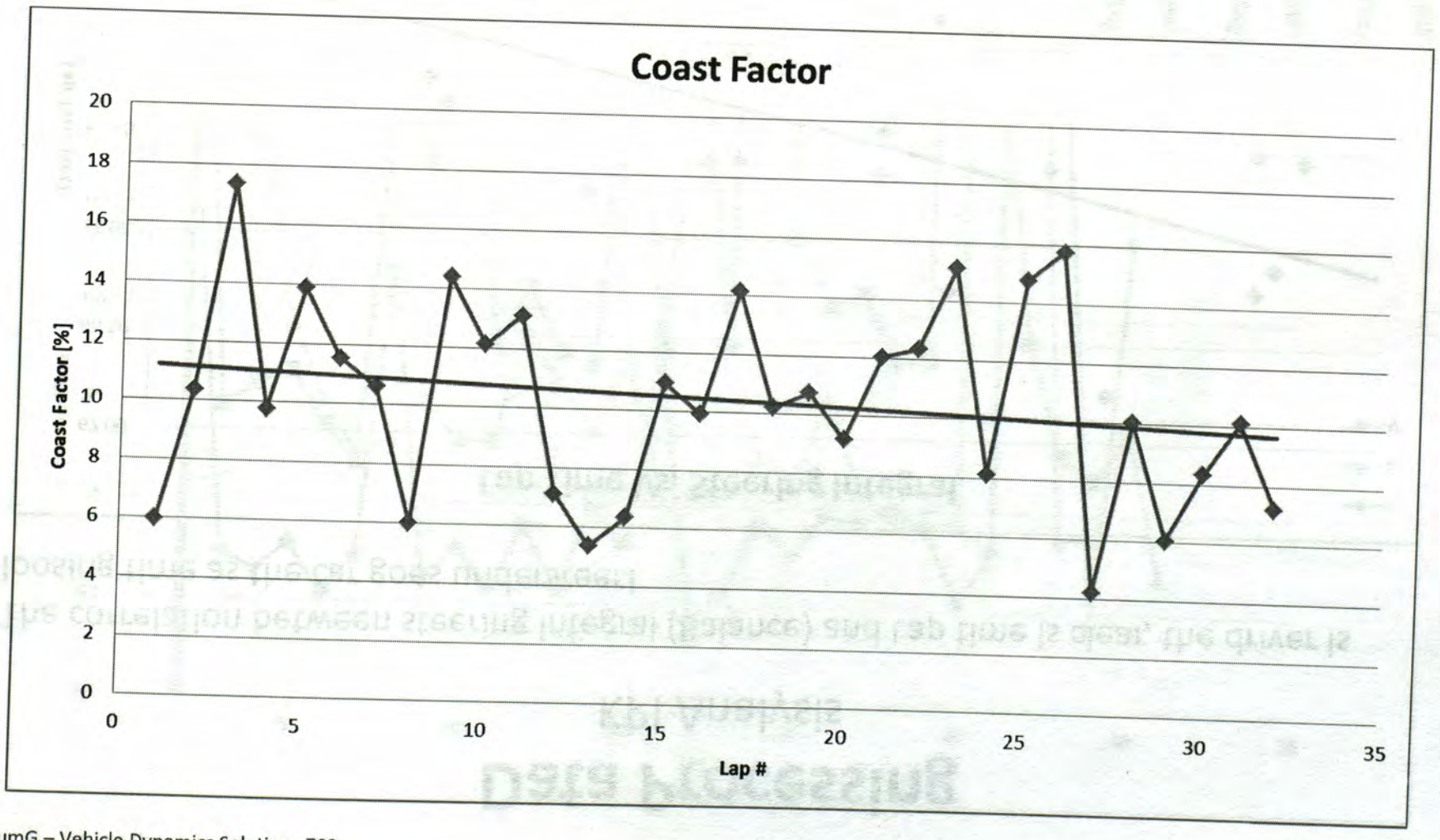
Brake Point Application KPI



Data Processing

KPI Analysis

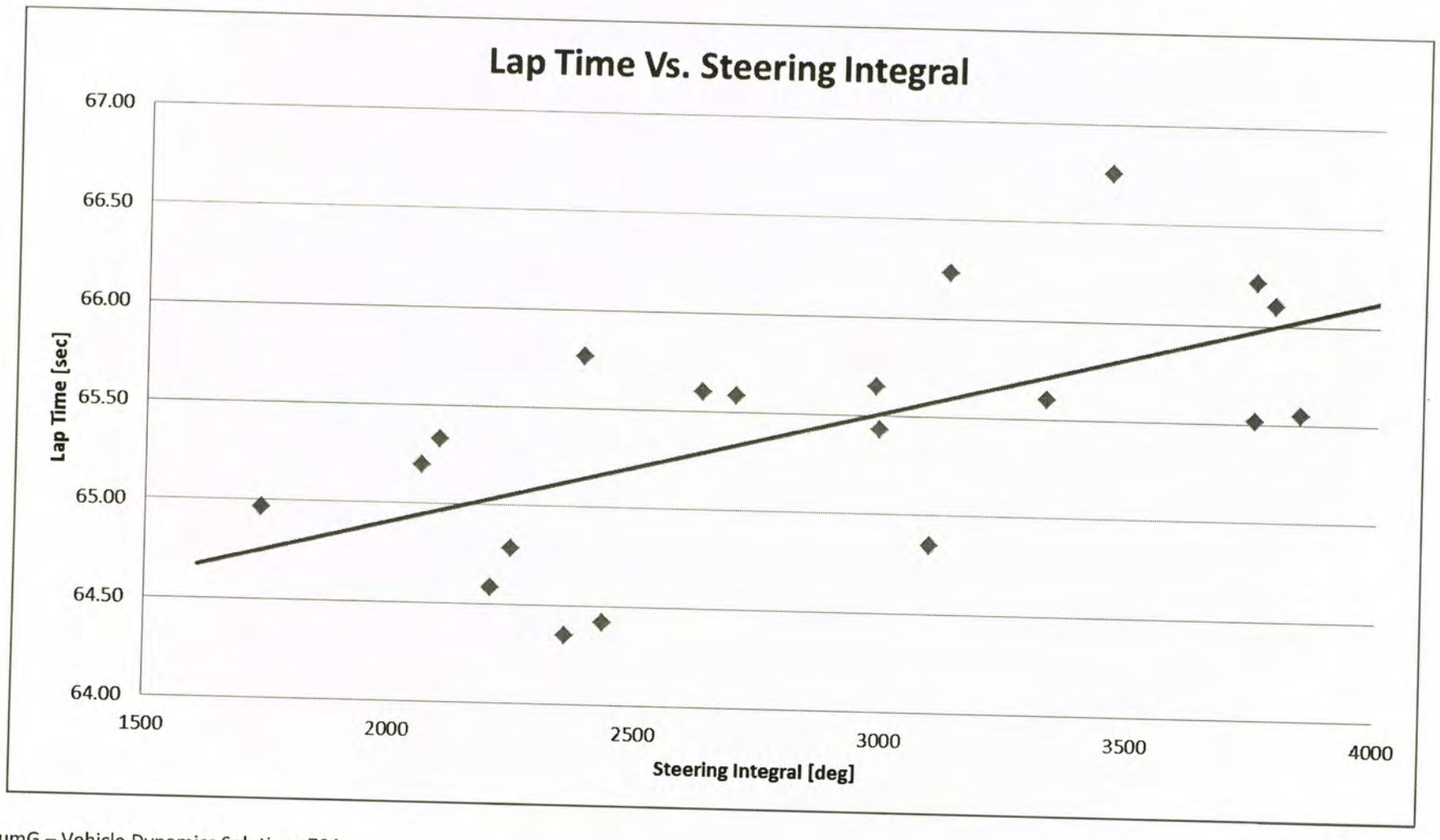
Coasting is quite constant.



Data Processing

KPI Analysis

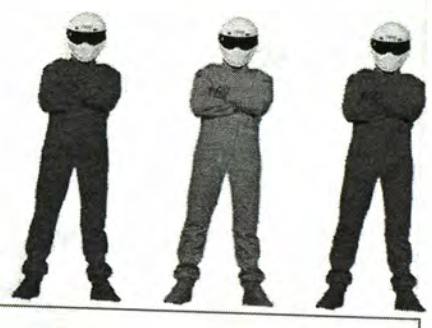
The correlation between steering integral (Balance) and Lap time is clear, the driver is loosing time as the car goes understeer!



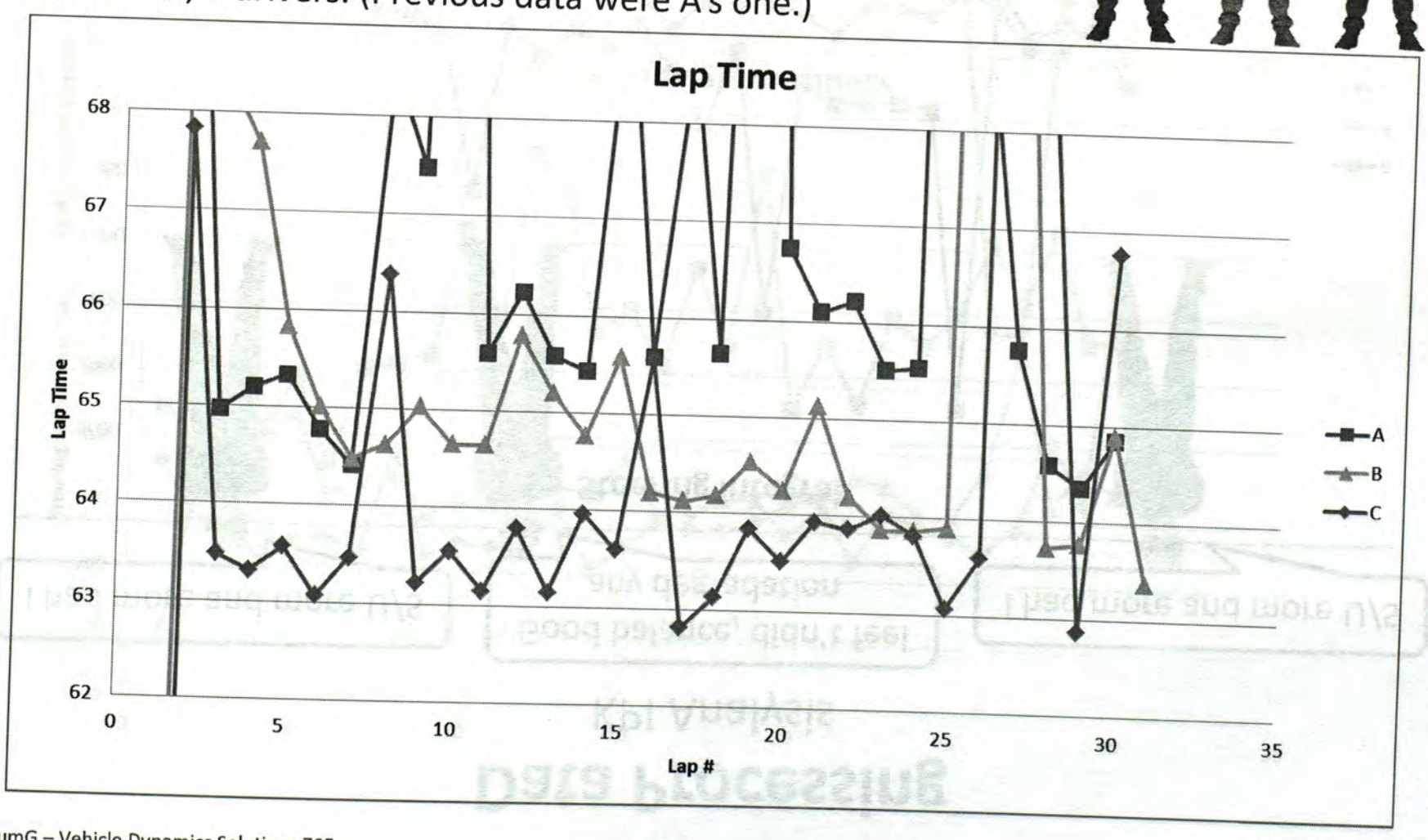
Data Processing

KPI Analysis

3 Drivers = 3 Styles

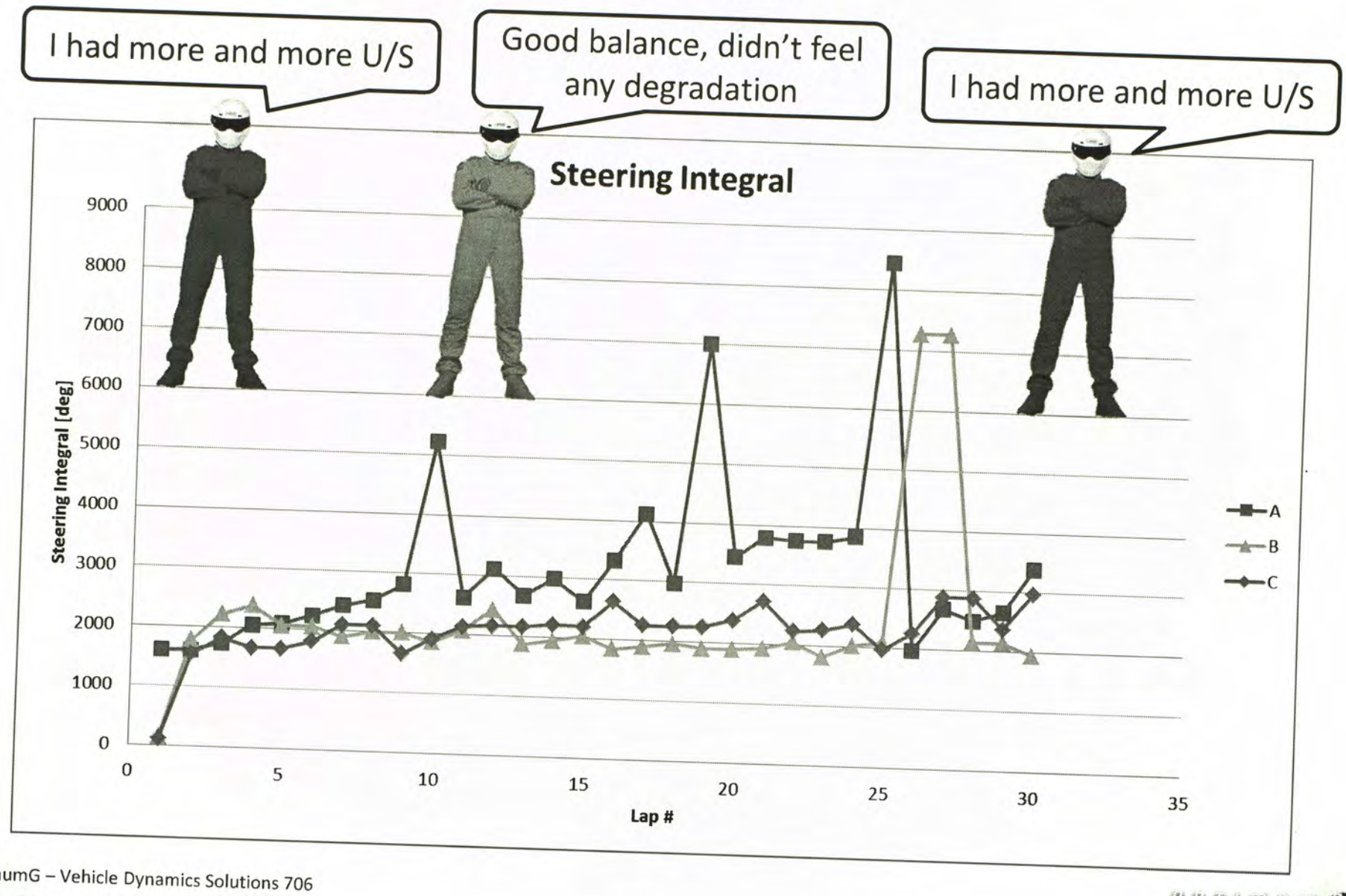


Same car, 3 drivers. (Previous data were A's one.)



Data Processing

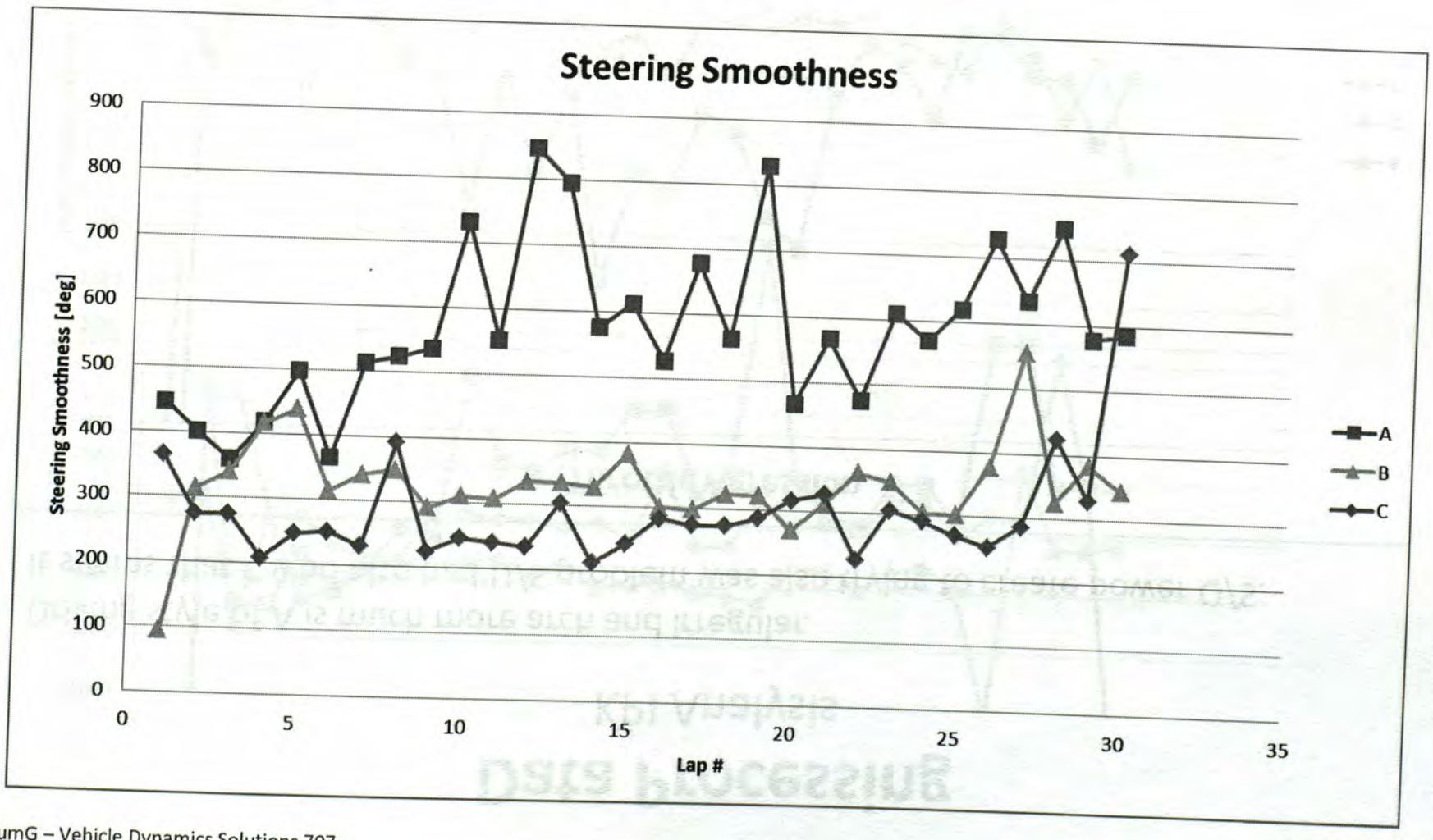
KPI Analysis



Data Processing

KPI Analysis

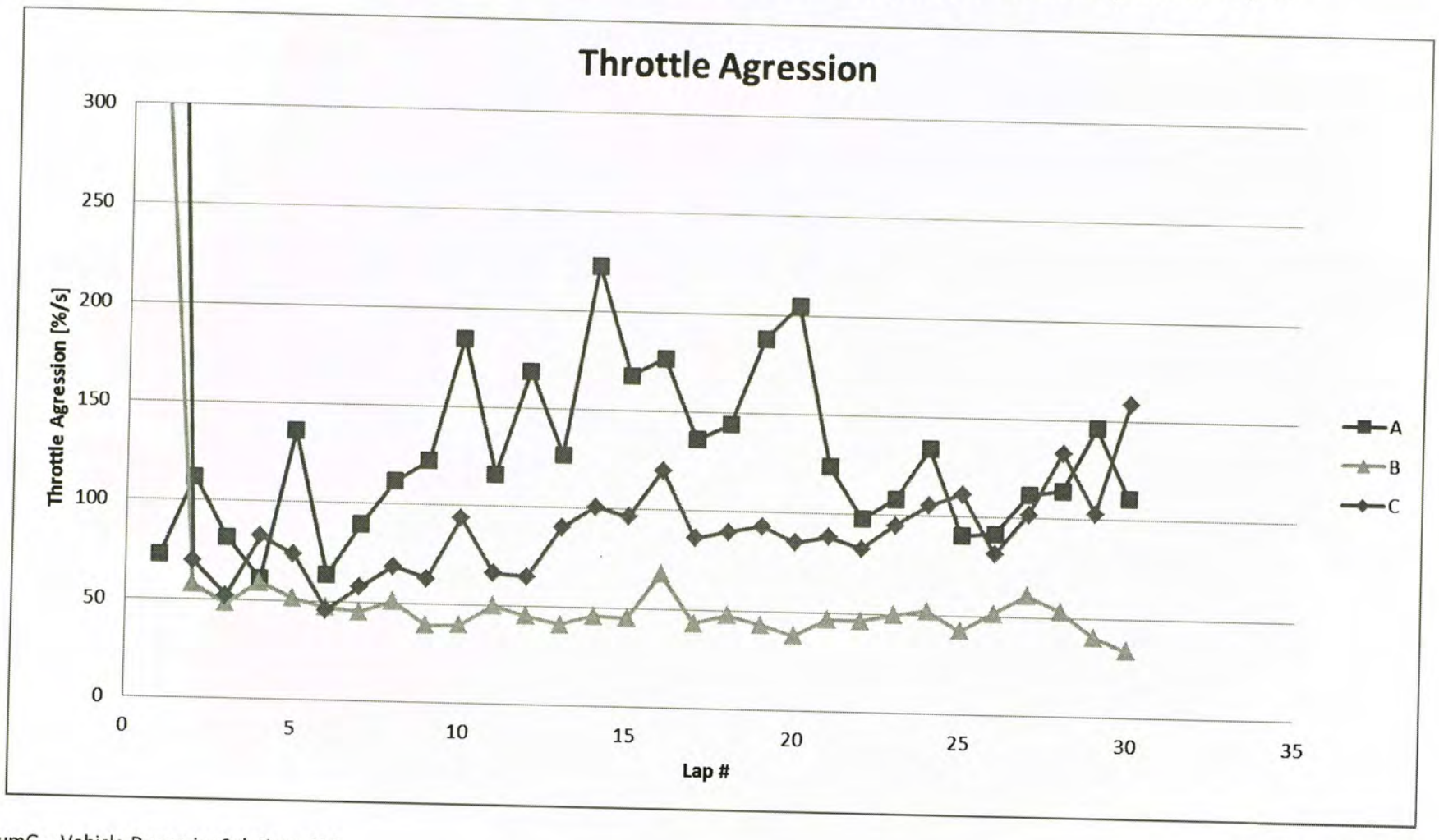
Driving style of **A** is much more arch and irregular.



Data Processing

KPI Analysis

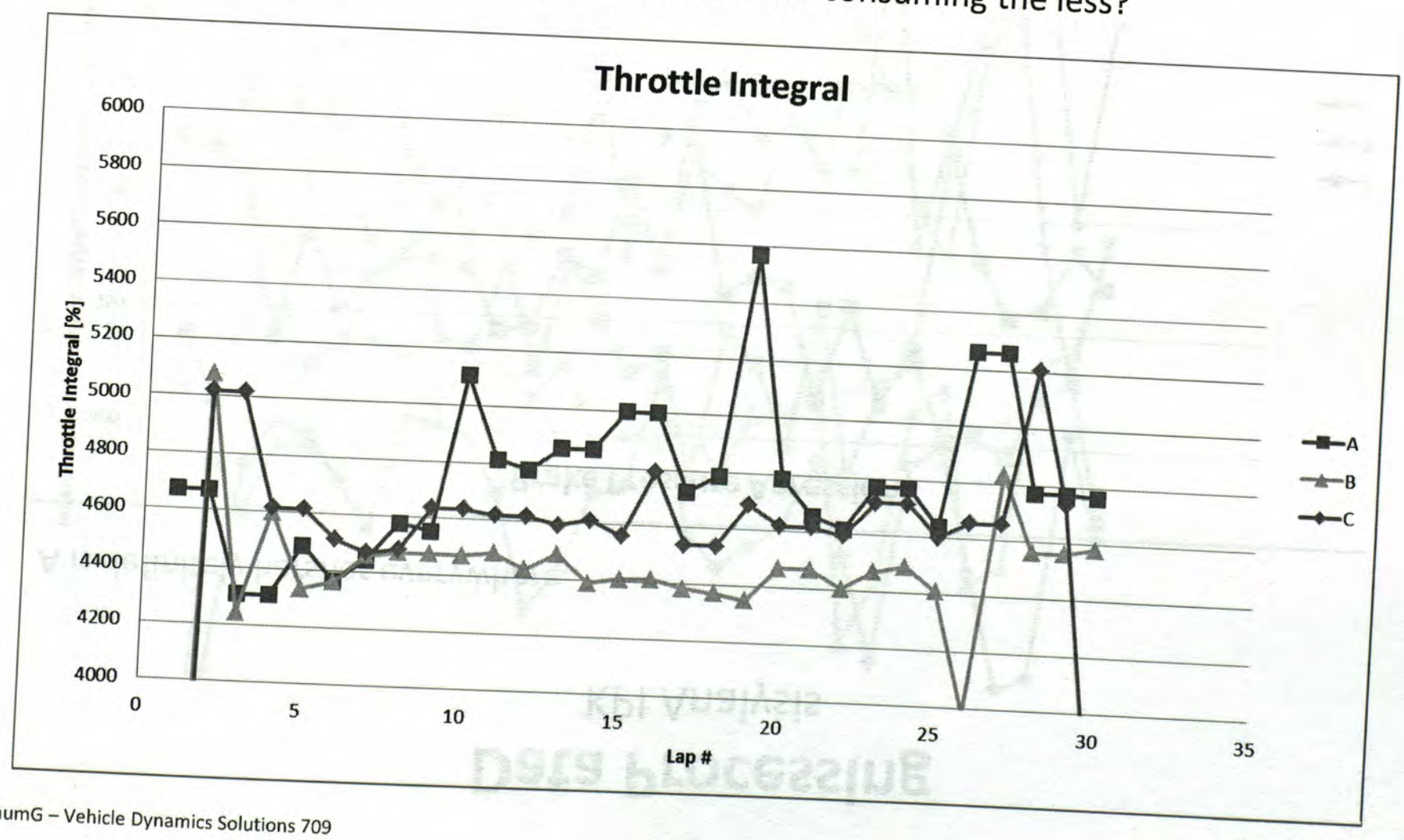
Driving style of **A** is much more arch and irregular.
It seams that **C** who also had U/S problem was also trying to create power O/S.



Data Processing

KPI Analysis

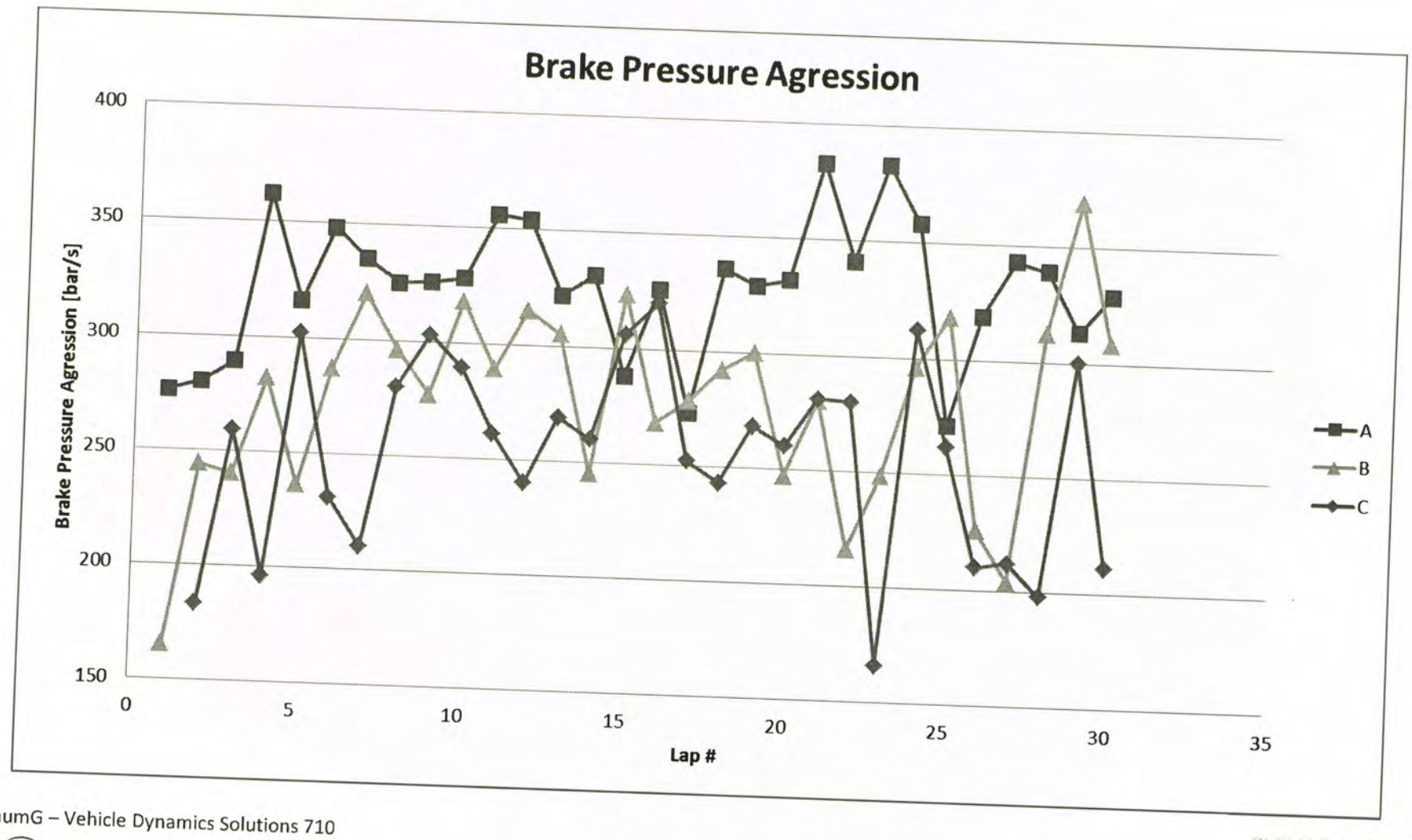
Driving style of **A** uses more throttle, **B** is the one consuming the less?



Data Processing

KPI Analysis

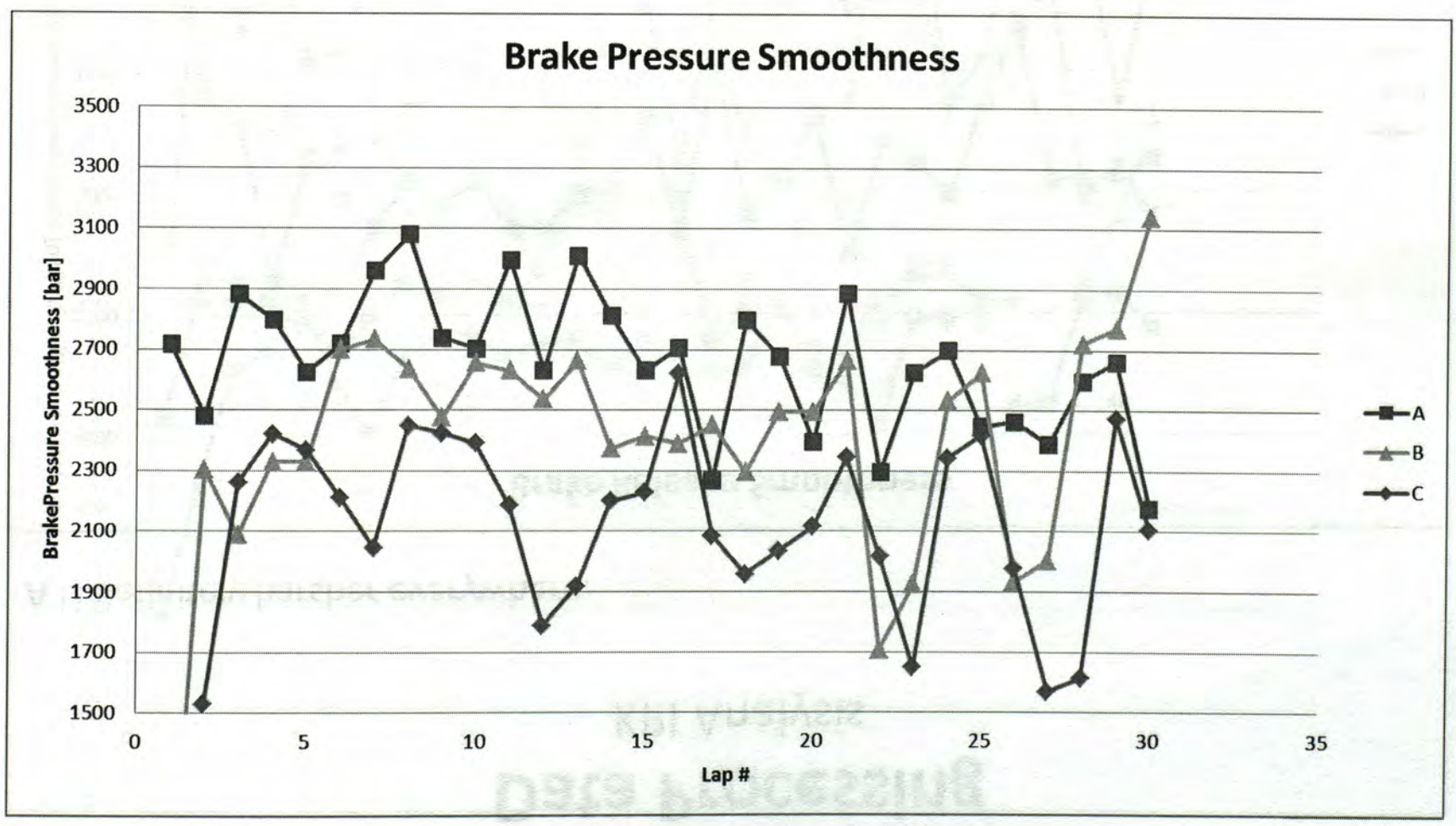
A is definitely harsher everywhere.



Data Processing

KPI Analysis

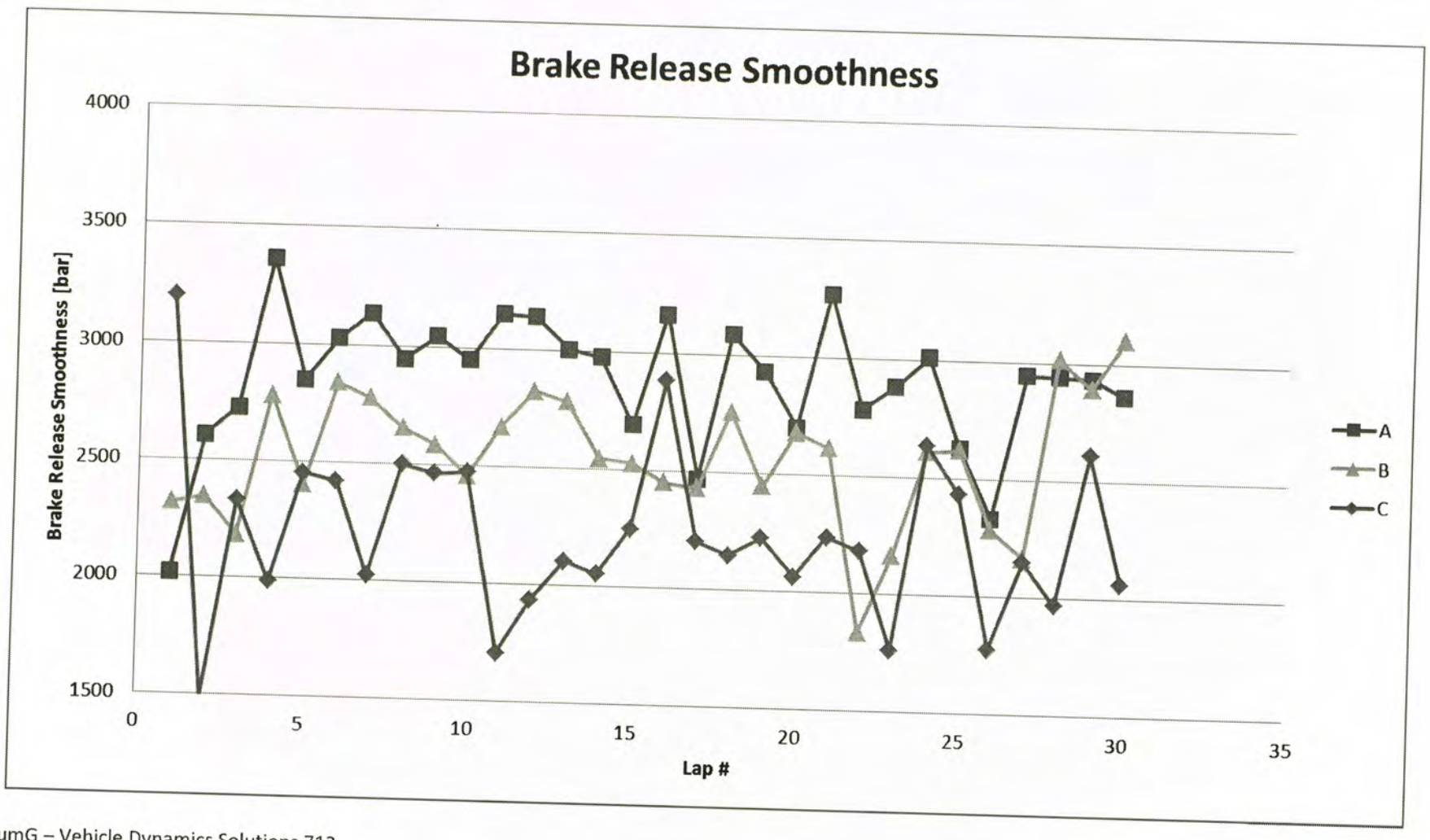
A is definitely harsher everywhere.



Data Processing

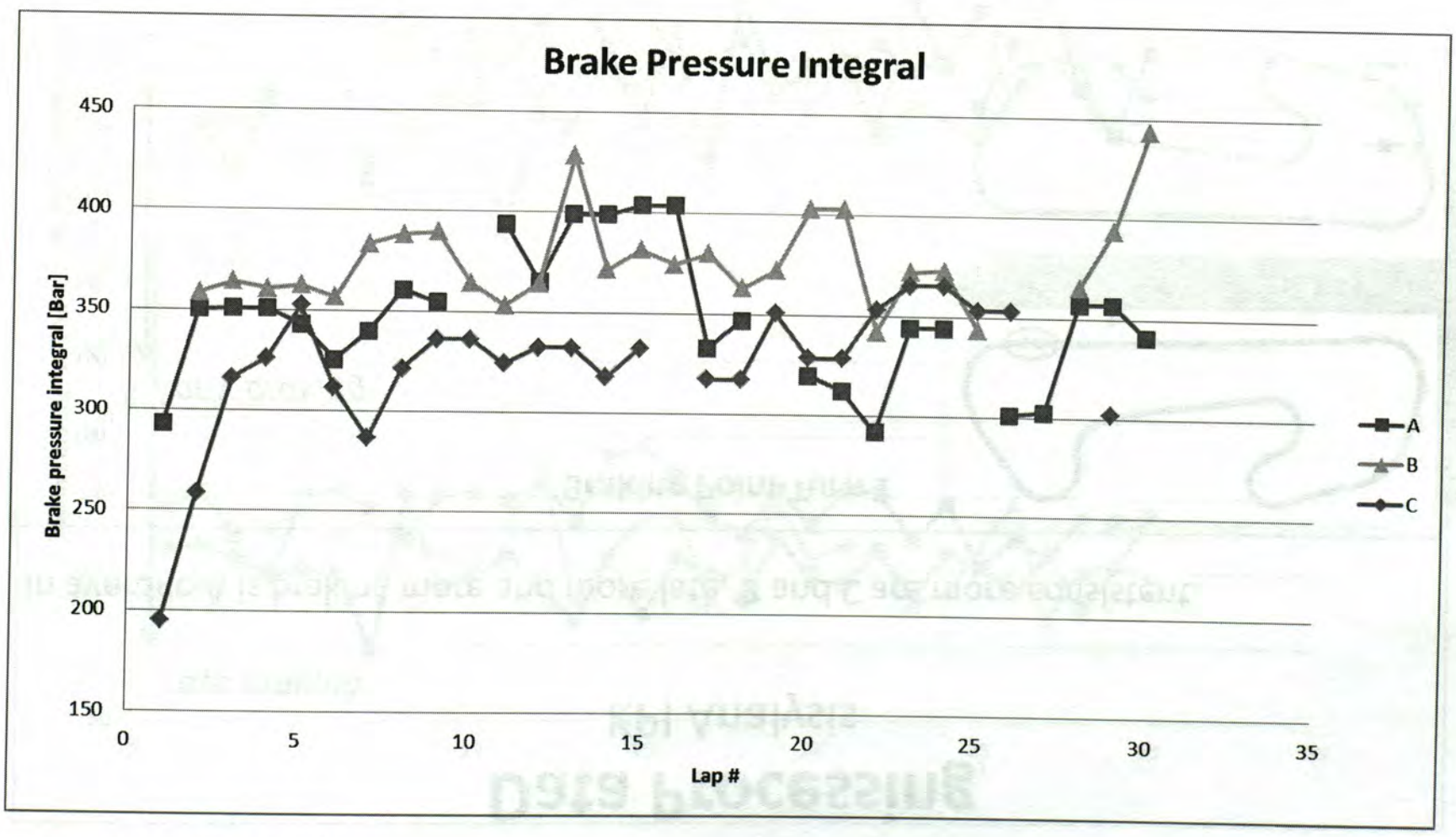
KPI Analysis

A is definitely harsher everywhere.



Data Processing

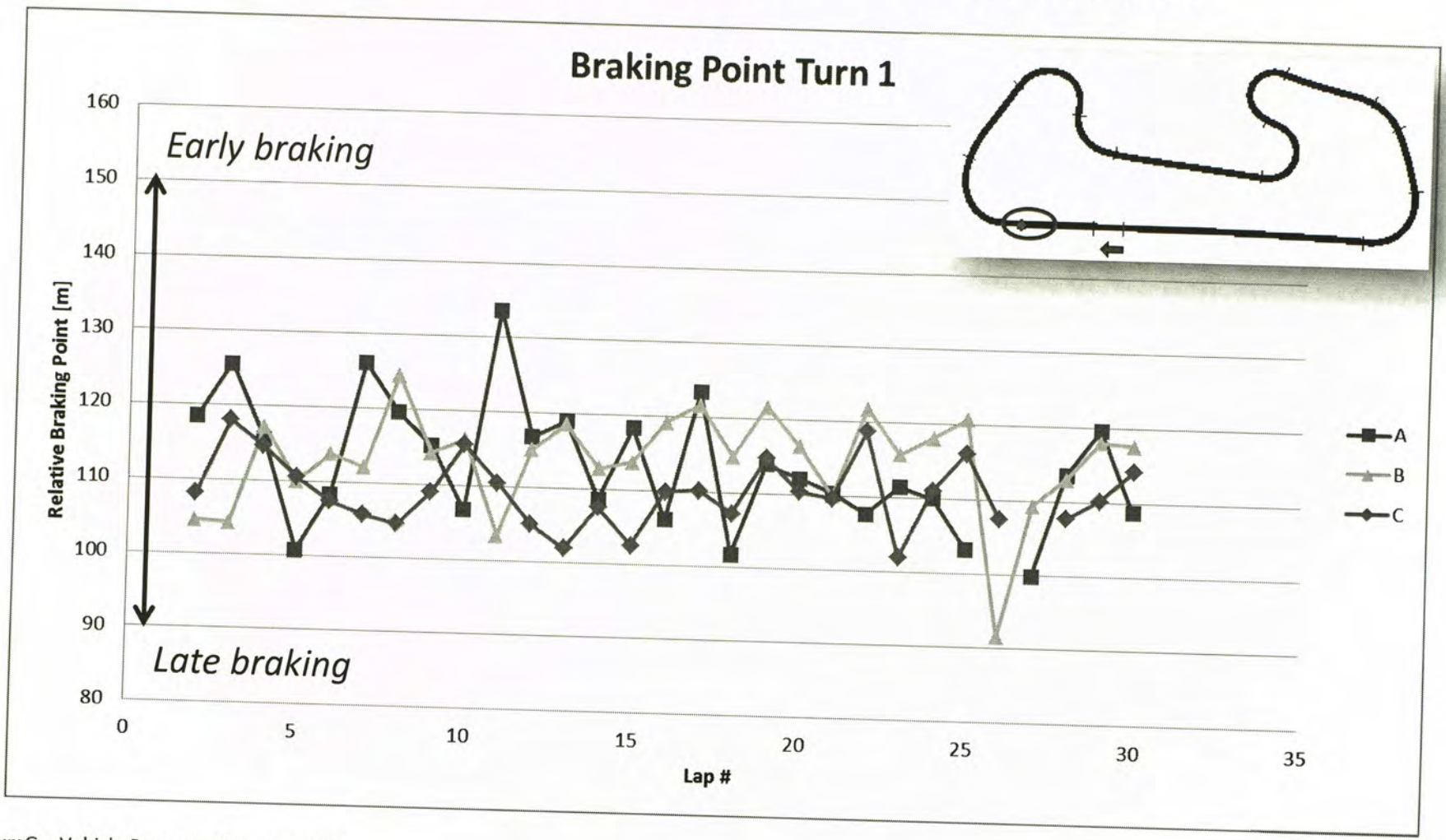
KPI Analysis



Data Processing

KPI Analysis

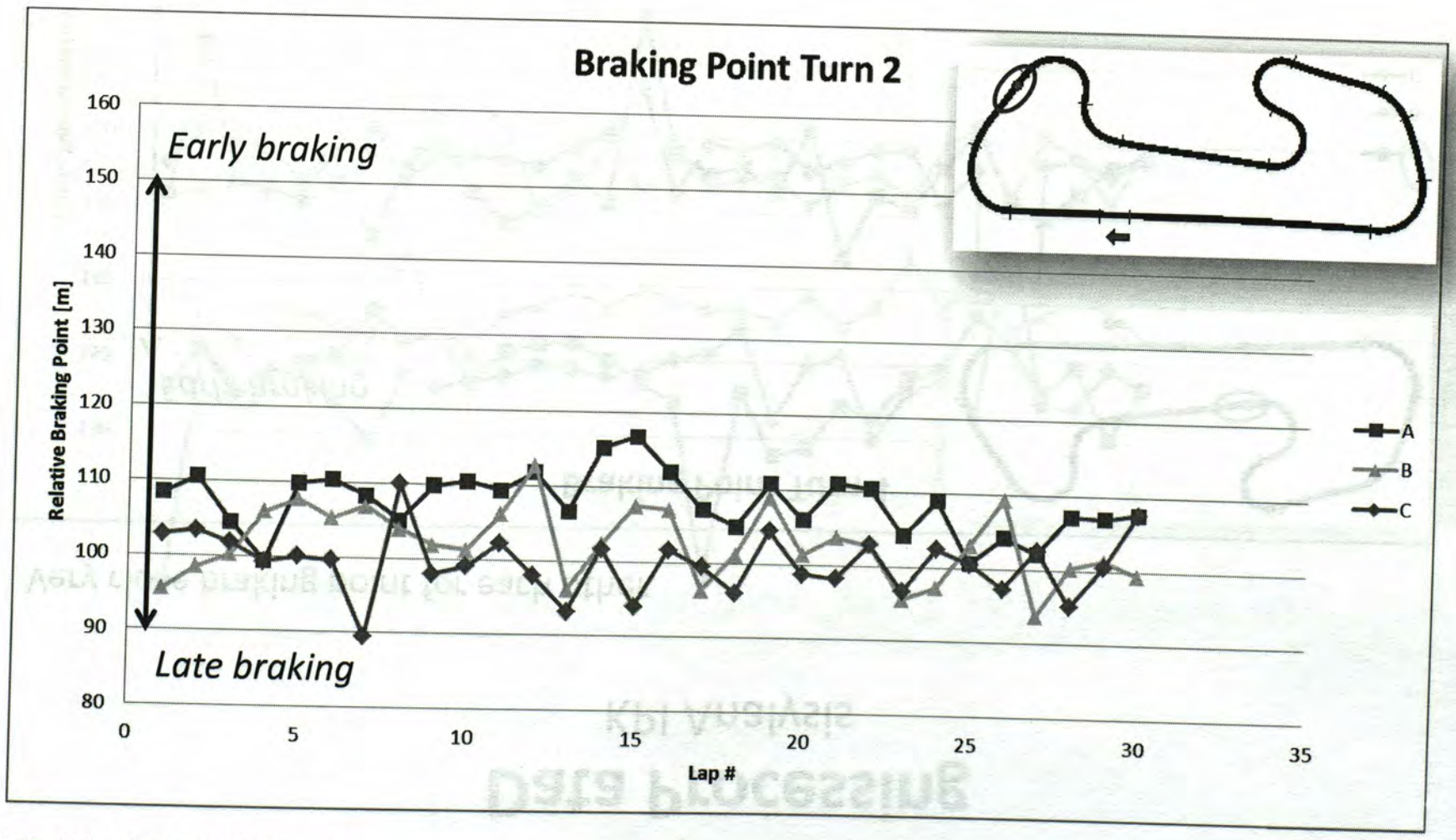
In average A is braking more and more late, **B** and **C** are more consistent.



Data Processing

KPI Analysis

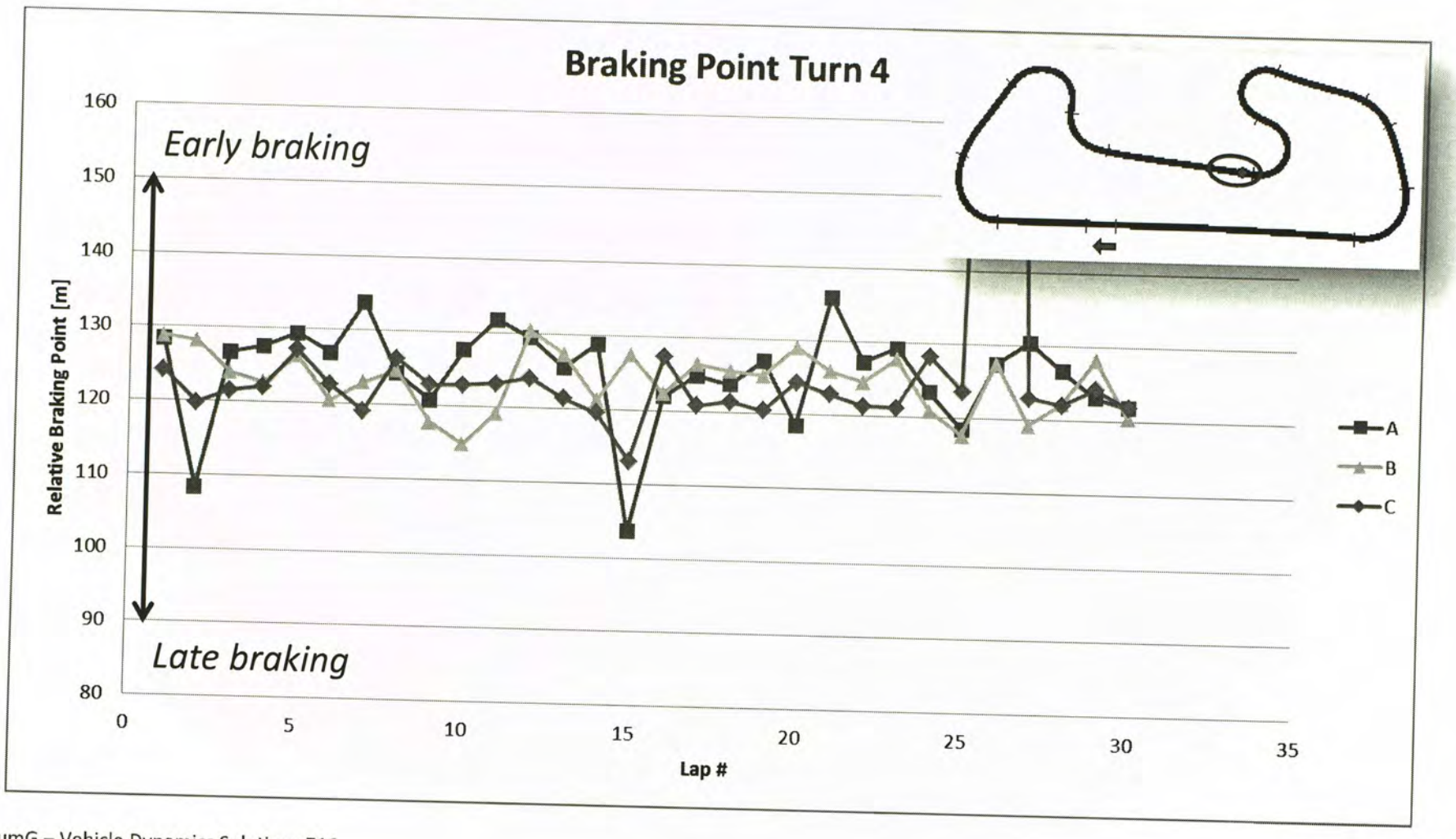
More consistent braking zone.



Data Processing

KPI Analysis

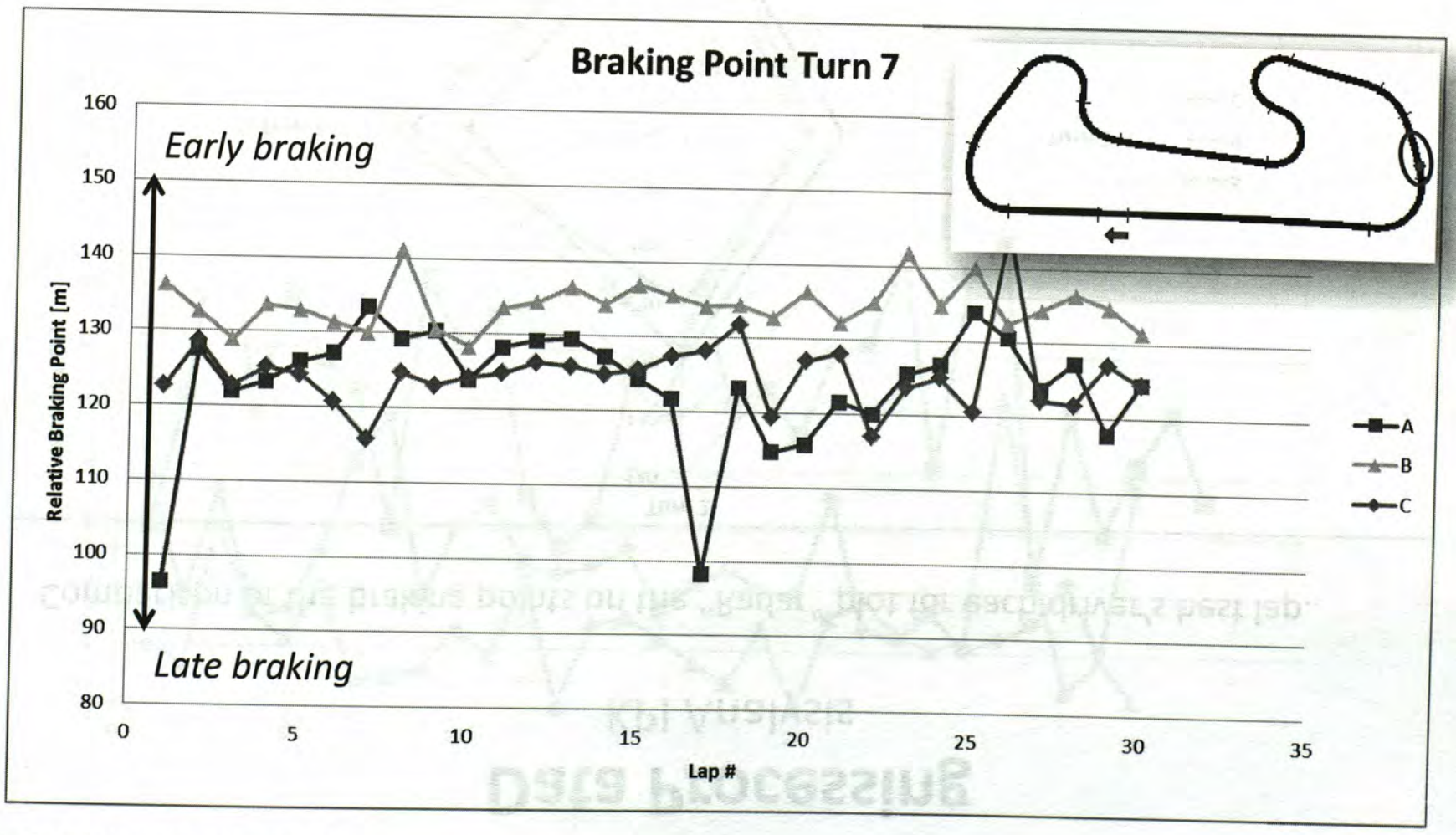
Very close braking point for each other.



Data Processing

KPI Analysis

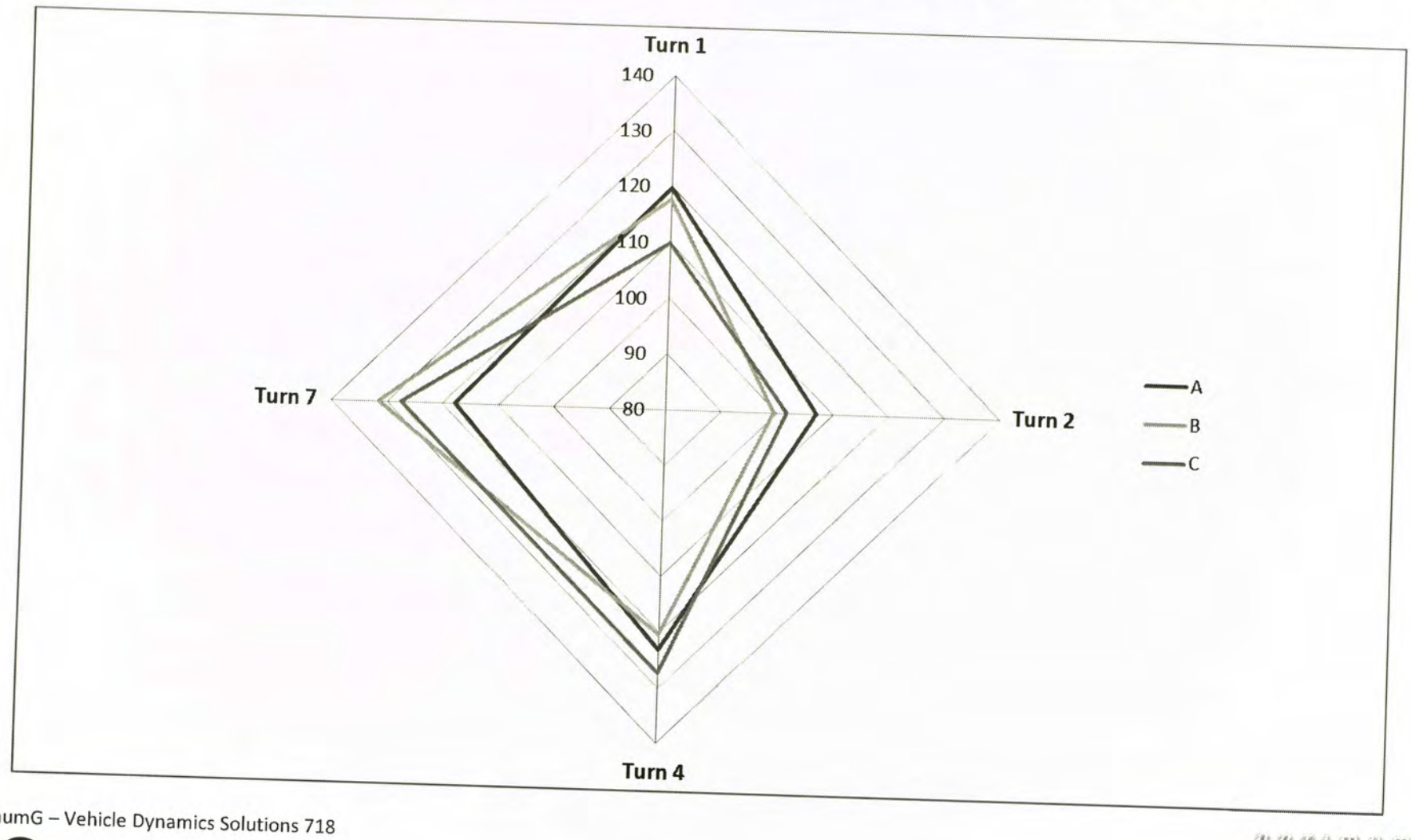
Now the tendency is that **A** is braking later.



Data Processing

KPI Analysis

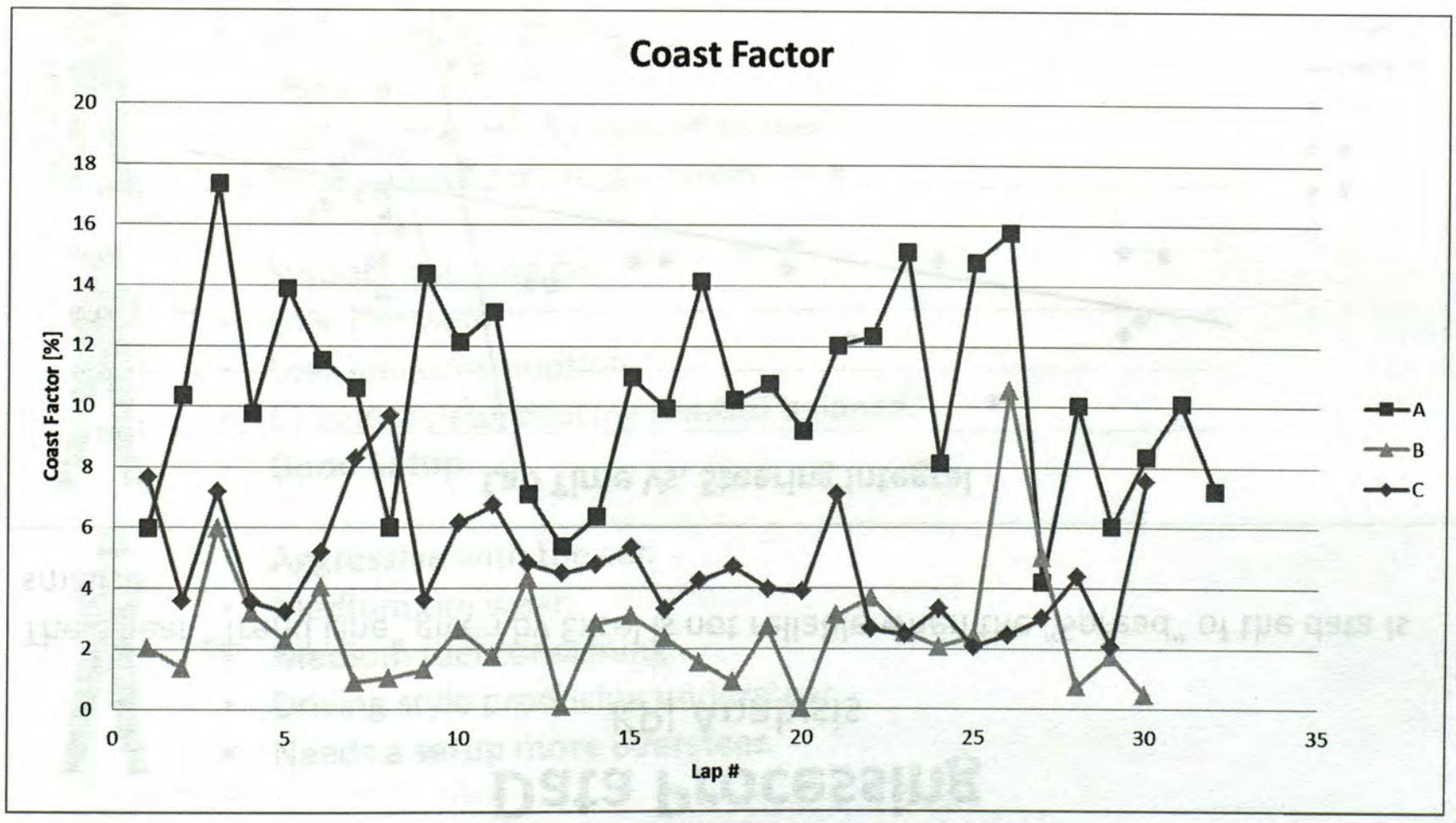
Comparison of the braking points on the "Radar" plot for each driver's best lap.



Data Processing

KPI Analysis

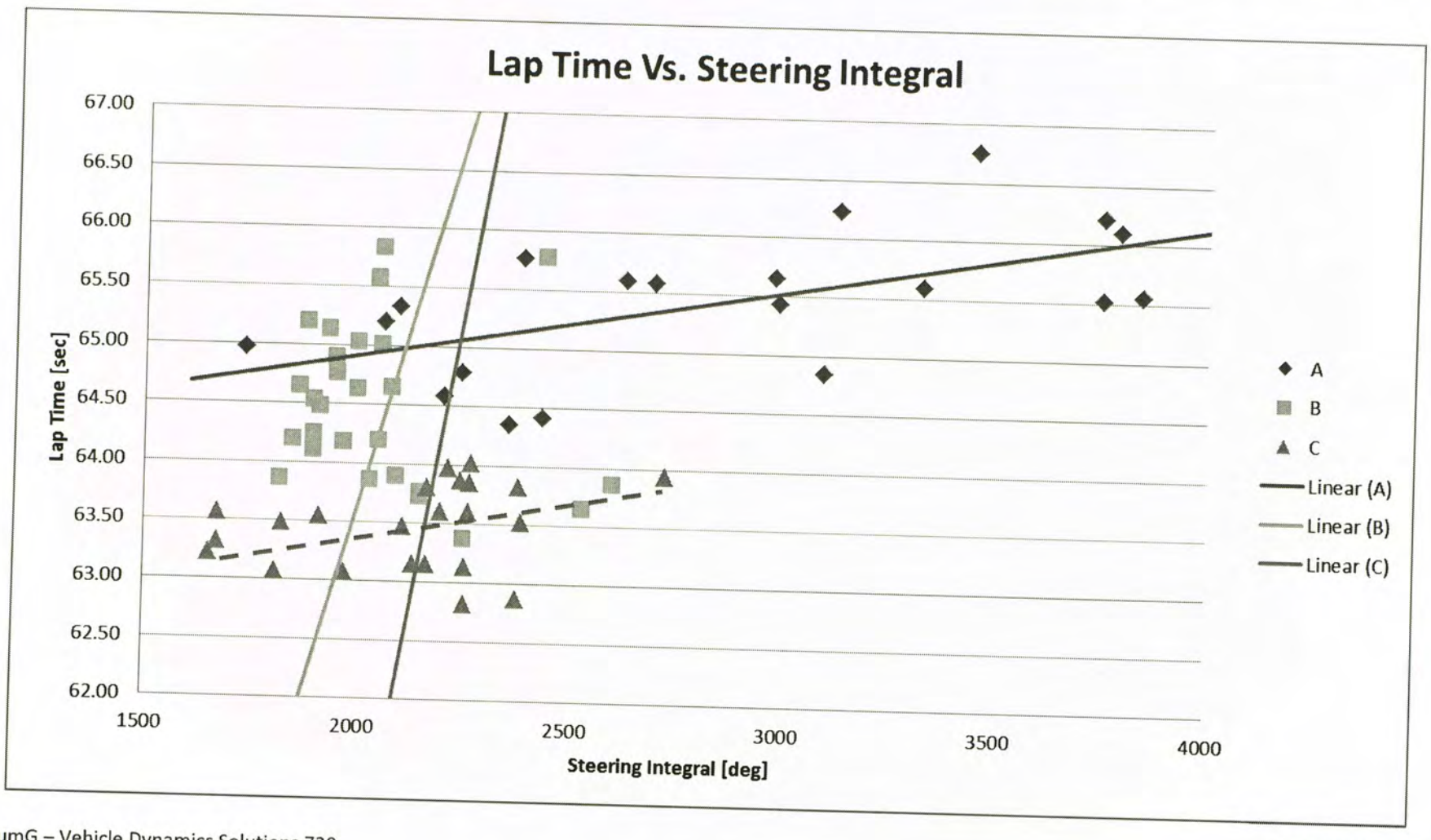
Despite **A** is the most aggressive driver, he is the one coasting the most.



Data Processing

KPI Analysis

The Linear "Trend Line" given by Excel is not reliable when the "Spread" of the data is small.



Data Processing

KPI Analysis

- **Very aggressive** with the car.
- **High** front tire wear.
- **High** fuel consumption.
- Driving style producing **understeer.**
- Needs a **setup more oversteer.**

- **Smooth** with the car.
- **Low** Tire wear.
- **Low** fuel consumption.
- Driving style producing **neutral balance.**
- **Good Setup.**

- **Aggressive** with the car.
- **Medium** tire wear.
- **Medium** fuel consuming.
- Driving style producing **understeer**.
- Needs a **setup more oversteer.**

Quotes

They did not know it was impossible; that is why they made it

Mark Twain

None of us are as smart as all of us"

Japanese proverb

Whatever you do, or dream you can, begin it. Boldness has genius and power and magic in it."

Goethe

Our deepest fear is not that we are inadequate.
Our deepest fear is that we are powerful beyond measure.
It is our light, not our darkness that most frightens us.
We ask ourselves: Who am I to be brilliant, gorgeous, talented, fabulous?
Actually, who are you not to be?
You are a child of God. Your playing small does not serve the world.
There is nothing enlightened about shrinking so that other people won't feel insecure around you.
We are all meant to shine, as children do.
We were born to make manifest the glory of God that is within us.
It's not just in some of us; it's in everyone.
And as we let our own light shine, we unconsciously give other people permission to do the same.
As we are liberated from our own fear, our presence automatically liberates others."

Marianne Williamson

Quotes

The man who thinks he can

If you think you are beaten, you are
If you think you dare not, you don't
If you'd like to win, but think you can't
Its most a cinch you won't

If you think you'll lose, you're lost
For out in the world we find
Success begins with a fellow's will
It is all in the state of mind

If you think you're outclassed, you are
You've got to think high to rise
You've got to be sure of yourself before
You can ever win a prize

Life's battles don't always go
To the stronger or faster man
But sooner or later, the man who wins
Is the man who thinks He can

I've been up and down and over and out
But I know one thing
Each time I find myself flat on my face,
I pick myself up and get back in the race.

Frank Sinatra. That' s life

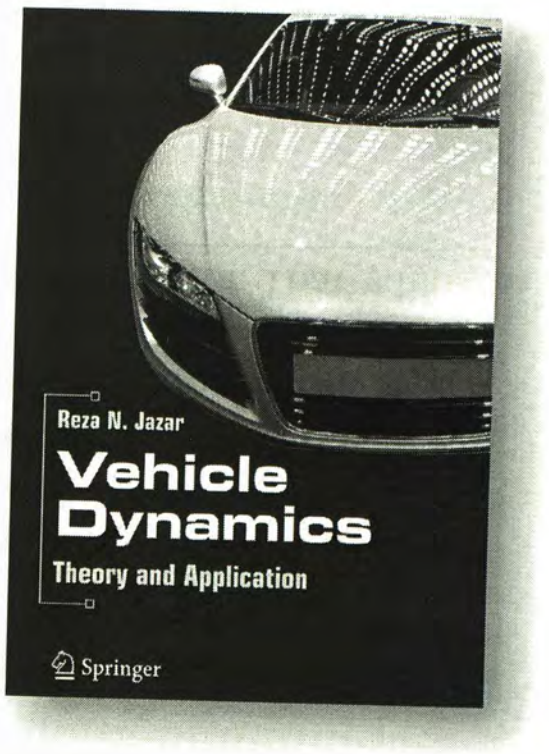
Suggested Reading

General

- Automotive Handbook [Bosch]

Aerodynamics

- Aerodynamics of Road Vehicles [Wolf Heinrich Hocho]
- Race Car Aerodynamics, designing for speed [Joseph Katz]
- Road Vehicle Aerodynamics [A.J. Scibor-Rylski]
- Competition Car Aerodynamics [Simon Mcbeath]
- Theory of Wing Sections [Abbot and Doenhoff]



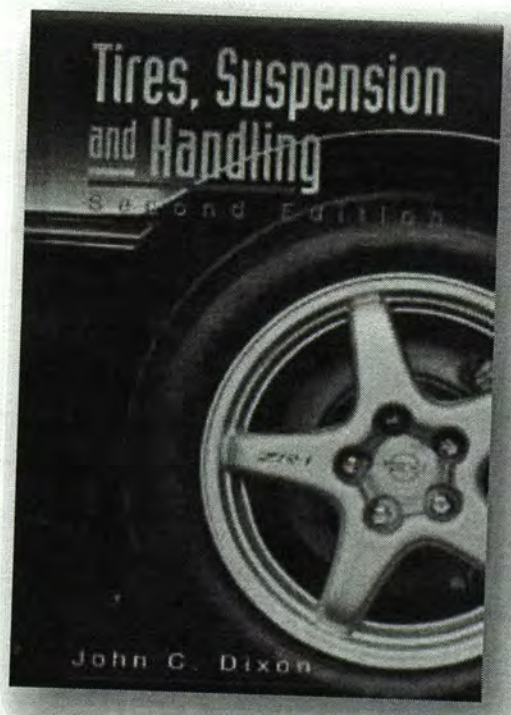
Data Acquisition

- Data Power [Buddy Fey]
- Competition Car Data logging [Simon Mcbeath]
- Analysis Techniques for Racecar Data Acquisition [Jorge Segers]

Suggested Reading

Vehicle Dynamics, Chassis and Suspension Basics

- Engineer to win [Carroll Smith]
- Prepare to Win [Carroll Smith]
- Race to Win [Carroll Smith]
- Tune to win [Carroll Smith]



Core Vehicle Dynamics

- Vehicle Dynamics: Theory and Application [Reza N. Jazar]
- Theory of Ground Vehicles [J. Y. Wong]
- Fundamentals of Vehicle Dynamics [Thomas D. Gillepsie]
- Race Car Vehicle Dynamics [Milliken & Milliken]
- The Automotive Chassis [J Reimplell & H Stoll]
- The Shock Absorber Handbook [John C. Dixon]
- Tires, Suspension and Handling [John C. Dixon]
- Suspension Analysis and Computational Geometry [John C. Dixon]
- Car Suspension and Handling [Bastow, Howard, and Whitehead]

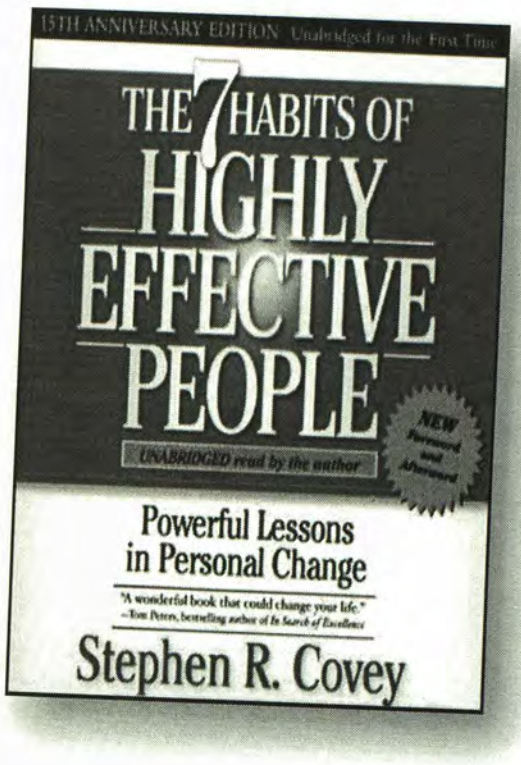
Suggested Reading

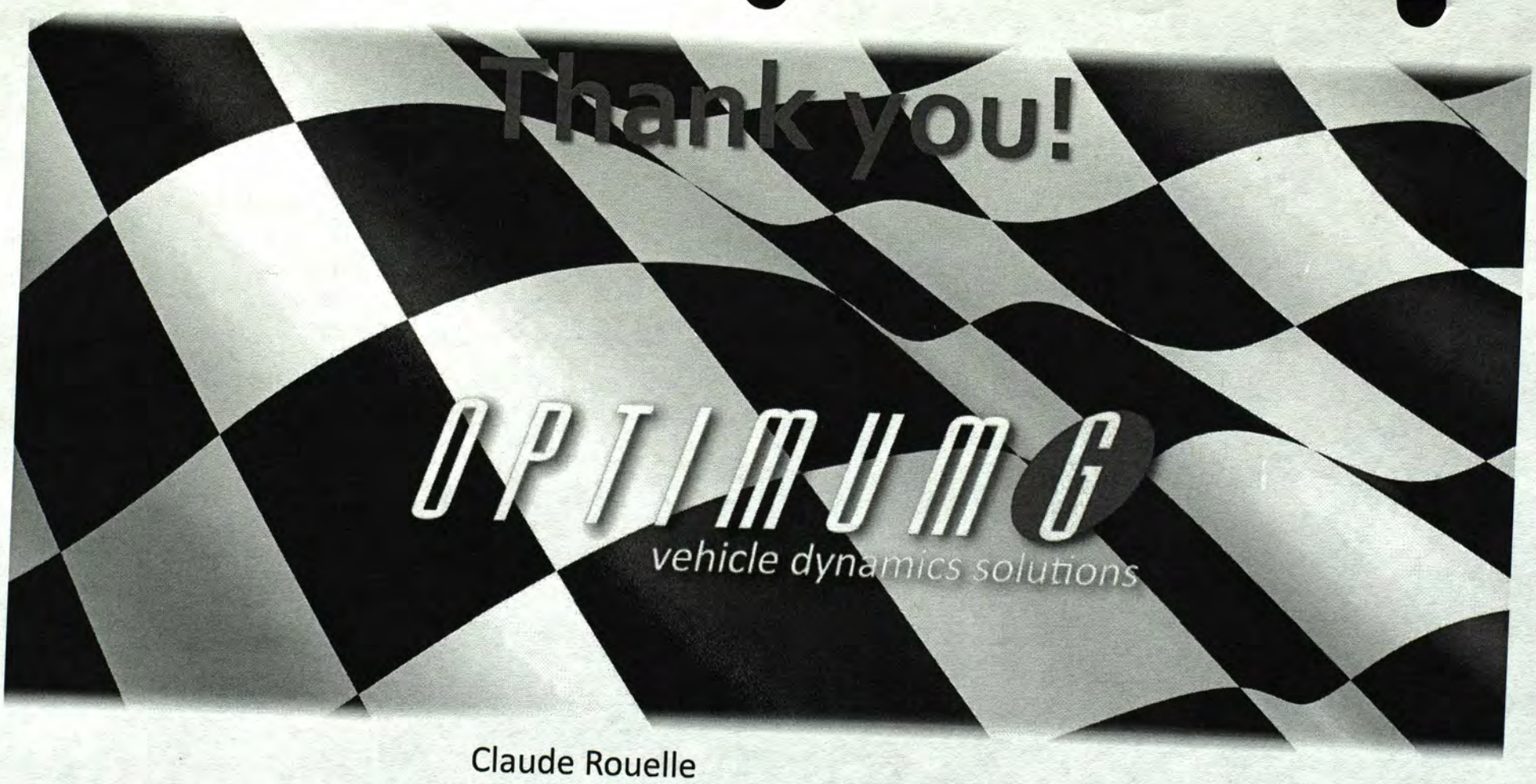
Periodicals

- Race Car Engineering
- Race Tech Magazine

Non engineering books which could change your life

- Who moved my cheese? (Spencer Johnson)
- The Alchemist (Paulo Coelho)
- How To Win Friends and Influence People (Dale Carnegie)
- The 7 Habits of Highly Effective People (Steven Covey)





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