Probability and Statistics

In this video, you will learn foundational concepts in Probability and Statistics.

What is Probability?

Probability is a branch of **theoretical** Mathematics that focuses on predicting the outcome of events that have yet to happen.

Probability does not tell us exactly what will happen, it is just a prediction.

How do we compute a probability?

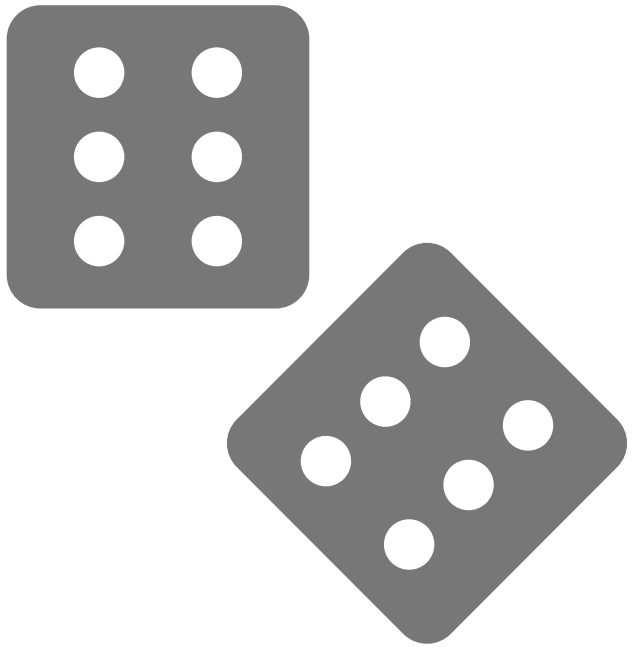
The probability of a simple event can be computed using the formula:

$$P = \frac{\text{Number of possible events}}{\text{Number of possible outcomes}}$$

Where:

- The number of possible events is the number of times a certain event can happen.
- The number of possible outcomes is the total number of results we can obtain in our experiment.

Probability Example: Rolling a Fair Die



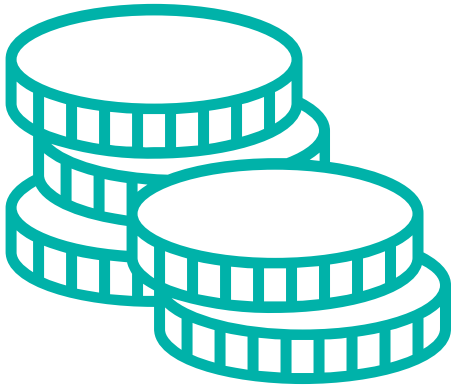
We want to compute the probability of getting 7 when rolling a fair die.

- The number of *possible events* is 0 because we cannot obtain 7 when rolling a die.
- The number of *possible outcomes* is 6 because we could get any number between 1 and 6.

Then, the probability of this event is:

$$P = \frac{0}{6} = 0$$

Probability Example: Tossing a Coin



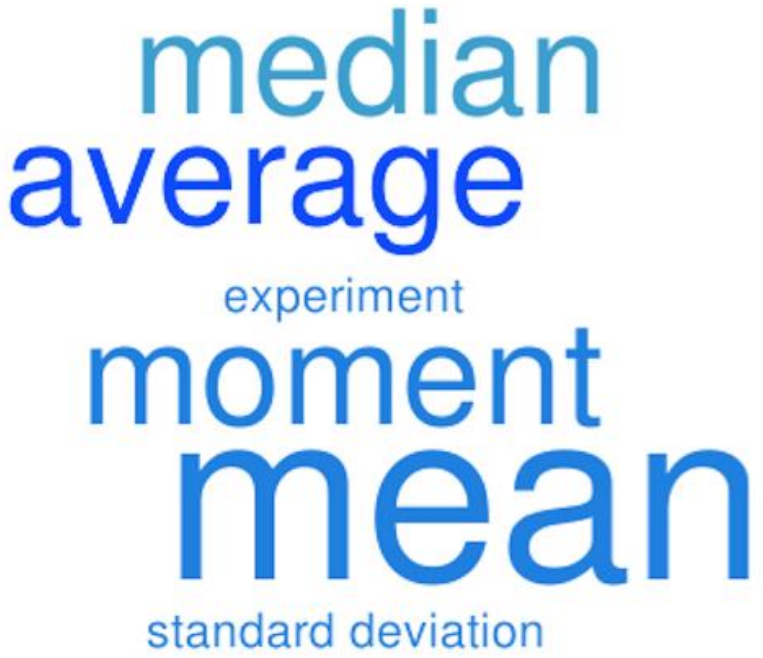
We want to compute the probability of getting heads or tails when tossing a coin:

- The number of *possible events* is 1
- The number of *possible outcomes* is 2 because we could either get heads or tails.

The probability of this event is:

$$P = \frac{1}{2} = 0.5$$

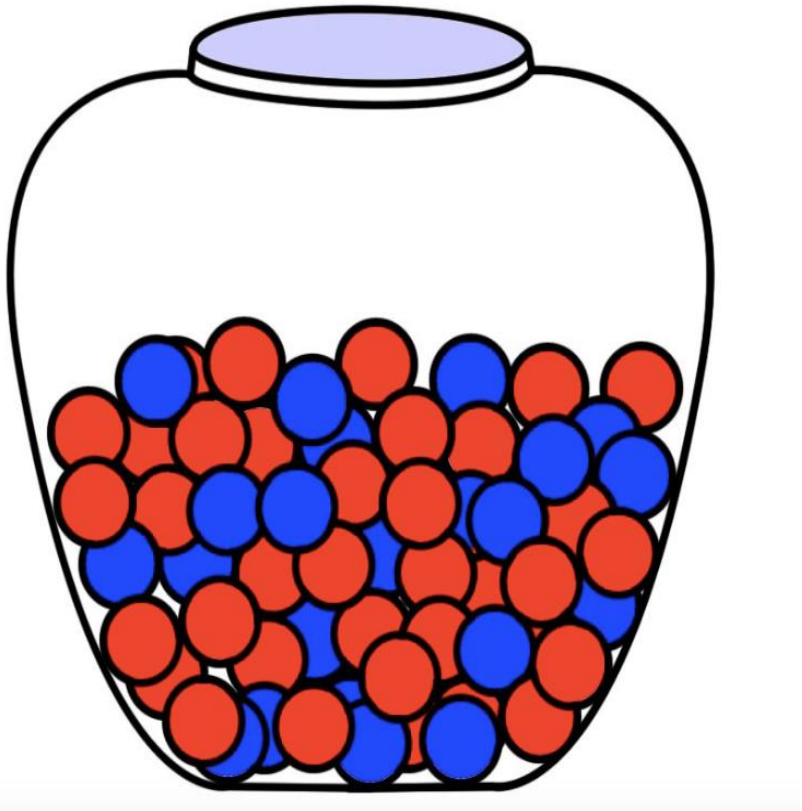
What is Statistics?



Statistics is a branch of **applied** Mathematics that focuses on studying the outcome of events that already happened, by analyzing the frequency of past events.

We **apply probability theory** to draw conclusions from data (events that have already happened).

Statistic Example: Blue and Red Marbles



We want to determine whether the chances of drawing a blue or red marble from a container are the same.

- A Statistician is asked to draw 100 marbles in total.
- He extracts 49 blue marbles and 51 red ones.

Therefore, he may conclude that the chances of extracting a blue or a red marble are the same.

Probability vs Statistics Example

Let's try to understand the difference between the two disciplines with an example:
How likely are you to get a 6 when rolling 1 die?

Probability

- Assumes the die is fair, as this is a theoretical problem.
- Determines that each face has a probability of 1/6.

Statistics

- Does not assume the die is fair or unfair.
- Observes the results for a while and keeps track of how often each number comes up.
- Determines if observations are consistent with the assumption of equal-probability faces.

In other words, **probability theory draws conclusions a priori** based on mathematical formulas, while **statistics draws conclusions a posteriori** based on real-life observation using the same rules as probability.

Probability vs Statistics Example

Decide whether a coin is fair or not

If we toss a coin, what is the likelihood we will get heads?

Number of Coin Tosses	Probability	Statistics
10	50%	40% to 60%
100	50%	45% to 55%

As we *increase* the number of coin tosses, the statistician may observe that the chances of getting an equal number of heads or tails are getting closer and may conclude that the coin is fair.

Summary

Probability

- Focuses on predicting the outcome of future events.
- Foundational for statistics.
- Expressed between 0 and 1.

Statistics

- Focuses on the outcomes of past events to draw conclusions.
- Uses the same rules as probability but applies them to past outcomes.