

CovidTo

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Predicting Covid Cases in Toronto Neighbourhoods: An Analysis using Linear Regression

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Executive Summary

This study used multiple linear regression to predict covid rates in Toronto's 140 neighborhoods using demographic data from Stats Canada. The correlation of these features to the target variable were analyzed in order to limit the number of potential predictors. Features which were highly correlated to each other were removed and an analysis of the p-values were done recursively to eliminate further predictors. Using the IQR method a few neighborhoods were removed that were considered outliers. The final model used only three variables to predict covid rates in the neighborhoods with an R^2 value of 0.605. Specifically these variables in the Stats Canada demographic database are known as Black (Visible Minority), Occupations in manufacturing and utilities, and Journey to work Between 12 p.m. and 4:59 a.m. The model was found to be $Rate = 1038 + 157.7 * Black\% + 140.8 * ManufactJob + 170.1 * WorkNights$. Such a model could be useful in order to determine where to target prescriptive or preemptive action.

Introduction

The purpose of this study is to build a linear regression model that will predict the number of covid-19 cases for any Toronto neighborhood based on its demographic data. Predicting covid-19 cases is useful in that an understanding of which socioeconomic factors that influence the growth of disease in a community will help with our understanding of the virus. This knowledge will enable resources to be better targeted in the future to help prevent transmission during this, or any future pandemic.

Load Libraries

The relevant data science libraries are loaded into the Jupyter notebook.

Data

A population Census is held across Canada every 5 years. It collects data about age and sex, families and households, language, immigration and internal migration, racial diversity, Aboriginal peoples, housing, education, income, and labor. The City of Toronto Neighborhood Profiles use this Census data to provide a portrait of the demographic, social and economic characteristics of the people and households in each City of Toronto neighborhood.^[1]

The data is made available through Toronto's Open Data portal (<https://open.toronto.ca/>).^[2]

Two datasets from this site were used for this study. One that shows the number of covid cases by Toronto neighborhood (<https://open.toronto.ca/dataset/covid-19-cases-in-toronto/>) and the other contains demographic features of each of these neighborhoods (<https://open.toronto.ca/dataset/neighbourhood-profiles/>). There are over 2,300 features shown for each neighborhood including:

- Aboriginal peoples
- Age and sex
- Education
- Families, households and marital status
- Housing

- Immigration and ethnocultural diversity
- Income
- Journey to work
- Labour
- Language
- Language of work
- Mobility and migration
- Population and dwelling counts
- Type of dwelling

Load Files

Pandas `read_csv` function is used to load the two files into dataframes.

Data Cleaning

The demographic data is a huge dataset containing over 2300 rows, each one representing a different demographic feature. In order to create one usable dataset the following tasks were performed:

1. The dataset was transposed so that the columns, which are the Toronto neighborhoods become rows and each feature becomes a column.
2. The columns which are presented as absolute numbers, for example *Number of Latin Americans*, were changed to represent percentages of each number compared to the neighborhood population.
3. Every column is given a unique identifying name.
4. A dataframe was created to capture the column descriptions which are quite lengthy.
5. Null rows were dropped - these were summary rows which did not relate to each neighborhood.
6. All the numeric columns were converted to floats - there were no categorical columns.
7. The two dataframes (demographic data and covid case data) were merged on the neighborhood id column to create one dataframe.

Now that the data is transposed there are over 2300 columns. Let's keep track of the meanings of these columns so we can later use this information to interpret the results. In the meantime we will refer to the columns by their numeric names. A sample from this table is shown below.

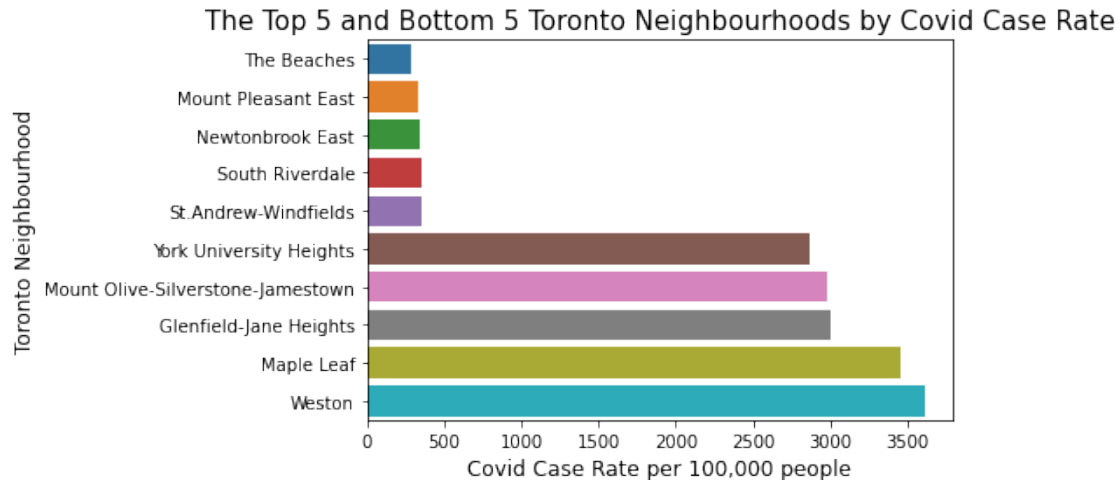
level_0	index	_id	Category	Topic
0	Col_0	1	Neighbourhood Information	Neighbourhood Information
1	Col_1	1	Neighbourhood Information	Neighbourhood Information
2	Col_2	3	Population	Population and dwellings
3	Col_3	4	Population	Population and dwellings
4	Col_4	6	Population	Population and dwellings

We can confirm that all the data has been converted to floats. We need to join the datasets so we have one dataset with all the features and target.

Data Exploration

There's a huge difference in the rate of covid-19 cases in each Toronto neighborhood.

The top 5 neighborhoods have **dramatically higher case rates** than the bottom five neighborhoods.



The rate in the neighborhood of Weston has a covid case rate of almost 13 times that of the Beaches neighborhood.

Neighbourhood	Rate per 100,000 people
The Beaches	282.839523
Weston	3612.716763

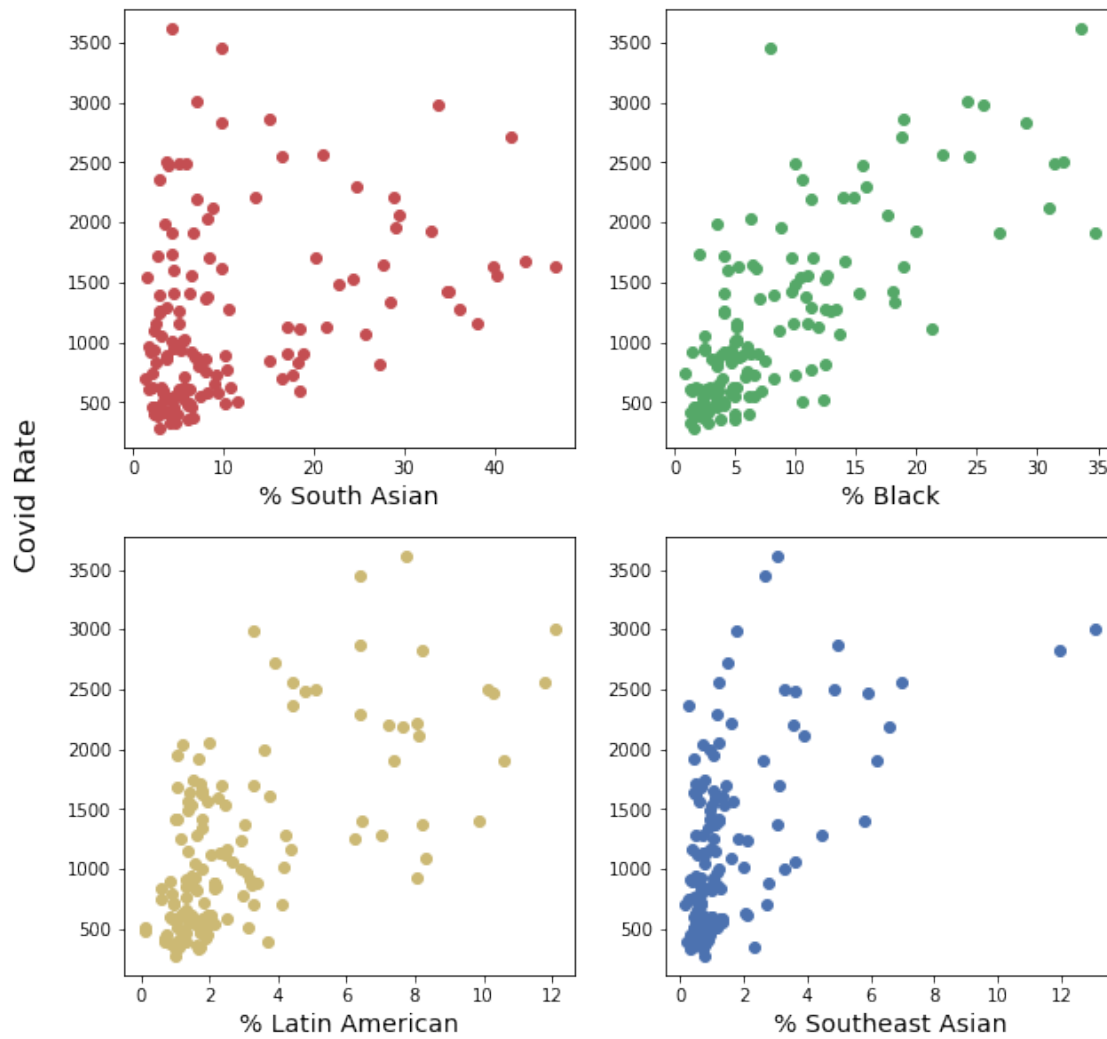
Various media reports have shown that certain demographic factors can influence covid rates^[3]. Reports have shown that the virus has disproportionately affected communities based on the following factors:

- Racial Profile
- Density
- Employment
- Income

Let's examine some columns that correspond to these factors and see how they relate to covid rates.

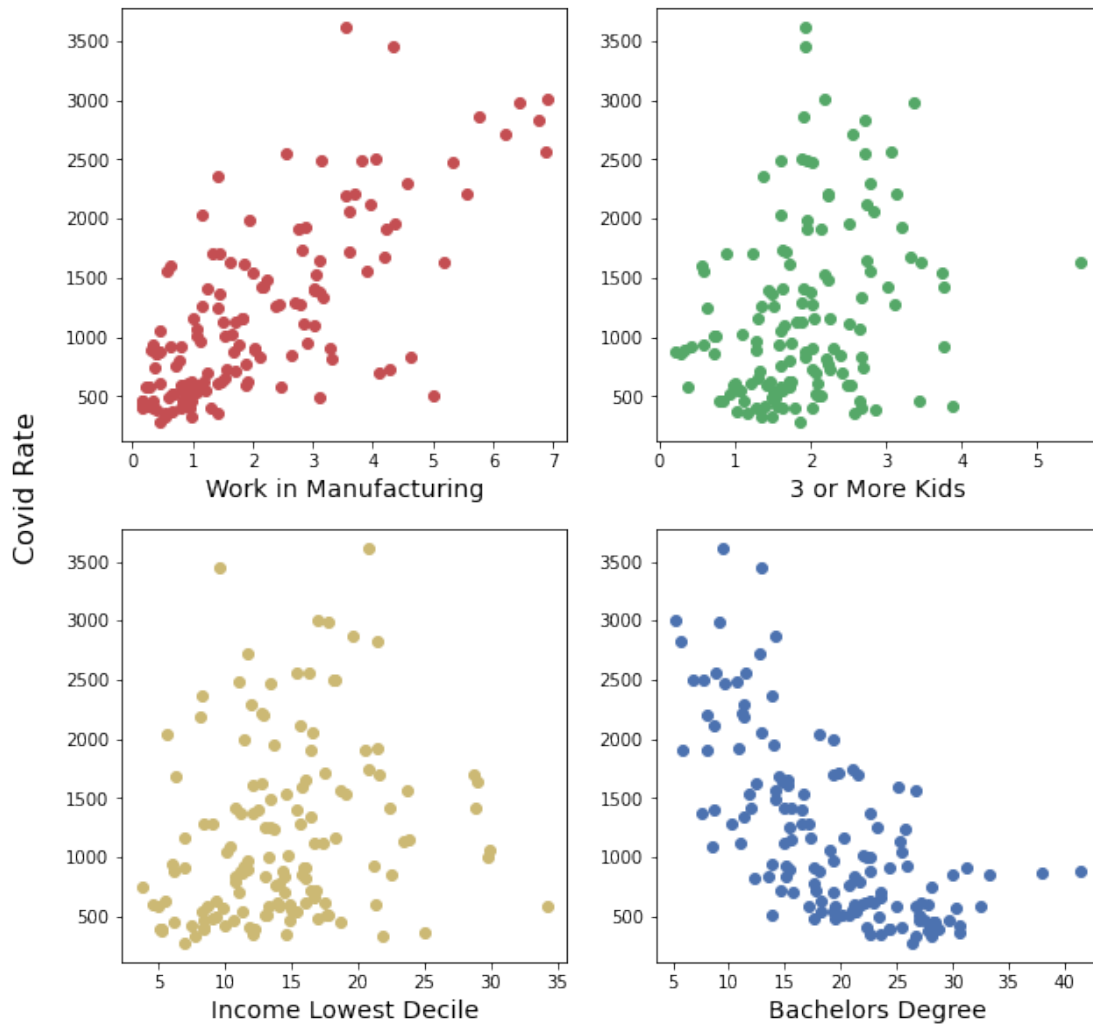
The following graph shows how covid rates vary depending on the proportion of different racial groups in each Toronto neighborhood. There seems to be a moderate positive relationship between covid rates and the percent of Blacks in each neighborhood and also Latin Americans.

Covid Rates of Toronto Neighborhoods vs Percent of certain Races



Now lets check some other potential predictors, including income, density and employment.

Covid Rates of Toronto Neighborhoods vs Percent of Demographic Factors



There seems to be a positive trend between the proportion of the population that work in manufacturing jobs in a neighborhood vs. covid rates, and a negative trend between the proportion who have Bachelor's Degrees.

Feature Selection

There are so many features (over 2,300) so in order to narrow it down we will examine the correlation of every feature to the target (Covid rate per 100,000 people). One of the most common ways to identify potential features is to examine the correlation between columns of X and y (Corr)^[4].

We will select the top 10 features based on this correlation and use this as our new pared down dataset.

The top 10 features in order of descending correlation to "Rate per 100,000" column. The top features are all seemingly related to Black, Caribbean and African origins except for a Labor

related column which is Occupations in manufacturing and utilities.

index	Rate per 100,000 people	Category	Characteristic
Rate per 100,000 people	1.000000	NaN	NaN
Col_1269	0.744447	Visible minority	Black
Col_1105	0.741047	Immigration and citizenship	Jamaica
Col_1099	0.737303	Immigration and citizenship	Americas
Col_1377	0.731488	Ethnic origin	Jamaican
Col_329	0.727007	Language	Niger-Congo languages
Col_1855	0.726108	Labour	9 Occupations in manufacturing and utilities
Col_1135	0.724949	Immigration and citizenship	Nigeria
Col_1414	0.713611	Ethnic origin	Central and West African origins
Col_1329	0.712460	Ethnic origin	Caribbean origins

Let's look at the top correlation in each Category of predictors. The category with the top correlated predictor is "Visible Minority" (**0.75**) and "Housing" is the category with the lowest correlated predictor (**0.63**).

It seems logical that predictors within the same category would be correlated, so let's find the top correlated predictor within each category.

index	Rate per 100,000 people	Category	Characteristic
Col_1269	0.744447	Visible minority	Black
Col_1105	0.741047	Immigration and citizenship	Jamaica
Col_1377	0.731488	Ethnic origin	Jamaican
Col_329	0.727007	Language	Niger-Congo languages
Col_1855	0.726108	Labour	9 Occupations in manufacturing and utilities
Col_105	0.690762	Families, households and marital status	3 or more children
Col_1049	0.684972	Income	In the third decile
Col_1635	0.677412	Education	Bachelor's degree
Col_1907	0.666669	Journey to work	Between 12 p.m. and 4:59 a.m.
Col_1594	0.629968	Housing	2 household maintainers

Statistics and Distributions

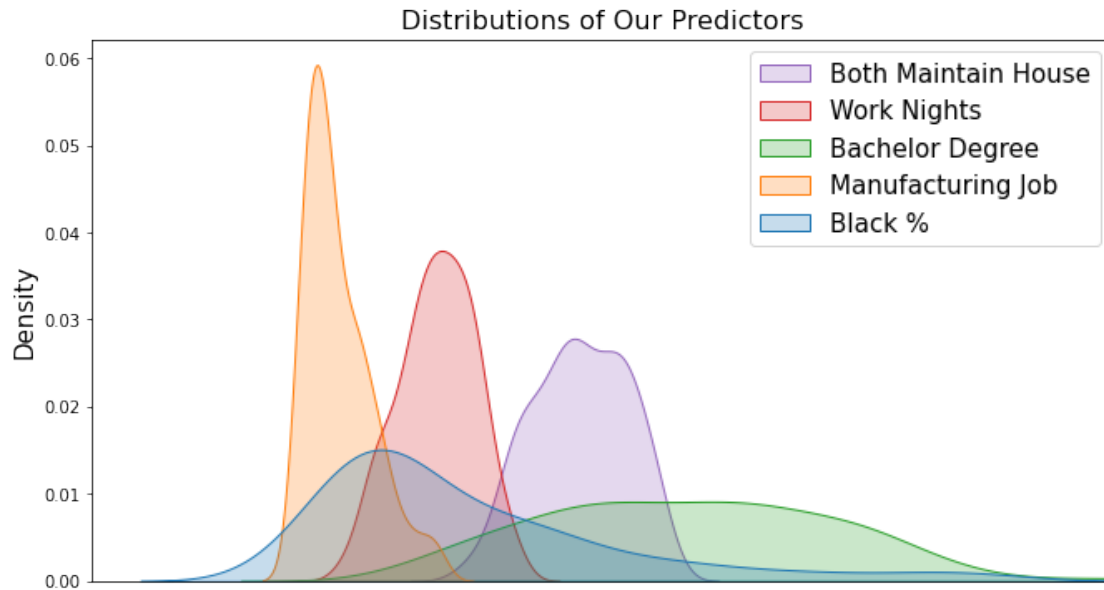
Let's examine the main statistics, such as mean and median, of our dataset. We can see these stats using the describe() function.

The statistics don't look out of the usual. We can see that the percent of Black people in each neighborhood vary from less than 1% to almost 35%. The mean number of people with Bachelor's degrees is 19% and one neighborhood has more than 41% of it's residents having the degree.

No values appear especially unusual and nothing appears completely out of the ordinary here.

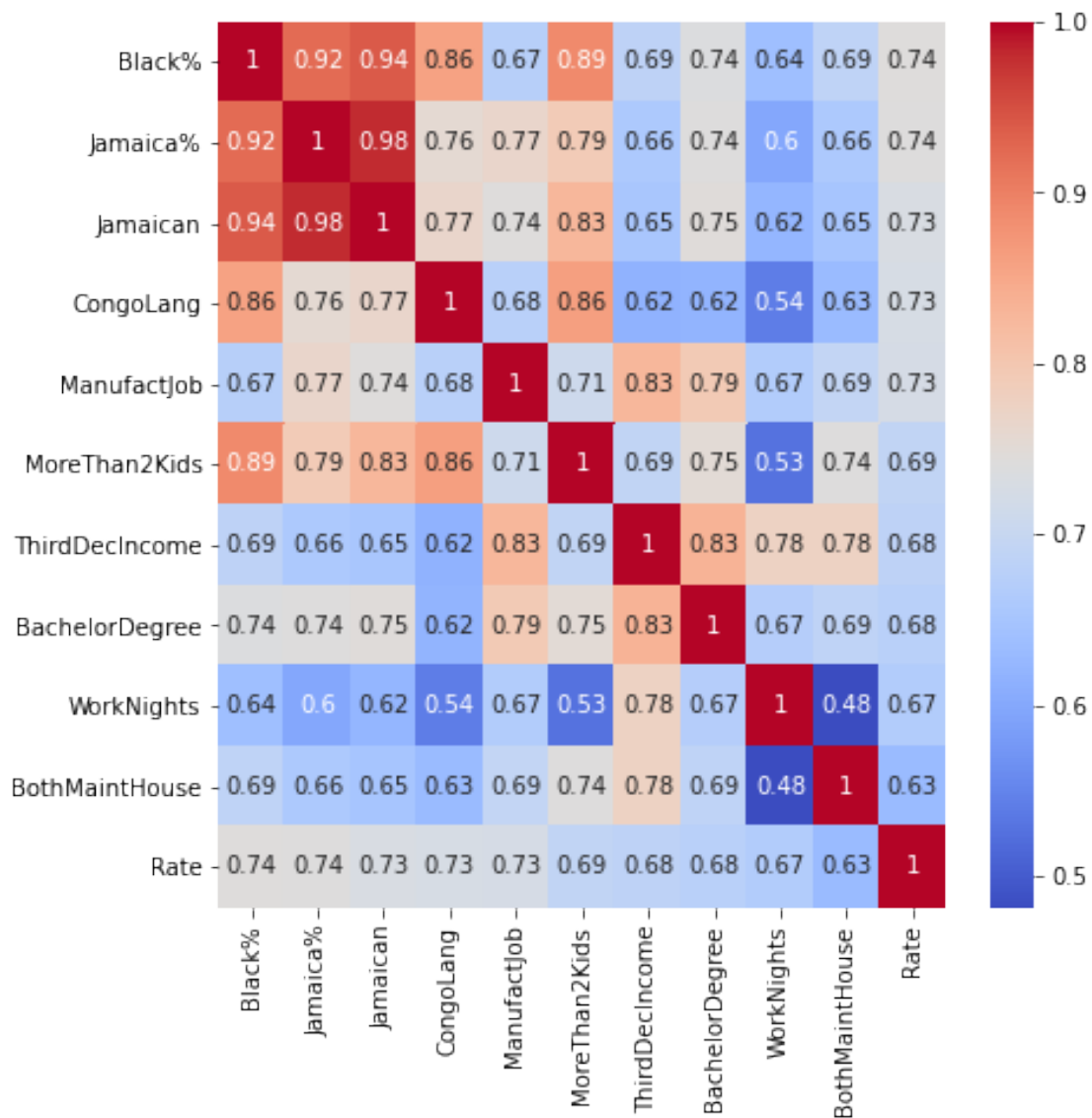
	Black%	ManufactJob	BachelorDegree	WorkNights	BothMaintHouse	Rate
count	140.000000	140.000000	140.000000	140.000000	140.000000	140.000000
mean	8.634214	2.084357	19.209571	6.882000	13.895786	1181.327621
std	7.611374	1.615108	7.164430	1.867379	2.337760	740.678441
min	0.860000	0.160000	5.280000	2.210000	8.210000	282.839523
25%	3.492500	0.855000	13.935000	5.647500	12.162500	589.051786
50%	5.650000	1.590000	19.285000	6.980000	13.945000	933.667577
75%	11.322500	3.032500	24.385000	8.217500	15.797500	1604.924084
max	34.760000	6.890000	41.390000	10.820000	18.090000	3612.716763

Let's look at the distributions of our predictors. We can see from the graph below that they are all fairly normally distributed and unimodal. The *Black %* feature is more skewed to the right than the other features. It's good that they are normally distributed as this is a prerequisite for building a linear regression model.



Remove Correlated Predictors

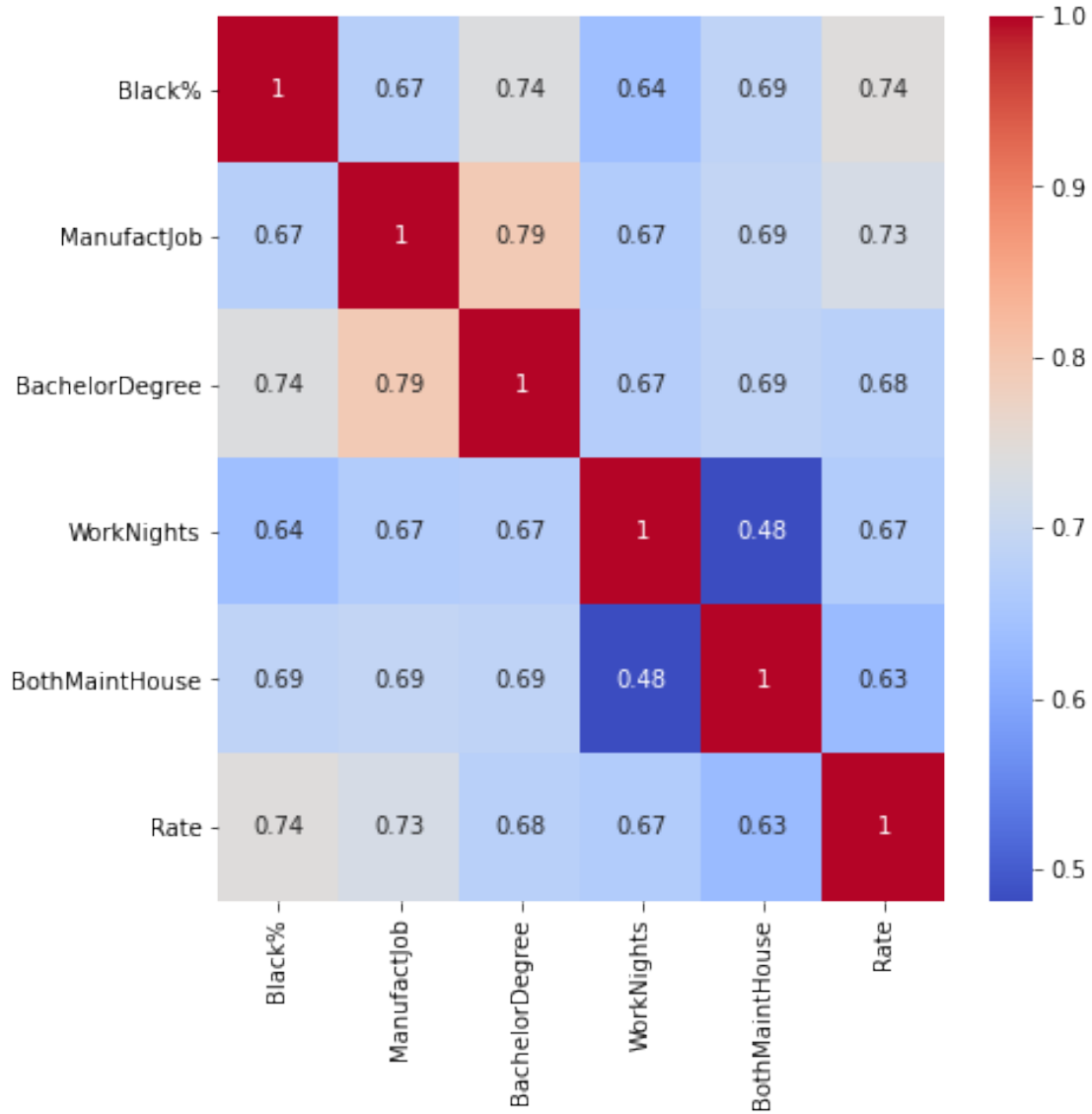
If certain features are highly correlated with other ones then there are redundant features and they can be removed to make the model more simple. The following heatmap shows that there are several highly correlated features, for example *Black* with *Jamaica%*, *Jamaican*, *CongoLang* and *MoreThan2Kids*.



It makes sense than language, ethnic origin, language and visible minority categories of data are highly correlated. Let's remove the *Jamaica%*, *Jamaican*, and *CongoLang* predictors.

In the interest of simplicity we can reduce features further. Black and Morethan2Kids is highly correlated, as well as ManufactJob and ThirdDecIncome so we will reduce the features that are least correlated with Rate.

Let's look at the heatmap of feature correlations. We can now see that we have all the correlations are under **0.80**. Let's use these remaining features to build a model.



Method & Results

Linear Regression

Linear regression is a linear way of modeling the relationship between a dependent variable (target) and one or more independent variables (features). If there one feature it is called simple linear regression, and if there is more than one it is called multiple linear regression.^[5]

The equation for multiple linear regression is as follows:

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \epsilon$$

where, for i=n observations:

y_i = dependent variable

x_i = explanatory variables

β_0 = y-intercept (constant term)

β_p = slope coefficients for each explanatory variable

ϵ = the model's error term (also known as the residuals)

Simple Linear Regression

Let's start with a simple linear regression model. We will use only one feature to predict the one target (Covid Rate). We will use the column which is most highly correlated to Covid Rate which is the percent of Black people in each neighborhood.

[628]: `LinearRegression()`

Here's the coefficient of the model:

[812]: 535.7894650023125

Here is the intercept:

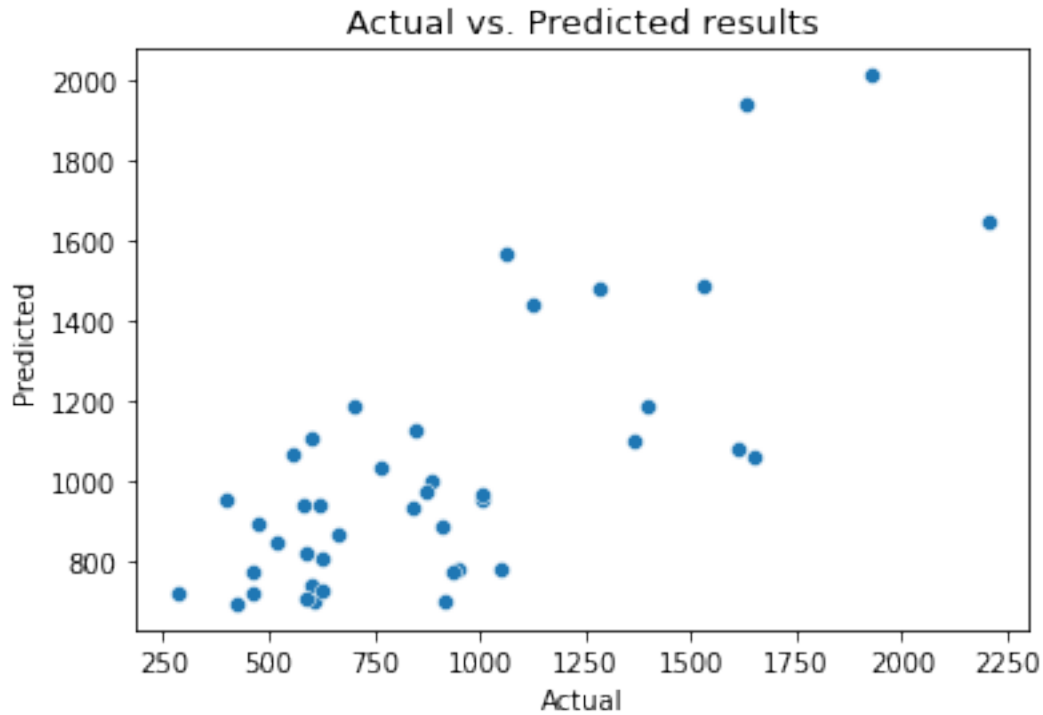
[813]: 1217.1073039109747

This is the equation of our current model:

`Rate = 1217.11 + 535.79*Black%`

Let's compare the actual vs. predicted values. They don't appear to be that close.

Actual	Predicted
821.054175	1615.920010
945.744151	1085.344638
411.211816	791.169031
916.681012	493.833270
1602.536636	898.410806
875.561257	917.066319
838.113558	1191.260502
910.879545	1163.272389
372.764405	464.154978
513.462743	929.264058



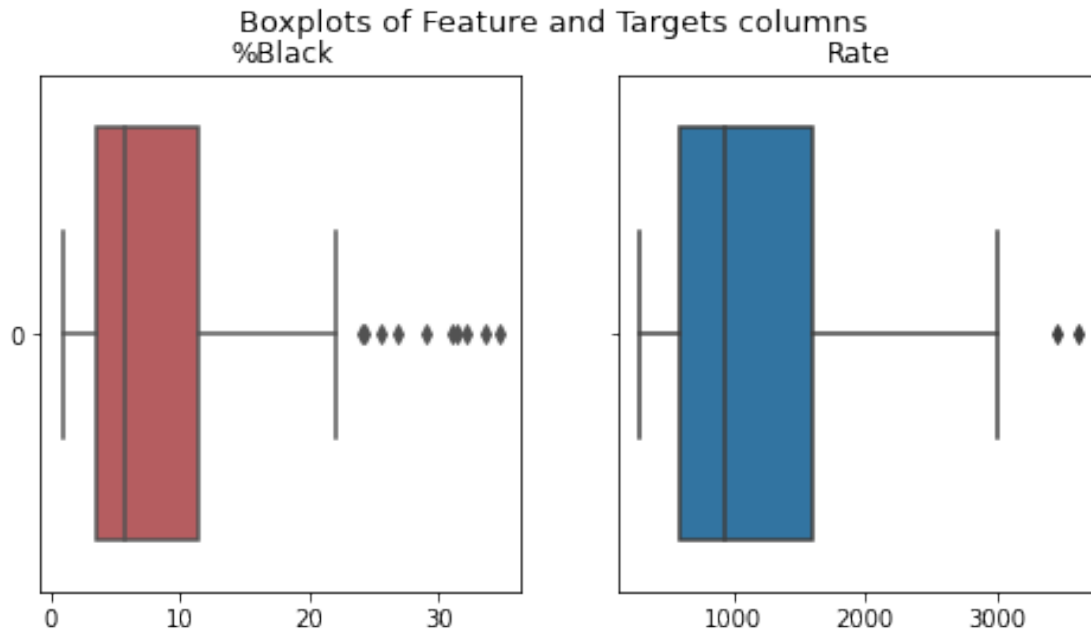
Let's look at the results of the model. The R^2 value is **0.49**. This is quite good for a social study such as this one.

The R-squared value is: 49.09
 The Root MSE is: 313.20868156422546
 The Intercept is: 1217.1073039109747

Outliers

In order to improve the model we could use more training data, however as we only have 140 neighborhoods we do not have that much data to begin with. We can also look at outliers.

The following boxplots show the outliers for the X (%Black) and y (Rate) columns.



The figure above shows us that there are outliers in both the feature and target columns. Let's remove the outliers and recreate the model and see if there's an improvement.

This code will remove all the rows that are outliers according to IQR. We call shape to determine that 11 rows have been removed.

We have removed 11 outliers.

Let's run the model again to see if there's an improvement.

The R-squared value is: 36.37
 The Root MSE is: 538.6686246770221
 The Intercept is: 1008.2957708286475

Let's look at the results of the new model. The R^2 value is now 0.36.

So removing the outliers actually made things worse.

Multiple Linear Regression

Now let's see if we can improve the model by using more than one feature. We will use the five features we discussed earlier: %Black, ManufactJob, Bachelor Degree, WorkNights and BothMaint-House.

Here's the coefficients of the model:

Parameter	Coefficient
Black%	243.434794
ManufactJob	268.551318
BachelorDegree	19.125431
WorkNights	175.770549
BothMaintHouse	-47.114529

Here is the intercept:

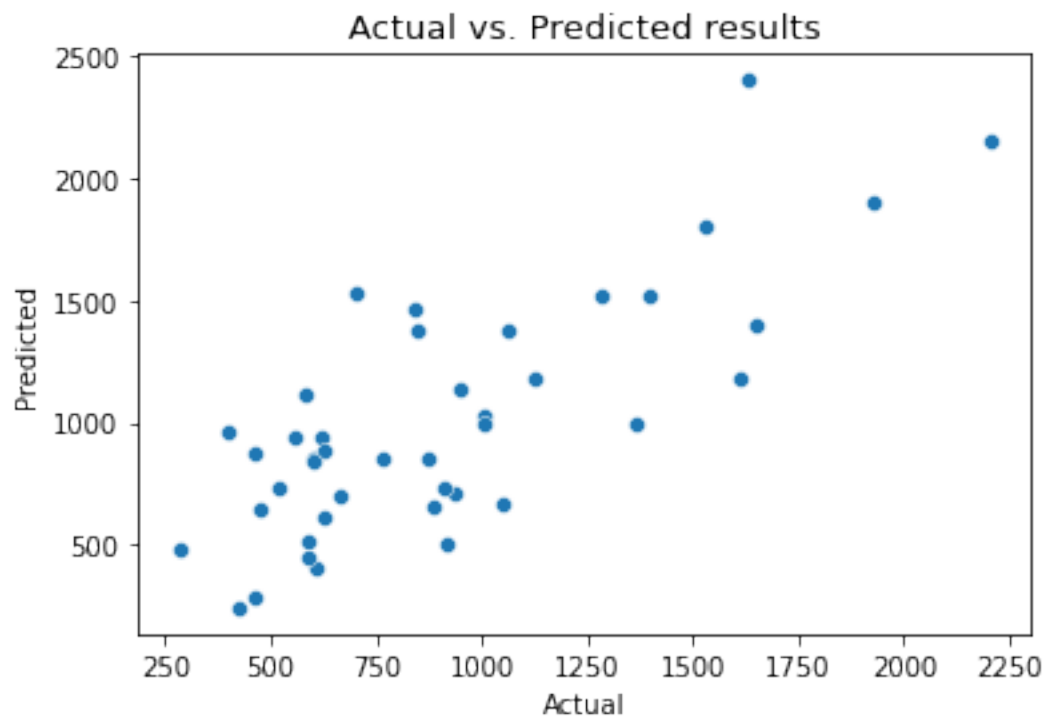
[643]: 1212.2741827886584

This is our new equation.

$$Rate = 1212.23 + 243.43 * Black\% + 268.55 * ManufactJob + 19.13 * BachelorDegree + 175.77 * WorkNights + -47.11 * BothMaintHouse$$

Let's compare the actual vs. predicted values. They don't appear to be that bad.

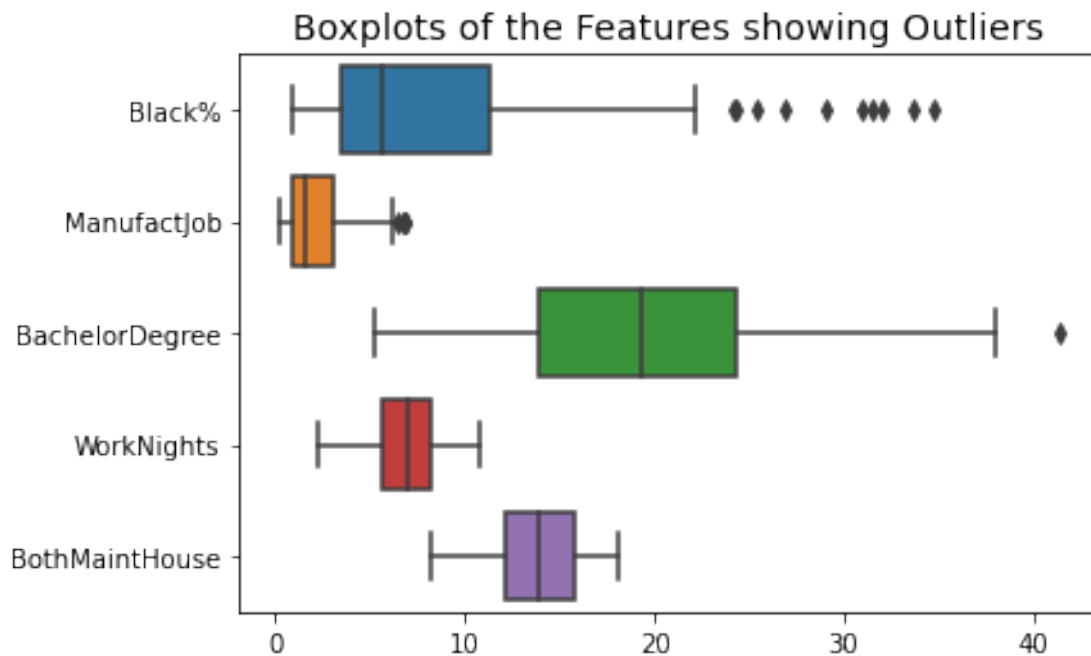
Actual	Predicted
1050.743209	739.361354
282.839523	576.095576
554.900465	775.974512
1064.519115	1333.753145
1280.614695	1739.366572
477.299185	634.470852
461.163450	391.203223



Let's look at the results of the model. The R^2 value is **0.43**. It's lower than simply using one predictor.

The R-squared value is: 43.47
The Root MSE is: 330.03675517892543
The Intercept is: 1212.2741827886584

We can try to remove outliers in each column but I suspect it won't help the accuracy of the model. We have already seen that Rate has outliers, the figure below also shows there are a couple outliers in BachelorDegree and ManufactJob.

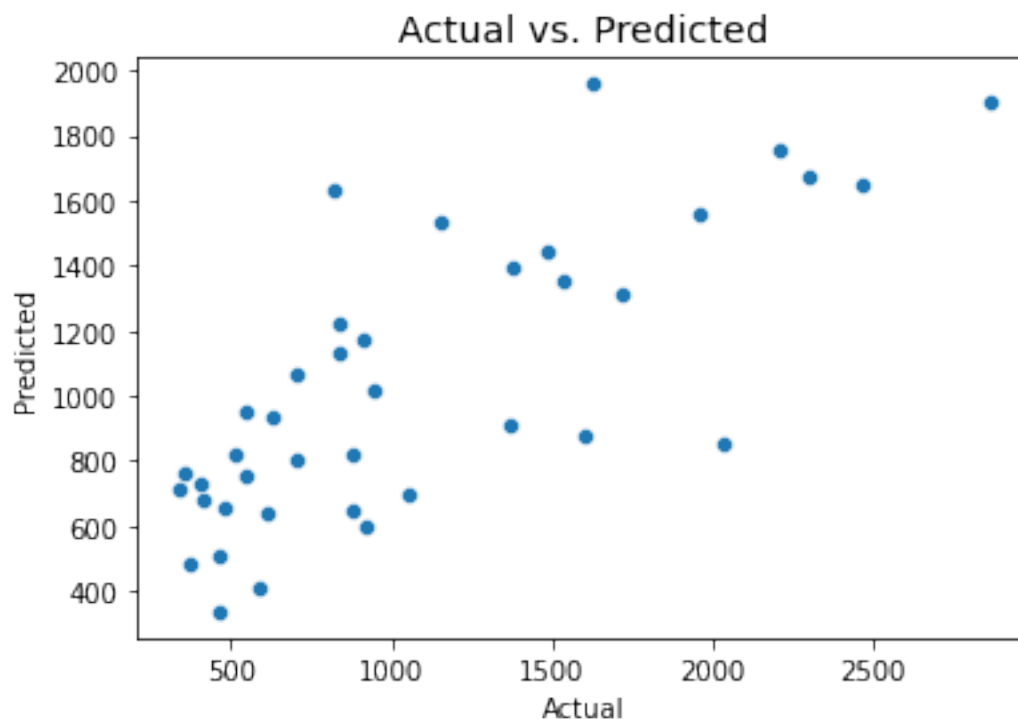


The R-squared value is: 56.82
The Root MSE is: 432.34644674183556
The Intercept is: 1031.6761147565676

These are very good results as the R^2 value has improved to almost **57%**. This is considered very good for a social science study.

We removed 13 outliers from the dataframe

Let's look at the new Predicted vs Actual plot. It's not bad, although it seems to be less accurate for the mid level values.



Recursive Feature Elimination (RFE)

Let's try a technique called recursive feature elimination (RFE) and see how it compares to our current model. RFE is part of python's sklearn package.

RFE is described as follows:

Given an external estimator that assigns weights to features (e.g., the coefficients of a linear model), the goal of recursive feature elimination (RFE) is to select features by recursively considering smaller and smaller sets of features.

Now let's check what the RFE recommended columns are.

Category	Topic	Characteristic
Language	Mother tongue	Akan (Twi)
Language	Mother tongue	Edo
Income	Income of individuals in 2015	\$20,000 to \$29,999
Income	Income of individuals in 2015	\$20,000 to \$29,999
Immigration and citizenship	Recent immigrants by selected place of birth	Other places of birth in Americas
Immigration and citizenship	Recent immigrants by selected place of birth	Nigeria
Education	Highest certificate, diploma or degree	Certificate of Apprenticeship or Certifi...

This is interesting. Three have one factor that has to do with Education, two with languages, two with income and two with immigration.

Let's check the R^2 when we use these columns.

[656]: 44.81

The R^2 value of this model is almost **45%**. That's pretty good.

The following table shows the correlation between columns. A couple features are **highly correlated** so let's remove those, and remove outliers and try again.

Col_1170	Col_1190	Col_332	Col_1631	Col_958	Rate per 100,000 people
1.000000	0.662406	0.784227	0.611573	0.541691	0.704420
0.662406	1.000000	0.761809	0.334436	0.342120	0.684025
0.784227	0.761809	1.000000	0.446778	0.419054	0.657404
0.611573	0.334436	0.446778	1.000000	0.644684	0.630197
0.541691	0.342120	0.419054	0.644684	1.000000	0.618513
0.704420	0.684025	0.657404	0.630197	0.618513	1.000000

28 columns with outliers were removed.

[661]: 2.93

The R^2 has now dropped significantly to 2.93.

P-value for Feature Reduction

One way to select predictor variables is using the p-value.

P-value gives us the probability of finding an observation assuming that a particular hypothesis is true. It's this probability that we use to decide whether to reject H_o , the Null hypothesis. We can use this to determine which features we can remove from the dataset.

As we remove features this effects the p-values for each feature in the dataset. One by one we will remove a feature with a high value (non significant $p > 0.5$) and recalculate the p-values.

The following table shows the p-values for each predictor.

0	1	2	3	4	5	6
	coef	std err	t	P> t	[0.025	0.975]
Black%	34.9662	11.486	3.044	0.003	12.228	57.704
ManufactJob	132.2317	38.480	3.436	0.001	56.057	208.406
BachelorDegree	7.6066	8.356	0.910	0.364	-8.934	24.147
WorkNights	95.0617	28.633	3.320	0.001	38.379	151.744
BothMaintHouse	-16.7414	16.746	-1.000	0.319	-49.892	16.409

The p-values indicate that there are potentially two features that do not contribute to the model. Let's remove one at a time and check the results. Here's the values after one predictor is removed.

0	1	2	3	4	5	6
	coef	std err	t	P> t	[0.025	0.975]
Black%	33.9600	11.425	2.972	0.004	11.345	56.575
ManufactJob	122.1344	36.821	3.317	0.001	49.249	195.019
WorkNights	94.1754	28.597	3.293	0.001	37.570	150.781
BothMaintHouse	-3.8738	8.974	-0.432	0.667	-21.637	13.890

Now there's only one feature left with a high p-value, let's remove it. Now we check the p-values and see that there are no insignificant values.

0	1	2	3	4	5	6
	coef	std err	t	P> t	[0.025	0.975]
Black%	36.0324	10.333	3.487	0.001	15.580	56.484
ManufactJob	126.8286	35.063	3.617	0.000	57.429	196.228
WorkNights	83.0461	12.330	6.735	0.000	58.641	107.452

Now all our features are significant. Let's check the R^2 value and the MSE of the new model.

The R-squared value is: 60.52
The Root MSE is: 413.42407505558594
The Intercept is: 1038.1824194302878

The new coefficients are:

	Coefficient
Black%	157.692253
ManufactJob	140.833170
WorkNights	170.074059

Great news, we have simplified the model and increased the R^2 value, in terms of a percent, to **over 60%**.

This could be considered a very good R^2 value for a social sciences type of study as this one.^[6]

Our final model is:

$$Rate = 1038 + 157.7 * Black\% + 140.8 * ManufactJob + 170.1 * WorkNights$$

Discussion

This study used multiple linear regression to predict covid rates in Toronto's 140 neighborhoods using demographic data from Stats Canada. The data initially contained over 2,300 potential predictors. The correlation of these features to the target variable were analyzed in order to limit the number of potential predictors. Features which were highly correlated to each other were removed and an analysis of the p-values were done recursively to eliminate further predictors. Using the IQR method a few neighborhoods were removed that were considered outliers. The final model used only three variables to predict covid rates in the neighborhoods with an R^2 value of 0.605.

Specifically these variables in the Stat Canada demographic database are known as *Black (Visible Minority)*, *Occupations in manufacturing and utilities*, and *Journey to work Between 12 p.m. and 4:59 a.m.*. It has been suggested that certain racialized communities are affected by covid for various reasons including a higher proportion with underlying health conditions such as hypertension, the increased likelihood of living in more densely populated neighborhoods, and the lower proportion that can work from home.^[3]

Further research is warranted and could include a larger analysis of neighborhoods across Canada, or in other countries. A linear model was appropriate for this analysis, as the predictors used were

not highly correlated to each other, they had a linear relationship with the target variable, and they were all fairly normally distributed. A model such as the one built for this study could help to target resources where they are most needed, or to preemptively determine where the most severe outbreaks will occur in order to take preventative action.

References

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Appendix

I noticed that the graph of Bachelor Degree vs. Rate appears to follow a 2nd degree polynomial. Linear regression assumes the relationship between the dependent and independent variables are linear, but sometimes the relationship is more complex. We need to generate a higher level equation. In this case the equation would be:

$$Y = \beta_0 + \beta_1 x + \beta_2 x^2$$

This is still a linear model however we are now fitting a **quadratic** equation.

We must now use the PolynomialFeatures class provided by sklearn. We then train the model using Linear Regression.

Let's check what the R^2 value would be if we just assume a linear relationship. It's **-22.51** which is not that good. It shows a moderately weak negative relationship between the independent and dependent variables.

[934]: -22.51

Now let's fit the pattern to a 2nd degree polynomial and check the R^2 value again.

498.5250780689128
0.5437237661821185

We can see that the R^2 value as a percent is now **54.4**. That is a large improvement.

Let's visualize this relationship with a best fitting line.

[936]: Text(0.5, 0, 'Percent with Bachelor Degrees')

