A Cascade-Resilient Liquidation Architecture for CDP-Backed Stablecoins

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Abstract

Liquidation engines constitute the primary line of defence for over-collateralised stable-coins. We combine square-root market-impact theory, central-bank stress tests, and reliability-engineering cascade models to design an institution-grade safety net that (i) suppresses fire-sale feedback, (ii) bounds bad debt at the 99.9% VaR level, and (iii) remains fully on-chain auditable. An indicative backtest of the ETH flash crash on 11 December 2024 shows an 84.6% reduction in bad debt versus a 5% fixed-spread liquidator while cutting peak price impact by 10.4 percentage points.

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1 From Specification to Formal Upgrade

The StableUnit protocol (StableUnit DAO 2025) provides a heuristic liquidation mechanism. This paper formalizes its design and introduces risk-aware, cascade-resilient upgrades. The table below presents a direct mapping between specification phrases and improvements introduced here.

Table 1: Design responses to fragile elements in the StableUnit specification.

StableUnit Specification	Improved Design (This Work)
Liquidations are done through market sales.	§4 - §5: replaced with liquidity-aware tranching and sealed-bid auctions to reduce price impact.
Price has a discount that gets bigger per block.	§7: replaced by fixed 99.9% VaR haircut (1 – δ) to avoid spiral discounting.
Liquidator has 60 seconds to sell the collateral.	§4: 60-second window formalised using a rotating 21-slot queue with congestion-aware timing and slashing.
Trader bot monitors is Liquidateble().	§6: architecture explicitly splits into <i>Trigger</i> bot and <i>Fill bot</i> , clarifying role separation.
Whitelist of stablecoins to exchange the collateral.	§7: wrapped in zk-attested three-oracle median to defend against price feed manipulation.
Risks cascade liquidation chain reaction.	§3: introduces $\kappa \sigma$ stress detection and GARCH-based volatility triggers.
Bot uses AAVEs Flashloans.	§4: tranche size Q_i adapts to Depth _{1%} , mitigating flash-loan risk if liquidity vanishes.
Caller receives 0.1% of the collateral.	§5-§9: replaced by 2nd-price auction with 1% refundable bond and a diminishing-marginal bounty function, reducing protocol costs and limiting whale rewards.
No other liquidator has the right to buy 60 seconds.	$\S5$ + Code listing: commit-reveal removes gas-sniping risk even after exclusivity window.
Protocol is in no rush to exchange for USD Pro.	§8: formalised buffer mechanism swaps at reserve price and burns USD Pro post-conversion.

2 System Overview

The liquidation engine is upgraded from a monolithic, discount-driven process into a modular architecture that combines stress detection, depth-aware liquidation, and auction-theoretic efficiency. At its core, the protocol must decide within a single EVM call whether to:

- (a) execute a spot sale via tranche slicing with tight slippage control, or
- (b) escalate into a capital-preserving batch auction under cascade stress.

Each upgrade responds directly to heuristic or fragile elements in the original StableUnit specification (see Table 1) and is anchored in risk metrics like volatility, depth, and Value-at-Risk.

Core Components:

- 1. Stress Regime Detector (§3): monitors realized volatility and spot/TWAP deviation to trigger auction mode only during cascading conditions.
- 2. Liquidity-Adaptive Tranching (§4): converts collateral into execution-sized slices based on real-time AMM depth and volatility, reducing slippage and flash-loan exposure.
- 3. Sealed-Bid Vickrey Auction (§5): replaces priority gas wars with sealed-bid commitment, bid-bond deterrence, and second-price clearingeliminating MEV and griefing.
- **4.** TriggerFill Separation & Keeper Queue (§6): formalizes the bot logic from the original spec as a ve-style prioritized 21-slot keeper ring with congestion-aware timing.
- 5. Dynamic Keeper Bounties (§9): replaces fixed 0.1% liquidator fees with a diminishing-marginal payout curve, aligning incentives across vault sizes while capping micro-vault APRs.
- **6. VaR-Driven Reserve Price** (§7): implements a median-based oracle mechanism with 99.9% Value-at-Risk haircut, protecting against fire-sale spirals and oracle deviations.
- 7. Contagion Heat-Map & Cooldowns ($\S10$): broadcasts real-time liquidation pressure using k-shell decomposition; peers may initiate self-pauses based on outer-shell exposure.
- **8.** Buyback Pipeline (§8): buffers filled stablecoins, converts to USD Pro only at fair market conditions, and burns USD Pro to close debtimproving capital efficiency and transparency.
- 9. Governance & Param Control (§11): central parameters $(\kappa, Y, \psi, \delta)$ reside in a timelocked config module; supporting notebooks are pinned to Arweave and changes are publicly emitted via ParamChange(hash) events.

Together, these components offer a cascade-resilient, auction-driven liquidation protocol that meets the demands of volatile market conditions without sacrificing decentralization or composability.

3 Stress-regime detection

3.1 Deterministic core

Stress is declared if

stress =
$$(|P_{\text{oracle}} - \text{TWAP}_{24h}| > \kappa \hat{\sigma}_t) \lor (\hat{\sigma}_t > Y), \qquad \kappa = 1.65.$$
 (1)

Here $\hat{\sigma}_t$ denotes the empirical 24-hour realized volatility (Andersen et al. 2003), computed as

$$\widehat{\sigma}_t = \sqrt{\frac{1}{n} \sum_{i=1}^n r_{t-i}^2}, \qquad r_t = \ln\left(\frac{P_t}{P_{t-1}}\right), \tag{2}$$

where r_t is the log return and P_t is the mid-price at time t.

(Tian and Zhu 2025) show $\kappa = 1.65$ minimises false positives on ETH/USD (Figure 6).

3.2 Adaptive forecast (optional)

Replace $\sigma_{\rm real}$ by a GARCH(1,1) forecast,

$$\sigma_{t+1}^2 = \omega + \alpha \epsilon_t^2 + \beta \sigma_t^2,$$

pushed via oracle every five minutes. Appendix C of Tian and Zhu reports a $30\,\%$ reduction in false negatives.

4 Liquidity-Adaptive Tranching

To reduce slippage, flash-loan exposure, and MEV vulnerability, the protocol divides liquidation inventory into dynamic tranches rather than executing full spot sales in a single transaction. Each tranche size Q_i adapts to both real-time liquidity and market volatility.

The tranche formula is:

$$Q_i = \min \left[Q_{\text{rem}}, \ \psi \text{ Depth}_{1\%} e^{-\gamma \widehat{\sigma}_t} \right], \qquad \psi = 0.12, \quad \gamma = 2.1.$$
 (3)

Here, $\hat{\sigma}_t$ denotes as in §3 the empirical 24-hour realized volatility at time t, and Depth_{1%} refers to the executable on-chain liquidity within a $\pm 1\%$ band around the mid-price P_t . Formally:

$$\mathrm{Depth}_{1\%} := \min \left(\int_{P_t \cdot 0.99}^{P_t \cdot 1.01} \mathrm{SellQty}(p) \, dp, \, \int_{P_t \cdot 0.99}^{P_t \cdot 1.01} \mathrm{BuyQty}(p) \, dp \right),$$

where SellQty(p) and BuyQty(p) represent the quantity available at price level p from automated market makers (AMMs).¹

As shown in Almgren and Chriss (2000), execution cost scales with \sqrt{Q} , making tranche-based execution significantly more efficient. Quarter-slicing reduces expected slippage by nearly 50% in typical AMM conditions. Reinforcement learning (RL)-based dynamic adjustments (H. Zhang, Chen, and Yang 2023) are theoretically compatible and may be proposed for deployement. This module also aligns with the cascade model taxonomy proposed in Zhao et al. (2025), supporting fault-tolerant execution under stress conditions.

5 Auction Mechanics and Keeper Incentives

Recent empirical work shows a clear shift in DeFi from fixed-spread mechanisms uch as those used by Aave and Compound (Aave Protocol Documentation 2025; Compound Protocol Documentation 2025) toward auction-based liquidations, exemplified by MakerDAOs tenddent auction (MakerDAO Documentation 2025). These auctions yield approximately 80% lower on-chain price impact and significantly reduce contagion risk by incentivizing liquidator competition. Chainlinks Smart Value Recapture (SVR) (chainlink-svr-2024) complements this trend by integrating oracle-driven pre-auctions to internalize MEV during liquidation events.

We formalize the liquidation mechanism as a sealed-bid, second-price auction (Vickrey 1961), in which truth-telling is a dominant strategy (Myerson 1994), to minimize winners curse and MEV risk. Bids are submitted by keeper bots off-chain and revealed on-chain after a delay; the protocol then selects the highest eligible bid and settles at the second-highest price. A 1% refundable bid bond deters griefing and MEV sniping (Tian and Zhu 2025).

¹On Uniswap V3, this corresponds to simulating a swap via the quoter.quoteExactInputSingle() method across the ±1% price range and summing executable ticks on both sides of the pool (Uniswap Protocol Documentation 2025).

Empirical simulations suggest this auction design reduces liquidation costs by 22 % compared to fixed-spread execution. To prevent excessive rent extraction, keeper reward is capped at:

$$fee_{max} = 0.5 \,\delta V$$
,

where δ is the reserve-price haircut (VaR) and V is the collateral value.

- Format sealed-bid, second-price (Vickrey- α).
- Bid bond 1% refundable.
- Reveal window k=10 blocks (120 s).

Let $\mathcal{B} = \{b_1, \dots, b_n\}$ be the set of valid bids. The clearing price is

$$P_{\text{clear}} = \max \left\{ b_j \in \mathcal{B} \mid b_j \ge P_{\text{reserve}}, \ b_j \le b_{(k)} \right\},$$

where $b_{(k)}$ is the k-th highest bid. This bounds the winner's curse risk.

Listing 1: Solidity pseudo-code: auction guard

6 TriggerFill Separation and Keeper Prioritisation

Inspired by StableUnits dual-bot architecture (StableUnit DAO 2025), we formalize the protocol's separation of duties:

- 1. Trigger bot monitors CDPs via isLiquidatablePosition and initiates liquidation.
- 2. Fill bot a keeper from a ranked staking queue, with a 60 s exclusive execution window.

Priority is assigned using SuDAO's vote-escrow (ve-style) staking model, where keeper eligibility increases with lock duration and stake amount. Each CDP is hashed to one of 21 keeper slots. Missed fills trigger a slashing penalty of 10% of current voting power.

To make this precise, we model keeper selection as:

TriggerBot
$$(t) \in \mathbb{B}$$
 monitors isLiquidatablePosition,
FillBot $_k(t) = \arg\max_{j \in [1,21]} \mathrm{VP}_j(t)$ (ve-priority).

Here, \mathbb{B} is the keeper bot set, and $\operatorname{VP}_j(t) = \operatorname{stake}_j \cdot f(T_j)$ denotes voting power from ve-style staking, where $f(T_j)$ increases with lock duration (e.g. $f(T_j) = T_j/T_{\max}$).

To reflect load-driven latency, expected fill time decays with active TVL:

$$\mathbb{E}[\tau_{\text{fill}}] = 60 \,\text{s} \cdot \exp\left(-\lambda \sum \text{TVL}_{\text{active}}\right).$$

This extends the exclusivity model of StableUnit DAO (2025) with congestion-aware timing.

7 Reserve price & oracle integrity

To enhance resilience against manipulation and outages, we recommend periodically revisiting the selected oracle ensemble (Deng et al. 2024). Let $\mathcal{O}_t = \{O_1, O_2, \dots, O_n\}$ be the approved oracle set at time t. Following Eskandari et al. (2021) we define the reserve price as the median oracle:

$$P_{\text{reserve}} = \underset{O \in \mathcal{O}_t}{\text{median}} [P_O] \cdot (1 - \delta),$$

where δ is a 99.9 % Value-at-Risk (VaR) haircut: majors $\delta = 2.3\%$, long-tail $\delta = 5.7\%$ according to (Tian and Zhu 2025, Fig. 10).

To formalise this:

$$\delta_{\text{VaR}} = \inf \left\{ d \in \mathbb{R}^+ : \Pr \left(L > d \cdot V \right) \le 0.001 \right\}$$

where L is the liquidation loss random variable, and V is collateral value.

The median operator is preferred for aggregation, as its deviation from the true price satisfies:

$$\Pr\left(|P_{\text{median}} - P_{\text{true}}| > \epsilon\right) \le 2\Phi\left(-\frac{\epsilon\sqrt{3}}{\sigma}\right),$$

where σ denotes the standard deviation across oracle sources (Eskandari et al. 2021). This assumes independent, symmetric noise and supports zk-proof attestations in secure oracle frameworks.

8 Buyback and Debt Repayment

After a successful liquidation, received stablecoins are temporarily buffered in the StableUnitBuyBack module. Unlike immediate conversion heuristics, this module executes conversion and repayment only when reserve-price conditions are satisfied.

- Accumulates whitelisted stablecoins from filled auctions;
- Repurchases USD Pro via AMM LPs or OTC, with near-zero slippage;
- Burns USD Pro to repay the corresponding CDP debt;
- Sends any surplus to a profit distribution contract.

This approach improves capital efficiency while reducing slippage. Future enhancements may include deploying protocol-owned liquidity (POL) on Balancer or Uniswap for deeper USD Promarkets.

9 Keeper Incentives and Bounty Curve

Recent protocols (Aave Protocol Documentation 2025; Llama Risk 2025a; Compound Protocol Documentation 2025; MakerDAO Documentation 2025) continue to rely on flat liquidation incentives. Simulated improvements to responsiveness via size-aware tips have been explored in Kirillov and Chung (2022).

In this work, to align protocol costs more closely with execution realities, we propose a size-sensitive marginal bounty schedule:

bounty rate
$$(V) = \min \left[\beta, \frac{\alpha}{\sqrt{V}} \right], \quad V = \text{vault value in } \$.$$

This formulation preserves strong incentives for small vaults while naturally tapering payouts on large positions, challenging the historical skew toward whales in favor of strengthening protocol reserves.

Governance sets:

- α : the reference bounty factor (e.g., calibrated so that $V_0 = \$50,000$ yields 0.1%),
- β : a hard cap (e.g., 0.25%) to prevent runaway APRs on micro-vaults.

This form ensures:

- Bounties grow sublinearly with vault size.
- Marginal outflow to keepers falls as $1/\sqrt{V}$.
- Rewards remain competitive across vault sizes without overpaying on large ones.

Implementation. The curve can be implemented in the keeper module as:

Listing 2: Reward curve for vault liquidations

```
function _bountyRate(uint256 vaultUsd) internal view returns (uint256) {
   uint256 rate = alphaRay / sqrt(vaultUsd);
   return rate > betaRay ? betaRay : rate;
}
```

Empirical benchmarks and governance calibration are discussed in the Appendix.

10 Cross-protocol contagion defence

Signed JSON heat-maps report live liquidation pressure. Peers may impose a $60 \,\mathrm{s}$ to $120 \,\mathrm{s}$ cooldown to prevent feedback loops. Criticality is computed using k-shell decomposition (Battiston and Puliga 2016); if outer-shell exposure exceeds $5 \,\%$ of TVL, the protocol auto-pauses.

11 Governance

The proposed architecture delegates control of key risk parameters $(\kappa, Y, \psi, \delta)$ to an on-chain governance module, RiskConfig, which is secured by a 7-day timelock to allow community review and prevent instant changes.

To ensure transparency and auditability, all stress-test notebooks and VaR analyses are permanently stored using decentralized, content-addressed systems such as Arweave or IPFS.²

12 Stress test: ETH flash crash 11 Dec 2024

To estimate the protocol's resilience under stress, we simulate a replay of the ETH flash crash of 11 December 2024 with approximately \$250M in TVL and 35,000 CDPs.

Table 2 compares liquidation outcomes against a baseline fixed-spread engine.

²Risk analyses and stress-test notebooks are pinned using decentralized storage: Arweave for permanent ledger-backed publishing (Williams et al. 2023), and IPFS for content-addressable versioning (Benet 2014).

	Table 2:	BoC replay	(\$250	M TVL.	35	000	CDPs).
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Metric	5% spread	Proposed	Improvement / pp
Peak 5-min impact (%)	-14.2	-3.8	10.4
Bad debt / TVL (%)	13.6	2.1	-84.6
Protocols hit	8	1	-87.5
Clearance time	$50 ext{ s}$	6 min	

13 Conclusion

This work proposes a cascade-resilient liquidation architecture grounded in mathematically sound mechanisms including stress detection, liquidity-aware tranching, VaR-based reserve pricing, and sealed-bid auctions. These modules formalize and improve upon the heuristic logic of the original StableUnit design.

While the components are backed by peer-reviewed literature and closed-form models, the full architecture must be validated through simulation, testnet deployment, and formal integration into StableUnit or comparable infrastructure prior to mainnet deployment.

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