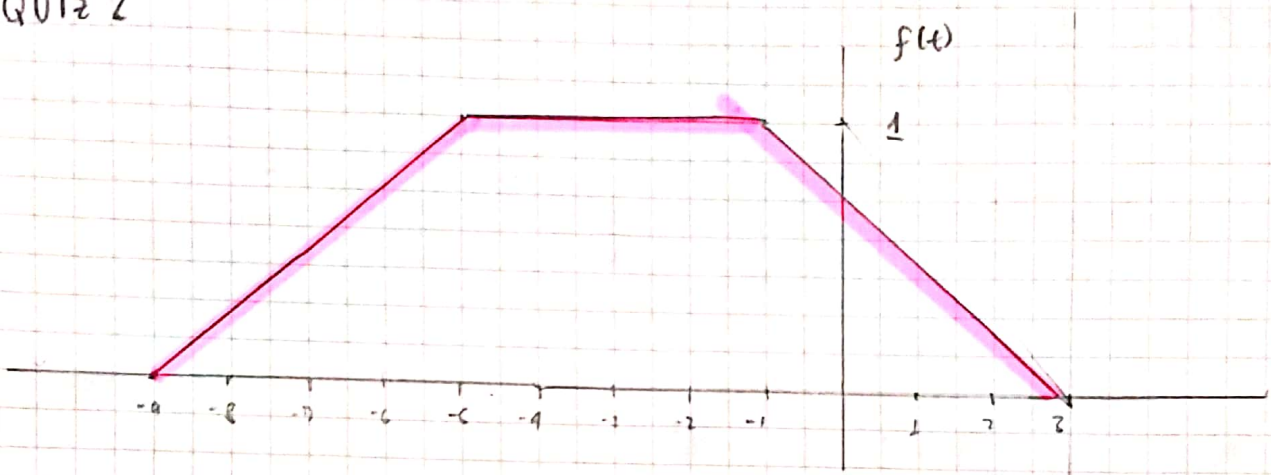


QUIZ 2



$$f(t) = \begin{cases} \frac{1}{4}t + \frac{9}{4} & -9 \leq t < -5 \\ 1 & -5 \leq t < -1 \\ -\frac{1}{4}t + \frac{3}{4} & -1 \leq t \leq 3 \end{cases}$$

$$F(\omega) = \int_{-9}^{-5} \left(\frac{1}{4}t + \frac{9}{4} \right) e^{-i\omega t} dt + \int_{-5}^{-1} (1) e^{-i\omega t} dt + \int_{-1}^3 \left(-\frac{1}{4}t + \frac{3}{4} \right) e^{-i\omega t} dt$$

(1.)

$$F(\omega) = \frac{1}{4} \left[\frac{-1}{i\omega} t e^{-i\omega t} - \frac{1}{i^2 \omega^2} e^{-i\omega t} \right] \Big|_{-9}^{-5} - \frac{9}{4i\omega} e^{-i\omega t} \Big|_{-9}^{-5}$$

$$F(\omega) = \frac{5}{4i\omega} e^{5i\omega} - \frac{1}{4i^2 \omega^2} e^{5i\omega} - \frac{9}{4i\omega} e^{9i\omega} + \frac{1}{4i^2 \omega^2} e^{9i\omega} - \frac{9}{4i\omega} e^{5i\omega} + \frac{9}{4i\omega} e^{9i\omega}$$

$$(2) f(\omega) = \int_{-3}^{-1} e^{-i\omega t} dt$$

$$= -\frac{1}{i\omega} \left[e^{-i\omega t} \right]_{-3}^{-1} = -\frac{1}{i\omega} e^{i\omega} + \frac{1}{i\omega} e^{5i\omega}$$

$$(3) f(\omega) = \int_{-1}^3 \left(-\frac{1}{4}t + \frac{3}{4} \right) e^{-i\omega t} dt$$

$$= -\frac{1}{4} \left[-\frac{1}{i\omega} t e^{-i\omega t} - \frac{1}{i^2 \omega^2} e^{-i\omega t} \right]_{-1}^3 - \frac{3}{4i\omega} e^{-i\omega t} \Big|_{-1}^3$$

$$= \frac{3}{4i\omega} e^{-3i\omega} + \frac{1}{4i^2 \omega^2} e^{-3i\omega} + \frac{1}{4i\omega} e^{i\omega} - \frac{1}{4i^2 \omega^2} e^{i\omega}$$

$$- \frac{3}{4i\omega} e^{-i\omega} + \frac{3}{4i\omega} e^{i\omega}$$

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$$f(\omega) = \frac{5e^{5i\omega}}{4i\omega} - \frac{9e^{5i\omega}}{4i\omega} - \frac{e^{5i\omega}}{4i^2 \omega^2} + \frac{e^{9i\omega}}{4i^2 \omega^2} - \frac{e^{i\omega}}{i\omega}$$

$$+ \frac{e^{5i\omega}}{i\omega} + \frac{e^{-3i\omega}}{4i^2 \omega^2} + \frac{e^{i\omega}}{4i\omega} - \frac{e^{i\omega}}{4i^2 \omega^2} + \frac{3e^{i\omega}}{4i\omega}$$

$$f(\omega) = \frac{-e^{5i\omega}}{i\omega} - \frac{e^{5i\omega}}{4i^2 \omega^2} + \frac{e^{9i\omega}}{4i^2 \omega^2} - \frac{e^{i\omega}}{i\omega} + \frac{e^{5i\omega}}{i\omega}$$

$$+ \frac{e^{-3i\omega}}{4i^2 \omega^2} + \frac{e^{i\omega}}{i\omega} - \frac{e^{i\omega}}{4i^2 \omega^2}$$

$$f(\omega) = -\frac{e^{5i\omega}}{4i^2\omega^2} + \frac{e^{9i\omega}}{4i^2\omega^2} + \frac{e^{-3i\omega}}{4i^2\omega^2} - \frac{e^{i\omega}}{4i^2\omega^2}$$

$$f(\omega) = \frac{1}{4i^2\omega^2} \left(-e^{-5i\omega} + e^{9i\omega} + e^{-3i\omega} - e^{i\omega} \right)$$

$$f(\omega) = \frac{1}{4i^2\omega^2} \left(\left(e^{9i\omega} + e^{-3i\omega} \right) - \left(e^{5i\omega} + e^{i\omega} \right) \right)$$

$$= \frac{1}{4i^2\omega^2} \left(e^{3i\omega} \left(e^{6i\omega} + e^{-6i\omega} \right) - e^{3i\omega} \left(e^{2i\omega} + e^{-2i\omega} \right) \right)$$

$$= \frac{1}{4i^2\omega^2} \left(\left(e^{6i\omega} + e^{-6i\omega} \right) - \left(e^{2i\omega} + e^{-2i\omega} \right) \right)$$

$$= \frac{e^{3i\omega}}{4i^2\omega^2} \left(2\cos(6\omega) - 2\cos(2\omega) \right)$$

$$f(\omega) = \frac{e^{3i\omega}}{4\omega^2} \left(\cancel{2} + 4\sin^2(\omega) - \cancel{2} - 4\sin^2(3\omega) \right)$$

$$f(\omega) = -\frac{e^{3i\omega}}{\omega^2} \left(\sin^2(\omega) - \sin^2(3\omega) \right)$$

$$f(\omega) = -e^{3i\omega} \left[\sin^2(\omega) - 9\sin^2(3\omega) \right]$$