Finite phase coherence time of a quantum field created by an ideal Bose gas

Dr. Alexej Schelle[1]

[1]*Freelancer (Senior) Lecturer @ IU Internationale Hochschule,*
email : alexej.schelle@gmail.com

A quantitative quantum field approach for a very weakly interacting, dilute Bose gas is presented. Within the presented model, which assumes the constraint of particle number conservation at constant average energy in the canonical ensemble, both coherent oscillations, as well as decay times of quantum coherence for a quantum field created by the atomic cloud of a Bose-Einstein condensate, are modeled simultaneously by a unique complex time variable and two different characteristic frequencies for the oscillation and decoherence of the field. Within the present theory, it is illustrated that the occurrence of coherence and a macroscopic ground state population has its origin in finite coherence times of the ensemble of quantum particles in the Bose gas, which - in contrast to the incoherent interactions between the different particles - leads to the preparation of a thermodynamically stable many-body quantum state with coherent superpositions of discrete and quantized condensate and non-condensate atom number states at constant total atom number.

I. INTRODUCTION

As a fundamental property of quantum mechanics, coherence demonstrates the wave nature of quantum particles by the ability of the particles to interfere when their wavelength becomes of the order of their average distance below the critical temperature [1]. In classical physics, when a particle is moved from point A to a second point B, the particle has a well-defined position and velocity at each instant of time during the cycle from A to B, and indeed this time is well-defined by the path length between the two positions and the average velocity of the traveling particle [2]. Quantum mechanically, however, the dynamics of particles are more complicated, because of uncertainty and localization of position and velocity for particles in an external confinement, thus in the setup as described above, the quantum particle may only be located at two different positions and therefore exhibit no probability to be located in the space between the two points A and B. As a consequence of quantum effects from quantization of the corresponding single-particle wave functions, if the particle tunnels from point A to point B, there remains a time interval during which it is quantum mechanically not well-defined whether the particle is located at position A or B, so that the quantum state of the particle may thus be in a coherent superposition of a state with localization in point A and a state with localization in point B, with a well-defined phase relation between these two states which allows to quantify the oscillations of the transition probability for the particle to move from point A to B, and vice versa. The time scale during which these two states are quantum mechanically correlated and the particle's wave function can interfere and coherently oscillate is called coherence time [3].

Calculations and analyses of coherence times typically rely on the knowledge of the spectrum of the given quantum system. In a Bose-Einstein condensate below the critical temperature, most of the particles in the Bose gas search to populate the ground state of the system, so that a macroscopic occupation of the ground state mode with a typically centered spatial distribution occurs below the critical temperature for Bose-Einstein condensation with the characteristics of a coherent matter wave [4], since all the particles' wave functions tend to interfere at the center of the external confinement. This happens because each of the particles can in principle tunnel from the ground state to a (multimode) excited state of the single-particle spectrum, because of the large energy uncertainty below the critical temperature. As a consequence of such single-particle coherence from tunneling as described above, each particle is thus in general in a superposition of a state with the localization characteristics of the single-particle ground state, and one or multiple of the excited single-particle states. The large coherence time achieved this way is a precursor for Bose-Einstein condensation, where the coherence time t is required to be sufficiently large to allow the different single-particle wave functions to interfere constructively and coherently - i. e. in particular larger than the oscillation time of particles among the different single-particle quantum states a particle in the Bose gas may coherently populate.

Within the framework of quantum mechanics, quantum processes obey time-reversal symmetry due to well-defined phase relations, but only on the time scale of coherence, or more precisely, as long as the quantum system is not coupled to an external thermal environment. Thus, it is the coherence time that defines a natural unit for the forward propagation of time in a quantum system like a Bose-Einstein condensate which adjusts its direction of time due to decoherence by the maximization of entropy. However, in the singularity of a Bose-Einstein condensate, nothing ad hoc guarantees that the laws of standard thermodynamics, like the entropy maximization principle, still hold. This is not least, because the process of decoherence is not necessarily dominant in the parameter range of Bose-Einstein condensation, which typically leads to long-range order with only one char-
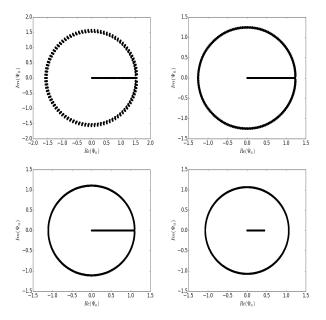
FIG. 1: (color online) Figures show the spectrum of fugacity values for a Bose-Einstein condensate with $N = 1000$ particles in a harmonic trap with trap frequencies $\omega_x, \omega_y = 2\pi \times 42.0$ Hz, and $\omega_z = 2\pi \times 120.0$ Hz at different temperatures, $T = 5.0$ nK (upper left), $T = 10.0$ nK (upper right), $T = 22.5$ nK (lower left) and $T = 35.0$ nK (lower right). Below the critical temperature $T_c = 26.9$ nK, there's only one Boltzmann equilibrium state (Bose-Einstein condensation), which corresponds to $\text{Re}(z) \rightarrow 1$ $(t \rightarrow 0)$. In this limit, unconnected parts of the spectrum are strongly coupled. Whereas the fugacity ring spectrum is gapless below the critical temperature, it obeys a gap and tends to get continuously close above T_c ($T = 35.0$ nK) with a large range of possible and coupled equilibrium states, obeying the same absolute value of the fugacity tends to one.

acteristic many-body quantum state and large coherence times, in particular for ideally closed setups [5]. In such case, the single-particle entropy may remain constant, or even decrease in the evolution of the condensation process if standard thermodynamic properties break down in the singularity.

So far, no quantitative estimates for the coherence time of a quantum particle have been derived from first principles in physical units within standard field theories for Bose-Einstein condensation which make use of particle number conservation in the canonical ensemble. In the following analysis, the coherence time is quantified for a quantum particle in a nearly ideal Bose-Einstein condensate confined in a harmonic trap, starting from a number-conserving quantum field theory for Bose-Einstein condensation [7, 8], assuming sufficiently dilute atomic gases. In this context, it is shown that gauging the quantum field is a necessary and sufficient condition for spontaneous symmetry breaking of time reversal and phase gauge symmetry of the quantum field below the critical

temperature, provided that the equilibrium state of the gas follows a Boltzmann equilibrium, which maximizes the entropy of the system in the present model. Time reversal symmetry and the corresponding phase gauge symmetry of a quantum particle in a Bose-Einstein condensate turn out to be spontaneously broken due to fluctuations of the corresponding entropy which does not rely on coherent particle interactions in this parameter regime, coupling symmetric and asymmetric parts of the underlying fugacity spectrum starting for temperatures at the critical temperature. The fact that the coherence time and the maximum entropy at the Boltzmann equilibrium are finite below the critical temperature can be considered as a signature for spontaneous symmetry breaking and therefore particularly indicates the quantum mechanical character of the Boltzmann equilibrium statistics in the present parameter range.

In the present approach, complex time is formally modeled as a scaled two-dimensional complex variable - which arises intrinsically from the constraint of particle number conservation. Standard time reversal symmetry is recovered in the classical limit of large temperatures. To connect the mathematical framework of Bose-Einstein condensates at thermal equilibrium to the physics of coherence and oscillation times of a single particle in a Bose gas at finite temperature, the concept of imaginary time turns out to be a suitable mathematical tool to describe both the quantitative scaling of coherence times as well as their uncertainty and broken gauge symmetry aspects induced by many-particle fluctuations below the critical temperature [6, 7, 9]. Whereas the definition of (purely) imaginary time often lacks a clear physical interpretation in other frameworks, such as the calculation of the Hawking radiation in black holes [9], or the definition and calculation of a Eukledian time in special relativity [10], in the present framework, the formal splitting of the time variable into two variables derived from scaling the standard time parameter t with the corresponding oscillation and decay frequency which arise from the underlying complex fugacity spectrum, imaginary time can be interpreted as the time scale during which a quantum particle in the gas relaxes towards an equilibrium quantum state imposing the assumption of a Boltzmann equilibrium (i. e. coupling elementary quantum states to the external environment). As shown in the sequel of this work, this is because the real part of complex time effectively converges to zero, whereas the imaginary part defines the relaxation time towards the Boltzmann equilibrium with maximal entropy at the given temperature. In contrast to (purely) imaginary time that defines the decay characteristics in the vertical direction of the complex plane, from the conservation of particle number and average energy in the framework of the canonical ensemble, the particles in the gas simultaneously exhibit both finite coherence times as well as oscillations between different regions A and B in space (e. g. spatial

domains of different single-particle wave functions) below the critical temperature also at the Boltzmann equilibrium, an effect that can be energetically understood as the coupling of symmetric and asymmetric parts of the underlying complex fugacity spectrum. Thus, the foundation of coherent particle coupling among different condensate and non-condensate atom number states in a Bose-Einstein condensate as described in the present model is due to a process related to elementary time evolution which defines a finite coherence time that is larger or equal to the complex time variable which defines the (upper bound of a) time scale for a particle to interact with its (thermal) environment. Therefore, each of the particles is coherently distributed among different single-particle quantum states, which leads to a coherent field distribution with different possible condensate and non-condensate number states at a constant total particle number in the Bose gas, which finally defines a macroscopic ground state population from coherent wave interference at a stable Boltzmann equilibrium of maximum entropy below the critical temperature.

II. THEORY

Within a non-local quantum field theoretical framework, a Bose-Einstein condensate at finite temperature can be described quantitatively by the following field ansatz,

$$\psi = \psi_0 + \psi_\perp = \sum_{\mathbf{k}} c_{\mathbf{k}} e^{\frac{-i\mu_{\mathbf{k}} t}{\hbar}}, \tag{1}$$

where ψ_0 represents the spatially integrated condensate field, and $\psi_\perp$ the non-condensate field in the framework of solid numbers in the standard complex plane [7]. In the absence of external perturbations (below the critical temperature), and assuming isotropic relative spatial phase dependence, the total field $\psi = 0$ vanishes so that the total symmetry of the system remains preserved. This is because the two condensate and non-condensate field components can be decomposed into uncorrelated single field components. We assume a spatially uniform distribution of the relative phase between the different single-particle wave functions. In contrast, as will be shown in the sequel of the present numerical analysis, below the critical temperature, where different atom number states get effectively coherently coupled because of finite coherence times, the resulting quantum states are of the form $|\mu\rangle = \sum_{\mathbf{k}} c_{\mathbf{k}} |\{N_{\mathbf{k}}\}\rangle$ which leads to the fact that $\psi_0 = -\psi_\perp \neq 0$ - called spontaneously broken gauge symmetry.

To this end, the concept of imaginary time provides a solid starting point for the analysis of coherence time and its uncertainty - since it defines a relation between

the time evolution of a single particle in the Bose gas and the equilibration of the many-particle system at finite temperature T, i. e. the interaction of the quantum particle with the surrounding quantum gas. It is the statistics of the present quantum field model which then describes the collective dynamics of the quantum field for a Bose-Einstein condensate through relating the parameters of single particles to the many-particle characteristics of the Bose gas. Whereas the concept of imaginary time is rather a pure mathematical convention in many quantum theoretical applications, in the current framework, the underlying mathematical relation (Wick rotation)

$$\beta\hbar = it \tag{2}$$

hence defines a relation between the absolute temperature and the coherent dynamics of a quantum particle in the presence of the $N - 1$ other particles in the Bose gas by its absolute value $|t| = \beta\hbar$. The time parameter t can be interpreted as the absolute value of time in units of unit time of the equilibrated quantum gas (which is defined by the lowest single-particle energy of a quantum particle in the external confinement, since all values lower than the single-particle ground state energy are effectively zero, because of the finite energy uncertainty), or equivalently as the single-particle entropy in units of the Boltzmann constant k_B.

As a formal ansatz, in Eq. (2), one may use that μt is not a real, but a complex number, that is $\mu t \in \mathbb{C}$. Hence, in the framework of this quantum field theory as presented, the definition of scaled time consists of a real part t_1 which describes the coherent phase evolution of the quantum particle at equilibrium (inverse oscillation frequency in the stationary state), corresponding to the ring of the fugacity spectrum in Fig. 1, whereas t_2, the imaginary part (of μt) describes decay processes of the particle in the atomic cloud at or close to equilibrium (compare linear part of the fugacity spectrum in Fig. 1), that is

$$\mu t = t_1 + it_2, \tag{3}$$

which can be quantified by solving the equation for the constraint of particle number conservation numerically, see Eq. (5). Please note that the introduction of a two-dimensional (scaled) time is not entirely necessary for further calculations and results. In principle, the quantities t_1 and t_2 can also be represented by corresponding rates Γ_1 and Γ_2 weighted with a one-dimensional time variable t [16]. However, the notation of two different time variables t_1 and t_2 (that is labeling the oscillation rate Γ_1 and the decay rate Γ_2, respectively) can bear
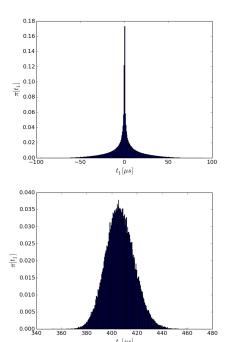
FIG. 3: (color online) Figure shows the (scaled) coherence time t_2 as a function of temperature T for a particle in the trap geometry $\omega_x, \omega_y = 2\pi \times 42.0$ Hz and $\omega_z = 120.0$ Hz. For intermediate values of temperature, the coherence time is around $(300 - 450)$ μs. Above the critical temperature, it approaches a value of around 200 μs.

FIG. 2: (color online) Figure shows 10^5 realizations of the imaginary parts (upper panel - labeled t_1) and real parts (lower figure - labeled t_2) of complex time parameterized by $t = t_1 + it_2$ as defined in Eq. (4) from random Markov sampling for a non-isotropic trap geometry $\omega_x, \omega_y = 2\pi \times 42.0$ Hz and $\omega_z = 120.0$ Hz at temperature $T = 5.0$ nK. The observation that t_2 is non-zero at thermal equilibrium indicates the coupling of the equilibrium state to quantum states with $t_2 \neq 0$ (non-classical correlations).

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some formal advantages, in particular in the framework of string theory [13]. Therefore, we will keep this notion for the analysis of this study. The absolute value of the parameter t can also be interpreted as the total entropy of a quantum particle in units of the Boltzmann constant k_B, after a single particle excitation in the presence of the atomic background.

Above the critical temperature, the real part of the fugacity spectrum shows a gap between the ring of constant absolute time, indicating that the symmetry of the quantum field can in principle not be broken without externally induced quantum fluctuations of energy and the corresponding entropy. Decreasing temperature by external cooling to decrease the gap between the outer ring and the inner linear (symmetry breaking) part of the fugacity spectrum finally leads to a gapless fugacity spectrum. However, due to the uncertainty of complex time, there is also a finite coupling probability between symmetric and asymmetric parts of the fugacity spectrum, in the parameter range, where the fugacity spectrum obeys a gap between the two principally unconnected spectra close above the critical temperature,
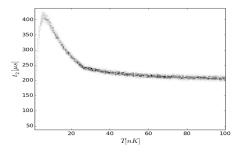
which leads to spontaneous symmetry breaking. Thus, a well-defined gauge of the quantum field (i. e. particle number conservation that leads to $\psi = 0$) assuming Boltzmann equilibrium turns out to be a necessary and sufficient condition for spontaneous symmetry breaking of the quantum field [12], since the coherence time is finite starting from temperatures approaching the critical temperature for Bose-Einstein condensation (see Fig. 3).

The probabilistic modeling of the dilute Bose gas of N particles below the critical temperature is described by the equation of conditional probability

$$\frac{p(\mu_A)}{p(\mu_B)} = \frac{e^{-\beta|\mu_A|}}{e^{-\beta|\mu_B|}} = \frac{e^{t_B}}{e^{t_A}}, \qquad (4)$$

in the present numerical framework, which defines the relative probability of a quantum particle to switch from a state A with corresponding chemical potential μ_A to a state B with chemical potential μ_B for a Bose gas at thermal equilibrium, where the chemical potential is defined by the intrinsic equation

$$\Delta N = \sum_{j \neq 0} z^j(\mu) \left[\prod_{k \neq x,y,z} \frac{1}{1 - e^{-j\beta\hbar\omega_k}} - 1 \right] + \mathcal{O}(a). \qquad (5)$$

In Eq. (5), $\Delta N = (N - N_0)$ is the difference between N the total number of particles, and N_0 the number of condensate particles. Further, μ denotes the chemical potential of the Bose gas, and ω_k the trap frequency in mode direction $k = x, y, z$. For numerical calculations, the single-particle ground state energy (level) $0.5\hbar(\omega_x + \omega_y + \omega_z)$ can be set to (effectively) zero.

Equation (4) does not necessarily describe a coherent forward or backward propagation in time, but only the probability for a transition (or new event) from a state

with (the characteristics) of a quantum state with entropy t_A to a state with entropy t_B, that is there is a non-vanishing probability that a transition with $t_A \geq t_B$ is accepted. Most likely, however, a transition is only accepted, if the absolute value $t_B \geq t_A$, which means that the particle tends to populate states with large entropy (and coherence time) in the Markov evolution process of the present quantum theory. In the present framework, the representation of a complex time arises naturally from Eq. (5), rather than from purely mathematical definition [13] and the system is allowed to exchange energy with the environment. In this setup, i. e. assuming the thermodynamical constraints of the canonical ensemble, coupling of unconnected parts of the fugacity spectrum isn't possible without coherently coupled single-particle states as indicated by the finite (scaled) coherence time t_2. Since the variable t can also be interpreted as the thermodynamic entropy in units of the Boltzmann constant, imposing Eq. (4) shows that the equilibrium state is a thermal equilibrium many-body quantum state of maximum entropy.

III. QUANTITATIVE ANALYSIS

For quantitative numerical modeling and analysis, the Bose gas is assumed to be dilute and obey a constant number of particles at finite temperature in a harmonic trapping potential to numerically calculate the coherence time (real part of complex time as defined in Eq. (2)) of a quantum particle in the Bose gas in SI units, which corresponds to the range of few hundred microseconds at the given parameter space. Interactions in the Bose gas are assumed to be negligible in the limit of a nearly ideal gas, where the s-wave scattering length a of the particles in the gas effectively tends to zero [15]. From Eq. (2), we learn that the Bose gas then obeys a stable (Boltzmann) equilibrium for absolute values of complex time in the limit, where $t_1 \to 0$ and $|t_2| \ll \hbar\beta$ (which corresponds to a small decoherence rate as compared to the upper bound $\hbar\beta$ for equilibration in the Bose gas). This indicates that the main decoherence mechanism in the Bose gas is thermalization.

Applying only the constraint of particle number conservation as defined by the conservation equation in Eq. (5), numerical sampling of complex time leads to a distribution of typical time scales for t_1 (scaled oscillation time) and t_2 (scaled coherence time), shown in Fig. 2. From the sampling of complex time, it is observed that t_1, the real part of complex time, is distributed around zero (with a vanishing width of approximately 0.03 μs) in the given parameter range (as defined in Fig. 2 - middle panel). The real part t_1 defines the average oscillation frequency in the fugacity spectrum of Fig. 2, which means that the quantum particle (with constant particle number at finite temperature) quickly couples to excited

single particle quantum states, related to $t_1 \sim 0 \pm 0.03$ μs - in the form of a standing wave. The distribution of the time scale t_2, the directional imaginary part of complex time (that is the effective coherence time for the particle in units of the decay constant Γ_2), is shown in the lower panel of Fig. 2. The typical range of the time scale t_2 is between 300 μs and 450 μs (see Fig. 3). Comparing the distribution of the time scale t_2 to the fugacity spectrum in Fig. 1, and to the time scale t_1 (upper figure), this shows that the coherent coupling of the single particle quantum states decay on a time scale of about $t_2 = (300 - 450)$ μs. To estimate the time scale in a physical framework, it is assumed that the unit time scale of the defined model is defined by $\tau = 2\pi \times (\omega_x \omega_y \omega_z)^{-1/3}$, since any frequency of the equilibrated system below that value is effectively zero in the thermally stable quantum system, because of the finite energy uncertainty.

To study the relaxation to thermal equilibrium (in the lower panel of Fig. 2), the conditional probability condition in Eq. (4) is imposed additionally to the constraint of particle number conservation in Eq. (5) and the condition of constant temperature in the Monte Carlo sampling algorithm for the time scales t_1 and t_2 in Eq. (3). From these two constraints and the sampling of complex time, one may calculate and analyze the absolute value of complex time, which corresponds to the single particle entropy (finite quantum coupling to the atomic vapor), related to both the coherence time scale t_2 and the oscillation time scale t_1, as shown in Fig. 2 for conditional probabilities. From Figs. 2, 3 one can extract that the absolute value of the time measure t is distributed around a mean value of approximately (405 ± 15) μs at a given typical temperature of $T = 5.0$ nK. The distribution of the absolute value of time t has a Gaussian shape and tends to be always distributed around finite quantitative values as compared to no conditional probability (not shown). This smoothing is due to the condition that the system has to satisfy the ergodicity assumption of the Boltzmann equilibrium in the present model, where not only the condition of constant temperature and the conservation of particle number, but also the condition of a stable Boltzmann equilibrium is imposed onto the applied sampling method or algorithm, respectively.

Calculating the entropy in units of the Boltzmann constant at constant temperature highlights Gaussian fluctuations (below as well as close above the critical temperature for Bose-Einstein condensation) in Fig. 4. Starting from temperatures not too far above the critical temperature, where the fugacity spectrum obeys a gap between the (symmetric) equilibrium state and the excited states (which are symmetry breaking), from the scaling of the entropy with temperature (not shown graphically), it can be understood that the process of spontaneous symmetry breaking is due to these non-zero fluctuations of the quantum entropy (particle coherence among different single-particle quantum states), occurring as a purely
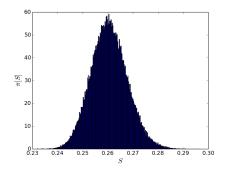
FIG. 4: (color online) Figure shows $5 * 10^5$ realizations of the maximum entropy (absolute value of the distribution of complex time t in units of the Boltzmann constant k_B, compare Eq. (4)) for a quantum particle in a Bose-Einstein condensate from Monte Carlo sampling using the ergodicity assumption. A Gaussian distribution is observed for non-isotropic trap geometries $\omega_x, \omega_y = 2\pi \times 42.0$ Hz and $\omega_z = 120.0$ Hz and temperature $T = 5.0$ nK, indicating the randomness of the quantum field and its fluctuations at thermal equilibrium.

quantum mechanical effect which decays on the time scale $t_2 < \hbar\beta$ and does not require interactions to the lowest order perturbation theory of the present setup. Thus, the definition of a well-defined gauge for the quantum field ($\psi = 0$) and the Boltzmann equilibrium is not only a necessary, but also a sufficient condition for breaking phase gauge symmetry of the quantum field in the given parameter regime. In the present study, this is not the case for a uniform distribution, since as indicated numerically, the finite coherence time can tend to zero in the case of a uniform distribution (not shown explicitly). However, this does not entirely prove that the Boltzmann equilibrium itself is a necessary condition for Bose-Einstein condensation, but only that defining a gauge is a necessary and sufficient condition for Bose-Einstein condensation due to finite coherent times - if conditional probabilities are assumed to follow the thermal Boltzmann equilibrium. Finally, the fact that the entropy is maximized and finite at thermal equilibrium indicates the non-classical correlations due to quantum coherence on the (scaled) time scale t_2 also at the thermal Boltzmann equilibrium.

IV. DISCUSSION

In the present theory, it is illustrated that the constraint of particle number conservation naturally leads to the composition of real-valued and purely imaginary valued time to an extended complex-valued number, which defines time in terms of a two-dimensional complex variable to fully capture the system in its full complexity - interacting particles below the critical temperature. It is the spectrum of this complex time with the corresponding fugacity spectrum that naturally defines the direction

of a composite (complex-valued) time, that is two simultaneous processes described by t_1 which describes coherent oscillations of the quantum particle's wave function and t_2 which defines the (de-)coherence time of a single quantum particle in the presence of the $N-1$ other particles, in the form of a cone propagating in the positive imaginary direction of complex space on average. In that representation, i. e. after coupling a single particle to the rest of the gas within the time scale $t_2 \leq \hbar\beta$, complex time doesn't obey time-reversal symmetry anymore, indicating the spontaneous breaking of the gauge symmetry for the quantum field, defined in terms of a variable for the chemical potential which is proportional to complex time. In the sequel of a quantitative analysis of complex time, it turns out that the real part of time is typically distributed around zero (that is almost all particles share the same coherent phase), whereas the imaginary part of complex time mostly entails the system's relaxation properties (finite coherence times, because of spontaneous fluctuations and decoherence). In the limit of vanishing temperatures, the process of spontaneous symmetry breaking (characterized by the well-defined relative phase relation between condensate and non-condensate particles) is directly and reproducibly measurable as condensate formation, respectively, as reported for instance in Ref. [4]. Measuring the relative phase between possible condensate and non-condensate states of a quantum particle in the Bose-Einstein condensate, especially in an intermediate temperature range below zero and the critical temperature, Bose-Einstein condensation in nearly-ideal Bose gases is direct evidence for spontaneous symmetry breaking from pure quantum fluctuations, as discussed in the sequel of the present theory.

It is the absolute value of complex time, which defines the entropy of the quantum particle in the Bose-Einstein condensate in units of the Boltzmann constant k_B. The finite value as well as the width of the entropy distribution indicates the coupling of several quantum states which do not belong to an exact Boltzmann equilibrium state as originally predicted in the framework of classical (equilibrium) many-particle mechanics, but are modified by the uncertainty and fluctuations of the particle's entropy in the atomic cloud at finite temperature. The real-valued part of time t_1 is directly related to the fugacity ring spectrum in the complex plane, whereas the coherence time t_2 describes the coupling of quantum states in the symmetry-breaking part of the fugacity spectrum. At the Boltzmann equilibrium, the distribution of entropy values ideally occurs in connection to quantum states in the neighborhood of $\text{Re}(z) = 1$ and $\text{Im}(z) = 0$ (fugacity equals one). Thus, the finite value of the entropy and its uncertainty indicates and quantifies the coupling of symmetric and asymmetric parts of the fugacity spectrum, which thus appear only structurally unconnected above the critical temperature. Finally, the different shape of

the entropy distribution from a flat distribution with a full range from $\mathrm{Re}(z) = 0$ to $\mathrm{Re}(z) = 1$ to a Gaussian distribution for conditional Boltzmann probabilities indicates that ergodicity leads to finite coherence times in the Boltzmann limit of maximum entropy, ensuring the zero gauge of the total spatially integrated quantum field at spatially uniform relative phase distributions of the single-particle wave functions for the quantum field to be a sufficient condition for Bose-Einstein condensation.

V. CONCLUSION

In conclusion, using a representation of complex time, the coherence time of a quantum particle in a Bose-Einstein condensate is on the order of a few hundred microseconds in the typical parameter regime of Bose gases at thermal equilibrium in harmonic traps below the critical temperature. In the presented quantum field theory, the definition of a complex-valued time arises naturally from the constraint of particle number conservation, mathematically defined by complex poles of an intrinsic equation for the fugacity (and the chemical potential) of the Bose-Einstein condensate. Within the presented model, gauging the quantum field in the Boltzmann equilibrium is a necessary and sufficient condition for spontaneous symmetry breaking for a nearly ideal Bose gas. Moreover, it is possible to argue and numerically verify that the process of spontaneous symmetry breaking is induced by time and entropy fluctuations enhanced by the reduction of the atomic density below the critical density for Bose-Einstein condensation, which couples (unconnected) symmetric and asymmetric parts of the fugacity spectrum. In the classical limit, entropy fluctuations tend to be distributed around a ring in the complex spectrum of the fugacity, recovering the time-reversal symmetry of the quantum field.

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