

Finite entropy fluctuations of a quantum particle in a Bose-Einstein condensate

Alexej Schelle¹

¹*Quantitative Analyst, Freelancer Associate @ IU Internationale Hochschule,
email : alexej.schelle@gmail.com*

A quantitative quantum field approach for a nearly-ideal, dilute Bose gas using a complex two-dimensional representation of time is presented. A quantitative measure for the coherence time of a quantum particle in the limit of very dilute atomic gases is derived and it is illustrated that the process of spontaneous symmetry breaking of the quantum field for the particle with a well-defined gauge has its origin in the coupling of symmetric and asymmetric parts of the underlying fugacity spectrum, which induces finite single-particle coherence times and thus breaks time reversal symmetry and the corresponding phase gauge symmetry of the quantum field below the critical temperature. The coupling and gauge symmetry breaking can be understood as due to spontaneous quantum fluctuations of two-dimensional time and the corresponding finite entropy at equilibrium, which does not rely on coherent particle interactions. In this context, defining a gauge for the quantum field at the Boltzmann equilibrium is a necessary and sufficient condition for maximization of entropy and spontaneous symmetry breaking at thermal equilibrium in the parameter regime of the present model. The concept of two-dimensional time finally converges to the standard scheme of purely imaginary time with time reversal symmetry in the classical limit of large temperatures.

I. INTRODUCTION

As a fundamental property of quantum mechanics, coherence demonstrates the wave nature of quantum particles by the ability of the particles to interfere with themselves when their wave length becomes of the order of their average distance below the critical temperature [1]. In classical physics, when a particle is moved from a point A to a second point B, the particle has a well-defined position and velocity at each instant of time during the cycle from A to B, and indeed this time is well-defined by the path length between the two positions and the average velocity of the traveling particle [2]. Quantum mechanically, however, the dynamics of a particle is more complicated, because of uncertainty and localization of position and velocity for particles in an external confinement, thus in the setup as described above, the quantum particle may only be located at two different positions and therefore exhibit no probability to be located in the space between the two points A and B. As a consequence of this quantum effects from quantization of the corresponding single particle wave functions, if the particle tunnels from point A to the point B, there remains a time interval during which it is quantum mechanically not well-defined whether the particle is located at position A or B, so that the quantum state of the particle may thus be in a coherent superposition of a state with localisation in point A and a state with localisation in point B, with a well-defined phase relation between these two states which allows to quantify the transition probability for the particle to move from point A to B, and vice versa. The time scale during which these two states are correlated and the particle can interfere with itself is called coherence time [3].

Calculations and analyses of coherence times typically rely on knowledge about the spectrum of the given quan-

tum system. In a Bose-Einstein condensate below the critical temperature, most of the particles in the Bose gas search to populate the ground state of the system, so that a macroscopic occupation of the ground state mode occurs below the critical temperature for Bose-Einstein condensation with the characteristics of a coherent matter wave [4]. However, each particle can in principle tunnel from the ground state to a (multimode) excited state of the single particle spectrum, also below the critical temperature. As a consequence of such single particle coherence from tunneling as described above, each particle is thus in general in a superposition of a state with the localisation characteristics of the single particle ground state, and one or multiple of the excited single particle states within a certain coherence time.

Since the quantum state of a particle may also change in time due to interactions with other particles, the total (coherent) phase of the particle is only defined up to a global phase, which may change in time due to interactions. Hence, very generally, besides the coherence time as discussed before, there is a second important time scale for a quantum particle, which is defined by the oscillation frequency of the particle's phase, that is the time scale during which the particle (which exhibits the phase coherence characteristics similar to a quantum optical coherent state) changes its phase due to interactions. Classical equations of motion do exhibit time reversal symmetry, however, in quantum mechanics, the process of decoherence leads to an increase (maximization) of entropy, which defines the direction of time in the framework of quantum thermodynamics.

Within the framework of quantum mechanics, thus, quantum processes also obey time-reversal symmetry due to well-defined phase relations, but only on the time scale of coherence. Thus, it is the coherence time which defines a natural unit for the forward propagation of time in a

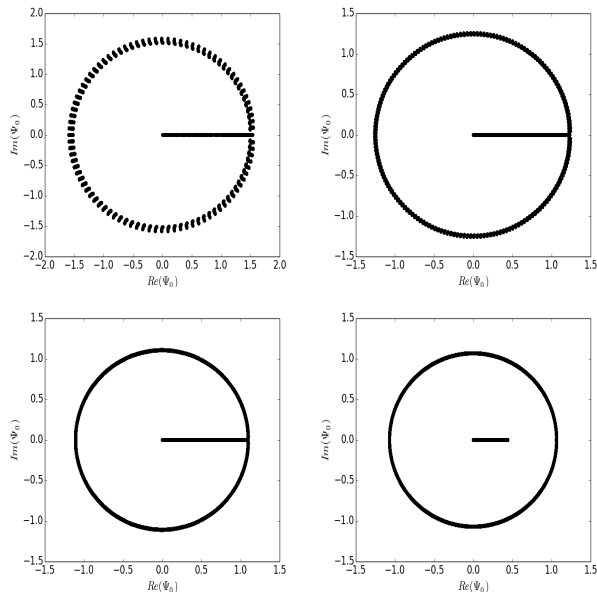


FIG. 1: (color online) Figures show the spectrum of fugacity values for a Bose-Einstein condensate with $N = 1000$ particles in a harmonic trap with trap frequencies $\omega_x, \omega_y = 2\pi \times 42.0$ Hz, and $\omega_z = 2\pi \times 120.0$ Hz at different temperatures, $T = 5.0$ nK (upper left), $T = 10.0$ nK (upper right), $T = 22.5$ nK (lower left) and $T = 35.0$ nK (lower right). Below the critical temperature $T_c = 26.9$ nK, there's only one Boltzmann equilibrium state (Bose-Einstein condensation), which corresponds to $\text{Re}(z) \rightarrow 1$ ($\tau \rightarrow 0$). In this limit, unconnected parts of the spectrum are strongly coupled. Whereas the fugacity ring spectrum is gapless below the critical temperature, it obeys a gap and tends to get continuous close above T_c ($T = 35.0$ nK) with a large range of possible and coupled equilibrium states, obeying the same absolute value of the fugacity tends to one. Above T_c , the coupling of unconnected parts of the fugacity spectrum (gauge symmetry breaking) is due to fluctuations of two-dimensional time and the corresponding entropy, as indicated by the finite time uncertainty and entropy in Figs. 2, 3 and 4.

quantum system which adjusts its direction of time due to decoherence. However, in the singularity of a Bose-Einstein condensate, nothing ad hoc guarantees that the laws of standard thermodynamics, like the entropy maximization principle, still hold. This is not least, because the process of decoherence is not dominant in the parameter range of Bose-Einstein condensation, which typically leads to long-range order with only one characteristic many-body quantum state and large coherence times. Furthermore, the Bose gas exchanges energy with the environment, so that entropy may remain constant (also for ideally closed physical setups [5]), or even decrease in the evolution of the condensation process, if standard thermodynamic properties break down in the singularity.

So far, no quantitative value for the coherence time of a quantum particle has been derived from first prin-

ciples in physical units within standard field theories for Bose-Einstein condensation. It is therefore shown that the coherence time can be approximately quantified for a quantum particle in a nearly-ideal Bose-Einstein condensate confined in a harmonic trap, starting from a number-conserving quantum field theory for Bose-Einstein condensation [7, 8], assuming sufficiently dilute atomic gases. In this context, it is shown that gauging the quantum field is a necessary and sufficient condition for spontaneous symmetry breaking of time reversal and phase gauge symmetry of the quantum field below the critical temperature, provided that the equilibrium state of the gas follows a Boltzmann equilibrium, which maximises the entropy in the present model. Time reversal symmetry and the corresponding phase gauge symmetry of a quantum particle in a Bose-Einstein condensate turn out to be spontaneously broken due to fluctuations of two-dimensional time and the corresponding entropy which does not rely on coherent particle interactions in this parameter regime, coupling symmetric and asymmetric parts of the underlying fugacity spectrum starting at the critical temperature. The fact that the coherence time and the maximum entropy at the Boltzmann equilibrium are finite below the critical temperature can be considered as a signature for spontaneous symmetry breaking and therefore particularly indicates the quantum mechanical character of the Boltzmann statistics in the present parameter range.

In the present approach, time is modeled as a two-dimensional complex vector - which arises intrinsically from the constraint of particle number conservation. Standard time reversal symmetry is recovered in the classical limit of large temperatures. In order to connect the mathematical framework of Bose-Einstein condensates at thermal equilibrium to the physics of coherence and oscillation times of a single particle in a Bose gas at finite temperature, the concept of imaginary time turns out to be a suitable mathematical tool to describe both the quantitative scaling of coherence times as well as their uncertainty and broken gauge symmetry aspects induced by many-particle fluctuations below the critical temperature [6, 7, 9]. Whereas the definition of (purely) imaginary time often lacks a clear physical interpretation in other frameworks, such as calculation of the Hawking radiation in black holes [9], or the definition and calculation of an Eukledian time in special relativity [10], in the present framework, two-dimensional complex time is derived from the underlying complex fugacity spectrum and can be interpreted as the time scale during which a quantum particle in the gas relaxes towards an equilibrium quantum state imposing the assumption of a Boltzmann equilibrium, simultaneously exhibiting both coherence and oscillations between two points A and B, and a well-defined mathematical distribution of entropy as a function of external as well as internal parameters with the fundamental characteristics of entropy and time un-

certainty in quantum mechanics. In order to solve the equations for the two-dimensional time and their corresponding fluctuations numerically, a Metropolis algorithm for the calculation of both coherence and oscillation times of a particle in a Bose-Einstein condensate is applied [11], and it is shown that the entropy distribution of the particle follows a Gaussian profile, maximizing its entropy for the condition of a Boltzmann equilibrium.

II. THEORY

The concept of imaginary time provides a solid starting point for the analysis of coherence time and its uncertainty. Whereas this concept is rather a pure mathematical convention in many quantum theoretical applications, in the current framework, the underlying mathematical relation (Wick rotation)

$$\beta\mu = i\tau \quad (1)$$

defines a relation between the absolute temperature, the chemical potential and the coherence time of a quantum particle in the presence of the $N - 1$ other particles in the Bose gas by its absolute value $|\tau| = \beta|\mu|$. The parameter τ can be interpreted as the absolute value of time in units of unit time (defined by the energy of the quantum particle), or equivalently as the single particle entropy in units of the Boltzmann constant k_B .

As a new ansatz in quantum field theory for Bose-Einstein condensation, in Eq. (1), τ is not a real, but a complex number, that is $\tau \in \mathbb{C}$. Hence, in the framework of this quantum field theory as presented, the definition of time consists of a real part t_1 which describes the coherent phase evolution of the quantum particle at equilibrium (inverse oscillation frequency in the stationary state), corresponding to the ring of the fugacity spectrum in Fig. 1, whereas t_2 , the imaginary part (of τ) describes decay processes of the particle in the atomic cloud at or close to equilibrium (compare linear part of the fugacity spectrum in Fig. 1, that is

$$\tau = t_1 + it_2, \quad (2)$$

which can be quantified by solving the equation for the constraint of particle number conservation numerically. The absolute value of τ can be interpreted as the total entropy of a quantum particle in units of the Boltzmann constant k_B , after a single particle excitation in the presence of the atomic background. Above the critical temperature, the real part of the fugacity spectrum has a gap between the ring of constant absolute time,

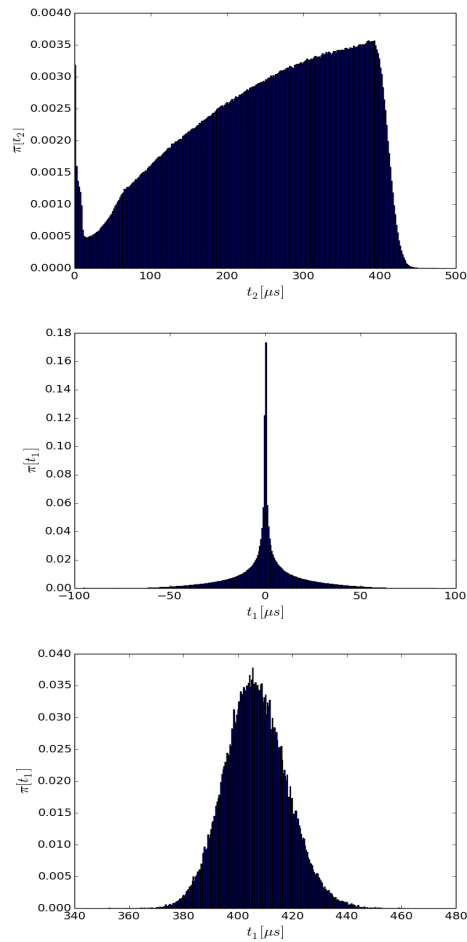


FIG. 2: (color online) Figure shows 10^5 realisations of the imaginary parts (upper panel) and real parts (middle figure) of two-dimensional time $\tau = t_1 + it_2$ as defined in Eq. (1) from random Markov sampling with (upper figure) and without (lower figure) conditional Boltzmann probability for the chemical potential, but with the constraint of particle number conservation, for a non-isotropic trap geometry $\omega_x, \omega_y = 2\pi \times 42.0$ Hz and $\omega_z = 120.0$ Hz at temperature $T = 5.0$ nK. Lower panel shows the distribution of the coherence time t_2 including the (conditional) Boltzmann equilibrium. The observation that t_2 is non-zero at thermal equilibrium indicates the coupling of the equilibrium state to quantum states with $t_2 \neq 0$ (non-classical correlations). From these fluctuations of the coherence time, the gauge symmetry of the system is broken spontaneously - what can be understood for example by comparing the fugacity spectrum for $T = 35.0$ nK in Fig. 1 to the scaling of the entropy in Fig. 4

indicating that the symmetry of the quantum field can in principle not be broken without quantum fluctuations of time and the corresponding entropy. Decreasing temperature by external cooling in order to decrease the gap between the outer ring and the inner linear (symmetry breaking) part of the fugacity spectrum finally leads to

a gapless fugacity spectrum. However, due to the uncertainty of two-dimensional time, there is also a finite coupling probability between symmetric and asymmetric parts of the fugacity spectrum, in the parameter range, where the fugacity spectrum obeys a gap between the two principally unconnected spectra close above the critical temperature, which leads to spontaneous symmetry breaking.

Thus, a well-defined gauge of the quantum field assuming Boltzmann equilibrium turns out to be a necessary and sufficient condition for spontaneous symmetry breaking of the quantum field [12], since the coherence time is finite starting from temperatures approaching the critical temperature for Bose-Einstein condensation (see Fig. 3) - what would not be the case, if the phase gauge symmetry were not broken. However, in order to observe spontaneous symmetry breaking of the quantum field from a gauge of the quantum field on a measurable scale, it requires that the coherence time is on the order of the time uncertainty (that is $\tau \geq 390 \mu\text{s}$ or larger, see Fig. 4). The spectrum of the fugacity itself always obeys a broken phase gauge symmetry with a gap between the Boltzmann equilibrium and multiple excited states, that is also at temperatures larger than the critical temperature. The broken gauge symmetry of the quantum field particularly remains in the limit of temperatures close above the critical temperature (see Fig. 4). In this context, it hence turns out that the symmetry breaking mechanism of the quantum field is due to the quantum mechanical coupling of the ring spectrum to the linear symmetry broken part of the fugacity spectrum, caused by quantum fluctuations of two-dimensional complex time, which are smaller than the time uncertainty for temperatures larger than the critical temperature and rapidly increase as temperature approaches zero. Since the phase gauge symmetry of the quantum field is spontaneously broken starting at the critical temperature, the imaginary part of time t_2 (coherence time) has a preferred direction in one direction of the imaginary axis (which defines a direction of time evolution) below the critical temperature, indicating that spontaneous phase symmetry breaking is equivalent to breaking the time reversal symmetry of the quantum field. Typically, the rotational direction of the coherent time evolution scales as $\exp[i\tau]$ and thus the decay as $\exp[-t_2]$, so that the preferred direction of imaginary time is the positive imaginary axis with arbitrary real values (similar to the definition and direction of space time in Minkowski space). The directional preference of time in one of all possible time directions correlates with the broken gauge symmetry of the quantum field [7], which represents the fugacity spectrum weighted with phase factors of the corresponding quantum occupations rotated by an angle π .

Probabilistic modeling of a dilute Bose gas of N particles below the critical temperature is well described by the equation of conditional probability

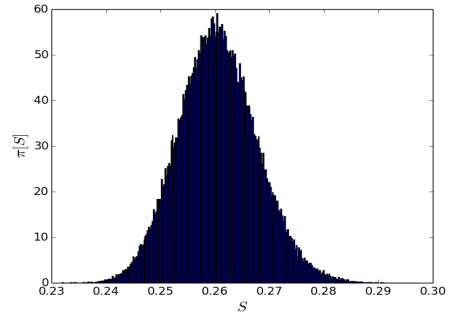


FIG. 3: (color online) Figure shows 5×10^5 realizations of the maximum entropy (absolute value of the distribution of two-dimensional time τ in units of the Boltzmann constant k_B) for a quantum particle in a Bose-Einstein condensate from Monte Carlo sampling using the ergodicity assumption. A Gaussian distribution is observed for non-isotropic trap geometries $\omega_x, \omega_y = 2\pi \times 42.0 \text{ Hz}$ and $\omega_z = 120.0 \text{ Hz}$ and temperature $T = 5.0 \text{ nK}$, indicating the randomness of the quantum field and its fluctuations at thermal equilibrium. The finite entropy below the critical temperature indicates that the Boltzmann equilibrium derived from detailed balance is shifted by a value of about 30%, because of quantum fluctuations.

$$\frac{p(\mu_A)}{p(\mu_B)} = \frac{e^{-\beta|\mu_A|}}{e^{-\beta|\mu_B|}} = \frac{e^{\tau_B}}{e^{\tau_A}}, \quad (3)$$

which defines the relative probability of a quantum particle to move from a state A with corresponding chemical potential μ_A to a state B with chemical potential μ_B for a Bose gas at thermal equilibrium, where the chemical potential is defined by the intrinsic equation

$$\Delta N = \sum_{j \neq 0} z^j(\mu) \left[\prod_k \frac{1}{1 - e^{-j\beta\hbar\omega_k}} - 1 \right] + \mathcal{O}(a). \quad (4)$$

In Eq. (4), ΔN is the difference between N the total number of particles, and N_\perp the number of non-condensate particles. Further, μ denotes the chemical potential of the Bose gas, and ω_k the trap frequency in mode direction $k = x, y, z$.

Equation (3) does not necessarily describe a coherent forward or backward propagation in time, but only the probability for a transition (or new event) from a state with (the characteristics) of a quantum state with entropy τ_A to a state with entropy τ_B , that is there is a non-vanishing probability that a transition with $\tau_A \geq \tau_B$ is accepted. Most likely, however, a transition is only accepted, if the absolute value $\tau_B \geq \tau_A$, which means that the particle tends to populate states with large entropy (and coherence time) in the Markov evolution process of the present quantum theory. From the definition of

the chemical potential(s) in Eq. (4), time is in principle quantized. Only the neglect of energy discreteness in the thermodynamic limit (large particle number and small frequencies) leads to a variable $\tau = t_1 + it_2$ which gets continuous. In the present framework, the representation of two-dimensional time arises naturally from Eq. (4), rather than from purely mathematical definition [13] and the system is allowed to exchange energy with the environment. In this setup, coupling of unconnected parts of the fugacity spectrum isn't possible without fluctuations and uncertainty of two-dimensional time. Since τ is also interpreted as the thermodynamic entropy in units of the Boltzmann constant, imposing Eq. (3) leads to a thermal equilibrium quantum state of maximum entropy.

III. QUANTITATIVE ANALYSIS

In the following, we assume the Bose gas to be dilute and obey a constant number of particles at finite temperature in a harmonic trapping potential in order to numerically calculate the coherence time (real part of two-dimensional time as defined in Eq. (1)) of a quantum particle in the Bose gas in SI units, which corresponds to the range of few hundred microseconds at the given parameter range. Interactions in the Bose gas are assumed to be negligible in the limit of a nearly ideal gas, where the s-wave scattering length a of the particles in the gas effectively tends to zero [15]. From Eq. (1), we learn that the Bose gas then obeys a stable (Boltzmann) equilibrium for values of two-dimensional time in the limit, where $t_1 \rightarrow 0$ and $t_2 \rightarrow 0$. Indeed, from Fig. 1, we can verify from the fugacity spectrum, which shows all possible values (poles) of the intrinsic Eq. (4), that the Boltzmann equilibrium corresponds to the fugacity spectrum, where the real part of the fugacity tends to one and the imaginary part to zero (corresponding to the case, where t_1 and t_2 are distributed around zero).

Applying only the constraint of particle number conservation as defined by the conservation equation in Eq. (4), numerical sampling of two-dimensional complex time leads to a distribution of typical time scales for t_1 (oscillation time) and t_2 (coherence time), shown in Fig. 2. From the sampling of two-dimensional time, it is observed that t_1 , the real part of complex two-dimensional time, is distributed around zero (with a vanishing width of approximately $0.03 \mu\text{s}$) in the given parameter range (as defined in Fig. 2 - middle panel). The real part t_1 defines the average oscillation frequency in the fugacity spectrum of Fig. 2, which means that the quantum particle (with constant particle number at finite temperature) does not oscillate, but mainly remains in quantum states which are related to $t_1 \sim 0 \pm 0.03 \mu\text{s}$ (and fugacity with imaginary part zero). The distribution of the time scale t_2 , the directional imaginary part of two-dimensional time (that is the coherence time for the particle to be in a state with

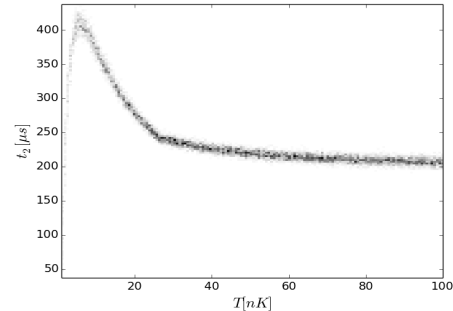


FIG. 4: (color online) Figure shows the scaling of the maximum entropy (absolute value of two-dimensional time τ) as a function of temperature for a quantum particle in a Bose-Einstein condensate from sampling with a conditional Boltzmann distribution for non-isotropic trap geometries $\omega_x, \omega_y = 2\pi \times 42.0 \text{ Hz}$ and $\omega_z = 120.0 \text{ Hz}$, indicating the quantum mechanical coupling of the symmetric fugacity ring spectrum with the asymmetric linear part of the fugacity spectrum, below as well as above the critical temperature. The total entropy τ is defined in units of the Boltzmann constant k_B . At temperature $T \sim 5 \text{ nK}$, maximum entropy is achieved exceeding the limit of time uncertainty from background fluctuations (which is on the order of $390 \mu\text{s}$). In this temperature range, it should be possible to directly measure the effect of spontaneous symmetry breaking in the form of condensate depletion (also in the absence of interactions). For increasing temperatures much larger than the critical temperature, the entropy (and coherence time) converges to zero with the characteristics of a classical Boltzmann equilibrium with $\tau = 0$.

phase coherence between condensate and non-condensate subspaces), is shown in the lower panel of Fig. 2. The typical range of the time scale t_2 is around $0 \mu\text{s}$ to $450 \mu\text{s}$ in the present parameter range, with a slight extension to values between $300 \mu\text{s}$ and $450 \mu\text{s}$ (upper panel). Comparing the distribution of the time scale t_2 to the fugacity spectrum in Fig. 1, it turns out that from the conservation of particle number, the quantum particle below the critical temperature shows spontaneous fluctuations between condensate and non-condensates quantum states, which are related to coherence times $t_2 = (0 - 450) \mu\text{s}$.

In order to study the relaxation to thermal equilibrium (in the lower panel of Fig. 2), the conditional probability condition in Eq. (3) is imposed additionally to the constraint of particle number conservation in Eq. (4) and the condition of constant temperature in the Monte Carlo sampling algorithm for the time scales t_1 and t_2 in Eq. (2). From these two constraints and the sampling of two-dimensional time, we may calculate and analyse the absolute value of complex time, which corresponds to the single particle entropy (finite quantum coupling to the atomic vapor), related to both the coherence time t_2 and the oscillation time t_1 , as shown in Fig. 2 for conditional probabilities. In Fig. 3, one can extract that the absolute value of time τ is distributed around a mean value of ap-

proximately $(405 \pm 15) \mu s$. In contrast to the distributions shown in the upper panel of Fig. 2, the absolute value of time τ has a Gaussian shape and tends to be always distributed around finite quantitative values as compared to no conditional probability. This smoothing is due to the condition that the system has to satisfy the ergodicity assumption of the Boltzmann equilibrium, where not only the condition of constant temperature and the conservation of particle number, but also the condition of a Boltzmann equilibrium is imposed onto the applied sampling method or algorithm, respectively.

Calculating the entropy in units of the Boltzmann constant at constant temperature highlights Gaussian fluctuations (below as well as close above the critical temperature for Bose-Einstein condensation) in Fig. 3. Starting from temperatures not too far above the critical temperature, where the fugacity spectrum obeys a gap between the (symmetric) equilibrium state and the excited states (which are symmetry breaking), from the scaling of the entropy with temperature in Fig. 4, it can be understood that the process of spontaneous symmetry breaking is due to these non-zero fluctuations of the quantum entropy, occurring as a purely quantum mechanical effect which do not require interactions in the present setup. Thus, the definition of a well defined gauge for the quantum field ($\psi = 0$) and the Boltzmann equilibrium is not only a necessary, but also a sufficient condition for breaking time reversal and equivalently phase gauge symmetry of the quantum field of the given parameter regime. In the present study, this is not the case for a uniform distribution, since as indicated numerically, the finite coherence time can tend to zero in case of a uniform distribution. However, this does not entirely prove that the Boltzmann equilibrium itself is a necessary condition for Bose-Einstein condensation, but only that defining a gauge is a necessary and also a sufficient condition for Bose-Einstein condensation, if the equilibrium is assumed to be the thermal Boltzmann equilibrium. Finally, the fact that the entropy is maximized and finite at thermal equilibrium indicates non-classical correlations at the thermal Boltzmann equilibrium.

IV. DISCUSSION

From a mathematical perspective, it is straight forward to define the concept of spontaneously broken gauge symmetry in quantum physics, however, the meaning and description of this process in terms of a variable describing complex two-dimensional time, a complex number with two subspaces defined by the variables t_1 and t_2 , is not as trivial. Normally, purely imaginary time is used only as a formal mathematical concept in order to solve conceptual problems of theoretical physics rather than to quantify physical observables. Quantum physical processes usually depend on real time, which has a well-defined

physical forward direction as defined by the process of decoherence, the positive direction of real valued continuous positive numbers. The direction of purely imaginary time, in contrast, has no preferred physical direction, which means that purely imaginary time can either pass in positive or negative imaginary direction and is therefore typically assumed to be a one-dimensional quantity in the absence of decoherence, that is used for theoretical descriptions of ideally closed quantum systems or singularities of the latter.

In the present theory, it is illustrated that the constraint of particle number conservation naturally leads to the composition of real valued and purely imaginary valued time to a extended complex valued number, which defines time in terms of a two-dimensional variable in order to fully capture the system in its full complexity - interacting particles below the critical temperature. It is the spectrum of this two-dimensional time with corresponding fugacity spectrum which naturally defines the direction of a composite (complex valued) time, that is two simultaneous time scales - t_1 which describes oscillations of the quantum particle's wave function and t_2 which defines the coherence time of a quantum particle in the presence of the $N - 1$ other particles, in the form of a cone propagating in positive imaginary direction of complex space on average. In that representation, complex time doesn't obey time-reversal symmetry anymore, indicating the spontaneous breaking of the gauge symmetry for the quantum field, defined in terms of a variable for the chemical potential which is proportional to complex time. In the sequel of a quantitative analysis of two-dimensional complex time, it turns out that the real part of time is typically distributed around zero (that is almost all particles share the same coherent phase), whereas the imaginary part of complex time mostly entails the system's relaxation properties (finite coherence times, because of spontaneous fluctuations and decoherence). In the limit of vanishing temperatures, the process of spontaneous symmetry breaking (well-defined relative phase between condensate and non-condensate particles) should be directly and reproducibly measurable, respectively, as reported for instance in [14]. Measuring the relative phase between possible condensate and non-condensate states of a quantum particle in the Bose-Einstein condensate, especially in an intermediate temperature range below zero and the critical temperature (see Fig. 4), where the entropy gets maximal, condensate depletion in a nearly-ideal Bose gas is a direct evidence for spontaneous symmetry breaking from pure quantum fluctuations, as discussed in the sequel of the present theory. Because of spontaneous symmetry breaking, measurements of the experimental condensate fraction lead to the observation that the measured condensate fraction appears actually lower than the fraction expected from calculation for an ideal gas, also in the limit of vanishing interactions, indicating the condensate depletion

from symmetry breaking due to spontaneous quantum fluctuations which do not rely on coherent particle interactions in this parameter regime. For temperatures much larger than the critical temperature, the coherence time tends to zero at thermal equilibrium and thus complex time tends to converge to the characteristics of non-directional purely imaginary time obeying time-reversal symmetry in the classical limit.

It is the absolute value of (two-dimensional) complex time, which defines the entropy of the quantum particle in the Bose-Einstein condensate in units of the Boltzmann constant k_B . The finite value as well as the width of the entropy distribution indicates the coupling of several quantum states which do not belong to an exact classical Boltzmann equilibrium state, but are modified by the uncertainty and fluctuations of the particle's entropy in the atomic cloud at finite temperature. As a matter of fact, the real valued part of time t_1 is directly related to the fugacity ring spectrum in the complex plane, whereas the coherence time t_2 describes the coupling of quantum states in the symmetry breaking part of the fugacity spectrum. At the Boltzmann equilibrium, the distribution of entropy ideally occurs over quantum states in the neighborhood of $\text{Re}(z) = 1$ and $\text{Im}(z) = 0$ (fugacity equals one). Thus, the finite value of the entropy and its uncertainty indicates and quantifies the coupling of symmetric and asymmetric parts of the fugacity spectrum, which thus appear only structurally unconnected in the limit close above the critical temperature. Finally, the different shape of the entropy distribution from a flat distribution with a full range from $\text{Re}(z) = 0$ to $\text{Re}(z) = 1$ to a Gaussian distribution for conditional Boltzmann probabilities indicates that ergodicity leads to finite coherence times in the Boltzmann limit of maximum entropy, ensuring the gauge of the quantum field to be a sufficient condition for Bose-Einstein condensation.

V. CONCLUSION

In conclusion, using a representation of two-dimensional time, the coherence time of a quantum particle in a Bose-Einstein condensate is on the order of few hundred microseconds in the typical parameter regime of Bose gases at thermal equilibrium in harmonic traps below the critical temperature. In the presented quantum field theory, the definition of a two-dimensional complex time arises naturally from the constraint of particle number conservation, mathematically defined by complex poles of an intrinsic equation for the fugacity (and the chemical potential) of the Bose-Einstein condensate. Within the presented model, gauging the quantum field in the Boltzmann equilibrium turns out to be a necessary and sufficient condition for spontaneous symmetry breaking for a nearly-ideal Bose gas. Moreover, it is possible to argue and numerically verify that the process of sponta-

neous symmetry breaking is induced by time and entropy fluctuations from the reduction of the atomic density below the critical density for Bose-Einstein condensation, which couples (unconnected) symmetric and asymmetric parts of the fugacity spectrum. In the classical limit, the fluctuations of entropy tends to be distributed around a ring in the complex spectrum of the fugacity, recovering time reversal symmetry of the quantum field.

Starting from the analysis of this article and the works presented in [7, 8], it is interesting to further apply the formalism of this quantum field theory for the description of interference of two weakly coupled Bose-Einstein condensates, as well as to better understand properties of weak localisation in the interference signal of the interacting atomic cloud with the condensate part of the gas, especially in case studies, where the external potential is (additionally) disordered and interactions in terms of the s-wave scattering length are included in the model for numerical calculations. Interesting initial discussions on complex time with Dominique Delande and Benoît Grémaud, current work on entropy models with Hannes Lüling, Karsten Miermans and Franz Elsner from hema.to GmbH, as well as useful comments on the manuscript by Malte Tichy from Blue Yonder Inc., is acknowledged.

-
- [1] C. Cohen-Tannoudji, J. Dupont-Roc, G. Grynberg, 1998, Atom-Photon Interactions: Basic Process and Applications, Savoirs Actuels, Editions du CNRS Paris
 - [2] W. Nolting, 2006, Grundkurs Theoretische Physik 1 : Klassische Mechanik (Springer-Lehrbuch) ,
 - [3] Roy J. Glauber, 1963, The Quantum Theory of Optical Coherence **130**, 2529
 - [4] M. R. Andrews, C. G. Townsend, G. Miesner, D., S. Durrfee, D. M. Kurn, W. Ketterle, 1997, Science, 275, 637-641
 - [5] Y. Castin, S. Sinatra, 2018, Comptes Rendus Physique 19, Temps de cohérence d'un condensat de Bose-Einstein dans un gaz isolé harmoniquement piégé
 - [6] A. A. Abrikosov, L. P. Gorkov, I. E. Dzyaloshinski, 1963, Courier Dover Publications 130,
 - [7] A. Schelle, 2017, Fluctuations and Noise Letters 16, Spontaneously broken gauge symmetry in a Bose gas with constant particle number, 1
 - [8] A. Schelle, T. Wellens, D. Delande, A. Buchleitner 2011, Phys. Rev. A 83, Number-conserving master equation theory for a dilute Bose-Einstein condensate
 - [9] Stephen W. Hawking, 1975, Commun. Math. Phys. 43, Particle creation by black holes, 199-220
 - [10] Stephen W. Hawking, 1978, Physical Review D. 18(6), Quantum gravity and path integrals (1978), 1747-1753
 - [11] W. Krauth, 2006, Oxford Master Series in Statistical, Computational, and Theoretical Physics 18(6), Quantum gravity and path integrals
 - [12] S. Elizur, 1975, Phys. Rev. D 12 3978-3982, Impossibility of spontaneously breaking local symmetries
 - [13] I. Dinov, V. Milen, 2021, Boston/Berlin: De Gruyter ISBN 9783110697803, Data Science - Time Complexity,

Inferential Uncertainty, and Spacekime Analytics

- [14] A. Tenart, G. Hercé, JP. Bureik, Nat. Phys. 17 1364–1368 (2021), Observation of pairs of atoms at opposite momenta in an equilibrium interacting Bose gas
- [15] A. Sinatra, Y. Castin, C. Lobo, Journal of Modern Optics 47 (2000), A Monte Carlo formulation of the Bogolubov theory