Kibble-Zurek mechanism at exceptional points

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Defect production for a ramp across a quantum critical point is described by the Kibble-Zurek mechanism, finding applications in diverse fields of physics. Here we focus on its generalization to a ramp across an exceptional point (EP) of a non-hermitian Hamiltonian. While adiabatic time evolution brings the system into an eigenstate of the final non-hermitian Hamiltonian, a PT-symmetric ramp to an EP or a passage from PT-symmetric to broken PT-symmetric states through an EP, produces a defect density scaling as $\tau^{-(d+z)\nu/(z\nu+1)}$ in terms of the usual critical exponents and $1/\tau$ the speed of the drive. Defect production is suppressed compared to the conventional hermitian case as the defect state can decay back to the ground state close to the EP; formally, only the part of the instantaneous excited state perpendicular to the ground state contributes to defect formation. The new scaling should in principle be accessible in a system described by a Lindblad master equation with an additionally imposed continuous measurement.

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Introduction. The Kibble-Zurek mechanism of universal defect production represents a paradigmatic phenomenon in nonequilibrium many-body physics [1–4]. While the adiabatic theorem ensures, that a system can follow its ground state upon dynamically changing a Hamiltonian parameter sufficiently slowly as long as the spectrum remains gapped, this is not anymore the case when crossing a continuous phase transition[5]. As a consequence, excitations are generated and the number of defects n exhibits universal behavior with a scaling that is determined solely by the universality class of the underlying phase transition:

$$n \sim \tau^{-d\nu/(z\nu+1)} . \qquad (1)$$

Here, $1/\tau$ denotes the rate at which the parameter is dynamically varied, d the spatial dimension, and ν as well as z the correlation length and dynamical critical exponent, respectively.

The Kibble-Zurek mechanism in quantum many-body systems is a consequence of the unitary character of its real-time evolution. However, recent developments suggest rich features appearing for non-hermitian Hamiltonians [6–9] impossible to realize within conventional unitary dynamics, which has been motivated also by experiments that have reported quantum dynamics with such Hamiltonians[10, 11]. While the eigenvalues of a non-hermitian Hamiltonian can still be interpreted as energy bands, already the meaning of its eigenvectors cannot be treated conventionally way as they are not orthogonal, and therefore possess finite overlap in the absence of any additional perturbation. Particularly important in this context are exceptional points [12] (EPs), where the complex spectrum becomes gapless, which can be regarded as the non-hermitian counterpart of conventional quantum critical points[5]. At EPs, two (or more) complex eigenvalues and *eigenstates* coalesce, which then do not form a complete basis.

In this work we study the defect production at EPs upon slowly changing a system parameter in the spirit of the Kibble-Zurek mechanism. We identify a channel for defect suppression unique to EPs which is absent for Hermitian dynamics. Due to non-orthogonality of wavefunctions, only a small fraction of the excited state, which points perpendicular to the ground state, accounts for defect production. Remarkably, however, we find that the number of defects n, differs from the unitary Kibble-Zurek result in Eq. (1) albeit still obeys universal behavior with d in Eq. (1) replaced by the modified effective dimension $d_{\text{eff}} = d + z$, involving also the dynamical critical exponent associated with the EP. As a consequence, defect production is reduced but still obeys a scaling law.

We study the defect production for a set of different, but complementary, protocols of parameter sweeps, which allow us to address different aspects of defect production in non-hermitian systems. We interpret our results in terms of a physical realization of our studied dynamics through a Lindblad master equation with an additionally imposed continuous measurement.

The model and observables. We consider Hamiltonians of the form [13–16]

$$H = \sum_p H_p, \quad H_p = p\sigma_x + \Delta\sigma_y + i\Gamma\sigma_z \qquad (2)$$

which can be decomposed into different momentum sectors p. For $\Gamma = 0$ the problem is Hermitian. For nonzero Γ the above Hamiltonian becomes non-hermitian with a spectrum given by $E_\pm(p) = \pm\sqrt{p^2 + \Delta^2 - \Gamma^2}$. When $\Delta > \Gamma$, H_p is PT-symmetric with real eigenvalues [8] for each p. For $\Gamma > \Delta$ on the other hand H_p has, in general, complex eigenvalues, at sufficiently large $p > \sqrt{\Gamma^2 - \Delta^2}$, however, the spectrum becomes real again. Non-Hermitian Hamiltonians of the kind in Eq. (2) can be emulated by optical waveguides[6, 10, 14], distributed-feedback structures[17], microcavities[11] or electric circuits[18]. Eq. (2) accounts for the low energy dynamics of the quantum Ising chain in an imaginary transverse field[19, 20] or an imaginary mass fermion system[16].

For the purpose of this work we consider time-dependent parameters $\Delta(t)$ and/or $\Gamma(t)$ yielding a Hamiltonian $H(t)$. Initially, before we start our parameter sweeps, we choose the Hamiltonian always to be Hermitian, i.e. $\Gamma = 0$, so that the initial condition as the *ground state* of the Hamiltonian is well-defined. At time $t = 0$ we start our time-dependent protocol over a time span τ. The time evolution follows from

$$i\partial_t|\Psi(t)\rangle = H(t)|\Psi(t)\rangle, \tag{3}$$

with $|\Psi(t)\rangle = \otimes_p|\Psi_p(t)\rangle$ for a given mode p. In general, the norm of the wave function is not conserved when time evolution is driven by a non-hermitian Hamiltonian, so that an additional prescription for performing measurements in such states has to be given. When interpreting such dynamics as a result of dissipation in the framework of a Lindblad master equation with an additional continuous measurement, expectation values of an operator $\mathcal{O}$ have to be evaluated as[15, 21, 22]

$$\langle\mathcal{O}(t)\rangle = \frac{\langle\Psi(t)|\mathcal{O}|\Psi(t)\rangle}{\langle\Psi(t)|\Psi(t)\rangle}, \tag{4}$$

as we will discuss in more detail below. In the following we will quantify the defect production via

$$\langle\sigma_\alpha(t)\rangle = \frac{1}{N}\sum_p \sigma_\alpha(p), \;\; \sigma_\alpha(p) = \frac{\langle\Psi_p(t)|\sigma_\alpha|\Psi_p(t)\rangle}{\langle\Psi_p(t)|\Psi_p(t)\rangle} \tag{5}$$

with $\alpha = y, z$ and N denoting the number of considered momentum states.

Hermitian dynamics. We investigate several relevant scenarios for the time dependent Δ and Γ. Let us start with the hermitian Kibble Zurek mechanism, which requires $\Delta = 0$, $\Gamma = -i\Delta_0 t/\tau$. This is the conventional Landau-Zener problem[23, 24] starting exactly from the critical point, thus it represents only a half crossing. This yields $\langle\sigma_y(\tau)\rangle = 0$, while

$$\langle\sigma_z(\tau)\rangle + \frac{\Delta_0}{\pi}\ln(2W/\Delta_0) \sim \tau^{-1/2}, \tag{6}$$

with W the high energy cutoff. Defect production is effective when the adiabatic condition is violated[25, 26], namely when $d\ln\Delta/dt \sim \Delta$. This gives the transition time $\tau^{1/2}$, and the defect density scales inversely with this. This also follows from the scaling behaviour of the matrix element

$$\sigma_z(p,\tau) = \sigma_z^{eq}(p) + f_{LZ}\left(\frac{p}{\Delta_0}(\tau\Delta_0)^{1/2}\right), \tag{7}$$

which is typical for the hermitian Landau-Zener problem, $\sigma_z^{eq}(p) = -\Delta/\sqrt{p^2+\Delta^2}$, $f_{LZ}(x)$ is a universal scaling function in the near-adiabatic limit, which decays exponentially with x. For a general quantum critical point, the momentum resolved defect density is

$$n(p,\tau) = \tilde{f}_{LZ}\left(p^z\tau^{z\nu/(z/nz+1)}\right), \tag{8}$$

and the defect density follows the Kibble-Zurek scaling[1, 2] as

$$n(\tau) = \int d^dp\; n(p,\tau) \sim \tau^{-d\nu/(z\nu+1)} \tag{9}$$

with d, z and ν being the spatial dimension, dynamical critical exponent and the exponent of the correlation length, respectively.

Gapless quench. $\Delta = \Gamma = \frac{\Delta_0}{2}t/\tau$. The eigenvalues of the Hamiltonian are always $\pm|p|$, irrespective of the value of Δ_0, therefore the system is critical. Nevertheless, the dynamics is non-trivial and as Δ and Γ evolves with time, defects are produced in spite of the fact that the instantaneous eigenvalues do not change. Eq. (3) can be solved exactly. Starting from the ground state at $t = 0$, the $p < 0$ solution is trivial and the wavefunction only picks up phase factor as $\exp(-ipt)[1,1]^T/\sqrt{2}$. On the other hand, the $p > 0$ ground state at $t = 0$ evolves to

$$\Psi_p(\tau) = \begin{bmatrix} 1 - \frac{i\Delta_0}{2p} \\ -1 - \frac{i\Delta_0}{2p}\end{bmatrix}\frac{\exp(ip\tau)}{\sqrt{2}} + \begin{bmatrix}1\\1\end{bmatrix}\frac{i\Delta_0\sin(p\tau)}{2\sqrt{2}p^2\tau}, \tag{10}$$

where the second term is generated by the time dependent protocol. For $\tau \to \infty$, this agrees with the right eigenfunction of the final Hamiltonian (up to normalization). Note only does the instantaneous eigenvalue remain unchanged, i.e. $E_\pm(p) = \pm|p|$, but also the time evolution is only sensitive to the instantaneous eigenenergies, namely the wavefunction contains $\exp(\pm ipt)$ exponential factors only. This is then used in Eq. (4) to yield $\langle\sigma_y(\tau)\rangle + \Delta_0\ln(2W/\Delta_0)/2\pi \sim \tau^{-2}$ and

$$\langle\sigma_z(\tau)\rangle \sim \tau^{-1}. \tag{11}$$

In this case, there is an adiabatic time evolution and the $\tau \to \infty$ solution coincides with the instantaneous expectation value after the time evolution. Since the instantaneous spectrum remains unchanged and gapless throughout, the above scaling originates entirely from the wavefunction itself and not from the usual argumentation of Kibble-Zurek scaling. This is analogous to quenching along a gapless line[27] by altering the velocity of critical excitations, though only the wavefunction changes in our case, the spectrum remains intact for all times.

In order to appreciate the role of wavefunction normalization in Eq. (4), we evaluated it without the denominator: $\langle\sigma_z(\tau)\rangle$ approaches a constant, while $\langle\sigma_y(\tau)\rangle \sim \ln(\tau)$, without any well defined adiabatic limit for $\tau \to \infty$.

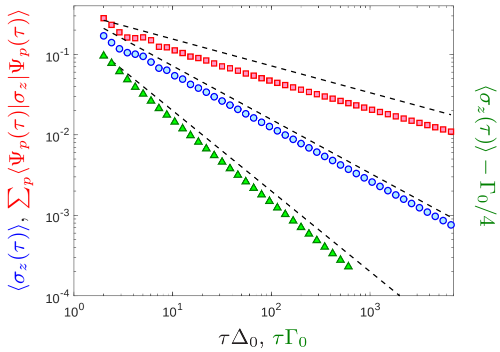

FIG. 1. The numerically determined defect density for normalized (blue circles) and unnormalized wavefunction (red squares) for the PT-symmetric ramp. We also show the defect density for the full non-hermitian drives as well (green triangles), measured from its adiabatic value. The black dashed lines denote the $\tau^{-1/3}$, $\tau^{-2/3}$ and τ^{-1} scaling. The cutoff $|p| < W = 10\Delta_0$ and $10\Gamma_0$ does not alter the dynamics, other values yield similar scaling.

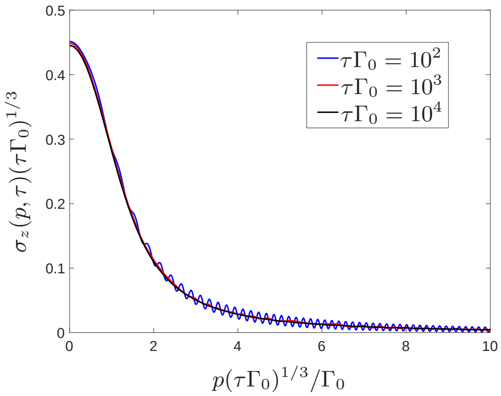

FIG. 2. The scaling of the numerically determined momentum resolved defect density, $f_{PT}(x)$ in the near adiabatic limit is shown for several values of τ for the PT-symmetric ramp.

PT-symmetric ramp. $\Delta = \Delta_0$, $\Gamma = \Delta_0 t/\tau$. This is the fully PT symmetric Kibble-Zurek, the time evolution ends exactly at the EP when the spectrum would become complex. The spin expectation values are $\langle\sigma_y(\tau)\rangle + \Delta_0\ln(W/\Delta_0)/\pi \sim \tau^{-2/3}$, and

$$\langle\sigma_z(\tau)\rangle \sim \tau^{-2/3}, \tag{12}$$

as shown in Fig. 1 from the numerics. The wavefunction for $t = \tau \to \infty$ agrees with the non-normalized right eigenfunction of the final non-hermitian Hamiltonian, similarly to the gapless quench.

The gap in the spectrum reads as $\Delta_0\sqrt{1 - t^2/\tau^2} \approx \Delta_0\sqrt{2}\sqrt{(\tau - t)/\tau}$ for $t \sim \tau$. The distance from the critical point is $\hat{t} = \tau - t$, which is used to obtain the critical exponents $z\nu = 1/2$ from the scaling of the gap[5], $\Delta \sim |\hat{t}|^{z\nu}$. Then, the transition time[25, 26] separating a/diabatic dynamics is determined from $\Delta^2 \sim d\Delta/d\hat{t}$, which gives the transition time $\hat{t}_{tr} \sim \tau^{1/3}$, in agreement with the Kibble-Zurek scaling[1, 2] $t_{tr} \sim \tau^{z\nu/(z\nu+1)}$. Note that similar critical exponents apply to the hermitian Rabi model as well[28] since the spectrum is the same for both cases.

Since the spectrum is gapless at the critical point as $|p|$, this defines $z = 1$, leaving us with $\nu = 1/2$ for the exponent of the correlation length. Therefore, the Kibble-Zurek scaling of the defect density in one dimension, $d = 1$ predicts $\sim \tau^{-d\nu/(z\nu+1)} = \tau^{-1/3}$ scaling. However, this exponent is different from Eq. (12). We demonstrate by inspecting the momentum resolved defect density, that the correct exponent is indeed 2/3 and its deviation from the Kibble-Zurek prediction arises from the wavefunction normalization. This sets the stage to generalize the Kibble-Zurek mechanism for a non-hermitian drive as well.

First of all, the numerical data indicates that the momentum resolved defect density, namely $\sigma_z(p,\tau)$, follows the scaling relation

$$\sigma_z(p,\tau) = \frac{1}{(\tau\Gamma_0)^{1/3}} f_{PT}\left(\frac{p}{\Gamma_0}(\tau\Gamma_0)^{1/3}\right) \tag{13}$$

with $f_{PT}(x)$ the universal scaling function shown in Fig. 2. Upon integrating this with respect to p by changing variable $x = p(\tau\Gamma_0)^{1/3}/\Gamma_0$, the $\tau^{-2/3}$ scaling of the defect density follows. This expression is also generalized to arbitrary critical point for the momentum resolved defect density as

$$n(p,\tau) = \frac{1}{\tau^{z\nu/(z\nu+1)}}\tilde{f}_{PT}\left(p^z\tau^{z\nu/(z\nu+1)}\right), \tag{14}$$

which, after performing a d-dimensional momentum integral, gives $n \sim \tau^{(d+z)\nu/(z\nu+1)}$. Notice the the prefactor of the scaling functions in Eqs. (13) and (14).

There are two complementary explanations for this modified scaling. In an a/diabatic picture, excitations are created by populating the excited state albeit only its perpendicular component to the ground state represents defect production. As we approach the EP, we enter into the diabatic regime at the transition time $\hat{t}_{tr}$ when the dynamics gets frozen and defect production kicks in. The perpendicular component of the excited state to the ground state at this instance is $\sin(\theta_p)$ with θ_p the angle the ground and excited states make as they are not orthogonal in general[29]. For Eq. (2), this is evaluated for small momentum states, living close to the EP, as $\sin(\theta_{p\approx 0}) = \sqrt{\Delta^2 - \Gamma^2}/\Delta \sim 1/\hat{t}_{tr}$ at the adiabatic-diabatic transition, namely the angle becomes proportional to the energy gap. This results in a $\tau^{-z\nu/(z\nu+1)}$ suppression factor for the defect density [30].

In a more dynamical picture, defects are created directly in the state perpendicular to the ground state, which decays to the ground state with a rate $1/\hat{t}_{tr}$, which reduces the hermitian Kibble-Zurek scaling by the probability to remain in the perpendicular state, $1/\hat{t}_{tr}$. At an

EP, there is only a single eigenstate and any other state decays to that[31]. Close to an EP, the state perpendicular to the ground state initially decays towards the ground state [32], which is followed by revival and periodic oscillation with frequency $E_+(p)$. However, when the driving rate, $\partial_t E_+(p)/E_+(p)$ is larger than the revival frequency, the system does not have enough time for revival and only the initial decay is probed. This gives the very same condition as the a/diabatic transition and the decay time is $\hat{t}_{tr}$.

Mathematically, the prefactor in Eq. (14) arises from the explicit normalization of the wavefunction as $d\langle\Psi_p(t)|\Psi_p(t)\rangle/dt = 2\Gamma(t)\langle\Psi_p(t)|\sigma_z|\Psi_p(t)\rangle$. States with large momentum p are hardly affected by the time dependent term and the corresponding wavefunction norm hardly changes. The low energy states are the most influenced by the non-hermitian and non-adiabatic time evolution. At short times ($t \ll \tau$), both the small matrix element and the $\Gamma(t)$ prefactor block its growth, but at a distance $\hat{t}_{tr}$ from the critical point, adiabaticity breaks down. Afterwards, diabatic time evolution takes place, and the norm of the wavefunction gets enhanced by $\hat{t}_{tr} \sim \tau^{1/3}$.

Full non-hermitian drive. $\Delta = 0$, $\Gamma = \Gamma_0 t/\tau$, which represents the non-hermitian Kibble-Zurek problem and is equivalent to quenching the imaginary tachyon mass[16]. The instantaneous spectrum contains EPs located at $|p| = \Gamma$ from the dispersion $E_\pm(p)$. By expanding it around the EP, the spectrum scales as $E_\pm(p \gtrsim \Gamma) = \pm\sqrt{2\Gamma}\sqrt{p-\Gamma}$ in the PT symmetric regime, and as $E_\pm(p \lesssim \Gamma) = \pm i\sqrt{2\Gamma}\sqrt{\Gamma - p}$ in the broken PT symmetry sector. Altogether the dynamical critical exponent is defined as $z = 1/2$, while $\nu = 1$. During the time evolution, all instantaneous eigenvalues are imaginary for $\Gamma(t) > |p|$ and real for $\Gamma(t) < |p|$ and are separated by an EP. This critical point is moved around in momentum space during the time evolution at a speed of Γ_0/τ, producing defects. This results in $\langle\sigma_y(\tau)\rangle = 0$ and

$$\langle\sigma_z(\tau)\rangle - \Gamma_0/4 \sim \tau^{-1}, \tag{15}$$

which is depicted in Fig. 1 from the numerical solution of Eq. (3). Here, it is even more crucial to properly normalize the wavefunction as in Eq. (4). Without the normalization, the spin expectation value changes exponentially in time due to the imaginary energy eigenvalues.

The numerically obtained adiabatic value for $\langle\sigma_z(\tau \to \infty)\rangle$ is also corroborated from diagonalizing the non-hermitian Hamiltonian analytically, and using its normalized right eigenfunction gives

$$\langle\sigma_z\rangle_{eq} = \frac{1}{2\pi}\int_{-\Gamma_0}^{\Gamma_0} dp \frac{\sqrt{\Gamma_0^2 - p^2}}{\Gamma_0} = \frac{\Gamma_0}{4}. \tag{16}$$

Only states with imaginary eigenvalue contribute ($|p| < \Gamma_0$). The PT-symmetric states ($|p| > \Gamma_0$) give zero contribution, since not only their eigenvalues but also the corresponding wavefunctions are real.

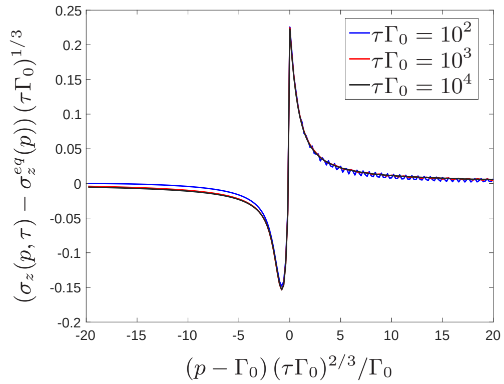

FIG. 3. The scaling of the numerically determined momentum resolved defect density, $f_{nh}(x)$ in the near adiabatic limit is shown for several values of τ for the full non-hermitian drive around the equilibrium EP.

By taking a closer look at near adiabatic dynamics, we calculate the momentum resolved spin expectation value, $\sigma_z(p,\tau)$ numerically, which is illustrated, together with its critical scaling, in Fig. 3. There is a clear difference in the contribution of states with imaginary or real instantaneous eigenvalues to this expectation value. This also differs significantly from the Landau-Zener transition probability of the corresponding hermitian system[3]. From the numerical data, the defect density obeys the scaling function

$$\sigma_z(p,\tau) = \sigma_z^{eq}(p) + \frac{1}{(\tau\Gamma_0)^{1/3}} f_{nh}\left(\frac{(p-\Gamma_0)^{1/2}}{\Gamma_0^{1/2}}(\tau\Gamma_0)^{1/3}\right) \tag{17}$$

with $f_{nh}(x)$ the corresponding universal scaling function, depicted in Fig. 3 and $\sigma_z^{eq}(p) = \mathrm{Re}\sqrt{1-(p/\Gamma_0)^2}$. Upon integrating this with respect to p, the $1/\tau$ scaling of the defect density follows.

This allows us to conjecture that for a general EP, the momentum resolved defect density satisfies the same form as Eq. (14), which agrees with Eq. (17) with $z = 1/2$. Therefore, the induced defect density vanishes as $\tau^{-(d+z)\nu/(z\nu+1)}$ upon traversing the EP adiabatically, similarly to the PT-symmetric ramp.

Concluding discussion. Throughout this work, the non-hermitian Hamiltonian was used for the evolution. From a more physical perspective such a dynamics can result from a Lindblad master equation in combination with a continuous measurement [15, 33, 34]. More specifically, consider a system, described just by the hermitian part of our Hamiltonian, coupled to an environment inducing a radiative decay in each of the individual two-level systems for each momentum p. In terms of a Lindblad equation this results in quantum jump operators equal to σ^-, which can be interpreted such that with a rate Γ the two-level system decays incoherently upon emitting a photon. Equivalently, one can map the Lindblad dynamics onto a quantum jump trajectory picture, where between quantum jumps, i.e., spontaneous emission events, the dynamics is solely given by the non-hermitian Hamiltonian in Eq. (2). Upon continuously monitoring the system such that we only consider those realizations, where no emission event has taken place, we end up with an evolution precisely captured by our non-hermitian Hamiltonian. Note, that this continuous measurement also ensures that the wave function is always properly normalized. In addition the measurement is, however, modifying the state of our system, since the absence of emission events, gradually forces the system over time towards the ground state of the two-level system. Otherwise an emission event becomes too likely [33]. While the anti-hermitian contribution therefore forces the system towards the ground state of the two-level system, the hermitian part counteracts this tendency by coherently transferring population back into the excited state. At an EP both of these processes compete most strongly to generate a nontrivial attractor of the dynamics, which is the single eigenstate of the non-hermitian system. This is the additional channel for defect annihilation that we identified in our analysis. Far away from the EP the hermitian part of the Hamiltonian dominates for our setups, so that the dynamics is almost fully coherent. A distinct non-hermitian Kibble Zurek scaling describing a different physical realization with different expectation value was studied in Ref. [35].

To sum up, the universal features of non-hermitian dynamics across EPs were investigated. We find that the adiabatic time evolution drives the initial wavefunction to a right eigenstate of the final non-hermitian Hamiltonian up to normalization, indicating that an adiabatic theorem probably exists for the systems under consideration. For a near adiabatic crossing of an EP, defects are produced, whose density obeys a generalized Kibble-Zurek scaling as $\tau^{-(d+z)\nu/(z\nu+1)}$. The modified exponent arises from the non-orthogonality of the wavefunctions, where only the perpendicular component (to the ground state) of the excited state accounts for defect production.

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[1] T. W. B. Kibble, *Topology of cosmic domains and strings*, J. Phys. A **9**, 1387 (1976).
[2] W. H. Zurek, *Cosmological experiments in superfluid helium?*, Nature **317**, 505 (1985).
[3] J. Dziarmaga, *Dynamics of a quantum phase transition: Exact solution of the quantum ising model*, Phys. Rev. Lett. **95**, 245701 (2005).
[4] A. Polkovnikov, K. Sengupta, A. Silva, and M. Vengalattore, *Colloquium : Nonequilibrium dynamics of closed interacting quantum systems*, Rev. Mod. Phys. **83**, 863 (2011).
[5] S. Sachdev, *Quantum Phase Transitions* (Cambridge Univ. Press, Cambridge, 1999).
[6] R. El-Ganainy, K. G. Makris, M. Khajavikhan, Z. H. Musslimani, S. Rotter, and D. N. Christodoulides, *Non-hermitian physics and pt symmetry*, Nat. Phys. **14**, 11 (2018).
[7] I. Rotter and J. P. Bird, *A review of progress in the physics of open quantum systems: theory and experiment*, Rep. Prog. Phys. **78**, 114001 (2015).
[8] C. M. Bender and S. Boettcher, *Real spectra in non-hermitian hamiltonians having $\mathcal{PT}$ symmetry*, Phys. Rev. Lett. **80**, 5243 (1998).
[9] M. V. Berry, *Physics of nonhermitian degeneracies*, Czech. J. Phys. **54**, 1039 (2004).
[10] J. M. Zeuner, M. C. Rechtsman, Y. Plotnik, Y. Lumer, S. Nolte, M. S. Rudner, M. Segev, and A. Szameit, *Observation of a topological transition in the bulk of a non-hermitian system*, Phys. Rev. Lett. **115**, 040402 (2015).
[11] T. Gao, E. Estrecho, K. Y. Bliokh, T. C. H. Liew, M. D. Fraser, S. Brodbeck, M. Kamp, C. Schneider, S. Höfling, Y. Yamamoto, F. Nori, Y. S. Kivshar, *et al.*, *Observation of non-hermitian degeneracies in a chaotic exciton-polariton billiard*, Nature **526**, 554 (2015).
[12] W. D. Heiss, *The physics of exceptional points*, J. Phys. A: Math. Theor. **45**, 444016 (2012).
[13] G. Dattoli, A. Torre, and R. Mignani, *Non-hermitian evolution of two-level quantum systems*, Phys. Rev. A **42**, 1467 (1990).
[14] T. E. Lee, *Anomalous edge state in a non-hermitian lattice*, Phys. Rev. Lett. **116**, 133903 (2016).
[15] H. Carmichael, *An Open Systems Approach to Quantum Optics* (Springer-Verlag, Berlin, 1993).
[16] T. E. Lee, U. Alvarez-Rodriguez, X.-H. Cheng, L. Lamata, and E. Solano, *Tachyon physics with trapped ions*, Phys. Rev. A **92**, 032129 (2015).
[17] S. Longhi, *Optical realization of relativistic non-hermitian quantum mechanics*, Phys. Rev. Lett. **105**, 013903 (2010).
[18] T. Stehmann, W. D. Heiss, and F. G. Scholtz, *Observation of exceptional points in electronic circuits*, J. Phys. A: Math. Gen. **37**, 7813 (2004).
[19] C. Li, G. Zhang, X. Z. Zhang, and Z. Song, *Conventional quantum phase transition driven by a complex parameter in a non-hermitian $\mathcal{PT}$* − symmetric *ising model*, Phys. Rev. A **90**, 012103 (2014).
[20] T. Deguchi and P. K. Ghosh, *The exactly solvable quasi-hermitian transverse ising model*, J. Phys. A: Math. Theor. **42**, 475208 (2009).
[21] Y. Ashida and M. Ueda, *Full-counting many-particle dynamics: Nonlocal and chiral propagation of correlations*, Phys. Rev. Lett. **120**, 185301 (2018).
[22] E. M. Graefe, H. J. Korsch, and A. E. Niederle, *Mean-field dynamics of a non-hermitian bose-hubbard dimer*, Phys. Rev. Lett. **101**, 150408 (2008).

[23] L. D. Landau, *A theory of energy transfer ii*, Phys. Z. Sowjet. **2**, 46 (1932).
[24] G. Zener, *Non-adiabatic crossing of energy levels*, Proc. R. Soc. London, Ser. A **137**, 696 (1932).
[25] C. De Grandi, V. Gritsev, and A. Polkovnikov, *Quench dynamics near a quantum critical point*, Phys. Rev. B **81**, 012303 (2010).
[26] A. Polkovnikov, *Universal adiabatic dynamics in the vicinity of a quantum critical point*, Phys. Rev. B **72**, 161201 (2005).
[27] U. Divakaran, A. Dutta, and D. Sen, *Quenching along a gapless line: A different exponent for defect density*, Phys. Rev. B **78**, 144301 (2008).
[28] M.-J. Hwang, R. Puebla, and M. B. Plenio, *Quantum phase transition and universal dynamics in the rabi model*, Phys. Rev. Lett. **115**, 180404 (2015).
[29] M. V. Berry and R. Uzdin, *Slow non-hermitian cycling: exact solutions and the stokes phenomenon*, Journal of Physics A: Mathematical and Theoretical **44**, 435303 (2011).
[30] For the hermitian case, orthogonality ensures that $\sin(\theta) = 1$.
[31] M. V. Berry, *Pancharatnam, virtuoso of the poincaré sphere: an appreciation*, Current Science **67**, 220 (1994).
[32] An unnormalized state perpendicular to the ground state evolves as $|\perp(t)\rangle = 2i\cot(\theta_p)\sin(E_+(p)t)|GS\rangle + \exp(iE_+(p)t)|\perp\rangle$ due to a time independent non-hermitian Hamiltonian, where θ is the angle between the ground state and the excited state, $\langle GS|\perp\rangle = 0$ and θ_p vanishes at EP. For $t \sim \pi/2E_+(p)$, it develops a large component parallel to the ground state with length $\cot(\theta_p)$.
[33] A. J. Daley, *Quantum trajectories and open many-body quantum systems*, Advances in Physics **63**, 77 (2014).
[34] Y. Ashida, S. Furukawa, and M. Ueda, *Parity-time-symmetric quantum critical phenomena*, Nat. Commun. **8**, 15791 (2017).
[35] S. Yin, G.-Y. Huang, C.-Y. Lo, and P. Chen, *Kibble-zurek scaling in the yang-lee edge singularity*, Phys. Rev. Lett. **118**, 065701 (2017).