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Predictions of the pseudo-complex theory of Gravity for EHT observations: II. Theory and robust predictions

P. O. Hess,[1,4]★ Th. Boller,[2,4] A. Müller[3], and H. Stöcker[4]
[1] Instituto de Ciencias Nucleares, UNAM, Circuito Exterior, C.U., A.P. 70-543, 04510, Mexico D.F., Mexico
[2] Max-Planck Institute for Extraterrestial Physics, Giessenbachstrasse, D-85748 Garching, Germany
[3] Excellence Cluster Universe, Technical University Munich, Giessenbachstrasse 1, 85741 Garching, Germany
[4] Frankfurt Institute for Advanced Studies, Johann Wolfgang Goethe Universität, Ruth-Moufang-Str.1, 60438 Frankfurt am Main, Germany

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ABSTRACT

We present a resumé on the modified theory of gravity, called *pseudo-complex General Relativity* (pc-GR). The consequences and predictions of this theory will be discussed in the context of the possible observational results of the *Even Horizon Telescope*, expected soon. We compare our results to other recently published contributions. Our main result is that the modified theory predicts a dark ring followed by a bright ring in the emissivity profile of the accretion disc. Even when no disc exists, the same features will be present for a cloud of gas falling in. The results depend only on the orbital frequency of a particle in a circular orbit, as a function in r and do not depend on the type of accretion disc, i.e., the predictions are robust. We conclusively demonstrate that there is no light ring, defined as a photon geodesic orbit, and different from the orbits of particles, present for a spin parameter a larger than 0.5 m.

Key words: General Relativity – Event Horizon Telescope – Sgr. A*

1 INTRODUCTION

In our first contribution (Boller 2019, hereafter Paper I) we presented observational predictions of a modified theory of Gravity. In the present contribution, the theory is explained in greater detail.

General Relativity (GR) has passed many observational tests Will (2006) in weak gravitational fields. The first observation of gravitational waves Abbott (2016) can be considered as a first test in the strong gravitational field. Recently bright spots near the black hole in SrgA* were observed GRAVITY (2018) and simulations, assuming that GR is still valid, indicate that they appear close to the *Innermost Stable Circular Orbit* (ISCO), at six times the gravitational radius, though the resolution limit is about 100μas. The *Event Horizon Telescope* EHT (2016) collaboration are expected to publish their results on the observation of SgrA* and the black hole at the center of M87 in 2019. In Giddings (2018) the oscillations of a black hole are investigated as a consequence of quantum fluctuations. Distortions of the black hole shadow and the shape of a light ring are discussed. They predict that a bright light ring should be present near the event horizon. In Schönenbach (2013, 2014), based on Hess (2009), it is assumed that any mass provokes vacuum fluctuations nearby, as explained in the book by Birrell (1982), describing the quantization up to one-loop gravity. In Visser (1996) these quantum fluctuations were calculated within the Schwarzschild metric, with no re-coupling to the background metric. The fluctuations explode at the Schwarzschild radius. In Schönenbach (2013, 2014) the vacuum fluctuations are treated in a phenomenological way, assuming a particular increase with the mass, such that it is finite at the Schwarzschild radius. The consequences for the light emission of accretions discs were computed in Schönenbach (2014). Their main result the that a dark ring exists in addition to a bright inner one in the emissivity profile of the accretion disc. It is a consequence of the dependence of the angular velocity of point particles in a circular orbit on the distance and will be explained in the next section. This feature is a consequence of vacuum fluctuations, but not of oscillations of a black hole, as proposed in Giddings (2018). The theory in Schönenbach (2013, 2014) is static. Also the recent observation of bright spots (GRAVITY 2018) can be reinterpreted such that the circular orbit is near $\frac{4}{3}$ of the gravitational radius (m). The theory is phenomenological, it treats the quantum fluctuations as an asymmetric perfect fluid, and involves a parameter describing the coupling of matter to the vacuum fluctuations,

★ E-mail: hess@nucleares.unam.mx

also called *dark-energy*. The theory will be briefly summarized in this contribution. Though it is phenomenological, it may shed some light on the effects of quantization on the standard theory. We will also show that there is not always a light ring in pc-GR present for a large spin parameter a.

Finally, we point out a different path to extend the standard theory, using canonical transformations (Struckmeier 2017). The Einstein equation has a contribution which represents the known form but also has additional terms which might represent the contribution of the energy-momentum tensor in pc-GR.

The paper is organized as follows: In Section 2 the main ingredients of the extended theory are presented, the implications for particles in a circular orbit are discussed as well as the accretion disc emission profiles. After that, a simulation of a gas clouds falling into the black hole is presented. For any emitting material a dark ring followed by a bright one will be one of the main observable results, which will be presented in section 3 for SgrA*. The properties of the emissivity profile of the accretion disc as predicted by pc-GR may be visible in EHT observations. In this case, the pc-GR theory can tested directly via the observational tests discussed in Paper I.

2 THE THEORY, CONSEQUENCES AND PREDICTIONS

In Hess (2009) the pc-GR was introduced, which is briefly described in the following sections, for more details please see for example Hess (2015): The theory extends the real space-time components to the so-called pseudo-complex ones, namely $x^\mu \rightarrow X^\mu = x^\mu + I y^\mu$, with $I^2 = 1$. That this is the only viable algebraic extension for the coordinates was shown in Kelly (1986), because only in this extension there are no ghost and/or tachyon solutions. Any function (or set of coordinates) can be expressed in terms of the so-called zero-divisor basis $\sigma_\pm = \frac{1}{2}(1 \pm I)$, i.e., $F(X) = F_+(X_+)\sigma_+ + F_-(X_-)\sigma_-$ and $X^\mu = X_+\sigma_+ + X_-\sigma_-$. Because $\sigma_-\sigma_+ = 0$, the two zero-divisor components commute. One obtains the Einstein equations for each component, i.e. $G^\pm_{\mu\nu} - \frac{1}{2} g^\pm_{\mu\nu} R^\pm = 8\pi T^\pm_{\mu\nu}$ $(c = G = 1)$. In each component the theory appears the standard structure of a theory of General Relativity. all principles and symmetries remain satisfied. The connection is obtained requiring that the orbit of a particle is real (ds^2 is real), which introduces a constraint whose solution (Hess 2017) requires an energy-momentum tensor of an anisotropic ideal fluid. In the original version of the pc-GR (Hess 2009), a modified variation principle was proposed, which is not necessary when the constraint is taken into account. Finally, the equations are mapped to their real part.

The appearance of a non-zero energy-momentum tensor implies the conjecture that *the mass not only curves space-time but also changes vacuum properties*, as is in fact shown in semi-classical calculations (Birrell 1982; Visser 1996). In Visser (1996) the density of vacuum fluctuations behave proportional to $1/\left(1 - \frac{2m}{r}\right)^2$, which is singular at the Schwarzschild radius. Because the calculations do not include back-reaction effects of the vacuum fluctuations to the metric, the behavior near the horizon is not physical and requires a modification. A full quantized theory should include this effect automatically. Due to the lack of a functional quantized theory of gravity we are left to treat it in a phenomenological way, assuming for the dark energy a non-isotropic perfect and classical fluid (Schönenbach 2014). The dark energy density is modeled by a $\frac{B}{8\pi r^5}$ dependence, which falls off strong enough in order not to be detected yet by solar system observations. The r-dependence can be questioned and other dependencies are possible, as discussed in Nielsen (2018), where pc-GR where tested on the results of gravitational wave events (as discussed in Hess (2016)). The parameter B is chosen such that the metric component g_{00} does not become zero, thus no event horizon appears in this theory. The reasoning for this choice is twofold: i) At the Schwarzschild radius the gravitational field is extremely strong, compared to the field in the solar system, and one has to count on the possibility that GR has to be extended in order to include further effects (like quantum effects). ii) The second reason is of philosophical nature and depends on the eye of the beholder, namely that the event horizon in GR excludes part of the space to be accessible to a *nearby observer* in the exterior and has several, for us, undesirable consequences like the information paradox. Both observations indicate that maybe GR reaches its limits or rather that something is missing. Therefore, our motivation to investigate what happens when there is no event-horizon for a so-called black hole.

Recent presentations of pc-GR are given in Schönenbach (2014); Hess (2017) who apply pc-GR to thin, optically thick accretion discs. Though a thin optical disc is not always realized (except perhaps around SgrA*), the effects discussed in this contribution apply equally to other disc models, which is also important for the black hole in the center of M87.

The modified Kerr metric is given by Schönenbach (2014)

$$
\begin{aligned}
g^{\mathrm{K}}_{00} &= \frac{r^2 - 2mr + a^2\cos^2\vartheta + \frac{B}{2r}}{r^2 + a^2\cos^2\vartheta} \\
g^{\mathrm{K}}_{11} &= -\frac{r^2 + a^2\cos^2\vartheta}{r^2 - 2mr + a^2 + \frac{B}{2r}} \\
g^{\mathrm{K}}_{22} &= -r^2 - a^2\cos^2\vartheta \\
g^{\mathrm{K}}_{33} &= -(r^2 + a^2)\sin^2\vartheta - \frac{a^2\sin^4\vartheta\left(2mr - \frac{B}{2r}\right)}{r^2 + a^2\cos^2\vartheta} \\
g^{\mathrm{K}}_{03} &= \frac{-a\sin^2\vartheta \;\; 2mr + a\frac{B}{2r}\sin^2\vartheta}{r^2 + a^2\cos^2\vartheta} \quad ,
\end{aligned}
\tag{1}
$$

where the index K refers to *Kerr*. For $B = 0$ the Kerr solution of GR is obtained. For $B > \frac{64}{27}\, m^3$ there is no event horizon, though for practical purposes we just use $B = \frac{64}{27}\, m^3$ which gives a zero for the minimal value of g^{K}_{00} at r equal to two thirds of the Schwarzschild radius.

The rotational parameter a is in units of m and ranges as usual from $0m$ to $1m$. Often the rotational parameter in observations is redefined without units. One distinct feature of the extended theory is that the orbital frequency of a particle in a circular orbit is always lower than in GR, though noticeable differences only appear near the Schwarzschild radius (Schönenbach (2014). Furthermore, the orbital frequency shows a maximum at $r = 1.72\ m$, which is independent of the value of a, after which it falls off toward the center and reaches zero at r approximately $\frac{2}{3}$ of the Schwarzschild radius, where the surface of the star is estimated to be.

This observation is a consequence of what is depicted in Fig. 1 where the effective potential is plotted versus the radial distance. At small r the potential is repulsive, similar to the Yukawa potential in nuclear physics, and as in nuclear physics this marks the size of the system. As a consequence, the radius of the so-called black hole will be smaller than in the Schwarzschild case ($a = 0$) and larger in the case of a Kerr black hole. The fall off is explained by the increase of the dark energy, which effectively reduces the gravitational constant (it is a function in r, see also (Hess 2015). In Schönenbach (2014) it is shown that for a $\lesssim 0.4$ m the last stable orbit is at a lower value of r but follows approximately the behavior of GR (see Fig. 2 for a more detailed explanation). Because particles can now reach further inside, more gravitational energy is released, and as a consequence the disc appears to be brighter. Above $a = 0.4\ m$ stable orbits are possible up to the surface. Circular stable orbits are possible up to the maximum of the orbital frequency. Because at the maximum the gradient in the orbital frequency of neighboring orbits is small, less friction is present, resulting in less heating. Thus, a dark ring appears near the position $r = 1.72\ m$. Further inside, the gradient becomes significantly stronger and a bright ring occurs (see Paper I). Note, that these features are independent on the detailed structures of the disc model.

We use the model proposed by Page (1974) for the accretion disc. It assumes a thin disc which is optically thick at the equator and the loss of energy is only through the emission of light. However, we acknowledge that other accretion disc models may also be valid. The model exploits the laws of conservation of mass, energy, and angular momentum. Properties such as the viscosity must be implicitly included in the parameters of the model (for example the accretion rate). This has to be extracted from somewhere else.

Though the model was applied exclusively to GR up to now, it is easy to extend it to other metrics, such as the one used in pc-GR. This is because the procedure discussed in Page (1974) is formulated in such a way that it is independent of the kind of metric that is used. For the simulation we use the GYOTO routine Vincent (2011), which applies the ray tracing method and allows the use of several models, including that used by Page and Thorne, permitting an arbitrary metric. In Vincent (2011) the theoretical background of the ray tracing technique is clearly described. It is also possible to change the subroutines in GYOTO to be consistent with the model of Page (1974). Such an extension for pc-GR is available in Schönenbach (2014). When the model of Page (1974) is not applicable and other models have to be used, the differences to GR, nevertheless, will be similar in structure because the effects only depend on the orbital frequency as a function in r.

Under the hypothesis that $B = \frac{64}{27}m^3$, simulations of accretion discs have been performed in Schönenbach (2014); Hess (2015). However, in these works, the total flux was calculated in *bolometric units*, the mass was normalized to the mass of the black hole in the Galactic Center, and the accretion rate was set to an arbitrary value. The EHT on the other hand observes well defined objects at a fixed wavelength of approximately 1.2 mm which corresponds to a frequency of 250 GHz EHT (2016). Also the field of view was chosen arbitrarily and the temperature of the disc near the inner edge has not been evaluated.

The results of the simulation depends on several well determined parameters, such as the accretion rate, the rotational parameter a, the field of view, and the mass of the black hole. The specific uncertainties for these parameters are calculated in relative units. Nevertheless, the important part is the *relative comparison* to GR. In pc-GR the intensities are much larger than in GR (see Paper I).

We have performed simulations from 10^o to 80^o in steps of ten degrees. Nevertheless, here we present only a very restricted selection of the simulations that we have performed. The corresponding FITS files will be made available online for all orientations within GR and pc-GR[1].

For the remainder of this section, we would like to discuss the existence of a light ring, which is essential in the predictions made in Giddings (2018). A light ring is defined by a geodesic photon orbit ($ds^2 = 0$) and should not be confused with the orbit of a particle ($ds^2 > 0$). In addition, the light ring should also not be confused with the bright ring our theory predicts, which has a completely different origin. To this end we recommend Berti (2014), where in Eq. (2.4.74) the condition of a light ring is given. After changing the variable $y = \frac{r}{m}$ this condition reads

$$2g_{00}^K(y_c) \quad = \quad y_c(g_{00}^K(y_c))' \quad , \tag{2}$$

where the prime refers to the derivative with respect to y, and y_c is the critical value.

In Fig. 3 the result is plotted. While in the standard theory there is only one solution at $y > m$ (in fact there are three, one at negative y and a second at $y < m$, i.e., both at nonphysical distances), now we have for $B = \frac{64}{27}m^3$ two solutions at distances with a physical meaning. One, that starts at $y = \frac{4}{3}$ and increases for $a > 0$ and a second one which starts at $y = 2.54$ and decrease for $a > 0$. Both solutions meet at approximately $a = 0.55m$, after which there is *no light ring*. Because the black hole at SgrA* probably rotates with a value of $a > 0.5m$ Genzel (2003), in the modified theory there is no light ring. Thus, we predict that the light ring predicted by the standard theory of GR will not be observed in SgrA*, but rather a dark ring along with an inner bright ring.

3 SGR A*

The main parameters for Sgr A* are its mass and the accretion rate. The mass of the black hole is known to be about 4.3 million solar masses (Gillessen 2009) and an upper limit of the accretion rate is estimated in Quataert (1999) with the value of 8×10^{-5} solar masses per year. Under the supposition that an accretion disc forms around SgrA*, the same value for the accretion rate will be used. For SgrA* there is no proof for the existence of an accretion disc. In the case where no accretion disc exists, when a cloud approaches the black hole it will pass through regions where an accretion disc would show different intensities. We expect that the changes in intensity will be qualitatively reflected when the cloud approaches the black hole.

[1] https://vizier.u-strasbg.fr/viz-bin/VizieR

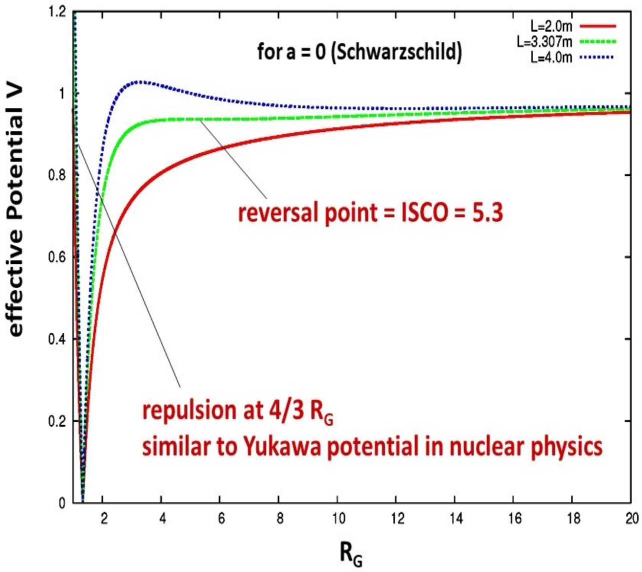

Figure 1. Effective potential in pc-GR for the case of a non-rotating black hole. The innermost stable orbit, which is given by the second derivative of the angular momentum of orbiting particles, is at 5.3 m. This value is lower than that predicted by the standard theory of GR, where the innermost stable orbit is 6.0 m. This difference is also shown in Fig. 2. An additional difference between pc-GR and standard GR is that the effective potential as predicted by pc-GR increases at distances below $\frac{4}{3}$ m. This is similar to the Yukawa potential in nuclear physics.

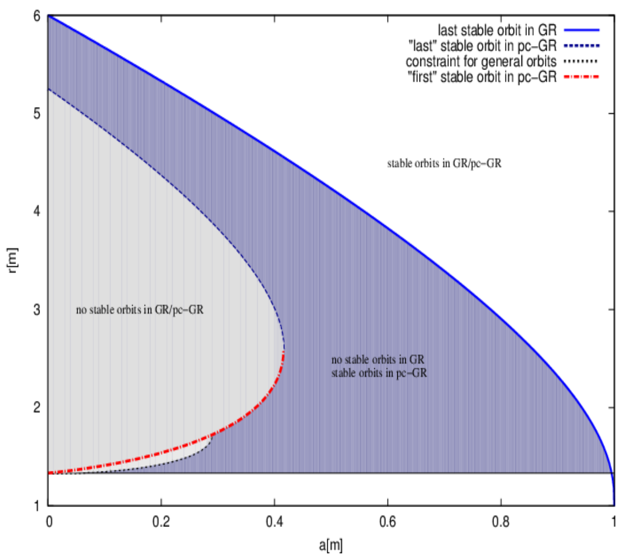

Figure 2. Distances of the last stable orbits versus spin as predicted by standard GR (dark blue) and pc-GR (lighter blue area). For standard GR the last stable orbit for $a = 0$ is at $r = 6m$, and for a maximally spinning black hole it is at $r = 1$ m. The grey area indicates the region of unstable orbits. For small a the pc-GR result follows the general trend of the standard GR result, but at smaller distances. For $a \gtrsim 0.44$, pc-GR permits stable orbits up to the surface. For more details see Schönenbach (2013); Hess (2015).

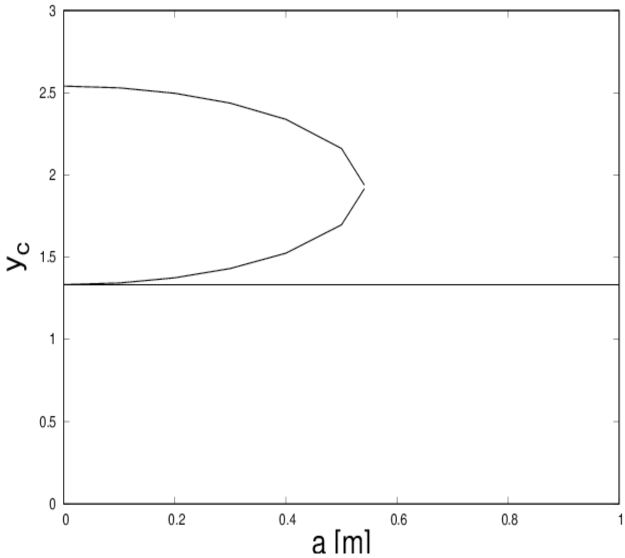

Figure 3. The position of the critical radius (position of the light ring), as a function of the spin parameter a. The variable y_c is in units of m. This figure was produced assuming $B = \frac{64}{27}$. Note, that for a limited range of a, there are two solutions for the light ring. After approximately $a = 0.55$ m the two solutions meet and there is no light ring for larger a. The horizontal line is at $y_c = \frac{4}{3}m$, i.e., two-thirds of the Schwarzschild radius. At this approximate position, pc-GR predicts the surface.

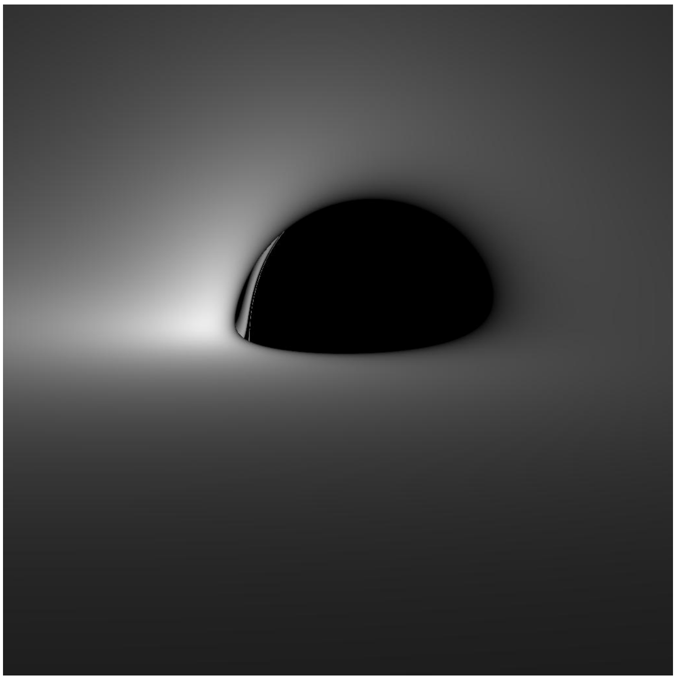

Figure 4. Counter clockwise rotating geometrically thin accretion disc around the rotating compact object SgrA* viewed from an inclination of 80^o. The figure shows the original disc model by Page and Thorne 1974, for the standard theory ($B = 0$) and at $a = 0.9$ m. The specific intensity is calculated at 250 GHz and normalized to the brightest spot in pc-GR for $a = 0.9$ m (see Figure 5).

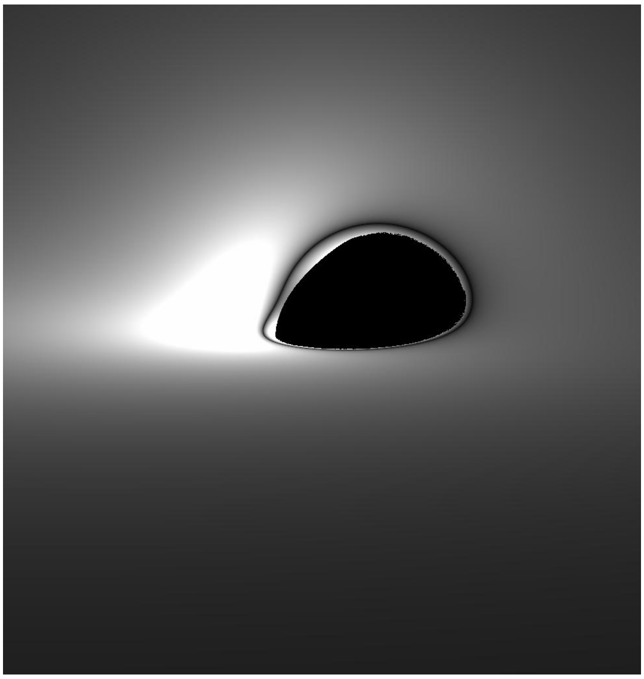

Figure 5. Counter clockwise rotating geometrically thin accr tion disc around the rotating compact object SgrA* viewed fro an inclination of 80^o. The figure shows the original disc mod by Page and Thorne 1974, for the pc-GR and at $a = 0.9$ m. Th specific intensity is calculated at 250 GHz. Note the appearance of an outer dark ring and an inner bright ring.

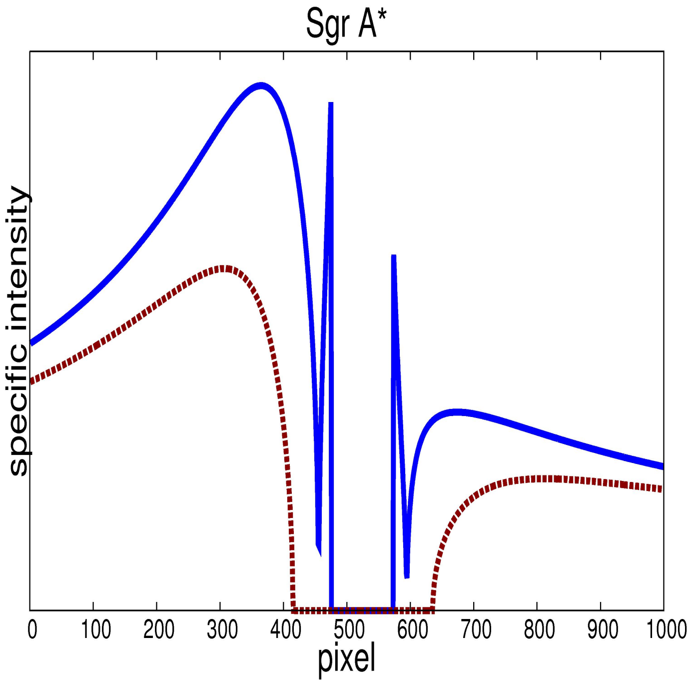

Figure 6. Comparison between standard GR (dashed red line) and pc-GR (solid blue line) of the intensities for SgrA*. Scales are not shown. These curves were calculated assuming $a = 0.9$ m. The inclination angle was set to 80^o. A dark ring is seen at zero intensity, while the bright ring is represented by a large peak further in. Due to resolution effects in the plot, the inner curve does not acquire an exact zero. For $a = 0.9m$, standard GR does not show sharp peaks in the predicted intensity. For pc-GR, the inner bright ring goes sharply to zero, approaching the surface. The horizontal axis corresponds to the range of approximately 22m. Compare this with the discussion of Fig. 3 in Paper I, where the time-dependent behaviour of the orbital motion of particles is discussed.

We assume a spatial resolution of 0.1 microarcsec over a field of 1000 pixels, resulting into a field of view of 100 μas.

The simulations provide figures for the *specific intensity* at the fixed frequency mentioned above. The unit for the specific intensity is erg $\mathrm{cm}^{-2}\mathrm{Hz}^{-1}\mathrm{sr}^{-1}$ (GYOTO manual 2015).

The version of the ray tracing routine we used, provided by Vincent (2011), developed numerical instabilities for very large distances. Because the specific intensity is independent of the distance, the simulations were performed at a distance of $1000m$. The field of view was correspondingly adjusted, which for SgrA $*$ at a distance of 1000 m is about 0.022 rad.

Figures 4 and 5 show the simulation for a disc at 80^o inclination angle for GR and pc-GR, respectively. As a rotational parameter we use $a = 0.9m$, taking into account that according to Genzel (2003) this parameter has to be larger than 0.5 m (in Genzel (2003) the rotational parameter is redefined without units).
This value is above 0.4 m, i.e., stable circular orbits exist up to the surface and therefore the appearance of the accretion disc will be very similar for all cases of $a < 0.5$ m to what is seen in Figures 4 and 5. Note, that the intensity in pc-GR is significantly larger than in GR. Also, at a distance (about 1.72 m) a dark ring occurs, which is a consequence of the maximum of the orbital frequency of a particle in a circular orbit Schönenbach (2013). Near this distance, friction is low (and therefore viscosity) and the emission of light reaches a minimum (see also Section 2). This is also noted in Figure 6, where the intensity is plotted versus the pixel number. The distance between two pixels corresponds to approximately 0.022m. The intensity reaches a minimum at a distance of 1.72 m and for lower radial distances the intensity increases again, which corresponds to the bright ring shown in Figure 5. In GR this behavior is not seen.

4 CONCLUSIONS

We summarized the basics of a modified theory of General Relativity. One of the main predictions is that the accretion disc emission will show both an outer dark ring and an inner bright ring. Though a thin accretion disc was assumed in this work, the results we obtained are robust and can also be applied for other types of emission. This is because these features depends only on the r-dependence of the orbital frequency of particles in a circular orbit. Another main difference is that in pc-GR the specific intensity is larger than in GR (see also the related discussion in Paper I).

Another prediction is that the surface is at approximately $\frac{4}{3}m$, i.e. in the Schwarzschild case ($a = 0$) the size of the black hole is smaller than in the standard theory and in the Kerr case with $a = 1m$ it is larger. In addition, for spin parameters $a > 0.55\,m$ pc-GR predicts that no light ring will be present.

These predictions and others (see also Paper I) should be verifiable by the *Event Horizon Telescope* and they are the same for the black hole in the center of M87.

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