№1

$x_1 = 1,\ p_1 = \frac{8}{12};\ x_2 = 2,\ p_2 = \frac{4}{12}\cdot\frac{8}{11};\ x_3 = 3,\ p_3 = \frac{4}{12}\cdot\frac{3}{11}\cdot\frac{8}{10};$

$x_4 = 4,\ p_4 = \frac{4}{12}\cdot\frac{3}{11}\cdot\frac{2}{10}\cdot\frac{8}{9};\ x_5 = 5,\ p_5 = \frac{4}{12}\cdot\frac{3}{11}\cdot\frac{2}{10}\cdot\frac{1}{9}\cdot\frac{8}{8}$

x_i	1	2	3	4	5
p_i	$\frac{2}{3}$	$\frac{8}{33}$	$\frac{4}{55}$	$\frac{8}{495}$	$\frac{1}{495}$

$\leftarrow$ Закон распред.

$$M(X) = \sum_i x_i p_i = 1\cdot\frac{2}{3} + 2\cdot\frac{8}{33} + 3\cdot\frac{4}{55} + 4\cdot\frac{8}{495} + 5\cdot\frac{1}{495} =$$

$$= \frac{13}{9}$$

$$D(X) = M(X^2) - (M(X))^2$$

$$(M(X))^2 = \frac{13^2}{9^2} = \frac{169}{81};$$

$$M(X^2) = \sum_i x_i^2 p_i = 1\cdot\frac{2}{3} + 4\cdot\frac{8}{33} + 9\cdot\frac{4}{55} + 16\cdot\frac{8}{495} + 25\cdot\frac{1}{495} =$$

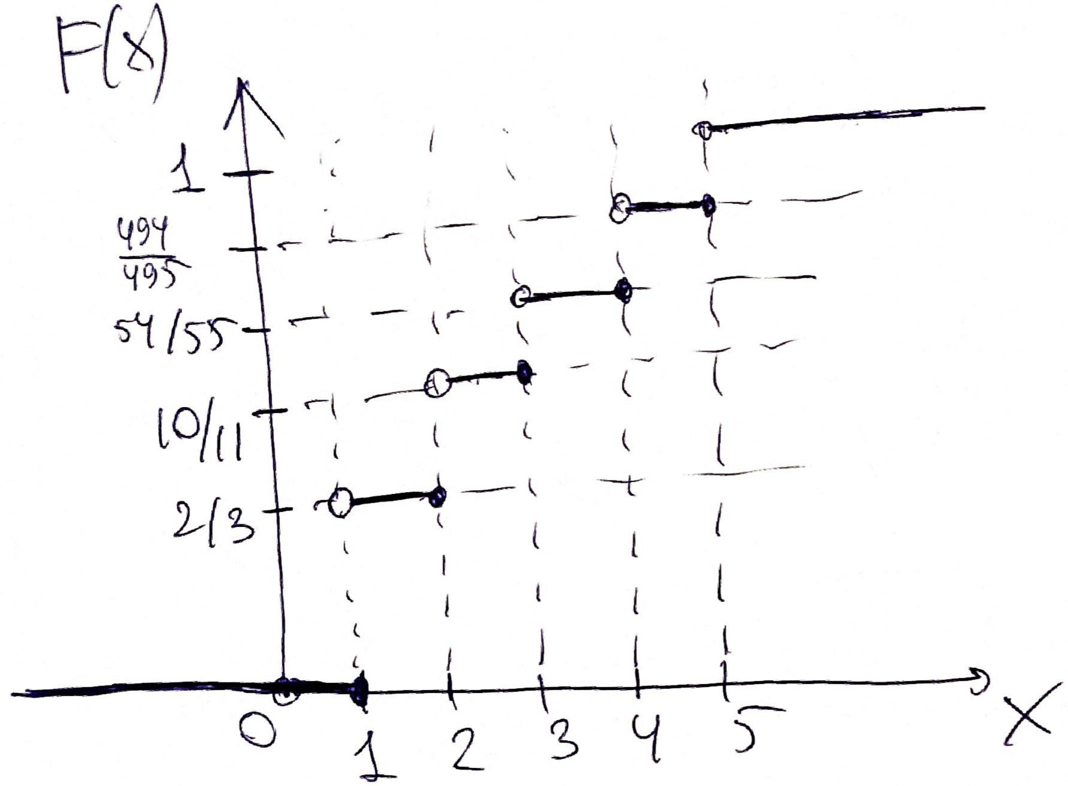
$$= \frac{13}{5}$$

$$D(X) = \frac{13}{5} - \frac{169}{81} = \frac{208}{405}$$

$$\delta(X) = \sqrt{D(X)} = \sqrt{\frac{208}{405}} = \frac{4\sqrt{65}}{45}$$

$$P(x \leq 2) = p_1 + p_2 = \frac{2}{3} + \frac{8}{33} = \frac{10}{11}$$

$$F(x)=\begin{cases} 0, & x\le 1 \\ 2/3, & 1<x\le 2 \\ 10/11, & 2<x\le 3 \\ 54/55, & 3<x\le 4 \\ 494/495, & 4<x\le 5 \\ 1, & x>5 \end{cases}$$



№2

$$f(x) = \begin{cases} a(3x-x^2), & x \in [0;3] \\ 0, & \text{otherwise} \end{cases}$$

ф. плотн расп.

$a-?$ $F(x)$ $M(x)$ $D(x)$ σ (с.к.о) $-?$ графики $-?$ F, f

P(случ знач >1)

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = \int_{0}^{3} a(3x-x^2)\,dx = \left(\frac{3x^2}{2}a - \frac{ax^3}{3}\right)\Bigg|_0^3 = a\left(\left(\frac{3\cdot 3^2}{2} - \frac{3^3}{3}\right) - \left(\frac{0}{2} - \frac{0}{3}\right)\right) = a\left(\frac{3^3}{2} - \frac{3^3}{3}\right) = a\left(\frac{3^4 - 3^3\cdot 2}{6}\right) =$$

$$3^3(3-2) = 3^3 \Rightarrow \frac{3\cdot\cancel{3}\cdot 3}{\cancel{6}_2}$$

$$= \frac{9}{2}a = 1 \Rightarrow a = \frac{2}{9}$$

2) $F(x) = \int_{-\infty}^{x} f(t)\,dt$

1. $x \in (-\infty; 0)$, $f(x) = 0 \Rightarrow F(x) = \int_{-\infty}^{x} 0\,dt = 0$

2. $x \in [0;3]$ $f(x) = \frac{2}{9}(3x - x^2) \Rightarrow$

$$\Rightarrow F(x) = \int_{-\infty}^{0} 0\,dt + \int_{0}^{x} \frac{2}{9}(3t - t^2)\,dt = \frac{2}{9}\left(\frac{3t^2}{2} - \frac{t^3}{3}\right)\Bigg|_0^x =$$

$$= \frac{2}{9}\left(\frac{3x^2}{2} - \frac{x^3}{3}\right) = \frac{\cancel{2}}{\cancel{9}_3}\cdot\frac{\cancel{3}x^2}{\cancel{2}} - \frac{2}{9}\cdot\frac{x^3}{3} = \frac{x^2}{3} - \frac{2x^3}{27}$$

3. $x \in (3;+\infty) \Rightarrow F(x) = \int_{-\infty}^{0} 0\,dt + \int_{0}^{3} \frac{2}{9}(3t-t^2)\,dt + \int_{3}^{x} 0\,dt =$

$$= \frac{2}{9}\left(\frac{3t^2}{2} - \frac{t^3}{3}\right)\Bigg|_0^3 = \frac{2}{9}\left(\frac{3\cdot 3^2}{2} - \frac{3^3}{3}\right) = \frac{2}{9}\cdot\frac{9}{2} = 1$$

Тогда

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x^2}{3} - \frac{2x^3}{27}, & x \in [0;3] \\ 1, & x > 3 \end{cases} \quad ; \quad f(x) = \begin{cases} \frac{2}{9}(3x-x^2), & x \in [0;3] \\ 0, & \text{otherwise} \end{cases}$$

$$M(x) = \int_{-\infty}^{\infty} x\cdot f(x)\,dx = \int_{-\infty}^{0} 0\,dx + \int_{0}^{3} x\cdot\frac{2}{9}(3x-x^2)\,dx + \int_{3}^{+\infty} 0\,dx =$$

$$= \frac{2}{9}\int_0^3 (3x^2 - x^3)\,dx = \frac{2}{9}\left(\frac{\cancel{3}x^3}{\cancel{3}} - \frac{x^4}{4}\right)\Bigg|_0^3 = \frac{2}{9}\left(3^3 - \frac{3^4}{4}\right) =$$

$* \ \frac{3^3\cdot 4 - 3^4}{4} = \frac{3^3(4-3)}{4} = \frac{27}{4}$

$$= \frac{\cancel{2}}{\cancel{9}}\cdot\frac{\cancel{27}^{3}}{\cancel{4}_{2}} = \frac{3}{2}$$

$$D(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - M^2(x) = /\text{опустим нули}/ = \frac{2}{9}\int_0^3 x^2(3x-x^2)\,dx - \frac{9}{4} =$$

$* \ 3x^3 - x^4$

$$= \frac{2}{9}\left(\frac{3x^4}{4} - \frac{x^5}{5}\right)\Bigg|_0^3 - \frac{9}{4} = \frac{2}{9}\left(\frac{3^5}{4} - \frac{3^5}{5}\right) - \frac{9}{4} =$$

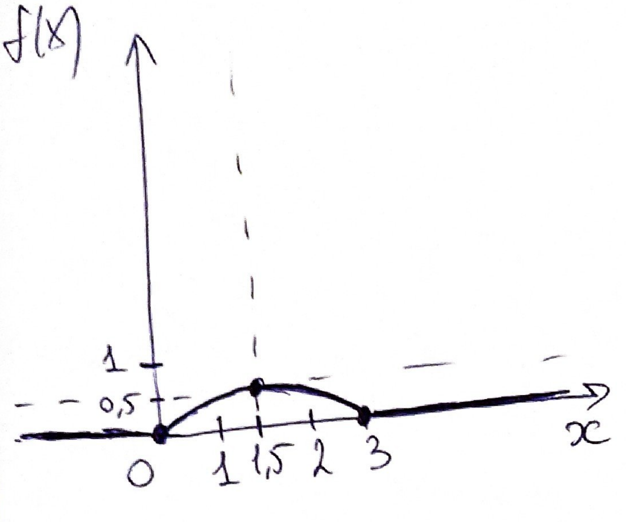
$* \ \frac{3^5(5-4)}{20} = \frac{3\cdot 3\cdot 3\cdot 3\cdot 3}{20}$

$$= \frac{\cancel{2}}{\cancel{3^2}}\cdot\frac{\cancel{3^2}\cdot 3^3}{\cancel{20}_{10}} - \frac{9}{4} = \frac{27}{10} - \frac{9}{4} = \frac{54}{20} - \frac{45}{20} = \frac{9}{20}$$

лист 2

$$\delta(X) = \sqrt{D(X)} = \sqrt{\frac{9}{20}} = \frac{3}{\sqrt{20}} = \frac{3}{2\sqrt{5}}$$

СКО



$$\frac{2}{9}(3x - x^2) = 0 \quad \Big|\cdot\frac{9}{2}$$

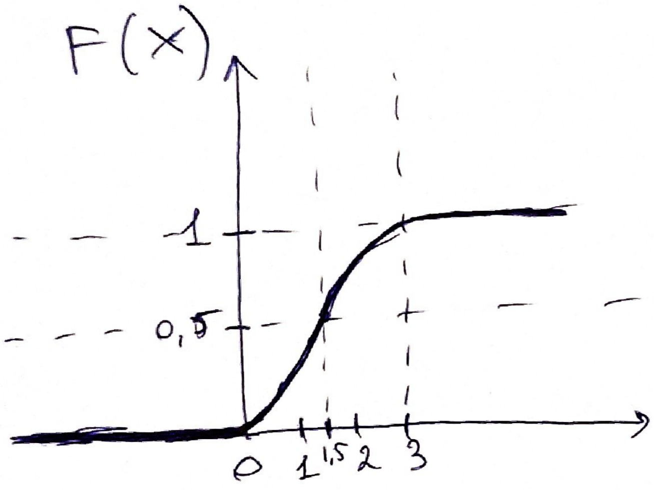
$$x(3-x) = 0 \Rightarrow x = 0,\ x = 3$$

$$y = 0$$

$$y = \frac{2}{3}x - \frac{2}{9}x^2$$

$$x_0 = -\frac{\frac{2}{3}}{2\cdot\left(-\frac{2}{9}\right)} = \frac{\cancel{2}}{\cancel{3}}\cdot\frac{\overset{3}{\cancel{9}}}{\underset{2}{\cancel{4}}} = 1{,}5$$

$$y_0 = \frac{\cancel{2}}{\cancel{3}}\cdot\frac{3}{2} - \frac{\cancel{2}}{\cancel{9}}\cdot\frac{\cancel{9}}{\underset{2}{\cancel{4}}} = 1 - \frac{1}{2} = 0{,}5$$



$x<0,\ y=0 \quad x>3,\ y=1$

$0 \le x \le 3 \quad y = \frac{x^2}{3} - \frac{2x^3}{27}$

Кубическая парабола, которая пройдет через $(1{,}5;\ 0{,}5)$.

$$P(x>1) = 1 - F(1),\ 1 \in [0;3] \Rightarrow F(x) = \frac{x^2}{3} - \frac{2x^3}{27},$$

$$F(1) = \frac{1}{3} - \frac{2}{27} = \frac{9-2}{27} = \frac{7}{27}$$

$$P(x>1) = 1 - \frac{7}{27} = \frac{20}{27}$$

Ответ: $a = \frac{2}{9}$; $F(x) = \begin{cases} 0, & x<0 \\ x^2/3 - \frac{2x^3}{27}, & 0 \le x \le 3 \\ 1, & x>3 \end{cases}$; $M(X) = \frac{3}{2}$; $D(X) = \frac{9}{20}$;

$\delta(X) = \frac{3}{2\sqrt{5}}$; $P(x>1) = \frac{20}{27}$; Графики выше на этом листе

№4. $p = q = \frac{1}{2}$ => при 5000 бросках ожидаем 2500 орлов и 2500 решек

~~б~~ орлов (уср)

Откл. на 70 => $2500-\mu$, $2500+\mu$

$\mu = 70$ => 2430 2570 — интервал, в котором будет находится колво выпавших орлов.

Воспользуемся формулой $P = \Phi\left(\frac{X-\mu}{\sigma}\right)$,

Φ — функ стандартного нормального распред

$\sigma = \sqrt{np(1-p)} = \sqrt{5000 \cdot 0{,}5^2} \approx 35{,}36$

$P = \Phi\left(\frac{2570-2500}{35{,}36}\right) - \Phi\left(\frac{2430-2500}{35{,}36}\right) =$

$= \Phi(1{,}98) - \Phi(-1{,}98) =$ / см. по таблице / $=$

$= 0{,}97615 - 0{,}02385 = 0{,}9523$

Ответ: 0,9523