Analytic formulas for marginal feature attributions of oblivious decision trees

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Explaining outcomes of a complex model

Setup

- the features are random variables $\mathbf{X} = (X_1, \ldots, X_n)$;
- the input-output function of the model $\mathbf{X} \mapsto f(\mathbf{X})$;
- f can be a linear model, a random forest, a neural net etc.

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ECOA/Regulation B require lenders to inform applicants of the primary reasons for decline or other adverse actions.

Game-theoretic local feature attributions

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Three ingredients:

- features $\mathbf{X}$ and model f;
- games $S \mapsto v(S; \mathbf{X}, f)(\mathbf{x})$ $(S \subseteq \{1, \ldots, n\})$ assigned to every point $\mathbf{x}$;
- game value h quantifying contribution of i^{th} feature as $h_i[v]$ at given $\mathbf{x}$.

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  Examples) f is a piecewise constant/linear function implemented by a tree ensemble/a ReLU network.

- games $S \mapsto v(S; \mathbf{X}, f)(\mathbf{x})$ ($S \subseteq \{1, \ldots, n\}$) assigned to every point $\mathbf{x}$;

  Examples) the conditional game $S \mapsto \mathbb{E}[f(\mathbf{X}) \mid \mathbf{X}_S = \mathbf{x}_S]$ "true to the data" and the marginal game $S \mapsto \mathbb{E}[f(\mathbf{x}_S, \mathbf{X}_{-S})]$ "true to the model".

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  Example) the Shapley value:

$$\varphi_i[v] := \sum_{S \subseteq \{1,\ldots,n\} \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} \left(v(S \cup \{i\} - v(S))\right).$$

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  - Remedy: Grouping features based on dependencies and using coalitional variants of the Shapley value (e.g. the Owen value) unifies the two frameworks and yields more stable explanations [Miroshnikov-Kotsiopoulos-F.-Ravi Kannan 2022].

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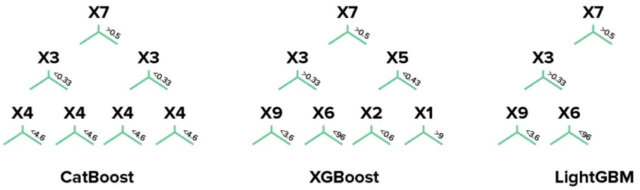
Different boosting libraries construct trees differently.



Picture from Medium.

- The CatBoost library utilizes **oblivious (symmetric) decision trees** as base learners [Dorogush-Ershov-Gulin 2018].
- Despite this constraint, ensembles of symmetric trees demonstrate competitive predictive power [Ferov-Modrý 2016], [Hancock-Khoshgoftaar 2020].

Main result: a model-specific and inherently-interpretable approach

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- Based on this analytic solution, we designed **an algorithm for estimating marginal feature attributions** of $\mathcal{T}$ according to certain precomputed look-up tables.
- The algorithm is **fast** (the computation complexity is $O(|\mathcal{T}| \cdot d)$) and **accurate** (variance of error $\propto \frac{1}{|D|}$).

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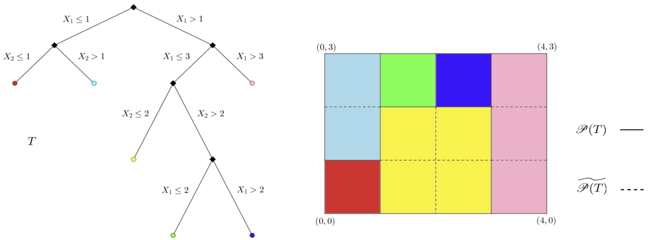
- For a tree T, marginal feature attributions based on a linear game value are piecewise constant, but only with respect to a grid partition $\widetilde{\mathscr{P}(T)}$, which is often finer than the tree's partition $\mathscr{P}(T)$. They coincide when T is symmetric.



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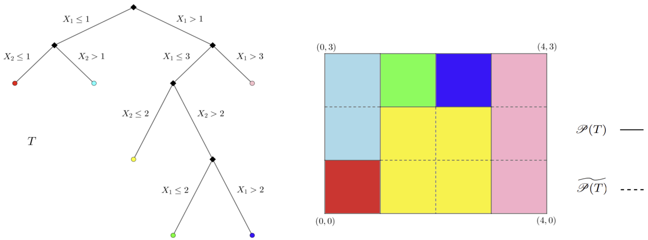

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- Game value computations can be simplified by exploiting the symmetry.