HW1

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1

$$(1+2i)(z-i)+(4i-3)(1-2z)+1+7i=0;$$

$$z-i+2iz+2+4i-3-8iz+6z+1+7i=0;$$

$$7z+10i-6iz=0;$$

$$z=\frac{10i}{6i-7}=\frac{-10i(6i+7)}{36+49}=\frac{12}{17}-\frac{14i}{17};$$

$$\text{Ответ: }\frac{12}{17}-\frac{14i}{17}.$$

2

$$\begin{cases}(3-i)z+(4+2i)w=1+3i, & \mid *(1+i)\\ (4+2i)z-(2-3i)w=7;\end{cases}$$

$$\begin{cases}(4+2i)z+(4+2i)(1+i)w=(1+3i)(1+i),\\ (4+2i)z-(2-3i)w=7;\end{cases}$$

$$(4+2i)(1+i)w+(2-3i)w=(1+3i)(1+i)-7;$$

$$w=\frac{(1+3i)(1+i)-7}{(4+2i)(1+i)+(2-3i)}=\frac{-9+4i}{4+3i}=$$
$$=\frac{(-9+4i)(4-3i)}{25}=\frac{-36+27i+16i+12}{25}=-\frac{24}{25}+\frac{43i}{25};$$

$$(4+2i)z-(2-3i)(-\frac{24}{25}+\frac{43i}{25})=7;$$

$$25z(4+2i)+(2-3i)(24-43i)=175;$$

$$100z+50iz+48-86i-72i-129=175;$$

$$z(100+50i)=256+158i;$$

$$z=\frac{256+158i}{(100+50i)}=\frac{128+79i}{(50+25i)}=\frac{(128+79i)((50-25i))}{3125}=$$
$$=\frac{(128+79i)((2-i))}{125}=\frac{256-128i+158i+79}{125}=$$
$$=\frac{335+30i}{125}=\frac{67+6i}{25}=\frac{67}{25}+\frac{6i}{25};$$

$$\text{Ответ: }\left(\frac{67}{25}+\frac{6i}{25};-\frac{24}{25}+\frac{43i}{25}\right).$$

3

1)

$$z = -\sqrt{3} + i;$$
$$tg\ \varphi = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3};$$
$$\varphi = -\frac{\pi}{6} + \pi = \frac{5\pi}{6};$$
$$r = |z| = \sqrt{-\sqrt{3}^2 + 1} = 2;$$
$$z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right);$$
$$\text{Ответ: } 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right).$$

2)

$$z = 1 = 1 + 0i;$$
$$\varphi = 0;$$
$$|z| = \sqrt{1^2 + 0^2} = 1 = r;$$
$$z = 1(\cos 0 + i\sin 0);$$
$$\text{Ответ: } 1(\cos 0 + i\sin 0).$$

3) -

4)

$$z = (\sqrt{3} - i)^1 00;$$
$$]\ \ w = \sqrt{3} - i; \qquad |w| = 2;$$
$$tg\ \varphi_w = -\frac{\sqrt{3}}{3}; \qquad \varphi_w = -\frac{\pi}{6};$$
$$z = w^{100} = 2^{100}\left(\cos\frac{-100\pi}{6} + i\sin\frac{-100\pi}{6}\right) =$$
$$= 2^{100}\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right);$$
$$\text{Ответ: } 2^{100}\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right).$$

4

1)

$$z^3 = -1;$$
$$z = \sqrt[3]{-1} = \sqrt[3]{1(\cos\pi + i\sin\pi)} =$$
$$= 1\left(\cos\frac{\pi + 2\pi k}{3} + i\sin\frac{\pi + 2\pi k}{3})\right), \quad k = 0, 1, 2.$$

2)

$$z^8 = 1 + i;$$
$$z = \sqrt[8]{1 + i} = \sqrt[8]{\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)} =$$
$$= \sqrt[16]{2}\left(\cos\frac{\frac{\pi}{4} + 2\pi k}{8} + i\sin\frac{\frac{\pi}{4} + 2\pi k}{8}\right), \quad k = 0, 1, 2, 3, 4, 5, 6, 7.$$

3)

$$z^5 = 1 + \sqrt{3}i;$$
$$z = \sqrt[5]{1 + \sqrt{3}i} = \sqrt[5]{2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)} =$$
$$= \sqrt[5]{2}\left(\cos\frac{\frac{\pi}{3} + 2\pi k}{5} + i\sin\frac{\frac{\pi}{3} + 2\pi k}{5}\right), \quad k = 0, 1, 2, 3, 4.$$

4)

$$z^6 = \overline{z}^3;$$

$$z^2 = \overline{z};$$

Пусть $z = a + bi$, тогда $\overline{z} = a - bi$, $a, b \in \mathbb{R}:$

$$(a+bi)^2 = a - bi;$$

$$a^2 - b^2 + 2abi = a - bi;$$

$$i(2ab + b) = b^2 - a^2 + a;$$

Такое возможно, если $2ab + b = 0$ => $b = 0$ или $a = -0.5$;

Если $b = 0$, то $a = 0$ или $a = 1$;

Если $a = -0.5$, => $b^2 + 0.25 = 0$ => нет решений;

Ответ: 0; 1.

5

$$\frac{(-2-2\sqrt{3})^7 i^4}{(-8-8i)^3(-1+\sqrt{3}i)^3} = \frac{4\left(\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}\right)1\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}{8\sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)} =$$

$$= \frac{2^{14}\left(\cos\frac{-14\pi}{3} + i\sin\frac{-14\pi}{3}\right)(\cos 2\pi + i\sin 2\pi)}{2^{10,5}\left(\cos\frac{-9\pi}{4} + i\sin\frac{-9\pi}{4}\right)2^3(\cos 2\pi + i\sin 2\pi)} =$$

$$= \sqrt{2} * \frac{\cos\frac{-2\pi}{3} + i\sin\frac{-2\pi}{3}}{\cos\frac{-\pi}{4} + i\sin\frac{-\pi}{4}} = \sqrt{2} * \frac{-0.5 - 0.5\sqrt{3}i}{0.5\sqrt{2} - 0.5\sqrt{2}i} =$$

$$= \frac{1+\sqrt{3}i}{i-1} = \frac{(1+3i)(i+1)}{(i-1)(i+1)} = \frac{\sqrt{3}-1}{2} - \frac{\sqrt{3}+1}{2}i;$$

Ответ: $\frac{\sqrt{3}-1}{2} - \frac{\sqrt{3}+1}{2}i.$

6

$$\left(\frac{2}{z} - 1\right)^n = 1;$$

Если $n = 0$, то $z \neq 0$;
Если $n \neq 0$, то $z = 1$.

7

$$\frac{1}{i^{11}} - \frac{1}{i^{41}} + \frac{1}{i^{75}} - \frac{1}{i^{1023}} = -\frac{1}{i} - \frac{1}{i} - \frac{1}{i} + \frac{1}{i} = -\frac{2}{i} = 2i;$$

Ответ: $2i$.

8

$$z = \sqrt{3}e^{\frac{2\pi i}{3}} = \sqrt{3}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = \sqrt{3}\left(-0.5 + 0.5\sqrt{3}i\right) = -0.5\sqrt{3} + 1.5i;$$

Ответ: $-0.5\sqrt{3} + 1.5i$.