D3-2

№1794

$$\int \sqrt{x}\,\ln^2 x\,dx - ?$$

$$I_n = \int \sqrt{x}\,\ln^n x\,dx = \int \frac{1}{n+1}\cdot\sqrt{x}^3\cdot\frac{(n+1)\ln^n x}{x}\,dx =$$

$$= \frac{1}{n+1}\int x^{\frac{3}{2}}\,d\ln^{n+1} x = \frac{1}{n+1}\left(x^{\frac{3}{2}}\ln^{n+1}x - \int \ln^{n+1}x\,dx^{\frac{3}{2}}\right) =$$

$$= \frac{1}{n+1}\left(x^{\frac{3}{2}}\ln^{n+1}x - \frac{3}{2}\int\sqrt{x}\,\ln^{n+1}x\,dx\right) = \frac{1}{n+1}\left(x^{\frac{3}{2}}\ln^{n+1}x - \frac{3}{2}I_{n+1}\right)$$

$$\Rightarrow I_{n+1} = \frac{2}{3}\left(x^{\frac{3}{2}}\ln^{n+1}x - (n+1)I_n\right)$$

$$I_0 = \int\sqrt{x}\,dx = \frac{2}{3}\sqrt{x^3} + C$$

$$I_1 = \frac{2}{3}\left(x^{\frac{3}{2}}\ln x - I_0\right) = \frac{2}{3}\left(x^{\frac{3}{2}}\ln x - \frac{2}{3}\sqrt{x^3}\right) =$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}} + C$$

$$I_2 = \int\sqrt{x}\,\ln^2 x\,dx = \frac{2}{3}\left(x^{\frac{3}{2}}\ln^2 x - 2I_1\right) =$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln^2 x - \frac{4}{3}\left(\frac{2}{3}x^{\frac{3}{2}}\ln x - \frac{4}{9}x^{\frac{3}{2}}\right) + C =$$

$$= \frac{2}{3}x^{\frac{3}{2}}\ln^2 x - \frac{8}{9}x^{\frac{3}{2}}\ln x + \frac{16}{27}x^{\frac{3}{2}} + C$$

№1799

$$\int x^2\sin 2x\,dx - ?$$

$$\int \sin 2x\, dx = -\frac{\cos 2x}{2} + C$$

$$\int \sin 2x\, dx = x\sin 2x - \int x\, d\sin 2x =$$

$$= x\sin 2x - 2\int x\cos 2x\, dx = x\sin 2x - \int \cos 2x\, dx^2 =$$

$$= x\sin 2x - x^2\cos 2x + \int x^2\, d\cos 2x =$$

$$= x\sin 2x - x^2\cos 2x - 2\int x^2 \sin 2x\, dx$$

$$\Rightarrow \int x^2 \sin 2x\, dx = \frac{1}{2}\left(x\sin 2x - x^2\cos 2x - \int \sin 2x\, dx\right) =$$

$$= \frac{1}{2}\left(x\sin 2x - x^2\cos 2x + \frac{\cos 2x}{2}\right) + C$$

№1804

$$\int x \operatorname{arctg} x\, dx = \frac{1}{2}\int \operatorname{arctg} x\, dx^2 =$$

$$= \frac{1}{2}x^2 \operatorname{arctg} x - \frac{1}{2}\int x^2 \operatorname{arctg} x = \frac{1}{2}x^2\operatorname{arctg} x -$$

$$- \frac{1}{2}\int \frac{x^2}{x^2+1}\, dx = \frac{1}{2}x^2 \operatorname{arctg} x - \frac{1}{2}\int 1\, dx + \frac{1}{2}\int \frac{dx}{x^2+1} =$$

$$= \frac{1}{2}\operatorname{arctg} x \cdot x^2 - \frac{x}{2} + \frac{1}{2}\operatorname{arctg} x + C.$$

№1808

$$\int x \ln\left(\frac{1+x}{1-x}\right) dx = \int \left(1 + x\ln\left(\frac{1+x}{1-x}\right) - \frac{1-x^2}{2}\cdot\frac{2}{1-x^2}\right) dx =$$

$$= \int 1\, dx - \int \left(\left(\frac{1-x^2}{2}\right)' \ln\left(\frac{1+x}{1-x}\right) + \left(\frac{1-x^2}{2}\right)\left(\ln\frac{1+x}{1-x}\right)'\right) dx =$$

$$= x + \ln\left(\frac{1+x}{1-x}\right) \cdot \frac{x^2-1}{2} + C$$

№ 1819

$$\int x^2 \ln\left(\frac{1-x}{1+x}\right) dx = \int\left(x^2 \ln\left(\frac{1-x}{1+x}\right) + \frac{x^3-1}{3} \cdot \frac{2}{x^2-1} -\right.$$

$$\left. - \frac{2}{3} \frac{x^3-1}{x^2-1}\right) dx = \int\left(\frac{x^3-1}{3}\right)' \ln\left(\frac{1-x}{1+x}\right) + \left(\frac{x^3-1}{3}\right)\left(\ln\frac{1-x}{1+x}\right)' dx -$$

$$- \frac{2}{3}\int \frac{x^2+x+1}{x+1} dx = \frac{x^3-1}{3} \ln\left(\frac{1-x}{1+x}\right) - \frac{2}{3}\int x\,dx - \frac{2}{3}\int \frac{dx}{x+1} =$$

$$= \frac{x^3-1}{3} \ln\left(\frac{1-x}{1+x}\right) - \frac{x^2}{3} - \frac{2}{3} \ln|x+1| + C$$

№ 1820

$$\int x^2 \sqrt{a^2+x^2} \quad -$$

№ 1840

$$\int \frac{x+1}{x^2+x+1} dx = \int \frac{\left(x+\frac{1}{2}\right)}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx + \frac{1}{2} \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} =$$

$$= \frac{1}{2} \ln\left(x^2+x+1\right) + \frac{1}{\sqrt{3}} \operatorname{arctg}\left(\frac{2x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right) + C$$

№ 1842

$$\int \frac{x^3 dx}{x^4 - x^2 + 2} = \frac{1}{2}\int \frac{x^2 dx^2}{x^4 - x^2 + 2} =$$

$$= \frac{1}{2}\int \frac{\left(x^2-\frac{1}{2}\right) d\left(x^2-\frac{1}{2}\right)}{\left(x^2-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2} + \frac{1}{4}\int \frac{d\left(x^2-\frac{1}{2}\right)}{\left(x^2-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{7}}{2}\right)^2} =$$

$\frac{1}{4}\ln|x^4-x^2+2| + \frac{1}{2\sqrt{7}}\operatorname{arctg}\left(\frac{2}{\sqrt{7}}\left(x^2-\frac{1}{2}\right)\right)+C$

№1851

$$\int\frac{x\,dx}{\sqrt{5+x-x^2}} = \int\frac{x\,dx}{\sqrt{\frac{21}{4}-\left(x-\frac{1}{2}\right)^2}} =$$

$$= \int\frac{\left(x-\frac{1}{2}\right)d\left(x-\frac{1}{2}\right)}{\sqrt{\frac{21}{4}-\left(x-\frac{1}{2}\right)^2}} + \frac{1}{2}\int\frac{d\left(x-\frac{1}{2}\right)}{\sqrt{\frac{21}{4}-\left(x-\frac{1}{2}\right)^2}} =$$

$$= -\sqrt{\frac{21}{4}-\left(x-\frac{1}{2}\right)^2} + \frac{1}{2}\arcsin\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{21}}{2}}\right)+C =$$

$$= -\sqrt{5+x-x^2} + \frac{1}{2}\arcsin\left(\frac{2x-1}{\sqrt{21}}\right)+C$$

№1853.1

$$\int\frac{\cos x\,dx}{\sqrt{1+\sin x+\cos^2 x}} = \int\frac{d\sin x}{\sqrt{2+\sin x-\sin^2 x}} =$$

$$= \int\frac{dt}{\sqrt{2+t-t^2}} = \int\frac{dt}{\sqrt{\left(\frac{3}{2}\right)^2-\left(t-\frac{1}{2}\right)^2}} = \arcsin\left(\frac{t-\frac{1}{2}}{\frac{3}{2}}\right)+C =$$

$$= \arcsin\left(\frac{2\sin x-1}{3}\right)+C$$