HW 4

1. $\int \frac{dx}{1+\cos^2 x} = \int \frac{dx}{\sin^2 x + 2\cos^2 x} =$

$= \int \frac{d\,tg\,x}{tg^2 x + 2} = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{tg\,x}{\sqrt{2}} + C$

1. $\int x^x (1+\ln x)\, dx = \int e^{x \ln x} (1 + \ln x)\, dx =$

$\Delta \quad (e^{x\ln x})' = e^{x \ln x} (x \ln x)' =$

$= e^{x \ln x}\left(\ln x + \frac{x}{x}\right) = e^{x\ln x}(1 + \ln x)$

$= e^{x \ln x} + C$

2. $\int \frac{x^2+1}{x^4+3x^2+1}\, dx = \int \frac{x^2+1}{\left(x^2+\frac{3+\sqrt{5}}{2}\right)\left(x^2+\frac{3-\sqrt{5}}{2}\right)} dx$

$= \int \frac{A\left(x^2+\frac{3+\sqrt{5}}{2}\right) + B\left(x^2+\frac{3-\sqrt{5}}{2}\right)}{\left(x^2+\frac{3+\sqrt{5}}{2}\right)\left(x^2+\frac{3-\sqrt{5}}{2}\right)}\, dx =$

$$\begin{cases} A + B = 1 \\ A \cdot \frac{3+\sqrt{5}}{2} + B \cdot \frac{3-\sqrt{5}}{2} = 1 \end{cases}$$

$A = \frac{5-\sqrt{5}}{10} \qquad B = \frac{5+\sqrt{5}}{10}$

$A = \frac{1}{2}, \quad B = \frac{1}{3}, \quad C = -\frac{1}{6}$

$$= \frac{1}{2}\int \frac{dt}{t^2-t+1} + \frac{1}{3}\int \frac{dt}{t+1} - \frac{1}{6}\int \frac{(2t-1)dt}{t^2-t+1} =$$

$$= \frac{1}{2}\int \frac{dt}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} + \frac{1}{3}\int \frac{dt}{t+1} - \frac{1}{6}\ln(t^2-t+1) =$$

$$= \frac{1}{\sqrt{3}}\, arctg\left(\left(\sqrt[6]{x}-\frac{1}{2}\right)\cdot\frac{2}{\sqrt{3}}\right) + \frac{1}{3}\ln(\sqrt[6]{x}+1) - \frac{1}{6}\ln(\sqrt[3]{x}-\sqrt[6]{x}+1) + C$$

3. $\int \frac{\sqrt{x}+1}{x^2-\sqrt{x}}\,dx = \int \frac{t+1}{t^4-t}\,dt^2 =$

$$= \int \frac{2t(t+1)}{t^4-t}\,dt = 2\int \frac{t+1}{t^3-1}\,dt =$$

$$= 2\int \frac{t+1}{(t-1)(t^2+t+1)}\,dt =$$

$$= \int \frac{A(t-1)+B(t^2+t+1)+C(t-1)(2t+1)}{(t-1)(t^2+t+1)}\,dt =$$

$A(t-1)$
$B(t^2+t+1)$
$C(t-1)(2t+1) = C(2t^2-t-1)$

$$\begin{cases} B+2C=0 \\ A+B-C=2 \\ -A+B-C=2 \end{cases} \qquad A=0,\ C=-\frac{2}{3},\ B=\frac{4}{3}$$

$$\stackrel{(=)}{} \frac{5-\sqrt{5}}{10}\int\frac{dx}{x^2+\frac{3-\sqrt{5}}{2}} + \frac{5+\sqrt{5}}{10}\int\frac{dx}{x^2+\frac{3+\sqrt{5}}{2}} =$$

$$= \frac{5-\sqrt{5}}{10}\cdot\sqrt{\frac{2}{3-\sqrt{5}}}\,\operatorname{arctg}\left(\sqrt{\frac{2}{3-\sqrt{5}}}\,x\right) +$$

$$+ \frac{5+\sqrt{5}}{10}\sqrt{\frac{2}{3+\sqrt{5}}}\,\operatorname{arctg}\left(\sqrt{\frac{2}{3+\sqrt{5}}}\,x\right) + C$$

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2. $$\int\frac{\sqrt[3]{x^2}}{1+\sqrt{x}}\,dx = \int\frac{t^4}{1+t^3}\,dt^6 =$$

$$= \int\frac{6t^9}{t^3+1}\,dt = \int\frac{6t^9+6t^6-6t^6-6t^3+6t^3+6-6}{t^3+1}\,dt$$

$$= \int\left(6t^6-6t^3+6-\frac{6}{t^3+1}\right)dt =$$

$$= \frac{6}{7}t^7 - \frac{3}{2}t^4 + 6t - 6\int\frac{dt}{t^3+1} \quad (=)$$

$$* \int\frac{dt}{t^3+1} = \int\frac{1}{(t+1)(t^2-t+1)}\,dt =$$

$$= \int\frac{A(t+1)+B(t^2-t+1)+C(t+1)(2t-1)}{(t+1)(t^2-t+1)}\,dt \quad (=)$$

$A(t+1)$
$B(t^2-t+1)$
$C(2t^2+t-1)$

$$\begin{cases} B+2C=0 \\ A-B+C=0 \\ A+B-C=1 \end{cases}$$

$$= \frac{4}{3}\int \frac{dt}{t-1} - \frac{2}{3}\int \frac{(2t+1)\,dt}{t^2+t+1} =$$

$$= \frac{4}{3}\ln(t-1) - \frac{2}{3}\ln(t^2+t+1) + C =$$

$$= \frac{4}{3}\ln(\sqrt{x}-1) - \frac{2}{3}\ln(x+\sqrt{x}+1) + C$$