Identifier Namespaces in Mathematical Notation

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Supervisor: Prof. Dr. Volker Markl

Outline

1. Motivation
2. Related Work
3. Namespace Discovery
4. Implementation and Evaluation
   - Java Language Processing: Proof of Concept
   - Parameter Selection
   - Experiment Results
   - Building Hierarchies
5. Conclustions

[[Namespace]]

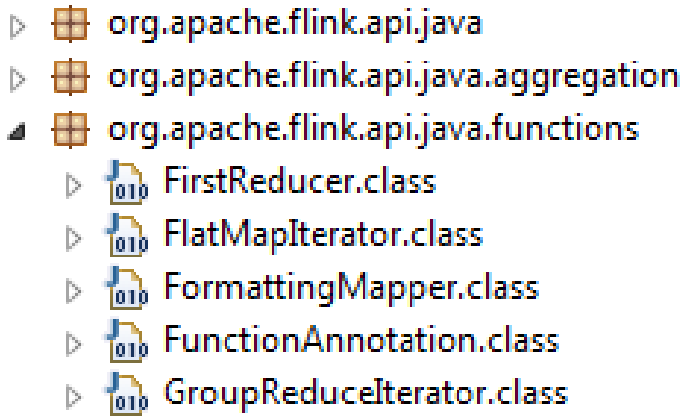
> “ In programming, **namespaces** are employed for the **grouping symbols and identifiers** around a particular functionality and to **avoid name collisions** between multiple identifiers that share the same name ”

No namespaces (C, old PHP)


```
$foo = new Zend_CodeGenerator_Php_Class();
```

http://framework.zend.com/apidoc/1.11/

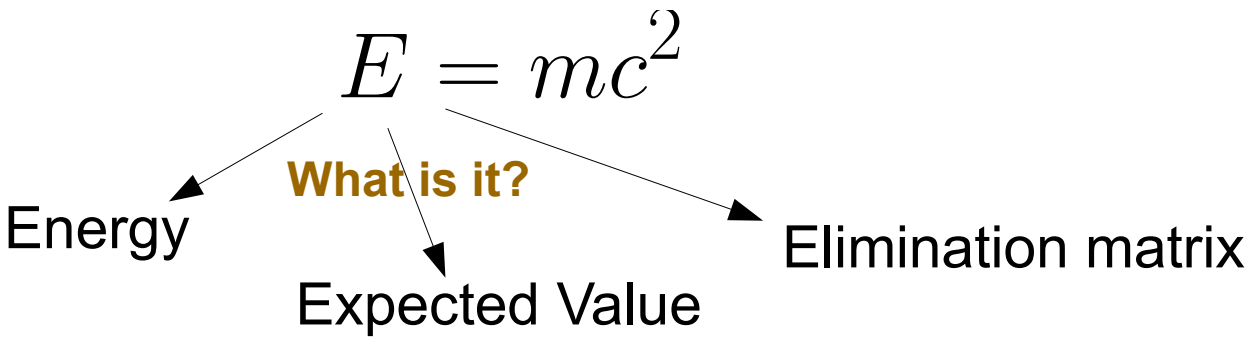
With namespaces (C++, Java, C#, python)



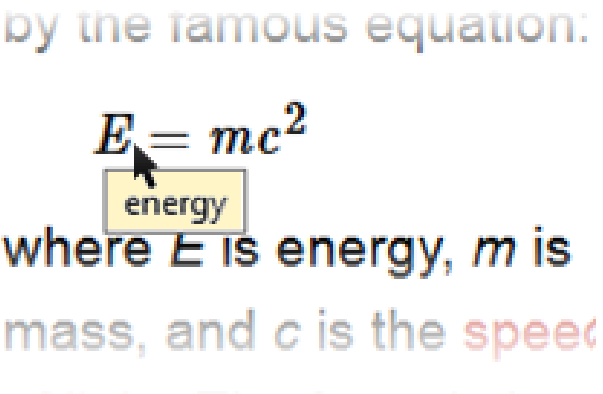

```
import o.a.f.api.java.ExecutionEnvironment;
ExecutionEnvironment.getExecutionEnvironment()
```

https://flink.apache.org/

Namespaces in Mathematics



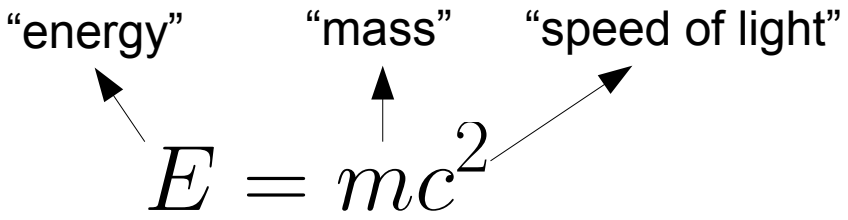
- Can introduce namespaces to mathematical identifiers
  - `import Physics/General/Relativity and Gravitation/{E, m, c}`
- It can give:
  - Identifier disambiguation
  - Better user experience
  - Additional context

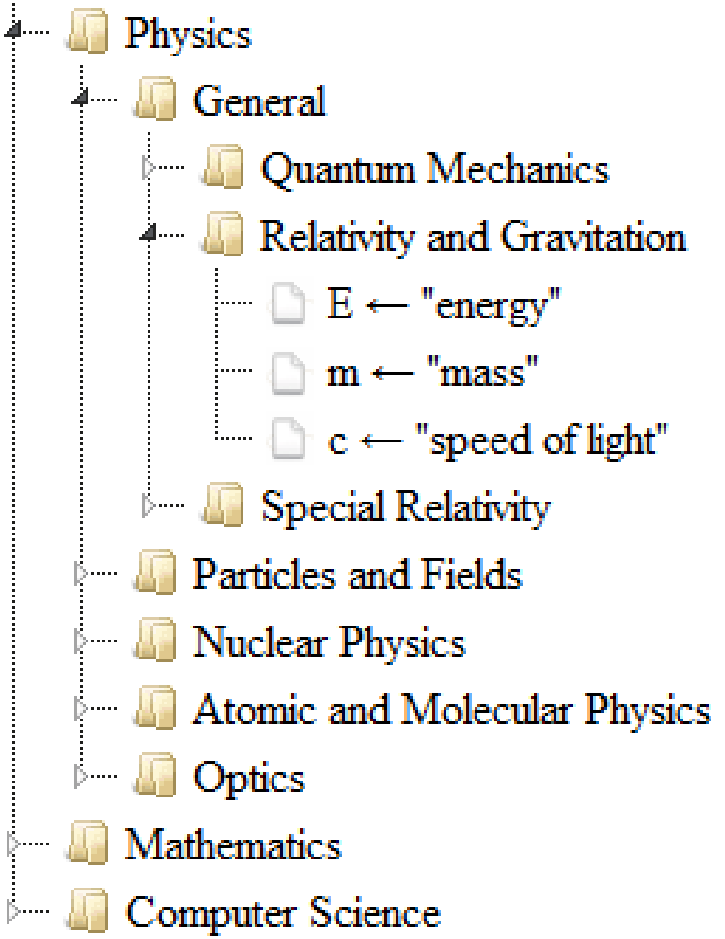


credit: [MLP]

Namespaces in Mathematics

- Problem: How to organize identifiers into namespaces?
- Manual assignment would take a lot of time
- Our method: **automatic namespace discovery** from a collection of documents






```
import Physics/General/Relativity and Gravitation/{E, m, c}
```

Outline

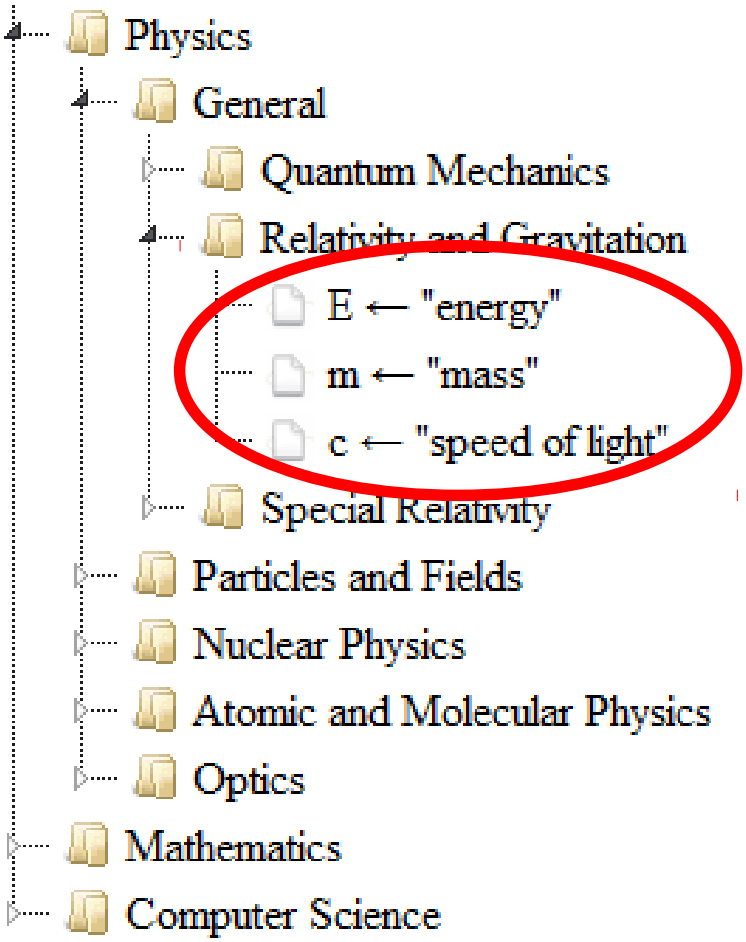
Definition Extraction

How to get the definitions? Extract them!

[[Mass–energy equivalence]]

> The equivalence of energy *E* and mass *m* is reliant on the speed of light *c* and is described by the famous equation:
>
> $$E = mc^2$$

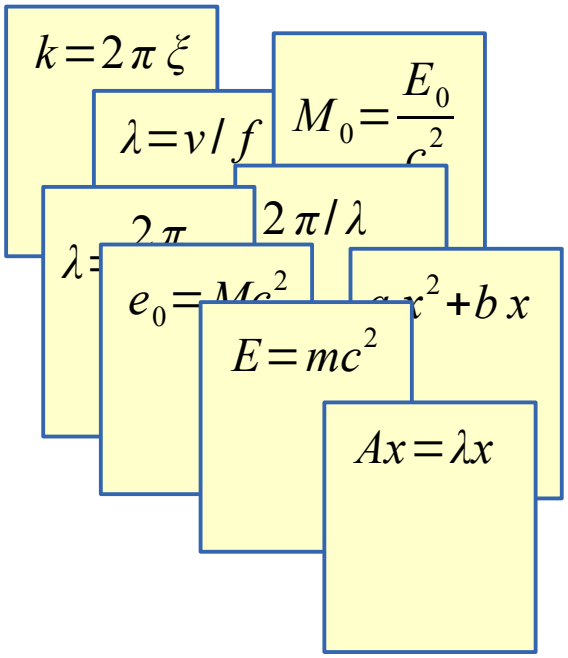
ID	Definition
E	energy
m	mass
c	speed of light



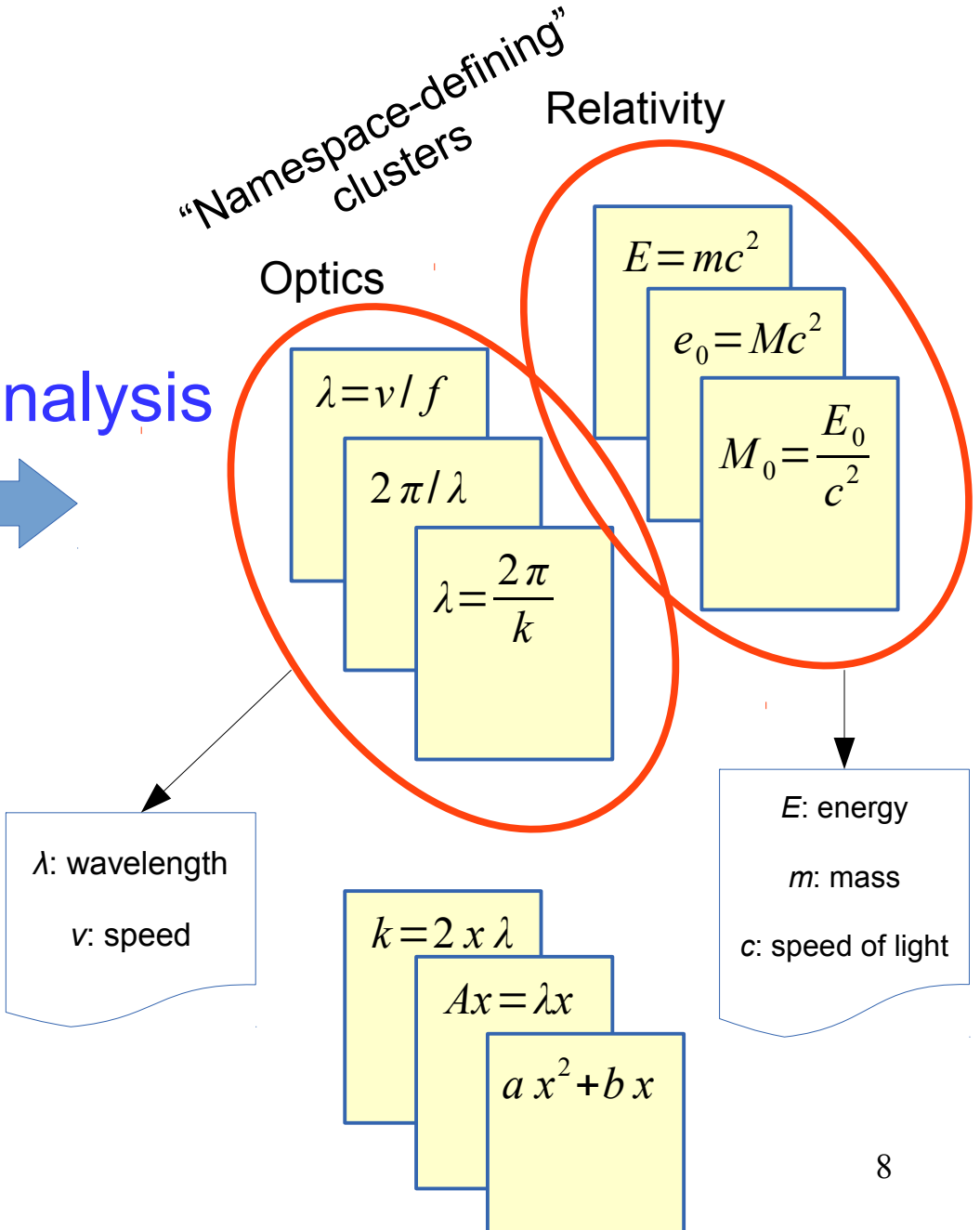
[MLP]

Namespace Discovery

Want to find groups of documents that use identifiers in the same way



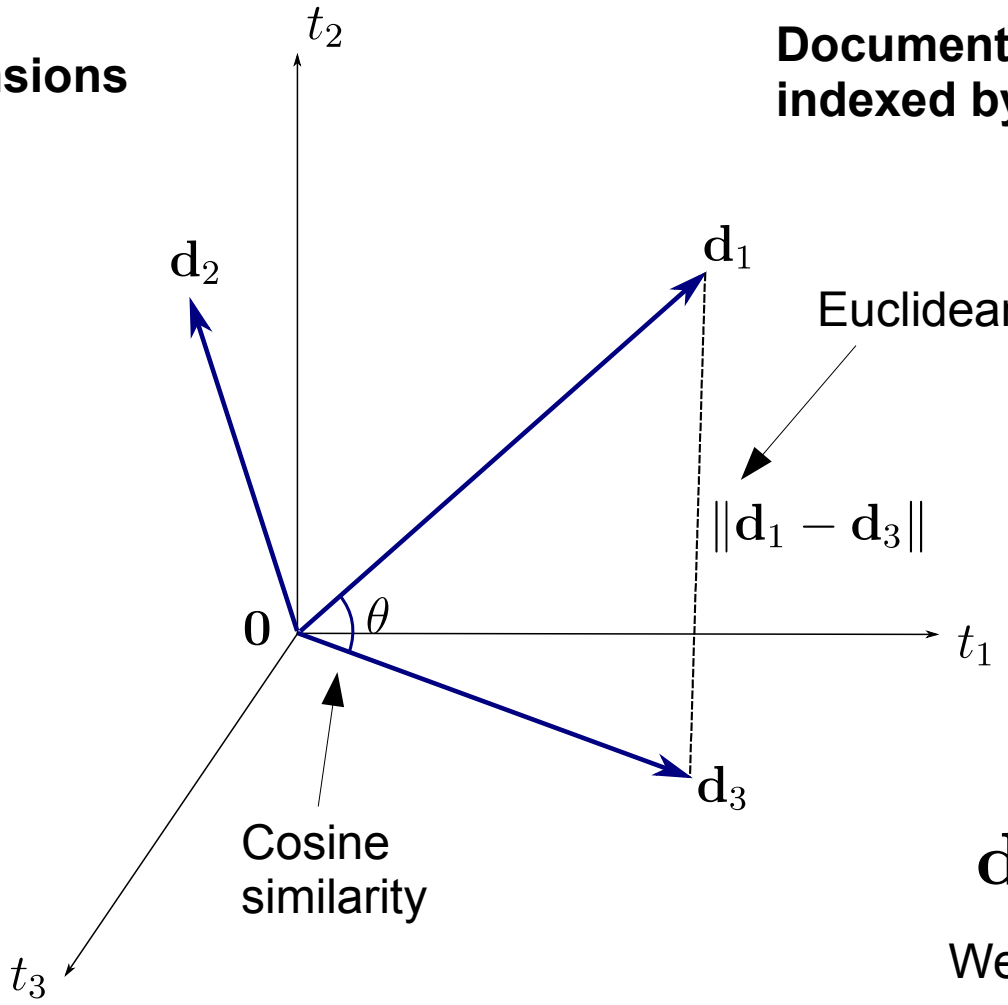
Cluster analysis



Vector Space Model

Terms are dimensions

Documents are vectors indexed by terms



$$\mathbf{d}_i = (w_1, w_2, w_3)$$

Weights *w* are TF or TF-IDF

[IR]

Identifier VSM

d_1

	Definition
E	energy
m	mass
c	speed of light

d_2

	Definition
m	mass
c	speed of light

d_3

	Definition
E	energy

d_4

	Definition
m	integer
c	constant

Build identifier-document matrix

TF of terms

	d_1	d_2	d_3	d_4
E	1	0	1	0
m	1	1	0	1
c	1	1	0	1

No definitions

	d_1	d_2	d_3	d_4
E	1	0	1	0
m	1	1	0	1
c	1	1	0	1
energy	1	0	1	0
mass	1	1	0	0
speed of light	1	1	0	0
integer	0	0	0	1
constant	0	0	0	1

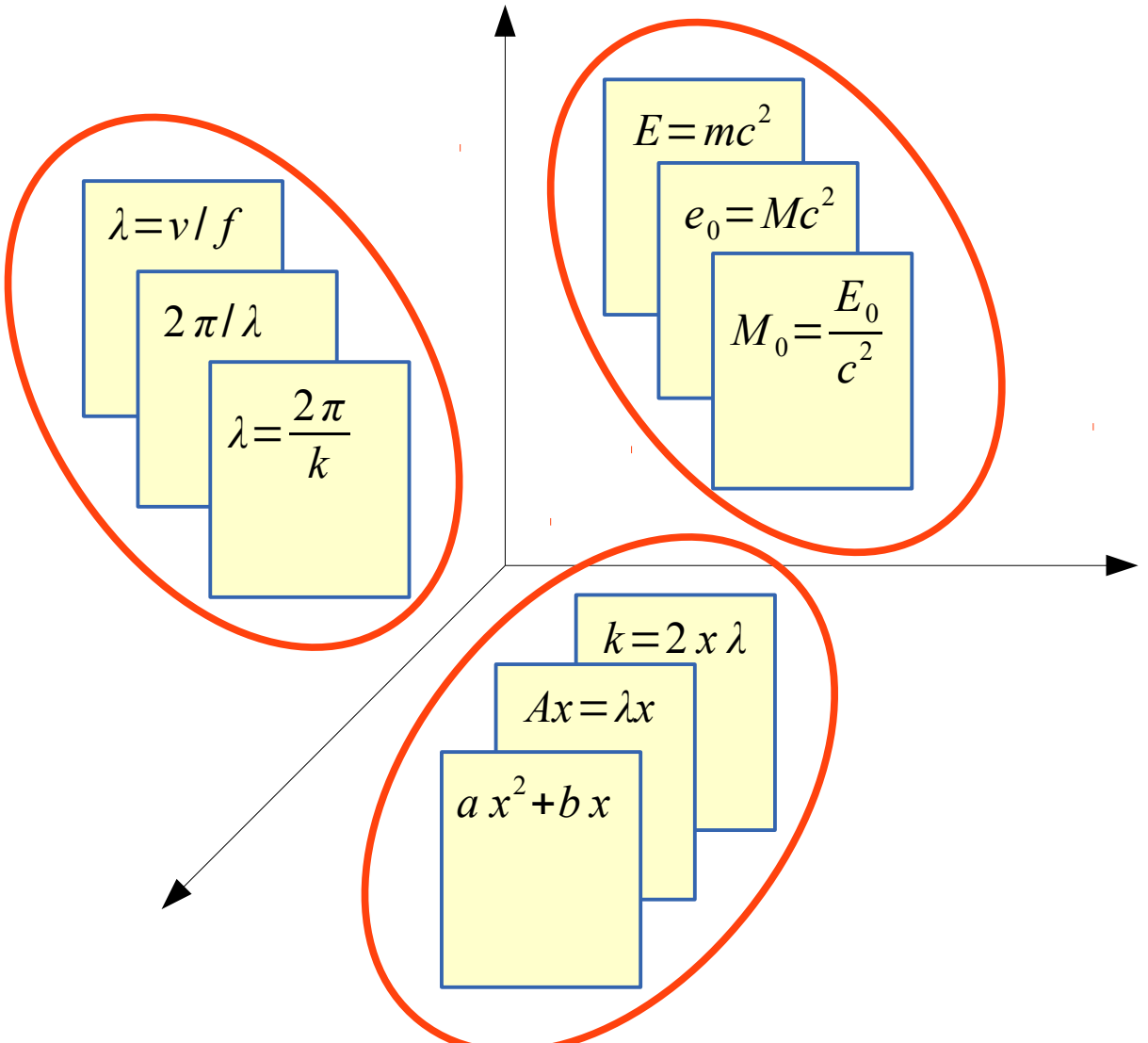
"Weak" association

	d_1	d_2	d_3	d_4
E_energy	1	0	1	0
m_mass	1	1	0	0
c_speed of light	1	1	0	0
m_integer	0	0	0	1
c_constant	0	0	0	1

"Strong" association

Document Clustering

Once represented documents as vectors, can cluster them

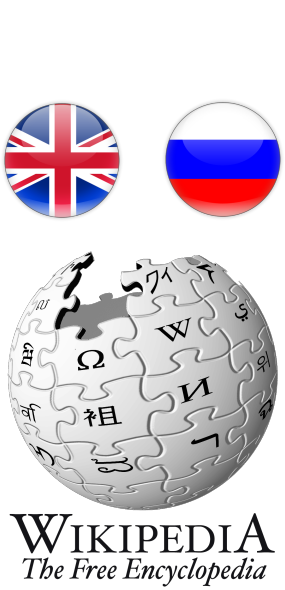


Can use:

- K-Means [IR]
- DBSCAN [SNN]
- LSA [LSI]

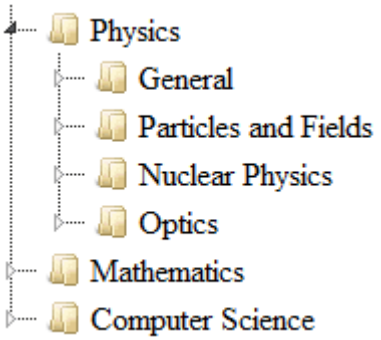
Outline

Implementation



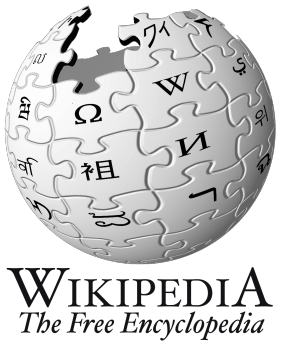
Definition extraction

Namespace discovery



our main contribution

Images: [[File:Wikipedia-logo-en-big.png]] http://www.freeflagicons.com https://flink.apache.org/material.html http://scikit-learn.org/stable/



Definition extraction

Namespace discovery

- Physics
  - General
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

wikiFilter

Filter pages with <math>

XPath

Extract identifiers

Standford NLP

POS Tagging

MLP Ranking

Rank definition candidates

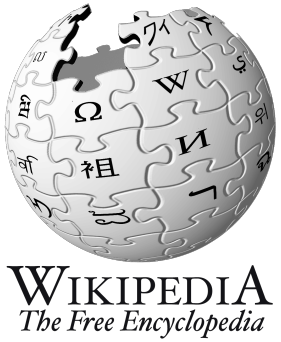
Output

[[Mass-enrgy equivalence]]

IDs: [*E, m, c, μ*, ...]

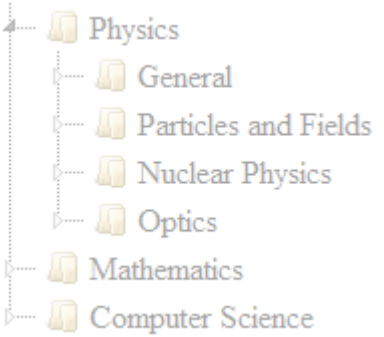
	Def	Rank
E	enegy	0.99
m	mass	0.99
c	speed of light	0.90

[MLP] Based on open source project by Robert Pagel



Definition extraction

Namespace discovery



Data Cleaning

Filter pages with `<math>`

Extract identifiers

POS Tagging

Rank definition candidates

Discard font features

$$\hat{\beta} \rightarrow \beta$$
$$\bar{X} \rightarrow X$$
$$\vec{v} \rightarrow v$$
$$\mathbf{v} \rightarrow v$$
$$\mathfrak{R} \rightarrow R$$
$$\mathbb{R} \rightarrow R$$
$$\mathcal{H} \rightarrow H$$
$$\hbar \rightarrow h$$

Discard false identifiers

Operators	Stop words	
cos	if	если
sin	iff	
exp	when	когда
max	then	тогда
min	or	или
trace	and	и

Definition extraction

Namespace discovery

- Physics
  - General
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

Represent in VSM

Cluster analysis

Filter clusters

Extract namespaces

Organize to hierarchy

Output

- Physics
  - General
    - Quantum Mechanics
    - Relativity and Gravitation
      - E ← "energy"
      - m ← "mass"
      - c ← "speed of light"
    - Special Relativity
  - Particles and Fields
  - Nuclear Physics
  - Atomic and Molecular Physics
  - Optics
- Mathematics
- Computer Science

Definition extraction

Namespace discovery

- Physics
  - General
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

Represent in VSM

Cluster analysis

Filter clusters

Extract namespaces

Organize to hierarchy

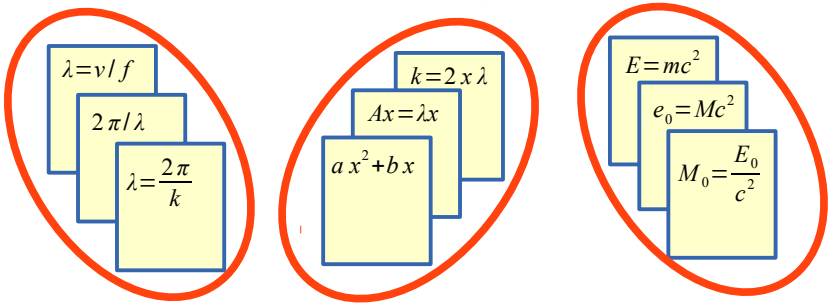
scikit learn

`TfidfVectorizer(min_df=2)`

`Kmeans` and `MiniBatchKMeans`

DBSCAN

`randomized_svd` and `NMF`



Definition extraction

Namespace discovery

- Physics
  - General
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

Represent in VSM

Cluster analysis

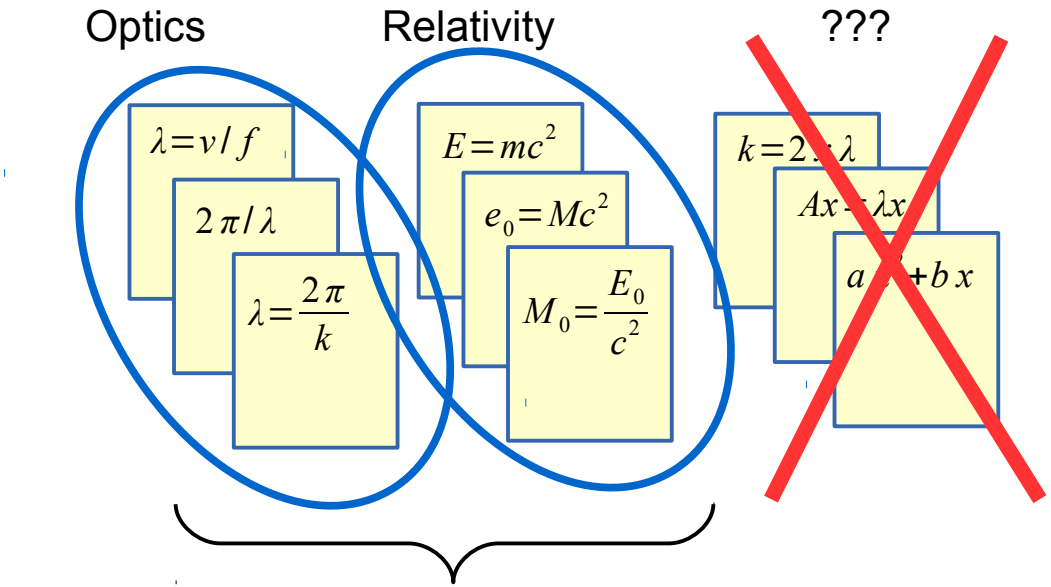
Filter clusters

Extract namespaces

Organize to hierarchy

All obtained clusters are "*homogenious*": witin-cluster similarity is maximal.

But not all are about the same domain. We keep only clusters with documents from the same category



"namespace-defining" clusters

Definition extraction

Namespace discovery

- Physics
  - General
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

Represent in VSM

Cluster analysis

Filter clusters

Extract namespaces

Organize to hierarchy

$E = mc^2$

	Def	R
E	energy	0.99
m	mass	0.99
c	speed of light	0.90
c	speed of light	1.00

$E_0 = M_0 c^2$

	Def	R
E_0	energy	0.90
M_0	mass	0.99
c	speed of light	0.90

$M_0 = \frac{E_0}{c^2}$

	Def	R
E_0	energy	0.99
M_0	mass	0.95
c	speed of light	0.90
c	energy	0.80

Relativity

	Def	R
E	energy	0.99
m	mass	0.99
c	speed of light	3.70
e_0	energy	0.99
M_0	mass	1.94
E_0	energy	1.89
c	energy	0.80

Definition extraction

Namespace discovery

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- Mathematics
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Represent in VSM

Cluster analysis

Filter clusters

Extract namespaces

Organize to hierarchy

$E = mc^2$

	Def	R
E	energy	0.99
m	mass	0.99
c	speed of light	0.90
c	speed of light	1.00

$E_0 = M_0 c^2$

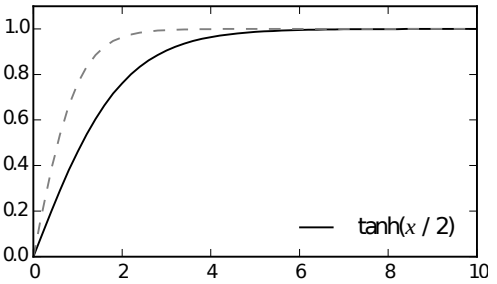
	Def	R
E_0	energy	0.90
M_0	mass	0.99
c	speed of light	0.90

$M_0 = \frac{E_0}{c^2}$

	Def	R
E_0	energy	0.99
M_0	mass	0.95
c	speed of light	0.90
c	energy	0.80

Relativity

	Def	R
E	energy	0.46
m	mass	0.46
c	speed of light	0.87
e_0	energy	0.46
M_0	mass	0.60
E_0	energy	0.57
c	energy	0.20



Definition extraction

Namespace discovery

- Physics
  - General
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  - Optics
- Mathematics
- Computer Science

Represent in VSM

Cluster analysis

Filter clusters

Extract namespaces

Organize to hierarchy

	Def	R
E	energy	0.99
m	mass	0.99
c	speed of light	3.70
c	speed of light in vacuum	0.99
m	mass	1.94
m	total mass	1.89

	Def	R
E	energy	0.99
c	***speed of light**	4.69
m	***mass**	4.82

Fuzzy grouping

FuzzyWuzzy https://github.com/seatgeek/fuzzywuzzy

Definition extraction

Namespace discovery

- Physics
  - General
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

Represent in VSM

Cluster analysis

Filter clusters

Extract namespaces

Organize to hierarchy

Where does hierarchy come from?

- Physics
  - General
    - Quantum Mechanics
    - Relativity and Gravitation
    - Special Relativity
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

PACS

MSC

ACM

Definition extraction

Namespace discovery

- Physics
  - General
  - Particles and Fields
  - Nuclear Physics
  - Optics
- Mathematics
- Computer Science

Represent in VSM

Cluster analysis

Filter clusters

Extract namespaces

Organize to hierarchy

00—General
01. Communication, education, history, and philosophy
02. Mathematical methods in physics
03. Quantum mechanics, field theories, and special relativity
04. General relativity and gravitation
05. Statistical physics, thermodynamics, and no ar dynamical systems
06. Metrology, measurements, and laboratory pr dures
07. Instruments, apparatus
physics and astronomy

10—The Physics of Elemen
11. General theory of fields
12. Specific theories and in
13. Specific reactions and p
14. Properties of specific p

20—Nuclear Physics
21. Nuclear structure
23. Radioactive decay and i
24. Nuclear reactions: gene
25. Nuclear reactions: spec
26. Nuclear astrophysics

PACS

04.	**General relativity and gravitation** *(for astrophysical aspects, gravitation; for relativistic aspects of cosmology, see 98.80.Jk; fo 03.30.+p)*
04.20.-q	**Classical general relativity** *(see also 02.40.-k Geometry, diffe topology)*
04.20.Cv	Fundamental problems and general formalism
04.20.Dw	Singularities and cosmic censorship
04.20.Ex	Initial value problem, existence and uniqueness of solutions
04.20.Fy	Canonical formalism, Lagrangians, and variational principles
04.20.Gz	Spacetime topology, causal structure, spinor structure
04.20.Ha	Asymptotic structure
04.20.Jb	Exact solutions
04.25.-g	**Approximation methods; equations of motion**
04.25.D-	Numerical relativity

https://www.aip.org/publishing/pacs/pacs-2010-regular-edition

Definition extraction

Namespace discovery

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- Mathematics
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Represent in VSM

Cluster analysis

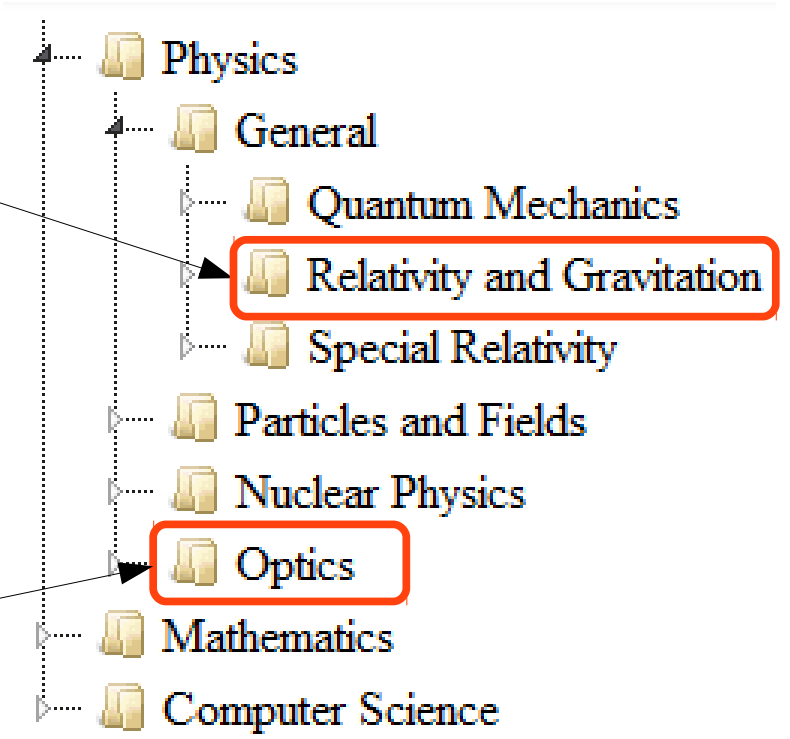
Filter clusters

Extract namespaces

Organize to hierarchy

	Def	R
E	energy	0.99
c	***speed of light**	4.69
m	***mass**	4.82

	Def	R
λ	wavelength	0.99
k	wavenumber	4.12
f	frequency	2.28



Extract keywords from namespaces ⟷ Extract keywords from categories

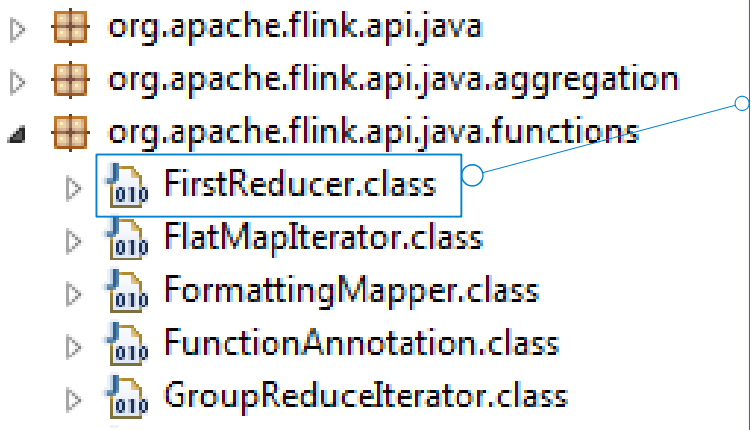
Calculate cosine between them

Outline

Java Language Processing

How to evaluate the quality?

- Hard! No ground truth, unsupervised settings
- Use data where ground truth is known: source code!



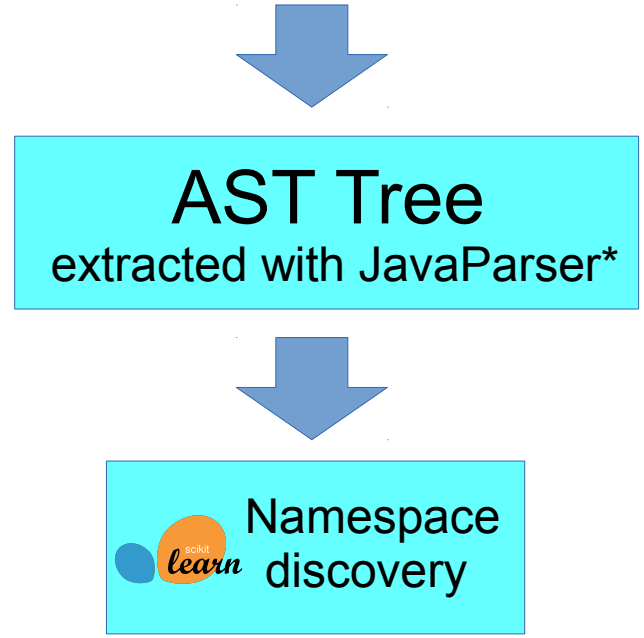

```
package org.apache.flink.api.java.functions;

                "definition"     identifier

public class FirstReducer<T> implements ... {

  private final int count;
  // ...

  @Override
  public void reduce(Iterable<T> values, Collector<T> out) {
    int emitCnt = 0;

    for (T val : values) {
      out.collect(val);
      // ...
    }
  }
}
```

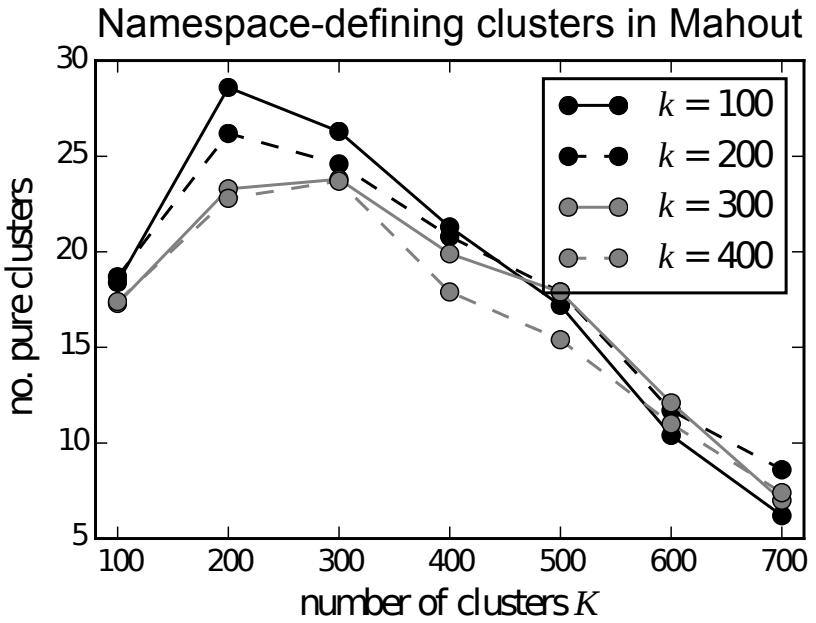
[[File:Java_logo_and_wordmark.svg]]

Java Language Processing

Apache Mahout

- 1560 Java Classes
- 46k variable declarations
- 150 packages





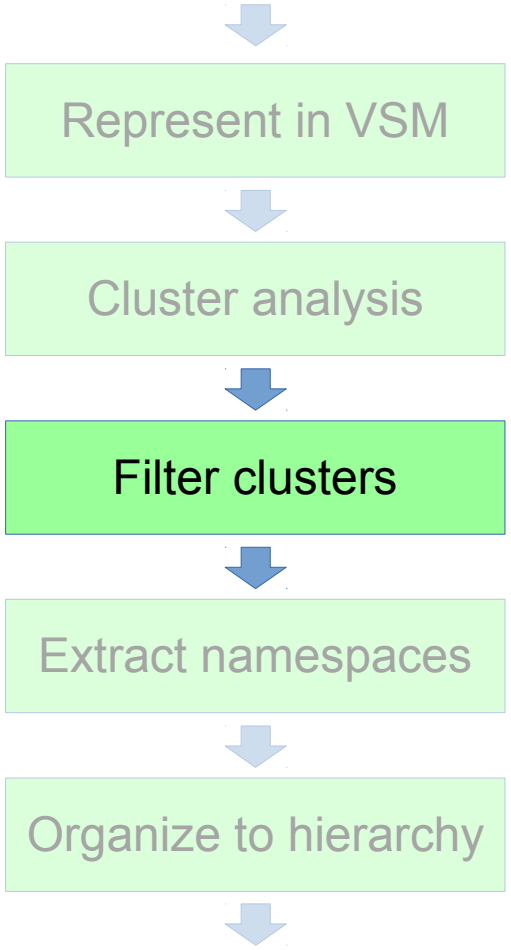
http://mahout.apache.org/images/

* https://github.com/javaparser/javaparser

✓ **Method works**

Outline

Experimental Setup



Objective: want to find as many namespace-defining clusters as possible

Cluster is *namespace-defining* if it

- has at least purity *p and*
- contains at least *n* documents

Purity *p* vs size *n* tradeoff:

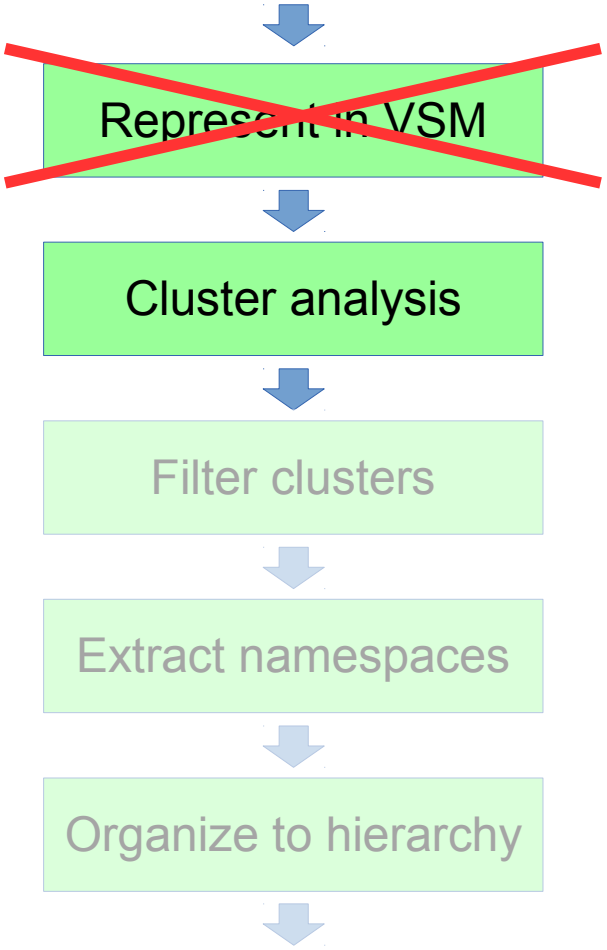
- Larger *p* – only pure clusters, smaller *p* – allow some slack
- Larger *n* – only big well-connected clusters are taken into account

Our settings: p ≥ 80% and n ≥ 3

Parameter Tuning

- Represent in VSM
  - Indentifier VSM: nodef, weak, strong
  - Weighting: TF, TF-IDF, logTF-IDF
- Cluster analysis
  - DBSCAN
    - base similarity function, ε, MinPts
  - K-Means
    - number of clusters K
  - Latent Semantic Analysis
    - matrix decomposition: SVD or NMF
    - rank of reduced matrix k
- Filter clusters
- Extract namespaces
- Organize to hierarchy

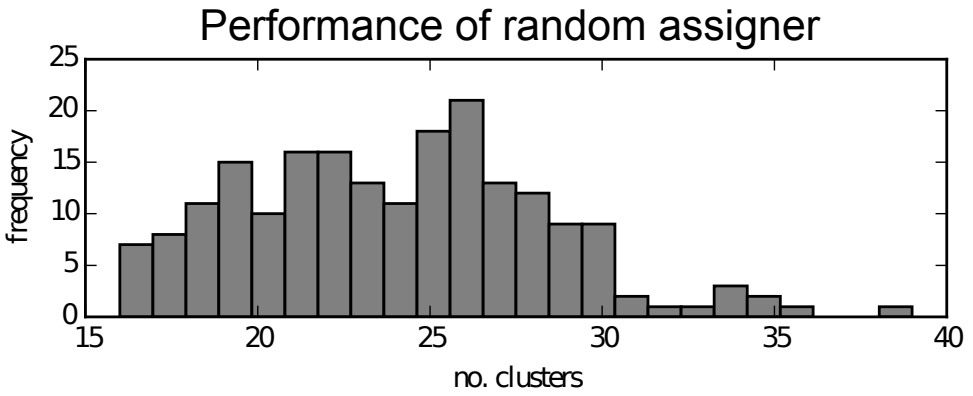
Baseline



Random cluster assigment

Algorithm:

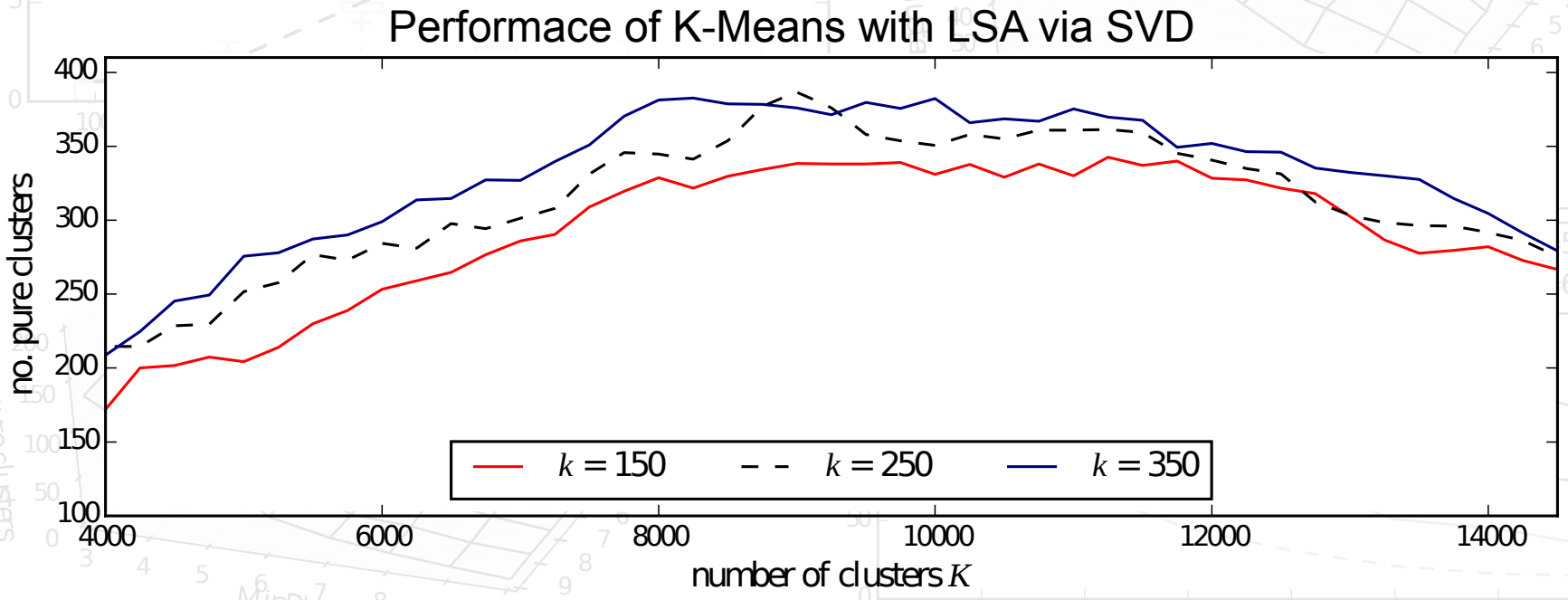
- let k = 0
- take 3 unseen documents at random
- assign them to cluster k
- increment k
- repeat until no documents left



Parameter Tuning

Best result is obtained with:

- Weak association
- LDA via SVD with k = 350 + K-Means with K = 9750



$$E = mc^2$$

What is it?

- Energy
- Expected Value
- Elimination matrix

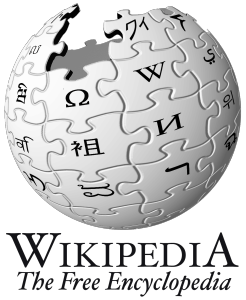
	E	m	c	λ	σ	μ
Linear algebra	matrix	matrix	scalar	eigenvalue	related permutation	algebraic multiplicity
General relativity	energy	mass	speed of light	length	shear	reduced mass
Coding theory	encoding function	message	transmitted codeword		natural isomorphisms	
Optics		order fringe	speed of light in vacuum	wavelength	conductivity	permeability
Probability	expectation	sample size		affine parameter	variance	mean vector

Outline

Conclusions

- We are first to approach the problem of namespace discovery
- Conclusion: Automatic namespace discovery is possible
- Can use established methods like VSM and Document clustering
- Best result: 414 namespaces, 10 times better than random guessing
- Works for other languages, not only English

Future Work



Definition extraction

Namespace discovery

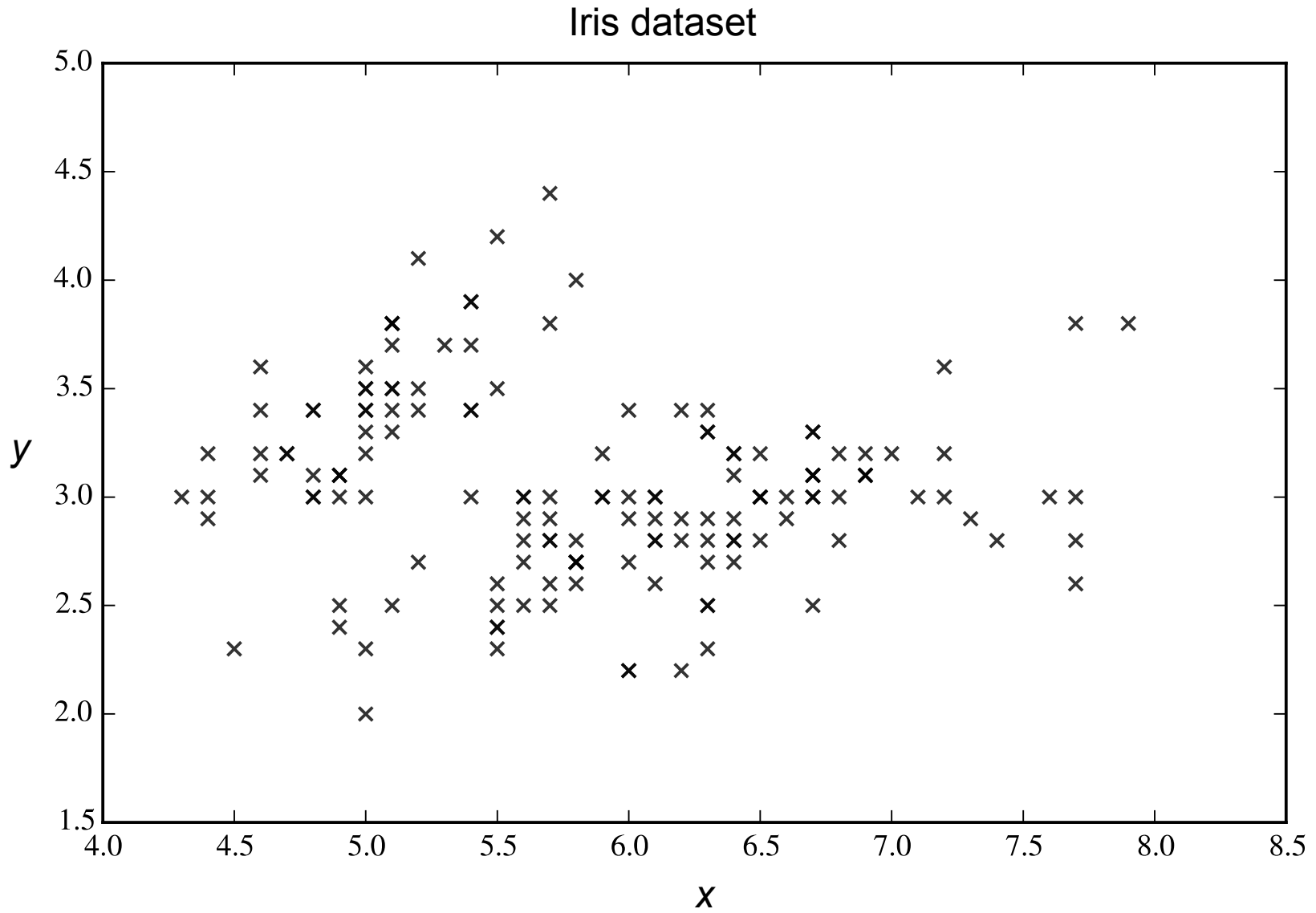
- Other datasets:
  - arXiv
  - StackExchange Q/A network: mathematics, cross-validated, physics, …
- ML methods for identifier extraction may give better results
- Other ways to embed definitions: 3-D tensors
- Expect advanced clustering algorithms to perform better
  - Split and Join operations in Scatter/Gather
  - Spectral Clustering
  - Cluster Ensembles
  - Topic Modeling: LDA

Questions?

References

- [MLP] Pagel, Robert, and Schubotz, Moritz. "Mathematical Language Processing Project.", 2014.
- [IR] Manning, Christopher et al. “Introduction to Information Retrieval”, 2008.
- [SSN] Ertöz, Levent, et al. "Finding clusters of different sizes, shapes, and densities in noisy, high dimensional data.", 2003.
- [LSI] Deerwester, Scott, et al. "Indexing by Latent Semantic Analysis.", 1990.

Back-up slide: Clustering Algorithms



Back-up slide: LSA

- Natural lanuage data is “noisy”
  - Synonymy: “graph” vs “chart”
  - Polysemy: “trunk” (part of elephant vs part of car)
- Denoise with dimensionality reduction
  - SVD: $D = U\Sigma V^T \qquad D \approx U_k \Sigma_k V_k^T$
  - NMF: $D = UV^T \qquad D \approx U_k V_k^T$
- Not only denoises but also reveals the latent structure of data