1. Discussion of Problem Set #1

2. Screening on Subway

The Chicago CTA has decided that it needs to do more to maximize its revenue. As such it has hired you to design its new price and service scheme. There are two types of customers, High-class and Low-class. They have preferences over the fare P and the degree of bad smell in the train car they ride in, denoted by B. They have told you that they are able to charge different fares depending on the car a customer rides in (i.e., to have different classes of service).

The type of a customer is not observable; the fraction of high-class customers is λ. Customers' utility functions are $u_i(P, B) = v - \theta_i P - B$, for $i = H, L$, where $\theta_L > \theta_H > 0$. All customers get utility (normalized) of 0 from walking (their next best alternative) instead of taking the CTA train.

Making train cars smell bad is not costless (workers need to be hired to make the cars smell worse): the CTA incurs a cost of $\gamma B > 0$ per customer who rides in a car that has smell level B.

1. Write down the problem you would solve for determining the CTA's profit-maximizing scheme. Assume throughout that the CTA cannot charge negative prices; i.e., that $P \geq 0$. Assume also that the CTA wants to serve both high and low class customers.
2. Solve the maximization problem.
   (a) Are there any constraints that are redundant?
   (b) Are there any constraints that must bind?
   (c) Can we say something about B_L and B_H right away? Why or why not?
   (d) What is the CTA's profit-maximizing scheme? How does it depend on the parameters of the problem?