Moral Hazard

Alexey Makarin

April 23 & 26, 2021

Moral Hazard relative vs. Adverse Selection

Both are about asymmetric information, but differ in two aspects.

1. *Nature of information.*
   - MH is about hidden action and the agent controls it
   - AS is about hidden information and the agent can't change it (nature decides).

Moral Hazard relative vs. Adverse Selection

Both are about asymmetric information, but differ in two aspects.

1. *Nature of information.*
    - MH is about hidden action and the agent controls it
    - AS is about hidden information and the agent can't change it (nature decides).
2. *Timing of information asymmetry.*
    - In MH, when the contract is signed, the information is symmetric because the agent has not yet made a hidden action which would become his private information later in the interaction.
    - In AS, when the contract is signed, the information is already asymmetric because the type of the agent is already defined by nature.

A Standard Moral Hazard Story

- **Principal (P)** hires **agent (A)** to perform a certain task.
- Principal wants the agent to exert effort but does not observe agent's effort, which the agent is choosing freely.
- Principal observes a noisy informative signal about agent's effort (e.g., output of the project) and can condition the agent's compensation on this signal in an attempt to motivate him.
- Examples of moral hazard?

Examples from Ancient History: Financial Crisis, 2007–08

Several actors had misaligned incentives, arguably increasing systemic risk and contributing to the global financial crisis:

Examples from Ancient History: Financial Crisis, 2007–08

Several actors had misaligned incentives, arguably increasing systemic risk and contributing to the global financial crisis:

- *Asset managers:* had an incentive to take on more risk when managing portfolios. Skewed reward structure: paid if returns are high, shielded from losses.

Examples from Ancient History: Financial Crisis, 2007–08

Several actors had misaligned incentives, arguably increasing systemic risk and contributing to the global financial crisis:

- *Asset managers:* had an incentive to take on more risk when managing portfolios. Skewed reward structure: paid if returns are high, shielded from losses.
- *Mortgage loan originators:* had an incentive to understate the risk of loans since those were often sold to mortgage pools (to form basis for mortgage-backed securities).

Examples from Ancient History: Financial Crisis, 2007–08

Several actors had misaligned incentives, arguably increasing systemic risk and contributing to the global financial crisis:

- *Asset managers:* had an incentive to take on more risk when managing portfolios. Skewed reward structure: paid if returns are high, shielded from losses.
- *Mortgage loan originators:* had an incentive to understate the risk of loans since those were often sold to mortgage pools (to form basis for mortgage-backed securities).
- *Large banks:* "too big to fail." Expected a government bailout in an event of failure, so took on bigger risks than would otherwise. "Privatize the profits, socialize the losses."

Examples from Ancient History: Financial Crisis, 2007–08

Several actors had misaligned incentives, arguably increasing systemic risk and contributing to the global financial crisis:

- *Asset managers:* had an incentive to take on more risk when managing portfolios. Skewed reward structure: paid if returns are high, shielded from losses.
- *Mortgage loan originators:* had an incentive to understate the risk of loans since those were often sold to mortgage pools (to form basis for mortgage-backed securities).
- *Large banks:* "too big to fail." Expected a government bailout in an event of failure, so took on bigger risks than would otherwise. "Privatize the profits, socialize the losses."

All are stark examples of moral hazard and its potential for harm.

Two Outcome Model: Agent

- Agent's utility function: $v(w) - e$
- Agent is risk-averse: $v'(\cdot) > 0$, $v''(\cdot) < 0$
- Agent's risk-aversion is critical for having a principal at all:
  - Since principal is less sensitive to risk (typically, risk-neutral), so optimal solution won't be to sell the firm to the agent.
  - Possible justification: agent is poor and has limited capital, while principal is rich and sufficiently diversified
- Agent's outside option yields utility $\bar{U}$

Two Outcome Model: Principal

- Suppose $e \in \{0, 1\}$.
- Success brings the principal x^S while failure brings x^F
- If $e = 1$, then prob. of success is P and prob. of failure is $1 - P$.
- If $e = 0$, then prob. of success is p and prob. of failure is $1 - p$.
- With unobservable effort, principal's expected utility function is:

$$R = \begin{cases} P(x^S - w^S) + (1 - P)(x^F - w^F) & \text{if } e = 1 \\ p(x^S - w^S) + (1 - p)(x^F - w^F) & \text{if } e = 0 \end{cases}$$

- Since principal moves first, ties break in her favor.

Two Outcome Model: Constraints

- How to make the agent exert effort?
- As before, need to make sure that agent is better off working hard (*incentive compatibility constraint*):

$$Pv(w^S) + (1-P)v(w^F) - 1 \geq pv(w^S) + (1-p)v(w^F)$$

- Also, need to make sure that agent does not want to quit (*individual rationality constraint*):

$$Pv(w^S) + (1-P)v(w^F) - 1 \geq \bar{U}$$

Solving the Two Outcome Model

Lemma 1. IR binds.

In a principal agent model with two actions, the IR constraint binds (i.e., is satisfied with equality) for the optimal contract.

Proof:

Solving the Two Outcome Model

Lemma 1. IR binds.

In a principal agent model with two actions, the IR constraint binds (i.e., is satisfied with equality) for the optimal contract.

Proof: Suppose IR does not bind:

$$Pv(w^S) + (1 - P)v(w^F) - 1 > \bar{U}$$

Solving the Two Outcome Model

Lemma 1. IR binds.

In a principal agent model with two actions, the IR constraint binds (i.e., is satisfied with equality) for the optimal contract.

Proof: Suppose IR does not bind:

$$Pv(w^S) + (1-P)v(w^F) - 1 > \bar{U}$$

Lower w^S and w^F by ε small enough such that IR is still satisfied. Rewrite IC in the following way:

$$(P-p)v(w^S-\varepsilon) - 1 \geq (P-p)v(w^F-\varepsilon)$$

Since v is concave and $w^S > w^F$, IC still holds. So, principal was able to raise her profits without violating constraints.

Solving the Two Outcome Model

Lemma 2. IC binds.

In a principal agent model with two actions, the IC constraint binds (i.e., is satisfied with equality) for the optimal contract.

Proof:

Solving the Two Outcome Model

Lemma 2. IC binds.

In a principal agent model with two actions, the IC constraint binds (i.e., is satisfied with equality) for the optimal contract.

Proof: Suppose IC does not bind:

$$(P-p)v(w^S)-1>(P-p)v(w^F)$$

Can lower w^S by δ and increase w^F by $\delta P/(1-P)$ such that principal's expected profit is unchanged and with δ small enough such that IC is still satisfied.

Solving the Two Outcome Model

Lemma 2. IC binds.

In a principal agent model with two actions, the IC constraint binds (i.e., is satisfied with equality) for the optimal contract.

Proof: Suppose IC does not bind:

$$(P-p)v(w^S)-1>(P-p)v(w^F)$$

Can lower w^S by δ and increase w^F by $\delta P/(1-P)$ such that principal's expected profit is unchanged and with δ small enough such that IC is still satisfied. Then note that, due to concavity of v, IR gets relaxed:

$$Pv(w^S-\delta)+(1-P)v(w^F+\delta P/(1-P))-1>\bar{U}$$

But then we can lower wages by ε (see Lemma 1). Hence, the original contract $\{w^S,\ w^F\}$ was not profit maximizing.

Solution

Two equations, two unknowns:

$$(P - p)[v(w^S) - v(w^F)] = 1$$
$$Pv(w^S) + (1 - P)v(w^F) = \bar{U} + 1$$

Solution

Two equations, two unknowns:

$$(P - p)[v(w^S) - v(w^F)] = 1$$
$$Pv(w^S) + (1 - P)v(w^F) = \bar{U} + 1$$

After some algebra, get:

$$w^F = v^{-1}\left(\bar{U} - \frac{p}{P - p}\right)$$
$$w^S = v^{-1}\left(\bar{U} + \frac{1 - p}{P - p}\right)$$

Solution

Two equations, two unknowns:

$$(P - p)[v(w^S) - v(w^F)] = 1$$
$$Pv(w^S) + (1 - P)v(w^F) = \bar{U} + 1$$

After some algebra, get:

$$w^F = v^{-1}\left(\bar{U} - \frac{p}{P - p}\right)$$
$$w^S = v^{-1}\left(\bar{U} + \frac{1 - p}{P - p}\right)$$

So, the optimal contract is such that $v(w^S) > \bar{U} + 1 > v(w^F)$.

Recap

- We solved the basic principal agent model with moral hazard.

Recap

- We solved the basic principal agent model with moral hazard.
- Principal incentivizes agent to exert effort by setting a high 'success' wage and a low 'failure' wage.

Recap

- We solved the basic principal agent model with moral hazard.
- Principal incentivizes agent to exert effort by setting a high 'success' wage and a low 'failure' wage.
- This simple model helps rationalize some features of payment structures in firms, such as bonuses, performance incentives, incentive stock options (ISOs), etc.

Recap

- We solved the basic principal agent model with moral hazard.
- Principal incentivizes agent to exert effort by setting a high 'success' wage and a low 'failure' wage.
- This simple model helps rationalize some features of payment structures in firms, such as bonuses, performance incentives, incentive stock options (ISOs), etc.
- Now let's try to solve a more general model and see if anything changes or if we can get any extra insights.

Multiple Outcomes Model: Agent

- Owner (principal) is hiring a manager (agent) to run a company
- Agent's utility function: $v(w) - g(e)$
  - Agent is risk-averse: $v'(\cdot) > 0$, $v''(\cdot) < 0$,
  - Agent doesn't like high effort: $g(e_H) > g(e_L)$
  - Agent's reservation utility is $\bar{U}$
- Suppose $e \in \{e_L, e_H\}$.

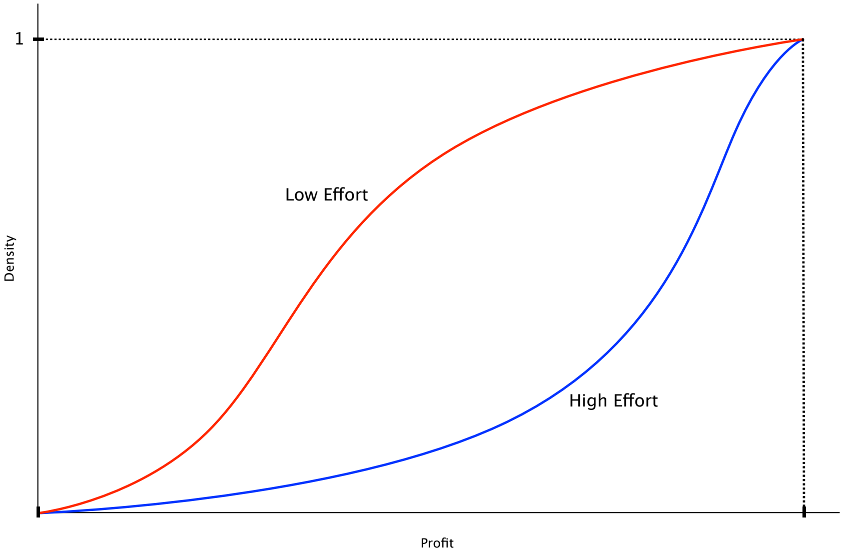
Multiple Outcomes Model: Principal

- Profit of the project is random: $\pi_1 < \ldots < \pi_n$
- $p(\pi_k|e)$ — probability that π_k is realized when agent chooses e.
- Assume that $p(\cdot|e_H)$ *first-order stochastically dominates* $p(\cdot|e_L)$:

$$\sum_{k=1}^{m} p(\pi_k|e_H) \leq \sum_{k=1}^{m} p(\pi_k|e_L)\ ,\ \forall m \in \{1, \ldots, n\}$$

- Principal: choose wage schedule $w(\pi_k)$ to maximize $\mathbb{E}(\pi - w)$

FOSD Illustrated



Benchmark: Observable Effort

- Suppose effort is observable. What then?
- Principal can condition wage on both effort and profits:

$$w(e, \pi_k) = w_k(e) \in \{w_1(e_L), \ldots, w_n(e_L); w_1(e_H), \ldots, w_n(e_H)\}$$

Benchmark: Observable Effort

- Suppose effort is observable. What then?
- Principal can condition wage on both effort and profits:

$$w(e, \pi_k) = w_k(e) \in \{w_1(e_L), \ldots, w_n(e_L); w_1(e_H), \ldots, w_n(e_H)\}$$

- Principal's problem:

$$\max_{e \in \{e_L, e_H\}, w_k(e)} \sum_{k=1}^{n} [\pi_k - w_k(e)] p(\pi_k | e)$$

$$\text{s.t.} \quad \sum_{k=1}^{n} v(w_k(e)) p(\pi_k | e) - g(e) \geq \bar{U}$$

Benchmark: Observable Effort

- Suppose effort is observable. What then?
- Principal can condition wage on both effort and profits:

$$w(e, \pi_k) = w_k(e) \in \{w_1(e_L), \ldots, w_n(e_L); w_1(e_H), \ldots, w_n(e_H)\}$$

- Principal's problem:

$$\max_{e \in \{e_L, e_H\}, w_k(e)} \sum_{k=1}^{n} [\pi_k - w_k(e)] p(\pi_k | e)$$
$$\text{s.t.} \quad \sum_{k=1}^{n} v(w_k(e)) p(\pi_k | e) - g(e) \geq \bar{U}$$

- Solve in two steps:
  1. Given e, what is the best $w(e)$?
  2. What is the best e?

Benchmark: Step 1: Finding Best $w(e)$

$$L(\hat{w}_1, \ldots, \hat{w}_n, \lambda) = \sum_{k=1}^{n} \hat{w}_k p(\pi_k|\hat{e}) - \lambda \left[\sum_{k=1}^{n} v(\hat{w}_k) p(\pi_k|\hat{e}) - g(\hat{e}) - \bar{U} \right]$$

Benchmark: Step 1: Finding Best $w(e)$

$$L(\hat{w}_1, \ldots, \hat{w}_n, \lambda) = \sum_{k=1}^{n} \hat{w}_k p(\pi_k|\hat{e}) - \lambda \left[\sum_{k=1}^{n} v(\hat{w}_k) p(\pi_k|\hat{e}) - g(\hat{e}) - \bar{U} \right]$$

$$\frac{\partial L}{\partial w_k} = p(\pi_k|\hat{e}) - \lambda v'(\hat{w}_k) p(\pi_k|\hat{e}) = 0 \ , \ \forall k = 1, \ldots, n$$

Benchmark: Step 1: Finding Best $w(e)$

$$L(\hat{w}_1, \ldots, \hat{w}_n, \lambda) = \sum_{k=1}^{n} \hat{w}_k p(\pi_k|\hat{e}) - \lambda \left[\sum_{k=1}^{n} v(\hat{w}_k) p(\pi_k|\hat{e}) - g(\hat{e}) - \bar{U} \right]$$

$$\frac{\partial L}{\partial w_k} = p(\pi_k|\hat{e}) - \lambda v'(\hat{w}_k) p(\pi_k|\hat{e}) = 0 \text{ , } \forall k = 1, \ldots, n$$

$$\implies v'(\hat{w}_1) = \ldots = v'(\hat{w}_n) = \frac{1}{\lambda} > 0$$

Benchmark: Step 1: Finding Best $w(e)$

$$L(\hat{w}_1, \ldots, \hat{w}_n, \lambda) = \sum_{k=1}^{n} \hat{w}_k p(\pi_k|\hat{e}) - \lambda \left[\sum_{k=1}^{n} v(\hat{w}_k) p(\pi_k|\hat{e}) - g(\hat{e}) - \bar{U} \right]$$

$$\frac{\partial L}{\partial w_k} = p(\pi_k|\hat{e}) - \lambda v'(\hat{w}_k) p(\pi_k|\hat{e}) = 0 \ , \ \forall k = 1, \ldots, n$$

$$\implies v'(\hat{w}_1) = \ldots = v'(\hat{w}_n) = \frac{1}{\lambda} > 0$$

- Note #1: $\lambda > 0$, so the participation constraint is binding

Benchmark: Step 1: Finding Best $w(e)$

$$L(\hat{w}_1, \ldots, \hat{w}_n, \lambda) = \sum_{k=1}^{n} \hat{w}_k p(\pi_k|\hat{e}) - \lambda \left[\sum_{k=1}^{n} v(\hat{w}_k) p(\pi_k|\hat{e}) - g(\hat{e}) - \bar{U} \right]$$

$$\frac{\partial L}{\partial w_k} = p(\pi_k|\hat{e}) - \lambda v'(\hat{w}_k) p(\pi_k|\hat{e}) = 0 \ , \ \forall k = 1, \ldots, n$$

$$\implies v'(\hat{w}_1) = \ldots = v'(\hat{w}_n) = \frac{1}{\lambda} > 0$$

- Note #1: $\lambda > 0$, so the participation constraint is binding
- Note #2: Since $v'(\cdot)$ is decreasing ($v'' < 0$), the only solution is equal wage across all states: $\hat{w}_1 = \ldots = \hat{w}_n = \hat{w}^*$

Benchmark: Step 1: Finding Best $w(e)$

$$L(\hat{w}_1, \ldots, \hat{w}_n, \lambda) = \sum_{k=1}^{n} \hat{w}_k p(\pi_k|\hat{e}) - \lambda \left[\sum_{k=1}^{n} v(\hat{w}_k) p(\pi_k|\hat{e}) - g(\hat{e}) - \bar{U} \right]$$

$$\frac{\partial L}{\partial w_k} = p(\pi_k|\hat{e}) - \lambda v'(\hat{w}_k) p(\pi_k|\hat{e}) = 0 \ , \ \forall k = 1, \ldots, n$$

$$\implies v'(\hat{w}_1) = \ldots = v'(\hat{w}_n) = \frac{1}{\lambda} > 0$$

- Note #1: $\lambda > 0$, so the participation constraint is binding
- Note #2: Since $v'(\cdot)$ is decreasing ($v'' < 0$), the only solution is equal wage across all states: $\hat{w}_1 = \ldots = \hat{w}_n = \hat{w}^*$
- Note #3: Because of #1 and #2, can rewrite the IR constraint as $v(\hat{w}^*) - g(e) = \bar{U}$, so $\hat{w}^*(e) = v^{-1}(\bar{U} + g(e))$ is the optimal wage conditional on e being implemented

Benchmark: Step 2: Finding Best e

- Suppose e_L is optimal.
  - Then the wage is $\hat{w}_L^* = v^{-1}(\bar{U} + g(e_L))$
  - The owner's payoff is $\sum_{k=1}^{n} \pi_k p(\pi_k|e_L) - v^{-1}(\bar{U} + g(e_L))$

Benchmark: Step 2: Finding Best e

- Suppose e_L is optimal.
  - Then the wage is $\hat{w}_L^* = v^{-1}(\bar{U} + g(e_L))$
  - The owner's payoff is $\sum_{k=1}^{n} \pi_k p(\pi_k | e_L) - v^{-1}(\bar{U} + g(e_L))$
- Suppose e_H is optimal.
  - Then the wage is $\hat{w}_H^* = v^{-1}(\bar{U} + g(e_H))$
  - The owner's payoff is $\sum_{k=1}^{n} \pi_k p(\pi_k | e_H) - v^{-1}(\bar{U} + g(e_H))$

Benchmark: Step 2: Finding Best e

- Suppose e_L is optimal.
  - Then the wage is $\hat{w}_L^* = v^{-1}(\bar{U} + g(e_L))$
  - The owner's payoff is $\sum_{k=1}^{n} \pi_k p(\pi_k | e_L) - v^{-1}(\bar{U} + g(e_L))$
- Suppose e_H is optimal.
  - Then the wage is $\hat{w}_H^* = v^{-1}(\bar{U} + g(e_H))$
  - The owner's payoff is $\sum_{k=1}^{n} \pi_k p(\pi_k | e_H) - v^{-1}(\bar{U} + g(e_H))$

Proposition. Benchmark Solution.

In the principal-agent model with observable managerial effort, an optimal contract specifies that the principal chooses effort e^* that maximizes $\sum_{k=1}^{n} \pi_k p(\pi_k | e) - v^{-1}(\bar{U} + g(e))$ and pays the manager a fixed wage $\hat{w}^* = v^{-1}(\bar{U} + g(e))$. This is the uniquely optimal contract if $v'' < 0$, i.e., if the manager is risk-averse.

Taking Stock

- The optimal contract with observable effort achieved two goals:
  - It specified the optimal level of effort by the manager.
  - It insured manager against risk.
- When effort is not observable, the two goals may be in conflict.
- To disentangle risk-sharing and optimal effort choice, we first study the case of a risk neutral manager with unobserved effort.

Taking Stock

- The optimal contract with observable effort achieved two goals:
  - It specified the optimal level of effort by the manager.
  - It insured manager against risk.
- When effort is not observable, the two goals may be in conflict.
- To disentangle risk-sharing and optimal effort choice, we first study the case of a risk neutral manager with unobserved effort.
- Note: The owner must (weakly) prefer the outcome of the game with observed effort relative to that of the game with unobserved effort. (Why?)

Risk Neutral Agent, Unobservable Effort

The owner's problem is:

$$\max_{w_1,\dots,w_n} \sum_{k=1}^{n} (\pi_k - w_k) p(\pi_k|e)$$

$$\text{s.t.} \quad \sum_{k=1}^{n} w_k p(\pi_k|e) - g(e) \geq \bar{U}$$

$$e = \arg\max_{\tilde{e}} \sum_{k=1}^{n} w_k p(\pi_k|\tilde{e}) - g(\tilde{e})$$

Risk Neutral Agent, Unobservable Effort

Proposition.

If agent is risk-neutral and effort is unobservable, the optimal contract generates the same effort choice and expected utilities for the manager and the owner as when the effort is observable.

Proof:

Risk Neutral Agent, Unobservable Effort

Proposition.

If agent is risk-neutral and effort is unobservable, the optimal contract generates the same effort choice and expected utilities for the manager and the owner as when the effort is observable.

<u>Proof:</u> Suppose the owner offers wage schedule $w_k = \pi_k - \alpha$. If manager accepts, then he chooses e to maximize expected utility:

$$\sum_{k=1}^{n} w_k p(\pi_k|e) - g(e) = \sum_{k=1}^{n} \pi_k p(\pi_k|e) - g(e) - \alpha$$

Note: solution to manager's problem solves owner's problem too!

Risk Neutral Agent, Unobservable Effort

Proposition.

If agent is risk-neutral and effort is unobservable, the optimal contract generates the same effort choice and expected utilities for the manager and the owner as when the effort is observable.

Proof: Suppose the owner offers wage schedule $w_k = \pi_k - \alpha$. If manager accepts, then he chooses e to maximize expected utility:

$$\sum_{k=1}^{n} w_k p(\pi_k|e) - g(e) = \sum_{k=1}^{n} \pi_k p(\pi_k|e) - g(e) - \alpha$$

Note: solution to manager's problem solves owner's problem too! Let e^* denote the optimal effort level chosen by manager given this contract. Denote α^* be the one that makes IR bind:

$$\sum_{k=1}^{n} \pi_k p(\pi_k|e^*) - g(e^*) - \bar{U} = \alpha^*$$

Check: Under wage schedule $w_k = \pi_k - \alpha^*$, the owner and risk-neutral manager get the same payoff as with observable effort.

Risk Averse Agent, Unobservable Effort

- Let's now (finally!) solve the main problem, i.e., with risk-averse agent and unobservable effort.
- The owner's problem in this case is:

$$\min_{w_1,\ldots,w_n} \sum_{k=1}^{n} w_k p(\pi_k|e)$$

$$\text{s.t. } \sum_{k=1}^{n} v(w_k)p(\pi_k|e) - g(e) \geq \bar{U}$$

$$e = \arg\max_{\tilde{e}} \sum_{k=1}^{n} v(w_k)p(\pi_k|\tilde{e}) - g(\tilde{e})$$

Main Problem: Implementing e_L

- Suppose owner wants to induce e_L.
- Then set a fixed wage $w^* = v^{-1}(\bar{U} + g(e_L))$ independent of k.
- Since wage is set independently of effort, manager chooses $e = e_L$.

Main Problem: Implementing e_H

- Suppose owner wants to induce e_H.
- Can rewrite IC constraint as:

$$\sum_{k=1}^{n} v(w_k)p(\pi_k|e_H) - g(e_H) \geq \sum_{k=1}^{n} v(w_k)p(\pi_k|e_L) - g(e_L)$$

Lemma

When the owner wants to induce high effort, both IR and IC bind.

Main Problem: Implementing e_H

Lemma

When the owner wants to induce high effort, both IR and IC bind.

<u>*Proof:*</u> Our strategy would be to show that Lagrange multipliers on both constraints are positive.

$$L(w_k) = -\sum_{k=1}^{n} w_k p(\pi_k|e_H) + \lambda \left[\sum_{k=1}^{n} v(w_k)p(\pi_k|e_H) - g(e_H) - \bar{U}\right] + \\ + \mu \left[\sum_{k=1}^{n} v(w_k)p(\pi_k|e_H) - g(e_H) - \sum_{k=1}^{n} v(w_k)p(\pi_k|e_L) + g(e_L)\right]$$

That is, need to show that $\lambda > 0$ and $\mu > 0$.

Main Problem: Implementing e_H

<u>*Proof (cont'd):*</u> Take FOC:

$$\frac{\partial L}{\partial w_k} = -p(\pi_k|e_H) + \lambda v'(w_k)p(\pi_k|e_H) + \mu v'(w_k)[p(\pi_k|e_H) - p(\pi_k|e_L)] = 0$$

Divide both sides by $v'(w_k)p(\pi_k|e_H)$:

$$-\frac{1}{v'(w_k)} + \lambda + \mu\left[1 - \frac{p(\pi_k|e_L)}{p(\pi_k|e_H)}\right] = 0 \ , \ \forall k$$

Main Problem: Implementing e_H

Proof (cont'd): Suppose that $\lambda = 0$. Then:

$$\frac{1}{v'(w_k)} = \mu \left[1 - \frac{p(\pi_k|e_L)}{p(\pi_k|e_H)} \right] \ , \forall k$$

However, for $k = 1$, this equality cannot hold. From FOSD, we have that $p(\pi_1|e_L) \geq p(\pi_1|e_H)$. At the same time, it must be that $1/v'(w_1) > 0$. Therefore, $\lambda > 0$.

Main Problem: Implementing e_H

Proof (cont'd): Suppose that $\lambda = 0$. Then:

$$\frac{1}{v'(w_k)} = \mu \left[1 - \frac{p(\pi_k|e_L)}{p(\pi_k|e_H)}\right] \ , \forall k$$

However, for $k = 1$, this equality cannot hold. From FOSD, we have that $p(\pi_1|e_L) \geq p(\pi_1|e_H)$. At the same time, it must be that $1/v'(w_1) > 0$. Therefore, $\lambda > 0$.

Suppose that $\mu = 0$. Then:

$$\frac{1}{v'(w_1)} = \ldots = \frac{1}{v'(w_n)} = \lambda$$

Since $v'' < 0$, this can only happen if $w_1 = \ldots = w_n$. But this violates the IC constraint since e_H is costlier than e_L. Therefore, $\mu > 0$.

Observations

1. We know that $\pi_1 < \ldots < \pi_n$. Does this imply that $w_1 < \ldots < w_n$, as in the simple two-state model?

Observations

1. We know that $\pi_1 < \ldots < \pi_n$. Does this imply that $w_1 < \ldots < w_n$, as in the simple two-state model?
   - **No.** For this, it must be that the following condition holds:

$$\frac{p(\pi_1|e_L)}{p(\pi_1|e_H)} \geq \frac{p(\pi_2|e_L)}{p(\pi_2|e_H)} \geq \ldots \geq \frac{p(\pi_n|e_L)}{p(\pi_n|e_H)}$$

     This is known as *monotone likelihood ratio property*. MLRP implies FOSD but not the other way around.

Observations

1. We know that $\pi_1 < \ldots < \pi_n$. Does this imply that $w_1 < \ldots < w_n$, as in the simple two-state model?
   - **No.** For this, it must be that the following condition holds:

$$\frac{p(\pi_1|e_L)}{p(\pi_1|e_H)} \geq \frac{p(\pi_2|e_L)}{p(\pi_2|e_H)} \geq \ldots \geq \frac{p(\pi_n|e_L)}{p(\pi_n|e_H)}$$

     This is known as *monotone likelihood ratio property*. MLRP implies FOSD but not the other way around.

2. Does unobservability of effort increase owner's compensation cost of implementing e_H?

Observations

1. We know that $\pi_1 < \ldots < \pi_n$. Does this imply that $w_1 < \ldots < w_n$, as in the simple two-state model?
   - **No.** For this, it must be that the following condition holds:

$$\frac{p(\pi_1|e_L)}{p(\pi_1|e_H)} \geq \frac{p(\pi_2|e_L)}{p(\pi_2|e_H)} \geq \ldots \geq \frac{p(\pi_n|e_L)}{p(\pi_n|e_H)}$$

     This is known as *monotone likelihood ratio property*. MLRP implies FOSD but not the other way around.

2. Does unobservability of effort increase owner's compensation cost of implementing e_H?
   - **Yes.** From Jensen's inequality:

$$\mathbb{E}v(w|e_H) = \bar{U} + g(e_H) \implies v[\mathbb{E}(w|e_H)] > \bar{U} + g(e_H) = v(w_H^*)$$

     So, $\mathbb{E}(w|e_H) > w_H^*$ where w_H^* is the wage that implements e_H in benchmark analysis with observable effort.

Conclusion

- Moral hazard is a key concept in information economics.

Conclusion

- Moral hazard is a key concept in information economics.
- In this lecture, we studied a model of moral hazard in two versions: one with two states (success or failure) and one with many states.

Conclusion

- Moral hazard is a key concept in information economics.
- In this lecture, we studied a model of moral hazard in two versions: one with two states (success or failure) and one with many states.
- Agent's risk aversion prevents agent and owner from achieving first-best with unobservable effort.

Conclusion

- Moral hazard is a key concept in information economics.
- In this lecture, we studied a model of moral hazard in two versions: one with two states (success or failure) and one with many states.
- Agent's risk aversion prevents agent and owner from achieving first-best with unobservable effort.
- Wage monotonicity across states in a model with many states requires the MLRP assumption

Conclusion

- Moral hazard is a key concept in information economics.
- In this lecture, we studied a model of moral hazard in two versions: one with two states (success or failure) and one with many states.
- Agent's risk aversion prevents agent and owner from achieving first-best with unobservable effort.
- Wage monotonicity across states in a model with many states requires the MLRP assumption
- Unobservability increases costs of implementing high but not low effort. Thus, sometimes, inefficiently low effort levels would be implemented.