Problem 1. Moral Hazard with Two Chefs [5 pts].

Liyan is opening a Russian restaurant on Piazza Fiume. She needs to hire *one* chef and there are two candidates: Georgii and Alexey. The probability that the restaurant is successful depends on which of them is hired and whether the hired chef puts in effort or not. Those probabilities are given by the following matrix:

	Effort	No Effort
Georgii	p	q
Alexey	$p-\varepsilon$	r

where $0 < r < q < p < 1$ and $\varepsilon > 0$. *Note that Georgii is better than Alexey whether effort is exerted or not.* Other than that, they are completely identical. Liyan is risk-neutral. On the other hand, Georgii and Alexey are risk-averse: their payoffs are $v(w) - c$ with effort and $v(w)$ without effort, where w is the wage and $v(\cdot)$ is a concave function with $v' > 0, v'' < 0, v(-\infty) = -\infty, v(+\infty) = +\infty$, and $c > 0$. The wage can be positive or negative (interpreted as a fine). The reservation utilities for both Georgii and Alexey are normalized to zero.

Liyan cannot observe whether the hired chef puts in effort or not because she cannot judge the quality of Russian food, but she *does* observe if the restaurant is successful or not. Therefore, the wage payment can depend on restaurant's success. Let w_S and w_F be the wage payments when the restaurant is successful and when it is not, respectively. Furthermore, the restaurant yields gross profits of S and F when it is successful and when it is a failure, respectively. Suppose that $S - F$ is large enough so that it is always optimal for Liyan to induce effort from the chef she hires.

(a) Let (w_S^*, w_F^*) be the optimal wage scheme for Georgii. Provide the equations that characterize (w_S^*, w_F^*) and briefly explain why it must satisfy those equations. [2 pts.]

(b) Actually, Liyan is strictly better off hiring Alexey instead of Georgii when ε is small enough even though Georgii is a better chef. Explain why this is the case by illustrating how the optimal wage scheme for Georgii can be modified to attain a strictly higher net profit for Alexey. (*Hint: What happens if the optimal wage scheme for Georgii is offered to Alexey?*) Provide the intuition for this result. [3 pts.]

Problem 2. Moral Hazard on the Trading Floor [10 pts].

A company hires a trader to generate return that can be either high or low: $r \in \{r_H, r_L\}$. The level of return depends on the trader's effort e: The trader may either work ($e = 1$) or browse the Web ($e = 0$). When $e = 1$, the trader incurs a cost of c; browsing costs 0. Let $Pr(r = r_H | e = 1) \equiv p_1$ and $Pr(r = r_H | e = 0) \equiv p_0$, where $p_1 > p_0$. The trader's effort level is privately known, i.e., it is not observable to the company. The company offers a contract that specifies a wage of w_H if the return is r_H and a wage of w_L if the return is r_L. The company and the trader are both risk neutral. Assume that the reservation utility of the trader is 0. [*Hint: You should be able to answer all questions without setting up a Lagrangian.*]

1. Assume that the company wants to implement the high effort level (i.e., $e = 1$). Write down the optimization problem for the company. Show that the relevant constraint(s) must either bind or can be treated as binding without loss of generality. [2 pts.]

2. Based on your answer to part 1, solve for the wages that characterize the optimal contract. What is the trader's payoff in the optimal contract? (The wages paid to the trader are not restricted to be nonnegative.) Interpret the risk-sharing and incentive features of the optimal contract. [1 pt.]

3. Suppose that the wages specified in the contract offered by the company to the trader must respect a

minimal wage constraint: the company must pay at least m regardless of the return:

$$w_H \geq m \text{ and } w_L \geq m.$$

(a) Write down the new optimization problem for the company, assuming that the company intends to implement the high effort level. [1 pt.]

(b) Assume that $-p_0 c/(p_1 - p_0) < m < 0$. Determine which constraints must bind. Note that $m < 0$; again, there is no restriction that wages must be nonnegative. [2 pts.]

(c) Based on your answer to part (b), solve for the optimal wages. [1 pt.]

(d) Compute the trader's payoff in the optimal contract. Compare the worker's payoff to the payoff from part 2 and provide a brief explanation in terms of the constraints from part (a). [2 pts.]

4. How would you implement low effort level (i.e., $e = 0$)? [1 pt.]