

1. Prove all of the Lemmas and Corollaries for the screening model covered in class, starting from the slide “Analysis of Constraints.” [1 pt. per Lemma/Corollary]
2. Prove that in the screening model a profit maximizing firm never pools the two types of workers. [3 pts.]
3. (MWG 14.C.7) Assume that there are two types of consumers for a firm’s product,  $\theta_H$  and  $\theta_L$ . The proportion of type  $\theta_L$  consumers is  $\lambda$ . A type  $\theta$ ’s utility when consuming amount  $x$  of the good and paying a total of  $T$  for it is  $u(x, T) = \theta v(x) - T$ , where:

$$v(x) = \frac{1 - (1 - x)^2}{2}$$

The firm is the sole producer of this good, and its cost of production per unit is  $c > 0$ . (*Note: Also, assume the monopolist must charge a linear price:  $T(x) = px$ .*)

- (a) Consider a nondiscriminating monopolist. Derive his optimal pricing policy. Show that he serves both classes of consumers if either  $\theta_L$  or  $\lambda$  is “large enough.” [2 pt.]
  - (b) Consider a monopolist who can distinguish the two types (by some characteristic) but can only charge a simple price  $p_i$  to each type  $\theta_i$ . Characterize his optimal prices. [1 pt.]
  - (c) Suppose the monopolist cannot distinguish the types. Derive the optimal two-part tariff (a pricing policy consisting of a lump-sum charge  $F$  plus a linear price per unit purchased of  $p$ ) under the assumption that the monopolist serves both types. Interpret. When will the monopolist serve both types? [2 pt.]
  - (d) Compute the fully optimal non-linear tariff  $T(x)$ . How do the quantities purchased by the two types compare with the levels in (a) to (c)? (*Hint: Recall that in a screening problem with two types and an arbitrary tariff schedule, one can reformulate the firm’s problem by choosing only one menu per type with the appropriate constraints.*) [2 pt.]
4. What equilibria of the education signaling game in class can be ruled out by (iteratively) eliminating strongly or weakly dominated strategies? (*Note: Assume that firms’ off-equilibrium-path beliefs are “sensible:” if some action  $e = e^*$  by  $L$  is found to be dominated in one iteration, then, in the next iteration, it must be that  $\mu(e^*) = 1$ .*) [2 pts.]
  5. In the education signaling game, show that when the prior probability of type  $\theta = H$  is close enough to 1, the (ex-ante or interim/Pareto) welfare-maximizing equilibrium is the pooling equilibrium with zero education. [2 pts.]