Problem 1. Price Discrimination as a Screening Model. [10 pts]

Consider the following model of price discrimination. There is a seller-monopolist and a buyer. The buyer's willingness to pay for the seller's product is buyer's private information. The buyer's utility function is $u(q, P, \theta) = \theta v(q) - P$ where q is the quantity purchased and P is the total payment to the seller. The buyer's private willingness to pay is represented by θ. It can be low, $\theta = \theta_L$, with probability $\lambda \in (0, 1)$ or high, $\theta = \theta_H > \theta_L$, with probability $(1 - \lambda)$. We assume that $v(\cdot)$ is a concave function passing through the origin: $v(0) = 0$, $v'(q) > 0$, and $v''(q) < 0 \; \forall q$. The buyer's reservation utility is $\bar{u} = 0$. The monopolist's profit function is $\pi = P - cq$ where c is the constant marginal cost of producing a unit of product.

1. Suppose the monopolist perfectly observes the buyer's willingness to pay. What are the optimal offers $\{(q_L, P_L), (q_H, P_H)\}$ that the monopolist proposes to the buyer? Describe the properties of this *first-best* solution. [2 pts]
2. Now suppose we are back in the real world of information frictions and the monopolist does not know the buyer's willingness to pay. What are the optimal offers $\{(q_L, P_L), (q_H, P_H)\}$ that the monopolist proposes to the buyer?
   (a) Set up the monopolist's constrained maximization problem. [1 pt]
   (b) Identify the redundant and binding IC/IR constraints and simplify the maximization problem. *(Note: Do not refer to the results obtained during lecture. Provide the arguments from scratch.)* [4 pts]
   (c) Show that, in equilibrium, the monopolist offers the high-type buyer a Pareto-efficient contract while, to the low-type buyer, the monopolist offers a contract that is distorted downward. Provide the intuition for this result. [2 pts]
   (d) Under which conditions on model parameters does the monopolist offers the low-type buyer an exclusionary contract $(q_L, P_L) = (0, 0)$? Provide the intuition for your findings. [1 pt]

Problem 2. Price Discrimination with Linear Pricing. [7 pts]

(MWG 14.C.7) Assume that there are two types of consumers for a firm's product, θ_H and θ_L. The proportion of type θ_L consumers is λ. A type θ's utility when consuming amount q of the good and paying a total of P for it is $u(q, P) = \theta v(q) - P$, where:

$$v(q) = \frac{1 - (1 - q)^2}{2}$$

The firm is the sole producer of this good, and its cost of production per unit is $c > 0$. Also, assume the monopolist must charge a linear price: $P(q) = pq$.

(a) Consider a nondiscriminating monopolist. Derive his optimal pricing policy. Show that he serves both classes of consumers if either θ_L or λ is "large enough." [2 pt.]

(b) Consider a monopolist who can distinguish the two types (by some characteristic) but can only charge a simple price p_i to each type θ_i. Characterize his optimal prices. [1 pt.]

(c) Suppose the monopolist cannot distinguish the types. Derive the optimal two-part tariff (a pricing policy consisting of a lump-sum charge F plus a linear price per unit purchased of p) under the assumption that the monopolist serves both types. Interpret. When will the monopolist serve both types? [2 pt.]

(d) Compute the fully optimal non-linear tariff $P(x)$. How do the quantities purchased by the two types compare with the levels in (a) to (c)? (*Hint: Recall that in a screening problem with two types and an* *arbitrary tariff schedule, one can reformulate the firm's problem by choosing only one menu per type with the appropriate constraints.*) [2 pt.]

Problem 3. Equilibrium Refinement in Signaling Games. [3 pts]

What equilibria of the education signaling game in class can be ruled out by (iteratively) eliminating strongly or weakly dominated strategies? (*Note: Assume that firms' off-equilibrium-path beliefs are "sensible:" if some action* $e = e^*$ *by* L *is found to be dominated in one iteration, then, in the next iteration, it must be that* $\mu(e^*) = 1$.)