Problem 1. Price Discrimination as a Screening Model. [10 pts]

Consider the following model of price discrimination. There is a seller-monopolist and a buyer. The buyer's willingness to pay for the seller's product is buyer's private information. The buyer's utility function is $u(q, P, \theta) = \theta v(q) - P$ where q is the quantity purchased and P is the total payment to the seller. The buyer's private willingness to pay is represented by θ. It can be low, $\theta = \theta_L$, with probability $\lambda \in (0, 1)$ or high, $\theta = \theta_H > \theta_L$, with probability $(1 - \lambda)$. We assume that $v(\cdot)$ is a concave function passing through the origin: $v(0) = 0$, $v'(q) > 0$, and $v''(q) < 0$ $\forall q$. The buyer's reservation utility is $\bar{u} = 0$. The monopolist's profit function is $\pi = P - cq$ where c is the constant marginal cost of producing a unit of product.

1. Suppose the monopolist perfectly observes the buyer's willingness to pay. What are the optimal offers $\{(q_L, P_L), (q_H, P_H)\}$ that the monopolist proposes to the buyer? Describe the properties of this *first-best* solution. [2 pts]

2. Now suppose we are back in the real world of information frictions and the monopolist does not know the buyer's willingness to pay. What are the optimal offers $\{(q_L, P_L), (q_H, P_H)\}$ that the monopolist proposes to the buyer?

   (a) Set up the monopolist's constrained maximization problem. [1 pt]

   (b) Identify the redundant and binding IC/IR constraints and simplify the maximization problem. *(Note: Do not refer to the results obtained during lecture. Provide the arguments from scratch.)* [4 pts]

   (c) Show that, in equilibrium, the monopolist offers the high-type buyer a Pareto-efficient contract while, to the low-type buyer, the monopolist offers a contract that is distorted downward. Provide the intuition for this result. [2 pts]

   (d) Under which conditions on model parameters does the monopolist offers the low-type buyer an exclusionary contract $(q_L, P_L) = (0, 0)$? Provide the intuition for your findings. [1 pt]

Problem 2. Signaling via a Test. [7 pts]

Consider the following game. A worker learns his type $\theta \in \{H, L\}$ where $H > L > 0$. The probability that a worker is of type H is $\frac{1}{2}$. The worker then has the option of taking a test, which costs $c \geq 0$. With probability π_θ he Passes the test; and with probability $1 - \pi_\theta$ he Fails. Assume that $1 > \pi_H > \pi_L \geq 0$. The fact that the worker has taken the test is public information. After taking the test, two employers competitively bid for the workers services by making a wage offer to the worker, and then the worker decides whether to work and for whom. The value of a type-θ worker to a firm is θ. The firms are risk-neutral.

1. Suppose, first, that $c = 0$. Prove that in any pure strategy perfect Bayesian equilibrium (PBE) both types of workers do the same thing, i.e., any PBE is a pooling equilibrium. What kinds of pooling equilibria are there? That is, what are the actions taken by each type, and what are the wages as a function of whether they took the test and, if they did, the outcome of the test? [2 pts]

2. Is there a value of c such that a pure strategy separating PBE exists (i.e., a PBE such that one type takes the test and the other does not)? If so, find it. If not, prove that such a PBE does not exist. [2 pts]

3. For this part only, assume that $\pi_H = \frac{1}{2}$ and $\pi_L = 0$. Prove that for some values of $c > 0$ there is a PBE in which all type H workers strictly prefer to take the test, but only a fraction, ρ of the type L workers take the test (i.e., type-L workers play a mixed strategy). What are the resulting equilibrium wages for workers who take the test and Pass, who Fail the test, and who do not take the test? [3 pts]

Problem 3. Equilibrium Refinement in Signaling Games. [3 pts]

What equilibria of the education signaling game in class can be ruled out by (iteratively) eliminating strongly or weakly dominated strategies? (*Note: Assume that firms' off-equilibrium-path beliefs are "sensible:" if some action* $e = e^*$ *by* L *is found to be dominated in one iteration, then, in the next iteration, it must be that* $\mu(e^*) = 1$.)