Brief Introduction to Information Design

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May 25, 2020

Mechanism Design and Information Design

- **Mechanism Design**
  - Fix an economic environment and information structure
  - Design the rules of the game to get a desirable outcome
- **Information Design**
  - Fix an economic environment and rules of the game
  - Design an information structure to get a desirable outcome

Mechanism Design and Information Design

- **Mechanism Design**
  - Can compare particular mechanisms
    - E.g., first-price versus second-price auctions
  - Can work with space of all mechanisms
    - E.g., Myerson's optimal mechanism, efficient mechanisms
- **Information Design**
  - Can compare particular information structures
    - Linkage principle: Milgrom-Weber (1982)
    - Information sharing in oligopoly: Novshek-Sonnenschein (1982)
  - Can work with space of all information structures
    - E.g., "Bayesian Persuasion" by Kamenica-Gentzkow (2011)

This Lecture

1. Leading Examples (adapted from Kamenica-Gentzkow, 2011);
2. General framework (in words);
3. Application to FPSB.

Bank Run: One Depositor and No Initial Information

- A bank depositor is deciding whether to run from the bank if he assigns a probability greater than 1/2 to a bad state:

Payoff	θ_G	θ_B
Stay	1	-1
Run	0	0

- Depositor knows only the probability of bad state, which is 2/3.
- Then, the outcome distribution with no prior information:

Outcome	θ_G	θ_B
Stay	0	0
Run	$\frac{1}{3}$	$\frac{2}{3}$

- Probability of a run is 1.

Optimal Information Design with One Depositor

- The regulator cannot stop the depositor withdrawing.
  - But it can choose what information is made available to prevent withdrawals!
- Best information structure:
  - Tell the depositor that the state is bad exactly often enough so that he will stay if he doesn't get the signal:

Outcome	θ_G	θ_B
Stay (intermediate signal)	$\frac{1}{3}$	$\frac{1}{3}$
Run (bad signal)	0	$\frac{1}{3}$

- Think of the regulator making a recommendation to the depositor subject to an obedience constraint (i.e., that it is optimal for the depositor to follow it).
- Probability of run is $1/3$.

Lessons

1. Wlog, can restrict attention to information structures where each player's signal space is equal to his action space.
   - Similar to revelation principle in mechanism design:
     - Wlog, we can restrict attention to mechanisms where each player's message space is equal to his type space.

Bank Run: One Depositor with Initial Information

- If the state is good, with probability $1/2$, the depositor will already have observed a signal t_G saying that the state is good.
- Outcome distribution with no additional information:

Outcome	θ_G, t_G	θ_G, t_0	θ_B, t_0
Stay	$\frac{1}{6}$	0	0
Run	0	$\frac{1}{6}$	$\frac{2}{3}$

- Probability of run is $5/6$.

Optimal Information Design with One Depositor with Initial Information

- Best information structure:
  - tell the depositor that the state is bad exactly often enough so that he will stay if he doesn't get the signal:

Outcome	θ_G, t_G	θ_G, t_0	θ_B, t_0
Stay	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
Run	0	0	$\frac{1}{2}$

- Probability of run is $1/2$.

Is Initially More Informed Depositor Good or Bad?

- With no information design:
    - With no initial information, probability of run is 1.
    - With initial information, probability of run is $5/6$.
- With information design:
    - With no initial information, probability of a run is $1/3$.
    - With initial information, probability of a run is $1/2$.

Is Initially More Informed Depositor Good or Bad?

- With no information design:
  - In this example, more initial information is better for the regulator.
- With information design:
  - More initial information is always bad for the regulator!

Lessons

1. Wlog, can restrict attention to information structures where each player's signal space is equal to his action space.
2. Prior information limits the scope for information design.

Bank Runs: Two Depositors and No Initial information (and Strategic Complements)

- A bank depositor would like to run from the bank if the state is bad OR the other depositor is running:

State θ_G	*Stay*	*Run*
Stay	1	-1
Run	0	0

State θ_B	*Stay*	*Run*
Stay	-1	-1
Run	0	0

- Assume that the probability of the bad state is $\frac{2}{3}$.

Bank Runs: Two Depositors and No Initial information

- Outcome distribution with no information:

State θ_G	*Stay*	*Run*
Stay	0	0
Run	0	$\frac{1}{3}$

State θ_B	*Stay*	*Run*
Stay	0	0
Run	0	$\frac{2}{3}$

- Best information structure:
  - Tell the depositors that the state is bad exactly often enough so that they will stay if they don't get the signal:

State θ_G	*Stay*	*Run*
Stay	$\frac{1}{3}$	0
Run	0	0

State θ_B	*Stay*	*Run*
Stay	$\frac{1}{3}$	0
Run	0	$\frac{1}{3}$

  - In this case, it is optimal to transmit a public signal.

Bank Runs: Two Depositors, No Initial information, and Strategic Substitutes

- Previous example had strategic complements
- Strategic substitutes example: A bank depositor would like to run from the bank if the state is bad AND the other depositor is staying:

State θ_G	*Stay*	*Run*
Stay	1	1
Run	0	0

State θ_B	*Stay*	*Run*
Stay	-1	1
Run	0	0

- Continue to assume that the probability of the bad state is $\frac{2}{3}$.

Bank Runs: Two Depositors and No Initial information

- Outcome distribution with no information: mixed strategy equilibrium.
- Best information structure:
  - Tell the depositors that the state is bad exactly often enough so that they will stay if they don't get the signal:

State θ_G	*Stay*	*Run*
Stay	$\frac{1}{3}$	0
Run	0	0

State θ_B	*Stay*	*Run*
Stay	$\frac{4}{9}$	$\frac{1}{9}$
Run	$\frac{1}{9}$	0

  - In this case, it is optimal to transmit private signals.

Lessons

1. Wlog, can restrict attention to information structures where each player's signal space is equal to his action space.
2. Prior information limits the scope for information design.
3. Public signals optimal if actions are strategic complementarities; private signals optimal if actions are strategic substitutes

General Formulation (in words)

- Fix a game with incomplete information about payoff states.
- Ask what could happen in equilibrium for any additional information that players could be given.
- Equivalent to looking for joint distribution over payoff states, initial information signals and actions satisfying an obedience condition ("Bayes correlated equilibrium").

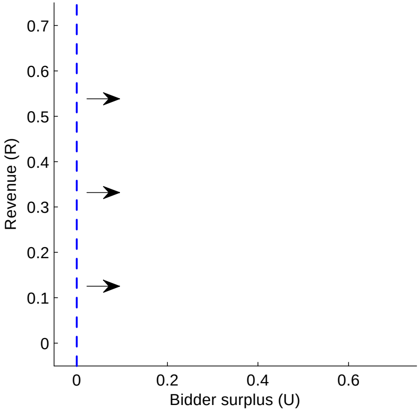
Application: First Price Auctions

- Typical cases: independent private values, common values, etc.
- But what if a planner sent signals about values of others? Can seller gain in revenue by revealing some information to players?
- Can one characterize the space of possible outcomes for all possible information structures?
- Information design approach gives you an answer: Bergemann, Brooks, and Morris (*Econometrica*, 2017)

Application: First Price Auctions

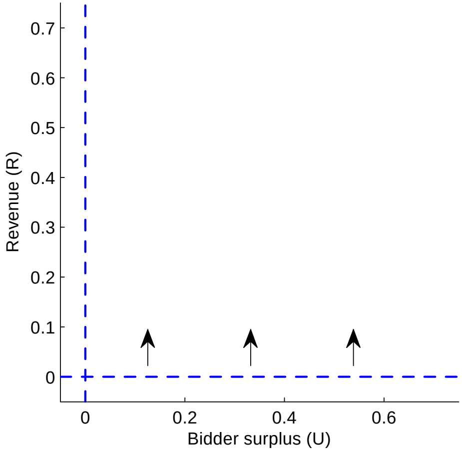
- 2 bidders with valuations uniformly distributed on $[0, 1]$.
- Independent Private Values
  - Each bidder bids half his value
  - Revenue is expectation of low value $= 1/3$
  - Total efficient surplus is expectation of high value $= 2/3$
  - Bidder surplus is $1/3$

Graphical Summary: Bounds 1



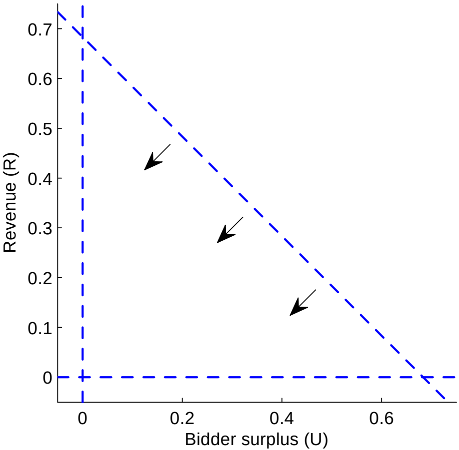
- Nonnegative bidder surplus.

Graphical Summary: Bounds 2



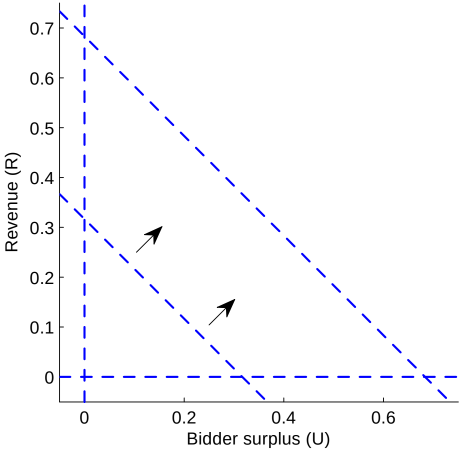
- Nonnegative revenues.

Graphical Summary: Bounds 3



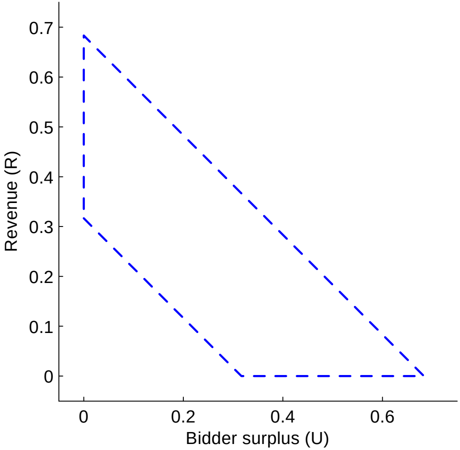
- Efficient social surplus: always give the object to the bidder with the highest valuation.

Graphical Summary: Bounds 4



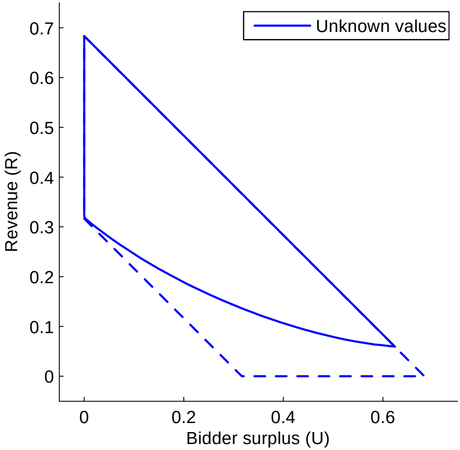
- Least efficient allocation: always give the object to the bidder with the lowest valuation.

Surplus Trapezoid



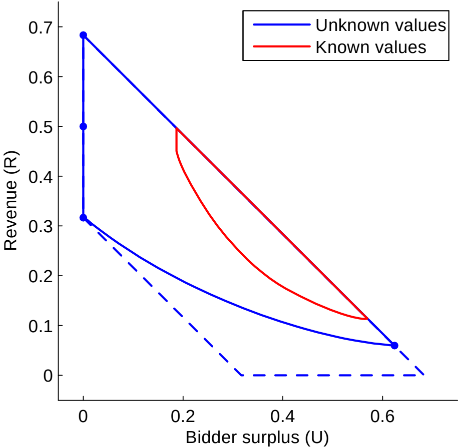
- So far: feasibility and participation constraints.

Incentives Imposes Restrictions: Unknown Values



- Incentive constraints (optimal bidding) add new constraints (even if you don't know your own value!)

Information Generates Incentives: Known Values



- Each bidder i knows his own value v_i.