1. Prove all of the Lemmas and Corollaries for the screening model covered in class, starting from the slide "Analysis of Constraints."

2. Prove that in the screening model a profit maximizing firm never pools the two types of workers.

3. (MWG 14.C.7) Assume that there are two types of consumers for a firm's product, θ_H and θ_L. The proportion of type θ_L consumers is λ. A type θ's utility when consuming amount x of the good and paying a total of T for it is $u(x,T) = \theta v(x) - T$, where:

$$v(x) = \frac{1 - (1-x)^2}{2}$$

   The firm is the sole producer of this good, and its cost of production per unit is $c > 0$. (*Note: Also, assume the monopolist must charge a linear price: $T(x) = px$.*)

   (a) Consider a nondiscriminating monopolist. Derive his optimal pricing policy. Show that he serves both classes of consumers if either θ_L or λ is "large enough."

   (b) Consider a monopolist who can distinguish the two types (by some characteristic) but can only charge a simple price p_i to each type θ_i. Characterize his optimal prices.

   (c) Suppose the monopolist cannot distinguish the types. Derive the optimal two-part tariff (a pricing policy consisting of a lump-sum charge F plus a linear price per unit purchased of p) under the assumption that the monopolist serves both types. Interpret. When will the monopolist serve both types?

   (d) Compute the fully optimal nonlinear tariff. How do the quantities purchased by the two types compare with the levels in (a) to (c)?

4. What equilibria of the education signaling game in class can be ruled out by (iteratively) eliminating strongly or weakly dominated strategies?

5. In the education signaling game, show that when the prior probability of type $\theta = H$ is close enough to 1, the (ex-ante or interim/Pareto) welfare-maximizing equilibrium is the pooling equilibrium with zero education.