Auctions

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May 15, 2020

Reserve Prices

What Is Reserve Price?

- So far, sellers played only a passive role in our analysis.
- Frequently, sellers don't want to sell the object if the price is too low — e.g., if it is lower than a certain threshold $r > 0$.
- Such price r is called the *reserve* (or *reservation*) price.
- Let us now examine how r affects the two auction formats.

Reserve Prices in SPSB

- Suppose that the seller sets some $r > 0$.
  - Bidders with $v < r$ can't make a positive profit, so they drop out.
  - Bidders with $v > r$ have the same optimal strategy as before.
- The expected payment of a bidder with value $v \geq r$ is:

$$m^{II}(v, r) = rG(r) + \int_r^v xg(x)dx$$

(since the winner pays the reserve price r whenever the second-highest bid is below r)

Reserve Prices in FPSB

- Suppose that the seller sets some $r > 0$.
  - Bidders with $v < r$ can't make a positive profit, so they drop out.
  - Bidders with $v > r$ have the same incentives, so can derive:

$$\beta^I = \mathbb{E}[\max\{Y_1, r\} | Y_1 < v] = r\frac{G(r)}{G(v)} + \frac{1}{G(v)}\int_r^v xg(x)dx$$

- The expected payment of a bidder with value $v \geq r$ is:

$$m^I(v, r) = G(v) \times \beta^I(v) = rG(r) + \int_r^v xg(x)dx$$

Which Reserve Price is Optimal?

- As before, FPSB and SPSB lead to the same expected revenue
- But now seller can manipulate r to maximize it.
- *Ex ante* expected payment of a particular bidder in $A \in \{I, II\}$ is:

$$\begin{aligned}
\mathbb{E}[m^A] &= \int_r^\omega m^A(v,r)f(v)dv = \int_r^\omega \left[rG(r) + \left(\int_r^v xg(x)dx\right)\right] f(v)dv \\
&= rG(r)[1-F(r)] + \int_r^\omega \left(\int_r^v xg(x)dx\right) f(v)dv \\
&= rG(r)[1-F(r)] + \int_r^\omega \left(\int_x^\omega f(v)dv\right) xg(x)dx \\
&= rG(r)[1-F(r)] + \int_r^\omega [1-F(x)]xg(x)dx
\end{aligned}$$

Which Reserve Price is Optimal?

- Suppose seller has valuation $v_0 \in [0, \omega)$. Then $r \geq v_0$. (Why?)
- Seller's expected profits are then:

$$\mathbb{E}\pi = N \times \mathbb{E}[m^A(V, r)] + [F(r)]^N v_0$$

- Taking FOC w.r.t. r:

$$\begin{aligned}\frac{\partial \mathbb{E}\pi}{\partial r} &= N[1 - F(r) - rf(r)]G(r) + N[F(r)]^{N-1} f(r) v_0 \\ &= N[1 - F(r)]G(r)[1 - (r - v_0)\lambda(r)]\end{aligned}$$

, where $\lambda(r) = f(r)/(1 - F(r))$ is the *hazard rate* function.

Optimal Reserve Price

- From the F.O.C., the optimal reserve price is determined by:

$$r^* - v_0 = \frac{1}{\lambda(r^*)}$$

- Note that $r^* > v_0$.
  - Intuitively, an increase in revenue from raising r is offsets the declined probability that the good remains unsold.
- Also note that r^* is independent of N!
  - Intuitively, reserve price comes into play only when there is a single bidder with a value that exceeds the reserve price.

Entry Fees

- Reserve price $r > 0$ results in bidders with $v < r$ dropping out.
- Alternatively, could set an *entry fee* — a fixed and non-refundable amount that bidders must pay prior to the auction.
- A reserve price r excludes bidders with $v < r$. The same set of bidders can be excluded with an entry fee $e = G(r) \times r$.
- Rest of analysis stays the same, so the two tools are equivalent.

Other Issues with Reserve Price

- Reserve prices increase revenue but decrease efficiency!
  - To see this, suppose that $v_0 = 0$.
  - Without reserve price, object is sold to the highest bidder who, in a symmetric model, has the highest valuation.
  - With a reserve price, object can remain unsold — inefficient.
- Reserve price requires credible commitment from the seller:
  - Seller leaves some money on the table by setting $r > v_0$.
  - Hence, needs to credibly commit not to sell the product with a lower price and not to lower the reserve price in the future.