1. Prove all of the Lemmas and Corollaries for the screening model covered in class, starting from the slide "Analysis of Constraints."

2. Prove that in the screening model a profit maximizing firm never pools the two types of workers.

3. (MWG 14.C.7) Assume that there are two types of consumers for a firm's product, θ_H and θ_L. The proportion of type θ_L consumers is λ. A type θ's utility when consuming amount x of the good and paying a total of T for it is $u(x,T) = \theta v(x) - T$, where:

$$v(x) = \frac{1-(1-x)^2}{2}$$

The firm is the sole producer of this good, and its cost of production per unit is $c > 0$. (*Note: Also, assume the monopolist must charge a linear price: $T(x) = px$.*)

   (a) Consider a nondiscriminating monopolist. Derive his optimal pricing policy. Show that he serves both classes of consumers if either θ_L or λ is "large enough."

   (b) Consider a monopolist who can distinguish the two types (by some characteristic) but can only charge a simple price p_i to each type θ_i. Characterize his optimal prices.

   (c) Suppose the monopolist cannot distinguish the types. Derive the optimal two-part tariff (a pricing policy consisting of a lump-sum charge F plus a linear price per unit purchased of p) under the assumption that the monopolist serves both types. Interpret. When will the monopolist serve both types?

   (d) Compute the fully optimal non-linear tariff $T(x)$. How do the quantities purchased by the two types compare with the levels in (a) to (c)? (*Hint: Recall that in a screening problem with two types and an arbitrary tariff schedule, one can reformulate the firm's problem by choosing only one menu per type with the appropriate constraints.*)

4. What equilibria of the education signaling game in class can be ruled out by (iteratively) eliminating strongly or weakly dominated strategies? (*Note: Assume that firms' off-equilibrium-path beliefs are "sensible:" if some action $e = e^*$ by L is found to be dominated in one interation, then, in the next iteration, it must be that $\mu(e^*) = 1$.*)

5. In the education signaling game, show that when the prior probability of type $\theta = H$ is close enough to 1, the (ex-ante or interim/Pareto) welfare-maximizing equilibrium is the pooling equilibrium with zero education.