

# Resolving Adverse Selection: Screening and Signaling

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April 20, 2020

# Motivation

- Adverse selection can be detrimental for markets
- How do markets cope with this issue?
- There are two standard mechanisms studied in the literature that help reduce adverse selection: *screening* and *signaling*
- **Screening**: uninformed party sets up a contract structure in such a way that certain types self-select into choosing different options
  - Example: Insurance company creates two types of contracts — one with high deductible and low premium, and one with low deductible and high premium
- **Signaling**: informed individuals develop a mechanism to signal their unobservable knowledge through observable actions
  - Example: Signaling on the job market, education

# This Lecture

- ① Labor Market Screening
- ② Labor Market Signaling

*Acknowledgment: For both models, I heavily borrow from Jeff Ely's graduate microeconomics class at Northwestern University.*

# Screening

# Labor Market Screening

- One firm, one worker.
- The worker has type  $\theta \in \{L, H\}$  and chooses effort  $e \geq 0$ .
- Output is determined by  $e$  but not by  $\theta$ .
- $F(e)$  is output when employed by the firm.
  - $F(0) = 0$ ,  $F'(e) > 0$ ,  $F''(e) < 0$ .
- $F(e) - w$  is firm's profit.

# Labor Market Screening

- The worker maximizes  $w - c(e|\theta)$ 
  - $c(0|\theta) = 0$
  - $c_e(\cdot|H) < c_e(\cdot|L)$
  - $c_{ee}(\cdot|\cdot) > 0$
- With some publicly known probability  $\lambda$ , the worker's type is  $L$ .
- Firm maximizes expected profits.
- We characterize PBE of the game.

# Timing

- ① Nature 'moves' first and the worker learns her type  $\theta$
- ② The firm moves next, unaware of the worker's type, and commits to a wage schedule  $w(e)$ .
- ③ The worker accepts or rejects the firm's offer. If accepts, chooses  $e$ , earns wage  $w(e)$ . If rejects, gets 0 utility.

# PBE: Recap from Francesco's Class

## Definition

A Perfect Bayesian Equilibrium (PBE) of an extensive form game  $\Gamma_E$  is a strategy profile  $\sigma$  and a system of beliefs  $\mu$  such that:

- 1 **Sequential Rationality:**  $\sigma$  is sequentially rational given  $\mu$  (i.e., at every info set  $H_i$ , the strategy of each player  $i$  maximizes her payoff given the strategy of other players and her beliefs);
- 2 **On-Equilibrium-Path Beliefs:** For any information set reached with positive probability given strategy  $\sigma$  (i.e., for any  $H$  such that  $Pr(H|\sigma) > 0$ ), beliefs must be formed according to Bayes' rule.
- 3 **Off-Equilibrium-Path Beliefs:** For any information set reached with null probability given strategy  $\sigma$  (i.e., for any  $H$  such that  $Pr(H|\sigma) = 0$ ), beliefs  $\mu(x)$  may be arbitrary but must be formed according to Bayes' rule whenever possible.



## Problem Reformulation

- By sequential rationality, the worker with type  $\theta$  will take  $w(e)$  as given and choose effort  $e$  to maximize

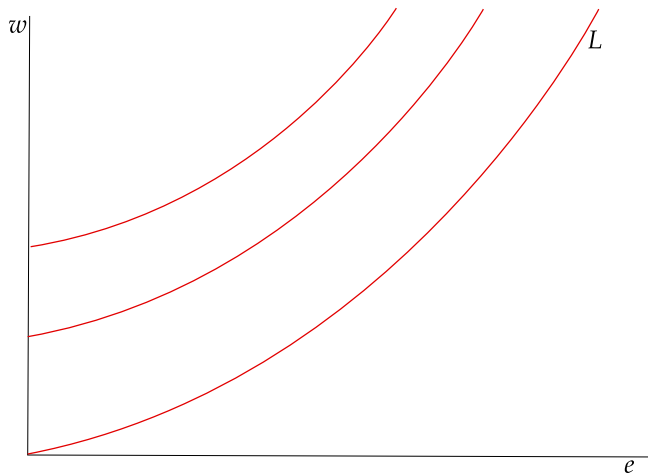
$$w(e) - c(e|\theta)$$

- Wlog, we can assume that ties are broken in the interest of the firm. (Hence, workers don't play mixed strategies.)
- Let  $e(\theta)$  be the effort choice of type  $\theta$ .
- The firm's expected profit is then:

$$\lambda[F(e(L)) - w(e(L))] + (1 - \lambda)[F(e(H)) - w(e(H))]$$

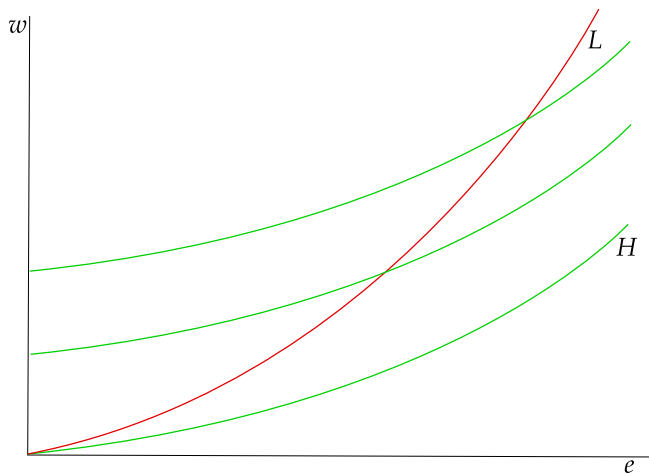
- Since the only aspect of the wage schedule that matters are pairs  $\{e(H), w(e(H))\}$  and  $\{e(L), w(e(L))\}$ , we might as well solve the firm's profit maximization problem by choosing those directly.
- But there are constraints that come from the requirement that  $e(H)$  and  $e(L)$  are chosen optimally by the worker.

# Incentive Compatibility



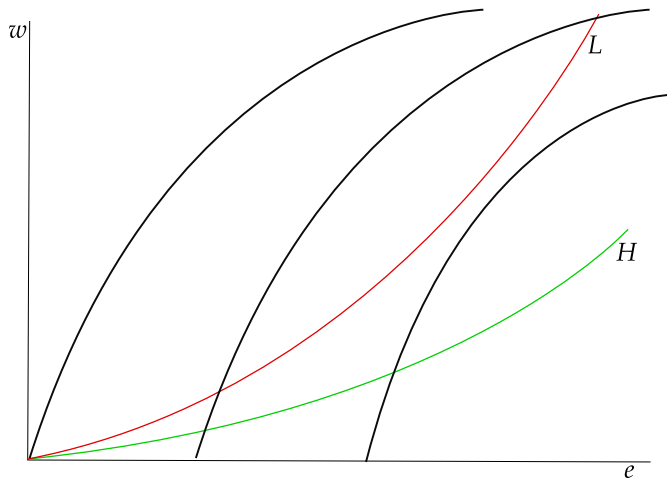
Indifference curves for type  $L$ .

# Incentive Compatibility



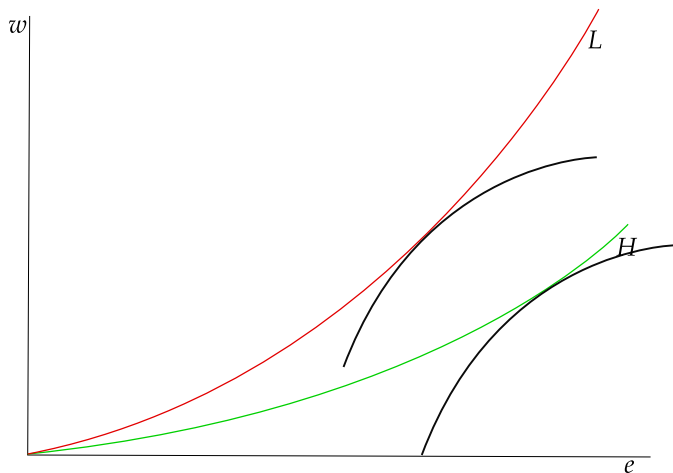
Indifference curves for type  $H$ .

# Incentive Compatibility



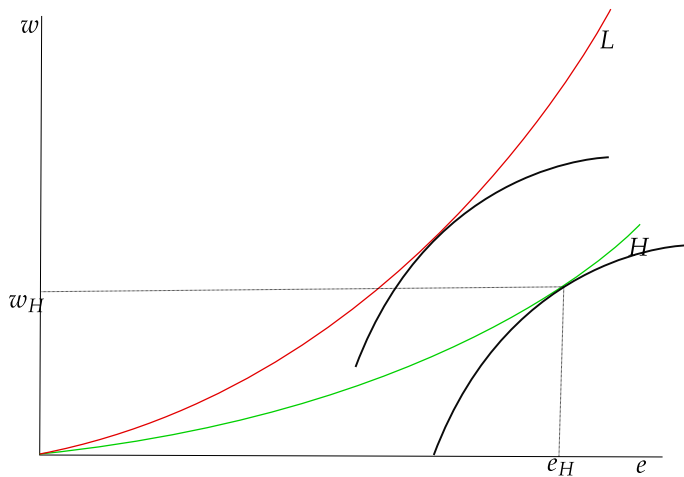
Iso-profit curves for the firm.

# Incentive Compatibility



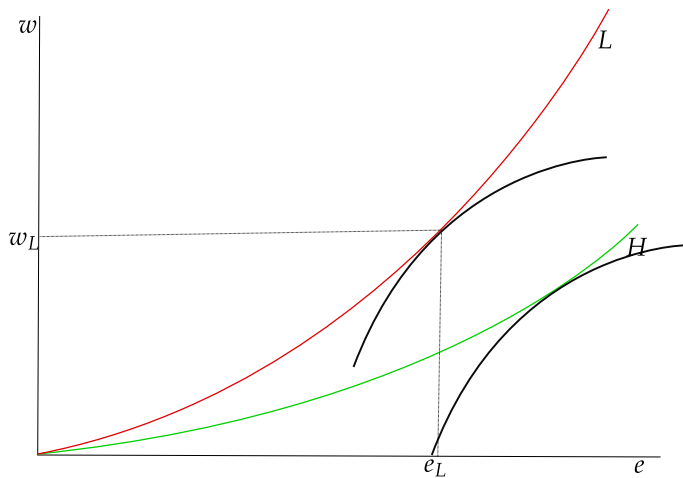
These are efficient points that give the two types zero utility.

# Incentive Compatibility



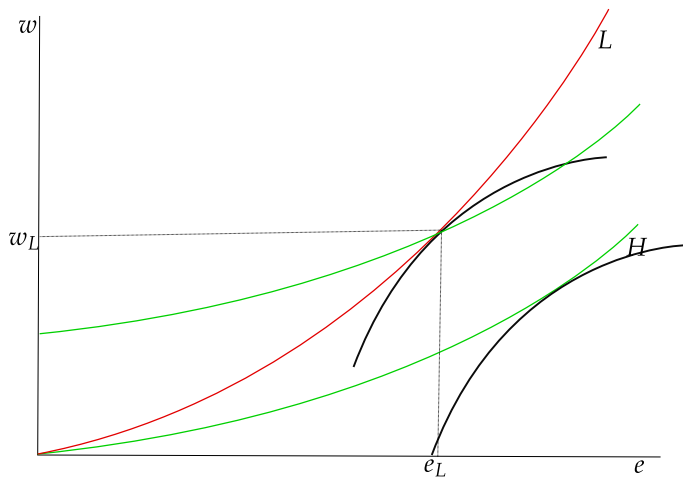
The contract  $(e_H, w_H)$ .

# Incentive Compatibility



The contract  $(e_L, w_L)$ .

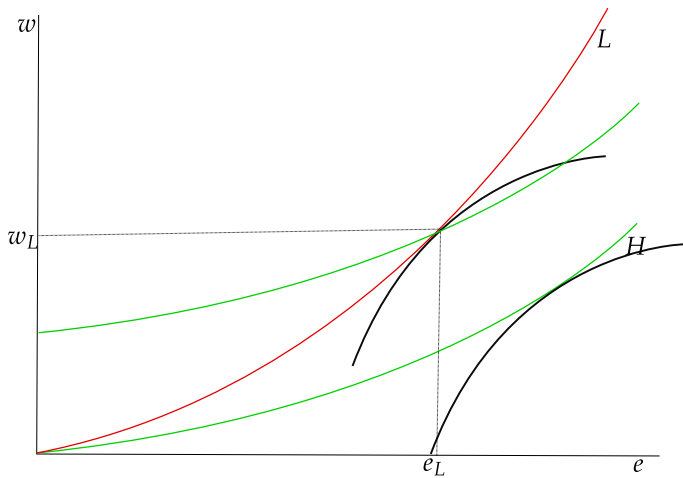
# Incentive Compatibility



This pair of contracts could not be chosen in an equilibrium.



## Incentive Compatibility



$$w_L - c(e_L|H) > w_H - c(e_H|H).$$

# Incentive Compatibility

We can characterize the set of contracts that could be chosen in an equilibrium

## Lemma

*We can (wlog) reformulate the problem into one in which the firm specifies a pair of contracts  $(e_L, w_L)$ ,  $(e_H, w_H)$  such that:*

$$\text{IC-L } w_L - c(e_L|L) \geq w_H - c(e_H|L)$$

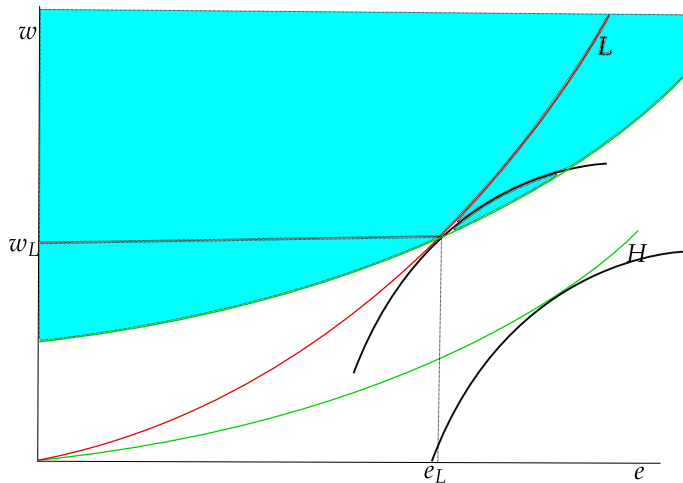
$$\text{IC-H } w_H - c(e_H|H) \geq w_L - c(e_L|H)$$

$$\text{IR-L } w_L - c(e_L|L) \geq 0$$

$$\text{IR-H } w_H - c(e_H|H) \geq 0$$

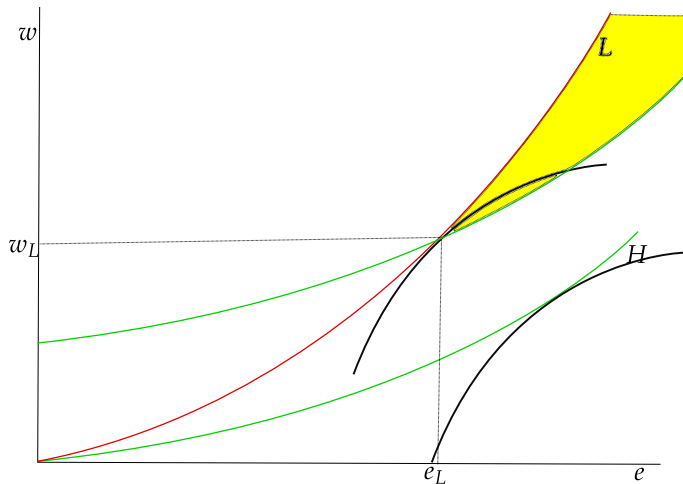
*The first two are called incentive compatibility constraints, and the last two are called individual rationality constraints.*

# Illustrating Incentive Compatibility



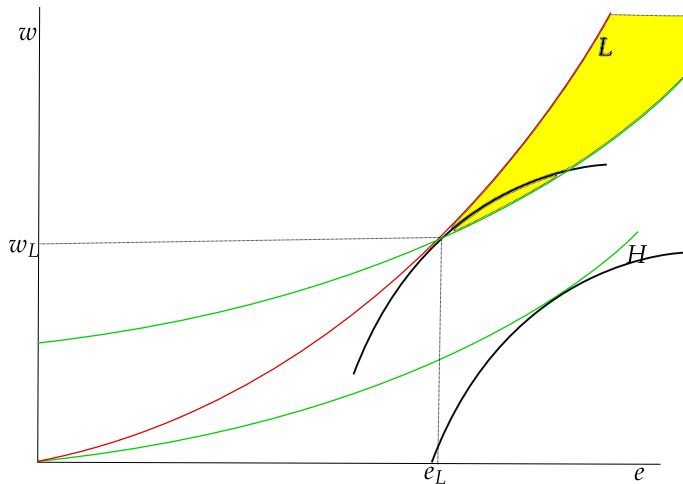
If we fix  $(e_L, w_L)$ , then  $(e_H, w_H)$  must lie in the shaded region. (IC-H)

# Illustrating Incentive Compatibility



But in order to also satisfy (IC-L) we must have  $(e_H, w_H)$  in the yellow shaded region.

# Illustrating Incentive Compatibility



Note that these contracts also satisfy the IR constraints.

# Constrained Profit Maximization

We can express the profit maximization problem of the firm as follows.

$$\begin{aligned} \max_{\{w_L, w_H, e_L, e_H\}} \quad & \mathbb{E}[F(e_\theta) - w_\theta] \\ \text{subject to} \quad & \\ & w_L - c(e_L|L) \geq w_H - c(e_H|L) \\ & w_H - c(e_H|H) \geq w_L - c(e_L|H) \\ & w_L - c(e_L|L) \geq 0 \\ & w_H - c(e_H|H) \geq 0 \end{aligned}$$

# Analysis of Constraints

## Lemma. Single Crossing Condition.

*If  $(e, w)$  is weakly preferred by type  $L$  to  $(e', w')$  and  $e \geq e'$ , then  $(e, w)$  is strictly preferred by type  $H$  to  $(e', w')$ .*

This follows from differences in cost functions between types.

## Corollary. IR-H is Redundant.

*If the IR-L constraints and the IC-H constraints are satisfied, then the IR-H constraint is also satisfied.*

## Corollary. Optimal Contract is Monotone.

*If  $(e_L, w_L), (e_H, w_H)$  satisfy the IC constraints, then  $e_H \geq e_L$ .*

# Analysis of the Optimum

By the previous lemma, we can restrict attention to contracts that are monotone, i.e.,  $e_H \geq e_L$ .

**Lemma. No Slack.**

*A constrained profit-maximizing contract satisfies the IR-L and IC-H constraints with equality.*

If not, then just lower wages and raise profits.

**Lemma. IC-L is Redundant.**

*If the contract is monotone and the IC-H constraint is satisfied with equality, then the IC-L constraint is satisfied.*



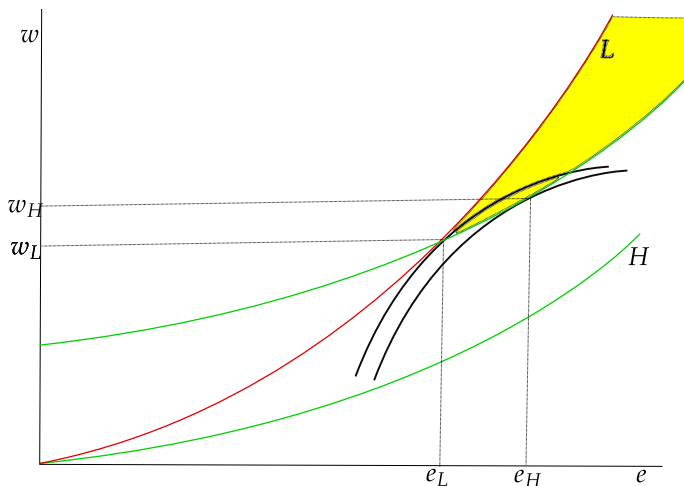
# Updated Constrained Profit Maximization

## Corollary

*The problem can be simplified to:*

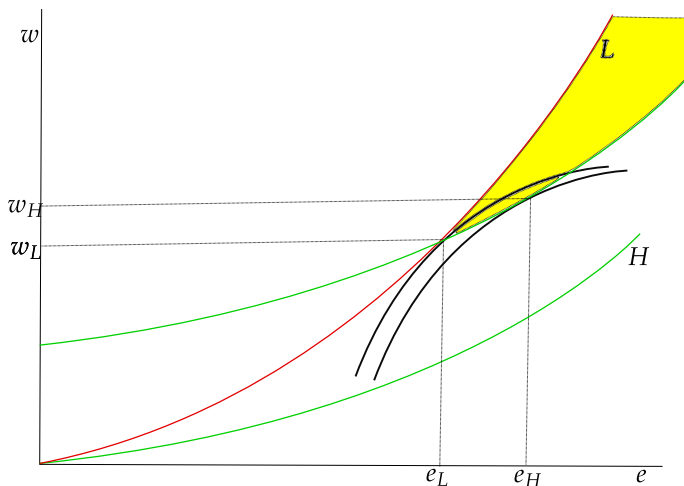
$$\begin{aligned} & \max_{\{w_L, w_H, e_L, e_H\}} \mathbb{E}[F(e_\theta) - w_\theta] \\ & \text{subject to} \\ & w_H - c(e_H|H) = w_L - c(e_L|H) \\ & w_L - c(e_L|L) = 0 \\ & e_H \geq e_L \end{aligned}$$

## A Guess



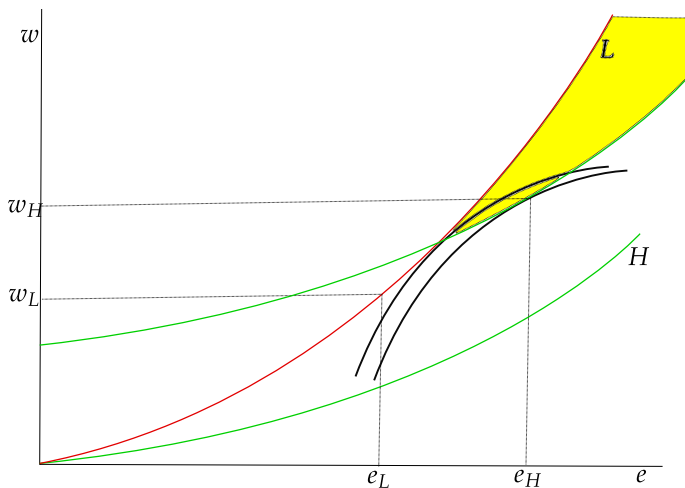
In this pair of contracts, we have retained the  $(e_L, w_L)$  contract from the first-best and then chose the  $(e_H, w_H)$  contract to maximize profits subject to the IC constraints.

## A Guess



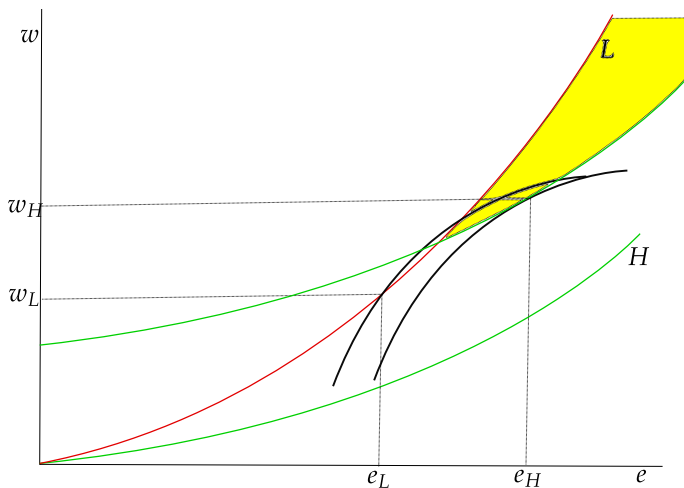
In fact, those conditions ensure that this pair of contracts is *constrained efficient* or *second-best*. But they are not profit-maximizing.

## A Guess



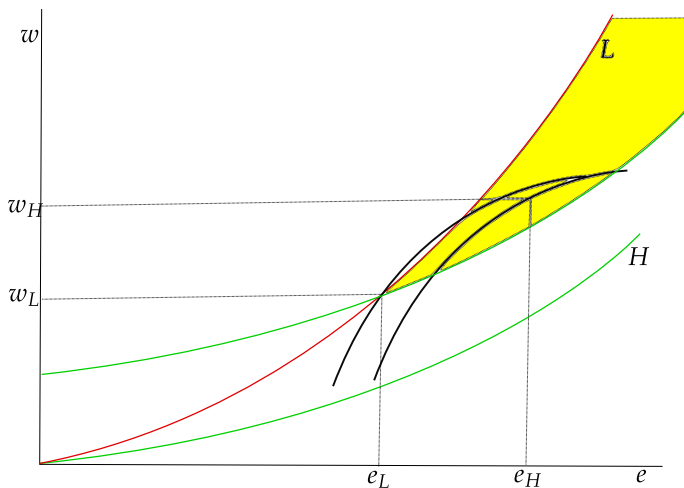
If we move the  $L$  contract down the  $L$ -indifference curve, we do not violate any constraints.

## A Guess



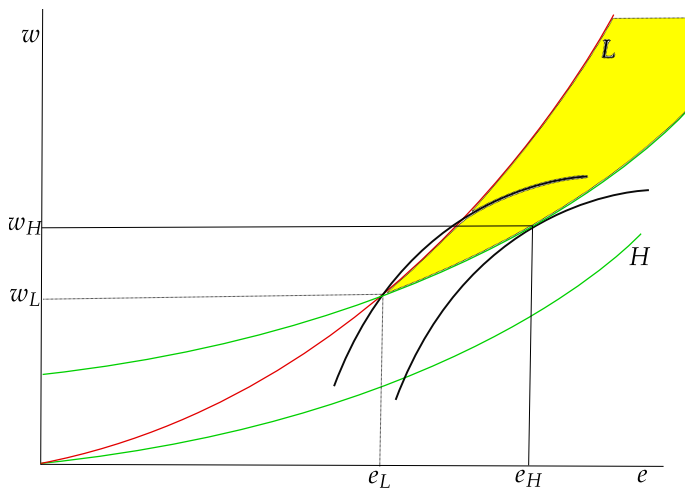
There is a loss in profit but that loss is second-order in the distance moved.

## A Guess



This relaxes the IR-H constraint.

## A Guess



And enables a first-order increase in profits from type  $H$ .

# Properties of the General Solution

## Proposition. No Pooling.

*It will never be optimal to offer a pooling contract, i.e.,  $e_H = e_L$ .*

Can always do better by offering a higher  $w_H$  and  $e_H$ .

## Proposition. Contract Efficiency.

*In the constrained profit-maximizing contracts, the contract chosen by type  $H$  is Pareto efficient while the contract chosen by type  $L$  is Pareto dominated (effort is too low.)*

Making the contract for  $L$  inefficient allows a firm to weaken as much as possible the incentive constraint of type  $H$ .



## Exclusion of Low Types

*Note:* It may be optimal to set  $e_L = 0$  and  $w_L = 0$ . In effect this would completely exclude type  $L$  in order to weaken as much as possible the incentive constraint of type  $H$ .

(Excluding type  $H$  would require also excluding type  $L$  and the result would be zero profits, so it is definitely not profit maximizing.)

# Signaling