Problem 1. Auctions with a Pareto Distribution [3 pts].

(*Krishna, 2.2.*) Suppose there are two bidders with private values that are distributed independently according to the distribution a Pareto distribution $F(x) = 1 - (x+1)^{-2}$ over $[0, \infty)$.

(a) Find symmetric equilibrium bidding strategies in a first-price auction. [1 pts.]

(b) Show by direct computation that expected revenues in first- and second-price auctions are equal. [2 pts.]

Problem 2. Auctioning Off a Bundle [7 pts].

A seller is selling a bundle consisting of two components. Bidder $i \in \{1, 2\}$ only knows the quality of component i, which is also the component he cares about more.

Specifically, bidder i's type t_i represents the quality of component i. The two bidders' types are independent, and each is drawn from a uniform distribution on $[0, 1]$. Bidder i's value for the bundle is $v_i = t_i + ct_j$, where $c \in [0, 1]$ represents the level of complementarity between the components. We consider auctions for the bundle in which each bidder i, after learning his type, places a bid $b_i \in [0, 1 + c]$.

(a) Suppose that the seller runs a first price auction. Find a symmetric Nash equilibrium in which each bidder's bid is a linear function of his type (i.e., where player i's bidding strategy is of the form $\beta_i(t_i) = k_i t_i$ for some $k_i \geq 0$), and show that there is only one such equilibrium. [2 pts.]

(b) Now suppose that the seller runs a second price auction. Find a symmetric Nash equilibrium in which each bidder's bid is a linear function of his type, and show that there is only one such equilibrium. [2 pts.]

(c) Compute the seller's expected revenue in the equilibria you found in parts (i) and (ii). If the seller's aim is to maximize expected revenue, which auction format does he prefer? [2 pts.]

(d) In the equilibria from parts (i) and (ii), compute bidder i's expected value for the bundle in two situations: (a) when he knows his type, but the auction has not yet been run; (b) when he knows his type, the auction has been run, and he learns that he has won. Give an intuitive explanation for the differences between your answers to (a) and (b). [1 pt.]

Problem 3. War of Attrition [10 pts].

(*Krishna, 3.1.*) Consider a two-bidder *war of attrition* in which the risk-neutral bidder with the highest bid wins the object but both bidders pay the losing bid. Bidders' values independently and identically distributed according to F.

(a) Use the revenue equivalence principle to derive a symmetric equilibrium bidding strategy. [5 pts.]

(b) Directly compute the symmetric equilibrium bidding strategy and the sellers' revenue when the bidders' values are uniformly distributed on [0,1]. *(Note: "Directly" means without using the revenue equivalence principle and instead starting from a bidder maximizing her expected utility function.)* [5 pts.]