From Auctions to Mechanism Design

Alexey Makarin

May 22, 2020

Balancing the Budget

- Frequently, it is desirable to consider mechanisms that do not require injection of funds from the mechanism designer—that is, the designer's budget is exactly balanced *ex post*.
- A mechanism is said to *balance the budget* if for every realization of values, net payments from agents sum to zero:

$$\sum_{i \in \mathcal{N}} M_i(\mathbf{v}) = 0 \quad \forall \mathbf{v}$$

- While VCG does not always have a balanced budget, it is still helpful in determining when this is feasible (more on this later).

The AGV Mechanism

- The *AGV (Arrow-d'Aspremont-Gérard-Varet)* mechanism, or the 'expected externality' mechanism, $(\boldsymbol{Q}^*, \boldsymbol{M}^A)$ is defined by:

$$M_i^A(\mathbf{v}) = \frac{1}{N-1} \sum_{j \neq i} \mathbb{E}_{\boldsymbol{V}_{-j}} \left[W_{-j}(v_j, \boldsymbol{V}_{-j}) \right] - \mathbb{E}_{\boldsymbol{V}_{-i}} \left[W_{-i}(v_i, \boldsymbol{V}_{-i}) \right]$$

  so that budget is balanced *ex post* for all $\mathbf{v}$:

$$\sum_{i \in \mathcal{N}} M_i^A(\mathbf{v}) = 0$$

Lemma.

The AGV mechanism is (Bayesian) incentive compatible.

Proof: Suppose other buyers report values $\mathbf{v}_{-i}$. Then by reporting a value of z_i, agent i earns:

$$\begin{aligned}
\mathbb{E}_{\boldsymbol{V}_{-i}}[Q_i^*(z_i, \mathbf{v}_{-i})v_i - M_i^A(z_i, \mathbf{v}_{-i})] &= \mathbb{E}_{\boldsymbol{V}_{-i}}[Q_i^*(z_i, \boldsymbol{V}_{-i})v_i + W_{-i}(z_i, \boldsymbol{V}_{-i})] \\
- \mathbb{E}_{\boldsymbol{V}_{-i}}\left[\frac{1}{N-1}\sum_{j \neq i} \mathbb{E}_{\boldsymbol{V}_{-j}}[W_{-j}(V_j, \boldsymbol{V}_{-j})]\right] &= \mathbb{E}_{\boldsymbol{V}_{-i}}[W(z_i, \boldsymbol{V}_{-i})] \\
- \mathbb{E}_{\boldsymbol{V}_{-i}}\left[\frac{1}{N-1}\sum_{j \neq i} \mathbb{E}_{\boldsymbol{V}_{-j}}[W_{-j}(V_j, \boldsymbol{V}_{-j})]\right]
\end{aligned}$$

Note that the second term does not depend on z_i. The first term is simply expected total welfare. It is maximized by setting $z_i = v_i$ because if $z_i > v_i$ or $z_i < v_i$, then the object could potentially be allocated inefficiently. ■

(Note that AGV may not satisfy individual rationality).

Existence of Efficient, IC, IR, and Balanced Mechanisms

Proposition

There exists an efficient, incentive compatible, and individually rational mechanism that balances the budget if and only if the VCG mechanism results in an expected surplus.

Proof: *Necessity* follows immediately: if VCG runs a deficit, then all other efficient, IC, and IR mechanisms run a deficit too. (Why?).

Need to show *sufficiency*. For that, let's construct an efficient, IC, and IR mechanism that balances the budget.

Existence of Efficient, IC, IR, and Balanced Mechanisms

Proof (cont'd): First, consider the AGV mechanism defined earlier. From revenue equivalence, there exist constants c_i^A such that:

$$U_i^A(v_i) = \mathbb{E}\left[W(v_i, \boldsymbol{V}_{-i})\right] - c_i^A$$

Second, consider the VCG mechanism defined earlier. From revenue equivalence, there exist constants c_i^V such that:

$$U_i^V(v_i) = \mathbb{E}\left[W(v_i, \boldsymbol{V}_{-i})\right] - c_i^V$$

Suppose that the VCG mechanism runs an expected surplus. Then:

$$\mathbb{E}\left[\sum_{i \in \mathcal{N}} M_i^V(\boldsymbol{V})\right] \geq 0 = \mathbb{E}\left[\sum_{i \in \mathcal{N}} M_i^A(\boldsymbol{V})\right] \Leftrightarrow \sum_{i \in \mathcal{N}} c_i^V \geq \sum_{i \in \mathcal{N}} c_i^A$$

Existence of Efficient, IC, IR, and Balanced Mechanisms

Proof (cont'd): Define $d_i = c_i^A - c_i^V \ \forall i > 1$. Let $d_1 = -\sum_{i=2}^N d_i$.

- Consider mechanism $(\boldsymbol{Q}^*, \overline{\boldsymbol{M}})$ defined by:

$$\overline{M}_i(\mathbf{v}) = M_i^A(\mathbf{v}) - d_i$$

- Clearly, $\overline{\boldsymbol{M}}$ balances the budget. (Why?)
- It is also IC since its payoffs differ from payoffs from an IC mechanism, $\boldsymbol{M}^A$, by an additive constant.
- Thus, need only to verify that $\overline{\boldsymbol{M}}$ is IR.

Existence of Efficient, IC, IR, and Balanced Mechanisms

Proof (cont'd): For all $i \neq 1$:

$$\overline{U}_i(v_i) = U_i^A(v_i) + d_i = U_i^A(v_i) + c_i^A - c_i^V = U_i^V(v_i) \geq 0$$

Left to check $i = 1$. By construction, $\sum_{i=1}^N d_i = 0$, so observe that:

$$d_1 = -\sum_{i>1} d_i = \sum_{i>1} \left(c_i^V - c_i^A\right) \geq \left(c_1^A - c_1^V\right)$$

$$\overline{U}_1(v_1) = U_1^A(v_1) + d_1 \geq U_1^A(v_1) + c_1^A - c_1^V = U_1^V(v_1) \geq 0 \quad \blacksquare$$

This proposition is quite useful in considering various efficient allocation problems. We now turn to an example of bilateral trade.

An Application to Bilateral Trade

- Seller has privately known cost $C \in [\underline{c}, \overline{c}]$ of producing a good.
- Buyer has privately known value $V \in [\underline{v}, \overline{v}]$ of consuming a good.
- C and V are independently distributed with full support on respective intervals; their prior distributions are commonly known.
- Note: incomplete information on both sides of the market!
- Suppose that $\underline{v} < \overline{c}$ and $\overline{v} < \underline{c}$, so supports overlap and sometimes it is efficient not to trade.
- Is there some way to guarantee that trade will take place whenever it should?

Bilateral Trade as a Mechanism

- A mechanism decides whether or not the good is traded.
- It also decides the amount P the buyer pays for the good and the amount R the seller receives.
- If the good is traded, the net gain to the buyer is $V - P$, and the net gain to the seller is $R - C$.
- At the moment, we do not restrict P or R to be positive or negative, nor do we assume that the budget is balanced—that is, $P = R$.
- A mechanism is efficient if whenever $V > C$, the object is produced and allocated to the buyer.

Proposition. Myerson–Satterthwaite theorem.

In the bilateral trade problem, there is no mechanism that is efficient, incentive compatible, individually rational, and at the same time balances the budget.

Proof: First, consider the VCG mechanism, whose operation in this context is as follows. The buyer announces a valuation V and the seller announces a cost C.

1. If $V \leq C$, the object is not exchanged and no payments are made.
2. If $V > C$, the object is exchanged. The buyer pays $\max\{C, \underline{v}\}$ and the seller receives $\min\{V, \overline{c}\}$. (Check!)

It is a weakly dominant strategy for the buyer to announce $V = v$ and for the seller to announce $C = c$. (Why?)

Unsurprisingly, VCG is efficient—object is transferred whenever $v > c$.

Proof (cont'd):

- This mechanism is IR:
  - A buyer with value $\underline{v}$ has an expected payoff of 0, and any buyer with value $v > \underline{v}$ has a nonnegative expected payoff.
  - Similarly, a seller with cost $\overline{c}$ has an expected payoff of 0, and any seller with cost $c < \overline{c}$ has a nonnegative expected payoff.
- VCG runs a deficit:
  - Whenever $V > C$, the fact that $\underline{v} < \overline{c}$ implies that seller's income $R = \min\{V, \overline{c}\}$ is *greater* than buyer's payment $P = \max\{C, \underline{v}\}$.
  - For any realization of V and C such that $V > C$, the deficit $R - P = V - C$, i.e., exactly the ex-post gains from trade.

Proof (cont'd):

- Now apply the revenue equivalence principle.
  - Consider some other mechanism that is IC and efficient.
  - By revenue equivalence, there is a constant K such that expected payment for any buyer with value v differs from his expected payment under the VCG mechanism by exactly K.
  - Similarly, there is a constant L such that the expected receipts of any seller with cost c differ from the expected receipts under the VCG mechanism by exactly L.
- Now check what IR implies for K and L:
  - Suppose the other mechanism is IR.
  - Since buyer with value $\underline{v}$ gets an expected payoff of 0 in in the VCG mechanism, it must be that $K \leq 0$.
  - Similarly, since a seller with costs $\overline{c}$ gets an expected payoff of 0, it must be that $L \geq 0$.

Proof (cont'd):

- Thus, expected deficit under the other mechanism is the expected deficit under the VCG mechanism plus $(L - K) \geq 0$.
- But since the VCG mechanism runs a deficit, every other mechanism also runs a deficit.
- Thus, in a bilateral trade problem, there does not exist an efficient mechanism that is IC, IR, and balances the budget. $\blacksquare$

Summary

Summary: Auctions and Mechanisms

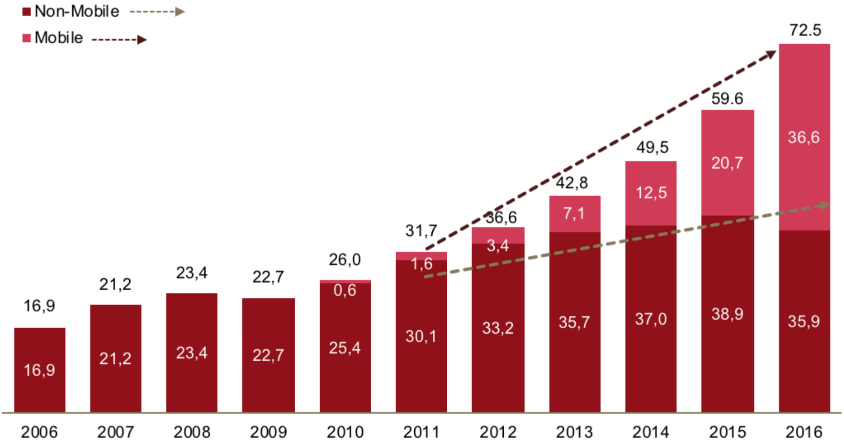
1. Auctions:
    - Introduction. History and modern use of auctions.
    - FPSB and SPSB auctions: strategies, revenues, etc.
    - Revenue equivalence: basic (FPSB vs. SPSB) and general (symmetric and increasing equilibria in any standard auction).
    - FPSB and SPSB with reserve prices, optimal reserve price.
2. Mechanisms:
    - Object allocation as a mechanism design problem.
    - Revelation principle.
    - Optimal mechanism: virtual valuations, individual reserve prices.
        - Optimal mechanism with symmetric buyers: SPSB with an optimal reserve price.
    - Efficient mechanisms: VCG, AGV, balanced budget.
        - Myerson-Satterthwaite theorem: in a bilateral trade problem, no mechanism is efficient, IC, IR, and budget-balanced.

Extra: Online Auctions

Use of Auctions on the Internet

- On the Internet (and in many other environments) people's attention ("eyeballs") is the key currency.
- Amazon has to decide what products to show when users search for an item, and which ads to show.
- Facebook has to decide which ads to show to its users.
- Google has to decide which search results to put on top, but also how to sell positions in the ranking.
- This is a big market. How ads/paid search positions are sold?
- Typically bid per click for certain key words, like "Rome," "master in economics," etc.

IAB Internet Advertising Revenue Report 2016, US$ bln



Internet Auctions

1994 “Banner ads”

- used the newspaper model to sell banners
- pay per impression
- no targeting
- prices negotiated in person

1997 search engine goto.com invents per click pricing

- per click pricing
- keyword targeting
- automated acceptance of updated bids
- generalization of first-price auctions

Why first-price auctions?

Problems with Generalized First Price Auctions

Generalized First Price Auction:

- The bidder with the highest bid gets the top slot and pays his bid.
- The second highest bidder gets the second slot and so on.

Unstable in practice: Prices and bids were displaying cyclical behavior going up and down. Bidders had incentives to revise their bids as fast as possible.

First internet auctions were designed by computer scientists and programmers, who until recently had little training or knowledge of auctions.

Now auctions are taught in best computer science programs as they are they key source of revenue for many internet companies (ads generate more than 90% of Google's revenue)

Rise of Second-Price Auctions

When economics analyzed data and theory of generalized first price auctions used to sell clicks, it turned out that there is no pure strategy equilibrium, that is optimal strategy is to submit random bids.

2002 Google "develops" a generalized second price auction (explanatory video by Hal Varian).

Format:

- The bidder with the highest bid gets the top slot and pays next-highest bid.
- The second highest bidder gets the second slot and pays the next-highest, etc.

Since then the format was dominant on the Internet and is used to sell most online ads. (However, in 2019, Google went back to a version of first-price auction, allegedly to increase transparency and boost competition.)