Resolving Adverse Selection: Screening and Signaling

Alexey Makarin

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Motivation

- Adverse selection can be detrimental for markets
- How do markets cope with this issue?
- There are two standard mechanisms studied in the literature that help reduce adverse selection: *screening* and *signaling*
- ***Screening***: uninformed party sets up a contract structure in such a way that certain types self-select into choosing different options
  - Example: Insurance company creates two types of contracts — one with high deductible and low premium, and one with low deductible and high premium
- ***Signaling***: informed individuals develop a mechanism to signal their unobservable knowledge though observable actions
  - Example: Signaling on the job market, education

This Week

1. Labor Market Screening
2. Labor Market Signaling

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2. **Labor Market Signaling**

Signaling

Setup

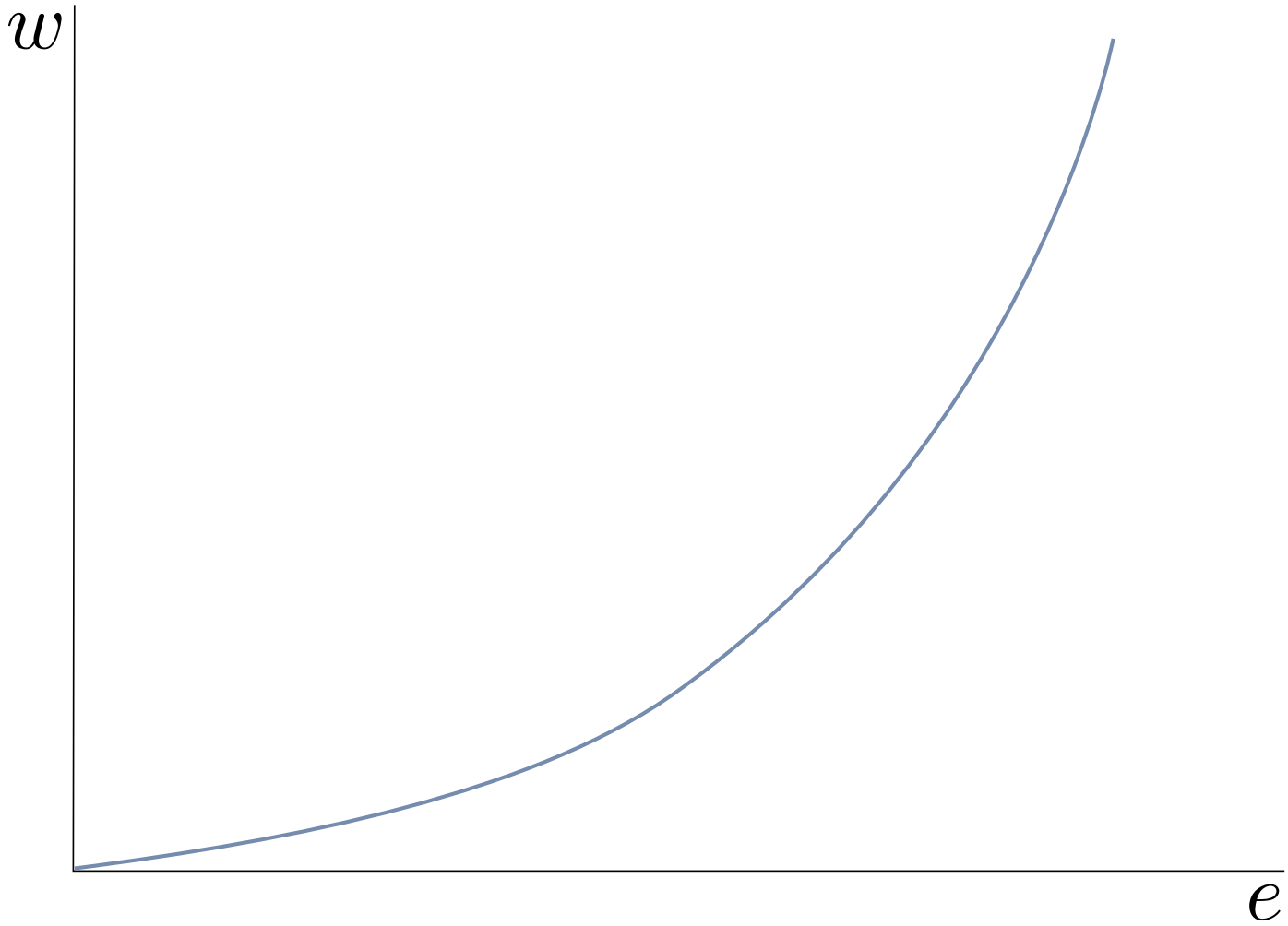
- A worker has ability θ.
- Only the worker knows θ.
- If employed by a firm the worker produces output θ.
- A firm's profit is
  - $\theta - w$ if it employs the worker at wage w
  - 0 otherwise
- There are more than one firm and firms are competitive.

Signaling

We will study the classic model of signaling by education level.

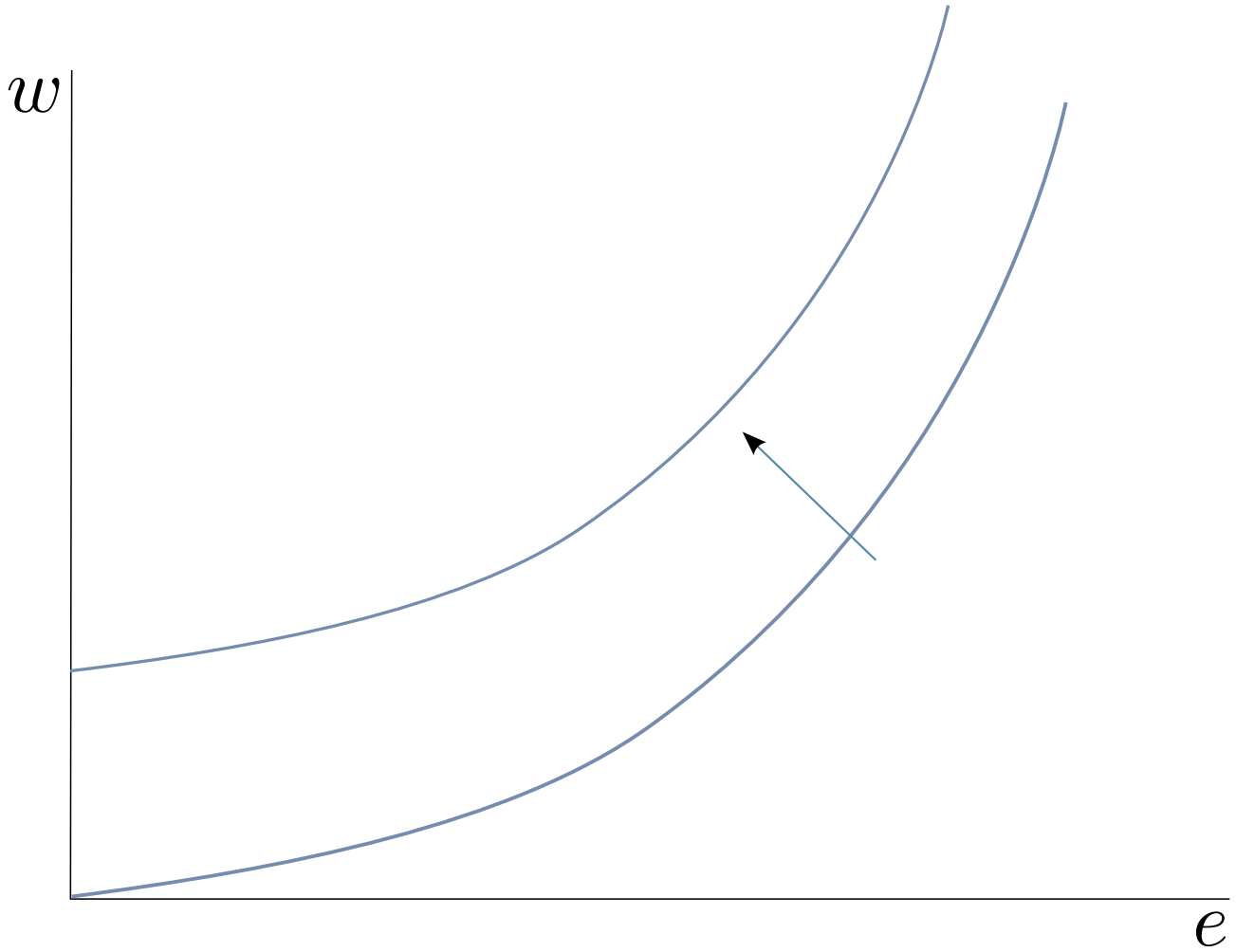
- Assume $\theta \in \{L, H\}$
- The worker moves first and chooses an observable education level e (a non-negative real number).
- Firms observe e but not θ.
- Firms then simultaneously make wage offers.
- Worker incurs cost $c(e|\theta)$. Utility is $w - c(e|\theta)$.
  - $c(0|\theta) = 0$
  - $c_e(\cdot|L) > c_e(\cdot|H)$.
  - $c_{ee}(\cdot|\cdot) > 0$.
- Worker can opt out from the labor market and get 0 utility

Indifference Curves



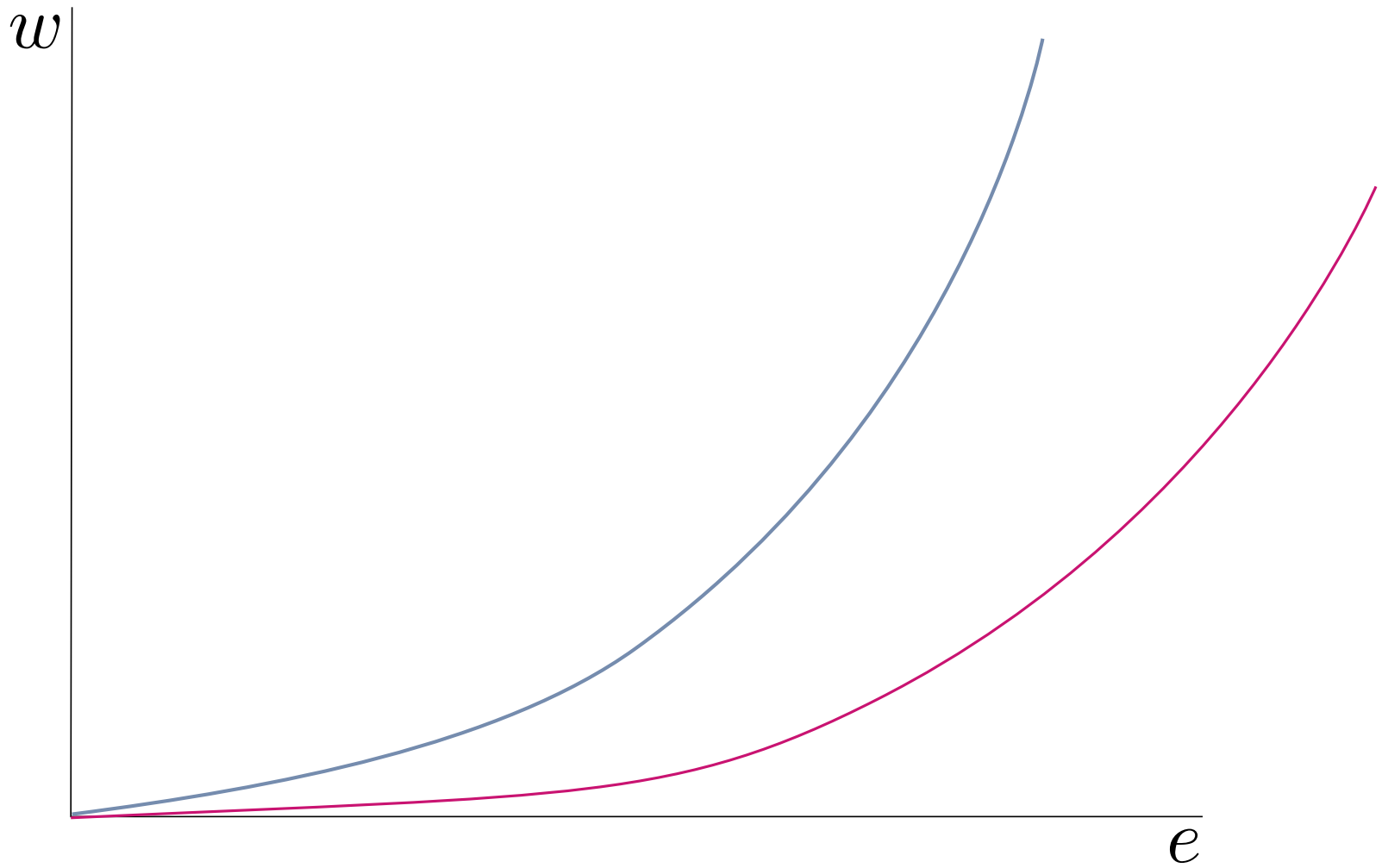
An indifference curve showing (e, w) pairs giving the same utility.

Indifference Curves



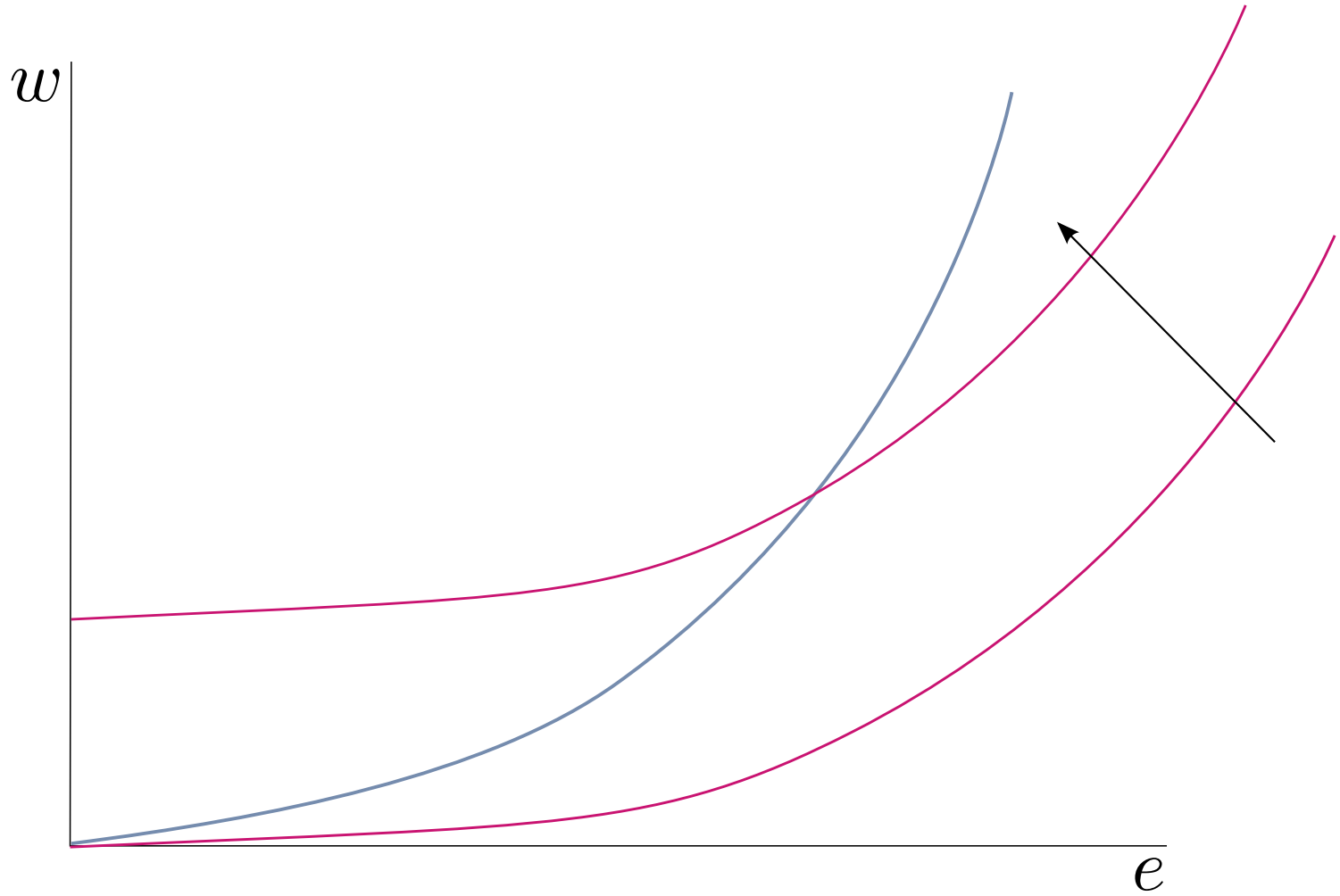
Shifts outward represent better bundles.

Indifference Curves



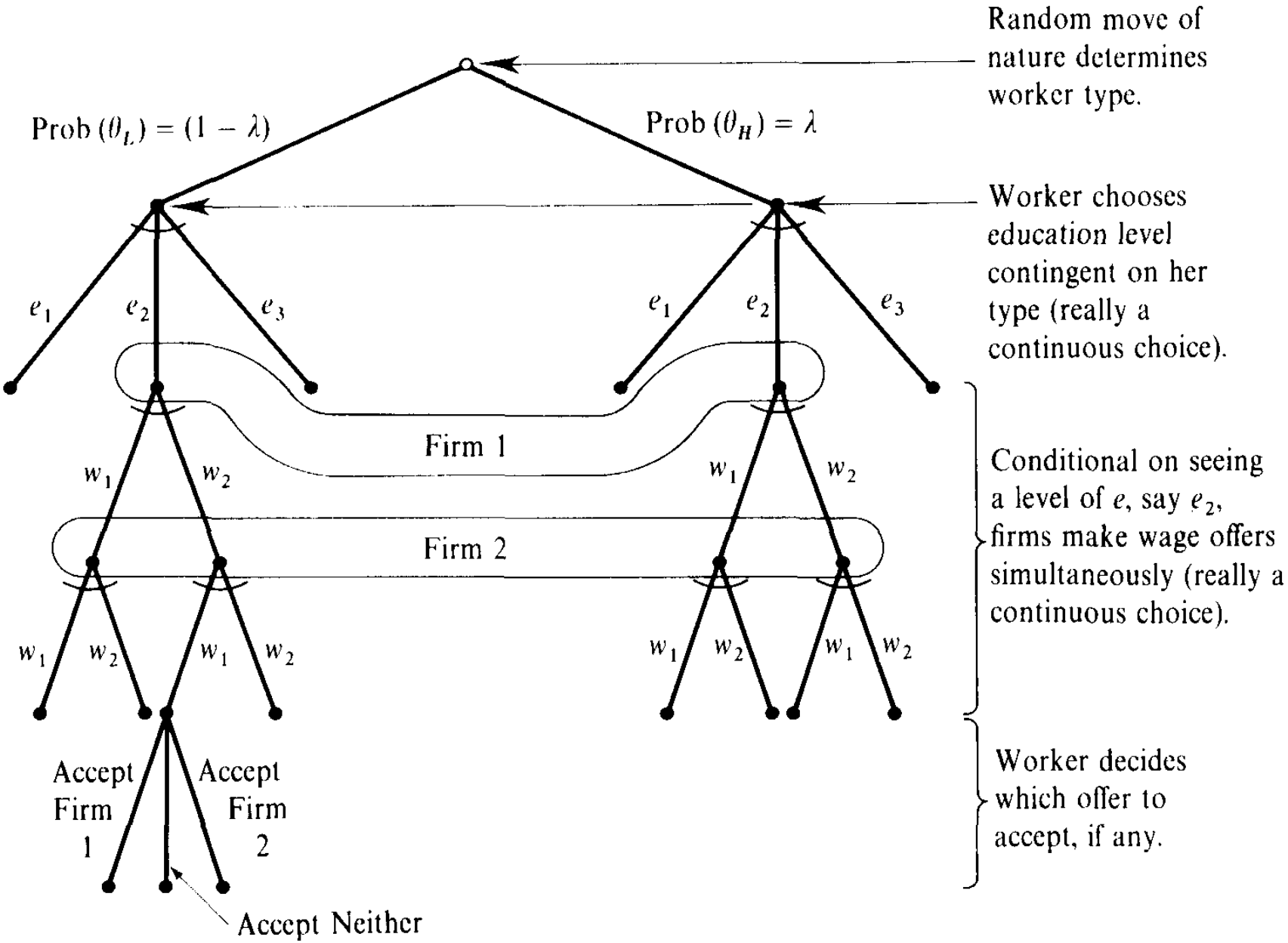
The red indifference curve is for type H while the grey is for type L

Indifference Curves



Single Crossing: Any pair of indifference curves for the two types cross exactly once.

Extensive Form of the Signaling Game



Analysis: Beliefs

- In a perfect Bayesian equilibrium, there will be a belief $\mu(e)$ about the worker's type θ.
- PBE: beliefs have to satisfy the consistency requirement:
  - If some $\hat{e}$ is chosen with a positive probability — Bayes' rule.
  - If $\hat{e}$ has zero probability of being chosen — $\mu(\hat{e})$ can be arbitrary.
- Focus on pure strategy equilibria: one education level per type
  - Mixed strategies are problematic for belief consistency in extensive form games with continuous actions.

Analysis: Wage

- Let $\mu(e)$ be the believed probability that the worker is type H.
- PBE: All firms will hold the same beliefs on equilibrium path.
  - Can have different beliefs off equilibrium path, but irrelevant here.
- Competition among firms leads to wage offers:

$$w(e) = \mathbb{E}_{\mu(e)}\theta = \mu(e)H + (1 - \mu(e))L$$

PBE Analysis

We can distinguish two types of equilibria.

- Separating equilbria in which each worker type chooses a different education level $e(H) \neq e(L)$.
- Pooling equilbiria in which $e(H) = e(L)$.

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- Separating equilbria in which each worker type chooses a different education level $e(H) \neq e(L)$.
- Pooling equilbiria in which $e(H) = e(L)$.
- (Partially separating with mixed strategies: e.g., $e(L) = e_L$ and H randomizes between e_L and some other e_H.)

Separating Equilibria

Lemma

In a separating equilibrium, type L chooses $e(L) = 0$.

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This is because PBE implies

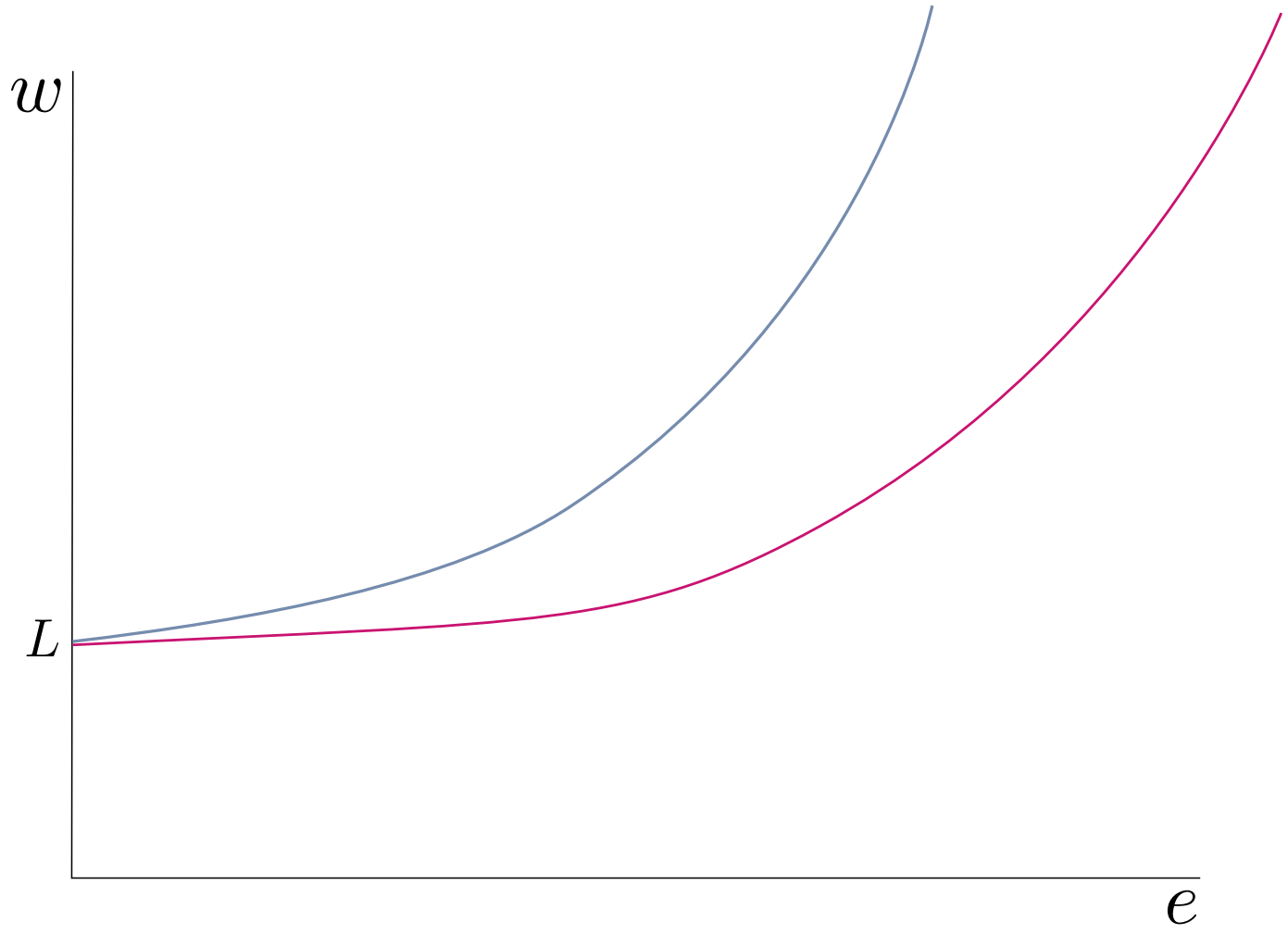
- In response to any education level e, competition will set the wage

$$w(e) = \mathbb{E}_{\mu(e)}\theta = \mu(e)H + (1 - \mu(e))L$$

- In a separating equilibrium, $\mu(e(L)) = 0$ and $w(e(L)) = L$.
- Thus, L will set $e(L) = 0$ since there e is pure waste for him/her

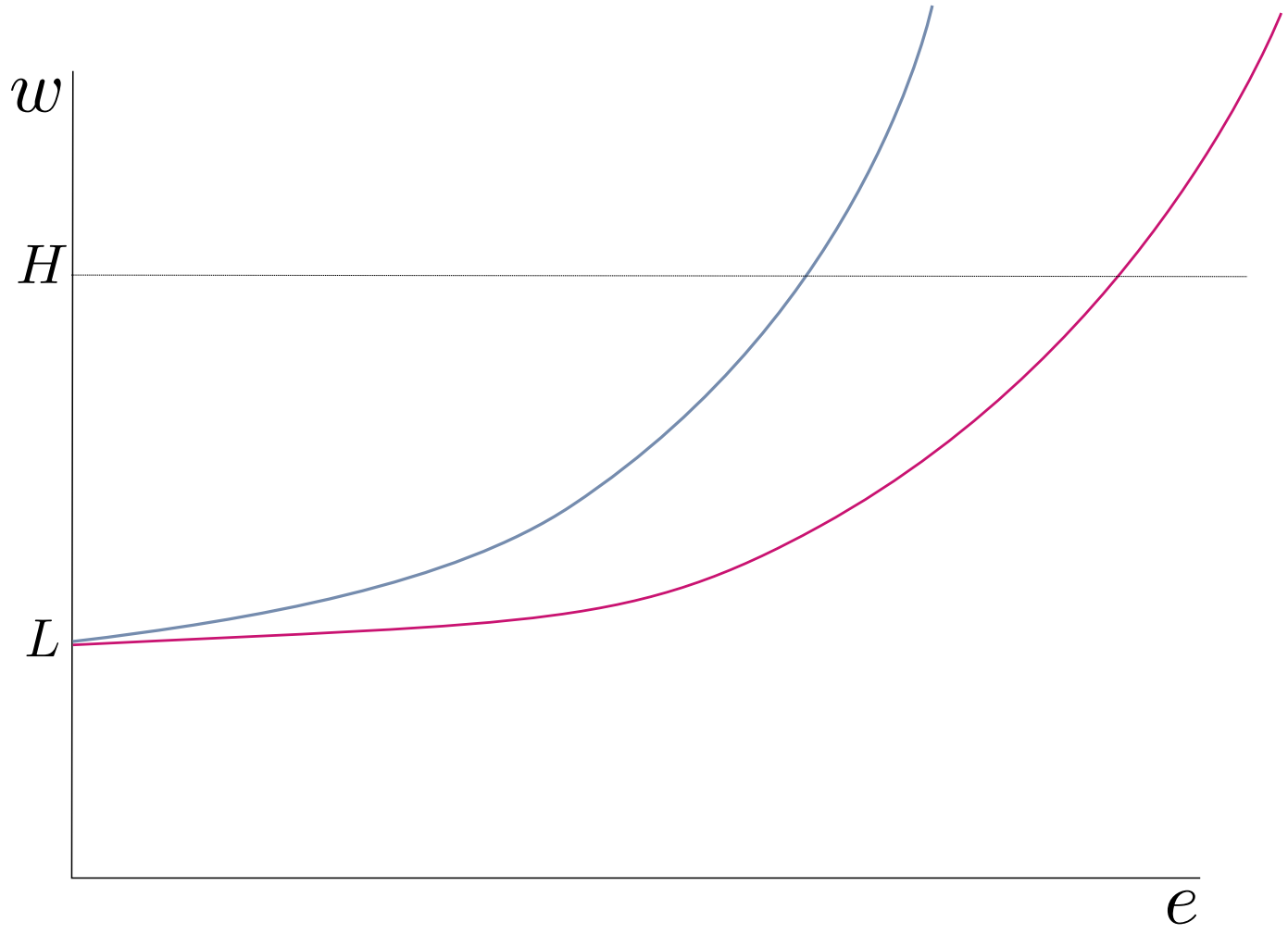
Likewise, $w(e(H)) = H$ in a separating equilibrium.

$e(L) = 0$ and $w(0) = L$



The diagram shows the utility that type L will get and also the utility that type H *would* get if type H chose education level 0.

$e(L) = 0$ and $w(0) = L$



But in a separating equilibrium, in fact type H will receive wage H.

Separating Equilibria: Incentive Compatibility

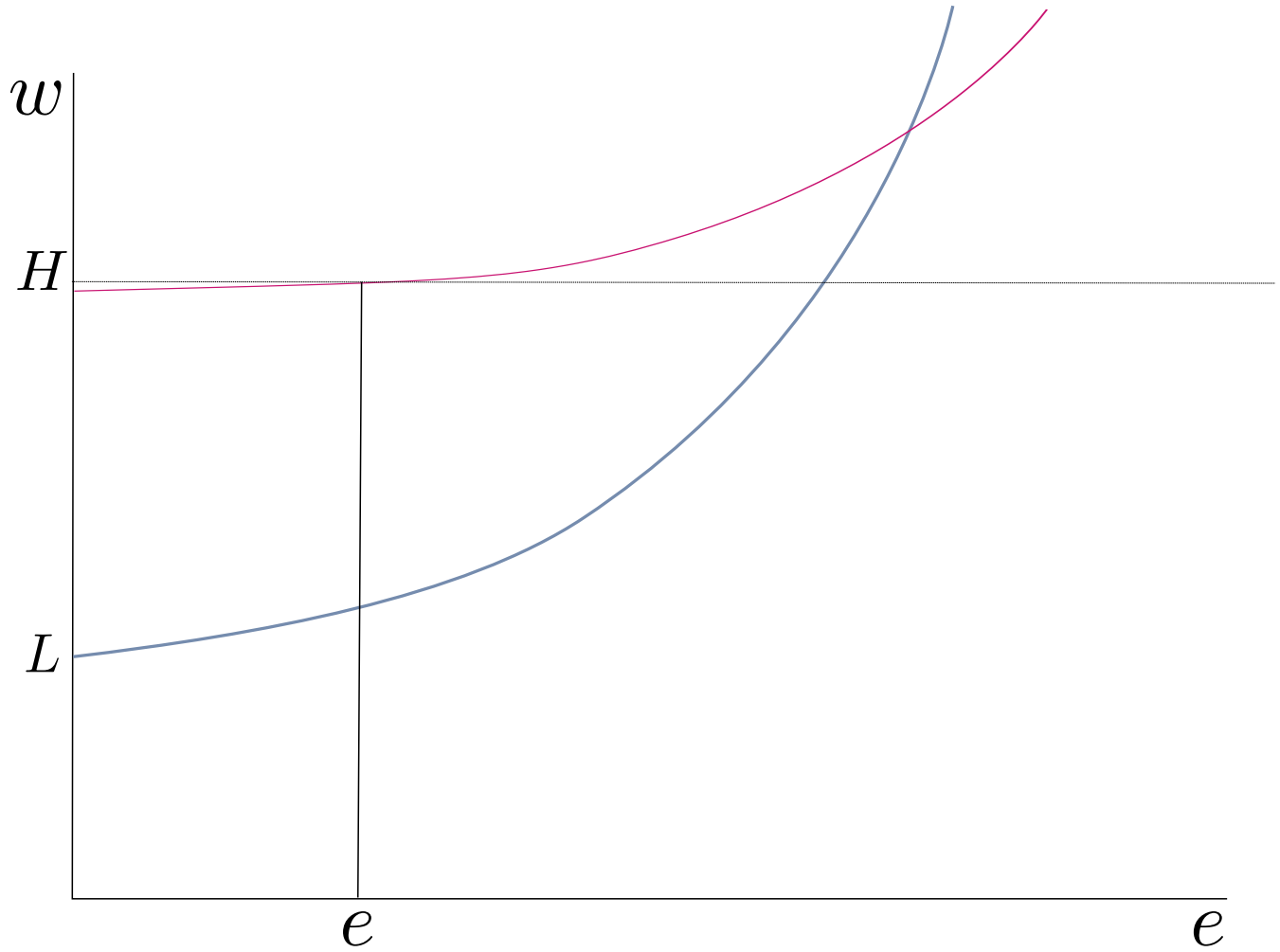
Lemma

In a a separating equilibrium, type H chooses some $e(H) > 0$ such that:

$$H - c(e(H)|H) \geq L \geq H - c(e(H)|L)$$

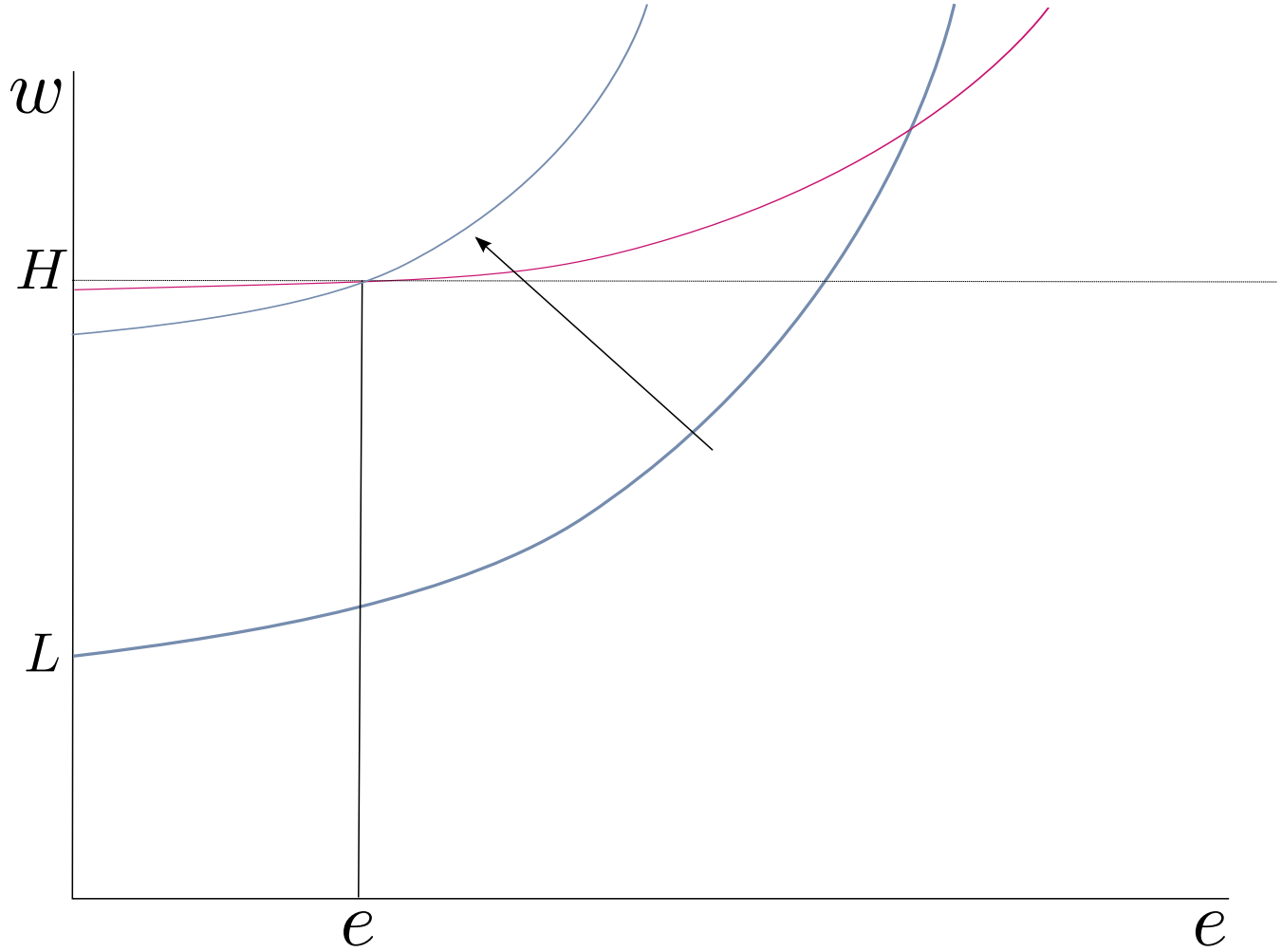
The first inequality says that type H prefers to choose education level $e(H)$ rather than $e = 0$ and the second says that type L prefers to choose education level $e = 0$ rather than $e(H)$. As you already know, these inequalities are called *incentive-compatibility constraints*.

Incentive Compatibility



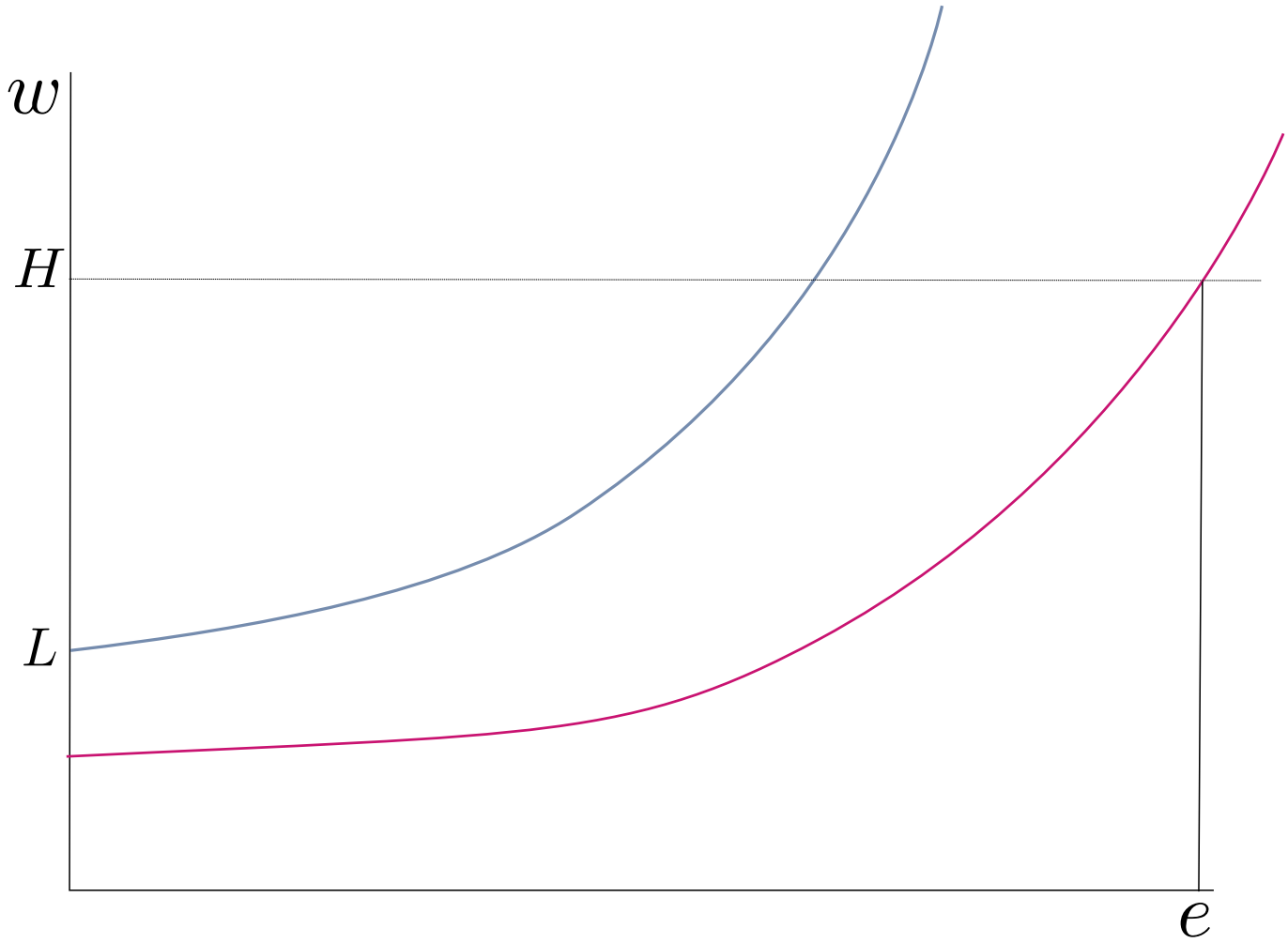
Suppose we tried to construct a separating equilibrium in which type H chooses this low level of education. This would give type H the utility level indicated.

Incentive Compatibility



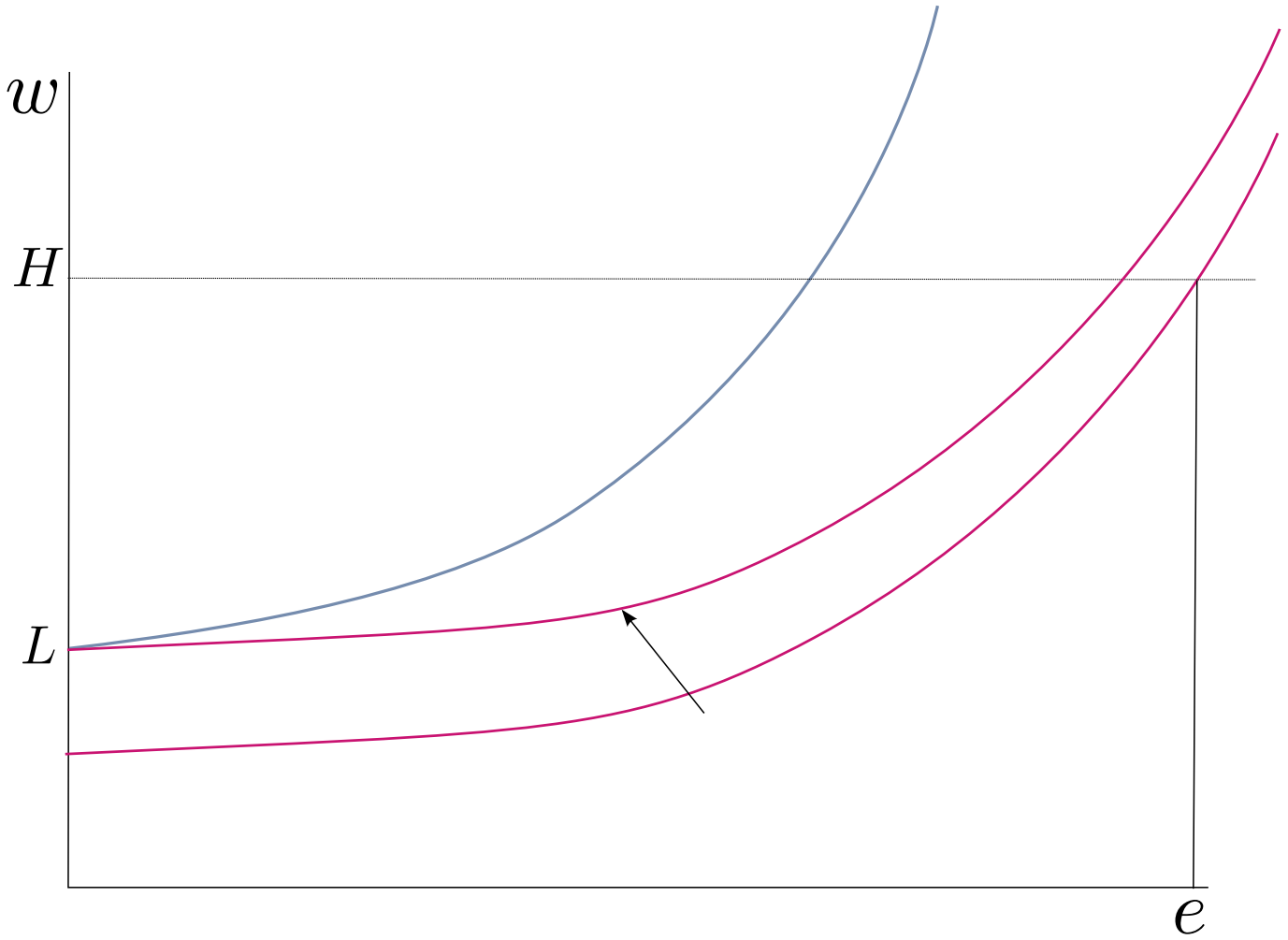
But this would induce type L to deviate and acquire education level e. Therefore this cannot be an equilibrium. (We have violated the second inequality.)

Incentive Compatibility



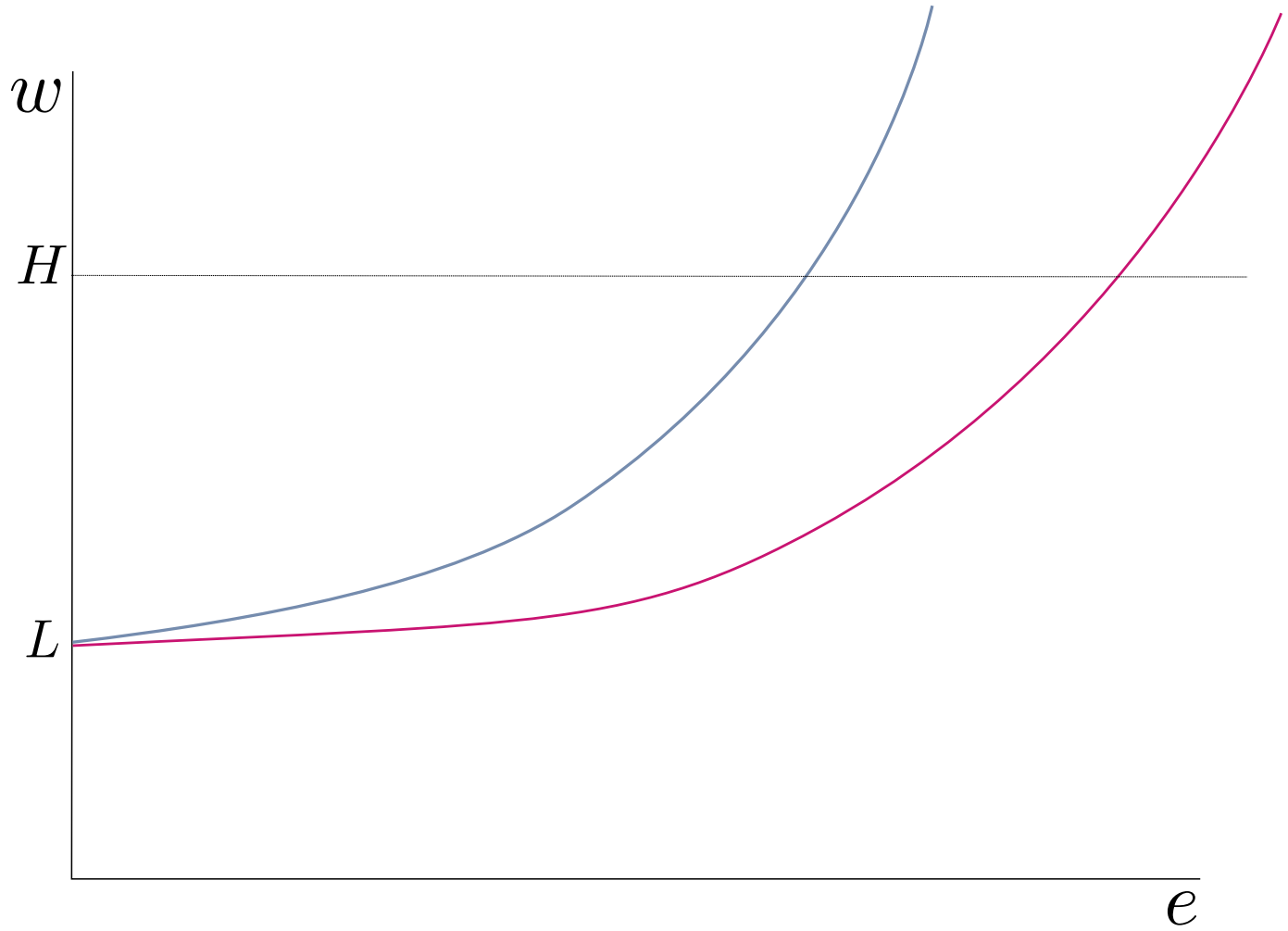
If we choose a high level of education this would be the utility of type H.

Incentive Compatibility



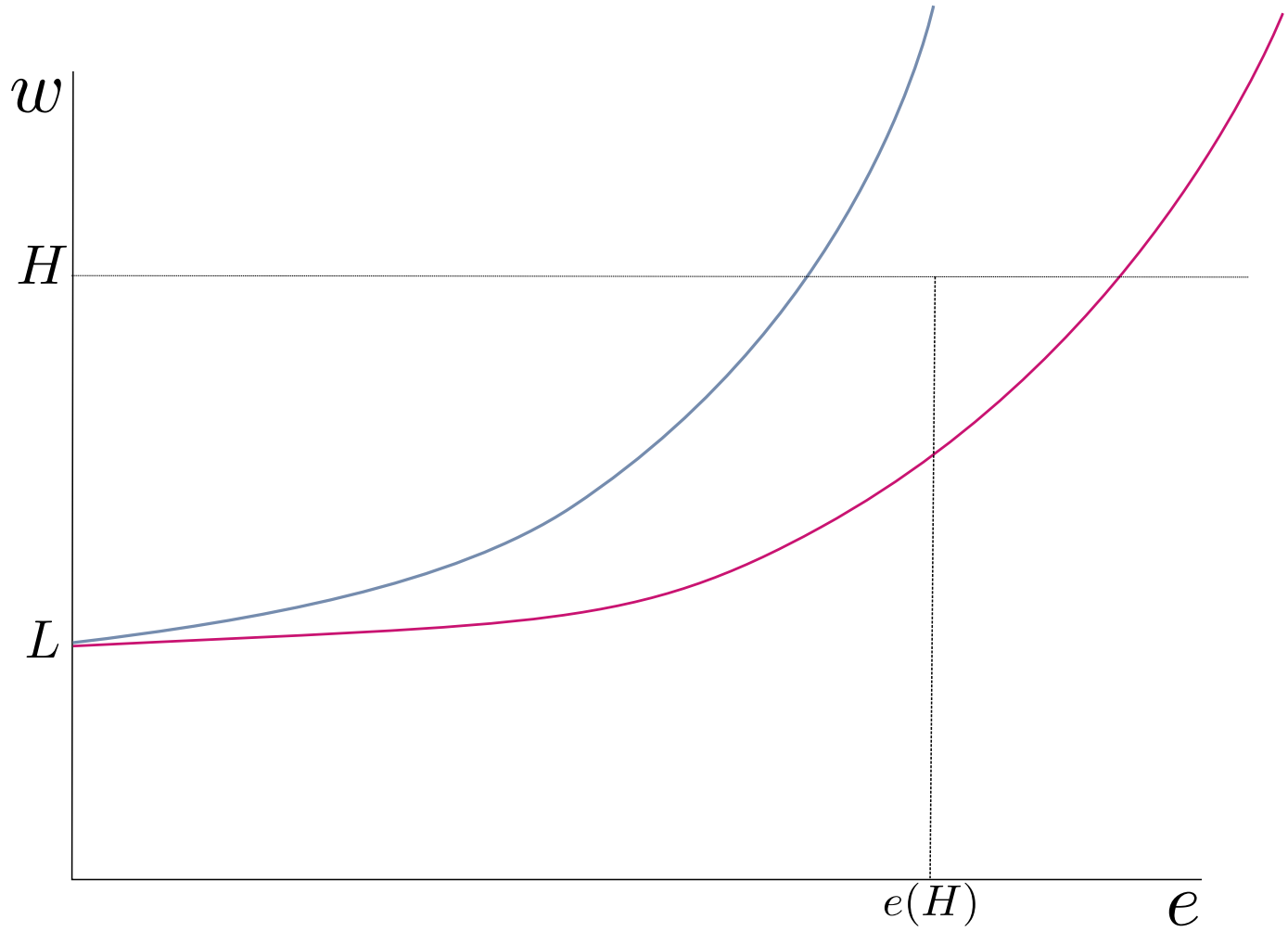
But rather than choose such a high level of education, type H would deviate and choose education level 0. (We have violated the first inequality.)

Incentive Compatibility



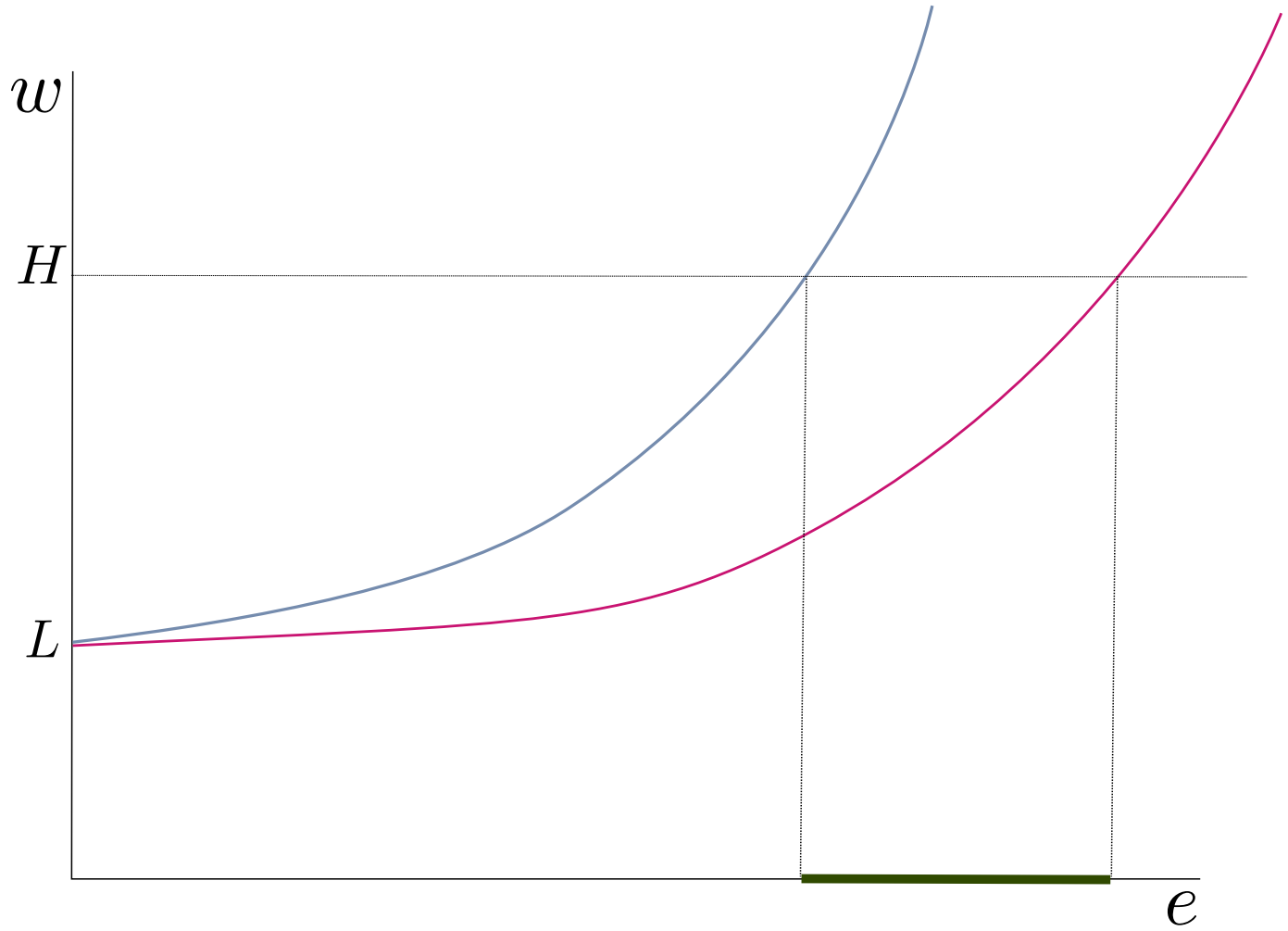
To see graphically how to choose $e(H)$ to satisfy the incentive-compatibility constraints, let's draw the two types' indifference curves through the point $(e, w) = (0, L)$.

Incentive Compatibility



Setting $e(H)$ at a medium level will be compatible with the incentives of both types.

Incentive Compatibility

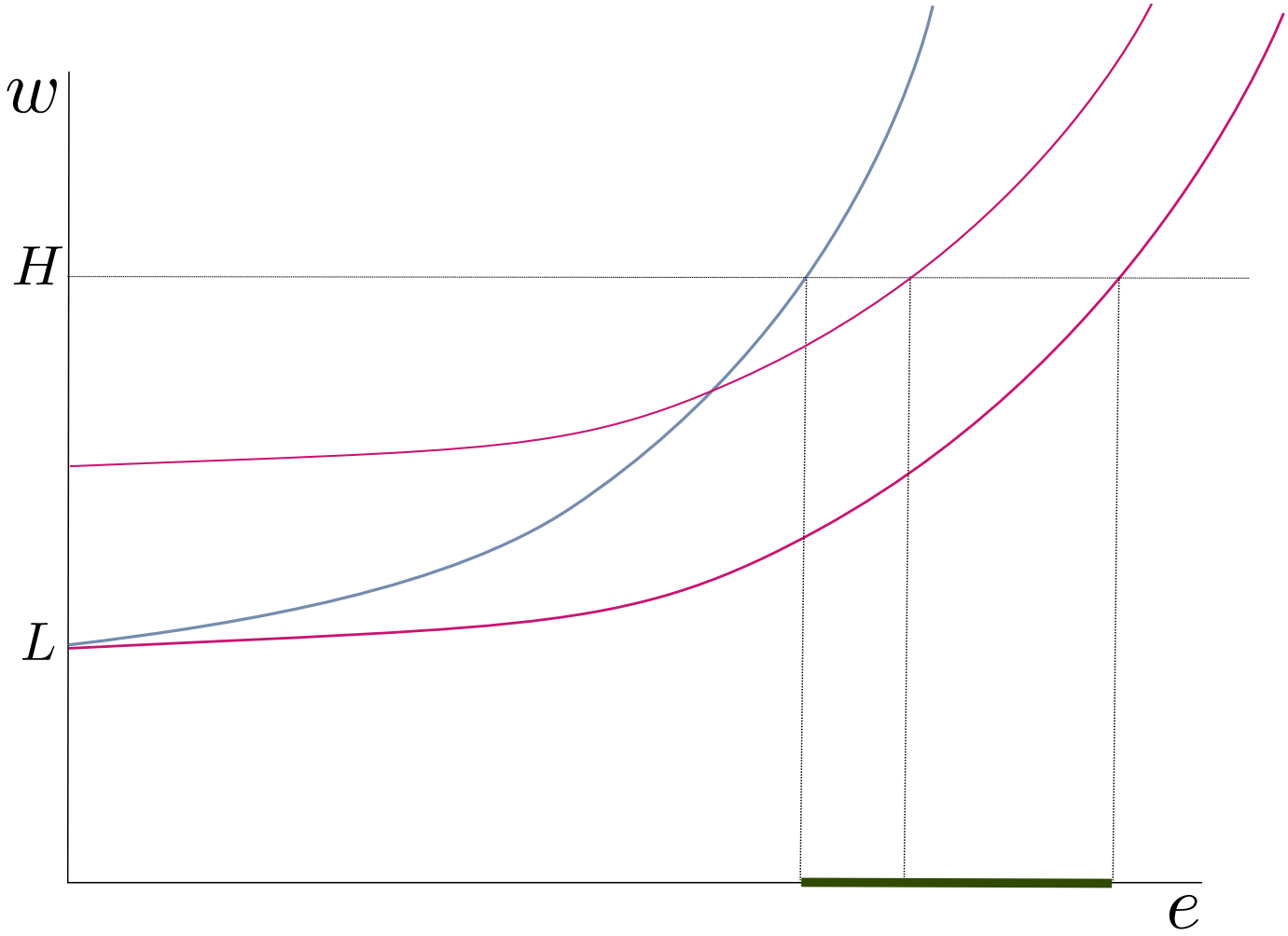


Any e in this range will satisfy the incentive-compatibility inequalities.

Separating Equilibria

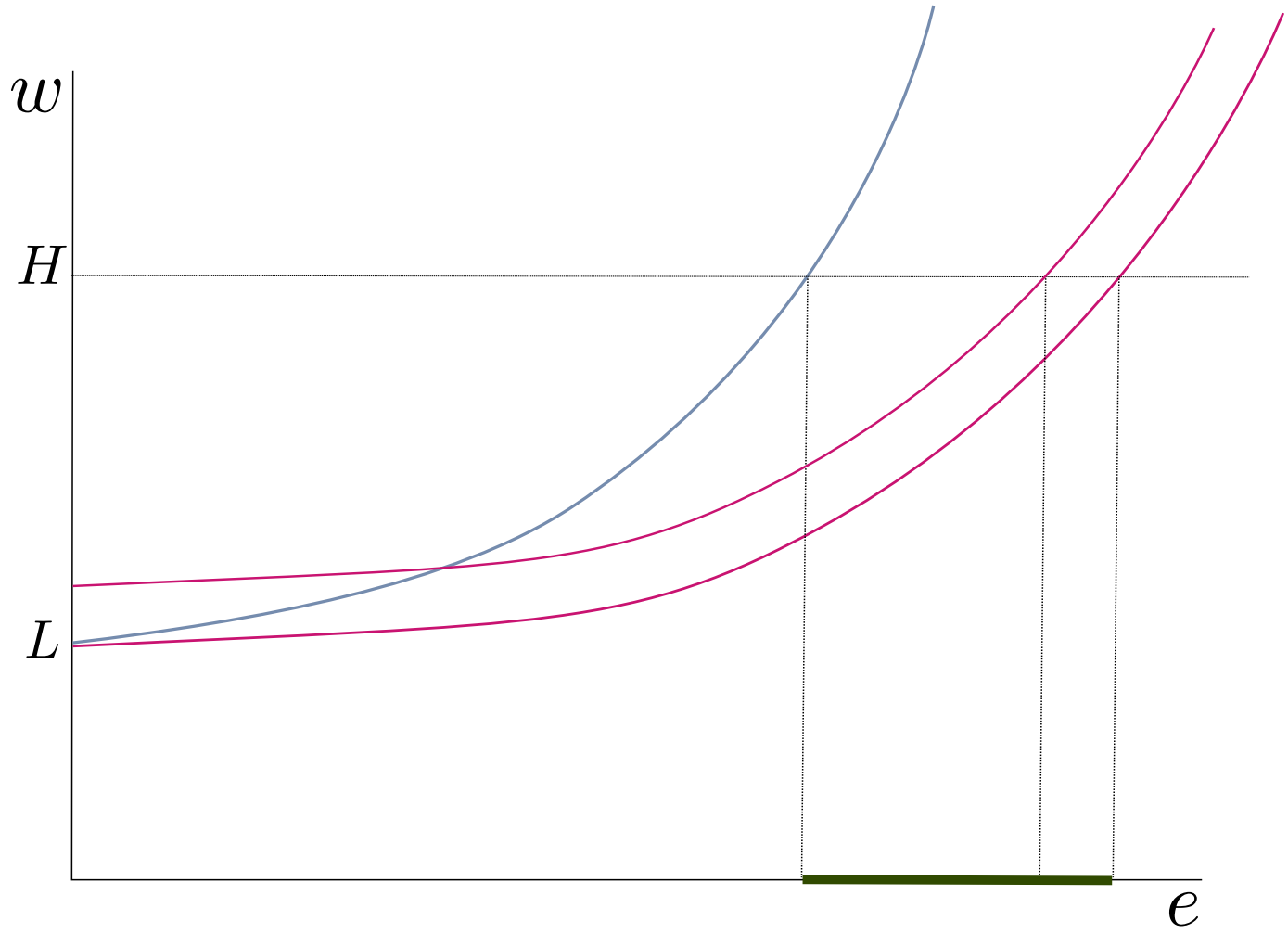
- The previous lemmas describe necessary conditions for a separating equilibrium.
- To show they are sufficient, all that remains is to specify out-of-equilibrium beliefs.
- What will a firm believe about a worker's ability when the worker chooses some $e \notin \{0, e(H)\}$?
- We can assume that the firm attaches probability 1 to L.
- They will therefore offer a wage of L to any worker with an education level $e \neq e(H)$.
- This means that any deviation by a worker of either type will be unprofitable.

Multiple Equilbria



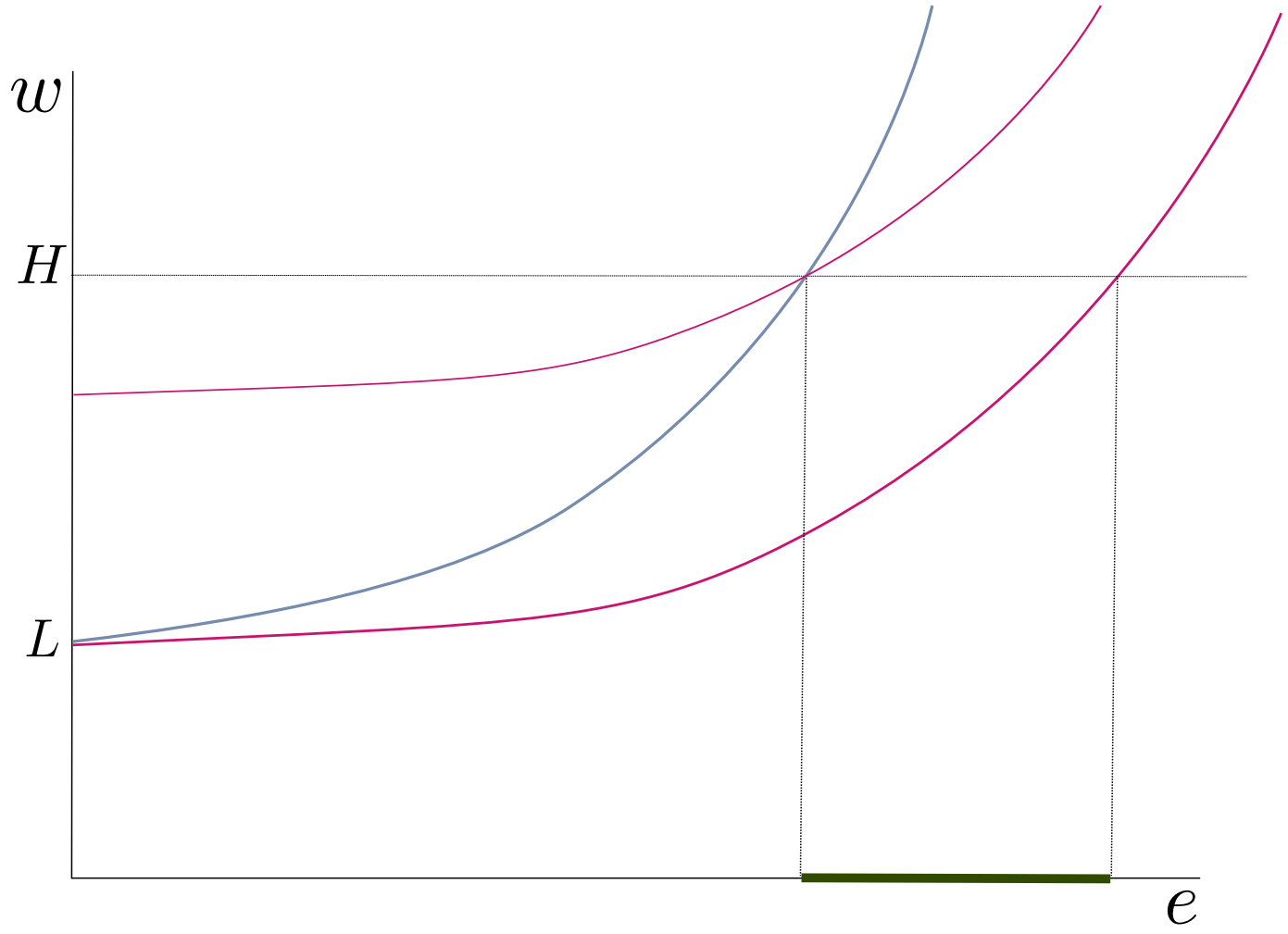
Some of these equilibria are better ...

Multiple Equilbria



. . . than others.

Multiple Equilbria

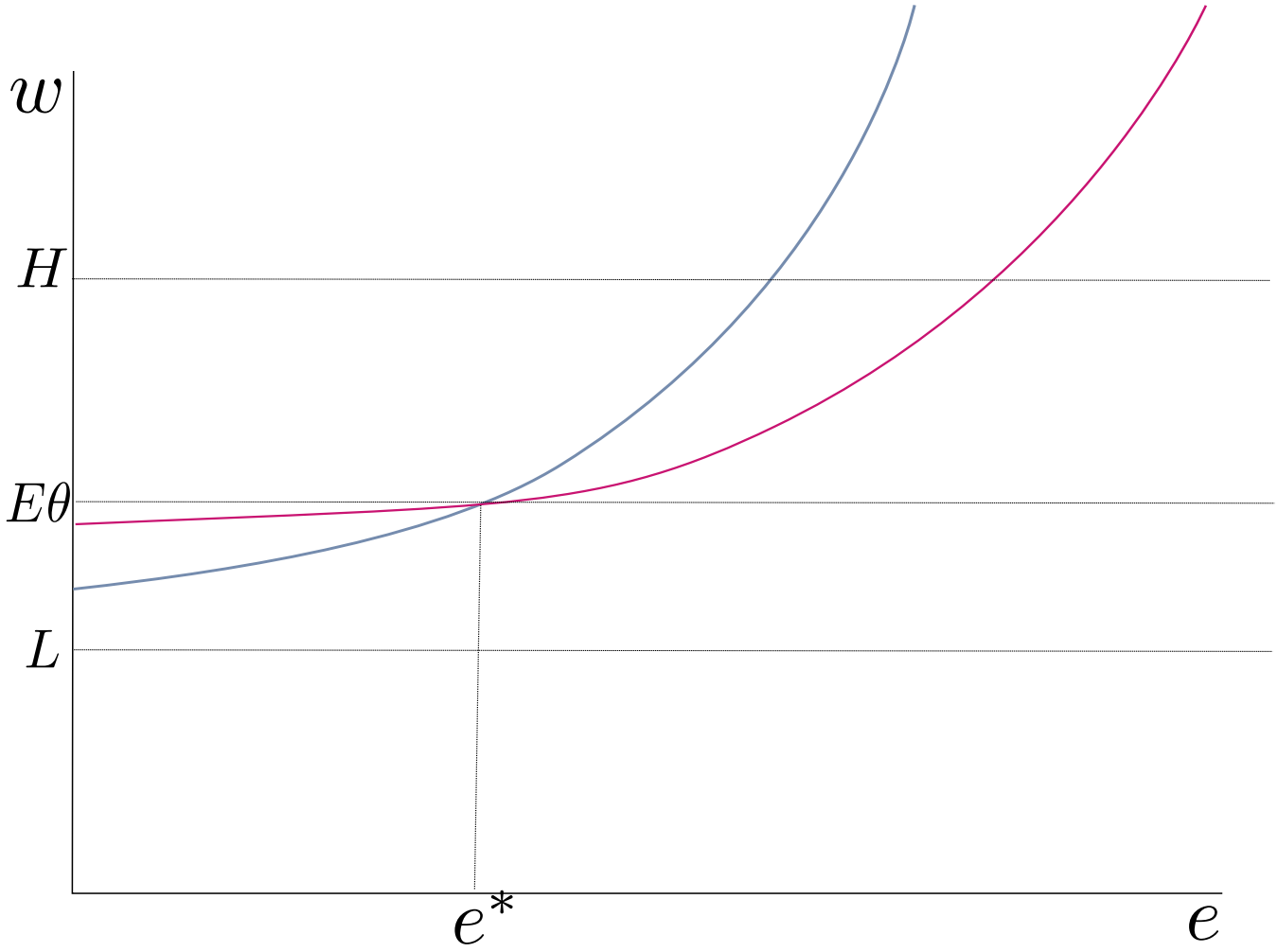


The best (in what sense?) is the one with the lowest level of education, i.e. the least costly signaling.

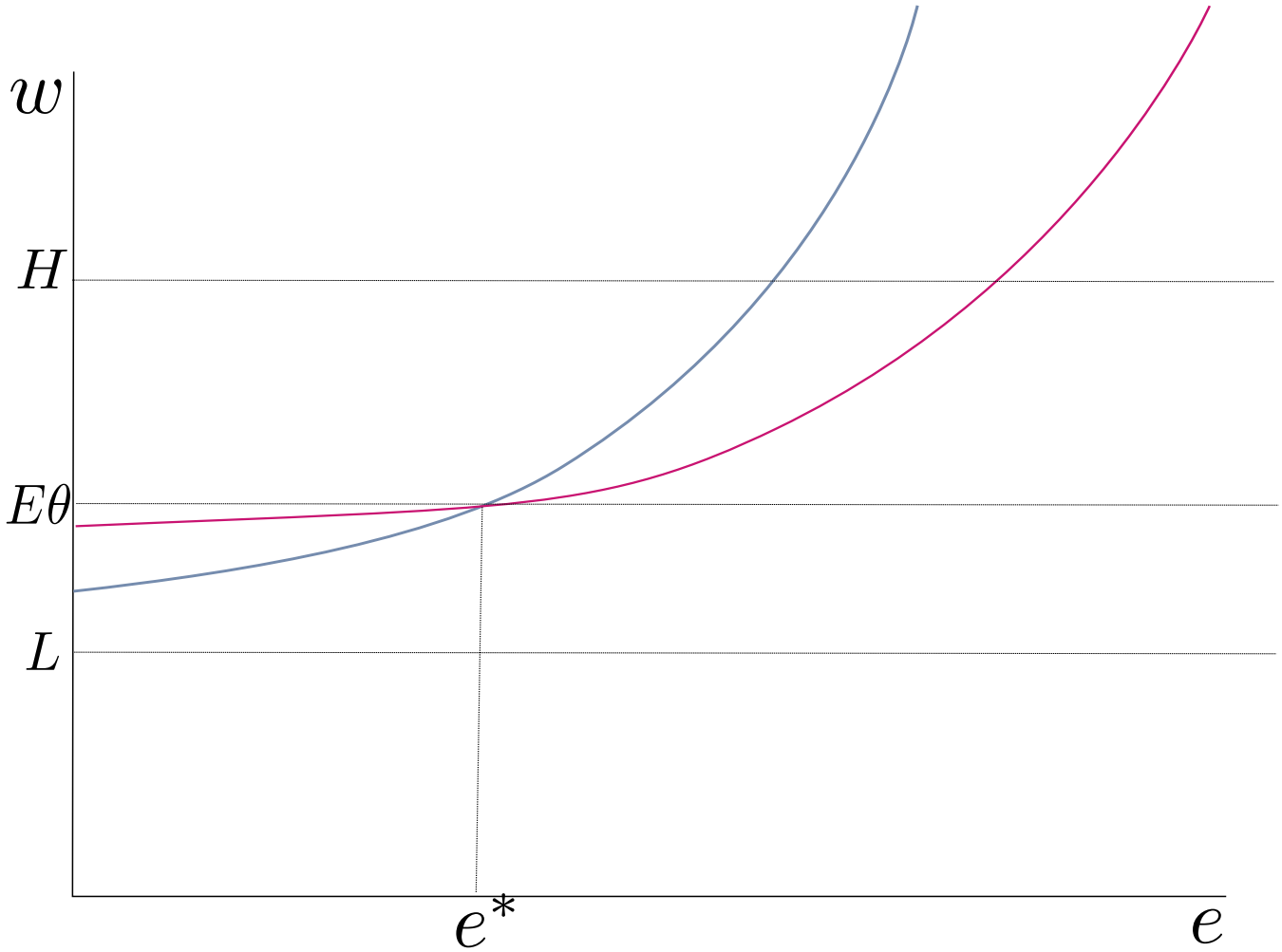
Pooling

- In a pooling equilibrium $e(L) = e(H) = e^*$.
- This means that $w(e^*) = \mathbb{E}\theta$.
- To make this an equilibrium, it is enough to set $w(e) = L$ for all $e \neq e^*$.

Pooling Equilibrium Illustrated

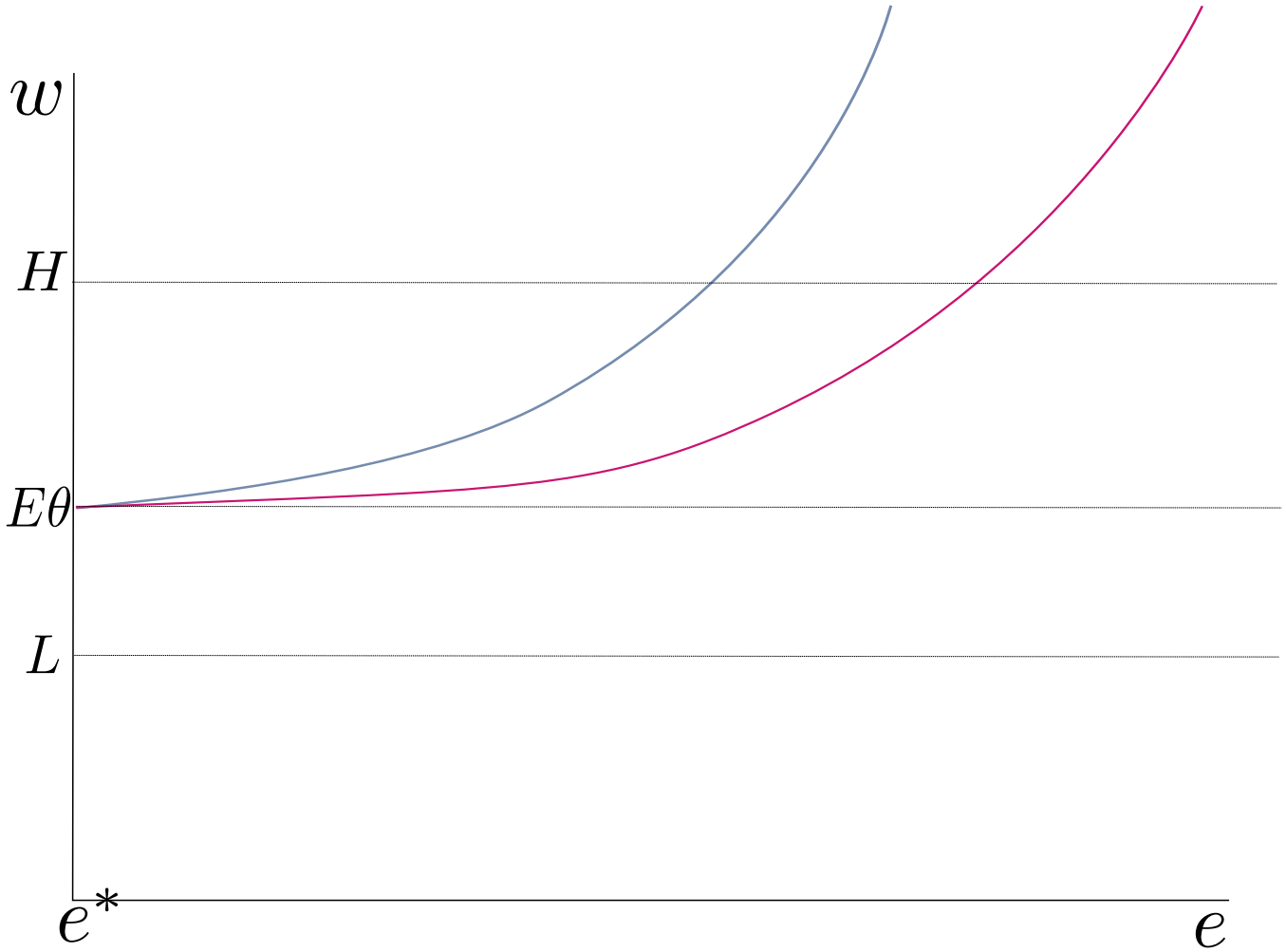


Efficiency of Equilibria



An example of a pooling equilibrium.

Efficiency of Equilibria



The best pooling equilibrium has zero education.

Multiplicity of Equilibria and Equilibrium Refinement

- Multiplicity of equilibria here — largely an artifact of freedom to choose beliefs off equilibrium path
- But sometimes these off-equilibrium-path beliefs make little sense
- Consider on the separating equilibria with e_H that is very high:
  - A low-type worker would never pick e_H and earn H if $L > H - c(e_H|H)$
  - So, why would a firm believe that worker who picks $e = e_H - \epsilon$ is a low type?

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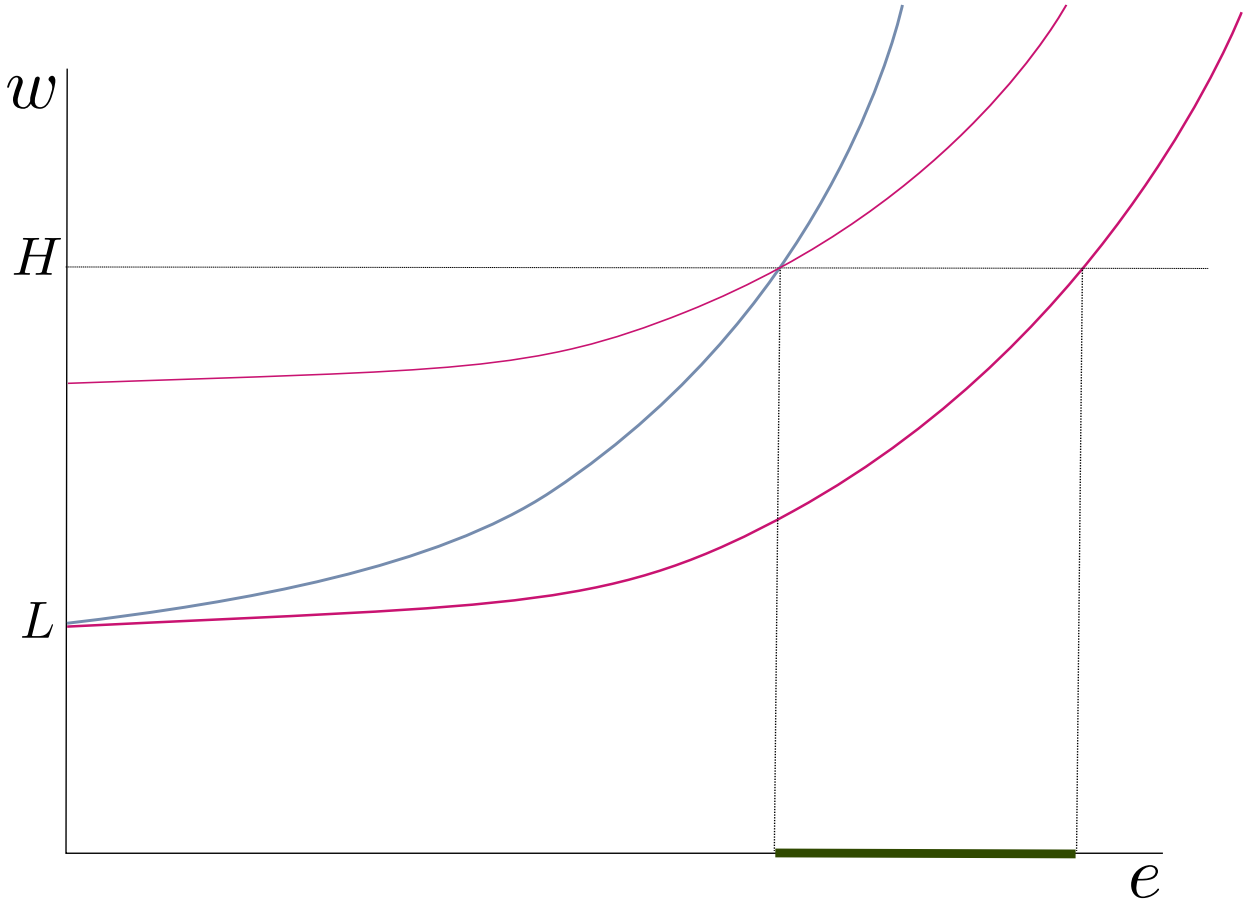
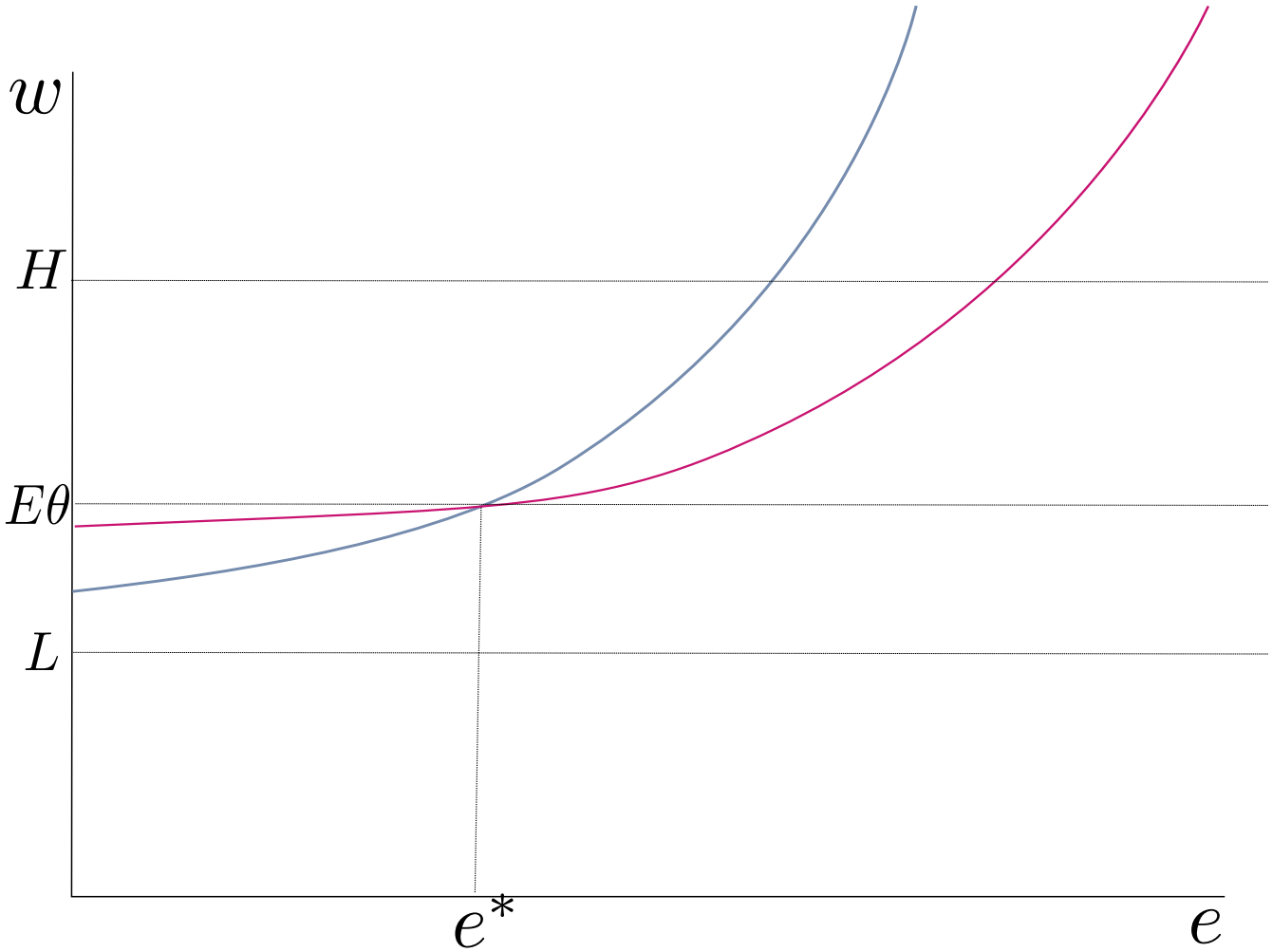
  - So, why would a firm believe that worker who picks $e = e_H - \epsilon$ is a low type?
- We can restrict belief formation and force firms to form correct beliefs in situations when certain worker actions are dominated
- Q: Which equilibria survive such reasonable refinements?

Thinking About 'Reasonable' Separating Equilibria



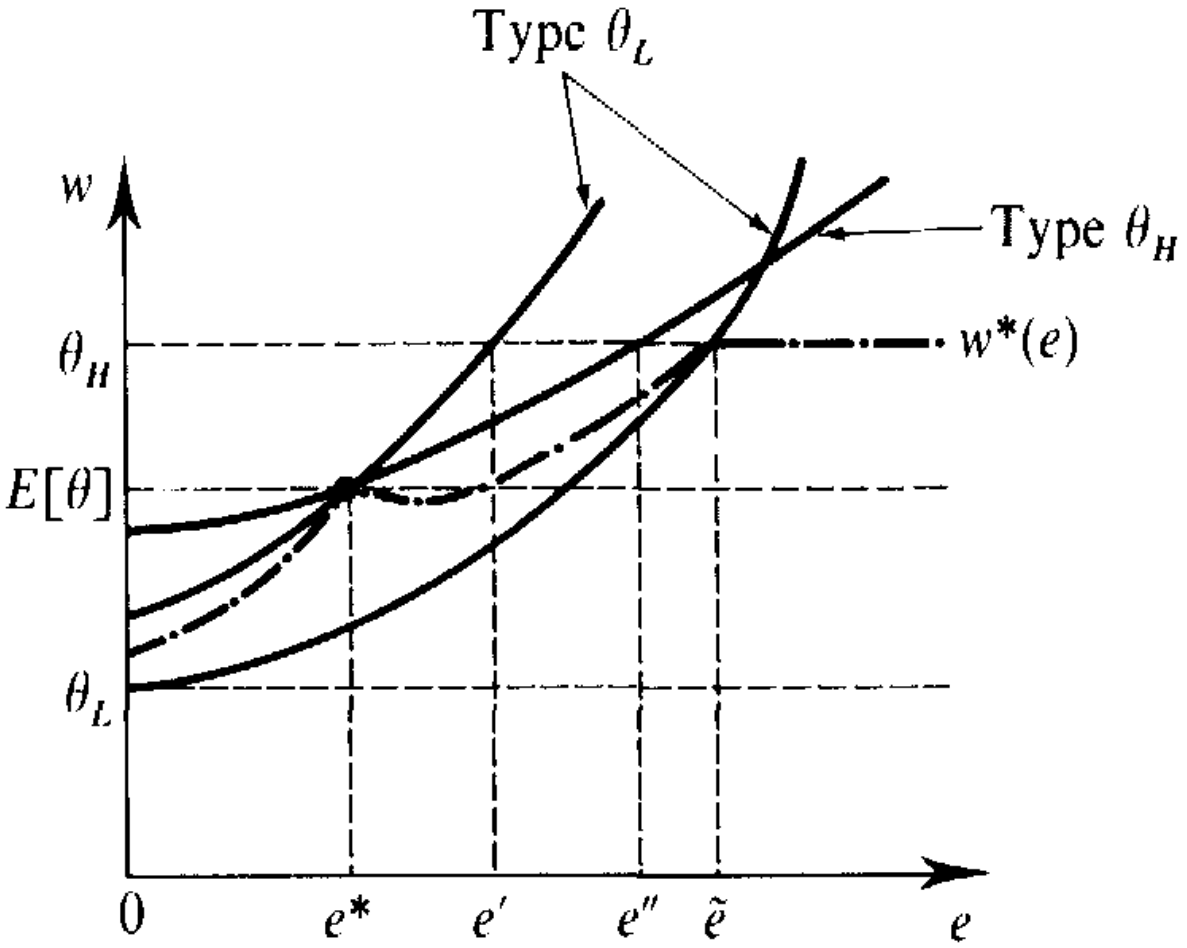
L-type won't choose e in the highlighted region. Thus, firms should have $\mu(e) = 1$ for those education levels. However, then H-type will choose the lowest e out of the highlighted education levels.

Thinking About 'Reasonable' Pooling Equilibria



H-type might prefer a separating equilibrium to some pooling equilibria.

Intuitive Criterion (Cho and Kreps, 1987)



Cho and Kreps (1987) intuitive criterion rules out *all* pooling equilibria.

Pareto-Improving Interventions

- In this model, education is wasteful, so there is likely scope for welfare-improving interventions.

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- First, if H-type prefers $(0, \mathbb{E}\theta)$ to (e_H, H), it may be Pareto-improving to ban signaling altogether. (Why?)

Pareto-Improving Interventions

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- First, if H-type prefers $(0, \mathbb{E}\theta)$ to (e_H, H), it may be Pareto-improving to ban signaling altogether. (Why?)
- Second, even if H prefers the separating equilibrium, there can be scope for Pareto improving forceful reduction in education.