## Problem 1. The Market for Lemons: Multiple Equilibria. [10 pts]

Consider the general Akerlof lemons model from Tuesday's section. The players are buyers and a seller. A seller has a used car whose quality $\theta \in [0,1]$ is his private information. $\theta$ represents his benefit from owning the car. It is drawn from cdf $F$ that satisfies $F(\theta) > 0$ for all $\theta > 0$. We can view this as the distribution of car quality among sellers in the population, with the actual seller being drawn at random. There are $N$ potential buyers. Each would obtain benefit $b(\theta) \geq \theta$ from owning a car of quality $\theta$. Thus the buyers always value the car at least as much as the seller.

Unless specified otherwise, the game proceeds as follows: (i) the seller learns the quality of her car, (ii) the buyers simultaneously make price offers $p_i, p_j \geq 0$; (iii) the seller can accept one of these offers or reject both. Buyer's utility is $b(\theta) - p$ for buying at price $p$; 0 for not buying. Seller's utility is $p - \theta$ for selling at price $p$; 0 for not selling.

1. Suppose $N = 2$, $F \sim U[0,1]$, and $b(\theta) = \alpha + \beta\theta$ with $\alpha \in (0,1)$ and $\beta \in (1-\alpha, 2-2\alpha)$. Show that $p_1 = p_2 = 2\alpha/(2-\beta)$ is the only possible subgame perfect equilibrium. Give an example of seller strategies that would support it and an example of seller strategies that wouldn't support it. [3 pts]
2. Suppose that $F \sim U[0,1]$ and that buyer's benefit from obtaining a car of quality $\theta \in [0,1]$ is given by:

$$b(\theta) = \begin{cases} \frac{1}{6} + \frac{1}{2}\theta & \text{if } \theta \in [0, \frac{1}{3}), \\ -1 + 4\theta & \text{if } \theta \in [\frac{1}{3}, \frac{2}{3}), \\ \frac{14}{9} + \frac{1}{6}\theta & \text{if } \theta \in [\frac{2}{3}, 1] \end{cases}$$

   (a) Suppose we evaluate this model using competitive equilibrium. That is, there are many buyers ($N$ is large), the seller extracts all the surplus, and the game specified above can be ignored for the purposes of this question. What prices are possible in equilibrium? [3 pts]

   (b) Suppose we consider the game-theoretic model above in which $N = 2$ buyers make simultaneous wage offers. Describe the subgame perfect equilibria. [4 pts]

## Problem 2. Adverse Selection in Insurance Markets. [12 pts]

Suppose there is a continuum of risk averse individuals endowed with wealth $w$. With a privately known probability, $\theta \sim U[0,1]$, they incur a loss of size $l$. Their *Bernoulli* utility function (per MWG definitions) is $u(x) = \sqrt{x}$ and, therefore, their *von Neumann-Morgenstern* utility function is $\mathbb{E}u(w) = \theta\sqrt{w-l} + (1-\theta)\sqrt{w}$. Perfectly competitive, risk-neutral firms offer an insurance contract at price $p$ that pays $l$ in case of a loss. Assume $w = 9$ and $l = 5$.

(a) Derive the demand function $p(\theta)$ on this market. [2 pts]

(b) Derive the marginal cost function $MC(\theta)$. Why is it *decreasing* with the number of insured? [1 pt]

(c) Plot the demand and the marginal cost functions on the same graph, with the share of insured individuals $(1-\hat{\theta})$ on the $x$-axis, where $\hat{\theta}$ is the type of the marginal buyer. [2 pt]

(d) Derive and plot the average cost function $AC(\theta)$ on this market. (*Hint: Average cost in this environment is equal to the expected insurance pay-out to an average buyer, i.e.,* $AC(\tilde{\theta}) = E[5\theta|\theta > \tilde{\theta}]$.) [1 pt]

(e) Find the competitive equilibrium price. Why does the competitive equilibrium require $p = AC$? What happens if $p > AC$ or $p < AC$ in a competitive market? [2 pts]

(f) Confirm that the market unravels. Calculate the welfare loss from lack of insurance on this market. [2 pt]

(g) Suppose that the government introduces a subsidy to the insurance companies. Now, for every \$5 loss, they now only pay \$3 and the other \$2 is paid out by the government. What happens to the competitive equilibrium on this market? Is the under-insurance problem still present? [2 pts]

## Problem 3. Close Examination of the Screening Model. [8 pts]

1. Prove five Lemmas and Corollaries for the screening model covered in class, starting from the slide "Analysis of Constraints" and stopping on "Analysis of the Optimum." [1 pt. per Lemma/Corollary]
2. Prove that in the screening model a profit maximizing firm never pools the two types of workers. [3 pts.]