RoME Master
in Economics

Information Economics: Spring 2021
Problem Set 3

Prof. Alexey Makarin
Due: 04/05/2021 16:45 CET

Problem 1. Moral Hazard and Summer RAship [5 pts].

Claudio won an ERC grant and decides to hire Aleksandr as a research assistant. Aleksandr has a vNM utility $\ln w - $ (the cost of effort) where w is the wage. Aleksandr's reservation utility is 1. Claudio can make a take-it-or-leave-it offer.

Once they sign a contract, Aleksandr chooses his effort level, Low or High. The low effort is costless while the high effort costs 1. The effort level is not verifiable (so the wage cannot be contingent on it) but the outcome of the project (Good or Bad) is verifiable. The probabilities of the good outcome are $2/3$ (the high effort level) and $1/3$ (the low effort level). Let w_G and w_B be the wage payments when the outcome is good and bad, respectively.

Throughout this question, Claudio's objective function is to minimize the expected wage payment subject to the constraint that he needs to induce Aleksandr to choose the high effort.

(a) What constraints must w_G and w_B satisfy? [1 pt]

(b) Which constraints are binding? Provide a detailed explanation why each of them binds or not. [2 pts]

(c) Derive the optimal contract. [1 pt]

(d) Concetta points out that Aleksandr needs to be paid at least $e (\approx 2.718)$ regardless of the outcome, according the new EIEF guidelines. Derive the optimal contract. In doing so, determine which constraints are binding or not with brief explanations. [1 pt]

Problem 2. Moral Hazard and Head-Hunting [10 pts].

(Final Exam 2020.) A risk-neutral small firm hires an agent whose effort is unobservable to the firm. The agent's effort $e \in \{0, 1\}$ determines the probability distribution of revenues $x \in \{x_L, x_H\}$, $x_H > x_L$, as follows: $Pr(x = x_H | e = 1) = 1/2$ and $Pr(x = x_H | e = 0) = 0$.

However, if the agent exerts high effort, then with probability $q \in (0, 1)$ he is scouted and hired away by a big firm before any revenue for the small firm is realized. If the agent is hired away, he gets utility of 1 (note that this includes his effort cost). If the agent stays at the firm and exerts effort level e for wage w, he gets utility $u(w) - e$, where $u(w)$ is continuous and strictly concave with $u(0) = 0$. The firm's profits are $x - w$ if the agent is not scouted, and 0 if he is. Assume that the agent's reservation utility is zero and that wages are non-negative. Suppose the firm offers a contract to the agent, which the agent may accept or reject. A contract is a profile of wages (w_L, w_H) to be paid to the agent after revenue is realized. If the agent accepts, he then chooses his effort.

(a) Suppose the firm wants to induce low effort. What do they set w_H to? Find the minimum w_L such that the agent is willing to comply with the firm's wish. Provide the intuition for your findings. [5 pts]

(b) Now suppose $q = 5/7$ and $u(w) = \sqrt{w}$. Find necessary and sufficient conditions on x_L and x_H such that determines whether the firm would wish to induce low effort, and interpret. (*Hint: If $w_L = w_H = 0$, what would the agent choose to do?*) [5 pts]

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Problem 3. Moral Hazard with Two Chefs [optional].

Liyan is opening a Russian restaurant on Piazza Fiume. She needs to hire *one* chef and there are two candidates: Georgii and Alexey. The probability that the restaurant is successful depends on which of them is hired and whether the hired chef puts in effort or not. Those probabilities are given by the following matrix:

	Effort	No Effort
Georgii	p	q
Alexey	$p - \varepsilon$	r

where $0 < r < q < p < 1$ and $\varepsilon > 0$. *Note that Georgii is better than Alexey whether effort is exerted or not.* Other than that, they are completely identical. Liyan is risk-neutral. On the other hand, Georgii and Alexey are risk-averse: their payoffs are $v(w) - c$ with effort and $v(w)$ without effort, where w is the wage and $v(\cdot)$ is a concave function with $v' > 0, v'' < 0, v(-\infty) = -\infty, v(+\infty) = +\infty$, and $c > 0$. The wage can be positive or negative (interpreted as a fine). The reservation utilities for both Georgii and Alexey are normalized to zero.

Liyan cannot observe whether the hired chef puts in effort or not because she cannot judge the quality of Russian food, but she *does* observe if the restaurant is successful or not. Therefore, the wage payment can depend on restaurant's success. Let w_S and w_F be the wage payments when the restaurant is successful and when it is not, respectively. Furthermore, the restaurant yields gross profits of S and F when it is successful and when it is a failure, respectively. Suppose that $S - F$ is large enough so that it is always optimal for Liyan to induce effort from the chef she hires.

(a) Let (w_S^*, w_F^*) be the optimal wage scheme for Georgii. Provide the equations that characterize (w_S^*, w_F^*) and briefly explain why it must satisfy those equations.

(b) Actually, Liyan is strictly better off hiring Alexey instead of Georgii when ε is small enough even though Georgii is a better chef. Explain why this is the case by illustrating how the optimal wage scheme for Georgii can be modified to attain a strictly higher net profit for Alexey. (*Hint: What happens if the optimal wage scheme for Georgii is offered to Alexey?*) Provide the intuition for this result.