1. Discussion of Problem Set #5

2. Issues with the VCG Mechanism

(*Final Exam 2020.*) In this problem, you will explore various issues with the VCG mechanism.

1. Suppose a VCG mechanism is applied to sell the objects in $\mathcal{O} = \{a; b\}$ to three buyers. A buyer can buy none, one, or both of the objects. The buyers' valuations are:

$$u_1(\varnothing) = 0, u_1(\{a\}) = 9, u_1(\{b\}) = 0, u_1(\{a, b\}) = 9$$
$$u_2(\varnothing) = 0, u_2(\{a\}) = 0, u_2(\{b\}) = 9, u_2(\{a, b\}) = 9$$
$$u_3(\varnothing) = 0, u_3(\{a\}) = 2, u_3(\{b\}) = 3, u_3(\{a, b\}) = 10$$

   Consider a version of the VCG payment rule: each buyer i pays m_i, which is the maximum welfare of the other buyers minus the realized welfare of the other buyers given the reported values.

   (a) Determine the assignment of objects to buyers and the payments, under truthful bidding. (4 pts.)

   (b) Discuss what is the issue with the VCG mechanism in this case. (1 pt.)

2. Consider the same setup but with different valuations:

$$u_1(\varnothing) = 0, u_1(\{a\}) = 3, u_1(\{b\}) = 0, u_1(\{a, b\}) = 3$$
$$u_2(\varnothing) = 0, u_2(\{a\}) = 0, u_2(\{b\}) = 4, u_2(\{a, b\}) = 4$$
$$u_3(\varnothing) = 0, u_3(\{a\}) = 2, u_3(\{b\}) = 3, u_3(\{a, b\}) = 10$$

   (a) Determine the assignment of objects to buyers and the payments, under truthful bidding. (1 pts.)

   (b) This example highlights another flaw of the VCG mechanism. State this flaw and discuss it. (4 pts.)

3. Consider a similar setup but with two buyers and the following valuations:

$$u_1(\varnothing) = 0, u_1(\{a\}) = 1, u_1(\{b\}) = 1, u_1(\{a, b\}) = 10$$
$$u_2(\varnothing) = 0, u_2(\{a\}) = 1, u_2(\{b\}) = 1, u_2(\{a, b\}) = 9$$

   (a) Determine the assignment of objects to buyers and the payments, under truthful bidding. (1 pts.)

   (b) Suppose buyer 2 thinks about entering the system as two different buyers, 2A and 2B. Can it be profitable fir her to do this? Discuss whether you think such splitting schemes are a flaw of the VCG mechanism. Can it be problematic in practice? (4 pts.)