Problem 1. Auctions with Reserve Prices as Mechanisms [4 pts].

Consider an N-buyer allocation problem with symmetric independent private values.

(a) What direct mechanism $(q^{\mathrm{I}}, m^{\mathrm{I}})$ is implemented in the symmetric equilibrium of a first-price auction with a reserve price of r? [0.5 pts.]

(b) What direct mechanism $(q^{\mathrm{II}}, m^{\mathrm{II}})$ is implemented in the symmetric equilibrium of a first-price auction with a reserve price of r? [0.5 pts.]

(c) What does the generalized revenue equivalence imply about the relation between m^{I} and m^{II}? [0.5 pts.]

(d) Suppose that agents' valuations being drawn from a uniform distribution on [0,1]. For this case, confirm your answer to part (c) by direct calculation. [2.5 pts.]

Problem 2. Incentive Compatible Mixed Auction [3 pts].

There is a single indivisible object with reservation price 0 who faces n potential buyers with valuations v_i, $i = 1, \ldots, n$. Buyer i's von Neumann Morgenstern utility function from obtaining the object at price p is $(v_i - p)$. The valuations v_i are independent random variables with differentiable distribution function F with positive density and with support $[0, \omega]$.

Suppose that the seller uses the following direct mechanism to allocate the object. Bidders report their valuations and with probability α the bidder with the highest reported valuation receives the object and with probability $(1 - \alpha)$ the bidder with the second highest reported valuation receives the object. For what values of α do transfers exist that make this mechanism incentive compatible?

Problem 3. Hiring with Project-Specific Human Capital [7 pts].

Ivan is revising his papers 1 and 2. To do so, he needs help from his RAs, Elena (his RA for paper 1) and Maxim (his RA for paper 2). Because of a regulation, he can hire only one of them and the hired one will work on both papers. RA i's cost of working is given by:

$$C_i = x_i + y + z$$

where y and z reflect how cumbersome working on the papers 1 and 2 is, respectively, and x_i represents how busy RA i is (i.e., the opportunity cost of working). x_E and x_M are uniformly distributed over $[0, 1]$. Both Elena and Maxim are risk neutral so their vNM utility is the wage minus the cost if hired, and 0 if not hired. Ivan is not allowed to pay or receive any money to/from a not-hired student.

Suppose that the values of y and z are commonly known. Each RA knows how busy s/he is but not the other (for instance, Elena knows the values of x_E, y, z but not x_M).

(a) Ivan is offering the following mechanism:

> Each RA reports the value of his x. The one who reports a lower value is hired, say i, at the wage of $x_j + y + z$, $(j \neq i)$.

Show that it is a (weakly) dominant strategy for each RA to report his x truthfully. (To make it simple, you can ignore the possibility that both RAs report exactly the same values). [1 pts.]

(b) Suppose that Ivan is not allowed to make the wage contingent on the report of the other RAs (but it can depend on the report of the hired one). If he wants to always hire the less busy one (i.e., the one whose x

is lower), how much wage, at least, does he pay when Elena is hired and her type is x_E in a Bayesian Nash equilibrium? *(Hint: Use the revenue equivalence theorem carefully.)* [3 pts.]

(c) Now assume that it is only the RA who knows how hard the work on the papers that s/he worked on (i.e., Elena knows x_E and y, Maxim knows x_M and z, and Ivan does not observe anything). Is it still possible for Ivan to always hire the less busy RA? Assume that x_E, x_M, y, and z are all independently and uniformly distributed over [0,1]. *(Hint: For two possible types of the same RA, check their incentives not to mimic each other. That is, write out two IC constraints and verify that they cannot hold simultaneously.)* [3 pts.]

Problem 4. Double Auction [6 pts].

A seller (agent 1) and a buyer (agent 2), each with private information, plan to use a so-called 'double auction' mechanism to allocate a good. The agent's valuations are independent and uniformly distributed on $v_i = [0, 1]$. The double auction works as follows: Each agent submits a bid $b_i \in [0, 1]$. If $b_1 > b_2$, then the seller keeps the good and no payments are made. If $b_1 \leq b_2$, then the buyer receives the good and pays the seller $(b_1 + b_2)/2$.

(a) Find a Bayesian Nash Equilibrium of this mechanism in which each player's strategy is linear in his type: that is, where $\beta_i(v_i) = \alpha_i + \gamma_i v_i$. [2 pts.]

(b) What direct mechanism (Q, M) does this BNE implement? Using your answer to part (i), explain why this direct mechanism is incentive compatible. (*Hint: Don't overthink the last part.*) [2 pts.]

(c) Now show directly that this direct mechanism is Bayesian incentive compatible. [2 pts.]