Resolving Adverse Selection: Screening and Signaling

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Motivation

- Adverse selection can be detrimental for markets
- How do markets cope with this issue?
- There are two standard mechanisms studied in the literature that help reduce adverse selection: *screening* and *signaling*
- ***Screening***: uninformed party sets up a contract structure in such a way that certain types self-select into choosing different options
  - Example: Insurance company creates two types of contracts — one with high deductible and low premium, and one with low deductible and high premium
- ***Signaling***: informed individuals develop a mechanism to signal their unobservable knowledge though observable actions
  - Example: Signaling on the job market, education

This Lecture

1. Labor Market Screening

2. Labor Market Signaling

Acknowledgment: For both models, I heavily borrow from Jeff Ely's graduate microeconomics class at Northwestern University.

Screening

Labor Market Screening

- One firm, one worker.
- The worker has type $\theta \in \{L, H\}$ and chooses effort $e \geq 0$.
- Output is determined by e but not by θ.
- $F(e)$ is output when employed by the firm.
  - $F(0) = 0$, $F'(e) > 0$, $F''(e) < 0$.
- $F(e) - w$ is firm's profit.

Labor Market Screening

- The worker maximizes $w - c(e|\theta)$
  - $c(0|\theta) = 0$
  - $c_e(\cdot|H) < c_e(\cdot|L)$
  - $c_{ee}(\cdot|\cdot) > 0$
- With some publicly known probability λ, the worker's type is L.
- Firm maximizes expected profits.
- We characterize PBE of the game.

Timing

1. Nature 'moves' first and the worker learns her type θ
2. The firm moves next, unaware of the worker's type, and commits to a wage schedule $w(e)$.
3. The worker accepts or rejects the firm's offer. If accepts, chooses e, earns wage $w(e)$. If rejects, gets 0 utility.

PBE: Recap from Francesco's Class

Definition

A Perfect Bayesian Equilibrium (PBE) of an extensive form game Γ_E is a strategy profile σ and a system of beliefs μ such that:

1. **Sequential Rationality**: σ is sequentially rational given μ (i.e., at every info set H_i, the strategy of each player i maximizes her payoff given the strategy of other players and her beliefs);
2. **On-Equilibrium-Path Beliefs**: For any information set reached with positive probability given strategy σ (i.e., for any H such that $Pr(H|\sigma) > 0$), beliefs must be formed according to Bayes' rule.
3. **Off-Equilibrium-Path Beliefs**: For any information set reached with null probability given strategy σ (i.e., for any H such that $Pr(H|\sigma) = 0$), beliefs $\mu(x)$ may be arbitrary but must be formed according to Bayes' rule whenever possible.

Problem Reformulation

- By sequential rationality, the worker with type θ will take $w(e)$ as given and choose effort e to maximize

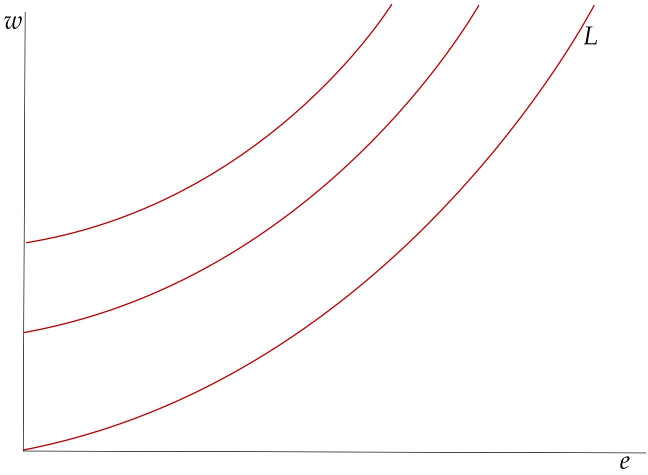
$$w(e) - c(e|\theta)$$

- Wlog, we can assume that ties are broken in the interest of the firm. (Hence, workers don't play mixed strategies.)
- Let $e(\theta)$ be the effort choice of type θ.
- The firm's expected profit is then:

$$\lambda[F(e(L)) - w(e(L))] + (1 - \lambda)[F(e(H)) - w(e(H))]$$

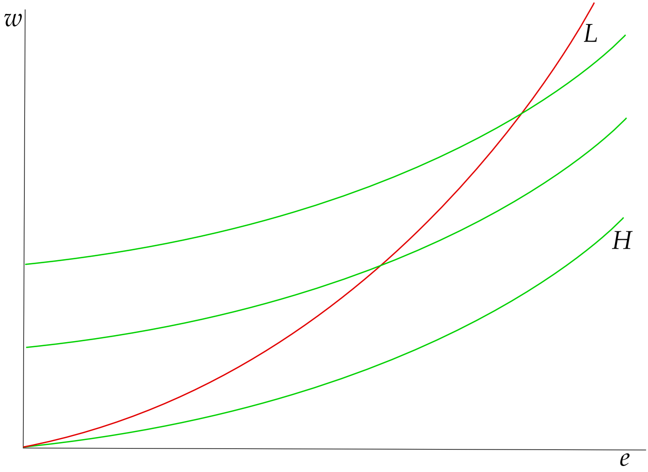
- Since the only aspect of the wage schedule that matters are pairs $\{e(H), w(e(H))\}$ and $\{e(L), w(e(L))\}$, we might as well solve the firm's profit maximization problem by choosing those directly.
- But there are constraints that come from the requirement that $e(H)$ and $e(L)$ are chosen optimally by the worker.

Incentive Compatibility



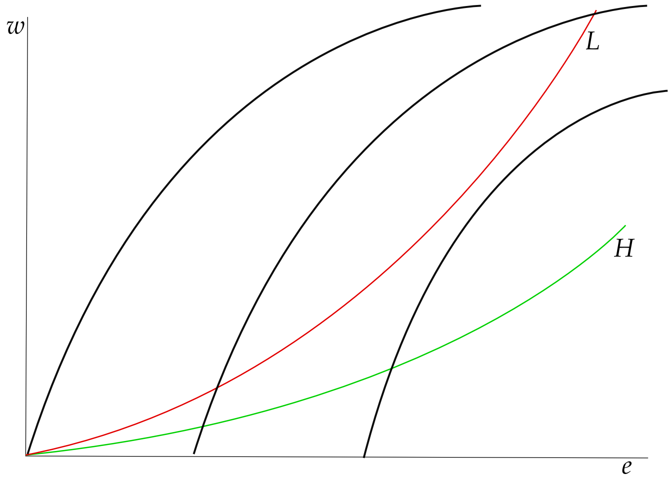
Indifference curves for type L.

Incentive Compatibility



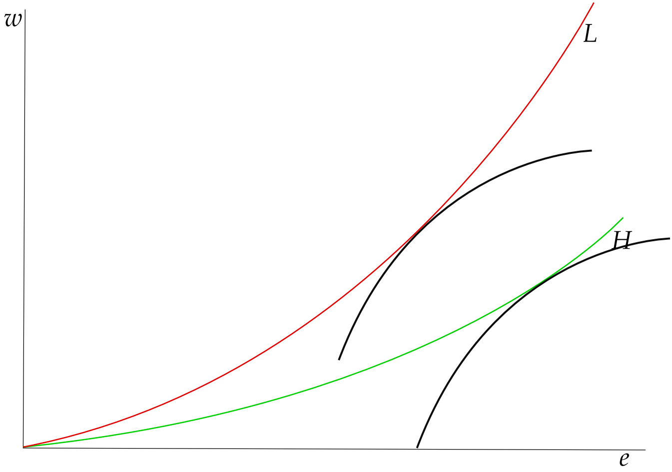
Indifference curves for type H.

Incentive Compatibility



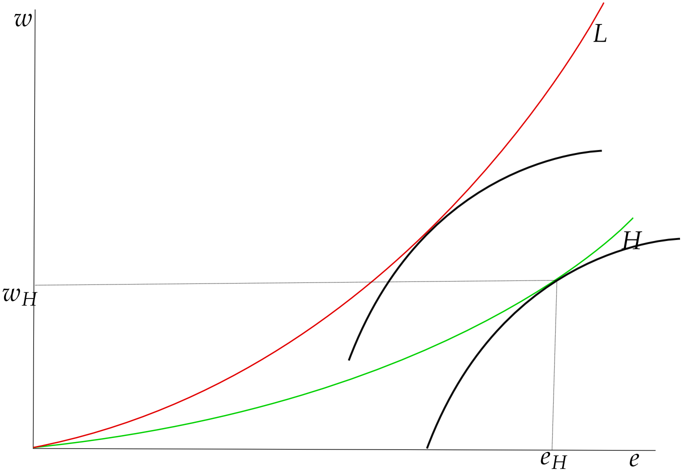
Iso-profit curves for the firm.

Incentive Compatibility



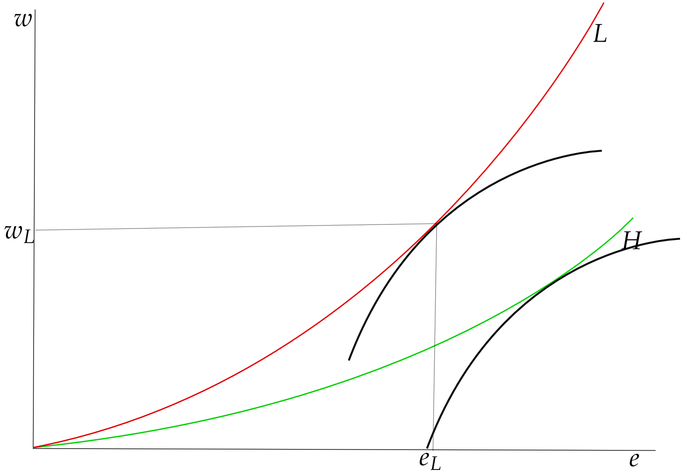
These are efficient points that give the two types zero utility.

Incentive Compatibility



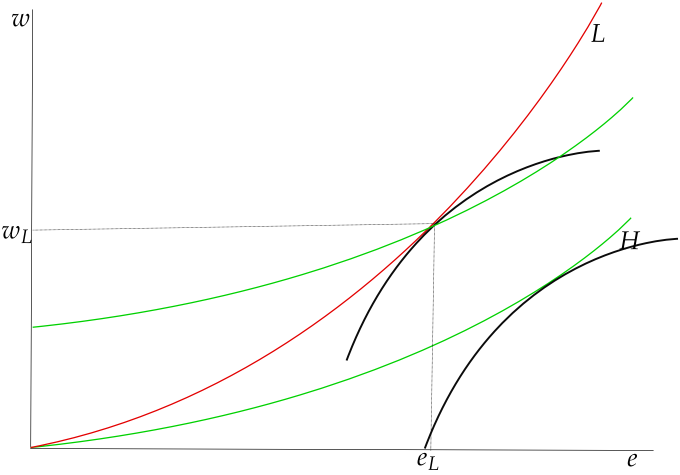
The contract (e_H, w_H).

Incentive Compatibility



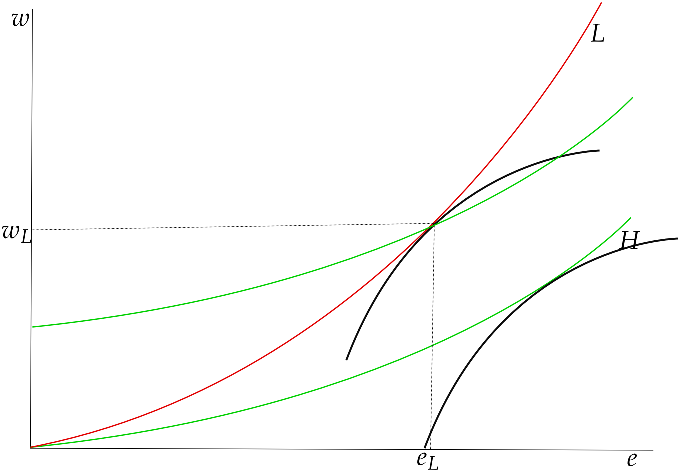
The contract (e_L, w_L).

Incentive Compatibility



This pair of contracts could not be chosen in an equilibrium.

Incentive Compatibility



$w_L - c(e_L|H) > w_H - c(e_H|H).$

Incentive Compatibility

We can characterize the set of contracts that could be chosen in an equilibrium

Lemma

We can (wlog) reformulate the problem into one in which the firm specifies a pair of contracts (e_L, w_L), (e_H, w_H) such that:

IC-L $w_L - c(e_L|L) \geq w_H - c(e_H|L)$

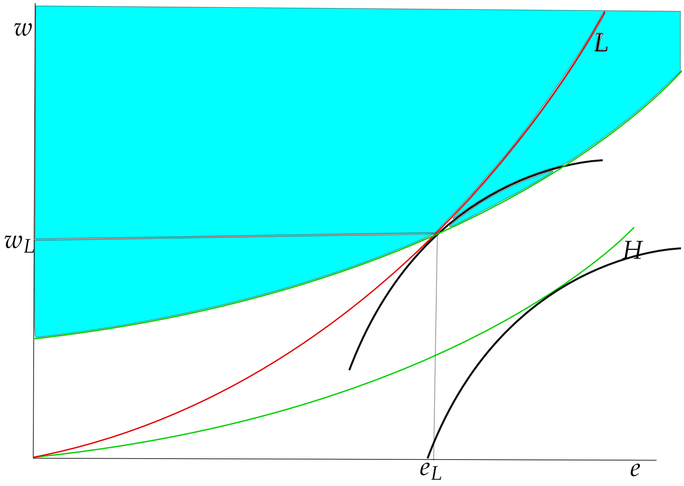
IC-H $w_H - c(e_H|H) \geq w_L - c(e_L|H)$

IR-L $w_L - c(e_L|L) \geq 0$

IR-H $w_H - c(e_H|H) \geq 0$

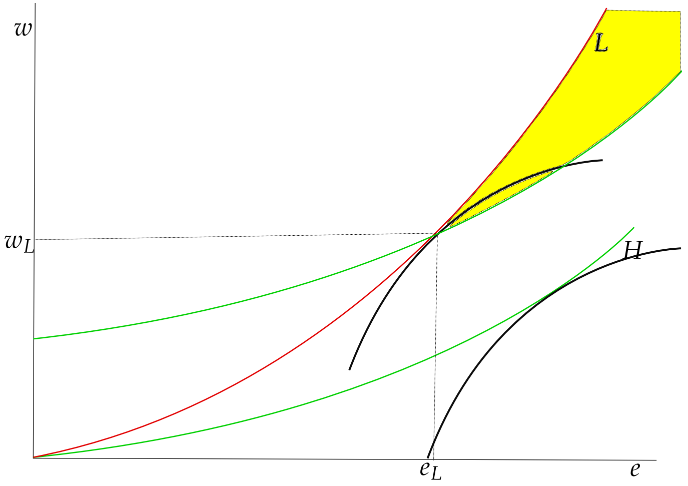
The first two are called incentive compatibility constraints, and the last two are called individual rationality constraints.

Illustrating Incentive Compatibility



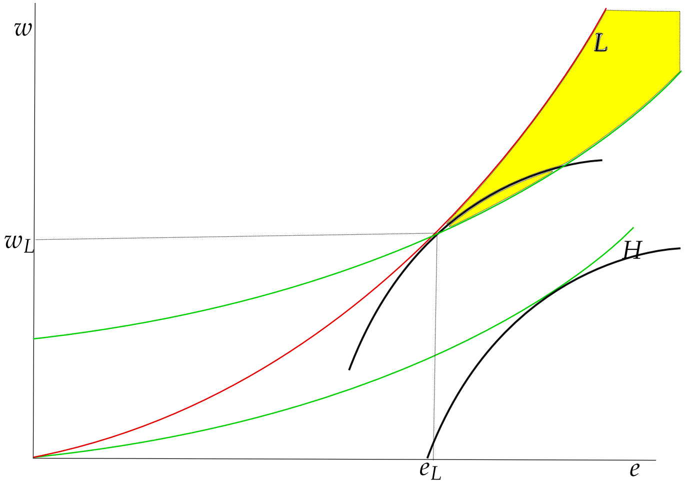
If we fix (e_L, w_l), then (e_H, w_H) must lie in the shaded region. (IC-H)

Illustrating Incentive Compatibility



But in order to also satisfy (IC-L) we must have (e_H, w_H) in the yellow shaded region.

Illustrating Incentive Compatibility



Note that these contracts also satisfy the IR constraints.

Constrained Profit Maximization

We can express the profit maximization problem of the firm as follows.

$$\begin{aligned}
\max_{\{w_L, w_H, e_L, e_H\}} & \mathbb{E}[F(e_\theta) - w_\theta] \\
\text{subject to} & \\
w_L - c(e_L|L) & \geq w_H - c(e_H|L) \\
w_H - c(e_H|H) & \geq w_L - c(e_L|H) \\
w_L - c(e_L|L) & \geq 0 \\
w_H - c(e_H|H) & \geq 0
\end{aligned}$$

Analysis of Constraints

Lemma. Single Crossing Condition.

If (e, w) is weakly preferred by type L to (e', w') and $e \geq e'$, then (e, w) is strictly preferred by type H to (e', w').

This follows from differences in cost functions between types.

Corollary. IR-H is Redundant.

If the IR-L constraints and the IC-H constraints are satisfied, then the IR-H constraint is also satisfied.

Corollary. Optimal Contract is Monotone.

If (e_L, w_L), (e_H, w_H) satisfy the IC constraints, then $e_H \geq e_L$.

Analysis of the Optimum

By the previous lemma, we can restrict attention to contracts that are monotone, i.e., $e_H \geq e_L$.

Lemma. No Slack.

A constrained profit-maximizing contract satisfies the IR-L and IC-H constraints with equality.

If not, then just lower wages and raise profits.

Lemma. IC-L is Redundant.

If the contract is monotone and the IC-H constraint is satisfied with equality, then the IC-L constraint is satisfied.

Updated Constrained Profit Maximization

Corollary

The problem can be simplified to:

$$\max_{\{w_L, w_H, e_L, e_H\}} \mathbb{E}[F(e_\theta) - w_\theta]$$

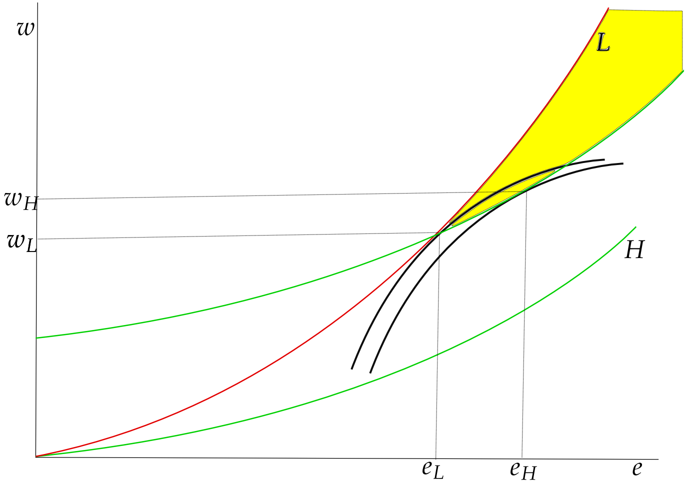
$$\text{subject to}$$

$$w_H - c(e_H|H) = w_L - c(e_L|H)$$

$$w_L - c(e_L|L) = 0$$

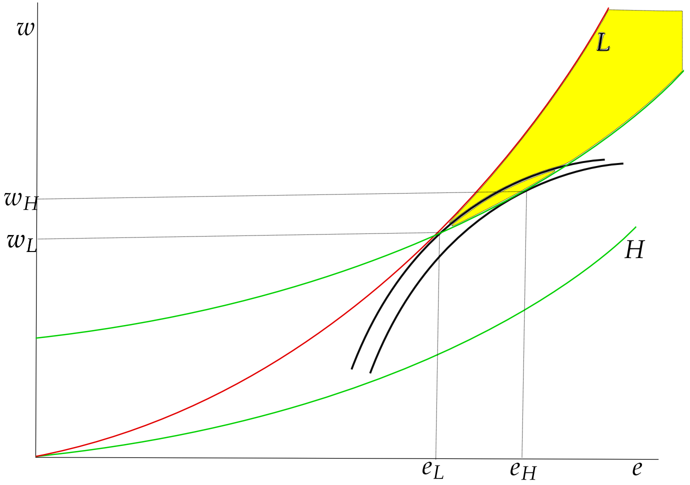
$$e_H \geq e_L$$

A Guess



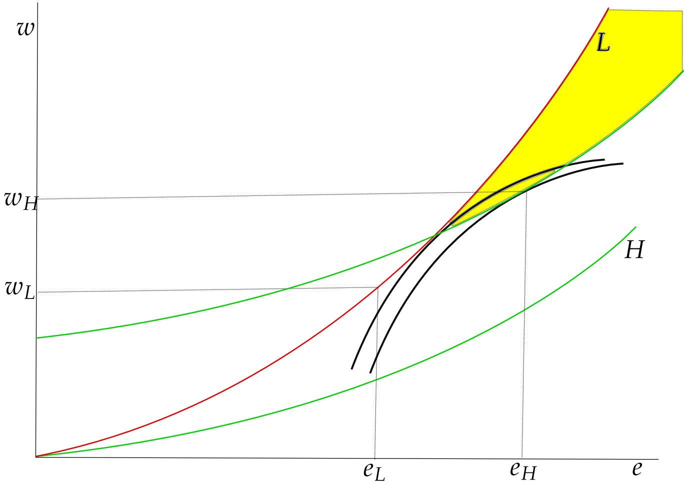
In this pair of contracts, we have retained the (e_L, w_L) contract from the first-best and then chose the (e_H, w_H) contract to maximize profits subject to the IC constraints.

A Guess



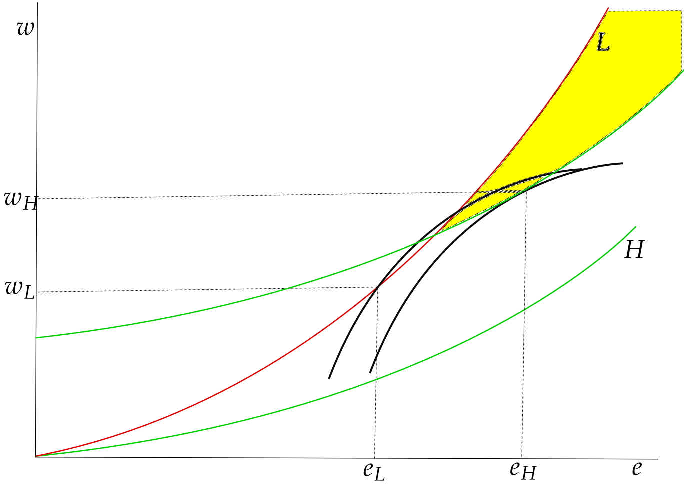
In fact, those conditions ensure that this pair of contracts is *constrained efficient* or *second-best.* But they are not profit-maximizing.

A Guess



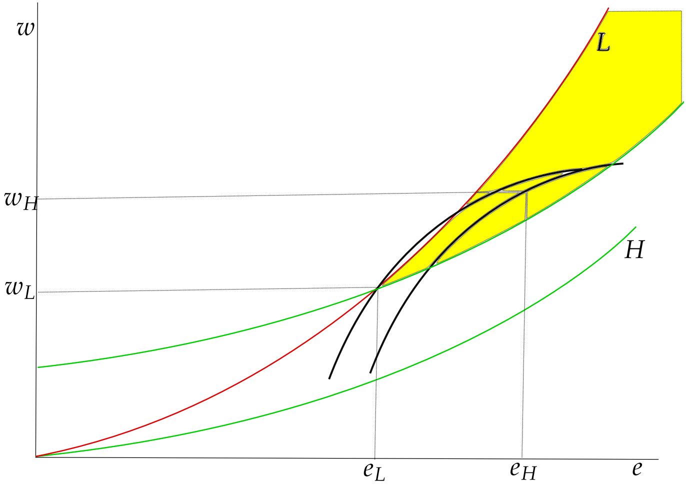
If we move the L contract down the L-indifference curve, we do not violate any constraints.

A Guess



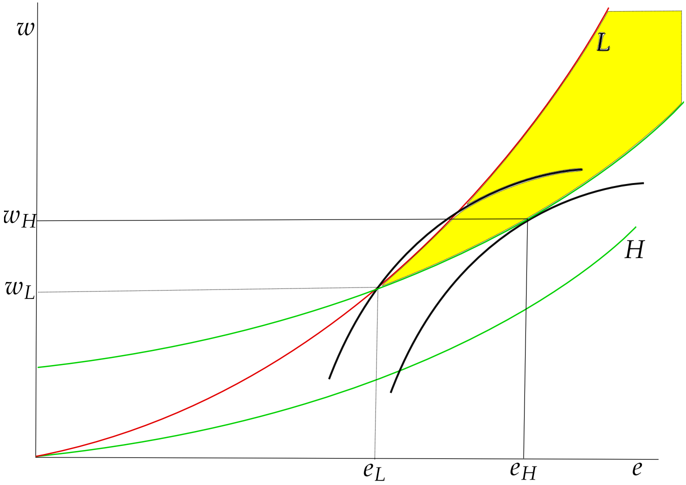
There is a loss in profit but that loss is second-order in the distance moved.

A Guess



This relaxes the IR-H constraint.

A Guess



And enables a first-order increase in profits from type H.

Properties of the General Solution

Proposition. No Pooling.

It will never be optimal to offer a pooling contract, i.e., $e_H = e_L$.

Can always do better by offering a higher w_H and e_H.

Proposition. Contract Efficiency.

In the constrained profit-maximizing contracts, the contract chosen by type H is Pareto efficient while the contract chosen by type L is Pareto dominated (effort is too low.)

Making the contract for L inefficient allows a firm to weaken as much as possible the incentive constraint of type H.

Exclusion of Low Types

Note: It may be optimal to set $e_L = 0$ and $w_L = 0$. In effect this would completely exclude type L in order to weaken as much as possible the incentive constraint of type H.
(Excluding type H would require also excluding type L and the result would be zero profits, so it is definitely not profit maximizing.)

Signaling