Finish Lecture 1.

Problem 1. The Market for Lemons: A General Set-Up.

The players are two buyers and a seller. A seller has a used car whose quality $\theta \in [0, 1]$ is his private information. θ represents his benefit from owning the car. It is drawn from cdf F that satisfies $F(\theta) > 0$ for all $\theta > 0$. We can view this as the distribution of car quality among sellers in the population, with the actual seller being drawn at random. There are two potential buyers. Each would obtain benefit $b(\theta) \geq \theta$ from owning a car of quality θ. Thus the buyers always value the car at least as much as the seller.

The game proceeds as follows: (i) the seller learns the quality of her car, (ii) the buyers simultaneously make price offers $p_i, p_j \geq 0$; (iii) the seller can accept one of these offers or reject both.

Buyer's utility is $b(\theta) - p$ for buying at price p; 0 for not buying. Seller's utility is $p - \theta$ for selling at price p; 0 for not selling.

1. Derive buyer's expected benefit conditional on obtaining the car.
2. Suppose $F \sim U[0, 1]$ and $b(\theta) = \beta\theta$ with $\beta \in (1, 2)$. Solve for a subgame perfect equilibrium.
3. Suppose $F \sim U[0, 1]$ and $b(\theta) = \alpha + \beta\theta$ with $\alpha \in (0, 1)$ and $\beta \in (1 - \alpha, 2 - 2\alpha)$. Solve for a subgame perfect equilibrium.

Problem 2. Adverse Selection in Insurance Markets.

Suppose there is a continuum of risk averse individuals endowed with wealth w. With a privately known probability, $\theta \sim U[0, 1]$, they incur a loss of size l. Their *Bernoulli* utility function (per MWG definitions) is $u(x) = \sqrt{x}$ and, therefore, their *von Neumann-Morgenstern* utility function is $\mathbb{E}u(w) = \theta\sqrt{w - l} + (1 - \theta)\sqrt{w}$. Perfectly competitive, risk-neutral firms offer an insurance contract at price p that pays l in case of a loss. Assume $w = 9$ and $l = 5$.

(a) Derive the demand function $p(\theta)$ on this market. [2 pts]

(b) Derive the marginal cost function $MC(\theta)$. Why is it *decreasing* with the number of insured? [1 pt]

(c) Plot the demand and the marginal cost functions on the same graph, with the share of insured individuals $(1 - \hat{\theta})$ on the x-axis, where $\hat{\theta}$ is the type of the marginal buyer. [1 pt]

(d) Derive and plot the average cost function $AC(\theta)$ on this market. (*Hint: Average cost in this environment is equal to the expected insurance pay-out to an average buyer, i.e.,* $AC(\tilde{\theta}) = E[3\theta|\theta > \tilde{\theta}]$.) [1 pt]

(e) Find the competitive equilibrium price. Why does the competitive equilibrium require $p = AC$? What happens if $p > AC$ or $p < AC$ in a competitive market? [2 pts]

(f) Confirm that the market unravels. Calculate the welfare loss from lack of insurance on this market. [1 pt]

(g) Suppose that the government introduces a subsidy to the insurance companies. Now, for every \$5 loss, they now only pay \$3 and the other \$2 is paid out by the government. What happens to the competitive equilibrium on this market? Is the under-insurance problem still present? [2 pts]

Problem 3. Adverse Selection in Corporate Finance.

Consider entrepreneurs selling their young companies to outside long-term investors. Companies can be good or bad. That is, the net present value of future cash flows to long-term investors (θ) can be high (θ_H) or low ($\theta_L < \theta_H$). Entrepreneurs privately know their company's θ. Long-term investors know that a company is good with probability $\mu < 1$. Entrepreneurs urgently needs cash (they have other great start-up ideas and need money). That is, for them, the present value of the company is $\delta\theta$, where $\delta \leq 1$. For simplicity, assume long-term investors are competitive and have deep pockets (ready to pay up to θ_H). Assume that entrepreneurs have to sell 100% of the company, so their action is either $s = 1$ (sell) or $s = 0$ (do not sell).

(a) Suppose there is a credible analyst or a credit rating agency that evaluates the company and publicly reveals θ to everyone. What happens? Which companies are sold and at what price? (*Hint: Note that we assumed that there are many more investors than companies.*) [0.5 pts]

(b) Suppose now that the companies' quality is unknown. Consider a so-called *pooling* equilibrium, i.e., in which both types of entrepreneurs manage to sell their companies at the same price. What is the price that investors pay in this situation? Is this price consistent with the equilibrium behavior of entrepreneurs? If not always, then under which conditions? [1 pt]

(c) Continue with the assumption that the companies' quality is unknown. Consider a so-called *separating* equilibrium, i.e., in which one type of entrepreneurs sells their companies while the other type does not.[1] Which type of entrepreneurs sells in such an equilibrium? At what price? Do all participants act optimally in this equilibrium—if not always, then under which conditions? [1 pt]

(d) Are the two equilibria in (b) and (c) mutually exclusive or do they coexist for some parameters? If they coexist, what determines the outcome? [0.5 pts]

Now assume that the quality of the firm θ is uniformly distributed on the interval $[0, 1]$ and focus on the pooling equilibrium.

(e) Derive the condition under which the pooling equilibrium does not exist—that is, when the market is frozen and there is no trade (the market is illiquid). [2 pts]

[1] In general, "separating" here means that the action (in this case, sell or not) reveals the agent's type, so it allows outsiders to separate good and bad types by observing agents' actions.