1. Discussion of Problem Set #4

2. Incentive Compatible Mixed Auction

There is a single indivisible object with reservation price 0 who faces n potential buyers with valuations v_i, $i = 1, \ldots, n$. Buyer i's von Neumann Morgenstern utility function from obtaining the object at price p is $(v_i - p)$. The valuations v_i are independent random variables with differentiable distribution function F with positive density and with support $[0, \omega]$.

Suppose that the seller uses the following direct mechanism to allocate the object. Bidders report their valuations and with probability α the bidder with the highest reported valuation receives the object and with probability $(1 - \alpha)$ the bidder with the second highest reported valuation receives the object. For what values of α do transfers exist that make this mechanism incentive compatible?

3. Auctions and Implementability

Alexey has one ticket for EURO 2020 (now, sadly, EURO 2021) but he found out he cannot make it, so he is giving it away to one of his three colleagues. Each colleague's valuation (type) for the ticket is independently and uniformly distributed over $[0, 1]$ and it is known only to themselves. The colleagues are risk-neutral.

Alexey would like to give the ticket to one who values the ticket less so that such a colleague has a chance to learn the excellence of watching football live.

(a) Show that the following allocation is *not implementable* in Bayesian Nash equilibria:

> *For any realization of valuations, the ticket is awarded to the highest type or the lowest type with equal probabilities.*

(b) Show that the following allocation *is* implementable in Bayesian Nash equilibria:

> *For any realization of valuations, the ticket is awarded to the highest type or the second highest type with equal probabilities.*

To implement the above allocation, what will be the expected payment of each type if type 0 never pays or receives money?

(c) Consider the following auction:

> *Each colleague submits a non-negative bid simultaneously and the winner is either the one making the highest bid or the second highest bid with equal probabilities. The winner pays his own bid.*

Suppose this auction has a symmetric equilibrium with a strictly increasing bidding function $b(\cdot)$ (actually, it does, but you do not have to prove it). Detive $b(\cdot)$ by using the result obtained in (b).