An Exact Model of the Neighbor Discovery Time for Schedule-Based Asynchronous Duty Cycling

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***Abstract*—In Wireless Sensor Networks, if two neighbor nodes operate under schedule-based asynchronous duty cycles, it is nontrivial to determine when they will discover each other, since the discovery opportunities may be distributed irregularly throughout the cycle, and also because nodes present random time offsets to each other. Moreover, the probability of message loss must be considered for useful estimations. This paper presents a method to find the expected neighbor discovery time when opportunities are distributed in arbitrary time slots, and subject to message loss. As a case study, the method was employed to solve a real problem in WSN design.**

***Index Terms*—Energy efficiency, wireless communications, wireless sensor networks.**

I. Introduction

Duty cycling the radio interface of battery-operated sensor motes is a necessity. These devices are severely energy constrained and, typically, need to operate for months or years with no possibility of battery replacement [1], [2].

Of all categories of duty cycling, schedule-based asynchronous duty cycling are probably the simpler to implement. They require no synchronization hardware or protocol, no complex computations and no tampering of MAC or physical layers of shelf motes or operating systems for WSNs. All that is required is that nodes follow a duty cycle schedule that presents the rotation closure property [3], i.e. schedules that ensure that two neighbor nodes will eventually have common active slots, despite their lack of synchronization, even if the slots are not border-aligned [3], [4].

However, when two neighbor nodes operate under schedule-based asynchronous duty cycles, the discovery opportunities may be scattered irregularly, according to the pattern formed by the intersection of active slots in both nodes. Typically, these individual schedules are already irregular enough. They follow block designs [4], complicated arrangements as the grid and the torus quorum [3], [5] or patterns based on prime numbers, as [6] and [7]. The *co-schedule* formed by the intersection of such schedules may be even more complex. In other words, if there was a discovery opportunity at every *n* slots, it would be easy to model the neighbor discovery time. However, these opportunities are not that well behaved, and such modelling is not trivial.

This is particularly true because the lack of synchronization implies random offsets between nodes' schedules. Moreover, this problem becomes more complicated, since a non-null probability of message loss is to be expected, particularly in wireless networks. In current literature, estimations of the neighbor discovery time (*NDT*) assume the latency to be the schedule cycle length [8] and/or disregard message loss probability [6]. This paper presents a method and an expression to obtain the *NDT* when discovery opportunities are distributed in arbitrary time slots, while accounting for the message delivery probability. As an illustration, the method was applied to solve a real issue where a schedule must be selected to meet a given set of requirements.

The rest of this paper is organized as follows. Section II states our problem of finding the expected *NDT* for two nodes operating under a schedule-based asynchronous duty cycle. Section III presents our exact model, that solves the problem. A practical application of the model is presented in Section IV. Some interesting implications that come directly from the proposed model are presented in Section V, while Section VI presents our final remarks and concludes the paper.

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II. Problem Statement

Say A and B are two nodes operating under the same duty cycling schedule formed of w time slots, of which k are active slots. Assume that A and B are not synchronized and that A is θ time slots ahead of B, where $\theta < w$ may be any number of slots, with equal probability. Assume also that nodes send a beacon every active slot, and that said schedule is such that ensures that both nodes will be active simultaneously during at least one time slot per cycle. Define t_0 as the moment when A and B are placed within communication range of each other. If $p > 0$ is the probability of A receiving a message from B, find the expected number of time slots that will elapse until A hears the first message from B, i.e the neighbor discovery time (*NDT*) in slots. We consider that p captures all the effects that could result in frame loss, including collisions.

III. Solution: An Exact Model for the *NDT*

We start building our model with the definitions of schedule, co-schedule and rotation of a schedule:

Definition 1: Schedule in w — A *schedule in* w, $S[w]$, is a strictly monotonically increasing finite sequence bounded in the interval $[0, w-1]$: $S[w] = \{s_0, s_1, ..., s_{q-1} | \forall i, j < q : s_i, s_j \in [0, w-1], i > j \Rightarrow s_i > s_j\}$.

Definition 2: Rotation of a schedule in w by θ — The *rotation of a schedule* $S[w] = \{s_0, s_1, ..., s_{q-1}\}$ by θ, represented as $\rho(S[w], \theta)$ is: $\rho(S[w], \theta) = \{(s_i + \theta) \; modulo \; w \mid s_i \in S[w]\}$.

Definition 3: Co-schedule — Given a schedule $S[w]$, its co-schedule subject to an offset θ is $\chi[w]^\theta = S[w] \cap \rho(S[w], \theta)$.

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The elements of a schedule $S[w]$ represent the active slots in each cycle (of w slots). If two nodes, *A* and *B*, are operating under the same schedule in w, and are offset by θ slots, it follows that $S[w]_B = \rho(S[w]_A, \theta)$. In this case, their co-schedule, $\chi[w]^\theta$, is a sequence of discovery opportunities also in the interval $[0, w-1]$. Therefore, the co-schedule of two nodes operating in schedules in w will also repeat at every w slots. As a consequence, we should consider that t_0 — the initial moment from which we start computing the *NDT* — may occur at any slot in the co-schedule. Moreover, as prior knowledge of θ is unrealistic under practical conditions, we should find the *NDT* for all values of θ, i. e. for all co-schedules $\chi[w]^\theta$, and average them.

Algorithm 1 summarizes the procedure for finding the expected NDT. We need to consider all possible co-schedules that two schedules may result in (as a function of their offset θ) and then, all possible starting positions of each of these co-schedules, with respect to t_0. Computing the co-schedules is equivalent to finding the common elements of two vectors, while finding the *NDT* for each co-schedule in relation to an external time reference (Line 5) can be achieved by the method we present next.

```
1 E[NDT] ← 0;
2 for θ ← 0 to w − 1 do
3 |   χ[w]^θ ← S[w] ∩ ρ(S[w], θ);
4 |   for t_0 ← 0 to w − 1 do
5 |   |   Compute E[NDT]^θ for χ[w]^θ;
6 |   end
7 |   E[NDT] ← E[NDT] + E[NDT]^θ;
8 end
9 E[NDT] ← E[NDT]/w
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Algorithm 1: Finding *E[NDT]* for a given schedule.

To find the *NDT* for a given $\chi[w]^\theta$, we define Φ_i as the average number of slots until the i_{th} discovery opportunity, and p as the probability of message delivery. Clearly,

$$E[NDT]^\theta = p\Phi_0 + p(1-p)\Phi_1 + p(1-p)^2\Phi_2 + \dots \tag{1}$$

Assuming there are q discovery opportunities in $\chi[w]^\theta$, this implies that $\Phi_{n+qc} = \Phi_n + wc$, where $c = 0, 1, \dots$ is the cycle number, and Equation 1 can be rewritten as:

$$\begin{aligned} E[NDT]^\theta &= p\Phi_0 + p(1-p)\Phi_1 + \dots + p(1-p)^{q-1}\Phi_{q-1} \\ &+ p(1-p)^q(\Phi_0 + w) + p(1-p)^{q+1}(\Phi_1 + w) \\ &+ \dots + p(1-p)^{2q-1}(\Phi_{q-1} + w) + \dots \end{aligned} \tag{2}$$

$$E[NDT]^\theta = p\sum_{c=0}^{\infty}\sum_{i=0}^{q-1}(\Phi_i + wc)(1-p)^{cq+i} \tag{3}$$

The problem now is reduced to finding these Phi-coefficients, that we now define more formally:

Definition 4: Phi-coefficients of a co-schedule — Given a co-schedule $\chi[w]^\theta$, its i_{th} Phi-coefficient, Φ_i, is the average slot for the i_{th} discovery opportunity in $\chi[w]^\theta$, considering all possible time offsets of $\chi[w]^\theta$ with respect to t_0.

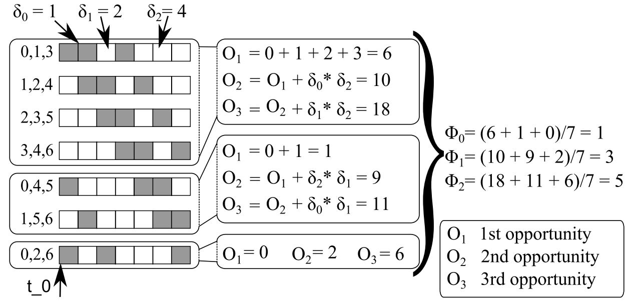

Fig. 1. An example of a co-schedule in 7, with the elements of its delta-set {1,2,4} and the Phi-coefficients {1,3,5}. Partial phi-coefficients (O_n) are computed for each of the three groups of rotations of the co-schedule in relation to t_0, and then summed up to find the Phi-coefficients.

We use Figure 1 to illustrate our method of obtaining the Phi-coefficients of a co-schedule. The figure shows all possible configurations of a co-schedule $\chi[7]^\theta$ = {2,3,5} with respect to t_0. We may see each of these possibilities as a rotation of the co-schedule in relation to t_0[1]. There are 7 (w) rotations that may be divided in three groups such that, each group starts with a rotation that has a discovery opportunity in t_0 and contains all other successive rotations of $\chi[7]^\theta$ with respect to t_0, until the next occurrence of an opportunity in the first slot.

Within each group, the three discovery opportunities follow a fixed relation, that is determined by the *delta-set* of the co-schedule — the difference between successive discovery opportunities, defined as follows:

Definition 5: Delta-set of a co-schedule — Given a co-schedule $\chi[w]^\theta = \{o_0, o_1, ..., o_{q-1}\}$ we define the *delta-set of* $\chi[w]^\theta$, as a sequence $\Delta\{\chi[w]^\theta\} = \{\delta_0, \delta_1, ..., \delta_{q-1} | \delta_i = o_i - o_{(i-1)\ modulo\ w}\}$.

Note that the delta-set of a co-schedule is independent of the initial time reference t_0, as it is given solely by the relative position of the successive discovery opportunities. Key to our method is the fact that all relations between discovery opportunities may be defined in terms of the delta-set of a co-schedule. In the topmost group in Figure 1, for instance, the second opportunity always occurs exactly one slot (δ_0) after the first opportunity. Likewise, the third opportunity happens two slots (δ_1) after the second. As for the first opportunity, it occurs in slots from zero to three, i. e. from 0 to $\delta_2 - 1$, successively.

In fact, similar relations may be observed in all three groups, the only difference being the order that each δ should be applied. The size of each group also comes from the delta-set (1, 2 and 4 rotations of $\chi[7]^\theta$). In short, all Phi-coefficients may be obtained entirely from the delta-set:

$$\Phi_0 = \frac{1}{2w}\sum_{i=0}^{q-1}\delta_i(\delta_i - 1) \quad \Phi_1 = \Phi_0 + \frac{1}{w}\sum_{i=0}^{q-1}\delta_i(\delta_{(i-1) modulo\ q})$$

$$\Phi_i = \Phi_0 + \frac{1}{w}\sum_{r=1}^{i}\sum_{j=0}^{q-1}\delta_i(\delta_{(j-r) modulo\ q})$$

Finally, Equation 4 gives the *NDT* of a co-schedule $\chi[w]^\theta$.

$$E[NDT]^\theta = p\sum_{c=0}^{\infty}\sum_{i=0}^{q-1}(\Phi_i + wc)(1-p)^{cq+i}, \tag{4}$$

[1]Although with different meanings, given their definitions, $\chi[w]$ and S[w] are both equivalent in mathematical terms, and we may expand the operation of rotation to include co-schedules as well.

where

$$\Phi_i = \begin{cases} \frac{1}{2w}\sum_{j=0}^{q-1} \delta_j(\delta_j - 1), & \text{if } i = 0 \\ \Phi_{i-1} + \frac{1}{w}\sum_{j=0}^{k-1} \delta_j \delta_{(j-i)\ modulo\ k}, & \text{otherwise.} \end{cases}$$

A more intuitive form for Equation 4 may be found by solving the infinite summations:

$$E[NDT]^\theta = \frac{p}{w}\sum_{i=0}^{q-1}(1-p)^i \left\{ \Phi_i \sum_{c=0}^{\infty}(1-p)^{cq} + w^2 \sum_{c=0}^{\infty} c(1-p)^{cq} \right\}$$

$$E[NDT]^\theta = \frac{p}{w}\sum_{i=0}^{q-1}(1-p)^i \left\{ \Phi_i \frac{1}{1-(1-p)^q} + w^2 \frac{(1-p)^q}{[1-(1-p)^q]^2} \right\}$$

$$E[NDT]^\theta = \left[\frac{1}{1-(1-p)^q} - 1\right] \cdot w + \sum_{i=0}^{q-1} \frac{p(1-p)^i}{1-(1-p)^q}\frac{\Phi_i}{w} \tag{5}$$

Equation 5, which is equivalent to Equation 4, is formed by two terms with clear meanings. The first term is the expected number of cycles (given the success probability for each cycle $[1-(1-p)^q]$) times the size of a cycle (w). The second term is the average of the expected distances to each of the q discovery opportunities weighted by their respective success probabilities given that the encounter will happen within a given cycle $\frac{p(1-p)^i}{1-(1-p)^q}$.

IV. Practical Application

In this section, we exemplify the adoption of our model by using it to solve a real problem. Suppose we need to deploy a WSN that should run for a given period of time and, in order to achieve this network longevity, nodes must operate in a duty cycle of 2% or less. Say these nodes will run an application that demands the average neighbor discovery time to be less than 4 seconds. Suppose also that preliminary measurements and simulations determined that, given the feasible network density and corresponding distances between the nodes, the probability of message reception will fall within the range $p \in [0.7, 1.0]$. Assume that, as in [4], projective planes are selected as a schedule mechanism and that the clock resolution of the motes in hand results in slots of 1 ms. Our problem is finding the most power efficient projective plane schedule that fulfils such requirements.

We start by using our model to obtain an NDT expression for projective planes. An example of a schedule based on the projective plane $\mathtt{P}\{7,3\}$ is provided in Figure 2. It can be described by two arguments. The first, v, is the number of elements or, in terms of a schedule, the number of slots in a cycle. The second argument, k, informs the number of active slots in the cycle. Two nodes operating under the same projective plane will have overlapping active time for at least the duration of one slot each cycle [9], and schedules based in projective planes are optimal (given a cycle length, they result in the lowest duty cycle [9]).

There are two possible classes of co-schedules for a projective plane. For an offset different than zero, there will be a single opportunity of discovery per cycle. On the other hand, if the offset is zero, there will be k discovery opportunities per cycle. In a cycle of v slots, there are v possible offsets and, therefore, assuming offsets are equally probable, the case with the zero offset happens with probability $1/v$, resulting in the formula:

Fig. 2. A $\mathtt{P}\{7,3\}$, its rotations and an example of co-schedule, $S[7]_A = \{1,2,4\}$, $S[7]_B = \{0,4,5\}$, $\theta = 3$ and $\chi[7]^3 = 4$.

TABLE I
Projective Planes With Duty Cycle of Less Than 2%

proj. plane	DC	proj. plane	DC	proj. plane	DC
$\mathtt{P}\{2863,54\}$	1.89%	$\mathtt{P}\{4557,68\}$	1.49%	$\mathtt{P}\{6643,82\}$	1.23%
$\mathtt{P}\{3541,60\}$	1.69%	$\mathtt{P}\{5113,72\}$	1.41%	$\mathtt{P}\{6973,84\}$	1.20%
$\mathtt{P}\{3783,62\}$	1.64%	$\mathtt{P}\{5403,74\}$	1.37%	$\mathtt{P}\{8011,90\}$	1.12%
$\mathtt{P}\{4161,65\}$	1.56%	$\mathtt{P}\{6321,80\}$	1.27%	$\mathtt{P}\{9507,98\}$	1.03%

$$E[NDT] = \frac{1}{v}E[NDT_{offset=0}] + \frac{v-1}{v}E[NDT_{offset\neq 0}]$$

The co-schedule for the offset zero is the schedule itself (which, incidentally, is the co-schedule used in Figure 1), therefore:

$$E[NDT_{offset=0}] = \left[\frac{1}{1-(1-p)^k} - 1\right] \cdot v + \sum_{i=0}^{k-1} \frac{p(1-p)^i}{1-(1-p)^k}\frac{\Phi_i}{v}$$

The co-schedule for all other offsets is given by a single opportunity of discovery per cycle, that opportunity being uniformly distributed across the cycle. Therefore:

$$E[NDT_{offset\neq 0}] = \left(\frac{1}{p} - 1\right)v + \frac{v-1}{2}$$

$$\begin{aligned} E[NDT] &= \frac{1}{v}\left\{\left[\frac{1}{1-(1-p)^k} - 1\right] \cdot v + \sum_{i=0}^{k-1} \frac{p(1-p)^i}{1-(1-p)^k}\frac{\Phi_i}{v}\right\} \\ &+ \frac{v-1}{v}\left[\left(\frac{1}{p} - 1\right)v + \frac{v-1}{2}\right] \\ E[NDT] &= \frac{p}{v^2}\sum_{i=0}^{k-1}(1-p)^i\Phi_i + \frac{(v-1)(1-p)}{p} + \frac{(v-1)^2}{2v} \\ &+ \frac{1}{1-(1-p)^k} - 1 \end{aligned}$$

Table I lists all 12 projective planes that present a duty cycle of 2% or less. Figure 3 shows the *NDT* as a function of delivery probability for these projective planes, as determined by our model. Projective planes $\mathtt{P}\{9507,98\}$ and $\mathtt{P}\{8011,90\}$ were not included since they would require $p > 1$ to achieve the demanded *NDT*. The figure shows that $\mathtt{P}\{4161,65\}$, $\mathtt{P}\{3783,62\}$, $\mathtt{P}\{3541,60\}$ and $\mathtt{P}\{2863,54\}$, all adhere to the requirements — achieving an average *NDT* of less than 4 seconds for delivery probability of 0.7 (or better). The selected schedule should be $\mathtt{P}\{4161,65\}$, since it is the one with the lowest duty cycle among the four.

In order to confirm the optimality of this model, we implemented the above twelve schedules in the R statistical environment [10] and simulated neighbor discoveries between two nodes with random offsets. We performed 40,000 repetitions

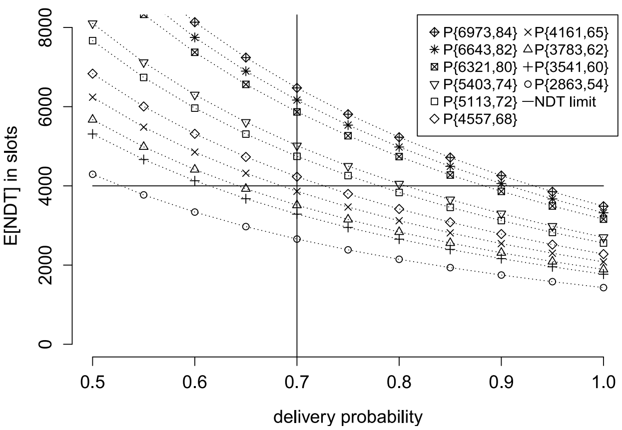

Fig. 3. The expected *NDT* as a function of delivery probability for the candidate schedules.

of the discovery for each value of delivery probability p (from 0.05 to 1.0 with increments of 0.05). The results show the predictions are always within 99% of the experimental results for all values of p, which is expected given the variability of the simulations.

V. Discussion

Some interesting conclusions on the design of asynchronous schedules can be drawn from the proposed model.

1) Two schedules of same length and duty cycle do not necessarily result in the same NDT. Henceforth, there is merit in designing schedules to minimize the NDT. However, minimizing the NDT without considering the delivery probability p (as in [4]) is not enough. A schedule that performs better when the link quality is high, may be surpassed when this quality drops.
2) The way active slots are distributed within the schedule is important, for it affects the way discovery opportunities are distributed within the co-schedule. Moreover, co-schedules are dependent on the offset, meaning that a given offset may result in more or less discovery opportunities per cycle, and this will affect the NDT.
3) Not only the number of discovery opportunities count. The interval between these opportunities, i.e. the way they occur within a cycle, also affects the NDT. If two schedules present the same duty cycle and result in the same number of discovery opportunities per cycle (averaged for all offsets), the one where these opportunities are more regularly distributed will produce the shorter NDT for good link quality.

In summary, schedules should be designed to maximize the number of discovery opportunities per cycle in the co-schedule, without sacrificing the duty cycle. Also, these opportunities should be as evenly distributed as possible. The intuition behind this assertive is simple — as the concentration of opportunities increase, there will be also an increase in the average time until a discovery opportunity occurs.

VI. Final Remarks

We presented an exact model for the *NDT* in schedule-based asynchronous duty cycling and then applied it to a real problem in the design of a WSN. The method consists of two parts: computation of all co-schedules in w — a straightforward algorithm with complexity O(w); and calculating a set of q coefficients, where q is the average number of discovery opportunities in the co-schedules (a step with complexity O(q^2)). Therefore, the whole procedure results in an algorithm of complexity O($w \times q^2$).

Although the model has been discussed under the premise of symmetric duty cycles (all nodes operating under the same schedule, with random offsets), it is in fact generic and might be used for asymmetric duty cycle as well. Exactly as in the case of symmetric schedules, it suffices to determine all the co-schedules and then apply the provided formula. We intend to study the effects of border-alignment in the NDT, although preliminary results, obtained from real implementation in sensor motes, suggest that the proposed model still holds.

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