UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Master Degree Project in Finance

Modeling CVA for Interest Rate Swaps in a CIR-framework

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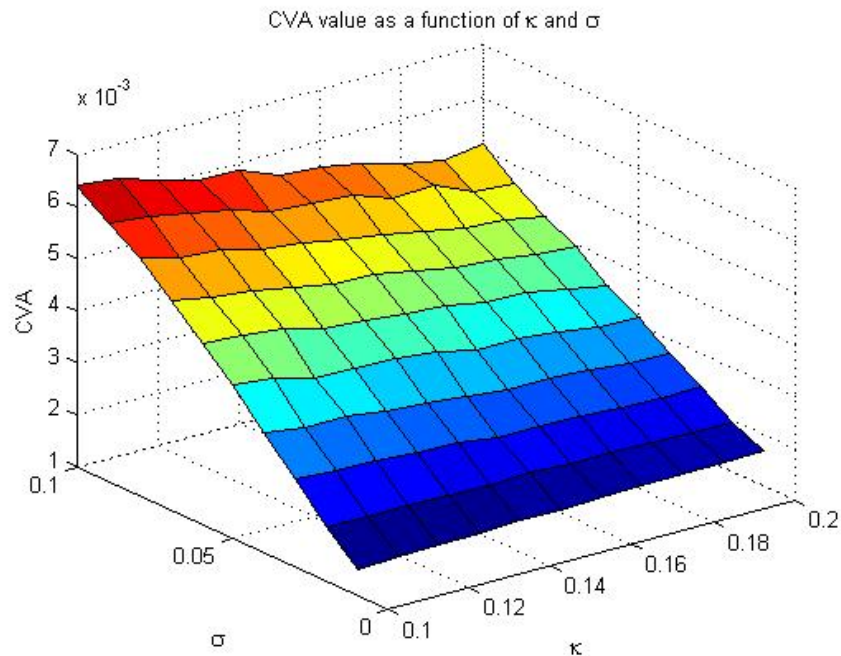# Modeling CVA for interest rate swaps in a CIR-framework

Master Thesis in Finance

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CVA value as a function of κ and σ

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Abstract

Knowing the true Counterparty Credit Risk (CCR) and accurately account for it, is vital in maintaining a stable financial system. The Basel committee noted that during the financial crisis of 2008-2009, about 70% of losses related to CCR actually came from volatility in the Credit Value Adjustment (CVA) instead of actual defaults. This thesis is examining the properties of CVA, how to measure CCR and why it is important to be able to accurately model it. The model risk for CVA is investigated for an interest rate swap contract in a CIR-framework; the sensitivity of the CVA with respect to the underlying parameters in the given setting is studied. The modeling of the CVA is shown to come with great uncertainties to many of the included terms. It is shown that the final CVA value is sensitive to changes in the underlying parameters describing the interest rate as well as to variations in the other terms included in the CVA model.

Content

Acronyms

BP – Basis Points

BCBS - Basel Committee on banking supervision

BIS - Bank of International Settlements

CCE - Counterparty Credit Exposure

CCR - Counterparty Credit Risk

CDS - Credit default swaps

CIR - Cox-Ingersoll-Ross

CR-CVA - Counterparty-risk Credit-value adjustment formula

CVA - Credit Value Adjustment

DRCC - Default Risk Capital Charge

EAD - Exposure at Default

EE - Expected Exposure

EPE - Expected Positive Exposure

FRA – Forward Rate Agreement

IRB - Internal Ratings Based

IRS – Interest Rate Swap

LGD - Loss Given Default

MtM - Mark-to-Market

NPV - Net Present Value

OTC - Over the Counter

PD - Probability of Default

PFE - Potential Future Exposure

VaR - Value at Risk

List of Figures

1. Introduction

In the first part of the 1900's century, the financial industry was increasingly regulated. However, deregulation in the beginning of the 1990's in US and Europe, lead to an uncontrolled growth of the financial industry worldwide. As an example, the banks of Iceland grew from a small industry to six times the GDP of the country in less than 10 years. During the turmoil of 2008, governments had to bailout several banks in Europe and in the US. Banks had taken unreasonable risks, mainly through possession of toxic assets that stressed their balance sheets (Calabresi, 2009).

A gradual decrease in the quality of the capital held by financial institutions, combined with inadequate liquidity buffers made the banking system vulnerable. This resulted in a reduced belief in the banking sector and the worries were transmitted to the entire financial system (Caruana, 2011). Hence it is of high importance to banks and the rest of the world that the financial sector can accurately measure their risk exposure. In response to the dramatic aftermath of the 2008-2009 financial crisis, the already existing Basel accords were further developed and its new design is now being implemented. In this thesis, we will study an important concept from the Basel III accord called Credit Value Adjustment (CVA), which can be explained briefly as the difference between the risk free value of an asset and the value where the risk of default is included, the true value.

In a climate where several European countries are experiencing financial difficulties and many banks are under pressure, knowing the true Counterparty Credit Risk (CCR) and accurately account for it is vital in maintaining a stable financial system. The Basel committee noted that during the financial crisis of 2008-2009, about 70% of losses related to CCR actually came from volatility in the CVA instead of actual defaults (Douglas, 2012). Hence, CCR is an important topic, therefore investigating CVA and CCR for bilateral derivatives is highly relevant. The field of CCR as a research area is growing, and will continue to grow. Furthermore, since this is a relatively new topic that is evolving and developing every day, the field of research is still open for new advancements.

In this thesis we will investigate the CVA, a measure of CCR under the Basel III framework. The aim is to model the CVA and investigate the features of the advanced CVA model provided in Basel III (Basel Committee on Banking Supervision, 2011). We will in this thesis focus on CVA for an interest rate swap, which in short is an agreement between two parties to exchange each other's interest rate cash flows, based on a notional amount from a floating to a fixed rate or vice versa. We assume that the interest rate follows a CIR-process which is independent of the default time of the counterparty. This is an assumption that has not been made in our referenced papers. The characteristics that define an interest rate which follows a CIR-process will be further explained in Section 4 of the thesis. Investigating how the CVA changes with the parameters in this setup will help us get an understanding of the model risk and also an insight in how sensitive the model is to changes in the parameters under the given assumptions. To do this, a CVA model is built by simulating an interest rate path following a CIR-process on which an interest rate swap is written. From the interest rate swap the so called Expected Exposure (EE), which can be explained as a weighted average of the exposure estimated for a future time, is

derived. Finally, by including the Expected Exposure together with the other terms of the CVA formula, a CVA value is calculated. The sensitivity in the model is analyzed with respect to the underlying parameters in the CIR-model and the Credit Default Swap (CDS) spread. A CDS, which will be further explained in Section 4, is a financial swap agreement, which for the buyer of it, works as an insurance against a default or a related credit event for a given entity. The software used for the model simulations is MATLAB and the theory will be built on literature and research from papers and books on the topic. The books and papers on which we have based the majority of our research are: Brigo, 2006; Brigo, 2008; Broadie, 2006 and Filipovic, 2009.

The rest of this thesis is organized as follows. In Section 2 we will introduce the Basel accords, explaining how they have developed over the years as a reaction to global financial events such as the financial crisis of 2008-2009. Furthermore, the content of Basel III, the latest update of the accords, will be discussed, by comparing it with Basel I and II. In Section 3, we will introduce credit risk where the focus will be put one special version of credit risk, namely counterparty credit risk. In Section 4 the models and instruments we used when calculating our CVA value will be explained. In Section 5 we give an explanation to what CVA is and its role in Basel III together with an introduction of it components. Following the previous part, Section 6 explains the individual steps in the process of calculating the expected exposure. In Section 7 the results of our study will be presented and discussed in Section 8.

2. The Basel Accords

In this section we will introduce the Basel accords, explaining how they have developed over the years from Basel I to the latest updates in Basel III.

The Basel Committee on Banking Supervision (BCBS) was founded in 1974 with the purpose of constructing guidelines and standards for banking regulations for authorities to implement in countries. It aims to create a convergence in financial regulations worldwide. What BCBS provides is guidelines and recommendations; hence they have no factual legal force.

2.1 The Basel accords, history leading up to Basel III

The 70's was a period full of financial stress, during which several liquidity related defaults occurred. A famous case is the default of Herstatt Bank in 1974, which followed as a result of flaws in capital requirements and because of a lack of standardized framework (Moles, et al., 2012). This resulted in that the Bank of International Settlements (BIS) constructed a foundation to the regulatory agreements, today referred to as the Basel Accords. In 1988 the BCBS, operating under BIS, agreed upon the first Basel Accord named Basel I, with the purpose of reducing banks' market and credit risk exposure (Bank for International, 2009).

Basel I's framework consisted of a set of minimum capital requirements. A minimum capital requirement is an amount of capital held that enables the bank to sufficiently have a buffer against losses. Basel I divided balance sheet assets into five different groups, depending on its credit risk the

asset was assigned five different risk weights from zero to hundred with a gap of twenty. A requirement of having a minimum capital ratio at 8 percent, calculated using regulatory total capital and the risk-weighted capital was imposed. Simultaneously, CDS were introduced, implying that the banks were able to hedge their lending risk and lower their credit risk exposure (Bank for International Settlements, 2011).

In 2004 the Basel II accords was published as an extension of Basel I, which included a more risk sensitive approach for reduction of credit risk, market risk and operational risk. The framework rests on three fundamentals, referred to as the three pillars:

- minimum capital requirements
- supervision review
- market discipline

The first pillar introduces capital requirements for the bank to reduce market, credit and operational risk. In order to measure the credit risk exposure, there were two recommendations of estimation, called the Internal Ratings-Based (IRB) and the Standardized Approach. The former, commonly used for major banks, allowed the banks to estimate their own internal rates for risk exposure. The latter is more risk sensitive and could be estimated in the same way as stated in Basel I, with a minimum capital ratio of 8 percent. Moreover, for measuring the operational risk, there were three different measures proposed: the Basic Indicator, Standardized and Internal Measurement Approach (Basel Committee on Banking Supervision, 2011).

The second pillar provides guidance for the bank's risk management and the method to deal with supervisory review and transparency. In the third pillar, the accord focuses on extending the market discipline through making it compulsory for banks to reveal and publish information that concerns the risk profile and the banks' capital adequacy.

Basel II regulated how to satisfy the requirement of the minimum capital ratio that is calculated from the regulated total capital and the risk-adjusted assets. This requirement has to be fulfilled in order to be considered as an adequately capitalized bank (Saunders, 2012).

$$Total\ riskbased\ capital = \frac{Total\ capital\ (Tier\ 1 + Tier\ 2)}{Risk\ adjusted\ assets} > 8\%$$

$$Tier\ 1\ capital = \frac{Core\ capital\ (Tier1)}{Risk\ adjusted\ assets} > 4\%$$

The bank's capital can as shown above be divided into categories of Tier I and Tier II where the total capital equals the sum of the Tiers less deductions. Tier I represent the core capital of the bank, consisting of the book value of common equity and perpetual preferred stock, which is a type of preferred stock with no maturity date; whereas Tier II is treated as the secondary capital resource. The

latter includes loan losses reserve and subordinated debt instruments. The risk-adjusted assets, which are the denominator of the capital ratio above, are compounded by risk-adjusted on-balance-sheet assets and risk-adjusted off-balance-sheet assets (Saunders, 2012).

2.2 Basel III

Basel II did not capture the risk exposure of the banks in a satisfying way. This has had the consequence that a new and improved comprehensive regulatory framework has been developed. Basel III was under development even before the second version was fully implemented. The financial crisis of 2008-2009 had too serious effects for no actions to be taken. Basel III is a framework that will be gradually implemented up until the 31st of December 2019, where after the minimum capital requirements are assumed to be completely met by the banks. Basel III will require increased quantitative and qualitative capital possessed by the banks, increased liquidity buffers and reduced unstable funding structures (Basel Committee on Banking Supervision, 2011).

2.3 Counterparty Credit Risk in Basel III

Basel III introduces a credit risk reform, which is taken into use at the writing moment. It refers to the Total Counterparty Credit Risk Capital Charge, which belongs to the risk adjusted assets and is described as the CVA-capital charge together with the Default Risk Capital Charge (DRCC).

The DRCC is constructed by the multiplication of the Exposure at Default (EAD), which is the total amount that an entity is exposed to at the time of default, with a risk weight. There are four different methods presented by the BIS to determine the EAD of Over-the-Counter (OTC) derivatives. Trades made OTC take place without the supervision of an exchange and directly between two entities.

1. Original Exposure Method
2. Current Exposure Method
3. Standardized Method
4. Internal Model Method

The methods differ in their risk sensitivity and using a less sensitive method generates larger capital requirement. Hence, the banks have incentive to use the most sensitive methods in the calculations. To calculate the risk weights BIS provides two methods: the IRB approach and the standardized approach. Their names entail their differences, the standardized approach is using rating from external sources and the IRB is based on internal credit ratings (Basel Committee on Banking Supervision, 2011).

The market risk capital charge for movements in the CVA caused by movements in the credit worthiness of counterparty is referred to as the CVA-capital charge. This part is a new addition to the Total CCR capital charge in Basel III. The Basel committee noted that during the latest financial crisis, about 70% of losses related to CCR actually came from volatility in the CVA instead of actual defaults. As a reaction to this notion BIS added the CVA- capital charge to the DRCC in Basel III (Douglas, 2012).

3. Introduction to Credit Risk

In this section the topic of credit risk will be introduced, followed by a targeted description of one specific version of credit risk, namely counterparty credit risk. Lastly we discuss how credit derivatives can be useful tools to reduce this risk.

3.1 Credit Risk

Credit risk can be defined as the risk that a borrower doesn't honor its payments.

Credit risk can usually be decomposed into the following risks (Schönbucher, 2003):

- Arrival risk – is the risk that a default will occur within a given time period.
- Timing risk – is the risk related to the precise time-point of the arrival risk's occurrence.
- Recovery risk – is the risk related to the size of the loss when a default takes place.
- Default dependency risk – This is the risk that several obligors simultaneously default within a specific time period. It could also be referred to as the correlation risk, which is a crucial factor to consider in a credit portfolio setting.

The credit risk or credit worthiness of a company or even a whole country is usually assessed and given a rating by a bureau or a rating agency such as Moody's and Standards & Poor (Hull, 2012). The credit risk rating is based on the probability of default (PD) of an entity and is categorized in different brackets ranging from AAA/Aaa (Standard & Poor's /Moody's) which is the highest rating followed by AA/Aa, A/A, BBB/Baa, BB/Ba and CCC/Caa. Each bracket is associated with a PD where a higher rating implies a lower PD (Standard & Poor's, 2009). However, the rating bureaus also divided each bracket into subcategories (such as Aa1, Aa2… or A+, A…) to decrease the coarseness of the scale of the credit rating. From the ratings, a risk premium is added to the interest rate of a loan or a bond that is issued by its entity. The challenge for credit agencies is to get a proper estimate of the PD, since the PD of an entity varies over time. What is interesting to mention is that for a bond with a high credit rating, the default probability tend to increase over time whereas the default probability tend to decrease over time for a bond with a relatively lower credit rating (Bodie, o.a., 2012). The reason behind this is that for poor rating bonds, the first couple of years maybe critical whereas it is possible that the financial health of the high rating bonds will decline with time.

3.2 Counterparty Credit Risk

The risk that one party after entering into a financial contract will default on it prior to its expiration is called counterparty credit risk (CCR). Hence CCR is the risk that the obligor will not be able to meet the demands required by the contract, such as fulfill payment duties. This risk is evident when trades are made Over-the-Counter (OTC), because then, unlike trades made via an exchange that are backed by a clearing house, it's hard to govern the financial status of the counterparty.

Note that CCR is closely related to other forms of credit risk. However, there are features that separate it, for example: the CCR is a bilateral risk, meaning that both counterparties can default.

The counterparty risk for a transaction is calculated by applying CVA, which first was specified within the Basel II accords (Bank For International Settlements, 2005) as well as in the IAS39 accounting standards (International Accounting Standards Board, 2009). It is defined as the difference between the risk free value of the portfolio and the value where the risk of default is included, the true value. However, up to 2008 many institutions neglected the CVA part in the Basel accords, reasoning that since their credit exposure is against big and successful companies who have a low risk of default, the credit risk can be neglected. The fact is, even in early 2008 almost 3 years after the Basel II was published, American banks where still only implementing the Basel I accords. Recently however, many banks have recognized that CCR can be substantial and thus cannot be omitted or ignored after events such as the bankruptcy of Lehman Brothers. Furthermore, BIS noted that during the financial crisis of 2008-2009, about 70% of losses related to CCR actually came from volatility in the Credit Value Adjustment (CVA) instead of actual defaults. Hence, it is crucial to include the counterparty risk when calculating the true value of a portfolio and CVA on the market value of CCR.

3.3 The usage of swaps to manage credit risk

Since we are modeling CVA for an interest rate swap and using CDS spreads as another term in the CVA calculation, we will therefore in this section give an introduction to the usage of swaps to manage credit risk.

A swap is a contractual agreement where two parties accept to exchange fixed payments against floating payments (Fusar, 2008). In another words, a swap traditionally is the exchange of one security with another between two entities to hedge certain risk such as for example interest rate risk or exchange rate risk. The swap market began 1981 in the US and the notional amount outstanding of swaps in the OTC derivative market was $415.2 trillion in 2006, more than 8.5 times the gross world product during that year according to BIS at the end of 2006. There are different types of swaps existing in the market, among them, the most common swaps are currency swaps, interest rate swaps, commodity swaps, equity swaps, and credit default swaps (Kozul, 2011).

4. Modeling Framework

In order to provide a full understanding of the CVA calculations, this section gives an introduction to the theoretical background for the models and financial instruments used when calculating the different terms in the CVA.

4.1 The Credit Default swap

A credit default swap is a financial agreement constructed to be an insurance against a default or a related credit event. It does so by transferring the credit exposure from one party to another. An illustration of this will follow below.

Assume that a company C with random default time τ, issued a bond. There is another company A who want to buy protection against credit losses due to a default in company C within T years for an amount

of N currency units. So company A turns to company B that promises A to cover the credit default loss by company C at the cost of a fee $\frac{S(T)N}{4}$ quarterly until time T unless the default occurs when $\tau < T$ (see Figure 4.1 and 4.2). The constant S(T) is called the T-year CDS-spread or CDS-premium and is quoted in basis points per annum. In the situation of default, the protection seller B pays the protection buyer A the nominal insured times the loss ratio of company C. The constant S(T) is determined so that the expected discounted cash flows between A and B are equal. Hence, the discounted expected cash flows between A and B are given by the so called default leg (B to A) and premium leg (A to B).

1. The discounted expected payment from B to A if company C default is called default leg, which is expressed as $NE\left[1_{\{\tau \leq T\}}D(\tau)(1 - \phi)\right]$.
2. The discounted expected payment from A to B is called premium leg, which is calculated by using $N\sum_{n=1}^{nT} E\left[D(t_n)\Delta_n 1_{\{\tau > t_n\}} + D(\tau)(\tau - t_{n-1})1_{\{t_{n-1} < \tau \leq t_n\}}\right]$.
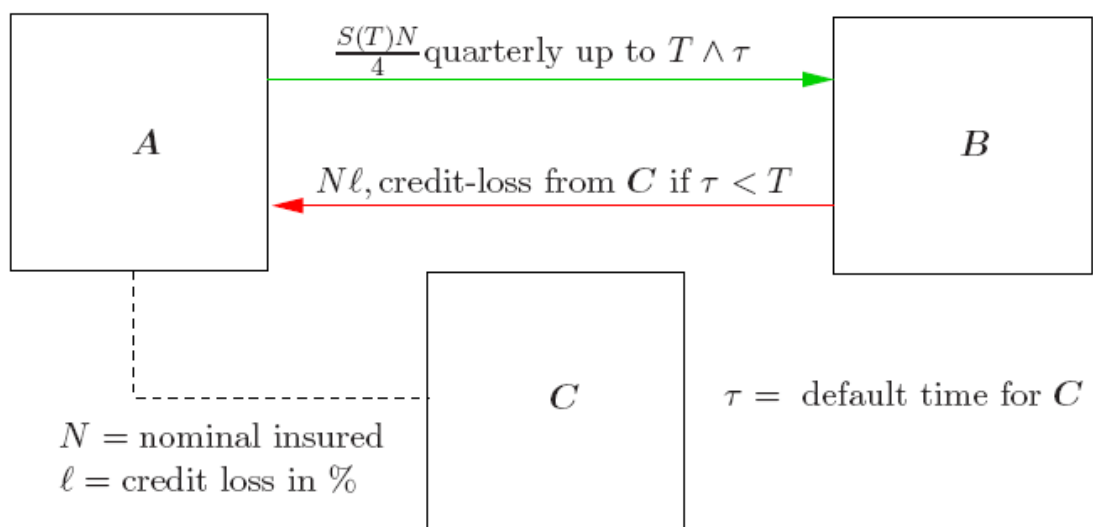

Figure 4.1: **Structure of a CDS contract (Herbertsson, 2012).**
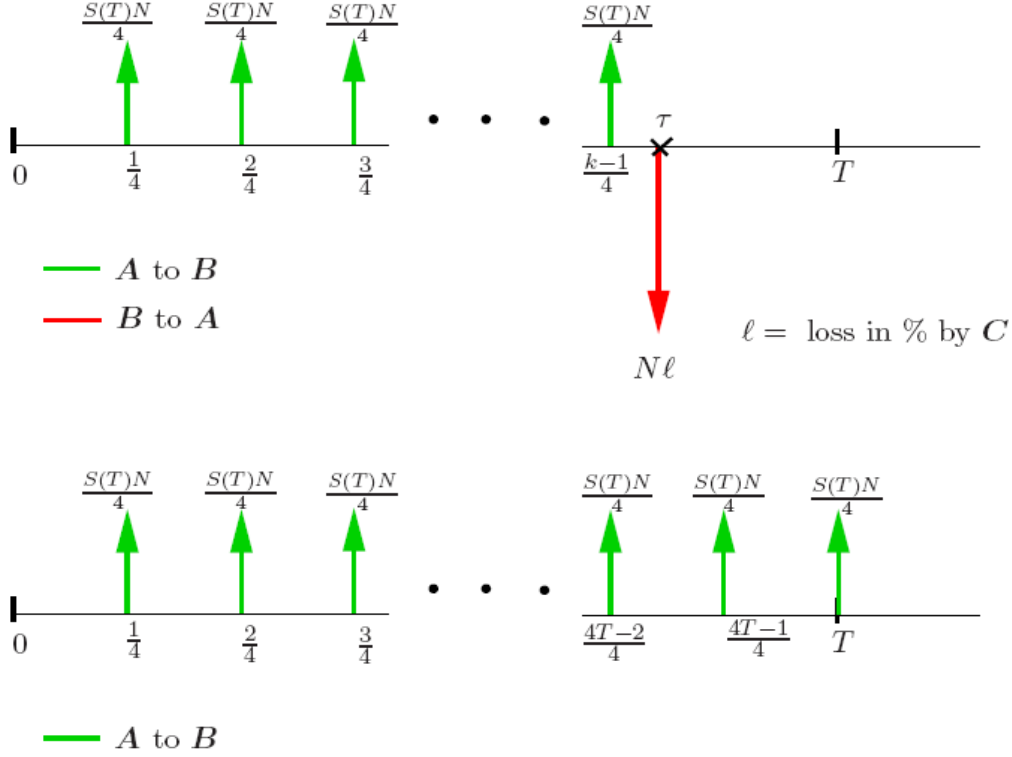
Figure 4.2: Different scenarios with no default before time T, and default before time T (Herbertsson, 2012).

By dividing the default leg with the premium leg, we get the $T-$year CDS spread $S(T)$ as

$$S(T) = \frac{E\left[1_{\{\tau \leq T\}} D(\tau)(1-\phi)\right]}{\sum_{n=1}^{nT} E\left[D(t_n)\Delta_n 1_{\{\tau > t_n\}} + D(\tau)(\tau - t_{n-1})1_{\{t_{n-1} < \tau \leq t_n\}}\right]}. \quad (4.1)$$

The CDS spread is expressed in basis points (bp) per annum and the expectation is under the risk neutral measure, ϕ is the recovery in the case of default and $D(t)$ is the discount factor, $D(t) = exp\left(-\int_0^t r(s)ds\right)$ where r_t is the short term risk free interest rate and a deterministic function of time, $r_t = r(t)$.

However, the equation above can be simplified when we assume that:

1. The interest rate is a deterministic function of time, $D(t) = exp\left(-\int_0^t r(s)ds\right)$, as well as independent of the default time τ. ($r(t)$ is the risk free interest rate as a function of time t.)
2. The credit loss $l = 1 - \phi$ is constant where ϕ is the recovery rate;

Then the Equation (4.1) can be rewritten as

$$S(T) = \frac{(1 - \phi) \int_0^T B_s dF(s)}{\sum_{n=1}^{nT} B_{tn} \Delta_n \left(1 - F(t_n) + \int_{t_{n-1}}^{tn} B_s(s - t_{n-1}) dF(s)\right)} \qquad (4.2)$$

where $F(t) = P[\tau \leq t]$ is the distribution function of the default time for the obligor and $B_t = E[D(t)]$.

Also when we assume that the interest rate r_t is a deterministic function of time, so that $r_t = r(t)$. Then τ and r_t are independent and $B_t = E[D(t)] = exp\left(-\int_0^t r(s)ds\right)$. Moreover, let ϕ be constant, and let $f_\tau(t)$ be the density of τ, i.e. $f_\tau(t) = \frac{dF(t)}{dt}$, then we have

$$E\left[1_{\{\tau \leq T\}} D(\tau)(1 - \phi)\right] = \int_0^\infty 1_{\{t \leq T\}} D(t)(1 - \phi) f_\tau(t) dt = (1 - \phi) \int_0^T D(t) f_\tau(t) dt.$$

Furthermore

$$E\left[D(t_n)\frac{1}{4} 1_{\{\tau > t_n\}}\right] = D(t_n)\frac{1}{4} E\left[1_{\{\tau > t_n\}}\right] = D(t_n)\frac{1}{4} P[\tau > t_n] = D(t_n)\frac{1}{4}(1 - F(t_n))$$

and

$$E\left[D(\tau)(\tau - t_{n-1}) 1_{\{t_{n-1} < \tau < t_n\}}\right] = \int_0^\infty 1_{\{t_{n-1} < t < t_n\}}(t - t_{n-1}) D(t) f_\tau(t) dt = \int_{t_{n-1}}^{t_n} D(t)(t - t_{n-1}) f_\tau(t) dt$$

is the accrued premium, which is a final payment done by A to B at the default time. The size of this premium is related to the time interval between the last payment and the default. Hence we can write the Equation (4.2) as

$$S(T) = \frac{(1 - \phi) \int_0^T D(t) f_\tau(t) dt}{\sum_{n=1}^{4T} \left(D(t_n)\frac{1}{4}(1 - F(t_n)) + \int_{t_{n-1}}^{t_n} D(s)(s - t_{n-1}) f_\tau(s) ds\right)}. \qquad (4.3)$$

The CDS-spread formula in Equation (4.3) can be simplified if we make two more assumptions:

1. Ignore the accrued premium term in the premium leg.
2. Assume that the loss is paid at time $t_n = \frac{n}{4}$, at the end of each quarter instead of at time τ when default occurs in the interval $\left[\frac{n-1}{4}, \frac{n}{4}\right]$, that is, when $\frac{n-1}{4} < \tau \leq \frac{n}{4}$.

Then Equation (4.2) can be rewritten into the following simplified expression (Herbertsson, 2012):

$$S(T) = \frac{(1 - \phi) \sum_{n=1}^{4T} D(t_n)(F(t_n) - F(t_{n-1}))}{\sum_{n=1}^{4T} D(t_n)(1 - F(t_n))\frac{1}{4}} \qquad (4.4)$$

where $F(t) = P[\tau \leq t]$.

Hence, from Equation (4.4) we see that in order to calculate the CDS spread $S(T)$, we need a model for the default time τ, more specific, we need and explicit expression for the default distribution $F(t) = P[\tau \leq t]$. Thus, in the next section, we will therefore discuss one such model for τ, namely a so called intensity based model.

4.2 Intensity based model

In this section we will study the so called intensity based model, which is needed when calculating the CDS spread.

To start with, assume that we have a probability measure P, and $\mathcal{F}_t$ represents the information available at time t. Moreover, we assume $(X_t)_{t>0}$ to be a d-dimensional stochastic process i.e. $X_t = (X_{t,1}, X_{t,2}, X_{t,3}, \cdots X_{t,d})$ where d is an integer and $X_{t,1}, X_{t,2}, X_{t,3}, \cdots X_{t,d}$ typically models different kind of economic or financial factors. Hence, in the function $\lambda: R^d \to [0, \infty]$, we have the stochastic process $\lambda_t(\omega) = \lambda(X_t(\omega))$. Furthermore, let E_1 be an exponential distributed random variable with parameter 1 that is independent of the process $(X_t)_{t>0}$. Then one can define the random variable τ as (Herbertsson, 2012):

$$\tau = \inf\left\{t \geq 0: \int_0^t \lambda(X_s)ds \geq E_1\right\}. \quad (4.4)$$

Hence, τ is the first time the increasing process $\int_0^t \lambda(X_s)ds$ reaches the random level E_1, see in the Figure 4.4 (Herbertsson, 2012)
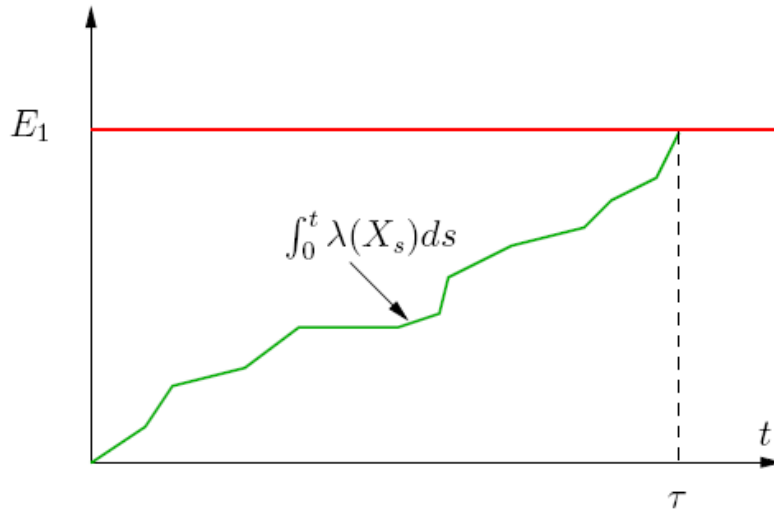

Figure 4.3: **The construction of τ via E_1 and the process X_s.**

From Equation (4.4), we can further derive (Herbertsson, 2012)

$$P[\tau_i > t] = E\left[\exp\left(-\int_0^t \lambda(X_s)ds\right)\right] \quad (4.5)$$

and

$$f_\tau(t) = E\left[\lambda(X_t)\exp\left(-\int_0^t \lambda(X_s)ds\right)\right] \quad (4.6)$$

where $f_\tau(t)$ is the density of the random variable τ.

Most importantly, if τ is a default time constructed as in Equation (4.4), we say that τ has the default intensity $\lambda(X_t)$ with respect to the information $(\mathcal{F}_t)_{t>0}$. The intuitive meaning of this is as follow.

Consider a single obligor with default time τ, and we assume as discussed before that λ_t is a stochastic process and $\lambda_t > 0$ for all t and $\mathcal{F}_t$ be the market information at time t. Then we want to have the intuitive relation between $\tau, \lambda_t, \mathcal{F}_t$

$$P[\tau \in [t, t + \Delta t)|\mathcal{F}_t] \approx \lambda_t \Delta t \quad if \ \tau > t \quad (4.7)$$

see also in Figure 4.4. Thus, the probability of having a default in the small time period $[t, t + \Delta t)$ conditional on the information $\mathcal{F}_t$, given that τ has not yet happened up to time t, is approximately equal to $\lambda_t \Delta t$, where Δt is "small" enough.

Hence, λ_t should be the arrival intensity of τ, given the information $\mathcal{F}_t$. To be more specific, λ_t is denoted as the default intensity of τ, with respect to the information $\mathcal{F}_t$ (Herbertsson, 2012)
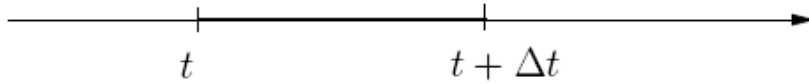
Figure 4.4: **Given the information, τ will arrive in $[t, t + \Delta t)$ with probability $\lambda_t \Delta t$.**

Finally, one can prove that the construction of τ in Equation (4.4) leads to the relationship in Equation (4.7), that is λ_t is the intensity of the random variable τ.

The Intensity $\lambda(X)$ can for example be considered in three different cases, namely:

- Intensity $\lambda(X)$ can be a deterministic constant;
- Intensity $\lambda(X_t)$ can be a deterministic function of time t, $\lambda(t)$;
- Intensity $\lambda(X_t)$ can be a stochastic process;

When the intensity $\lambda(X)$ is a deterministic constant, we have $\lambda(X_s)$ equal to λ, then the Equation (5.5) can be simplified to

$$P[\tau > t] = E\left[exp\left(-\int_0^t \lambda(X_s)ds\right)\right] = E[e^{-\lambda t}] = e^{-\lambda t}$$

that is, τ is exponentially distributed with parameter λ.

When $\lambda(X_t)$ is a deterministic function of time t, which means that λ is not constant, we have

$$P[\tau > t] = E\left[exp\left(-\int_0^t \lambda(s)ds\right)\right] = exp\left(-\int_0^t \lambda(s)ds\right) \quad (4.8)$$

and

$$f_\tau(t) = \lambda(t)exp\left(-\int_0^t \lambda(s)ds\right). \quad (4.9)$$

Another important case of deterministic default intensity $\lambda(t)$ is a so called piecewise constant default intensity. Let $\tilde{T}_1, \tilde{T}_2, \cdots, \tilde{T}_J$ be $\mathcal{J}$ different time points, then we can define a piecewise default intensity $\lambda(t)$ as

$$\lambda(t) = \begin{cases} \lambda_1 & if \quad 0 \le t < \tilde{T}_1 \\ \lambda_2 & if \quad \tilde{T}_1 \le t < \tilde{T}_2 \\ \quad \vdots \\ \lambda_J & if \quad \tilde{T}_{J-1} \le t < \tilde{T}_J \end{cases} \quad (4.10)$$

for some positive constants $\lambda_1, \lambda_2, \cdots, \lambda_J$. Figure 4.5 illustrate a piecewise constant default intensity $\lambda(t)$ in Equation (4.10). From Equation (4.8), we get that the default distribution $P[\tau > t]$ is given by

$$P[\tau > t] = 1 - exp\left(-\int_0^t \lambda(s)ds\right) \quad (4.11)$$
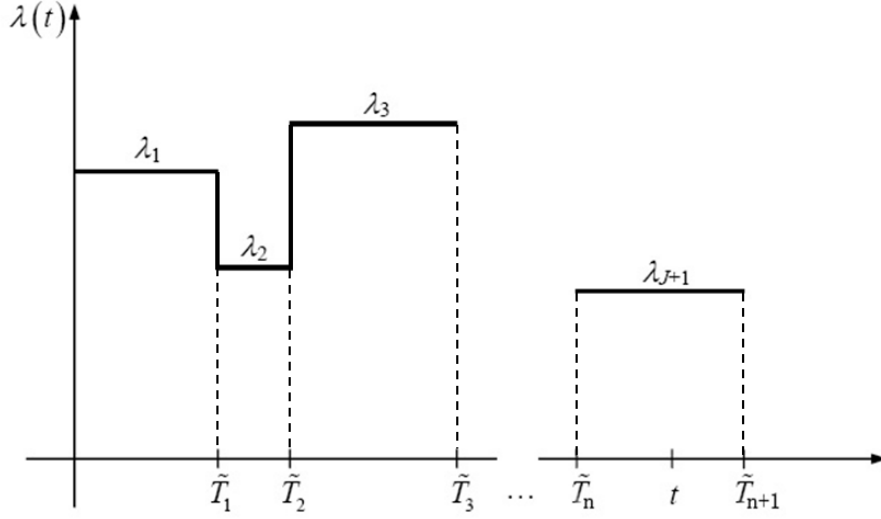
which together with Equation (4.10) yields that

$$
P[\tau > t] = \begin{cases}
1 - e^{-\lambda_1 t} & if & 0 \le t < \tilde{T}_1 \\
1 - e^{-\lambda_1 \tilde{T}_1 - (t-\tilde{T}_1)\lambda_2} & if & \tilde{T}_1 \le t < \tilde{T}_2 \\
& \vdots & \\
1 - e^{-\sum_{j=1}^{J-1} \lambda_j (\tilde{T}_j - \tilde{T}_{j-1}) - (t-\tilde{T}_{J-1})\lambda_J} & if & \tilde{T}_{J-1} \le t < \tilde{T}_J
\end{cases}
$$

where we define $\tilde{T}_0$ as $\tilde{T}_0 = 0$.

An example of stochastic default intensity λ_t is a Vasicek-process, given by

$$
d\lambda_t = \alpha(\mu - \lambda_t)dt + \sigma dW_t
$$

where W_t is a Brownian motion under risk-neutral measure P. Then we get that the survival probability up to time t is given by (Björk, 2009)

$$
P[\tau > t] = exp(A(t) - B(t)\lambda_0)
$$

for

$$
B(t) = \frac{1 - e^{-\alpha t}}{\alpha}
$$

$$
A(t) = (B(t) - t)\left(\mu - \frac{\sigma^2}{2\alpha^2}\right) - \frac{\alpha^2}{4\alpha}B(t)^2.
$$

Moreover, since $f_\tau(t) = -\frac{P[\tau > t]}{dt}$, we conclude that the density $f_\tau(t)$ is given by

20

$$f_\tau(t) = \left[e^{-\alpha t} \left(\frac{\sigma^2}{2\alpha^2} + \lambda_0 \right) - (e^{-\alpha t} - 1) \left(\mu - \frac{\sigma^2}{2\alpha^2} \right) \right] e^{A(t) - B(t)\lambda_0}.$$

4.3 Valuation of an Interest Rate Swap

In this section we will explain the features of an interest rate swap, first in a general way followed by a more theoretical manner.

An interest rate swap is an agreement between two parties, A and B, to exchange a fixed leg interest rate cash flow stream for a floating equivalent under a given period (see Figure 4.6). One party pays the fixed stream while the other pays the floating. The floating rate is based on an interest rate, for example the LIBOR. Depending on movements in the underlying interest rate the value of the swap contract changes with time. A common reason to enter into an interest rate swap contract is to manage risks related to interest rates (Asgharian, et al., 2007).

An interest rate swap contract specifies the following properties:

- Swap rate (the annual fixed interest rate cash flow stream)
- The interest rate on which the floating rate is taken from
- Maturity
- Payment frequency
- Notional amount

A simple illustration of the payments streams is displayed in Figure 4.6.
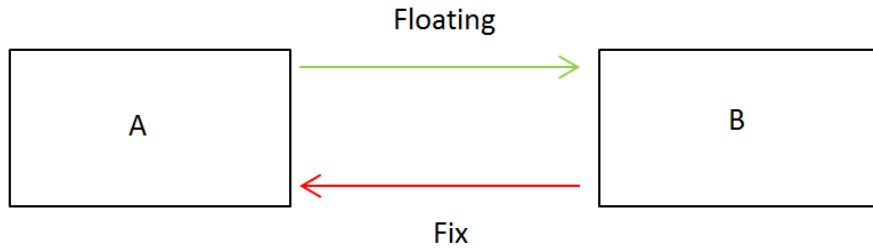
Figure 4.6: **Structure of an interest rate swap.**

In the following two subsections we are discussing the valuation of an interest rate swap. The interest rate swap can be viewed as a portfolio of generalized Forward Rate Agreements (FRA). Hence to get a proper understanding of the value of the Interest rate swap we start by introducing the FRA.

4.3.1 Forward Rate Agreement

In this section we will closely follow the setup and notation from (Brigo, et al., 2006).

Forward rates are interest rates that can be locked in today for an investment in a future time period. The forward rate can be defined through a prototypical Forward Rate Agreement (FRA). An FRA is an over-the-counter contract between two parties that determines the rate of interest to be paid or received on an obligation beginning at a future start date. It is characterized by three time instants:

- t - the time at which the rate is considered
- T_1 - the expiry date
- T_2 - the time of maturity

where $t \leq T_1 \leq T_2$.

The holder of the FRA receives an interest-rate payment at time T_2 for the period between T_1 and T_2. At the maturity T_2, a payment based on the fixed rate K_{FRA} is exchanged against a floating payment based on the spot rate $L(T_1, T_2)$. To put it in a simple way, the contract allows one to lock in the interest rate between time T_1 and time T_2 at a value of K_{FRA}, for a contract with simply compounded rates. This means that the expected cash flows must be discounted from T_2 to T_1. At time T_2 one receives $\delta(T_1, T_2)K_{FRA}N$ units of cash and simultaneously pays the amount $\delta(T_1, T_2)L(T_1, T_2)N$. Here N is the contract's nominal value and $\delta(T_1, T_2)$ denotes the year fraction for the contract period $[T_1, T_2]$. Thus, the value of the FRA, at time T_2 can, for the seller of the FRA (fixed rate receiver), be expressed as (Brigo, et al., 2006)

$$N \cdot \delta(T_1, T_2)(K_{FRA} - L(T_1, T_2)). \qquad (4.12)$$

Further, $L(T_1, T_2)$ can also be written as

$$L(T_1, T_2) = \frac{1 - P(T_1, T_2)}{\delta(T_1, T_2)P(T_1, T_2)} \qquad (4.13)$$

and this enables us to rewrite Equation (4.12) as

$$N \cdot \delta(T_1, T_2)\left[K_{FRA} - \frac{1 - P(T_1, T_2)}{\delta(T_1, T_2)P(T_1, T_2)}\right] = N\left[\delta(T_1, T_2)K_{FRA} - \frac{1}{P(T_1, T_2)} + 1\right]. \qquad (4.14)$$

To find the value of the FRA at time t, the cash flow exchanged in Equation (4.14) must be discounted back to time t, that is, we want to compute the quantity

$$N \cdot P(t, T_2)\left[\delta(T_1, T_2)K_{FRA} - \frac{1}{P(T_1, T_2)} + 1\right]. \qquad (4.15)$$

To do this we first note that according to classical, no arbitrage interest rate theory, the implied forward rate between time t and T_2 can be derived from two consecutive zero coupon bonds due to the equality (Filipovic, 2009)

$$P(t, T_2) = P(t, T_1)P(T_1, T_2). \qquad (4.16)$$

This gives that $P(t, T_1) = \frac{P(t, T_2)}{(T_1, T_2)}$. Hence, by using Equation (4.16) we then get that

$$N \cdot P(t, T_2) \left[\delta(T_1, T_2)K_{FRA} - \frac{1}{P(T_1, T_2)} + 1\right]$$

$$= N[P(t, T_2)\delta(T_1, T_2)K_{FRA} - P(t, T_1) + P(t, T_2)].$$

Thus, we have that the value of the above FRA contract at time t is given by

$$FRA(t, T_1, T_2, \delta(T_1, T_2), N, K_{FRA}) = N[P(t, T_2)\delta(T_1, T_2)K_{FRA} - P(t, T_1) + P(t, T_2)]. \quad (4.17)$$

Only one value of K_{FRA} gives the FRA a value of 0 at time t. By solving for this value of K_{FRA}, we get that the appropriate FRA rate to use in the contract is the simply compounded forward interest rate prevailing at time t for the expiry $T_1 > t$ at maturity $T_2 > T_1$, which is defined as

$$F_s(t; T_1, T_2) = \frac{P(t, T_1) - P(t, T_2)}{\delta(t, T_2)P(t, T_2)} = \frac{1}{\delta(T_1, T_2)}\left(\frac{P(t, T_1)}{P(t, T_2)} - 1\right). \quad (4.18)$$

Rewriting the value of Equation (4.17) in terms of the simply compounded forward interest rate in Equation (4.18) gives

$$FRA(t, T_1, T_2, \delta(T_1, T_2), N, K_{FRA}) = N \cdot P(t, T_2)\delta(T_1, T_2)\left(K_{FRA} - F_s(t; T_1, T_2)\right). \quad (4.19)$$

4.3.2 Interest rate swap

In this subsection we introduce the interest rate swap which is a generalization of the FRA. A prototypical payer interest rate swap exchanges cash flows between two indexed legs, starting from a future time. At every time point T_i, within one, by the swap contract specified period $T_{\alpha+1}, \dots T_\beta$, the fixed leg pays $N\delta_i K_{IRS}$, where K_{IRS} is a fixed interest rate, N is the nominal value and δ_i is the year fraction between T_{i-1} and T_i, $(\delta_i = T_i - T_{i-1})$. The floating leg pays $N\delta_i L(T_{i-1}, T_i)$ corresponding to the interest rate $L(T_{i-1}, T_i)$ resetting at the preceding instant T_{i-1} for the maturity given by T_i. For simplicity, in this case, we are considering that the fixed-rate payments and floating-rate payments occur at the same dates and with the same year fractions. Hence cash flows only take place at the date of the coupons $T_{\alpha+1}, T_{\alpha+2}, T_{\alpha+3} \dots T_\beta$. The individual who pays the fixed leg and receives the floating, B in Figure 4.6, is the payer while the individual on the opposite side is termed the receiver, A in Figure 4.6.

The discounted payoff at time $t < T_\alpha$ from B's side can be expressed as

$$\sum_{i=\alpha+1}^{\beta} D(t, T_i) N\delta_i (L(T_{i-1}, T_i) - K_{IRS})$$

and the discounted payoff at time $t < T_\alpha$ from A's side can be expressed as

$$\sum_{i=\alpha+1}^{\beta} D(t, T_i) N\delta_i \left(K_{IRS} - L(T_{i-1}, T_i)\right).$$

23

Seeing this last contract, from A's side, as a portfolio of FRAs, every individual FRA can be valued using the Formulas (4.17) and (4.19). This implies that the value of the interest rate swap $\Pi_{\text{reciever}}(t)$, is given by (see also in Brigo, et al., 2006)

$$\Pi_{\text{reciever}}(t) = \sum_{i=\alpha+1}^{\beta} \text{FRA}(t, T_{i-1}, T_i, \delta_i, N, K)$$

$$\Pi_{\text{reciever}}(t) = N \sum_{i=\alpha+1}^{\beta} \delta_i \cdot P(t, T_i)\big(K_{\text{IRS}} - F_s(t; T_{i-1}, T_i)\big).$$

So using Equation (4.18) in the above expression implies that

$$\Pi_{\text{reciever}}(t) = N \sum_{i=\alpha+1}^{\beta} \left(\delta_i \cdot K_{\text{IRS}} \cdot P(t, T_i) - \frac{\delta_i \cdot P(t, T_i)}{\delta(T_{i-1}, T_i)} \left(\frac{P(t, T_{i-1})}{P(t, T_i)} - 1 \right) \right)$$

which can be simplified into

$$\Pi_{\text{reciever}}(t) = N \sum_{i=\alpha+1}^{\beta} \left(\delta_i \cdot K_{\text{IRS}} \cdot P(t, T_i) - \big(P(t, T_{i-1}) - P(t, T_i) \big) \right).$$

The sum above can be separated into two sums

$$N \sum_{i=\alpha+1}^{\beta} \big(\delta_i \cdot K_{\text{IRS}} \cdot P(t, T_i) \big) + N \sum_{i=\alpha+1}^{\beta} \big(P(t, T_i) - P(t, T_{i-1}) \big).$$

The second sum of the two, can be simplified

$$N \sum_{i=\alpha+1}^{\beta} \big(P(t, T_i) - P(t, T_{i-1}) \big) = N \cdot P\big(t, T_\beta\big) - N \cdot P(t, T_\alpha).$$

This is because of when adding up the terms from $i = \alpha + 1$ to $i = \beta$ all terms in the sum cancel out apart from $N \cdot P\big(t, T_\beta\big)$ and $-N \cdot P(t, T_\alpha)$. Adding the sums back together yields Formula (4.20), the formula for the value of and interest rate swap in time $t \leq T_\alpha$, from the receiver's point of view.

$$\Pi_{\text{reciever}}(t) = -N \cdot P(t, T_\alpha) + N \cdot P\big(t, T_\beta\big) + N \sum_{i=\alpha+1}^{\beta} \delta_i \cdot K_{\text{IRS}} \cdot P(t, T_i). \quad (4.20)$$

If we want to look at the value of the swap from the side of the payer instead of the receiver, then the value is simply obtained by changing the sign of the cash flows

$$\Pi_{\text{reciever}}(t) = -\Pi_{\text{payer}}(t).$$

We can write the total value of the swap at $t \leq T_\alpha$ seen from the payer as (Filipovic, 2009)

$$\Pi_{\text{payer}}(t) = N\left(P(t, T_\alpha) - P(t, T_\beta) - K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)\right). \quad (4.21)$$

To clarify the interpretation of Formula (4.21), its features will now be further discussed. The interest rate swap can be decomposed into two legs, a floating and a fixed. These two legs can be seen as two fundamental prototypical contracts. The floating leg, $N \cdot P(t, T_\alpha)$ in Formula (4.21), can be thought of as a floating rate note, while the fixed leg, $-N\left(P(t, T_\beta) + K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)\right)$ in Formula (4.21), can be seen as a coupon bearing bond. This gives that the Interest rate swap could be seen as an agreement for exchanging the floating rate note (floating leg) for the coupon bearing bond (fixed leg).

A coupon bearing bond is a contract that guarantees a payment of a deterministic amount of cash at future times $T_{\alpha+1}, T_{\alpha+2}, T_{\alpha+3} \ldots T_\beta$. Generally, the cash flows are defined as $N \cdot \delta_i \cdot K_{\text{IRS}}$ for $i < \beta$ and $N \cdot \delta_\beta \cdot K_{\text{IRS}} + N$ for $i = \beta$. Here K_{IRS} is the fixed interest rate and N is the nominal bond value. Discounting the cash flows back to present time t from the payment times T_i, the value of the coupon bearing bond is (Brigo, et al., 2006)

$$N\left(P(t, T_\beta) + K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)\right).$$

where $N \cdot K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)$ are the future discounted cash flows from the coupon payments, and $N \cdot P(t, T_\beta)$ is the discounted reimbursement of the notional value of the bond. $N\left(P(t, T_\beta) + K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)\right)$ is the second part of Formula (4.21).

The floating leg in the interest rate swap, $N \cdot P(t, T_\alpha)$ in Formula (4.21), can be thought of as a floating rate note. A floating rate note is a contract guaranteeing a payment at a future time $T_{\alpha+1}, T_{\alpha+2}, T_{\alpha+3} \ldots T_\beta$ of the interest rate that resets at the earlier time $T_\alpha, T_{\alpha+1}, T_{\alpha+2} \ldots T_{\beta-1}$. Moreover, the note pays a cash flow at T_β consisting of the reimbursement of the notional value. The value of a floating rate note is obtained by changing the sign of the formula for the $\Pi_{\text{reciever}}(t)$ in Equation (4.20) with a fixed leg of zero and then adding it to the present value of the cash flow paid at time T_β. This gives (Brigo, et al., 2006)

$$N \cdot P(t, T_\alpha) - N \cdot P(t, T_\beta) - 0 + N \cdot P(t, T_\beta) = N \cdot P(t, T_\alpha).$$

The reason for this convenient final formula is that the entire floating rate note can be replicated through a self-financing portfolio. This shows that a floating rate note is always equal to N units of cash at its reset dates, in our case every quarter of a year. In other words, the floating rate is always equal to its notional amount when $t=T_i$. One could express this as "a floating rate note always trades at par" (Björk, 2009).

The forward swap rate $K_{\alpha,\beta}(t)$ at time t for the sets of times T and year fraction δ is the rate in the fixed leg of the interest rate swap in Formula (4.21) that makes the swap a fair contract at time t (Brigo, et al., 2006). The formula is displayed below

$$K_{\alpha,\beta}(t) = \frac{P(t, T_\alpha) - P(t, T_\beta)}{\sum_{i=\alpha+1}^{\beta} \delta_i \cdot P(t, T_i)}. \qquad (4.22)$$

In our calculations we assume that the interest rate swap contract is written at time $t = T_\alpha$, this reduces Equation (4.22) to

$$K_{\alpha,\beta}(t) = \frac{1 - P(t, T_\beta)}{\sum_{i=\alpha+1}^{\beta} \delta_i \cdot P(t, T_i)}.$$

4.4 The CIR Model

In this thesis we will use a model for the short term interest rate given by the CIR model (Cox, et al., 1985) on the term structure of interest rate.

The CIR model is developed to provide a general framework for determining the term structure of interest rates. It can be used for the pricing of derivative products and risk free securities. The model has its origin in Vasicek's model from 1977 but with the change that it introduces a square root term in the diffusion coefficient on the instantaneous short rate dynamics. The CIR model have been highly recognized and grown to be a benchmark within interest rate theory. Reasons for this is its analytical compliance and, different from Vasicek's model, that it assumes the short term instantaneous interest rates to always be positive, hence making it more handy to use. However the assumption of an always positive short term instantaneous interest rates is not always true in the financial climate of today. The CIR model specifies an instantaneous interest rate r_t which follows a stochastic differential equation given by

$$dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dZ_t$$

where κ is a strictly positive parameter illustrating the speed of adjustment to the long term mean θ. The term $\sigma\sqrt{r_t}$ is the standard deviation factor of the process and σ is often called the volatility. The condition $2\theta\kappa > \sigma^2$ puts a positive restriction on r_t, hence $r_t \geq 0$ and θ is the equilibrium interest rate. Lastly, Z is a wiener process modeling the random market risk factor. (Cox, et al., 1985)

5. CVA and CVA-capital charge

In this section we will give a description of CVA and its capital charge introduced in Basel III.

5.1 Introduction to Credit Value Adjustment

In this subsection, we will closely follow the notation and setup presented in (Brigo, o.a., 2009).The Credit Value Adjustment (CVA) for a derivative is defined as the difference between the risk free value of the derivative and the value where the risk of default is included, the true value.

To further introduce the concept of CVA an example will follow. Unilateral counterparty risk is assumed; hence one of the parties in the bilateral contract is seen as risk free, we call this party the investor. However the other party has a risk of default. Hence, it is only the investor who is facing a counterparty risk.

We start with letting τ denote the default time and ϕ be the recovery rate in the case of default. Let $\Pi^D(t,T)$ denote the discounted value of the bilateral financial contract at time t from the investor's point of view. Note that $\Pi^D(t,T)$ is the sum of all future discounted cash flows to the investor in the contract over the period t to T. So due to the counterparty risk, $\Pi^D(t,T)$ is a random variable when t>0.

Continuing, we denote default free version of $\Pi^D(t,T)$ as $\Pi(t,T)$. Hence $\Pi(t,T) = \Pi^D(t,T)$ if there would be no risk of default.

The Net Present Value (NPV) of the risk-free cash flow is defined as

$$NPV(t,T) = \mathbb{E}[\Pi(t,T)|\mathcal{F}_t].$$

Here $\mathcal{F}_t$ is denoted as all the information available at time t. With the above definitions in mind now we can define $\Pi^D(t,T)$ as

$$\Pi^D(t,T) = 1_{\{\tau>T\}}\Pi(t,T) + 1_{\{t\leq\tau\leq T\}}\left[\Pi(t,\tau) + D(t,\tau)\left(\phi\big(NPV(\tau,T)\big)^+ - \big(-NPV(\tau,T)\big)^+\right)\right].$$

In the formula above we can see, if no default happens, $\Pi(t,T) = \Pi^D(t,T)$. However if a default would occur before time T, then it could be in either of the following scenarios: $NPV(\tau,T) > 0 \ or \ NPV(\tau,T) \leq 0$.

If the investor is in the position of having a positive exposure to the counterparty at the default, the investor can, as shown in the formula above, only expect to recover $\phi\big(NPV(\tau,T)\big)^+$. However if the situation is the reversed, i.e. the counterparty's exposure to the investor at default is positive, then the investor have to pay $-NPV(\tau)$.

The Counterparty-risk Credit-value adjustment formula (CR-CVA) is defined as follow. The value of $\mathbb{E}[\Pi^D(t,T)|\mathcal{F}_t]$ is always non-negative, and the cash flows associated with the scenario where default is an option are always smaller compared to the corresponding of the default free version (Brigo, 2008).

$$\mathbb{E}[\Pi^D(t,T)|\mathcal{F}_t] = \mathbb{E}[\Pi(t,T)|\mathcal{F}_t] - (1-\phi)\mathbb{E}\left[1_{\{t\le\tau\le T\}}D(t,\tau)\big(NPV(\tau)\big)^+|\mathcal{F}_t\right].$$

The CVA can be decomposed into three terms, the loss given default, the expected exposure and the probability of default. Closer focus will be put on each of these terms in Section 5.4, 5.5 and 5.6.

5.2 CVA under Basel III

In this thesis we are calculating the CVA using the advanced formula for CVA which will be introduced in the coming subsections. Hence, what we are calculating is one CVA value which later can be used as an input in a VaR-model to calculate the CVA-capital charge defined in Basel III. Nevertheless, when calculating the CVA-capital charge which is a new feature to Basel III the CVA calculations must be based on the formulas we use and introduce below in the Section 5.2.3. Hence as an important addition we will in this section explain the CVA-capital charge which our results can be used to calculate. Also we will introduce the terms expected exposure, probability of default and loss given default which is included in our CVA calculations.

5.2.1 CVA Capital Charge

Due to worsening in the counterparty's credit quality, the CVA capital charge was added to the CCR capital charge to measure the risk of Mark-to-Market (MtM) losses. There exist two different methods to calculate the CVA capital charge, the standard and the advanced, which one to use depends on what method a bank is approved for in calculating capital charge for counterparty default risk and certain interest rate risk. Banks that use the advanced method to calculate CVA capital charge need to have IMM approval for counterparty credit risk as well as the approval to use the market risk internal models approach for the specific interest-rate risk of bonds (Basel Committee on Banking Supervision, 2011).

5.2.2 Standard Model

For the banks that do not have the IMM approval for counterparty credit risk, they must calculate its CVA capital charge using the following formula stated in the Section 104 of Basel III (Basel Committee on Banking Supervision, 2011)

$$K = 2.33 \cdot \sqrt{h} \cdot \beta$$

where

$$\beta^2 = \left[0.5 \cdot \sum_{i=1}^{N} w_i\big(M_i EAD_i^{total} - M_i^{hedge}B_i\big) - \sum_{ind} w_{ind} \cdot M_{ind} \cdot B_{ind}\right]^2$$

$$+ \sum_{i=1}^{N} 0.75 \cdot w_i^2 \cdot \big(M_i \cdot EAD_i^{total} - M_i^{hedge}B_i\big)^2$$

where

- h is the one-year risk horizon (in units of a year), h=1
- w_i is the weight applicable to counterparty 'i', which should be weighted according to the external rating or internal rating according to Basel III.
- EAD_i^{total} is the exposure at default of counterparty 'i' who does not granted the approval for IMM, and the non-IMM banks the exposure should be discounted by using the factor $[1 - exp(-0.05 \cdot M_i)]/(0.05 \cdot M_i)$.
- B_{ind} is the full notional of one or more index CDS used for hedge the CVA risk, and should be discounted by using the factor $[1 - exp(-0.05 \cdot M_{ind})]/(0.05 \cdot M_{ind})$.
- w_{ind} is the weight applicable to index hedges. The bank must map indices according to one of the weights w_i based on the average spread of index 'ind'.
- M_i is the effective maturity of the transactions with counterparty 'i'.
- M_i^{hedge} is the maturity of the hedge instrument with notional B_i (the M_i^{hedge} and B_i needs to be add together when there are several positions).
- M_{ind} is the maturity of the index hedge 'ind', which is the notional weight average maturity in case of several hedge position.

5.2.3 Advanced method

In order to calculate a VaR on CVA, banks uses their own specific interest rate risk VaR model for bonds by modeling changes in the CDS-spreads of counterparties. The VaR is calculated on the aggregated CVAs of the banks OTC derivatives. The model will not measure the sensitivity of CVA to changes in other market factors because of the restriction for the model to changes in the counterparties' credit spread. The CVA capital charge is composed by the sum of a stressed and a non-stressed VaR component, and due to its property, the calibration of the expected exposure is normally done using the credit spread calibration of the worst one-year-period contained in the three-year-period.

In Basel III it reads that, the CVA for CVA capital charge calculations must for every counterparty be based on the formula below, regardless of which accounting valuation method a bank uses to determine CVA. This is also the formula we use for our CVA calculations. The following formula is taken from Section 98 of Basel III (Basel Committee on Banking Supervision, 2011)

$$CVA = (LGD_{MKT}) \sum_{i=1}^{T} \left(\frac{EE_{t_{i-1}} * D_{t_{i-1}} + EE_{t_i} * D_{t_i}}{2} \right) * Max \left[0; exp \left(\frac{-s_{t_{i-1}} * t_{t_{i-1}}}{LGD_{MKT}} \right) - exp \left(\frac{-s_{t_i} * t_{t_i}}{LGD_{MKT}} \right) \right]$$

where,

- t_i is the i-th revaluation time bucket, start from i=1.
- T is the contractual maturity with the counterparty.
- s_i is the CDS-spread of the counterparty at time t_i.
- LGD_{MKT} is the loss given default of the counterparty, based on the spread of a market instrument of the counterparty, which must be assessed instead of an internal estimate.

- EE_i is the expected exposure to the counterparty at time t_i.
- D_i is the default risk-free discount factor at time t_i, and $D_0 = 1$.

5.3 Exposure

Counterparty Credit Exposure (CCE), or simply exposure, is the amount a company could lose if its counterparty defaults (Cesari, 2012). The Mark-to-Market (MtM) value of the OTC contract, which states the current market value of an asset, is the value we need to use to calculate the counterparty credit exposure. The MtM value can be either positive or negative; hence there is an asymmetry of potential losses. For example, suppose the MtM value is negative at default, then the company will owe its counterparty; while if it is the opposite case when the MtM value is positive, the counterparty will be unable to make future commitments and lose the amount of the MtM value. With the property that the company loses if the MtM is positive and gain nothing when the MtM is negative, we can define the expression of the exposure as:

$$Exposure\ at\ time\ t = \max(0, \Pi_t)$$

where Π_t is the MtM value of the contract at time t and i represents the different scenarios. In Figure 5.1, we display 10 simulations of exposure for interest rate swaps with the length of 10 years.
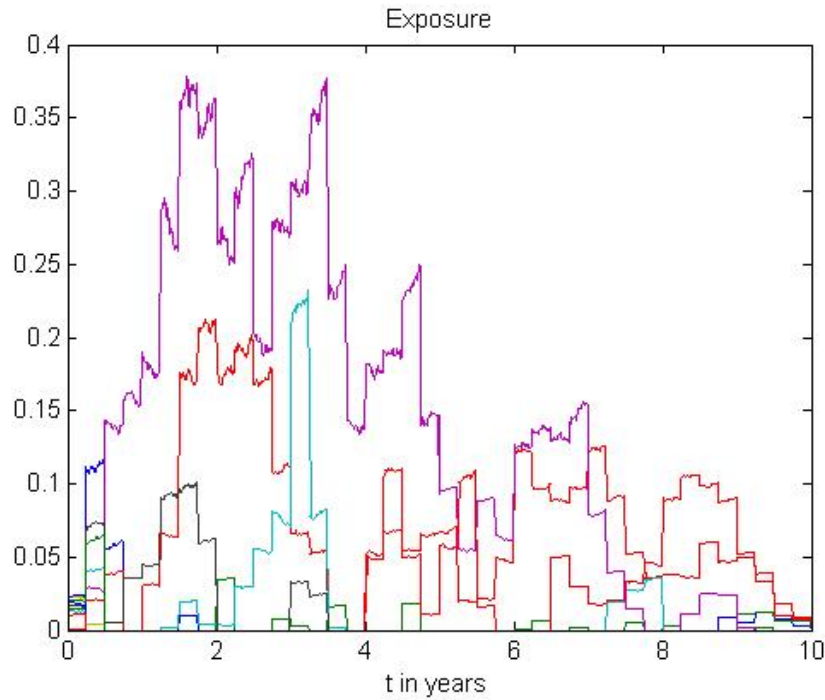

Figure 5.1: Ten simulations of Exposure for interest rate swaps.

5.3.1 Quantitative measure of Exposure

There are several ways to quantify exposure and according to the definition by the Basel Committee on Banking Supervision, three of them are widely used:

- Expected Exposure (EE): it is the expectation of loss given default based on a zero recovery rate. In other word, it is the average exposure after considering different scenarios. Thus, it can be calculated using the formula

$$EE_t = \frac{1}{N}\sum_{i=1}^{N}\max(0,\Pi_t^i)$$

  where N is the number of simulated interest rate paths, and Π_t^i is the exposure at time t for simulation number i. Note that EE_t is a Monte Carlo estimation of $E[\max(0,\Pi_t)]$.

- Potential Future Exposure (PFE): the definition for PFE is very much similar to Value at Risk (VaR), which is the worst possible exposure with a certain confidence interval (usually 99%). What is different between the two is that the PFE has a longer time horizon, usually years; while the VaR's time horizon is accounted in days

- Expected Positive Exposure (EPE): is slightly different to the previous two concepts since it is defined to be the time average of the EE_t, the formula is shown below:

$$EPE = \frac{1}{M}\sum_{t=1}^{M}max(0,EE_t)$$

  where t is the time now and M is the maturity. What needs to be mention is that the EE_t and EPE are both weighted value of a point in time.

5.4 Loss given default

Loss Given Default (LGD) can be expressed as one minus the recovery rate. In other terms, it is how much a company will lose if a default occurs. In theory, the LGD can take every value form 0-100%, zero when a default doesn't involve a loss and 100 % when everything is lost with the default. This is a value that is very hard to find a precise estimate for. The problem that is encountered when trying to estimate a good LGD value is that the sample data is often too small. This means that more subjective methods must be used to find a good estimation of the LGD. Depending on the features attached to the data, quantitative methods allows for an implicit or explicit estimation of the value.

The market LGD approach is an explicit method that looks at market prices of bonds directly after default, comparing these with their original par value. After discounting the recoveries and costs that can be observed, the value of the company can be determined and from that the LGD can be extracted (Engelmann, 2011).

By investigating the probability distribution of recoveries from 1970-2003 for all available bonds and loans data from Moody's, the LGD can be approximately estimated on average to be 60%, which is the value we will use in our CVA calculations. (Schuermann, 2004).

5.5 Probability of Default

Probability of default is the likelihood that a default will occur during a specific period of time. In a financial setting, PD is an estimation of the likelihood that a financial institution is incapable of or unwilling to fulfill its debt obligations. The PD depends on microeconomic factors i.e. counterparties as well as the macroeconomic factors such as an economic down turn. When the overall economic environment is unhealthy, PD will be relatively high compared to normal years. However, it also depends on the strategy the counterparties use to deal with different situation.

Probability of default is an important concept since a correct estimation can prevent potential loss for companies. It is generally difficult to estimate and model since the defaults are relatively rare, statistical cases are not enough to be sufficient. However, there are two methods existing to estimate the credit worthiness of an entity: the basic model which was discussed in Section 5.2.2 and the advanced model, which was discussed Section 5.2.3. For the estimation of the PD in our model we use the formula

$$Max\left[0; exp\left(\frac{-s_{t_{i-1}} * t_{t_{i-1}}}{LGD_{MKT}}\right) - exp\left(\frac{-s_{t_i} * t_{t_i}}{LGD_{MKT}}\right)\right]. \quad (5.1)$$

This term comes from the approximation (see also in Herbertsson 2012)

$$S(T) \approx \lambda \cdot LGD_{MKT} \quad Hence \quad \lambda \approx \frac{S(T)}{LGD_{MKT}}.$$

Thus

$$P[\tau > t] = exp\left(-\frac{S(T) \cdot t}{LGD_{MKT}}\right)$$

and this implies that

$$P[t_{i-1} < \tau \le t_i] = P[\tau \le t_i] - P[\tau \le t_{i-1}] = P[\tau > t_{i-1}] - P[\tau > t_i]$$
$$= exp\left(\frac{S(T) * t_{t_{i-1}}}{LGD_{MKT}}\right) - exp\left(\frac{S(T) * t_{t_i}}{LGD_{MKT}}\right)$$

where $t_{i-1} < t_i$. The "max" in Equation (5.1) guarantees that the value of PD is non-negative, since in this formula $S(T)$ may be a function of t_{t_i}.

The term s_{t_i} is the CDS-spread. In our thesis we have created three different CDS-spreads by simulating three different risk scenarios; High; medium and Low. This structure is inspired by Brigo's article in 2008.The complete table with the CDS-spreads will be shown in Section 7.

6. Calculating Expected Exposure

In this section we explain the steps taken to calculate our CVA values for an interest rate swap in a CIR-framework.

6.1 The interest rate path

To model CVA for an interest rate swap in a CIR-framework we need to model the expected exposure. To get a value for the expected exposure in the swap contract we start by simulating interest rate paths following a CIR-process.

For the simulation we take support in an algorithm described in (Broadie, 2006). If the interest rate follows a CIR-process including the parameters κ, θ, σ and r_t, the value of r_t given r_s, where $s < t$, is given by multiplying a scaling factor with a non-central chi-squared distributed random variable.

$$r_t = c_t \chi_d'^2(a) \qquad (6.1)$$

In Equation (6.1), a denotes the non-centrality chi-squared parameter, d is the degrees of freedom and c_t is a scaling factor. The parameters are dependent on the distance between s and t, κ, σ, θ and the initial value of the short term rate r_0.

$$a = \frac{4\kappa e^{-\kappa(t-s)}}{\sigma^2(1 - e^{-\kappa(t-s)})} * r_s$$

$$d = \frac{4\theta\kappa}{\sigma^2}$$

$$c_t = \frac{\sigma^2(1 - e^{-\kappa(t-s)})}{4\kappa}.$$

The CIR-model is estimated with the restriction that $2\theta\kappa > \sigma^2$, and if this holds, then it means that d will always be bigger than one.

When d >1 a non-central chi-squared random variable can be written as a sum of a non-central chi-squared random variable with one degree of freedom and a normal chi-squared random variable with d-1 degrees of freedom (Broadie, 2006). So for this case, with an estimation that is restricted subject to d >1, the interest rate simulation for every period is modeled as follow:

$$r_t = c_s(\chi_1'^2(a) + \chi_{d-1}^2)$$

where

$$\chi_1'^2(a) = (Z + \sqrt{a})^2$$

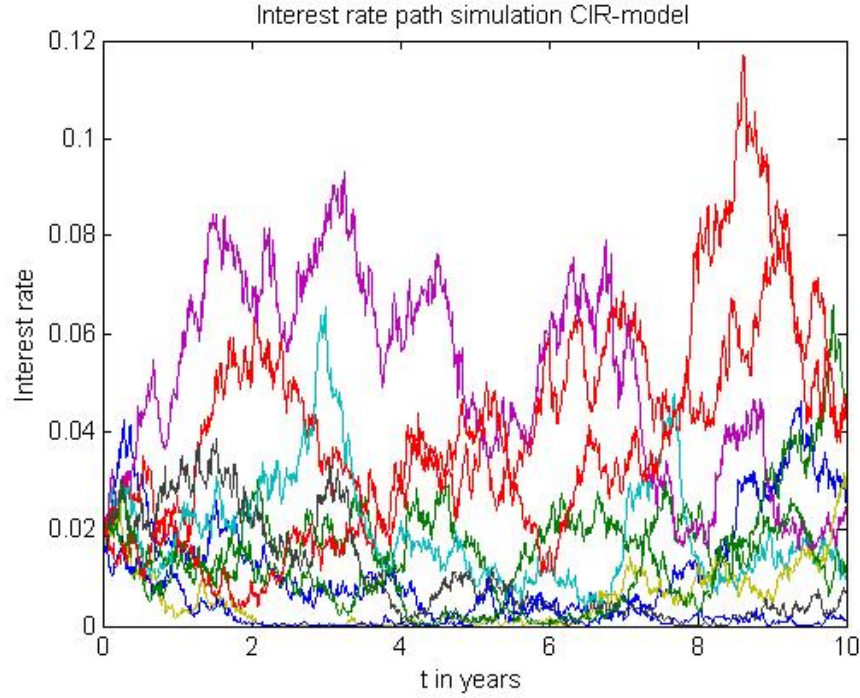and Z is a standard normal distributed random variable.

Figure 6.1 show 10 simulated interest rate paths over a period of ten years when the interest rate r_t follows a CIR process with the parameters κ = 0.1, θ= 0.03, σ = 0.1, r_0= 0.02.

6.2 Bond Price

Subject to assumptions of continuous trading and absence of transaction costs, it is possible to model the arbitrage-free price of a default-free zero coupon bond $P(t,T)$ in the following way (Björk, 2009)

$$P(t,T) = A(t,T)e^{-G(t,T)\mathrm{r}}$$

$$A(t,T) = \left(\frac{\alpha e^{\beta(T-t)}}{\beta(e^{\alpha(T-t)} - 1) + \alpha}\right)^{\gamma}$$

$$B(t,T) = \frac{\alpha e^{(T-t)} - 1}{\beta(e^{\alpha(T-t)} - 1) + \alpha}$$

where

$$\alpha = \sqrt{k^2 + 2\sigma^2}, \qquad \beta = \frac{k + \alpha}{2}, \qquad \gamma = \frac{2k\theta}{\sigma^2}.$$
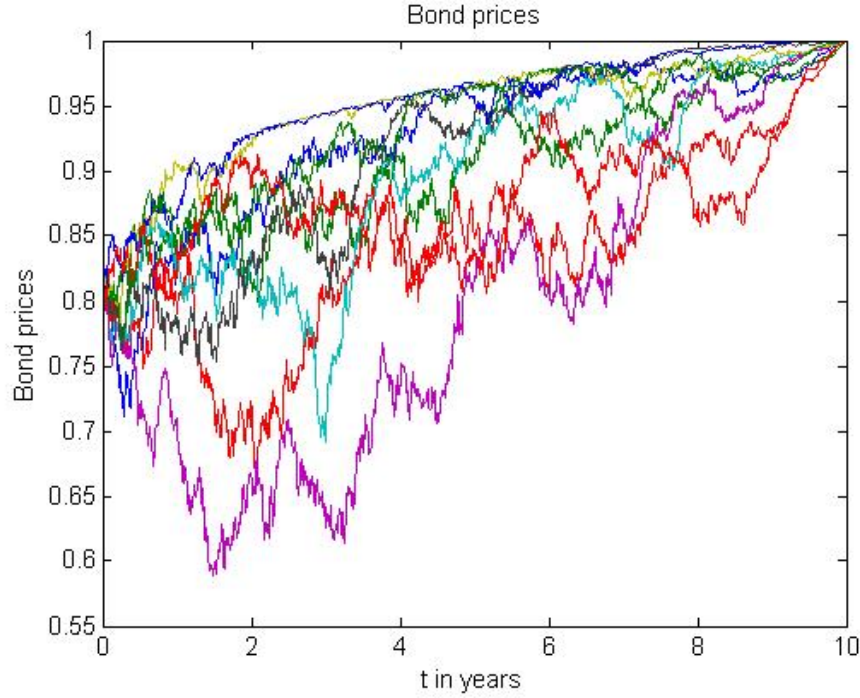
Figure 6.2: Simulation of Bond Prices.

In Figure 6.2 we show the bond price $P(t,T)$ as a function of time t, given the realization of the interest rate r_t, with the same parameters and the same simulation of r_t as in Figure 6.1.

6.3 The Swap Contract

The value of the swap contract $\Pi_{\text{payer}}(t)$ is calculated at close time points up until maturity. It is calculated with the formula below

$$\Pi_{\text{payer}}(t) = N\left(P(t, T_\alpha) - P(t, T_\beta) - K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)\right)$$

where the calculation is based on the bond prices $P(t, T)$ from the previous subsection. In this subsection we will let $\alpha = \alpha(t)$, i.e. the index α will change as t runs from 0 to T_β. More specific, for each t, $0 \le t \le T_\beta$, we will let $\alpha(t)$ be the time point $T_{\alpha(t)}$ closest to t but still above or equal to t, i.e. $T_{\alpha(t)-1} < t \le T_{\alpha(t)}$.

To ease the interpretation of the formula, the $\Pi_{\text{payer}}(t)$ can be decomposed into its individual terms. The factor N is the notional amount, hence the scaling factor for the interest rate swap contract. The derivation of the term $N \cdot P(t, T_\alpha)$ is further explained in Section 4. The term could be described as a floating rate note. It is always equal to N when $t = T_i$, so for all $i = \alpha, \alpha + 1, \alpha + 2, ..., \beta$, $N \cdot P(t, T_\alpha) = N$, when $\alpha = \alpha(t)$ and $T_{\alpha(t)-1} < t \le T_{\alpha(t)}$. This will be explained in more detail below.

The term $N\left(P(t, T_\beta) - K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)\right)$ is all the future discounted cash flows at time t from the fixed leg payment stream. Here $N \cdot K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i)$ are the future discounted cash flows from the coupon payments, this sum decreases in size as the contract approaches maturity. The term $N \cdot P(t, T_\beta)$ is the discounted reimbursement of the notional value in the fixed leg, this goes to N as the contract approaches maturity. Here K is the swap rate that makes the contract fair at time $t = 0$, or in other words the swap rate that makes the present value of the floating and fixed leg equal at time $t = 0$. Finally, δ is the year fraction between T_{i-1} and T_i (Brigo, et al., 2006) (Björk, 2009).

The formula above is used to calculate the value of the interest rate swap at time t from $t = 0$ to $t = T_\beta$ with close steps. The swap rate K is calculated once, at $t = 0$. The formula above calculates the value of an interest rate swap at time t for $t \leq T_\alpha$. Hence, to implement the formula at $t > 0$ the calculations assumes that T_α which initially is at $t = 0$ follows the coupon payments. Hence, by letting $\alpha = \alpha(t)$, the index α will change as a function of t from 0 to T_β. Thus, at $t \leq 0$ the present value of the contract is calculated by discounting all future cash flows from $t \leq 0$ to $t = T_\beta$ for a contract with the length of ten years. However, when $0 < t \leq 1.25$ ($t = 1.25$ is the time for the first coupon payment, which are paid quarterly until maturity) and $T_\alpha = 1.25$ the swap value is calculated for a swap contract with the length of 9.75 years. This is done until $t = T_\beta$, and since α is a function of t the calculations are made on increasingly shorter swap contracts as t increases. However K is constant as the swap rate calculated at $t = 0$.

Thus, when $t = 10$ no more coupon payments are left to be exchanged, hence $N \cdot K \cdot \delta \sum_{i=\alpha+1}^{\beta} P(t, T_i) = 0$, $N \cdot P(t, T_\alpha) = N$ and $N \cdot P(t, T_\beta) = N$, which gives the swap contract a value of $N - N - 0 = 0$ at $t = 10$.

More details on the derivation of $\Pi_{\text{payer}}(t)$ are to find in Section 4. In Figure 6.3, we display 10 simulations of the swap value over 10 years.
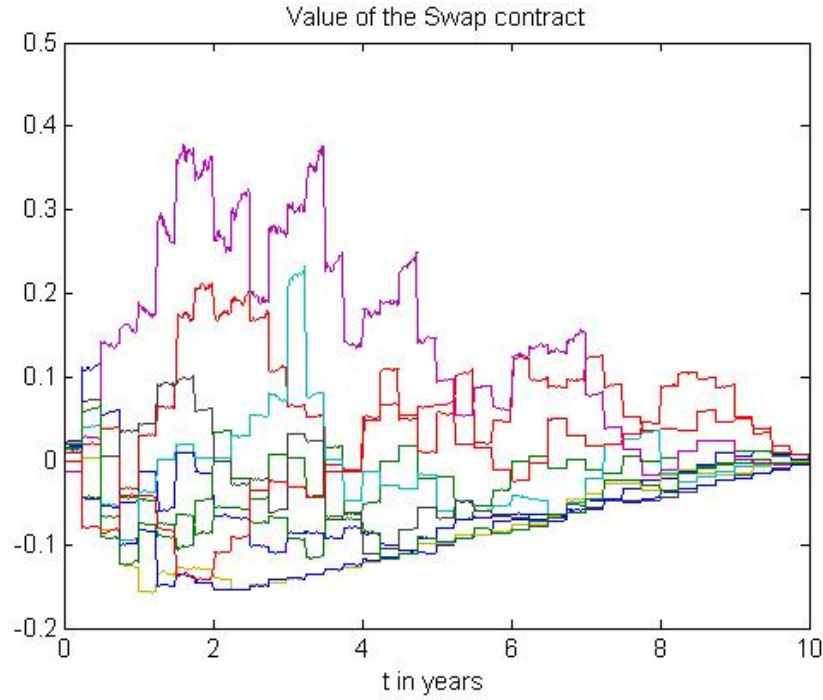
Figure 6.3: Simulation of the value of the swap contracts.

The interest rate swap is a function of the interest rate and since we model the interest rate as a CIR-process, the value of the swap in every time point is a function of θ, σ, r and κ. The quarterly jumps in Figure 6.3 come from the resetting of the floating rate and that the expected value of the sum of all future coupons is reduced after every coupon payment.

6.4 Expected Exposure

From the value of the swap contract calculated above, we can derive the expected exposure using the formula

$$EE_t = \frac{1}{N} \sum_{i=1}^{N} \max(0, \Pi_t^i).$$

For this we use the method explained in Section 5. It is displayed in Figure 6.4.
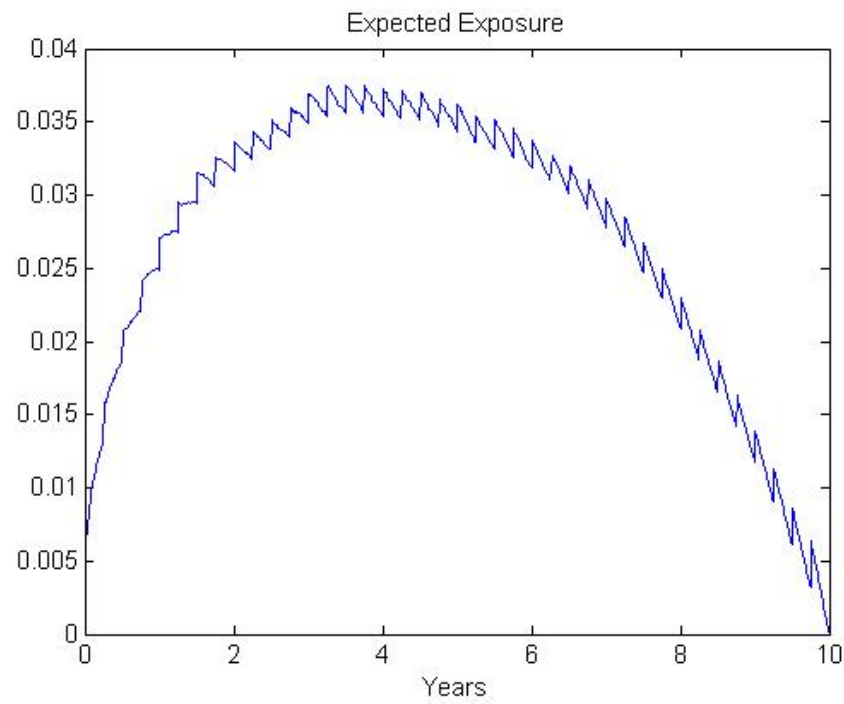
Now we have calculated all the terms needed for our CVA value.

7. Results

Our CVA is calculated using the following formula with a notional amount of 1 and a value of the other terms as specified later in this section.

$$CVA = (LGD_{MKT}) \sum_{i=1}^{T} \left(\frac{EE_{t_{i-1}} * D_{t_{i-1}} + EE_{t_i} * D_{t_i}}{2} \right) * Max \left[0; exp \left(\frac{-s_{t_{i-1}} * t_{t_{i-1}}}{LGD_{MKT}} \right) - exp \left(\frac{-s_{t_i} * t_{t_i}}{LGD_{MKT}} \right) \right]$$

For the CDS we have created CDS spreads by simulating 3 different scenarios; High; medium and Low. This setup is inspired by Brigo's article in 2008 (Brigo, 2008).

Year	Low Risk		Medium Risk		High Risk	
	Intensity	CDS	Intensity	CDS	Intensity	CDS
1	0.0035	22.759956	0.0205	133.59204	0.0508	332.3057
2	0.0046	26.295304	0.022	138.39149	0.0521	336.4322
3	0.0051	28.530952	0.0233	142.70249	0.0516	336.7854
4	0.008	34.185844	0.0243	146.38399	0.053	339.0235
5	0.0095	39.418711	0.0235	147.63146	0.0541	341.6065
6	0.011	44.411884	0.0254	150.31184	0.055	344.1345
7	0.0126	49.326422	0.0268	153.35912	0.0548	345.7788
8	0.0142	54.164755	0.0286	156.8823	0.056	347.7539
9	0.0158	58.926599	0.0299	160.38565	0.0587	350.7132
10	0.0174	63.610364	0.031	163.74592	0.059	353.1898

Table 7.1: Different CDS spreads representing different risk scenarios.

The discount rate D_t is set to be a constant interest rate of 0.02.

The Expected Exposure EE_t is estimated with 10000 interest rate paths with, κ = 0.1, θ= 0.03, σ = 0.1, r_0= 0.02, where one parameter is changed for every scenario in Figure 7.1 while the others are kept constant. This is done to see how sensitive the CVA is to changes in the individual parameters in the CIR-framework. The CVA is calculated with a quarterly exposure frequency (i in the formula). The exposure frequency is defined as how many steps the summation term in the CVA calculation is divided into on a yearly basis. This gives that our summation term contains 40 steps.

The LGD is set to 60% for all simulations. The reason behind the chosen level of LGD is motivated in Section 5.
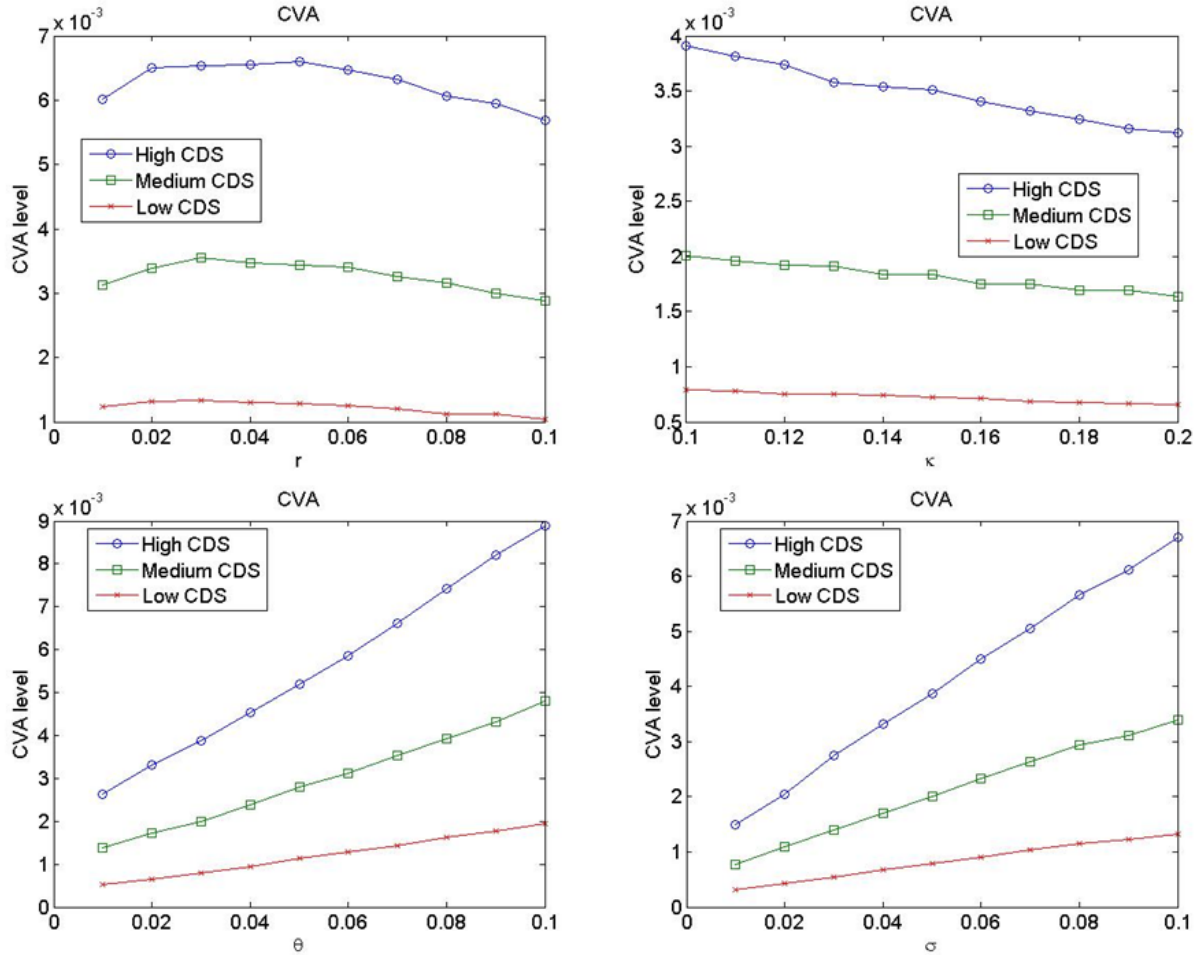
Figure 7.1: The change in CVA as a function of $\kappa, \theta, \sigma, r_0$ in three CDS scenarios.

Although the calculated CVA value is relatively low, one should realize that these CVA values are calculated on a notional of one, normally the notional is much larger, ranging up to billions. So if $N = 10^8$, i.e. 100 million dollars, then a CVA value of e.g. 4×10^{-3} means a CVA value of $4 \times 10^5 = 400000$ dollars, which is a non-negligible amount. However the relationships we are investigating are not contingent on the size of the notional, hence for simplicity it is set to one.

Looking at the pictures above, it can be seen that the CVA is affected by the CDS level. Furthermore, we discovered that the CVA values seem to be sensitive to changes in the parameter θ the most, followed by σ, which is intuitively clear, since σ derives the volatility in the exposure.

To further look at the final CVA values, we plotted the CVA in a bivariate structure to show how it changes with different combinations of the parameters $\kappa, \theta, \sigma, r_0$. The Figure 7.2 is constructed using the "high" CDS-spread.
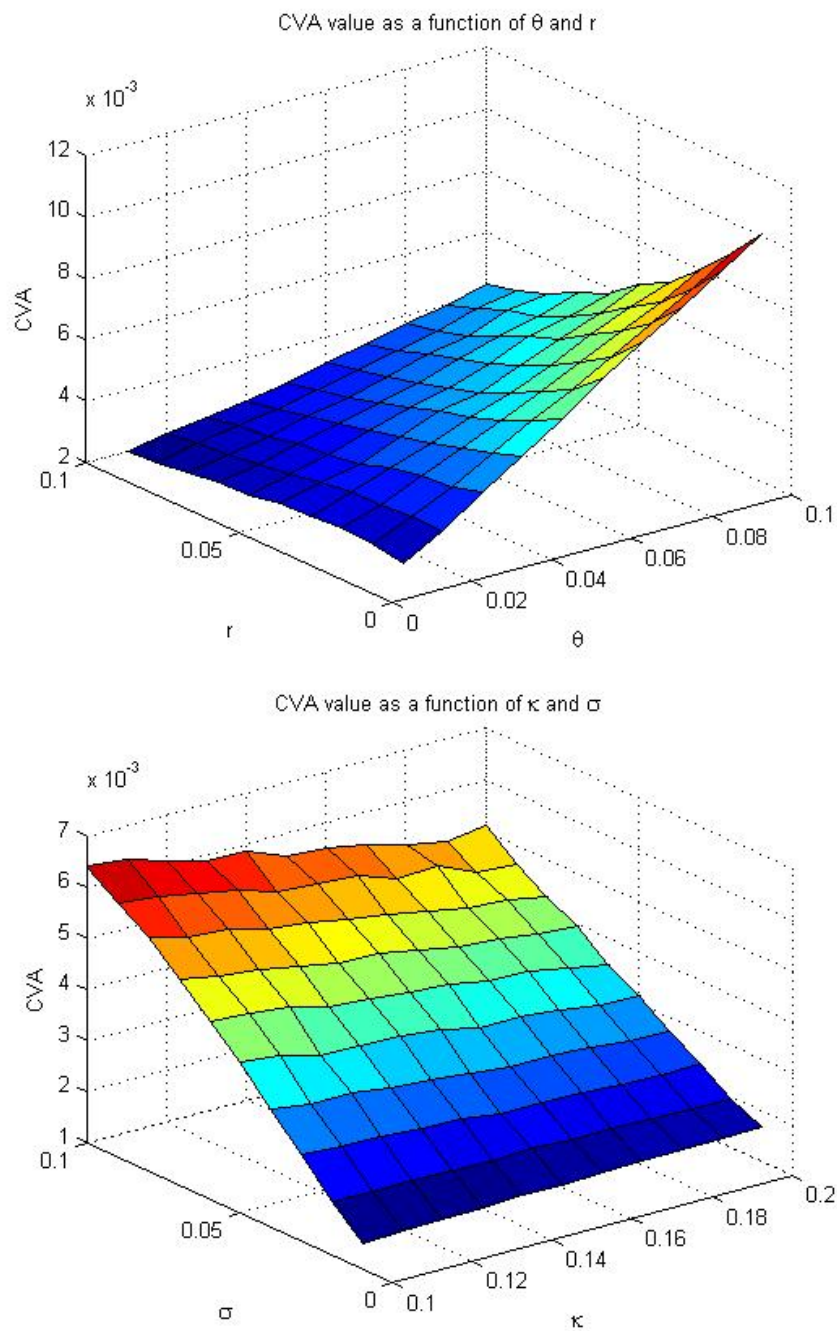
Figure 7.2: **CVA as a function of $\kappa, \theta, \sigma, r_0$.**

Instead of using a flat interest rate for the discount term D_t, we have also calculated the CVA in the CIR-framework where D_t follows the simulated Interest rate. Figure 7.3 illustrate a comparison between the results.
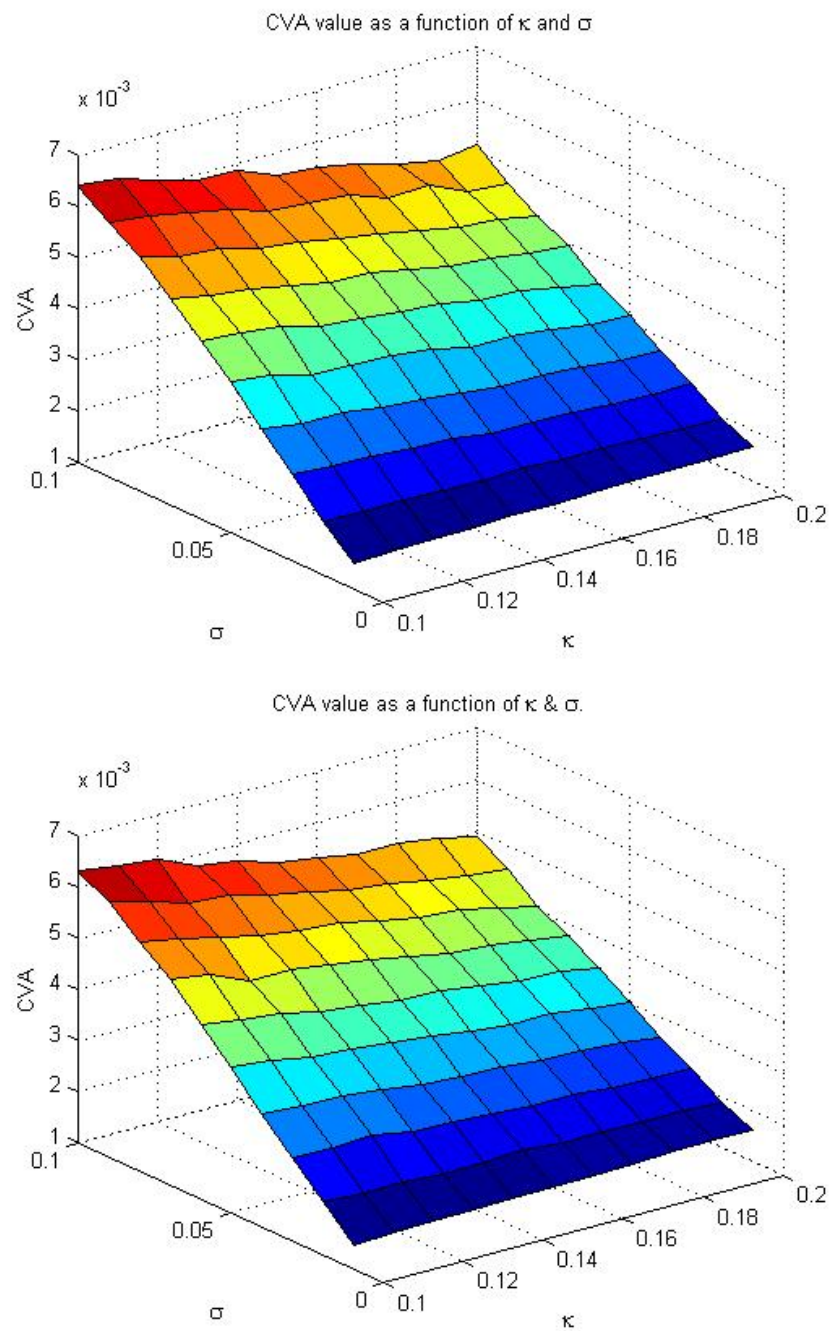
Figure 7.3: Comparison of CVA values as a function of κ and σ using different discount rate scenarios, where the upper one's discount rate is dependent on the CIR simulation.
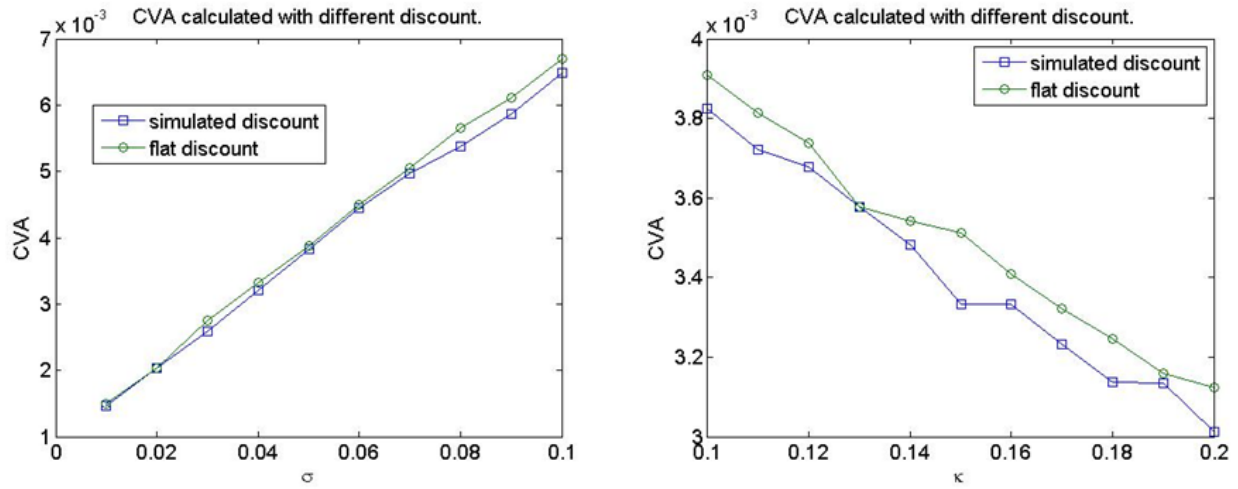
Figure 7.4: **Comparison of CVA as a function of different discount rates.**

From Figure 7.4, it can be concluded that the two different discount scenarios seem to give relatively similar CVA values.

8. Discussion and conclusion

By looking at the model risk for CVA on an interest rate swap in the CIR-framework and by investigating how sensitive the CVA-value is with respect to the underlying parameters, we can conclude that the level of the underlying parameters has a big impact on the final CVA value.

The estimated value of the EE will change with respect to $\kappa, \theta, \sigma, r_0$, driving the simulated interest rate path. The parameters that affect the CVA value the most are θ and σ. That the CVA calculation is the most sensitive to changes in θ and σ is fairly intuitive considering the meaning of the included parameters. The long term mean of the process θ will have a great impact on the level of the CIR-process, in our case the interest rate. Also considering that σ is the volatility of the process, an increase in this parameter will lead to an increased uncertainty about the future interest rate, hence, a bigger risk will be associated with a high σ. The level of r_0 will measured over a longer period not have as big of an impact on the interest rate. Further, if κ is high it means that the process quickly will return to its mean, however the speed of the return to the long term mean does clearly not have a dramatic impact on the CVA.

Even if the CVA value is small and a small change of its value doesn't seem too intimidating, it could have big effects. Since we have done our estimations with a notional of one, the real CVA must be scaled with the proper amount of the notional. It is also discovered by our research that the different CDS levels have a great impact on the final level of the CVA.

Comparing our work to previous research from Brigo (2008) and Douglas (2012), we can conclude that the results from our CVA calculations give similar CVA values as these papers. With similar input variables our output matches the results of previous efforts to calculate CVA for an interest rate swap. They as us, emphasize the importance of being aware of the volatility in the CVA and account for it in a sound manner. The scientific contribution of our work is the finding that parameters $\kappa, \theta, \sigma, r_0$ have an impact on the CVA for an interest rate swap in a CIR framework.

Lastly, since there is significant insecurity in the modeling of every term in the CVA model, it is very difficult to say with confidence that the estimated CVA value calculated in our model is accurate. Nevertheless if the CVA is not taken seriously, there will be a great chance that what happened in the financial crisis of 2008-2009 might happen again. During that crisis losses could be traced to volatility in CVA. So now when the importance of accurately measuring the CVA has been recognized, we believe that the research within this field will grow further. To conclude, it is evident that the final CVA value is sensitive to changes in the underlying parameters $\kappa, \theta, \sigma, r_0$ as well as to variations in the other terms included in the CVA model, which indicates an evident model risk.

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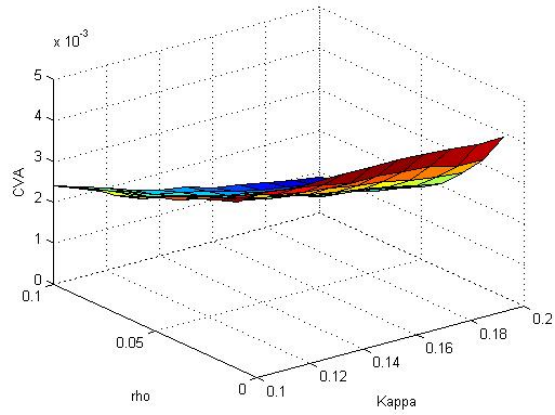
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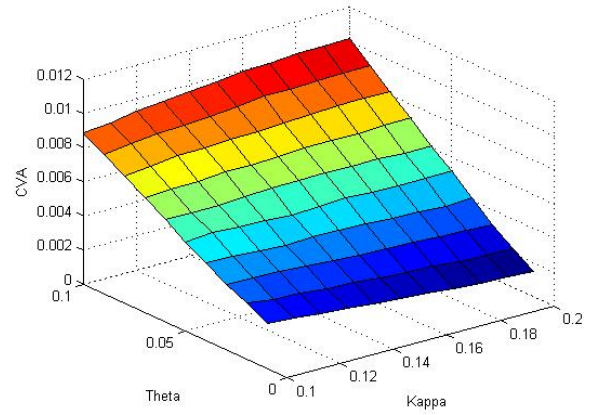
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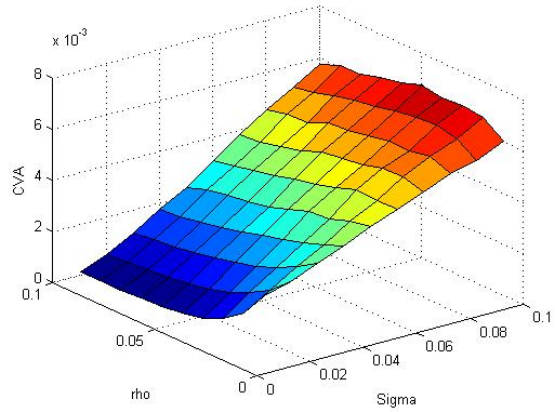
Appendix

CVA value for different combinations of the Instantaneous short rate & Kappa

CVA value for different combinations of Kappa & Theta

CVA value for different combinations of Sigma & the Instantaneous short rate

CVA value for different combinations of Theta & Sigma

CVA value for different combinations of Kappa & Theta.

CVA value for different combinations of the Instantaneous short rate & Kappa.

CVA value for different combinations of Sigma & the Instantaneous short rate.

CVA value for different combinations of Theta & Sigma.