

# **CVA AND WRONG WAY RISK**

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## **ABSTRACT**

This paper proposes a simple model for incorporating wrong-way and right-way risk into CVA (credit value adjustment) calculations. These are the calculations made by a dealer to determine the reduction in the value of its derivatives portfolio because of the possibility of a counterparty default. The model relates the hazard rate of the counterparty to the value of the transactions outstanding between the dealer and the counterparty. Numerical results for portfolios of 25 instruments dependent on five underlying market variables are presented. The paper finds that wrong-way and right-way risk have a significant effect on the Greek letters of CVA as well as on CVA itself. It also finds that the nature of the effect depends on the collateral arrangements.

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## **CVA AND WRONG WAY RISK**

### **1. Introduction**

It has for some time been standard practice for derivatives dealers to adjust the reported value of their derivatives transactions with a counterparty to reflect the possibility of losses being incurred because of a default by the counterparty. The adjustment is known as the credit value adjustment or CVA.

A derivatives dealer has one CVA for each counterparty. These CVAs are themselves derivatives and must be managed similarly to other derivatives. CVAs are particularly complex derivatives. In fact, the CVA for a counterparty is more complex, and more difficult to value, than any of the transactions between the dealer and the counterparty. This is because the CVA for a counterparty is, as we will see, contingent on the net value of the portfolio of derivatives outstanding with that counterparty.

Calculating CVAs is very computationally intensive. Gregory (2009) provides an excellent discussion of the issues. Statistics published about Lehman Brothers give a sense of the scale of the problem. Reuters (2008) reported that, at the time of its failure Lehman, had about 1.5 million derivatives transactions outstanding with 8,000 different counterparties. This means that it had to calculate 8,000 different CVAs and the number of derivative transactions on which each CVA was dependent averaged about 200.

Another measure, more controversial than CVA, is debit value adjustment or DVA.<sup>1</sup> The DVA calculated by the dealer for a counterparty is an estimate of the costs to the counterparty of the possibility that the dealer might default. The possibility that it might default is in theory a benefit to the dealer and some accounting standards require the book value of the derivatives outstanding with a counterparty to be calculated as their no-default value minus the CVA plus the DVA.<sup>2</sup> The

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<sup>1</sup> DVA is sometimes also called debt value adjustment.

<sup>2</sup> There is an interdependence here in that if the dealer defaults subsequent defaults by the counterparty are irrelevant and vice versa. See Hull and White (2001) for a discussion of this point in the context of credit default swaps.

reason why DVA is controversial is that there is no way a dealer can monetize DVA without actually defaulting.

It is interesting to note that when the credit spread of a derivatives dealer increases, DVA increases. If the dealer uses DVA accounting, this in turn leads to an increase in the reported value of the derivatives on the books of the dealer and a corresponding increase in its profits. Some banks reported several billion dollars of profits from this source in 2008. Not surprisingly, DVA gains and losses have now been excluded from the definition of common equity in determining regulatory capital. In this paper, we will focus on CVA, but many of the points we make are equally applicable to DVA.

Market variables that affect the no-default value of a dealer's outstanding transactions with a counterparty also affect the dealer's CVA for that counterparty. We will refer to these market variables as the "underlying market variables." In addition, CVA is affected by the counterparty's term structure of credit spreads (the "counterparty credit spreads"). CVA therefore gives rise to two types of exposures. One arises from potential movements in the underlying market variables; the other from potential movements in counterparty credit spreads.

In December 2010, the Basel Committee on Banking Supervision published a new regulatory framework for banks known as Basel III.<sup>3</sup> It requires a dealer's CVA risk arising from changes in a counterparty's credit spreads to be identified and included in the calculation of capital for market risk. However, the dealer's CVA risk arising from underlying market variables are not included in this calculation. Some dealers have developed their own sophisticated systems for managing both types of risk. These dealers feel that Basel III proposals are inadequate because, if a dealer hedges against the underlying market variables, the hedging trades will lead to an increase, not a decrease, in its required capital.<sup>4</sup>

Since the crisis, governments have moved to require "standardized" over-the-counter derivatives to be cleared through central clearing parties. This paper focuses on how counterparty credit risk

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<sup>3</sup> See Basel Committee on Banking Supervision (2010)

<sup>4</sup> See, for example, Pengelly (2011)

is handled for those transactions that continue to be cleared bilaterally.<sup>5</sup> This includes all derivatives transactions classified as “non-standard.” It also includes some categories of derivatives, such as forward foreign exchange contracts, that are excluded from the central clearing legislation.

Transactions between a dealer and a counterparty are typically governed by an International Swaps and Derivatives Association (ISDA) Master Agreement. This specifies that all transactions between the two parties are to be netted and considered as a single transaction in the event that there is an early termination. The circumstances under which one side can send an early termination notice to the other side and the procedures that are then used are specified in the ISDA Master Agreement.

Collateralization has become an important feature of over-the-counter derivatives markets. An ISDA Master Agreement typically has a Credit Support Annex (CSA) which specifies the rules governing the collateral that has to be posted by the two sides. In particular, it specifies a variety of terms including the threshold, the independent amount, the minimum transfer amount, haircuts that will apply to assets that are posted as collateral, etc. Suppose that the two sides are Party A and Party B and Party B is required to post collateral. The threshold is the unsecured credit exposure to Party B that Party A is willing to bear. If the value of the derivatives portfolio to Party A is less than the threshold, no collateral is required from Party B. If the value of the derivatives portfolio to Party A is greater than the threshold, the required collateral is equal to the difference between the value and the threshold. The independent amount plays the same role as the initial margin in a futures contract and can be regarded as a negative threshold. Failure to post the required collateral by Party B would allow Party A to terminate its outstanding transactions with Party B after what is known as a “cure period” has elapsed.

In the calculation of CVA, it is usually assumed that the counterparty’s probability of default is independent of the dealer’s exposure to the counterparty. A situation where there is a positive dependence between the two, so that the probability of default by the counterparty tends to be high (low) when the dealer’s exposure to the counterparty is high (low), is referred to as “wrong-

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<sup>5</sup> How derivatives dealers will assess their credit exposure arising from transactions that are cleared through a CCP remains to be seen. The dealer is exposed to a) a default by any of the other clearing house members and b) a default by the clearing house itself.

way risk.” A situation where there is negative dependence, so that the probability of default by the counterparty tends to be high (low) when the dealer’s exposure to the counterparty is low (high) is referred to “right-way risk.”

A subjective judgment of the amount of wrong-way or right-way risk in transactions with a counterparty requires a good knowledge of the counterparty’s business, in particular the nature of the risks facing the business. It also requires knowledge of the transactions the counterparty has entered into with other dealers.<sup>6</sup> The latter is difficult to know precisely but the extra transparency provided by post-crisis legislation may help.

One situation in which wrong-way risk tends to occur is when a company is selling credit protection to the dealer. (AIG and monolines are obvious examples here.) This is because credit spreads are correlated. When credit spreads are high the value of the protection to the dealer is high and as a result the dealer has a large exposure to the company. At the same time the credit spreads of the company are also likely to be high indicating a relatively high probability of default for the company. Similarly, right-way risk tends to occur when a company is buying credit protection from the dealer.

A situation in which a company is speculating by entering into many similar trades with one or more dealers is likely to lead to wrong-way risk for the dealers. This is because the company’s financial position and therefore its probability of default is likely to be affected adversely if the trades move against the company. If the company enters into transactions to partially hedge an existing exposure there should in theory be right-way risk. This is because, when the transactions move against the counterparty, it will be benefitting from the unhedged portion of its exposure so that its probability of default will tend to be relatively low.<sup>7</sup>

This paper explains the way CVA is calculated and the advanced approach for calculating CVA risk capital under Basel III. The paper then proposes a model for incorporating wrong-way risk into CVA calculations. Other authors who have attempted to tackle the wrong-way risk problem

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<sup>6</sup> A complication is that sometimes a company will trade with many different market participants in an attempt to conceal its true exposure.

<sup>7</sup> An exception could be when the counterparty is liable to run into liquidity problems. Although the assets being hedged have increased in value, the counterparty might be unable to post collateral when required. An example here is Ashanti Goldfields in September 1999. See for example Hull (2010, page 395) for a description of what happened.

include Cespedes et al (2010) and Sokol (2010). These authors propose an “exposure sampling” approach where the process followed by the exposure is approximated with a one-factor Markov process and a Gaussian (or other) copula is used to model the dependence between this process and the time to default.

Our model is simpler than the exposure sampling approach and involves a relatively small adjustment to the method used to calculate CVA when the usual assumption of no dependence between exposure and probability of default is made. We specify a deterministic model relating the hazard rate of the counterparty to the value of the dealer’s transactions with the counterparty. Numerical results from the implementation of the model are presented.

## 2. Calculating CVA

Suppose that  $T$  is the longest-maturity derivative outstanding between a derivatives dealer and one of its counterparties. Define  $v(t)$  as the value of a derivative that pays off the dealer’s net exposure to the counterparty at time  $t$  and  $R$  as the recovery rate (assumed to be constant). If we assume that the probability of a counterparty default at time  $t$  is independent of  $v(t)$  then, as discussed by, for example, Hull and White (1995), Canabarro and Duffie (2003), and Picault (2005),

$$CVA = (1 - R) \int_{t=0}^T q(t) v(t) dt \quad (1)$$

where  $q(t)$  is the probability density function of the risk-neutral time to default for the counterparty.

If no collateral is posted the net exposure,  $E_{NC}(t)$ , at time  $t$  is given by

$$E_{NC}(t) = \max(w(t), 0) \quad (2)$$

where  $w(t)$  is the value to the dealer of the derivatives portfolio at time  $t$ .<sup>8</sup> If transactions are collateralized with zero threshold and an early termination is declared as soon as the counterparty

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<sup>8</sup> We assume that collateral is posted continuously and that there is no minimum transfer amount. Results can be modified to relax these assumptions.

fails to post the required collateral, then in the event of default at time  $t$  the available collateral is  $\max(w(t), 0)$ . If there is a threshold,  $K$ , the available collateral is  $\max(w(t) - K, 0)$ .

In practice, a period of time elapses between the time when the counterparty ceases to post collateral and an early termination event being declared by the dealer. This is to allow any disputes about the value of the portfolio between the dealer and the counterparty to be resolved and is referred to the “cure period.” Suppose that the length of the cure period is  $c$ . The collateral available if there is termination event at time  $t$  is

$$C(t) = \max(w(t - c) - K, 0) \quad (3)$$

An independent amount,  $I$ , can be treated as a negative threshold so that

$$C(t) = \max(w(t - c) + I, 0)$$

The net exposure,  $E_{NET}(t)$ , at time  $t$  is

$$E_{NET}(t) = \max(E_{NC}(t) - C(t), 0) \quad (4)$$

The variable  $v(t)$  is the value of a derivative that pays off  $E_{NET}(t)$  at time  $t$ . It can be calculated as the expected value of  $E_{NET}(t)$  in a risk-neutral world discounted at the risk-free rate.

To approximate the integral in equation (1) we can choose times  $t_i$  ( $0 \leq i \leq n$ ) with  $t_0 = 0$ ,  $t_n = T$  and  $t_0 < t_1 < t_2 < \dots < t_n$  and set

$$CVA = (1 - R) \sum_{i=1}^n q_i v_i \quad (5)$$

where  $q_i$  is the probability of default between times  $t_{i-1}$  and  $t_i$  and  $v_i = v(t_i^*)$  with

$t_i^* = 0.5(t_{i-1} + t_i)$ . Note that  $q_i$  is the unconditional risk-neutral probability of default between

times  $t_{i-1}$  and  $t_i$  (as seen at time zero). It is not the probability of default conditional on no earlier default.<sup>9</sup>

The  $q_i$ 's are usually calculated from credit spreads. Sometimes a complete term structure of credit spreads for the counterparty can be observed in the market; sometimes it has to be estimated using credit spread data for other companies. If  $s_i$  is the credit spread for a maturity of  $t_i$ , an estimate of the average risk-neutral hazard rate between times 0 and  $t_i$  is approximately  $s_i/(1-R)$  so that the probability of no default between times 0 and  $t_i$  is  $\exp[-s_i t_i/(1-R)]$ . It follows that

$$q_i = \exp\left(-\frac{s_{i-1}t_{i-1}}{1-R}\right) - \exp\left(-\frac{s_i t_i}{1-R}\right) \quad (6)$$

Using a delta/gamma approximation, the impact on CVA of a small change  $\Delta s$  in all the  $s_i$ 's is

$$\begin{aligned} \Delta(\text{CVA}) = & \sum_{i=1}^n \left[ t_i \exp\left(-\frac{s_i t_i}{1-R}\right) - t_{i-1} \exp\left(-\frac{s_{i-1} t_{i-1}}{1-R}\right) \right] v_i \Delta s \\ & + \frac{1}{2(1-R)} \sum_{i=1}^n \left[ t_{i-1}^2 \exp\left(-\frac{s_{i-1} t_{i-1}}{1-R}\right) - t_i^2 \exp\left(-\frac{s_i t_i}{1-R}\right) \right] v_i (\Delta s)^2 \end{aligned} \quad (7)$$

This equation enables the dependence of CVA on counterparty credit spreads to be included in the bank's model for calculating market risk capital. Equations (5), (6), and (7) correspond to the equations used in the Basel III advanced approach for determining capital for CVA risk.<sup>10</sup>

DVA can be handled similarly. If  $R$  is the recovery rate of the dealer,  $v(t)$  is the value of a derivative that pays off the counterparty's exposure to the dealer at time  $t$ , and  $q(t)$  is the probability density function of the dealer's time to default, then the right hand side of equation (1) gives the DVA. The counterparty's net exposure to the dealer, after taking collateral posted by the dealer into account, can be calculated in an analogous way to that indicated above and the

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<sup>9</sup> A confusion concerning the use of conditional and unconditional default probabilities in calculating CVA was pointed out by Rebonato et al (2010).

<sup>10</sup> Charging capital for the CVA risk associated with credit spreads is likely to encourage dealers to hedge spread risk by buying credit default swaps. This might have the unintended consequence of increasing credit spreads.



approximations indicated in equations (5) and (6) can be used with  $q_i$  defined as the dealer's risk-neutral probability of default between times  $t_{i-1}$  and  $t_i$ ,  $v_i$  defined as the net exposure of the counterparty to the dealer at time  $t_i^*$ , and  $s_i$  defined as the dealer's credit spread for a maturity  $t_i$ .

### 3. Monte Carlo Simulation

In practice, the  $v_i$ 's are almost invariably calculated using Monte Carlo simulation. The market variables affecting the no-default value of a dealer's derivatives with a counterparty are simulated between times 0 and  $T$  in a risk-neutral world.<sup>11</sup> One approach is to arrange the simulation so that the value of the dealer's portfolio with the counterparty is calculated at times  $t_i^* - c$  and  $t_i^*$  ( $1 \leq i \leq n$ ). (This means, On each simulation trial at each time  $t_i^* - c$  ( $1 \leq i \leq n$ ), the collateral is determined using equation (3). Then at time  $t_i^*$  the value of the dealer's portfolio with the counterparty is determined and the net exposure of the dealer to the counterparty is calculated using equation (4). The variable  $v_i$  is estimated as the present value of the average of the calculated net exposures at time  $t_i^*$  ( $1 \leq i \leq n$ ).<sup>12</sup> The variable  $q_i$  is calculated using equation (6).

In practice, some approximations are usually made so that computations are feasible. The model used to value the portfolio during the Monte Carlo simulation may be simpler than either

- a) the model used to simulate the underlying market variables, or
- b) the model used by the dealer for marking to market its book.

Also, the risk-neutral measure for the  $v_i$ 's may be different from that used for the  $q_i$ 's.

The Monte Carlo simulation is computationally quite time consuming. Some dealers calculate CVA once a day, others less frequently. It is normal for the values of the market variables used in the simulation and the portfolio values to be stored. If a new transaction is contemplated with a counterparty, this stored data can then be used to quickly calculate the value of this transaction at each valuation time for each simulation trial. This enables the portfolio value at each valuation

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<sup>11</sup> When interest rates are stochastic a convenient numeraire is the value of a risk-free zero coupon bond maturing at the next valuation time. The Monte Carlo simulation can then be implemented by assuming that a) the returns on market variables between valuation dates and b) the discount rates between valuation dates equal the yield on the numeraire bond.

<sup>12</sup> Only a small amount of additional computation time is necessary to calculate DVA at the same time as CVA.

time for each simulation trial to be updated relatively quickly to reflect the impact of the new transaction.<sup>13</sup>

Dealers also often estimate “peak exposures” which are high percentiles (e.g., 95%) of the exposures at times  $t_i^*$  ( $1 \leq i \leq n$ ). The “maximum peak exposure” is the maximum of the exposures calculated at these times. An interesting point (usually ignored in practice) is that these estimates should in theory be made using the real-world measure to calculate exposures, not the risk-neutral measure.

#### 4. CVA Exposures

Calculating the sensitivity of CVA to a small parallel shift in a counterparty’s credit spread is straightforward using equation (7).

Calculating the first and second partial derivatives of CVA with respect to the underlying market variables is liable to be more time consuming. Consider a market variable  $x$  with initial value  $x_0$ . It is necessary to calculate the effect on the paths sampled of changing  $x_0$  to  $x_0 + \varepsilon$  and  $x_0 - \varepsilon$  for a small  $\varepsilon$  when all random number streams are kept the same. When the variable follows geometric Brownian motion this is not too difficult. A small percentage change at time zero leads to the same small percentage change at all future times on all simulation trials. (This is true both when the volatility is deterministic and when the volatility is stochastic.) For other variables such as those following mean reverting processes, the impact of a change at time zero on the change at future times is liable to depend on the path followed by the market variable.

Suppose that  $v_i^+$  and  $v_i^-$  are the values calculated for  $v_i$  when the initial value of the market variable is  $x_0 + \varepsilon$  and  $x_0 - \varepsilon$ , respectively. From equation (5)

$$\frac{\partial \text{CVA}}{\partial x} = \frac{1 - R}{2\varepsilon} \sum_{i=1}^n q_i (v_i^+ - v_i^-)$$

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<sup>13</sup> Note that the impact of a new transaction on CVA can be positive or negative.

$$\frac{\partial^2 \text{CVA}}{\partial x^2} = \frac{1-R}{\varepsilon^2} \sum_{i=1}^n q_i (v_i^+ + v_i^- - 2v_i)$$

These equations enable CVA risks relating to the underlying market variables to be assessed and hedged. As already mentioned, under Basel III CVA exposures arising from the underlying market variables are not included in the calculation of market risk capital.

## 5. A Model for Wrong-Way and Right-Way Risk

The situation where  $q$  is positively dependent on  $v$  is referred to as “wrong-way” risk. In this case, there is a tendency for a counterparty to default when the dealer’s exposure is relatively high. The situation where  $q$  is negatively dependent on  $v$  is referred to as “right-way” risk. In this case, there is a tendency for a counterparty to default when the dealer’s exposure is relatively low.

A simple way of dealing with wrong-way risk is to use what is termed the “alpha” multiplier to increase  $v(t)$  in the version of the model in which  $v(t)$  and  $q(t)$  are assumed to be independent. The effect of this is to increase CVA by the alpha multiplier. The Basel II rules set alpha equal to 1.4 or allows banks to use their own models, with a floor for alpha of 1.2. This means that, at minimum, the CVA has to be 20% higher than that given by the model where  $q(t)$  and  $v(t)$  are independent. If a bank does not have its own model for wrong way risk it has to be 40% higher. Estimates of alpha reported by banks range from 1.07 to 1.10.

The usual approach to modeling wrong-way risk is to reflect it in the way the  $v$ ’s are calculated. The alpha multiplier approach just mentioned is one example of this approach. Another sometimes used is to set  $v(t)$  equal to the present value of the exposure that is  $k$  standard deviations above the average exposure for some  $k$ .

We will use a different approach. Instead of changing the calculation of  $v(t)$  we change the calculation of  $q(t)$  so that  $q(t)$  depends on the path followed the value of the portfolio up until time  $t$ .

Define  $h(t)$  as the hazard rate of the counterparty at time  $t$ . As before we define  $w(t)$  as the value to the dealer of its portfolio with the counterparty at time  $t$ . A simple model of wrong-way/right-way is to define  $h(t)$  as a deterministic function of  $w(t)$ . Two models we have used are:

$$h(t) = \exp[a(t) + bw(t)] \quad (8)$$

and

$$h(t) = \ln[1 + \exp(a(t) + bw(t))] \quad (9)$$

Each of these models has the property that  $h(t)$  is a monotonic function of  $w(t)$  with  $h(t) > 0$ . The parameter  $b$  measures the sensitivity of  $h(t)$  to  $w(t)$ . The function  $a(t)$ , as will be explained in more detail later, is determined using an iterative search procedure so that the average survival probability, calculated across all simulations, up to any time  $T$  matches that calculated from credit spreads. When  $b > 0$ ,  $h(t)$  is an increasing function of  $w(t)$ , which corresponds to wrong-way risk; when  $b < 0$ ,  $h(t)$  is a decreasing function of  $w(t)$ , which corresponds to right-way risk. In the model given by equation (8),  $h(t)$  increases exponentially as  $w(t)$  takes more extreme values.<sup>14</sup> In the model in equation (9)  $h(t)$  increases linearly as  $w(t)$  takes on more extreme values.<sup>15</sup> For moderate values of  $w(t)$  the two models are very similar. In the results we present, we use the model in equation (9), which is illustrated in Figure 1.

There are two approaches to determining the parameter  $b$ . One is to base it on historical data. Data is collected for past dates on a) credit spreads for the counterparty and b) what the value of the current portfolio with the counterparty would have been. The credit spreads can be converted into hazard rates and the parameter  $b$  can be estimated from whichever of equations (8) and (9) are used. The disadvantage of this is that the counterparty's credit spread may have been heavily influenced by other factors in the past.

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<sup>14</sup> When  $b > 0$  the relevant extreme values are high; when  $b < 0$  they are low.

<sup>15</sup> Either model can be modified to incorporate a cap on  $h(t)$ .

The other approach involves some subjective judgement and it best illustrated with an example. Suppose that the current value to the dealer of the portfolio,  $w(0)$ , is \$3 million and the counterparty's five-year credit spread is 300 basis points. Assuming a recovery rate of 40%, this means that the average five-year hazard rate is 5% per year. Also, suppose that it is estimated that, if  $w(0)$  increased to \$20 million, the spread would rise to 600 basis points, corresponding to an average five-year hazard rate of 10%. Assuming the term structure of hazard rates is flat,<sup>16</sup> when  $w(0) = 3$  million,  $h(0) = 5\%$  and when  $w(0) = 20$  million,  $h(0) = 10\%$ . Solving a pair of simultaneous equations for  $a(0)$  and  $b$ , we find that at time zero, for the model in equation (8),  $b$  is 0.0408 per million dollars while, for the model in equation (9),  $b$  is 0.0423 per million dollars.

The implementation of the model requires only a small change to the procedure for calculating CVA described in Section 3. The values of  $a(t_i^*)$  for  $1 \leq i \leq n$  must be determined so that the average survival probability, across all simulations, up to time  $t_i^*$  equals the survival probability calculated from the term structure of credit spreads. This means that we require

$$\frac{1}{m} \sum_{j=1}^m \left[ \sum_{i=1}^k \exp(-h_{ij} \Delta t) \right] = \exp\left(-\frac{s_k t_k}{1-R}\right) \quad \text{for } 1 \leq k \leq n$$

where  $h_{ij}$  and  $w_{ij}$  are the values of  $h(t_i^*)$  and  $w(t_i^*)$  on the  $j$ th simulation trial and  $m$  is the number of simulation trials.

For the model in equation (8)

$$h_{ij} = \exp\left(a(t_i^*) + bw_{ij}\right)$$

For the model in equation (9)

$$h_{ij} = \ln\left[1 + \exp\left(a(t_i^*) + bw_{ij}\right)\right]$$

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<sup>16</sup> Given the imprecision of any attempt to quantify wrong-way risk this is a reasonable assumption.

First,  $k$  is set equal to 1 and an iterative search is used to determine  $a(t_1^*)$  from the  $w_{1j}$ . This determines the  $h_{1j}$ . Second,  $k$  is set equal to 2 and an iterative search is carried out to determine  $a(t_2^*)$  from the  $w_{2j}$  values and the  $h_{1j}$ . This determines the  $h_{2j}$ ; and so on.

## 6. Numerical Results

We first illustrate the model in equation (9) by assuming that the dealer has a simple portfolio consisting of a single one-year forward foreign exchange transaction.<sup>17</sup> We assume that the principal is \$100 million, the domestic and foreign risk free rates are 5%, the initial exchange rate is 1.0, the delivery exchange rate specified in the forward contract is also 1.0, and the volatility of the exchange rate is 15%. We suppose that the counterparty's credit spread (all maturities) is 125 basis points. Table 1 considers the case where  $b$  is 0.03 per million dollars and the dealer's position is long. Four different situations are considered: no collateral, a threshold of \$10 million with a cure period of 15 days, a threshold of zero with a cure period of 15 days, and an independent amount of \$5 million with a cure period of 15 days. Table 2 provides similar results to Table 1 for the case where  $b$  is 0.03 per million dollars and the dealer is short; Table 3 does so for the case where  $b = -0.03$  per million dollars and the dealer is long; Table 4 does so for the case where  $b = -0.03$  per million dollars and the dealer is short.

The results illustrate that wrong-way and right-way risk have a material effect on the deltas and gammas of CVA as well as on CVA itself. This is true for both the deltas and gammas with respect to the exchange rate and the deltas and gammas with respect to the credit spread. The magnitude of the effect is difficult to predict. Indeed, in some cases, even the direction of effect can be difficult to predict. As shown by the tables, sometimes the direction of the effect depends on the collateral arrangements.

In general, the impact of wrong-way and right-way risk on CVA depends in a complex way on CVA itself and the collateral arrangements. To illustrate this, we randomly generated 250 portfolios. Each portfolio consists of 25 options on one of five different assets. The asset prices

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<sup>17</sup> The results from using the model in equation (8) are similar.

are assumed to follow geometric Brownian motion with pairwise correlations of 0.36. Each option has the following properties.

- (a) It is equally likely to be long or short
- (b) It is equally likely to be a call or a put
- (c) The underlying is equally likely to any one of the five assets
- (d) All maturities between one and five years are equally likely
- (e) All strike prices within 30% of the current asset price are equally likely
- (f) The underlying principal is \$25 million.

The assets do not provide any income. They have an initial price of \$25 and a volatility of 25%. The risk-free rate is 5%, the credit spread of the counterparty (all maturities) is 125 basis points, and the recovery rate is 40%.

Figure 2 provides a scatter plot of the relationship between the dollar change in CVA caused increasing  $b$  from 0 to 0.01 per million dollars and the CVA for  $b=0$  for the 250 portfolios when there is no collateralization. It can be seen that the change in CVA tends to increase as CVA increases. The reason is that as CVA increases there is a tendency for both the mean and standard deviation of the value of the portfolio at future times to increase. The gap between the exposures on high- $w(t)$  paths and low- $w(t)$  paths increases. Wrong-way risk causes the hazard rate to increase dramatically on the high- $w(t)$  paths and decrease (modestly) on the low- $w(t)$  paths.

Figure 3 provides a similar plot to Figure 2 for the situation where the threshold is \$10 million and the cure period is 15 days. The average relationship between the change in CVA and CVA is in this case is one where the change first increases and then decreases. To understand the reason for this, we ignore the cure period and focus on the threshold. The impact of the threshold is to restrict the net exposure to less than \$10 million. For low  $w(t)$ 's this restriction has little effect. However, as  $w(t)$  increases, the net exposure is increasingly impacted by the \$10 million restriction. The gap between the exposure on high- $w(t)$  paths and low- $w(t)$  paths is much less than in the no collateral case. The highest theoretical CVA is achieved when the value of  $w(t)$  is

certain to be above \$10 million at all times. In this case, the net exposure is the same on all  $w(t)$  paths and wrong-way risk has no effect. This explains the pattern observed in Figure 3. The cure period does have an effect on the results, but its effect is less than the effect of the threshold.

Figure 4 provides a similar plot to Figures 2 and 3 for the situation where the threshold is zero and the cure period is 15 days. In this case the net exposure is entirely as a result of the cure period. When  $w(t)$  is positive, the net exposure increases as the standard deviation of the change in  $w(t)$  during the cure period increases. High standard deviations of  $w(t)$  tend to be associated with high  $w(t)$ 's. High  $w(t)$ 's are in turn associated with increases in the hazard rate when there is wrong-way risk. This leads to the pattern shown in Figure 4.

Figure 5 provides a plot for the situation where there is an independent amount equal to \$5 million and a cure period of 15 days. As in the case of Figure 4 the impact of the cure period on the exposure at time  $t$  for a particular  $w(t)$  depends on the standard deviation of the change in  $w(t)$  during the cure period. Indeed, for positive exposures to be generated, the standard deviation has to be sufficiently high that there is a reasonable chance of a \$5 million increase in  $w(t)$  during the cure period. The reasons for the pattern observed are similar to those for Figure 4.

The impact of right-way risk can be examined by changing  $b$  from 0 to  $-0.01$  instead of from 0 to  $+0.01$ . The results are similar to those shown in Figures 2 to 5, except that the change in CVA is negative instead of positive. We have carried out other experiments involving portfolios of interest rate swaps and portfolios. The results are similar to those in Figures 2 to 5.

## 7. Conclusions

We have proposed a simple model for handling wrong-way/right-way risk in the calculation of CVA. The model can be implemented by making a small change to usual method for calculating CVA. It is necessary for the dealer to make a single estimate describing the sensitivity of the counterparty's credit spread to the value of its portfolio with the counterparty.

Tests of the model show that wrong-way and right-way risk have a significant effect on the Greek letters of CVA as well as on CVA itself. Because CVA is such a complex derivative, it is difficult to estimate these effects without a model. Indeed, even the sign of the effect can be counterintuitive.



Further tests involving the random generation of portfolios indicate that when there is no collateral or when collateral is posted with zero threshold or when collateral is posted with an independent amount, the dollar impact of wrong-way risk on CVA tends to increase as CVA increases. The situation where the threshold is materially positive is more interesting. As CVA increases, the impact of wrong way risk tends to first increase and then decrease.

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**Table 1:** Impact of wrong-way risk on CVA for a long forward contract to buy 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15%. The credit spread is 125 basis points for all maturities and the recovery rate is 40%.  $K$  is the threshold in \$ millions,  $c$  is the cure period in days, and  $b$  is the parameter in equation (9).

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = -5$ $c = 15$
CVA (\$ millions) for $b = 0$	0.048	0.036	0.011	0.002
Impact of $b = 0.03$ per \$mm on:				
CVA	54.8%	41.7%	37.3%	53.5%
Delta wrt Exch Rate	32.0%	15.6%	12.8%	39.3%
Gamma wrt Exch Rate	2.6%	-25.4%	17.7%	-0.7%
Delta wrt Spread	53.8%	41.2%	36.8%	52.8%
Gamma wrt Spread	181.8%	124.3%	122.8%	184.3%

**Table 2:** Impact of wrong-way risk on CVA for a short forward contract to sell 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15% . The credit spread is 125 basis points for all maturities and the recovery rate is 40%.  $K$  is the threshold in \$ millions,  $c$  is the cure period in days, and  $b$  is the parameter in equation (9).

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = -5$ $c = 15$
CVA (\$ millions) for $b = 0$	0.048	0.039	0.011	0.001
Impact of $b = 0.03$ per \$mm on:				
CVA	40.5%	34.0%	27.6%	28.9%
Delta wrt Exch Rate	16.2%	7.7%	-1.9%	-341.9%
Gamma wrt Exch Rate	-7.0%	-21.4%	16.4%	26.5%
Delta wrt Spread	40.0%	33.7%	27.4%	28.8%
Gamma wrt Spread	114.8%	91.0%	77.0%	70.7%

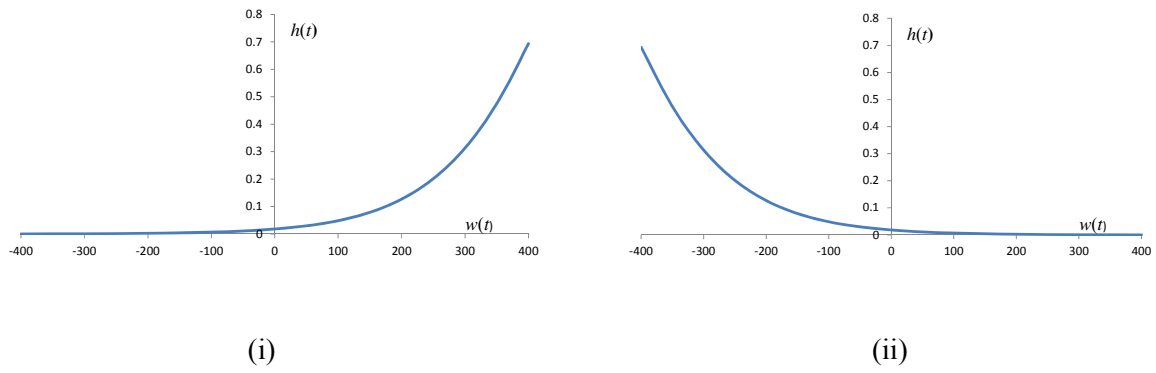
**Table 3:** Impact of right-way risk on CVA for a long forward contract to buy 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15%. The credit spread is 125 basis points for all maturities and the recovery rate is 40%.  $K$  is the threshold in \$ millions,  $c$  is the cure period in days, and  $b$  is the parameter in equation (9).

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = -5$ $c = 15$
CVA (\$ millions) for $b = 0$	0.048	0.036	0.011	0.002
Impact of $b = -0.03$ per \$mm on:				
CVA	-37.5%	-32.7%	-29.1%	-35.7%
Delta wrt Exch Rate	-26.7%	-18.8%	-14.8%	-28.9%
Gamma wrt Exch Rate	-8.2%	11.7%	-16.0%	6.2%
Delta wrt Spread	-37.2%	-32.5%	-28.9%	-35.6%
Gamma wrt Spread	-79.2%	-74.5%	-72.1%	-77.3%

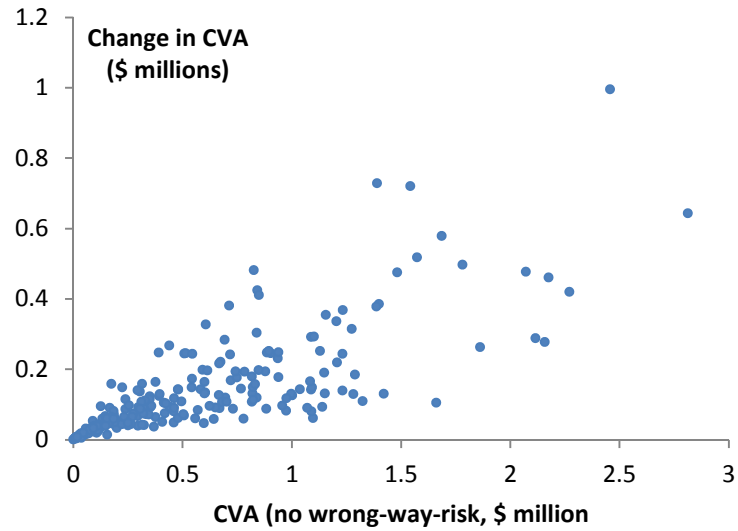
**Table 4:** Impact of right-way risk on CVA for a short forward contract to buy 100 million units of a foreign currency in one year. The current exchange rate is 1.0, the domestic and foreign risk-free interest rates are both 5%, and the volatility of the exchange rate is 15%. The credit spread is 125 basis points for all maturities and the recovery rate is 40%.  $K$  is the threshold in \$ millions,  $c$  is the cure period in days, and  $b$  is the parameter in equation (9).

	No Collateral	$K = 10$ $c = 15$	$K = 0$ $c = 15$	$K = -5$ $c = 15$
CVA (\$ millions) for $b = 0$	0.048	0.039	0.011	0.001
Impact of $b = -0.03$ per \$mm on:				
CVA	-33.9%	-30.8%	-25.9%	-26.9%
Delta wrt Exch Rate	-19.3%	-13.6%	-4.9%	209.1%
Gamma wrt Exch Rate	0.9%	14.4%	-16.7%	-37.5%
Delta wrt Spread	-33.6%	-30.6%	-25.7%	-26.7%
Gamma wrt Spread	-78.8%	-75.5%	-71.3%	-69.0%

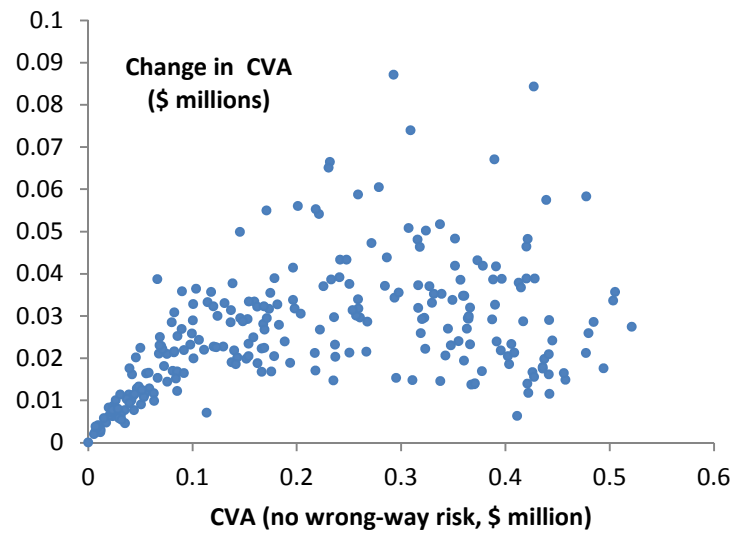
**Figure 1:** The model in equation (9) when (i)  $a(t) = -4$  and  $b = 0.01$  and (ii)  $a(t) = -4$  and  $b = -0.01$



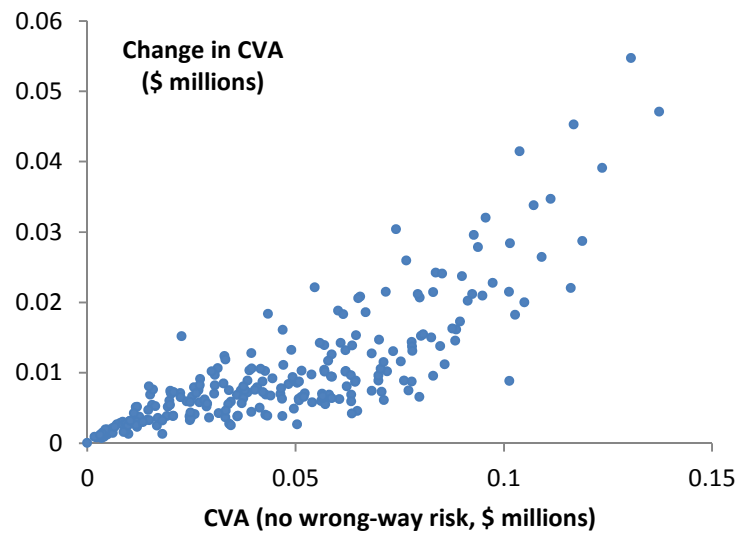
**Figure 2:** Impact of wrong-way risk for 250 portfolios of options when there is no collateral. The horizontal axis shows CVA when  $b=0$ . The vertical axis shows the change in CVA when  $b$  is increased from 0 to 0.01 per million.



**Figure 3:** Impact of wrong-way risk for 250 portfolios of options when the threshold is \$10 million and the cure period is 15 days. The horizontal axis shows CVA when  $b=0$ . The vertical axis shows the change in CVA when  $b$  is increased from 0 to 0.01 per million



**Figure 4:** Impact of wrong-way risk for 250 portfolios of options when the threshold is zero and the cure period is 15 days. The horizontal axis shows CVA when  $b=0$ . The vertical axis shows the change in CVA when  $b$  is increased from 0 to 0.01.





**Figure 5:** Impact of wrong-way risk for 250 portfolios of when there is an independent amount of \$5 million and a cure period of 15 days. The horizontal axis shows CVA when  $b=0$ . The vertical axis shows the change in CVA when  $b$  is increased from 0 to 0.01 per million

