How to calculate CVA with “Wrong Way Risk”

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Introduction

In the area of counterparty credit risk “Wrong Way Risk” is a hot topic. It is required by Basel III and there are lively debates in various trade forums about the right approach. The discussion today is, however, still very academic and therefore of limited practical use.

So what exactly is “Wrong way risk”? According to ISDA “Wrong way risk occurs when exposure to a counterparty is adversely correlated with the credit quality of that counterparty. Wrong way risk, as an additional source of risk, is rightly of concern to banks and regulators.”

Calculating CVA without handling correlation between exposure and default is common practice but it may underestimate the CVA value in the case of the Wrong Way Risk. However, this correlation is in reality very difficult to capture in a large portfolio with different products, currencies and other calculation complexities.

Here we are suggesting a simple approach for simulating the range of CVA values obtained when varying the Wrong Way Risk correlation. Therefore this is a practical method for making a conservative assumption on CVA.

The idea here is to calculate the CVA value by varying the dependency between exposure and time to default. The method consists of correlating exposure behaviour to the default behaviour during the simulation, using a joint distribution built with the marginal distributions of the exposure and the time to default. We repeat this calculation for various stressed values of the correlation. The most conservative result will correspond to the maximum CVA. This allows us to answer the question: How much does the CVA capital charge cost in the case of extreme Wrong Way Risk?

CVA Definition

The CVA is defined as the difference between the risk-free portfolio value and the true portfolio value, taking into account the possibility of a counterparty default. In other words, the CVA is the market value of counterparty credit risk and is given by the risk neutral expectation of the discounted loss $CVA = (1-R)\int_0^T E\left[D(t)\cdot e(t)/\tau = t)\right]dP(t)$ where R is the recovery rate, $D(t)$ the discount factors, $e(t)$ the positive exposures, τ the time to default and $P(t)$ the probability of default distribution.

The full model is very complex to compute and requires the future evolution of market data to be simulated, often for more than 15 years, with the future pricing of the portfolio under these market data scenarios. Additionally it requires simulation of the time of default for each counterparty, simulation of correlation between default and exposure, and finally modeling of collateral and netting. Modeling the impact of all these joint effects is a significant challenge and we would be interested to hear back from institutions who managed to do this.

Common approximation of CVA, ignoring Wrong Way Risk

In order to make the calculation of CVA practical, the common approximation made is to ignore the dependency between exposure and default. Then CVA can then be calculated as

$CVA \approx (1-R)\sum_{t_k \le T}\Delta_k E\left[D(t_k)e(t_k)\right]\left[P(t_k)-P(t_{k-1})\right]$ where $\Delta_k = t_k - t_{k-1}$. In other words, with this approximation it is enough to sum up small increments of time, in which each increment calculates the discounted expected positive future exposure multiplied by the cumulative default probability, and the sum of each increment is then multiplied by the loss given default.

But because Wrong Way Risk is ignored in this approach, we now need to find a way that makes it possible for us to include it.

What about Wrong Way Risk?

In general we know that default events and exposures are inter-dependent. Thus it may happen that the counterparty exposure is at its highest when its probability of default is also at its peak. This case is referred to as Wrong Way Risk. If we de-couple the expected exposure and the default probability as done in the common CVA approximation we will under-estimate the real CVA value in the case of Wrong Way Risk.

In order to calculate CVA with Wrong Way Risk we correlate both the exposure and default behaviour during the simulation using a joint distribution built with the marginal distributions of the exposures and time to default and the given correlation. We define by $e^*(t_k)$ the expected exposure conditional on default at time t_k, i.e. $e^*(t_k) = E\left[D(t_k)\cdot e(t_k) / \tau = t_k\right]$. We then simulate $e^*(t_k)$ at each step t_k, using the random variables $e_k = f(Z_{e_k})$ for exposure, $\tau_k = g(Z_{\tau_k})$ for the time to default and writing $Z_{\tau_k} = \rho \cdot Z_{e_k} + \sqrt{1-\rho^2}\cdot Z$ to correlate e_k and τ_k. Here ρ is the correlation parameter that we will vary between -100% and +100%, and where the special case of $\rho = 0$ converges to the common CVA approximation that ignores Wrong Way Risk.

In this approach we do not have to determine ρ explicitly. We simply make several CVA calculations with different assumptions of the correlation parameter. This gives us a feel of the possible Wrong Way Risk impact and we can determine the worst-case outcome.

In pictures

We have calculated CVA for three real counterparties, which we will here refer to as “ABC”, “XYZ” and “AEF”

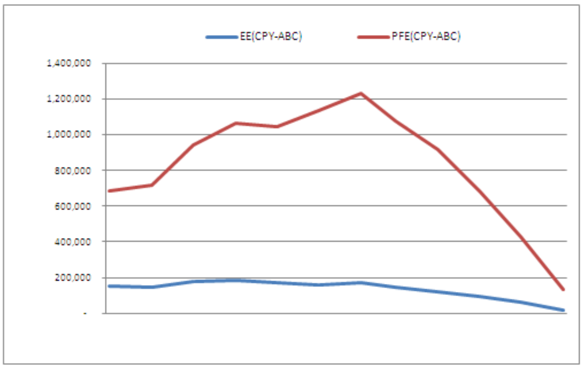
The left side charts show in blue line the evolution of the Expected Exposure (EE) and in the red line the Potential Future Exposure at 99% confidence (PFE) for each of the three counterparties.

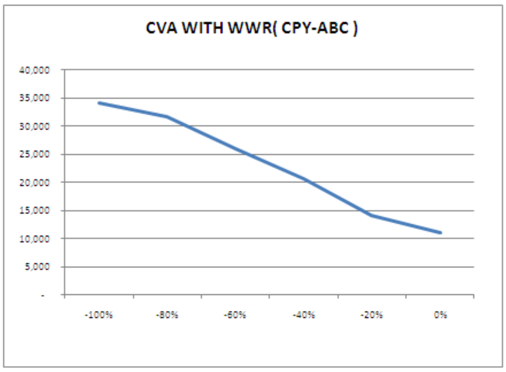
The right side charts show the computed CVA as a function of the correlation. The charts clearly show that when the effect of exposure to default correlation is included, the CVA can more than double compared with the CVA calculated with the common approximation.

Counterparty “ABC”

The left side chart shows the evolution of the Expected Exposure (EE) and the Potential Future Exposure at 99% confidence (PFE) for the counterparty “ABC”.

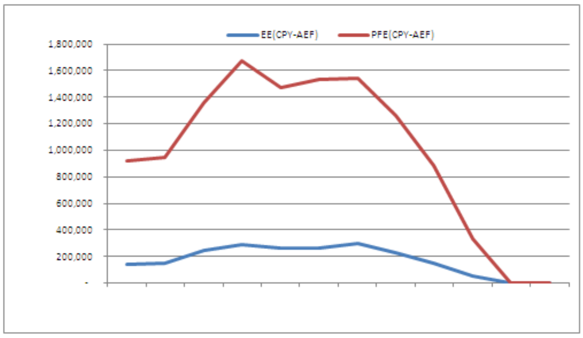
The right side chart shows the computed CVA as a function of the correlation of the counterparty “ABC”. The chart shows the CVA, in the case of extreme Wrong Way Risk, is reached when the correlation is equal to -100%. For this correlation value the CVA capital charge costs 35,000 EUR while the uncorrelated CVA capital charge costs only 11,000 EUR.

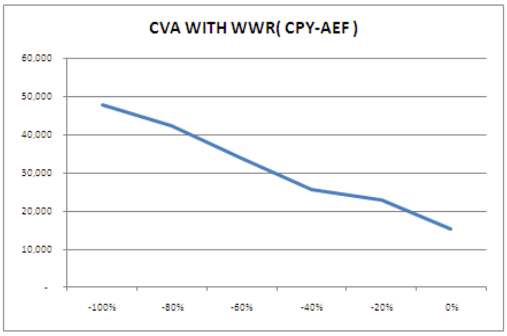




Counterparty “AEF”
The left side chart shows the evolution of the Expected Exposure (EE) and the Potential Future Exposure at 99% confidence (PFE) for the counterparty “AEF”.

The right side chart shows the computed CVA as a function of the correlation of the counterparty “AEF”. For this correlation value the CVA capital charge costs 29,000 EUR while the uncorrelated CVA capital charge costs only 15,000 EUR.





Counterparty “XYZ”
The left side charts show the evolution of the Expected Exposure (EE) and the Potential Future Exposure at 99% confidence (PFE) of the counterparty “XYZ”.

The right side chart shows the computed CVA as a function of the correlation of the counterparty “XYZ”. For this correlation value the CVA capital charge costs 105,000 EUR while the uncorrelated CVA capital charge costs only 39,000 EUR.