Par Credit Default Swap Spread Approximation from Default Probabilities

Purpose: There have been considerable client inquiries on how default probabilities are calculated from credit default swap spreads using our pricing analytic[1]. This is a quantitative process that is not easy to explain intuitively. As such, rather than explain the calculation of default probabilities from credit default swap spreads, this paper focuses on the reverse – approximating par credit default swap spreads from default probabilities.

Definition: A credit default swap is an agreement in which one party buys protection for losses occurring due to a credit event of a reference entity up to the maturity date of the swap. The buyer of protection on a bond, a loan, or a class of bonds or loans, will typically buy protection on the notional of the asset and, upon the occurrence of a credit event, would deliver an obligation of the reference credit in exchange for the protection payout. The protection buyer pays a periodic fee for this protection up to the maturity date, unless a credit event triggers the contingent payment. If such trigger happens, the buyer of protection only needs to pay the accrued fee up to the day of the credit event (standard credit default swap).

Determining the Par Spread: A credit default swap has two valuation legs: fee and contingent. For a par spread, the net present value of both legs must equal to zero.

The valuation of the fee leg is approximated by:

$$PV\ of\ No\ Default\ Fee\ Pmts = S_N \cdot Annuity_N =$$

$$S_N \sum_{i=1}^{N} DF_i \cdot PND_i \cdot \Delta_i$$

where, S_N is the Par Spread for maturity N

DF_i is the Riskless Discount Factor from T_o to T_i

PND_i is the No Default Probability from T_o to T_i

Δ_i is the Accrual Period from T_{i-1} to T_i

If accrual fee is paid upon default, then the valuation of the fee leg is approximated by:

$$PV\ of\ No\ Default\ Fee\ Pmts + PV\ of\ Default\ Accruals =$$

$$S_N \cdot Annuity_N + S_N \cdot Default\ Accrual_N =$$

$$S_N \sum_{i=1}^{N} DF_i \cdot PND_i \cdot \Delta_i + S_N \sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i) \cdot \frac{\Delta_i}{2}$$

where, $(PND_{i-1} - PND_i)$ is the Probability of a Credit Event occurring during period T_{i-1} to T_i

$\frac{\Delta_i}{2}$ is the Average Accrual from T_{i-1} to T_i

The valuation of the contingent leg is approximated by:

$$PV\ of\ Contingent = Contingent_N =$$

$$(1-R) \sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i)$$

where, R is the Recovery Rate of the reference obligation

Therefore, for a par credit default swap,

$$Valuation\ of\ Fee\ Leg = Valuation\ of\ Contingent\ Leg$$

or

$$S_N \sum_{i=1}^{N} DF_i \cdot PND_i \cdot \Delta_i + S_N \sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i) \cdot \frac{\Delta_i}{2} =$$

$$(1-R) \sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i)$$

or

$$S_N = \frac{(1-R) \sum_{i=1}^{N} DF_i \cdot (PND_{i-1} - PND_i)}{\sum_{i=1}^{N} DF_i \cdot PND_i \cdot \Delta_i + DF_i \cdot (PND_{i-1} - PND_i) \cdot \frac{\Delta_i}{2}}$$

Example:

Recovery 30%

Period (i)	Yld$_i$	DF$_i$	PND$_i$	Annuity$_N$	Default Accrual$_N$	Contingent$_N$	Approx S_N (A/360)
0.25	2.35%	99.41%	96.43%	0.240	0.004	0.025	1008
0.5	2.33%	98.84%	93.05%	0.470	0.008	0.048	988
0.75	2.39%	98.25%	89.76%	0.690	0.012	0.071	995
1	2.52%	97.63%	86.56%	0.901	0.016	0.093	998
1.25	2.70%	96.98%	83.91%	1.104	0.019	0.111	972
1.5	2.87%	96.28%	81.51%	1.300	0.022	0.127	945
1.75	3.05%	95.55%	79.41%	1.490	0.025	0.141	915
2	3.22%	94.78%	77.30%	1.673	0.028	0.155	896
2.25	3.37%	93.99%	75.79%	1.851	0.030	0.165	863
2.5	3.52%	93.17%	74.32%	2.024	0.032	0.175	837
2.75	3.67%	92.31%	72.87%	2.192	0.034	0.184	813
3	3.82%	91.44%	71.45%	2.355	0.036	0.193	794
3.25	3.92%	90.55%	70.66%	2.515	0.037	0.198	763
3.5	4.02%	89.64%	69.90%	2.672	0.038	0.203	737
3.75	4.12%	88.72%	69.14%	2.825	0.039	0.208	714
4	4.22%	87.79%	68.37%	2.975	0.040	0.213	695
4.25	4.30%	86.85%	68.22%	3.123	0.040	0.214	665
4.5	4.37%	85.91%	68.06%	3.269	0.040	0.215	639
4.75	4.45%	84.96%	67.91%	3.413	0.040	0.216	615
5	4.52%	84.00%	67.76%	3.555	0.040	0.217	594

[1]Our pricing analytic is available on Orbit (www.morgancredit.com) or on Bloomberg ([ticker] [coupon] [maturity]<Corp>CDSW<Go>).