FACULTY OF SOCIAL SCIENCES
Department of Economics
University of Copenhagen

Master Thesis

Nicholaj Mosegaard Svendsen

Counterparty Credit Risk

Credit Value Adjustment and Wrong-Way Risk

Supervisor: Henrik Olejasz Larsen

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Abstract

The objective of this thesis was to investigate how to price counterparty credit risk through credit value adjustment (CVA). CVA is the market price of counterparty credit risk, defined as the difference between the risk free value of a derivative or a portfolio of derivatives and the value of the derivative or portfolio of derivatives that takes counterparty credit risk and thus the possibility that a counterparty may default, into account.

This thesis takes the viewpoint of an institution entering into interest rate swaps with a sovereign. This allows the focus to be on unilateral CVA, since sovereigns historically have only entered one-way credit support annexes (CSA), and thus eliminated counterparty credit risk on their part. If the sovereign defaults on the contract, the institution suffers a loss. On the other hand, if the institution defaults on the loss, the sovereign should not suffer a loss. That will depend on the CSA. A discussion of credit risk mitigation through primarily netting and collateral is carried out in the thesis. Unilateral CVA and one-way CSA are on retreat, but the transition towards two-way CSA is an ongoing process.

Interest rate swaps are by notional the far most common derivative in the financial markets, and widely used as a risk management instrument by sovereigns. The decline in credit quality of sovereigns following the financial crisis and the credit crisis especially makes for an interesting investigation of counterparty credit risk in derivative contracts between an institution and a sovereign. Germany, Spain and Greece are chosen as counterparties.

The starting point of the thesis is a brief review of the current derivatives market, followed by a theoretical section on how to price interest rate swaps, credit default swaps and swaptions. The theoretical section also covers how to price CVA, to give an understanding of the concept before applying it to interest rate swaps.

CVA consists of a market risk and a credit risk component. To determine the market risk component of CVA, a model for credit exposure is needed. The one-factor Hull White model is used, because it can be fitted to the initial yield curve. This is done by calibrating the model to market data using the Hull White trinomial tree and using swaption quotes as calibration instruments. The calibration is performed by minimizing the difference between swaption prices obtained by the Hull White model and prices computed using Black's model.

After the volatility parameters for the Hull White model are obtained, a Monte Carlo simulation of the model is performed. This results in the exposure profiles needed in order to calculate CVA.

The credit risk component is implied from credit default swap (CDS) spreads as quoted in the market. In order to obtain the cumulative probability of default, the function for the hazard rate is bootstrapped. A brief discussion of CDS used as an indicator for credit quality is carried out.

With the market risk and credit risk component at hand, the CVA calculation is straightforward, given the assumptions made in the thesis. The calculation of CVA applying to the three different counterparties is carried out. The different CVA charges stem alone from the different CDS quotes, since the exposure profiles are identical. The CVA charge increases as the credit quality of the counterparty decreases.

The market risk component of CVA is analyzed, showing that exposures are determined by interest rate volatility and the length of the swap. Netting effects were also studied, finding that netting can be greatly beneficial, reducing exposures significantly on a diversified portfolio.

Finally methods to include wrong way risk in the calculation of CVA are investigated. This inevitably entails combining expected exposures and default risk. This can be done through a correlation approach using a joint market-credit model utilising an ordered scenario copula model. A general counterintuitive conclusion is reached, namely that wrong-way risk increases as the credit quality of the counterparty increases. This is followed by a discussion of calibration issues in models that tries to incorporate wrong-way risk. The method for calculating CVA in Basel III is presented. Finally a discussion of central counterparties which might be the future for no longer over the counter traded derivatives is presented.

Preface

This master thesis has been submitted to the Department of Economics at the University of Copenhagen. The subject was chosen out of an interest in financial markets and CVA being a hot topic in finance.

I would like to thank my supervisor Henrik Olejasz Larsen for valuable guidance and the comments he has provided throughout the process of writing this thesis.

Moreover I would like to thank Rasmus Søndergaard Pedersen and the Department of Economics for providing access to data. I would also like to thank Christina Mosegaard Svendsen, Tina Maria Hansen and Tobias Lee Dahle-Morville for proofreading. Any remaining faults or omissions are naturally my own. Finally I would like to thank Tina Maria Hansen for being indulgent and patient.

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Nicholaj Mosegaard Svendsen

Contents

1. Introduction

In the wake of the financial crisis there has been an increased focus on counterparty credit risk. The pricing of counterparty credit risk through credit value adjustment (CVA) received a great deal of attention, quickly becoming the new normal in global financial markets. Counterparty credit risk was often ignored before the financial crisis, but with the collapse of Bear Sterns and the bankruptcy of Lehman Brothers, this changed forever. It quickly became apparent that no counterparty could be considered risk-free, leading to huge losses on marked to market positions. Counterparty credit risk has become an integral part of Basel III, and institutions are required to calculate CVA as part of their regulatory capital requirement.

This thesis is inspired by changes in financial markets following the financial crisis, specifically the assessment of counterparty credit risk through CVA. In order to determine the fair value of a derivative, CVA must be subtracted from the risk-free value of the derivative. CVA is calculated by some combination of credit exposure and default probability. This could be interesting to examine more closely.

In this thesis counterparty credit risk is quantified through CVA on derivative contracts with sovereigns as seen from an institution. Sovereigns typically enter into one-way credit support annexes (CSA) and therefore rarely post collateral. The sovereigns' counterparty is required to post collateral, thereby eliminating counterparty credit risk as seen from the sovereign. The analysis of CVA is therefore carried on without taking collateral into consideration and calculates unilateral and not bilateral credit value adjustment. This is because a one-way CSA reduces the difference between unilateral and bilateral CVA. Furthermore, sovereigns that use derivatives are typically restricted to the use of interest rate swaps, which are utilized to manage debt obligations. This allows the analysis to focus exclusively on counterparty credit risk present in these products. Germany, Spain and Greece are chosen as counterparties in this thesis due to their varying credit quality.

To begin with this thesis will give a brief introduction of derivatives and the concept of counterparty credit risk. Here it will be briefly outlined how counterparty credit risk can be mitigated. Thereafter, it will be explained how to value various derivative products and CVA. These formulas will be used in the numerical analysis of counterparty credit risk. The Hull

White one-factor model of the short rate will be introduced and analyzed as well as calibrated to market data. The calibrated model will be used in order to perform a Monte Carlo simulation of exposure profiles for the respective counterparties. Credit default swap spreads will be bootstrapped using reduced form modelling to yield default probabilities for the counterparties that will be analyzed in the thesis. When both exposures and default probabilities are obtained, CVA can be calculated. Hereafter the thesis will calculate CVA on interest rate swaps in order to quantify counterparty credit risk. This will provide an understanding of how default probability and exposure influence counterparty credit risk and give an explanation to why counterparty credit risk soared during the financial crisis. Furthermore, this thesis will try to illustrate the benefits from netting by comparing CVA calculated on a portfolio with and without netting. It is often assumed that exposure and default probability are independent in the calculation of CVA, this assumption is almost always violated. A positive dependence is called wrong way risk and it could be interesting to investigate how wrong way risk affects counterparty risk. This thesis will give an example on how wrong way risk can be modelled so the calculation of CVA take wrong way risk into account. Finally, the thesis will briefly discuss CVA as defined in Basel III as well as provide a discussion of central clearing.

1.1 Outline

Section 1 offers an introduction to the thesis. Section 2 provides an overview of derivative markets and briefly introduces some derivative products. Section 3 covers counterparty credit risk, including counterparty risk mitigation, valuation of interest rate swaps, swaptions and credit default swaps. Finally, a formula for CVA is derived. Section 4 covers the market risk part of CVA, calibrates the Hull White model to match market quotes and uses it to simulate exposures. Section 5 calibrates hazard rates from credit default swap quotes, thereby obtains default probabilities implied by the market. Section 6 combines the market risk and the credit risk components of CVA, obtained from the last two sections, and calculates CVA. The impact of netting on interest rate swaps is studied. CVA as described in

Basel III is discussed, as well as central clearing and its' effect on counterparty risk. A method to include wrong way risk in the calculation of CVA is presented. Section 7 concludes.

2. The Derivatives Market

A derivative is a financial contract whose value depends on the performance of some underlying market factors such as, interest rates, currency exchange rates and equity prices. Derivatives are one of the most used financial instruments and include a wide range of financial contracts including, options, swaps, futures and forwards contracts.

- A future or a forward is a contract between two parties to buy or sell an asset at a specified time and at a given price.
- An option is a contract that gives the owner the right, but not the obligation to sell or buy the underlying asset at a given price at/or before a specified future time.
- A swap is a contract in which two counterparties exchange future cash flows on financial instruments.

One of the differences between a future and a forward contract is that the future contract is exchange traded and the forward contract is traded over-the-counter (OTC). Therefore, futures have clearing houses that guarantee the transactions which makes the likelihood of a default on a future contract highly unlikely. The forward contract is traded OTC, therefore, there is a probability of a counterparty defaulting on the contract. Another difference between a forward and a future contract is that future contracts are marked to market daily and settlement can occur several times during the life of the contract. Forward contracts only have one settlement typically at the end of the contract. This means that not alone is a future contract highly unlikely to default it also has minimal exposure and thus contain close to no counterparty risk. A forward contract contains counterparty risk due to the fact that the counterparty can default on the contract and exposures are likely not zero at all times.

Derivatives can be traded either on an exchange or OTC. Derivatives traded on an exchange are typically standardised simple contracts, e.g. futures. Forwards and swaps are mainly traded OTC, because of the flexibility it offers. Options are commonly traded at exchanges as well as OTC and whether the option is traded OTC or at an exchange depends on the underlying asset.

The focus of this thesis is on interest rate options, which are primarily traded OTC. A swaption is a combination of a swap and an option, giving the owner the right, but not the obligation, to enter into an underlying swap. During recent years, there has been an increasing focus on counterparty risk, which has been reflected in a growth in central clearing of derivatives. Central clearing will be further discussed in section 6.5.

The OTC market for derivatives has grown tremendously in the past decades and it is considerably larger than the exchange market for derivatives. The notional amount outstanding totalled USD693 trillion at the end of June 2013 as shown in figure 2.1a – an incomprehensible number, BIS (2013). As can be seen in figure 2.1c, interest rate derivatives accounts for the majority of the notional amounts outstanding totalling USD577 trillion. Foreign exchange and credit derivatives account for almost the rest of the notional amounts outstanding. As depicted in figure 2.1b the gross market values as a share of notional amounts outstanding show that exposure on derivative products are fundamentally different from exposure on an equivalent bond or loan. In the case of e.g. an interest rate swap, it is the exchange of floating against fixed coupons, involving only cash flows. Therefore it has no principal in contrast to a bond or a loan, which reduces the risk substantially. Furthermore, it is only the difference between the floating and fixed coupon that is exchanged. In the case of a counterparty default, the swap contract is marked to market. If it has a positive value for the institution, it suffers a loss – dependent on the exposure and the recovery rate of the counterparty. If it has a negative value, the institution loses nothing.

Figure 2.1a

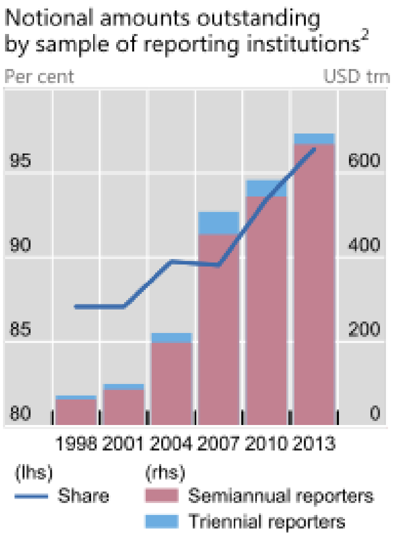


Figure 2.1b

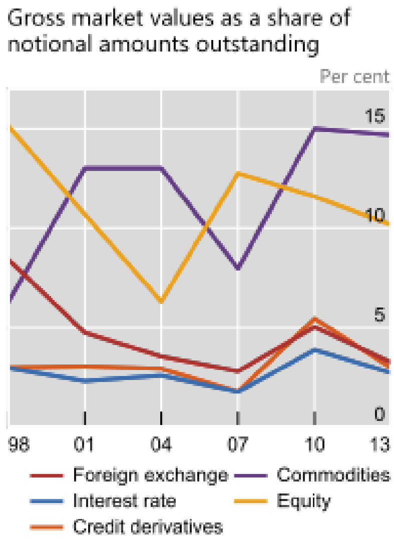
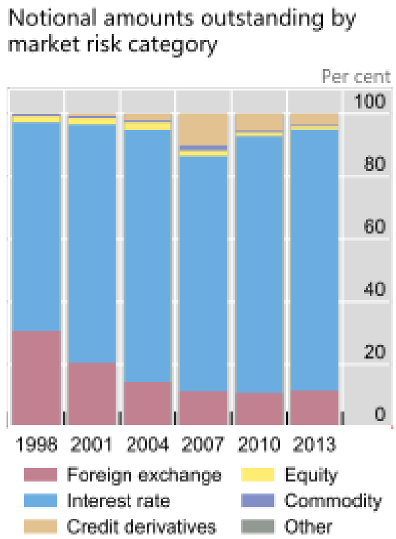


Figure 2.1c



[1] Adjusted for inter-dealer double-counting. [2] Share refers to the percentage of semiannual reporters in the global total.

Source: BIS (2013), Page 2

3. Counterparty Credit Risk

Counterparty credit risk, or counterparty risk, is the credit risk between two counterparties in a derivatives trade. Counterparty risk is of major importance in OTC derivatives, since counterparty risks in exchange traded derivatives are mitigated, as mentioned in section 2 in the explanation of future versus forward. Counterparty risk is distinguished from other forms of credit risk, because the underlying contracts future value is unforeseeable in both magnitude and sign. Counterparty risk is defined by some combination of market risk and credit risk;

- Market risk is represented by the credit exposure.
- Credit risk is defined by the likelihood of default of the counterparty.

Credit value adjustment (CVA) values the counterparty risk in derivative trades. The recovery value is ignored in exposure calculations by convention. The recovery value will have an influence on the level of counterparty risk, since a recovery rate of 1 (which is equivalent to 100 percent recovered) would correspond to no counterparty risk.

The credit exposure (or just exposure) is the loss on a derivative contract in the event of a defaulting counterparty. A positive value of a derivative contract results in exposure, since the institution will suffer a loss if the counterparty defaults. A negative value of a derivative contract will have zero exposure, as the contract is cancelled in the event of a default of the counterparty. Therefore, in order to quantify the exposure of a derivative contract, the current and future exposure must be calculated. It should be noted that the exposure is conditional on counterparty default, since the exposure is only relevant if the counterparty defaults. The default probability measures how likely the counterparty is to default over a given time horizon.

The calculation of CVA should account for all counterparty credit risks in a derivative contract. The factors that define CVA are;

- The exposure
- The default probability of the counterparty
- The default probability of the institution itself
- Netting

- Collateral
- Hedging

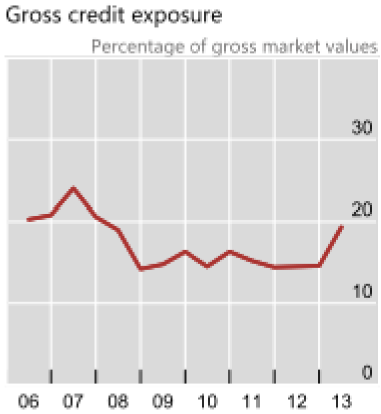
The terms exposure and default probability are defined above. In the case of bilateral CVA, both the default probability of the counterparty and the institution influences the CVA charge. The focus here will be on unilateral CVA. CVA can e.g. be hedged by buying credit default swaps referenced on the counterparty. Hedging counterparty risk will not be examined, but note that hedging counterparty risk is a very complex task due to the highly uncertain future exposures.

3.1 Counterparty Credit Risk Mitigation

The International Swaps and Derivatives Association (ISDA) is a global trade organization for OTC derivatives. ISDA maintain the industry standard documentation, the ISDA Master Agreement[1], which specifies the general terms and conditions of the agreed derivatives contract between parties such as, netting and collateral among others. Counterparty credit risk can be mitigated in a number of ways. One of the ways to reduce counterparty risk is to reduce the exposure. Another way is to reduce the loss in the event of a counterparty default (thereby also reducing exposure). This is done by netting and collateralization. The effects of netting and collateralization are illustrated in figure 3.1 below. The gross credit exposure constitutes less than 20 percent of the gross market values.

[1] Can be found at http://www.isda.org/publications/isdamasteragrmnt.aspx

Figure 3.1



Source BIS (2013), Page 6

3.1.1 Netting

The concept of netting allows positive and negative values of derivative contracts in a netting set to off-set each other, thereby reducing credit risk by reducing exposure.

Payment netting covers the scenario where an institution is to make payments to and receive payments from the same counterparty. If a USD110m payment on a fixed swap is to be made and a USD100m payment on a floating swap is received, then if the contracts are under the same netting set the institution would make a payment worth USD10m, reducing credit risk. Gregory (2012)[2] reports that on the day Lehman Brothers declared bankruptcy, KfW made an automated transfer of EUR300m, despite the fact that Lehman Brothers would obviously not honour their obligations, showing the drawbacks of not utilizing the benefits from netting.

Closeout netting is the most relevant scenario when mitigating counterparty credit risk and covers the situation where the counterparty has defaulted on a contract. Closeout netting gives the institution the right to terminate transactions with the concerned counterparty and stop any payments. Most importantly, it gives the institution the right to offset positive and negative mark to market values of transactions in the event of counterparty default. The closeout amount is the result of the summed transaction values, and thus represents a more fitting measure of the exposure under the netting set. The growth in OTC derivatives

[2] Page 47

markets can largely be accounted to netting, because netting ensures the credit exposure in the market grows at a lower rate than the growth of the notional amount outstanding, according to Gregory (2012)[3].

3.1.2 Collateralization

Collateralization is another tool to mitigate counterparty risk. Collateralization in a derivatives contract means that one or both parties are required to post collateral when the underlying asset moves against the relevant counterparties favour. The collateral offsets the exposure in the sense that if a counterparty defaults, the other party will keep the collateral to offset losses. The use of collateral depends on the credit quality of the institution. According to a market survey by ISDA (2010), sovereigns very rarely post collateral, whereas dealer banks are often required to post collateral. The fact that sovereigns rarely post collateral reflects that higher rated counterparties can obtain favourable terms in collateral agreements. It is possible to extend the ISDA Master Agreement with a credit support annex (CSA), which ensures enforceability of the agreed collateral postings, as well as standardising the agreement with regards to eligible collateral, timing of collateral transfers, calculation of collateral, haircuts, thresholds and downgrade triggers among others.

3.2 Quantification Of Counterparty Credit Risk

This thesis takes the viewpoint of an institution entering into a derivatives contract with a sovereign. This allows the analysis to be done without taking collateral into account, because sovereigns very rarely post collateral and historically have only entered one-way CSAs. The Danish sovereign has begun a transition from one-way to two-way CSAs, which illustrates that the extended use of one-way CSAs by sovereigns is on retreat, SLOG (2013).
The focus on sovereigns allows the analysis to concentrate strictly on interest rate swaps, since they are a widely used risk management instrument by sovereigns and interest rate swaps are the only derivative Denmark uses, SLOG (2013). As mentioned above interest rate

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swaps are the most used derivative in the financial markets, and thus makes it an interesting and relevant case.

CVA is the risk free value of a derivative minus the risky value of the derivative. As mentioned in section 3 above, calculation of CVA involves calculating exposures and default probabilities on the derivatives in question. Calculating exposures and default probabilities rely on the ability to value the derivatives contracts correctly. For that reason this section examines the derivatives needed for the analysis and calculation of CVA, namely interest rate swaps, swaptions and credit default swaps.

3.2.1 Interest Rate Swaps

An interest rate swap (IRS) is a derivatives contract between two parties exchanging a series of fixed interest rate payments for a series of floating interest rate payments over a given period of time. The counterparty paying the floating rate has entered into a receiver swap, whereas the counterparty paying the fixed rate has entered into a payer swap. An IRS thus has a fixed and a floating leg. Valuing the IRS consists of pricing each leg individually and subtracting the value of one leg from the other, depending on whether it is a receiver or payer swap.

The floating leg of a swap is linked to some xIBOR (currency x Interbank Offered Rate). The payment frequency of the floating leg depends on the tenor of the xIBOR, which is 6 month (6M) on the EURIBOR (The Euro Interbank Offered Rate) used in this thesis. The outline in Linderstrøm (2012) is followed. The present value of the floating leg for a set of coverages $\delta_{S+1}^{Float},...,\delta_E^{Float}$ and dates $T_S,...,T_E$ spaced apart by the tenor δ_i^{Float} is given by

$$PV_t^{Float} = \sum_{i=S+1}^{E} \delta_i^{Float} F(t,T_{i-1},T_i) N_i P(t,T_i) \tag{3.1}$$

$F(t,T_{i-1},T_i)$ is the interest rate between period T_{i-1} and T_i, N_i is the notional amount and $P(t,T_i)$ is the discount factor.

The fixed leg is valued in a similar fashion

$$PV_t^{Fixed} = \sum_{i=S+1}^{E} \delta_i^{Fixed} K N_i P(t,T_i) \tag{3.2}$$

Where $\delta_{S+1}^{Fixed},...,\delta_{E}^{Fixed}$ is a set of coverages and dates $T_S,...,T_E$ spaced apart by the tenor δ_i^{Fixed}. K is the fixed rate paid in the swap.

Valuing the legs thus consist of summing all discounted payments on the swap.

For the payer swap the floating leg is an asset and the fixed leg is a liability, it is given that

$$PV_t^{Payer} = \sum_{i=S+1}^{E} \delta_i^{Float} F(t,T_{i-1},T_i) N_i P(t,T_i) - \sum_{i=S+1}^{E} \delta_i^{Fixed} K N_i P(t,T_i) \tag{3.3}$$

It follows that $PV_t^{\mathrm{Re}ceiver} = -PV_t^{Payer}$.

By convention it is customary to trade swaps at zero net present value, the par swap rate $R(t,T_S,T_E)$ can be defined as the fixed rate that ensures that $PV_t^{Float} = PV_t^{Fixed}$

$$R(t,T_S,T_E) = \frac{\delta_i^{Float} F(t,T_{i-1},T_i) N_i P(t,T_i)}{\delta_i^{Fixed} K N_i P(t,T_i)} \tag{3.4}$$

3.2.2 Swaptions

A European swaption (on an IRS) gives the right, but not the obligation, to enter into an IRS at a specified future time at a specified fixed rate. European swaptions (from now on swaptions) are used to calibrate an interest rate model in section 4.3. This section leads to the Black (1976) formula for pricing swaptions.

Following Linderstrøm (2012) and the notation as in section 3.2.1, the options exercise time and the swaps start time is denoted by T_S. The swaps maturity time is denoted by T_E. The value of a payer swaption with physical settlement is the discounted positive values of the par swap rates minus the fixed rate. Cash settlement is not considered in this thesis because cash settlement does not guarantee arbitrage free pricing, see Mercurio (2008).

$$PV_{T_S}^{Payer} = \sum_{i=S+1}^{E} \delta_i P(T_S,T_i) \max(R(t,T_S,T_E) - K, 0) \tag{3.5}$$

Where $\sum_{i=S+1}^{E} \delta_i P(T_S,T_i)$ is the annuity factor. In order to use quoted market swaptions as a calibration instrument, the valuation is made in a risk neutral world. A numeraire trick is utilized by using $\sum_{i=S+1}^{E} \delta_i P(T_S, T_i)$ as the numeraire, see e.g. Hull (2012) or Hansen (2009). The market is thereby made arbitrage free and the expectation of equation (3.5) yields the fair price of the swaption

$$\begin{aligned} PV_T^{Payer} &= \delta_i P(T_S, T_i) E_t^A \left[\frac{\delta_i P(T_S, T_i) \max(R(t, T_S, T_E) - K, 0)}{\delta_i P(T_S, T_i)} \right] \\ &= \delta_i P(T_S, T_i) E_t^A \left[\max(R(t, T_S, T_E) - K, 0) \right] \end{aligned} \tag{3.6}$$

This leads to the Black (1976) formula for pricing payer swaptions

$$PV_T^{Payer} = \delta_i P(T_S, T_i) \left[R(t, T_S, T_E) \Phi(d_1) - K\Phi(d_2) \right] \tag{3.7}$$

Where

$$\begin{aligned} d_1 &= \frac{\ln(R(t, T_S, T_E) / K) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} \\ d_2 &= \frac{\ln(R(t, T_S, T_E) / K) - \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \end{aligned} \tag{3.8}$$

Φ denotes the cumulative standard normal distribution and σ the volatility. In order to value a swaption the forward swap rate and the annuity factor as seen from time t must be calculated and used in equation (3.7) together with the volatility. Note that the volatility is assumed to be constant, and that the price movement of the forward follows a lognormal distribution. The receiver swaption can be valued by utilizing the put-call parity for swaptions as in Linderstrøm (2012)

Forward Starting Payer Swap(*K*) = Payer Swaption(*K*) – Receiver Swaption(*K*) (3.9)

$$PV_T^{\text{Receiver}} = \delta_i P(T_S, T_i) \left[K\Phi(-d_2) - R(t, T_S, T_E) \Phi(-d_1) \right] \tag{3.10}$$

3.2.3 Credit Default Swaps

A credit default swap (CDS) is a derivative that has many similarities with an insurance, in the sense that the protection seller will compensate the protection buyer in the event of a default of the reference security. There are however many differences between a traditional insurance contract and a CDS. One of the main differences is, that on a CDS the protection buyer does not need to own the underlying referenced security. A CDS contract can thus be used for speculation on debt. The CDS is the main credit derivative instrument. It can be used for hedging or speculating purposes. It is however important to bear in mind that CDSs themselves are a source of counterparty risk when using CDSs as a hedging tool. CDSs can efficiently transfer credit risk, but they can also be treacherous in the sense that they can increase counterparty risk instead of mitigating it. The wrong way risk embedded in a CDS makes it a dangerous instrument if not handled correctly, Gregory (2012)[4].

The protection buyer makes a stream of payments to the protection seller (called the fee leg), until maturity or default of the referenced security. This thesis will not differentiate between soft and hard credit events, and the enforceability of soft credit events. Neither will it differentiate between physical and cash settlement, since all CDS contracts were converted to cash settlement following the CDS big-bang of 2009, Linderstrøm (2012) and Gregory (2012)[5]. The other leg in a CDS contract is called the protection leg, where the protection buyer pays the loss given default (LGD) to the protection seller in the case of default. The recovery rate (R) is defined as $LGD = 1 - R$.

As in Linderstrøm (2012), an intensity model is used to be able to price CDSs. τ is denoted as the default time and the indicator function is defined as $N_t = 1_{\{\tau \le t\}}$. The indicator function can be modelled using intensities and points out whether a default has occurred. If $Q(\tau \le t)$ denotes the probability of default, intensity λ_t is given by

$$\lambda_t = \lim_{h \downarrow 0} Q(\tau \le t + h \mid t < \tau) \tag{3.11}$$

Where $\lambda_t \cdot \Delta t$ is the probability of a default occurring over the interval $(t, t + \Delta t)$, which means that the process for the intensity has to be non-negative. The deterministic time varying intensity function that will be employed is given by

$$Q(\tau \le T) = 1 - \exp\left(-\int_0^T \lambda(u) du\right) \tag{3.12}$$

It follows that the probability of survival is given by $1 - Q(\tau \le t) = Q(T < \tau)$. The intensity process can be modelled as a stochastic process, also known as a Cox process. However, as mentioned in Linderstrøm (2012), stochasticity of the intensity function is not needed when pricing linear credit derivatives. It follows that the credit risky zero coupon bond without

[4] Page 18 and chapter 10
[5] Chapter 10

recovery, denoted by $B(t,T)$ paying 1 if $T < \tau$ and 0 if default has occurred, can be valued as

$$\begin{aligned} B(t,T) &= E_t^Q\left[\exp\left(-\int_t^T r_s + \lambda_s\right)ds\right] \\ &= E_t^Q\left[\exp\left(-\int_t^T r_s ds\right)\right]E_t^Q\left[\exp\left(-\int_t^T \lambda_s ds\right)\right] \\ &= P(t,T)Q(T < \tau) \end{aligned} \tag{3.13}$$

Following Linderstrøm (2012) the simplifying assumptions, that default can only occur on coupon dates and that the function of the hazard rate is piecewise constant are made. The latter is elaborated in section 5.1. As in the case with interest rate swaps, the value of the CDS is determined by the value of each leg and thus pricing a CDS involves pricing both legs. The fee leg is valued for a CDS contract with starting time T_S, maturity T_E and the spread defined as C

$$\begin{aligned} PV_t^{Fee} &= C \cdot \sum_{S+1}^{E} \delta_i P(t,T_i)Q(T_i < \tau) \\ &= C \cdot \sum_{S+1}^{E} \delta_i P(t,T_i)\left[1 - \exp\left(-\int_0^T \lambda(u)du\right)\right] \end{aligned} \tag{3.14}$$

Valuing the protection leg is more complex, since both the loss given default and the default time are stochastic. It is however custom to assume a constant recovery rate of 40 %. Thereby a constant loss given default of 60 %. The present value of the protection leg is then given by

$$\begin{aligned} PV_t^{\mathrm{Pr}otection} &= LGD \cdot \sum_{S+1}^{E} P(t,T_i)Q(T_{i-1} \le \tau < T_i) \\ &= (1-R) \cdot \sum_{S+1}^{E} P(t,T_i)\left[\exp\left(-\int_t^{T_{i-1}} \lambda(u)du\right) - \exp\left(-\int_t^{T_i} \lambda(u)du\right)\right] \end{aligned} \tag{3.15}$$

This implies that the par spread of a CDS contract, that causes the protection leg and the fee leg to have the same value at inception, is given by

$$G = \frac{(1-R)\sum_{S+1}^{E} P(t,T_i)\left[\exp(-\int_t^{T_{i-1}} \lambda(u)du) - \exp(-\int_t^{T_i} \lambda(u)du)\right]}{\sum_{S+1}^{E} \delta_i P(t,T_i)\exp(-\int_t^{T_i} \lambda(u)du)} \tag{3.16}$$

The intensities represents risk neutral probabilities, which means that the intensities can be adjusted to match market prices. This is the method of bootstrapping or calibrating the intensity curve, a method applied in section 5.2.

3.2.4 Credit Value Adjustment

CVA measures the counterparty risk in a derivatives contract. In order to do that the exposure and likelihood of default must be known. This section provides a derivation of the CVA formula later used in the calculation of CVA in IRS agreements with sovereigns.

The derivations follow Gregory (2012)[6]. The starting point of the derivation is to find an expression for the risky value of a netted set of derivatives positions $\tilde{V}(t,T)$ with maturity T. The risk free value of the netted set of derivatives is given by $V(t,T)$ and default time as τ. Consequently $V(s,T)$ is the future uncertain mark to market value for $(t < s \leq T)$.

If the counterparty does not default before time T the risky value is the same as the risk free value of the position and the payoff is given by

$$1_{\tau>T} V(t,T) \tag{3.17}$$

Where $1_{\tau>T}$ is an indicator function of whether default has occurred (as was the case when valuing CDS). In this case it takes the value of 1 if no default has occurred until time T.

If there is a default before time T the payoff consists of two parts - the payoffs before the default time and the payoff at default. Therefore, the payoff in case of default is given by first the cash flows received until the default time, which equals the risk free value up until that point

$$1_{\tau\leq T} V(t,\tau) \tag{3.18}$$

And second the payoff at default

$$1_{\tau\leq T} \left(R \max(V(\tau,T),0) + \min(V(\tau,T),0) \right) \tag{3.19}$$

Where the first part of the equation indicates that the institution, in the case of default, will receive the recovery rate R multiplied with the risk free value of the derivatives, if the mark

[6] Appendix 12A and 12B

to market value is positive. The last part of the equation show that the institution will suffer the full loss if the mark to market value of the positions is negative.

In order to value the payoffs, a probability measure is needed. This thesis uses the risk neutral measure, which is most relevant in pricing applications such as CVA, Gregory (2012)[7]. However, the CVA calculation in Basel III uses the physical measure, which is more relevant for risk management. Please note that the Basel III CVA formula uses both the physical and the risk-neutral measure, in the sense that the exposures are estimated from historical data, whereas the default probabilities are estimated using market data, BCBS (2012). Under the risk neutral probability measure the future value of the risky derivatives is

$$\tilde{V}(t,T) = E^Q\left[1_{\tau>T}V(t,T) + 1_{\tau\le T}V(t,\tau) + 1_{\tau\le T}\left(R\max(V(\tau,T),0) + \min(V(\tau,T),0)\right)\right] \quad (3.20)$$

Using the fact that

$$\min(V(\tau,T),0) = V(\tau,T) - \max(V(\tau,T),0) \quad (3.21)$$

It is given that

$$\begin{aligned}\tilde{V}(t,T) &= E^Q\left[1_{\tau>T}V(t,T) + 1_{\tau\le T}V(t,\tau) + 1_{\tau\le T}\left(R\max(V(\tau,T),0) + V(\tau,T) - \max(V(\tau,T),0)\right)\right] \\ &= E^Q\left[1_{\tau>T}V(t,T) + 1_{\tau\le T}V(t,\tau) + 1_{\tau\le T}\left((R-1)\max(V(\tau,T),0) + V(\tau,T)\right)\right]\end{aligned} \quad (3.22)$$

And using the fact that

$$1_{\tau>T}V(t,T) + 1_{\tau\le T}V(t,T) = V(t,T) \quad (3.23)$$

It is given that

$$\tilde{V}(t,T) = V(t,T) - E^Q\left[(1-R)1_{\tau\le T}\max(V(\tau,T),0)\right] \quad (3.24)$$

Remembering that CVA is defined as the difference between the risk free and the risky derivative, we have that

$$\tilde{V}(t,T) = V(t,T) - CVA \quad (3.25)$$

Or

$$CVA = E^Q\left[(1-R)1_{\tau\le T}\max(V(\tau,T),0)\right] \quad (3.26)$$

If the assumption that $V(s,T)$ includes discounting is dropped, the CVA formula can be written as

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$$CVA = E^{Q}\left[(1-R)1_{\tau\le T}\max(V(\tau,T),0)\frac{\beta(t)}{\beta(\tau)}\right] \tag{3.27}$$

Where $\beta(s)$ is the value of the money market account at time s, as in Gregory (2012)[8].

If the common assumption that the recovery rate is independent of exposures is made, CVA can be written as

$$CVA = (1-R)E^{Q}\left[1_{\tau\le T}\max(V(\tau,T),0)\frac{\beta(t)}{\beta(\tau)}\right] \tag{3.28}$$

This is common practice, since modelling a stochastic dependence between the exposures, default probabilities and recovery rates is a highly complex and demanding task. This is sometimes referred to as double wrong way risk. In order to make the quantification of counterparty risk manageable to solve, the above assumption is made.

The formula for CVA is derived by stating

$$V^{*}(u,T) = V(u,T) \mid \tau = u \tag{3.29}$$

Where $V^{*}(u,T)$ is the exposure at a future date $V(u,T)$, where the default is known to occur. Therefore CVA can be written as

$$CVA(t,T) = (1-R)E^{Q}\left[1_{u\le T}\max(V^{*}(u,T),0)\right] \tag{3.30}$$

If the further simplifying assumption that the exposure and the default probability are independent is made, thus ignoring wrong way risk, the formula for CVA can be found by taking the expectation to equation (3.30)

$$CVA(t,T) = (1-R)E^{Q}\left[\int_{t}^{T}P(t,T_i)\max(V^{*}(u,T),0)dF(t,u)\right] \tag{3.31}$$

Where $P(t,T_i)$ is the discount factor and $F(t,u)$ is the cumulative probability of default.

Wrong way risk is the case where there is a positive dependence between exposure and default probability, thereby making it more likely for counterparties to default when the exposure is high. Wrong way risk is not considered a major factor in simple products such as interest rate swaps, but credit derivatives such as credit default swaps can contain a great deal of wrong way risk. A more thorough discussion of wrong way risk and a method to include wrong way risk in the calculation of CVA will be presented in section 6.4. For now the

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common assumption of independence between exposures and default probabilities is made, in order to quantify the two elements that constitute CVA – besides the recovery rate.
The expected exposure is defined as the mean of exposure, and the exposure is defined as

$$E_i(t) = \max(V_i(t), 0) \tag{3.32}$$

Which means that the formula for CVA under the risk neutral measure, assuming that defaults are deterministic, can be written as

$$CVA(t,T) = (1-R)\left[\int_t^T P(t,T_i)EE(u,T)dF(t,u)\right] \tag{3.33}$$

In practice CVA is calculated by approximating the integral with a finite sum over the valuation dates

$$CVA(t,T) = (1-R)\sum_{i=1}^{m} P(t,T_i)EE(t,t_i)\left[F(t,t_i) - F(t,t_{i-1})\right] \tag{3.34}$$

As long as m is sufficiently large, this is a good approximation – and the one used in the MATLAB implementation in section 6. CVA can also be written as

$$CVA(t,T) = (1-\mathrm{R})\sum_{i=1}^{m} P(t,T_i)EE(t_i)\, PD(t_{i-1},t_i) \tag{3.35}$$

Where

$$PD(t_{i-1},t_i) = F(t,t_i) - F(t,t_{i-1}) \tag{3.36}$$

A number of assumptions are made in the estimation of CVA. It is assumed that there is no settlement risk, no liquidity risk and no systemic risk. These are very common assumptions to make when trying to value counterparty credit risk. The replacement costs are valued by marking them to market risk-free, which ignores possible bid-ask spreads, Gregory (2012)[9]. The recovery rate is assumed to be constant in this thesis, it can however vary significantly. As shown in Gregory (2012)[10], recovery rates for CDS auctions in 2008 varied from close to 0 to close to 1.
Since the thesis takes the viewpoint of an institution entering into an IRS deal with a sovereign, collateral is ignored in the calculation of CVA. This is done because, as argued in section 3.1.2 and 3.2, sovereigns very rarely post collateral and thus collateral can be

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ignored when valuing CVA as seen from the institution. Sovereigns do, however, require their counterparties to post collateral to mitigate their own counterparty risk.

CVA can be both unilateral and bilateral (BCVA). This thesis focuses exclusively on pricing unilateral CVA. Nevertheless, a quick note on BCVA. BCVA is the calculation of CVA under the assumption that the institution as well as the counterparty can default on the contract. This breaks with the going-concern principle. BCVA has the benefit compared to CVA, that it creates symmetry between the two counterparties. Symmetry makes objective fair value calculation easier ensuring that the counterparties agree on the fair value of the trade in question. With unilateral pricing, the two counterparties will both require a credit risk premium and thus reaching an agreement on the fair value of a trade is difficult. BCVA can be calculated given the assumption of no wrong way risk as

$$\begin{aligned} BCVA = {} & (1-R)\sum_{j=1}^{n} EE(t_j)[1-PD_I(0,t_{j-1})]PD_C(t_{j-1},t_j) \\ & +(1-R)\sum_{j=1}^{n} NEE(t_j)[1-PD_C(0,t_{j-1})]PD_I(t_{j-1},t_j) \end{aligned} \quad (3.37)$$

As in Gregory (2012)[11], where $EE(t_j)$ is the discounted expected exposure, $[1-PD_I(0,t_{j-1})]$ is the probability, the institution has not defaulted yet. $PD_C(t_{j-1},t_j)$ is the probability of the counterparty defaulting in the given interval, $NEE(t_j)$ is the discounted negative expected exposure, $[1-PD_C(0,t_{j-1})]$ is the probability the counterparty has not defaulted yet and $PD_I(t_{j-1},t_j)$ is the probability of the institution defaulting in the interval. The institution calculates a CVA charge on themselves in order to calculate BCVA. Consequently the institutions' CVA charge can decrease when their own credit quality declines. Thereby creating a counterintuitive effect due to the fact that a sufficiently large decrease in the CVA charge can lead to booked profits, most recently experienced during the euro crisis, Gregory (2012)[12]. Or the reverse where low credit risk has a negative impact on earnings due to BCVA. The second term in equation (3.36) is also called debit value adjustment (DVA). DVA is controversial since the institution makes a gain on negative exposures and their own risk of defaulting. However, gains can only be realised in the event of a default of the institution,

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which means that DVA cannot be realised. Basel III requires DVA to be fully deducted from the calculation of the capital requirement, BCBS (2011).

Following the brief introduction of BCVA and discussion of DVA, it can be argued that BCVA is not as relevant in the case studied in this thesis, as in trades with two equal counterparties. This is because one-way CSAs make the difference between BCVA and CVA less significant, as the exposure on the contract seen from the counterparty will always be close to zero. Especially in the case of sovereigns, emphasized by the CSA applying to Danish interest rate swap contracts. The Danish CSA has a threshold level of zero and a minimum transfer amount of DKK500,000. This accentuates that negative exposures will be at very low levels, thereby minimizing the effect from the DVA in equation (3.36). The probability of the institutions' own default will however still make the BCVA charge smaller than the CVA charge. This thesis values counterparty risk from a going-concern consideration, and therefore focuses on unilateral CVA.

4. Market Risk

As previously mentioned CVA consists of a market risk and a credit risk component. This section determines the market risk component. The market risk component of CVA is the discounted exposure. If the standard assumption that the expected recovery value R is constant is made and it is assumed that the credit exposure and default probability are independent, then as shown earlier the CVA formula can be written as

$$CVA = (1-\mathrm{R})\sum_{i=1}^{m} P(t,T_i)EE(t_i)PD(t_{i-1},t_i) \tag{4.1}$$

Where $1-\mathrm{R}$ is the loss given default (LGD), $P(t,T_i)$ is the discount factor, EE is the expected exposure and PD is the probability of default, as defined earlier.

In order to determine the market risk component of CVA, a model for credit exposure is needed to estimate exposures. This is done by simulating interest rates, which makes it possible to value the derivatives in question, and hence the expected exposure in the simulation framework. Monte Carlo methods are used due to their degree of flexibility, which is also the reason practitioners have adopted this approach.

One of the challenges of computing CVA is that CVA analysis has long time horizons compared to VaR, or other risk measures. This greatly increases the computation time for simulations, which is one of the drawbacks of computational simulation per se. An estimator is often used to price the underlying swaps to ease the computation time. Computational time is described as one of the largest weaknesses in CVA setups according to Pykhtin and Zhu (2007). Another challenge in the computation of CVA is that it consists of both a market and a credit risk component, which are likely not independent. And to make matters worse the recovery rate is likely to be dependent as well. This complicates the derivation of CVA significantly, and for the sake of parsimony I will treat them as if they were independent in order to determine the market risk component.

4.1 Hull White model

One of the historically most used interest rate models is the one-factor Hull White model. It was first proposed by Hull and White (1990) in a paper that explored extensions of the Vasicek (1977) model, which is why they originally referred to it as the extended-Vasicek model. The Hull White model can be characterized as the Ho-Lee model with mean reversion, or the Vasicek model with a time-dependent reversion level, which enables it to fit the initial term structure. The Hull White model is an affine term structure model, as are all Vasicek types of models. This means that the continuously compounded spot rate is an affine function in the short rate

$$R(t,T) = \alpha(t,T) + \beta(t,T)r(t) \tag{4.2}$$

The pioneering work by Vasicek (1977) deriving an arbitrage free price for interest rate derivatives follows the arguments used by Black and Scholes (1973). This led to a series of time-homogeneous models taking advantage of the fact, that the short rate dynamics only depends on constant coefficients and that the entire term structure could be described as a one-dimensional dynamic. The last method is used in the Hull White one factor model as well. The small number of parameters in the classical time-homogeneous models led to a poor fit to market data, which led Hull White to explore possible extensions to the Vasicek model. The need to be able to fit the observed yield curve led Hull and White into introducing a time-varying parameter in the Vasicek model.

Multi-factor models are able to fit the yield curve better and thus capture parallel shifts, twists and butterfly movements, while a one-factor model is only able to capture parallel moves with limited steepening and flattening movements. This is partly due to the fact that all rates are perfectly correlated in one-factor models, whereas observed interest rates are known to exhibit some decorrelation. This makes multi-factor models an obvious choice, since they have flexibility to match market prices. However multi-factor models come with other drawbacks, as they complicate implementation as well as reduce analytical tractability considerably. Increasing the number of risk factors can overcomplicate the model as well as lead to unrealistic scenarios and increase computation time as well. With the need to address wrong way risk in the calculation of CVA, and thus co-dependencies between market and credit risk factors, a parsimonious model will often be preferred, according to Gregory (2012)[13]. Even though interest rate models with two or three factors may depict the yield curve more realistically, the trade off faced is that it will greatly complicate implementation. Therefore, this thesis proceeds with the Hull White model - being aware of the limitations in the selected model, including the theoretical possibility of negative rates, due to the assumption of normally distributed instantaneous spot rates. However there is not a single interest rate model fitting all purposes. Even moving to non Gaussian models or models including more risk factors, difficulties capturing typical market shapes can still be experienced, in the sense that stochastic volatility models may be needed to account for the volatility smile, see Gregory (2012)[14] or Brigo and Mercurio (2006). Hull White (1994) have also developed a two-factor model, however if not explicitly stated, this thesis refers to the one-factor Hull White model.

The short rate in the Hull White model follows a Brownian motion with time dependent mean-reversion. The one-factor model only has one source of risk, the short rate r, which is assumed to follow the stochastic differential equation (SDE):

$$dr = [\theta(t) - ar]dt + \sigma dz \tag{4.3}$$

In a risk neutral world, dr is the change in the short rate, after a small change in time, dt. a is the mean reversion rate and σ is the volatility of the short rate. Both a and σ are positive constants. The two volatility parameters determine the relative volatility of short and long rates, and the overall volatility level. A high value of a will cause short term volatility to dampen quickly, thereby reducing long term volatility. dz is a Weiner process, and $\theta(t)$ is the time-dependent drift function that allows the model to fit the term structure. The SDE can be solved by applying Itô's lemma:

$$d(re^{at}) = e^{at}dr + are^{at}dt = (\theta dt + \sigma dz)e^{at} \tag{4.4}$$

Integrate over both sides:

$$\begin{aligned} &r(t)e^{at} - r(s)e^{as} = \int_s^t \theta(u)e^{au}du + \sigma\int_s^t e^{au}dz(u) \\ &\Rightarrow r(t) = r(s)e^{-a(t-s)} + \int_s^t \theta(u)e^{-a(t-u)}du + \sigma\int_s^t e^{-a(t-u)}dz(u) \end{aligned} \tag{4.5}$$

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The drift function $\theta(t)$ can be calculated from the initial term structure. As explained in Brigo and Mercurio (2006)[15], it can be shown that:

$$F(t,T) = -\frac{\partial}{\partial T}\left[\ln P(t,T)\right] \tag{4.6}$$

And because bond prices in the Hull White model are given by

$$\mathrm{P}(\mathrm{t},\mathrm{T}) = \mathrm{A}(\mathrm{t},\mathrm{T})e^{-rB(t,T)} \tag{4.7}$$

It is given that

$$\begin{aligned} F(t,T) &= -\frac{\partial}{\partial T}\left[\ln \mathrm{A}(\mathrm{t},\mathrm{T}) - rB(t,T)\right] \\ \frac{\partial F(t,T)}{\partial r} &= -\frac{\partial}{\partial r}\frac{\partial}{\partial T}\left[\ln \mathrm{A}(\mathrm{t},\mathrm{T}) - rB(t,T)\right] = e^{-a(T-t)} \end{aligned} \tag{4.8}$$

Using Itô's lemma

$$dF(0,T) = ... + \sigma e^{-a(T-t)}dz \tag{4.9}$$

As explained in the solution manual to Hull (2012)[16], this has zero drift in a world that is forward risk neutral with respect to $\mathrm{P}(\mathrm{t},\mathrm{T})$. In this world the market price of risk is $-\sigma B(t,T)$. If we move to a world where the market price of risk is zero, the drift increases to $\sigma^2 e^{-a(T-t)}B(t,T)$. If we integrate this from $t=0$ to $t=T$ it can be seen that the forward rate between time 0 and time T grows with

$$\frac{\sigma^2}{2a^2}(1-e^{-aT})^2 \tag{4.10}$$

In this world where the market price of risk is zero, the futures price has a growth rate equal to zero. The forward price equals the futures price at time T, so it follows that at time zero the futures price must exceed the forward price with

$$\frac{\sigma^2}{2a^2}(1-e^{-aT})^2 \tag{4.11}$$

[15] Chapter 3

[16] See problem 30.14

$G(0,t)$ is defined as the instantaneous futures rate for maturity t, so we have that

$$G(0,t) - F(0,t) = \frac{\sigma^2}{2a^2}(1 - e^{-at})^2 \tag{4.12}$$

And that

$$G_t(0,t) - F_t(0,t) = \frac{\sigma^2}{a}(1 - e^{at})e^{-at} \tag{4.13}$$

In the risk-neutral world the expected value of r at time t is the futures rate, $G(0,t)$. So we must have that the expected growth of r at time t is $G_t(0,t) - a\left(r - G(0,t)\right)$, so that $\theta(t) - ar = G_t(0,t) - a\left(r - G(0,t)\right)$, which means that

$$\theta(t) = G_t(0,t) + a\left(G(0,t)\right) \tag{4.14}$$

It follows that the drift function is defined as

$$\begin{aligned}\theta(t) &= F_t(0,t) + aF(0,t) + \frac{\sigma^2}{a}(1 - e^{-at})e^{-at} + \frac{\sigma^2}{2a^2}(1 - e^{-at})^2 \\ &= F_t(0,t) + aF(0,t) + \frac{\sigma^2}{2a}(1 - e^{-2at})\end{aligned} \tag{4.15}$$

Where the market observed instantaneous forward rate at time t is $F(0,t)$ and $F_t(0,t)$ is the partial derivative with respect to time.

$$F(0,T) = -\frac{\partial \ln P(0,T)}{\partial T} \tag{4.16}$$

$$F_t(0,t) = \frac{\partial F(0,t)}{\partial T} \tag{4.17}$$

$P(0,T)$ is the maturity T discount factor. Which means that $r(t)$ can be rewritten as

$$r(t) = r(s)e^{-a(t-s)} + \alpha(t) - \alpha(s)e^{-a(t-s)} + \sigma\int_s^t e^{-a(t-u)}dz(u) \tag{4.18}$$

Where

$$\alpha(t) = F(0,t) + \frac{\sigma^2}{2a^2}(1 - e^{-at})^2 \tag{4.19}$$

As in Brigo and Mercurio (2006) it should be evident that $r(t)$ conditional on F_s is normally distributed with mean and variance given by

$$E\left\{r(t)\middle|F_s\right\}=r(s)e^{-a(t-s)}+\alpha(t)-\alpha(s)e^{-a(t-s)}$$
$$Var\left\{r(t)\middle|F_s\right\}=\frac{\sigma^2}{2a}\left[1-e^{-2a(t-s)}\right] \tag{4.20}$$

And following the outline in Brigo and Mercurio (2006) it can further be shown that

$$\int_t^T r(u)du\,|F_t \sim N\left(B(t,T)\left[r(t)-\alpha(t)\right]+\ln\frac{P(0,t)}{P(0,T)}+\frac{1}{2}\left[V(0,T)-V(0,t)\right],V(t,T)\right) \tag{4.21}$$

Where

$$B(t,T)=\frac{1}{a}\left[1-e^{-a(T-t)}\right]$$
$$V(t,T)=\frac{\sigma^2}{a^2}\left[T-t+\frac{2}{a}e^{-a(T-t)}-\frac{1}{2a}e^{-2a(T-t)}-\frac{3}{2a}\right] \tag{4.22}$$

And it is given that

$$\mathrm{P}(\mathrm{t},\mathrm{T})=\mathrm{A}(\mathrm{t},\mathrm{T})e^{-rB(t,T)} \tag{4.23}$$

Where

$$\mathrm{A}(\mathrm{t},\mathrm{T})=\frac{P(0,T)}{P(0,t)}\exp\left\{B(t,T)F(0,t)-\frac{\sigma^2}{4a}(1-e^{-2at})B(t,T)^2\right\} \tag{4.24}$$

Or

$$\ln\mathrm{A}(\mathrm{t},\mathrm{T})=\frac{P(0,T)}{P(0,t)}+B(t,T)F(0,t)-\frac{\sigma^2}{4a^3}(e^{-aT}-e^{-at})^2(e^{2at}-1) \tag{4.25}$$

Even though the model has many analytical properties, such as being able to price European options, the thesis proceeds by representing the Hull White model in a trinomial tree. This enables the calibration of the model parameters $a(t)$, $\sigma(t)$ and $\theta(t)$ to market prices. This is done by constructing a recombining trinomial tree to represent movements in the instantaneous spot rate under the dynamics of the Hull White model. The construction of the trinomial tree is a two-stage procedure, as described in Hull and White (1994). The first stage is to construct a preliminary tree. The second step is to adjust the parameters of the model in order to have the tree reproduce market data.

The model parameters can be estimated from either historical data or calibrated by fitting the model to market data. The two methods are fundamentally different, as parameters estimated from historical data use the real-world or physical measure whereas parameters estimated from arbitrage-based pricing uses the risk-neutral measure. The first approach is typically used in risk management when estimating potential future exposure (PFE), whereas the latter is used in pricing applications such as CVA. This is the reason why this thesis proceeds with the risk-neutral measure. A discussion of parameter estimation techniques when using historical interest rate data can be found in Meng, Kaplin and Levy (2013). To calibrate the model to market data involves fitting the model to observed prices and volatilities in the market. The selection of calibration instruments depends on the problem at hand.

4.2 Data

The starting point for the construction of the Hull White tree is to extract the zero coupon yield curve from market data. This is done by bootstrapping the zero curve from the market quotes. I have chosen the 6 month EURIBOR, various EURIBOR forwards as well as interest rate swaps against the 6 month EURIBOR with different maturities. An argument for choosing EONIA instead could be made, since this seems to be the new standard for interest rate swaps reflecting the increased credit risk in XIBOR rates following the financial crisis. However many existing swaps are based on EURIBOR, for example all swaps in Denmark's portfolio are based on EURIBOR or CIBOR, even though the Danish government will only enter swaps based on EONIA or CITA in the future SLOG (2013).

One could also make an argument for using a dual-curve setup, to appropriately account for the non-trivial spread between XIBOR and EONIA rates. However, this approach will not be followed here. The market data needed to strip the yield curve can be found in table 4.1 below.

Table 4.1: Market Data Mid Prices

FRA	mid	IRS	
6M*	0,34%	2Y	0,65%
1X7	0,37%	3Y	0,89%
2X8	0,41%	4Y	1,16%
3X9	0,44%	5Y	1,41%
4X10	0,47%	6Y	1,62%
5X11	0,51%	7Y	1,80%
6X12	0,54%	8Y	1,96%
7X13	0,57%	9Y	2,10%
8X14	0,60%	10Y	2,23%
9X15	0,63%	15Y	2,63%
6M* is the 6		20Y	2,74%
month EURIBOR		30Y	2,74%

Source: Bloomberg EUR006M, EUSAXY Curncy and EUFRXY Curncy, where XY is the time interval, September 2, 2013

As mentioned above the observed market quotes need to be bootstrapped to obtain the zero coupon yield curve, since the observed rates are par rates. This can become a complex non-linear problem, which is why it is solved by formulating the curve calibration problem as a least squares optimization problem, as explained in Linderstrøm (2012) and discussed in Boor (2001).

$$\underset{P}{Min}\left\| B(P)-A \right\|^2 \tag{4.26}$$

Where $A=\{a_1,...,a_N\}^T$ is the set of market quotes, $P=\{p_1,...,p_N\}^T$ is a set of parameters and $B(P)=\{b_1,...,b_N\}^T$ is a set of model quotes calculated using equation (4.4).

The optimization has the following first order condition:

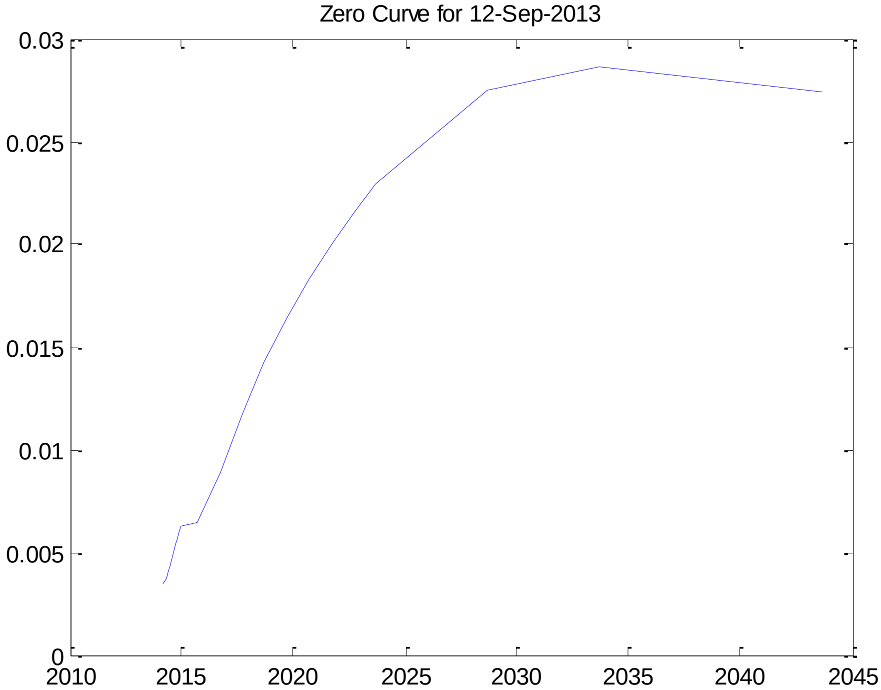
$$\left(B(P)-A\right)^{\mathrm{T}}\frac{\partial B(P)}{\partial P}=0 \tag{4.27}$$

Where $\frac{\partial B(P)}{\partial P}$ denotes the Jacobian matrix:

$$\underset{N\times M}{\frac{\partial B(P)}{\partial P}}=\begin{pmatrix} \frac{\partial b_1}{\partial p_1} & \cdots & \frac{\partial b_1}{\partial p_M} \\ \vdots & \ddots & \vdots \\ \frac{\partial b_N}{\partial p_1} & \cdots & \frac{\partial b_N}{\partial p_M} \end{pmatrix} \tag{4.28}$$

Hermite cubic spline is chosen as interpolation method, because that method ensures a smooth yield curve. The non-differential linear interpolation method seemed to do fairly well from eyeball statistics based on the zero coupon rate curve. However non-differential interpolation methods perform rather poorly when used to calibrate forward rates. Smooth forward xIBOR curves are needed for precise valuation of the interest rate swaps, which is why the Hermite cubic spline method is preferred and the linear interpolation method is discarded. See Hagan and West (2006) for a more thorough review of interpolation methods. The zero coupon curve calibrated from the market data in table 4.1 is depicted in figure 4.1. The MATLAB code can be found in Appendix A.1. Note that this is done using the built in MATLAB function *pyld2zero*, which calls the *zbtprice* function to bootstrap the zero curve from the par bonds. This is done by using the theoretical par bond arbitrage and yield interpolation. Normally the *zbtprice* function uses the *interp1q* function for the bootstrapping procedure, which is a quick one dimensional interpolation. In this thesis this is changed and instead the *interp1* function is called, this allows a broader variety of interpolation methods to be chosen. The *spline* method in MATLAB was chosen, namely because it has the nice properties, of being differentiable, and that the first and second derivatives are continuous. The hump in the short end of the zero curve is due to the shift from FRA to IRS quotes.

Figure 4.1



The only other input data needed to be able to calibrate the model to market data is a set of swaption quotes. More precisely a swaption volatility matrix that contains the Black implied volatilities with different exercise dates and underlying swap maturities. Black's model is used to price the European swaptions, Black (1976). These prices are then compared to the prices obtained from the Hull White model, and minimizing the difference between the observed market prices and the predicted prices of the model calibrates the model. The swaption data used is depicted in table 4.2 below.

Table 4.2: Swaption Volatilities

Option Expiries	Swap Maturities 1	2	3	4	5	7	10	15	20
1	78.63	63.74	57.33	51.77	47.82	39.58	32.99	28.35	27.02
2	65.10	52.05	46.81	42.85	40.14	34.70	30.26	27.12	26.03
3	50.45	43.40	40.18	37.47	35.31	31.15	28.03	25.73	25.11
4	42.80	36.81	35.25	33.32	31.62	28.62	26.51	24.71	24.30
5	36.99	33.16	31.54	30.11	28.34	26.68	25.51	24.02	23.55
7	29.26	27.34	26.33	25.53	24.93	23.76	23.44	22.88	22.16
10	23.71	22.86	22.57	22.37	22.25	22.02	22.25	21.12	20.93
15	21.56	22.57	22.69	22.81	22.97	22.12	22.22	20.42	19.32
20	22.43	23.96	23.95	23.94	23.86	22.49	21.77	19.79	17.43

Source: Bloomberg EUSVXXYY, where XXYY indicate expiry and maturity, September 2, 2013

4.3 The trinomial tree

This thesis proceed by following the two-stage procedure for constructing trinomial trees presented in Hull and White (1994) and Hull and White (1996). Recall that the Hull White model for the instantaneous short rate is

$$dr = [\theta(t) - ar]dt + \sigma dz \tag{4.29}$$

As mentioned earlier, the first stage is to construct a preliminary tree for r. This is done by defining a new variable r^* obtained from r by setting $\theta(t) = 0$ and the initial value of r to zero. It is given that

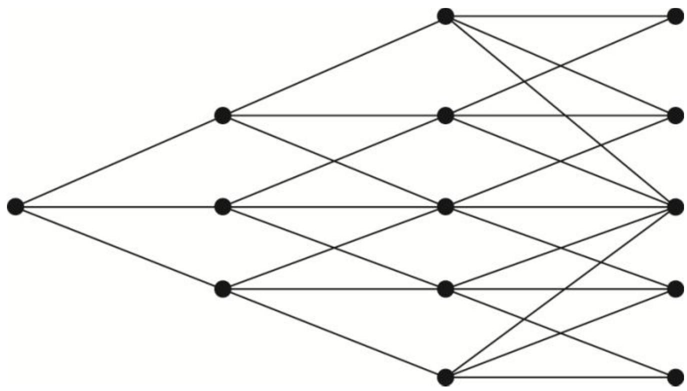
$$dr^* = -ar^* dt + \sigma dz \tag{4.30}$$

This process is symmetrical around the initial value of r. The objective is to construct a preliminary tree similar to the tree shown in figure 4.2.

4.3.1 Stage one

Since the drift rate has been set to zero, the central node at each time step has $r^* = 0$. The vertical distance between the nodes on the tree is set to $\Delta r^* = \sqrt{3V}$, where V is the variance of the change in r in time Δt. Hull and White (1994) explains that from the viewpoint of error minimization this is a good choice of Δr^*. In order to build a preliminary tree as shown below, it must determined which branching method that will apply at each node. The three different branching methods are depicted in figure 4.3.

Figure 4.2: Initial HW Tree



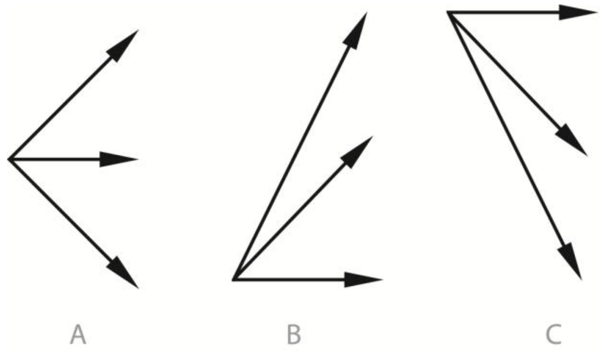
The probabilities at each node are calculated to match the mean and standard deviation of the change in $r*$ for the process of $dr*$. Node (i, j) is defined as the node for which $t = i\Delta t$ and $r = j\Delta r$. Furthermore p_u, p_m and p_d are defined as the probabilities for upward, middle or downward branches coming from a node. If r is at node (i, j), the expected change in mean and variance from the next time step is $j\Delta rM$ and V respectively. The probabilities must sum to unity and the probability for all three possible movements must be positive. The probabilities in the case of branching methods A, B and C are given by

$$p_u = \frac{1}{6} + \frac{j^2M^2 + jM}{2} \qquad p_u = \frac{1}{6} + \frac{j^2M^2 - jM}{2} \qquad p_u = \frac{7}{6} + \frac{j^2M^2 - 3jM}{2}$$

$$\text{A: } p_m = \frac{2}{3} - j^2M^2 \qquad \text{B: } p_m = -\frac{1}{3} - j^2M^2 + 2jM \quad \text{C: } p_m = -\frac{1}{3} - j^2M^2 - 2jM$$

$$p_d = \frac{1}{6} + \frac{j^2M^2 - jM}{2} \qquad p_d = \frac{7}{6} + \frac{j^2M^2 - 3jM}{2} \qquad p_d = \frac{1}{6} + \frac{j^2M^2 + jM}{2}$$

Figure 4.3: Branching Methods



Most of the time, the appropriate branching method is A. However, to ensure that all probabilities are positive, at some point it is necessary to switch from branching method A to C when j is sufficiently large. The same is the case when j is sufficiently small, given that $a > 0$. $j_{\min}$ and $j_{\max}$ are defined as values of j where the branching switch from process A to B and C respectively. Hull and White (1994) show that the probabilities are always positive if $j_{\max}$ is set equal to an integer between $-0.184/M$ and $-0.816/M$ and $j_{\min} = -j_{\max}$. According to Hull and White (1994) it is most efficient to set $j_{\max}$ equal to the smallest integer greater than $-0.184/M$, and again $j_{\min} = -j_{\max}$.

Choosing initial values for the parameters a and σ completes the first step, as a trinomial tree for the simplified process as illustrated in figure 4.2 above is now constructed.

4.3.2 Stage two

The second stage is to introduce the correct time-varying drift $\theta(t)$. In order to do this the preliminary tree is converted from a tree for r^* into a tree for r. This is done by displacing the nodes at time $i\Delta t$ by an amount α_i, so that

$$\alpha(t) = r(t) - r^*(t) \tag{4.31}$$

The value in the new tree at node (i, j) equals the value in the old tree at node (i, j) plus α_i. The new tree is illustrated in figure 4.4 below. The probabilities in the tree are unchanged, and the values of the α_i's are chosen to match the initial term structure observed in the market. In practice this is done by defining $Q_{i,j}$ as the present value of a security that pays off 1 if node (i, j) is reached and zero otherwise. The $Q_{i,j}$'s and α_i's can now be calculated by using forward induction. Formally this is done by assuming that the $Q_{i,j}$'s for $i \leq m\,(m \geq 0)$ have been determined. As explained in Hull (2012) it is given that $Q_{0,0} = 1$. The next step is now to determine α_m so the tree correctly prices a discount bond maturing at $(m+1)\Delta t$ at time 0. The interest rate at node (m, j) is $\alpha_m + j\Delta r$ which means that the price of the discount bound maturing at $(m+1)\Delta t$ is given by

$$P_{m+1} = \sum_{j=-n_m}^{n_m} Q_{m,j} \exp\left[-(\alpha_m + j\Delta r)\Delta t\right] \tag{4.32}$$

Where n_m is the number of nodes on each side of the central node at time $m\Delta t$. This equation has the solution

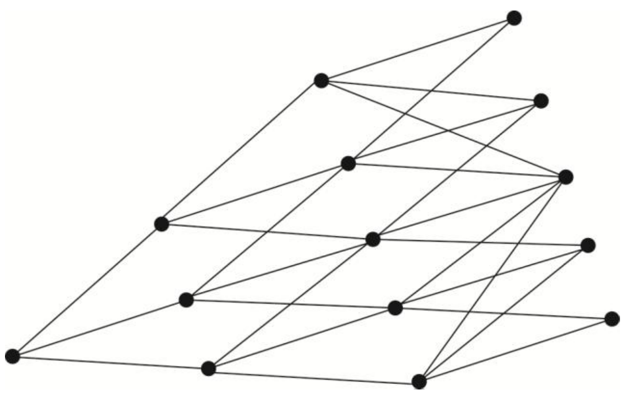
$$\alpha_m = \frac{\sum_{j=-n_m}^{n_m} Q_{m,j}\, \mathrm{e}^{-j\Delta r\Delta t} - \ln P_{m+1}}{\Delta t} \tag{4.33}$$

When α_m has been calculated, then $Q_{i,j}$ for $i = 1 + m$ can be calculated since

$$Q_{m+1,j} = \sum_k Q_{m,k} q(k, j) \exp\left[-(\alpha_m + k\Delta r)\Delta t\right] \tag{4.34}$$

Where $q(k, j)$ is the probability of a move from node (m,k) to node $(m+1, j)$. This concludes the tree building procedure.

Figure 4.4: HW Tree



The MATLAB implementation of the Hull White trinomial tree can be found in Appendix A.1, and follows an example on how to price swaptions from the MATLAB website (MATLAB example A). Note that the swaptions are valued in the Hull White model using the financial

toolbox provided by MATLAB. Specifically the function *swaptionbyhw* values the swaptions, that depends on the *hwtree* function as input and the *hwtengine* which is where the actual Hull White tree as outlined above is constructed. The *hwtree* takes a volatility process specification and an interest rate specification, as well as a specification of the time structure as inputs when building the Hull White trinomial tree. The calibration consists of minimizing the difference between observed market prices and the model's predicted prices. As mentioned before the market prices are calculated in Black's model using Black's implied volatility matrix. The model prices are found by pricing the receiver swaptions at each node j through each time step of the tree. $Swaption\,value = \max\{0, value\,of\;\; swap\}$.
This means that the underlying swaps must be evaluated at each node, which is done by discounting the fixed leg and the floating leg cash flows. This process is repeated at each node back through the tree to obtain the present value of the receiver swaptions. When prices for all swaptions are found, they can be compared to the observed market prices.
A least squares non linear minimization is used to find the Hull White volatility parameters minimizing the difference between the observed prices and the model prices. The optimization is achieved by iterations.
The starting parameters are set to $a = 0.1$ and $\sigma = 0.01$, as well as lower and upper bounds denoted by *lb* and *ub* respectively. These can be varied and can have an effect on the calibration. However the values chosen seem to be the convention. The calibration of the Hull White model results in the following volatility parameters. The output as well as the MATLAB code can be found in Appendix B.1 and A.1.

$$a = 0.0425$$
$$\sigma = 0.0104$$

4.4 Monte Carlo Simulation

After obtaining the volatility parameters of the Hull White model, it is possible to set up the Monte Carlo simulation scheme. Monte Carlo simulation has become the standard approach for simulating exposures, Gregory (2012)[17]. The advantages of using this approach are that it copes with many of the complexities arising when simulating exposure profiles such as path dependency, netting, collateralisation and transaction specifics.

One of the drawbacks of Monte Carlo methods is that it is more complex. Therefore, more time consuming. According to Pykhtin and Zhu (2007), the Monte Carlo simulation is likely to be the bottleneck of the CVA calculation and to speed up the computations one may need to simplify the underlying pricing functions by using approximations in the estimation of exposures. This thesis uses approximations in the pricing functions in the implementation, but it seems that practitioners have moved on from Monte Carlo methods to American Monte Carlo methods, sometimes called least squares Monte Carlo, Risk Magazine (2012). Remembering that the volatility parameters obtained are calibrated by arbitrage-based pricing, all estimated exposures uses the risk-neutral probability measure.

The Hull White model was defined as

$$dr = [\theta(t) - ar]dt + \sigma dz \tag{4.35}$$

When a path for the short rate has been simulated, the full yield curve at each simulation date is generated, in order to have each scenario contain the full term structure. It is given that

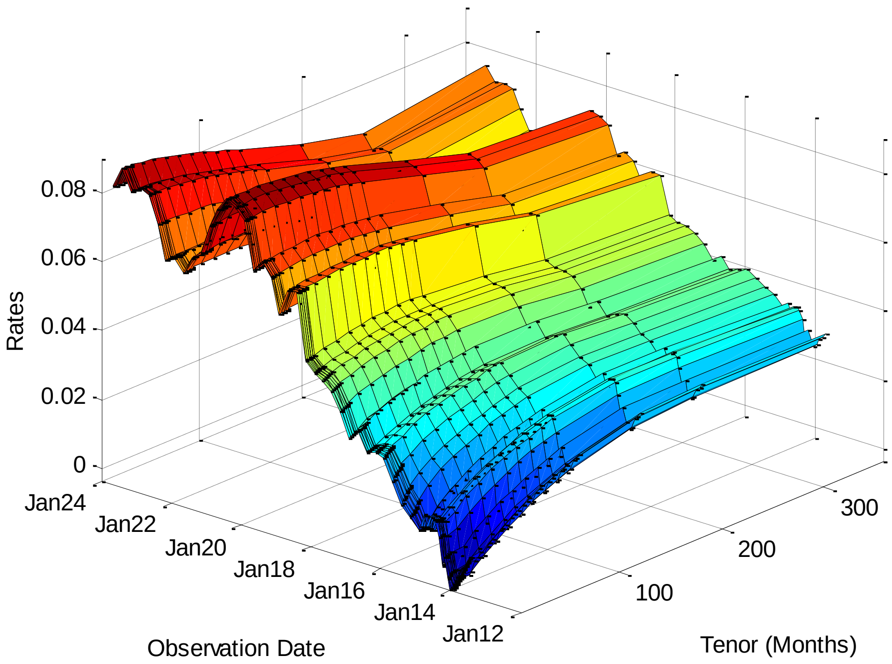
$$\begin{aligned} \ln \mathrm{A}(\mathrm{t},\mathrm{T}) &= \frac{P(0,T)}{P(0,t)} + B(t,T)F(0,t) - \frac{\sigma^2}{4a^3}(e^{-aT} - e^{-at})^2(e^{2at} - 1) \\ B(t,T) &= \frac{1}{a}\left[1 - e^{-a(T-t)}\right] \\ R(t,T) &= -\frac{1}{T-t}\ln A(t,T) + \frac{1}{T-t}B(t,T)r(t) \end{aligned} \tag{4.36}$$

The MATLAB implementation follows an example provided from the MATLAB website (MATLAB example B). The parameters obtained from the calibration above are plugged into the *HullWhite1F* function in MATLAB as well as the initial yield curve. The interest rate curve for each scenario is simulated at each valuation date using the built in Hull White model in MATLAB. The implementation can be found in Appendix A.2. Figure 4.5 below shows the interest rate curve evolution for scenario 20.

[17] Chapter 9

Figure 4.5

Scenario 20 Yield Curve Evolution



As mentioned in section 3.2 this thesis takes the viewpoint of a bank entering into interest rate swaps with sovereigns, where Germany, Spain and Greece was chosen as counter-parties. The swap values are close to zero value at the day of initiation, which means that they are traded at (close to) par. The chosen portfolio is shown in table 4.3 below. It is inspired by the portfolio held by the Danish Government SLOG (2013). Therefore, it contains only payer interest rate swaps, as seen from the institution. Later the portfolios will be altered to include receiver swaps in order to analyze the effects of netting. The contracts held against each counterparty are identical to show the effect from default risk on the calculated CVA.

Table 4.3: Portfolio A

Swaps		Fixed rate	Principal (USDm)
1Y	Payer	0.54%	7.5
2Y	Payer	0.65%	14
3Y	Payer	0.89%	20
4Y	Payer	1.17%	35
5Y	Payer	1.42%	40
6Y	Payer	1.64%	65
7Y	Payer	1.83%	50
8Y	Payer	2.00%	37.5
9Y	Payer	2.16%	25
10Y	Payer	2.30%	12.5

To determine the market risk component of CVA the swap portfolio have to be priced for each scenario, at each simulation date. The pricing is done using the price approximation function, *hswapapprox*. Approximations in a Monte Carlo setup are common because of the need of relatively quick computation speed as discussed in the introduction in section 4. Restrictions are only one zero curve for all swaps, each leg of the swaps must have the same reset value and the day count convention is set to Actual/365. I am of the opinion that the approximation used does not affect the results considerably, because the pricing is done on arbitrary portfolios of swaps. And especially since a dual curve setup was not adopted. Therefore, separation of forward and discounting curves cannot be done, which is one of the restrictions in the approximation function.

As explained in the computations on the MATLAB website for MATLAB example B, the computed mark to market prices of the swaps are aggregated in a cube containing all future contract values for all simulation dates and all scenarios. The cube is a three dimensional matrix with the rows representing the simulation dates, the columns the swaps and each page a different scenario. In figure 4.6 below, the mark to market swap prices for scenario 32 and 39 are depicted, to show how the prices evolve in two out of the thousand scenarios. The value of the swaps goes to zero as maturity is reached.

Figure 4.6

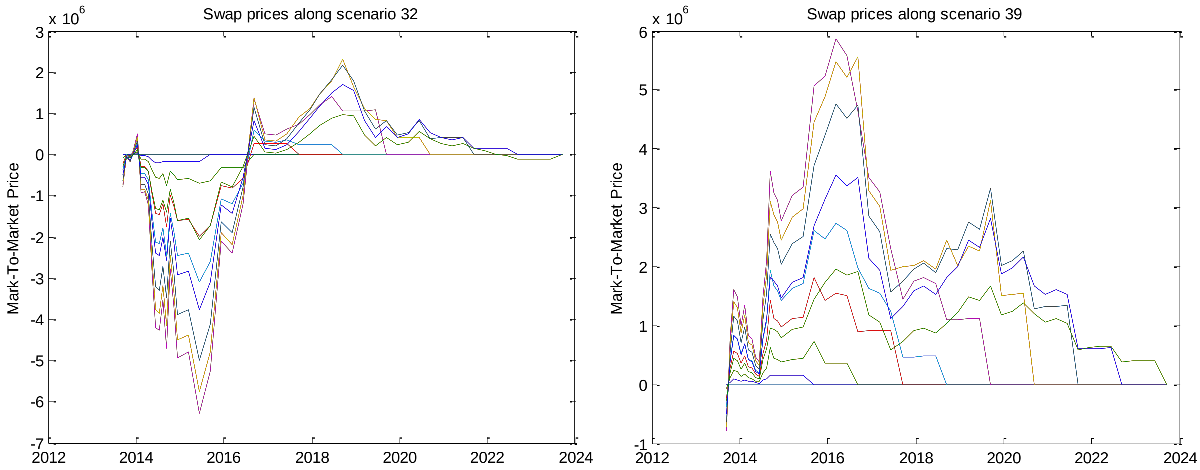
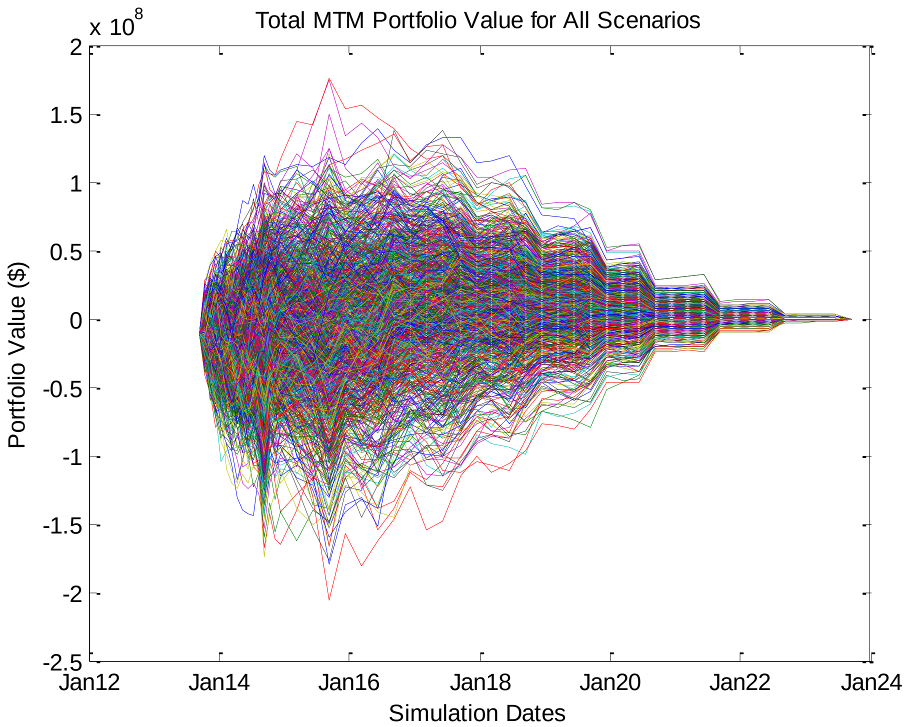


Figure 4.7 shows the mark to market portfolio value for all scenarios. The values of the swaps depend on the evolution of the interest rate, which is different in each scenario, resulting in different mark to market prices that can be either negative and/or positive during the life of the contract considered. As the two figures above portrait, the value of the swaps approach zero as they approach maturity. Swaps with longer maturities have more risk due to an increased lifetime and a greater number of future payments. The increased time horizon will result in greater fluctuations of the interest rate, and the increased annuity will increase the swap’s sensitivity towards the interest rate. The opposite argument can be used to explain why the mark to market prices goes towards zero as the swaps get closer to maturity.

Figure 4.7



With swap market values for all scenarios in hand, this thesis proceeds to compute exposures. One of the central features of counterparty risk is that an institution makes a loss on a contract with a positive mark to market value, but does not gain on negative values in the event of counterparty default. This is the case in the unilateral framework. One could argue that most contracts are bilateral by nature. In that case one must account for both the institution's and the counterparty's default, which would cause the need to quantify negative exposures. Negative exposures can lead to gains on contracts due to declining credit quality of the institution itself. This creates a counterintuitive effect as booked profits rise when credit quality declines. I refer to Gregory (2012) for a further discussion of debt value adjustment (DVA). As mentioned earlier the focus is on CVA in the unilateral case, because as argued earlier it is the most relevant measure for existing swap contracts with sovereigns.

For a normal distribution with mean μ (expected future values) and standard deviation σ, the future value of a portfolio is given by $V = \mu + \sigma Z$, where Z is a standard normal variable.

The exposure E_i of a contract is defined as the maximum value V_i of contract i at time t

$$E_i(t) = \max(V_i(t), 0) = \max(\mu + \sigma Z, 0) \tag{4.37}$$

The total exposure versus a counterparty is given by the sum of the individual exposures

$$E_{cp}(t) = \sum E_i(t) = \sum \max(V_i(t), 0) \tag{4.38}$$

In the case of netting agreements, the future values of the swaps offset one another. This means that the aggregated effect of all contracts must be determined. The total exposure of all contracts versus a counterparty when accounting for netting agreements is given by

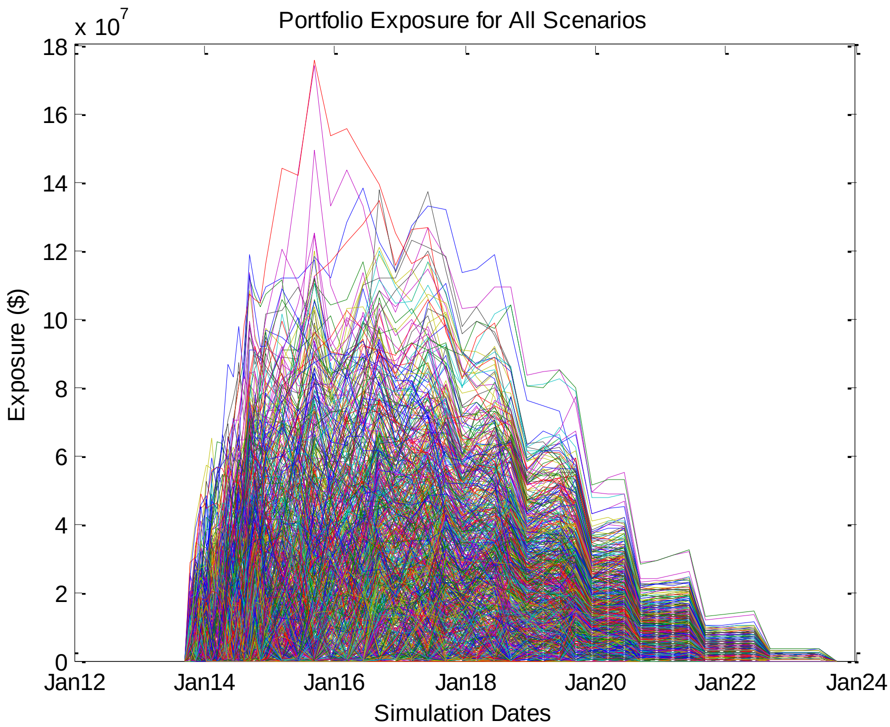
$$E_{cp_{na}}(t) = \max(\sum V_i(t), 0) \tag{4.39}$$

The total exposure is calculated using both of the above listed formulas, choosing the appropriate one dependent on netting agreements and summing the values arising from them to obtain the exposure.

The MATLAB implementation can be found in Appendix A.2. Figure 4.8 below shows the total portfolio exposure calculated using the equations for exposure stated above. The exposure logically approaches zero as the swaps reaches maturity because the value of the contracts goes towards zero. It can be shown that the maximum exposure of a swap contract occurs at one-third of the swaps lifetime[18], given assumptions of a flat yield curve and that the future value of the contract follows a normal distribution. The latter assumption is satisfied in this case, since the Hull White model is used to simulate exposures. The first assumption is not satisfied, but the exposures seem to peak at around one-third in figure 4.8. However, some of the contracts have rather short maturities, which means that this is only eyeball statistics. The exposures are all calculated under the risk neutral probability measure, given the fact that the Hull White model was calibrated using market data and not historical data.

[18] The derivation can be found in Gregory (2012), Appendix 8C e.g.

Figure 4.8



4.5 Credit exposure measures

For the CVA calculations, it is necessary to derive the expected exposures. Since there are several other useful metrics for credit exposure, some of the most commonly used measures to quantify counterparty exposure are defined below in accordance with the definitions in Basel II, BCBS (2006).

Potential future exposure (PFE), sometimes called peak exposure, is the same measure used for Value-at-Risk (VaR). PFE is defined as the maximum expected exposure at future times at a given confidence level α. It defines the exposure that will be exceeded with the probability of $1-\alpha$. For a normal distribution it is defined, as in Gregory (2012)[19] by

$$PFE_{\alpha} = \mu + \sigma\Phi^{-1}(\alpha) \tag{4.40}$$

Where $\Phi^{-1}(\cdot)$ is the inverse of the cumulative normal distribution function. With a confidence level of 95 percent, the exposure is 1.64 standard deviations above the expected future value μ.

[19] The derivations of the credit measures can be found in Gregory (2012), Appendix 8B e.g.

Maximum potential future exposure (MPFE) is simply defined as the maximum potential future exposure across all times.

Expected exposure (EE) is the average of all exposure values.

$$EE = \int_{-\mu/\sigma}^{\infty} (\mu + \sigma x)\varphi(x)dx = \mu\Phi(\mu/\sigma) + \sigma\varphi(\mu/\sigma) \qquad (4.41)$$

Where $\varphi(\cdot)$ is a normal distribution and $\Phi(\cdot)$ the cumulative normal distribution function. As can be seen from the equation above, expected exposure increases with both the mean and the standard deviation. The $\max\{\cdot,0\}$ operator is used in all paths at all time steps to calculate the overall exposure, as explained above. Expected exposure is determined by taking the mean of the overall exposure across all paths and time steps.

Expected positive exposure (EPE) is the weighted average of the expected exposure across all times, which is why it is sometimes called the time average expected exposure.

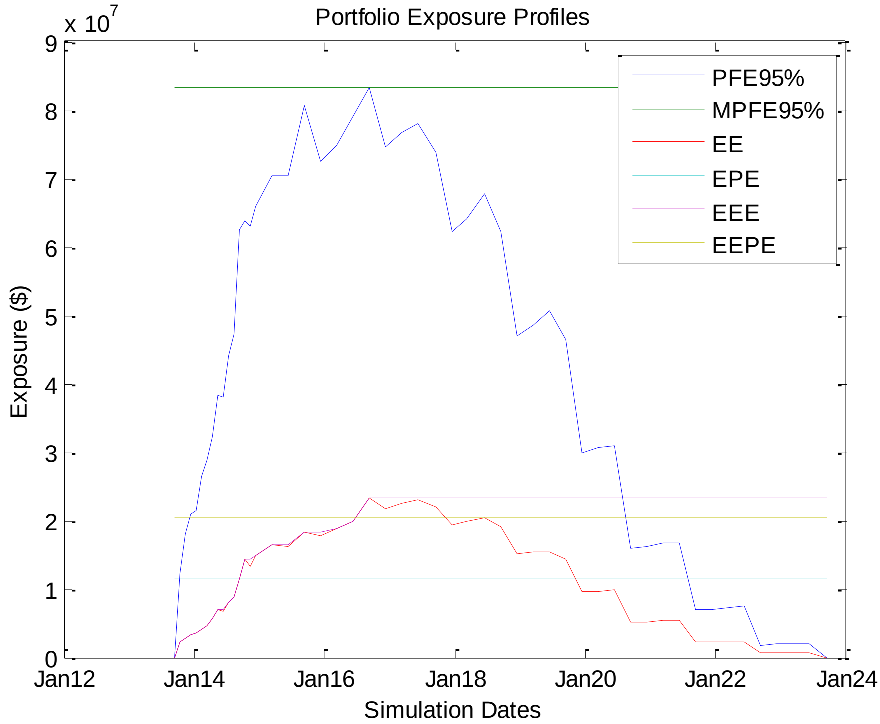
Effective expected exposure (EEE) is defined as the maximum expected exposure across all times.

Effective expected positive exposure (EEPE) is the weighted average of the effective expected exposure.

The expected exposure for an interest rate swap is zero at inception when traded at par, because it has no value for either party, and the same applies when maturity is reached since all payments have been made. However during the lifetime of the swap it can have both positive and negative value for one party, as illustrated above in figure 4.6. Two opposite effects influence the exposure on the swap, namely the volatility of the underlying interest rate, which increases with time and the remaining payments on the swap, which decreases as time goes by, reducing the amount outstanding.

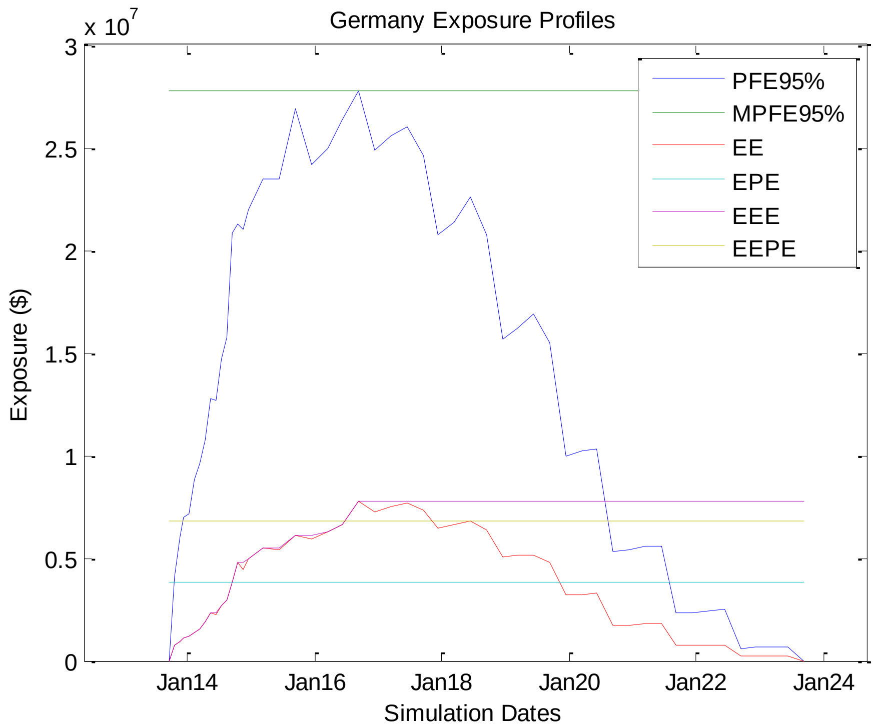
The exposure profiles for the entire portfolio are depicted in figure 4.9 below. As expected due to arguments discussed above, the expected exposure and potential future reaches maximum values at the time where around a third of the life time of the swaps has passed. Effective expected exposure is clearly constrained to be the non-decreasing expected exposure as seen in figure 4.9. The expected exposure for the portfolio peaks at around USD22.4m or at 7.31 percent of notional.

Figure 4.9



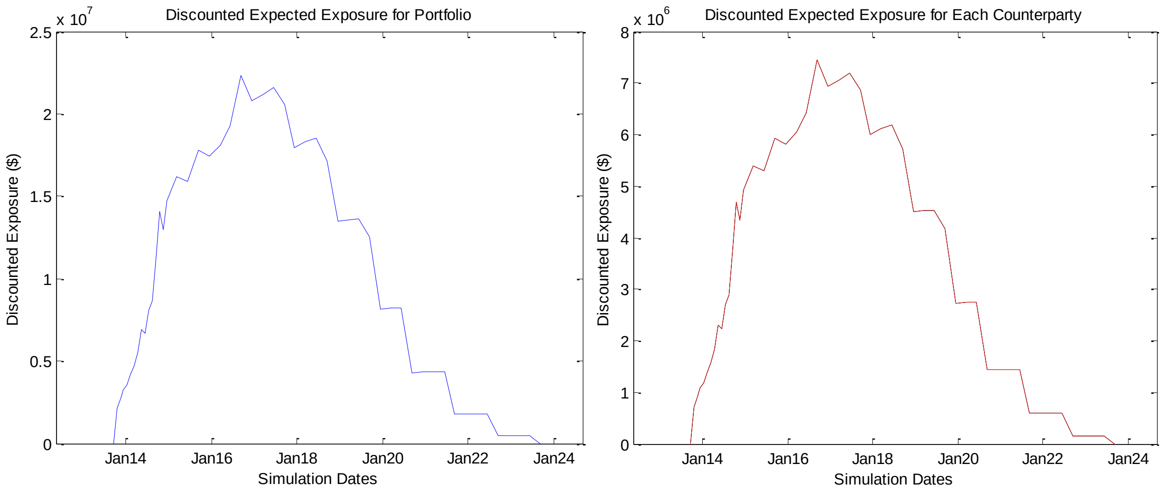
Because the swap portfolio for each counterparty is identical, the exposure profiles are identical as well. Therefore, only one of the three counterparty specific exposure profiles are presented. The exposure profiles for Germany represents the exposure profiles for Greece and Spain as well, depicted in figure 4.10 below. If the counterparty specific exposure profiles are summed, results are the exposure profiles for the entire portfolio presented in figure 4.9. The expected exposure for Germany, Spain and Greece peaks at around USD7.7m or at 2.50 percent of the total notional. In the 5% worst case scenarios simulated, the exposure peaks around USD27.7m or at 9.05 percent of total notional.

Figure 4.10



Several exposure measures have been presented in detail, however, for the CVA calculations, there is a need to calculate the discounted expected exposures. This is done by using the discount factors from all simulated scenarios. This is done in MATLAB using the *Binary Singleton Expansion Function* which multiplies the matrices containing the computed discount factors and the matrices containing the computed expected exposures for all counterparties. Summing the discounted expected exposures for each counterparty, results in the discounted expected exposure profile for the portfolio as a whole. The discounted expected exposure for the portfolio as well as the discounted expected exposures for the individual counterparties are depicted in figure 4.11. The discounted expected exposure for each counterparty is identical, as the discount factors are identical and the expected exposures are identical as discussed earlier. Due to discounting, the expected exposures used for the CVA calculations are lower than the exposures without discounting, which is evident when comparing figure 4.11 and figure 4.9 and 4.10. The discounted expected exposures for each counterparty is 2.43 percent of notional or around USD7.4m, whereas the discounted expected exposure for the entire portfolio is lowered to 7.28 percent of notional or around USD7.28m. This concludes the market risk section.

Figure 4.11



5. Credit Risk

Up until now there has been a focus on the market risk component of CVA and the measurement of counterparty credit exposure. In the next section, the focus belies on the credit risk component of CVA.

Credit risk measures the probability of a counterparty defaulting on a contract, whereas market risk measures the expected loss. Once again, it has to be considered whether the physical of risk-neutral parameters should be estimated. Physical default probabilities can be estimated from historical data or from an equity-based approach as in Merton's (1974) classical framework (also known as structural modelling). Risk-neutral default probabilities are estimated from market data and use a reduced-form modelling approach. As in the market risk section the risk-neutral approach is chosen. This is because the risk-neutral probability measure is most relevant for pricing applications and the CDS spread used in the Basel III CVA definition is defined as a market implied parameter, BCBS (2012). The risk-neutral probabilities are obtained from the CDS spreads observed in the market, in accordance with Basel III. The arbitrage-free default probabilities are typically larger than the default probabilities estimated from historical data. This is due to liquidity risk premium and spread risk premium being applied to the risk-neutral default probability, see Gregory (2012)[20] and O'Kane and Turnbull (2003).

5.1 Bootstrapping

Earlier it was shown how to value the premium and the protection leg on a CDS. The par spread is defined as the spread which ensures the present value of the premium leg equals the present value of the protection leg, which in turn means that the par spread, as in e.g. Hull and White (2000), is given by

$$G = \frac{(1-R)\sum_{S+1}^{E} P(t,T_i)\left[\exp(-\int_t^{T_{i-1}} \lambda(u)du) - \exp(-\int_t^{T_i} \lambda(u)du)\right]}{\sum_{S+1}^{E} \delta_i P(t,T_i)\exp(-\int_t^{T_i} \lambda(u)du)} \tag{5.1}$$

[20] Chapter 10

The definition of the par spread gives a direct relationship between survival probabilities and the credit default swap spread observed in the market. If a standard simplifying assumption is made and the recovery rate is fixed at 40 percent, then it is possible to calibrate the intensity curve. The calibration follows the outline in O'Kane and Turnbull (2003) by assuming that the hazard rate is a piecewise flat function of maturity time with flat extrapolation. As mentioned in Linderstrøm (2012) many different interpolation methods could be chosen, but it would be computationally intensive. Furthermore it would inherit the risk of yielding negative hazard rates, which would clearly be incorrect since possibilities cannot be negative.

For Germany, Spain and Greece CDS spreads for 6M, 1Y, 2Y, 3Y, 4Y, 5Y, 6Y, 7Y, 8Y, 9Y and 10Y maturities are listed in table 5.1. As stated previously, a piecewise flat function for the hazard rate is assumed, i.e. the hazard rate term structure consists of 10 sections, $\lambda_{0,½}, \lambda_{½,1}, \lambda_{1,2}, \lambda_{2,3}, \lambda_{3,4}, \lambda_{4,5}, \lambda_{5,6}, \lambda_{7,8}, \lambda_{8,9}, \lambda_{9,10}$

The calibration or bootstrapping of the hazard rates consist of calculating the survival probabilities by solving equation 5.1 This is an iterative procedure solved in MATLAB by the *fzero* function. If $\tau = T - t$ as in O'Kane and Turnbull (2003), then

$$\exp(-\int_t^T \lambda(u)du) =$$

$$\begin{cases} \exp(-\lambda_{0,½}\tau) & 0 < \tau \le ½ \\ \exp(-\lambda_{0,½} - \lambda_{½,1}(\tau - ½)) & ½ < \tau \le 1 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - \lambda_{1,2}(\tau - 1)) & 1 < \tau \le 2 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - \lambda_{2,3}(\tau - 2)) & 2 < \tau \le 3 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - 2\lambda_{2,3} - \lambda_{3,4}(\tau - 3)) & 3 < \tau \le 4 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - 2\lambda_{2,3} - 2\lambda_{3,4} - \lambda_{4,5}(\tau - 4)) & 4 < \tau \le 5 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - 2\lambda_{2,3} - 2\lambda_{3,4} - 2\lambda_{4,5} - \lambda_{5,6}(\tau - 5)) & 5 < \tau \le 6 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - 2\lambda_{2,3} - 2\lambda_{3,4} - 2\lambda_{4,5} - 2\lambda_{5,6} - \lambda_{6,7}(\tau - 6)) & 6 < \tau \le 7 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - 2\lambda_{2,3} - 2\lambda_{3,4} - 2\lambda_{4,5} - 2\lambda_{5,6} - 2\lambda_{6,7} - \lambda_{7,8}(\tau - 7)) & 7 < \tau \le 8 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - 2\lambda_{2,3} - 2\lambda_{3,4} - 2\lambda_{4,5} - 2\lambda_{5,6} - 2\lambda_{6,7} - 2\lambda_{7,8} - \lambda_{8,9}(\tau - 8)) & 8 < \tau \le 9 \\ \exp(-\lambda_{0,½} - 2\lambda_{½,1} - 2\lambda_{1,2} - 2\lambda_{2,3} - 2\lambda_{3,4} - 2\lambda_{4,5} - 2\lambda_{5,6} - 2\lambda_{6,7} - 2\lambda_{7,8} - 2\lambda_{8,9} - \lambda_{9,10}(\tau - 9)) & 9 < \tau \le 10 \\ & \tau > 10 \end{cases}$$

Or as stated more formally in Beumee, Brigo, Schiemert and Stoyle (2009)

$$h(t) = \exp(-\int_0^t \lambda(u)du) \tag{5.2}$$

Where $h(t)$, as above, is the piecewise flat hazard rate, consequently the default times τ are exponentially distributed. It is given that

$$H(t) = \int_0^t \lambda(u)du \tag{5.3}$$

Where $H(t)$ is the cumulated intensity function that satisfies

$$Surv(t) = \exp(-H(t)) \tag{5.4}$$

And therefore,

$$P(s < \tau \leq t) = \exp(-H(s)) - \exp(-H(t)) \tag{5.5}$$

5.2 Data

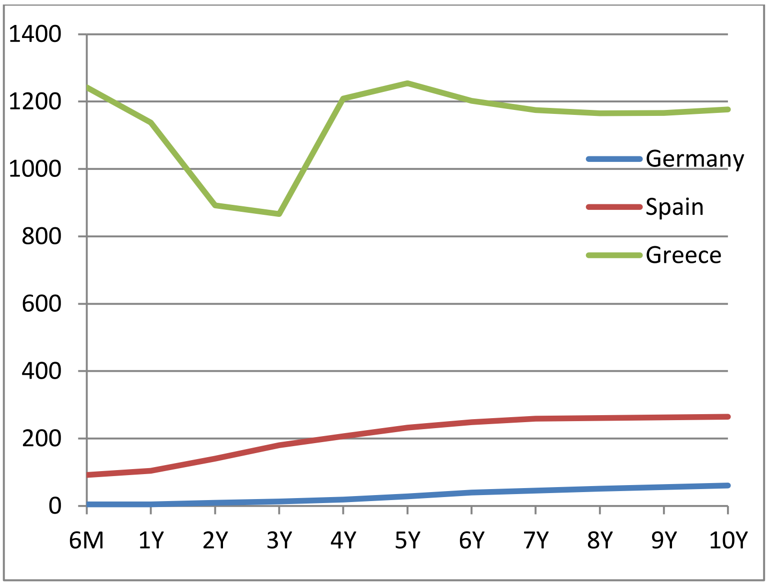
The default swap spreads observed in the market for the three counterparties are listed as mid quotes in table 5.1, and a graphic illustration is provided in figure 5.1. The three counterparties have very different spreads, which reflect the differences in credit quality. Germany is least likely to default and Greece is most likely to default due to highly stressed CDS levels. The reason for choosing exactly these counterparties is that they represent a wide range of credit quality, as represented by their different CDS spreads. This is a key aspect when measuring counterparty credit risk.

Table 5.1: CDS Mid Market Quotes

	Germany	Spain	Greece
6M	4.47	92.11	1241.69
1Y	5.00	104.00	1137.70
2Y	9.00	140.00	892.01
3Y	13.00	180.49	866.35
4Y	18.50	207.11	1209.06
5Y	28.00	232.50	1254.02
6Y	39.50	248.50	1201.98
7Y	45.50	259.00	1174.90
8Y	51.50	261.00	1164.78
9Y	55.50	263.00	1165.92
10Y	60.50	264.99	1176.11

Source: Bloomberg CDBRXY, CSPAXY and CGGBXY, where X denotes currency and Y years, September 2, 2013

Figure 5.1: CDS Mid Market Quotes



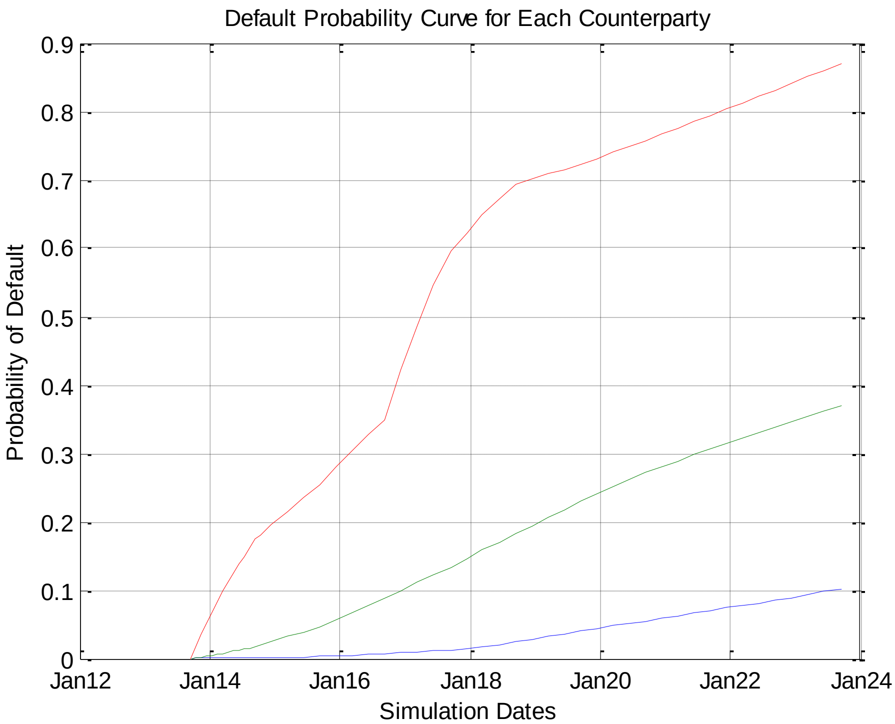
Source: Bloomberg CDBRXY, CSPAXY and CGGBXY, where X denotes currency and Y years, September 2, 2013

As illustrated in figure 5.1, the CDS par spread for Germany and Spain increases monotonically, whereas the par spread for Greece has a more uncharacteristic shape. It starts by declining, up until the three year mark and then quickly rises to a level above the six months par spread, at the five year mark. It then slowly declines, with the exception of an increase from the nine to the ten year CDS spread mark. Declining CDS spreads are not uncommon for distressed entities, reflecting that the credit quality in the future is expected to improve, conditional on the entity surviving up until that point. The hump shaped CDS spread curve of Greece is however highly unusual, in the sense that the implied probability of default decline in the short term, increase in the medium term and then decline in the long term. The CDS spread curve illustrates the dire straits Greece was in financially, with the probability of default declining in the medium long run. It is also explained by the fact that CDS instruments are most liquid around the five year mark, whereas other maturities are relatively illiquid.

With the CDS par spreads at hand, this thesis proceeds with the process of bootstrapping the hazard rate term structure as outlined above. As this requires numerical methods, it is conducted in MATLAB (Appendix A.2). This is done by calling the *cdsbootstrap* function with *ZeroData*, *CDSDates*, *CDSSpreads*, *Settle* and *simulationDates* as input. The *cdsbootstrap* function bootstraps the default probabilities by minimizing the difference between the observed spread in the market and the spread calculated with the standard model[21]. The function *cdsstdsurvprob* converts default probabilities to hazard rates and the hazard rates to survival probabilities, as shown in equation (5.4). The cumulative probability of default equals one minus the survival probability.

Figure 5.2 shows the cumulative default probability curve for each counterparty, with Greece in the top, Spain in the middle and Germany in the bottom. It should be noted that the market implied default probabilities for Greece are higher than those for Spain and Germany. The probabilities of defaulting within the first two years are given by 0.3%, 4.6% and 25.4%, for Germany, Spain and Greece respectively. The probabilities of defaulting increase to 2.4%, 18.3% and 69,4% for default happening within the first five years. The probability of defaulting within ten years are given for Germany, Spain and Greece respectively by 10.2%, 37.0% and 86.9%. This emphasizes the difference in credit quality of the counterparties – and the CVA charges are expected to differ accordingly when valued on the same contracts.

Figure 5.2



[21] The ISDA CDS standard model can be found at http://www.cdsmodel.com/cdsmodel/

5.3 Discussion of the use of CDS spreads

Inferring default risk from CDS quotes in the market is relatively straightforward. However a number of concerns regarding the trustworthiness of using CDSs as indicators of sovereign risk persist, as reported by Reuters (2013)[22]. The authors point that the EU ban on naked CDS positions has caused liquidity and trading volumes to fall dramatically. Lower liquidity increases the bid-offer spread, which increases the uncertainty when calibrating default probabilities. As a result, the price discovery value of CDSs have decreased. A naked CDS position means that the owner of the CDS contract does not own the underlying debt security. The absence of naked positions make the CDS more comparable to normal insurance contracts, where the owner of the insurance need to own the insured product. Naked positions have been equated with speculation, which might be because CDS spreads rose considerably when the European sovereign crisis began in May 2010. This resulted in increased hedging needs by banks, which in turn increased the CDS spreads even further, creating a downward spiral. As mentioned by Gregory (2012)[23], the naked CDS positions can be linked to counterparty risk hedging. Gregory (2012)[24] argue that a ban on naked positions makes the CDS market inefficient, as illustrated above by the decline in liquidity and volumes. Further, he argues that the ban possibly reduces the value of CDSs used as protection against credit exposure, as future exposure is uncertain and a protection buyer might want to buy more protection than the current exposure allows. However, CVA hedging by nature appears to increase spreads and thereby increase the need for further hedging, further increasing spreads and consequently hedging needs. The Bank of England (2010)[25] underlines that the ban on naked CDS positions is not without it's merits:

"...given the relative illiquidity of sovereign CDS markets a sharp increase in demand from active investors can bid up the cost of sovereign CDS protection. CVA desks have come to account for a large proportion of trading in the sovereign CDS market and so their hedging activity has reportedly been a factor pushing prices away from levels solely reflecting the underlying probability of sovereign default."

[22] "Sovereign CDS loses relevance", published 7/8/13
[23] See chapter 16
[24] Chapter 16
[25] See page 81 of the quarterly bulletin 2010 Q2

Since all CVA desks want to re-hedge in the same direction when market conditions change, they tend to amplify volatility and illiquidity during stressed times, creating delivery squeezes in stressed times. In order to alleviate this, a ban on naked sovereign CDS positions was implemented by the European Union. According to Gregory (2012)[26], the use of CDS as an instrument in CVA calculation and hedging is a double-edged sword:

- On the one hand, it enables risk-neutral calculation of CVA through market implied hazard rates and is the primary hedging tool.
- On the other hand, it increases complexity of the CVA calculation when used to hedge counterparty credit risk considerably, as CDSs are prone to contain wrong way risk.

The bid-ask spreads for Greece are shown in table 5.2. They are quite severe, which as mentioned above, reduces the price discovery value and thus make the hazard rates computed from the spreads more vague. The bid-ask spreads for Germany and Spain are not as dramatic. While bizarre bid-ask spreads for Greece are explained by severe financial distress in the sovereign. The method of using CDSs to bootstrap hazard rates still seem to be market standard, since this is the most natural way to estimate market implied default probabilities. The ban has caused sovereign CDS spreads to widen, but at the same time, CVA hedging pushed prices away from levels reflecting the underlying default probabilities. There is a need to address the issues of price discovery in CDS contracts. A possible solution could be an extended use of credit curve mapping, even though the use of CDS spreads is required by Basel III whenever available, BCBS (2012).

[26] Chapter 16

Table 5.2: Bid ask spreads

	Greece	
	bid	ask
6M	838,12	1645,26
1Y	799,01	1476,39
2Y	686,28	1097,74
3Y	728,11	1004,59
4Y	1101,6	1316,51
5Y	1172,82	1335,21
6Y	1117,68	1286,27
7Y	1085,24	1264,56
8Y	1067,71	1261,85
9Y	1059,44	1272,39
10Y	1058,07	1294,14

Source: Bloomberg CGGBXY, where X denotes currency and Y years, September 2, 2013

6. Credit Value Adjustment

Section 4 and 5 focused on assessing the market risk and the credit risk component of CVA. This chapter proceeds by combining the two components in order to determine counterparty credit risk, which is done by pricing CVA.

6.1 CVA Without Netting

Recalling from section 3.2.4 that CVA can be calculated as

$$CVA(t,T) = (1-\mathrm{R})\sum_{i=1}^{m} P(t,T_i)EE(t_i)\,PD(t_{i-1},t_i) \tag{6.1}$$

All notation as described earlier. Section 4 focused on calculating the market risk component, the discounted exposures, while section 5 calculated the cumulative default probability implied from CDS quotes. Counterparty credit risk can then be priced by the use of the CVA formula, given the assumption of a constant recovery rate.

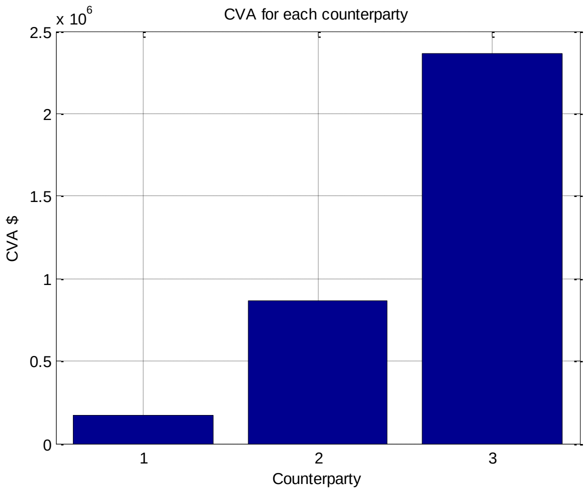
The discounted expected exposure as well as the cumulative probability of default are summed for each counterparty and the respective CVAs are given by

Table 6.1: CVA

CVA in	Germany	Spain	Greece
USD	168,937	864,435	2,364,262

As mentioned in section 4.5, the exposure profiles for the three counterparties are identical. The different CVA values consequently stems from the credit risk component, where it was noted that Greece was much more likely to default than Spain, and Spain more likely to default than Germany. With the exposure profiles and default probabilities estimated in this thesis, the CVA charged on the IRSs entered with Greece is 14 times larger than the CVA charge on the IRSs entered with Germany. This is visualized in figure 6.1 below, showing the considerable difference in CVA against each counterparty. Counterparty 1, 2 and 3 are Germany, Spain and Greece respectively.

Figure 6.1



The different CVA charges clearly show the credit quality's impact on the CVA, and thus on counterparty credit risk. Remembering that the total notional on the swaps outstanding with each counterparty was USD306,000,000 the CVA for each counterparty as a percentage of notional value is shown in table 6.2 below.

Table 6.2: CVA as percentage of notional amount

	Germany	Spain	Greece
CVA % of notional	0.0552	0.2825	0.7726

The CVA charge for Germany, Spain and Greece are thus given by 5.52 bp., 28.25 bp. and 77.26 bp., where basis points are denoted as bp. A basis point is equal to one hundredth of a percent. Once again it is confirmed that CVA is much larger on the IRS with Greece, therefore the value of the risky swap with Greece is lower than the IRS with Germany. The CVA charge applied to the risky IRS made with Spain is also significantly larger than the one applying to Germany.

5.52 bp. in the case of Germany might not seem like a noteworthy amount, however remembering the unfathomable number of USD693 trillion total notional amounts outstanding mentioned in chapter 2, even a CVA of 5.52 bp. becomes a considerable number, in this case around USD383 billion. This illustrates that there are significant counterparty credit risk in the derivatives markets, especially considering that Germany is the least likely counterparty to default in this study – and is by all means considered as a very low risk counterparty.

6.2 CVA With Netting

So far the analysis has ignored credit risk mitigation. An institution is likely to have offsetting positions, which means that credit exposures are mitigated by the means of netting. As pointed out in section 3.2, collateral is ignored due to the fact that sovereigns seldom post collateral. The analysed portfolio contained only payer swaps as it was inspired by the actual portfolio held by the Danish Government SLOG (2013) that only enters IRS agreements. In the following section, the effect on CVA stemming from netting is examined. The notional amount on each swap remains the same, however, the portfolios held against each counterparty are changed as it now consist of a combination of receiver and payer swaps. This will allow an analysis of the effects from netting agreements, but does not reflect actual portfolios held by sovereigns. It does reflect the fact that institutions often hold offsetting positions. The probability of counterparty default remains unchanged and thus the results from section 5.2 apply. Note that CVA on Portfolio B will differ slightly from Portfolio A in both the case with and without netting. This is because of an upward sloping yield curve, the difference is however negligible. The new portfolio is presented in table 6.3 below.

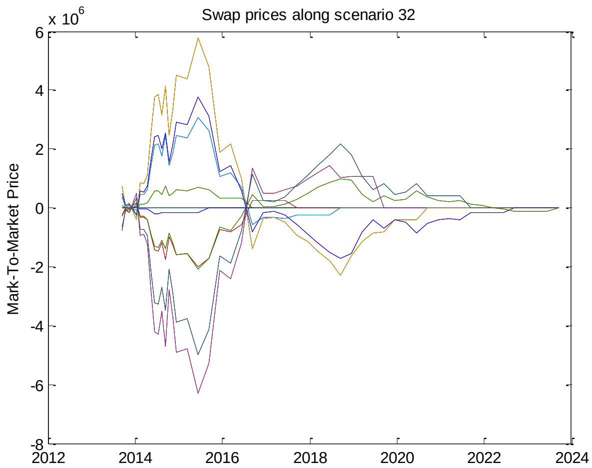
Table 6.3: Portfolio B

Swaps		Fixed rate	Principal (USDm)
1Y	Receiver	0.54%	7.5
2Y	Payer	0.65%	14
3Y	Receiver	0.89%	20
4Y	Payer	1.17%	35
5Y	Receiver	1.42%	40
6Y	Payer	1.64%	65
7Y	Receiver	1.83%	50
8Y	Payer	2.00%	37.5
9Y	Receiver	2.16%	25
10Y	Payer	2.30%	12.5

6.2.1 Exposures

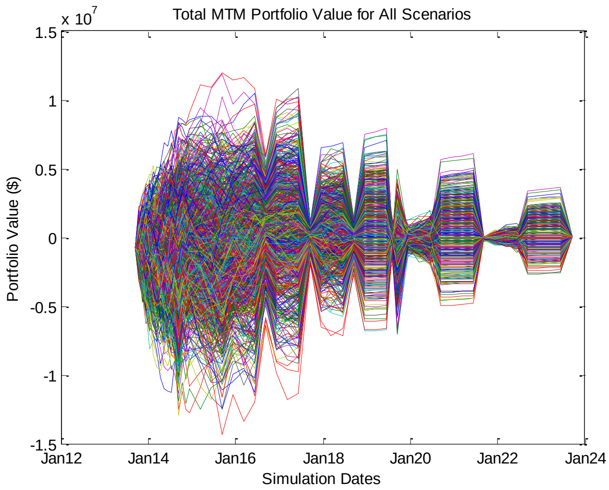
The MATLAB code is exactly the same used in section 4.4 and 4.5 and can be found in Appendix A.2. The Hull White parameters are unchanged as well. The only difference is that another portfolio of swaps is used as input. The output from the MATLAB implementation can be found in Appendix B.3. If we compare the prices in scenario 32 with netting presented in figure 6.2 to the case without netting in figure 4.6, it is quite clear that the combination of receiver and payer IRSs represents offsetting positions.

Figure 6.2



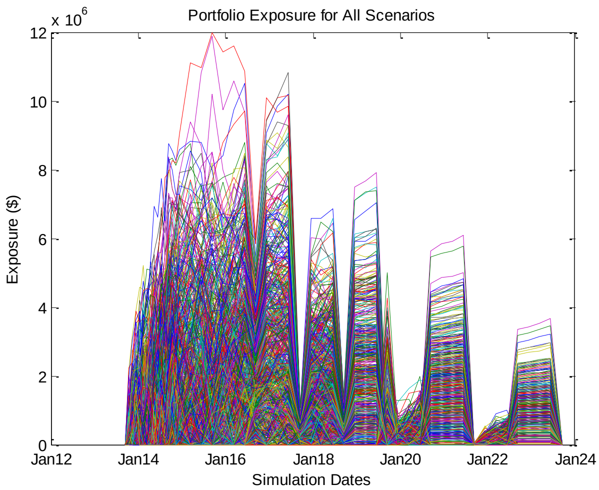
This can be seen more clearly in figure 6.3 below, where the total mark to market value of the portfolio for all scenarios experiences considerable jumps towards zero during the life of the swaps. The jumps are caused by the difference in maturity on the receiver and payer swaps. The effects from the offsetting positions become more apparent the closer longer dated swaps are to maturity. The short lived swaps are not too sensitive with respect to the interest rate and they only have a few remaining payments, which means that the mark to market on short lived swaps does not differ much from zero when incepted at par. A more thorough discussion of these effects was carried out in section 4.4 and 4.5. Another essential observation is that the mark to market values of the portfolio are considerably smaller in figure 6.3 compared to figure 4.7 in section 4.4. Notice that the scale on the axis has changed. This is due to the offsetting positions in the swap holdings due to netting.

Figure 6.3



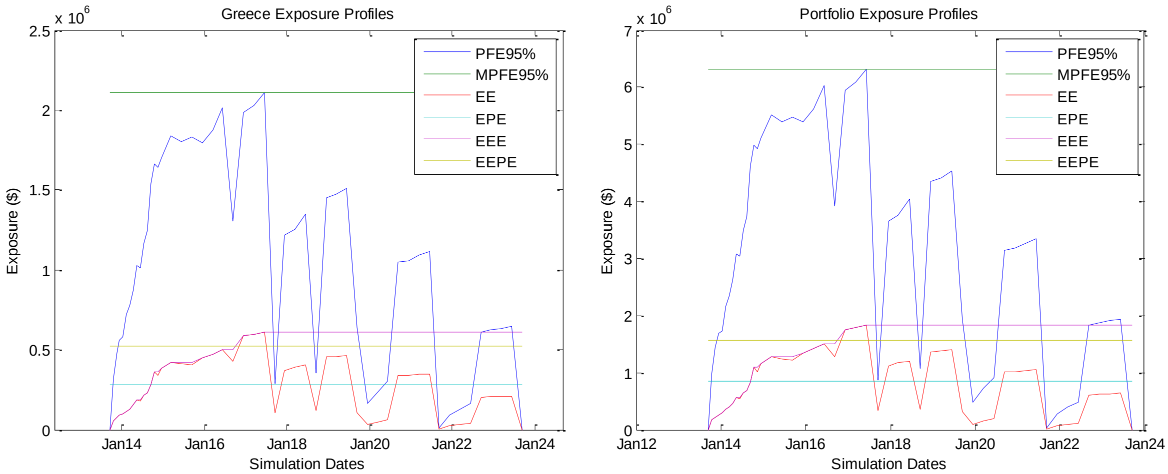
As expected the exposure on the portfolio as a whole has decreased considerably as well. Notice that the scale on the axis is almost seven times larger in figure 4.8 compared to figure 6.4 below. The exposure declines when the mark to market values jumps toward zero as expected.

Figure 6.4



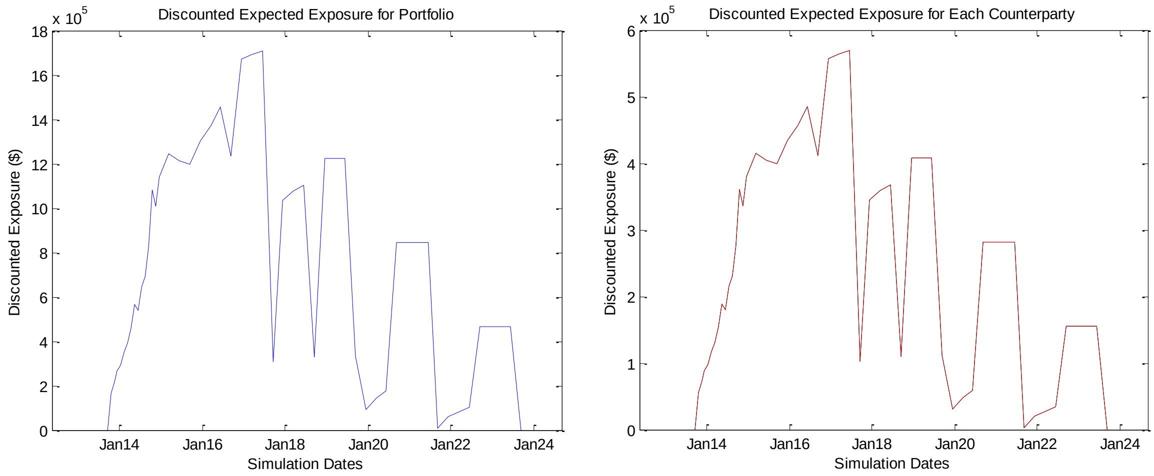
The counterparty specific exposure profiles for Greece are depicted below. The exposure profiles for all three counterparties are identical for the same reasons discussed in section 4.5. Once again it is noticed that the exposure has declined significantly. The expected exposure for the three counterparties now peaks at only USD0.6m, compared to USD7.7m in the case without netting. In percentage of the notional amount outstanding that is 0.2 compared to 2.5 percent in the case without netting. The expected exposure with netting is thus approximately 13 times smaller compared to the case without netting. The expected exposure and peak exposure profiles naturally depict the same jumps as the mark to market values of the swaps given the definition of these two measures described in section 4.5.

Figure 6.5



The portfolio exposure profiles in figure 6.5 display the same tendencies as the counterparty specific exposure profiles, here shown for Greece. Please note, that the exposure declines with netting and is independent from the choice of credit exposure measure. The expected exposure for the portfolio now peaks at only USD1.8m compared to USD22.4m in the case without netting, or 0.59 percent of notional compared to 7.31 percent.

Figure 6.6

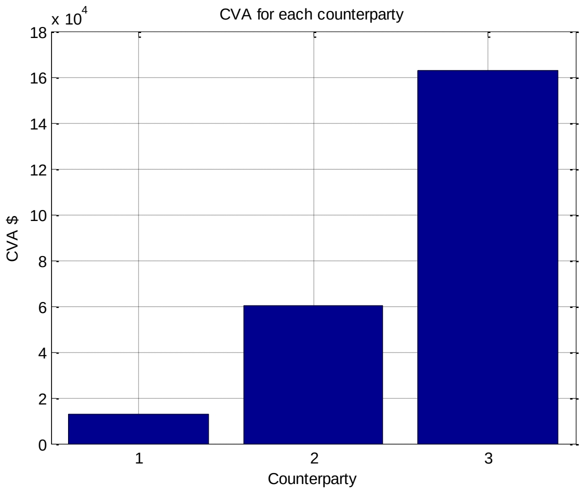


The discounted expected exposures depicted in figure 6.6, which are used in the calculation of CVA, are considerably lower as well when compared to the discounted expected exposures shown in figure 4.11. The counterparty specific discounted expected exposure peaks at 0.19 percent, compared to 2.43 percent in the case without netting.

6.2.2 Credit Value Adjustment

CVA is calculated as the sum of the product of discounted expected exposures and default probabilities given the assumptions made in section 3.2.4. CVA is expected to decline considerably given the fall in exposures, when default probabilities remain unchanged. This can be confirmed by comparing figure 6.1 that shows CVA without netting with figure 6.7 below that shows CVA with netting. Counterparty 1, 2 and 3 are Germany, Spain and Greece respectively.

Figure 6.7



As before when calculating CVA in the case without netting, the discounted expected exposures as well as the probabilities of default are summed for each counterparty. The respective CVAs are shown in table 6.4

Table 6.4: CVA

	Germany	Spain	Greece
CVA in USD	13,123	60,292	162,780

The effects from netting are quite remarkable and underline the fact that netting mitigates counterparty credit risk significantly, as discussed in section 3.1. CVA with netting for Germany, Spain and Greece respectively amount to 7.8, 7.0 and 6.9 percent of CVA without netting. It is a very significant reduction in counterparty risk. Note that the effect is greater for counterparties with better credit rating. This stems from the fact that the exposure has a greater influence on CVA when the default probability of the given counterparty is low, compared to the case where the default probability is high. This follows directly from the formula for CVA.

Perhaps it is more rewarding to describe CVA as a percentage of notional as done in table 6.5. For Germany, Spain and Greece CVA only constitute of 0.43 bp., 1.97 bp. and 5.32 bp. With netting the CVA applying to Greece is lower than the CVA applying to Germany without netting, which perpetuates the beneficial effects from netting.

It is however important to bear in mind that the portfolio chosen to illustrate the benefits from netting is very diversified, thus eliminating a substantial amount of exposure, which is ultimately reflected in the CVA calculations performed in this section.

Table 6.5: CVA as percentage of notional amount

	Germany	Spain	Greece
CVA % of notional	0.0043	0.0197	0.0532

6.3 Basel III

There has been an increased focus on regulation and capital requirements following the financial crisis. The crisis made it especially clear that the existing Basel II framework did not sufficiently account for counterparty credit risk. Over half of the Basel III requirements are related to counterparty credit risk, BCBS (2010).

The focus in this thesis will primarily be on the CVA aspect of Basel III. Basel II only accounts for losses due to default, but as stated by Gregory (2012)[27] only a third of the losses related to counterparty credit risk was due to actual defaults. The rest of the losses occurred because of the uncertainty of default on marked to market positions. Because of this, Basel III introduces a capital charge on CVA volatility, CVA VaR. The quantification of CVA VaR poses a considerable challenge, since both CVA and VaR are complex to determine. Therefore usually calculated using a Monte Carlo simulation scheme, as mentioned earlier.

CVA for internal model method (IMM) approved models is defined in BCBS (2010) as

$$CVA = LGD_{MKT} \sum_{i=1}^{T} \max\left(0; \exp\left(-\frac{s_{i-1}t_{i-1}}{LGD_{MKT}}\right) - \exp\left(-\frac{s_i t_i}{LGD_{MKT}}\right)\right)\left(\frac{EE_{i-1}D_{i-1} + EE_i D_i}{2}\right) \quad (6.2)$$

LGD_{MKT} is the loss given default as defined earlier. The loss given default in Basel III is not constant (as assumed in the calculation of CVA in this thesis), but instead inferred from

[27] Page 388

expectations in the market. The first bracket inside the summation is an approximation for the default probability, where CDS spreads are used as in section 5.1. Note that it is an approximation between the hazard rate and the spread, which does not account for the shape of the credit curve prior to time t_{i-1}, Gregory (2012)[28]. This approximation would not be suitable for inferring default probabilities from e.g. Greece as done in this thesis using piecewise constant hazard rates, since the approximation is more inaccurate for more sloped curves. The last bracket in equation (6.2) describes the discounted expected exposures. The division by two smoothes the exposures, thus makes the calculated exposures more precise when few time intervals are used.

Equation (6.2) from Basel III and equation (6.1) used to calculate CVA in this thesis are practically the same small differences aside. There are key differences in the exposures used in the calculations presented in this thesis compared to the ones used in Basel III. The expected exposures used in this thesis were risk neutral, whereas the expected exposures used in the CVA capital charge in Basel III are based on historical data. Thus misalignments in CVA calculations are very common, due to the inconsistencies in pricing CVA used for trading or hedging differs from the CVA calculated for capital requirements in Basel III, Gregory (2012)[29].

Using equation (6.2) as required in Basel III unambiguously leads to mixing up risk neutral and physical probabilities, since exposures must be calculated from historical data. While default probabilities as well as the loss given default must be inferred from the market. This coupled with the simplification of default probabilities used in equation (6.2) can have unintended and undesirable consequences, since an institution will have to choose between hedging CVA using their own setup or CVA as defined in Basel III, ultimately leading to regulatory capital arbitrage. Furthermore, the CVA charge is likely to be cyclical as it is calculated from CDS spreads. Credit risk cannot be hedged as effectively as market risk. Therefore, institutions will have little control over their CVA charge in stressed times, Gregory (2012)[30]. The CVA capital charge is, however, proposed to be dropped in trades with

[28] Page 206
[29] Page 389
[30] Page 396

sovereigns in CRD IV, according to Gregory (2012)[31]. The reasoning behind this is that sovereigns rarely post collateral and trade in large notional IRS, thereby creating a considerable amount of exposure for their counterparties. This ultimately needs to be hedged in order to ease the capital requirement stemming from the calculation of CVA. The hedging is performed by buying CDSs, thus creating the problem described in section 5.3. Increased hedging leads to increased CDS spreads, which leads to a need for further hedging.

6.4 Wrong Way Risk

The thesis has ignored wrong way risk so far. Wrong way risk is a positive dependence between exposure and default probability. This section will present a method to quantify wrong way risk. The calculation of CVA as in section 6.1 and 6.2 assuming independence between exposure and default probability no longer applies in the presence of wrong way risk. As argued in section 3.2.4 wrong way risk is not considered a major factor when valuing CVA on IRSs, but since CDS spreads are used in the calculation of CVA wrong way risk will be present.

There are two major concerns when trying to model wrong way risk, namely a lack of historical data and misspecification of the relationship, Gregory (2012)[32]. There might not be historical data fitting the current market, as illustrated by the unprecedented European sovereign crisis in late 2009. With regards to misspecification, correlation only measures a linear relationship between exposure and default probability, but it might be difficult to measure higher order dependence.

Equation (6.1) for CVA can still be used if the exposure is calculated conditional on default having occurred

$$CVA(t,T) = (1-\mathrm{R})\sum_{j=1}^{m} P(t,T_j)EE(t_j \mid t_j = \tau_C)\, PD(t_{i-1},t_i) \qquad (6.3)$$

[31] Page 396

[32] Chapter 15

$EE(t_j \mid t_j = \tau_C)$ is the expected exposure at time t_j conditional on it being the counterparty default time τ_C, as in Gregory (2012)[33]. In the setting analysed in this thesis, this would mean that the expected exposures for the three different counterparties would differ, as their probability of default differ. The unconditional exposure in the presence of wrong way risk is different from the conditional exposure.

There are many feasible methods to model wrong way risk. This thesis will use the correlation approach presented in Garcia-Cespedes et al. (2010) and Rosen and Saunders (2012). In the correlation approach, the probability of default is hold fixed in order to calculate the conditional expected exposure, seen in equation (6.3). A bivariate normal distribution with some specified correlation parameter drives default times and exposures. A positive correlation means that an early default time leads to higher exposure in the presence of wrong way risk and vice versa if there is right way risk. One of the advantages of the correlation approach is that it uses existing exposure calculations made without taking wrong way risk into account. Instead Hull and White (2012) hold the exposure fixed and calculates the default probabilities also called a parametric approach. Both methods are viable and perhaps it should be encouraged to use both, as shown in Gregory (2012)[34] the two approaches can yield different results, implying that a misspecification of the market-credit relationship is very likely.

The correlation approach is used by applying the method to a forward contract for illustrative purposes, as in Gregory (2012)[35]. The future value of a forward contract is assumed to follow (6.4), where V_t is the future value of the contract, μ is the drift, σ is the volatility and W_t is a standard Brownian motion.

$$dV_t = \mu dt + \sigma dW_t \tag{6.4}$$

The future value at time s follows a normal distribution

$$V_s \sim N\left(\mu s, \sigma\sqrt{s}\right) \tag{6.5}$$

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[34] Chapter 15
[35] Appendix 15A

That is the (unconditional) expected exposure for an underlying mark to market value V_t can be derived following equation (3.41) as

$$EE_s = \mu s \Phi\left(\frac{\mu}{\sigma}\sqrt{s}\right) + \sigma\sqrt{s}\varphi\left(\frac{\mu}{\sigma}\sqrt{s}\right) \tag{6.6}$$

The value of the contract at time s is given by

$$V(s) = \mu s + \sigma\sqrt{s}Y \tag{6.7}$$

Where Y is a Gaussian random variable as in Gregory (2012)[36]. If the default time of the counterparty is denoted by τ and the counterparty's default probability up to time s is denoted by $F(s)$. Then the probability of default for the counterparty can be defined as in section 5.1. With the exception that a constant hazard rate is assumed in this example, instead of a process for the hazard rate, to ease the computation

$$F(s) = 1 - \exp(-hs) \tag{6.8}$$

Default is driven by a Gaussian variable Z

$$\tau = F^{-1}\left(\Phi(Z)\right) \tag{6.9}$$

The two Gaussian variables driving default and exposure are linked through a correlation parameter ρ, and ε is an independent Gaussian variable

$$Y = \rho Z + \sqrt{1-\rho^2}\varepsilon \tag{6.10}$$

The expected exposure conditional on default can be calculated as

$$\begin{aligned} EE(s \mid \tau = s) &= E\left[\max\left(0, V(s)\right) \mid Z = \Phi^{-1}\left(F(\tau)\right)\right] \\ &= \int_{-\mu(t)/\sigma(t)}^{\infty} \left[\mu'(s) + \sigma'(s)\right]\varphi(x)dx \end{aligned} \tag{6.11}$$

Where

$$\mu'(t) = \mu(t) - \rho\sigma(t)\Phi^{-1}(F(\tau)) \tag{6.12}$$

And

$$\sigma'(t) = \sqrt{1-\rho^2}\sigma(t) \tag{6.13}$$

Which means that (6.10) can be written as

$$EE(s \mid \tau = s) = \mu'(t)\Phi\left(\frac{\mu'(s)}{\sigma'(s)}\right) + \sigma'(t)\varphi\left(\frac{\mu'(s)}{\sigma'(s)}\right) \tag{6.14}$$

[36] Appendix 15A

In this example the effects from wrong way risk on the exposure profile can be computed. Figure 6.8, 6.9, 6.10 and 6.11 below shows wrong way risk in different scenarios, where the outline in Spreadsheet 15.1 in Gregory (2012)[37] has been used.

Figure 6.8: Forward exposure, Mu = 0, Sigma = 10%, h = 2%, Rho = 25%

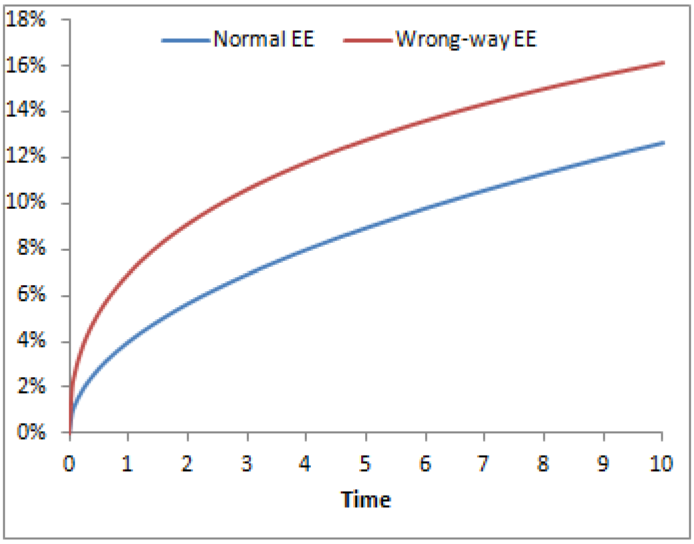
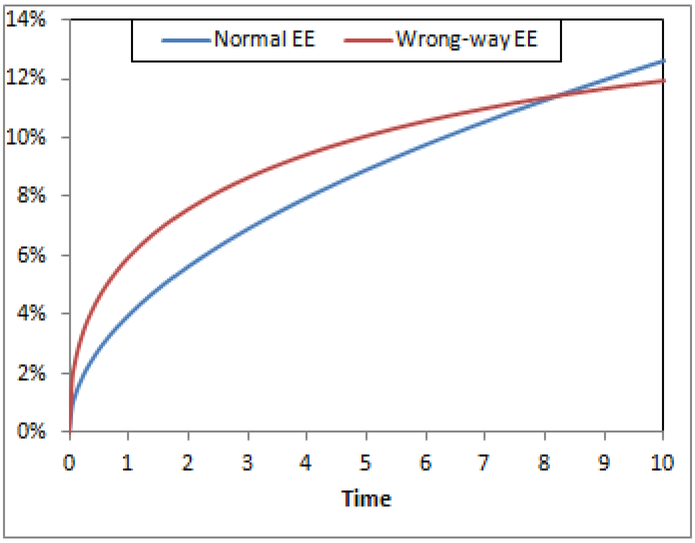


Figure 6.9: Forward exposure, Mu = 0, Sigma = 10%, h = 7.5%, Rho = 25%



[37] Can be found on http://www.cvacentral.com/

Figure 6.10: Forward exposure, Mu = 0, Sigma = 10%, h = 2%, Rho = 50%

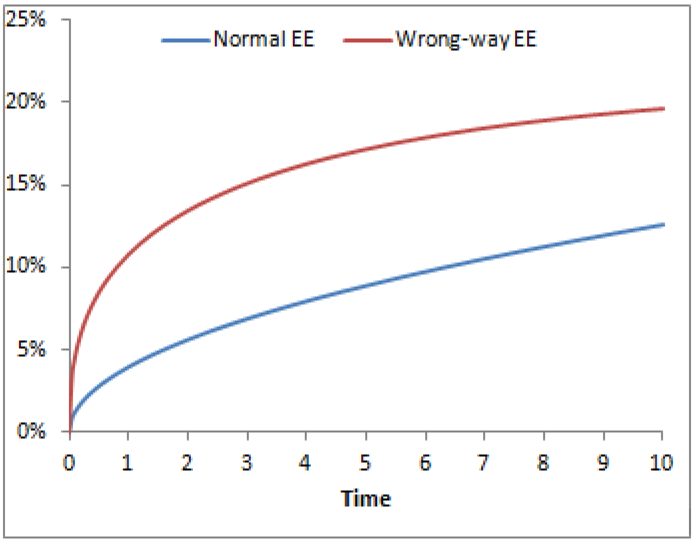
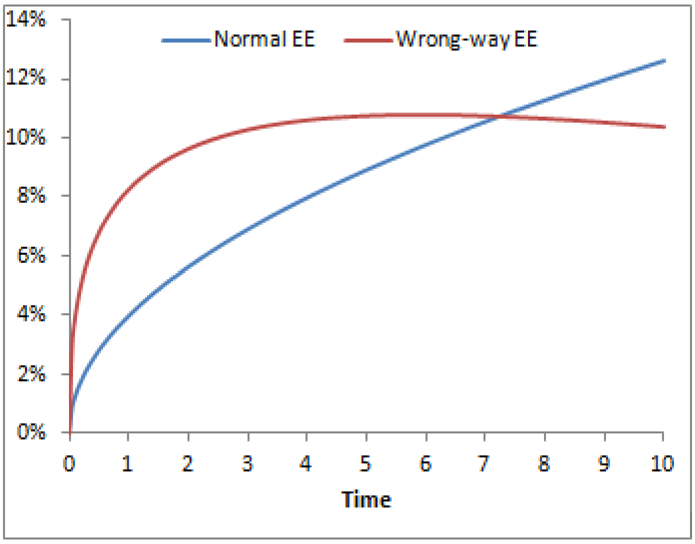


Figure 6.11: Forward exposure, Mu = 0, Sigma = 10%, h = 7.5%, Rho = 50%



Varying the volatility naturally effect the expected exposures. This is not shown, since it is not as interesting as varying the correlation parameter and the hazard rate. This is due to the expected exposure not accounting for wrong way risk remain the same when only varying the correlation parameter or the hazard rate. This can be seen in figure 6.10 compared to figure 6.8, expected exposure rises when the correlation parameter is increased, since the correlation parameter represents wrong way risk.

Figure 6.9 and 6.11 seem counterintuitive since expected exposure accounting for wrong way risk decreases with the credit quality of the counterparty, when comparing expected exposures with figure 6.8 and 6.10 representing the higher credit quality counterparty. The reasoning behind this is, that a default occurring for a counterparty with better credit quality is less likely and therefore has a greater effect when it occurs.

It is also observed that the expected exposure accounting for wrong way risk is lower than the normal expected exposure in both figure 6.9 and 6.11. A default of a stressed counterparty is not unthinkable in the far future, which is illustrated by the implied cumulative default probabilities in figure 5.2. This implies that conditional exposures in the short run are more likely to be affected by wrong way risk, Gregory (2012)[38].

A lot of assumptions have been made in the example above. One of the assumptions is that correlation is modelled using a Gaussian copula function. Any copula function could be chosen, as noted by Garcia-Cespedes et al. (2010). A comprehensive list of copula functions are examined in Brigo and Mercurio (2006)[39], an argument can be made for moving to non-Gaussian copulas to allow upper and lower tail dependence.

The correlation approach could be implemented in the simulation of exposures presented in this thesis. There are, however, difficulties arising from this method. It concerns the correlation parameter, which is difficult to calibrate. Correlation estimates can be found in Rosen and Saunders (2010). Though as noted by Gregory (2012)[40], the calibration of a market and credit correlation is very complicated. There is a major risk of misspecification to take into account, given that relevant historical data might not exist. This underlines the fact that wrong way risk is not easy to model – and why this route was not chosen in this thesis.

Two approaches to model wrong way risk were briefly presented in this thesis, namely the correlation approach and the parametric approach. They have different maximum values of CVA, meaning that a correlation of a 100 percent cannot be taken as a limiting case, according to Gregory[41], due to the parametric approach has a maximum value of CVA exceeding that of the correlation approach.

In the case of interest rate derivatives studied in this thesis, a negative relationship between default rates and interest rates are generally observed, Longstaff and Schwartz (1995). This can be explained by the monetary policy typically followed by central banks in times of

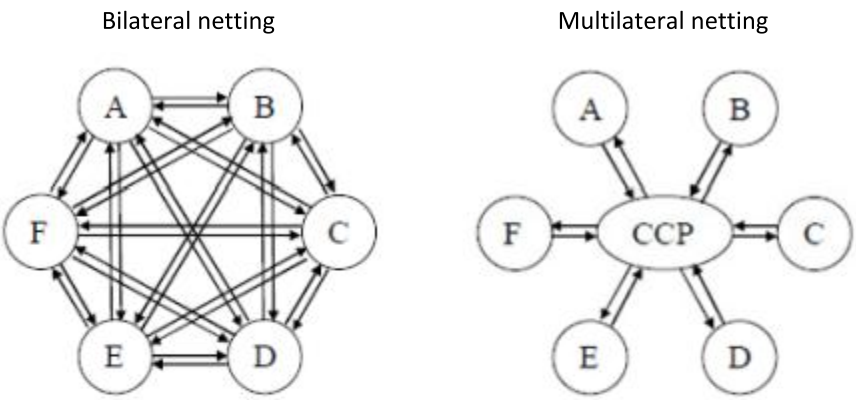
[38] Page 314
[39] Chapter 21
[40] Page 318
[41] Page 317

economic downturns. The probability of default might also be high when interest rates are high, thereby creating wrong way risk in both bad and good economic climates for interest rate products, as noted by Gregory (2012)[42]. This indicates that in order to correctly model wrong way risk in interest rate models, it is necessary to model the co-dependency between interest rate volatility and default probability. Not a particularly tangible task.

6.5 Central Counterparties

Central counterparties (CCPs) are a way to mitigate counterparty risk. This has not yet been mentioned in this thesis. If OTC trades go through a central counterparty, counterparty risk is reduced by means of central clearing. With central clearing there is multilateral netting across all members of the CCP, consequently no institution has counterparty risk towards a single counterparty. The CCP ensures clearing. This is illustrated in figure 6.12 below.

Figure 6.12



Source: Gregory (2012) page 95

The advantages of central clearing are, as stated in Gregory (2012)[43]:

- Multilateral netting (explained above).
- Operational and legal efficiency through centralisation.

[42] Page 323
[43] Page 114

- Loss mutualisation. Losses are distributed across all members of the CCP, reducing the impact on each counterparty and thereby reducing the probability of systemic effects. Shared losses prevent domino effects.
- Increased transparency. The CCP will have all information on trades and exposure on trades, making it easier for regulators to quantify and stress test positions.
- Increased liquidity because of multilateral netting mitigating counterparty risk and easier market entry.
- Default management. If a counterparty defaults, a central auction can result in lower price disruptions during a crisis.

CCPs come with drawbacks. One of these drawbacks is well known from insurance theory, namely moral hazard. Moral hazard makes it less profitable for an institution to actively monitor their own counterparty risk. A CCP is similar to an insurance in the sense that any losses suffered due to a counterparty default are shared across all members. Therefore, it is no surprise that CCPs suffer from adverse selection as well, which can occur if the institution knows more about the risks in a product than the CCP.

Other drawbacks are that OTC products are highly customised and illiquid, which makes it difficult to clear them in a CCP, and trading OTC derivatives through a CCP increases costs through increased margins.

A CCP is meant to decrease systemic risk, however, this is not necessarily the case, as argued by Gregory (2012)[44]. A CCP centralises the OTC markets. Therefore, a failure of a CCP would be a systemic and financially disastrous event. CCPs are often argued to reduce systemic risk from SIFIs (Systemically Important Financial Institutions), nevertheless a CCP is a SIFI by definition. CCPs do not have the same flaws as SIFIs, in the sense that a CCP does not have an incentive to behave recklessly due to an implicit promise of support from central banks because of their SIFI status. CCPs do not necessarily reduce systemic risk and counterparty risk is not removed, but rather distributed and converted into other types of risk, e.g.

[44] Chapter 6 and 7

operational, legal and liquidity risk, Gregory (2012)[45]. Thereby the need to calculate CVA is effectively removed for derivatives traded through a CCP.

With regards to the Basel III CVA capital charge, some critics believe that the CVA VaR methodology was driven by political pressure aimed at pushing OTC markets towards central clearing, since there is no CVA capital charge on centrally cleared derivatives, Gregory (2012)[46].

Gregory (2012)[47] further argues that the transition to central clearing may be stalled for many OTC traded derivatives, since a CCP only function when the members of the CCP post collateral. As mentioned in section 3.1.2 and 3.2, sovereigns rarely post collateral. Therefore, IRSs with sovereigns are likely to continue trading OTC. The non-collateral posting counterparties accounts for most of the exposure as well, which implies that a transition towards two-way CSA's and collateral postings for both counterparties is needed in order to mitigate counterparty credit risk on a larger scale. The Danish sovereign has begun a transition from one-way toward two-way CSAs, as argued in section 3.2.

[45] Chapter 6 and 7

[46] Page 393 and page 429

[47] Page 429

7. Conclusion

There has been an increased focus on pricing counterparty credit risk in recent years. This thesis has priced counterparty credit risk for an institution entering into interest rate swaps with three different sovereigns.

The starting point of the thesis was to examine how credit value adjustment is quantified. Credit value adjustment (CVA) was shown to depend on exposure and probability of default. In order to determine exposure and probability of default, the pricing of interest rate swaps, credit default swaps and swaptions were investigated.

The thesis started with quantifying the market risk component of CVA. This required numerical methods because of the complexity of CVA. A Hull White one-factor model was calibrated to market data using a Hull White tree. After obtaining volatility parameters for the Hull White one-factor model, exposure profiles for an arbitrarily chosen portfolio were obtained using Monte Carlo simulation.

The credit risk component of CVA was determined using CDS quotes observed in the market for the same three counterparties. The default probabilities were obtained by bootstrapping the hazard rates, which was an iterative procedure solved in MATLAB.

CVA was calculated for the three counterparties. The counterparty credit risk differed substantially between the three counterparties, with CVA being considerably larger for counterparties with lower credit quality. CVA with netting was calculated using an altered portfolio including receiver swaps. It was shown that netting mitigates counterparty risk significantly.

CVA as defined in Basel III was examined. It was argued that the CVA calculated in this thesis differs from CVA as defined in Basel III. Basel III mixes the risk neutral and physical probability measure and this thesis calculated CVA using risk neutral probabilities.

A method to include wrong way risk in the calculation of CVA was also presented. In that context an example of how to model wrong way risk on a forward contract was presented, showing that wrong way risk can reduce expected exposure on contracts with counterparties that have low credit quality.

Finally central clearing was reviewed arguing that central clearing can eliminate counterparty risk by converting it to other, and perhaps less dangerous, type of risk. It was argued that CCPs will only have an impact on standardized derivative contracts and on contracts where both counterparties posts collateral.

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9. Appendices

Appendix A

A.1 Calibration of the Hull White model MATLAB code

Contruction of zero curve

```
Settle = datenum('12-Sep-2013');

ParRates = [0.00344
         0.00373
         0.004065
         0.004385
         0.00473
         0.00505
         0.00535
         0.00569
         0.00599
         0.00629
         0.006473
         0.00892
         0.0116
         0.01408
         0.01618
         0.018
         0.01959
         0.02102
         0.02229
         0.026315
         0.027435
         0.027385];

CurveDates = [datenum('12-Mar-2014')
          datenum('12-Apr-2014')
          datenum('12-May-2014')
          datenum('12-Jun-2014')
          datenum('12-Jul-2014')
          datenum('12-Aug-2014')
          datenum('12-Sep-2014')
          datenum('12-Oct-2014')
          datenum('12-Nov-2014')
          datenum('12-Dec-2014')
          datenum('12-Sep-2015')
          datenum('12-Sep-2016')
          datenum('12-Sep-2017')
          datenum('12-Sep-2018')
          datenum('12-Sep-2019')
          datenum('12-Sep-2020')
          datenum('12-Sep-2021')
          datenum('12-Sep-2022')
          datenum('12-Sep-2023')
          datenum('12-Sep-2028')
          datenum('12-Sep-2033')
          datenum('12-Sep-2043')];
```

```
Compounding = 2;

[ZeroRates, CurveDates] = pyld2zeroTEST(ParRates, CurveDates, Settle,
Compounding)

plot(CurveDates,ZeroRates)
datetick
title(['Zero Curve for ' datestr(Settle)]);

function [ZeroRatesZP, CurveDates] = pyld2zeroTEST(ParRates, CurveDates,
Settle, ...
    varargin)
if (nargin < 3)
    error(message('finance:pyld2zero:tooFewInputs'));
end

if nargin < 4 || isempty(varargin{1})
    Compounding = 2;
else
    Compounding = varargin{1};
end

if nargin < 5 || isempty(varargin{2})
    Basis = 0;
else
    Basis = varargin{2};
end

if nargin < 6 || isempty(varargin{3})
    OutputCompounding = Compounding;
else
    OutputCompounding = varargin{3};
end

if nargin < 7 || isempty(varargin{4})
    OutputBasis = Basis;
else
    OutputBasis = varargin{4};
end

if any([size(Compounding), size(Basis), size(OutputCompounding), ...
        size(OutputBasis)] ~= 1)
    error (message('finance:pyld2zero:nonScalarInputs'))
end

if (all(Compounding ~= [-1 1 2 3 4 6 12 365]))
    error(message('finance:pyld2zero:invalidCompounding'))

elseif Compounding == -1
    error(message('finance:pyld2zero:unsupportedCompounding'));
end

if ~all(isvalidbasis(Basis))
    error(message('finance:pyld2zero:invalidBasis'))
end
```

```matlab
if (all(OutputCompounding ~= [-1 1 2 3 4 6 12 365]))
    error(message('finance:pyld2zero:invalidOutputCompounding'))
end

if ~all(isvalidbasis(OutputBasis))
    error(message('finance:pyld2zero:invalidOutputBasis'))
end

 [yr,mnth,dy,hr,mn,sec] = datevec(CurveDates);
if any(hr>0) || any(mn>0) || any(sec>0)
    warning(message('finance:pyld2zero:datesHaveTimes'))
    CurveDates = datenum(yr,mnth,dy);
end

 [CurveDates, SortIndex] = sort(CurveDates);
ParRates = ParRates(SortIndex);

tag = 0;

if min(CurveDates) < Settle
    error(message('finance:pyld2zero:settleValueTooLarge'))
else
    if CurveDates(1) == Settle
        CurveDates = CurveDates(2:end);
        ParRates = ParRates(2:end);
        tag = 1;
    end
end

NumBonds = length(ParRates);
Col = ones(NumBonds, 1);

if OutputCompounding~=2 && ~(isisma(Basis))

    if OutputCompounding == -1
        ParRates = 2*(exp(ParRates/2) - 1);
    else
        ParRates = 2*((1+ParRates/OutputCompounding).^...
            (OutputCompounding/2) - 1);
    end
end

ParPrices = 100*Col;

Bonds = [CurveDates ParRates 100*Col OutputCompounding*Col OutputBasis*Col
1*Col];

 [ZeroRatesZP, CurveDates] = zbtpriceTEST(Bonds, ParPrices, ...
    Settle, Compounding);

if tag
    ZeroRatesZP(2:end+1) = ZeroRatesZP;
    CurveDates(2:end+1) = CurveDates;
    CurveDates(1) = Settle;
end
```

```
function [ZeroRates, CurveDates] = zbtpriceTEST(Bonds, Prices, Settle, ...
    OutputCompounding, varargin)
if (nargin < 3)
    error(message('finance:zbtprice:tooFewInputs'));
end

[NumBonds, NumCols] = size(Bonds);
if (NumCols < 2)
    error(message('finance:zbtprice:tooFewColumns'))
elseif (NumCols > 6)
    error(message('finance:zbtprice:tooManyColumns'))
end

if (nargin > 4 || ~isempty(varargin))
    warning(message('finance:zbtprice:noLongerSupported'));
end

Col = ones(NumBonds,1);
DefaultCols = [100*Col, 2*Col, 0*Col, 1*Col];
Bonds = [Bonds, DefaultCols(:, (NumCols-1):4)];

Maturity = Bonds(:,1);
CouponRate  = Bonds(:,2);
Face      = Bonds(:,3);
Period    = Bonds(:,4);
Basis     = Bonds(:,5);
EndMonthRule  = Bonds(:,6);

if all(isisma(Basis))
    CompFreq = 1;
elseif all(~isisma(Basis))
    CompFreq = 2;
else
    error(message('finance:zbtyield:bondsInconsistentBasis'));
end

Prices = Prices(:);
if length(Prices)~=NumBonds
    error(message('finance:zbtprice:pricesDoNotMatchBonds', length( Prices
), NumBonds))
end

Settle = finargdate(Settle);
if any(Settle ~= Settle(1))
    error(message('finance:zbtprice:bondsMustSettleOnSameDay'));
else
    Settle = Settle(1);
end

if nargin<4 || isempty(OutputCompounding)
    OutputCompounding = 2;
end

 [CFAmounts, CFDates, CFTimes] = ...
    cfamounts(CouponRate, Settle, Maturity, ...
```

```
    Period, Basis, EndMonthRule, [], [], [], [], Face);

NumInst = length(Prices);

CFAmounts = [-Prices              CFAmounts];
CFDates =   [Settle*ones(NumInst,1) CFDates];
CFTimes =   [zeros(NumInst,1)       CFTimes];

 [CFSet, Dates, Times] = cfport(CFAmounts, CFDates, CFTimes);
NumTimes = length(Times);

 [Maturity, I] = max(CFDates .* (CFAmounts~=0) ,[],2);

CFEnd = CFAmounts( (1:NumInst)' + NumInst*(I-1) );
TFEnd = CFTimes(   (1:NumInst)' + NumInst*(I-1) );

 [EndDates, I, InstOrder] = unique(Maturity,'legacy');
NumPoints = length(EndDates);

 [JPoint, InstOrder] = ndgrid(1:NumPoints, InstOrder);
MatMap = (JPoint == InstOrder);

CFSet = MatMap*CFSet;
CFEnd = MatMap*CFEnd;

TFEnd = TFEnd(I);

[CurveDates, EndInd] = intersect(Dates, EndDates,'legacy');
EndTimes = Times(EndInd);

RateMap = zeros(NumTimes, NumPoints);

RateMap(1:EndInd(1)-1,1) = 1;

for j=1:NumPoints,
    EndRates = zeros(NumPoints,1);
    EndRates(j) = 1;
    RateMap(EndInd(1):end,j) = interp1(EndTimes, EndRates,
Times(EndInd(1):end),'spline');
end

EndRates = 2*( (-CFEnd./CFSet(:,1)).^(1./TFEnd) - 1 );
EndRates = max(EndRates, 0.04);

    function [Error, dEidRj] = zerobootsub(EndRates)

        Rates = RateMap * EndRates;
        Disc    = (1 + Rates./CompFreq).^(-Times);
        DelDisc = (1 + Rates./CompFreq).^(-Times-1) .* (-Times./CompFreq);
        DelDisc(1) = 0;

        Error  = CFSet*Disc;
        dEidRj = CFSet*(DelDisc(:,ones(1,size(RateMap,2))).*RateMap);
    end
```

```
fsolveOpt = optimset('Jacobian','on', 'Display','off', ...
    'TolFun',1e-12,  'TolX',1e-12, ...
    'MaxIter', 20);

[ZeroRates, ~, ExitFlag] = fsolve(@zerobootsub,EndRates,fsolveOpt);

if ( ExitFlag <= 0 )
    warning(message('finance:zbtprice:solutionConvergenceFailure'));
end

if OutputCompounding~=2

    if OutputCompounding == -1
        ZeroRates = CompFreq*log(1 + ZeroRates/CompFreq);
    else
        ZeroRates = OutputCompounding * ...
            ((1 + ZeroRates/CompFreq).^(CompFreq/OutputCompounding) - 1);
    end
end
end
```

Construct the swaption volatility matrix and compute prices using Black's model

```
irdc =
IRDataCurve('Zero',Settle,CurveDates,ZeroRates,'Compounding',2,'Basis',2,'I
nterpMethod','spline');

RateSpec =
intenvset('Rates',ZeroRates,'EndDates',CurveDates,'StartDate',Settle,'Basis
',2);

InstrumentExerciseDate = datenum('12-Sep-2014');
InstrumentMaturity = datenum('12-Sep-2034');
InstrumentStrike = .0;

SwaptionBlackVol = [78.63   63.74   57.33   51.77   47.82   39.58   32.99
28.35   27.02
65.1    52.05   46.81   42.85   40.14   34.7    30.26   27.12   26.03
50.45   43.4    40.18   37.47   35.31   31.15   28.03   25.73   25.11
42.8    36.81   35.25   33.32   31.62   28.62   26.51   24.71   24.3
36.99   33.16   31.54   30.11   28.34   26.68   25.51   24.02   23.55
29.26   27.34   26.33   25.53   24.93   23.76   23.44   22.88   22.16
23.71   22.86   22.57   22.37   22.25   22.02   22.25   21.12   20.93
21.56   22.57   22.69   22.81   22.97   22.12   22.22   20.42   19.32
22.43   23.96   23.95   23.94   23.86   22.49   21.77   19.79   17.43]/100;
ExerciseDates = [1:5 7 10 15 20];
Tenors = [1:5 7 10 15 20];

EurExDatesFull = repmat(daysadd(Settle,ExerciseDates*360,1)',...
    length(Tenors),1);
EurMatFull = reshape(daysadd(EurExDatesFull,...
    repmat(360*Tenors,1,length(ExerciseDates)),1),size(EurExDatesFull));

relidx = find(EurMatFull <= InstrumentMaturity);
```

```
SwaptionBlackPrices = zeros(size(SwaptionBlackVol));
SwaptionStrike = zeros(size(SwaptionBlackVol));

for iSwaption=1:length(ExerciseDates)
    for iTenor=1:length(Tenors)
        [~,SwaptionStrike(iTenor,iSwaption)] = swapbyzero(RateSpec,[NaN 0],
Settle, EurMatFull(iTenor,iSwaption),...
            'StartDate',EurExDatesFull(iTenor,iSwaption),'LegReset',[1
2],'Basis',2);
        SwaptionBlackPrices(iTenor,iSwaption) = swaptionbyblk(RateSpec,
'call', SwaptionStrike(iTenor,iSwaption),Settle, ...
            EurExDatesFull(iTenor,iSwaption), EurMatFull(iTenor,iSwaption),
SwaptionBlackVol(iTenor,iSwaption));
    end
end

nPeriods = 20;
DeltaTime = 1;
nTrials = 1000;

Tenor = (1:20)';

SimDates = daysadd(Settle,360*DeltaTime*(0:nPeriods),1)
SimTimes = diff(yearfrac(SimDates(1),SimDates))
```

Calibrate the model

```
TimeSpec = hwtimespec(Settle,daysadd(Settle,360*(1:30),6), -1);
HW1Fobjfun = @(x) SwaptionBlackPrices(relidx) - ...
    swaptionbyhw(hwtree(hwvolspec(Settle,['12-Sep-2013';'12-Sep-2014';'12-
Sep-2015';'12-Sep-2016';'12-Sep-2017';'12-Sep-2018';'12-Sep-2020';'12-Sep-
2023';'12-Sep-2028';'12-Sep-2033'],x(2),...
    ['12-Sep-2013';'12-Sep-2014';'12-Sep-2015';'12-Sep-2016';'12-Sep-
2017';'12-Sep-2018';'12-Sep-2020';'12-Sep-2023';'12-Sep-2028';'12-Sep-
2033'],x(1),'spline'), RateSpec, TimeSpec), 'call',
SwaptionStrike(relidx),...
    EurExDatesFull(relidx), 0, Settle, EurMatFull(relidx),'Basis',2);
options = optimset('disp','iter','MaxFunEvals',1000,'TolFun',1e-5);

x0 = [.1 .01];
lb = [0 0];
ub = [1 1];
HW1Fparams = lsqnonlin(HW1Fobjfun,x0,lb,ub,options);

HW_alpha = HW1Fparams(1)
HW_sigma = HW1Fparams(2)
```

Simulate the term structure

```
HW1F = HullWhite1F(RateSpec,HW_alpha,HW_sigma)

HW1FSimPaths = HW1F.simTermStructs(nPeriods,'NTRIALS',nTrials,...
    'DeltaTime',DeltaTime,'Tenor',Tenor,'antithetic',true);
trialIdx = 20;
figure
```

```
surf(Tenor,SimDates,HW1FSimPaths(:,:,trialIdx))
datetick y keepticks keeplimits
title(['Evolution of the Zero Curve for Trial:' num2str(trialIdx) ' of Hull
White Model'])
xlabel('Tenor (Years)')
```

A.2 Exposure simulation, default calibration and CVA calculation

Import file containing swap information

```
swapFile = 'test.xlsx';
swapData = dataset('XLSFile',swapFile,'Sheet','Swap Portfolio');

swaps = struct(...
    'Counterparty',[],...
    'NettingID',[],...
    'Principal',[],...
    'Maturity',[],...
    'LegRate',[],...
    'LegType',[],...
    'LatestFloatingRate',[],...
    'FloatingResetDates',[]);

swaps.Counterparty = swapData.CounterpartyID;
swaps.NettingID    = swapData.NettingID;
swaps.Principal    = swapData.Principal;
swaps.Maturity     = swapData.Maturity;
swaps.LegType      = [swapData.LegType ~swapData.LegType];
swaps.LegRate      = [swapData.LegRateReceiving swapData.LegRatePaying];
swaps.LatestFloatingRate = swapData.LatestFloatingRate;
swaps.Period       = swapData.Period;
swaps.LegReset     = ones(size(swaps.LegType));

numSwaps = numel(swaps.Counterparty);
numCounterparties = max(swaps.Counterparty);

Settle   = datenum('12-Sep-2013');

Tenor  = [6 7 8 9 10 11 12 13 14 15 2*12 3*12 4*12 5*12 6*12 7*12 8*12 9*12
10*12 15*12 20*12 30*12]';

ZeroDates = datemnth(Settle,Tenor);
Compounding = 2;
Basis = 0;
RateSpec = intenvset('StartDates', Settle,'EndDates', ZeroDates,...
    'Rates', ZeroRates,'Compounding',Compounding,'Basis',Basis);

RateCurveObj = IRDataCurve('Zero',Settle,ZeroDates,ZeroRates,...
    'Compounding', Compounding,'Basis', Basis, 'InterpMethod', 'spline');

figure;
plot(ZeroDates, ZeroRates, 'o-');
xlabel('Date');
datetick('keeplimits');
ylabel('Zero rate'); grid on;
title('Yield Curve at Settle Date');
```

```
numScenarios = 1000;

simulationDates = datemnth(Settle,0:15);
simulationDates = [simulationDates
datemnth(simulationDates(end),3:3:107)]';
numDates = numel(simulationDates);

floatDates = cfdates(Settle-360,swaps.Maturity,swaps.Period);
swaps.FloatingResetDates = zeros(numSwaps,numDates);
for i = numDates:-1:1
    thisDate = simulationDates(i);
    floatDates(floatDates > thisDate) = 0;
    swaps.FloatingResetDates(:,i) = max(floatDates,[],2);
end
```

Setup the Hull White model and simulate scenarios

```
Alpha = 0.0425;
Sigma = 0.0104;

r0 = RateCurveObj.getZeroRates(Settle+1,'Compounding',-1);
t0 = Settle;

FwdRates =
RateCurveObj.getForwardRates(t0+1:max(swaps.Maturity),'Compounding',-1);
hullwhite1 = hwv(Alpha,@(t,x) hw1LevelFun(t0,t,FwdRates,Alpha,Sigma),...
    Sigma,'StartState',r0)

calibration.RateCurveObj = RateCurveObj;
calibration.Tenor = Tenor;
calibration.ShortRateModel = hullwhite1;
calibration.Alpha = Alpha;
calibration.Sigma = Sigma;

prevRNG = rng(0);

 [scenarios, dfactors] =
hgenerateScenario(calibration,simulationDates,numScenarios);

rng(prevRNG);

dfactors = ones(numDates,numScenarios);
for i = 2:numDates
    tenorDates = datemnth(simulationDates(i-1),Tenor);
    rateAtNextSimDate = interp1(tenorDates,squeeze(scenarios(i-1,:,:)),...
        simulationDates(i),'linear','extrap');

    dfactors(i,:) = zero2disc(rateAtNextSimDate,...
        repmat(simulationDates(i),1,numScenarios),simulationDates(i-1),-
1,3);
end
dfactors = cumprod(dfactors,1);

i = 20;
```

```
figure;
surf(Tenor, simulationDates, scenarios(:,:,i))
axis tight
datetick('y','mmmyy');
xlabel('Tenor (Months)');
ylabel('Observation Date');
zlabel('Rates');
set(gca,'View',[-49 32]);
title(sprintf('Scenario %d Yield Curve Evolution\n',i));

values = hcomputeMTMValues(swaps,simulationDates,scenarios,Tenor);

i = 32;
figure;
plot(simulationDates, values(:,:,i));
datetick;
ylabel('Mark-To-Market Price');
title(sprintf('Swap prices along scenario %d', i));

i = 39;
figure;
plot(simulationDates, values(:,:,i));
datetick;
ylabel('Mark-To-Market Price');
title(sprintf('Swap prices along scenario %d', i));

figure;
totalPortValues = squeeze(sum(values, 2));
plot(simulationDates,totalPortValues);
title('Total MTM Portfolio Value for All Scenarios');
datetick('x','mmmyy')
ylabel('Portfolio Value (USD)')
xlabel('Simulation Dates')
```

Compute exposures

```
instrument_exposures = zeros(size(values));
unnettedIdx = swaps.NettingID == 0;
instrument_exposures(:,unnettedIdx,:) = max(values(:,unnettedIdx,:),0);

for i = 1:numCounterparties

    nettedIdx = swaps.NettingID == i;
    numInst = sum(nettedIdx);

    nettingSetValues = values(:,nettedIdx,:);
    nettedExposure = max(sum(nettingSetValues,2),0);
    positiveIdx = repmat(nettedExposure > 0,[1 numInst]);

    instrument_exposures(:,nettedIdx,:) = nettingSetValues .* positiveIdx;

end

exposures = zeros(numDates,numCounterparties,numScenarios);
for i = 1:numCounterparties
```

```
    cpSwapIdx = swaps.Counterparty == i;
    exposures(:,i,:) = sum(instrument_exposures(:,cpSwapIdx,:),2);

end

figure;
totalPortExposure = squeeze(sum(exposures,2));
plot(simulationDates,totalPortExposure);
title('Portfolio Exposure for All Scenarios');
datetick('x','mmmyy')
ylabel('Exposure (USD)')
xlabel('Simulation Dates')

expPort = squeeze(sum(exposures,2));

PEcp   = prctile(exposures,95,3);
PEport = prctile(expPort,95,2);

MPEcp    = max(PEcp);
MPEport = max(PEport);

EEcp   = mean(exposures,3);
EEport = mean(expPort,2);

simTimeInterval = yearfrac(Settle, simulationDates, 1);
simTotalTime = simTimeInterval(end)-simTimeInterval(1);
EPEcp    = 0.5*(EEcp(1:end-1,:)+EEcp(2:end,:))' *
diff(simTimeInterval)/simTotalTime;
EPEport = 0.5*(EEport(1:end-1)+EEport(2:end))' *
diff(simTimeInterval)/simTotalTime;

EffEEcp = zeros(size(EEcp));
for i = 1:size(EEcp,2)

    m = EEcp(1,i);
    for j = 1:numel(simulationDates)
        if EEcp(j,i) > m
            m = EEcp(j,i);
        end
        EffEEcp(j,i) = m;
    end

end

EffEEport = zeros(size(EEport));
m = EEport(1);
for i = 1:numel(simulationDates)
    if EEport(i) > m
        m = EEport(i);
    end
    EffEEport(i) = m;
end

EffEPEcp    = 0.5*(EffEEcp(1:end-1,:)+EffEEcp(2:end,:))' *
diff(simTimeInterval)/simTotalTime;
```

```
EffEPEport = 0.5*(EffEEport(1:end-1)+EffEEport(2:end))' *
diff(simTimeInterval)/simTotalTime;

figure;
plot(simulationDates,PEport,...
    simulationDates,MPEport*ones(size(PEport)),...
    simulationDates,EEport,...
    simulationDates,EPEport*ones(size(PEport)),...
    simulationDates,EffEEport,...
    simulationDates,EffEPEport*ones(size(PEport)))
legend({'PFE95%','MPFE95%',...
    'EE','EPE','EEE','EEPE'})

datetick('x','mmmyy')
title('Portfolio Exposure Profiles');
ylabel('Exposure (USD)')
xlabel('Simulation Dates')

cpIdx = 1;
figure;
plot(simulationDates,PEcp(:,cpIdx),...
    simulationDates,MPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))),...
    simulationDates,EEcp(:,cpIdx),...
    simulationDates,EPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))),...
    simulationDates,EffEEcp(:,cpIdx),...
    simulationDates,EffEPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))))
legend({'PFE95%','MPFE95%',...
    'EE','EPE','EEE','EEPE'})
datetick('x','mmmyy','keeplimits')
title('Germany Exposure Profiles');
ylabel('Exposure (USD)')
xlabel('Simulation Dates')

cpIdx = 2;
figure;
plot(simulationDates,PEcp(:,cpIdx),...
    simulationDates,MPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))),...
    simulationDates,EEcp(:,cpIdx),...
    simulationDates,EPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))),...
    simulationDates,EffEEcp(:,cpIdx),...
    simulationDates,EffEPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))))
legend({'PFE95%','MPFE95%',...
    'EE','EPE','EEE','EEPE'})
datetick('x','mmmyy','keeplimits')
title('Spain Exposure Profiles');
ylabel('Exposure (USD)')
xlabel('Simulation Dates')

cpIdx = 3;
figure;
plot(simulationDates,PEcp(:,cpIdx),...
    simulationDates,MPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))),...
    simulationDates,EEcp(:,cpIdx),...
    simulationDates,EPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))),...
    simulationDates,EffEEcp(:,cpIdx),...
    simulationDates,EffEPEcp(cpIdx)*ones(size(PEcp(:,cpIdx))))
legend({'PFE95%','MPFE95%',...
    'EE','EPE','EEE','EEPE'})
```

```
datetick('x','mmmyy','keeplimits')
title('Greece Exposure Profiles');
ylabel('Exposure (USD)')
xlabel('Simulation Dates')

discExp = zeros(size(exposures));
for i = 1:numScenarios
    discExp(:,:,i) = bsxfun(@times,dfactors(:,i),exposures(:,:,i));
end

discEE = mean(discExp,3);

figure;
plot(simulationDates,sum(discEE,2))
datetick('x','mmmyy','keeplimits')
title('Discounted Expected Exposure for Portfolio');
ylabel('Discounted Exposure (USD)')
xlabel('Simulation Dates')

figure;
plot(simulationDates,discEE)
datetick('x','mmmyy','keeplimits')
title('Discounted Expected Exposure for Each Counterparty');
ylabel('Discounted Exposure (USD)')
xlabel('Simulation Dates')
```

Calibrate default probabilities

```
CDS = dataset('XLSfile',swapFile,'Sheet','CDS Spreads')
CDSDates = datenum(CDS.Date);
CDSSpreads = double(CDS(:,2:end));

ZeroData = [RateSpec.EndDates RateSpec.Rates];

DefProb = zeros(length(simulationDates), size(CDSSpreads,2));
for i = 1:size(DefProb,2)
    probData = cdsbootstrap(ZeroData, [CDSDates CDSSpreads(:,i)],...
        Settle, 'probDates', simulationDates);
    DefProb(:,i) = probData(:,2);
end

figure;
plot(simulationDates,DefProb)
title('Default Probability Curve for Each Counterparty');
xlabel('Date');
grid on;
ylabel('Cumulative Probability')
datetick('x','mmmyy')
ylabel('Probability of Default')
xlabel('Simulation Dates')
```

Calculate CVA

```
Recovery = 0.4;
CVA = (1-Recovery) * sum(discEE(2:end,:) .* diff(DefProb));
```

```
for i = 1:numel(CVA)
    fprintf('CVA for counterparty %d = USD%.2f\n',i,CVA(i));
end

figure;
bar(CVA);
title('CVA for each counterparty');
xlabel('Counterparty');
ylabel('CVA USD');
grid on;
```

Bootstrap CDS

```
function [ProbData,HazData] =
cdsbootstrap(ZeroData,MarketData,Settle,varargin);

('recoveryrate',.4,@(x) (isnumeric(x) &&...
                        all(x>=0) && all(x<=1) ));
p.addParamValue('period',4,@(x) all(ismember(x,[1 2 3 4 6 12])));
p.addParamValue('basis',2,@(x) all(isvalidbasis(x)));
p.addParamValue('busdayconvention',{'actual'},...
            @(x) all(ismember(lower(x),{'actual','follow','previous',...
                        'modifiedfollow','modifiedprevious'})));
p.addParamValue('payaccruedpremium',true,@islogical);
p.addParamValue('timestep',10,@(x) (isnumeric(x) &&...
                        (x>0) && (mod(x,1)==0)));
p.addParamValue('probdates',[]);
p.addParamValue('zerocompounding',2,@(x) (isscalar(x) &&...
                         ismember(x,[-1 1 2 3 4 6 12])));
p.addParamValue('zerobasis',0,@(x) isscalar(x) && isvalidbasis(x));

try
   Settle = datenum(Settle);
catch ME
    newME = MException('fininst:cds:SettleInvalid',...
        'Invalid settlement date');
    newME = addCause(newME,ME);
    throw(newME)
end

try
   if isa(ZeroData,'IRDataCurve')
      ZeroData.getDiscountFactors(Settle+1);
   else
      zero2disc(interp1(ZeroData(:,1),ZeroData(:,2),Settle+1,...
         'linear','extrap'),Settle+1,Settle);
   end
catch ME
   newME = MException('fininst:cds:ZeroDataInvalid',...
     'Invalid zero data input');
   newME = addCause(newME,ME);
   throw(newME)
end

try
   MarketDates = datenum(MarketData(:,1));
catch ME
```

```
    newME = MException('fininst:cds:MarketDatesInvalid',...
        'Invalid dates in market data input');
    newME = addCause(newME,ME);
    throw(newME)
end

if any(Settle > MarketDates)
   error(message('fininst:cds:MarketDatesBeforeSettle'));
end

if size(MarketData,2) == 2
    Spread = MarketData(:,2);
    isUpfront = false;
elseif size(MarketData,2) == 3
    Upfront = MarketData(:,2);
    ContractSpread = MarketData(:,3);
    isUpfront = true;
else
    newME = MException('fininst:cds:MarketDataInvalid',...
        'Error in market data inputs');
    throw(newME)
end

try
    p.parse(varargin{:});
catch ME
    newME = MException('fininst:cds:optionalInputError',...
        'Error in optional parameter value inputs');
    newME = addCause(newME,ME);
    throw(newME)
end

Recovery = p.Results.recoveryrate(:);
Period = p.Results.period(:);
Basis = p.Results.basis(:);
if ischar(p.Results.busdayconvention)
   BusDayConvention = {p.Results.busdayconvention};
else
   BusDayConvention = p.Results.busdayconvention(:);
end
PayAccruedPremium = p.Results.payaccruedpremium(:);
TimeStep = p.Results.timestep;

if ~isempty(p.Results.probdates)
   try
      ProbDates = datenum(p.Results.probdates);
   catch ME
    newME = MException('fininst:cds:ProbDatesInvalid',...
        'Invalid optional probability dates input');
    newME = addCause(newME,ME);
    throw(newME)
   end
else
   ProbDates = MarketDates;
end

ZeroCompounding = p.Results.zerocompounding;
ZeroBasis = p.Results.zerobasis;
```

```
DefProbCurveBasis = 2;

N = length(MarketDates);
mismatch = false;
if length(Recovery)==1
   Recovery = repmat(Recovery,N,1);
elseif length(Recovery)~=N
   mismatch = true;
end
if length(Period)==1
   Period = repmat(Period,N,1);
elseif length(Period)~=N
   mismatch = true;
end
if length(Basis)==1
   Basis = repmat(Basis,N,1);
elseif length(Basis)~=N
   mismatch = true;
end
if length(BusDayConvention)==1
   BusDayConvention = repmat(BusDayConvention,N,1);
elseif length(BusDayConvention)~=N
   mismatch = true;
end
if length(PayAccruedPremium)==1
   PayAccruedPremium = repmat(PayAccruedPremium,N,1);
elseif length(PayAccruedPremium)~=N
   mismatch = true;
end

if (mismatch)
   error(message('fininst:cds:SizeMismatchMarketData'));
end

b2 = [];
for jdx=1:N
   if isUpfront
      errorFun = @(x) Upfront(jdx)*1e7 - cdsprice(ZeroData,...
      [MarketDates(1:jdx) [b2;x]], Settle,...
      MarketDates(jdx),ContractSpread(jdx),'RecoveryRate',Recovery(jdx),...
      'Period',Period(jdx),'Basis',Basis(jdx),'BusDayConvention',...
      BusDayConvention(jdx),'PayAccruedPremium',PayAccruedPremium(jdx),...

'TimeStep',TimeStep,'ZeroCompounding',ZeroCompounding,'ZeroBasis',ZeroBasis
);
   else
      errorFun = @(x) Spread(jdx) - cdsspread(ZeroData,...
         [MarketDates(1:jdx) [b2;x]], Settle, MarketDates(jdx),...
         'RecoveryRate',Recovery(jdx),'Period',Period(jdx),...
         'Basis',Basis(jdx),'BusDayConvention',BusDayConvention(jdx),...
         'PayAccruedPremium',PayAccruedPremium(jdx),'TimeStep',TimeStep,...
         'ZeroCompounding',ZeroCompounding,'ZeroBasis',ZeroBasis);
   end
   bnew = fzero(errorFun,[0 1]);
   b2 = [b2;bnew];
end
DefaultProbData = b2;
```

```
 [SurvProbData,HazardRates] = internal.fininst.cdsstdsurvprob(...
   yearfrac(Settle,ProbDates,DefProbCurveBasis),DefaultProbData,...
   yearfrac(Settle,MarketDates,Basis));

if (any(HazardRates<0))
   warning(message('fininst:cds:NegativeHazardRates'));
end

ProbData = [ProbDates 1-SurvProbData];
HazData = [MarketDates HazardRates];

function [prob,b] = cdsstdsurvprob(t,DefProb,time)

prob = ones(size(t));
time0 = [0;time];
dtime = diff(time0);

Q = 1- DefProb;
b = zeros(size(Q));
b(1) = -log(Q(1))/time(1);
for jdx=2:length(Q)
    b(jdx) = (-log(Q(jdx)) - diff([0;time(1:jdx-1)]')*b(1:jdx-
1))./(time(jdx) - time(jdx-1));
end

for jdx=1:length(time)
   if jdx < length(time)
      tmpidx = t > time0(jdx) & t <= time0(jdx+1);
   else
      tmpidx = t > time0(jdx);
   end
   H = 0;
   if (jdx>1)
      H = dtime(1:jdx-1)'*b(1:jdx-1);
   end
   H = H + (t(tmpidx) - time0(jdx))*b(jdx);
   prob(tmpidx) = exp(-H);
end
```

Appendix B

B.1 MATLAB output from the calibration of the Hull White model

```
                                         Norm of      First-order
 Iteration  Func-count     f(x)          step          optimality   CG-iterations
     0          3         63.0716                       1.32e+04
     1          6         6.69635      0.0199541            4.69           0
     2          9         3.76202       0.114558             598           0
     3         12         3.36077      0.0385007            21.2           0
     4         15         3.27875       0.027189            18.3           0
     5         18         3.27596     0.00574774            0.77           0
     6         21         3.27593    0.000626463         0.00853           0

Local minimum possible.

lsqnonlin stopped because the final change in the sum of squares relative to
its initial value is less than the selected value of the function tolerance.

<stopping criteria details>


HW_alpha =

    0.0425


HW_sigma =

    0.0104


HW1F =

  HullWhite1F with properties:

    ZeroCurve: [1x1 IRDataCurve]
        Alpha: @(t,V)inAlpha
        Sigma: @(t,V)inSigma
```

B.2 MATLAB output from the simulation of the Hull White model without netting

```
hullwhite1 =

   Class HWV: Hull-White/Vasicek
   ----------------------------------------
     Dimensions: State = 1, Brownian = 1
   ----------------------------------------
      StartTime: 0
     StartState: 0.00987946
    Correlation: 1
          Drift: drift rate function F(t,X(t))
      Diffusion: diffusion rate function G(t,X(t))
     Simulation: simulation method/function simByEuler
          Sigma: 0.0104
          Level: function @(t,x)hw1LevelFun(t0,t,FwdRates,Alpha,Sigma)
          Speed: 0.0425

CDS =

    Date                  cp1       cp2        cp3
    '3/12/2014'         4.465    92.105     1241.7
    '9/12/2014'             5       104     1137.7
    '9/12/2015'             9       140     892.01
    '9/12/2016'            13    180.49     866.35
    '9/12/2017'          18.5    207.11     1209.1
    '9/12/2018'            28     232.5       1254
    '9/12/2019'          39.5     248.5       1202
    '9/12/2020'          45.5       259     1174.9
    '9/12/2021'          51.5       261     1164.8
    '9/12/2022'          55.5       263     1165.9
    '9/12/2023'          60.5    264.99     1176.1

CVA for counterparty 1 = $168936.85
CVA for counterparty 2 = $864435.08
CVA for counterparty 3 = $2364261.61
```

B.3 MATLAB output from the simulation of the Hull White model with netting

```
hullwhite1 =

   Class HWV: Hull-White/Vasicek
   ----------------------------------------
     Dimensions: State = 1, Brownian = 1
   ----------------------------------------
      StartTime: 0
     StartState: 0.00987946
    Correlation: 1
          Drift: drift rate function F(t,X(t))
      Diffusion: diffusion rate function G(t,X(t))
     Simulation: simulation method/function simByEuler
          Sigma: 0.0104
          Level: function @(t,x)hw1LevelFun(t0,t,FwdRates,Alpha,Sigma)
          Speed: 0.0425

CDS =

    Date                  cp1       cp2       cp3
    '3/12/2014'         4.465    92.105    1241.7
    '9/12/2014'             5       104    1137.7
    '9/12/2015'             9       140    892.01
    '9/12/2016'            13    180.49    866.35
    '9/12/2017'          18.5    207.11    1209.1
    '9/12/2018'            28     232.5      1254
    '9/12/2019'          39.5     248.5      1202
    '9/12/2020'          45.5       259    1174.9
    '9/12/2021'          51.5       261    1164.8
    '9/12/2022'          55.5       263    1165.9
    '9/12/2023'          60.5    264.99    1176.1

CVA for counterparty 1 = $13122.86
CVA for counterparty 2 = $60292.03
CVA for counterparty 3 = $162780.17
```