MATH598B Homework 1

Alex Fruge

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Suppose we have a vector space $\mathbb{R}^d$, with n vectors in this space.

1 Question 1

What is the maximum number n of orthogonal vectors we can put into this vector space? Explain why, in detail.

Initial Guess: The maximum number of orthogonal vectors that can fit into $\mathbb{R}^d$ is d, no intuition why yet.
Answer work
Answer

2 Question 2

Suppose we concatenate the n orthogonal vectors, each in $\mathbb{R}^d$, into a single matrix. What properties will this matrix have? Be thorough.
Answer work
Answer

3 Question 3

Suppose $n \gg d$. Assuming they are packed "optimally", what bounds can we place on the expected dot product of two random vectors from our set? To answer this, you should do some mix of reading papers, trying to derive a bound, and doing numerical experiments. Include your code, cite your sources.
Answer work
Answer

4 Question 4

Why do you think thinking about this is relevant for the subject of the course? What do you think these vectors are supposed to represent, and why might we care about them being "optimally packed"?
Answer work
Answer