Using TMB (Template Model Builder) to predict the winner of a ping pong match

Alex Fun
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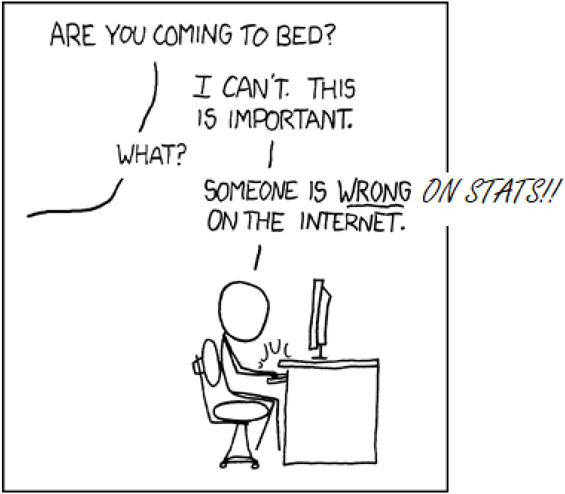
20180712 (3.50 pm @ useR!2018/P7)

Part 1: The stats.stackexchange problem



(adapted from www.xkcd.com/386)

Cross Validated

QUESTIONS TAGS USERS BADGES UNANSWERED ASK QUESTION

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6	15	21	Me
7	16	21	Me

asked 4 months ago

viewed 13,670 times

active 3 months ago

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(https://stats.stackexchange.com/questions/329521)

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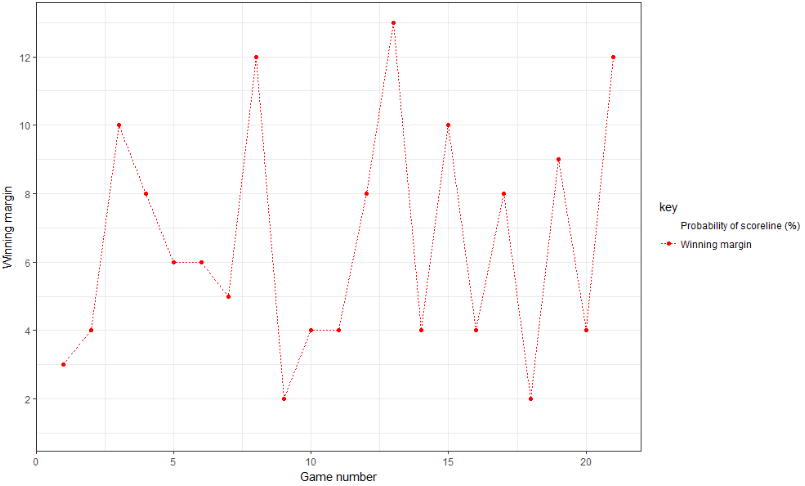

But what if the assumption is wrong?

- Postulate that in a truly competitive game, the original poster has a much higher (than 59%) chance of winning a point

Part 2: Why use automatic derivatives in optimisation?

Optimisation

- Objective function (e.g. log-likelihood) and decision variables (e.g. regression coefficients).
- Gradient based vs non-gradient based (not relevant here).

Optimisation: Newton's method

Suppose one wishes to find the value of (the decision variables) $\mathbf{x}$ such that $y = f(\mathbf{x})$ is maximised. Start with an initial guess $\mathbf{x}_0$ for the optimal values, and iteratively build better guesses $\mathbf{x}_1, \mathbf{x}_2, \ldots$, via:

$$\mathbf{x}_{n+1} = \mathbf{x}_n - \frac{f'(\mathbf{x}_n)}{f''(\mathbf{x}_n)}$$

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If $\mathbf{x}$ is a vector:

$$\mathbf{x}_{n+1} = \mathbf{x}_n - [\mathbf{H}f(\mathbf{x}_n)]^{-1}\nabla f(\mathbf{x}_n)$$

(https://en.wikipedia.org/wiki/Newton%27s_method_in_optimization)

Optimisation: Newton's method

Nabla is the gradient and **H** is the hessian:

$$\nabla = \left(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_m} \right)$$

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1\,\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_1\,\partial x_m} \\ \dfrac{\partial^2 f}{\partial x_2\,\partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} & \cdots & \dfrac{\partial^2 f}{\partial x_2\,\partial x_m} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial^2 f}{\partial x_m\,\partial x_1} & \dfrac{\partial^2 f}{\partial x_m\,\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_m^2} \end{bmatrix}$$

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Wow, much derivatives to find! (LaTeX from Wikipedia)

Calculating derivatives: Analytically

- Logistic regression:

$$\log(\frac{p_i}{1-p_i}) = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots$$

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- After some algebra (or consultation of https://stats.stackexchange.com/a/16541):

$$\begin{aligned}\frac{\partial \ell}{\partial \beta_j} &= ... \\ &= \sum_i y_i x_{ij} - \sum_i p_i x_{ij},\end{aligned}$$

$$\frac{\partial^2 \ell}{\partial \beta_j \partial \beta_k} = \dots$$

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(ellipses are left as an exercise for the reader.)

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- https://justindomke.wordpress.com/2009/02/17/automatic-differentiation-the-most-criminally-underused-tool-in-the-pe

Part 3: TMB

What is TMB

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  https://github.com/kaskr/adcomp/wiki
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- R package that calculates first and second derivatives of any objective function written in C++.
- Very, very fast $\leftrightarrow$ uses state of the art C++ packages like Eigen, CppAD, ...
- Used where more ‘complex’ log-likelihood functions are required. E.g.:
  - the glmmTMB package allows you to “fit linear and generalized linear mixed models with various extensions, including zero-inflation”.
  - A Laplace approximation for the likelihood is implemented to deal with models containing random effects.

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- The aim is to find values for the following parameters that maximise the likelihood of observing the game scorelines:
  - p_0: probability of point win when original poster is not playing at their best.
  - β: changes probability of winning in the next game depending on the scoreline this game.
  - α_0 and α_1: coefficients governing the averaging between p_0 and p_i.

Modelling: Probability to win a game with a certain scoreline

- In a game, suppose player one wins points with probability p, and let x and y be the total number of points player one and two won, respectively.
- Suppose player one reaches 21 points first with a margin of two:

$$\Pr(21 : y) = \binom{20 + y}{y} p^{21} (1 - p)^{y}$$

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- In a game, suppose player one wins points with probability p, and let x and y be the total number of points player one and two won, respectively.
- Suppose player one reaches 21 points first with a margin of two:

$$\Pr(21 : y) = \binom{20 + y}{y} p^{21}(1 - p)^{y}$$

- Else:

$$\Pr(x : y) = 2^{y-20}\binom{40}{20} p^{x}(1 - p)^{y}$$

(whuber: https://stats.stackexchange.com/a/329554)

Code: R

```r
R_obj_fun_template <- function(par, me, you, score_diff) {

    p_0 <- par[1]
    alpha_0 <- par[2]
    alpha_1 <- par[3]
    beta <- par[4]

    n_obs <- length(score_diff)

    p <- p_0 # initialise initial model fitted probability
    nll <- 0 # initialise model negloglik

    for (i in seq_along(you)) {

        if (i > 1) {
            # mixing coefficient
            alpha <- plogis(alpha_0 + alpha_1 * score_diff[i - 1]);

            # update p with model
            p <-  alpha * p_0 + (1 - alpha) *  plogis(qlogis(p) + beta);


        }

        # increment nll
        nll <- nll - log(get_prob_of_scoreline(me[i], you[i], p));

    }



    return(nll)
}

optim_in_R$fn <- function(par) R_obj_fun_template(par, df$Me, df$Opponent, df$score_diff)
optim_in_R$par <- c(0.592246, 0, 0, 0)
# quasi-Newton optimisation with approximations to the Hessian
# i.e. second derivatives are never required
optim_in_R$method <- 'BFGS'
```

Code: R

```r
get_prob_of_scoreline <- function(me, you, prob_win_point){

    # warning, not vectorised!
    stopifnot(all(length(me) == 1, length(you) == 1, length(prob_win_point) == 1))

    min_me_you <- min(me, you)


    if (you >= 20) {
        out_prob <- choose(40, 20) * 2^(min_me_you - 20) * prob_win_point^me * (1 - prob_win_point)^you

    } else {
        out_prob <- choose(20 + min_me_you, min_me_you) * prob_win_point^me * (1 - prob_win_point)^you

    }

    return(out_prob);
}
```

Code: Rcpp

```
R_obj_fun_template_Rcpp <- cppFunction(
    "double R_obj_fun_templatep_Rcpp(NumericVector par, IntegerVector me, IntegerVector you, IntegerVector score_diff) {



    // parameters:
    double p_0 = par[0];
    double alpha_0 = par[1];
    double alpha_1 = par[2];
    double beta = par[3];

    int n_obs = me.size();

    double p = p_0; // initialise initial model fitted probability
    double nll = 0; // initialise model negloglik


    for (int i = 0; i < n_obs; i++) {

        if (i > 0) {
            // mixing coefficient
            double alpha = Rf_plogis(alpha_0 + alpha_1 * score_diff[i - 1], 0.0, 1.0, 1, 0);

            // update p with model

            p =  alpha * p_0 + (1 - alpha) *  Rf_plogis(Rf_qlogis(p, 0.0, 1.0, 1, 0) + beta, 0.0, 1.0, 1, 0);


        }
        nll += -log(get_prob_of_scoreline_Rcpp(me[i], you[i], p));


    }


    return nll;

}",
    includes = "double get_prob_of_scoreline_Rcpp(int me, int you, double prob_win_point) {
    double out_prob;
```

Code: TMB

```
template<class Type>
Type objective_function<Type>::operator() () {


    DATA_VECTOR(you);
    DATA_VECTOR(me);
    DATA_VECTOR(score_diff);


    // parameters:
    PARAMETER(p_0);
    PARAMETER(alpha_0);
    PARAMETER(alpha_1);
    PARAMETER(beta);

    int n_obs = me.size();

    Type p = p_0; // initialise initial model fitted probability
    Type nll = 0; // initialise model negloglik
    vector<Type> p_vec_out(n_obs);

    for (int i = 0; i < n_obs; i++) {

        if (i > 0) {
            // mixing coefficient
            Type alpha = invlogit(alpha_0 + alpha_1 * score_diff[i - 1]);

            // update p with model
            p =  alpha * p_0 + (1 - alpha) *  invlogit(logit(p) + beta);


        }
        nll += -log(get_prob_of_scoreline(me[i], you[i], p));
        // store into vector to be returned to R
        p_vec_out[i] = p;
    }


    REPORT(p_vec_out);
    return nll;

}
```

Example usage

```r
## Get TMB objective function which includes .$gr (gradient) and .$he (hessian) elements.
optim_in <- MakeADFun(data = list(me = df$Me, you = df$Opponent, score_diff = df$Me - df$Opponent),
                      parameters = list(p_0 = 0.592246, alpha_0 = 0, alpha_1 = 0, beta = 0),
                      DLL = 'reg_20180312_init_obj_fun',
                      silent = T)

# see that optim_in has the right elements to be passed into R's optimisation function optim.
str(optim_in)
# List of 10
#     $ par     : Named num [1:4] 0.592246 0 0 0
#     ..- attr(*, "names")= chr [1:4] "p_0" "alpha_0" "alpha_1" "beta"
#     $ fn      :function (x = last.par, ...)
#     $ gr      :function (x = last.par, ...)
#     $ he      :function (x = last.par, atomic = usingAtomics())
#     $ hessian : logi FALSE
#     $ method  : chr "BFGS"
#     $ retape  :function (set.defaults = TRUE)
#     $ env     :<environment: 0x000000001bd5b158>
#     $ report  :function (par = last.par)
#     $ simulate:function (par = last.par, complete = FALSE)


# optimise using optim
```

Results: Speed of optimisation

```
> microbenchmark(TMB = do.call(optim, optim_in),
+                TMB_numerical = do.call(optim, optim_in_no_jac),
+                Rcpp = do.call(optim, optim_in_Rcpp),
+                base_R = do.call(optim, optim_in_R))
Unit: milliseconds
          expr        min         lq       mean     median         uq
           TMB   4.028838   4.326319   4.737954   4.430616   4.755356
 TMB_numerical  16.559796  17.426758  18.959108  17.864092  18.819943
          Rcpp  15.682760  16.367006  18.124086  16.978561  18.956832
        base_R 553.979831 571.790193 607.814579 581.838190 609.109290
```

Results: Coefficients

```
> (optim_out <- do.call(optim, optim_in))
$`par`
        p_0     alpha_0     alpha_1        beta
  0.5461695 -33.9765827   3.7675724   0.1189786

$value
[1] 57.25596

$counts
function gradient
     161       68
```

Results: Coefficients

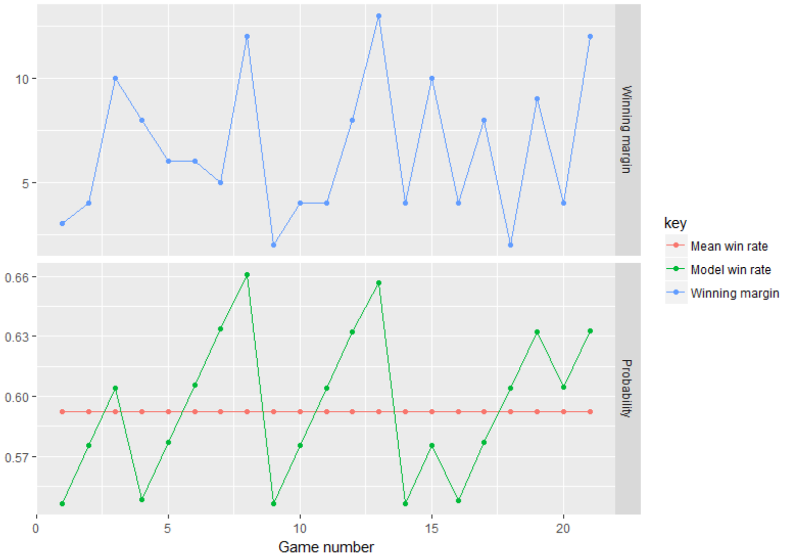
- Recall that s_i is the observed score difference in the i-th game.
- The probability of the original poster winning the $i+1$-th game, given by the model, is:

$$p_{i+1} = 0.546\alpha + (1-\alpha)(\mathrm{expit}(\mathrm{logit}(p_i) + 0.119))$$

where the mixing coefficient α is:

$$\alpha = \mathrm{expit}(-34 + 3.77 s_i)$$

Results: fitted game win probs



The last word

The last word

Your thoughts on whether points and games are independent? Not to mention stationary over time (cumulated games and experience playing each other)? – Mark L. Stone Mar 21 at 15:58

1 I don't think the points and games were independent. The more I won, the more complacent I got and my opponent took advantage of that. – richard Mar 21 at 19:28

Thank you

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