Data and analysis of “Experimental high-dimensional Greenberger-Horne-Zeilinger entanglement with superconducting transmon qutrits”

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Super function

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In[1]:= σabs[a_, b_, da_, db_] := Sqrt[((D[Sqrt[x^2 + b^2], x]) //. x → a)^2 da^2 +
            ((D[Sqrt[a^2 + x^2], x] //. x → b))^2 db^2]
        σarg[a_, b_, da_, db_] := Sqrt[((D[2 ArcTan[b / (Sqrt[x^2 + b^2] + x)], x]) //. x → a)^2
            da^2 + ((D[2 ArcTan[x / (Sqrt[a^2 + x^2] + a)], x]) //. x → b)^2 db^2]

In[3]:= TomoGHZ[n_, pres_, mes01_, mes12_, mes02_, diag_] := (
          (* DATA *)
          {{mes1[0, 1], mes2[0, 1], mes3[0, 1],
              mes4[0, 1], mes5[0, 1], mes6[0, 1], mes7[0, 1], mes8[0, 1]},
            {mes1[1, 2], mes2[1, 2], mes3[1, 2], mes4[1, 2], mes5[1, 2],
              mes6[1, 2], mes7[1, 2], mes8[1, 2]},
            {mes1[0, 2], mes2[0, 2], mes3[0, 2], mes4[0, 2], mes5[0, 2],
              mes6[0, 2], mes7[0, 2], mes8[0, 2]}} = {mes01, mes12, mes02};
          (* shots *)
          countsdiag = Round[n diag ] // N;
          counts01 = Round[n mes01] // N;
          counts12 = Round[n mes12] // N;
          counts02 = Round[n mes02] // N;
          (* Standard deviation assuming binomial distribution: p=
            counts/totalshots = mes01,mes12,mes02 *)
          SDdiag = Round[Sqrt[n diag (1 - diag)]] // N;
          SDmes[0, 1] = Round[Sqrt[n mes01 (1 - mes01)]] // N;
          SDmes[1, 2] = Round[Sqrt[n mes12 (1 - mes12)]] // N;
          SDmes[0, 2] = Round[Sqrt[n mes02 (1 - mes02)]] // N;
          Print["Total number of shots = ", n];
          Print["Counts diagonal: ", IntegerPart[countsdiag] // MatrixForm,
            " ± ", IntegerPart[SDdiag] // MatrixForm];
          Print["Counts 01 subspace: ", IntegerPart[counts01] // MatrixForm,
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 " ± ", IntegerPart[SDmes[0, 1]] // MatrixForm];
Print["Counts 12 subspace: ", IntegerPart[counts12] // MatrixForm,
 " ± ", IntegerPart[ SDmes[1, 2]] // MatrixForm];
Print["Counts 02 subspace: ", IntegerPart[counts02] // MatrixForm,
 " ± ", IntegerPart[SDmes[0, 2]] // MatrixForm];
(* REAL AND IMAGINARY PARTS *)
ev = {1, -1, -1, 1, -1, 1, 1, -1};
(* Formula (combination of expectation values *)
Do[rexp[i, j] = 1/8 (mes1[i, j] - mes2[i, j] - mes3[i, j] - mes4[i, j]).ev;
 iexp[i, j] = 1/8 (mes5[i, j] - mes6[i, j] - mes7[i, j] - mes8[i, j]).ev;,
 {i, 0, 1}, {j, i + 1, 2}];
(* Propagate standard deviation *)
(* SD each expectation value *)
Do[SDmes1[i, j] = Sqrt[Sum[(SDmes[i, j][[1, k]])^2, {k, 1, 8}]]/n;
 SDmes2[i, j] = Sqrt[Sum[SDmes[i, j][[2, k]]^2, {k, 1, 8}]]/n;
 SDmes3[i, j] = Sqrt[Sum[SDmes[i, j][[3, k]]^2, {k, 1, 8}]]/n;
 SDmes4[i, j] = Sqrt[Sum[SDmes[i, j][[4, k]]^2, {k, 1, 8}]]/n;
 SDmes5[i, j] = Sqrt[Sum[SDmes[i, j][[5, k]]^2, {k, 1, 8}]]/n;
 SDmes6[i, j] = Sqrt[Sum[SDmes[i, j][[6, k]]^2, {k, 1, 8}]]/n;
 SDmes7[i, j] = Sqrt[Sum[SDmes[i, j][[7, k]]^2, {k, 1, 8}]]/n;
 SDmes8[i, j] = Sqrt[Sum[SDmes[i, j][[8, k]]^2, {k, 1, 8}]]/n,
 {i, 0, 1}, {j, i + 1, 2}];
Print["Expectation values (in () the theoretical value)"];
Print[" 01, 12, 02, (theory) subspaces respectively"];
Print["⟨xxx⟩ = ", mes1[0, 1].ev, " ± ", Round[SDmes1[0, 1], pres],
 " | ", mes1[1, 2].ev, " ± ", Round[SDmes1[1, 2], pres], " | ",
 mes1[0, 2].ev, " ± ", Round[SDmes1[0, 2], pres], " | ", " (0.667)"];
Print["⟨yyx⟩ = ", mes2[0, 1].ev, " ± ", Round[SDmes2[0, 1], pres],
 " | ", mes2[1, 2].ev, " ± ", Round[SDmes2[1, 2], pres], " | ",
 mes2[0, 2].ev, " ± ", Round[SDmes2[0, 2], pres], " | ", " (-0.667)"];
Print["⟨yxy⟩ = ", mes3[0, 1].ev, " ± ", Round[SDmes3[0, 1], pres],
 " | ", mes3[1, 2].ev, " ± ", Round[SDmes3[1, 2], pres], " | ",
 mes3[0, 2].ev, " ± ", Round[SDmes3[0, 2], pres], " | ", " (-0.667)"];
Print["⟨xyy⟩ = ", mes4[0, 1].ev, " ± ", Round[SDmes4[0, 1], pres],
 " | ", mes4[1, 2].ev, " ± ", Round[SDmes4[1, 2], pres], " | ",
 mes4[0, 2].ev, " ± ", Round[SDmes4[0, 2], pres], " | ", " (-0.667)"];
Print["⟨yyy⟩ = ", mes5[0, 1].ev, " ± ", Round[SDmes5[0, 1], pres],
 " | ", mes5[1, 2].ev, " ± ", Round[SDmes5[1, 2], pres], " | ",
 mes5[0, 2].ev, " ± ", Round[SDmes5[0, 2], pres], " | ", " (0)"];
Print["⟨xxy⟩ = ", mes6[0, 1].ev, " ± ", Round[SDmes6[0, 1], pres],
 " | ", mes6[1, 2].ev, " ± ", Round[SDmes6[1, 2], pres], " | ",
 mes6[0, 2].ev, " ± ", Round[SDmes6[0, 2], pres], " | ", " (0)"];
Print["⟨xyx⟩ = ", mes7[0, 1].ev, " ± ", Round[SDmes7[0, 1], pres],
 " | ", mes7[1, 2].ev, " ± ", Round[SDmes7[1, 2], pres], " | ",
 mes7[0, 2].ev, " ± ", Round[SDmes7[0, 2], pres], " | ", " (0)"];
Print["⟨yxx⟩ = ", mes8[0, 1].ev, " ± ", Round[SDmes8[0, 1], pres],
 " | ", mes8[1, 2].ev, " ± ", Round[SDmes8[1, 2], pres], " | ",
 mes8[0, 2].ev, " ± ", Round[SDmes8[0, 2], pres], " | ", " (0)"];
(* SD real and imaginary parts *)
Do[SDreal[i, j] =
  1/8 Sqrt[SDmes1[i, j]^2 + SDmes2[i, j]^2 + SDmes3[i, j]^2 + SDmes4[i, j]^2];
 SDreal[j, i] = SDreal[i, j];
 SDimg[i, j] =
  1/8 Sqrt[SDmes5[i, j]^2 + SDmes6[i, j]^2 + SDmes7[i, j]^2 + SDmes8[i, j]^2];
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 SDimg[j, i] = SDimg[i, j];, {i, 0, 1}, {j, i + 1, 2}];
(* construct the matrix *)
offdiag = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};
Do[offdiag[[i + 1, j + 1]] = Chop[(rexp[i, j] + I iexp[i, j])];
 offdiag[[j + 1, i + 1]] = Conjugate[offdiag[[i + 1, j + 1]]], {i, 0, 1}, {j, i + 1, 2}];
SDoffdiagreal = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};
SDoffdiagimg = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};
Do[SDoffdiagreal[[i + 1, j + 1]] = SDreal[i, j];
 SDoffdiagreal[[j + 1, i + 1]] = SDreal[j, i];
 SDoffdiagimg[[i + 1, j + 1]] = SDimg[i, j];
 SDoffdiagimg[[j + 1, i + 1]] = SDimg[j, i], {i, 0, 1}, {j, i + 1, 2}];
ρ = DiagonalMatrix[diag] + offdiag;
SDdiag = SDdiag / n;
SDρreal = DiagonalMatrix[SDdiag] + SDoffdiagreal;
SDρimg = SDoffdiagimg;
Print["ρ = ", Round[ρ, pres] // MatrixForm];
Print["SD Re(ρ) = ", Round[SDρreal, pres] // MatrixForm];
Print["SD Im(ρ) = ", Round[SDρimg, pres] // MatrixForm];
(* FIDELITY *)
fidelity =
 1 / 3 (Sum[diag[[i]], {i, 1, 3}] + Sum[offdiag[[i, j]], {i, 1, 3}, {j, 1, 3}]);
(* using only the real part *)
fidelityreal =
 1 / 3 (Sum[diag[[i]], {i, 1, 3}] + 2 Sum[rexp[i, j], {i, 0, 1}, {j, i + 1, 2}]);
SDfid = 1 / 3 Sqrt[Sum[SDdiag[[i]]^2, {i, 1, 3}] +
    4 Sum[SDreal[i, j]^2, {i, 0, 1}, {j, i + 1, 2}]];
Print["Fidelity = ", Round[Chop[fidelityreal], pres], " ± ", Round[SDfid, pres]];

(* Absolute and argument values of the density matrix *)
(* Absolute values ρ elements *)
offdiagabs = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};
SDoffdiagabs = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};
Do[offdiagabs[[i + 1, j + 1]] = Chop[Abs[(rexp[i, j] + I iexp[i, j])]];
 SDoffdiagabs[[i + 1, j + 1]] = σabs[rexp[i, j], iexp[i, j], SDreal[i, j], SDimg[i, j]];
 SDoffdiagabs[[j + 1, i + 1]] = SDoffdiagabs[[i + 1, j + 1]];
 offdiagabs[[j + 1, i + 1]] = offdiagabs[[i + 1, j + 1]], {i, 0, 1}, {j, i + 1, 2}];
(* Argument values ρ elements *)
offdiagarg = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};
SDoffdiagarg = {{0, 0, 0}, {0, 0, 0}, {0, 0, 0}};
Do[offdiagarg[[i + 1, j + 1]] = Chop[Arg[(rexp[i, j] + I iexp[i, j])]];
 SDoffdiagarg[[i + 1, j + 1]] = σarg[rexp[i, j], iexp[i, j], SDreal[i, j], SDimg[i, j]];
 SDoffdiagarg[[j + 1, i + 1]] = SDoffdiagarg[[i + 1, j + 1]];
 offdiagarg[[j + 1, i + 1]] = -offdiagarg[[i + 1, j + 1]], {i, 0, 1}, {j, i + 1, 2}];
(* Print results *)
ρabs = DiagonalMatrix[diag] + offdiagabs;
SDabs = (SDdiag IdentityMatrix[3] + SDoffdiagabs);
ρarg = offdiagarg;
SDarg = SDoffdiagarg;
Print["Abs(ρ) = ", Round[ρabs, pres] // MatrixForm];
Print["SD Abs(ρ) = ", Round[SDabs, pres] // MatrixForm];
Print["Arg(ρ) = ", Round[ρarg, pres] // MatrixForm];
Print["SD Arg(ρ) = ", Round[SDarg, pres] // MatrixForm];
(* phases estimation *)
Print["ϕ1 = ", Round[-ρarg[[1, 2]], pres], " ± ", Round[SDarg[[1, 2]], pres]];
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  Print["ϕ2 = ", Round[-ρarg[[1, 3]], pres], " ± ", Round[SDarg[[1, 3]], pres]];
  Print["(ϕ1-ϕ2) direct (from ρ) = ",
   Round[-ρarg[[2, 3]], pres], " ± ", Round[SDarg[[2, 3]], pres]];
  Print["(ϕ1-ϕ2) indirect (from the difference)= ",
   Round[-ρarg[[1, 2]] + ρarg[[1, 3]], pres], " ± ",
   Round[Sqrt[SDarg[[1, 2]]^2 + SDarg[[1, 3]]^2], pres]];
 )
```

Results experiment on ibmq_rome transmons [Q1,Q2,Q3]

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In[4]:= n = 512; (* shots *) pres = 0.001; (* precision *)
```

Raw data

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In[5]:= diag = {0.34, 0.283, 0.205};
        {mesu01, mesu12, mesu02} =
          {{{0.172, 0.021, 0.021, 0.17, 0.02, 0.164, 0.172, 0.006}, {0.027, 0.164, 0.17, 0.023,
              0.143, 0.021, 0.02, 0.191}, {0.033, 0.164, 0.137, 0.021, 0.152, 0.023, 0.014, 0.127},
            {0.025, 0.162, 0.158, 0.014, 0.164, 0.016, 0.014, 0.166},
            {0.09, 0.082, 0.092, 0.117, 0.082, 0.072, 0.09, 0.084},
            {0.098, 0.125, 0.104, 0.1, 0.066, 0.104, 0.072, 0.074},
            {0.107, 0.064, 0.096, 0.078, 0.094, 0.078, 0.076, 0.088},
            {0.084, 0.119, 0.086, 0.08, 0.094, 0.084, 0.086, 0.094}},
           {{0.113, 0.027, 0.035, 0.113, 0.029, 0.096, 0.137, 0.039},
            {0.035, 0.098, 0.104, 0.031, 0.111, 0.031, 0.029, 0.1},
            {0.016, 0.127, 0.064, 0.059, 0.127, 0.037, 0.035, 0.113},
            {0.035, 0.113, 0.088, 0.043, 0.107, 0.027, 0.029, 0.121},
            {0.109, 0.078, 0.043, 0.086, 0.057, 0.066, 0.08, 0.072},
            {0.047, 0.105, 0.09, 0.07, 0.08, 0.061, 0.066, 0.078},
            {0.047, 0.104, 0.098, 0.055, 0.08, 0.078, 0.08, 0.1},
            {0.047, 0.104, 0.078, 0.068, 0.082, 0.055, 0.053, 0.09}},
           {{0.1, 0.016, 0.039, 0.094, 0.031, 0.164, 0.154, 0.043},
            {0.033, 0.121, 0.148, 0.025, 0.109, 0.018, 0.016, 0.107},
            {0.049, 0.121, 0.145, 0.039, 0.129, 0.023, 0.035, 0.137},
            {0.035, 0.137, 0.121, 0.021, 0.135, 0.027, 0.033, 0.102},
            {0.078, 0.078, 0.08, 0.064, 0.07, 0.068, 0.045, 0.07},
            {0.07, 0.07, 0.051, 0.07, 0.059, 0.088, 0.09, 0.066},
            {0.074, 0.066, 0.066, 0.086, 0.062, 0.127, 0.098, 0.059},
            {0.1, 0.066, 0.059, 0.076, 0.061, 0.074, 0.109, 0.043}}};

In[7]:= TomoGHZ[n, pres, mesu01, mesu12, mesu02, diag]
```

Total number of shots = 512

Counts diagonal: $\begin{pmatrix} 174 \\ 145 \\ 105 \end{pmatrix} \pm \begin{pmatrix} 11 \\ 10 \\ 9 \end{pmatrix}$

$$\text{Counts 01 subspace:} \quad \begin{pmatrix} 88 & 11 & 11 & 87 & 10 & 84 & 88 & 3 \\ 14 & 84 & 87 & 12 & 73 & 11 & 10 & 98 \\ 17 & 84 & 70 & 11 & 78 & 12 & 7 & 65 \\ 13 & 83 & 81 & 7 & 84 & 8 & 7 & 85 \\ 46 & 42 & 47 & 60 & 42 & 37 & 46 & 43 \\ 50 & 64 & 53 & 51 & 34 & 53 & 37 & 38 \\ 55 & 33 & 49 & 40 & 48 & 40 & 39 & 45 \\ 43 & 61 & 44 & 41 & 48 & 43 & 44 & 48 \end{pmatrix} \pm \begin{pmatrix} 9 & 3 & 3 & 8 & 3 & 8 & 9 & 2 \\ 4 & 8 & 8 & 3 & 8 & 3 & 3 & 9 \\ 4 & 8 & 8 & 3 & 8 & 3 & 3 & 8 \\ 4 & 8 & 8 & 3 & 8 & 3 & 3 & 8 \\ 6 & 6 & 7 & 7 & 6 & 6 & 6 & 6 \\ 7 & 7 & 7 & 7 & 6 & 7 & 6 & 6 \\ 7 & 6 & 7 & 6 & 7 & 6 & 6 & 6 \\ 6 & 7 & 6 & 6 & 7 & 6 & 6 & 7 \end{pmatrix}$$

$$\text{Counts 12 subspace:} \quad \begin{pmatrix} 58 & 14 & 18 & 58 & 15 & 49 & 70 & 20 \\ 18 & 50 & 53 & 16 & 57 & 16 & 15 & 51 \\ 8 & 65 & 33 & 30 & 65 & 19 & 18 & 58 \\ 18 & 58 & 45 & 22 & 55 & 14 & 15 & 62 \\ 56 & 40 & 22 & 44 & 29 & 34 & 41 & 37 \\ 24 & 54 & 46 & 36 & 41 & 31 & 34 & 40 \\ 24 & 53 & 50 & 28 & 41 & 40 & 41 & 51 \\ 24 & 53 & 40 & 35 & 42 & 28 & 27 & 46 \end{pmatrix} \pm \begin{pmatrix} 7 & 4 & 4 & 7 & 4 & 7 & 8 & 4 \\ 4 & 7 & 7 & 4 & 7 & 4 & 4 & 7 \\ 3 & 8 & 6 & 5 & 8 & 4 & 4 & 7 \\ 4 & 7 & 6 & 5 & 7 & 4 & 4 & 7 \\ 7 & 6 & 5 & 6 & 5 & 6 & 6 & 6 \\ 5 & 7 & 6 & 6 & 6 & 5 & 6 & 6 \\ 5 & 7 & 7 & 5 & 6 & 6 & 6 & 7 \\ 5 & 7 & 6 & 6 & 6 & 5 & 5 & 6 \end{pmatrix}$$

$$\text{Counts 02 subspace:} \quad \begin{pmatrix} 51 & 8 & 20 & 48 & 16 & 84 & 79 & 22 \\ 17 & 62 & 76 & 13 & 56 & 9 & 8 & 55 \\ 25 & 62 & 74 & 20 & 66 & 12 & 18 & 70 \\ 18 & 70 & 62 & 11 & 69 & 14 & 17 & 52 \\ 40 & 40 & 41 & 33 & 36 & 35 & 23 & 36 \\ 36 & 36 & 26 & 36 & 30 & 45 & 46 & 34 \\ 38 & 34 & 34 & 44 & 32 & 65 & 50 & 30 \\ 51 & 34 & 30 & 39 & 31 & 38 & 56 & 22 \end{pmatrix} \pm \begin{pmatrix} 7 & 3 & 4 & 7 & 4 & 8 & 8 & 5 \\ 4 & 7 & 8 & 4 & 7 & 3 & 3 & 7 \\ 5 & 7 & 8 & 4 & 8 & 3 & 4 & 8 \\ 4 & 8 & 7 & 3 & 8 & 4 & 4 & 7 \\ 6 & 6 & 6 & 6 & 6 & 6 & 5 & 6 \\ 6 & 6 & 5 & 6 & 5 & 6 & 6 & 6 \\ 6 & 6 & 6 & 6 & 5 & 8 & 7 & 5 \\ 7 & 6 & 5 & 6 & 5 & 6 & 7 & 5 \end{pmatrix}$$

Expectation values (in () the theoretical value)

01, 12, 02, (theory) subspaces respectively

$\langle xxx \rangle = 0.61 \pm 0.035 \mid 0.329 \pm 0.032 \mid 0.383 \pm 0.033 \mid \ (0.667)$

$\langle yyx \rangle = -0.577 \pm 0.035 \mid -0.287 \pm 0.031 \mid -0.393 \pm 0.032 \mid \ (-0.667)$

$\langle yxy \rangle = -0.489 \pm 0.034 \mid -0.284 \pm 0.033 \mid -0.386 \pm 0.034 \mid \ (-0.667)$

$\langle xyy \rangle = -0.581 \pm 0.034 \mid -0.295 \pm 0.031 \mid -0.379 \pm 0.033 \mid \ (-0.667)$

$\langle yyy \rangle = 0.029 \pm 0.035 \mid 0.091 \pm 0.033 \mid -0.043 \pm 0.033 \mid \ (0)$

$\langle xxy \rangle = 0.005 \pm 0.037 \mid -0.109 \pm 0.033 \mid 0.072 \pm 0.032 \mid \ (0)$

$\langle xyx \rangle = -0.003 \pm 0.035 \mid -0.122 \pm 0.034 \mid 0.132 \pm 0.034 \mid \ (0)$

$\langle yxx \rangle = -0.059 \pm 0.035 \mid -0.131 \pm 0.032 \mid 0.13 \pm 0.033 \mid \ (0)$

$$\rho = \begin{pmatrix} 0.34 & 0.282 + 0.011\, i & 0.193 - 0.047\, i \\ 0.282 - 0.011\, i & 0.283 & 0.149 + 0.057\, i \\ 0.193 + 0.047\, i & 0.149 - 0.057\, i & 0.205 \end{pmatrix}$$

$$\text{SD Re}(\rho) = \begin{pmatrix} 0.021 & 0.009 & 0.008 \\ 0.009 & 0.02 & 0.008 \\ 0.008 & 0.008 & 0.018 \end{pmatrix}$$

$$\text{SD Im}(\rho) = \begin{pmatrix} 0. & 0.009 & 0.008 \\ 0.009 & 0. & 0.008 \\ 0.008 & 0.008 & 0. \end{pmatrix}$$

Fidelity $= 0.692 \pm 0.015$

$$\text{Abs}(\rho) = \begin{pmatrix} 0.34 & 0.282 & 0.198 \\ 0.282 & 0.283 & 0.16 \\ 0.198 & 0.16 & 0.205 \end{pmatrix}$$

$$\text{SD Abs}(\rho) = \begin{pmatrix} 0.021 & 0.009 & 0.008 \\ 0.009 & 0.02 & 0.008 \\ 0.008 & 0.008 & 0.018 \end{pmatrix}$$

$$\text{Arg}(\rho) = \begin{pmatrix} 0. & 0.038 & -0.24 \\ -0.038 & 0. & 0.362 \\ 0.24 & -0.362 & 0. \end{pmatrix}$$

$$\text{SD Arg}(\rho) = \begin{pmatrix} 0. & 0.031 & 0.041 \\ 0.031 & 0. & 0.051 \\ 0.041 & 0.051 & 0. \end{pmatrix}$$

$\phi 1 = -0.038 \pm 0.031$

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ϕ2 = 0.24 ± 0.041

(ϕ1-ϕ2) direct (from ρ) = -0.362 ± 0.051

(ϕ1-ϕ2) indirect (from the difference)= -0.278 ± 0.052
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Measurement error mitigated data

In[8]:=
```
diagm = {0.352, 0.286, 0.206};
{mesum01, mesum12, mesum02} =
  {{{0.177, 0., 0., 0.174, 0., 0.167, 0.175, 0.}, {0.002, 0.168, 0.174, 0.,
     0.144, 0., 0., 0.193}, {0.009, 0.169, 0.14, 0., 0.157, 0., 0., 0.125},
    {0., 0.167, 0.161, 0., 0.169, 0., 0., 0.168}, {0.08, 0.072, 0.081, 0.106, 0.072,
     0.062, 0.08, 0.075}, {0.088, 0.117, 0.096, 0.088, 0.056, 0.092, 0.06, 0.065},
    {0.099, 0.052, 0.086, 0.069, 0.085, 0.063, 0.065, 0.079},
    {0.073, 0.111, 0.077, 0.069, 0.086, 0.073, 0.073, 0.086}},
   {{0.115, 0.005, 0.017, 0.112, 0.01, 0.095, 0.139, 0.021},
    {0.017, 0.096, 0.104, 0.011, 0.111, 0.014, 0.013, 0.097},
    {0., 0.128, 0.062, 0.041, 0.127, 0.019, 0.016, 0.114},
    {0.016, 0.114, 0.088, 0.026, 0.108, 0.01, 0.009, 0.122},
    {0.105, 0.065, 0.029, 0.08, 0.045, 0.056, 0.074, 0.06},
    {0.032, 0.101, 0.084, 0.06, 0.074, 0.05, 0.054, 0.071},
    {0.035, 0.099, 0.094, 0.039, 0.075, 0.066, 0.068, 0.094},
    {0.034, 0.097, 0.071, 0.057, 0.077, 0.042, 0.042, 0.084}},
   {{0.098, 0., 0.022, 0.093, 0.012, 0.168, 0.159, 0.025},
    {0.015, 0.121, 0.151, 0.005, 0.11, 0., 0., 0.108},
    {0.032, 0.121, 0.145, 0.022, 0.129, 0.002, 0.014, 0.139},
    {0.017, 0.138, 0.123, 0.004, 0.136, 0.005, 0.016, 0.102},
    {0.069, 0.07, 0.073, 0.055, 0.063, 0.056, 0.033, 0.063},
    {0.063, 0.06, 0.039, 0.063, 0.045, 0.083, 0.084, 0.051},
    {0.068, 0.054, 0.055, 0.08, 0.049, 0.124, 0.092, 0.046},
    {0.093, 0.056, 0.047, 0.07, 0.05, 0.069, 0.104, 0.028}}};
```

In[10]:=
```
TomoGHZ[n, pres, mesum01, mesum12, mesum02, diagm]
```

Total number of shots = 512

Counts diagonal: $\begin{pmatrix} 180 \\ 146 \\ 105 \end{pmatrix} \pm \begin{pmatrix} 11 \\ 10 \\ 9 \end{pmatrix}$

Counts 01 subspace: $\begin{pmatrix} 91 & 0 & 0 & 89 & 0 & 86 & 90 & 0 \\ 1 & 86 & 89 & 0 & 74 & 0 & 0 & 99 \\ 5 & 87 & 72 & 0 & 80 & 0 & 0 & 64 \\ 0 & 86 & 82 & 0 & 87 & 0 & 0 & 86 \\ 41 & 37 & 41 & 54 & 37 & 32 & 41 & 38 \\ 45 & 60 & 49 & 45 & 29 & 47 & 31 & 33 \\ 51 & 27 & 44 & 35 & 44 & 32 & 33 & 40 \\ 37 & 57 & 39 & 35 & 44 & 37 & 37 & 44 \end{pmatrix} \pm \begin{pmatrix} 9 & 0 & 0 & 9 & 0 & 8 & 9 & 0 \\ 1 & 8 & 9 & 0 & 8 & 0 & 0 & 9 \\ 2 & 8 & 8 & 0 & 8 & 0 & 0 & 7 \\ 0 & 8 & 8 & 0 & 8 & 0 & 0 & 8 \\ 6 & 6 & 6 & 7 & 6 & 5 & 6 & 6 \\ 6 & 7 & 7 & 6 & 5 & 7 & 5 & 6 \\ 7 & 5 & 6 & 6 & 6 & 5 & 6 & 6 \\ 6 & 7 & 6 & 6 & 6 & 6 & 6 & 6 \end{pmatrix}$

Counts 12 subspace: $\begin{pmatrix} 59 & 3 & 9 & 57 & 5 & 49 & 71 & 11 \\ 9 & 49 & 53 & 6 & 57 & 7 & 7 & 50 \\ 0 & 66 & 32 & 21 & 65 & 10 & 8 & 58 \\ 8 & 58 & 45 & 13 & 55 & 5 & 5 & 62 \\ 54 & 33 & 15 & 41 & 23 & 29 & 38 & 31 \\ 16 & 52 & 43 & 31 & 38 & 26 & 28 & 36 \\ 18 & 51 & 48 & 20 & 38 & 34 & 35 & 48 \\ 17 & 50 & 36 & 29 & 39 & 22 & 22 & 43 \end{pmatrix} \pm \begin{pmatrix} 7 & 2 & 3 & 7 & 2 & 7 & 8 & 3 \\ 3 & 7 & 7 & 2 & 7 & 3 & 3 & 7 \\ 0 & 8 & 5 & 4 & 8 & 3 & 3 & 7 \\ 3 & 7 & 6 & 4 & 7 & 2 & 2 & 7 \\ 7 & 6 & 4 & 6 & 5 & 5 & 6 & 5 \\ 4 & 7 & 6 & 5 & 6 & 5 & 5 & 6 \\ 4 & 7 & 7 & 4 & 6 & 6 & 6 & 7 \\ 4 & 7 & 6 & 5 & 6 & 5 & 5 & 6 \end{pmatrix}$

Counts 02 subspace: $\begin{pmatrix} 50 & 0 & 11 & 48 & 6 & 86 & 81 & 13 \\ 8 & 62 & 77 & 3 & 56 & 0 & 0 & 55 \\ 16 & 62 & 74 & 11 & 66 & 1 & 7 & 71 \\ 9 & 71 & 63 & 2 & 70 & 3 & 8 & 52 \\ 35 & 36 & 37 & 28 & 32 & 29 & 17 & 32 \\ 32 & 31 & 20 & 32 & 23 & 42 & 43 & 26 \\ 35 & 28 & 28 & 41 & 25 & 63 & 47 & 24 \\ 48 & 29 & 24 & 36 & 26 & 35 & 53 & 14 \end{pmatrix} \pm \begin{pmatrix} 7 & 0 & 3 & 7 & 2 & 8 & 8 & 4 \\ 3 & 7 & 8 & 2 & 7 & 0 & 0 & 7 \\ 4 & 7 & 8 & 3 & 8 & 1 & 3 & 8 \\ 3 & 8 & 7 & 1 & 8 & 2 & 3 & 7 \\ 6 & 6 & 6 & 5 & 5 & 5 & 4 & 5 \\ 5 & 5 & 4 & 5 & 5 & 6 & 6 & 5 \\ 6 & 5 & 5 & 6 & 5 & 7 & 7 & 5 \\ 7 & 5 & 5 & 6 & 5 & 6 & 7 & 4 \end{pmatrix}$

Expectation values (in () the theoretical value)

01, 12, 02, (theory) subspaces respectively

$\langle xxx \rangle$ = 0.693 ± 0.034 | 0.408 ± 0.03 | 0.459 ± 0.031 | (0.667)

$\langle yyx \rangle$ = −0.677 ± 0.033 | −0.353 ± 0.029 | −0.47 ± 0.029 | (−0.667)

$\langle yxy \rangle$ = −0.582 ± 0.031 | −0.355 ± 0.03 | −0.464 ± 0.032 | (−0.667)

$\langle xyy \rangle$ = −0.665 ± 0.031 | −0.371 ± 0.029 | −0.457 ± 0.031 | (−0.667)

$\langle yyy \rangle$ = 0.028 ± 0.033 | 0.116 ± 0.031 | −0.056 ± 0.029 | (0)

$\langle xxy \rangle$ = −0.006 ± 0.034 | −0.134 ± 0.031 | 0.098 ± 0.029 | (0)

$\langle xyx \rangle$ = −0.006 ± 0.033 | −0.154 ± 0.033 | 0.16 ± 0.032 | (0)

$\langle yxx \rangle$ = −0.072 ± 0.034 | −0.154 ± 0.031 | 0.155 ± 0.032 | (0)

$$\rho = \begin{pmatrix} 0.352 & 0.327 + 0.014\,i & 0.231 - 0.059\,i \\ 0.327 - 0.014\,i & 0.286 & 0.186 + 0.07\,i \\ 0.231 + 0.059\,i & 0.186 - 0.07\,i & 0.206 \end{pmatrix}$$

$$\text{SD Re}(\rho) = \begin{pmatrix} 0.021 & 0.008 & 0.008 \\ 0.008 & 0.02 & 0.007 \\ 0.008 & 0.007 & 0.018 \end{pmatrix}$$

$$\text{SD Im}(\rho) = \begin{pmatrix} 0. & 0.008 & 0.008 \\ 0.008 & 0. & 0.008 \\ 0.008 & 0.008 & 0. \end{pmatrix}$$

Fidelity = 0.778 ± 0.014

$$\text{Abs}(\rho) = \begin{pmatrix} 0.352 & 0.327 & 0.239 \\ 0.327 & 0.286 & 0.199 \\ 0.239 & 0.199 & 0.206 \end{pmatrix}$$

$$\text{SD Abs}(\rho) = \begin{pmatrix} 0.021 & 0.008 & 0.008 \\ 0.008 & 0.02 & 0.007 \\ 0.008 & 0.007 & 0.018 \end{pmatrix}$$

$$\text{Arg}(\rho) = \begin{pmatrix} 0. & 0.043 & -0.248 \\ -0.043 & 0. & 0.359 \\ 0.248 & -0.359 & 0. \end{pmatrix}$$

$$\text{SD Arg}(\rho) = \begin{pmatrix} 0. & 0.026 & 0.032 \\ 0.026 & 0. & 0.039 \\ 0.032 & 0.039 & 0. \end{pmatrix}$$

$\phi 1$ = −0.043 ± 0.026

$\phi 2$ = 0.248 ± 0.032

($\phi 1 - \phi 2$) direct (from ρ) = −0.359 ± 0.039

($\phi 1 - \phi 2$) indirect (from the difference)= −0.291 ± 0.041