

Regression Methods Final Project

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```
library(tidyverse)

## Warning: package 'tidyverse' was built under R version 4.5.2
## Warning: package 'readr' was built under R version 4.5.2
## Warning: package 'forcats' was built under R version 4.5.2
## -- Attaching core tidyverse packages ----- tidyverse 2.0.0 --
## v dplyr     1.1.4      v readr     2.1.6
## v forcats   1.0.1      v stringr   1.5.2
## v ggplot2   4.0.0      v tibble    3.3.0
## v lubridate  1.9.4      v tidyr    1.3.1
## v purrr     1.1.0
## -- Conflicts ----- tidyverse_conflicts() --
## x dplyr::filter() masks stats::filter()
## x dplyr::lag()   masks stats::lag()
## i Use the conflicted package (<http://conflicted.r-lib.org/>) to force all conflicts to become errors
library(olsrr)

## Warning: package 'olsrr' was built under R version 4.5.2
##
## Attaching package: 'olsrr'
##
## The following object is masked from 'package:datasets':
##
##     rivers
library(car)

## Loading required package: carData
##
## Attaching package: 'car'
##
## The following object is masked from 'package:dplyr':
##
##     recode
##
## The following object is masked from 'package:purrr':
##
##     some
library(psych)

## Warning: package 'psych' was built under R version 4.5.2
```

```

## 
## Attaching package: 'psych'
##
## The following object is masked from 'package:car':
## 
##      logit
## 
## The following objects are masked from 'package:ggplot2':
## 
##      %+%, alpha

#Country: the name of the country
#Year: the year of the report (from 2005 to 2021)
#Life Ladder
#GDP per capita: the gross domestic product per capita in purchasing power parity (PPP) adjusted dollar
#Social support: the perceived social support (ability to count on others) in the country
#Freedom to make life choices: the perceived freedom to make life choices in the country.
#Generosity: the perceived generosity of people in the country.
#Perceptions of corruption: the perceived level of corruption in the government and business sectors #o.

happiness <- read.csv("World Happiness Report 2005-2021.csv")

happiness |> #166 different countries
  group_by(country = Country.name) |>
  count()

## # A tibble: 166 x 2
## # Groups:   country [166]
##   country     n
##   <chr>    <int>
## 1 Afghanistan     13
## 2 Albania         14
## 3 Algeria          10
## 4 Angola            4
## 5 Argentina        16
## 6 Armenia           15
## 7 Australia         15
## 8 Austria           14
## 9 Azerbaijan        14
## 10 Bahrain          11
## # i 156 more rows

describe(happiness) #Confidence.in.national.government missing 216 obs, Perceptions.of.corruption missin

##                                     vars     n    mean      sd median trimmed    mad
## Country.name*                      1 2089  84.45  48.30   83.00  84.49 60.79
## Year                                2 2089 2013.73  4.46 2014.00 2013.80  5.93
## Life.Ladder                          3 2089   5.47  1.12   5.41   5.47  1.21
## Log.GDP.per.capita                  4 2062   9.38  1.14   9.46   9.43  1.39
## Social.support                       5 2076   0.81  0.12   0.83   0.83  0.11
## Healthy.life.expectancy.at.birth    6 2031  63.18  6.95  64.98  63.84  6.46
## Freedom.to.make.life.choices        7 2057   0.75  0.14   0.77   0.76  0.15
## Generosity                           8 2009   0.00  0.16  -0.02  -0.01  0.15
## Perceptions.of.corruption           9 1976   0.75  0.19   0.80   0.78  0.13
## Positive.affect                     10 2065   0.65  0.11   0.66   0.66  0.12
## Negative.affect                     11 2073   0.27  0.09   0.26   0.26  0.08

```

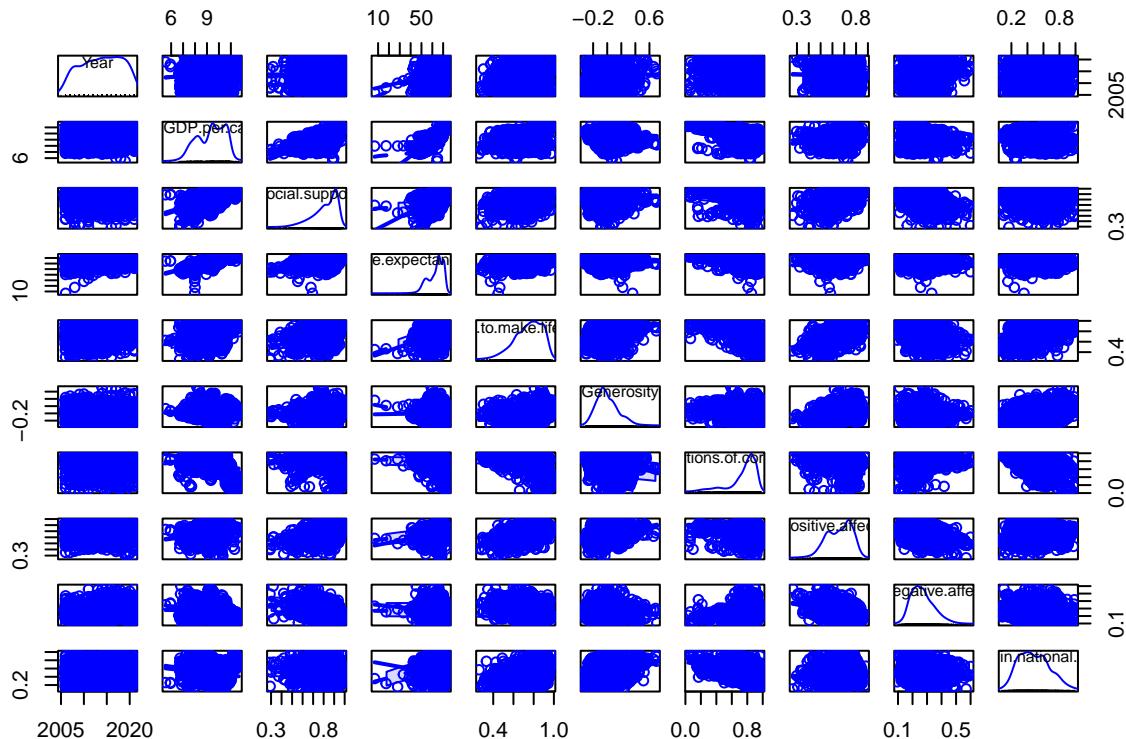
```

## Confidence.in.national.government    12 1873     0.48  0.19    0.47    0.48  0.21
##                                         min      max range skew kurtosis   se
## Country.name*                      1.00 166.00 165.00  0.00    -1.20 1.06
## Year                            2005.00 2021.00 16.00 -0.10    -1.08 0.10
## Life.Ladder                      2.18   8.02  5.84  0.03    -0.65 0.02
## Log.GDP.per.capita                5.53  11.67  6.14 -0.32    -0.78 0.03
## Social.support                   0.29   0.99  0.70 -1.09     1.06 0.00
## Healthy.life.expectancy.at.birth  6.72  74.35 67.63 -1.15     3.02 0.15
## Freedom.to.make.life.choices     0.26   0.99  0.73 -0.65    -0.08 0.00
## Generosity                      -0.34   0.71  1.04  0.80     0.90 0.00
## Perceptions.of.corruption       0.04   0.98  0.95 -1.50     1.84 0.00
## Positive.affect                  0.18   0.88  0.70 -0.40    -0.35 0.00
## Negative.affect                 0.08   0.70  0.62  0.73     0.81 0.00
## Confidence.in.national.government 0.07   0.99  0.92  0.33    -0.53 0.00

happiness <- na.omit(happiness) #Removed NAs to allow for anova reduced model comparison

scatterplotMatrix(happiness[, c(2,4:12)]) #Some variables show patterns of interaction

```



```

timeladder <- lm(Life.Ladder~Year+Log.GDP.per.capita+Social.support+Healthy.life.expectancy.at.birth+Fr

summary(timeladder)

##
## Call:
## lm(formula = Life.Ladder ~ Year + Log.GDP.per.capita + Social.support +
##     Healthy.life.expectancy.at.birth + Freedom.to.make.life.choices +
## 
```

```

##      Generosity + Perceptions.of.corruption + Positive.affect +
##      Negative.affect + Confidence.in.national.government, data = happiness)
##
## Residuals:
##      Min       1Q    Median     3Q    Max
## -1.76266 -0.31790  0.03539  0.32900  1.73643
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)            23.987105   6.443053  3.723 0.000203 ***
## Year                  -0.012732   0.003213 -3.963 7.70e-05 ***
## Log.GDP.per.capita    0.342676   0.023146 14.805 < 2e-16 ***
## Social.support         1.637769   0.173183  9.457 < 2e-16 ***
## Healthy.life.expectancy.at.birth 0.027968   0.003269  8.556 < 2e-16 ***
## Freedom.to.make.life.choices 0.982182   0.148527  6.613 5.00e-11 ***
## Generosity             0.474490   0.087803  5.404 7.42e-08 ***
## Perceptions.of.corruption -1.146152   0.095015 -12.063 < 2e-16 ***
## Positive.affect        1.936139   0.160672 12.050 < 2e-16 ***
## Negative.affect         0.169925   0.188564  0.901 0.367631
## Confidence.in.national.government -0.733337   0.093021 -7.884 5.57e-15 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5295 on 1736 degrees of freedom
## Multiple R-squared:  0.7869, Adjusted R-squared:  0.7857
## F-statistic: 641.1 on 10 and 1736 DF, p-value: < 2.2e-16
anova(timeladder) #Analysis of Variance and Type-1 p-values

```

```

## Analysis of Variance Table
##
## Response: Life.Ladder
##                               Df Sum Sq Mean Sq F value Pr(>F)
## Year                         1 11.56  11.56 41.2314 1.741e-10 ***
## Log.GDP.per.capita           1 1399.94 1399.94 4993.5244 < 2.2e-16 ***
## Social.support                1 124.75  124.75 444.9738 < 2.2e-16 ***
## Healthy.life.expectancy.at.birth 1 37.15  37.15 132.5008 < 2.2e-16 ***
## Freedom.to.make.life.choices 1 102.80  102.80 366.6938 < 2.2e-16 ***
## Generosity                    1 24.55  24.55 87.5678 < 2.2e-16 ***
## Perceptions.of.corruption    1 27.03  27.03 96.4011 < 2.2e-16 ***
## Positive.affect               1 51.48  51.48 183.6406 < 2.2e-16 ***
## Negative.affect               1  0.75   0.75  2.6699  0.1024
## Confidence.in.national.government 1 17.42  17.42 62.1505 5.575e-15 ***
## Residuals                     1736 486.69  0.28
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
vif(timeladder) #No values above 10

```

##	Year	Log.GDP.per.capita
##	1.232979	4.470511
##	Social.support	Healthy.life.expectancy.at.birth
##	2.771800	3.514736
##	Freedom.to.make.life.choices	Generosity
##	2.609419	1.257058

```

##          Perceptions.of.corruption           Positive.affect
##                           1.897562                      1.819813
##          Negative.affect Confidence.in.national.government
##                           1.485697                      2.004402

X <- model.matrix(timeladder)
cor(X) #Checks correlations between predictors, hinting that there may be significant interactions between the variables

## Warning in cor(X): the standard deviation is zero

##                                     (Intercept)      Year Log.GDP.per.capita
## (Intercept)                               1          NA             NA
## Year                                    NA 1.000000000 0.10259471
## Log.GDP.per.capita                      NA 0.10259471 1.000000000
## Social.support                          NA 0.01303793 0.70348775
## Healthy.life.expectancy.at.birth        NA 0.18314551 0.82907463
## Freedom.to.make.life.choices            NA 0.24585824 0.37196884
## Generosity                             NA -0.01102567 -0.01643428
## Perceptions.of.corruption              NA -0.10403717 -0.36946006
## Positive.affect                        NA 0.01531360 0.23901810
## Negative.affect                         NA 0.23822771 -0.25794891
## Confidence.in.national.government     NA 0.02972279 -0.18893556

##                                     Social.support
## (Intercept)                               NA
## Year                                    0.01303793
## Log.GDP.per.capita                      0.70348775
## Social.support                          1.000000000
## Healthy.life.expectancy.at.birth        0.61191261
## Freedom.to.make.life.choices            0.41741510
## Generosity                             0.06551809
## Perceptions.of.corruption              -0.23377831
## Positive.affect                        0.43839205
## Negative.affect                         -0.42597884
## Confidence.in.national.government     -0.17744541

##                                     Healthy.life.expectancy.at.birth
## (Intercept)                               NA
## Year                                    0.18314551
## Log.GDP.per.capita                      0.82907463
## Social.support                          0.61191261
## Healthy.life.expectancy.at.birth        1.000000000
## Freedom.to.make.life.choices            0.38820239
## Generosity                             0.02257912
## Perceptions.of.corruption              -0.31980999
## Positive.affect                        0.23387226
## Negative.affect                         -0.13024104
## Confidence.in.national.government     -0.17908242

##                                     Freedom.to.make.life.choices Generosity
## (Intercept)                               NA          NA
## Year                                    0.2458582 -0.01102567
## Log.GDP.per.capita                      0.3719688 -0.01643428
## Social.support                          0.4174151  0.06551809
## Healthy.life.expectancy.at.birth        0.3882024  0.02257912
## Freedom.to.make.life.choices            1.0000000  0.32666339
## Generosity                             0.3266634  1.000000000
## Perceptions.of.corruption              -0.4946633 -0.28975067

```

```

## Positive.affect          0.5777809  0.32478394
## Negative.affect         -0.2562483 -0.10086338
## Confidence.in.national.government 0.4034902  0.29360883
##                                         Perceptions.of.corruption Positive.affect
## (Intercept)                                NA          NA
## Year                                     -0.1040372  0.0153136
## Log.GDP.per.capita                      -0.3694601  0.2390181
## Social.support                           -0.2337783  0.4383921
## Healthy.life.expectancy.at.birth        -0.3198100  0.2338723
## Freedom.to.make.life.choices            -0.4946633  0.5777809
## Generosity                               -0.2897507  0.3247839
## Perceptions.of.corruption                1.0000000 -0.2973773
## Positive.affect                          -0.2973773  1.0000000
## Negative.affect                         0.2768005 -0.3046321
## Confidence.in.national.government      -0.4582092  0.1144486
##                                         Negative.affect
## (Intercept)                                NA
## Year                                      0.2382277
## Log.GDP.per.capita                      -0.2579489
## Social.support                            -0.4259788
## Healthy.life.expectancy.at.birth        -0.1302410
## Freedom.to.make.life.choices            -0.2562483
## Generosity                               -0.1008634
## Perceptions.of.corruption                0.2768005
## Positive.affect                          -0.3046321
## Negative.affect                         1.0000000
## Confidence.in.national.government      -0.1294733
##                                         Confidence.in.national.government
## (Intercept)                                NA
## Year                                      0.02972279
## Log.GDP.per.capita                      -0.18893556
## Social.support                            -0.17744541
## Healthy.life.expectancy.at.birth        -0.17908242
## Freedom.to.make.life.choices            0.40349022
## Generosity                               0.29360883
## Perceptions.of.corruption                -0.45820918
## Positive.affect                          0.11444858
## Negative.affect                         -0.12947333
## Confidence.in.national.government      1.000000000
# ^ Possible starting point for multi-collinearity investigation

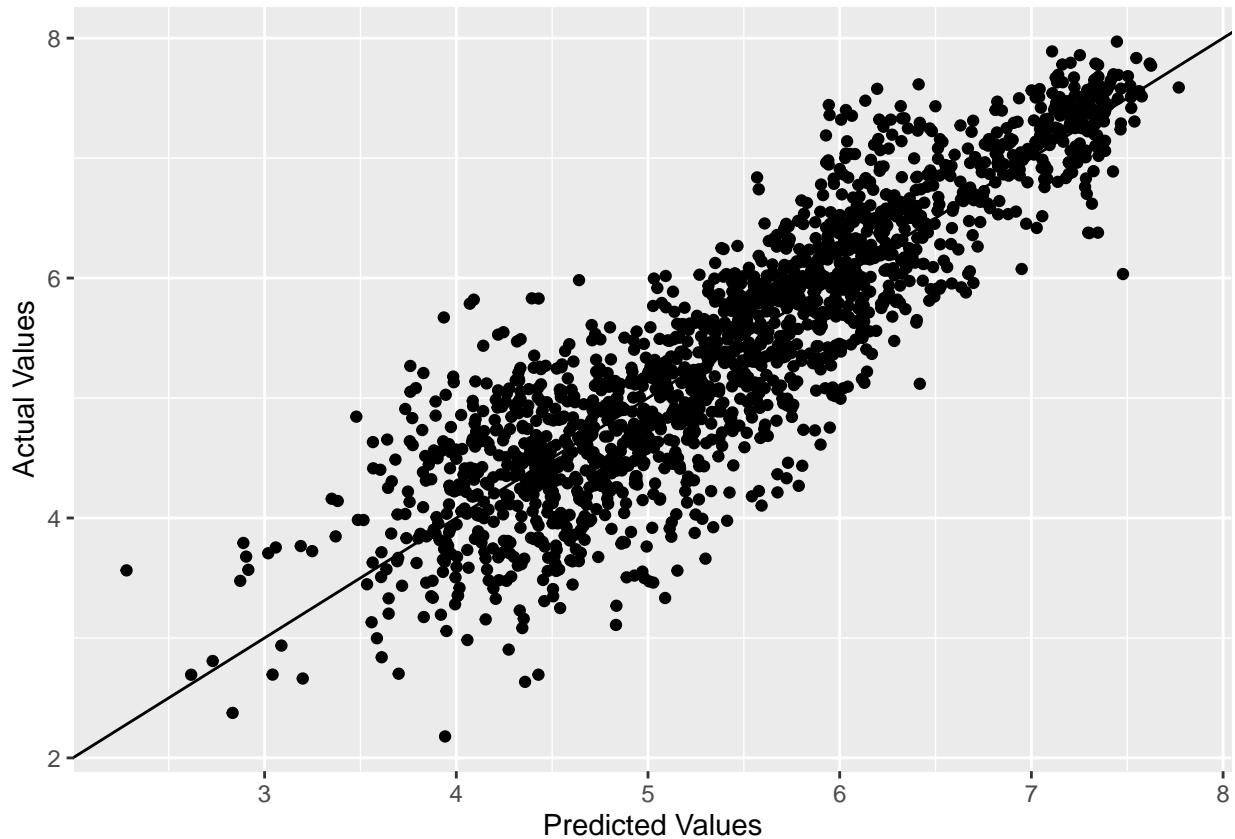
XnoNAs <- X[-1, -1]
as.data.frame(cor(XnoNAs)) |>
  summarise(across(everything(), mean)) #Taking out the NAs (intercept from the X matrix) and averaging

##      Year Log.GDP.per.capita Social.support Healthy.life.expectancy.at.birth
## 1 0.1708366          0.2411118     0.2409325                  0.2637511
##   Freedom.to.make.life.choices Generosity Perceptions.of.corruption
## 1                 0.2982377  0.1621413             -0.1288939
##   Positive.affect Negative.affect Confidence.in.national.government
## 1           0.233771   -0.00927711                  0.0710831

happiness |>
  ggplot(aes(x = predict(timeladder), y = Life.Ladder)) +

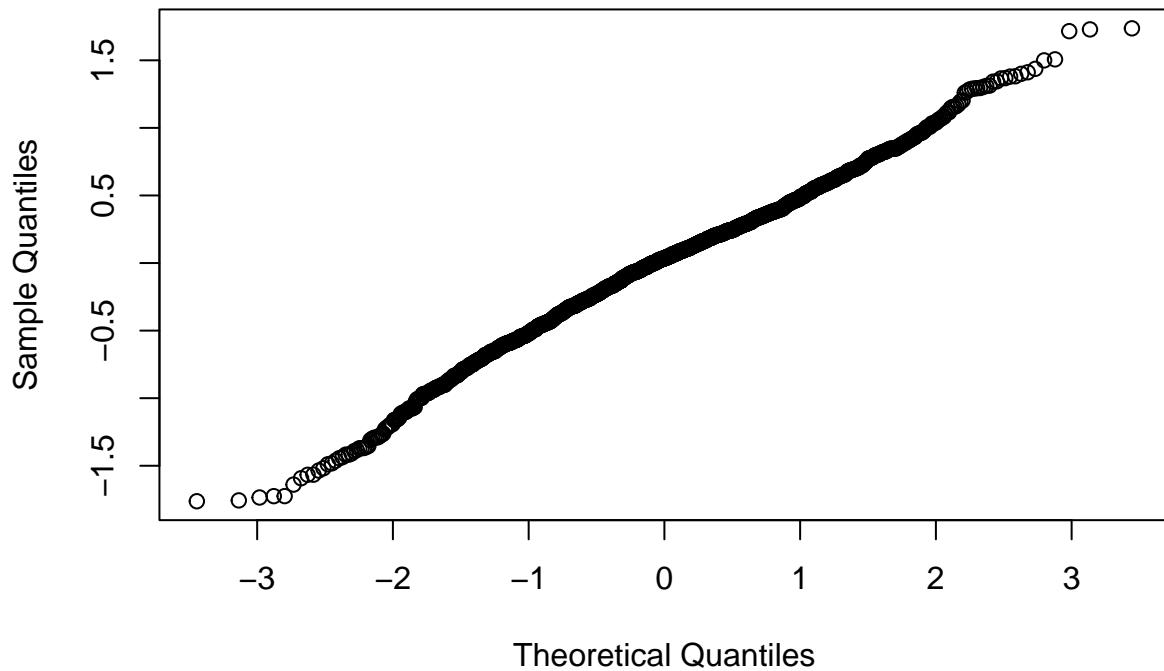
```

```
geom_point() +
geom_abline(intercept = 0, slope = 1) +
labs(x = 'Predicted Values', y = 'Actual Values')
```

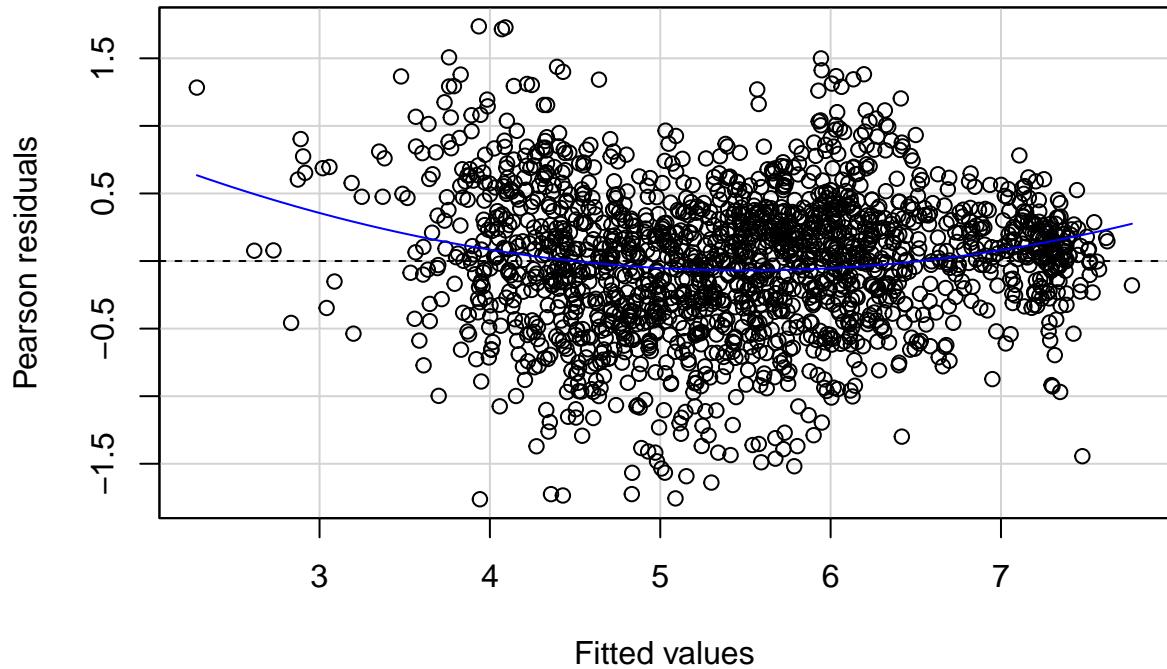


```
qqnorm(timeladder$residuals) #Plots raw residual quantiles against theoretical normal distribution
```

Normal Q-Q Plot



```
residualPlot(timeladder) #Plots standardized residuals ([y - y_hat] / sqrt(Var)) against fitted
```

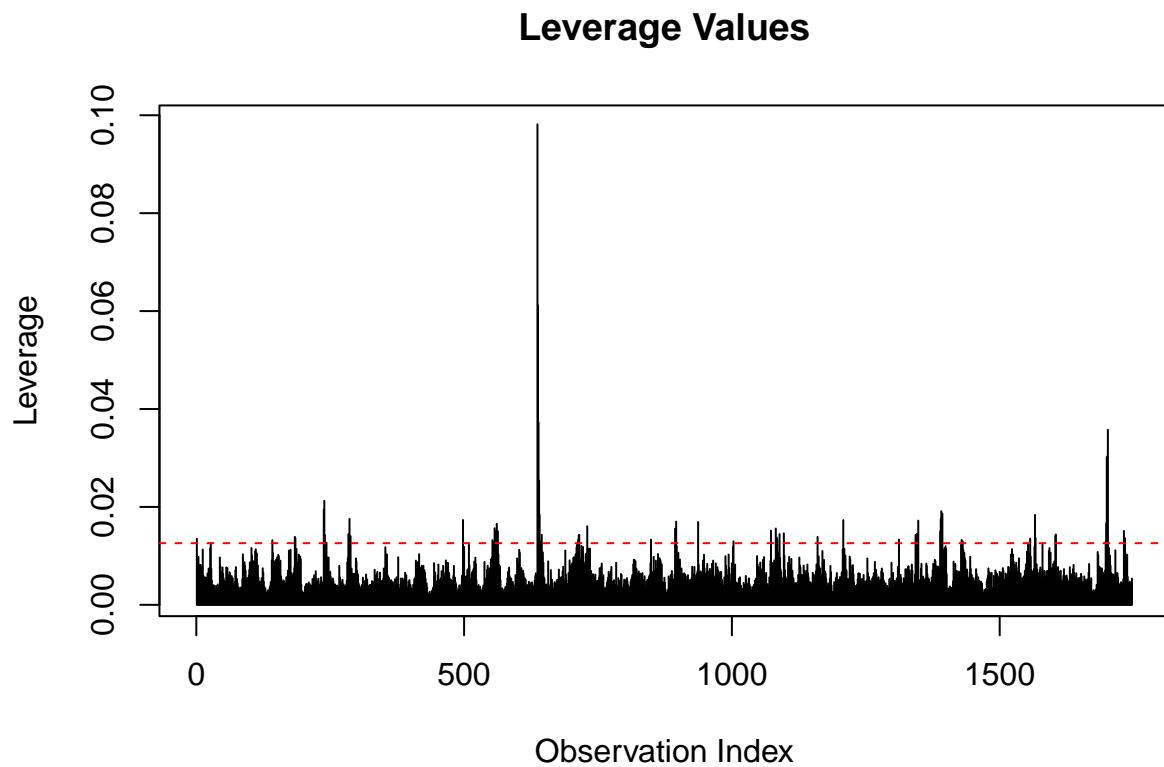


```

leverage_values <- hatvalues(timeladder)

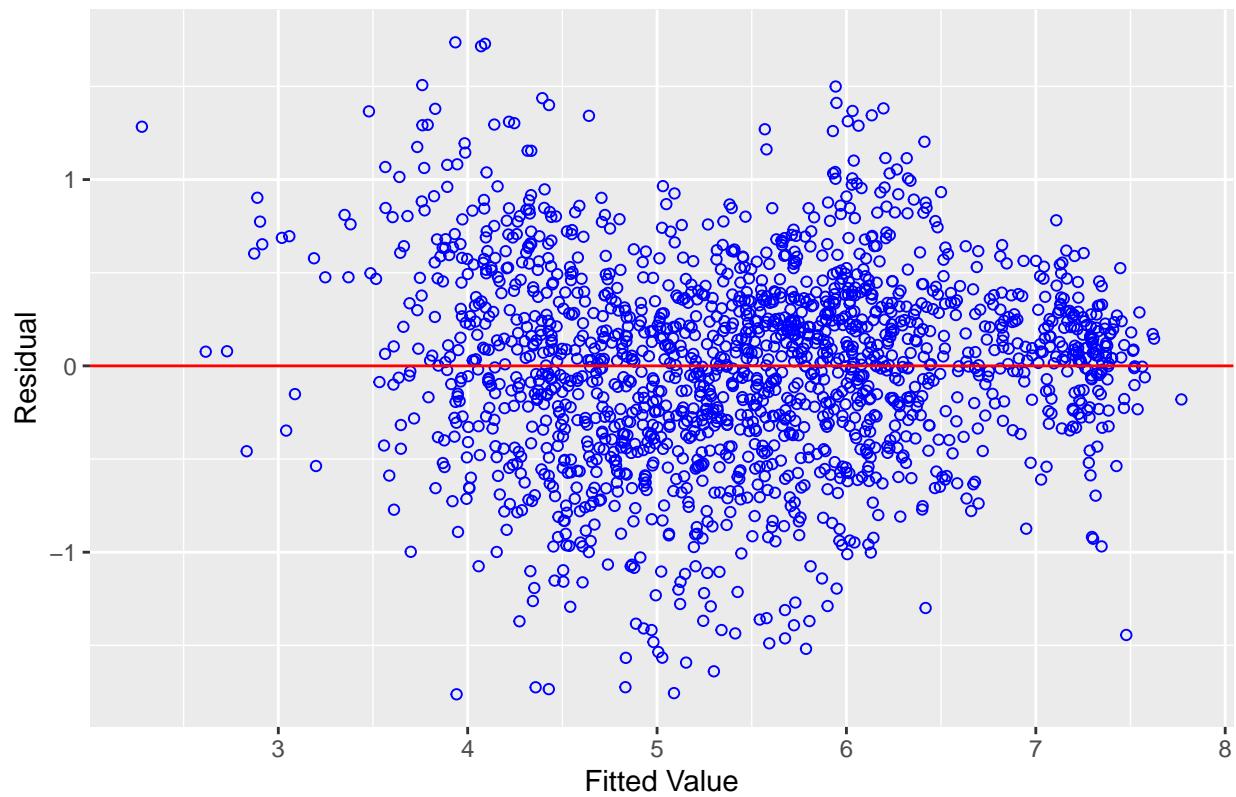
plot(leverage_values, type = "h", main = "Leverage Values",
      xlab = "Observation Index", ylab = "Leverage")
abline(h = 2 * mean(leverage_values), col = "red", lty = 2) #Add a reference line for high leverage

```

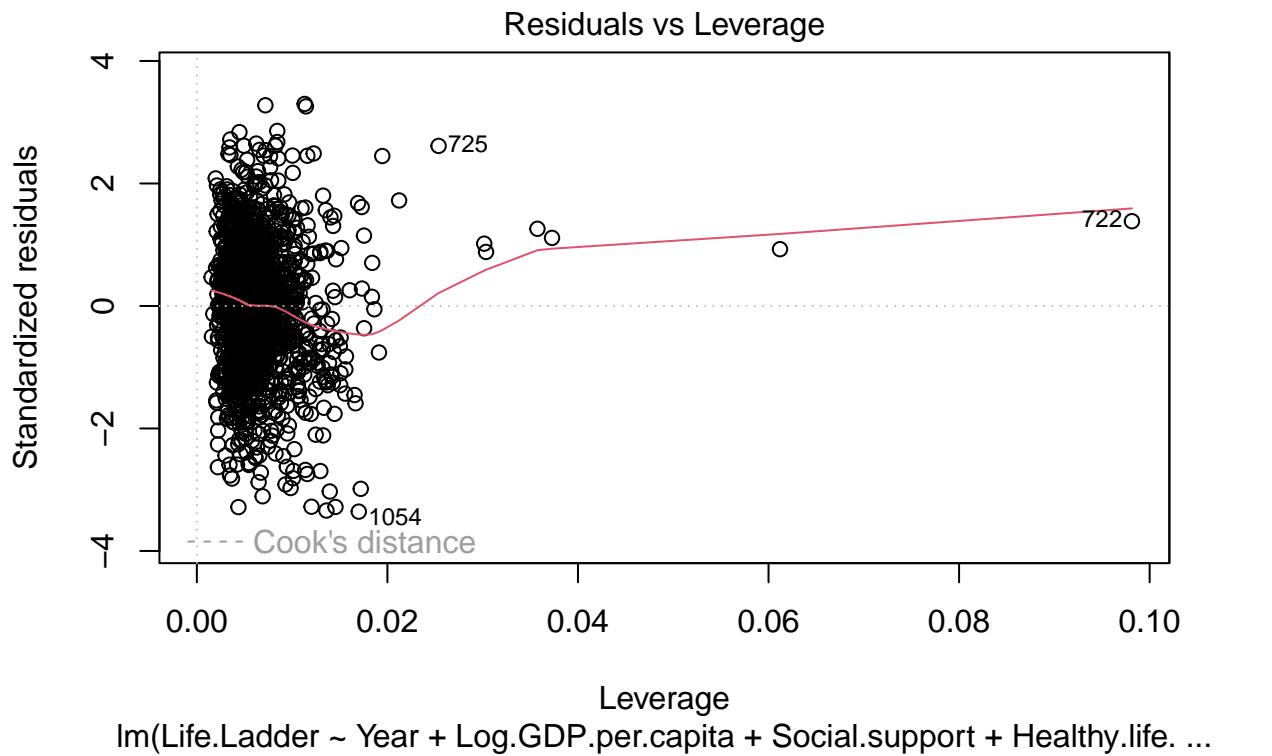


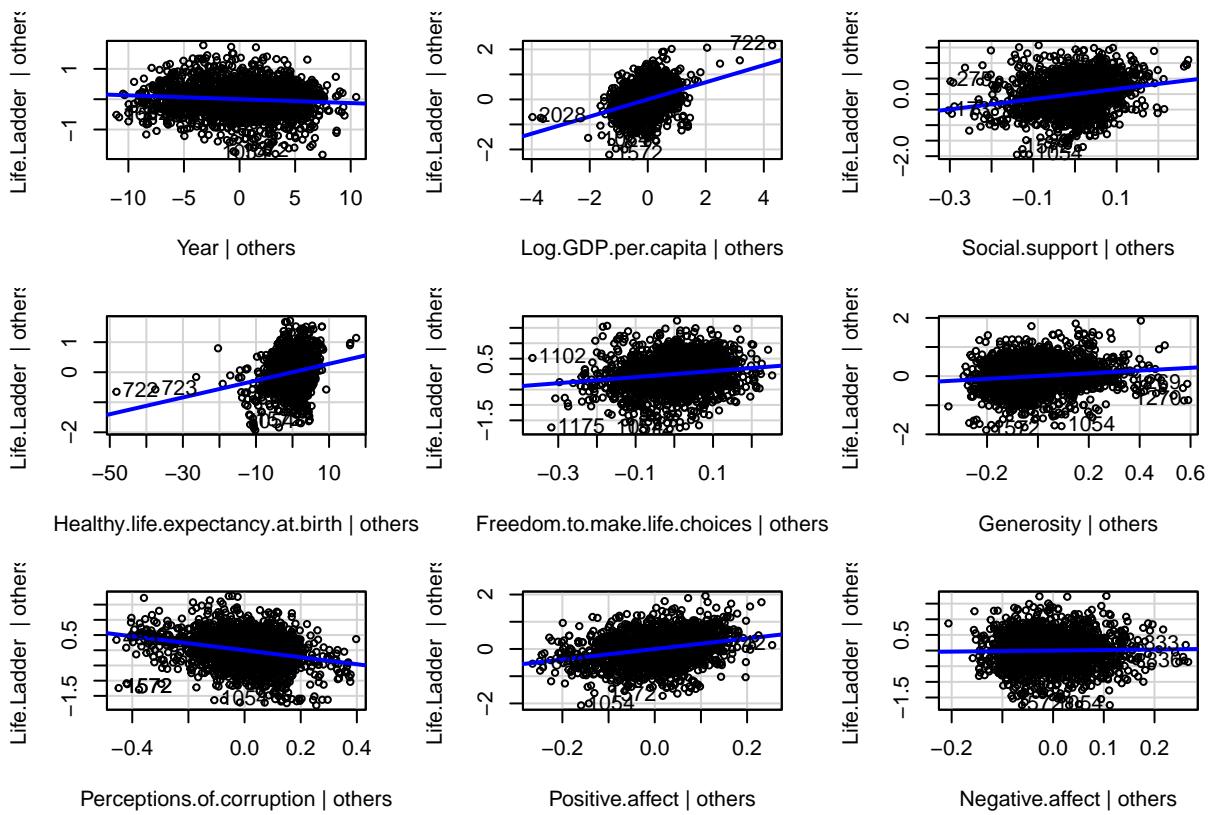
```
ols_plot_resid_fit(timeladder) #Another residual vs fitted plot
```

Residual vs Fitted Values

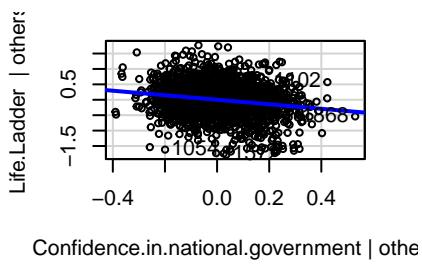


```
plot(timeladder, which = 5) #Residuals vs Leverage plot
```





Added-Variable Plots



```
#OLS Steps with AIC metric (Long outputs)

ols_step_forward_aic(timeladder, details = TRUE) #Removes Negative.affect
```

```
## Forward Selection Method
## -----
## Candidate Terms:
##
## 1. Year
## 2. Log.GDP.per.capita
## 3. Social.support
## 4. Healthy.life.expectancy.at.birth
## 5. Freedom.to.make.life.choices
## 6. Generosity
## 7. Perceptions.of.corruption
## 8. Positive.affect
## 9. Negative.affect
## 10. Confidence.in.national.government
##
## Step      => 0
## Model     => Life.Ladder ~ 1
## AIC       => 5430.112
##
## Initiating stepwise selection...
```

Table: Adding New Variables						
Predictor	DF	AIC	SBC	SBIC	R2	Adj. R2
## Log.GDP.per.capita	1	3751.504	3767.901	-1208.408	0.61787	0.61765
## Healthy.life.expectancy.at.birth	1	4090.996	4107.393	-869.506	0.53591	0.53564
## Social.support	1	4160.616	4177.013	-799.997	0.51704	0.51676
## Freedom.to.make.life.choices	1	4826.658	4843.055	-134.871	0.29289	0.29248
## Positive.affect	1	4912.233	4928.630	-49.395	0.25739	0.25696
## Perceptions.of.corruption	1	4982.652	4999.049	20.946	0.22684	0.22640
## Negative.affect	1	5251.673	5268.070	289.691	0.09813	0.09761
## Generosity	1	5367.627	5384.024	405.537	0.03624	0.03569
## Confidence.in.national.government	1	5420.165	5436.562	458.028	0.00682	0.00625
## Year	1	5423.248	5439.645	461.108	0.00506	0.00449
## Step => 1						
## Added => Log.GDP.per.capita						
## Model => Life.Ladder ~ Log.GDP.per.capita						
## AIC => 3751.504						
## Step => 2						
## Added => Positive.affect						
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect						
## AIC => 3171.719						
Table: Adding New Variables						
Predictor	DF	AIC	SBC	SBIC	R2	Adj. R2
## Perceptions.of.corruption	1	3067.249	3094.577	-1891.922	0.74230	0.74186
## Healthy.life.expectancy.at.birth	1	3075.521	3102.849	-1883.684	0.74108	0.74063
## Social.support	1	3088.874	3116.203	-1870.386	0.73909	0.73864
## Freedom.to.make.life.choices	1	3099.621	3126.949	-1859.683	0.73748	0.73703
## Generosity	1	3111.759	3139.088	-1847.594	0.73565	0.73520
## Negative.affect	1	3167.896	3195.224	-1791.683	0.72702	0.72655
## Confidence.in.national.government	1	3172.758	3200.087	-1786.840	0.72626	0.72579
## Year	1	3173.456	3200.785	-1786.145	0.72615	0.72568

```

## -----
## 
## Step    => 3
## Added   => Perceptions.of.corruption
## Model   => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption
## AIC     => 3067.249
##
##                                     Table: Adding New Variables
## -----
## Predictor          DF   AIC      SBC      SBIC      R2   Adj. R2
## -----
## Social.support      1   2943.269  2976.063  -2015.581  0.76023  0.75968
## Healthy.life.expectancy.at.birth  1   2967.143  2999.937  -1991.834  0.75693  0.75637
## Confidence.in.national.government 1   3033.929  3066.723  -1925.397  0.74746  0.74688
## Generosity         1   3038.858  3071.652  -1920.494  0.74674  0.74616
## Freedom.to.make.life.choices    1   3039.486  3072.280  -1919.868  0.74665  0.74607
## Year               1   3067.488  3100.282  -1892.010  0.74256  0.74197
## Negative.affect     1   3068.419  3101.213  -1891.084  0.74242  0.74183
##
## 
## Step    => 4
## Added   => Social.support
## Model   => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC     => 2943.269
##
##                                     Table: Adding New Variables
## -----
## Predictor          DF   AIC      SBC      SBIC      R2   Adj. R2
## -----
## Healthy.life.expectancy.at.birth 1   2847.817  2886.077  -2110.613  0.77324  0.77259
## Generosity         1   2913.995  2952.255  -2044.867  0.76448  0.76381
## Confidence.in.national.government 1   2916.982  2955.242  -2041.899  0.76408  0.76340
## Freedom.to.make.life.choices    1   2923.630  2961.890  -2035.294  0.76318  0.76250
## Negative.affect     1   2937.383  2975.642  -2021.629  0.76131  0.76062
## Year               1   2945.021  2983.280  -2014.040  0.76026  0.75957
##
## 
## Step    => 5
## Added   => Healthy.life.expectancy.at.birth
## Model   => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC     => 2847.817
##
##                                     Table: Adding New Variables
## -----
## Predictor          DF   AIC      SBC      SBIC      R2   Adj. R2
## -----
## Generosity         1   2821.547  2865.272  -2136.773  0.77688  0.77611
## Confidence.in.national.government 1   2826.819  2870.544  -2131.542  0.77620  0.77543
## Freedom.to.make.life.choices    1   2838.379  2882.104  -2120.071  0.77472  0.77394
## Year               1   2844.176  2887.901  -2114.320  0.77397  0.77319
## Negative.affect     1   2849.172  2892.897  -2109.362  0.77332  0.77254
##
## 
## Step    => 6

```

```

## Added => Generosity
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC => 2821.547
##
##                                     Table: Adding New Variables
##
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## -----
## Confidence.in.national.government  1  2791.452  2840.642 -2166.659  0.78094  0.78006
## Freedom.to.make.life.choices       1  2816.267  2865.458 -2142.066  0.77781  0.77691
## Year                         1  2818.674  2867.865 -2139.680  0.77750  0.77660
## Negative.affect                 1  2822.998  2872.189 -2135.395  0.77695  0.77605
##
## Step => 7
## Added => Confidence.in.national.government
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC => 2791.452
##
##                                     Table: Adding New Variables
##
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## -----
## Freedom.to.make.life.choices       1  2760.813  2815.469 -2197.007  0.78499  0.78400
## Year                         1  2789.086  2843.742 -2169.023  0.78149  0.78048
## Negative.affect                 1  2793.423  2848.080 -2164.729  0.78094  0.77994
##
## Step => 8
## Added => Freedom.to.make.life.choices
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC => 2760.813
##
##                                     Table: Adding New Variables
##
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## -----
## Year                         1  2747.875  2807.997 -2209.779  0.78683  0.78572
## Negative.affect                 1  2762.793  2822.915 -2195.032  0.78500  0.78388
##
## Step => 9
## Added => Year
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC => 2747.875
##
##                                     Table: Adding New Variables
##
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## -----
## Negative.affect     1  2749.058  2814.646 -2208.574  0.78692  0.78570
##
## 
```

```

## No more variables to be added.
##
## Variables Selected:
##
## => Log.GDP.per.capita
## => Positive.affect
## => Perceptions.of.corruption
## => Social.support
## => Healthy.life.expectancy.at.birth
## => Generosity
## => Confidence.in.national.government
## => Freedom.to.make.life.choices
## => Year

##
##
## Stepwise Summary
## -----
## Step Variable AIC SBC SBIC R2 Adj. R2
## -----
## 0 Base Model 5430.112 5441.043 469.535 0.00000 0.00000
## 1 Log.GDP.per.capita 3751.504 3767.901 -1208.408 0.61787 0.61765
## 2 Positive.affect 3171.719 3193.582 -1787.449 0.72611 0.72579
## 3 Perceptions.of.corruption 3067.249 3094.577 -1891.922 0.74230 0.74186
## 4 Social.support 2943.269 2976.063 -2015.581 0.76023 0.75968
## 5 Healthy.life.expectancy.at.birth 2847.817 2886.077 -2110.613 0.77324 0.77259
## 6 Generosity 2821.547 2865.272 -2136.773 0.77688 0.77611
## 7 Confidence.in.national.government 2791.452 2840.642 -2166.659 0.78094 0.78006
## 8 Freedom.to.make.life.choices 2760.813 2815.469 -2197.007 0.78499 0.78400
## 9 Year 2747.875 2807.997 -2209.779 0.78683 0.78572
## -----
## Final Model Output
## -----
## Model Summary
## -----
## R 0.887 RMSE 0.528
## R-Squared 0.787 MSE 0.279
## Adj. R-Squared 0.786 Coef. Var 9.669
## Pred R-Squared 0.784 AIC 2747.875
## MAE 0.408 SBC 2807.997
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
## AIC: Akaike Information Criteria
## SBC: Schwarz Bayesian Criteria
##
## ANOVA
## -----
## Sum of Squares DF Mean Square F Sig.
## -----

```

```

## Regression    1797.206      9      199.690    712.359    0.0000
## Residual     486.918    1737      0.280
## Total        2284.124    1746
## -----
## 
## 
##                                     Parameter Estimates
## 
## 
##             model   Beta   Std. Error   Std. Beta     t     Sig   lower
## 
##             (Intercept) 22.533      6.237           3.613  0.000  10.299
##             Log.GDP.per.capita 0.341      0.023       0.346 14.778  0.000  0.296
##             Positive.affect 1.925      0.160       0.179 12.017  0.000  1.611
##             Perceptions.of.corruption -1.134      0.094      -0.182 -12.058  0.000 -1.318
##             Social.support 1.587      0.164       0.169  9.684  0.000  1.266
##             Healthy.life.expectancy.at.birth 0.028      0.003       0.181  8.812  0.000  0.022
##             Generosity 0.478      0.088       0.068  5.446  0.000  0.306
##             Confidence.in.national.government -0.741      0.093      -0.125 -8.001  0.000 -0.923
##             Freedom.to.make.life.choices 0.979      0.148       0.118  6.595  0.000  0.688
##             Year      -0.012      0.003      -0.046 -3.862  0.000 -0.018
## 
## 
## ols_step_backward_aic(timeladder, details = TRUE) #Removes Negative.affect as well
## Backward Elimination Method
## -----
## 
## Candidate Terms:
## 
## 1. Year
## 2. Log.GDP.per.capita
## 3. Social.support
## 4. Healthy.life.expectancy.at.birth
## 5. Freedom.to.make.life.choices
## 6. Generosity
## 7. Perceptions.of.corruption
## 8. Positive.affect
## 9. Negative.affect
## 10. Confidence.in.national.government
## 
## 
## Step    => 0
## Model   => Life.Ladder ~ Year + Log.GDP.per.capita + Social.support + Healthy.life.expectancy.at.bi
## AIC     => 2749.058
## 
## Initiating stepwise selection...
## 
##                                     Table: Removing Existing Variables
## 
##             Predictor     DF     AIC     SBC     SBIC      R2   Adj. R2
## 
##             Negative.affect 1  2747.875  2807.997 -2209.779  0.78683  0.78572
##             Year            1  2762.793  2822.915 -2195.032  0.78500  0.78388
##             Generosity      1  2776.203  2836.325 -2181.775  0.78334  0.78222
##             Freedom.to.make.life.choices 1  2790.520  2850.642 -2167.620  0.78156  0.78043
##             Confidence.in.national.government 1  2808.509  2868.631 -2149.833  0.77930  0.77815

```

```

## Healthy.life.expectancy.at.birth      1    2819.224    2879.346    -2139.238    0.77794    0.77679
## Social.support                      1    2834.816    2894.938    -2123.818    0.77595    0.77479
## Positive.affect                     1    2887.396    2947.518    -2071.813    0.76910    0.76791
## Perceptions.of.corruption          1    2887.677    2947.799    -2071.534    0.76906    0.76787
## Log.GDP.per.capita                 1    2954.778    3014.900    -2005.146    0.76002    0.75878
## -----
## 
## Step      => 1
## Removed   => Negative.affect
## Model     => Life.Ladder ~ Year + Log.GDP.per.capita + Social.support + Healthy.life.expectancy.at.birth
## AIC       => 2747.875
## 
##                                     Table: Removing Existing Variables
## -----
## Predictor                         DF   AIC      SBC      SBIC      R2      Adj. R2
## 
## Year                             1    2760.813    2815.469    -2197.007    0.78499    0.78400
## Generosity                       1    2775.452    2830.109    -2182.518    0.78319    0.78219
## Freedom.to.make.life.choices    1    2789.086    2843.742    -2169.023    0.78149    0.78048
## Confidence.in.national.government 1    2809.101    2863.757    -2149.210    0.77897    0.77795
## Healthy.life.expectancy.at.birth 1    2822.286    2876.942    -2136.157    0.77729    0.77627
## Social.support                    1    2837.738    2892.394    -2120.859    0.77532    0.77428
## Positive.affect                  1    2885.396    2940.052    -2073.670    0.76910    0.76804
## Perceptions.of.corruption       1    2886.315    2940.972    -2072.759    0.76898    0.76792
## Log.GDP.per.capita              1    2952.778    3007.434    -2006.933    0.76002    0.75892
## -----
## 
## 
## No more variables to be removed.
## 
## Variables Removed:
## 
## => Negative.affect
## 
## 
##                                     Stepwise Summary
## -----
## Step     Variable           AIC      SBC      SBIC      R2      Adj. R2
## 
## 0       Full Model        2749.058    2814.646    -2208.574    0.78692    0.78570
## 1       Negative.affect    2747.875    2807.997    -2209.781    0.78683    0.78572
## 
## 
## Final Model Output
## 
## 
##                                     Model Summary
## -----
## R                   0.887      RMSE            0.528
## R-Squared           0.787      MSE             0.279
## Adj. R-Squared      0.786      Coef. Var       9.669
## Pred R-Squared      0.784      AIC            2747.875
## MAE                0.408      SBC            2807.997

```

```

## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
## AIC: Akaike Information Criteria
## SBC: Schwarz Bayesian Criteria
##
##                               ANOVA
##
##          Sum of
##          Squares      DF   Mean Square      F      Sig.
## -----
## Regression    1797.206       9     199.690    712.359    0.0000
## Residual      486.918     1737      0.280
## Total         2284.124     1746
## -----
##                               Parameter Estimates
##
##          model      Beta   Std. Error   Std. Beta      t      Sig   lower
## -----
##             (Intercept) 22.533     6.237
##             Year      -0.012     0.003     -0.046    -3.862    0.000   -0.018
##             Log.GDP.per.capita 0.341     0.023     0.346    14.778    0.000    0.296
##             Social.support 1.587     0.164     0.169     9.684    0.000    1.266
##             Healthy.life.expectancy.at.birth 0.028     0.003     0.181     8.812    0.000    0.022
##             Freedom.to.make.life.choices 0.979     0.148     0.118     6.595    0.000    0.688
##             Generosity 0.478     0.088     0.068     5.446    0.000    0.306
##             Perceptions.of.corruption -1.134     0.094    -0.182   -12.058    0.000   -1.318
##             Positive.affect 1.925     0.160     0.179    12.017    0.000    1.611
##             Confidence.in.national.government -0.741     0.093    -0.125   -8.001    0.000   -0.923
## -----
## ols_step_both_aic(timeladder, details = TRUE) #Also removes Negative.affect

## Stepwise Selection Method
## -----
## 
## Candidate Terms:
## 
## 1. Year
## 2. Log.GDP.per.capita
## 3. Social.support
## 4. Healthy.life.expectancy.at.birth
## 5. Freedom.to.make.life.choices
## 6. Generosity
## 7. Perceptions.of.corruption
## 8. Positive.affect
## 9. Negative.affect
## 10. Confidence.in.national.government
## 
## 
## Step      => 0
## Model    => Life.Ladder ~ 1
## AIC      => 5430.112

```

```

## Initiating stepwise selection...
##
##                                     Table: Adding New Variables
## -----
## Predictor             DF   AIC    SBC   SBIC      R2   Adj. R2
## -----
## Year                  1  5423.248  5439.645  461.108  0.00506  0.00449
## Log.GDP.per.capita   1  3751.504  3767.901 -1208.408  0.61787  0.61765
## Social.support        1  4160.616  4177.013 -799.997  0.51704  0.51676
## Healthy.life.expectancy.at.birth 1  4090.996  4107.393 -869.506  0.53591  0.53564
## Freedom.to.make.life.choices 1  4826.658  4843.055 -134.871  0.29289  0.29248
## Generosity            1  5367.627  5384.024  405.537  0.03624  0.03569
## Perceptions.of.corruption 1  4982.652  4999.049  20.946  0.22684  0.22640
## Positive.affect       1  4912.233  4928.630 -49.395  0.25739  0.25696
## Negative.affect       1  5251.673  5268.070  289.691  0.09813  0.09761
## Confidence.in.national.government 1  5420.165  5436.562  458.028  0.00682  0.00625
## -----
## Step      => 1
## Added     => Log.GDP.per.capita
## Model     => Life.Ladder ~ Log.GDP.per.capita
## AIC       => 3751.504
## 
##                                     Table: Adding New Variables
## -----
## Predictor             DF   AIC    SBC   SBIC      R2   Adj. R2
## -----
## Year                  1  3753.086  3774.949 -1207.702  0.61796  0.61752
## Social.support         1  3484.114  3505.976 -1475.968  0.67248  0.67210
## Healthy.life.expectancy.at.birth 1  3656.413  3678.276 -1304.130  0.63853  0.63811
## Freedom.to.make.life.choices 1  3389.653  3411.516 -1570.163  0.68972  0.68936
## Generosity            1  3553.509  3575.371 -1406.762  0.65921  0.65881
## Perceptions.of.corruption 1  3560.295  3582.158 -1399.994  0.65788  0.65749
## Positive.affect       1  3171.719  3193.582 -1787.449  0.72611  0.72579
## Negative.affect       1  3692.659  3714.521 -1267.978  0.63095  0.63053
## Confidence.in.national.government 1  3732.755  3754.618 -1227.983  0.62238  0.62195
## -----
## Step      => 2
## Added     => Positive.affect
## Model     => Life.Ladder ~ Log.GDP.per.capita + Positive.affect
## AIC       => 3171.719
## 
##                                     Table: Removing Existing Variables
## -----
## Predictor             DF   AIC    SBC   SBIC      R2   Adj. R2
## -----
## Log.GDP.per.capita   1  4912.233  4928.630 -49.395  0.25739  0.25696
## Positive.affect      1  3751.504  3767.901 -1208.408  0.61787  0.61765
## 
##                                     Table: Adding New Variables
## -----

```

```

## Predictor DF AIC SBC SBIC R2 Adj. R2
## -----
## Year 1 3173.456 3200.785 -1786.145 0.72615 0.72568
## Social.support 1 3088.874 3116.203 -1870.386 0.73909 0.73864
## Healthy.life.expectancy.at.birth 1 3075.521 3102.849 -1883.684 0.74108 0.74063
## Freedom.to.make.life.choices 1 3099.621 3126.949 -1859.683 0.73748 0.73703
## Generosity 1 3111.759 3139.088 -1847.594 0.73565 0.73520
## Perceptions.of.corruption 1 3067.249 3094.577 -1891.922 0.74230 0.74186
## Negative.affect 1 3167.896 3195.224 -1791.683 0.72702 0.72655
## Confidence.in.national.government 1 3172.758 3200.087 -1786.840 0.72626 0.72579
## -----
## Step => 3
## Added => Perceptions.of.corruption
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption
## AIC => 3067.249
##
## Table: Removing Existing Variables
## -----
## Predictor DF AIC SBC SBIC R2 Adj. R2
## -----
## Log.GDP.per.capita 1 4617.049 4638.912 -345.533 0.37356 0.37284
## Positive.affect 1 3560.295 3582.158 -1399.994 0.65788 0.65749
## Perceptions.of.corruption 1 3171.719 3193.582 -1787.449 0.72611 0.72579
## -----
## Table: Adding New Variables
## -----
## Predictor DF AIC SBC SBIC R2 Adj. R2
## -----
## Year 1 3067.488 3100.282 -1892.010 0.74256 0.74197
## Social.support 1 2943.269 2976.063 -2015.581 0.76023 0.75968
## Healthy.life.expectancy.at.birth 1 2967.143 2999.937 -1991.834 0.75693 0.75637
## Freedom.to.make.life.choices 1 3039.486 3072.280 -1919.868 0.74665 0.74607
## Generosity 1 3038.858 3071.652 -1920.494 0.74674 0.74616
## Negative.affect 1 3068.419 3101.213 -1891.084 0.74242 0.74183
## Confidence.in.national.government 1 3033.929 3066.723 -1925.397 0.74746 0.74688
## -----
## Step => 4
## Added => Social.support
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC => 2943.269
## -----
## Table: Removing Existing Variables
## -----
## Predictor DF AIC SBC SBIC R2 Adj. R2
## -----
## Log.GDP.per.capita 1 3655.938 3683.266 -1305.411 0.63904 0.63842
## Positive.affect 1 3234.936 3262.264 -1724.907 0.71634 0.71585
## Perceptions.of.corruption 1 3088.874 3116.203 -1870.386 0.73909 0.73864
## Social.support 1 3067.249 3094.577 -1891.922 0.74230 0.74186
## -----
## 
```

```
##
```

Table: Adding New Variables

```
##
```

```
## Predictor
```

```
DF
```

```
AIC
```

```
SBC
```

```
SBIC
```

```
R2
```

```
Adj. R2
```

```
##
```

```
## Year
```

```
1
```

```
2945.021
```

```
2983.280
```

```
-2014.040
```

```
0.76026
```

```
0.75957
```

```
## Healthy.life.expectancy.at.birth
```

```
1
```

```
2847.817
```

```
2886.077
```

```
-2110.613
```

```
0.77324
```

```
0.77259
```

```
## Freedom.to.make.life.choices
```

```
1
```

```
2923.630
```

```
2961.890
```

```
-2035.294
```

```
0.76318
```

```
0.76250
```

```
## Generosity
```

```
1
```

```
2913.995
```

```
2952.255
```

```
-2044.867
```

```
0.76448
```

```
0.76381
```

```
## Negative.affect
```

```
1
```

```
2937.383
```

```
2975.642
```

```
-2021.629
```

```
0.76131
```

```
0.76062
```

```
## Confidence.in.national.government
```

```
1
```

```
2916.982
```

```
2955.242
```

```
-2041.899
```

```
0.76408
```

```
0.76340
```

```
##
```

```
##
```

```
## Step => 5
```

```
## Added => Healthy.life.expectancy.at.birth
```

```
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
```

```
## AIC => 2847.817
```

```
##
```

```
##
```

Table: Removing Existing Variables

```
##
```

```
## Predictor
```

```
DF
```

```
AIC
```

```
SBC
```

```
SBIC
```

```
R2
```

```
Adj. R2
```

```
##
```

```
## Log.GDP.per.capita
```

```
1
```

```
3067.460
```

```
3100.254
```

```
-1892.038
```

```
0.74256
```

```
0.74197
```

```
## Positive.affect
```

```
1
```

```
3144.054
```

```
3176.848
```

```
-1815.828
```

```
0.73103
```

```
0.73041
```

```
## Perceptions.of.corruption
```

```
1
```

```
2997.141
```

```
3029.935
```

```
-1961.994
```

```
0.75272
```

```
0.75215
```

```
## Social.support
```

```
1
```

```
2967.143
```

```
2999.937
```

```
-1991.834
```

```
0.75693
```

```
0.75637
```

```
## Healthy.life.expectancy.at.birth
```

```
1
```

```
2943.269
```

```
2976.063
```

```
-2015.581
```

```
0.76023
```

```
0.75968
```

```
##
```

```
##
```

```
##
```

Table: Adding New Variables

```
##
```

```
## Predictor
```

```
DF
```

```
AIC
```

```
SBC
```

```
SBIC
```

```
R2
```

```
Adj. R2
```

```
##
```

```
## Year
```

```
1
```

```
2844.176
```

```
2887.901
```

```
-2114.320
```

```
0.77397
```

```
0.77319
```

```
## Freedom.to.make.life.choices
```

```
1
```

```
2838.379
```

```
2882.104
```

```
-2120.071
```

```
0.77472
```

```
0.77394
```

```
## Generosity
```

```
1
```

```
2821.547
```

```
2865.272
```

```
-2136.773
```

```
0.77688
```

```
0.77611
```

```
## Negative.affect
```

```
1
```

```
2849.172
```

```
2892.897
```

```
-2109.362
```

```
0.77332
```

```
0.77254
```

```
## Confidence.in.national.government
```

```
1
```

```
2826.819
```

```
2870.544
```

```
-2131.542
```

```
0.77620
```

```
0.77543
```

```
##
```

```
##
```

```
## Step => 6
```

```
## Added => Generosity
```

```
## Model => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
```

```
## AIC => 2821.547
```

```
##
```

```
##
```

Table: Removing Existing Variables

```
##
```

```
## Predictor
```

```
DF
```

```
AIC
```

```
SBC
```

```
SBIC
```

```
R2
```

```
Adj. R2
```

```
##
```

```
## Log.GDP.per.capita
```

```
1
```

```
3060.444
```

```
3098.703
```

```
-1899.334
```

```
0.74389
```

```
0.74315
```

```
## Positive.affect
```

```
1
```

```
3063.628
```

```
3101.887
```

```
-1896.169
```

```
0.74342
```

```
0.74269
```

```
## Perceptions.of.corruption
```

```
1
```

```
2933.415
```

```
2971.675
```

```
-2025.571
```

```
0.76185
```

```
0.76117
```

```
## Social.support
```

```
1
```

```
2941.821
```

```
2980.081
```

```
-2017.219
```

```
0.76070
```

```
0.76001
```

```
## Healthy.life.expectancy.at.birth
```

```
1
```

```
2913.995
```

```
2952.255
```

```
-2044.867
```

```
0.76448
```

```
0.76381
```

```
## Generosity
```

```
1
```

```
2847.817
```

```
2886.077
```

```
-2110.613
```

```
0.77324
```

```
0.77259
```

```

## -----
## 
##          Table: Adding New Variables
## -----
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## 
## Year           1  2818.674  2867.865 -2139.680  0.77750  0.77660
## Freedom.to.make.life.choices  1  2816.267  2865.458 -2142.066  0.77781  0.77691
## Negative.affect  1  2822.998  2872.189 -2135.395  0.77695  0.77605
## Confidence.in.national.government  1  2791.452  2840.642 -2166.659  0.78094  0.78006
## 
## 
## Step      => 7
## Added     => Confidence.in.national.government
## Model     => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC       => 2791.452
## 
##          Table: Removing Existing Variables
## -----
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## 
## Log.GDP.per.capita  1  3005.683  3049.408 -1954.016  0.75208  0.75122
## Positive.affect    1  3042.770  3086.495 -1917.194  0.74676  0.74588
## Perceptions.of.corruption  1  2934.713  2978.438 -2024.468  0.76195  0.76113
## Social.support     1  2904.694  2948.419 -2054.263  0.76600  0.76520
## Healthy.life.expectancy.at.birth  1  2877.269  2920.994 -2081.480  0.76965  0.76885
## Generosity        1  2826.819  2870.544 -2131.542  0.77620  0.77543
## Confidence.in.national.government  1  2821.547  2865.272 -2136.773  0.77688  0.77611
## 
## 
##          Table: Adding New Variables
## -----
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## 
## Year           1  2789.086  2843.742 -2169.023  0.78149  0.78048
## Freedom.to.make.life.choices  1  2760.813  2815.469 -2197.007  0.78499  0.78400
## Negative.affect  1  2793.423  2848.080 -2164.729  0.78094  0.77994
## 
## 
## Step      => 8
## Added     => Freedom.to.make.life.choices
## Model     => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC       => 2760.813
## 
##          Table: Removing Existing Variables
## -----
## Predictor      DF   AIC     SBC    SBIC      R2   Adj. R2
## 
## Log.GDP.per.capita  1  2971.339  3020.530 -1988.330  0.75718  0.75620
## Positive.affect    1  2909.214  2958.405 -2049.931  0.76566  0.76472
## Perceptions.of.corruption  1  2896.003  2945.194 -2063.029  0.76743  0.76649
## Social.support     1  2860.348  2909.539 -2098.374  0.77213  0.77121
## Healthy.life.expectancy.at.birth  1  2827.179  2876.370 -2131.251  0.77641  0.77551
## Generosity        1  2790.973  2840.164 -2167.133  0.78100  0.78012

```

```

## Confidence.in.national.government      1    2816.267    2865.458    -2142.066    0.77781    0.77691
## Freedom.to.make.life.choices          1    2791.452    2840.642    -2166.659    0.78094    0.78006
##
## -----
## Table: Adding New Variables
## -----
## Predictor      DF   AIC      SBC      SBIC      R2   Adj. R2
## -----
## Year           1    2747.875   2807.997   -2209.779   0.78683   0.78572
## Negative.affect 1    2762.793   2822.915   -2195.032   0.78500   0.78388
##
## Step      => 9
## Added     => Year
## Model     => Life.Ladder ~ Log.GDP.per.capita + Positive.affect + Perceptions.of.corruption + Social...
## AIC       => 2747.875
##
## Table: Removing Existing Variables
## -----
## Predictor      DF   AIC      SBC      SBIC      R2   Adj. R2
## -----
## Log.GDP.per.capita      1    2952.778   3007.434   -2006.933   0.76002   0.75892
## Positive.affect        1    2885.396   2940.052   -2073.670   0.76910   0.76804
## Perceptions.of.corruption 1    2886.315   2940.972   -2072.759   0.76898   0.76792
## Social.support         1    2837.738   2892.394   -2120.859   0.77532   0.77428
## Healthy.life.expectancy.at.birth 1    2822.286   2876.942   -2136.157   0.77729   0.77627
## Generosity             1    2775.452   2830.109   -2182.518   0.78319   0.78219
## Confidence.in.national.government 1    2809.101   2863.757   -2149.210   0.77897   0.77795
## Freedom.to.make.life.choices      1    2789.086   2843.742   -2169.023   0.78149   0.78048
## Year                   1    2760.813   2815.469   -2197.007   0.78499   0.78400
##
## -----
## Table: Adding New Variables
## -----
## Predictor      DF   AIC      SBC      SBIC      R2   Adj. R2
## -----
## Negative.affect 1    2749.058   2814.646   -2208.574   0.78692   0.78570
##
## -----
## No more variables to be added or removed.
##
## Variables Selected:
## -----
## => Log.GDP.per.capita
## => Positive.affect
## => Perceptions.of.corruption
## => Social.support
## => Healthy.life.expectancy.at.birth
## => Generosity
## => Confidence.in.national.government
## => Freedom.to.make.life.choices
## => Year

```

```

## Stepwise Summary
## -----
## Step   Variable           AIC      SBC      SBIC      R2      Adj
## -----
## 0     Base Model          5430.112  5441.043  469.535  0.00000  0.000
## 1     Log.GDP.per.capita (+) 3751.504  3767.901 -1208.408  0.61787  0.617
## 2     Positive.affect (+)  3171.719  3193.582 -1787.449  0.72611  0.726
## 3     Perceptions.of.corruption (+) 3067.249  3094.577 -1891.922  0.74230  0.742
## 4     Social.support (+)  2943.269  2976.063 -2015.581  0.76023  0.760
## 5     Healthy.life.expectancy.at.birth (+) 2847.817  2886.077 -2110.613  0.77324  0.773
## 6     Generosity (+)       2821.547  2865.272 -2136.773  0.77688  0.776
## 7     Confidence.in.national.government (+) 2791.452  2840.642 -2166.659  0.78094  0.780
## 8     Freedom.to.make.life.choices (+)  2760.813  2815.469 -2197.007  0.78499  0.784
## 9     Year (+)            2747.875  2807.997 -2209.779  0.78683  0.786
## -----
## Final Model Output
## -----
## Model Summary
## -----
## R           0.887      RMSE        0.528
## R-Squared    0.787      MSE         0.279
## Adj. R-Squared 0.786      Coef. Var  9.669
## Pred R-Squared 0.784      AIC        2747.875
## MAE         0.408      SBC        2807.997
## -----
## RMSE: Root Mean Square Error
## MSE: Mean Square Error
## MAE: Mean Absolute Error
## AIC: Akaike Information Criteria
## SBC: Schwarz Bayesian Criteria
## -----
## ANOVA
## -----
## Sum of
## Squares      DF      Mean Square      F      Sig.
## -----
## Regression  1797.206      9      199.690  712.359  0.0000
## Residual    486.918   1737      0.280
## Total       2284.124   1746
## -----
## Parameter Estimates
## -----
## model      Beta      Std. Error      Std. Beta      t      Sig      lower
## -----
## (Intercept) 22.533      6.237
## Log.GDP.per.capita 0.341      0.023      0.346      14.778  0.000      0.296
## Positive.affect 1.925      0.160      0.179      12.017  0.000      1.611
## Perceptions.of.corruption -1.134      0.094      -0.182     -12.058  0.000     -1.318
## Social.support 1.587      0.164      0.169      9.684   0.000      1.266

```

```

##  Healthy.life.expectancy.at.birth      0.028      0.003      0.181      8.812      0.000      0.022
##          Generosity                  0.478      0.088      0.068      5.446      0.000      0.306
## Confidence.in.national.government   -0.741      0.093     -0.125     -8.001      0.000     -0.923
## Freedom.to.make.life.choices       0.979      0.148      0.118      6.595      0.000      0.688
## Year                           -0.012      0.003     -0.046     -3.862      0.000     -0.018
## ---

#Negative.affect removal might indicate information is already captured i.e. by Positive.affect
#Also tells us which features consistently make the bottom of the list, and/or offer the least amount of information

timeladder_reduced <- lm(Life.Ladder ~ Log.GDP.per.capita + Social.support + Healthy.life.expectancy.at.birth + Perceptions.of.corruption + Positive.affect + Confidence.in.national.government, data = happiness)

summary(timeladder_reduced) #All Type-3 p-values insignificant, R Squared = 0.777, F-statistic 1031 on 6 DF

## 
## Call:
## lm(formula = Life.Ladder ~ Log.GDP.per.capita + Social.support +
##     Healthy.life.expectancy.at.birth + Perceptions.of.corruption +
##     Positive.affect + Confidence.in.national.government, data = happiness)
## 

## Residuals:
##      Min        1Q        Median        3Q        Max
## -1.90680 -0.32702  0.03446  0.33724  1.76644
## 

## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)    
## (Intercept)                 -1.687836  0.219315 -7.696 2.34e-14 ***
## Log.GDP.per.capita           0.337600  0.023437 14.405 < 2e-16 ***
## Social.support                1.799771  0.165228 10.893 < 2e-16 ***
## Healthy.life.expectancy.at.birth 0.031174  0.003211  9.709 < 2e-16 ***
## Perceptions.of.corruption    -1.250451  0.094941 -13.171 < 2e-16 ***
## Positive.affect                2.616863  0.142124 18.413 < 2e-16 ***
## Confidence.in.national.government -0.405660  0.084480 -4.802 1.71e-06 ***
## ---                        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## 

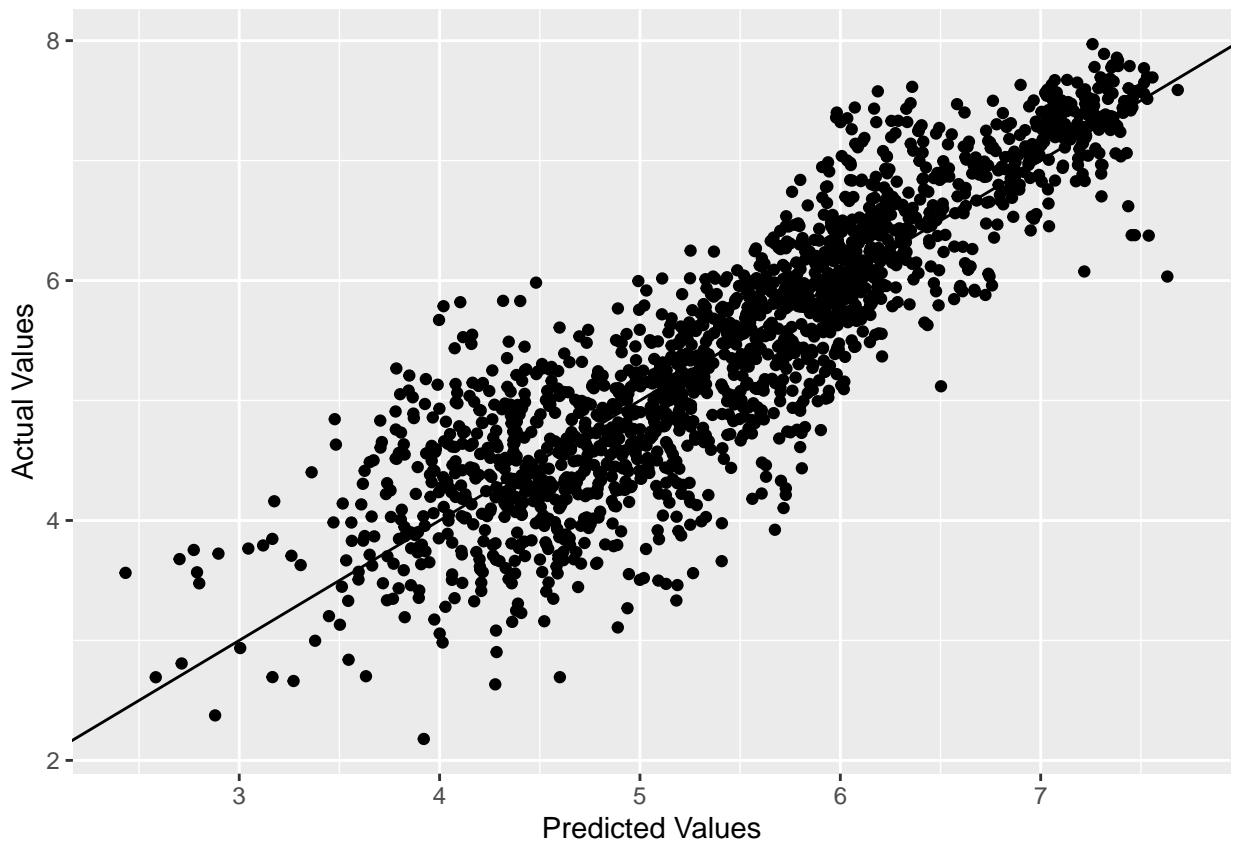
## Residual standard error: 0.542 on 1740 degrees of freedom
## Multiple R-squared:  0.7762, Adjusted R-squared:  0.7754 
## F-statistic:  1006 on 6 and 1740 DF,  p-value: < 2.2e-16

anova(timeladder_reduced, timeladder) #While the pvalue is significant, which implies that the full model is better than the reduced one

## Analysis of Variance Table
## 
## Model 1: Life.Ladder ~ Log.GDP.per.capita + Social.support + Healthy.life.expectancy.at.birth +
##     Perceptions.of.corruption + Positive.affect + Confidence.in.national.government
## Model 2: Life.Ladder ~ Year + Log.GDP.per.capita + Social.support + Healthy.life.expectancy.at.birth +
##     Freedom.to.make.life.choices + Generosity + Perceptions.of.corruption +
##     Positive.affect + Negative.affect + Confidence.in.national.government
## 
##   Res.Df   RSS Df Sum of Sq    F    Pr(>F)    
## 1    1740 511.18
## 2    1736 486.69  4    24.488 21.837 < 2.2e-16 ***
## ---                        
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

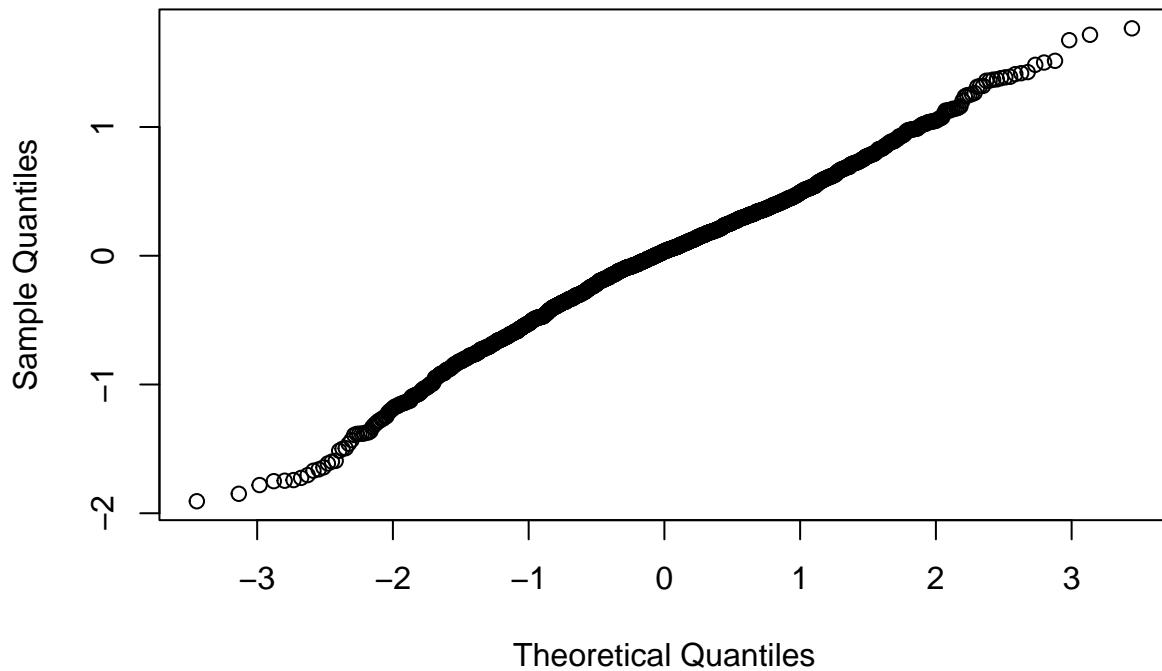
```

```
happiness |>
  ggplot(aes(x = predict(timeladder_reduced), y = Life.Ladder)) +
  geom_point() +
  geom_abline(intercept = 0, slope = 1) +
  labs(x = 'Predicted Values', y = 'Actual Values')
```

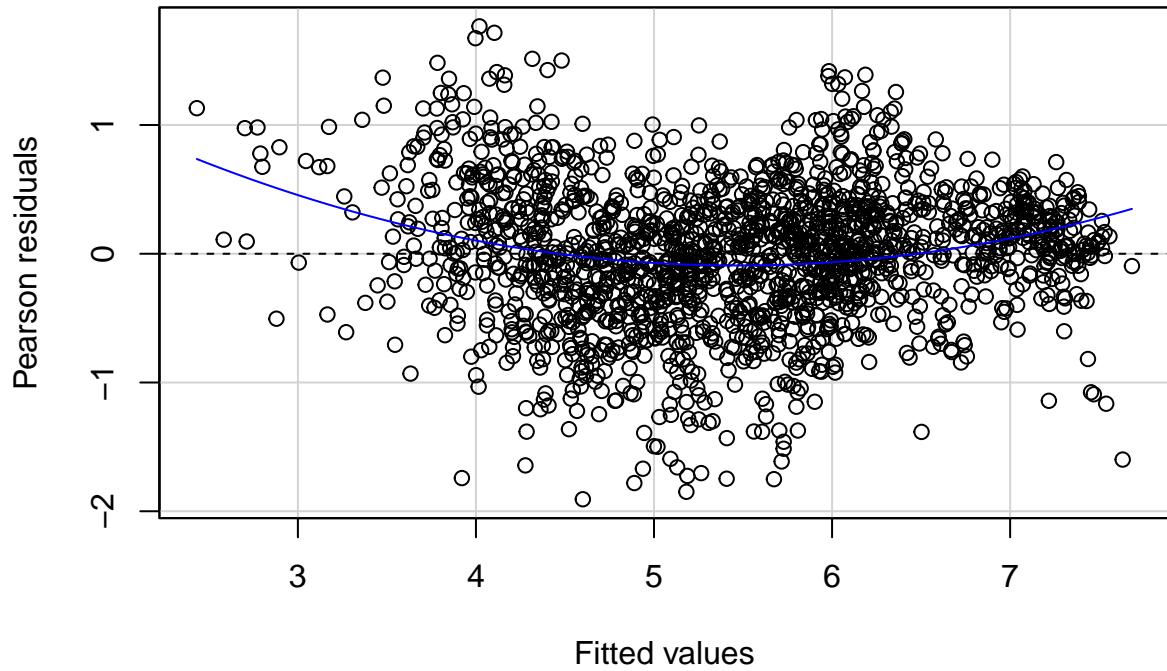


```
qqnorm(timeladder_reduced$residuals)
```

Normal Q-Q Plot



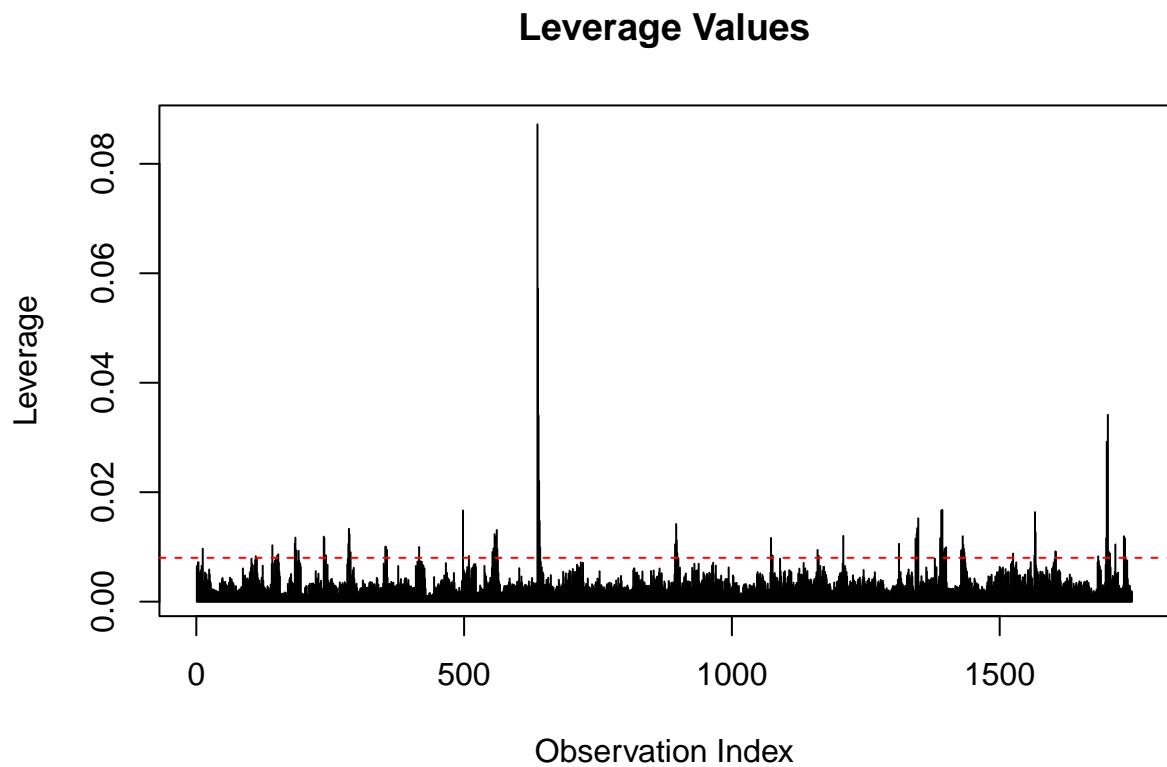
```
residualPlot(timeladder_reduced)
```



```

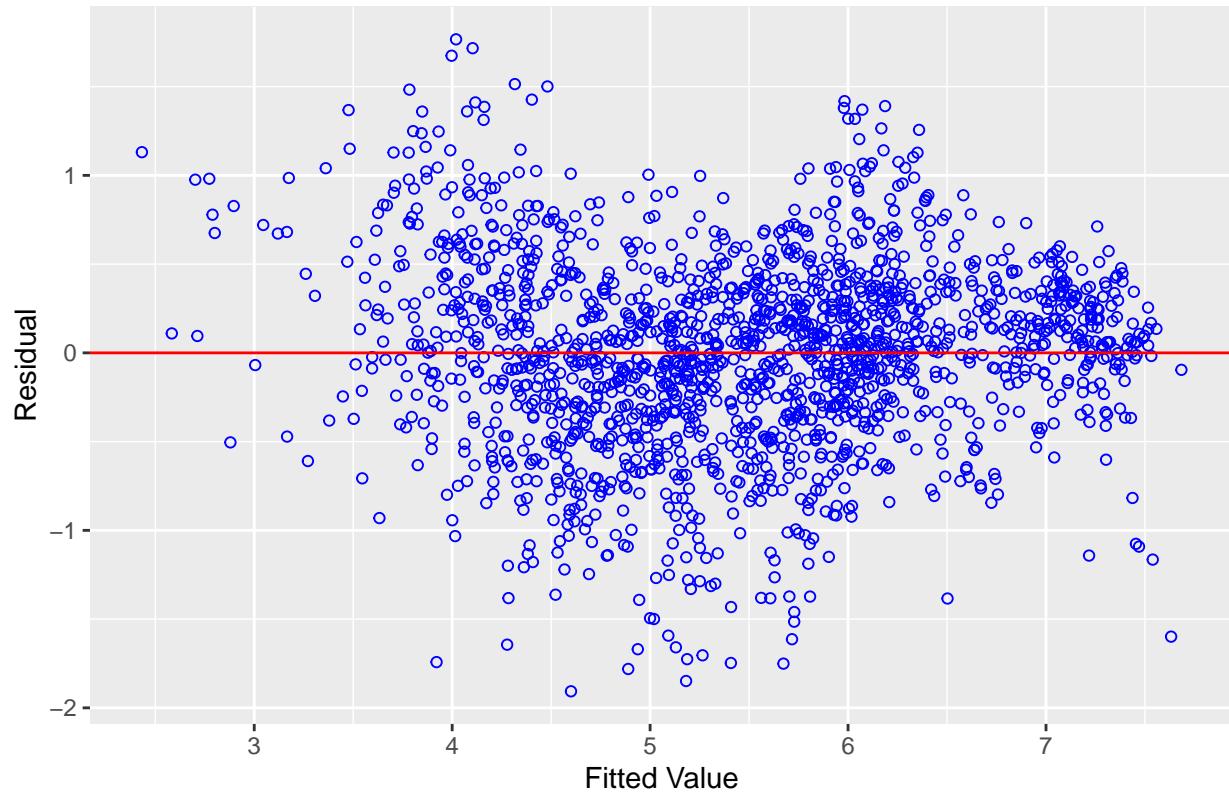
leverage_values_subset <- hatvalues(timeladder_reduced)

plot(leverage_values_subset, type = "h", main = "Leverage Values",
     xlab = "Observation Index", ylab = "Leverage")
abline(h = 2 * mean(leverage_values_subset), col = "red", lty = 2) #Add a reference line for high leverage
  
```

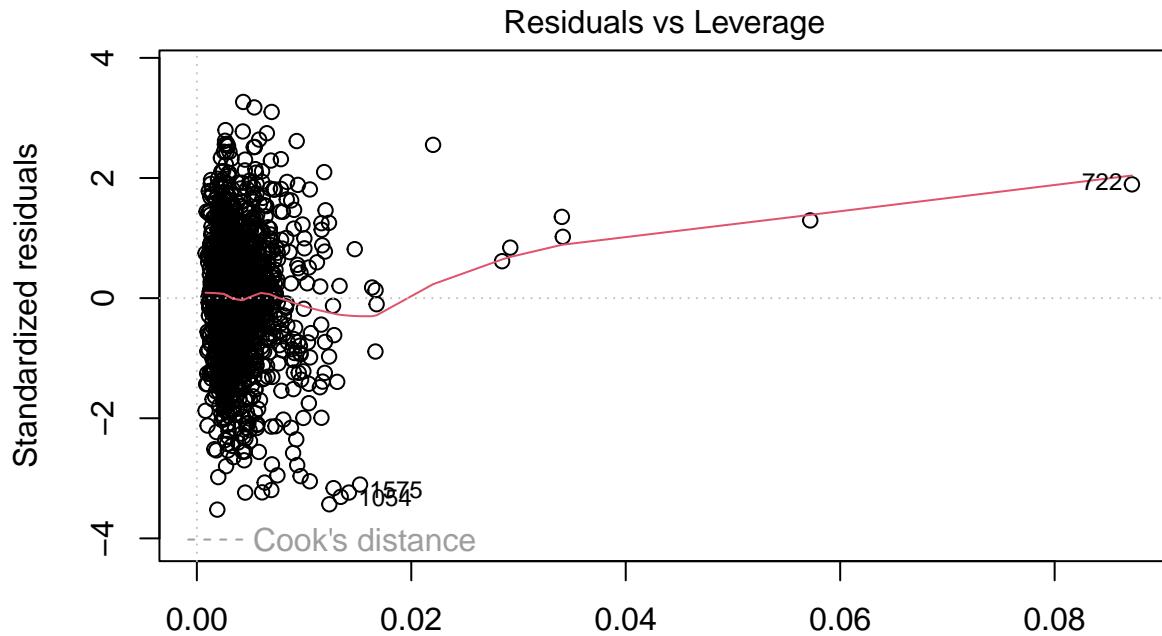


```
ols_plot_resid_fit(timeladder_reduced)
```

Residual vs Fitted Values



```
plot(timeladder_reduced, which = 5)
```



Leverage
lm(Life.Ladder ~ Log.GDP.per.capita + Social.support + Healthy.life.expecta ...

```

library(caret)

## Warning: package 'caret' was built under R version 4.5.2
## Loading required package: lattice
##
## Attaching package: 'caret'
## The following object is masked from 'package:purrr':
##      lift
train_control <- trainControl(method = "cv", number = 10)

# Train a model
model <- train(
  Life.Ladder~Log.GDP.per.capita+Social.support+Healthy.life.expectancy.at.birth+Perceptions.of.corrupt...
  data = happiness,
  method = "lm",
  trControl = train_control
)
# displaying k fold parameters
model

## Linear Regression
##
## 1747 samples
##      6 predictor

```

```

## 
## No pre-processing
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 1573, 1571, 1571, 1573, 1573, 1572, ...
## Resampling results:
##
##    RMSE      Rsquared   MAE
##    0.543028  0.776672  0.4175366
##
## Tuning parameter 'intercept' was held constant at a value of TRUE
model$metric

## [1] "RMSE"
# normalizing k fold parameters to our data range
lifeladder_range <- max(happiness$Life.Ladder) - min(happiness$Life.Ladder)
lifeladder_range

## [1] 5.792083
mean(model$resample$RMSE)/lifeladder_range

## [1] 0.09375349
mean(model$resample$MAE)/lifeladder_range

## [1] 0.07208747

haporig <- read.csv("World Happiness Report 2005-2021.csv")
excluded <- haporig %>% filter(Country.name %in% setdiff(unique(haporig$Country.name),unique(happiness$Country.name)))
excluded %>% group_by(Country.name) %>% summarise(Generosity.=sum(is.na(Generosity)),Life.Expectancy=sum(Life.Expectancy))

## # A tibble: 17 x 6
##   Country.name   Generosity. Life.Expectancy Corruption Confidence Total.obs
##   <chr>          <int>           <int>        <int>       <int>       <int>
## 1 Algeria          2              0            2          10         10
## 2 Bahrain          3              0            6          11         11
## 3 China             1              0           16          16         16
## 4 Cuba              1              0            1           0          1
## 5 Hong Kong S.A.R.~ 1             12            0           0         12
## 6 Kosovo            2             15            0           3         15
## 7 Libya             0              0            2           6          6
## 8 Maldives          0              0            1           1          1
## 9 North Cyprus      7              7            0           0          7
## 10 Oman             0              0            1           1          1
## 11 Saudi Arabia     1              0           13          16         16
## 12 Somaliland region 4              4            0           0          4
## 13 South Sudan      4              0            0           0          4
## 14 Sudan             0              0            0           5          5
## 15 Syria             0              0            0           7          7
## 16 Turkmenistan     0              0           10          10         10
## 17 United Arab Emir~ 3              0           11          14         14

# Overall correlation between happiness and Confidence
cor(happiness$Confidence.in.national.government,happiness$Life.Ladder)

## [1] -0.0825536

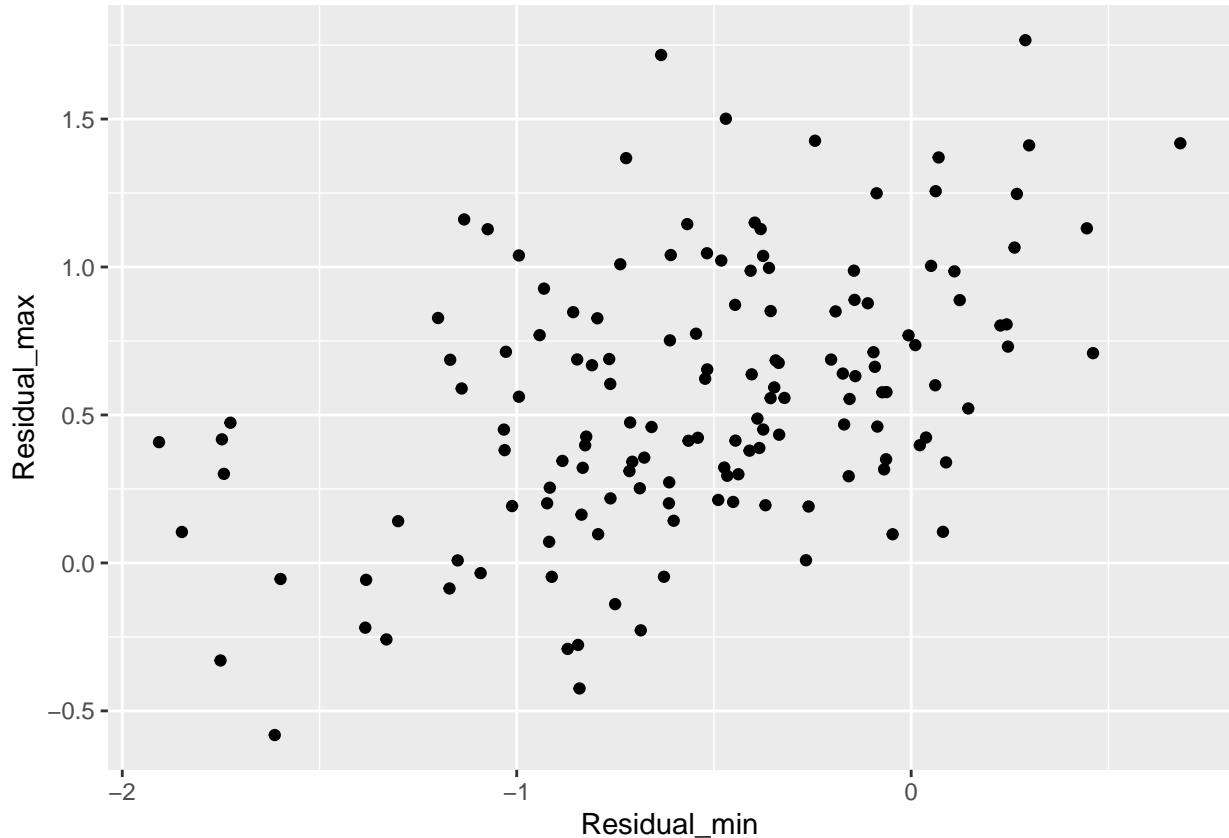
```

```

timeladder <- lm(Life.Ladder~Year+Log.GDP.per.capita+Social.support+Healthy.life.expectancy.at.birth+Frac...
timeladder_reduced <- lm(Life.Ladder~Log.GDP.per.capita+Social.support+Healthy.life.expectancy.at.birth+...

#Maximum vs minimum residual of countries
residualminmax <- happiness %>% select(Country.name,Year,Log.GDP.per.capita,Social.support,Healthy.life...
ggplot(residualminmax,aes(x=Residual_min,y=Residual_max)) + geom_point()

```



```

#Correlation for partial model
with(residualminmax,cor(Residual_min,Residual_max))

## [1] 0.4678802

# Correlation for full model
happiness %>% select(Country.name,Year,Log.GDP.per.capita,Social.support,Healthy.life.expectancy.at.bir...

## [1] 0.4233393

model2 <- lm(Life.Ladder ~ Negative.affect, data = happiness)
summary(model2)

##
## Call:
## lm(formula = Life.Ladder ~ Negative.affect, data = happiness)
## 
## Residuals:
##      Min       1Q   Median       3Q      Max 
## -2.93320 -0.80153  0.00554  0.87082  2.45872 
## 
```

```

## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)           6.65621   0.08953  74.35 <2e-16 ***
## Negative.affect -4.37423   0.31745 -13.78 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.087 on 1745 degrees of freedom
## Multiple R-squared:  0.09813,    Adjusted R-squared:  0.09761
## F-statistic: 189.9 on 1 and 1745 DF,  p-value: < 2.2e-16
model3 <- lm(Life.Ladder ~ Confidence.in.national.government, data = happiness)
summary(model3)

##
## Call:
## lm(formula = Life.Ladder ~ Confidence.in.national.government,
##      data = happiness)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -3.4818 -0.8510 -0.0760  0.8122  2.5841
##
## Coefficients:
##                               Estimate Std. Error t value Pr(>|t|)
## (Intercept)           5.71286   0.07377  77.44 < 2e-16 ***
## Confidence.in.national.government -0.48959   0.14149   -3.46 0.000552 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.14 on 1745 degrees of freedom
## Multiple R-squared:  0.006815,  Adjusted R-squared:  0.006246
## F-statistic: 11.97 on 1 and 1745 DF,  p-value: 0.0005525

```