Notes for "Hernán MA, Robins JM (2019). Causal Inference. Boca Raton: Chapman and Hall/CRC, forthcoming."

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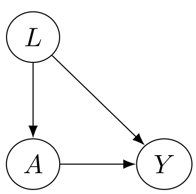
These notes are for personal understanding and they most certainly will have typos and errors as they develop. Please read with care.

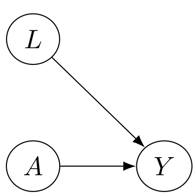
1 Chapter 1 - 3

Definition (Consistency): This is the assumption that $Y = Y^A$, where Y is the response and A is the binary treatment. The implications of this are: $\mathbb{E}(Y^{a=1}|A=1) = \mathbb{E}(Y|A=1)$ and likewise for $A = 0$.

Consider the following causal DAG:



L are covariates, A is the binary treatment, Y is the binary response. Suppose that a randomized control trial is performed. Then the resulting DAG will look like this:



Definition (Full Exchangeability): $(Y^{a=0}, Y^{a=1}) \perp\!\!\!\perp A$
Definition (Exchangeability): $Y^a \perp\!\!\!\perp A, \forall a$
Definition (Mean Exchangeability): $\mathbb{E}(Y^a|A=1) = \mathbb{E}(Y^a|A=0), \forall a$

Note that full exchangeability $\implies$ exchangeability $\implies$ mean exchangeability.

Remark: Randomized control trials produce exchangeability implying that association is equivalent to causation. To see this: By consistency assumption, we have $\mathbb{P}(Y = 1|A = 1) = \mathbb{P}(Y^{a=1} = 1|A = 1)$. Furthermore, by randomization we have exchangeability so $\mathbb{P}(Y^{a=1} = 1|A = 1) = \mathbb{P}(Y^{a=1} = 1)$. Hence we have $\mathbb{P}(Y = 1|A = 1) = \mathbb{P}(Y^{a=1} = 1)$, where the LHS can we estimated with the observed data. Likewise, a similar argument can be used to show that $\mathbb{P}(Y = 1|A = 0) = \mathbb{P}(Y^{a=0} = 1)$.

1.1 Inverse probability weighting estimators

Define $f(a|l)$ to be the conditional probability distribution of $A|L$. Then:

$$
\begin{aligned}
\mathbb{E}\left(\frac{I(A=a)Y}{f(A|L)}\right) &= \mathbb{E}\left(\mathbb{E}\left(\frac{I(A=a)Y}{f(A|L)}|A,L\right)\right) \\
&= \sum_l \sum_{a'} \mathbb{E}\left(\frac{I(A=a)Y}{f(a'|l)}|A=a',L=l\right)\mathbb{P}(A=a'|L=l)\mathbb{P}(L=l) \\
&= \sum_l \sum_{a'} \mathbb{E}\left(\frac{I(A=a)Y}{f(a'|l)}|A=a',L=l\right) f(a'|l)\mathbb{P}(L=l), \text{ by definition} \\
&= \sum_l \mathbb{E}(Y|A=a,L=l')\mathbb{P}(L=l') \\
&= \mathbb{E}(Y^{A=a}), \text{ by the adjustment formula}
\end{aligned} \tag{1}
$$