

Notes for "Hernán MA, Robins JM (2019). Causal Inference. Boca Raton: Chapman and Hall/CRC, forthcoming."

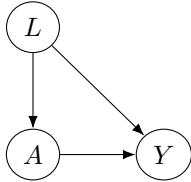
Alex Gao

These notes are for personal understanding and they most certainly will have typos and errors as they develop. Please read with care.

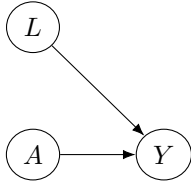
1 Chapter 1 - 3

Definition (Consistency): This is the assumption that $Y = Y^A$, where Y is the response and A is the binary treatment. The implications of this are: $\mathbb{E}(Y^{a=1}|A=1) = \mathbb{E}(Y|A=1)$ and likewise for $A=0$.

Consider the following causal DAG:



L are covariates, A is the binary treatment, Y is the binary response. Suppose that a randomized control trial is performed. Then the resulting DAG will look like this:



Definition (Full Exchangeability): $(Y^{a=0}, Y^{a=1}) \perp\!\!\!\perp A$

Definition (Exchangeability): $Y^a \perp\!\!\!\perp A, \forall a$

Definition (Mean Exchangeability): $\mathbb{E}(Y^a|A=1) = \mathbb{E}(Y^a|A=0), \forall a$

Note that full exchangeability \implies exchangeability \implies mean exchangeability.

Remark: Randomized control trials produce exchangeability implying that association is equivalent to causation. To see this: By consistency assumption, we have $\mathbb{P}(Y=1|A=1) = \mathbb{P}(Y^{a=1}=1|A=1)$. Furthermore, by randomization we have exchangeability so $\mathbb{P}(Y^{a=1}=1|A=1) = \mathbb{P}(Y^{a=1}=1)$. Hence we have $\mathbb{P}(Y=1|A=1) = \mathbb{P}(Y^{a=1}=1)$, where the LHS can be estimated with the observed data. Likewise, a similar argument can be used to show that $\mathbb{P}(Y=1|A=0) = \mathbb{P}(Y^{a=0}=1)$.