TMATH 126 Finale Review Workshop

Unit 1

Unless otherwise stated let $\vec{u} = \langle 2, -3, 5\rangle$ and $\vec{v} = \langle 4, -1, 2\rangle$

1a: Coordinates and Vectors in 2D and 3D

a Draw vectors $\vec{u}$ and $\vec{v}$ in 2D and 3D. Then draw $\vec{u} + \vec{v}$ using the tip-to-tail method.

b Draw and label $\vec{u} - \vec{v}$ on the same coordinate system.

1b: Dot Product, Cross Product

a Explain the applications of the dot and cross product. (ie what do they tell/give us)

b Find the scalar projection of $\vec{u}$ onto $\vec{v}$.

c Find the vector projection of $\vec{v}$ onto $\vec{u}$.

d Find the angle between $\vec{u}$ $\vec{v}$.

e Find the area of a parallelogram with the points $A(0, 1, 3)$, $B(1, 3, 5)$, $C(5, 7, 5)$, $D(4, 5, 3)$.

1c: Equations of Lines and Planes

a Given point $A(1, 1, 1)$ and $\vec{d} = \langle 10, -10, 50\rangle$ Write the vector equation of a line in 2D and 3D that contains point A moves is parallel to $\vec{d}$.

b Find the equation of a plane with points $(4, -3, 1)$, $(-3, -1, 1)$, and $4, -2, 8$.

c Determine if the planes given by $4x - 9y - z = 2$ and $x + 2y - 14z = -6$ are parallel, orthogonal, or neither.

d Determine where the line $\vec{r}(t) = \langle -2t, 2 + 7t, -1 - 4t\rangle$ and the plane given by $\langle 4x + 9y - 2z = -8$

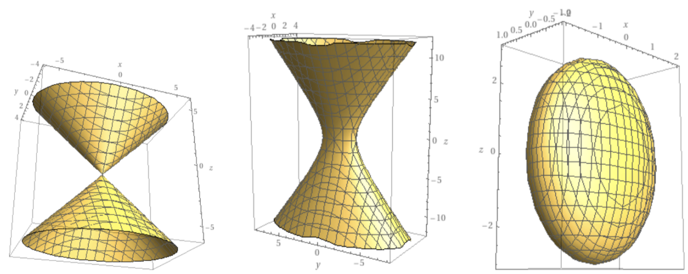
e Find the line of intersection of the planes given by $3x + 6y - 5z = -3$ and $-2x + 7y - z = 24$.

f Find the point on the plane $3x + 4y + z = 1$ that is closest to the point $(1, 0, 1)$.

1d: Equations of Cylinders and Other Simple Surfaces

Exercise 2 (Matching). *Match each equation to its graph and explain your choices.*

$9x^2 + 36y^2 + 4z^2 = 36$ $\quad$ $4x^2 + 9y^2 - 4z^2 = 0$ $\quad$ $36x^2 + 9y^2 - 4z^2 = 36$



Identify and sketch the following 3D surfaces.

a $x^2 + y^2 + z^2 = 1$

b $x = cost$ and $y = sint$

c $x^2 + \frac{y^2}{9} + \frac{z^2}{4}$

d $z = 4z^2 + y^2$

Unit 2

A particle moves along a path in space described by the vector-valued function

$$\vec{r}(t) = \langle 3\cos t, 3\sin t, t^2 \rangle, \quad 0 \leq t \leq 2\pi.$$

a Find the domain of $\vec{r}(t)$.

b Describe the motion of the particle in words. What shape does the projection of the path make in the xy-plane?

c Find $\vec{r}\,'(t)$ and set up an integral for the arc length of the curve from $t = 0$ to $t = 3\pi$. Then use a calculator or software to evaluate the integral.

d Find the equation of the line tangent to the curve at $t = \pi$.

e Let $z = f(x, y) = x^2 + y^2$. Describe the motion of the particle. Additionally what is the domain of $\vec{r}(t)$.

Unit 3

Consider the function $f(x, y) = 9 - x^2 - y^2$.

a Sketch at least three level curves of $f(x, y)$ for the values $f(x, y) = 0$, 4, and 8. Label each curve clearly.

b Describe the shape of the surface $z = f(x, y)$. What kind of surface is it? What do the level curves represent geometrically?

Consider the function

$$f(x,y) = x^3 - 3xy^2.$$

a Compute the first-order partial derivatives $f_x(x,y)$ and $f_y(x,y)$.

b Find the equation of the tangent plane to the surface at point $P = (1, 1, f(1, 1))$. Write the linear approximation of $f(x,y)$ near P.

c Calculate the gradient vector $\nabla f(1, 1)$. Find the directional derivative of f at point P in the direction of the vector $\mathbf{v} = \langle 2, 1 \rangle$. Determine the direction of fastest increase of f at P.

d Use the result from part (c) to write the equation of the line tangent to the surface $f(x,y)$ at the point $P = (1, 1, f(1, 1))$ in the direction of the vector $\mathbf{v} = \langle 2, 1 \rangle$.

e Find all critical points of $f(x,y)$ and classify them(local extrema, saddle points, etc). Use the second derivative test (Hessian matrix).

a consider the integral $\iint_D sin(y^2)\, dA$ where D is the region bounded by $y = x$ and the $y - axis$ from $x = 0$ to $x = 1$ (set up and solve the integral both ways, one of these integrals will not be possible)

b $\iint_D e^{\frac{x}{y}}\, dA$, Where D is bounded by $y = \sqrt{x}$ and $y = x^3$

a let $f(x) = \frac{16}{4-x^2} + 6cos(x)$

b Give the Taylor series for f based at $b = 0$ using one sigma sine.

c Find the largest open interval on which this taylor series converges.

d Find $T_3(x)$, the third Taylor polynomial for f based at $b = 0$. Then estimate the value of $\int_0^1 f(x)dx$ by replacing $f(x)$ with $T_3(x)$.