Distance Weighted Discrimination (Benito & Marron, 2004)

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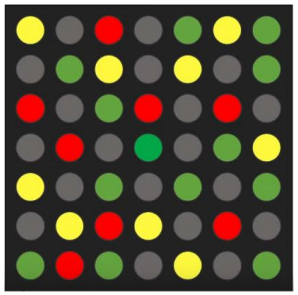
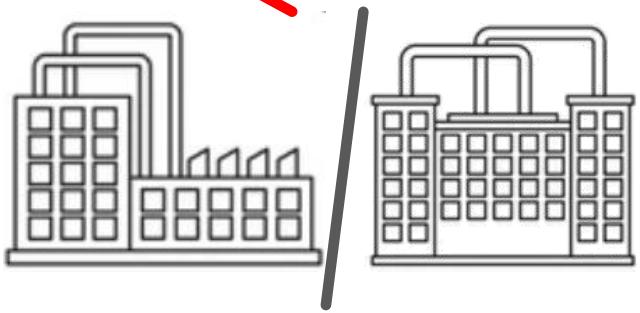
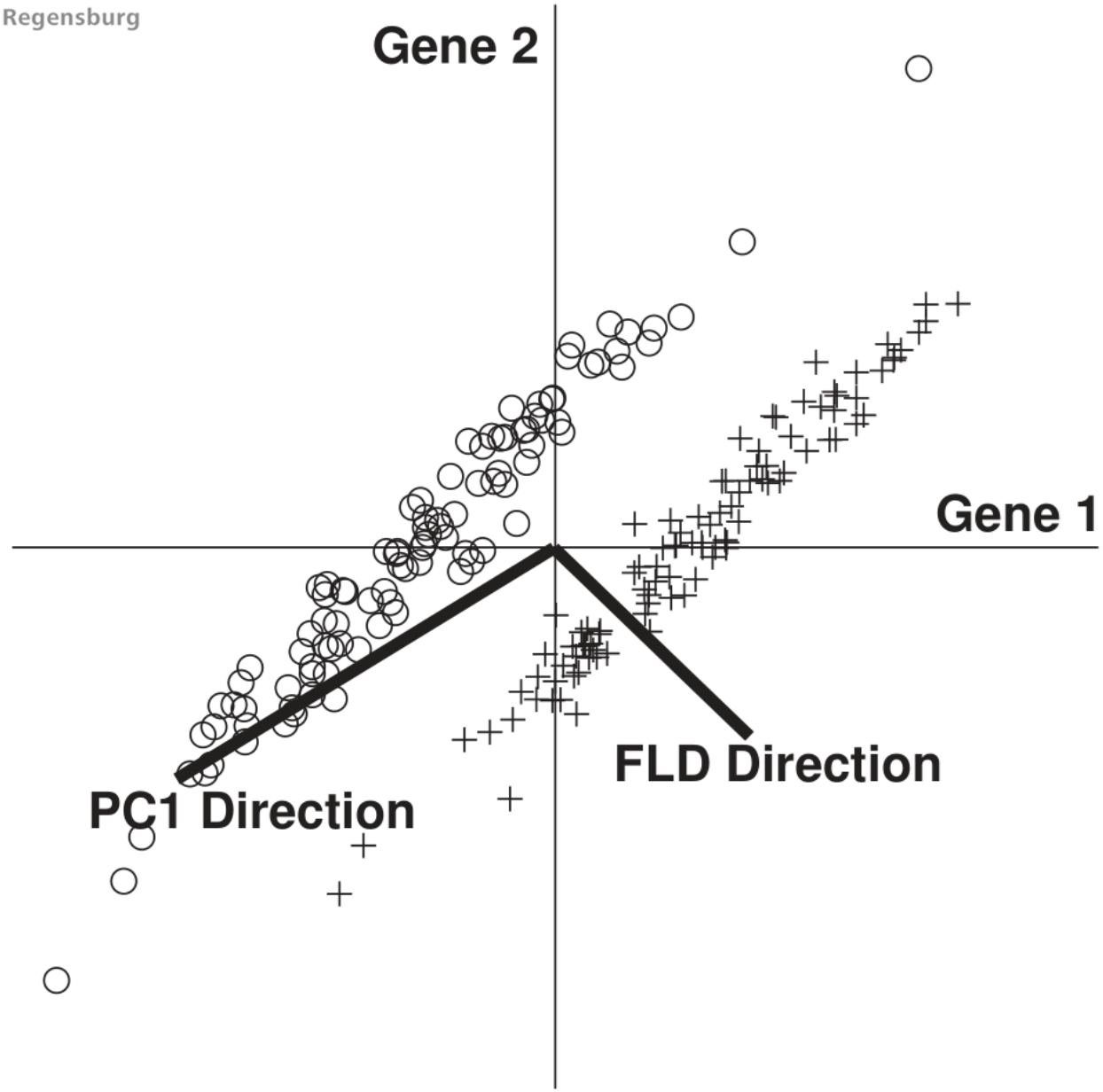
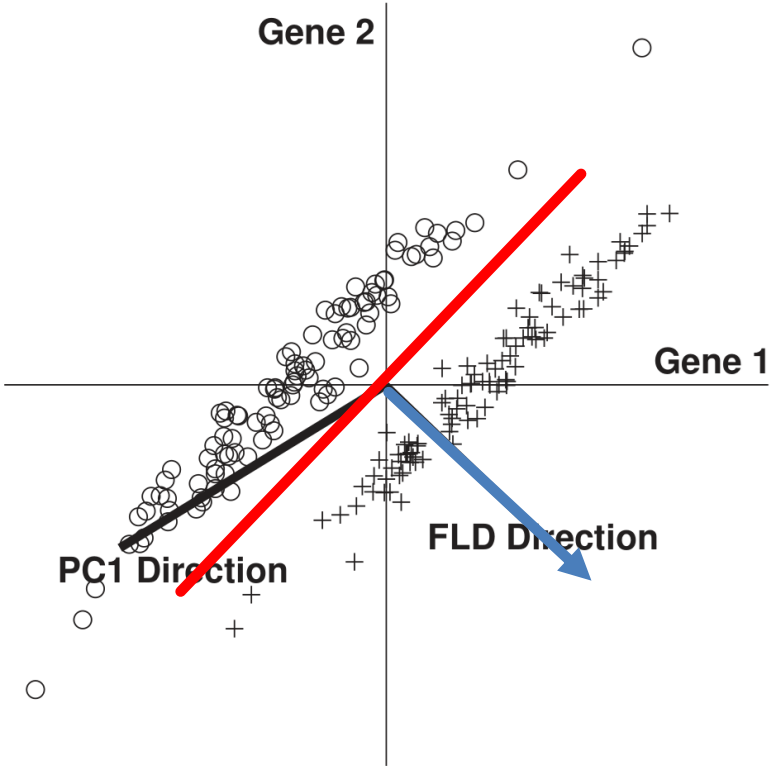





Gene 2
Gene 1
FLD Direction
PC1 Direction

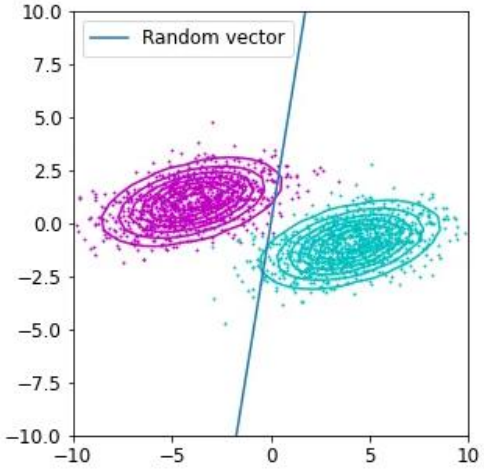


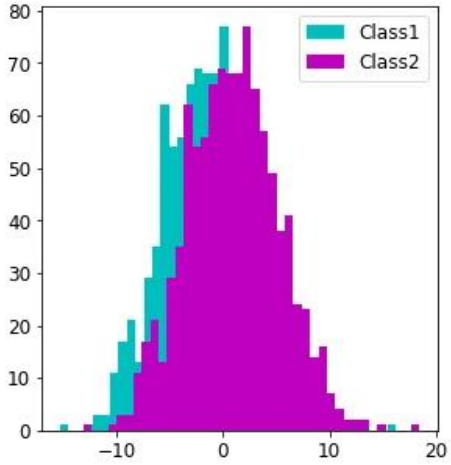
$$\vec{\Theta} \cdot \vec{x_1} + c = 0$$

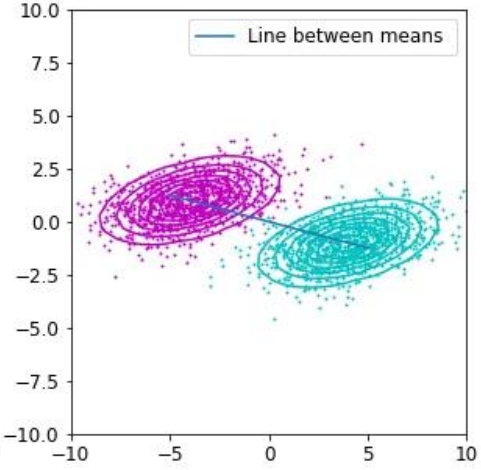
$$\vec{\Theta} \cdot \vec{x_2} + c = 0$$

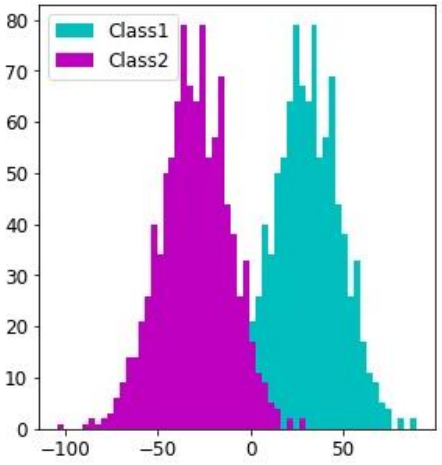
$$\vec{\Theta} \cdot (\vec{x_2} - \vec{x_1}) = 0$$

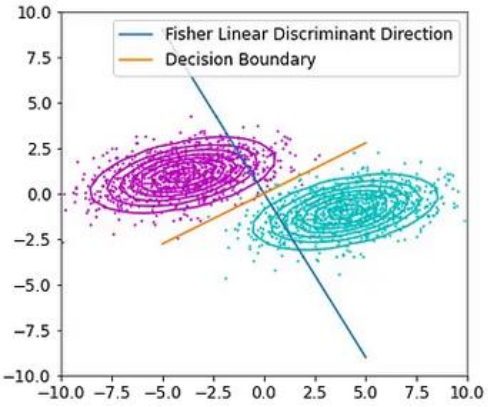
Fisher Linear Discriminant (FLD)

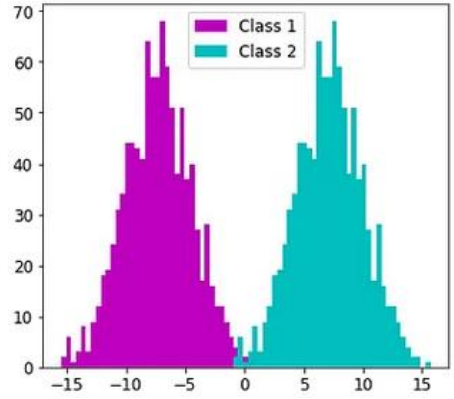












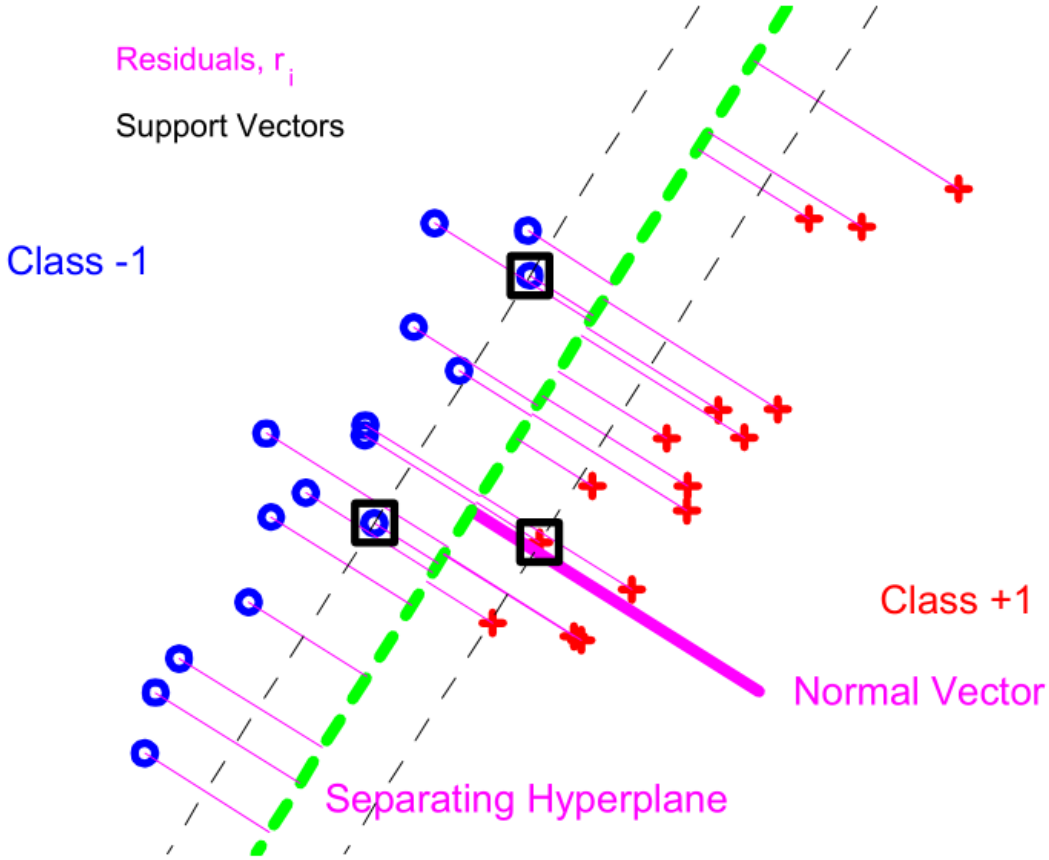
Fisher Linear Discriminant (FLD)

$$J(\boldsymbol{w}) = \frac{(m_1 - m_2)^2}{s_1^2 + s_2^2}$$

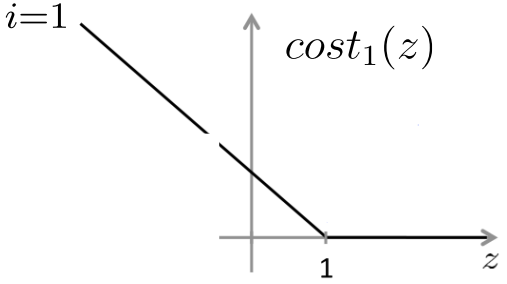
$$\frac{\partial J(w)}{\partial w} = 0$$

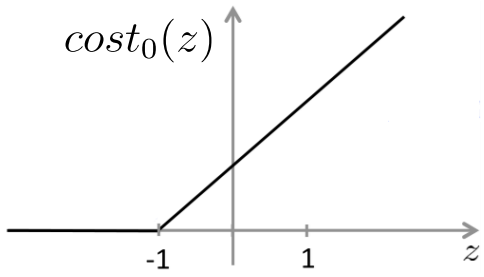
$$w \propto S_w^{-1}(\boldsymbol{m}_2 - \boldsymbol{m}_1) \qquad \text{with } S_w = \Sigma_2 + \Sigma_1$$

Support Vector Machine (SVM)



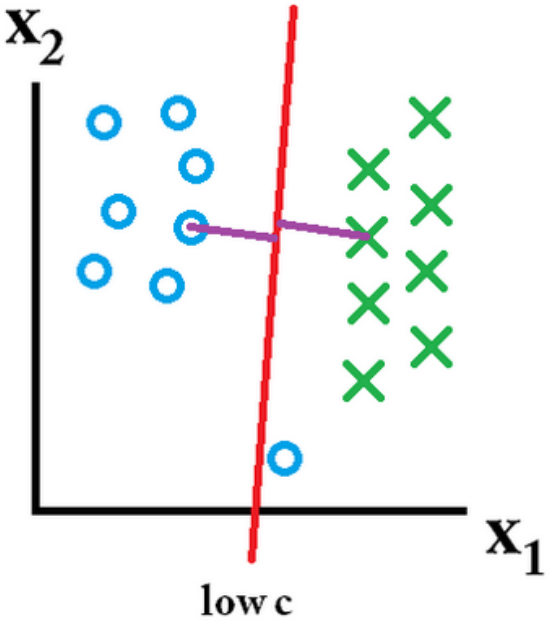
$$\min_w C \cdot \sum_{i=1}^{n} \left[y^{(i)} cost_1(w^T x^{(i)}) + (1 - y^{(i)}) cost_0(w^T x^{(i)}) \right] + \frac{1}{2} \sum_{j=1}^{d} w_j^2$$

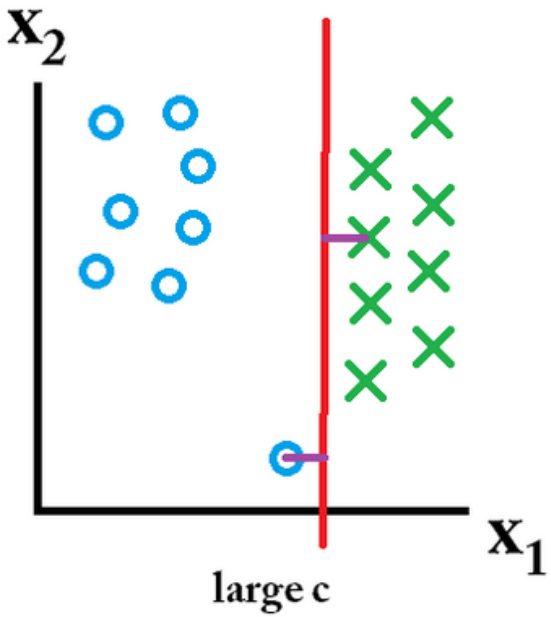




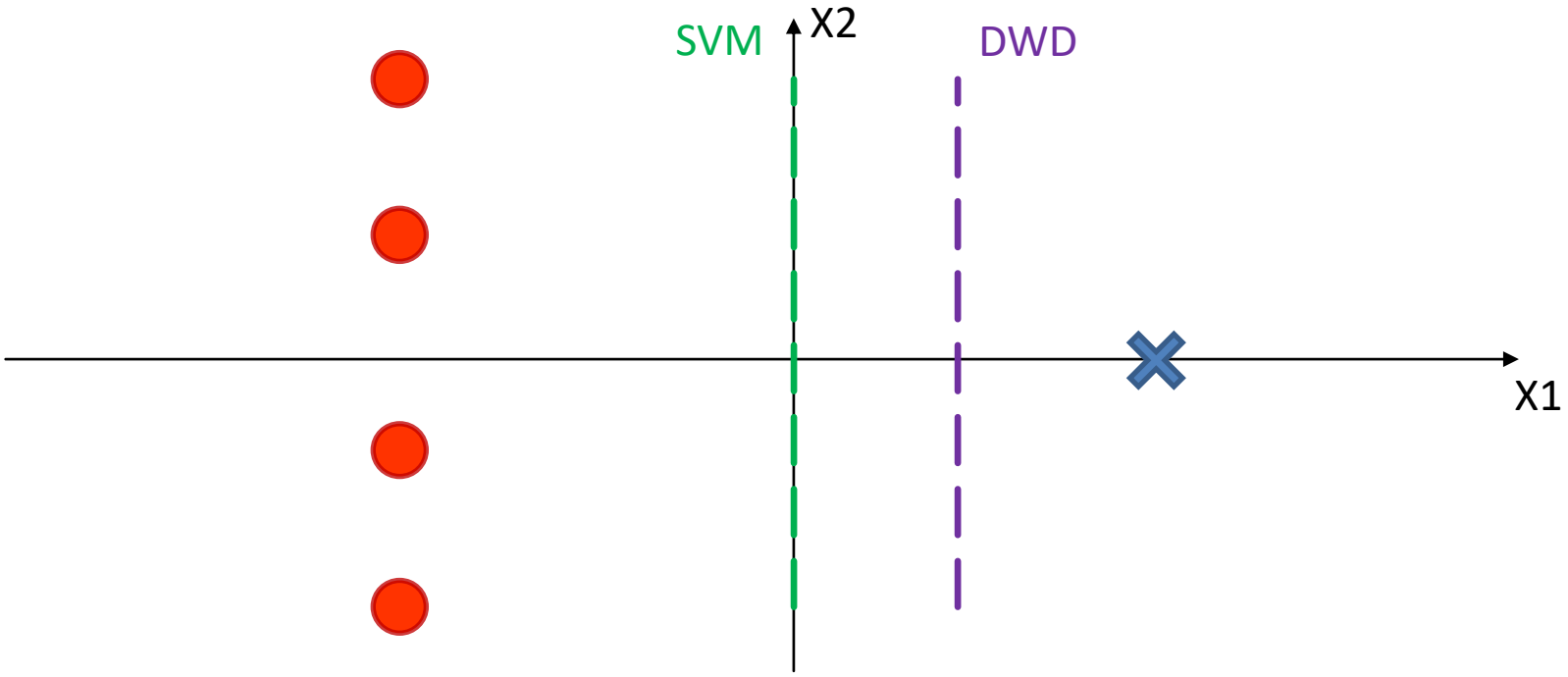
$w^T x^{(i)} \geq 1$

$w^T x^{(i)} \leq -1$



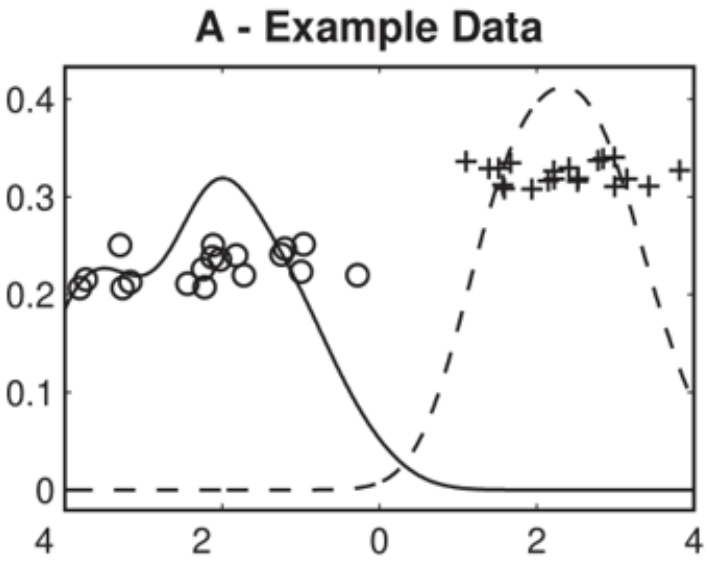


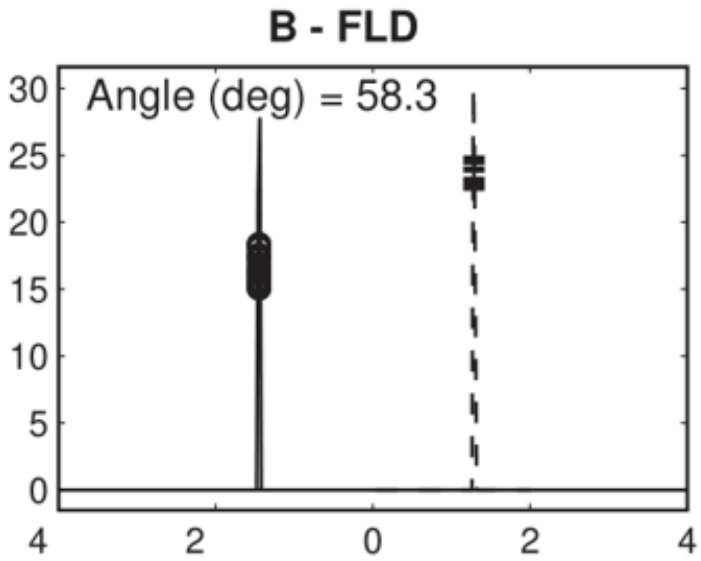
Distance Weighted Discriminant (DWD)

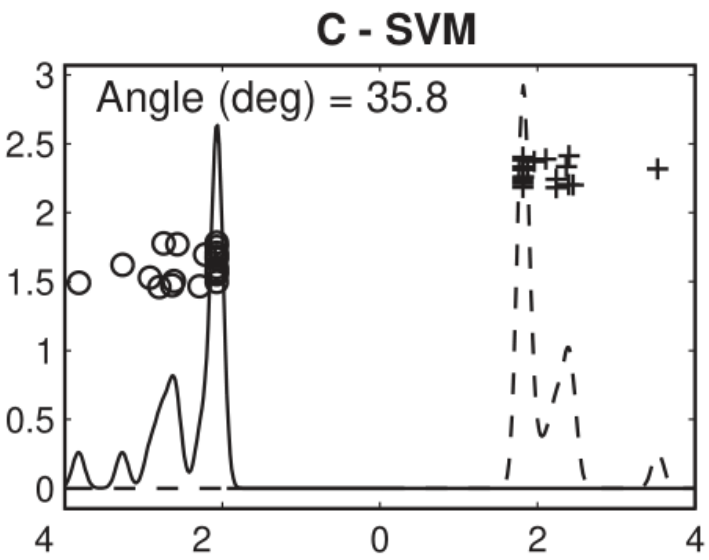


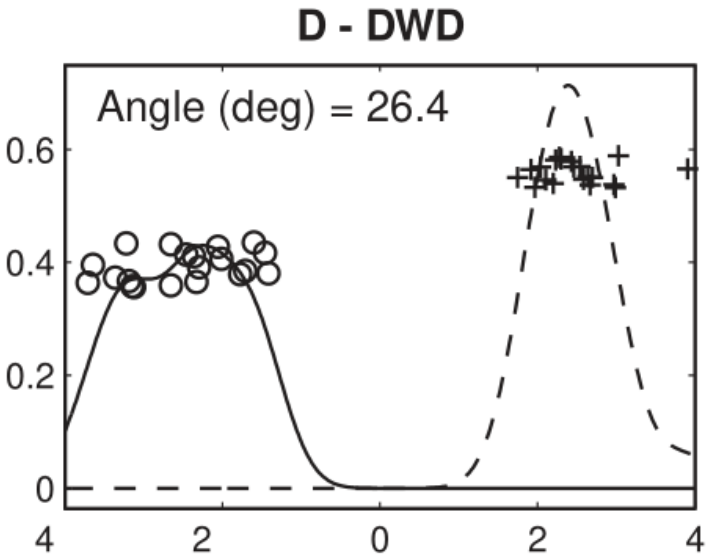
$$\min_{\omega_0,\boldsymbol{\omega}} \quad \left[\sum_{i=1}^{n} \frac{1}{d_i} + c\sum_{i=1}^{n} \eta_i\right],$$

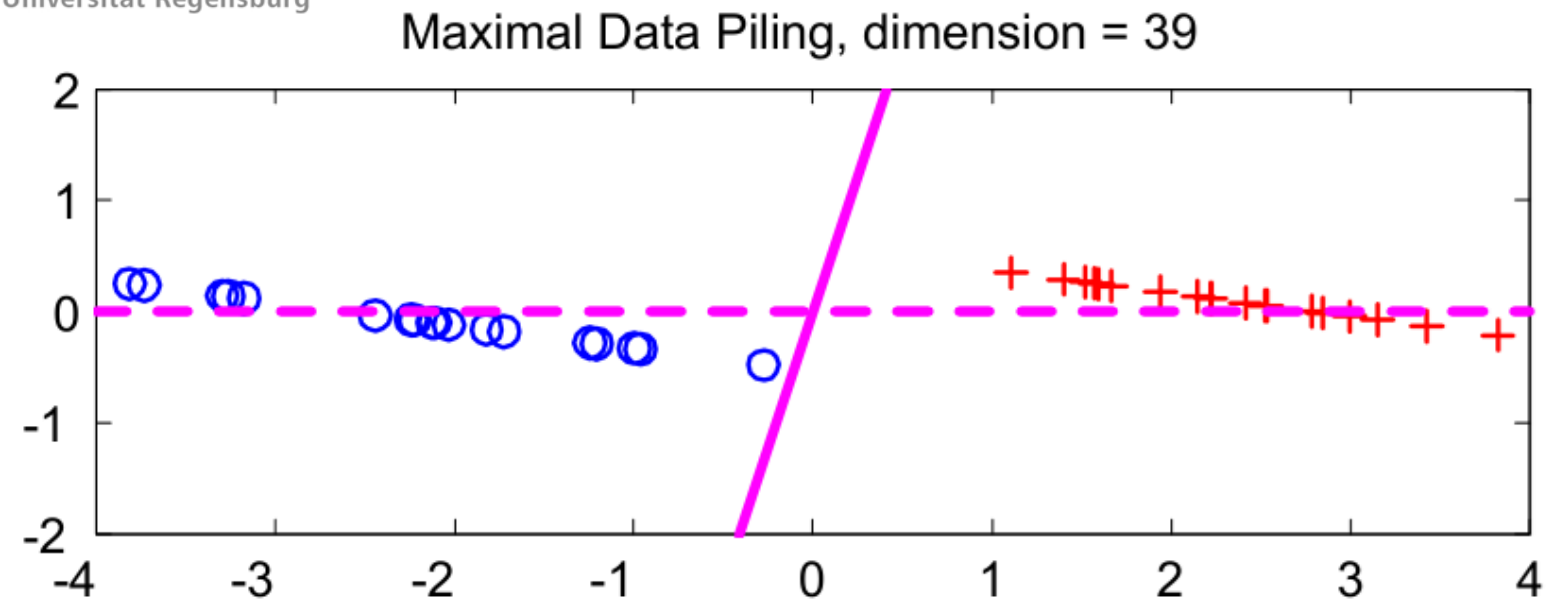
$$\text{subject to} \quad d_i = y_i(\omega_0 + \boldsymbol{x}_i^T\boldsymbol{\omega}) + \eta_i \geq 0,\ \eta_i \geq 0,\ \forall i,\ \text{and}\ \boldsymbol{\omega}^T\boldsymbol{\omega} = 1.$$

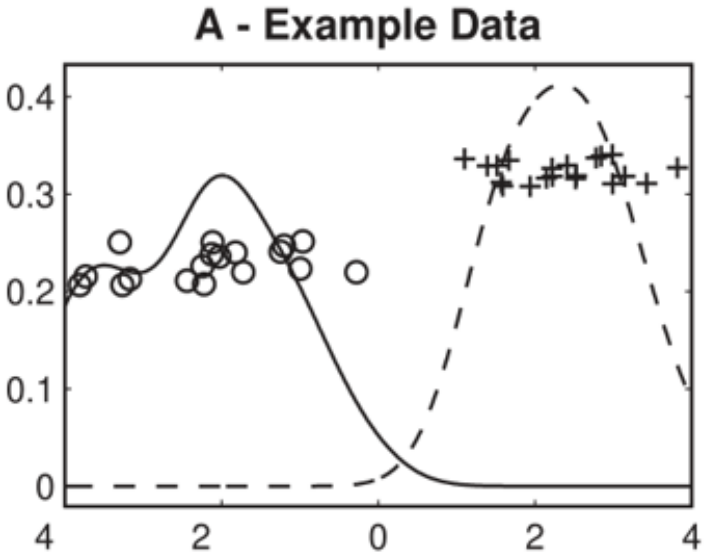

A - Example Data
0.4
0.3
0.2
0.1
0
4
2
0
2
4

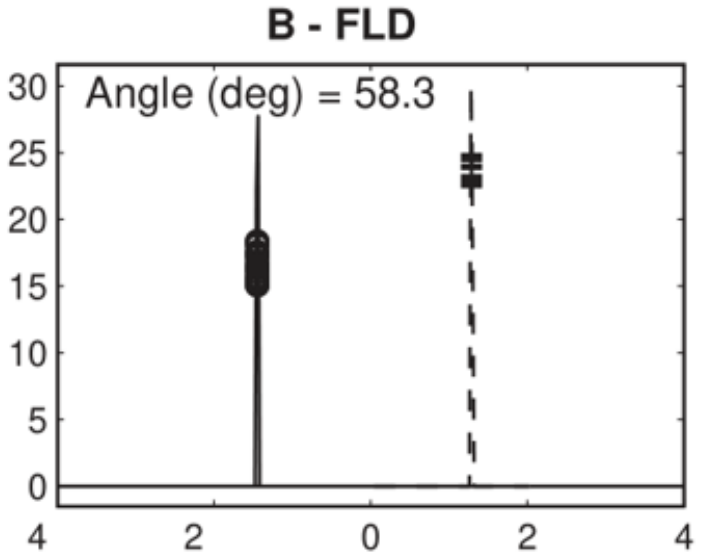

B - FLD
Angle (deg) = 58.3
30
25
20
15
10
5
0
4
2
0
2
4


C - SVM
Angle (deg) = 35.8
3
2.5
2
1.5
1
0.5
0
4
2
0
2
4

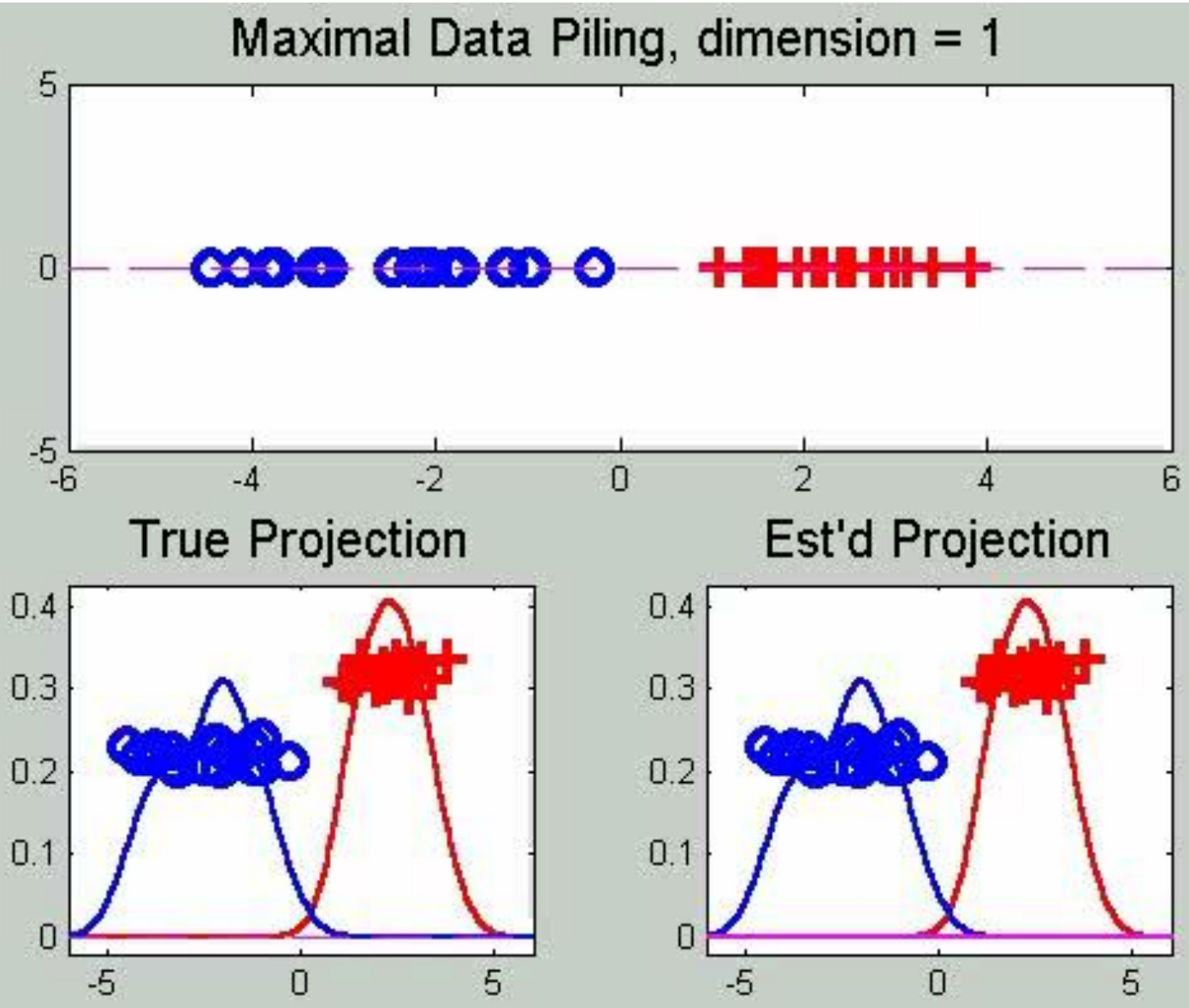

D - DWD
Angle (deg) = 26.4
0.6
0.4
0.2
0
4
2
0
2
4

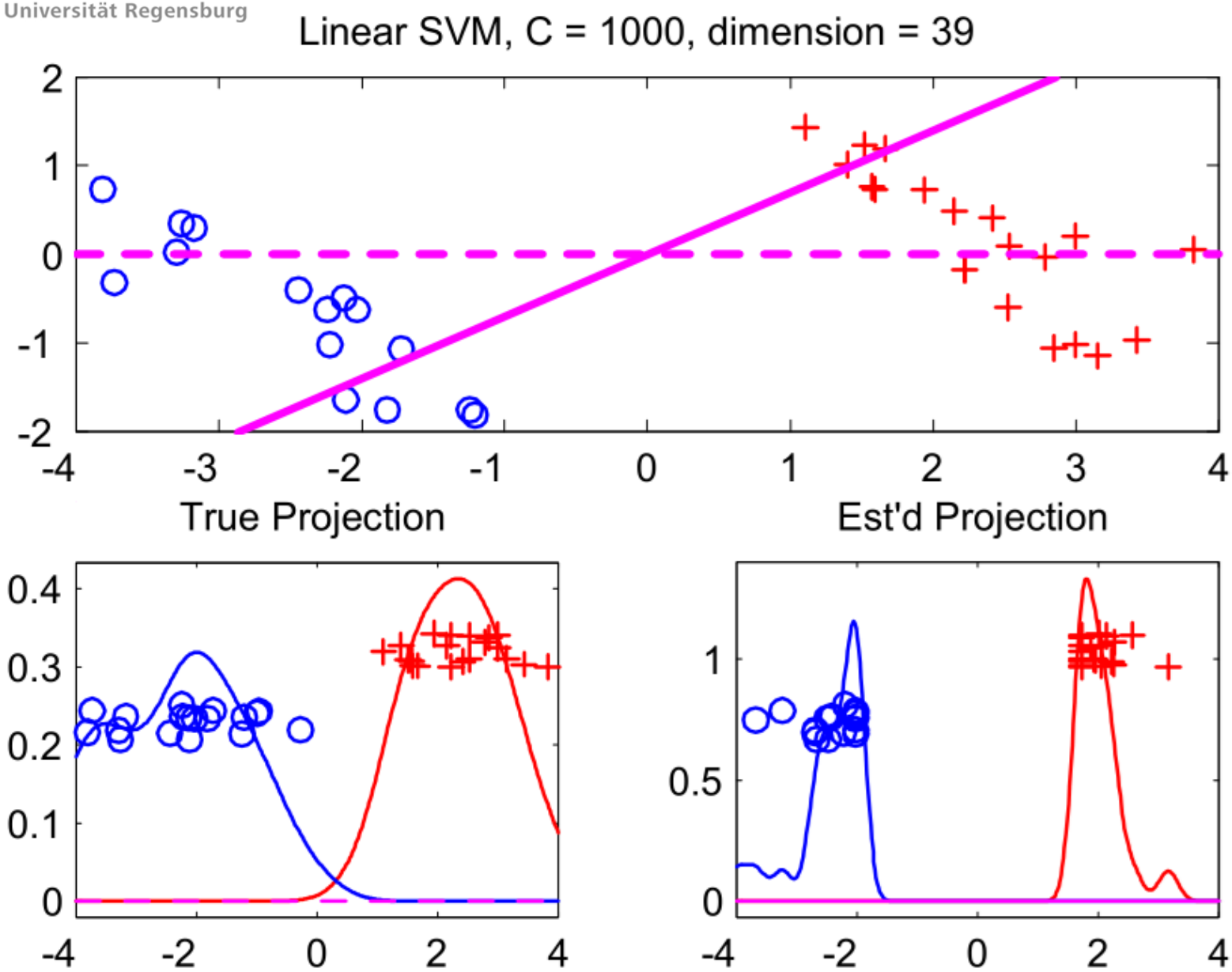

Maximal Data Piling, dimension = 39
2
1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4


A - Example Data
0.4
0.3
0.2
0.1
0
4
2
0
2
4

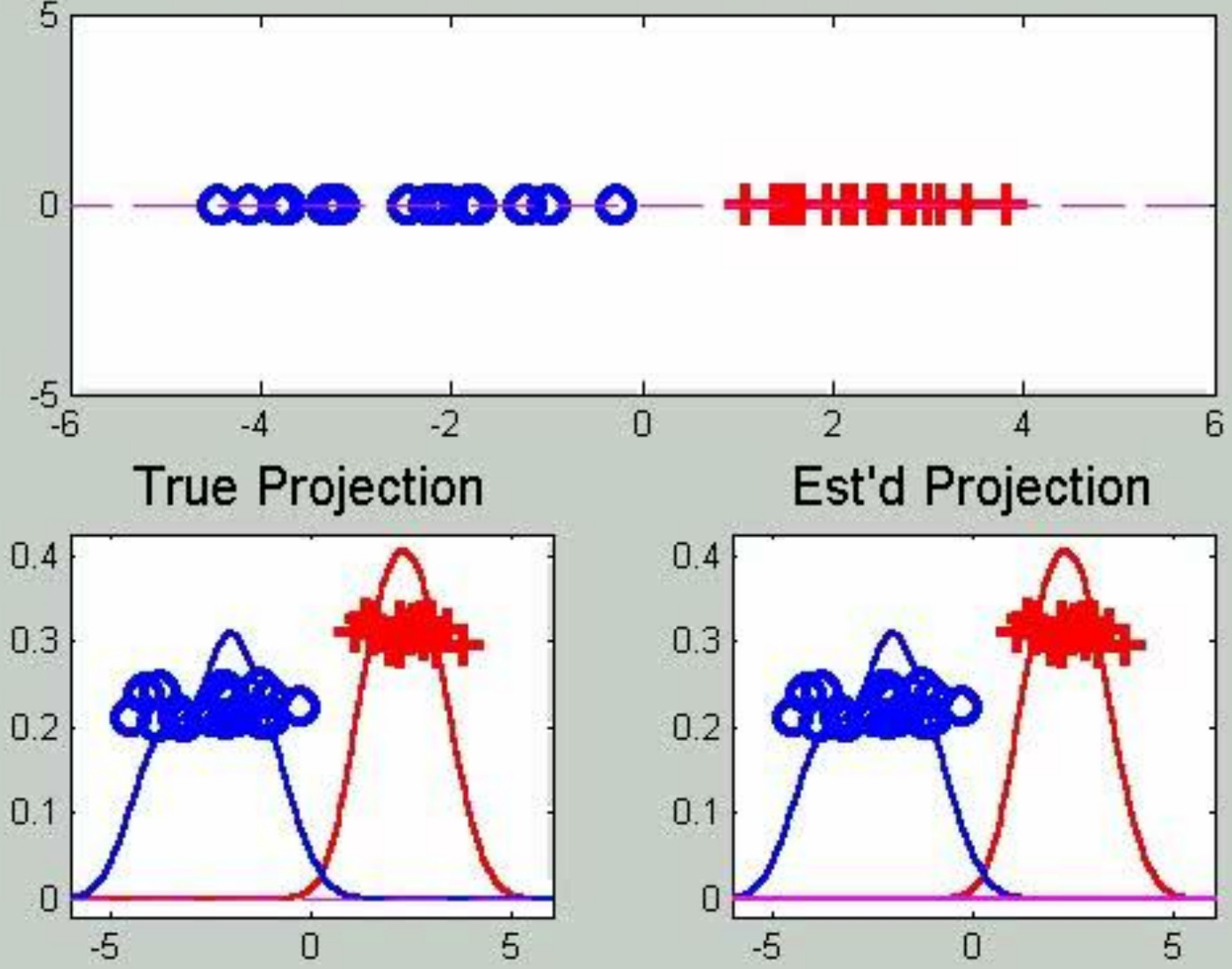

B - FLD
Angle (deg) = 58.3
30
25
20
15
10
5
0
4
2
0
2
4

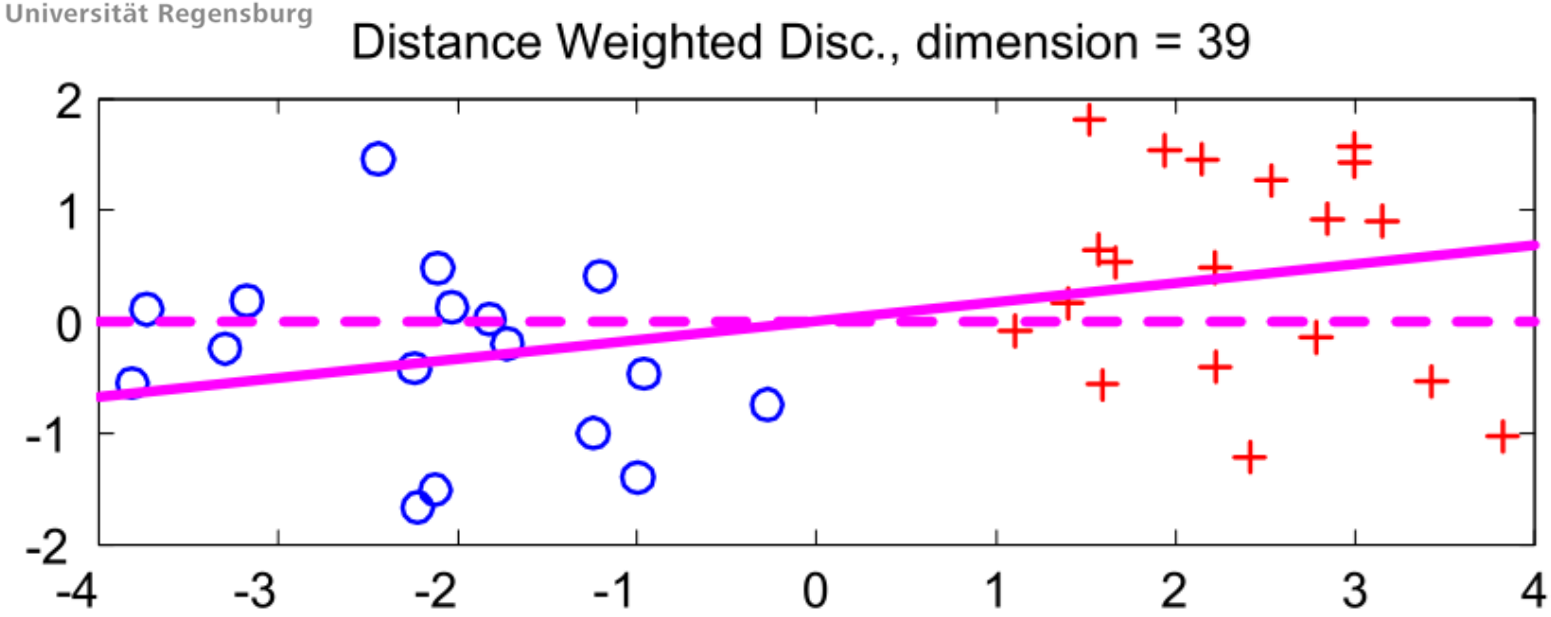
clip available at: http://www.marronwebfiles.sites.oasis.unc.edu/OldSoftware/HDD/DWD/ **-> e.g. DWD1figMDP.avi**

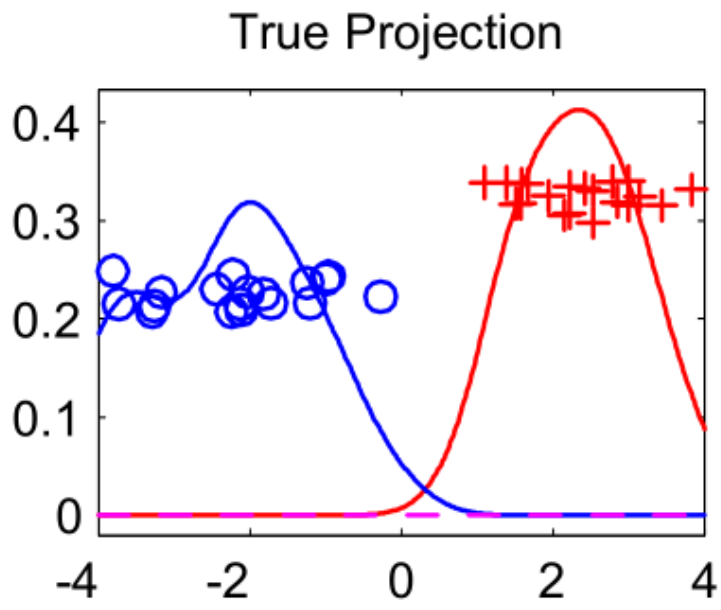


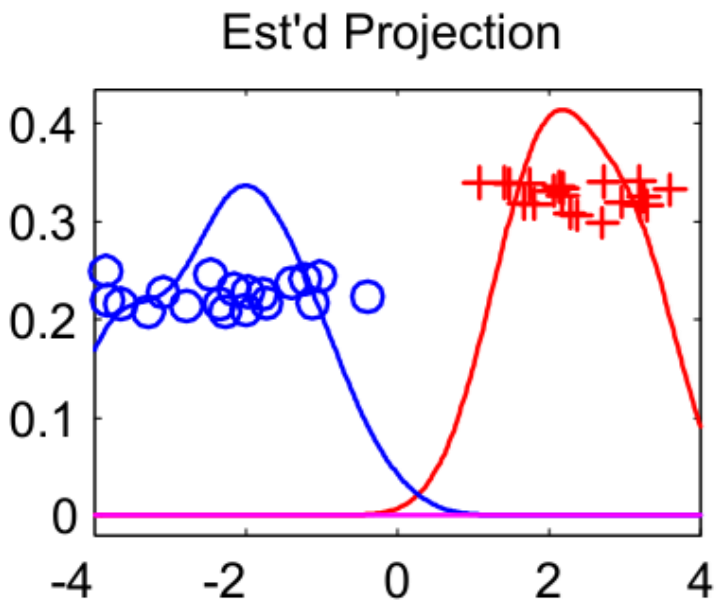

Linear SVM, C = 1000, dimension = 39
2
1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4
True Projection
Est'd Projection
0.4
0.3
0.2
0.1
0
-4
-2
0
2
4
1
0.5
0
-4
-2
0
2
4

clip available at: http://www.marronwebfiles.sites.oasis.unc.edu/OldSoftware/HDD/DWD/ **-> e.g. DWD1figSVM.avi**

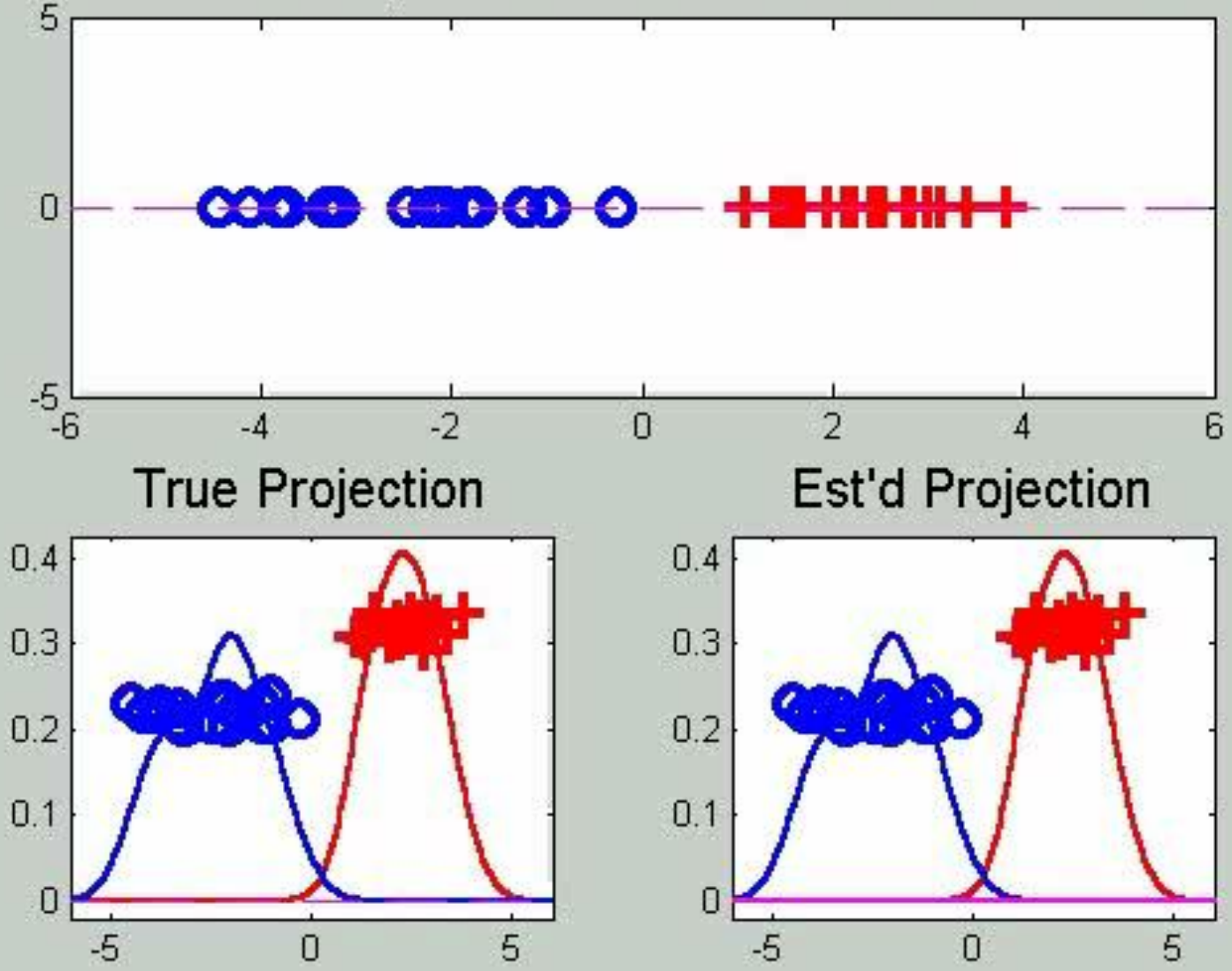


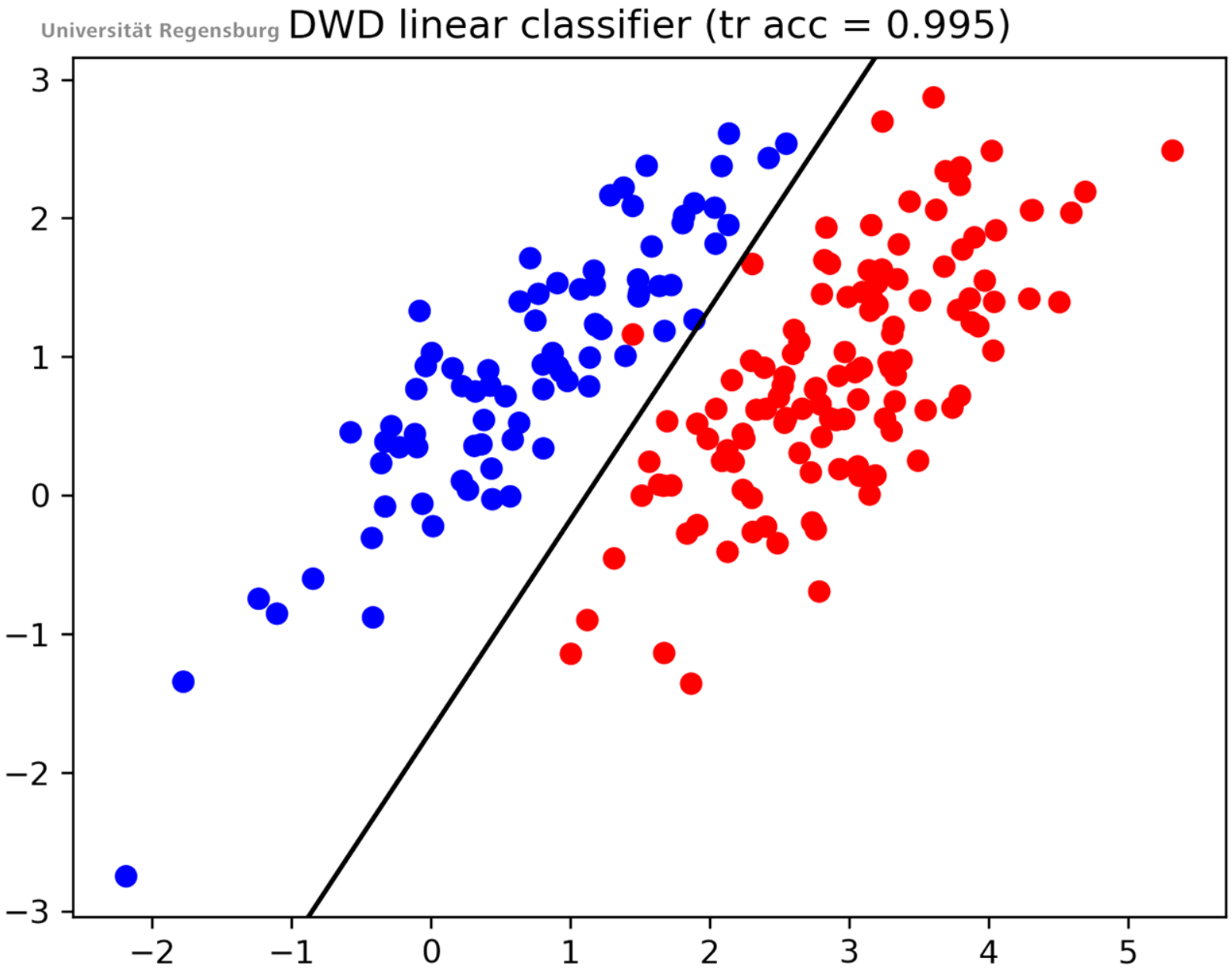

Distance Weighted Disc., dimension = 39
2
1
0
-1
-2
-4
-3
-2
-1
0
1
2
3
4


True Projection
0.4
0.3
0.2
0.1
0
-4
-2
0
2
4

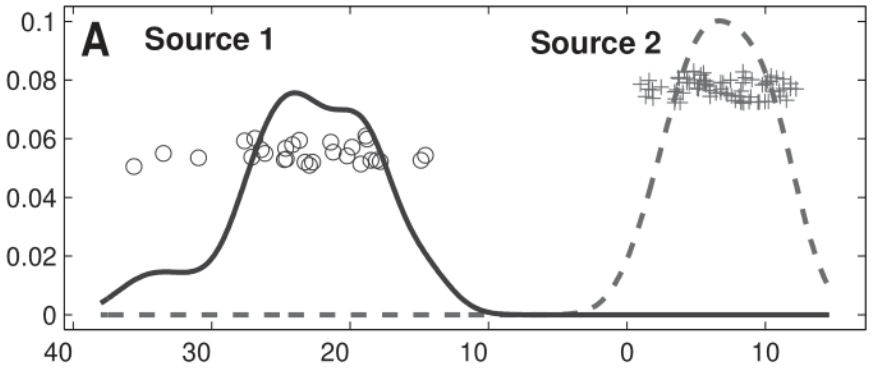

Est'd Projection
0.4
0.3
0.2
0.1
0
-4
-2
0
2
4

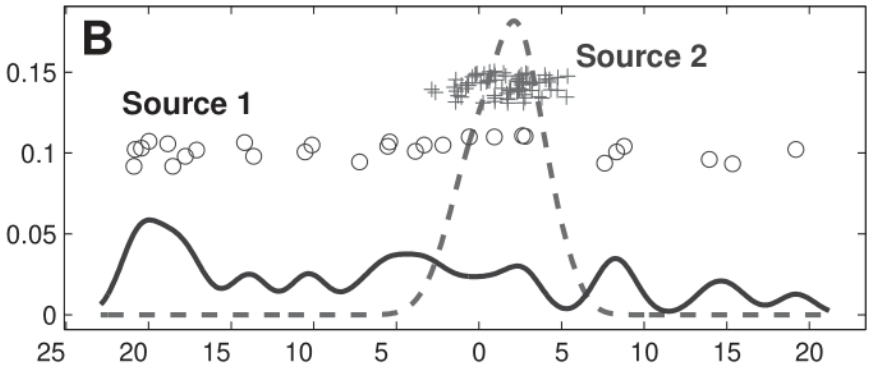
clip available at: http://www.marronwebfiles.sites.oasis.unc.edu/OldSoftware/HDD/DWD/ **-> e.g. DWD1figDWD.avi**



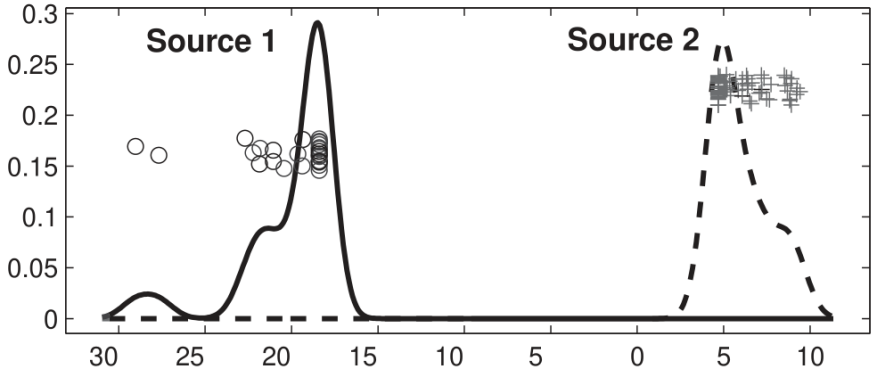

DWD linear classifier (tr acc = 0.995)
3
2
1
0
−1
−2
−3
−2
−1
0
1
2
3
4
5

DWD

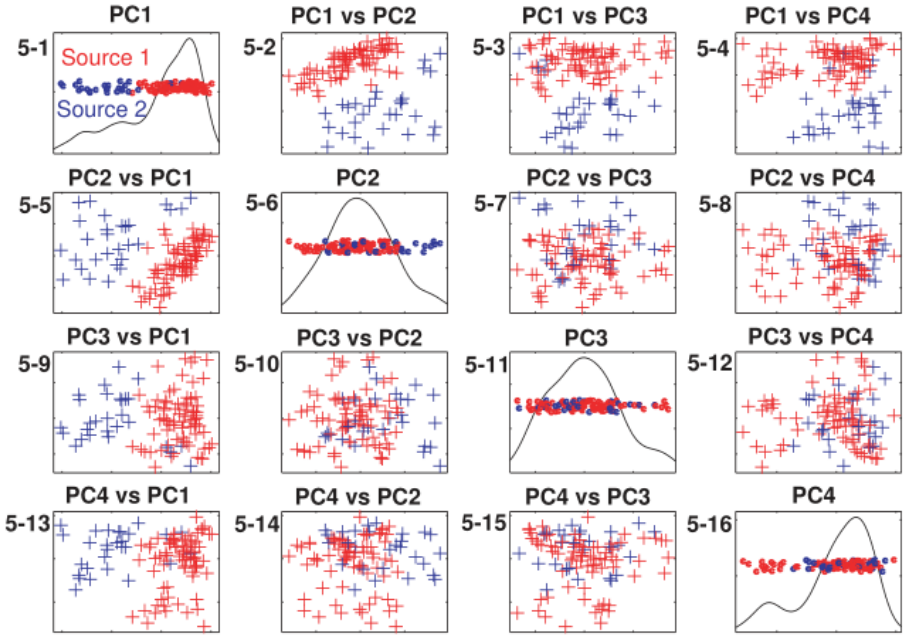

A
Source 1
Source 2
0.1
0.08
0.06
0.04
0.02
0
40
30
20
10
0
10


B
Source 2
Source 1
0.15
0.1
0.05
0
25
20
15
10
5
0
5
10
15
20

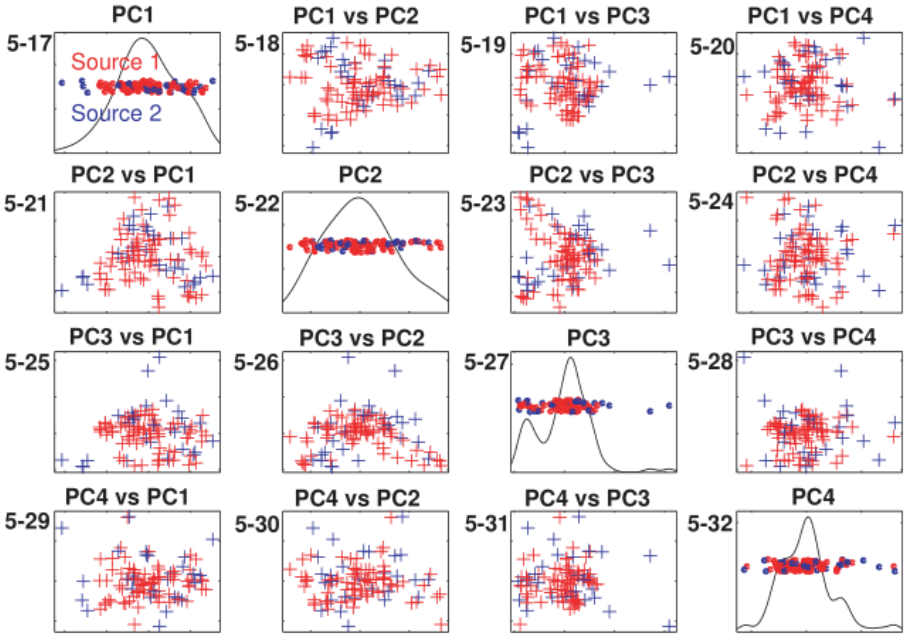
SVM


Source 1
Source 2
0.3
0.25
0.2
0.15
0.1
0.05
0
30
25
20
15
10
5
0
5
10

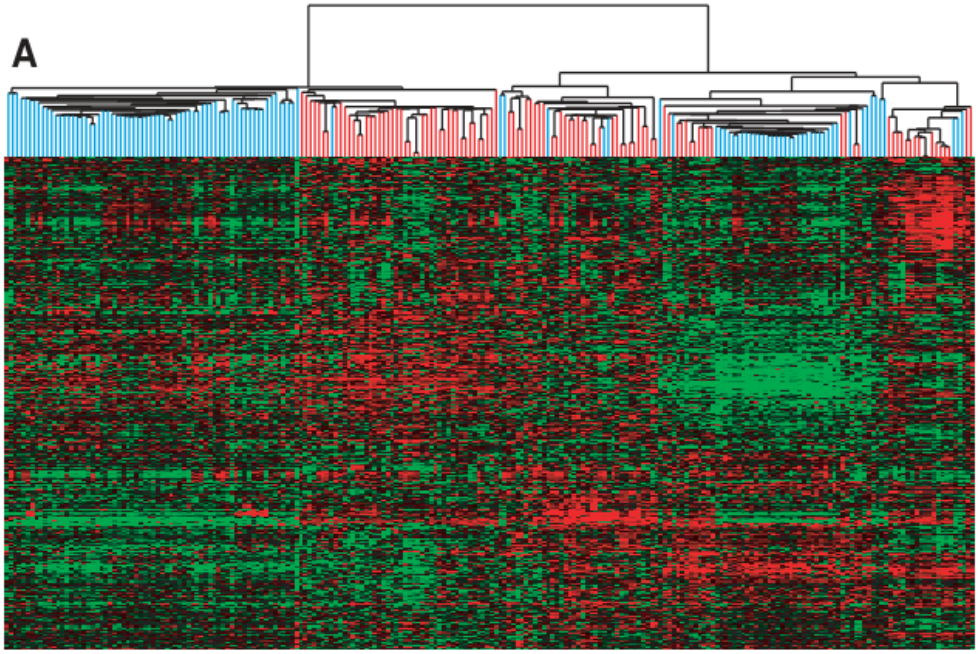
before DWD



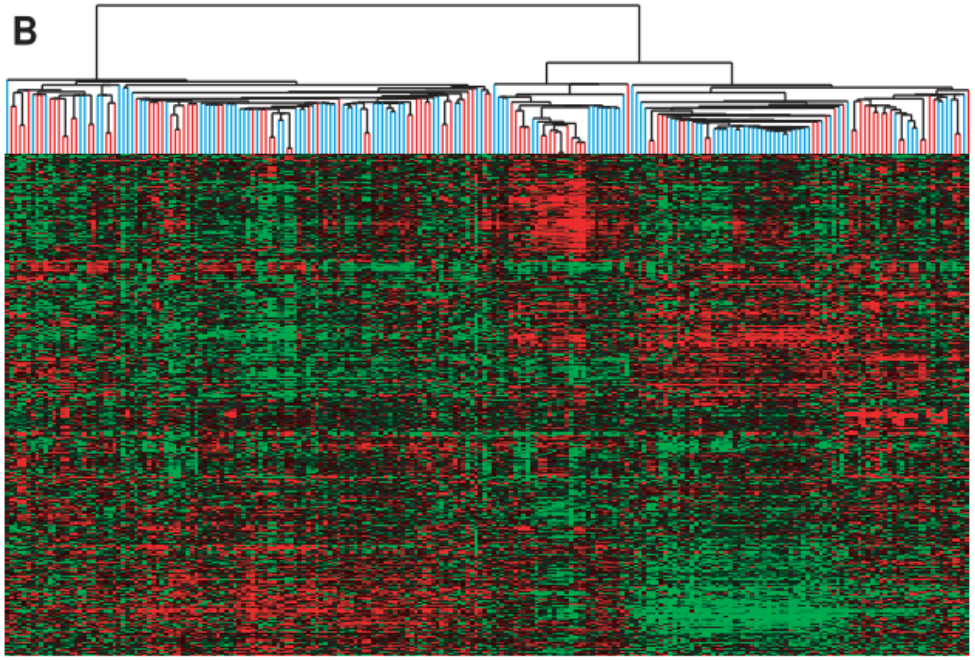
after DWD

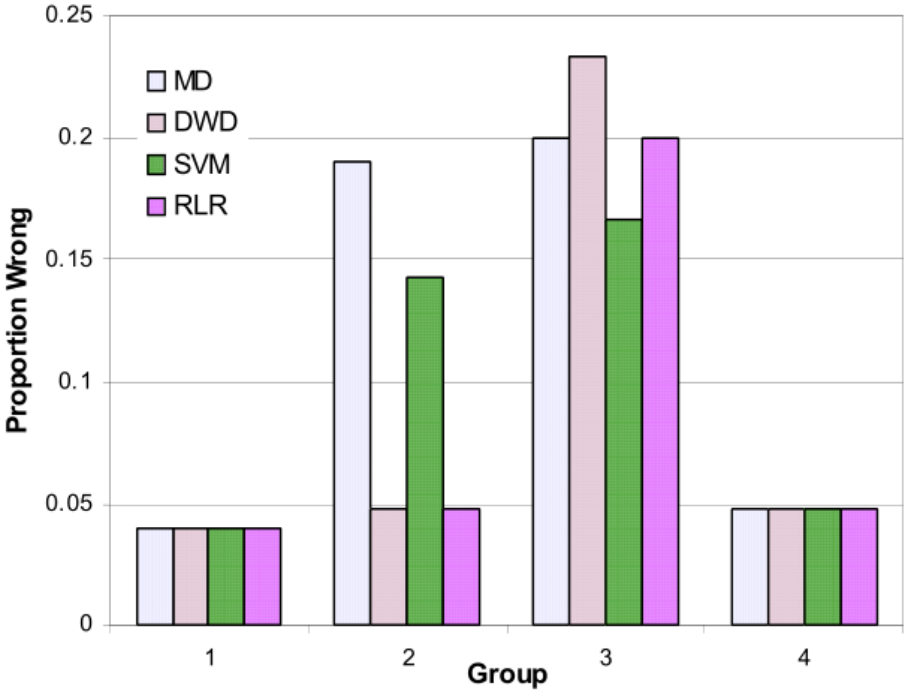


before DWD



after DWD





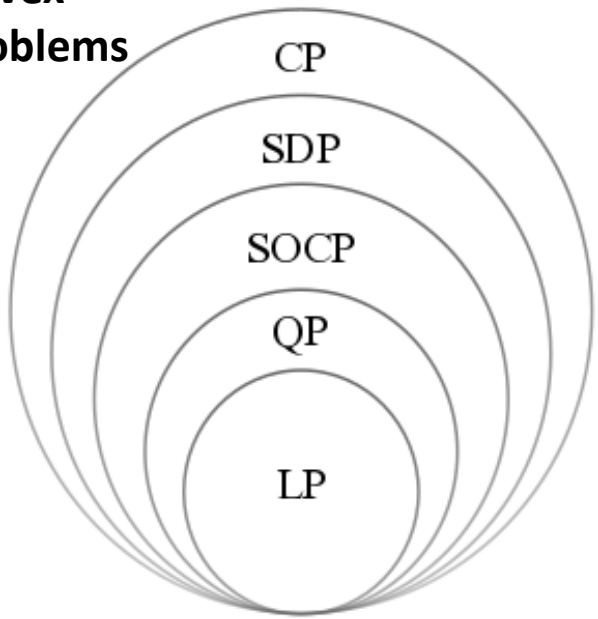
(Group 1) Luminal vs. other cancer types

(Group 2) Luminal A vs. Luminal B & C types

(Group 3) Normal vs. Erb & Basal cancer types

(Group 4) Erb vs. Basal cancer types

hierarchy of convex optimization problems



Thank you!