

Problem 3.2.

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a) • recursive function

recval $f: \text{int} \rightarrow \text{int} = (x: \text{int}) \rightarrow F(x),$
val main: int = f(1)

$F(x) := \text{if } (x \leq 0) \{ 1 \}$
 $\text{else } \{ x \cdot f(x-1) \}$

• while loop

val $f: \text{int} \rightarrow \text{int} = (x: \text{int}) \rightarrow F(x),$
val main: int = f(1)

$F(x) := \{$
 var result = 1,
 while (x > 0) {
 result = result * x,
 x = x - 1 }
 result
}

b) • $f(1) \rightsquigarrow ((x: \text{int}) \rightsquigarrow F(x)) \cdot 1 \rightsquigarrow F(1) \rightsquigarrow 1 \cdot \text{fact}(1-1) \rightsquigarrow 1 \cdot \text{fact}(0) \rightsquigarrow 1 \cdot F(0) \rightsquigarrow 1 \cdot 1 \rightsquigarrow 1$

• $f(1) \rightsquigarrow ((x: \text{int}) \rightsquigarrow F(x)) \cdot 1 \rightsquigarrow F(1)$

$F(1) \rightsquigarrow \text{result} = 1$

while (1 > 0) $\rightsquigarrow \{ \text{result} = 1 \cdot 1, \quad x = 1 - 1 \} \rightsquigarrow \dots$

$\dots \rightsquigarrow \text{while } (0 > 0) \rightsquigarrow \text{result} \rightsquigarrow 1$

Problem 3.3:

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$$\Gamma ::= \text{Decl}^*$$

$$\text{Decl} ::= \text{type } a = A$$

$$\text{Decl} ::= \text{type } b = B$$

→ productions to the grammar:

$$\text{Decl} ::= \text{type } u = A + B$$

→ typing rules:

$$\frac{\Gamma \vdash A : \text{type} \quad \Gamma \vdash B : \text{type}}{\Gamma \vdash A + B : \text{type}}$$

→ evaluation rules:

$$\frac{\Gamma \vdash \text{inj}_1(a:A) \rightarrow u:A+B; \quad \Gamma \vdash \text{inj}_2(b:B) \rightarrow v:A+B; \quad \Gamma \vdash f(a:A) \rightarrow c:C; \quad \Gamma \vdash g(b:B) \rightarrow c:C}{\text{cases}(u, f, g) \rightarrow f(a); \quad \text{cases}(v, f, g) \rightarrow g(b)}$$