1) GF(13)

a) $(7x^4 + 3x^3 + x^2 + 10) - (9x^4 + 6x^3 + 7x^2 + 8x + 2)$

$= -2x^4 - 3x^3 - 6x^2 - 8x + 8$

$= (-2 \bmod 13)x^4 + (-3 \bmod 13)x^3 + (-6 \bmod 13)x^2 + (-8 \bmod 13)x + (8 \bmod 13)$

$= \boxed{11x^4 + 10x^3 + 7x^2 + 5x + 8}$

b) $(7x^3 + 2x + 9) \times (2x^3 + x^2 + 8x + 7)$

$= 14x^6 + 7x^5 + 56x^4 + 49x^3 + 4x^4 + 2x^3 + 16x^2 + 14x + 18x^3 + 9x^2 + 72x + 63$

$= 14x^6 + 7x^5 + 60x^4 + 69x^3 + 25x^2 + 86x + 63$

$= \boxed{x^6 + 7x^5 + 8x^4 + 4x^3 + 12x^2 + 5x + 11}$

c) $\dfrac{12x^5 + 4x^4 + 30x^3 + 12x^2 + x}{3x^3 + 4x^2 + 3}$

$$
\begin{array}{r}
4x^2 - 2x \\
3x^3 + 4x^2 + 3 \overline{\big)\, 12x^5 + 4x^4 + 30x^3 + 12x^2 + x} \\
\underline{-12x^5 + 16x^4 \qquad + 12x^2} \\
-8x^4 + 30x^3 + x
\end{array}
$$

2) x^3+x+1

a) $(x^2+x+1) \times (x+1)$

$x^3+x^2+x^2+x+x+1$

x^3+2x^2+2x+1

$\boxed{x^3+1}$

b) $(x+1) - (x^2+x+1)$

$= -x^2$

$= \boxed{x^2}$

c) $\frac{x^2+x+1}{x^2+1}$

$$\begin{array}{r|l} & 1 \\ x^2+1 & x^2+x+1 \\ & -x^2 \quad +1 \\ \hline & x \end{array} \quad = \boxed{1 + \frac{x}{x^2+1}}$$