

Quantitative Finance – Home Assignment 2

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In the German market various index certificates are traded. From a legal perspective a certificate is a bond which guarantees a certain stream of payments to its holders. In the standard case (straight bond) 100% of the notional value is paid at the maturity of the bond and there are fixed annual coupon payments. For an index certificate the payment at maturity and/or the coupon payments depend on the development of the DAX.

This assignment is based on the following framework:

- All instruments are traded in a frictionless market with a continuously compounded riskless money market account with an annualized rate of 3%.
- The starting value of the DAX is 23,500.
- The DAX follows a Geometric Brownian Motion with a drift rate of 8.7% under the physical measure and a volatility given below in table 1.
- Each certificate has a maturity of five years and pays annual coupons (at the end of years 1, 2, 3, 4, and 5).

Task 1: Compute the value of a European Call option with a maturity of five years written on the DAX with a strike price of 25,000 using a Monte Carlo simulation with 60 time steps, i.e., $\frac{T}{M} = \frac{5}{60}$.

Task 2: Compute the arbitrage-free prices of the assigned certificate described below using a Monte Carlo simulation with 60 time steps, i.e., $\frac{T}{M} = \frac{5}{60}$.

The Monte Carlo simulation should make use of at least 10,000 runs with antithetic random numbers (i.e., 5,000 original random numbers plus 5,000 antithetics).

Each student is assigned to compute the price of one specific certificate (see table 1).

Certificate 1

Variance certificate

Coupon: Annual fix (values can be found in table 1)

Terminal payment: The terminal payoff equals the square-root of the difference between the maximum and the minimum value of the DAX until maturity, divided by 100.

Certificate 2

Bonus certificate

Coupon: The coupons C at dates t are 10% if the DAX is above a lower boundary (values can be found in table 1) and zero otherwise.

Terminal payment: 100%

Certificate 3**Knock-out certificate**

Coupon: Annual fix (values can be found in table 1)

Terminal payment: Zero if the value of the DAX is higher than a certain barrier (values can be found in table 1) at one of the coupon dates and otherwise it is the payoff of an European call option on the DAX with strike price of 28,000.

Certificate 4**Asian variance certificate**

Coupon: Annual fix (values can be found in table 1)

Terminal payment: The terminal payment is the difference between the maximum value of the DAX and its average value until maturity, divided by 100.

Table 1: Individual assignments

Student ID	Certificate	Coupon	Bonus boundary	Barrier	Volatility
12234772	1	5			14%
12319124	2	n.a.	30000		15%
12321013	3	2		29500	16%
12217747	4	4			17%
12312460	1	6			18%
12321078	2	n.a.	28000		14%
12219257	3	2.5		28500	15%
12500180	4	4.5			16%
12313572	1	7			17%
12220513	2	n.a.	26000		18%
12500107	3	3		27500	14%
12235839	4	5			15%
12306407	1	8			16%
12140322	2	n.a.	24000		17%
12316295	3	3.5		26500	18%
12500736	4	5.5			14%
12312432	1	9			15%
12500603	2	n.a.	22000		16%
12316432	3	4		25500	17%
12329171	4	6			18%