ELECTROMAGNETISM

Base Units

Charge (q) – Coulombs (C) or Amp Seconds (As)
Electric Field ($\vec{E}$) – Newtons per Coulomb (NC^{-1}) **or** Volts per Meter (Vm^{-1})
Magnetic Field ($\vec{B}$) – Tesla (T)
Magnetic Flux (ϕ_B) – Weber (Wb) **or** Tesla Square Metres (Tm^2)
Electric Potential (U_E) – Joules (J) **or** Volt Coulombs (VC)
Voltage (V) – Volts (V) **or** Joules per Coulomb (JC^{-1})
Current ($\vec{I}$) – Amperes (A) **or** Coulombs per Second (Cs^{-1})

Constants

Permittivity of Free Space $\varepsilon_0 = 8.854 \times 10^{-12}\ (A^2s^4kg^{-1}m^{-3})$
Permeability of Free Space $\mu_0 = 4\pi \times 10^{-7}\ (NA^{-2})$

Mass of an Electron $m_e = 9.109 \times 10^{-31}\ (kg)$
Mass of a Proton $m_p = 1.673 \times 10^{-27}\ (kg)$
Mass of a Neutron $m_n = 1.675 \times 10^{-27}\ (kg)$

Charge of an Electron $q_e = -1.602 \times 10^{-19}\ (C)$
Charge of a Proton $q_e = +1.602 \times 10^{-19}\ (C)$

Equations

Electrostatics

$$\vec{F} = q\vec{E}$$

The Force on a charged particle due to an electric field. The direction is given by the direction of the field however if the charge is negative, it experiences a force in the opposite direction to the direction of the field.

$$E = \frac{V}{d}$$

The formula for Electric field as a function of the scalar voltage field.

$$W = qV = q\vec{E} \cdot \vec{d} = qE_{\parallel}d$$

Work done on a charged particle in an electric field is the change in electric potential energy.

$$E_1 = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r^2}$$

The electric field at a radius due to a charged particle q_1.

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

The magnitude of force on a pair of charge particles at a radius (Coulomb's Law).

Circuits

$$V = IR$$

Ohm's Law

$$I = \frac{q}{t}$$

Current is the number of charges passing a point per second.

$$P = IV = I^2 R = \frac{V^2}{R}$$

The power output in a circuit is the number of charges passing per second (current) multiplied by the energy lost by each particle across a component in the circuit (voltage).

Magnetism

$$B = \frac{\mu_0 I}{2\pi r}$$

The magnitude of the electric field at a perpendicular radius from a current carrying conductor.

$$B = \mu_0 I \frac{N}{L}$$

The magnitude of the magnetic field inside a solenoid with N turns and length L.

$$\vec{F} = q\vec{v} \times \vec{B} = q\vec{v}_{\perp}\vec{B} = qvB\sin\theta$$

The force on a charged particle moving through a magnetic field. Due to the cross product, the force is always perpendicular to the velocity and, as such, will always induce some form of circular motion.

$$\vec{F} = l\vec{I} \times \vec{B} = l\vec{I}_{\perp}\vec{B} = lIB\sin\theta$$

The Motor Effect: The force on a current carrying conductor of length l *in a magnetic field.*

$$\frac{F}{l} = \frac{\mu_0}{2\pi} \cdot \frac{I_1 I_2}{r}$$

The force per length on a pair of current carrying conductors. Note that for net force, l *is the length of the two wires which are parallel (the common length).*

$\phi_B = \vec{B} \cdot \vec{A} = \vec{B}_{\parallel}\vec{A} = BA\cos\theta$

Magnetic Flux through an area. Note that $\vec{A}$ *denotes the area vector which is at* $90°$ *to the surface.*

Electromagnetism

$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

The net force on a charged particle due to both a magnetic and electric field.

$$\varepsilon = -N\frac{\Delta\phi_B}{\Delta t}$$

E.M.F. induced in a coil with N *turns (Faraday-Lenz Law).*

$\vec{\tau} = NI\vec{A} \times \vec{B} = NI\vec{A}_{\perp}\vec{B} = NIAB\sin\theta$

The torque on a rectangular current carrying coil with N turns in a magnetic field.

Transformers

$$\frac{V_P}{N_P} = \frac{V_S}{N_S}$$

Voltage per turn ratio for a transformer is constant.

$V_P I_P = V_S I_S$

The power output for the primary and secondary coils in an ideal transformer is the same.

Extension Equations

Electrostatics

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}\hat{r}$$

The force on a charged particle due to another charged particle. $\hat{r}$ denotes the unit radius vector. To find the force on particle q_1 the radius begins at particle q_2 and points to particle q_1.

$$W_E = \int_A^B \vec{F}_E \cdot d\vec{r} = q\int_A^B \vec{E} \cdot d\vec{r}$$

$$V = \frac{W_E}{q} = \int_A^B \vec{E} \cdot d\vec{r}$$

Integral form of work done on a charged particle in an electric field and Voltage. Note that the formulae use the dot product.

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Gauss' Law. The double integral with a circle denotes the fact that the integral is performed across a 3D surface in space.

$$\vec{E} = -\vec{\nabla}\mathrm{V}$$

The formula for electric field as a function of a voltage field.

Magnetism

$$\oiint \vec{B} \cdot d\vec{A} = 0$$

Due to there being no magnetic monopoles, the magnetic flux through any 3D surface is zero.

$$\phi_B = \oint \vec{B} \cdot d\vec{A}$$

$$\vec{\phi}_B = \hat{A} \oint \vec{B} \cdot d\vec{A}$$

Magnetic Flux through any 2D area outlined by some perimeter. Note that the area of a coil is not the physical area but rather the area the coil forms the perimeter of.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

Ampere's Law: The current through a conductor is proportional to the magnetic field around it.
(This equation only applies to current carrying conductors and an extra term was added by Maxwell to account for the displacement current between capacitors)

$$\vec{\mu} = nI\vec{A}$$

Magnetic dipole moment of a current carrying coil. The direction obeys the right-hand rule with respect to current.

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$U = -\vec{\mu} \cdot \vec{B}$$

Torque and potential energy for a magnetic dipole in a magnetic field.

Electromagnetism

$$\varepsilon = \int_0^l (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{l}$$

E.M.F. is the same as the net work done per charge.

$$\varepsilon = -\frac{\partial}{\partial t}\oint \vec{B} \cdot d\vec{A} = -\frac{\partial \phi_B}{\partial t}$$

E.M.F. induced due to a change in flux through the area outlined by a loop.

$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

The E.M.F. generated in a current carrying conductor, in general when moving through a magnetic field.

$$\vec{I} = \frac{dQ}{dt}$$

Differential form for current.

Maxwell-Heaviside Equations

Integral Form	**Vector Form**
$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$	$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$
$\oiint \vec{B} \cdot d\vec{A} = 0$	$\vec{\nabla} \cdot \vec{B} = \vec{0}$
$\oint \vec{E} \cdot d\vec{l} = -\frac{\partial \phi_B}{\partial t}$	$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t}$	$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$

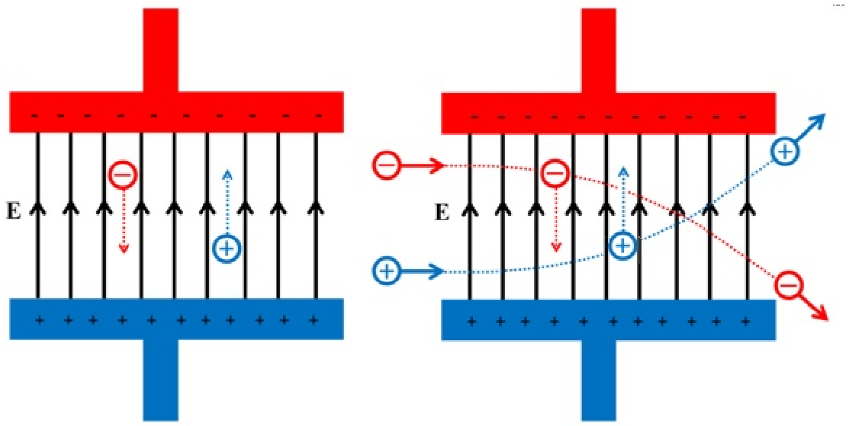
*$\rho = \frac{dq}{dV}$ (charge per volume), $J = \frac{dI}{dA}$ (current per area)

*See Appendices for notes on Del (∇) notation

Course Notes

Charges Moving in an Electric Field

In these types of problems, the same logic from projectile motion is required but instead of gravity acting as the acceleration, it is the electric field. As such, the problem should be broken up such that the y-direction is parallel to the field and the x-direction is perpendicular to the field.



Voltage

Voltage takes many forms in electromagnetism and can often be difficult to conceptualise. Voltage is often considered the 'pushing force' in electrical circuits but, really, it is a way of describing the potential for charges in a field to do work. This work done is why the voltage decreases across components of a circuit, the work was done therefore the potential to do work decreases.

Put simply, Voltage is the measure of the potential of the Electric field and can also be considered the potential energy per charge due to a field. As a result:

$$E = \frac{V}{d}$$

The Magnetic Field

The Magnetic Field is an interesting field to consider as it only exists in certain reference frames (it is fictitious). By performing a Lorentz transform on charged particle moving next to a current carrying conductor, you will find that there is a force on it due to an increased density of charges, caused by length contraction within the wire. The same is not true for a permanent magnet as the magnetism of atoms is due to the magnetic moments of the electrons in the shells.

Charges Moving in a Magnetic Field

For charges to experience a force in a magnetic field, they must be moving. This is due to the effects of special relativity and you can find more details on this on the Veritasium YouTube channel: (https://www.youtube.com/watch?v=1TKSfAkWWN0). This also means that the magnetic force is fictitious, as in, it only exists in certain frames of reference.

The force, because it is a result of the cross product of velocity and the field, will always result in a force which is perpendicular to both the field and velocity, the direction is given by the right-hand rule. As a result of it being perpendicular to the velocity, it can never produce energy (the force can never do work) and will always result in circular motion. (i.e. $\vec{F}_B = \frac{mv^2}{r}$)

Flux

There are two forms of flux in electromagnetism, electric and magnetic. However, the HSC course is only concerned with **magnetic flux** (ϕ_B). Flux is a measure of the amount of field which passes through the defined area and, as such, can be defined as the field which is perpendicular to the area, or the field which is parallel to the area vector.

The way of making sense of this is imagining you are looking at a piece of paper on a table from above. Now you begin to look at the piece of paper from a lower angle, slowly beginning to look at it so you can only see the very thin edge. The apparent decrease in size of the paper is the same as *effective area* and is analogous to amount of area a magnetic field can pass through.

The Area Vector ($\vec{A}$)

The area vector for any 2D surface has the magnitude of the area, and points perpendicular to the surface. Which side the vector points in is arbitrary, but it is important the definition remains consistent in the scenario i.e. if it is defined as pointing up at the start of a rotation, after half a revolution, it should be pointed down.

Induction

Induction is the process through which a Voltage or E.M.F. is induced across a circuit. This can happen due to two different effects:

1) A conductor moving in a magnetic field
2) A changing magnetic flux through a conductive loop.

In the first form of induction, the E.M.F. is produced by the force on each of the charged particles as the conductor moves. Because a conductor is made of protons and electrons, as it moves through the magnetic field, the charges experience a force. The E.M.F. generated is the work done by the force per charge and, for a rod, is given by the equation (where θ is the angle between v and B):

$$\varepsilon = vlB \sin\theta$$

In the second form of induction, it is slightly more abstract as to what causes the voltage as it is a **change in magnetic flux**. Due to Maxwell's equations defining flux through a 3D object as zero, the flux must instead be through a 2D surface. As a result, the area which the flux is generated by is the area traced by a conductive coil and the E.M.F. can be defined as follows for a coil with N turns:

$$\varepsilon = -N\frac{\Delta\phi_B}{\Delta t}$$

The change in flux can be generated in a few ways:

- Turning on/off a magnetic field source
- Moving a magnet or solenoid through the coil
- Moving the coil away/towards a magnet

Why does this effect exist?

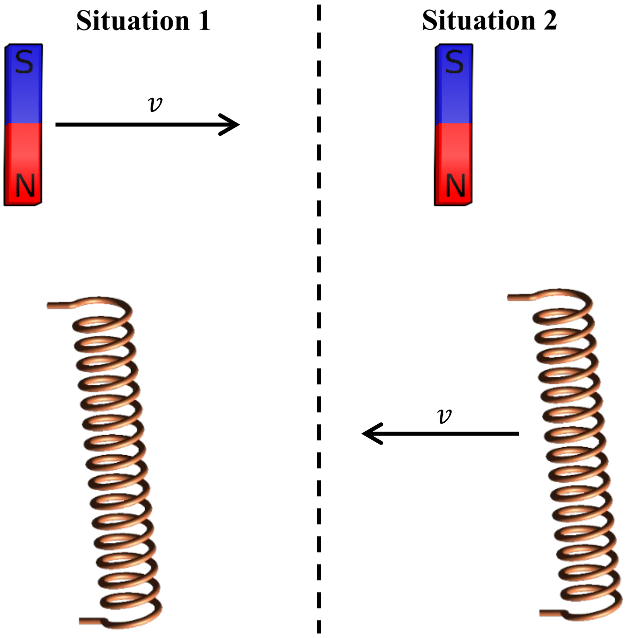
It's not entirely clear why this effect occurs, even to experienced physicists. However, there is a more fundamental law which may help to explain where this comes from.

A Note on the Difference Between Relative Motion and Changing Flux

Many people like to speculate that movement of a magnet across a coil is equivalent to movement of the coil across a magnet. Although these both result in the same phenomenon, they are not mathematically equivalent (i.e. situation 1 and 2 below are equivalent).

Many people like to say that when the coil is moved that the electrons in the coil are moving in a magnetic field and the magnet being moved across the coil can be treated as such. This is not completely true. If you shift into the coil's frame of reference, then yes this is true, and you can achieve the same result by doing so however in the lab frame only the magnet is moving. However, in the lab frame the magnetic field's value at each point in space is changing but the field itself cannot 'move' and as such, treating it this way from the lab's frame of reference is incorrect.

If you are unconvinced, try to prove that these two equations are, in the general case, equivalent as this is what is required to show that the two frames of references are mathematically equivalent. (1 is the equation which describes situation 1 and 2 for situation 2)



1

$$\varepsilon = -\frac{d\phi_B}{dt}$$

2

$$\varepsilon = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

The Motor Effect

This is the application of moving charges in a magnetic field. The motor effect is merely the notion that charges moving through a current carrying conductor will experience a force. As the charges experience a force, they will move, attracting the conductor with them. This is the mechanism through which the entire wire, not just the charges experience a force.

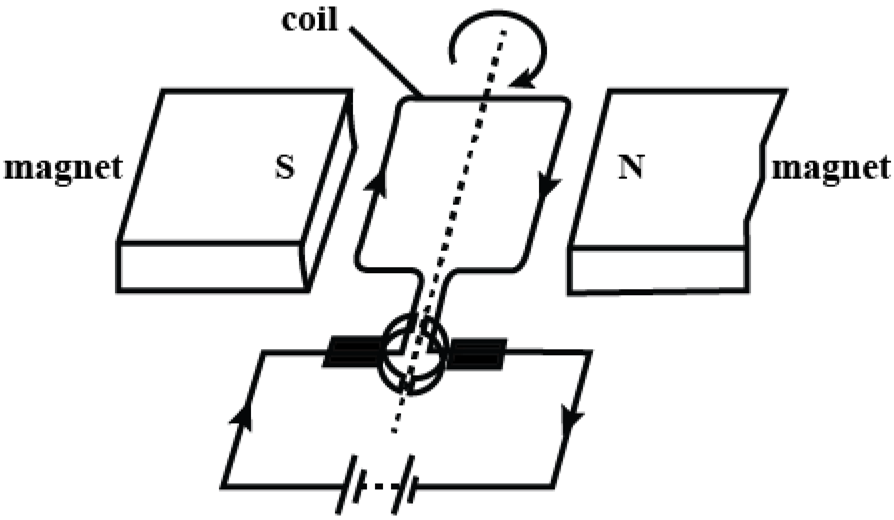
Deriving the Motor Effect

$$q\vec{v} \times \vec{B} = \frac{d}{dt} q\vec{s} \times \vec{B} = \left(\frac{dq}{dt}\vec{s} + q\frac{d\vec{s}}{dt}\right) \times \vec{B} = \left(\frac{dq}{dt}\vec{s} + \vec{0}\right) \times \vec{B} = s\overrightarrow{\frac{dq}{dt}} \times \vec{B} = l\vec{I} \times \vec{B}$$

Motors

DC Motors

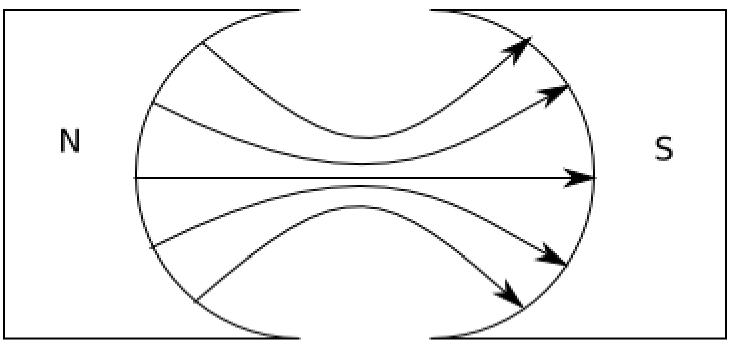
Motors are devices which take advantage of the Motor Effect to produce rotation.



There are two common questions which arise when considering the design of a motor:

1. How does one keep the torque uniform?
2. How does one keep the torque in the same direction?

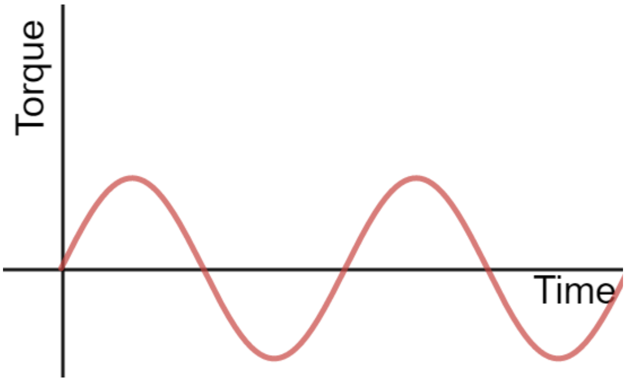
The first problem of keeping the torque at a uniform strength across the rotation has two solutions. The first way to improve this is by using curved magnets. The curved magnets produce a magnetic field which, at the outer edge, is pointed towards or away from the centre. This type of field is known as a **radial** magnetic field.



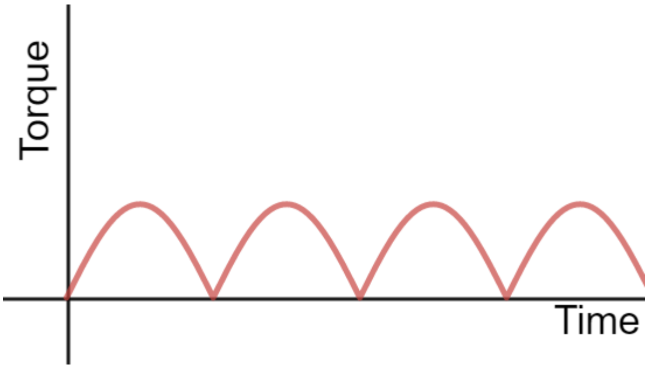
Because the field points in the same direction as the radius at the location of the arms, the angle between the force and the radius is a constant 90° at almost all times during the rotation. The end result is the torque produced is at a maximum value for all of the rotation (except when vertical).

Although this keeps the torque at a maximum for most of the rotation, the torque is not uniform due to the drop when the armature is vertical. To ensure the torque is completely uniform, there need to be more coils. If there are more coils, this drop of one armature as it reaches the vertical is counteracted by another coil at an offset. The more coils added to a motor, the more uniform the torque.

The second commonly considered problem with a motor is what happens when an arm of the armature (loop) changes from one side of the motor to the other (i.e. it becomes closer to the side with the North pole of the magnet).
This would normally result in a change to the direction of the torque on the armature due to the direction of each arm changing from slightly upwards to slightly downwards or vice versa.
This change in direction can be counteracted by a change in the current direction.



Torque where there is no change in current direction.



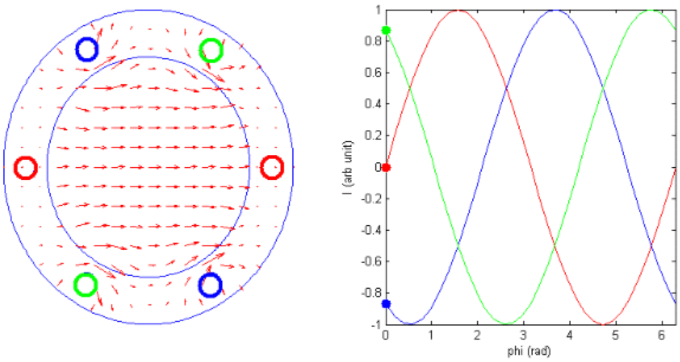
Torque where there is a change in current direction.

In a DC Motor, this change in current direction is achieved through a split-ring commutator, present in the image above. A split ring consists of semi-circular plates attached to each ending of the armature. The plates rotate with the motor and make contact with brushes, which are in turn connected to a voltage supply. As the motor rotates to this point, each plate breaks contact with a brush and connects to the other brush, changing the direction of the voltage across the armature. This then reverses the direction of the current and therefore the force. This acts as a sort of double reversal, keeping the torque in the same direction.

AC Induction Motors

An AC Induction motor works by utilising 3-phase AC power to create a rotating magnetic field inside the motor. This is analogous to the stator (external magnets) being rotated around the coil in a regular motor.

This induces a current in the coil which causes it to be ‘dragged’ by the field. The coil (or squirrel cage) inside the motor will be dragged with some torque due to the changing magnetic flux until it is rotating at the same speed as the field. The Squirrel Cage is a special type of coil which maximises magnetic flux and torque in this setup.



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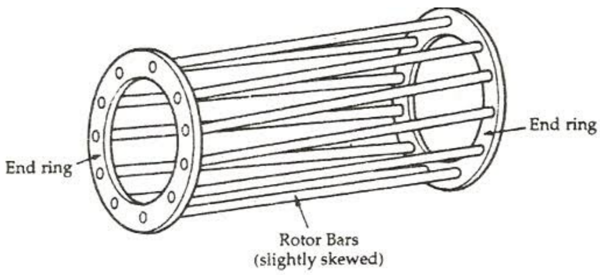


Generators
Generators work in reverse to motors, so instead of supplying power to turn a coil, a coil is turned, generating power.

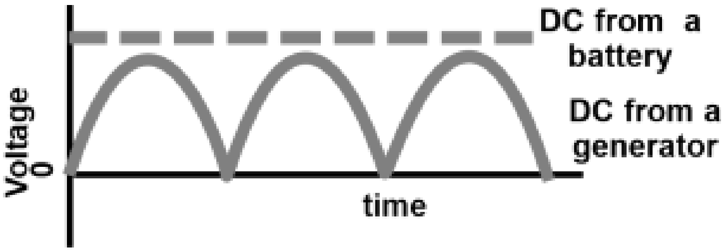
The act of spinning the coil through a magnetic field creates a force on the electrons in the moving wires, generating a voltage. The voltage generated by this is the same as the back E.M.F. created when the motor is spinning and is in the same direction as when the motor spins in a given direction.

Due to a current being generated in the coil as it is rotated, there is also a force acting as a torque in the opposite direction, resisting the rotation of the generator. This force is described by the equation:

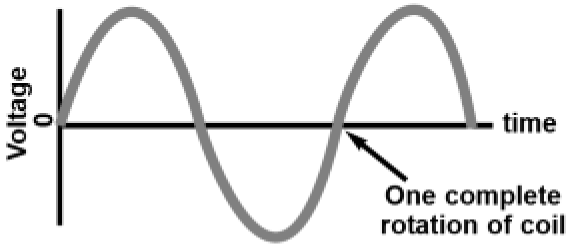
$$\vec{F} = l\vec{I} \times \vec{B}$$

This can be rationalised by considering that if there was no resisting torque, or the torque acted in the other direction, there would be an infinite amount of energy generated (i.e. once it was started it would never stop or it would infinitely generate more energy).

DC Generators
A DC Generator makes use of the same equipment as a DC motor. As with motors, the current direction flips every half turn and, as a result, the current generated would be AC.
To stop this from happening, a split-ring commutator is used, reversing the current again every half turn, keeping the current direction the same. In an ideal DC generator, the graph follows an $|\sin \omega t|$ graph but in reality, there is a sudden drop before it reaches its minima each time (as the commutator loses contact slightly before the transition).



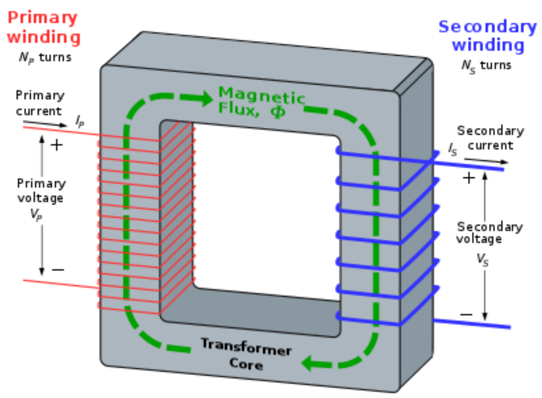
AC Generators
AC generators are actually rather simple as they use the same setup as a DC motor except without a commutator (use a slip ring instead). As already discussed, the direction of force in a DC motor swaps every half turn. In a generator, because the rotation is in a constant direction and being created by an external entity, this reversal still occurs but instead it is the direction of the voltage being generated which swaps. As a result, the direction of the current swaps every half turn which, by definition, is the generation of an Alternating Current.
This is carried away for use through a slip-ring commutator, which doesn't swap the direction of the current. A slip-ring commutator can be thought of as though you attached an alligator clip to each end of the coil and magically made them unable to tangle



Transformers

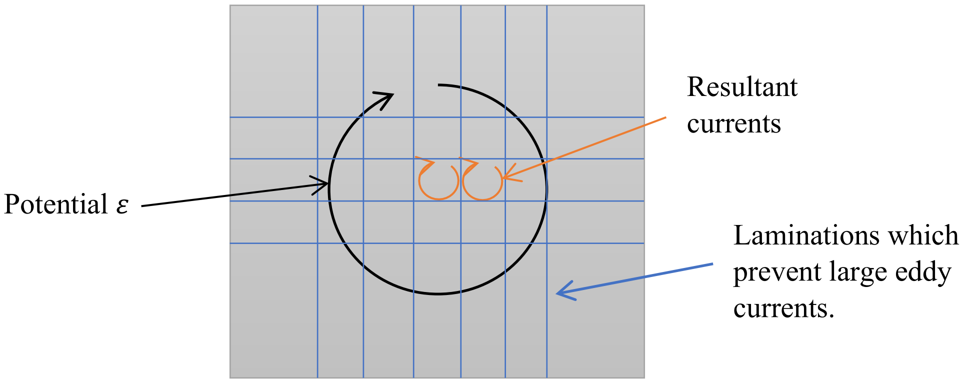
Transformers perform a simple but important task: changing the voltage in an AC circuit. Transformers are made from two coils and a circular or rectangular iron core which both coils are wound around.
One coil, called the primary coil, is connected to power and has an alternating current put through it (often 50Hz for Australia) which then generates an alternating magnetic field in the solenoid. This magnetic field then induces a magnetic field in the iron core, which then channels the magnetic field around the iron loop and through the other coil, called the secondary coil.
The changing magnetic flux through the secondary coil caused by this process creates an E.M.F. which also alternates at the same frequency. By changing the number of turns of the secondary coil, the amount of E.M.F. produced changes. By increasing the number of coils, the voltage increases from the primary to secondary coils (step up) and be decreasing the number of turns the voltage drops (step down).

Transformer equations rely on the assumption that energy is conserved in the voltage transformation from coil to coil. As a result, when a voltage step-up occurs, there must be a current step-down and vice versa.



Laminations

Laminations are used in transformers to prevent eddy currents from being induced in the iron core. Eddy currents can be a big problem for the efficiency of transformers due to the fact that they use alternating current at high frequencies, which creates a changing magnetic field. Due to the frequency at which this changing magnetic field oscillates, the E.M.F. it would create is massive, which would normally create large eddy currents which would ruin the efficiency of the transformer. To counteract this, plastic laminations are used, parallel to the direction of the magnetic field within the metal core. The laminations reduce the area of the core to small sections, which prevents large currents from forming by reducing the maximum E.M.F. in each section to a smaller value. This increases the effectiveness of the core at channelling the $\vec{B}$ field.



Extension Notes

Coulomb's Law

Coulomb's Law is typically written as $F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$, however, it is a well-known fact that this force has a direction. This direction can be denoted by the unit radius vector which follows the conventions listed in the Vector Conventions section in the appendices.

This new formula is written as follows:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}$$

Gauss' Law

Gauss has many laws to his name, some in physics, some in pure mathematics. However, the law we consider "Gauss' Law" in Electromagnetism is:

$$\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

or

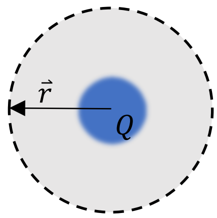
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

Symmetry

Symmetry is a key part of Gauss' law as it allows for the construction of equations where many of the variables are constant. Symmetry is the notion that for a 3D point in space, any point on a sphere around that point is indistinguishable from another point on that sphere. For a 1D line, any point on a circle around that line is indistinguishable from another point on that circle. This allows the fields in the integrals to be considered constant. This concept, although a neat mathematical trick, also tells us something about the nature of our universe. Because the universe obeys symmetry, we are allowed to use Gauss' Laws in these ways or, in other words, Gauss' Laws are the logical derivatives of what should occur if the universe is symmetric in certain ways.

Gauss' Law to Derive Coulomb's Law

To derive the formula for the field around a point charge, we must begin by considering a point charge in space with some charge Q. In 3D space, around a single point there is what is known as spherical symmetry. Put more simply, because of the nature of the universe all points at a constant radius from a point are indistinguishable from the perspective of that point.
Because of this, we must consider an imaginary sphere of radius r around that point charge.



We must then consider the equation $\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$. Because we have constructed this scenario based on symmetry, we know that $\vec{E}$ is constant and parallel to the area vector at all points on the surface of the sphere and can, therefore, be taken out of the integral as a constant.

$$E \oiint dA = E\ A = \frac{Q_{enc}}{\varepsilon_0}$$

We know the area of a sphere is $4\pi r^2$ and now all we have to do is rearrange the formula for $\vec{E}$.

$$E\ 4\pi r^2 = \frac{Q_{enc}}{\varepsilon_0}$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

To derive Coulomb's Law, all we have to do is multiply by q, which will find the equation for force.

$$F = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}$$

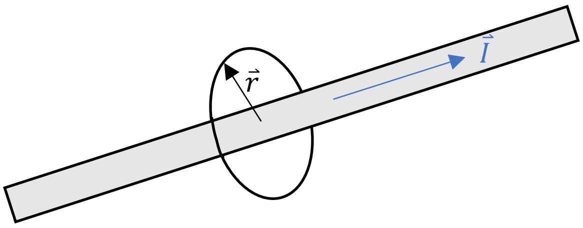
Maxwell's Equations to Derive Magnetic Field Around A Wire

To derive this equation, we consider Ampere's Law for magnetism: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \varepsilon_0 \frac{\partial \phi_E}{\partial t}$

In this case, we are considering the case where there is no changing electric flux as a constant electric field in the wire has already been established. As such, the equation to be considered is the following:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

To consider this scenario, we must construct a scenario with a conductor carrying some current I and an imaginary circle of some constant radius r around the wire.



The reason for this circle is that around a point on a wire, all points on this constructed circle are indistinguishable. If we rotate the circle, from the wire's perspective it will look identical. This allows us to consider B as a constant in the integral which results in the following:

$$B \oint dl = B\, l = \mu_0 I$$

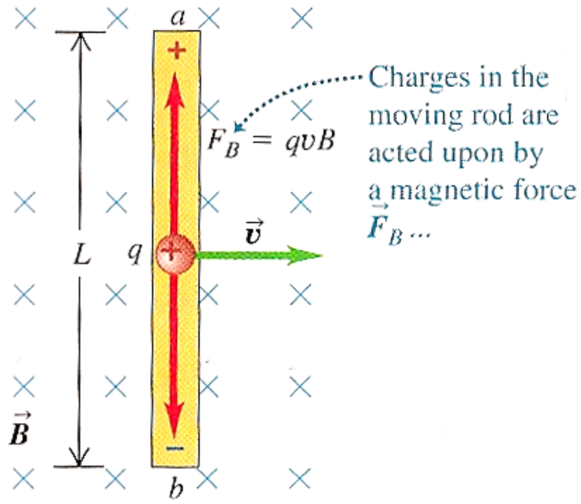
To solve for B all that must be done is establish what l is. In this case, because our shape was a circle, the length is the circumference of the circle, $2\pi r$.

$$B\, 2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

What you will notice about the equation we have just derived is that it is the formula given for the magnetic field around a wire.

Derivation of the induced EMF on a rod moving through a Magnetic Field



Since we know that there is a force on all charges in the rod, and that they are dispersed evenly along the rod we can say the rod has a charge density per length of each type of charge:

$$q_+ = \lambda_+ L$$

$$q_- = \lambda_- L$$

$$q_- = q_+$$

The force due to motion on the positive charges is up the page and the force on the negative charges is down the page. Work is the force done along the distance:

$$W = \int_0^L \vec{F} \cdot d\vec{L}$$

The total work is the work done on the positive charges and negative charges:

$$W_{net} = \int_0^L q_+ vB \cdot dL + \int_0^L q_- vB \cdot dL$$

$$= \int_0^L \lambda_+ L\, vB \cdot dL + \int_0^L \lambda_- L\, B \cdot dL$$

$$= \lambda_+\, vB \int_0^L L\, dL + \lambda_-\, vB \int_0^L L\, dL$$

$$= (\lambda_+\, vB + \lambda_-\, vB) \left[\frac{L^2}{2}\right]_0^L$$

$$= \frac{(\lambda_+ + \lambda_-) vBL^2}{2} \qquad \lambda_+ + \lambda_- = \frac{q_+}{L} + \frac{q_-}{L} = \frac{q_+ + q_-}{L}$$

$$\frac{q_+ + q_-}{L} \frac{vBL^2}{2} \qquad |q_+| = |q_-| = q$$

$$= qvBL$$

$$\varepsilon = vBL \qquad V = \frac{W}{q} = \varepsilon$$

The Electric Field Inside A Conductor

The electric field inside an ideal conductor in a circuit is always zero. The reason for this is that when an electric field is created at the end of a conductor, the electrons at that end begin to move. But when the electrons move, there is a net positive field where they used to be and a net negative field where they moved to. As a result, the other electrons in the conductor are pulled towards where the electrons used to be and pushed away from where they are now. This is then the reason why currents occur at a constant rate throughout the circuit rather than being different the further from the voltage supply you are (they are conductors because they conduct the field without causing it to decay).
Furthermore, it is the ratio of the efficiency of this chain reaction to the supplied field strength that gives rise to resistance. The more efficient a conductor the closer to zero the field inside it is and it is the inconsistencies in the field which give rise to resistive effects such as heat.

The Electric Field as A Function of a Voltage Field

$$\vec{E} = -\vec{\nabla}V = -\begin{bmatrix}\frac{\partial V}{\partial x} & \frac{\partial V}{\partial y} & \frac{\partial V}{\partial z}\end{bmatrix}$$

(See Appendices for more on gradient the Del operator)

All this definition states is that the electric field is the negative gradient in space of the voltage (i.e. if voltage increases in one direction, the electric field points in the other direction). In this case, the Electric field vector points in the direction of greatest decrease in voltage.

Solenoids and Inductors

In an electric circuit a solenoid is called an inductor and are commonly used in AC circuits. An inductor makes use of the Faraday-Lenz Law to store energy in the magnetic field and use that energy when the circuit is turned off. When a circuit is turned off, the current through the circuit quickly stops. As the current stops, the magnetic field produced in the inductor decreases at the same rate. The decrease in flux (with respect to time) induces a voltage and therefore a current in the inductor, maintaining the current for a small time after the circuit was turned off.
It is important to note this doesn’t last forever as the inductor can only induce a current while there is a rate of change of current. As soon as there is no current from the source at all, the magnetic flux will also experience no change and the induced current will stop. The purpose is not to maintain the current forever but to increase the efficiency of the circuit so that the maximum current is maintained for longer. This effect is especially important in AC Circuits.

Why Metal Rods?

Putting iron rods through the centre of solenoids is common practice for most physics departments and the comment is often made that it amplifies the magnetic field, but why?

What is happening in this scenario is that the iron is a material which rearranges its internal atomic magnetic fields very easily. When iron is placed in even a weak magnetic field, its own internal magnetic domains rearrange to align with the external magnetic field. The iron then creates its own magnetic field due to this alignment.

This effect is why putting iron inside a solenoid magnifies the magnetic field.