FROM THE UNIVERSE TO THE ATOM

Base Units

Mass (m) – Kilograms (kg)
Displacement ($\vec{s}$) – Metres (m)
Time (t) – Seconds (s)
Speed (v) – Metres per second (ms^{-1} **or** m/s)
Momentum (p) – Kilogram metres per second ($kg\ ms^{-1}$ **or** $kg\ m/s$)
Wavelength (λ) – Metres (m)
Frequency (f) – Hertz (Hz **or** s^{-1})
Energy (E) – Joules (J **or** $kg\ m^2\ s^{-2}$)
Luminosity (L) – Power ($J\ s^{-1}$)
Intensity (I) – Power per area ($J\ m^{-2}\ s^{-1}$ **or** $kg\ s^{-3}$)
Angular Momentum (L) – Kilogram square-metres per second ($kg\ m^2\ s^{-1}$)

Constants

The Speed of Light $c = 3.00 \times 10^8\ m\ s^{-1}$
Planck Constant $h = 6.626 \times 10^{-34}\ kg\ m^2\ s^{-1}$

Rydberg Constant (Hydrogen) $R = 1.097 \times 10^7\ m^{-1}$

Wein's Displacement Constant

Equations

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

The de Broglie wavelength of an object with mass.

$$\frac{1}{\lambda} = R\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right)$$

Rydberg equation for the wavelength of a photon ejected or absorbed by a hydrogen atom.

$$E = mc^2$$

The rest energy of an object with mass.

$$E_K = mc^2(\gamma - 1)$$

Kinetic Energy of a particle (technically general but mostly used for relativistic cases).

$$N_t = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{0.5}}} = N_0 e^{-\lambda t}$$

$$\lambda = \frac{\log_e 2}{t_{0.5}}$$

The Nuclear decay equations.

$$\hbar = \frac{h}{2\pi}$$

Another notation/form of the Plank Constant (it shows up a lot).

$$L = mr^2\omega = mvr = \frac{nh}{2\pi}$$

$$mvr = n\hbar, \qquad \hbar = \frac{h}{2\pi}$$

Bohr's postulate that Angular Momentum is quantised.

Extension Equations

$$\frac{v}{D} = H_0$$

Hubble's Law (though the Hubble constant (H_0) *has been shown to be different at different distances.*

$$L = \sigma A T^4, \qquad \sigma = 5.7 \times 10^{-8}\, W\, m^{-2}\, T^{-4}$$

Stefan-Boltzmann Law for stars.

$$L = \oiint \vec{I} \cdot d\vec{A}$$

The light flux emitted by an object is a constant and is its luminosity.

$$-\frac{\hbar^2}{2\mathrm{m}}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial\mathrm{t}}$$

$$\hat{H}\Psi = i\hbar\frac{\partial\Psi}{\partial\mathrm{t}}$$

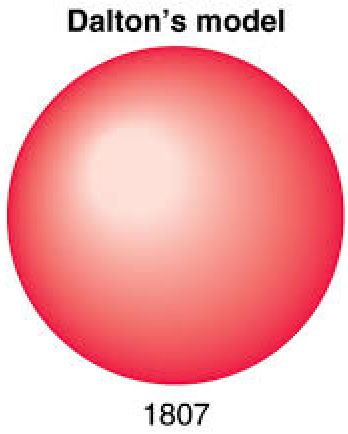
The General form for Schrödinger's Wave Equation. $\hbar$ *is a constant,* $i^2 = -1$, m *is the object's mass,* ∇^2 *is the Laplacian operator and* V *is the potential energy as a function of time and position.*

Course Notes

Models of The Atom

Dalton – 1808

Dalton expanded on the model proposed by the Greeks where he hypothesised that the Atom was made of a solid 'billiard ball' and was uncuttable. He explained different elements by proposing that each element had its own ball.



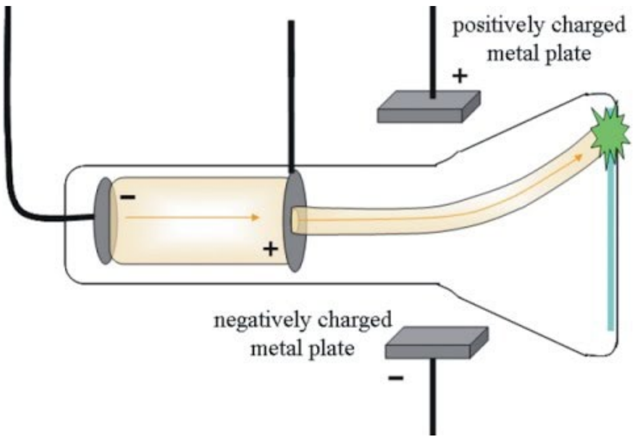
Thomson – 1904

Thomson experimented with cathode ray tubes from different materials, using electric and magnetic fields to determine its properties:

- The ray was negatively charged
- The thing which made up the ray could be forced out of any material

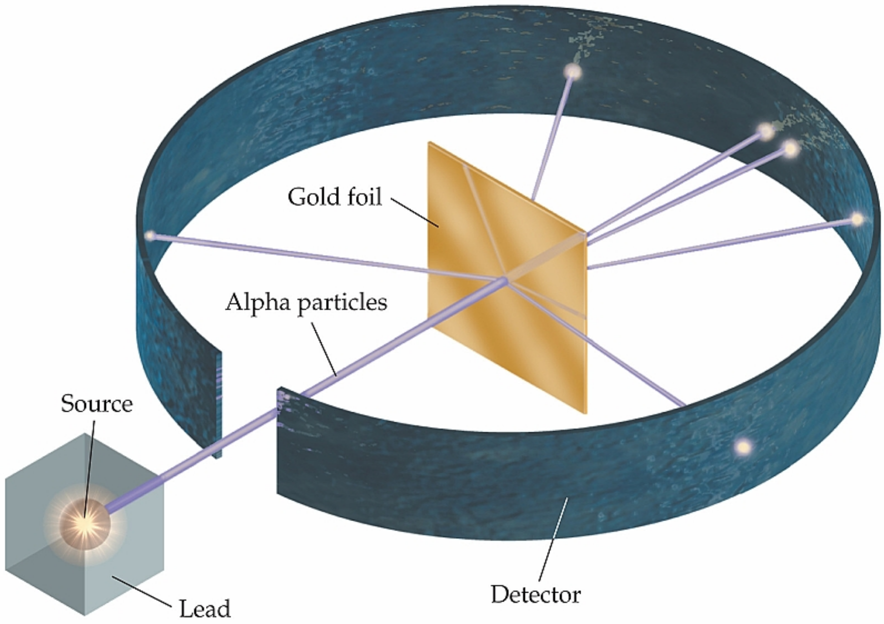
He was also able to balance the effects of the electric and magnetic fields to determine their charge to mass ratio but was unable to calculate either separately. In doing so he determined that the ray was made of small particles with some mass and charge.

From this he developed the 'Plumb Pudding' model which was an amendment to the Dalton model, with each atom being made of a different positively charged ball structure of uniform charge distribution and with negatively charged particles (electrons) distributed randomly throughout.



Rutherford – 1911

Rutherford discovered, with the gold foil experiment (Geiger-Marsden), that the atom is made of mostly empty space but has a centre with an extreme density of positive charge (a nucleus). This was concluded from the fact that most of the alpha particles passed straight through the atoms, some were deflected slightly, and a few were deflected back in the direction they came from.



The fact that most particles went straight through meant that they did not come near another positive charge and experienced nearly no force. However, the fact that ones that did get deflected got deflected by a large amount implied a large charge density where the alpha particle travelled (and a large force).

Bohr – 1913

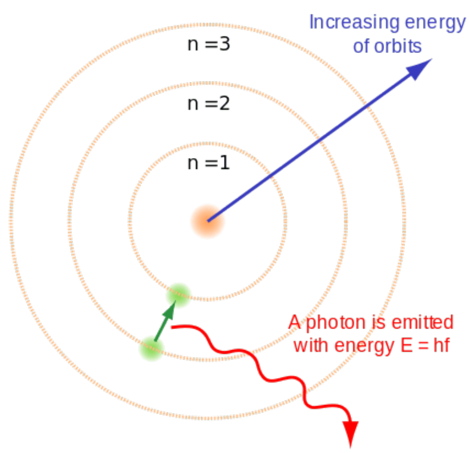
Bohr noted that, by Maxwell's equations, a charge in circular motion was accelerating and would therefore create light. This light should therefore be proportional to the acceleration and would cause the electron to fall into the nucleus emitting a smooth spectrum of light. **Neither of these occur.**

Bohr then went on to develop his quantised orbital theory of electron orbits where the electrons orbited at set radii and would 'jump' or 'fall' between them. To go along with this, he formalised 3 postulates:

1. Electrons orbit around a nucleus with a high density of positive charge per volume with centripetal acceleration from the coulomb force: $\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2} = \frac{mv^2}{r}$

2. Electrons orbit in a quantised number of orbitals where the angular momentum of the electrons exists in quantised states with $L = mr^2\omega = mvr = \frac{nh}{2\pi}$ where n is the integer orbit.

3. While in these orbits the electrons are in a 'stationary state', meaning they will not lose a small amount of energy as a photon and fall to some radius between the orbits.

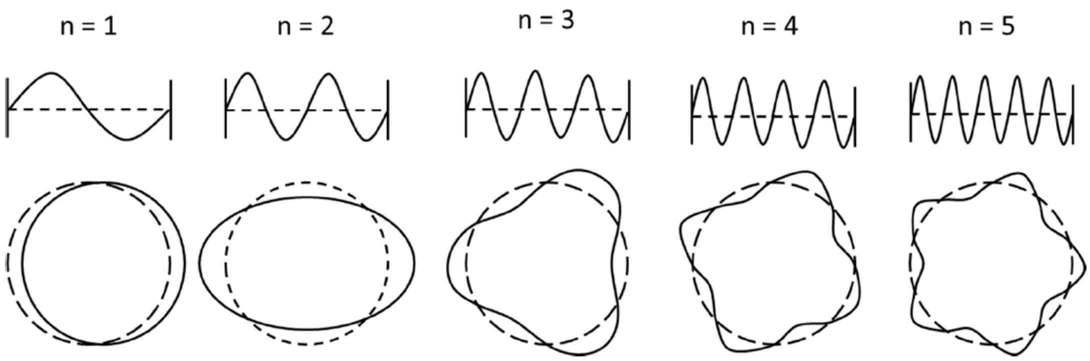


Theorisation of the Proton and Neutron – 1920s

Throughout the 1920s it was discovered that the mass of atoms was approximately integer multiples of the mass of the Hydrogen nucleus and that the atomic number was proportional to the charge on the nucleus. This knowledge, combined with the discovery of isotopes (atoms with different masses but the same atomic number), led to the theorisation of the Proton and Neutron (believed to have the same mass).

De Broglie – 1924

He explained Bohr's model by proposing that the electron could be a wave and a particle (invoking Einstein's wave-particle duality theory). De Broglie postulated that if the electron were a wave then it would reason that it would have an integer number of wavelengths. This explained why the electron only existed at certain radii, as these were the radii where the electron could inhabit an orbit where the circumference is an integer multiple of the wavelength.



See https://www.desmos.com/calculator/xww8n1r3kt for a de Broglie wave generator.

Schrödinger – 1926

Schrödinger took de Broglie's proposed wave theory and generalised it for all fundamental particles (particles which exhibit wave properties). He invented the wave function which stores all properties about that particle including its position, momentum, energy etc. This wave function however represents the probabilities of each of these properties being a certain value, not the values themselves.

Schrodinger's Wave Equation is written in general form, where Ψ is the wave function:

$$-\frac{\hbar^2}{2\mathrm{m}}\nabla^2\Psi + V\Psi = i\hbar\frac{\partial\Psi}{\partial \mathrm{t}}$$

V is the sum of all potentials on the particle (has units of energy) m is its mass, ∇^2 is the Laplacian, $\hbar = \frac{\mathrm{h}}{2\pi}$ and $i^2 = -1$

This theory is currently the accepted theory of subatomic particles.

But isn't wave motion accelerated motion?

Yes, wave oscillation is a form of acceleration. It wasn't until a year or so after Schrödinger published his equation that people began to realise that the thing doing the 'waving' was in fact probabilities, not the particle. As such, Bohr's 'stationary state' idea is correct (just not in the way he thought).

Chadwick – 1932

He found the first experimental evidence for the Neutron (in essence he discovered it, though he did not theorise it).

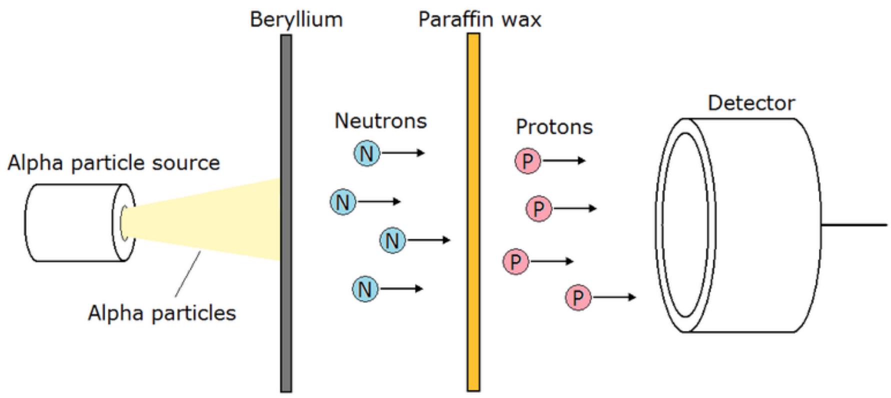
Neutrons had been theorised for a long time and theoretical physicists had just kind of accepted they existed. They provided a great explanation for why the mass of particles on the periodic table increased at a greater rate than their charge.

The Neutron Experiment

Alpha particles were emitted onto a Beryllium plate. A sheet of paraffin wax was then placed after the Beryllium and a charge detector detected what were found to be protons being emitted from the wax. However, the charge detector did not detect anything when placed between the beryllium and wax.

By putting a thick lead plate between the beryllium and the wax, the release of protons from the wax was stopped. Using this, Chadwick reasoned that there must be some neutral charge being emitted from the beryllium.

Using conservation of mass and momentum, Chadwick determined the ratio of the neutral particles mass to the proton and found that the masses were almost identical (the neutral particle was just 0.1% heavier). This allowed Chadwick to conclude that he had found the theorised Neutron.



The Standard Model

Terminology

Quark – A particle which, among other properties, possesses a colour charge (red, green or blue). As a result, they interact with Gluons and are affected by the strong interaction.

Lepton – A particle which does not interact via the strong interaction (i.e. it has no colour charge)

Boson – A particle responsible for force interactions.

Virtual Particle – A particle which is not directly observable but can still interact with other particles.

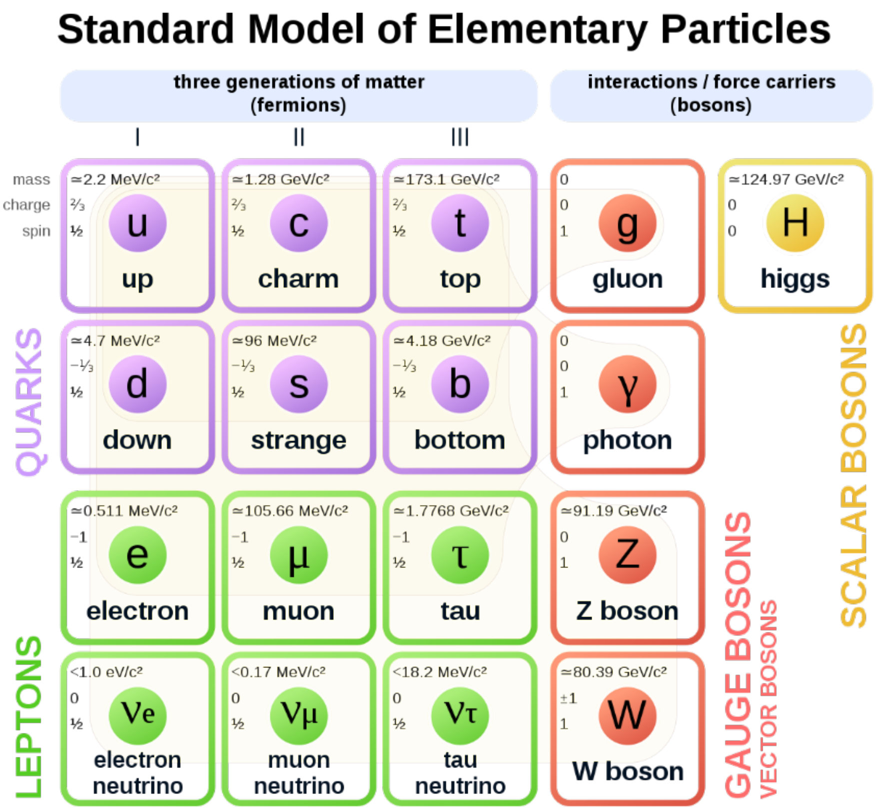
Hadrons – Particles made of two or more quarks with integer charges.

Baryons – Particles made of three quarks

Anti-Matter – A particle which is the complete opposite of another (opposite electric charge, opposite colour charge etc.)

Annihilation – Occurs when a particle collides with its anti-particle. The process creates an immense amount of energy in the form of photons.

Fundamental Forces – The minimum number of forces required to describe all phenomena in the universe (Strong, Weak, Electromagnetic, Gravity)

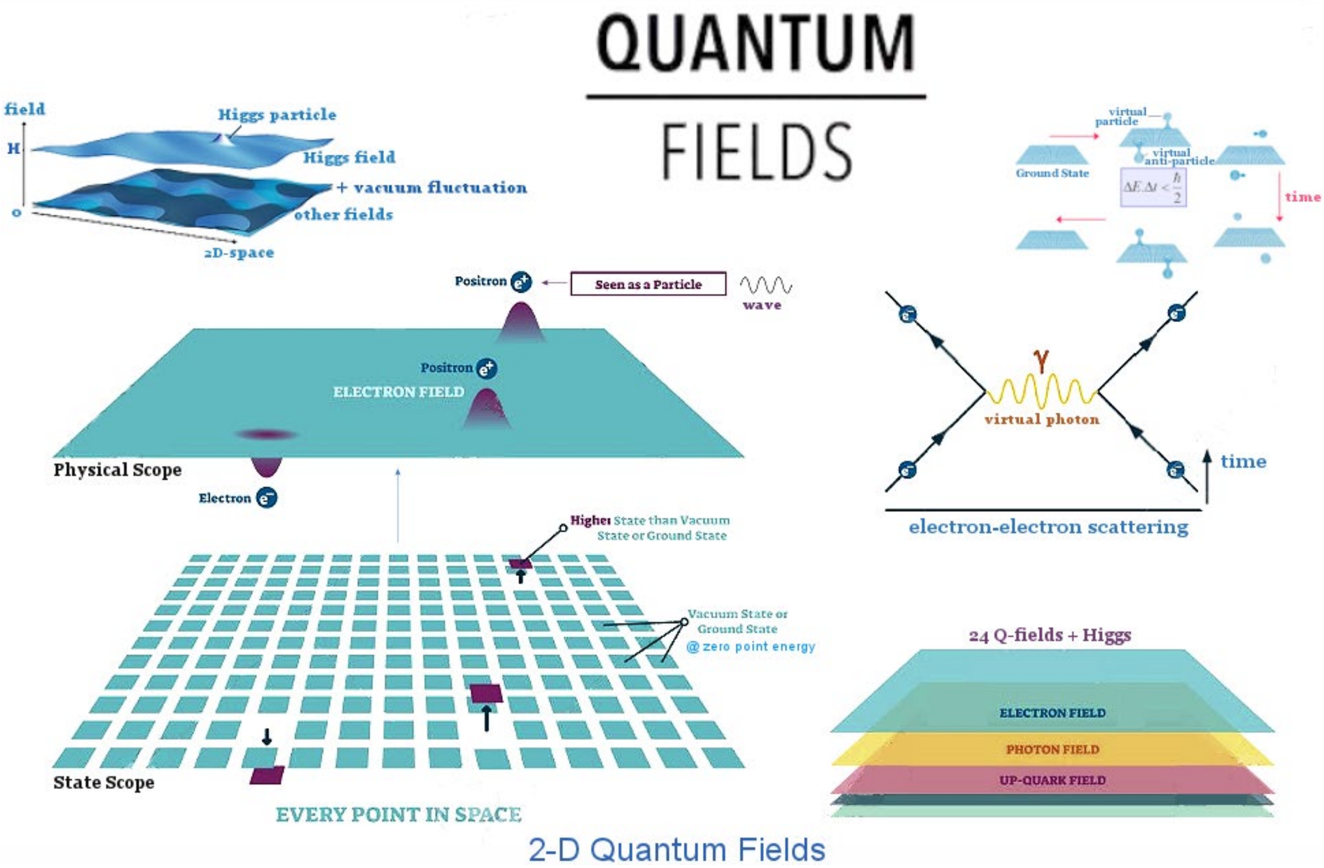


What is the Standard Model?

The standard model is the current model of particle physics which has been experimentally verified.

The standard model consists of fundamental particles with certain properties such as charge, mass etc.

Each of these fundamental particles is a little wiggle (wave) in its respective field (e.g. the electron is a small disturbance in the electron field). The reason particles have certain properties is a function of how much one field affects another field.



Anti-Particles

Anti-particles such as the anti-electron (positron) come up a lot in quantum physics. Although they have been observed in experiment, they were first predicted by Dirac when he was formulating his equation for the electron, where he found his equations always had two solutions. His two solutions consisted of opposite energies and opposite charges and he interpreted this as being a particle and its anti-particle.

Although the maths he performed was a little more complex, a basic understanding of this can be gleaned from the energy equation:

$$E^2 = {m_0}^2c^4 + p^2c^2$$

If we assume the velocity and momentum are zero, we find $E^2 = m^2c^4$

This does not rearrange to $E = mc^2$, rather it rearranges to $E = \pm mc^2$ and the negative solution is what Dirac interpreted as the anti-particle.

Conservation Laws

All conservation laws are due to invariances in some value. Where there is an invariance in some value during an interaction, some value is conserved (some are given below).

Conservation of Charge

In any interaction where there is Gauge symmetry, the total charge of the universe must be conserved where:

$$(+1) + (-1) = 0$$

Conservation of Colour

In any interaction, the total colour charge must be conserved where:

$$(B) + (G) + (R) = 0$$

$$(B) + \overline{B} = 0$$

Conservation of Momentum

In all interactions where there is translational symmetry (location of the event occurred does not change the interaction between the particles), momentum must be conserved.

Conservation of Angular Momentum (Spin)

In all interactions where there is rotational symmetry (the event would be the same if space were rotated), intrinsic angular momentum is conserved.

Conservation of Lepton Number

This is empirical and has never not been observed. This is an additive law where normal leptons such as electrons (e^-) and electron neutrinos (ν_e) have $+1$ lepton number and their anti-particles such as the positron (e^+) and anti-neutrino $(\overline{\nu}_e)$ have a lepton number of -1.

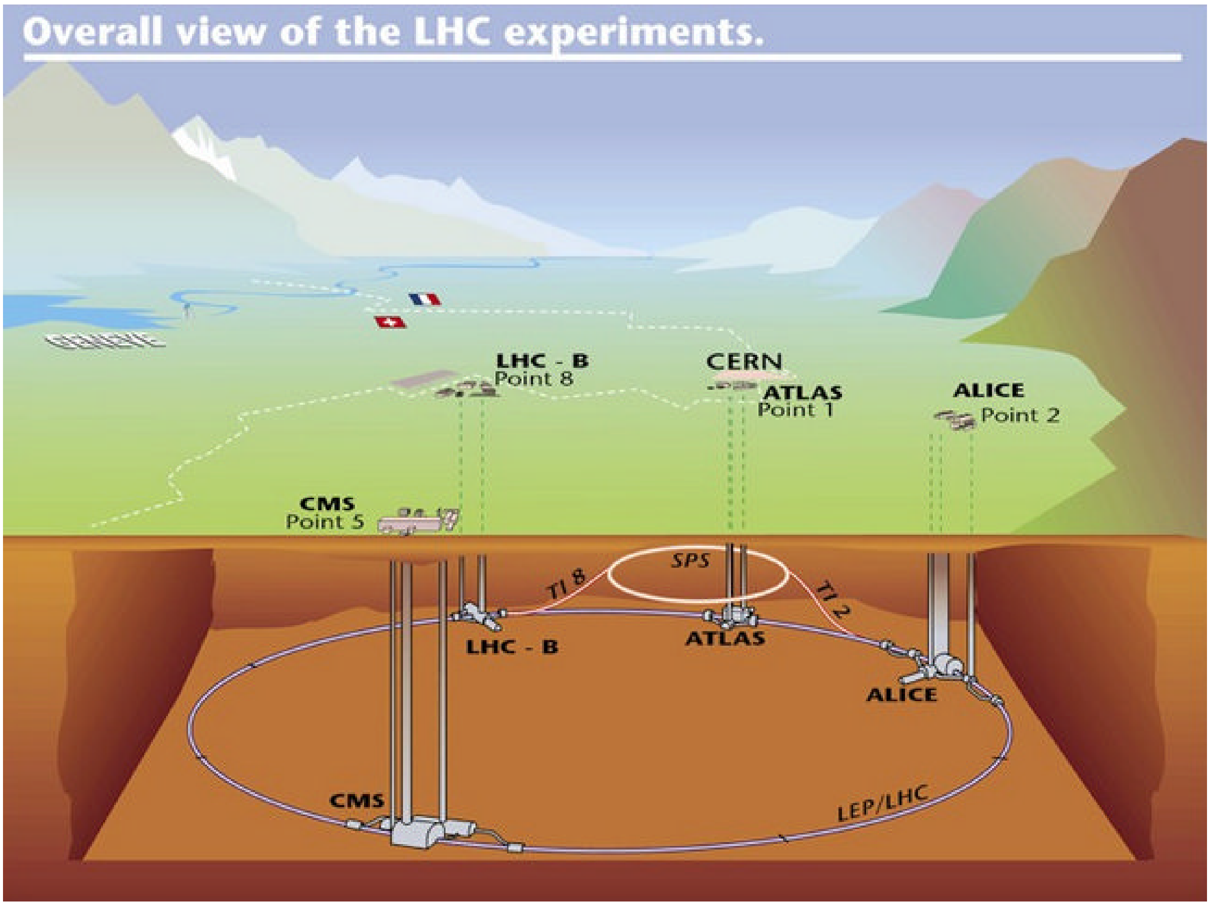
The LHC and other Evidence for the Standard Model

The LHC collides protons and other Hadrons together at very high speeds ($0.999\,999\,99c$).

The protons slam together at such high energies that they break apart and their constituent particles decay into lower energy particles.

The particles produced are random (to an extent) so they slam protons together many millions of times per second. They produce too much data to store so most of it is thrown out and only the possibly interesting ones are stored.

The detectors at the LHC are able to detect certain types of particles depending on the detector (there are 4 main detectors). The detected particles and their energies can then be used to reverse engineer the collision which occurred, and it has been shown that the collisions match what would be expected from protons made of three quarks colliding.



Quarks, Gluons & Colour Charge

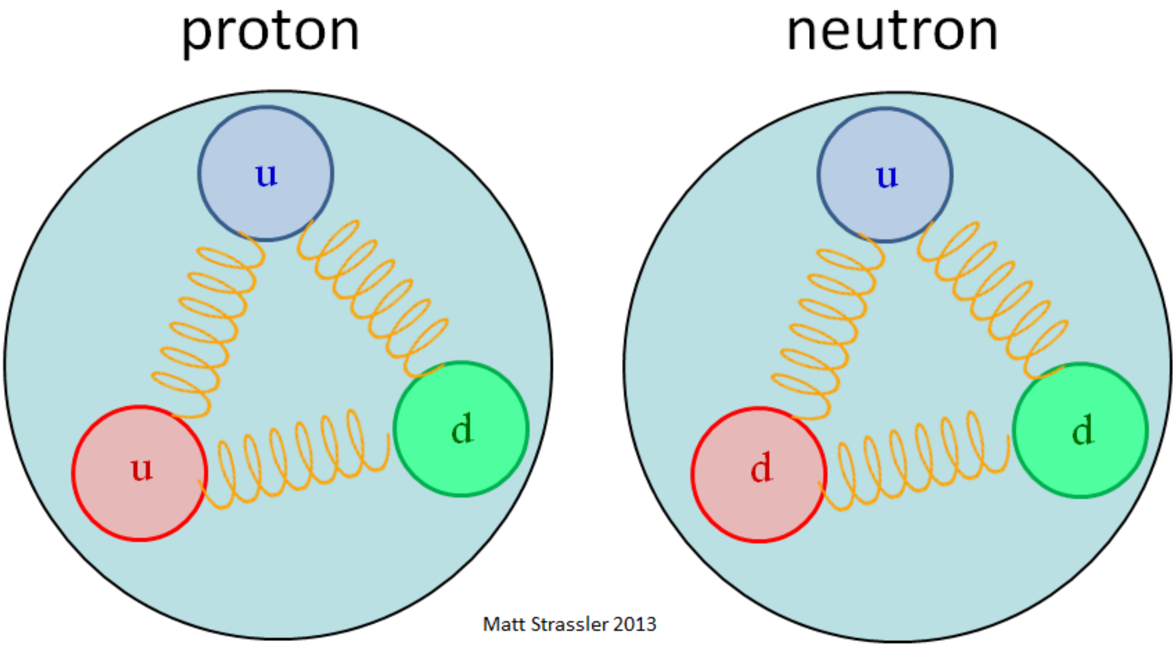
Gluons and Quarks possess a property called colour charge (it is not colourful but there are three types instead of two so physicists needed something more than $+\ -$) the colours are Red, Green and Blue (with a corresponding Anti-Red, Anti-Green and Anti-Blue for anti-particles). The charges are named as such since red + green + blue = white, so the charges cancel in sets of three. The study of these charges and the corresponding interactions is called Quantum Chromodynamics.

The 'flavour' of a quark is its classification (i.e. Up, Down, Charm, Strange etc.).

Though Quarks possess electric charge, the primary force which bonds them is mediated by their colour charge and the force particle corresponding to it: the gluon.

The gluon has no mass or electric charge, but it does have a colour charge and an anti-colour charge (never the same like blue and anti-blue though) and so it is one of the few particles which can interact with quarks without interacting with other particles.

Quarks are held together by flux tubes of gluons which attract the quarks together. The main property of these flux tubes that differs from lone gluons is that the colour charge of the gluons in the flux tubes cancel to be zero, keeping the net colour charge of the proton and neutron zero.



The analogy of springs is used for the flux tubes since the attractive force created by the gluons increases with the distance of the particles as with a spring.

Nuclear Physics

Terminology

Bonding Energy – The potential energy in the bond such that:

$$E_{bond} = -U$$

$$F_{bond} = -\nabla U = -\frac{dU}{dr}\hat{r}$$

Atomic Mass Unit ($\boldsymbol{u}$) – The average mass of the nucleons in a Carbon 12 nucleus such that:

$$u = \frac{m_{C-12}}{12}$$

Decay – The process by which one particle becomes another particle by emitting another particle.

Nucleon – A proton or neutron.

Half Life – The time a sample of some material takes to transmute into another material.

Control Rod – A material used to absorb neutrons in a nuclear reaction.

LHC – The Large Hadron Collider located under Switzerland and run by CERN.

Decay

Nuclear decay is a random (but statistically predictable) process whereby energetically unstable nuclei decay into more stable nuclei and release certain particles such that:

- Charge is conserved
- Energy is conserved
    - $U + E_K$
- Momentum is conserved

Alpha (α) Decay

Alpha decay is a result of the strong force between neighbouring nucleons being weaker than the electrostatic repulsion of the protons. The exact mechanism is hard to explain but fundamentally it is when the potential energy binding an alpha particle (α *or* ${}^{4}_{2}He$) to the nucleus is near zero. There is still a large potential bond between neighbouring nucleons (due to the strong interaction) however the chunk is only loosely held so it is ejected.

The leftover potential energy is lost as a photon and can be measured as a loss in mass.

The nuclear equation can be written as:

$${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + {}^{0}_{0}\gamma$$

Or

$${}^{A}_{Z}X \to {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha + {}^{0}_{0}\gamma$$

Beta (β) Decay

Beta decay occurs in two forms

1. β^-
2. β^+

β^- decay is where a neutron becomes a proton through the weak interaction and, by conservation of charge, an electron is released. During this process, an anti-neutrino is also given off.

$${}^{A}_{Z}X \to {}_{Z+1}^{\;\;A}Y + e^- + \bar{\nu}_e + {}^{0}_{0}\gamma$$

β^+ decay is where a proton becomes a neutron through the weak interaction and, by conservation of charge, a positron is released. During this process, a neutrino is also given off.

$${}^{A}_{Z}X \to {}_{Z-1}^{\;\;A}Y + e^+ + \nu_e + {}^{0}_{0}\gamma$$

The potential energy lost by this transmutation is the emitted as a photon.

Why neutrinos are emitted

Neutrinos are emitted in these interactions to conserve spin and lepton number. The neutrino conserves spin because it spins the opposite way to the electron (is spin down if the electron is spin up) making the net spin zero.

Neutrinos and electrons have lepton numbers of $+1$ while anti-neutrinos and positrons have lepton numbers of -1. The total sum of the products must equal the initial (which was zero).

Gamma (γ) Emission

Gamma emission is the emission of a photon (typically in the gamma spectrum but could be in any spectrum) and can occur due to two effects.

1. Nuclear decay (α or β)
2. Movement of protons within the nucleus

The first effect occurs due to conservation of energy and the lost potential energy part of nuclear interactions.

The second effect is part of the strong interaction where a proton can swap with a nearby neutron to reduce its potential energy (and therefore net force). In doing so it has lost energy, so a photon is emitted to conserve energy.

Penetrating Distance and Ionisation

Different forms of radiation are able to penetrate materials to varying distances. Generally, the further radiation can penetrate, the lower its ionisation potential. This is not true when comparing gamma radiation to X-rays as gamma rays possess both a greater penetration distance and ionisation energy.

α particles are most likely to cause ionisation as they strongly attract electrons that they are near to.

β^- (electrons) cause ionisation because they collide with the electrons in outer shells of atoms, transferring momentum and knocking the electron off.

β^+ (positrons) cause ionisation because they annihilate electrons in a collision.

γ rays have much greater energies ($E = hf$) than is required to ionise an atom and so, on collision with an electron, transfer their energy to the electron as kinetic energy, ionising the atom.

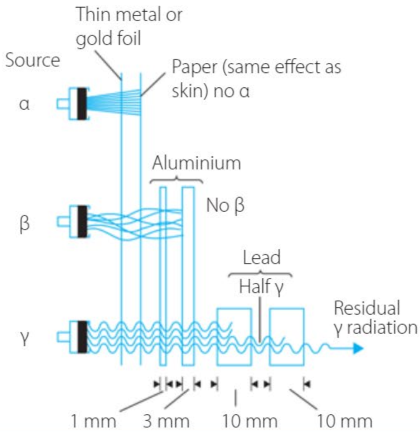


FIGURE 16.6 Gamma rays are much more penetrating than α or β particles. In turn, β particles are more penetrating than α particles.

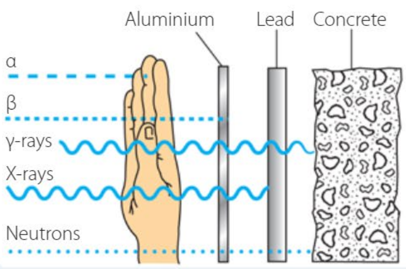


FIGURE 16.7 Penetrating power for different radiations is indicated by their relative absorptions in materials.

Half Life

Half Life describes the statistical nature of nuclear decay in one easy concept. All unstable nuclei (prone to either form of decay) will have some time after which half of a large sample will have decayed.

Since after every integer multiple of the half life time ($t_{0.5}$) the sample has halved in size, we can write that mathematically where N_t is the amount after some time and N_0 is the original amount:

$$N_t = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{0.5}}}$$

It is easy to see when $t = t_{0.5},\ \ N_t = \frac{N_0}{2}$ and when $t = 2t_{0.5},\ \ N_t = \frac{N_0}{4}$ etc.

Now we rearrange because the formula sheet is unnecessarily specific…

$$N_t = N_0 e^{\ln\left(\left(\frac{1}{2}\right)^{\frac{t}{t_{0.5}}}\right)}$$

$$N_t = N_0 e^{\frac{t}{t_{0.5}}\ln\left(\frac{1}{2}\right)}$$

$$N_t = N_0 e^{\frac{-t}{t_{0.5}}\ln(2)}$$

$$N_t = N_0 e^{-t\frac{\ln(2)}{t_{0.5}}}$$

Now we let $\frac{\ln(2)}{t_{0.5}} = \lambda$

$$N_t = N_0 e^{-\lambda t}$$

$$\lambda = \frac{\ln(2)}{t_{0.5}}$$

Fission

Fission is the process by which a nucleus is split into two smaller parts. Typically, this is done with heavy nuclei such as Uranium-235 $\left({}^{235}_{92}U\right)$ whereby the atom is made unstable by shooting low velocity neutrons at it and it splits into two parts, one a little heavier than the other. As a result of this, neutrons are also released. E.g.

$${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{92}_{36}Kr + {}^{141}_{56}Ba + 3{}^{1}_{0}n$$

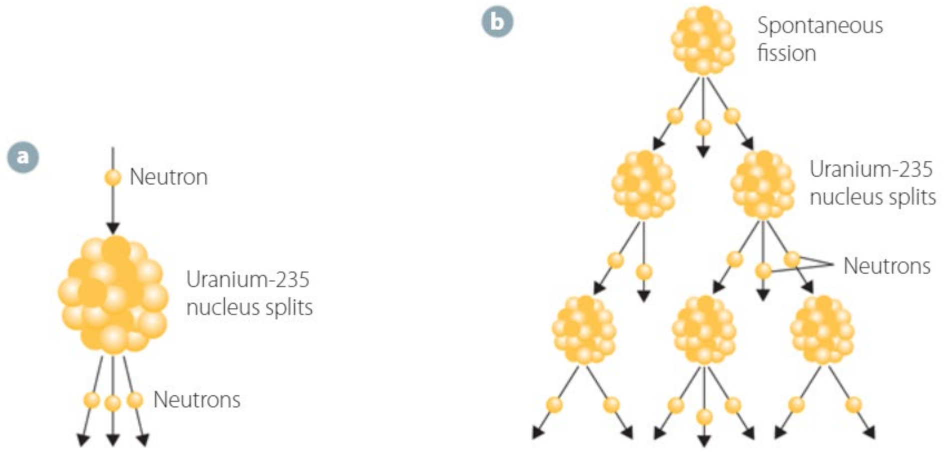


FIGURE 16.11 Nuclear fission. **a** A slow neutron causes a uranium-235 nucleus to split, releasing three fast neutrons. **b** A chain reaction occurs, if, for example, two of the released neutrons cause further nuclear fission in other uranium nuclei. Vast amounts of energy can be released.

A nuclear reaction such as that in a bomb occurs when more neutrons are produced per second than reactions are occurring (i.e. more than one neutron is produced per fission on average), this is a runaway reaction.

A nuclear reaction such as ones used in nuclear reactors is one where the number of neutrons produced per reaction is less than or approximately one, so it is ‘controlled’.

The Moderator

This is made of a material with a slightly higher mass than the neutron such as Hydrogen $({}^{1}_{1}H)$, Deuterium $({}^{2}_{1}H)$ or Tritium $({}^{3}_{1}H)$. The neutrons collide with these atoms and share their energy. This slows down the neutron so that it can be absorbed for more reactions.

Control Rods

If a reaction such as one in a nuclear reactor begins producing more neutrons than desired, control rods are inserted. Control rods are made of materials such as boron which more freely absorb neutrons (known as neutron poisons).

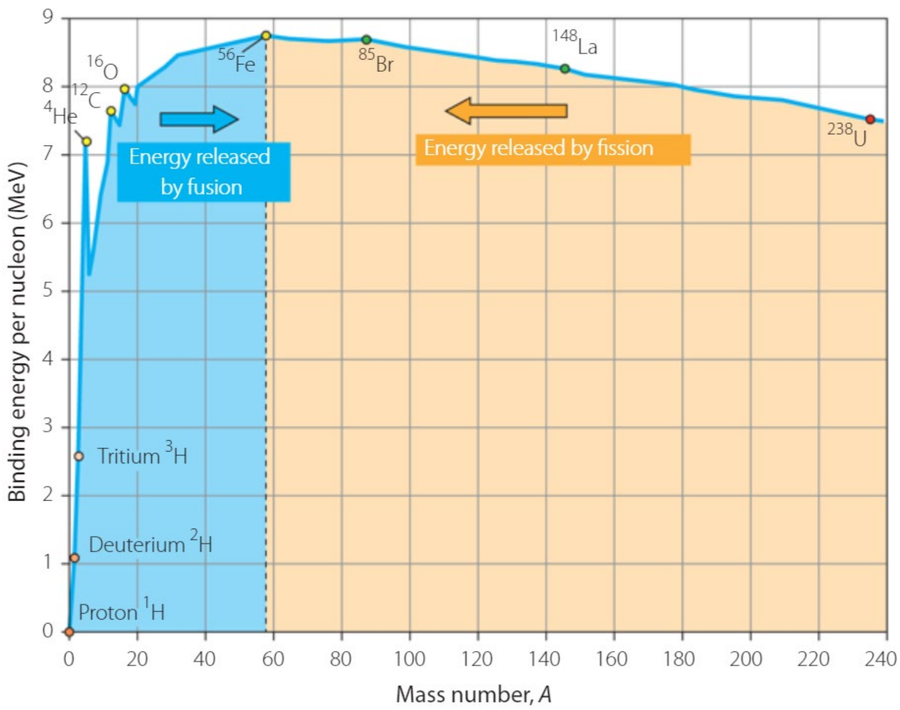
Enrichment

Since Uranium-235 is one of the few very reactive substances for nuclear reactions, it often needs to be separated out from the less reactive Uranium isotopes if a faster reaction is desired (the concentration of Uranium-235 required for a nuclear bomb is around 97%). Other Uranium isotopes can even absorb neutrons without reacting as they are more stable.

Fusion

Fusion is the process by which protons and neutrons are brought together with enough energy that they overcome their electric repulsion and are able to bond via the strong force. In the sun this is done by the pressure of gravity (and with a little help from quantum tunnelling) and in fusion reactors it occurs by colliding particles with enough kinetic energy that they bond but not so much that they obliterate each other.

Below is a graph of overall binding energies of nuclei.



Getting Energy from Fusion

Remembering that in actuality, the bonding energy is the negative potential energy, an increase in bonding energy is a net decrease in energy:

As atoms before iron are bonded, the net energy inside the nucleus decreases. By conservation of energy, an equivalent amount of energy must be released. The released energy is given by $E = -\Delta U$ (the change in energy between the particles). If E is negative, then extra energy is needed to bond them.

Forces, Energies and Emitted Particles

As a general rule, attractive forces produce negative potentials and repulsive forces produce positive potentials. When the net energy between particles decreases (bonding energy increases) there has been an increase in strength of the attractive force and vice versa.

When the energy between two quantum particles decreases, the energy must be emitted. The energy is often emitted as a photon. Sometimes, a particle can acquire a lower energy state (greater attractive force) by transmuting into another particle (like proton to neutron) and if this occurs, conservation of charge dictates that the equivalent charge must be emitted. Therefore, a positron and anti-neutrino will be released in such process such that they carry away the energy and charge.

Fusion Inside a Star – Hydrogen to Helium and the CNO Cycle

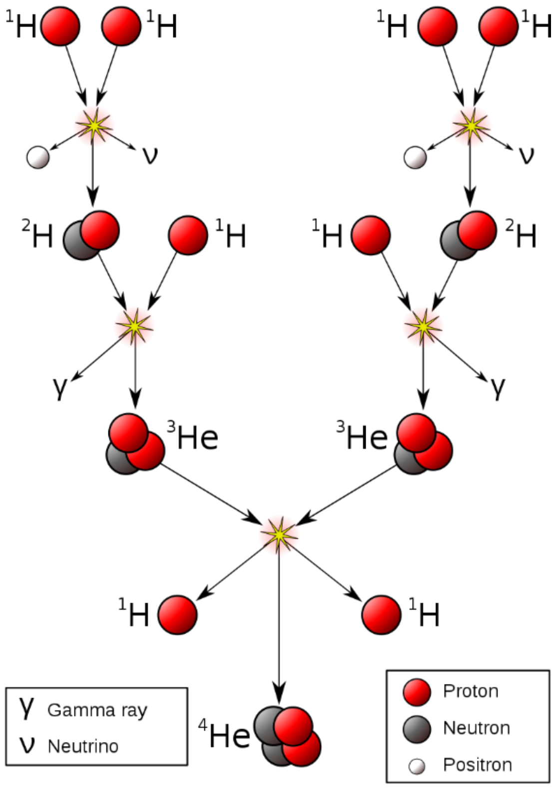
Proton-Proton Fusion – 26.73 MeV

Here's how helium can be made in the sun from 4 protons.

$${}^{1}_{1}H + {}^{1}_{1}H \rightarrow {}^{2}_{1}H + e^{+} + \nu_{e}$$

$${}^{2}_{1}H + {}^{1}_{1}H \rightarrow {}^{3}_{2}He + \gamma$$

$${}^{3}_{2}He + {}^{3}_{2}He \rightarrow {}^{4}_{2}He + 2{}^{1}_{1}H$$



The CNO Cycle – 25 MeV

The CNO cycle is another way that stars take 4 protons and make a helium nucleus.

$${}^{12}_{6}C + {}^{1}_{1}H \rightarrow {}^{13}_{7}N + \gamma$$

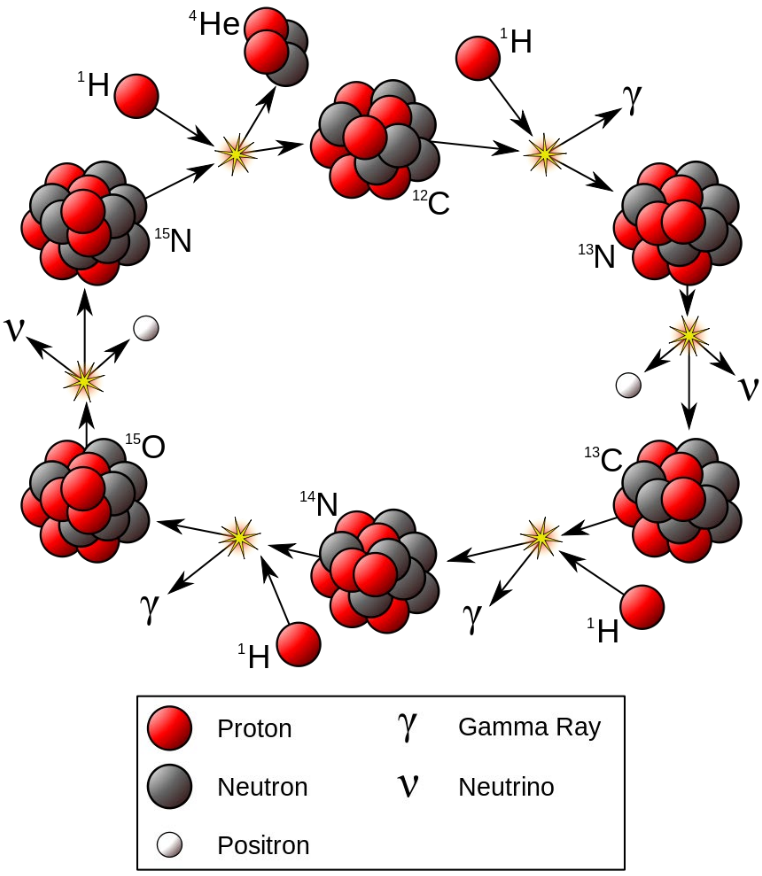
$${}^{13}_{7}N \rightarrow {}^{13}_{6}C + e^{+} + \nu_{e}$$

$${}^{13}_{6}C + {}^{1}_{1}H \rightarrow {}^{14}_{7}N + \gamma$$

$${}^{14}_{7}N + {}^{1}_{1}H \rightarrow {}^{15}_{8}O + \gamma$$

$${}^{15}_{8}O \rightarrow {}^{15}_{7}N + e^{+} + \nu_{e}$$

$${}^{15}_{7}N + {}^{1}_{1}H \rightarrow {}^{12}_{6}C + {}^{4}_{2}He$$

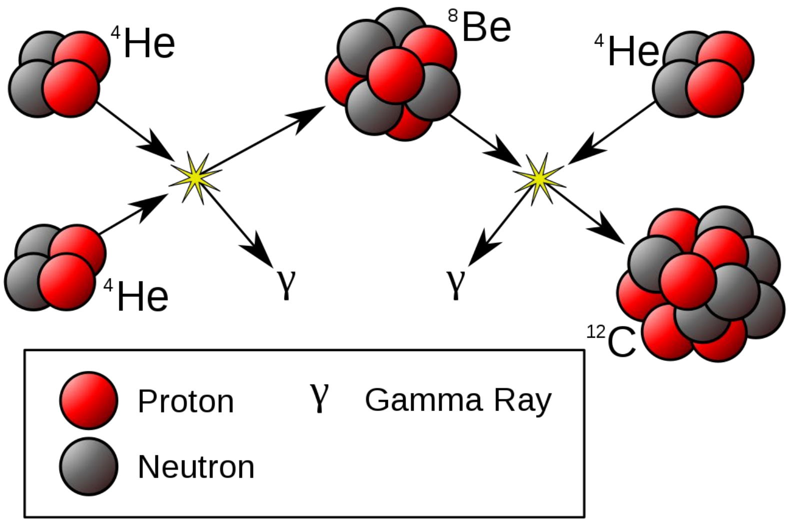


Triple Alpha Fusion – 7.725 MeV

Triple α fusion occurs in old, post main series stars.

${}^{4}_{2}He + {}^{4}_{2}He \rightarrow {}^{8}_{4}Be + \gamma$

${}^{8}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + \gamma$



What does $E = mc^2$ mean?

When Einstein wrote his paper, he showed that due to special relativity an object which loses some kinetic energy E lost mass $\frac{E}{c^2}$

Although it is written as $E = mc^2$ it is actually written $m = \frac{E}{c^2}$
What this shows is that mass is actually just a way we measure the net energy content of an object. The inherent mass an object has actually comes from the energy the Higgs field gives the particles it is made of. Furthermore, mass is not a real property, rather it is a manifestation of energy and is how we measure it.

The Big Bang and the Origins of the Elements

The Big Bang theory originates from General Relativity, where the solution to Einstein's equations shows that the universe must have a beginning.

The Big Bang Theory is the currently accepted theory that the universe began as infinitesimal point and expanded from that point. The theory describes the very expansion of space itself, that is if space is made of 4 dimensions (x, y, z, ct) then these dimensions are what expanded.

This early universe was very dense in energy and so matter was able to spring out of the energy of the vacuum in the form particle anti-particle pairs. Initially, the temperature of the universe was very great, so the particles were moving too fast to combine and quickly collided with their respective anti-particles.

The energy from these collisions was initially released as gamma rays. As the universe expanded doppler shift occurred, eventually lowering the energy of the photons

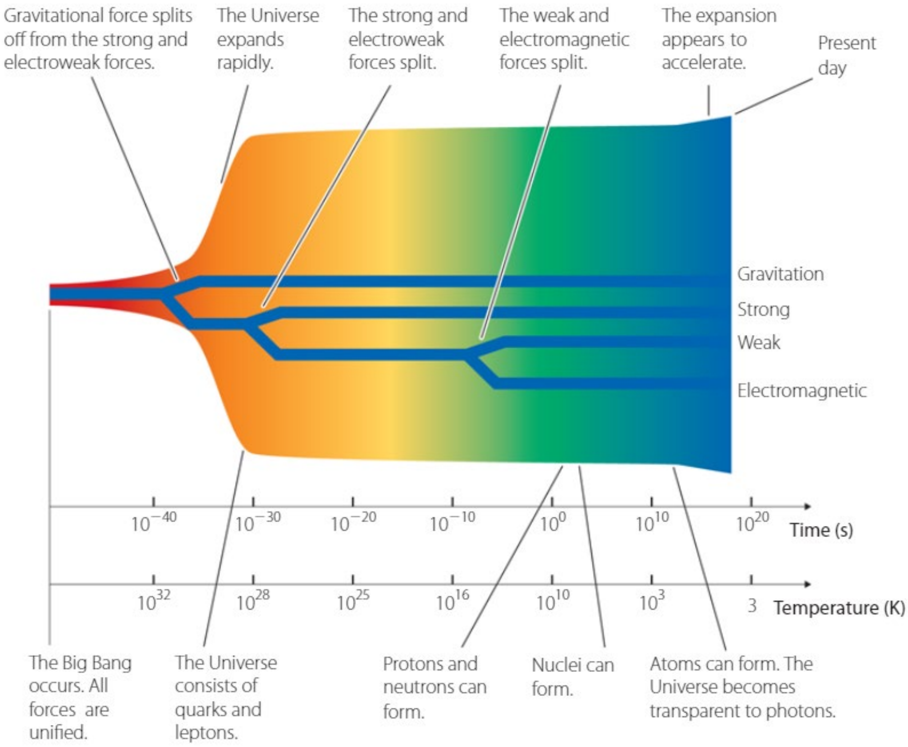
As the universe expanded, the temperature of the particles decreased, and they were able to combine to form protons and neutrons.

Although particles and anti-particles were created in equal parts, the asymmetry of the weak nuclear interaction is attributed as one of the possible reasons there is far less anti-matter in the universe today. This is still one of the great mysteries of physics and is yet to be fully modelled.

Timeline of the Big Bang

It is believed that, originally, Gravity; Electromagnetism; the Strong Force; and the Weak Force were all one force. It is believed that Gravity was the first to split as its own force, then the strong force, leaving the Electroweak force (the united electromagnetic and weak force which has been mathematically proven).

This is why Physicists would like to find a theory of everything so they can finally prove or disprove the theory.



Inflation

This is an un-proven theory which says that if the universe underwent a brief period where space doubled 100 times then the homogeneity of the universe can be explained. There is currently no evidence for inflation, though it is widely accepted.

The Cosmic Microwave Background and Cosmological Redshift

Initially, large amounts of radiation was emitted due to matter anti-matter annihilation in the form of gamma rays. As the universe expanded the space taken up by the photons expanded, increasing the wavelength of the photons. This is known as cosmological redshift and is also what redshifts the light coming from distant galaxies.

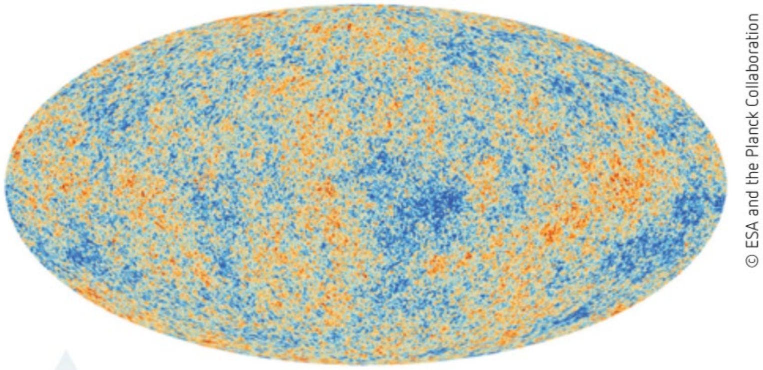


FIGURE 13.5 The remnant radiation from the Big Bang has been stretched by the expansion of the Universe. It is now observed in the microwave region of the electromagnetic spectrum.

The Hubble Constant and Spatial Expansion

Alexander Friedmann was the first to show that solutions to Einstein's field equations had possible solutions for both an expanding and contracting universe. Einstein did not readily accept this idea but with empirical evidence, eventually admitted that Friedmann was correct (after Friedmann had died from typhoid).

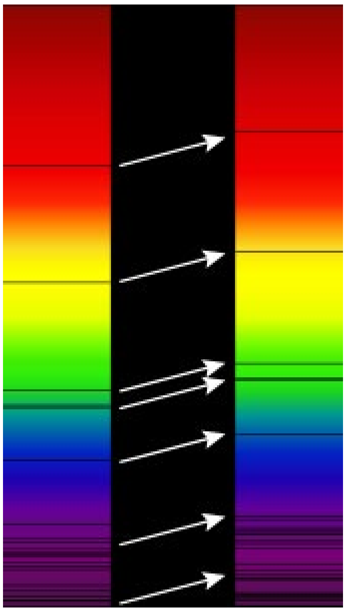
Georges Lemaitre found a similar solution and used empirical astronomical data to show that the universe was expanding. Due to the fact that all points in space are expanding away from each other, objects which are further away move away at a faster speed (since there is more points in space moving away from each other). Using the solution that the universe is expanding at a constant rate, Lemaitre concluded that $\frac{v}{D} = constant$.

Hubble was able to later use empirical evidence to show this relationship and measure the constant. We now call this relationship Hubble's Law $\frac{v}{D} = H_0$ where H_0 is Hubble's constant.

In reality the Hubble constant changes with distance, showing an accelerating speed of expansion.

Proof for Spatial Expansion

The evidence for this expansion is the emission spectra of stars, where spectral lines of similar stars are shifted, and the further the star is, the more it is shifted.



Absorption Spectra as described by Atomic Theory

The absorption spectrum of a star is given by the gaseous particles in its atmosphere. Photons which are emitted by the star that can be absorbed by atoms in the atmosphere are absorbed by those atoms, the re-emitted later with a random direction. The result of this is that a much lower intensity of the light reaches the earth.

Accelerating Spatial Expansion

It was believed that the most probable solution to Einstein's field equations was that space may be expanding, but that it was slowing down due to the negative potential energy of gravity. It turns out that it is possible (and allowed in the field equations) for the universe to have a positive energy content which would cause the universe to expand at an accelerating rate. This energy can be measured by making this assumption but is not directly predicted by any current theories.

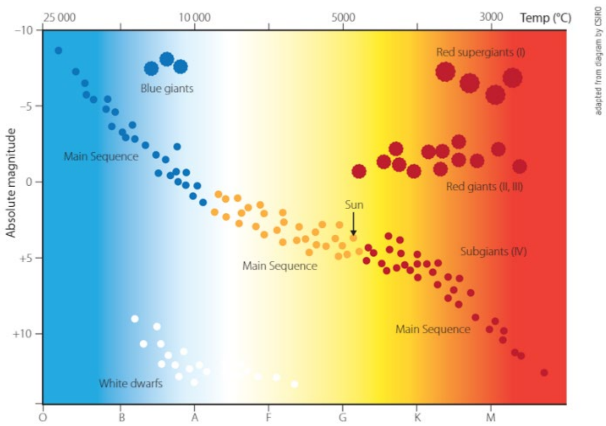
The accelerating rate of spatial expansion was measured by a physicist at ANU. What they found was that the Hubble constant for distant supernovae increased the further away you look, showing that the velocity at which objects move away from each other changes with time and with a positive gradient (i.e. $\frac{\Delta v}{\Delta t} > 0$).

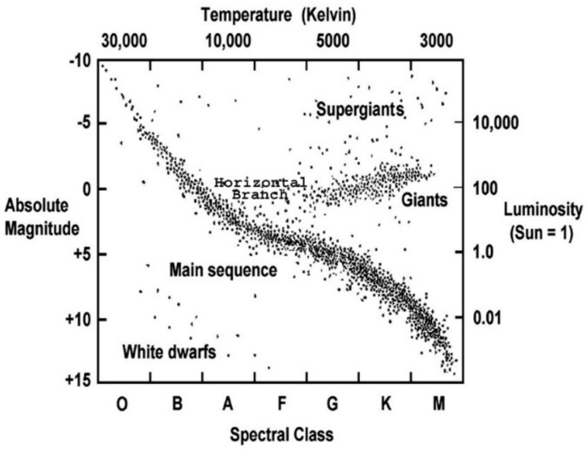
For more, you can watch the video from the man himself:
https://www.youtube.com/watch?v=55pcpTjd3BY&ab_channel=ANUTV

The mechanism for this expansion is called Dark Energy, though nothing is known about it.

The Hertzsprung-Russel (H-R) diagram

The colour, size and mass of a star can be associated with its temperature and luminosity.





The Axes:

- **Absolute Magnitude**: How bright the star would appear if it were 10 parsecs away (on a logarithmic scale)
- **Luminosity**: The power output of the star
- **Temperature**: The surface temperature of the star which causes blackbody radiation
- **Spectral Class**: The colour of the star defined at certain cut-off points.

High Mass stars are typically towards the top of the diagram since high mass stars can fuse more atoms per second and therefore are more luminous.

All stars emit light as a function of blackbody radiation (where the wavelength emitted depends on their surface temperature).

The red giants are giant because they are fusing lots of atoms producing a large outward pressure. However, the energy produced is spread across many layers, so the outer layer is cooler and therefore more red.

White dwarfs are the remains of the cores of stars which emit light due to blackbody radiation but no longer fuse nuclei. They have been compressed to the point where the only thing keeping them at a fixed radius is the outwards pressure of the electrons in their shells. The Pauli Exclusion Principle keeps them from compressing further so they maintain their size.

Neutron stars are white dwarfs where the gravity was so strong it allowed the electron shells to shrink and the electrons to combine with protons to make neutrons. Now the only thing keeping them apart is the pressure of the exclusion principle between the neutrons.

Extension Notes

Derivation of the Properties of a Star for the H-R Diagram

Measuring the properties of stars is a difficult process. The properties that are directly measurable from a star are:

- It's apparent brightness (the intensity of the light which reaches the observer)
- The angle it takes up in the sky
- The apparent colour (the redshifted peak wavelength which reaches the observer)
- The star's redshifted absorption spectrum

From these pieces of data it is possible to determine the star's temperature, actual colour, power output (luminosity), radius and its distance away from us. These are the properties which appear on the H-R diagram so it is particularly curious that these are not directly measurable.

The most accurate way of doing this is by comparing the absorption spectrum's shape to known element combinations and logically deriving how redshifted the light is and how hot it must be for certain elements to be in the atmosphere. However, this data has already been collected and used to form the Stefan-Boltzmann equation and Hubble's Law so the derivation below shows how these equations can be used to determine the properties of any star mathematically.

Conservation of Luminosity (Light Flux)

The light flux through a 3D are or luminosity of a star is conserved at any distance. This means that the light flux through a sphere of radius r outside the star is the luminosity:

$$\oiint \vec{I} \cdot d\vec{A} = L$$

$$I(4\pi R^2) = L$$

$$I = \frac{L}{4\pi R^2}$$

This is what gives rise the r^2 law in Year 11 physics ($I_1 {r_1}^2 = I_2 {r_2}^2$).

The Intensity of light which reaches the Earth determines the apparent brightness.

Identifying the Temperature of a Star without Doppler Shift

Due to the effects of cosmological redshift, the colour of stars can change, making Wein's law pretty useless if you're observing the colour of the star directly.

$$\lambda_{max} = \frac{b}{T}$$

The Stefan-Boltzmann Law is then used to determine the temperature of a star, allowing for the assessment of the doppler shift through Wein's law.

$$L = \sigma A_s T^4, \qquad \sigma = 5.7 \times 10^{-8}\ W\ m^{-2}\ T^{-4}$$

Where L is luminosity and A_s the surface area of the star.

Measuring the Temperature of stars

Now we know the theory behind light intensity, we can combine the known Stefan-Boltzmann Law (which was determined experimentally) and the Luminosity law to calculate the temperature of the star.

Since the surface area of the star is given by $A_s = 4\pi r^2$ where r is the radius of the star, we put this into the Stefan-Boltzmann Law.

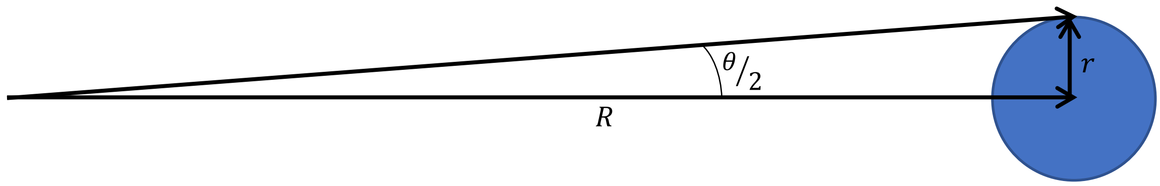
$$L = 4\pi r^2 \sigma T^4$$

We now take that and substitute it into our intensity equation:

$$I = \frac{4\pi r^2 \sigma T^4}{4\pi R^2}$$

$$I = \frac{r^2}{R^2} \sigma T^4$$

Now, we also know that the following is true, where $\frac{\theta}{2}$ is half the angle which the star takes up in the night sky:



$$\tan\frac{\theta}{2} = \frac{R}{r}$$

$$I = \sigma T^4 \tan^2\frac{\theta}{2}$$

$$T = \sqrt[4]{\frac{I}{\sigma\ \tan\frac{\theta}{2}}}$$

Since the intensity can be measured and the angle can be measured, the temperature of the star can be measured.

Applying the known temperature to find the degree of redshift

The following equations will be used

$$\lambda_{max} = \frac{b}{T}, \qquad b = 2.898 \times 10^{-3}\ Km$$

$$\frac{v_s}{H_0} = R, \qquad H_0 = 2.265 \times 10^{-18}\ s^{-1}$$

$$f' = f\frac{|v_w - v_o|}{|v_w - v_s|}, \qquad v_o = 0, \qquad v_w = c$$

$$v = f\lambda$$

It can therefore be derived that

$$\lambda' = \lambda\frac{|v_w - v_s|}{|v_w - v_o|}$$

$$\lambda' = \lambda\frac{|v_w - v_s|}{v_w}$$

$$\lambda' = \lambda\frac{c - v_s}{c}$$

$$\frac{\lambda'}{\lambda} = \left(1 - \frac{v_s}{c}\right)$$

$$\frac{v_s}{c} = 1 - \frac{\lambda'}{\lambda}$$

$$v_s = c\left(1 - \frac{\lambda'}{\lambda}\right)$$

λ' is the peak wavelength of the star which we can directly measure and by knowing T from before we can figure out the theoretical colour if the star were not moving $\left(\lambda = \frac{b}{T}\right)$ so now we know how fast the star is moving.

$$\frac{v_s}{H_0} = R$$

Now we know R we can use the earlier relationship $\tan\frac{\theta}{2} = \frac{r}{R}$

$$r = R\tan\frac{\theta}{2}$$

So now we know the star's size, its colour and its temperature, the last thing to find is its luminosity or its power output. We can do this two ways:

$$L = 4\pi\, I\, R^2 \qquad\qquad L = 4\pi\sigma r^2 T^4$$

Now we know all properties of the star and we can plot it on the H-R diagram, all by measuring its colour, brightness and size in the night sky.

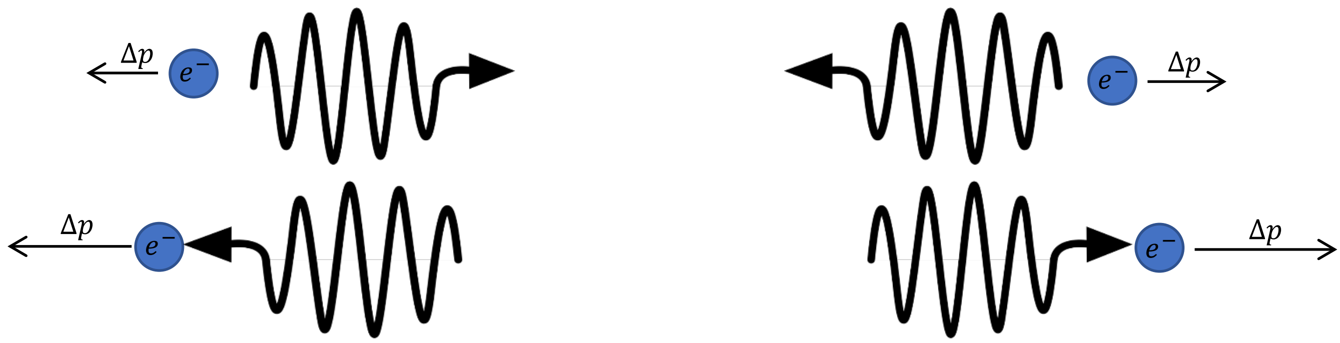
Quantum Forces

In Quantum Physics (specifically Quantum Electrodynamics and Quantum Chromodynamics) particles create a force by exchanging virtual particles. These particles determine the strength of the force and the range.

Virtual particles obey the rule $\Delta E \Delta t < \frac{\hbar}{2}$ where ΔE is the particles uncertainty in energy, Δt is its lifetime. Virtual particles such as electrons and positrons are able to 'pop' in and out of existence from the vacuum of space so long as they obey this rule and the conservation laws.

The Electromagnetic Force

The Particle Theory of electromagnetism is described by Quantum Electrodynamics. This theory predicted that photons which are undetectable are passed between electric particles and they carry the electromagnetic force. For repulsive forces, the photons leave the particles in the direction of the other, due to conservation of momentum, the particles must move apart. When the photons collide with the opposite particle, they again must move away.



The same is apparently true attractive forces except the photons carry negative momentum making the particles move towards each other. Such properties are allowed for virtual particles.

The Weak and Strong Force

The same is apparently true for the weak and strong force as with the electromagnetic force. The only difference is that the force carrying boson is different. In the case of the strong force and the weak force the boson has a mass and therefore its range is reduced.

By rearranging the uncertainty principle with $E = \gamma mc^2$, we find $\Delta t < \frac{\hbar}{2\Delta \gamma mc^2}$

Therefore, with a large mass, a larger uncertainty is allowed and therefore Δt must be smaller.

This is an (albeit lacking) explanation of why the Strong and Weak nuclear forces have a small range.

Evidence for Virtual Particles

The primary evidence for virtual particles is the Casimir Effect, where very close parallel plates (in a vacuum) will move together due to a greater number of virtual particles being able to 'pop' in and out of existence outside the plates than inside the plates. This pushes the plates together due to a greater pressure outside than inside the plates.

The Strong Force

The Strong Force is one of the four fundamental forces in the Standard Model of particle physics. The strong force is what binds three Quarks together into the proton (Up + Up + Down) or the neutron (Up + Down + Down). The strong force is mediated by gluons which 'glue' the quarks together with immense force and Mesons which are made up by Quark – Anti-Quark pairs.

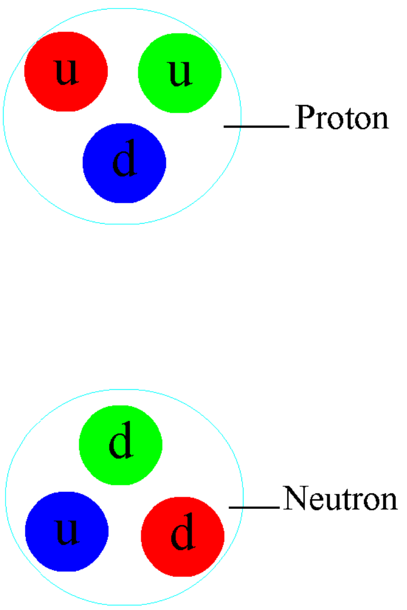
The strong force is what binds quarks. The nuclear force as it is sometimes called is what binds nearby Hadrons. This is a side effect of the strong interaction but is not the true strong force.

The nuclear force occurs when any two Hadrons (such as protons and neutrons) are near each other (less than the diameter of a proton). When a quark is pulled from one of the Hadrons, the potential energy created generates a new quark, which is sucked back into the Hadron, and an anti-quark which attaches to the detached quark to form a meson. This meson is then exchanged with the other Hadron, with the anti-quark annihilating and the new quark replacing the annihilated one. This same process occurs over and over.
This process is sort of like a covalent bond between atoms except the Hadrons share quarks not electrons and it is this process which bonds protons and neutrons.

In the process of sharing quarks (through the intermediary meson), protons and neutrons can swap places. This can happen through an up and down quark transmuting into the other during the swap.

The strong force interaction between nucleons is sometimes called the nuclear force (as it is a side effect of the strong force between quarks). There is a limit in the range of this interaction of about the diameter of a proton due to the limited range that the intermediary meson can travel before decaying.

The strong (nuclear) force is around 100 times stronger than the electric force between protons which may serve as an explanation as to why atoms with around 100 protons are unstable.



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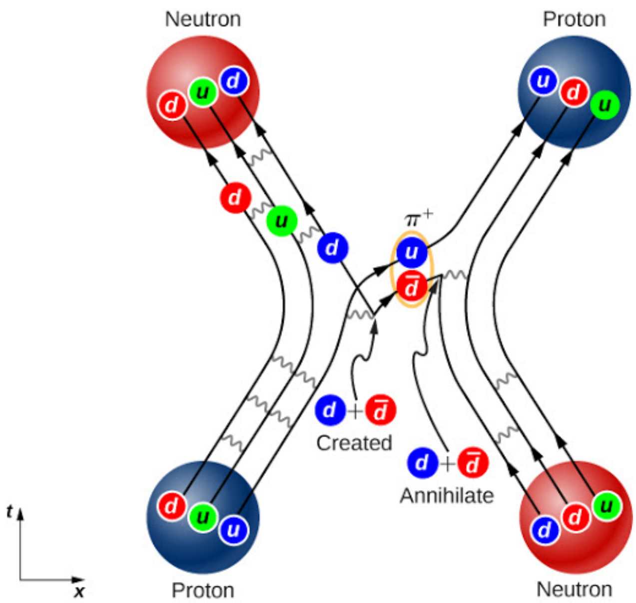

The Weak Force

The Weak Force is more probabilistic in nature than the Strong force and describes the likelihood for a particle such as a proton or neutron to change the 'flavour' of one of its quarks. Quantum physics allows for slight deviations in mass of particles and when a particle such as an up quark $\left(U^{+2/3}\right)$ has too much mass (such as from a positive potential energy or kinetic energy), it can decay into a W^+ boson and down quark $\left(D^{-1/3}\right)$ – remembering that while W bosons have mass, they are only force carrying particles for the Weak force and exist for short periods of time.

The time that a W boson exists for can be calculated using Heisenberg's uncertainty principle for virtual particles: $\Delta E \Delta t < \frac{\hbar}{2} \Rightarrow \Delta t < 10^{-24}\ s$.

The W^+ boson then decays into a positron (e^+) and neutrino (ν_e).

A similar process occurs with neutrons transmuting into protons except a down quark becomes an up quark and a W^- boson becomes an electron (e^-) and anti-neutrino. $(\bar{\nu}_e)$

The remaining mass of the W boson is lost in the form of energy as a photon (γ).

The transmutation from an Up to Down quark is what characterises the Weak force.

The reason a mass difference large enough to create a W boson might arise can be explained by the increased potential energy that arises when many nucleons (protons and neutrons) are near each other.

It is also worth noting that the weak force only acts on particles