Random experiment - An experiment, trial or observation that can be repeated numerous times under the same conditions e.g. flipping a coin

Sample space - A complete list of all possible outcomes of a random experiment is called the sample space and is denoted by S.

e.g. rolling a fair 6-sided dice $S = \{1, 2, 3, 4, 5, 6\}$

Event - An event is a property associated with the outcomes of a random experiment. It is represented by a subset of a sample space

e.g. rolling a fair 6-sided dice and getting an even number Even $= \{2, 4, 6\}$

Independent events - When two events are said to be independent of each other, this means that the probability that one event occurs in no way affects the probability of the other event occurring e.g. you roll a die and then flip a coin.

Mutually exclusive events - Two events that can not occur at the same time e.g. a flip of a coin can be heads or tails but not both

Complement of event A - The complement of an event A is denoted by A' and is the event that A does not occur

Notation

$\cap$ → Intersect (Overlap)

$\cup$ → Union ( ) → Inside circles

' → Complement (not)

Combined events

(i) The event that "A or B or both occur" is

called the union of A and B

$A \cup B$

(ii) The event that "both A and B occur" is

called the intersection of A and B

$A \cap B$

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Example:

A coin is tossed 3 times and the outcomes are to be recorded in the order that they occur.

S = {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

Some examples of events in this case are:

A = exactly two heads = {HHT, HTH, THH}

B = third toss is a head = {HHH, THH, TTH, HTH}

C = all three the same outcome = {HHH, TTT}

Should be union

Give a verbal description and list the elements of the events

(a) $A \cup B$ (b) $A \cap C$ (c) $\cancel{A \cup B'}$ $A \cap B'$

(a) {HHT, HTH, THH, HHH, TTH}

(b) None

(c) {HHT}

Venn Diagrams

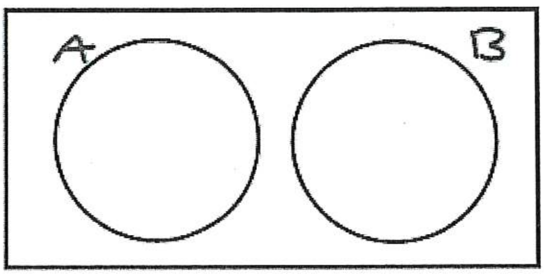
Venn Diagrams are used as visual representations of probability.

Examples:

1. Roll a fair die

   Event A is an even number

   Event B is an odd number



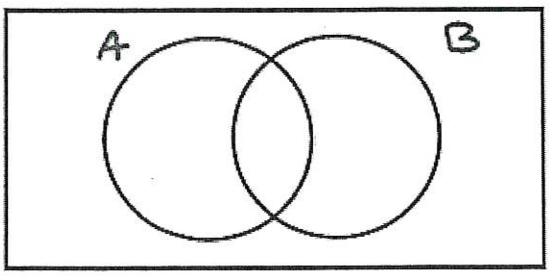
Mutually Exclusive

$P(A \cap B) = 0$

2. Roll a fair die

   Event A is a number less than 4

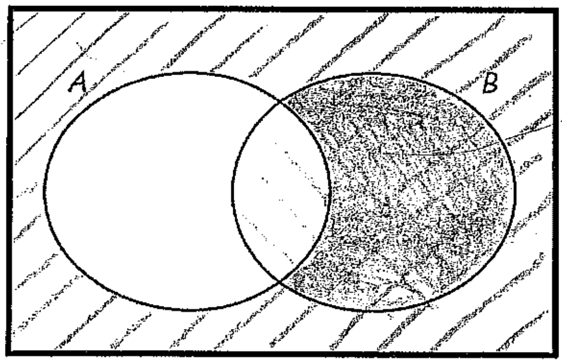
   Event B is an odd number



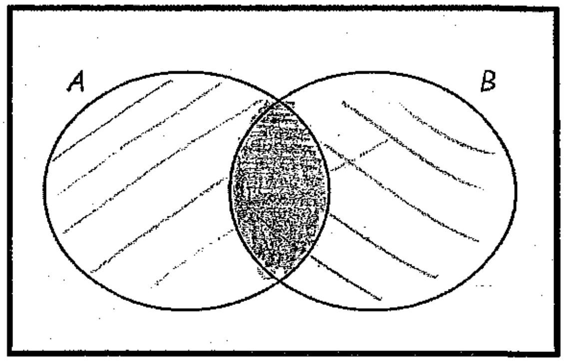
Venn diagrams

$A \cap B$ - only where both shaded parts overlap
$A \cup B$ - all of the shaded parts

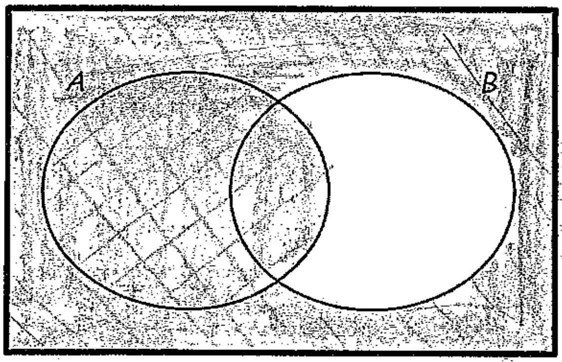
$A' \cap B$



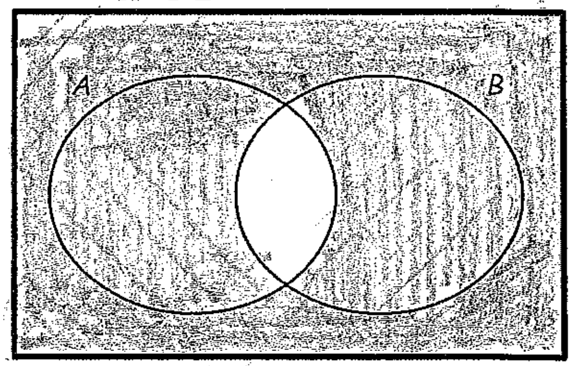
$A \cap B$



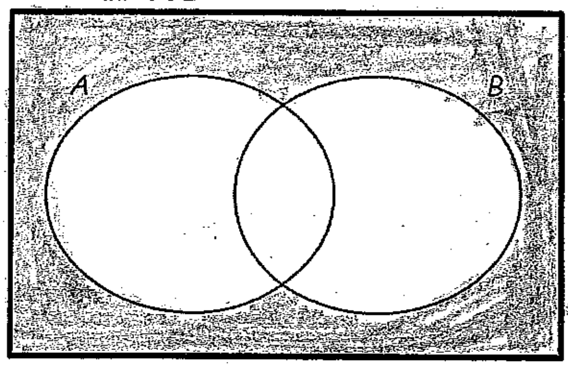
$A \cup B'$



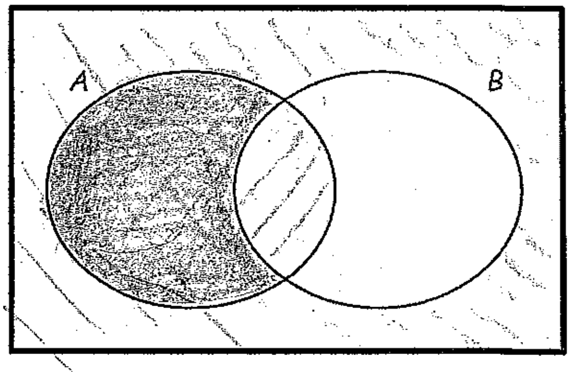
$A' \cup B'$



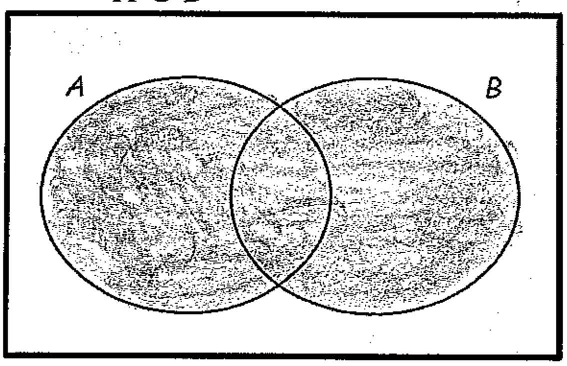
$A' \cap B'$



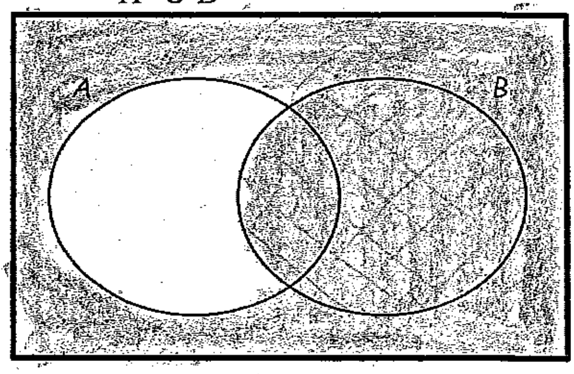
$A \cap B'$



$A \cup B$

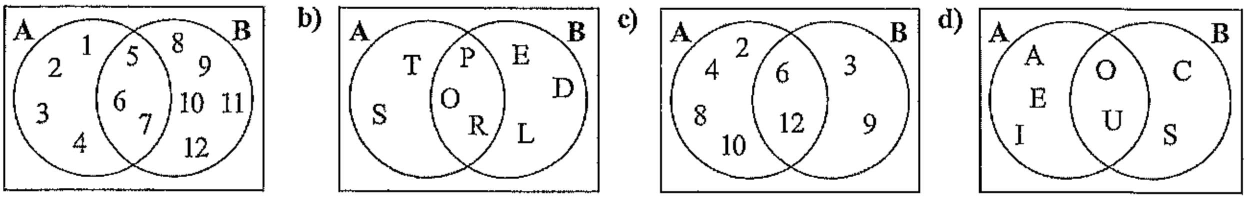


$A' \cup B$



Set Notation and Venn Diagrams

1. For each of the following, list the elements of (i) $A \cap B$ and (ii) $A \cup B$.



a)	i)	$A \cap B = \{$ 5, 6, 7 $\}$	ii)	$A \cup B = \{$ 1 2 3 4 5 6 7 8 9 10 11 12 $\}$
b)	i)	$A \cap B = \{$ P O R $\}$	ii)	$A \cup B = \{$ S T O P R E D L $\}$
c)	i)	$A \cap B = \{$ 6 12 $\}$	ii)	$A \cup B = \{$ 2 4 6 8 10 6 12 3 9 $\}$
d)	i)	$A \cap B = \{$ O U $\}$	ii)	$A \cup B = \{$ A E I O U C S $\}$

2. Find $A \cap B$ for each of the following, where the universal set is the set of all real numbers.

a) $A = \{x: 0 < x < 50\}$ $B = \{x: 30 < x < 100\}$ $A \cap B = \{$ $30 < x < 50$ $\}$

b) $A = \{x: 20 < x \leq 30\}$ $B = \{x: 30 \leq x < 100\}$ $A \cap B = \{$ $x = 30$ $\}$

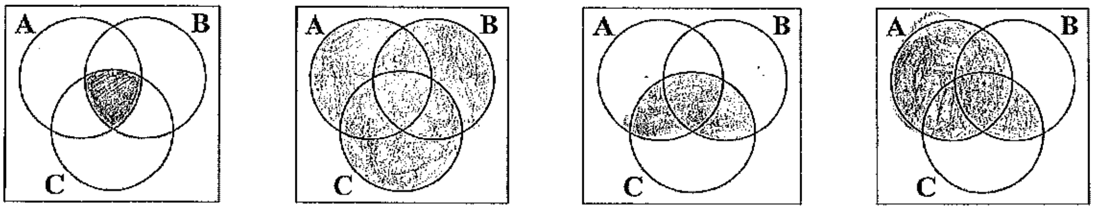
c) $A = \{x: x \leq 100\}$ $B = \{x: x \leq 50\}$ $A \cap B = \{$ $x \leq 50$ $\}$

d) $A = \{x: x < 50\}$ $B = \{x: x > 60\}$ $A \cap B = \{$mutually exclusive$\}$

e) $A = \{x: x > 20\}$ $B = \{x: x \leq 150\}$ $A \cap B = \{$ $20 < x \leq 150$ $\}$

3. Shade each of the following regions on the diagrams below.

a) $A \cap B \cap C$ b) $A \cup B \cup C$ c) $(A \cup B) \cap C$ d) $A \cup (B \cap C)$



4. For each of the following, list the elements of A', the complement of set A.

a) $\xi = \{1,2,3,4,5,6,7,8,9,10\}$ $A = \{\text{even numbers}\}$ $A' = \{$ 1, 3, 5, 7, 9 $\}$

b) $\xi = \{\text{prime numbers}\}$ $A = \{\text{odd numbers}\}$ $A' = \{$ 2 $\}$

c) $\xi = \{\text{factors of } 120\}$ $A = \{x: x < 20\}$ $A' = \{$ 20, 24, 30, 36, 40, 60, 120 $\}$

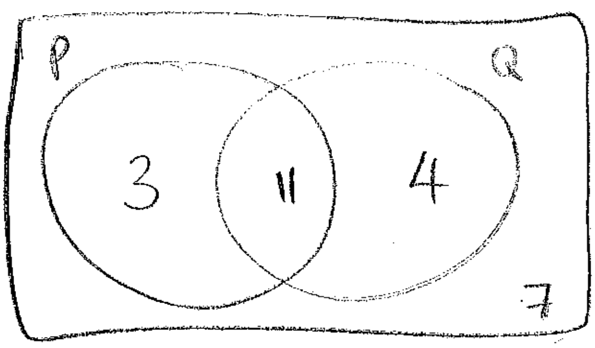
5. Given that $n(P \cap Q) = 11$, $n(P \cup Q)' = 7$, $n(Q') = 10$ and $n(\xi) = 25$, draw a Venn diagram to help you find:

a) $n(Q) =$ 15

b) $n(P') =$ 11

c) $n(P \cap Q)' =$ 14

d) $n(P \cup Q') =$ 21



Probability of Events: ADDITION LAW

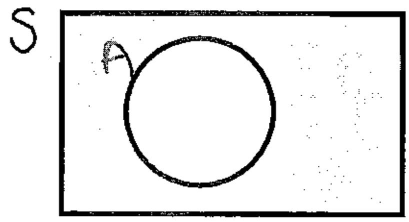
We use areas to represent probabilities.

Since P(S) = 1 the area of the rectangle is 1 unit.

Examples

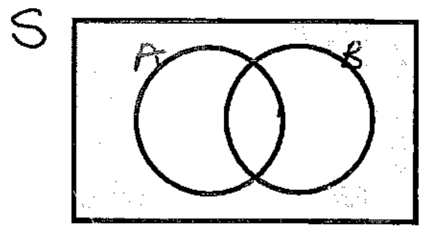
1. For any event A, the probability that A will not occur is given by

$P(A') = 1 - P(A)$



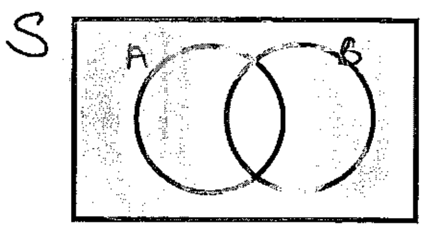
2. For any two events A and B

$P(A' \cap B') = 1 - P(A \cup B)$



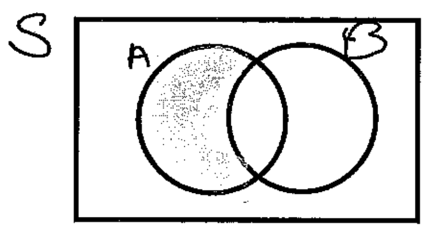
3. For any two events A and B

$P(A' \cup B') = 1 - P(A \cap B)$



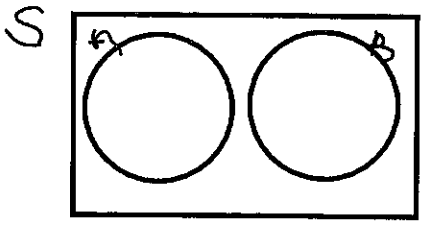
4. For any two events A and B

$P(A \cap B') = 1 - P(A' \cup B)$



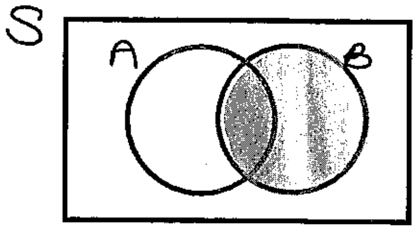
Rule 1: If events A and B are mutually exclusive events then

$$P(A \cup B) = P(A) + P(B)$$



Rule 2: Generally, for events A and B

* $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



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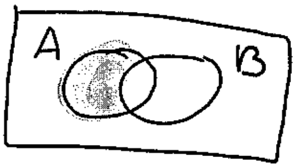
Numerical Probabilities

Examples

1. The 2 events A and B are such that

   $P(A) = 0.6$ $P(B) = 0.3$ $P(A \cap B) = 0.2$



   Evaluate

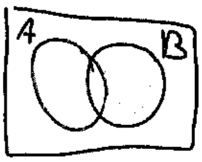
   (a) $P(A')$ (b) $P(A \cup B)$ (c) $P(A \cap B')$

   (d) the probability that only one of A and B will occur

a) $1 - 0.6 = 0.4 = P(A')$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.6 + 0.3 - 0.2$
$= \underline{0.7}$

c) $P(A \cap B')$
$= P(A) - P(A \cap B)$
$= 0.6 - 0.2$
$= \underline{0.4}$

d) 

$= P(A) + P(B) - 2P(A \cap B)$
$= \underline{0.5}$

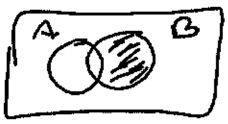
2. The 2 events A and B are such that

   $P(A) = P(B) = p$ $P(A \cap B) = 0.4$ $P(A \cup B) = 0.7$

   Evaluate

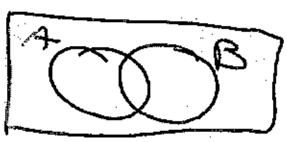
   (a) p (b) $P(A' \cap B)$ (c) $P(A' \cap B')$

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.7 = p + p - 0.4$
$0.7 = 2p - 0.4$
$1.1 = 2p$
$\underline{0.55 = p}$



b) $P(A' \cap B) = P(B) - P(A \cap B)$
$= 0.55 - 0.4 = 0.15$

c) $P(A' \cap B') = 1 - P(A \cup B)$
$= 1 - 0.7$
$= \underline{0.3}$



Numerical Probabilities

Examples

1. The 2 events A and B are such that
   $P(A) = 0.6 \qquad P(B) = 0.3 \qquad P(A \cap B) = 0.2$

   Evaluate

   (a) $P(A')$ (b) $P(A \cup B)$ (c) $P(A \cap B')$

   (d) the probability that only one of A and B will occur

2. The 2 events A and B are such that

   $P(A) = P(B) = p \qquad P(A \cap B) = 0.4 \qquad P(A \cup B) = 0.7$

   Evaluate

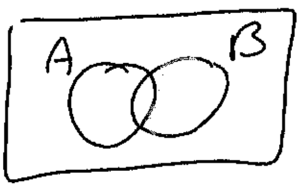
   (a) p (b) $P(A' \cap B)$ (c) $P(A' \cap B')$

2a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = p + p - 0.4$

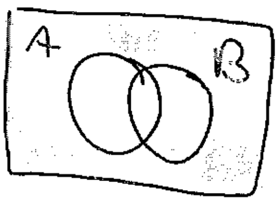
$1.1 = 2p$

$p = 0.55$

b) 

$= P(B) - P(A \cap B)$

$= 0.55 - 0.4 = \underline{0.15}$

c) 

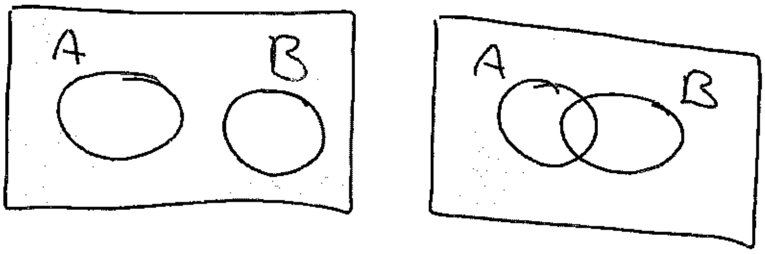
$P(A' \cap B') = 1 - P(A \cup B)$

$= 1 - 0.7 = \underline{0.3}$

3. Two events A and B are such that

$P(A) = 0.35$ $P(B) = 0.45$ $\underline{P(A' \cap B') = 0.6}$

Determine whether events A and B are mutually exclusive.



$P(A) + P(B) + P(A' \cap B') = 1$

if mutually exclusive

$0.35 + 0.45 + 0.6 = 1.4$

$\therefore$ not mutually exclusive

3. Two events A and B are such that

$P(A) = 0.35 \qquad P(B) = 0.45 \qquad P(A' \cap B') = 0.6$

Determine whether events A and B are mutually exclusive.

If mutually exc.

$P(A) + P(B) = 0.35 + 0.45$

$= \underline{0.8}$

$P(A' \cap B') =$

$0.6 \neq 0.2$ so not mutually exc.

Multiplication Law for Independent Events

Basically like GCSE :-)

Independent Events - When an event has no effect on another

For independent events A and B:

$$\boxed{P(A \cap B) = P(A) \times P(B)}$$

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Examples

1. The two events A and B are such that

   P(A) = 0.3 P(B) = 0.2 P(A U B) = 0.44

   (a) Show that A and B are independent

   (b) Calculate the probability of exactly one of the events occurring

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.3 + 0.2 - 0.44$$
$$= 0.06$$
$$P(A) \times P(B) = 0.3 \times 0.2 = 0.06 = P(A \cap B)$$

$\therefore$ independent

b)  $P(A \cup B) - P(A \cap B)$

$0.44 - 0.06 = \underline{0.38}$

2. The independent events A and B are such that

   P(A) = 0.6 P(B) = 0.3 $P(A \cap B) = 0.18$

   Find

   (a) P(A U B) [3]

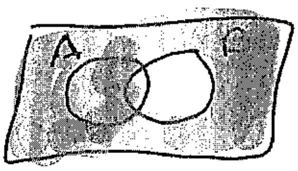
   (b) P(A U B') [3]

a) $P(A \cup B) = P(A) + P(B) - P(A \cup B)$

$= 0.6 + 0.3 - 0.18$

$= 0.9 - 0.18$

$= \underline{0.72}$

b) 

$= 1 - [P(B) - P(A \cap B)]$

$= 1 - [0.3 - 0.18]$

$= 1 - 0.12$

$= \underline{0.88}$

3. A and B are two events such that

   $P(A) = 0.35$ $P(B) = 0.25$ $P(A \cup B) = 0.5$

   Find

   (a) $P(A \cap B)$

   (b) $P(A')$

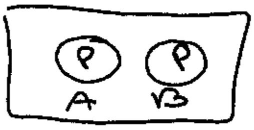
   (c) $P(A \cup B')$

a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.1$

b) 0.65

c) $0.85 = 1 - [P(B) - P(A \cap B)]$

4. The events A and B are such that $P(A) = P(B) = p$ and $P(A \cup B) = 0.64$
   (a) Given that A and B are mutually exclusive, find the value of p
   (b) Given instead that A and B are independent, show that $25p^2 - 50p + k = 0$ where k is a constant whose value should be found, and hence find the value of p

a) 

$$P(A \cup B) = P(A) + P(B)$$
$$0.64 = p + p$$
$$0.64 = 2p$$
$$\underline{p = 0.32}$$

b) $$P(A \cap B) = P(A) \times P(B)$$
$$= p \times p = p^2$$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.64 = p + p - p^2$$

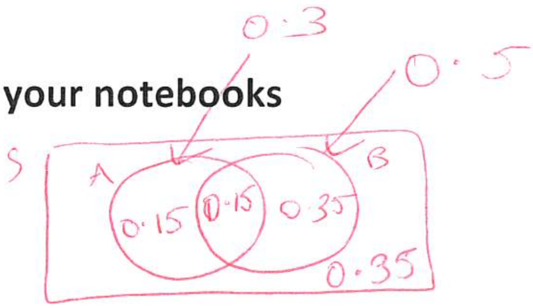
$$p^2 - 2p + 0.64 = 0 \quad \times 25$$
$$25p^2 - 50p + 16 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{50 \pm \sqrt{(-50)^2 - 4(25)(16)}}{2(25)}$$
$$= \frac{50 \pm \sqrt{900}}{50}$$

$$\frac{50+30}{50} \quad \text{or} \quad \frac{50-30}{50}$$
$$= \frac{80}{50} = \underline{\underline{1.6}} \qquad \frac{20}{50} = \underline{\underline{0.4}}$$

$$\therefore \underline{p = 0.4}$$

Exercise – Please answer in your notebooks



A and B are 2 events such that:

$P(A) = 0.3$

$P(B) = 0.5$

and $P(A \cap B) = 0.15$

Find:

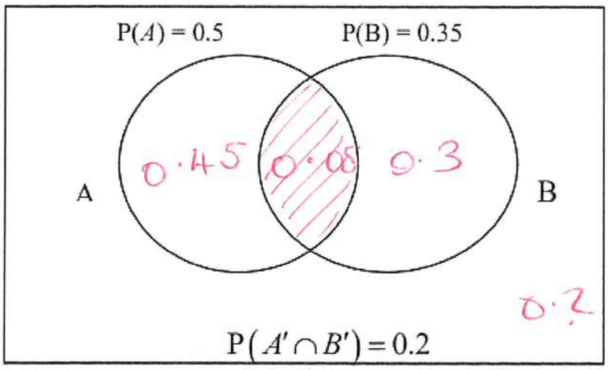
(i) $P(A')$ $1 - P(A) = 1 - 0.3 = \underline{0.7}$

(ii) $P(B')$ $1 - P(B) = 1 - 0.5 = \underline{0.5}$

(iii) $P(A \cup B)$ $P(A) + P(B) - P(A \cap B) = 0.3 + 0.5 - 0.15$

$= 0.8 - 0.15 = \underline{0.65}$

Probabilities relating to the events A and B are given in the diagram below:



$P(A \cup B) = 1 - P(A' \cap B')$

$= 1 - 0.2$

$= 0.8$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.5 + 0.35 - P(A \cap B)$

$0.8 = 0.85 - P(A \cap B)$

$P(A \cap B) = \underline{0.05}$

Find the probability that both A and B occur.

$\downarrow$

$P(A \cap B)$

S1 January 2007 question 2

The events A and B are such that $P(A) = 0.48$, $P(B) = 0.38$, $P(A \cap B) = 0.28$.

Calculate

(a) $P(A \cup B)$, $P(A) + P(B) - P(A \cap B) = \underline{0.58}$ [2]

$0.48 + 0.38 - 0.28 =$

(b) $P(A' \cap B')$, $1 - P(A \cup B) = 1 - 0.58 = \underline{0.42}$ [2]

~~(c) $P(B|A')$.~~ [4]

The two events A, B are such that $P(A) = 0.65$, $P(A \cup B) = 0.93$.

Evaluate $P(B)$ given that

(a) A and B are mutually exclusive, [2]

$P(A) + P(B) = P(A \cup B)$

$0.65 + P(B) = 0.93$

$P(B) = \underline{0.28}$

*(b) A and B are independent. [4]

$P(A \cap B) = P(A) \times P(B)$

$P(A) + P(B) - (P(A \cap B)) = P(A \cup B)$

$P(A) + P(B) - (P(A) \times P(B)) = P(A \cup B)$

$0.65 + P(B) - 0.65P(B) = 0.93$

$P(B)(1 - 0.65) = 0.28$

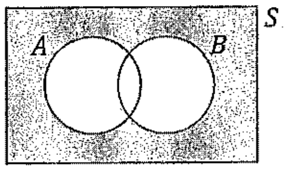
$P(B) = \dfrac{0.28}{0.35}$

$P(B) = 0.56$

Name ____________________

SOLUTIONS

1 (a) Probability of event A only = 0.45 − 0.25 = 0.2
Probability of event B only = 0.30 − 0.25 = 0.05



$P(A \cup B) = 0.2 + 0.25 + 0.05 = 0.5$

Alternatively, you could use

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.45 + 0.30 - 0.25 \\ &= 0.5 \end{aligned}$$

(b) $P(A' \cap B') = 1 - 0.5 = 0.5$

2 (a) (i) When throwing two dice, the sample space consists of 36 pairs of scores.
The number of pairs the same = 6
Hence, P(scores are equal) = 6/36 = 1/6

(ii) The possible scores where Amy's score is higher are:
(2, 1)
(3, 1), (3, 2)
(4, 1), (4, 2), (4, 3)
(5, 1), (5, 2), (5, 3), (5, 4)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5)

Hence, Probability that Amy's score is higher $= \frac{15}{36} = \frac{5}{12}$

(b) Sample space consists of:
(1, 3), (3, 1), (2, 2)

Probability scores are equal $= \frac{1}{3}$

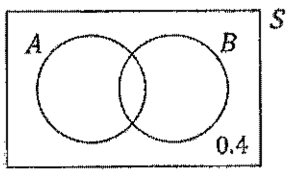
3 (a) Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ and $P(A \cap B) = P(A) \times P(B)$, we obtain

$$0.5 = 0.3 + P(B) - 0.3 \times P(B)$$
$$0.2 = 0.7\,P(B)$$

Hence, $P(B) = 0.2857$ (correct to 4 d.p.)

(b) $$\begin{aligned} \text{P(exactly one event occurs)} &= P(A \cup B) - P(A \cap B) \\ &= 0.5 - 0.3 \times 0.2857 \\ &= 0.5 - 0.0857 \\ &= 0.4143 \end{aligned}$$

4 (a)



From the Venn diagram, $P(A \cup B) = 1 - 0.4 = 0.6$

Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$,

we obtain $0.6 = 0.4 + 0.35 - P(A \cap B)$.

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Hence, $P(A \cap B) = 0.15$ and as there is an overlap between events A and B, the two events are not mutually exclusive.

(b) $P(A) \times P(B) = 0.4 \times 0.35 = 0.14$, and as $P(A \cap B) = 0.15$, we have $P(A) \times P(B) \neq P(A \cap B)$ proving that events A and B are not independent

5 (a) As the events are independent we can use $P(A \cap B) = P(A) \times P(B)$.

$$P(A \cap B) = P(A) \times P(B)$$
$$= 0.4 \times 0.3 = 0.12$$

(b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$$= 0.4 + 0.3 - 0.12 = 0.58$$

(c) The probability that neither A nor B occur is given by $1 - P(A \cup B)$.

Probability that neither A nor B occur $= 1 - 0.58$
$= 0.42$

6 (a) Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, we obtain

$$0.5 = 0.4 + 0.2 - P(A \cap B)$$
$$P(A \cap B) = 0.1$$

(b) $P(A) \times P(B) = 0.4 \times 0.2 = 0.08$

As $P(A \cap B) \neq P(A) \times P(B)$ the events A and B are not independent.

7 (a) Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, we obtain

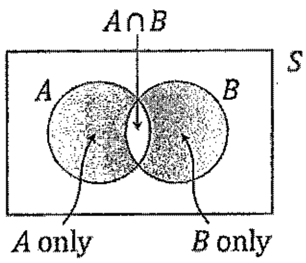
$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.2 + 0.4 - 0.52$$
$$= 0.08$$

If the events A and B are independent, then probability of both events occurring

$$= P(A) \times P(B) = 0.2 \times 0.4 = 0.08$$

As $P(A) \times P(B) = P(A \cap B)$, events A and B are independent.

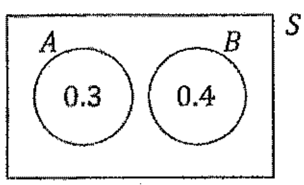
(b) $P(A \text{ only}) = P(A) - P(A \cap B)$
$$= 0.2 - 0.08$$
$$= 0.12$$



$P(B \text{ only}) = P(B) - P(A \cap B)$
$$= 0.4 - 0.08$$
$$= 0.32$$

$P(A \text{ or } B \text{ only}) = P(A \text{ only}) + P(B \text{ only}) = 0.12 + 0.32 = 0.44$

8 (a) Events A or B can occur but not both. There is no overlap.



$P(A \cup B) = P(A) + P(B)$
$$= 0.3 + 0.4 = 0.7$$

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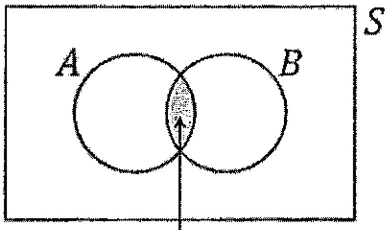
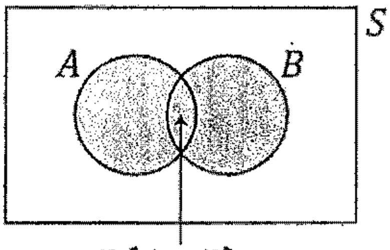
(b)





$$P(A \cap B) = P(A) \times P(B) = 0.3 \times 0.4 = 0.12$$
$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.3 + 0.4 - 0.12 \\ &= 0.58 \end{aligned}$$

9 (a) $\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ &= 0.2 + 0.3 \\ &= 0.5 \end{aligned}$

(b) $\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ &= 0.2 + 0.3 - 0.2 \times 0.3 \\ &= 0.2 + 0.3 - 0.06 \\ &= 0.44 \end{aligned}$

(c) $P(A \cup B) = P(B) = 0.3$