Contents

Discrete model of circular plate

Due to the symmetry of our structure, our study will be based on the half of it on figure 1. 4 finite elements were used to construct the discrete model.

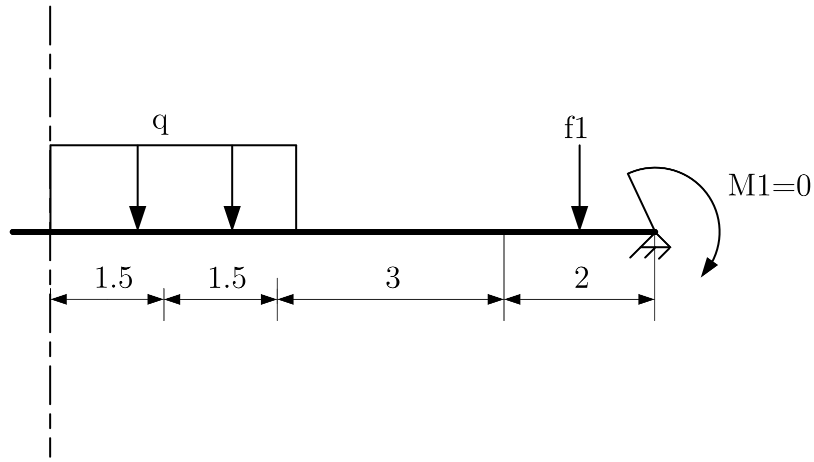

Figure 1: 1D Rod system in global coordinate system

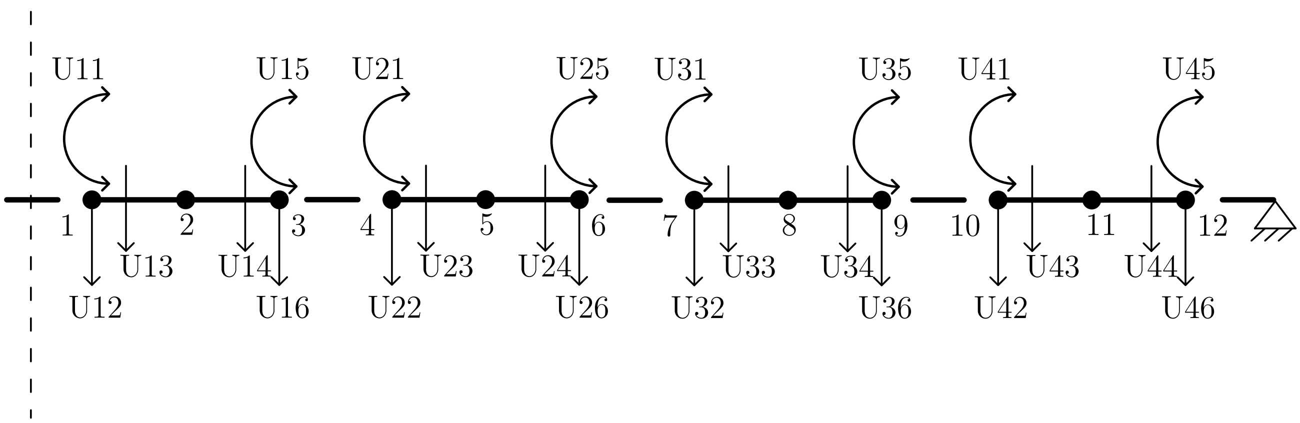

Figure 2: 1D Rod system in global coordinate system

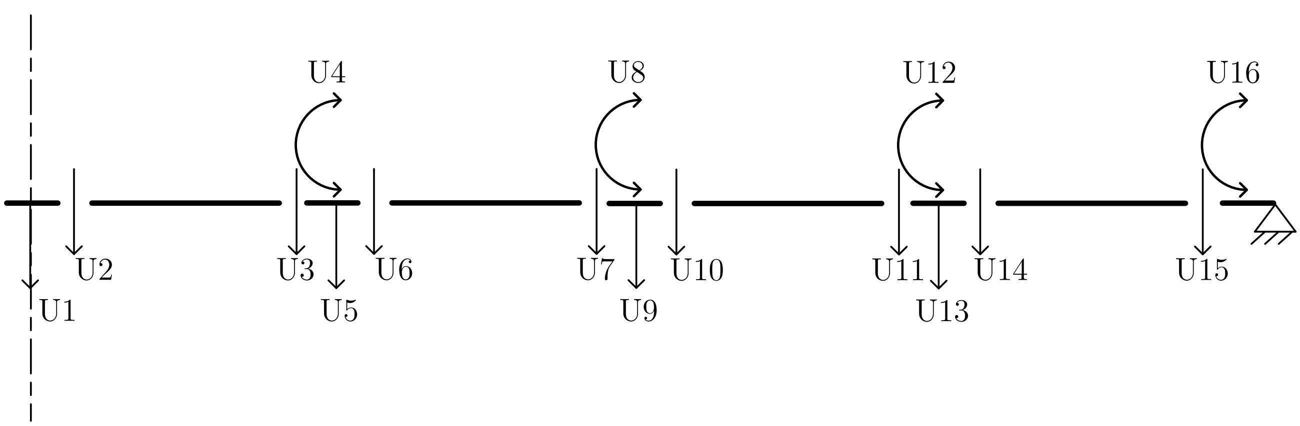

Figure 3: 1D Rod system in global coordinate system

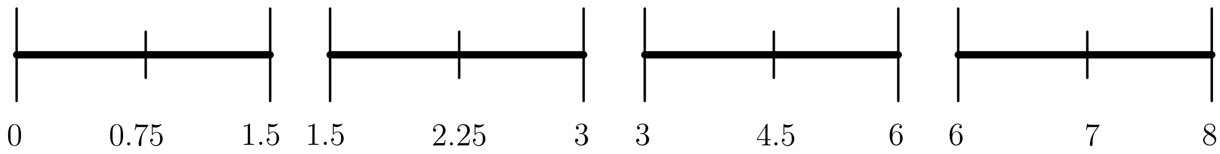

Figure 4: 1D Rod system in global coordinate system

Compatibility matrix of displacements

It's based on the discrete model. It represents a relationship among the local and global displacements. We used 4 finite elements for each one has 6 local displacement and 5 nodes have total number of global displacements m=16. The final compatibility matrix of displacements are shown on figure 5, where rows are global displacements and columns are local displacements.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5: Mitral valve structure

Coefficient matrix of equilibrium equations

$$[\overline{A}_k] = 2\pi \begin{bmatrix} \rho_{k,1} & 0 & 0 & 0 & 0 & 0 \\ 1.5\dfrac{\rho_{k,1}}{b_k} - 1 & 1 & -2\dfrac{\rho_{k,1}}{b_k} & 0 & \dfrac{\rho_{k,1}}{2b_k} & 0 \\ -\dfrac{\rho_{k,2}}{b_k} + 2 & -\dfrac{5}{6} & 2\dfrac{\rho_{k,2}}{b_k} - 2 & \dfrac{2}{3} & -\dfrac{\rho_{k,2}}{b_k} & \dfrac{1}{6} \\ -\dfrac{\rho_{k,2}}{b_k} & -\dfrac{1}{6} & 2\dfrac{\rho_{k,2}}{b_k}2 & -\dfrac{2}{3} & -\dfrac{\rho_{k,2}}{b_k} - 2 & \dfrac{5}{6} \\ 0 & 0 & 0 & 0 & -\rho_{k,3} & 0 \\ \dfrac{\rho_{k,3}}{2b_k} & 0 & -2\dfrac{\rho_{k,3}}{b_k} & 0 & 1 + 1.5\dfrac{\rho_{k,3}}{b_k} & -1 \end{bmatrix} \tag{1}$$

$A = [C]^T[\overline{A}]$ - equilibrium equation matrix, where:

$[C]^T$ - is transpose of compatibility matrix of displacements [C]

$[A]$ - is the matrix above in table

Flexibility matrix

$$[D_k] = \frac{2\pi b_k}{15K_k(1-\nu_k^2)} \begin{bmatrix} j_2 & -\nu_k j2 & 2j_1 & -2\nu_k j_1 & -\rho_{k,2} & \nu_k \rho_{k,2} \\ & j_2 & -2\nu_k j_1 & 2j_1 & \nu_k \rho_{k,2} & -\rho_{k,2} \\ & & 16\rho_{k,2} & -16\nu_k \rho_{k,2} & 2j_3 & -2\nu_k j_3 \\ & & & 16\rho_{k,2} & -2\nu_k j_3 & 2j_3 \\ & & & & j_4 & -\nu_k j_4 \\ symm. & & & & & j_4 \end{bmatrix} \tag{2}$$

where:

$K_k = \frac{E_k t_k^3}{12(1-\nu_k^2)}$ $j_1 = \rho_{k,2} - b_k$
$j_2 = 4\rho_{k,2} - 3b_k$
$j_3 = \rho_{k,2} + b_k$
$j_4 = 4\rho_{k,2} + 3b_k$

External load vector

$$F_k = \frac{2\pi b_k \rho_k}{3} \begin{Bmatrix} 3\rho_{k,2} - b_k \\ \\ 3\rho_{k,2} + b_k \end{Bmatrix} = \{\eta_k\}\rho_k \tag{3}$$

Internal forces and displacements

The equilibrium of finite element method used to solve the mentioned annular plate. The results were obtained from a MATLAB commands. Based on these calculations, the results shows the internal forces and the a distribution of global displacements along our structure. The results shows that the maximum displacement was U= 36.0471 mm in the direction gravity. The allowable displacement was U.allowable = L/250 = 64 mm so, the verification was correct based on the current geometry and material properties. Some parametric analysis were done for plate thickness as a very important parameter to reduce the displacement values.