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# Discrete model of circular plate

According to given initial conditions structure is circle width some amount of distributed load in center. Due to the symmetry of our structure, has to be studied only half of structure(fig.1).

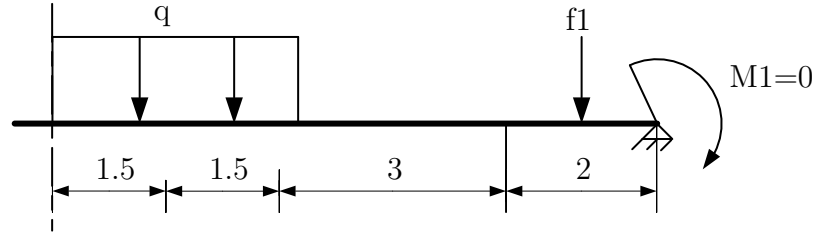


Figure 1: 1D Rod system in global coordinate system

Developed discrete model of given structure consist 4 finite elements(fig.2), local displacements and moments of each element shown on figure 3, while global ones are on figure 4.

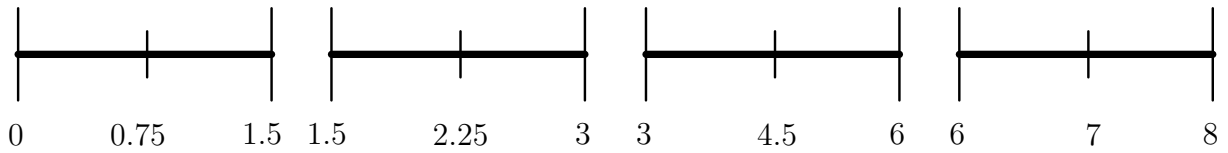


Figure 2: 1D Rod system in global coordinate system

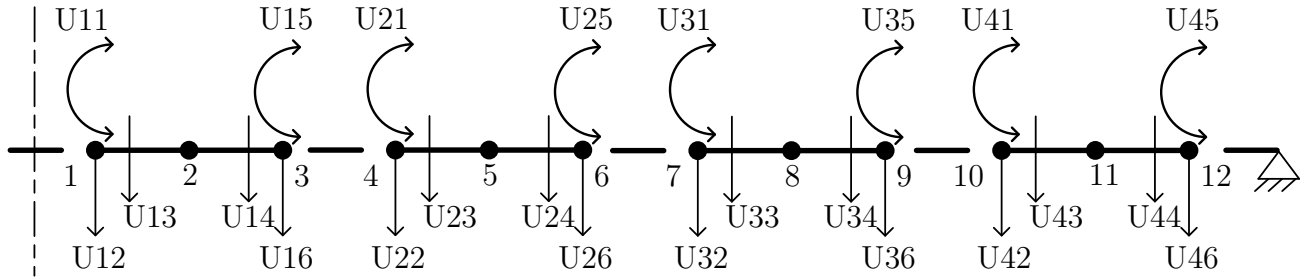


Figure 3: 1D Rod system in global coordinate system

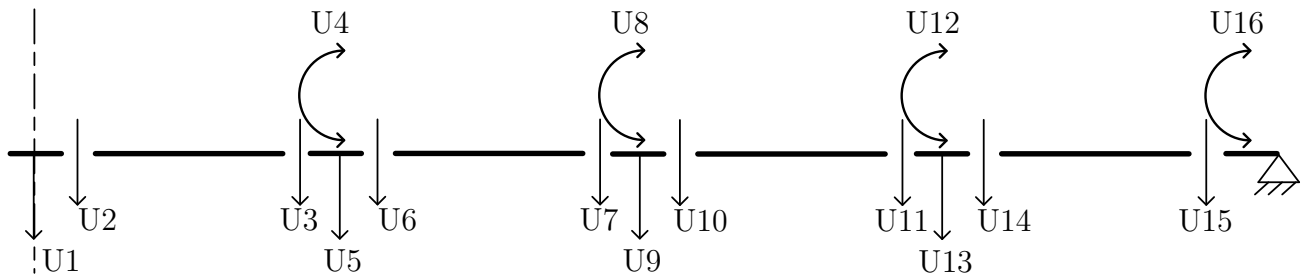


Figure 4: 1D Rod system in global coordinate system

# Compatibility matrix of displacements

It's based on the discrete model. It represents a relationship among the local and global displacements. We used 4 finite elements for each one has 6 local displacement and 5 nodes have total number of global displacements  $m=16$ . The final compatibility matrix of displacements are shown on figure 5, where rows are global displacements and columns are local displacements.

| C  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|----|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|
| 1  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 2  | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 3  | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 4  | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 5  | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 6  | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 7  | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 8  | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 9  | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 10 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 11 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 12 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 13 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 14 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 15 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1  | 0  | 0  | 0  | 0  | 0  | 0  |
| 16 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 1  | 0  | 0  | 0  | 0  | 0  |
| 17 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 1  | 0  | 0  | 0  | 0  |
| 18 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 1  | 0  | 0  | 0  |
| 19 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 1  | 0  | 0  |
| 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 1  | 0  |
| 21 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 22 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 23 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 1  |
| 24 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0  | 0  | 0  | 0  | 0  | 0  | 0  |

Figure 5: Mitral valve structure

| C = | Columns 1 through 15 |   |   |   |   |   |   |   |   |   |   |   |   |   |   | Column 16 |
|-----|----------------------|---|---|---|---|---|---|---|---|---|---|---|---|---|---|-----------|
|     | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 1   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 1                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |
| 0   | 0                    | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0         |

# Coefficient matrix of equilibrium equations

$$[\bar{A}_k] = 2\pi \begin{bmatrix} \rho_{k,1} & 0 & 0 & 0 & 0 & 0 \\ 1.5 \frac{\rho_{k,1}}{b_k} - 1 & 1 & -2 \frac{\rho_{k,1}}{b_k} & 0 & \frac{\rho_{k,1}}{2b_k} & 0 \\ -\frac{\rho_{k,2}}{b_k} + 2 & -\frac{5}{6} & 2 \frac{\rho_{k,2}}{b_k} - 2 & \frac{2}{3} & -\frac{\rho_{k,2}}{b_k} & \frac{1}{6} \\ -\frac{\rho_{k,2}}{b_k} & -\frac{1}{6} & 2 \frac{\rho_{k,2}}{b_k} & -\frac{2}{3} & -\frac{\rho_{k,2}}{b_k} - 2 & \frac{5}{6} \\ 0 & 0 & 0 & 0 & -\rho_{k,3} & 0 \\ \frac{\rho_{k,3}}{2b_k} & 0 & -2 \frac{\rho_{k,3}}{b_k} & 0 & 1 + 1.5 \frac{\rho_{k,3}}{b_k} & -1 \end{bmatrix} \quad (1)$$

$A = [C]^T [\bar{A}]$  - equilibrium equation matrix, where:

$[C]^T$  - is transpose of compatibility matrix of displacements  $[C]$

$[A]$  - is the matrix above in table

|                       |          |          |         |          |         |         |                       |          |         |          |         |         |         |          |         |          |          |
|-----------------------|----------|----------|---------|----------|---------|---------|-----------------------|----------|---------|----------|---------|---------|---------|----------|---------|----------|----------|
| A =                   |          |          |         |          |         |         |                       |          |         |          |         |         |         |          |         |          |          |
|                       |          |          |         |          |         |         |                       |          |         |          |         |         |         |          |         |          |          |
| Columns 1 through 9   |          |          |         |          |         |         |                       |          | 0       | -18.8496 | 0       | 18.8496 | 0       | 0        | 0       | 0        | 0        |
| -6.2832               | 6.2832   | 0        | 0       | 1.7671   | 0       | 0       | 0                     | 0        | 0       | 43.9823  | -6.2832 | 12.5664 | 6.2832  | -25.1327 | 0       | 21.9911  | 0        |
| 6.2832                | -5.2360  | 0        | 4.1888  | -6.2832  | 1.0472  | 0       | 0                     | 0        | 0       | 0        | 0       | -6.2832 | -5.2360 | 25.1327  | 4.1888  | -18.8496 | -31.4159 |
| -6.2832               | -1.0472  | 25.1327  | -4.1888 | -18.8496 | 5.2360  | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | -1.0472 | 50.2655  | -4.1888 | -31.4159 | -37.6991 |
| 0                     | 0        | 0        | 0       | -9.4248  | 0       | 9.4248  | 0                     | 0        | 0       | 0        | 0       | 28.2743 | 0       | -50.2655 | 0       | 43.9823  | 0        |
| 3.5343                | 0        | -25.1327 | 0       | 25.1327  | -6.2832 | 12.5664 | 6.2832                | -25.1327 | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | -6.2832 | -5.2360               | 25.1327  | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | -18.8496              | -1.0472  | 50.2655 | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | Columns 10 through 18 |          |         |          |         |         |         |          |         |          |          |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 7.0686                | 0        | 0       | -50.2655 | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| Columns 10 through 18 |          |          |         |          |         |         |                       |          | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 0        | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 0                     | 5.3014   | 0        | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| 4.1888                | -18.8496 | 1.0472   | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |
| -4.1888               | -31.4159 | 5.2360   | 0       | 0        | 0       | 0       | 0                     | 0        | 0       | 0        | 0       | 0       | 0       | 0        | 0       | 0        | 0        |

## Flexibility matrix

$$[D_k] = \frac{2\pi b_k}{15K_k(1 - \nu_k^2)} \begin{bmatrix} j_2 & -\nu_k j_2 & 2j_1 & -2\nu_k j_1 & -\rho_{k,2} & \nu_k \rho_{k,2} \\ & j_2 & -2\nu_k j_1 & 2j_1 & \nu_k \rho_{k,2} & -\rho_{k,2} \\ & & 16\rho_{k,2} & -16\nu_k \rho_{k,2} & 2j_3 & -2\nu_k j_3 \\ & & & 16\rho_{k,2} & -2\nu_k j_3 & 2j_3 \\ & & & & j_4 & -\nu_k j_4 \\ \text{symm.} & & & & & j_4 \end{bmatrix} \quad (2)$$

where:

$$\begin{aligned} K_k &= \frac{E_k t_k^3}{12(1-\nu_k^2)} j_1 = \rho_{k,2} - b_k \\ j_2 &= 4\rho_{k,2} - 3b_k \\ j_3 &= \rho_{k,2} + b_k \\ j_4 &= 4\rho_{k,2} + 3b_k \end{aligned}$$

|                       |         |         |         |         |         |         |         |           |           |         |         |         |         |         |
|-----------------------|---------|---------|---------|---------|---------|---------|---------|-----------|-----------|---------|---------|---------|---------|---------|
| D_ =                  |         |         |         |         |         |         | 0.0009  | -0.0005   | 0.0016    | 0       | 0       | 0       | 0       |         |
| Columns 1 through 9   |         |         |         |         |         |         | 0       | 0         | 0         | 0       | 0.0039  | -0.0012 | 0.0017  | -0.0005 |
| 0.0001                | -0.0000 | 0       | 0       | -0.0001 | 0.0000  | 0       | 0 0     | 0 0       | 0         | 0       | -0.0012 | 0.0039  | -0.0005 | 0.0017  |
| -0.0000               | 0.0001  | 0       | 0       | 0.0000  | -0.0001 | 0       | 0 0     | 0 0       | 0         | 0       | 0.0017  | -0.0005 | 0.0207  | -0.0062 |
| 0                     | 0       | 0.0017  | -0.0005 | 0.0004  | -0.0001 | 0       | 0 0     | 0 0       | 0         | 0       | -0.0005 | 0.0017  | -0.0062 | 0.0207  |
| 0                     | 0       | -0.0005 | 0.0017  | -0.0001 | 0.0004  | 0       | 0 0     | 0 0       | 0         | 0       | -0.0013 | 0.0004  | 0.0034  | -0.0010 |
| -0.0001               | 0.0000  | 0.0004  | -0.0001 | 0.0008  | -0.0002 | 0       | 0 0     | 0 0       | 0         | 0       | 0.0004  | -0.0013 | -0.0010 | 0.0034  |
| 0.0000                | -0.0001 | -0.0001 | 0.0004  | -0.0002 | 0.0008  | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0.0010  | -0.0003 | 0.0004    | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | -0.0003 | 0.0010  | -0.0001   | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0.0004  | -0.0001 | 0.0052    | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | -0.0001 | 0.0004  | -0.0016   | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | -0.0003 | 0.0001  | 0.0009    | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0.0000  | 0.0000  | 0.0000    | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| Columns 10 through 18 |         |         |         |         |         |         | 0       | 0         | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0 0     | 0 0       | 0         | 0       | 0       | 0       | 0       | 0       |
| 0                     | 0       | 0       | 0       | 0       | 0       | 0       | 0.0048  | 0 -0.0014 | 0 0.0023  | -0.0007 | -0.0013 | 0.0004  |         |         |
| -0.0001               | -0.0003 | 0.0001  | 0       | 0       | 0       | 0       | -0.0014 | 0 0.0048  | 0 -0.0007 | 0.0023  | 0.0004  | -0.0013 |         |         |
| 0.0004                | 0.0001  | -0.0003 | 0       | 0       | 0       | 0       | 0.0023  | 0 -0.0007 | 0 0.0214  | -0.0064 | 0.0031  | -0.0009 |         |         |
| -0.0016               | 0.0009  | -0.0003 | 0       | 0       | 0       | 0       | -0.0007 | 0 0.0023  | 0 -0.0064 | 0.0214  | -0.0009 | 0.0031  |         |         |
| 0.0052                | -       |         |         |         |         |         |         |           |           |         |         |         |         |         |

## External load vector

Vector of external forces for node described as:

$$\{F_k\} = \frac{2\pi b_k \rho_k}{3} \begin{Bmatrix} 3\rho_{k,2} - b_k \\ 3\rho_{k,2} + b_k \end{Bmatrix} = \{\eta_k\} \rho_k \quad (3)$$

$[F] = [F_o] + [C]^T[F_p]$ , where:

$[F_o] = -m12\pi r h o_{f,1}$

$m_1$  - value of bending moment which correspond  $u_1$

$\rho_{f,1}$  - coordinate where is  $m_1$

$[F_p]$  - represent the equivalent of distributed loads.

$[C]^T$  - is transpose of compatibility matrix of displacements  $[C]$

F =

```

0
47.1239
94.2478
0
0
188.4956
235.6194
0
0
0
0
0
0
753.9822
0
0
0

```

The results of global displacements are shown as following : Where: A - coefficient matrix of equilibrium equations A' - transpose coefficient matrix of equilibrium equations F - External load vector

Uglob =

```

0.0340
0.0383
0.0185
0.0020
0.0216
0.0235
0.0134
-0.0069
0.0143
0.0205
0.0224
-0.0037
0.0185
0.0179
0.0026
-0.0103

```

The results of internal forces are shown as following: Where: U - global displacements

Ulocal =

```

-158.2735
-155.1518
-31.3524
50.9813
-11.0997
-51.6406
-11.0997
-10.0672
3.5628
10.3970

```

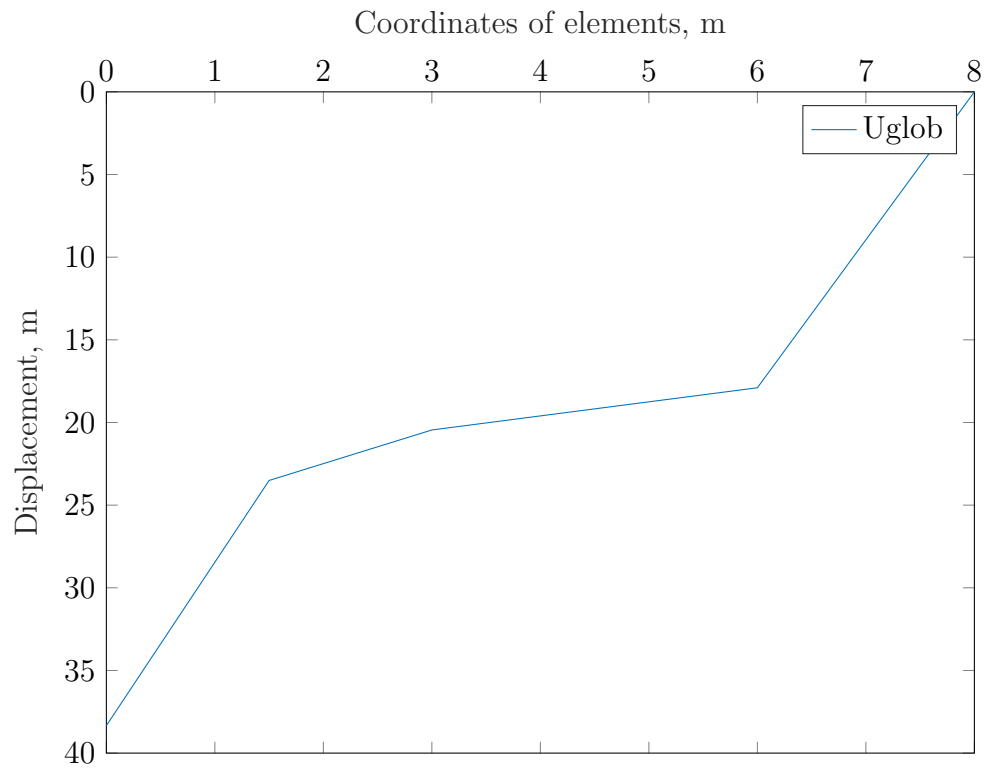


Figure 6: My first matlab2tikz figure

3.4310  
-2.2716  
3.4310  
-9.1624  
14.6141  
4.6054  
22.3579  
8.0553  
22.3579  
11.6794  
9.7933  
4.5640  
0.0000  
5.7628

## Internal forces and displacements

The equilibrium of finite element method used to solve the mentioned annular plate. The results were obtained from a MATLAB commands. Based on these calculations, the results shows the internal forces and the a distribution of global displacements along our structure. The results shows that the maximum displacement was  $U = 36.0471$  mm in the direction gravity. The allowable displacement was  $U_{allowable} = L/250 = 64$  mm so, the verification was correct based on the current geometry and material properties. Some parametric analysis were done for plate thickness as a very important parameter to reduce the displacement values.

```
Ulocal =  
-158.2735  
-155.1518  
-31.3524  
50.9813  
-11.0997  
-51.6406  
-11.0997  
-10.0672  
3.5628  
10.3970  
3.4310  
-2.2716  
3.4310  
-9.1624  
14.6141  
4.6054  
22.3579  
8.0553  
22.3579  
11.6794  
9.7933  
4.5640  
0.0000  
5.7628
```



# Code listing

## Main function

```
1 clc; clear;
2 format compact;
3 addpath(' ../matlab2tikz/ ');
4 % initial data
5 qLOAD =20; % KN/m
6 f=20; % KN
7 L=16; % span length m
8 h=0.05; % thickness of plate
9 E=210e6; % modulus of elastisty kPa
10 v=.3; % poisson's ratio
11 %
12 r=L/2; % radius of plate
13 B=[1.5 1.5 3 2];%vector of elements lengths, meters
14 no_FE=length(B); % Number of plate finite elements, as length of
   vector with elements lengths
15 b=B/2; % half of elements lengths
16 coords = zeros(no_FE,3);%filling matrix (number of elements by 3) by
   zeros
17 coords(1,2)=B(1)/2;% coordinate of half of first element
18 coords(1,3)=B(1);% coordinate of end of first element
19 for i=2:no_FE% loop over coordinates matrix, "i" is current element,
   "i-1" is previous element
20     coords(i,1)=coords(i-1,3);%start coordinate as end coordinate
   of previous element
21     coords(i,2)=coords(i-1,3) + B(i)/2;%half coordinate as end
   coordinate of previous element plus half length of current
   element
22     coords(i,3)= coords(i-1,3) + B(i);%end coordinate as end
   coordinate of previous element plus length of current element
23 end
24 delete coordsoutput.txt;
25 diary('coordsoutput.txt');
26 diary on ;
27 coords% printing coordinates matrix
28 diary off ;%to avoid print other commands.
29 no_of_local_dis=6; % Number of local displacements
30 no_of_global_dis=16; % Number of global displacements
31
32 % 2. Compatipality matrix C
33 % for the first FE
34 c_1stcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
35 c_1stcompatipality_matrix(2:6,1:5)=eye(5);
36 % for the Second FE
37 c_2ndcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
38 c_2ndcompatipality_matrix(1:6,4:9)=eye(6);
```

```

39 % for the third FE
40 c_3rdcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
41 c_3rdcompatipality_matrix(1:6,8:13)=eye(6);
42 % for the 4th FE
43 c_4thcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
44 c_4thcompatipality_matrix(1:5,12:16)=eye(5);
45 % the total compatipality matrix of displacements
46 delete Cmtxoutput.txt;
47 diary('Cmtxoutput.txt');
48 diary on ;
49 C=[c_1stcompatipality_matrix;c_2ndcompatipality_matrix;
      c_3rdcompatipality_matrix;c_4thcompatipality_matrix]
50 diary off ;%to avoid print other commands.
51
52 %4. Matrix of equilibrium equations A
53 for k=1:no_FE
54 A_matrix = getAmtx(coords(k,1), coords(k,2), coords(k,3), b(k));
55 A_(k*6-5:k*6,k*6-5:k*6)=2*pi*A_matrix;
56 end
57 delete Amtxoutput.txt;
58 diary('Amtxoutput.txt');
59 diary on ;
60 A=C'*A_
61 diary off ;%to avoid print other commands.
62
63
64
65 % % 5. Flexibility MATRIX OF D
66 for k=1:no_FE
67 Rok2=coords(k,2);
68 bk=b(k);
69 D_matrix = getDmtx(coords(k,2), b(k), v);
70 K.k=E*h^3/(12*(1-v^2));
71 D_(k*6-5:k*6,k*6-5:k*6)=(2*pi*bk/(15*K.k*(1-v^2)))*D_matrix;
72 end
73 delete Dmtxoutput.txt;
74 diary('Dmtxoutput.txt');
75 diary on ;
76 D_
77 diary off ;%to avoid print other commands.
78
79
80 % % 6. EXTERNAL LOAD VECTOR F
81 Fo=zeros(no_of_global_dis,1);
82 Rof=6;
83 Fo(13)=f*2*pi*Rof;
84 % Rof coordinate where f
85 % Fkp is nodal external load vector which is equivilant to
      distributed load of the kth element

```

```

86 q_Load_vector=[20 20 0 0];
87 for k=1:no_FE
88     bk=b(k);
89     q=q_Load_vector(k);
90     Rok2=coords(k,2);
91     Fk=(2*pi*bk/3)*q*[3*Rok2-bk;3*Rok2+bk];
92     Fp=[0;0;Fk;0;0];
93     Fp_(6*k-5:k*6,1)=Fp;
94 end
95
96 delete Foutput.txt;
97 diary('Foutput.txt');
98 diary on ;
99 F=Fo+C'*Fp_
100 diary off ;%to avoid print other commands.
101
102 delete Ugloboutput.txt;
103 diary('Ugloboutput.txt');
104 diary on ;
105 Uglob=inv(A*inv(D_)*A')*F
106 diary off ;%to avoid print other commands.
107
108 delete Ulocaloutput.txt;
109 diary('Ulocaloutput.txt');
110 diary on ;
111 Ulocal=inv(D_)*A'*Uglob
112 diary off ;%to avoid print other commands.
113
114
115
116 M_Ro=Ulocal(1:2:end);
117 M_fi=Ulocal(2:2:end);
118 delete um_mmoutput.txt;
119 diary('um_mmoutput.txt');
120 diary on ;
121 um_mm = 1000*[Uglob(2:4:end);0]
122 diary off ;%to avoid print other commands.
123 xcoord = [0;coords(1:end,3)];
124 plot(xcoord,um_mm,'DisplayName','Uglob');
125 xlabel('Coordinates of elements, m')
126 ylabel('Displacement, m')
127 set(gca,'XAxisLocation','top','YAxisLocation','left','ydir','reverse'
    );
128 matlab2tikz('um_mm.tex','showInfo', false);
129
130 delete u_allowableoutput.txt;
131 diary('u_allowableoutput.txt');
132 diary on ;
133 u_allowable=16/250*1000

```

134 **diary** off ;%to avoid print other commands.

### **getAmtx function**

```
1 function A_matrix = getAmtx(Rok1, Rok2, Rok3, bk)
2     A_matrix = zeros(6,6);
3     A_matrix(1,1)=Rok1;
4     A_matrix(2,1)=1.5*Rok1/bk-1;
5     A_matrix(2,2)=1;
6     A_matrix(2,3)=-2*Rok1/bk;
7     A_matrix(2,5)=Rok2/2*bk;
8     A_matrix(3,1)=-Rok2/bk+2;
9     A_matrix(3,2)=-5/6;
10    A_matrix(3,3)=2*Rok2/bk-2;
11    A_matrix(3,4)=2/3;
12    A_matrix(3,5)=-Rok2/bk;
13    A_matrix(3,6)=1/6;
14    A_matrix(4,1)=-Rok2/bk;
15    A_matrix(4,2)=-1/6;
16    A_matrix(4,3)=2*Rok2/bk+2;
17    A_matrix(4,4)=-2/3;
18    A_matrix(4,5)=-Rok2/bk-2;
19    A_matrix(4,6)=5/6;
20    A_matrix(5,5)=-Rok3;
21    A_matrix(6,1)=Rok3/2*bk;
22    A_matrix(6,3)=-2*Rok3/bk;
23    A_matrix(6,5)=1+1.5*Rok3/bk;
24    A_matrix(6,6)=-1;
25 end
```

### **getDmtx function**

```
1 function D_matrix = getDmtx(Rok2, bk, v)
2     j1=Rok2-bk;
3     j2=4*Rok2-3*bk;
4     j3=Rok2+bk;
5     j4=4*Rok2+3*bk;
6     D_matrix(1,1)=j2;
7     D_matrix(1,2)=-v*j2;
8     D_matrix(1,3)=2*j1;
9     D_matrix(1,4)=-2*v*j1;
10    D_matrix(1,5)=-Rok2;
11    D_matrix(1,6)=v*Rok2;
12    D_matrix(2,1)=D_matrix(1,2);
13    D_matrix(2,2)=j2;
14    D_matrix(2,3)=-2*v*j1;
15    D_matrix(2,4)=2*j1;
16    D_matrix(2,5)=v*Rok2;
17    D_matrix(2,6)=-Rok2;
18    D_matrix(3,1)=D_matrix(1,3);
```

```

19     D_matrix(3,2)=D_matrix(2,3);
20     D_matrix(3,3)=16*Rok2;
21     D_matrix(3,4)=-16*v*Rok2;
22     D_matrix(3,5)=2*j3;
23     D_matrix(3,6)=-2*v*j3;
24     D_matrix(4,1)=D_matrix(1,4);
25     D_matrix(4,2)=D_matrix(2,4);
26     D_matrix(4,3)=D_matrix(3,4);
27     D_matrix(4,4)=16*Rok2;
28     D_matrix(4,5)=-2*v*j3;
29     D_matrix(4,6)=2*j3;
30     D_matrix(5,1)=D_matrix(1,5);
31     D_matrix(5,2)=D_matrix(2,5);
32     D_matrix(5,3)=D_matrix(3,5);
33     D_matrix(5,4)=D_matrix(4,5);
34     D_matrix(5,5)=j4;
35     D_matrix(5,6)=-v*j4;
36     D_matrix(6,1)=D_matrix(1,6);
37     D_matrix(6,2)=D_matrix(2,6);
38     D_matrix(6,3)=D_matrix(3,6);
39     D_matrix(6,4)=D_matrix(4,6);
40     D_matrix(6,5)=D_matrix(5,6);
41     D_matrix(6,6)=j4;
42 end

```