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Discrete model of circular plate

According to given initial conditions structure is circle width some amount of distributed load in center. Due to the symmetry of our structure, has to be studied only half of structure(fig.1).

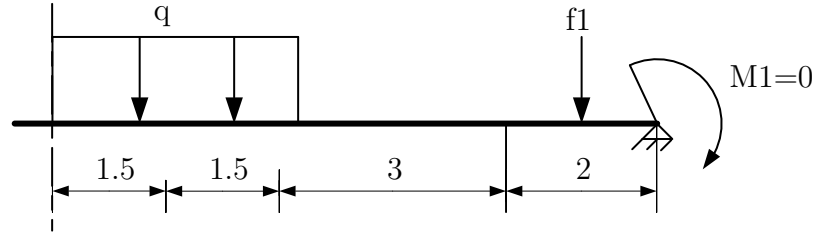


Figure 1: Schematic representation for half of structure

Developed discrete model of given structure consist 4 finite elements(fig.2), local displacements and moments of each element shown on figure 3, while global ones are on figure 4.

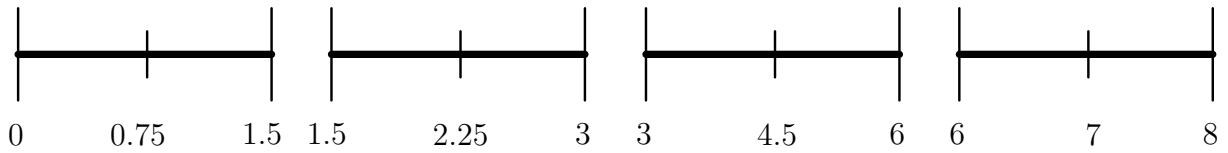


Figure 2: Coordinate scheme of finite elements

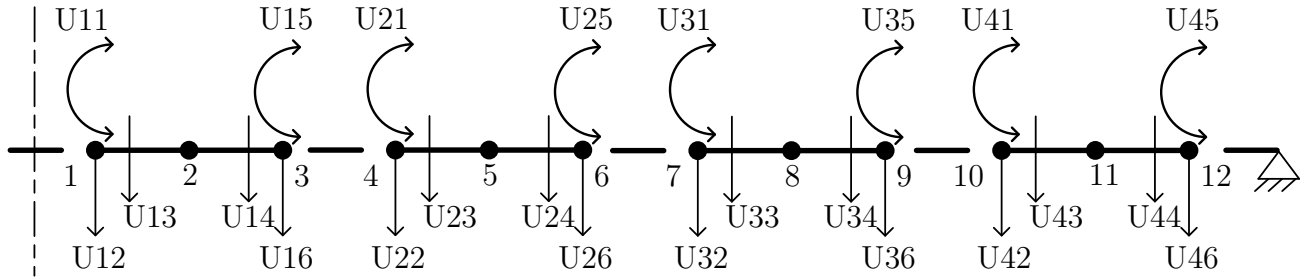


Figure 3: Local displacements and moments of each element

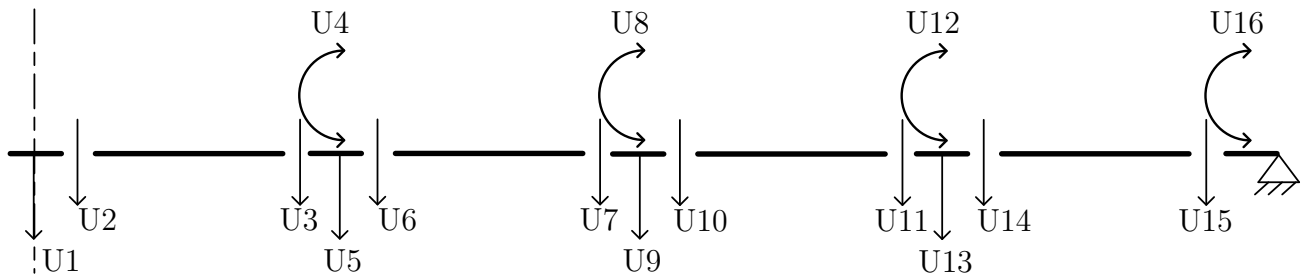


Figure 4: Global displacements and moments of each element

Compatibility matrix of displacements

It's based on the discrete model. It represents a relationship among the local and global displacements. We used 4 finite elements for each one has 6 local displacement and 5 nodes have total number of global displacements $m=16$. The final compatibility matrix of displacements are shown on figure 5, where rows are global displacements and columns are local displacements.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5: Compatibility matrix of displacements

Coefficient matrix of equilibrium equations

The matrix of the coefficients of the equilibrium equations for a ring element of a round plate is given in the table.

$$[\bar{A}_k] = 2\pi \begin{bmatrix} \rho_{k,1} & 0 & 0 & 0 & 0 & 0 \\ 1.5\frac{\rho_{k,1}}{b_k} - 1 & 1 & -2\frac{\rho_{k,1}}{b_k} & 0 & \frac{\rho_{k,1}}{2b_k} & 0 \\ -\frac{\rho_{k,2}}{b_k} + 2 & -\frac{5}{6} & 2\frac{\rho_{k,2}}{b_k} - 2 & \frac{2}{3} & -\frac{\rho_{k,2}}{b_k} & \frac{1}{6} \\ -\frac{\rho_{k,2}}{b_k} & -\frac{1}{6} & 2\frac{\rho_{k,2}}{b_k} & -\frac{2}{3} & -\frac{\rho_{k,2}}{b_k} - 2 & \frac{5}{6} \\ 0 & 0 & 0 & 0 & -\rho_{k,3} & 0 \\ \frac{\rho_{k,3}}{2b_k} & 0 & -2\frac{\rho_{k,3}}{b_k} & 0 & 1 + 1.5\frac{\rho_{k,3}}{b_k} & -1 \end{bmatrix} \quad (1)$$

$A = [C]^T[\bar{A}]$ - equilibrium equation matrix, where:

$[C]^T$ - is transpose of compatibility matrix of displacements $[C]$

$[A]$ - is the matrix above in table

Calculation of the equilibrium equation matrix from Matlab

[illegible]

Flexibility matrix

To reduce the size of the matrix and ease of placing it in the report, the equations were replaced by j_1, j_2, j_3, j_4 .

$$[D_k] = \frac{2\pi b_k}{15K_k(1 - \nu_k^2)} \begin{bmatrix} j_2 & -\nu_k j_2 & 2j_1 & -2\nu_k j_1 & -\rho_{k,2} & \nu_k \rho_{k,2} \\ & j_2 & -2\nu_k j_1 & 2j_1 & \nu_k \rho_{k,2} & -\rho_{k,2} \\ & & 16\rho_{k,2} & -16\nu_k \rho_{k,2} & 2j_3 & -2\nu_k j_3 \\ & & & 16\rho_{k,2} & -2\nu_k j_3 & 2j_3 \\ & & & & j_4 & -\nu_k j_4 \\ \text{symm.} & & & & & j_4 \end{bmatrix} \quad (2)$$

Where:

$$K_k = \frac{E_k t_k^3}{12(1 - \nu_k^2)} \quad j_1 = \rho_{k,2} - b_k$$

$$j_2 = 4\rho_{k,2} - 3b_k$$

$$j_3 = \rho_{k,2} + b_k$$

$$j_4 = 4\rho_{k,2} + 3b_k$$

Calculation of the flexibility matrix from Matlab

```
D_ =
Columns 1 through 14
0.0001    -0.0000         0         0   -0.0001    0.0000         0         0         0         0         0         0         0         0
-0.0000    0.0001         0         0    0.0000   -0.0001         0         0         0         0         0         0         0         0
0         0         0.0017   -0.0005    0.0004   -0.0001         0         0         0         0         0         0         0         0
0         0         -0.0005    0.0017   -0.0001    0.0004         0         0         0         0         0         0         0         0
-0.0001    0.0000    0.0004   -0.0001    0.0008   -0.0002         0         0         0         0         0         0         0         0
0.0000   -0.0001   -0.0001    0.0004   -0.0002    0.0008         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0.0010   -0.0003    0.0004   -0.0001   -0.0003    0.0001         0         0
0         0         0         0         0         0        -0.0003    0.0010   -0.0001    0.0004    0.0001   -0.0003         0         0
0         0         0         0         0         0         0.0004   -0.0001    0.0052   -0.0016    0.0009   -0.0003         0         0
0         0         0         0         0         0        -0.0001    0.0004   -0.0016    0.0052   -0.0003    0.0009         0         0
0         0         0         0         0         0        -0.0003    0.0001    0.0009   -0.0003    0.0016   -0.0005         0         0
0         0         0         0         0         0         0.0001   -0.0003   -0.0003    0.0009   -0.0005    0.0016         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0.0039   -0.0012
0         0         0         0         0         0         0         0         0         0         0         0         -0.0012    0.0039
0         0         0         0         0         0         0         0         0         0         0         0         0.0017   -0.0005
0         0         0         0         0         0         0         0         0         0         0         0         -0.0005    0.0017
0         0         0         0         0         0         0         0         0         0         0         0         -0.0013    0.0004
0         0         0         0         0         0         0         0         0         0         0         0         0.0004   -0.0013
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0         0         0         0         0         0         0         0         0         0         0         0         0         0
0.0017   -0.0005   -0.0013    0.0004         0         0         0         0         0         0         0         0         0         0
-0.0005    0.0017    0.0004   -0.0013         0         0         0         0         0         0         0         0         0         0
0.0207   -0.0062    0.0034   -0.0010         0         0         0         0         0         0         0         0         0         0
```

-0.0062	0.0207	-0.0010	0.0034	0	0	0	0	0	0
0.0034	-0.0010	0.0065	-0.0019	0	0	0	0	0	0
-0.0010	0.0034	-0.0019	0.0065	0	0	0	0	0	0
0	0	0	0	0.0048	-0.0014	0.0023	-0.0007	-0.0013	0.0004
0	0	0	0	-0.0014	0.0048	-0.0007	0.0023	0.0004	-0.0013
0	0	0	0	0.0023	-0.0007	0.0214	-0.0064	0.0031	-0.0009
0	0	0	0	-0.0007	0.0023	-0.0064	0.0214	-0.0009	0.0031
0	0	0	0	-0.0013	0.0004	0.0031	-0.0009	0.0059	-0.0018
0	0	0	0	0.0004	-0.0013	-0.0009	0.0031	-0.0018	0.0059

External load vector

Vector of external forces for node described as:

$$\{F_k\} = \frac{2\pi b_k \rho_k}{3} \begin{Bmatrix} 3\rho_{k,2} - b_k \\ 3\rho_{k,2} + b_k \end{Bmatrix} = \{\eta_k\} \rho_k \quad (3)$$

$[F] = [F_o] + [C]^T[F_p]$, where:

$[F_o] = f12\pi\rho_{f1}$

$f1$ – value of external force which correspond u_1

ρ_{f1} – coordinate where is m_1

$[F_p]$ – represent the equivalent of distributed loads.

$[C]^T$ – is transpose of compatibility matrix of displacements $[C]$

F =

```

0
47.1239
94.2478
0
0
188.4956
235.6194
0
0
0
0
0
0
753.9822
0
0
0

```

The results of global displacements are shown as following :

$$[U] = \{[A][D]^{-1}\}^{-1}[A]^T[F] \quad (4)$$

Where:

A – coefficient matrix of equilibrium equations

$[A]^T$ – transpose coefficient matrix of equilibrium equations

$[D]$: flexibility matrix

$[F]$ – External load vector

Uglob =

```

0.0340
0.0383
0.0185
0.0020
0.0216
0.0235
0.0134
-0.0069
0.0143
0.0205
0.0224
-0.0037
0.0185
0.0179
0.0026
-0.0103

```

The results of internal forces are shown as following:

$$[S] = [D]^{-1}[A]^T[U] \quad (5)$$

Where:

$[D]$ - flexibility matrix

$[A]^T$ - transpose coefficient matrix of equilibrium equations

$[U]$ - global displacements

Results of internal forces (S) consists of M_ρ and moments M_φ . Schema have 24 values of internal forces (S) it's divided into: 12 values of moment M_ρ and 12 values of moments M_φ . In the matlab command $M_\rho = S(1 : 2 : end)$ means to take the 1st value of (S) , 3rd , 5th , 7 th till end. All those values belong to moment M_ρ . For $M_\varphi = S(2 : 2 : end)$ means to take the 2nd value of (S), 4th , 6th.....till end.

M_Ro =

```
-158.2735
-31.3524
-11.0997
-11.0997
 3.5628
 3.4310
 3.4310
14.6141
22.3579
22.3579
 9.7933
 0.0000
```

M_fi =

```
-155.1518
 50.9813
-51.6406
-10.0672
10.3970
-2.2716
-9.1624
 4.6054
 8.0553
11.6794
 4.5640
 5.7628
```


Internal forces and displacements

The equilibrium of finite element method used to solve the mentioned annular plate. The results were obtained from a MATLAB commands. Based on these calculations, the results shows the internal forces and the a distribution of global displacements along our structure. The results shows that the maximum displacement was $U = 36.0471$ mm in the direction gravity. The allowable displacement was $U_{allowable} = L/250 = 64$ mm so, the verification was correct based on the current geometry and material properties. Some parametric analysis were done for plate thickness as a very important parameter to reduce the displacement values.

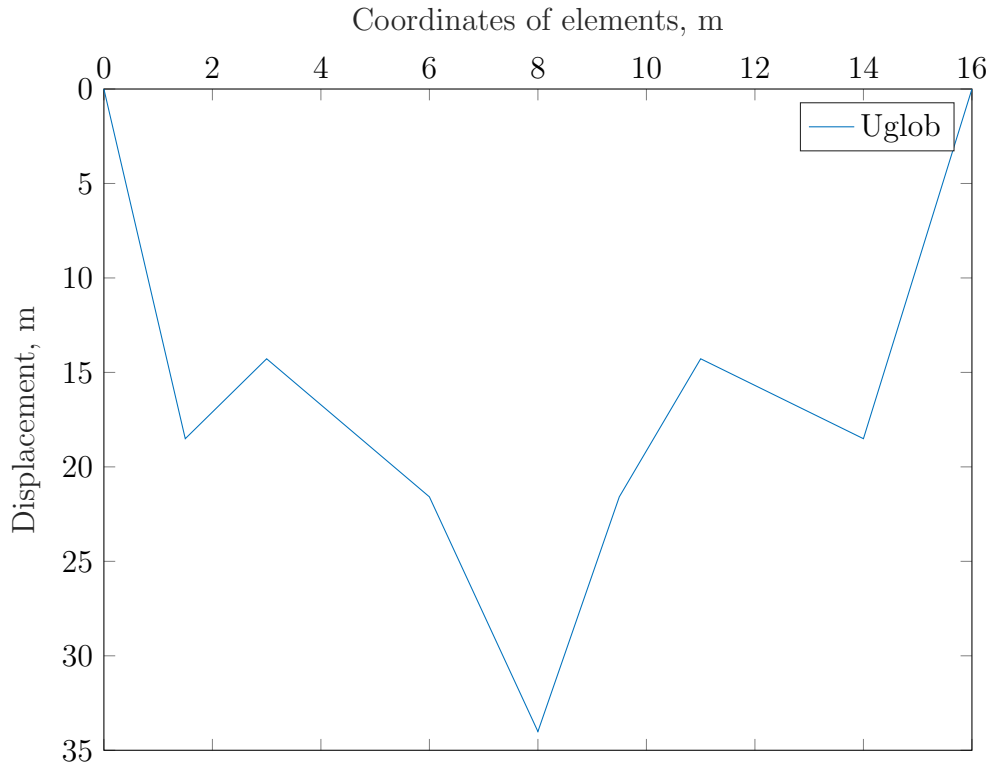


Figure 6: Global displacement across plate

Code listing

Main function

```
1 close all; clc; clear;
2 format compact;
3 addpath(' ../ matlab2tikz/ ');
4 % initial data
5 q_LOAD =20; % KN/m
6 f=20; % KN
7 L=16; % span length m
8 h=0.05; % thickness of plate
9 E=210e6; % modulus of elastisty kPa
10 v=.3; % poisson's ratio
11 %
12 r=L/2; % radius of plate
13 B=[1.5 1.5 3 2];%vector of elements lengths, meters
14 no_FE=length(B); % Number of plate finite elements, as length of vector with
    elements lengths
15 b=B/2; % half of elements lengths
16 coords = zeros(no_FE,3);%filling matrix (number of elements by 3) by zeros
17 coords(1,2)=B(1)/2;% coordinate of half of first element
18 coords(1,3)=B(1);% coordinate of end of first element
19 for i=2:no_FE% loop over coordinates matrix, "i" is current element, "i-1" is
    previous element
20     coords(i,1)=coords(i - 1,3);%start coordinate as end coordinate of previous
        element
21     coords(i,2)=coords(i - 1,3) + B(i)/2;%half coordinate as end coordinate of
        previous element plus half length of current element
22     coords(i,3)= coords(i - 1,3) + B(i);%end coordinate as end coordinate of
        previous element plus length of current element
23 end
24 delete coordsoutput.txt;
25 diary('coordsoutput.txt');
26 diary on ;
27 coords% printing coordinates matrix
28 diary off ;%to avoid print other commands.
29 no_of_local_dis=6; % Number of local displacements
30 no_of_global_dis=16; % Number of global displacements
31 % 2. Compatipality matrix C
32 % for the first FE
33 c_1stcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
34 c_1stcompatipality_matrix(2:6,1:5)=eye(5);
35 % for the Second FE
36 c_2ndcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
37 c_2ndcompatipality_matrix(1:6,4:9)=eye(6);
38 % for the third FE
39 c_3rdcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
40 c_3rdcompatipality_matrix(1:6,8:13)=eye(6);
41 % for the 4th FE
42 c_4thcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
43 c_4thcompatipality_matrix(1:5,12:16)=eye(5);
44 % the total compatipality matrix of displacements
45 delete Cmtxoutput.txt;
46 diary('Cmtxoutput.txt');
47 diary on ;
48 C=[c_1stcompatipality_matrix;c_2ndcompatipality_matrix;c_3rdcompatipality_matrix;
    c_4thcompatipality_matrix]
```

```

49 diary off ;%to avoid print other commands.
50 %% 4. Matrix of equilibrium equations A
51 for k=1:no_FE
52 A_matrix = getAmtx(coords(k,1), coords(k,2), coords(k,3), b(k));
53 A_(k*6-5:k*6,k*6-5:k*6)=2*pi*A_matrix;
54 end
55 delete Amtxoutput.txt;
56 diary('Amtxoutput.txt');
57 diary on ;
58 A=C'*A_
59 diary off ;%to avoid print other commands.
60 %% 5. Flexibility MATRIX OF D
61 for k=1:no_FE
62 Rok2=coords(k,2);
63 bk=b(k);
64 D_matrix = getDmtx(coords(k,2), b(k), v);
65 K.k=E*h^3/(12*(1-v^2));
66 D_(k*6-5:k*6,k*6-5:k*6)=(2*pi*bk/(15*K.k*(1-v^2)))*D_matrix;
67 end
68 delete Dmtxoutput.txt;
69 diary('Dmtxoutput.txt');
70 diary on ;
71 D_
72 diary off ;%to avoid print other commands.
73 %% 6. EXTERNAL LOAD VECTOR F
74 Fo=zeros(no_of_global_dis,1);
75 Rof=6;
76 Fo(13)=f*2*pi*Rof;
77 % Rof coordinate where f
78 % Fkp is nodal external load vector which is equivalent to distributed load of
    the kth element
79 q_Load_vector=[20 20 0 0];
80 for k=1:no_FE
81     bk=b(k);
82     q=q_Load_vector(k);
83     Rok2=coords(k,2);
84     Fk=(2*pi*bk/3)*q*[3*Rok2-bk;3*Rok2+bk];
85     Fp=[0;0;Fk;0;0];
86     Fp_(6*k-5:k*6,1)=Fp;
87 end
88 delete Foutput.txt;
89 diary('Foutput.txt');
90 diary on ;
91 F=Fo+C'*Fp_
92 diary off ;%to avoid print other commands.
93 delete Ugloboutput.txt;
94 diary('Ugloboutput.txt');
95 diary on ;
96 Uglob=inv(A*inv(D_)*A')*F
97 diary off ;%to avoid print other commands.
98 delete Ulocaloutput.txt;
99 diary('Ulocaloutput.txt');
100 diary on ;
101 Ulocal=inv(D_)*A'*Uglob
102 diary off ;%to avoid print other commands.
103 delete M_Rooutput.txt;
104 diary('M_Rooutput.txt');
105 diary on ;
106 M_Ro=Ulocal(1:2:end)

```

```

107 diary off ;%to avoid print other commands.
108 delete M_fioutput.txt;
109 diary('M_fioutput.txt');
110 diary on ;
111 M_fi=Ulocal(2:2:end)
112 diary off ;%to avoid print other commands.
113 delete um_mmoutput.txt;
114 diary('um_mmoutput.txt');
115 diary on ;
116 um_mm = 1000*[Uglob(1:4:end);0]
117 diary off ;%to avoid print other commands.
118 xcoord = [0;coords(1:end,3)];
119 figure(1);
120 plot(xcoord,um_mm,'DisplayName','Uglob');
121 xlabel('Coordinates of elements , m')
122 ylabel('Displacement , m')
123 set(gca, 'XAxisLocation', 'top', 'YAxisLocation', 'left', 'ydir', 'reverse');
124 matlab2tikz('um_mm.tex', 'showInfo', false);
125 delete u_allowableoutput.txt;
126 diary('u_allowableoutput.txt');
127 diary on ;
128 u_allowable=16/250*1000
129 diary off ;%to avoid print other commands.
130 fullCoord = [0;coords(1:end,3);coords(1:end,3)+coords(end)];
131 flippedU = zeros(length(um_mm),1);
132 j=length(um_mm);
133 for i=1:length(um_mm)
134     flippedU(i) = um_mm(j);
135     j=j-1;
136 end
137 fullU = [flippedU;um_mm(2:end)];
138 figure(2);
139 plot(fullCoord, fullU, 'DisplayName','Uglob');
140 xlabel('Coordinates of elements , m')
141 ylabel('Displacement , m')
142 set(gca, 'XAxisLocation', 'top', 'YAxisLocation', 'left', 'ydir', 'reverse');
143 matlab2tikz('fullU.tex', 'showInfo', false);

```

getAmtx function

```

1 function A_matrix = getAmtx(Rok1, Rok2, Rok3, bk)
2     A_matrix = zeros(6,6);
3     A_matrix(1,1)=Rok1;
4     A_matrix(2,1)=1.5*Rok1/bk-1;
5     A_matrix(2,2)=1;
6     A_matrix(2,3)=-2*Rok1/bk;
7     A_matrix(2,5)=Rok2/2*bk;
8     A_matrix(3,1)=-Rok2/bk+2;
9     A_matrix(3,2)=-5/6;
10    A_matrix(3,3)=2*Rok2/bk-2;
11    A_matrix(3,4)=2/3;
12    A_matrix(3,5)=-Rok2/bk;
13    A_matrix(3,6)=1/6;
14    A_matrix(4,1)=-Rok2/bk;
15    A_matrix(4,2)=-1/6;
16    A_matrix(4,3)=2*Rok2/bk+2;
17    A_matrix(4,4)=-2/3;
18    A_matrix(4,5)=-Rok2/bk-2;
19    A_matrix(4,6)=5/6;

```

```

20     A_matrix(5,5)=-Rok3;
21     A_matrix(6,1)=Rok3/2*bk;
22     A_matrix(6,3)=-2*Rok3/bk;
23     A_matrix(6,5)=1+1.5*Rok3/bk;
24     A_matrix(6,6)=-1;
25 end

```

getDmtx function

```

1 function D_matrix = getDmtx(Rok2, bk, v)
2     j1=Rok2-bk;
3     j2=4*Rok2-3*bk;
4     j3=Rok2+bk;
5     j4=4*Rok2+3*bk;
6     D_matrix(1,1)=j2;
7     D_matrix(1,2)=-v*j2;
8     D_matrix(1,3)=2*j1;
9     D_matrix(1,4)=-2*v*j1;
10    D_matrix(1,5)=-Rok2;
11    D_matrix(1,6)=v*Rok2;
12    D_matrix(2,1)=D_matrix(1,2);
13    D_matrix(2,2)=j2;
14    D_matrix(2,3)=-2*v*j1;
15    D_matrix(2,4)=2*j1;
16    D_matrix(2,5)=v*Rok2;
17    D_matrix(2,6)=-Rok2;
18    D_matrix(3,1)=D_matrix(1,3);
19    D_matrix(3,2)=D_matrix(2,3);
20    D_matrix(3,3)=16*Rok2;
21    D_matrix(3,4)=-16*v*Rok2;
22    D_matrix(3,5)=2*j3;
23    D_matrix(3,6)=-2*v*j3;
24    D_matrix(4,1)=D_matrix(1,4);
25    D_matrix(4,2)=D_matrix(2,4);
26    D_matrix(4,3)=D_matrix(3,4);
27    D_matrix(4,4)=16*Rok2;
28    D_matrix(4,5)=-2*v*j3;
29    D_matrix(4,6)=2*j3;
30    D_matrix(5,1)=D_matrix(1,5);
31    D_matrix(5,2)=D_matrix(2,5);
32    D_matrix(5,3)=D_matrix(3,5);
33    D_matrix(5,4)=D_matrix(4,5);
34    D_matrix(5,5)=j4;
35    D_matrix(5,6)=-v*j4;
36    D_matrix(6,1)=D_matrix(1,6);
37    D_matrix(6,2)=D_matrix(2,6);
38    D_matrix(6,3)=D_matrix(3,6);
39    D_matrix(6,4)=D_matrix(4,6);
40    D_matrix(6,5)=D_matrix(5,6);
41    D_matrix(6,6)=j4;
42 end

```