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Discrete model of circular plate

According to given initial conditions structure is circle width some amount of distributed load in center. Due to the symmetry of our structure, has to be studied only half of structure(fig.1).

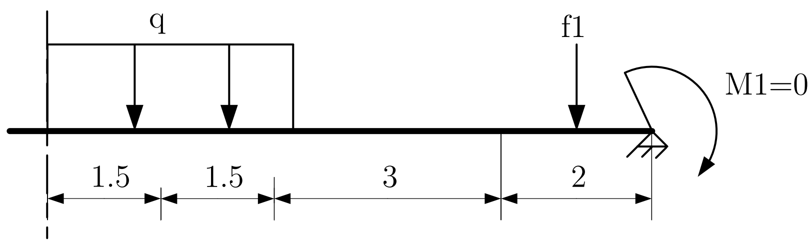


Figure 1: Schematic representation for half of structure

Developed discrete model of given structure consist 4 finite elements(fig.2), local displacements and moments of each element shown on figure 3, while global ones are on figure 4.

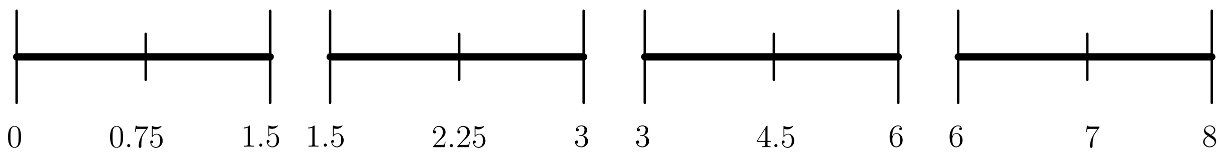


Figure 2: Coordinate scheme o finite elements

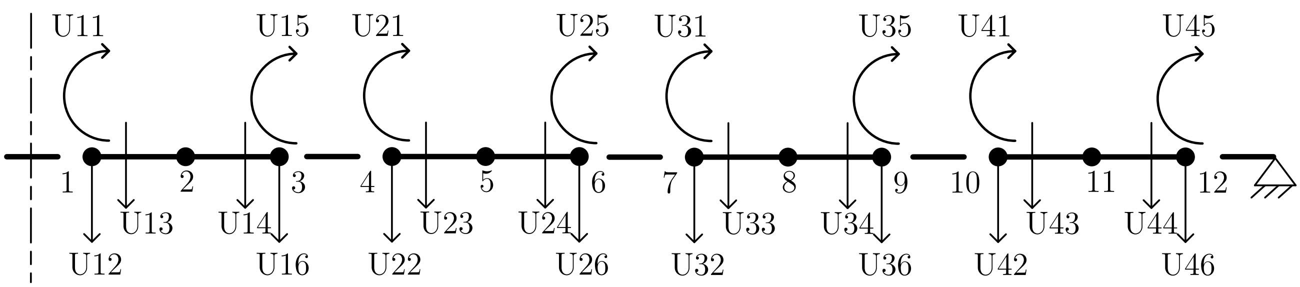


Figure 3: Local displacements and moments of each element

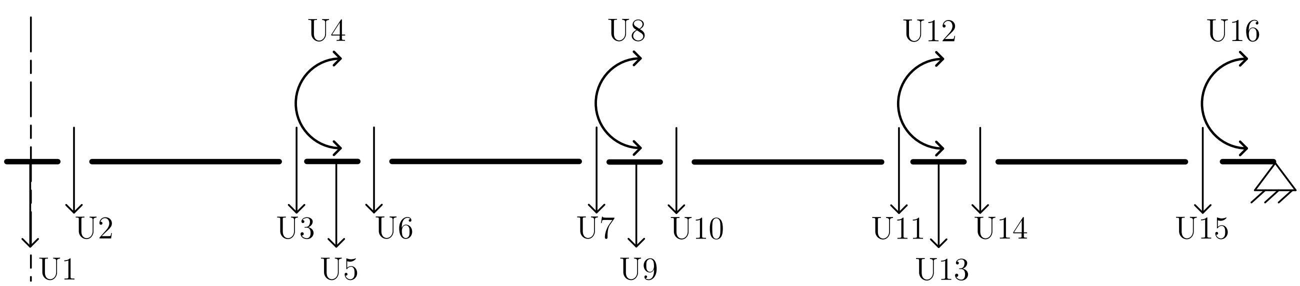


Figure 4: Global displacements and moments of each element

Compatibility matrix of displacements

It's based on the discrete model. It represents a relationship among the local and global displacements. We used 4 finite elements for each one has 6 local displacement and 5 nodes have total number of global displacements m=16. The final compatibility matrix of displacements are shown on figure 5, where rows are global displacements and columns are local displacements.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5: Compatibility matrix of displacements

Coefficient matrix of equilibrium equations

The matrix of the coefficients of the equilibrium equations for a ring element of a round plate is given in the table.

$$
[\overline{A}_k] = 2\pi \begin{bmatrix}
\rho_{k,1} & 0 & 0 & 0 & 0 & 0 \\
1.5\frac{\rho_{k,1}}{b_k} - 1 & 1 & -2\frac{\rho_{k,1}}{b_k} & 0 & \frac{\rho_{k,1}}{2b_k} & 0 \\
-\frac{\rho_{k,2}}{b_k} + 2 & -\frac{5}{6} & 2\frac{\rho_{k,2}}{b_k} - 2 & \frac{2}{3} & -\frac{\rho_{k,2}}{b_k} & \frac{1}{6} \\
-\frac{\rho_{k,2}}{b_k} & -\frac{1}{6} & 2\frac{\rho_{k,2}}{b_k}2 & -\frac{2}{3} & -\frac{\rho_{k,2}}{b_k} - 2 & \frac{5}{6} \\
0 & 0 & 0 & 0 & -\rho_{k,3} & 0 \\
\frac{\rho_{k,3}}{2b_k} & 0 & -2\frac{\rho_{k,3}}{b_k} & 0 & 1 + 1.5\frac{\rho_{k,3}}{b_k} & -1
\end{bmatrix} \tag{1}
$$

$A = [C]^T[\overline{A}]$ - equilibrium equation matrix, where:

$[C]^T$ - is transpose of compatibility matrix of displacements $[C]$

$[A]$ - is the matrix above in table

Calculation of the equilibrium equation matrix from Matlab

```
A =
  Columns 1 through 9
   -6.2832    6.2832         0         0    3.1416         0         0         0         0
    6.2832   -5.2360         0    4.1888   -6.2832    1.0472         0         0         0
   -6.2832   -1.0472   25.1327   -4.1888  -18.8496    5.2360         0         0         0
         0         0         0         0   -9.4248         0    9.4248         0         0
    6.2832         0  -25.1327         0   25.1327   -6.2832   12.5664    6.2832  -25.1327
         0         0         0         0         0         0   -6.2832   -5.2360   25.1327
         0         0         0         0         0         0  -18.8496   -1.0472   50.2655
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0   12.5664         0  -50.2655
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
  Columns 10 through 18
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0    9.4248         0         0         0         0         0         0         0
    4.1888  -18.8496    1.0472         0         0         0         0         0         0
   -4.1888  -31.4159    5.2360         0         0         0         0         0         0
         0  -18.8496         0   18.8496         0         0         0         0         0
         0   43.9823   -6.2832   12.5664    6.2832  -25.1327         0    9.4248         0
         0         0         0   -6.2832   -5.2360   25.1327    4.1888  -18.8496    1.0472
         0         0         0  -18.8496   -1.0472   50.2655   -4.1888  -31.4159    5.2360
         0         0         0         0         0         0         0  -37.6991         0
         0         0         0   12.5664         0  -50.2655         0   43.9823   -6.2832
         0         0         0         0         0         0         0         0         0
```

```
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
  Columns 19 through 24
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
   37.6991         0         0         0         0         0
   50.2655    6.2832  -75.3982         0   21.9911         0
  -31.4159   -5.2360   75.3982    4.1888  -43.9823    1.0472
  -43.9823   -1.0472  100.5310   -4.1888  -56.5487    5.2360
         0         0         0         0  -50.2655         0
```

Flexibility matrix

To reduce the size of the matrix and ease of placing it in the report, the equations were replaced by j1, j2, j3, j4.

$$[D_k] = \frac{2\pi b_k}{15K_k(1-\nu_k^2)} \begin{bmatrix} j_2 & -\nu_k j_2 & 2j_1 & -2\nu_k j_1 & -\rho_{k,2} & \nu_k \rho_{k,2} \\ & j_2 & -2\nu_k j_1 & 2j_1 & \nu_k \rho_{k,2} & -\rho_{k,2} \\ & & 16\rho_{k,2} & -16\nu_k \rho_{k,2} & 2j_3 & -2\nu_k j_3 \\ & & & 16\rho_{k,2} & -2\nu_k j_3 & 2j_3 \\ & & & & j_4 & -\nu_k j_4 \\ symm. & & & & & j_4 \end{bmatrix} \tag{2}$$

Where:

$K_k = \frac{E_k t_k^3}{12(1-\nu_k^2)}$ $j_1 = \rho_{k,2} - b_k$

$j_2 = 4\rho_{k,2} - 3b_k$

$j_3 = \rho_{k,2} + b_k$

$j_4 = 4\rho_{k,2} + 3b_k$

Calculation of the flexibility matrix from Matlab

```
D =
  Columns 1 through 9
    0.0001   -0.0000         0         0   -0.0001    0.0000         0         0         0
   -0.0000    0.0001         0         0    0.0000   -0.0001         0         0         0
         0         0    0.0017   -0.0005    0.0004   -0.0001         0         0         0
         0         0   -0.0005    0.0017   -0.0001    0.0004         0         0         0
   -0.0001    0.0000    0.0004   -0.0001    0.0008   -0.0002         0         0         0
    0.0000   -0.0001   -0.0001    0.0004   -0.0002    0.0008         0         0         0
         0         0         0         0         0         0    0.0010   -0.0003    0.0004
         0         0         0         0         0         0   -0.0003    0.0010   -0.0001
         0         0         0         0         0         0    0.0004   -0.0001    0.0052
         0         0         0         0         0         0   -0.0001    0.0004   -0.0016
         0         0         0         0         0         0   -0.0003    0.0001    0.0009
         0         0         0         0         0         0    0.0001   -0.0003   -0.0003
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
  Columns 10 through 18
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
```

0	0	0	0	0	0	0	0	0
-0.0001	-0.0003	0.0001	0	0	0	0	0	0
0.0004	0.0001	-0.0003	0	0	0	0	0	0
-0.0016	0.0009	-0.0003	0	0	0	0	0	0
0.0052	-0.0003	0.0009	0	0	0	0	0	0
-0.0003	0.0016	-0.0005	0	0	0	0	0	0
0.0009	-0.0005	0.0016	0	0	0	0	0	0
0	0	0	0.0039	-0.0012	0.0017	-0.0005	-0.0013	0.0004
0	0	0	-0.0012	0.0039	-0.0005	0.0017	0.0004	-0.0013
0	0	0	0.0017	-0.0005	0.0207	-0.0062	0.0034	-0.0010
0	0	0	-0.0005	0.0017	-0.0062	0.0207	-0.0010	0.0034
0	0	0	-0.0013	0.0004	0.0034	-0.0010	0.0065	-0.0019
0	0	0	0.0004	-0.0013	-0.0010	0.0034	-0.0019	0.0065
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Columns 19 through 24

0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0	0	0	0	0	0
0.0048	-0.0014	0.0023	-0.0007	-0.0013	0.0004
-0.0014	0.0048	-0.0007	0.0023	0.0004	-0.0013
0.0023	-0.0007	0.0214	-0.0064	0.0031	-0.0009
-0.0007	0.0023	-0.0064	0.0214	-0.0009	0.0031
-0.0013	0.0004	0.0031	-0.0009	0.0059	-0.0018
0.0004	-0.0013	-0.0009	0.0031	-0.0018	0.0059

External load vector

Vector of external forces for node described as:

$$\{F_k\} = \frac{2\pi b_k \rho_k}{3} \begin{Bmatrix} 3\rho_{k,2} - b_k \\ 3\rho_{k,2} + b_k \end{Bmatrix} = \{\eta_k\}\rho_k \tag{3}$$

$[F] = [F_o] + [C]^T[F_p]$, where:

$[F_o] = f_1 2\pi\rho_{f1}$

f_1 – value of external force which correspond u_1

ρ_{f_1} - coordinate where is m_1

$[F_p]$ - represent the equivalent of distributed loads.

$[C]^T$ - is transpose of compatibility matrix of displacements $[C]$

```
F =
         0
   47.1239
   94.2478
         0
         0
  188.4956
  235.6194
         0
         0
         0
         0
         0
  753.9822
         0
         0
         0
```

The results of global displacements are shown as following :

$$[U] = \{[A][D]^{-1}\}^{-1}[A]^T[F] \tag{4}$$

Where:

A - coefficient matrix of equilibrium equations

$[A]^T$ - transpose coefficient matrix of equilibrium equations

$[D]$: flexibility matrix

$[F]$ - External load vector

```
Uglob =
    0.1642
    0.1682
    0.1496
    0.0239
    0.1452
    0.1484
    0.1520
    0.0061
    0.1459
    0.1515
    0.0946
   -0.0318
```

```
  0.0792
  0.0781
  0.0037
 -0.0373
```

The results of internal forces are shown as following:

$$[S] = [D]^{-1}[A]^T[U] \tag{5}$$

Where:

$[D]$ - flexibility matrix
$[A]^T$- transpose coefficient matrix of equilibrium equations
$[U]$ - global displacements

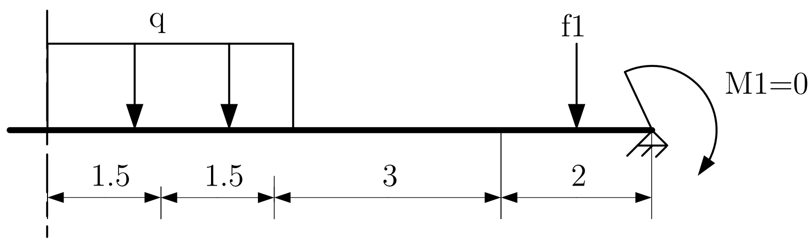
Results of internal forces (S) consists of M_ρ and moments M_φ. Schema have 24 values of internal forces (S) it's divided into: 12 values of moment M_ρ and 12 values of moments M_φ. In the matlab command $M_\rho = S(1:2:end)$ means to take the 1st value of (S) , 3rd , 5th , 7 th ..... till end. All those values belong to moment M_ρ. For $M_\varphi = S(2:2:end)$ means to take the 2nd value of (S), 4th , 6th.....till end.

```
M_Ro =
   34.5141
   66.5837
   78.1404
   78.1404
   65.6556
   54.6631
   54.6631
   41.7329
   36.0571
   36.0571
   15.8937
         0


M_fi =
   -4.5561
   54.9516
   64.1707
    7.7766
   14.6629
   37.2764
   27.1181
   19.3936
   33.4322
   27.8368
   15.2635
   15.4994
```

Internal forces and displacements

The equilibrium of finite element method used to solve the mentioned annular plate. The results were obtained from a MATLAB commands. Based on these calculations, the results shows the internal forces and the a distribution of global displacements along our structure. The results shows that the maximum displacement was U = 36.0471 mm in the direction gravity. The allowable displacement was U = L/250 = 64 mm so, the verification was correct based on the current geometry and material properties. Some parametric analysis were done for plate thickness as a very important parameter to reduce the displacement values.



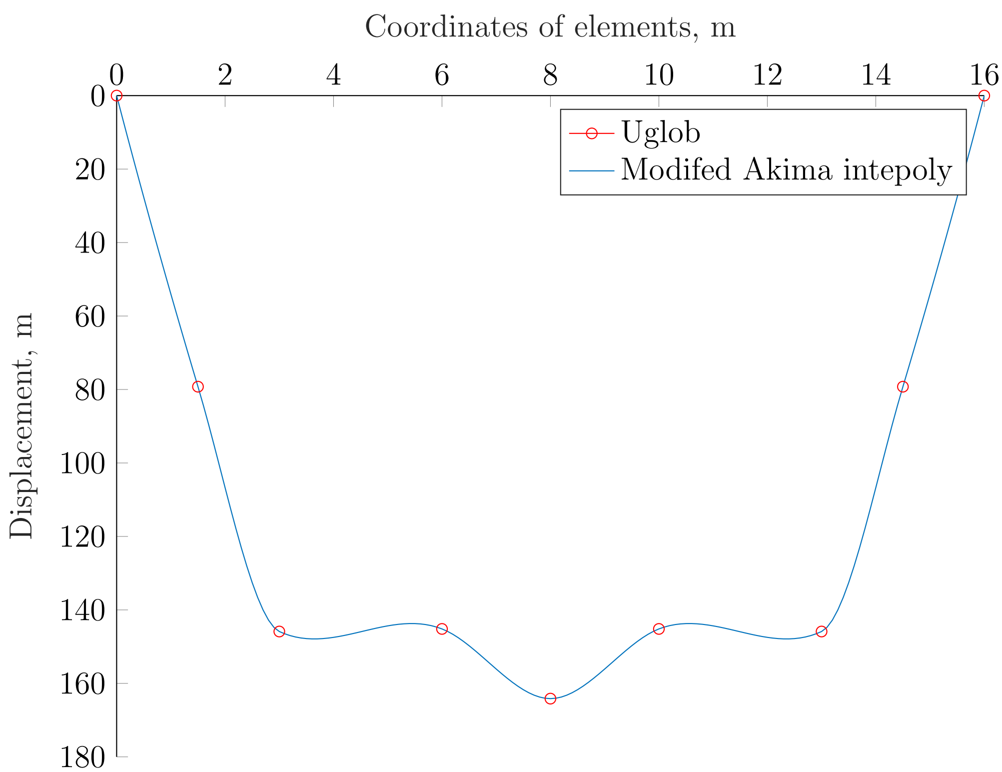


Figure 6: Global displacement across plate

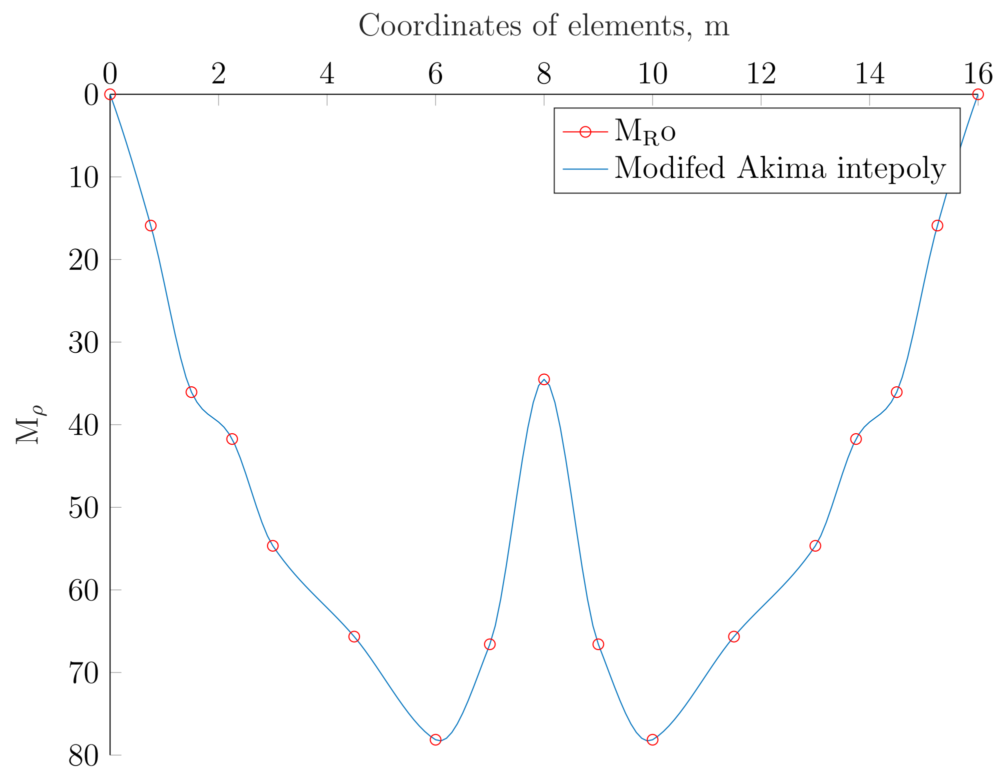


Figure 7: M_ρ across plate

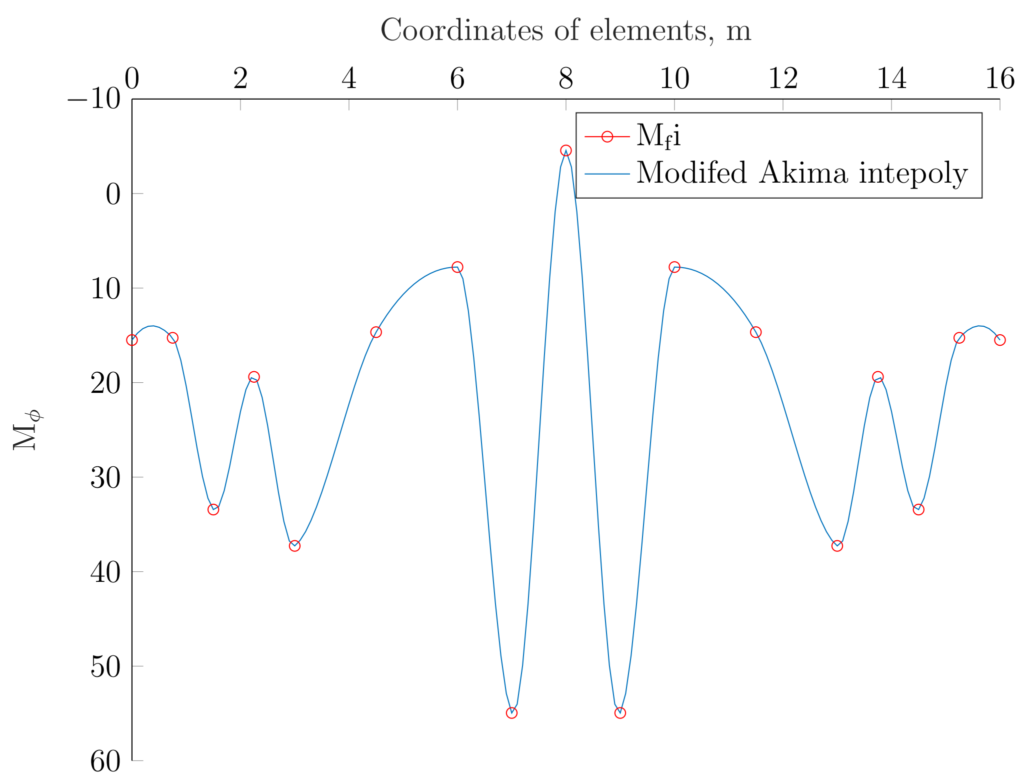


Figure 8: M_φ across plate

Code listing

Main function

```
close all; clc; clear;
format compact;
addpath('../matlab2tikz/');
% initial data
q_LOAD =20; % KN/m
f=20; % KN
L=16; % span length m
h=0.05; % thickness of plate
E=210e6; % modulus of elastisty kPa
v=.3; % poisson's ratio
%
r=L/2; % radius of plate
B=[1.5 1.5 3 2];%vector of elements lengths, meters
no_FE=length(B); % Number of plate finite elements, as length of vector with elements lengths
b=B/2; % half of elements lengths
coords = zeros(no_FE,3);%filling matrix (number of elements by 3) by zeros
coords(1,2)=B(1)/2;% coordinate of half of first element
coords(1,3)=B(1);% coordinate of end of first element
for i=2:no_FE% loop over coordinates matrix, "i" is current element, "i-1" is previous element
    coords(i,1)=coords(i - 1,3);%start coordinate as end coordinate of previous element
    coords(i,2)=coords(i - 1,3) + B(i)/2;%half coordinate as end coordinate of previous element plus half length of current element
    coords(i,3)= coords(i - 1,3) + B(i);%end coordinate as end coordinate of previous element plus length of current element
end
saveAsFileDairy('coords', coords);
no_of_local_dis=6; % Number of local displacements
no_of_global_dis=16; % Number of global displacements
% 2. Compatipality matrix C
% for the first FE
c_1stcompatipality_matrix=zeros(no_of_local_dis,no_of_global_dis);
c_1stcompatipality_matrix(2:6,1:5)=eye(5);
% for the Second FE
c_2ndcompatipality_matrix=zeros(no_of_local_dis,no_of_global_dis);
c_2ndcompatipality_matrix(1:6,4:9)=eye(6);
% for the third FE
c_3rdcompatipality_matrix=zeros(no_of_local_dis,no_of_global_dis);
c_3rdcompatipality_matrix(1:6,8:13)=eye(6);
% for the 4th FE
c_4thcompatipality_matrix=zeros(no_of_local_dis,no_of_global_dis);
c_4thcompatipality_matrix(1:5,12:16)=eye(5);
% the total compatipality matrix of displacements
C=[c_1stcompatipality_matrix;c_2ndcompatipality_matrix;c_3rdcompatipality_matrix;c_4thcompatipality_matrix];
%saveAsFileDairy('C', C);
%4. Matrix of equilibrium equantions A
for k=1:no_FE
A_matrix = getAmtx(coords(k,1), coords(k,2), coords(k,3), b(k));
A_(k*6-5:k*6,k*6-5:k*6)=2*pi*A_matrix;
end
A=C'*A_;
```

```
saveAsFileDairy('A', A);
%% 5. Flexibility MATRIX OF D
for k=1:no_FE
Rok2=coords(k,2);
bk=b(k);
D_matrix = getDmtx(coords(k,2), b(k), v);
K.k=E*h^3/(12*(1-v^2));
D_(k*6-5:k*6,k*6-5:k*6)=(2*pi*bk/(15*K.k*(1-v^2)))*D_matrix;
end
saveAsFileDairy('D', D_);
%% 6. EXTERNAL LOAD VICTOR F
Fo=zeros(no_of_global_dis,1);
Rof=6;
 Fo(13)=f*2*pi*Rof;
 % Rof coordinate where f
 % Fkp is nodal external load vector which is equivilant to distributed load of the kth element
 q_Load_vector=[20 20 0 0];
 for k=1:no_FE
     bk=b(k);
     q=q_Load_vector(k);
     Rok2=coords(k,2);
     Fk=(2*pi*bk/3)*q*[3*Rok2-bk;3*Rok2+bk];
     Fp=[0;0;Fk;0;0];
     Fp_(6*k-5:k*6,1)=Fp;
 end
F=Fo+C'*Fp_;
saveAsFileDairy('F', F);

Uglob=inv(A*inv(D_)*A')*F;
saveAsFileDairy('Uglob', Uglob);

S=inv(D_)*A'*Uglob;
saveAsFileDairy('S', S);

M_Ro=S(1:2:end);
saveAsFileDairy('M_Ro', M_Ro);

M_fi=S(2:2:end);
saveAsFileDairy('M_fi', M_fi);

um_mm = 1000*[Uglob(1:4:end);0];
saveAsFileDairy('um_mm', um_mm);
% xcoord = [0;coords(1:end,3)];
% figure(1);
% plot(xcoord,um_mm,'DisplayName','Uglob');
% xlabel('Coordinates of elements, m')
% ylabel('Displacement, m')
% set(gca,'XAxisLocation','top','YAxisLocation','left','ydir','reverse');
% matlab2tikz('um_mm.tex','showInfo', false);
u_allowable=16/250*1000;
saveAsFile('u_allowable', u_allowable);
fullCoord = getFullCoords(B, coords);
plotU(fullCoord, um_mm, 1, true);
plotMro(fullCoord, M_Ro, 2, true);
plotMfi(fullCoord, M_fi,3, true);
```

getAmtx function

```
function A_matrix = getAmtx(Rok1, Rok2, Rok3, bk)
    A_matrix = zeros(6,6);
    A_matrix(1,1)=Rok1;
    A_matrix(2,1)=1.5*Rok1/bk-1;
    A_matrix(2,2)=1;
    A_matrix(2,3)=-2*Rok1/bk;
    A_matrix(2,5)=Rok2/(2*bk);
    A_matrix(3,1)=-Rok2/bk+2;
    A_matrix(3,2)=-5/6;
    A_matrix(3,3)=2*Rok2/bk-2;
    A_matrix(3,4)=2/3;
    A_matrix(3,5)=-Rok2/bk;
    A_matrix(3,6)=1/6;
    A_matrix(4,1)=-Rok2/bk;
    A_matrix(4,2)=-1/6;
    A_matrix(4,3)=2*Rok2/bk+2;
    A_matrix(4,4)=-2/3;
    A_matrix(4,5)=-Rok2/bk-2;
    A_matrix(4,6)=5/6;
    A_matrix(5,5)=-Rok3;
    A_matrix(6,1)=Rok3/(2*bk);
    A_matrix(6,3)=-2*Rok3/bk;
    A_matrix(6,5)=1+1.5*Rok3/bk;
    A_matrix(6,6)=-1;
end
```

getDmtx function

```
function D_matrix = getDmtx(Rok2, bk, v)
    j1=Rok2-bk;
    j2=4*Rok2-3*bk;
    j3=Rok2+bk;
    j4=4*Rok2+3*bk;
    D_matrix(1,1)=j2;
    D_matrix(1,2)=-v*j2;
    D_matrix(1,3)=2*j1;
    D_matrix(1,4)=-2*v*j1;
    D_matrix(1,5)=-Rok2;
    D_matrix(1,6)=v*Rok2;
    D_matrix(2,1)=D_matrix(1,2);
    D_matrix(2,2)=j2;
    D_matrix(2,3)=-2*v*j1;
    D_matrix(2,4)=2*j1;
    D_matrix(2,5)=v*Rok2;
    D_matrix(2,6)=-Rok2;
    D_matrix(3,1)=D_matrix(1,3);
    D_matrix(3,2)=D_matrix(2,3);
    D_matrix(3,3)=16*Rok2;
    D_matrix(3,4)=-16*v*Rok2;
    D_matrix(3,5)=2*j3;
    D_matrix(3,6)=-2*v*j3;
    D_matrix(4,1)=D_matrix(1,4);
    D_matrix(4,2)=D_matrix(2,4);
    D_matrix(4,3)=D_matrix(3,4);
    D_matrix(4,4)=16*Rok2;
    D_matrix(4,5)=-2*v*j3;
    D_matrix(4,6)=2*j3;
    D_matrix(5,1)=D_matrix(1,5);
    D_matrix(5,2)=D_matrix(2,5);
    D_matrix(5,3)=D_matrix(3,5);
    D_matrix(5,4)=D_matrix(4,5);
    D_matrix(5,5)=j4;
    D_matrix(5,6)=-v*j4;
    D_matrix(6,1)=D_matrix(1,6);
    D_matrix(6,2)=D_matrix(2,6);
    D_matrix(6,3)=D_matrix(3,6);
    D_matrix(6,4)=D_matrix(4,6);
    D_matrix(6,5)=D_matrix(5,6);
    D_matrix(6,6)=j4;
end
```