Contents

Discrete model of circular plate

According to given initial conditions structure is circle width some amount of distributed load in center. Due to the symmetry of our structure, has to be studied only half of structure(fig.1).

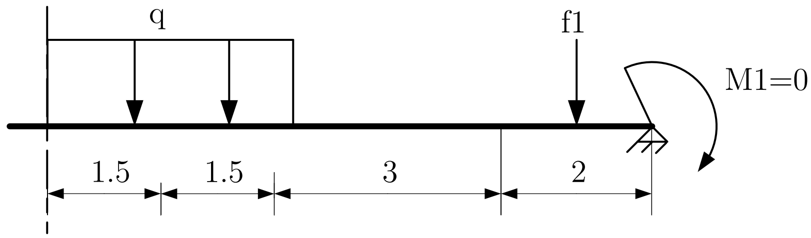

Figure 1: 1D Rod system in global coordinate system

Developed discrete model of given structure consist 4 finite elements(fig.2), local displacements and moments of each element shown on figure 3, while global ones are on figure 4.

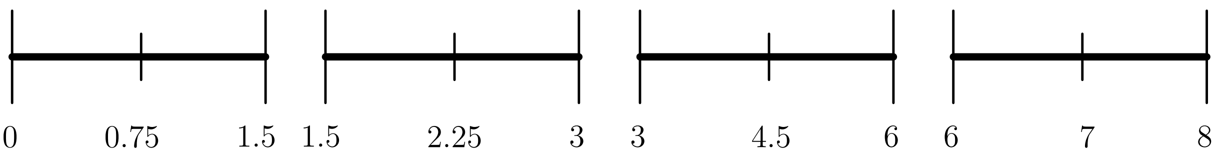

Figure 2: 1D Rod system in global coordinate system

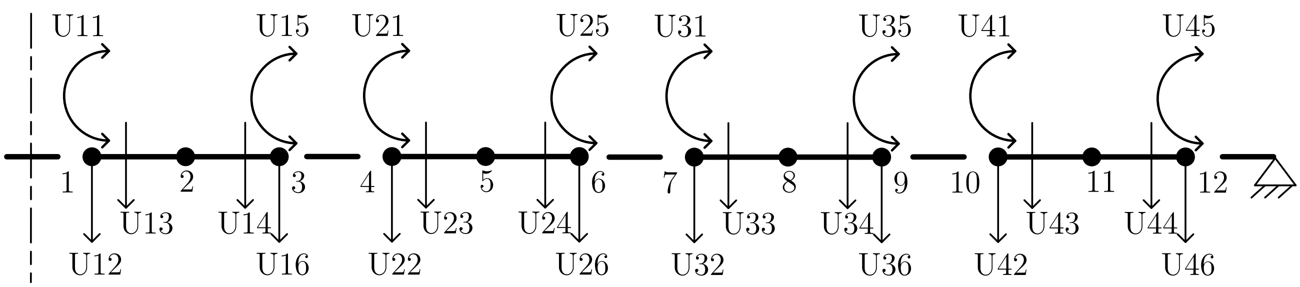

Figure 3: 1D Rod system in global coordinate system

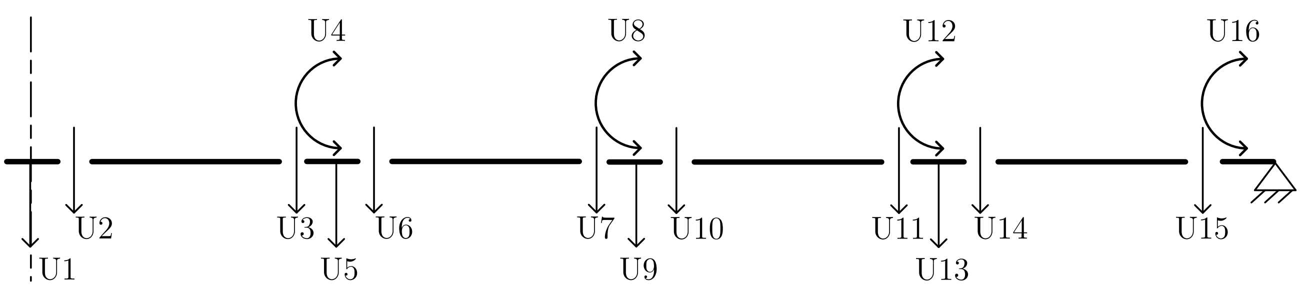

Figure 4: 1D Rod system in global coordinate system

Compatibility matrix of displacements

It's based on the discrete model. It represents a relationship among the local and global displacements. We used 4 finite elements for each one has 6 local displacement and 5 nodes have total number of global displacements m=16. The final compatibility matrix of displacements are shown on figure 5, where rows are global displacements and columns are local displacements.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
5	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
6	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
7	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
16	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
17	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
18	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
19	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
20	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
21	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0
22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 5: Mitral valve structure

Coefficient matrix of equilibrium equations

$$[\overline{A}_k] = 2\pi \begin{bmatrix} \rho_{k,1} & 0 & 0 & 0 & 0 & 0 \\ 1.5\dfrac{\rho_{k,1}}{b_k} - 1 & 1 & -2\dfrac{\rho_{k,1}}{b_k} & 0 & \dfrac{\rho_{k,1}}{2b_k} & 0 \\ -\dfrac{\rho_{k,2}}{b_k} + 2 & -\dfrac{5}{6} & 2\dfrac{\rho_{k,2}}{b_k} - 2 & \dfrac{2}{3} & -\dfrac{\rho_{k,2}}{b_k} & \dfrac{1}{6} \\ -\dfrac{\rho_{k,2}}{b_k} & -\dfrac{1}{6} & 2\dfrac{\rho_{k,2}}{b_k}2 & -\dfrac{2}{3} & -\dfrac{\rho_{k,2}}{b_k} - 2 & \dfrac{5}{6} \\ 0 & 0 & 0 & 0 & -\rho_{k,3} & 0 \\ \dfrac{\rho_{k,3}}{2b_k} & 0 & -2\dfrac{\rho_{k,3}}{b_k} & 0 & 1 + 1.5\dfrac{\rho_{k,3}}{b_k} & -1 \end{bmatrix} \tag{1}$$

$A = [C]^T[\overline{A}]$ - equilibrium equation matrix, where:

$[C]^T$ - is transpose of compatibility matrix of displacements $[C]$

$[A]$ - is the matrix above in table

```
A =
  Columns 1 through 9
   -6.2832    6.2832         0         0    1.7671         0         0         0         0
    6.2832   -5.2360         0    4.1888   -6.2832    1.0472         0         0         0
   -6.2832   -1.0472   25.1327   -4.1888  -18.8496    5.2360         0         0         0
         0         0         0         0   -9.4248         0    9.4248         0         0
    3.5343         0  -25.1327         0   25.1327   -6.2832   12.5664    6.2832  -25.1327
         0         0         0         0         0         0   -6.2832   -5.2360   25.1327
         0         0         0         0         0         0  -18.8496   -1.0472   50.2655
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0    7.0686         0  -50.2655
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
  Columns 10 through 18
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0    5.3014         0         0         0         0         0         0         0
    4.1888  -18.8496    1.0472         0         0         0         0         0         0
   -4.1888  -31.4159    5.2360         0         0         0         0         0         0
         0  -18.8496         0   18.8496         0         0         0         0         0
         0   43.9823   -6.2832   12.5664    6.2832  -25.1327         0   21.2058         0
         0         0         0   -6.2832   -5.2360   25.1327    4.1888  -18.8496    1.0472
         0         0         0  -18.8496   -1.0472   50.2655   -4.1888  -31.4159    5.2360
         0         0         0         0         0         0         0  -37.6991         0
         0         0         0   28.2743         0  -50.2655         0   43.9823   -6.2832
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
  Columns 19 through 24
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
   37.6991         0         0         0         0         0
   50.2655    6.2832  -75.3982         0   21.9911         0
  -31.4159   -5.2360   75.3982    4.1888  -43.9823    1.0472
```

```
-43.9823   -1.0472  100.5310   -4.1888  -56.5487    5.2360
       0         0         0         0  -50.2655         0
```

Flexibility matrix

$$[D_k] = \frac{2\pi b_k}{15K_k(1-\nu_k^2)} \begin{bmatrix} j_2 & -\nu_k j2 & 2j_1 & -2\nu_k j_1 & -\rho_{k,2} & \nu_k \rho_{k,2} \\ & j_2 & -2\nu_k j_1 & 2j_1 & \nu_k \rho_{k,2} & -\rho_{k,2} \\ & & 16\rho_{k,2} & -16\nu_k \rho_{k,2} & 2j_3 & -2\nu_k j_3 \\ & & & 16\rho_{k,2} & -2\nu_k j_3 & 2j_3 \\ & & & & j_4 & -\nu_k j_4 \\ symm. & & & & & j_4 \end{bmatrix} \tag{2}$$

Where:

$K_k = \frac{E_k t_k^3}{12(1-\nu_k^2)}$ $j_1 = \rho_{k,2} - b_k$
$j_2 = 4\rho_{k,2} - 3b_k$
$j_3 = \rho_{k,2} + b_k$
$j_4 = 4\rho_{k,2} + 3b_k$

```
D_ =
  Columns 1 through 9
    0.0001   -0.0000         0         0   -0.0001    0.0000         0         0         0
   -0.0000    0.0001         0         0    0.0000   -0.0001         0         0         0
         0         0    0.0017   -0.0005    0.0004   -0.0001         0         0         0
         0         0   -0.0005    0.0017   -0.0001    0.0004         0         0         0
   -0.0001    0.0000    0.0004   -0.0001    0.0008   -0.0002         0         0         0
    0.0000   -0.0001   -0.0001    0.0004   -0.0002    0.0008         0         0         0
         0         0         0         0         0         0    0.0010   -0.0003    0.0004
         0         0         0         0         0         0   -0.0003    0.0010   -0.0001
         0         0         0         0         0         0    0.0004   -0.0001    0.0052
         0         0         0         0         0         0   -0.0001    0.0004   -0.0016
         0         0         0         0         0         0   -0.0003    0.0001    0.0009
         0         0         0         0         0         0    0.0001   -0.0003   -0.0003
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
  Columns 10 through 18
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
   -0.0001   -0.0003    0.0001         0         0         0         0         0         0
    0.0004    0.0001   -0.0003         0         0         0         0         0         0
   -0.0016    0.0009   -0.0003         0         0         0         0         0         0
    0.0052   -0.0003    0.0009         0         0         0         0         0         0
   -0.0003    0.0016   -0.0005         0         0         0         0         0         0
    0.0009   -0.0005    0.0016         0         0         0         0         0         0
         0         0         0    0.0039   -0.0012    0.0017   -0.0005   -0.0013    0.0004
         0         0         0   -0.0012    0.0039   -0.0005    0.0017    0.0004   -0.0013
         0         0         0    0.0017   -0.0005    0.0207   -0.0062    0.0034   -0.0010
         0         0         0   -0.0005    0.0017   -0.0062    0.0207   -0.0010    0.0034
         0         0         0   -0.0013    0.0004    0.0034   -0.0010    0.0065   -0.0019
         0         0         0    0.0004   -0.0013   -0.0010    0.0034   -0.0019    0.0065
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
```

```
         0         0         0         0         0         0         0         0         0
         0         0         0         0         0         0         0         0         0
  Columns 19 through 24
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
         0         0         0         0         0         0
    0.0048   -0.0014    0.0023   -0.0007   -0.0013    0.0004
   -0.0014    0.0048   -0.0007    0.0023    0.0004   -0.0013
    0.0023   -0.0007    0.0214   -0.0064    0.0031   -0.0009
   -0.0007    0.0023   -0.0064    0.0214   -0.0009    0.0031
   -0.0013    0.0004    0.0031   -0.0009    0.0059   -0.0018
    0.0004   -0.0013   -0.0009    0.0031   -0.0018    0.0059
```

External load vector

Vector of external forces for node described as:

$$\{F_k\} = \frac{2\pi b_k \rho_k}{3} \begin{Bmatrix} 3\rho_{k,2} - b_k \\ 3\rho_{k,2} + b_k \end{Bmatrix} = \{\eta_k\}\rho_k \tag{3}$$

$[F] = [F_o] + [C]^T[F_p]$, where:

$[F_o] = -m12\pi rho_{f,1}$

m_1 – value of bending moment which correspond u_1

$\rho_{f,1}$ - coordinate where is m_1

$[F_p]$ - represent the equivalent of distributed loads.

$[C]^T$ - is transpose of compatibility matrix of displacements $[C]$

```
F =
         0
   47.1239
   94.2478
         0
         0
  188.4956
  235.6194
         0
         0
         0
         0
         0
  753.9822
         0
         0
         0
```

The results of global displacements are shown as following :

$$[U] = \{[A][D]^{-1}\}^{-1}[A]^T[F] \tag{4}$$

Where:

A - coefficient matrix of equilibrium equations

$[A]^T$ - transpose coefficient matrix of equilibrium equations

$[D]$: flexibility matrix

$[F]$ - External load vector

```
Uglob =
    0.0340
    0.0383
    0.0185
    0.0020
    0.0216
    0.0235
    0.0134
   -0.0069
    0.0143
    0.0205
    0.0224
   -0.0037
    0.0185
    0.0179
    0.0026
   -0.0103
```

The results of internal forces are shown as following:

$$[S] = [D]^{-1}[A]^T[U] \tag{5}$$

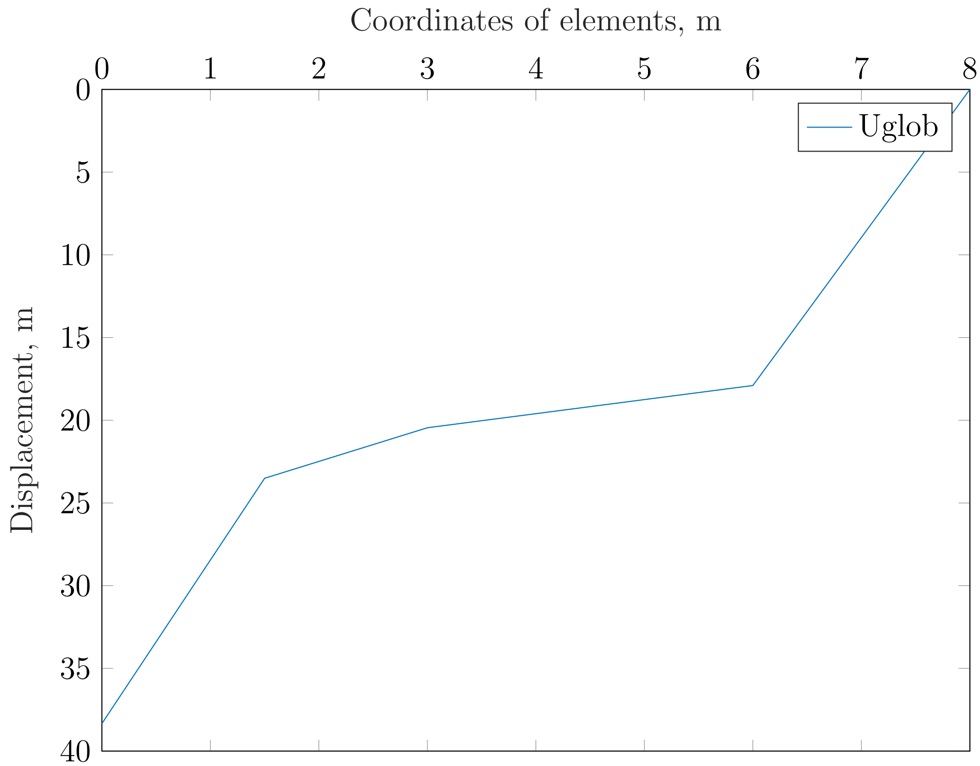

Figure 6: My first matlab2tikz figure

Where:

$[D]$ - flexibility matrix

$[A]^T$- transpose coefficient matrix of equilibrium equations

$[U]$ - global displacements

```
Ulocal =
 -158.2735
 -155.1518
  -31.3524
   50.9813
  -11.0997
  -51.6406
  -11.0997
  -10.0672
    3.5628
   10.3970
    3.4310
   -2.2716
    3.4310
   -9.1624
   14.6141
    4.6054
   22.3579
    8.0553
   22.3579
   11.6794
    9.7933
    4.5640
    0.0000
    5.7628
```

Internal forces and displacements

The equilibrium of finite element method used to solve the mentioned annular plate. The results were obtained from a MATLAB commands. Based on these calculations, the results shows the internal forces and the a distribution of global displacements along our structure. The results shows that the maximum displacement was U = 36.0471 mm in the direction gravity. The allowable displacement was $U_allowable$ = L/250 = 64 mm so, the verification was correct based on the current geometry and material properties. Some parametric analysis were done for plate thickness as a very important parameter to reduce the displacement values.

```
Ulocal =
 -158.2735
 -155.1518
  -31.3524
   50.9813
  -11.0997
  -51.6406
  -11.0997
  -10.0672
    3.5628
   10.3970
    3.4310
   -2.2716
    3.4310
   -9.1624
   14.6141
    4.6054
   22.3579
    8.0553
   22.3579
   11.6794
    9.7933
    4.5640
    0.0000
    5.7628
```

Code listing

Main function

```
clc; clear;
format compact;
addpath('../matlab2tikz/');
% initial data
q_LOAD =20; % KN/m
f=20; % KN
L=16; % span length m
h=0.05; % thickness of plate
E=210e6; % modulus of elastisty kPa
v=.3; % poisson's ratio
%
r=L/2; % radius of plate
B=[1.5 1.5 3 2];%vector of elements lengths, meters
no_FE=length(B); % Number of plate finite elements, as length of vector with elements lengths
b=B/2; % half of elements lengths
coords = zeros(no_FE,3);%filling matrix (number of elements by 3) by zeros
coords(1,2)=B(1)/2;% coordinate of half of first element
coords(1,3)=B(1);% coordinate of end of first element
for i=2:no_FE% loop over coordinates matrix, "i" is current element, "i-1" is previous element
    coords(i,1)=coords(i - 1,3);%start coordinate as end coordinate of previous element
    coords(i,2)=coords(i - 1,3) + B(i)/2;%half coordinate as end coordinate of previous element plus half length of current element
    coords(i,3)= coords(i - 1,3) + B(i);%end coordinate as end coordinate of previous element plus length of current element
end
delete coordsoutput.txt;
diary('coordsoutput.txt');
diary on ;
coords% printing coordinates matrix
diary off ;%to avoid print other commands.
no_of_local_dis=6; % Number of local displacements
no_of_global_dis=16; % Number of global displacements

% 2. Compatipality matrix C
% for the first FE
c_1stcompatipality_matrix=zeros(no_of_local_dis,no_of_global_dis);
c_1stcompatipality_matrix(2:6,1:5)=eye(5);
% for the Second FE
c_2ndcompatipality_matrix=zeros(no_of_local_dis,no_of_global_dis);
c_2ndcompatipality_matrix(1:6,4:9)=eye(6);
```

```
% for the third FE
c_3rdcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
c_3rdcompatipality_matrix(1:6,8:13)=eye(6);
% for the 4th FE
c_4thcompatipality_matrix=zeros(no_of_local_dis ,no_of_global_dis);
c_4thcompatipality_matrix(1:5,12:16)=eye(5);
% the total compatipality matrix of displacements
delete Cmtxoutput.txt;
diary('Cmtxoutput.txt');
diary on ;
C=[c_1stcompatipality_matrix;c_2ndcompatipality_matrix; c_3rdcompatipality_matrix;c_4thcompatipality_matrix]
diary off ;%to avoid print other commands.

%4. Matrix of equilibrium equantions A
for k=1:no_FE
A_matrix = getAmtx(coords(k,1), coords(k,2), coords(k,3), b(k));
A_(k*6-5:k*6,k*6-5:k*6)=2*pi*A_matrix;
end
delete Amtxoutput.txt;
diary('Amtxoutput.txt');
diary on ;
A=C'*A_
diary off ;%to avoid print other commands.



% % 5. Flexibility MATRIX OF D
for k=1:no_FE
Rok2=coords(k,2);
bk=b(k);
D_matrix = getDmtx(coords(k,2), b(k), v);
K.k=E*h^3/(12*(1-v^2));
D_(k*6-5:k*6,k*6-5:k*6)=(2*pi*bk/(15*K.k*(1-v^2)))*D_matrix;
end
delete Dmtxoutput.txt;
diary('Dmtxoutput.txt');
diary on ;
D_
diary off ;%to avoid print other commands.


% % 6. EXTERNAL LOAD VICTOR F
Fo=zeros(no_of_global_dis ,1);
Rof=6;
 Fo(13)=f*2*pi*Rof;
 % Rof coordinate where f
 % Fkp is nodal external load vector which is equivilant to distributed load of the kth element
```

```
 q_Load_vector=[20 20 0 0];
 for k=1:no_FE
      bk=b(k);
      q=q_Load_vector(k);
      Rok2=coords(k,2);
      Fk=(2*pi*bk/3)*q*[3*Rok2-bk;3*Rok2+bk];
      Fp=[0;0;Fk;0;0];
      Fp_(6*k-5:k*6,1)=Fp;
 end

delete Foutput.txt;
diary('Foutput.txt');
diary on ;
F=Fo+C'*Fp_
diary off ;%to avoid print other commands.

delete Ugloboutput.txt;
diary('Ugloboutput.txt');
diary on ;
Uglob=inv(A*inv(D_)*A')*F
diary off ;%to avoid print other commands.

delete Ulocaloutput.txt;
diary('Ulocaloutput.txt');
diary on ;
Ulocal=inv(D_)*A'*Uglob
diary off ;%to avoid print other commands.



M_Ro=Ulocal(1:2:end);
M_fi=Ulocal(2:2:end);
delete um_mmoutput.txt;
diary('um_mmoutput.txt');
diary on ;
um_mm = 1000*[Uglob(2:4:end);0]
diary off ;%to avoid print other commands.
xcoord = [0;coords(1:end,3)];
plot(xcoord,um_mm,'DisplayName','Uglob');
xlabel('Coordinates of elements, m')
ylabel('Displacement, m')
set(gca,'XAxisLocation','top','YAxisLocation','left','ydir','reverse'
    );
matlab2tikz('um_mm.tex','showInfo', false);

delete u_allowableoutput.txt;
diary('u_allowableoutput.txt');
diary on ;
u_allowable=16/250*1000
```

```
diary off ;%to avoid print other commands.
```

getAmtx function

```
function A_matrix = getAmtx(Rok1, Rok2, Rok3, bk)
    A_matrix = zeros(6,6);
    A_matrix(1,1)=Rok1;
    A_matrix(2,1)=1.5*Rok1/bk-1;
    A_matrix(2,2)=1;
    A_matrix(2,3)=-2*Rok1/bk;
    A_matrix(2,5)=Rok2/2*bk;
    A_matrix(3,1)=-Rok2/bk+2;
    A_matrix(3,2)=-5/6;
    A_matrix(3,3)=2*Rok2/bk-2;
    A_matrix(3,4)=2/3;
    A_matrix(3,5)=-Rok2/bk;
    A_matrix(3,6)=1/6;
    A_matrix(4,1)=-Rok2/bk;
    A_matrix(4,2)=-1/6;
    A_matrix(4,3)=2*Rok2/bk+2;
    A_matrix(4,4)=-2/3;
    A_matrix(4,5)=-Rok2/bk-2;
    A_matrix(4,6)=5/6;
    A_matrix(5,5)=-Rok3;
    A_matrix(6,1)=Rok3/2*bk;
    A_matrix(6,3)=-2*Rok3/bk;
    A_matrix(6,5)=1+1.5*Rok3/bk;
    A_matrix(6,6)=-1;
end
```

getDmtx function

```
function D_matrix = getDmtx(Rok2, bk, v)
    j1=Rok2-bk;
    j2=4*Rok2-3*bk;
    j3=Rok2+bk;
    j4=4*Rok2+3*bk;
    D_matrix(1,1)=j2;
    D_matrix(1,2)=-v*j2;
    D_matrix(1,3)=2*j1;
    D_matrix(1,4)=-2*v*j1;
    D_matrix(1,5)=-Rok2;
    D_matrix(1,6)=v*Rok2;
    D_matrix(2,1)=D_matrix(1,2);
    D_matrix(2,2)=j2;
    D_matrix(2,3)=-2*v*j1;
    D_matrix(2,4)=2*j1;
    D_matrix(2,5)=v*Rok2;
    D_matrix(2,6)=-Rok2;
    D_matrix(3,1)=D_matrix(1,3);
```

```
    D_matrix(3,2)=D_matrix(2,3);
    D_matrix(3,3)=16*Rok2;
    D_matrix(3,4)=-16*v*Rok2;
    D_matrix(3,5)=2*j3;
    D_matrix(3,6)=-2*v*j3;
    D_matrix(4,1)=D_matrix(1,4);
    D_matrix(4,2)=D_matrix(2,4);
    D_matrix(4,3)=D_matrix(3,4);
    D_matrix(4,4)=16*Rok2;
    D_matrix(4,5)=-2*v*j3;
    D_matrix(4,6)=2*j3;
    D_matrix(5,1)=D_matrix(1,5);
    D_matrix(5,2)=D_matrix(2,5);
    D_matrix(5,3)=D_matrix(3,5);
    D_matrix(5,4)=D_matrix(4,5);
    D_matrix(5,5)=j4;
    D_matrix(5,6)=-v*j4;
    D_matrix(6,1)=D_matrix(1,6);
    D_matrix(6,2)=D_matrix(2,6);
    D_matrix(6,3)=D_matrix(3,6);
    D_matrix(6,4)=D_matrix(4,6);
    D_matrix(6,5)=D_matrix(5,6);
    D_matrix(6,6)=j4;
end
```