Forces in rod

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Chapter 1

1D rod system

Object of work is system of 1D rods. Object has been divided to discrete elements n1, n2 and n3. All elements are connected to each other through nodes n2, n3 and to special points through n1 and n4 (figure 1). Mathematical model of discrete system is expressed by equations of motion for nodes. As elements is 1D, nodes will be 1D as well. All system acting in global coordinate system $\{X, Y, Z\}$.

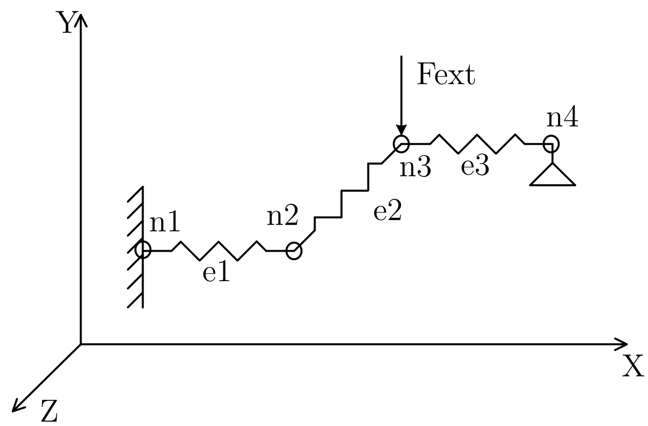

Figure 1.1: 1D Rod system in global coordinate system

1.1 Linear case

The motion of nodes can be expressed by Newton's equation of motion. If only the normal component of the translational motion is considered, the equation reduces to

$$F(x) - m\ddot{x} = 0 \tag{1.1}$$

where $F(x)$ is axial force, m – mass of node and $\ddot{x}$ is acceleration, initial conditions are: $x(0) = 0$ and $\dot{x}(0) = V_0$.

Mass of each node can be calculated, like sum of half mass of each element, which acting in node. $m_n = \sum_e m_e/2$

All vector varibles of node should be calculated in global coordinate system and of element in own local coordinate system(figure 1.1). For trasformation between coordinate systems direction cosine matrix (DCM)(1.2) can be used.

$$DCM = \begin{bmatrix} cos(Xx) & cos(Xy) & cos(Xz) \\ cos(Yx) & cos(Yy) & cos(Yz) \\ cos(Zx) & cos(Zy) & cos(Zz) \end{bmatrix} \tag{1.2}$$

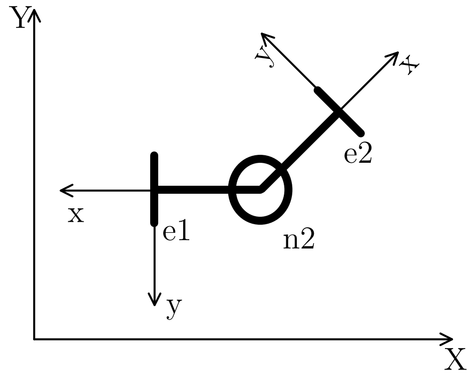

Figure 1.2: Extracting node from system

where X, Y, Z is global coordinate system and x, y, z is local coordinate system.

For investigating motion of system need to integrate (1.1). For this propose need to express all possible to act forces in node(1.3) for each node in relation to their application place.

$$\overrightarrow{F_n} = \sum \overrightarrow{F} = \overrightarrow{F_{ext}} + \overrightarrow{F_{elem}} \times [DCM] + \overrightarrow{F_{press}} \times [DCM] \tag{1.3}$$

F_{ext} is external load force, applied to node in global coordinates. Value of this force for each time step is loaded from list of loads.

F_{elem} is sum of internal forces of each elemen, which acting in node. From each element becomes only half of force to node, other half going to neigourh node. In case of 1D element system, internal force of each element can be express like axial force and it is equal to integral of stress over area:

$$N(x) = \int_A \sigma dA \tag{1.4}$$

For 1D rod system F_{elem} can be expressed like:

$$F_{elem} = \sum_e N_e(x)/2 \tag{1.5}$$

F_{press} is external pressure, can be described like force applied to element in local coordinates. Value of this force for each time step is loaded from list of loads.

Element force becomes from physical deformation of element. Geometry equation(1.6) describing relation between initial length of element and lenght in Δt state, by other words its describing deformation of element over time.

$$\varepsilon = \frac{dU}{dx} = \frac{l - l_0}{l_0} \tag{1.6}$$

According to Hook law $\sigma = \varepsilon E$ and geometry equation (1.6), inner force can be changed to

$$N(x) = \int_A \varepsilon E dA = EA \int \varepsilon = \frac{EA}{l_0} * (l - l_0) \tag{1.7}$$

where l current length of element, l_0 length of element at $t = 0$, E – Young's modulus for element material. To be able to integrate equation of motion, need to express deformation in equation (1.7) by differenses between displacements of nodes, to which element is connected:

$$N(x) = \frac{EA}{l_0} * (U_i - U_j) \tag{1.8}$$

Equations of motion for integrating can be described like:

$$\ddot{U}(\Delta t) = Fn/m \tag{1.9}$$

$$\dot{U}(\Delta t) = \dot{U}(t) + \ddot{U}(\Delta t)\Delta t \tag{1.10}$$

$$U(\Delta t) = U(t) + \dot{U}(\Delta t)\Delta t \tag{1.11}$$

Chapter 2

Nonlinear case