Biological Tissue Movement

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Simulation of fibrous tissue movement

Investigation behavior of biological tissue can be done by estimating movement of it. Tissues of biological origin, usually had a weak structure and little stiffness.

opisat podrobno strukturu i phiz svoystva, sunut kartinki

Such physical structure imposes restrictions on the possible tools in the measurement. Another possible problem is the limited number of measurement attempts. Such limitations of physical measurements call into question the possibility of such activities in general. An alternative to physical experiments is the numerical simulation of these experiments. There are a large number of different numerical modeling methods. All of them are derivatives of Finite Element Method(FEM) and Discrete Element Method(DEM).

raspisat chto takoe FEM and DEM in basic way, just to show difference of

Based on the properties of the material of biological tissues, the most appropriate method is Mass - Spring modeling(MSM). This method based on ideas of DEM and basic element here is very know in mechanic simple one dimensional(1D) beam.

have to show so pic about MSM, with explanation what is going on Computational complexity of MSM is much less compare to FEM-based methods, because of less number of equations to integrate on each time step. This important advantage and physics way have method describes basic element gave to MSM very wide using in computer games for calculating reality-looks hair or cloth movement in real time. Modelling by using MSM could be parallel calculated on each time step.[1] [2]

Mechanic equivalent of fibrous tissue could be described as system of 1D rods. Example of such flat 2D system are shown on figure 2. System consists of discrete elements e_1, e_2 and e_3.

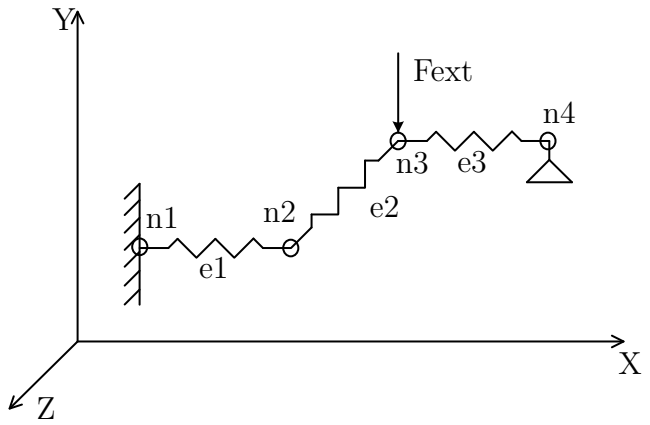

Figure 1: 1D Rod system in global coordinate system

All elements are connected to each other through nodes n_2, n_3 and to special points through n_1 and n_4. Each element e_n of system has own orientation in global coordinate system.

pic about loc glob coords of one element In case that node does not have external interrupt, like pressure or other applied force, schematic represent of node can be as on figure 2.

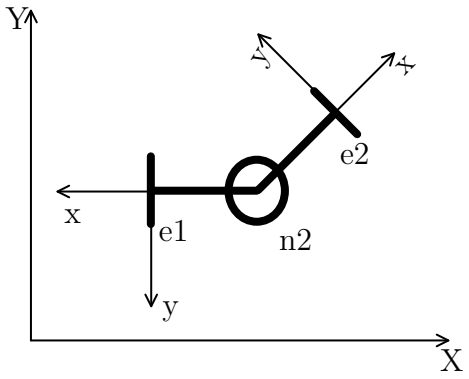

Figure 2: Extracted node from system

Mathematical model of discrete system is expressed by equations of nodes motion. As elements is 1D, nodes will be 1D as well. All system acting in global coordinate system $\{X, Y, Z\}$ and each element acting in own local coordinate system $\{x, y, z\}$.

Let's try to describe minimal possible way to get simulation of such structure as shown on figure 1. From schematic representation of node comes that all vector variables of node should be calculated in global coordinate system and element variables in own local coordinate system(figure 2). For transformation between coordinate systems direction cosine matrix (DCM)(1) can be used.

$$DCM = \begin{bmatrix} cos(X,x) & cos(X,y) & cos(X,z) \\ cos(Y,x) & cos(Y,y) & cos(Y,z) \\ cos(Z,x) & cos(Z,y) & cos(Z,z) \end{bmatrix} \tag{1}$$

where $\{X, Y, Z\}$ is global coordinate system and $\{x, y, z\}$ is local coordinate system.

According to primitive scheme of node 2, mass of each node can be calculated, like sum of half mass of each element, which acting in node. $m_n = \sum_e m_e/2$

Linear deformation

For investigating motion of any mechanical system need to integrate equation of motion(5). For this propose need to express all acting forces in node(2) for each node in relation to their application place.

$$F_n(X) = F_{ext}(X) + F_{elem}(x, y, z) \times [DCM] + F_{press}(x, y, z) \times [DCM] \tag{2}$$

F_{ext} is external load force, applied to node in global coordinates. Value of this force for each time step is loaded from list of loads.

F_{press} is external pressure and can be described like force applied to element in local coordinates. Value of this force for each time step is loaded from list of loads.

F_{elem} is sum of internal forces of each element, which acting in node. From each element counts only half of force to node, other half going to neighbour node. In case of 1D element system, internal force of each element can be express like axial force and it is equal to integral of stress over area:

$$N(x) = \int_A \sigma dA \tag{3}$$

For 1D rod system F_{elem} can be expressed like:

$$F_{elem} = \sum_e N_e(x)/2 \tag{4}$$

The motion of nodes can be expressed by Newton's equation of motion. As 1D element was choosed as discrete element, only the normal component of the translational motion is considered, the equation reduces to

$$F(x) - m\ddot{x} = 0 \tag{5}$$

where $F(x)$ is axial force, equal to $F_n(X)$ for 1D rod system, m – mass of node and $\ddot{x}$ is acceleration, initial conditions are: $x(0) = 0$ and $\dot{x}(0) = V_0$.

Equations of motion for Euler's scheme of integration can be described like:

$$\ddot{U}(\Delta t) = Fn(X)/m \tag{6}$$

$$\dot{U}(\Delta t) = \dot{U}(t) + \ddot{U}(\Delta t)\Delta t \tag{7}$$

$$U(\Delta t) = U(t) + \dot{U}(\Delta t)\Delta t \tag{8}$$

Element force becomes from physical deformation of element. In linear case of study, deformation of element much less compare to element dimensions. It is expressed by linear geometry equation(9), which showing relation between initial length of element and length in Δt state.

$$\varepsilon = \frac{dU}{dx} = \frac{l(\Delta t) - l_0}{l_0} \tag{9}$$

According to Hook law $\sigma = \varepsilon E$ and linear geometry equation (9), inner force can be changed to:

$$N(x) = \int_A \varepsilon E dA = EA \int \varepsilon = \frac{EA}{l_0} * (l(\Delta t) - l_0) \tag{10}$$

where $l(\Delta t)$ is current length of element, l_0 length of element at $t = 0$, E – Young's modulus for element material. To be able to integrate equation of motion, need to express deformation in equation (10) by differences between displacements of nodes, to which element is connected:

$$N(x) = \frac{EA}{l_0} * (U_i - U_j) \tag{11}$$

Nonlinear deformation

Nonlinearity in main mean that element can get huge deformation compare to element demesions. Equation of F_{elem} in this case would change to nonlinear form:

$$N(x) = \int\limits_t \int\limits_A \sigma dA dt \tag{12}$$

From this equantion comes that cross sectional area and stiffness coefficient will get nonlinearity.

Changing of cross sectional area over time for 1D element is changing its length over time. Linear geometry equation(9), showing linear relations between length, because difference in Δt state takes according initial length of element. In case of huge deformation need to recalculate length of element on each time step and take difference of displacement according to previous time step. In end of geometry equation become to nonlinear form:

$$\varepsilon = \frac{dU}{dx} = \frac{l(\Delta t) - l(t - \Delta t)}{l(\Delta t)} \tag{13}$$

Inner force(10) also become to nonlinear form:

$$N(x) = \int\limits_t \int\limits_A \varepsilon E dA = EA \int\limits_t \varepsilon = \frac{EA}{l(\Delta t)} * (l(\Delta t) - l(t - \Delta t)) \tag{14}$$

where l is current length of element, l_0 length of element at $t = 0$, E – Young's modulus for element material.

And nonlinear equantion of inner force for integration:

$$N(x) = \frac{EA}{l_0} * (U_i - U_j) \tag{15}$$

0.0.1 Nonlinear behavior model

Mooney-Rivlin models are popular for modeling the large strain nonlinear behavior of incompressible materials. According to thermodynamics laws, the 2nd Piola-Kirchhoff stress is the partial derivative of the Helmholtz free energy with respect to the elastic part of the Green strain tensor (with a density thrown in).

$$\boldsymbol{\sigma}^{\mathrm{PK2}} = \rho_o \frac{\partial \Psi}{\partial \mathbf{E}^{\mathrm{el}}}$$

The Helmholtz free energy contains thermal energy and mechanical strain energy. But in most every discussion of Mooney-Rivlin coefficients, the thermal part is neglected, leaving only the mechanical part, W. (Actually, W is declared to represent $\rho_o \Psi$, not just Ψ). Second, since all

of the deformation of a hyperelastic material is elastic by definition, it is sufficient to write $\mathbf{E}^{\mathrm{el}}$ simply as $\mathbf{E}$. This gives

$$\boldsymbol{\sigma}^{\mathrm{PK2}} = \frac{\partial W}{\partial \mathbf{E}}$$

But there is a challenge with this general approach. It is the determination of off-diagonal (shear) terms. As with the shear terms in Hooke's Law, they are not independent of the normal terms, but must be consistent with coordinate transformations that transform normal components into shears and vice-versa. And as with Hooke's Law, the resolution is to define the material behavior for the principal values and rely on coordinate transformations to give the appropriate corresponding behavior of the shear terms.

$$\sigma_i^{\mathrm{PK2}} = \frac{\partial W}{\partial E_i}$$

But alas, even this is not quite what is done in Mooney-Rivlin models. Instead, derivatives are taken with respect to ¡i¿stretch ratios¡/i¿, λ, which are the ratios of initial and final lengths in the principal directions, (L_F/L_o). So the stretch ratio is "one plus engineering strain," $\lambda = 1 + \epsilon_{Eng}$, and therefore $\lambda - 1 = \epsilon_{Eng} = \Delta L\,/L_o$.

According to Mooney-Rivlin model and geometry equation (**??**), inner force can be changed to

$$N(x) = \int_A \varepsilon E dA = EA \int \varepsilon = \frac{EA}{l_0} * (l - l_0) \tag{16}$$

where l is current length of element, l_0 length of element at $t = 0$, E – Young's modulus for element material. To be able to integrate equation of motion, need to express deformation in equation (16) by differenses between displacements of nodes, to which element is connected:

$$N(x) = \frac{EA}{l_0} * (U_i - U_j) \tag{17}$$

Equations of motion for integrating (6), (7),(8) would be same as in linear case**Picault2014**.

Integration schemes

Results

List of literature

[1] A. Rasmusson, J. Mosegaard, and T. S. Sørensen, "Exploring parallel algorithms for volumetric Mass-Spring-Damper models in CUDA," in *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 2008, ISBN: 3540705201. DOI: 10.1007/978-3-540-70521-5_6.

[2] R. M. Amorim, R. S. Campos, M. Lobosco, C. Jacob, and R. W. Dos Santos, "An electromechanical cardiac simulator based on cellular automata and mass-spring models," in *Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics)*, 2012, ISBN: 9783642333491. DOI: 10.1007/978-3-642-33350-7-45.