Forces in rod

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November 7, 2018

Chapter 1

1D rod system

Object of work is system of 1D rods. System consists of discrete elements n1, n2 and n3. All elements are connected to each other through nodes n2, n3 and to special points through n1 and n4 (figure 1.1). Mathematical model of discrete system is expressed by equations of motion for nodes. As elements is 1D, nodes will be 1D as well. All system acting in global coordinate system $\{X, Y, Z\}$ and each element acting in own local coordinate system $\{x, y, z\}$ (figure 1.2).

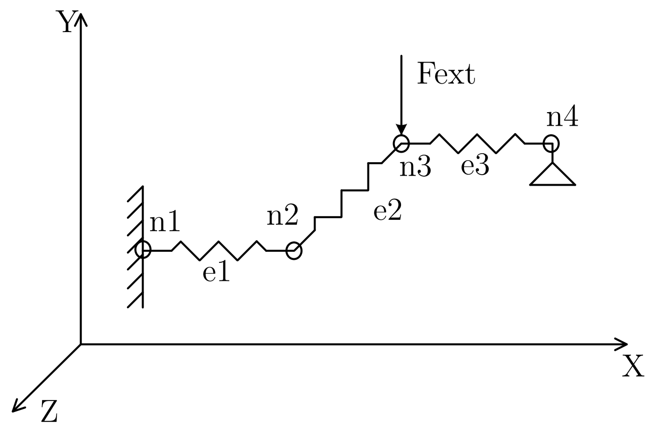

Figure 1.1: 1D Rod system in global coordinate system

In case that node does not have external interrupt, like pressure or other apllied force, schematic represent of node can be as on figure 1.2.

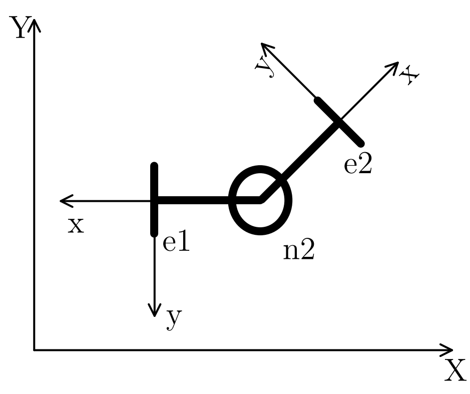

Figure 1.2: Extracting node from system

From schematic representation of node comes that all vector varibles of node should be calculated in global coordinate system and element varibles in own local coordinate system(figure 1.2). For trasformation between coordinate systems direction cosine matrix (DCM)(1.1) can be used.

$$DCM = \begin{bmatrix} cos(X,x) & cos(X,y) & cos(X,z) \\ cos(Y,x) & cos(Y,y) & cos(Y,z) \\ cos(Z,x) & cos(Z,y) & cos(Z,z) \end{bmatrix} \tag{1.1}$$

where X, Y, Z is global coordinate system and x, y, z is local coordinate system.

According to primitive scheme of node 1.2, mass of each node can be calculated, like sum of half mass of each element, which acting in node. $m_n = \sum_e m_e/2$

For investigating motion of system need to integrate (1.5). For this propose need to express all possible to act forces in node(1.2) for each node in relation to their application place.

$$F_n(X) = F_{ext}(X) + F_{elem}(x,y,z) \times [DCM] + F_{press}(x,y,z) \times [DCM] \tag{1.2}$$

F_{ext} is external load force, applied to node in global coordinates. Value of this force for each time step is loaded from list of loads.

F_{press} is external pressure and can be described like force applied to element in local coordinates. Value of this force for each time step is loaded from list of loads.

F_{elem} is sum of internal forces of each element, which acting in node. From each element counts only half of force to node, other half going to neighbour node. In case of 1D element system, internal force of each element can be express like axial force and it is equal to integral of stress over area:

$$N(x) = \int_A \sigma dA \tag{1.3}$$

For 1D rod system F_{elem} can be expressed like:

$$F_{elem} = \sum_e N_e(x)/2 \tag{1.4}$$

The motion of nodes can be expressed by Newton's equation of motion. As 1D element was choosed as discrete element, only the normal component of the translational motion is considered, the equation reduces to

$$F(x) - m\ddot{x} = 0 \tag{1.5}$$

where $F(x)$ is axial force, equal to $F_n(X)$ for 1D rod system, m – mass of node and $\ddot{x}$ is acceleration, initial conditions are: $x(0) = 0$ and $\dot{x}(0) = V_0$.

Equations of motion for Euler's scheme of integration can be described like:

$$\ddot{U}(\Delta t) = Fn(X)/m \tag{1.6}$$

$$\dot{U}(\Delta t) = \dot{U}(t) + \ddot{U}(\Delta t)\Delta t \tag{1.7}$$

$$U(\Delta t) = U(t) + \dot{U}(\Delta t)\Delta t \tag{1.8}$$

1.1 Linear case

Element force becomes from physical deformation of element. In linear case of study, deformation of element much less compare to element demesions and.

1.2 Nonlinear case

Nonlinearity in main mean that element can get huge deformation compare to element demesions. Equation of F_{elem} in this case would change to nonlinear form:

$$N(x) = \int_t \int_A \sigma dA dt \tag{1.9}$$

In this case cross sectional area and stiffness coefficient get nonlinearity.

1.2.1 Nonlinear cross sectional area

Changing of cross sectional area over time is expressed by geometry equation(1.10), which showing relation between initial length of element and length in Δt state, by other words its describing deformation of element over time.

$$\varepsilon = \frac{dU}{dx} = \frac{l - l_0}{l_0} \tag{1.10}$$

According to Hook law $\sigma = \varepsilon E$ and geometry equation (1.10), inner force can be changed to

$$N(x) = \int_A \varepsilon E dA = EA \int \varepsilon = \frac{EA}{l_0} * (l - l_0) \tag{1.11}$$

where l is current length of element, l_0 length of element at $t = 0$, E – Young's modulus for element material. To be able to integrate equation of motion, need to express deformation in equation (1.11) by differenses between displacements of nodes, to which element is connected:

$$N(x) = \frac{EA}{l_0} * (U_i - U_j) \tag{1.12}$$

1.2.2 Nonlinear stiffness coefficient