# Biological Tissue Movement

Oleksandr Hubanov
Vilnius Gediminas Technical University

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# Simulation of tissue movement

Investigation behavior of biological tissue can be done by estimating movement of it. Tissues usually had a weak structure and little stiffness,

Object of work is system of 1D rods. System consists of discrete elements n1, n2 and n3. All elements are connected to each other through nodes n2, n3 and to special points through n1 and n4 (figure 1). Mathematical model of discrete system is expressed by equations of motion for nodes. As elements is 1D, nodes will be 1D as well. All system acting in global coordinate system $\{X, Y, Z\}$ and each element acting in own local coordinate system $\{x, y, z\}$ (figure 2).

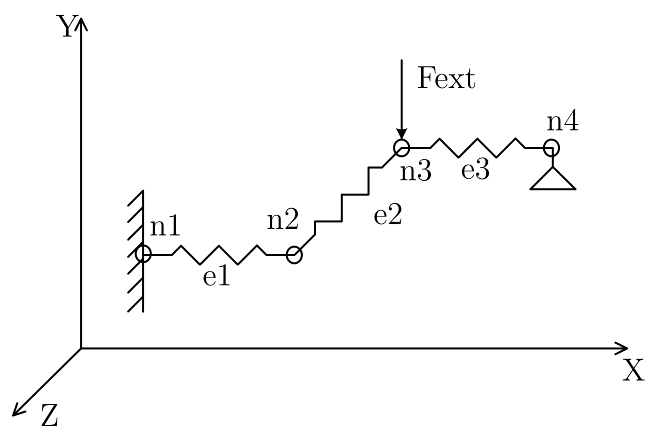


Figure 1: 1D Rod system in global coordinate system

Let's try to describe minimal possible way to get simulation of such structure as shown on figure 1. First of all need to understand size of In case that node does not have external interrupt, like pressure or other applied force, schematic represent of node can be as on figure 2.

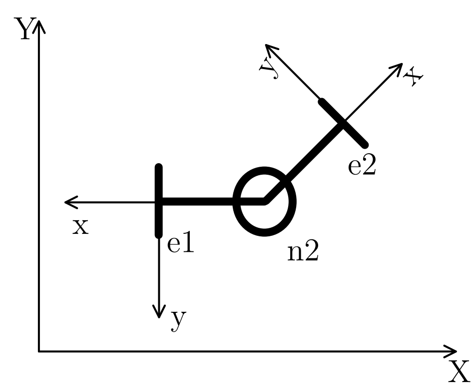


Figure 2: Extracting node from system

From schematic representation of node comes that all vector varibles of node should be calculated in global coordinate system and element varibles in own local coordinate system(figure

2). For trasformation between coordinate systems direction cosine matrix (DCM)(1) can be used.

$$DCM = \begin{bmatrix} cos(X,x) & cos(X,y) & cos(X,z) \\ cos(Y,x) & cos(Y,y) & cos(Y,z) \\ cos(Z,x) & cos(Z,y) & cos(Z,z) \end{bmatrix} \quad (1)$$

where X, Y, Z is global coordinate system and x, y, z is local coordinate system.

According to primitive scheme of node 2, mass of each node can be calculated, like sum of half mass of each element, which acting in node. $m_n = \sum_e m_e/2$

# Linear deformation

For investigating motion of system need to integrate (5). For this propose need to express all possible to act forces in node(2) for each node in relation to their application place.

$$F_n(X) = F_{ext}(X) + F_{elem}(x, y, z) \times [DCM] + F_{press}(x, y, z) \times [DCM] \quad (2)$$

$F_{ext}$ is external load force, applied to node in global coordinates. Value of this force for each time step is loaded from list of loads.

$F_{press}$ is external pressure and can be described like force applied to element in local coordinates. Value of this force for each time step is loaded from list of loads.

$F_{elem}$ is sum of internal forces of each element, which acting in node. From each element counts only half of force to node, other half going to neighbour node. In case of 1D element system, internal force of each element can be express like axial force and it is equal to integral of stress over area:

$$N(x) = \int_A \sigma dA \quad (3)$$

For 1D rod system $F_{elem}$ can be expressed like:

$$F_{elem} = \sum_e N_e(x)/2 \quad (4)$$

The motion of nodes can be expressed by Newton's equation of motion. As 1D element was choosed as discrete element, only the normal component of the translational motion is considered, the equation reduces to

$$F(x) - m\ddot{x} = 0 \quad (5)$$

where $F(x)$ is axial force, equal to $F_n(X)$ for 1D rod system, $m$ – mass of node and $\ddot{x}$ is acceleration, initial conditions are: $x(0) = 0$ and $\dot{x}(0) = V_0$.

Equations of motion for Euler's scheme of integration can be described like:

$$\ddot{U}(\Delta t) = Fn(X)/m \quad (6)$$

$$\dot{U}(\Delta t) = \dot{U}(t) + \ddot{U}(\Delta t)\Delta t \quad (7)$$

$$U(\Delta t) = U(t) + \dot{U}(\Delta t)\Delta t \quad (8)$$

Element force becomes from physical deformation of element. In linear case of study, deformation of element much less compare to element demesions. It is expressed by linear geometry equation(9), which showing relation between initial length of element and length in $\Delta t$ state.

$$\varepsilon = \frac{dU}{dx} = \frac{l(\Delta t) - l_0}{l_0} \tag{9}$$

According to Hook law $\sigma = \varepsilon E$ and linear geometry equation (9), inner force can be changed to:

$$N(x) = \int_A \varepsilon E dA = EA \int \varepsilon = \frac{EA}{l_0} * (l(\Delta t) - l_0) \tag{10}$$

where $l(\Delta t)$ is current length of element, $l_0$ length of element at $t = 0$, $E$ – Young's modulus for element material. To be able to integrate equation of motion, need to express deformation in equation (10) by differenses between displacements of nodes, to which element is connected:

$$N(x) = \frac{EA}{l_0} * (U_i - U_j) \tag{11}$$

## Nonlinear deformation

Nonlinearity in main mean that element can get huge deformation compare to element demesions. Equation of $F_{elem}$ in this case would change to nonlinear form:

$$N(x) = \int_t \int_A \sigma dA dt \tag{12}$$

From this equantion comes that cross sectional area and stiffness coefficient will get nonlinearity.

Changing of cross sectional area over time for 1D element is changing its length over time. Linear geometry equation(9), showing linear relations between length, because difference in $\Delta t$ state takes according initial length of element. In case of huge deformation need to recalculate length of element on each time step and take difference of displacement according to previous time step. In end of geometry equation become to nonlinear form:

$$\varepsilon = \frac{dU}{dx} = \frac{l(\Delta t) - l(t - \Delta t)}{l(\Delta t)} \tag{13}$$

Inner force(10) also become to nonlinear form:

$$N(x) = \int_t \int_A \varepsilon E dA = EA \int_t \varepsilon = \frac{EA}{l(\Delta t)} * (l(\Delta t) - l(t - \Delta t)) \tag{14}$$

where $l$ is current length of element, $l_0$ length of element at $t = 0$, $E$ – Young's modulus for element material.

And nonlinear equantion of inner force for integration:

$$N(x) = \frac{EA}{l_0} * (U_i - U_j) \tag{15}$$