US Unemployment Rate Time Series Stochastic and Deterministic Model Development

Authors

NAME	Student ID
Alexander Hudson	
Student 2	
Student 3	
Student 4	
Student 5	
Student 6	

Contents

Introduction

The purpose of this study is to analyse changes in the monthly unemployment rate in the United States from 1948 to 2019. For this, time series models will be developed and evaluated using data from the US Bureau of Labor Statistics, which was sourced through Kaggle. Model development will involve the use of deterministic and stochastic (ARIMA/SARIMA) models. Evaluation of the models will then be undertaken to determine the best model found which will then be used for forecasting.

Objectives

- To fit a deterministic model which best explains the variance in the data from four possible options, including, linear, quadratic, cosine (harmonic) and seasonal.
- To forecast unemployment rates using the optimal deterministic model found.
- To develop a set of possible stochastic ARIMA and SARIMA models using specification tools including, ACF (Autocorrelation Function), PACF (Partial Autocorrelation Function), EACF (Extended Autocorrelation) and BIC (Bayesian information criterion).
- To evaluate which is the more appropriate model using coefficient significance values, AIC and BIC, and residual analysis.
- To forecast unemployment rates using the optimal ARIMA and SARIMA models through residual analysis.

Method

Phase 1) Deterministic Modelling

- TSA package installed in R Studio.
- Unemployment dataset loaded and converted to a TS object.
- The TS object was then fitted into linear, quadratic, seasonal and harmonic models.
- Model diagnostics (residual analysis) were generated for each model to assess best fit.
- The seasonal model was selected to predict the next 12 months of the unemployment rate percentage.

Phase 2) Stochastic Modelling with ARIMA

- The TS object was transformed by taking the log of the dataset and then differenced once.
- PACF and ACF plots were generated for the log transformed and differenced TS object.
- EACF plots and BIC tables were subsequently generated to infer plausible ARIMA models.
- ADF tests were conducted before and after the transformation and differencing to ensure the dataset was stationary.
- The ARIMA models were then assessed by evaluating the significance of their parameters using ML and CSS evaluation methods.
- The models were then ranked using AIC and BIC methods. The top performing ARIMA models were then assessed further through residual analysis.
- The optimal ARIMA model was selected for forecasting purposes.

Phase 3) Stochastic Modelling with SARIMA

- The TS object was differenced once and seasonally differenced once more with a period of 12.
- ACF and PACF plots were generated for the differenced TS object.
- An EACF plot was generated after finding the seasonal values of the model (i.e., P and Q values) to infer plausible values for the ARIMA portion of the potential SARIMA models (i.e., p and q values).
- The parameters of the SARIMA models were assessed using both ML and CSS evaluation methods and were ranked using AIC and BIC.
- The best performing SARIMA models were assessed further through residual analysis.
- The optimal SARIMA model was then selected for forecasting purposes.

Materials

- R Programming
    - Packages: TSA, tseries, forecast, FitAR, lmtest, fUnitRoots

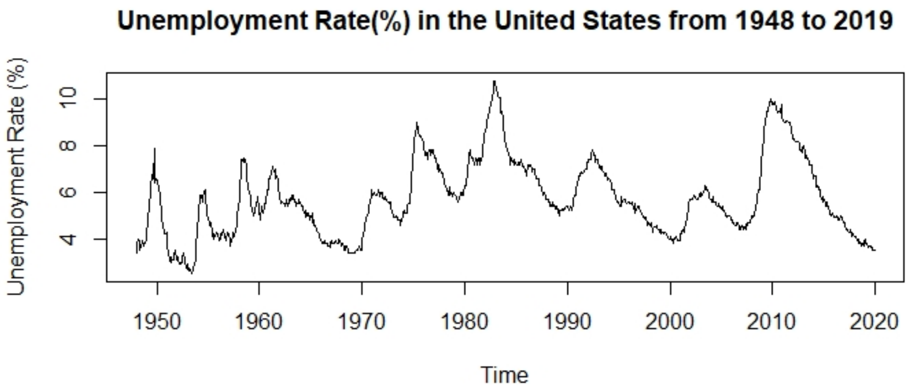
Results

Section 1) Deterministic Modelling

Section 1 focuses on developing deterministic models and finding the most appropriate model to fit the time series object. For this, linear, quadratic, seasonal and harmonic models were trialled, with R-squared and residuals examined to determine the most suitable model. Before applying different models to the TS object, it was important to analyse the time series plot generated. The following observations were noted from Figure 1.1:

- **Trend:** A slight general trend is visible, with several smaller trends visible as unemployment rate spike in several periods, before decreasing.
- **Seasonality:** Subtle repeating patterns seem to be in the data in which the unemployment rate changes from high to low over decades.
- **Change Point:** No clear change point is apparent in the time series.
- **Variance:** A change in variance is visible as the time series continues.
- **AR or MA:** The presence of both AR (Autocorrelation) and MA (Moving Average) activity are seen in this time series.

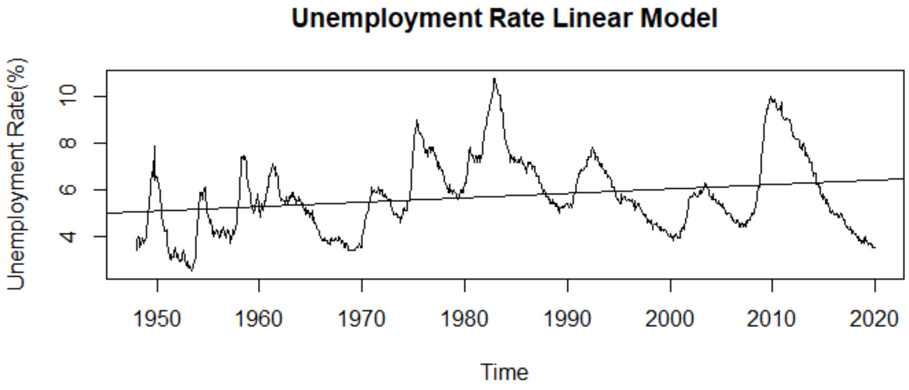
Figure 1.1 US Unemployment Time Series 1948 – 2019



1.1 Fitting a Linear Model

A fitted linear model to the data demonstrated an R-squared value of 0.058, with a statistically significant P-value (α<0.05) of 5.261e-13. Additionally, the model coefficients (i.e., intercept and t) were also statistically significant.

Figure 1.2 Linear Model



1.2 Linear Model Diagnostics

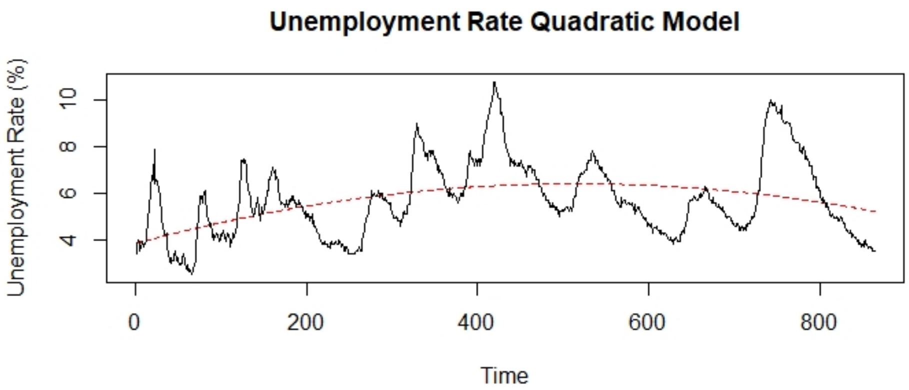
Upon review of the linear model diagnostics, found in Appendix 1 (see Appendix), it is clear a linear model is not appropriate. The following observations were documented:

- Standardised Residuals are good, with the majority falling between 3 and –3 and all falling between 4 and –4.
- The histogram of the standardised residuals looks symmetric, which is desired.
- The QQ plot looks normal, with some deviation at the tail ends of the reference line.
- Shapiro- Wilk Normality Test provided an undesirable P-value of 1.864e-12 (less than alpha) which corresponds to a time series that is not normally distributed.
- Review of the ACF plot shows a wave like pattern, which corresponds to significant autocorrelation remaining.

1.3 Fitting a Quadratic Model

The output summary of the quadratic model generated a model with an improved R-squared value of 0, and a statistically significant (α<0.05) P-value of <$2.2*10^{-16}$. The model coefficients (i.e., intercept, t and t^2) were also statistically significant.

Figure 1.3 Quadratic Model



1.4 Quadratic Model Diagnostics

Upon review of the linear model diagnostics, found in Appendix 2 (see Appendix), it is clear a quadratic model is not a good fit for the time series. The following observations were documented:

- Standardised Residuals are good, with the majority falling between 3 and –3 and all falling between 4 and –4.
- The histogram of the standardised residuals looks symmetric, being desirable.
- The QQ plot looks normal, with some deviation at the tail ends of the reference line.

- Shapiro- Wilk Normality Test recorded an undesirable P-value of <2.2e-16. Given that the P-Value is less than $\alpha = 0.05$, we reject the null hypothesis and conclude that the data is not normally distributed.
- Review of the ACF plot shows significant autocorrelation, like the linear model prior.

1.5 Fitting a Seasonal Model

The output summary of the seasonal model generated an R-squared value of 0.923 and a statistically significant ($\alpha<0.05$) P-value of $<2.2*10^{-16}$. Additionally, the model coefficients were statistically significant. Given that the data is monthly, 12 parameters are used as demonstrated below, with one representing each month.

$$Y = \beta 1 + \beta 2 + \ldots \beta 12 \tag{1.5.1}$$

1.6 Seasonal Model Diagnostics

Upon review of the seasonal model diagnostics, found in Appendix 3 (see Appendix), the following observations were documented:

- Standardised Residuals are good, falling between 4 and -4
- The histogram of the standardised residuals looks symmetrical
- The QQ plot looks normal, with deviations at the tail ends of the reference line.
- Shapiro- Wilk Normality Test recorded an undesirable P-value of $4.13*10^{-13}$. Given that the P-Value is less than α, we do reject the null hypothesis and conclude that the data is not normally distributed.
- Review of the ACF plot continues to show a significant wave like pattern, corresponding to a high level of autocorrelation

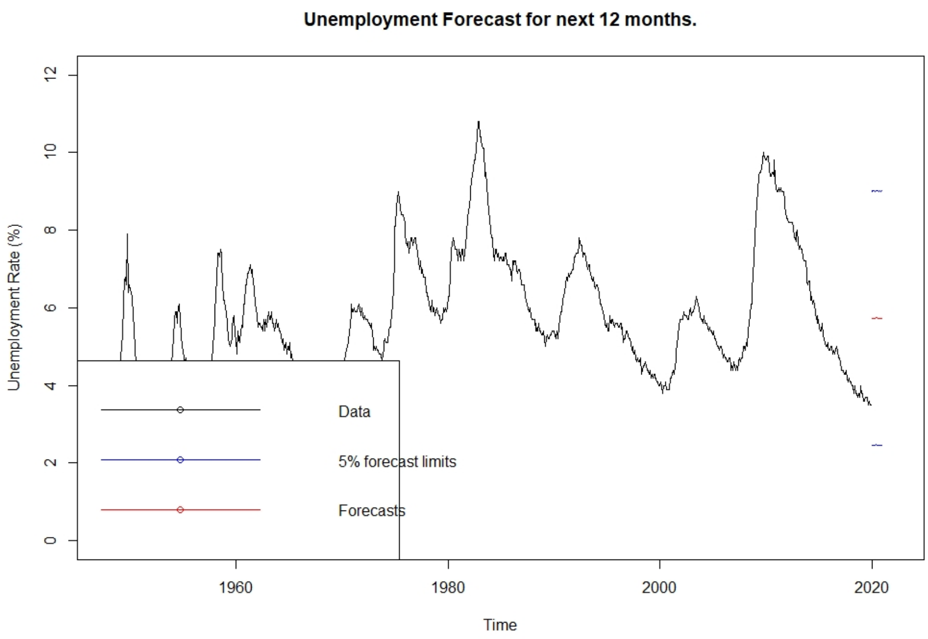
Despite the seasonal model having an improved R Squared value, the diagnostics are less than ideal. Other models were also examined, including harmonic, however, the output summary and residuals were poor. Upon review of all deterministic models evaluated, the preferred model is the seasonal model, despite its limitations.

1.7 Predicting changes to Ozone Layer Thickness using the Quadratic Model

Upon review of the diagnostic output from each of the models and considering the R-squared values, the seasonal model is the optimal model for the time series and for subsequent forecasts.

The forecast of the next 12 months of unemployment rates can be seen in figure 1.4 below.

Figure 1.4 Forecast Unemployment Rate for next 12 months



The forecast over the next 12 months does not deviate from an average of 5.73%. This may indicate that there are larger effects on the variance than seasonality alone.

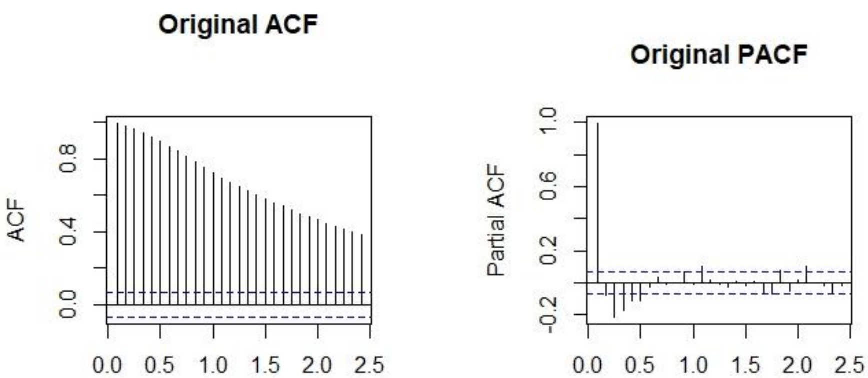
Section 2) ARIMA Models

To improve upon the deterministic models found in Section 1, Section 2 focuses on model development using a stochastic approach. Section 2 involves finding appropriate ARIMA (p, d, q) models to fit the time series object. For the purposes of ARIMA model development, seasonality will be ignored and revisited in Section 3. Several tools were used to develop potential ARIMA based models, including ACF, PACF, EACF and BIC tables. The models generated were assessed using coefficient significances, AIC/BIC ranking and residual analysis.

2.1 ACF & PACF of Unemployment TS Object

ACF and PACF plots were examined, as shown in Figure 2.1. A trend can be seen through the decaying wave-like pattern in the ACF plot. This indicates that the time series object is not stationary and requires transformation.

Figure 2.1 ACF & PACF of Time Series Data



The output of an ADF test, p = 0.024 was a borderline result, indicating that the time series was stationary (p-value less than alpha). However, this is contrary to that observed in a visual inspection, therefore a secondary PP test was used. This found a not significant P-value of p = 0.293, confirming that the time series object was non-stationary. To improve stationarity, the time series would need differenced.

2.2 Transformation & First Difference

The resulting time series after it has been differenced once wis demonstrated in Figure 2.2, with the plot now appearing to be stationary. This was confirmed by a subsequent ADF test, with a P-value less than alpha, $p < 0.01$. Additionally, due to the variance in the data, a log transformation was applied, with the plot shown in Figure 2.3 below.

Figure 2.2 First Difference of Time Series Data

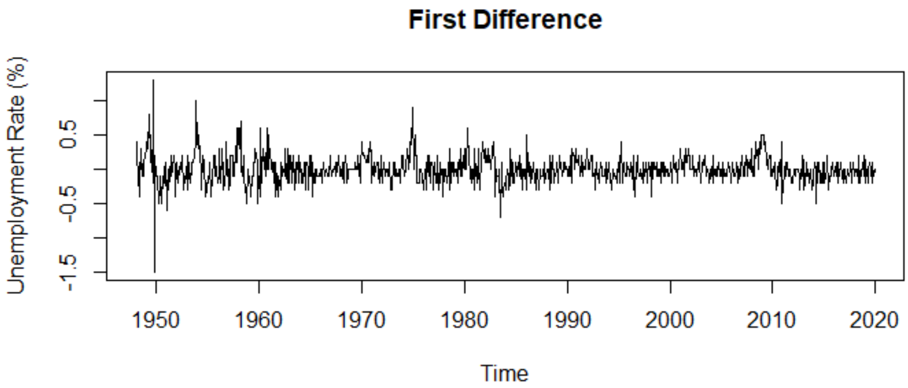
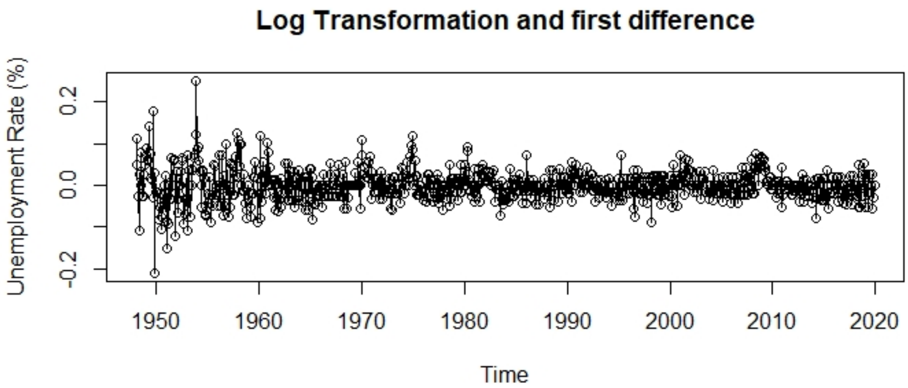


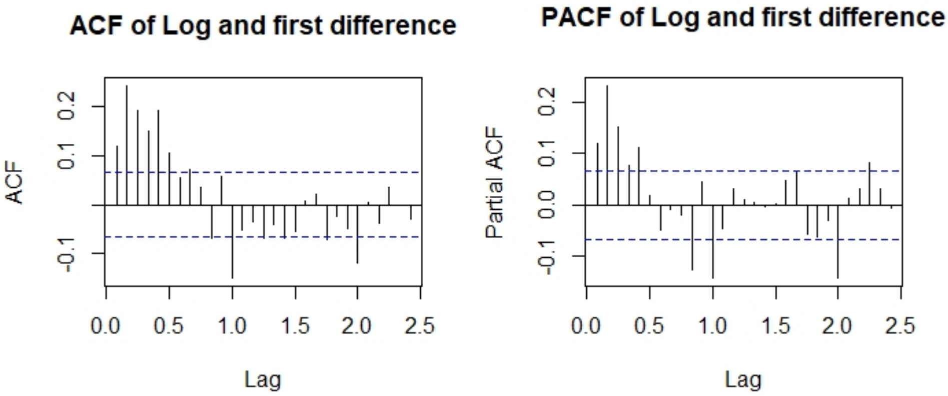
Figure 2.3 Log & First Difference of Time Series Data



2.3 ARIMA Model Output

Figure 2.4 shows the ACF and the PACF plots from the log transformed differenced time series object. It is important to note that there is no longer a decaying wave-like trend in the ACF plot. This indicates that the time series is stationary, confirming the outcome of ADF tests from sub-section 2.2. The examination of the ACF plot shows 6 or 7 significant lags, which translates to q = 6 or 7. The examination of the PACF plot indicates 5 significant lags, which corresponds to p = 5. Using this information, the potential ARIMA models (p, d, q) include, ARIMA (5, 1, 6), and ARIMA (5, 1, 7).

Figure 2.4 ACF & PACF of Transformed Time Series Data



For further model generation, the EACF table was used, as shown in figure 2.5 below. The top left vertex was identified at AR = 1 and MA = 5. The potential models generated from this include, ARIMA (1, 1, 5), ARIMA (2, 1, 5), ARIMA (1, 1, 6), and ARIMA (2, 1, 6). These models are different to those identified from ACF/PACF, although q = 6 appears to be a common factor among the models.

Figure 2.5 EACF

AR/MA	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	x	x	o	x	o	x	o	x	o	o
1	x	x	o	o	x	o	o	o	o	o	o	x	o	o
2	x	x	o	o	x	o	o	o	o	o	o	x	o	x
3	x	x	x	o	x	o	o	o	o	o	o	x	o	o
4	x	x	x	x	o	o	o	o	o	o	o	x	o	o
5	x	x	x	x	x	o	o	o	o	o	o	x	o	x
6	x	x	x	o	x	x	o	o	o	o	o	x	o	o
7	x	x	x	x	x	x	o	o	o	o	o	x	o	o

In Figure 2.6, ‘p’ values are represented by the ‘unemployment’ label and ‘q’ values by the error label, with the top models generated being the more suitable. This figure demonstrates considerable support for ‘p’ equal to 2 or 3 (p = 2, 3) and ‘q’ equal to 4 or 5 (q= 4, 5). From this, potential ARIMA model include, ARIMA (2,1,4), ARIMA (2,1,5), ARIMA (3,1,4), and ARIMA (3,1,5). A repeated model is found in ARIMA (2,1,5), with it appearing in both BIC and EACF.

Figure 2.6 BIC Table of Time Series Data

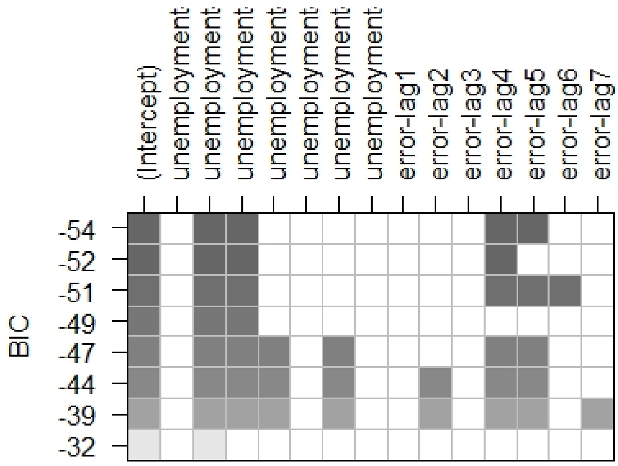


Table 2.1 summarises the results of the model generation phase of ARIMA model fitting. Notably, only one model is confirmed through more than one method.

Table 2.1 Summary of ARIMA Models

ARIMA (p, d, q) Model	Source
(5,1,6)	ACF/PACF
(5,1,7)	ACF/PACF
(1,1,5)	EACF
(2,1,5)	EACF, BIC
(1,1,6)	EACF
(2,1,6)	EACF
(2,1,4)	BIC
(3,1,4)	BIC
(3,1,5)	BIC

2.5 Parameter Significance and Ranking ARIMA Models with AIC/BIC

ML and CSS estimation methods were utilised to determine whether the parameters in the generated ARIMA models were significant. Most of the models analysed had at least one insignificant parameter. However, the parameters of ARIMA (2, 1, 4) were all significant for both ML and CSS. Further model evaluations were done using AIC and BIC tables as seen in Figure 2.7 and Figure 2.8, respectively. For the AIC table, the top three models were ARIMA (5, 1, 7), ARIMA (5, 1, 6), and ARIMA (2, 1, 4), with those for the BIC table being ARIMA (2, 1, 4), ARIMA (2, 1, 5) and ARIMA (1, 1, 5). ARIMA (2, 1, 4) is the only model to appear in the top three of both tables; Additionally, it has the fewest parameters, with seven.

Figure 2.7 AIC Table

```
             df       AIC
model_517_ml 13 -402.5869
model_516_ml 12 -397.8751
model_214_ml  7 -397.7132
model_216_ml  9 -381.4688
model_215_ml  8 -381.2891
model_315_ml  9 -380.8269
model_314_ml  8 -378.2606
model_115_ml  7 -376.1366
model_116_ml  8 -374.2269
```

Figure 2.8 BIC Table

```
             df       BIC
model_214_ml  7 -364.3903
model_215_ml  8 -343.2058
model_115_ml  7 -342.8137
model_516_ml 12 -340.7501
model_517_ml 13 -340.7015
model_314_ml  8 -340.1773
model_216_ml  9 -338.6251
model_315_ml  9 -337.9832
model_116_ml  8 -336.1435
```

2.6 Residual Analysis of ARIMA Models

Residual Analysis was conducted for the top models including ARIMA (2,1,4), ARIMA (2,1,5), ARIMA (1,1, 5) and ARIMA (5,1,7) and ARIMA (5,1,6) (the output can be seen in Appendices 4 to 13). Generally, most of the standardised residuals of these models fall between –4 and 4 with a few outliers. The Q-Q plots look mostly acceptable, although the right tail has values deviating from the line and the left has the outlier of 1950. Based on Shapiro Wilks, none of the models demonstrate normally distributed residuals. Finally, little autocorrelation is left in the ACF plots.

The main differences between the models can be seen in the Ljung-Box test. For this, the models ARIMA (2, 1, 4), ARIMA (5, 1, 7), ARIMA (1, 1, 5), and ARIMA (2, 1, 5) demonstrate lags above the reference line until the 9th, 10th, 10th, 11th lag, respectively. The model which demonstrated the best result on this test was the ARIMA (5, 1, 6) model, with lags above the reference line up to the 23rd lag. Based on the results of the Ljung-Box test results, ARIMA (5, 1, 6) appears superior to ARIMA (2, 1, 4), despite it appearing lower in the BIC rankings.

2.7 Error Measures

Based on Table 2.2, ARIMA (5, 1, 7) CSS model seems to generally have the lowest error rate as compared to the other viable models. It has the lowest root mean squared error, mean absolute error, mean absolute error and mean absolute standard error. Additionally, is seems to have the second lowest score for mean error and its mean percentage error is like the other models.

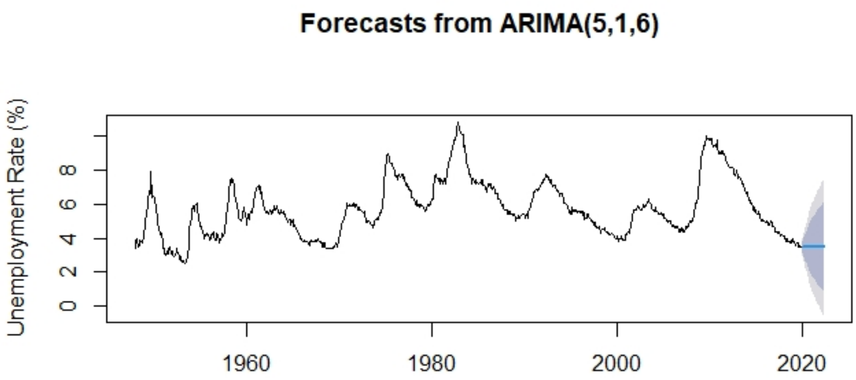
Table 2.2 Error Measures for ARIMA Models

	ME	RMSE	MAE	MPE	MAPE	MASE
ARIMA (2, 1, 4) ML	$0.131e^{-4}$	0.190	0.140	-0.020	2.587	0.163
ARIMA (2, 1, 4) CSS	$2.588e^{-4}$	0.191	0.138	-0.003	2.555	0.161
ARIMA (5, 1, 6) ML	$0.106e^{-4}$	0.189	0.138	-0.018	2.558	0.161
ARIMA (5, 1, 6) CSS	$1.486e^{-4}$	0.189	0.137	-0.011	2.526	0.159
ARIMA (5, 1, 7) ML	$3.148e^{-4}$	0.188	0.139	-0.017	2.570	0.161
ARIMA (5, 1, 7) CSS	$-2.777e^{-4}$	0.187	0.137	-0.019	2.510	0.159
ARIMA (1, 1, 5) ML	$-1.263e^{-4}$	0.193	0.140	-0.0003	2.582	0.162
ARIMA (1, 1, 5) CSS	$-6.860e^{-4}$	0.193	0.139	-0.019	2.573	0.162

2.8 Forecast using ARIMA Model

ARIMA (5, 1, 6) ML was decided to be the most suitable model in terms of its evaluation statistics and was used for forecasting. This is mainly due to its superior results on the Ljung-Box test, which demonstrated that the model was appropriately capturing the autocorrelation in the series. For visual clarity, the next 30 months were included in the forecast, as shown in Figure 2.9 below. The average forecast over the next 30 months is 3.47%. This differs from the estimate generated by the deterministic seasonal model of 5.73%. However, the forecast using ARIMA is more accurate, with the final unemployment rate in the time series is 3.5% for December 2019. Unlike the deterministic seasonal model, the ARIMA model continues from the final point of the dataset.

Figure 2.9 Forecast of Next 30 Months of Unemployment Rate



Section 3) Generating Possible SARIMA Models

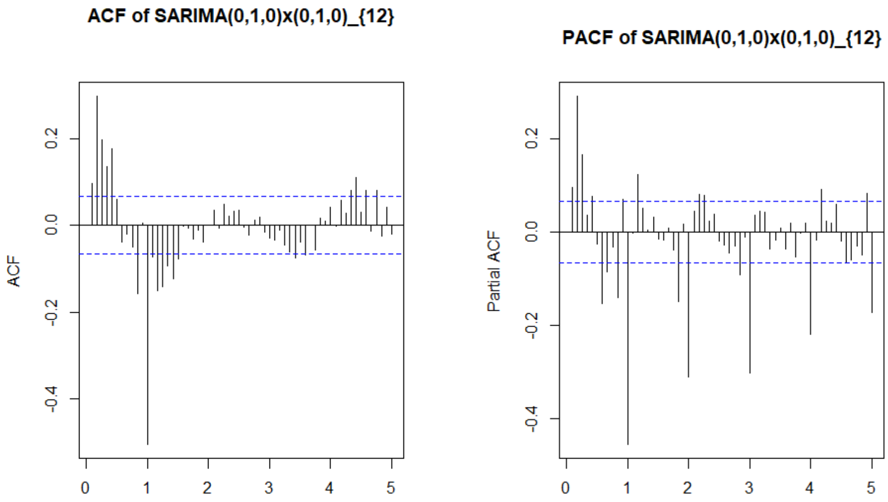
Section 3 is focused on identifying an optimal seasonal ARIMA model. No transformation is necessary; as such, the original time series object will be used. Tools including ACF, PACF, and EACF are used to find potential

SARIMA models after the time series is differenced once and then seasonally differenced with a period of 12. The model is simplified such that the seasonal MA and AR values (I.e., P and Q) are made known. Subsequently, a suitable model can be found by examining the output of ACF and PACF plots, as well checking the EACF plot.

3.1 The ACF and PACF of d=1 and D=1

Based on the ACF and PACF in Figure 2.1, a trend of autocorrelation can be observed with every 12 lags. This translates into the presence of seasonality with a period of 12. As such, the time series with specified period of 12 was seasonally differenced (i.e., D=1) once to reduce seasonality, then differenced (i.e., d=1) once to remove trend. As shown in the Figure 3.1, the differencing makes the seasonal trend more apparent. Thus, the seasonal trend will need to be removed by investigating the impact of setting Q = 1.

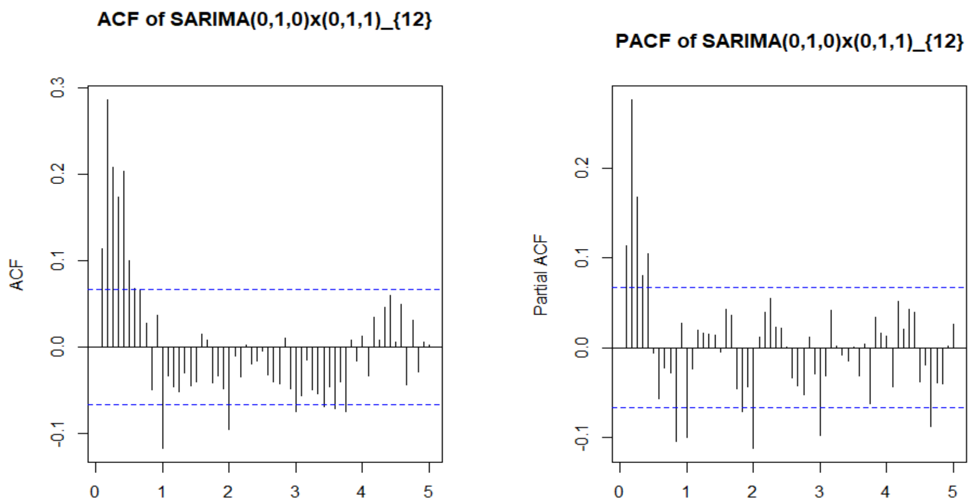
Figure 3.1 ACF and PACF plots of SARIMA (0,1,0) x (0,1,0) _ {12}



3.2 The ACF and PACF of SARIMA (0,1,0) x (0,1,1) _ {12}

Applying Q = 1 to the model shows more prominent spikes in the ACF plot, but also highlights the significant peaks in the PACF plot. Specifically, the significant spikes are for the first three seasonal lags of the PACF plot, therefore, we can apply P = 3 to the model.

Figure 3.2 ACF and PACF plots of SARIMA (0,1,0) x (0,1,1) _ {12}

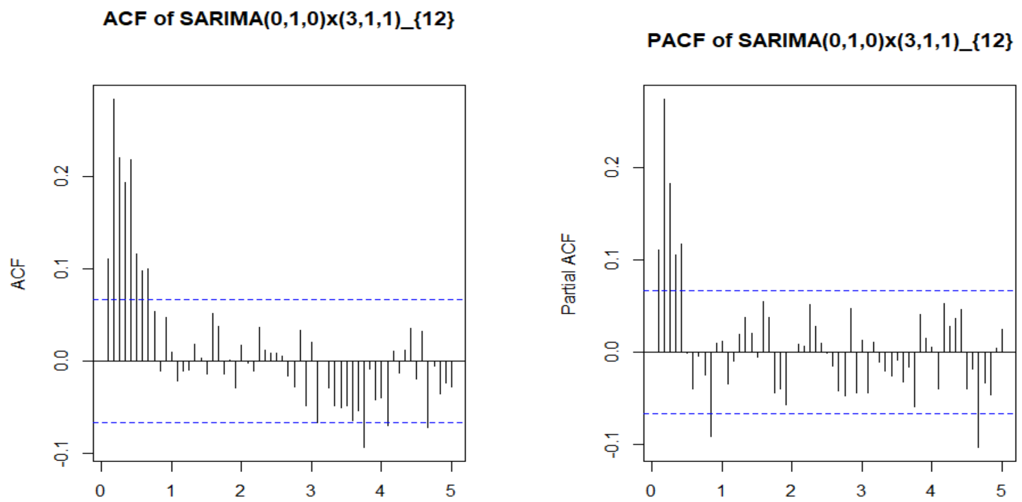


3.3 Model Output from SARIMA (0,1,0) x (3,1,1) _ {12}

With the seasonal ARIMA parameters now defined as (P, D, Q) = (3, 1, 1), the seasonal lags have been addressed. From here, potential models based on ACF and PACF can be found, like Section 2.3. The ACF plot in Figure 3.3

below shows eight significant spikes within the first seasonal lag, which corresponds to an ARIMA parameter of q = 8. The PACF in the same figures shows six significant spikes within the first seasonal lag, which translates to a parameter of p = 6. Thus, one potential model is SARIMA (6, 1, 8) x (3, 1, 1) _ {12}.

Figure 3.3 ACF and PACF plot of SARIMA (0, 1, 0) x (3, 1, 1) _ {12}



Examination of the EACF table in Figure 3.4 shows several points of origin; for simplicity, the chosen point of origin is (p, q) = (3, 5). Possible models include SARIMA (3, 1, 4) x (3, 1, 1) _ {12}, SARIMA (3, 1, 6) x (3, 1, 1) _ {12}, SARIMA (4, 1, 5) x (3, 1, 1) _ {12}, and SARIMA (4, 1, 6) x (3, 1, 1) _ {12}.

Figure 3.4 EACF of SARIMA(0,1,0)x(3,1,1)_{12}

```
AR/MA
  0 1 2 3 4 5 6 7 8 9 10 11 12
0 x x x x x x x x o o o  o  o
1 x x o o x o o o x o o  o  o
2 x o o o x o o o o x o  o  o
3 x x o x x o o o o o o  o  o
4 x x o x o o o o o o o  o  o
5 o x o x x x o o o o o  o  o
6 o x o o x o o o o o o  o  o
7 x x x o x x x o o o o  o  o
```

Table 3.1 demonstrates five potential SARIMA models which can be fitted to the time series object.

Table 3.1 Summary of SARIMA Models

SARIMA (p, d, q) x (3,1,1) _{12} Model	Source
(6,1,8)	ACF/PACF
(3,1,5)	EACF
(3,1,6)	EACF
(4,1,5)	EACF
(4,1,6)	EACF

3.4 Parameter Significance and SARIMA Model Ranking with AIC/BIC

ML and CSS methods were used to determine the significance of the parameters in the model. Although every model had insignificant parameters, ARIMA (3, 1, 5) x (3, 1, 1) _ {12} CSS had all but one parameter as significant, including seasonal parameters. Only SARIMA (4, 1, 5) x (3, 1, 1) _ {12} ML showed significance in their highest parameters (in this case, AR4 and MA5), although the CSS version did not demonstrate the same result. Therefore, these two models seem to be the most suitable on this metric.

Examination of AIC and BIC rankings of the ML models, shown in figures 3.5 and 3.6, indicates that SARIMA (4, 1, 5) x (3, 1, 1) _ {12} is optimal model ML model. It ranks the highest in the AIC table and the second highest in the BIC table. Another possible option is the SARIMA (3, 1, 5) x (3, 1, 1) _ {12} ML, which ranks the highest in the BIC and the third highest in the AIC. Additionally, it has the lowest degrees of freedom of all the models.

Figure 3.5 AIC Table of SARIMA Models

	df	AIC
sarima_415_ML	14	-361.1556
sarima_416_ML	15	-359.3760
sarima_315_ML	13	-358.5200
sarima_316_ML	14	-356.5154
sarima_618_ML	19	-355.3570

Figure 3.6 BIC Table of SARIMA Models

	df	BIC
sarima_315_ML	13	-296.8166
sarima_415_ML	14	-294.7059
sarima_316_ML	14	-290.0657
sarima_416_ML	15	-288.1799
sarima_618_ML	19	-265.1752

3.5 Residual Analysis of SARIMA Models

This section will examine the residuals for the worst performing of the SARIMA models, SARIMA (6, 1, 8) x (3, 1, 1) _ {12} CSS, and the best performing of the SARIMA models, SARIMA (4, 1, 6) x (3, 1, 1) _ {12} CSS. Residuals of the plots can be found in Appendix 14 and 15. Despite their performances, both models are very similar, both have:

- Standardised residuals between –4 and 4 with anomalies near the 1950s,
- No significant peaks within their ACF plots meaning no significant autocorrelation,
- QQ-plots and Histograms trending towards normal,
- Shapiro-Wilk Test results indicating non-normality,
- Ljung-Box test results showing p-values for every lag above the reference line.

The main difference between these models is that the SARIMA (4, 1, 6) x (3, 1, 1) _ {12} CSS model demonstrates better Ljung-Box results, although both are acceptable, and its residuals are closer to normality, although still not normal.

3.6 Error Measures

Based on the Table, the CSS model seems to consistently perform better than their ML counterparts. The two models worth noting are the ARIMA (4, 1, 5) x (3, 1, 1) {12} CSS and ARIMA (4, 1, 6) x (3, 1, 1) {12} CSS models, which have the lowest error scores for the root mean squared error, mean absolute error, mean absolute percentage error and mean absolute squared error. This demonstrates that these models seem to perform the best in terms of absolute error. Finally, it is worth noting that all the models seemed to perform relatively similar on squared or absolute measures, the main difference appeared to be between ML and CSS methods of estimation.

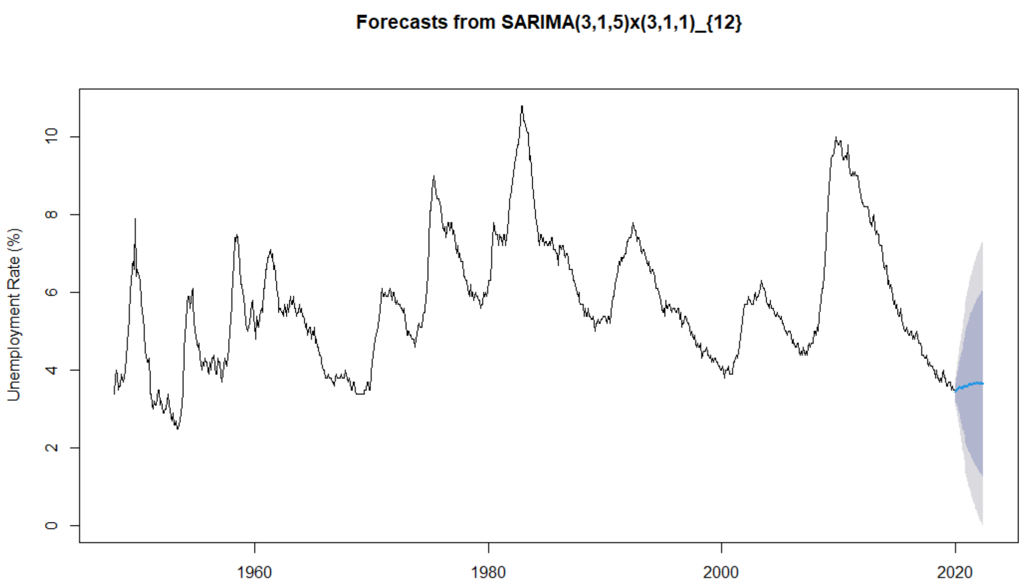
Table 3.2 Error Measures for SARIMA Models

	ME	RMSE	MAE	MPE	MAPE	MASE
ARIMA (3, 1, 5) x (3, 1, 1) {12} ML	$-5.560e^{-3}$	0.184	0.136	-0.128	2.530	0.158
ARIMA (3, 1, 5) x (3, 1, 1) {12} CSS	$-0.799e^{-3}$	0.175	0.127	-0.028	2.309	0.147
ARIMA (3, 1, 6) x (3, 1, 1) {12} ML	$-5.564e^{-3}$	0.184	0.136	-0.128	2.530	0.158
ARIMA (3, 1, 6) x (3, 1, 1) {12} CSS	$-0.236e^{-3}$	0.175	0.127	-0.019	2.313	0.148
ARIMA (4, 1, 5) x (3, 1, 1) {12} ML	$-5.373e^{-3}$	0.184	0.134	-0.120	2.507	0.156
ARIMA (4, 1, 5) x (3, 1, 1) {12} CSS	$0.256e^{-3}$	0.174	0.126	-0.008	2.292	0.146
ARIMA (4, 1, 6) x (3, 1, 1) {12} ML	$-0.379e^{-3}$	0.184	0.134	-0.121	2.508	0.156
ARIMA (4, 1, 6) x (3, 1, 1) {12} CSS	$0.285e^{-3}$	0.174	0.126	-0.009	2.292	0.146

3.7 Forecasting with SARIMA Models

SARIMA (3, 1, 5) x (3, 1, 1) _ {12} CSS is the best of the SARIMA models, and it will be used for forecasting purposes in this section. This model demonstrates significance on all but one parameter, is the highest ranked in the BIC and demonstrates good residuals, and minimal error. Additionally, this model has the fewest number of parameters making it the most parsimonious. Like section 2.7, the model will be used to forecast the next 30 months of unemployment rates. The average of this forecast is approximately 3.60%, which is slightly higher than the average found from the ARIMA model. However, like the ARIMA model, this model starts from the last unemployment rate in the series. Contrary to the ARIMA model, the SARIMA model also considers the potential rise in unemployment rates in 2020.

Figure 3.7 Forecast of Time Series Data



Discussion

The purpose of the study was to investigate how different models would best fit the time series dataset for US unemployment rates from 1948 to 2019. Broadly, two types of models were used with the dataset, being deterministic and stochastic. The deterministic approach involved fitting the TS object to an appropriate model. The models trialled included linear, quadratic, seasonal and harmonic. The stochastic approach aimed to fit a ARIMA or SARIMA model to the time series object, such that autocorrelation could properly be accounted for. Each model within each type was evaluated against similar model such that a most appropriate deterministic, ARIMA and SARIMA model were selected. These three best models were then used to forecast into the future by month. Overall, the SARIMA model was found to be the most appropriate, being able to adequately account for variance and autocorrelation, and minimise error.

From the deterministic models, most of the models struggled to capture variance in the series. The linear, quadratic and harmonic models all suffered from low R squared scores. However, the seasonal model managed to return a high R-squared score of 0.9229. Despite the positive results, the residuals associated with each of the models were unacceptable, which can be seen in the appendices 1, 2 and 3. All models suffered from high levels of autocorrelation remaining in the respective ACF plots. In addition, the outcome of the Shapiro-Wilk normality test indicated that all the deterministic models evaluated were not normally distributed. Based on all the evaluations, the seasonal model was found to be the most appropriate, although it suffered from all these like the other models. Despite the limitations of the residuals, the seasonal model was used for the purposes of forecasting, to determine the next 12 months of the unemployment rate in the United States. The forecast produced results which seemed to be consistently around an average of 5.73% across the 12-month period. Additionally, the final unemployment rate recorded in the time series was 3.5% in December 2019, as compared to the predicted 5.73% in the month of January 2020. This sudden increase in unemployment puts the accuracy of the forecast and by extension seasonal model into question.

To explore other options, a stochastic approach to time series modelling was applied. The first possible model was the ARIMA (p, d, q) model. For this, the time series object was transformed to reduce changing variance and differenced to remove the trend. An ADF test and the results from the ACF from the differenced time series object confirmed the trend was removed. To generate potential ARIMA models tools such as, ACF, PACF, EACF and BIC tables were used. The output included nine ARIMA models, one of which was found using both the EACF plot and the BIC table (see Table 2.1 Summary of ARIMA models). Since interpretations of the ACF and PACF plots are somewhat subjective, alternate models may be found by other investigators.

Once the models were found, evaluation took place though AIC/BIC scores and residual analysis to determine which model to forecast. The AIC/BIC ranking found ARIMA (2, 1, 4), (5, 1, 7), (5, 1, 6), (1, 1, 5) and (2, 1, 5) to be the most suitable models, with ARIMA (2, 1, 4) to be the only one to have all significant parameters. However, residual analysis would suggest that ARIMA (5, 1, 6) ML is the optimal model instead of ARIMA (2, 1, 4) ML. The key differentiator between the top models was the output of the Ljung-Box Test, in which ARIMA (5, 1, 6) ML demonstrated the most superior results (see section 2.6). Finally, error measures were considered in the evaluation of models, in which ARIMA (5, 1, 7) CSS demonstrated the best results. However, due to poor Ljung-Box test results, it was not considered the most appropriate model. Therefore, based on the overall evaluation results, ARIMA (5, 1, 6) was considered optimal model, primarily due to its parameter significance and ability to account for autocorrelation. In general, the residuals of the ARIMA models were a vast improvement to the residuals of the deterministic models. However, the greatest improvement was the ACF plots of the ARIMA models, where little to no autocorrelation remained compared to the wave-like pattern exhibited by the seasonal deterministic model. This observation could explain the improvement seen in the subsequent forecast found using ARIMA (5, 1, 6). The average forecast over the next 30 months using ARIMA (5, 1, 6) was 3.47%, with the model starting from the point time series ended. This is a significant improvement upon the deterministic seasonal model which demonstrated an average of 5.73% when compared to the final observation of the time series of 3.5%.

To improve upon the results found by the ARIMA model, a seasonal component was added to account for the changing variance by month. The development of SARIMA models involved a similar approach to that of ARIMA models, in which ACF, PACF, and EACF plots were used. However, in addition to normal differencing applied in ARIMA models, seasonal differencing was also applied. ACF and PACF plots were used initially to identify the seasonal components of potential models and the AR/MA components. Identification of AR/MA components was not a straightforward process using the EACF plot. Figure 3.4 highlights several options as the vertex including (p, q) = (1, 5) and (p, q) = (4, 4). Although the first of these options would generate models with fewer parameters, (p, q) = (3, 5) was selected as the most appropriate starting point, as it was not hindered by the presence of neighbouring 'x's'. The output of the EACF plot brought the total models to five, which included SARIMA (3, 1, 5) x (3, 1, 1) _ {12}, (3, 1, 6) x (3, 1, 1) _ {12}, (4, 1, 5) x (3, 1, 1_ {12}), (4, 1, 6) x (3, 1, 1) _ {12}, and (6, 1, 8) x (3, 1, 1) _ {12}. Note that BIC tables were not required for SARIMA modelling due to its likely generating a model with a high number of parameters.

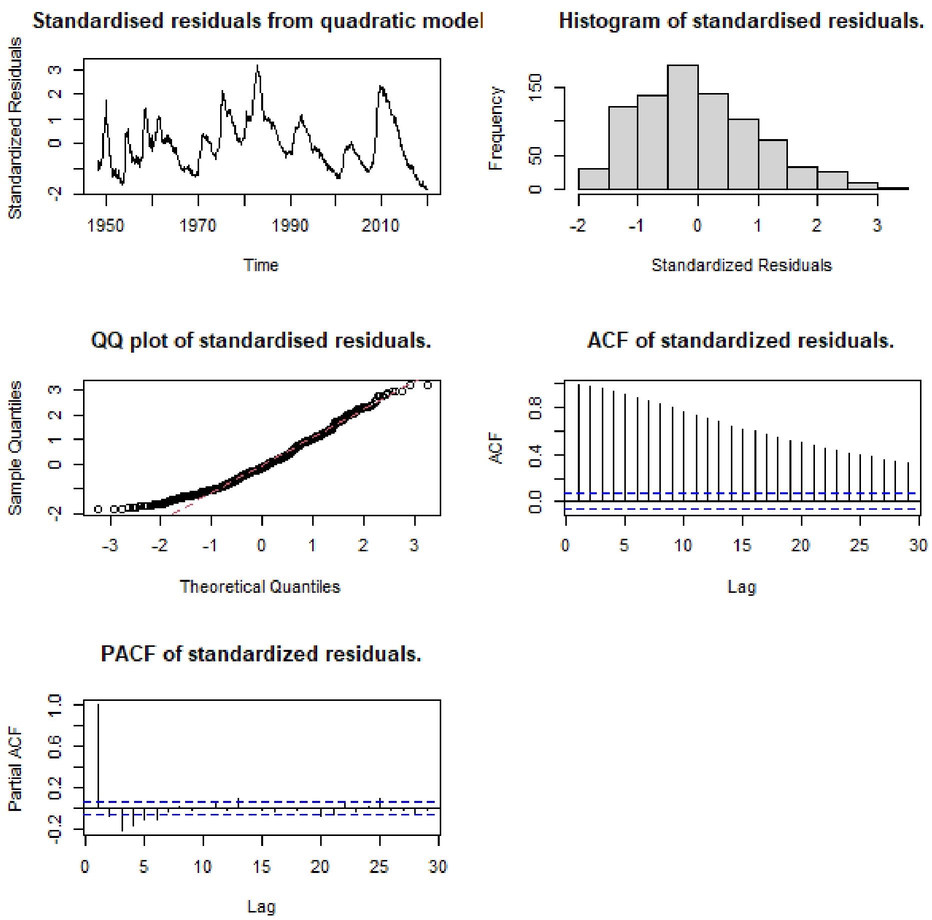
Once models were identified, evaluation needed to be undertaken to decide which model to use for forecasting. From the evaluation, SARIMA (3, 1 ,5) x (3, 1, 1) _ {12} CSS was decided to be the most appropriate model. Results from the BIC table demonstrated that ML version of the model was the best, with the AIC also considering it adequate. The CSS version of the model had mostly significant parameters and demonstrated lower error, with the results being similar to the models with the least error, even compared to its ML counterpart. Little difference was seen between models in the residual analysis, meaning it was difficult to use to decide the most appropriate model. Using the CSS model also theoretically makes sense since the data is not normally distributed, which the ML models assume. Generally, the SARIMA models appeared to have more parameters than the ARIMA models generated, although SARIMA demonstrated better residuals, particularly the Ljung-Box test (see Appendix 14 and 15). However, like the ARIMA models, little to no autocorrelation was present in the ACF plots of the residuals for the SARIMA models. SARIMA (3, 1, 5) x (3, 1, 1) _ {12} CSS was identified as the best model given that most of its parameters had shown significance, a feature that the other SARIMA models lacked. The average forecast over the next 30 months using the model was 3.60%, which was slightly higher than ARIMA (5, 1, 6). However, this was within a reasonable distance from the December 2019 rate of 3.5%, the last point in the time series. Inspection of Figure 3.7 also highlighted a steady rise in the forecast of unemployment rates, compared to the straight line seen in Figure 2.9 with ARIMA (5, 1, 6).

Conclusion

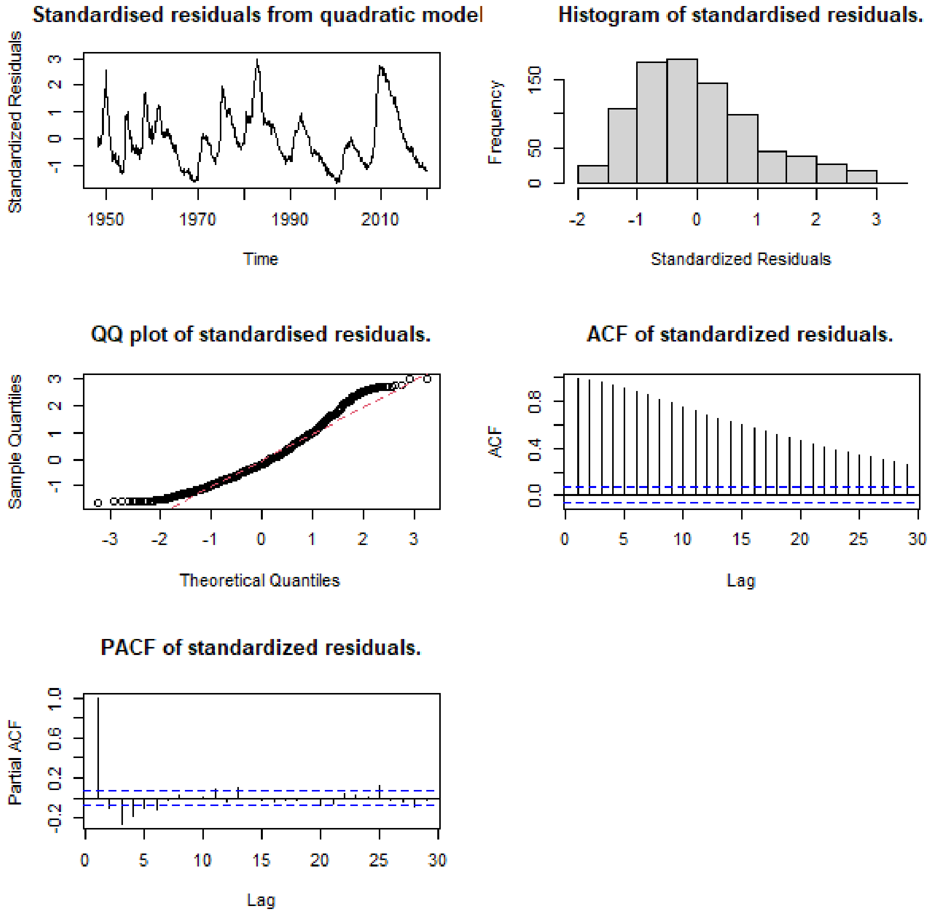
The purpose of this study was to analyse changes in the monthly unemployment rate in the United States using both deterministic and stochastic approaches. From the study, it was found that deterministic models were not a good fit for the time-series, with much autocorrelation remaining in the ACF plots. Although a seasonal model proved to be the superior deterministic model, it too suffered from poor residuals. In contrast, the SARIMA and ARIMA models were a vast improvement, reducing the presence of autocorrelation in the residuals while also being able to account for the variance in the series. However, it was the SARIMA models that had a slight edge over the ARIMA models with improved outputs from the Ljung-Box test. This meant that the model was better able to account for the progression of the series over time, which results in a more appropriate model. The corresponding forecasts found with SARIMA also highlighted improved ability to capture seasonality in the time series, with the final forecast indicating that the unemployment rate in the United States was projected to increase slowly over time.

Appendices

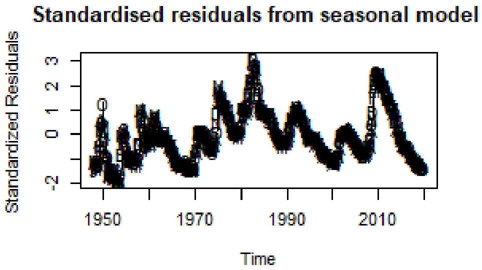
Appendix 1: Linear Model Residuals

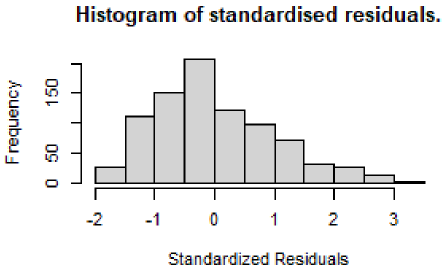


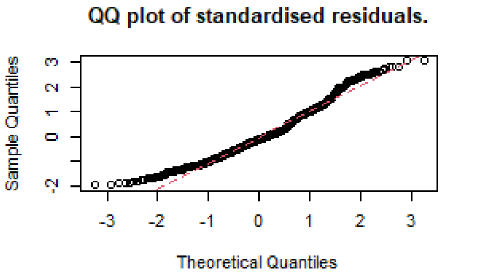
Appendix 2: Quadratic Model Residuals

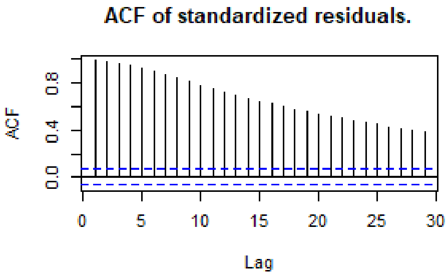


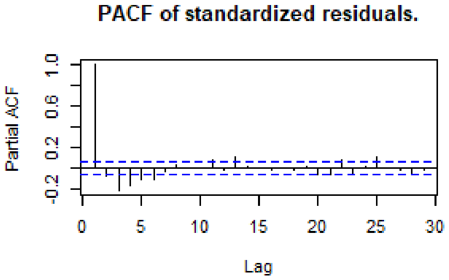
Appendix 3: Seasonal Model Residuals



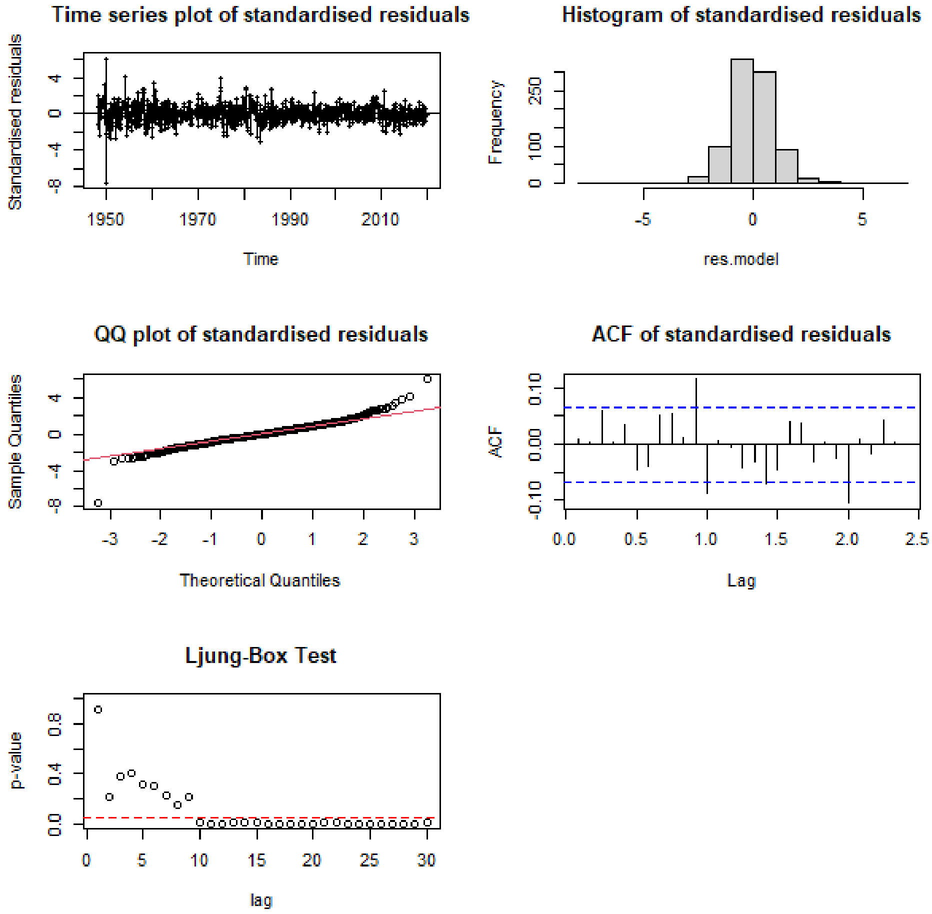




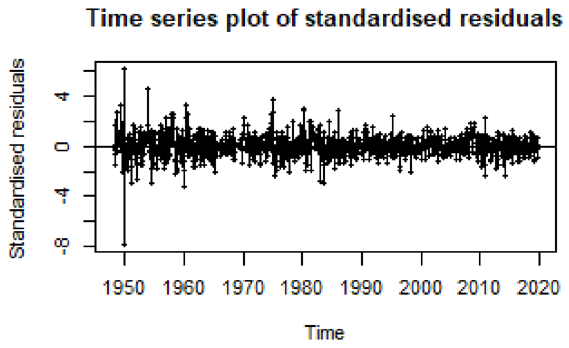


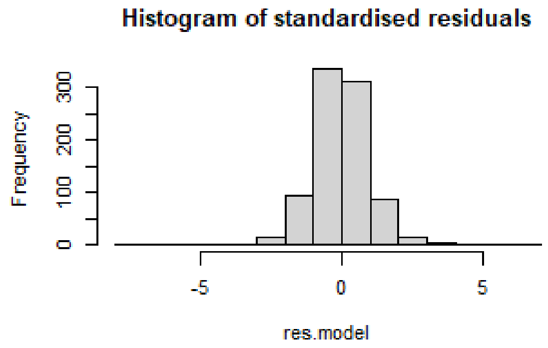


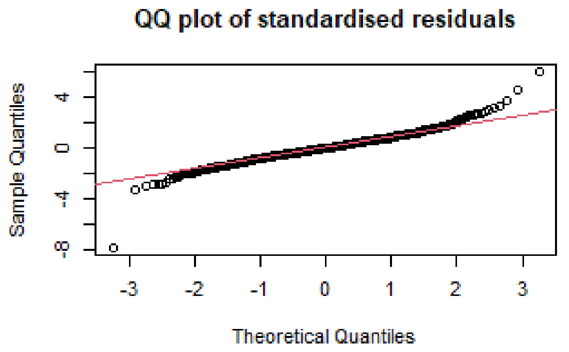
Appendix 4: ARIMA (2,1,4) ML Residual Output

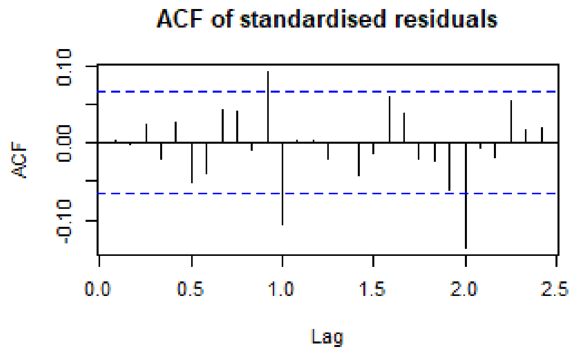


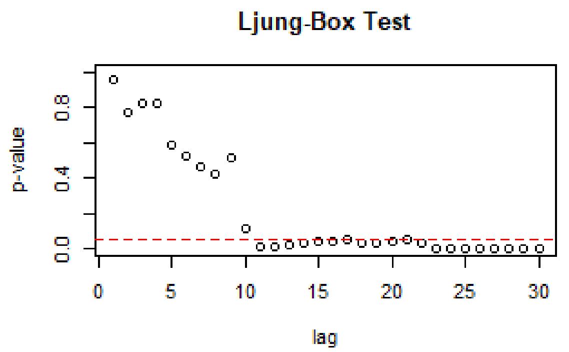
Appendix 5: ARIMA (2,1,4) CSS Residual Output



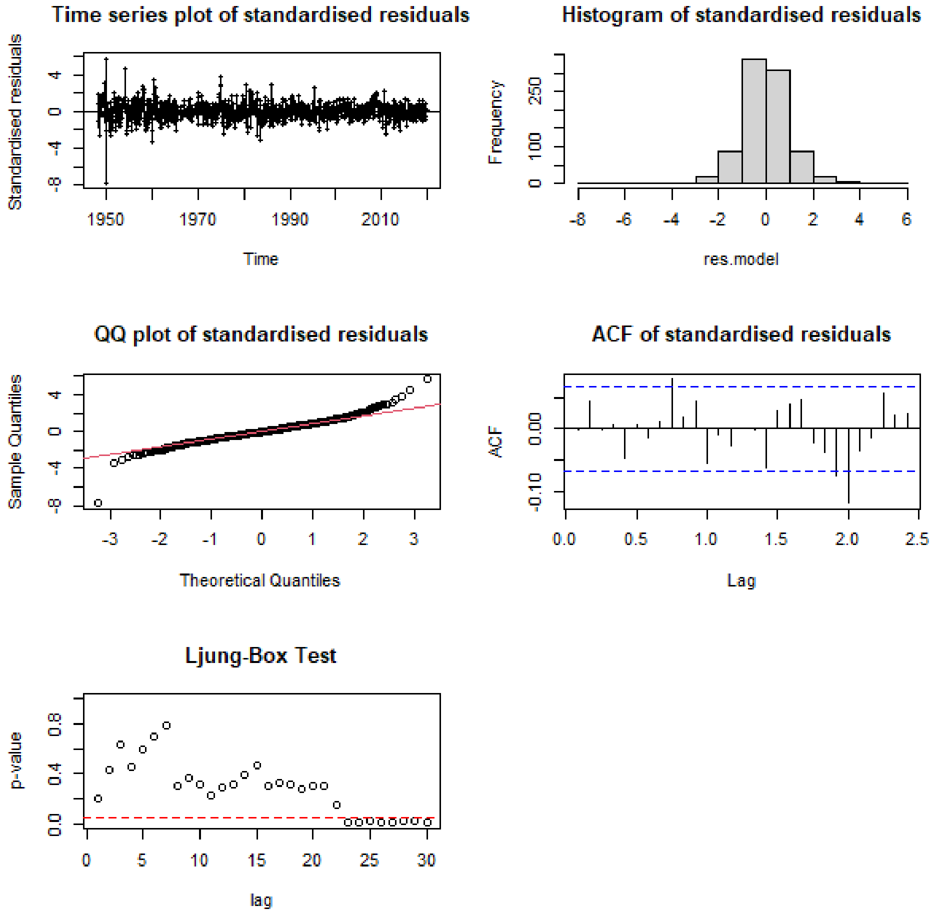




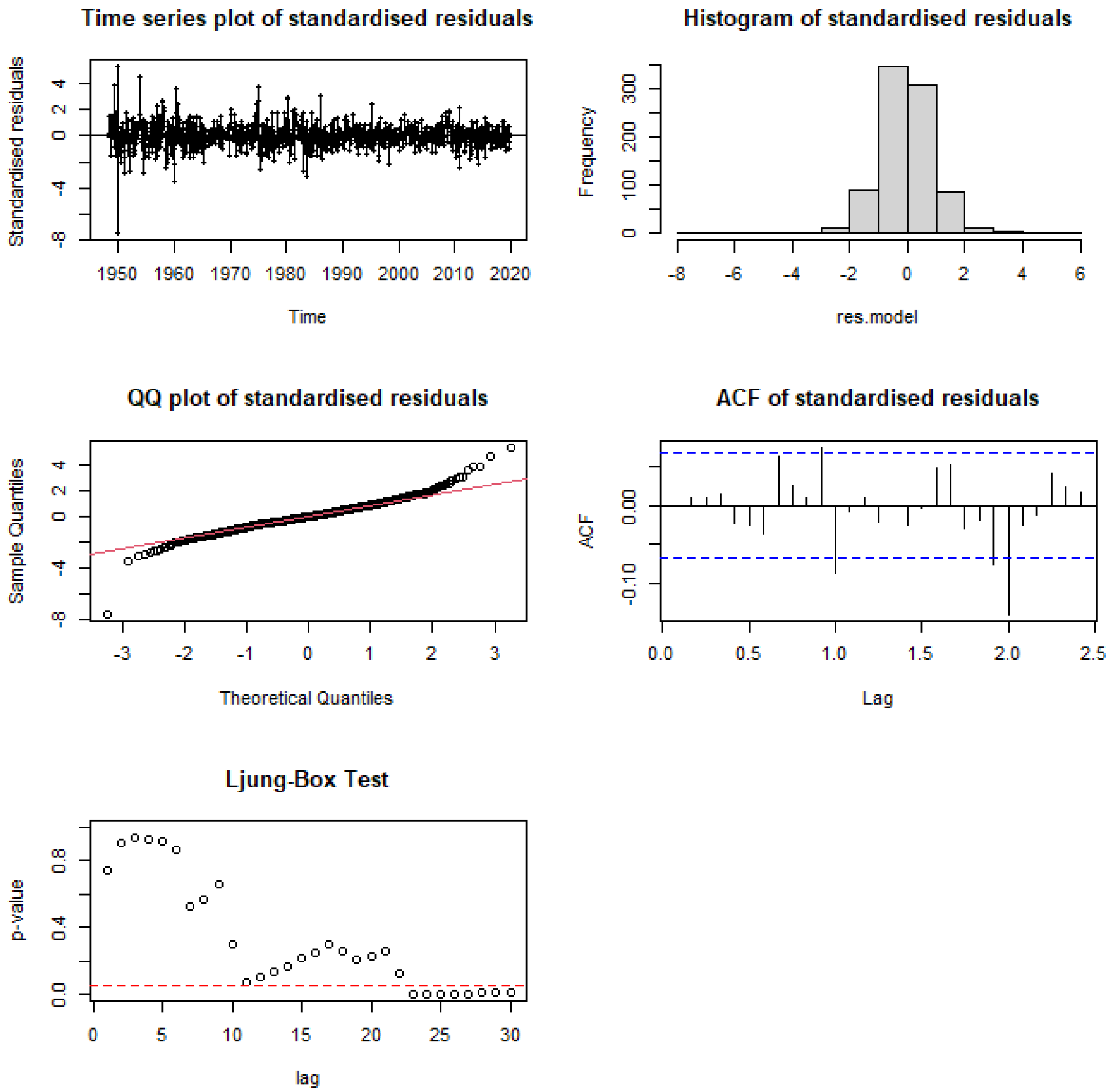




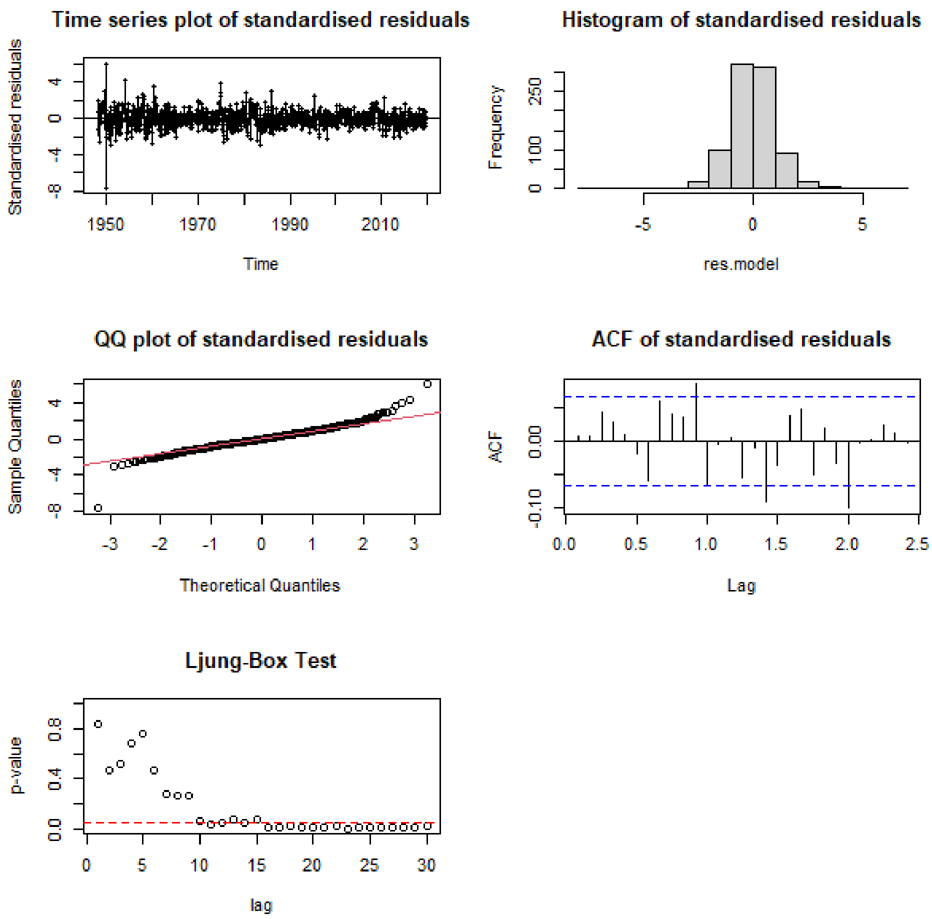
Appendix 6: ARIMA (5,1,6) ML Residual Output



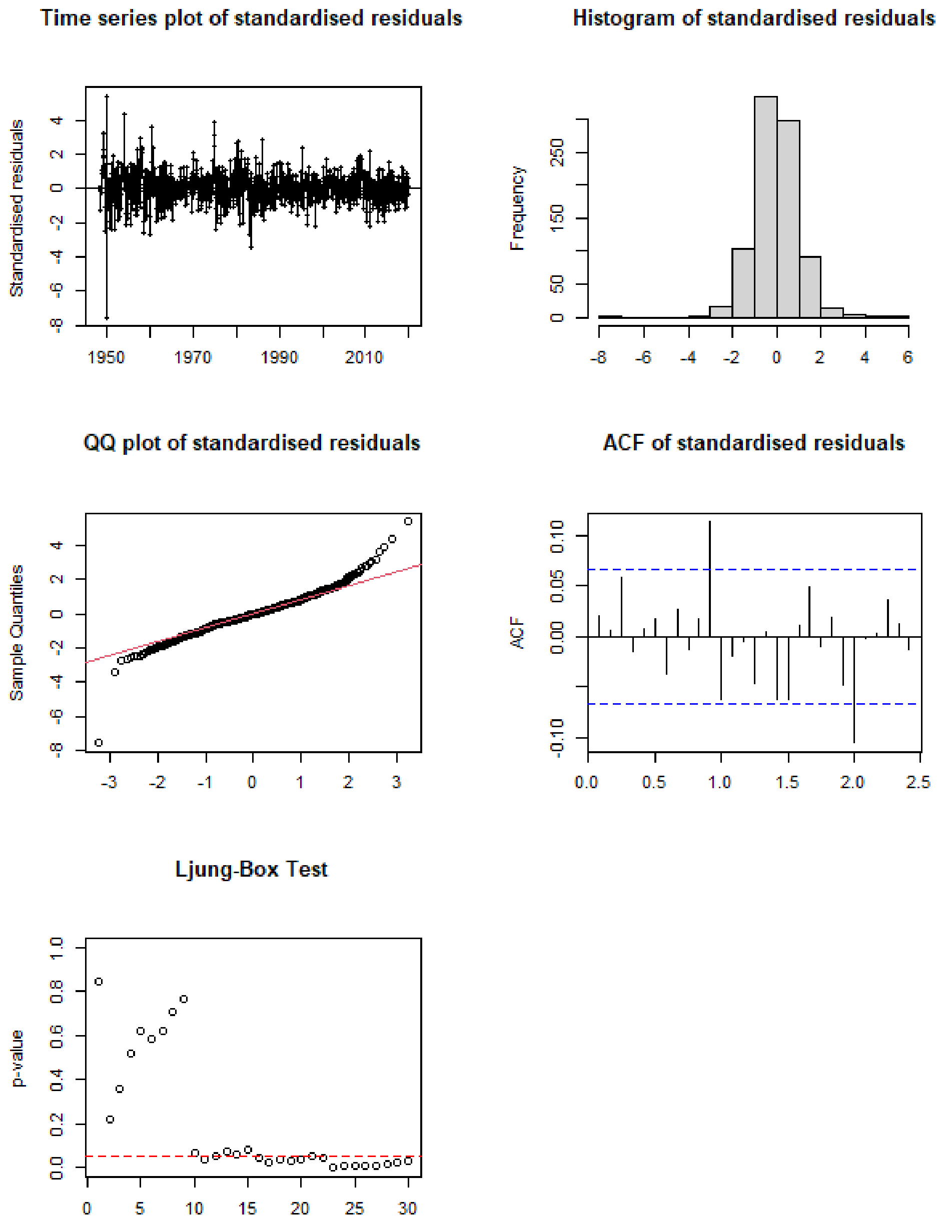
Appendix 7: ARIMA (5,1,6) CSS Residual Output



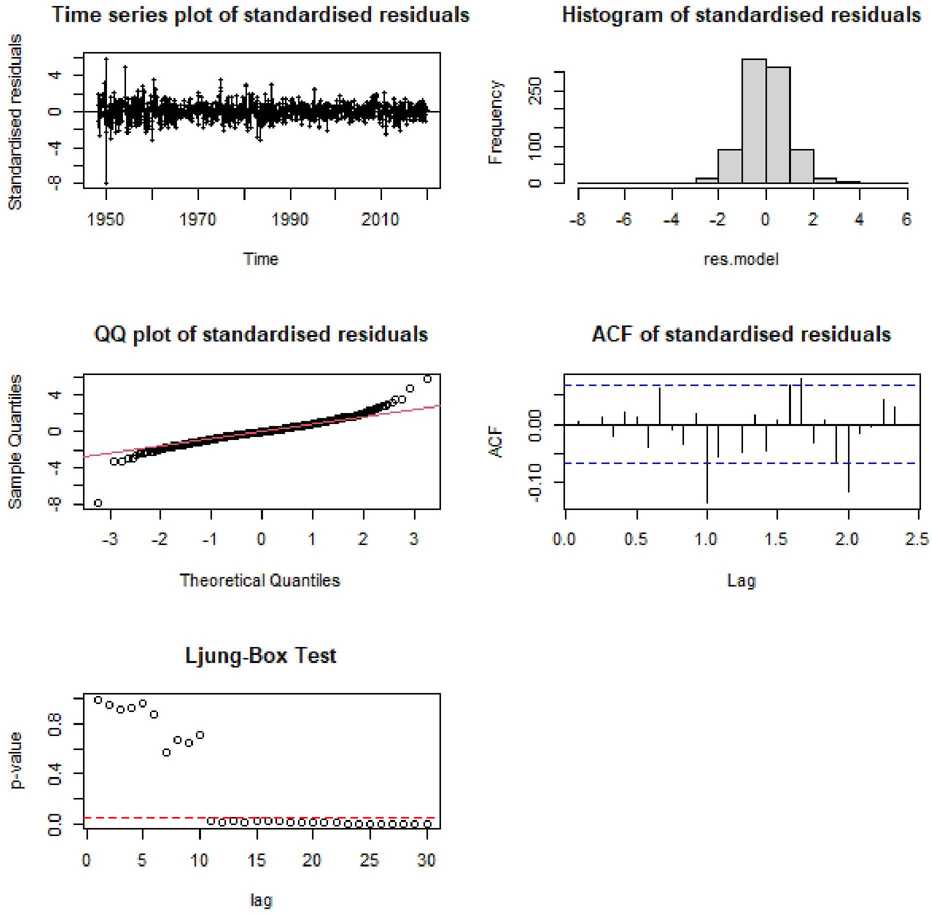
Appendix 8: ARIMA (5,1,7) ML Residual Output



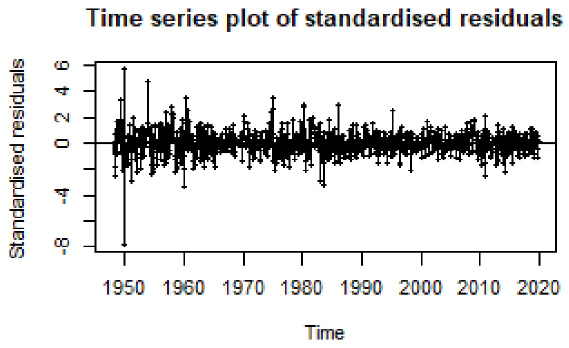
Appendix 9: ARIMA (5,1,7) CSS Residual Output

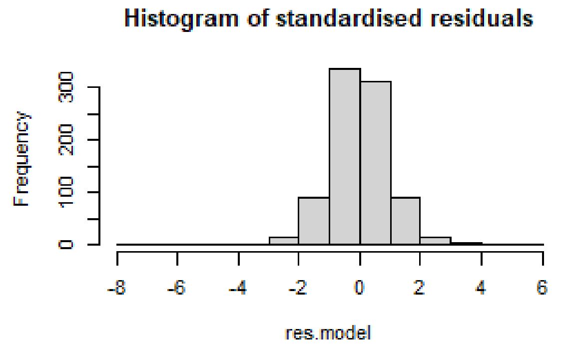


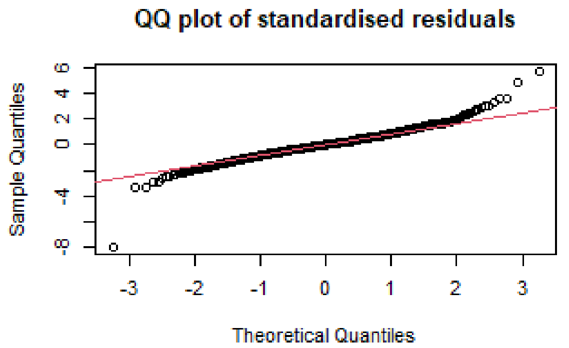
Appendix 10: ARIMA (2,1,5) ML Residual Output

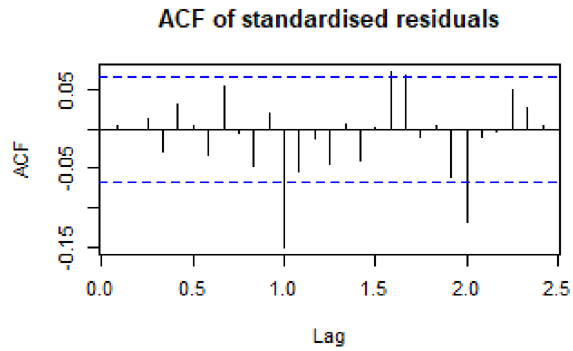


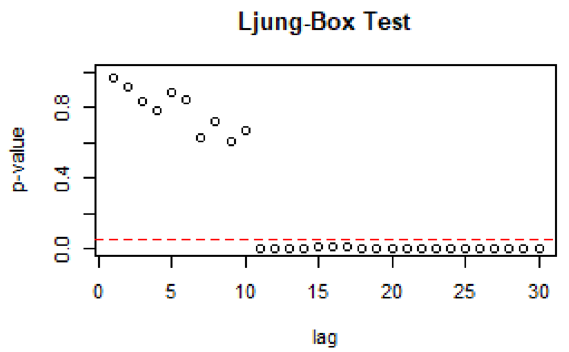
Appendix 11: ARIMA (2,1,5) CSS Residual Output



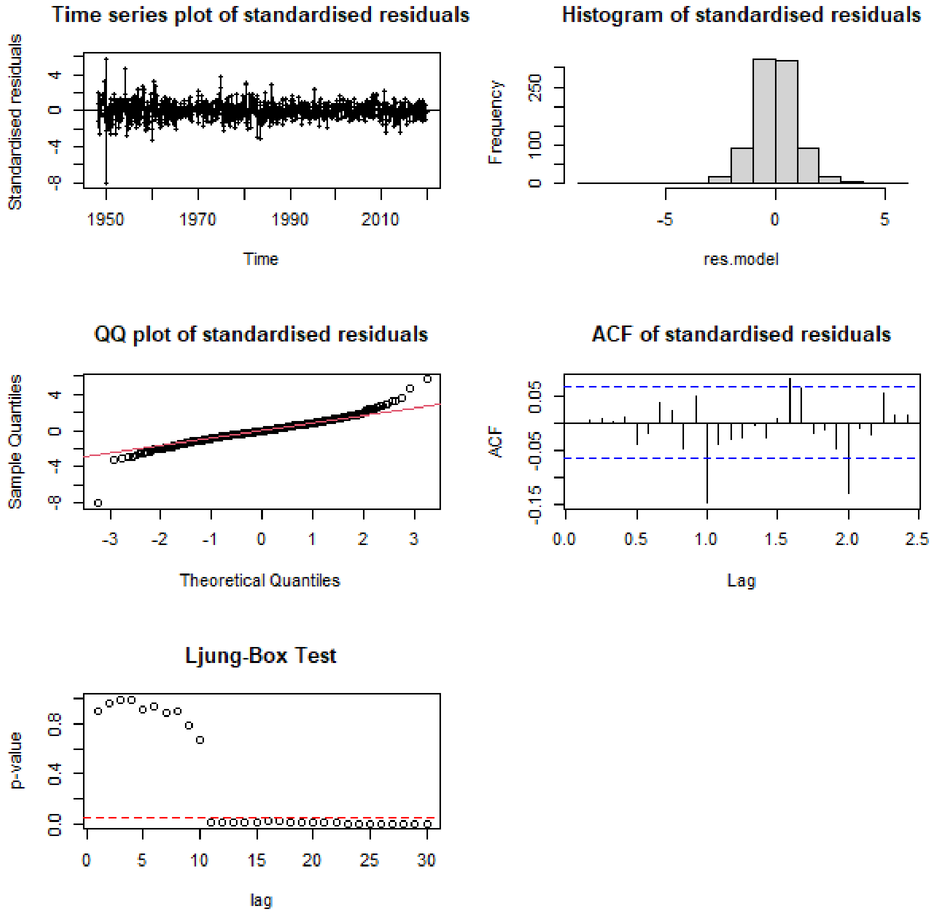




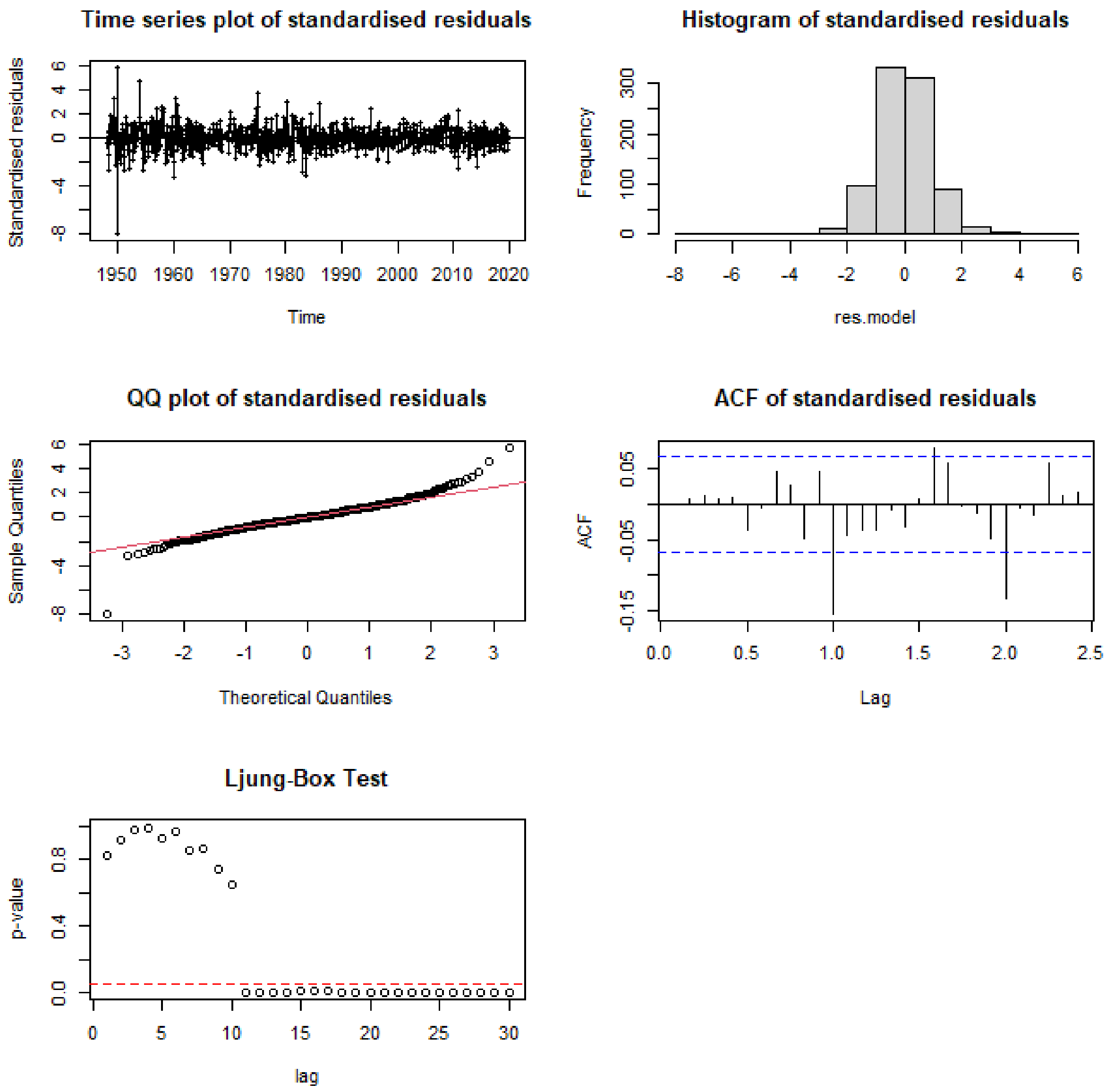




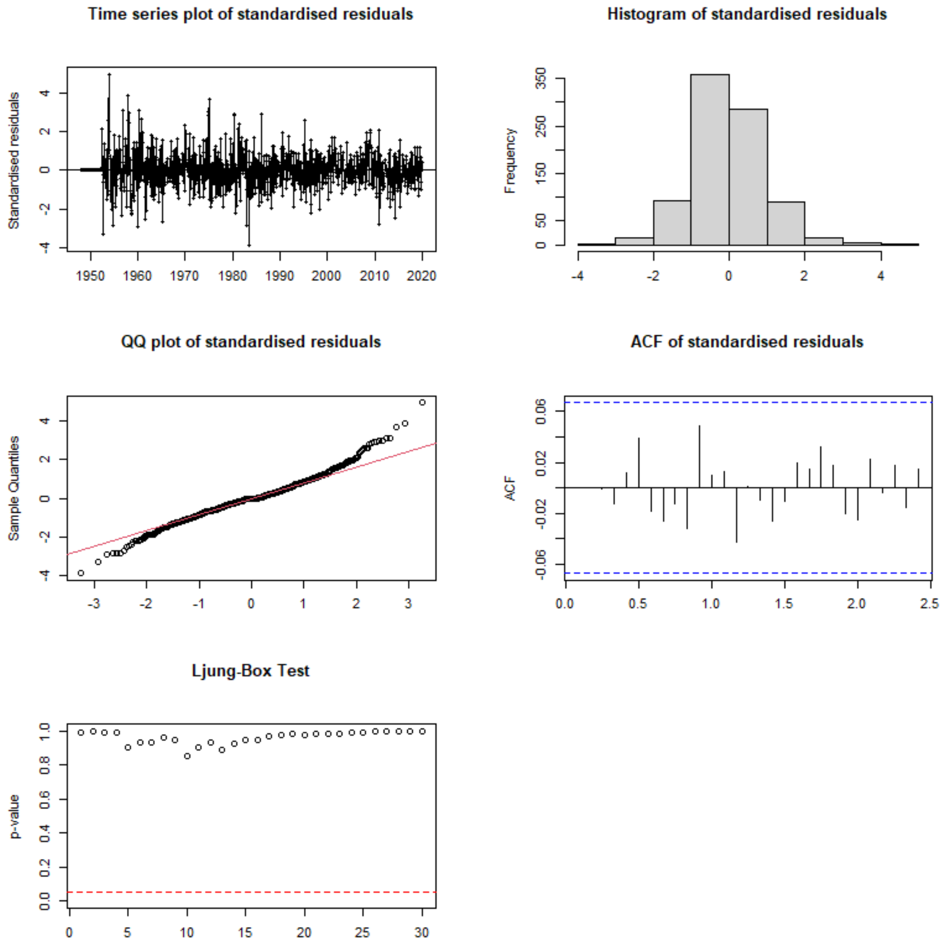
Appendix 12: ARIMA (1,1,5) ML Residual Output



Appendix 13: ARIMA (1,1,5) CSS Residual Output



Appendix 14: SARIMA(4,1,6)x(3,1,1)_{12} CSS Residual Output



Appendix 15: SARIMA(6,1,8)x(3,1,1)_{12} CSS Residual Output