Project 2: Finite Element Solution for Non-linear Elastic Materials

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1 Code Description

The code provides three different solutions to the elasticity model.

1.1 Linear Elastic Solution

The linear Elastic Solution constructs the following matrices

$$[N]_{2x16} = \begin{bmatrix} N_1 & 0 & & \dots & & N_8 & 0 \\ 0 & N_1 & & & & 0 & N_8 \end{bmatrix}$$

$$[G]_{3\times 16} = \begin{bmatrix} \frac{\partial}{\partial \xi} & 0 \\ 0 & \frac{\partial}{\partial \eta} \\ \frac{\partial}{\partial \eta} & \frac{\partial}{\partial \xi} \end{bmatrix} [N]$$

$$[J]_{3\times 3} = [G]_{3\times 16}\, [P]_{16\times 3}$$

$$[B]_{3\times 16} = [J]^{-1}_{3\times 3}\, [G]_{3\times 16} = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & & \dots & & \frac{\partial N_8}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial y} & & & & 0 & \frac{\partial N_8}{\partial y} \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & & & & \frac{\partial N_8}{\partial y} & \frac{\partial N_8}{\partial x} \end{bmatrix}$$

and then solves

$$[K]_{16\times 16} = \sum_{qp} [B]^T_{3\times 16}\, [D]_{3\times 3}\, [B]_{3\times 16}\, \omega_{qp}\, |J|$$

and

$$[f_\Omega] = \sum_{qp} [N]^T_{2\times 16}\, [b]_{2\times 1}\, \omega_{qp}\, |J|$$

Following assembly of all of the elements, it then applies dirichlet boundaries and solves

$$[u]_{n\times 1} = [K]^{-1}_{n\times n}\, [f]_{n\times 1}$$

where n is the amount of global nodes.

1.2 Newton-Raphson Solution

The Newton-Raphson solution uses the above definitions of the N, G, J, and B matrices, but uses the Newton-Raphson Method to approximate the correct D matrix.

1.3 Secant Solution

The Secant solution uses the above definitions of the N, G, J, and B matrices, but uses the Newton-Raphson Method to approximate the correct D matrix.

2 Salient Features

The code that was written is straightforward as far as the process is concerned. The most interesting feature of the code is the application of boundary conditions in an adhoc manner. Because of the symmetric of the problem, there must be a $t = 0$ boundary at both $x = 0$ and $y = 0$, and at $x = 2$, there is a $t = 20MPa$ boundary. This are applied by looking for mesh points falling on that line. For arbitrary geometries, a more general way of applying these condititions would be required.

3 Computational Effort Description

This code is not particularly computationally taxing, and is enumerated in Table 1.

	Linear	Newton-Raphson (100 iterations)	Secant (100 iterations)
Computation Time (s)	0.190064	20.479119	21.076019

Table 1: Computational Effort Metrics for Different Solving Methods

4 Results and Concerns

The concerns for this code are enumerated below:

- The linear code does not provide a physically accurate solution.
  - The boundaries between elements are not continuous, and they should be using the Galerkin Method.
  - The actual values do not show similarity to the analytical solution.
  - An error occurs in matlab's backdivision operator indicating that the matrix is close to singular or badly scaled.
- The Secant and Newton-Raphson methods neither converge nor show a physically accurate solution
  - The boundaries between elements are not continuous, and they should be using the Galerkin Method.
  - The actual values do not show similarity to the analytical solution.

- An error occurs in matlab's backdivision operator indicating that the matrix is close to singular or badly scaled.
- There is absolutely no change in the convergence criteria, which shows that I was unable to correctly use the Newton-Raphson method.

All of these errors show large issues within the code that would have to be debugged in order to provide accurate results.

Part I
Figures

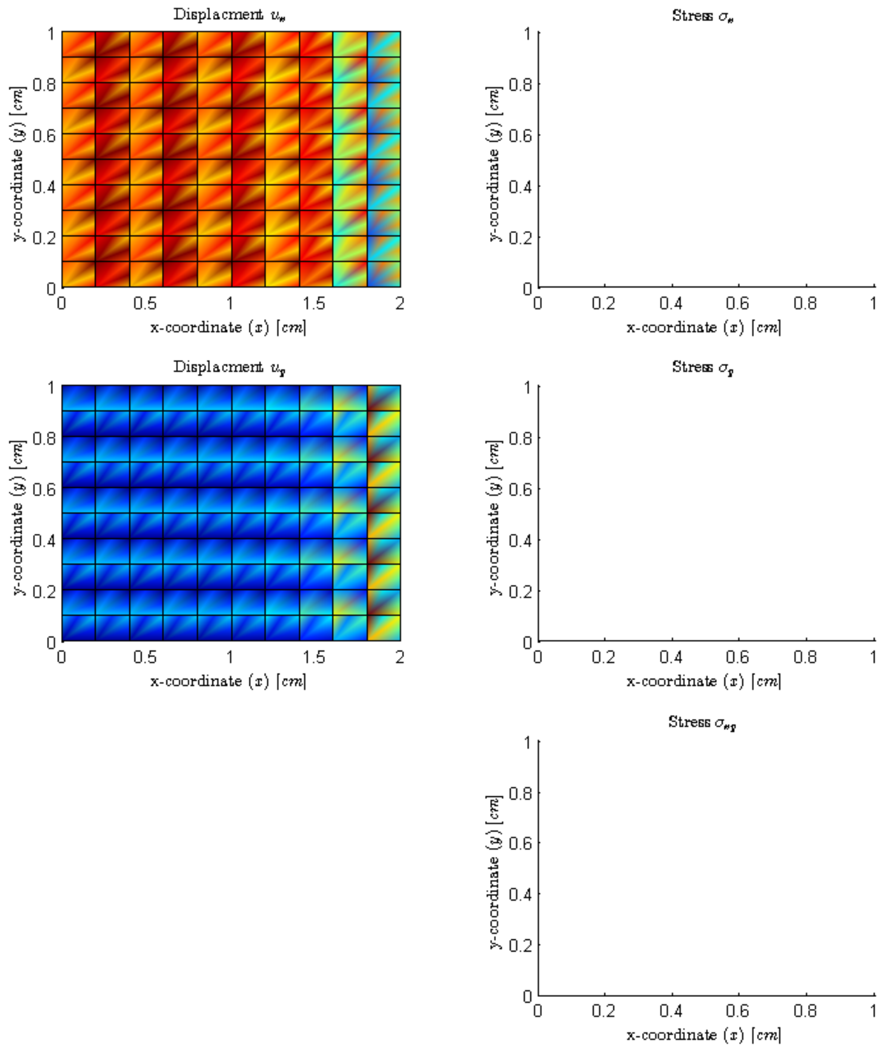

Figure 1: Linear Solution to FEM Approximation of Elasticity Problem

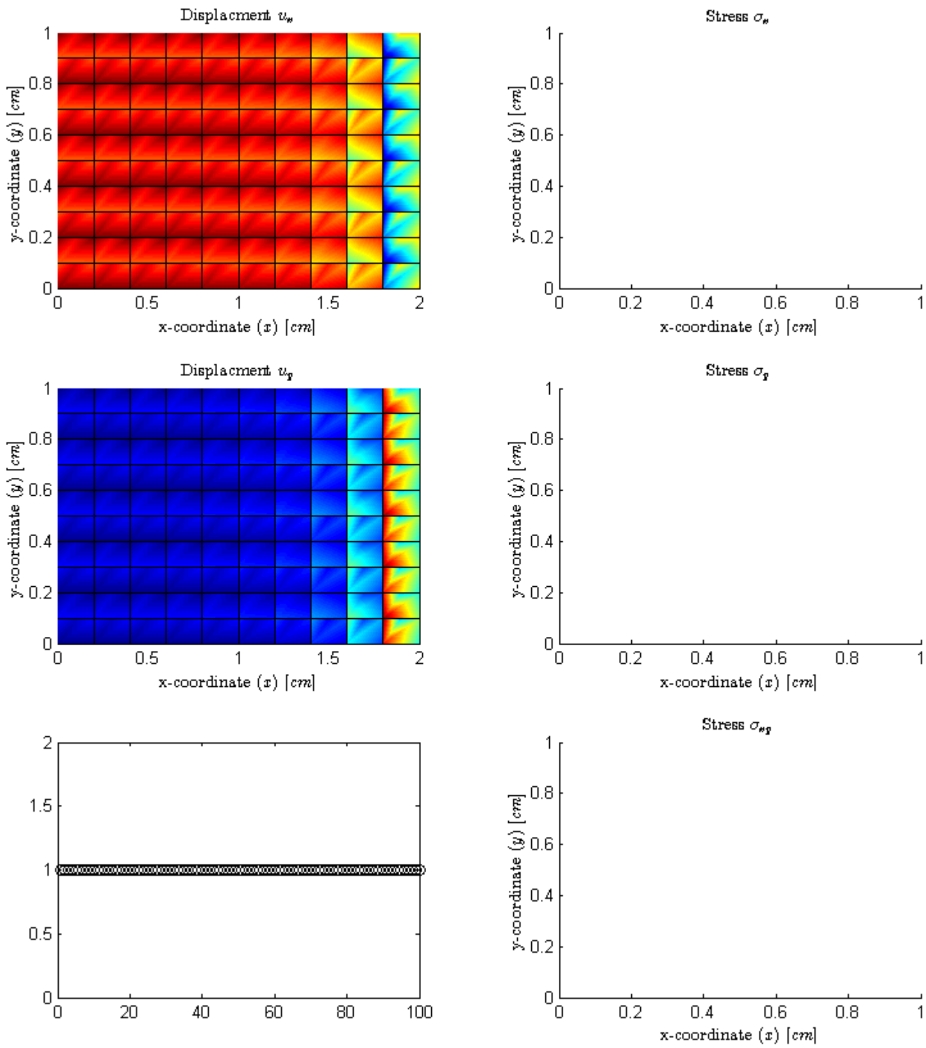

Figure 2: Newton-Raphson Solution to FEM Approximation of Elasticity Problem

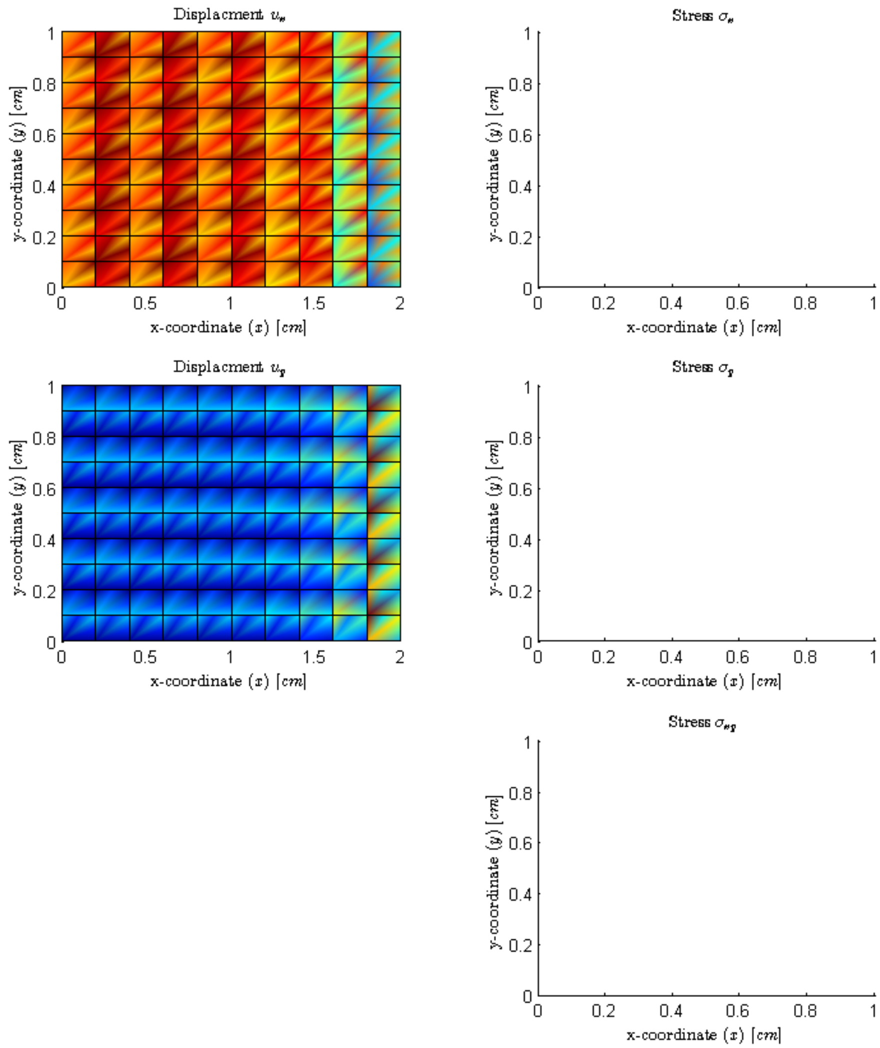

Figure 3: Secant Solution to FEM Approximation of Elasticity Problem