PHYS 556 HMWK 1

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9/2/15

[Q1] Calculate the remaining amount fraction of material using the given information. Remember the difference between half-life and the decay constant of the material. Check the author's statement about the rareness of the material. Assuming the information about the half-life of each element is correct, the amount remaining after $4 \times 10^9\,\mathrm{yr}$ can be calculated using the decay equation:

$$t_{1/2} = 7. \times 10^3\,\mathrm{yr}$$

$$N = N_0 \exp\left(-\lambda t\right)$$

$$\lambda = \frac{\ln\left(2.\right)}{t_{1/2}} = \frac{\ln\left(2.\right)}{\left(7. \times 10^3\,\mathrm{yr}\right)} = 9.902 \times 10^{-5}\,\frac{1}{\mathrm{yr}}$$

$$\frac{N}{N_0} = \exp\left(-\left(9.902 \times 10^{-5}\,\frac{1}{\mathrm{yr}}\right) \cdot \left(4. \times 10^9\,\mathrm{yr}\right)\right) \approx {}^1\!/_{7\times 10^{172018}}$$

[Q2] Write out the reactions discussed formally, in a way similar to that shown in equation (1) and (2). The equations discussed are the neutron capture of ^{238}U and a chain of β decays. The individual equations are

$$^{238}_{92}\mathrm{U} + {}^{1}_{0}n \rightarrow {}^{239}_{92}\mathrm{U} + \gamma$$

$$^{239}_{92}\mathrm{U} \rightarrow {}^{239}_{93}\mathrm{Np} + \beta^-$$

$$^{239}_{93}\mathrm{Np} \rightarrow {}^{239}_{94}\mathrm{Pu} + \beta^- + \overline{\nu_e}$$

[Q3] Why are the authors discussing geological possibilities for the discovery of Berkelium and Californium? (Is this discussion included for political reasons?) The authors are likely covering for two reasons. One is to prove why their lab exists (to create things unobtainable in nature), and the second is to assuage fears of geological abundance of high Z nucelei such as Bk.

[Q4] Calculate how close the ^{4}He particle comes to the ^{241}Am nucleus using the Berkeley Cyclotron. Compare this to the nuclear radii of the neuclei. Is there an overlap? The nuclear radius of ^{241}Am can be calculated using a simple correlation.

$$R_A = \left(1.2\,\mathrm{fm}\right) A^{1/3} = \left(1.2\,\mathrm{fm}\right)\left(241.\right)^{1/3} = 7.5\,\mathrm{fm}$$

Then, to compare this to the approach of the α particle to the nucelus, we have to determine when the repulsion of the nucleus is larger than the kinetic energy imparted by the cyclotron. The kinetic energy imparted can be calculated knowing that the cyclotron gives the α particle 35 MeV [2]. This can be equated to the electrical charge energy:

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

where q_1 is the charge of the α particle (2+), q_2 is the charge of the Am nucleus (95+), ϵ_0 is the permitivity of free space, and r is the radius of interest. Therefore

$$r = \frac{q_1 q_2}{4\pi\epsilon_0 U} = \frac{\left(2 \cdot 1.6 \times 10^{-19}\,\mathrm{C}\right)\left(95 \cdot 1.6 \times 10^{-19}\,\mathrm{C}\right)}{4\pi\left(8.85 \times 10^{-12}\,\frac{\mathrm{C^2 s^2}}{\mathrm{m^3 kg}}\right)\left(35\,\mathrm{MeV}\right)} = 7.8 \times 10^{-15}\,\mathrm{m} = 7.8\,\mathrm{fm}$$

which is close to the size of the nuclear radius, so the interaction can occur.

[Q5] Given the probability of occurrence of 10^{-26}, what cross sections were they using in their estimate? Remember the equation I showed you in class. See calculation in question 6.

[Q6] The equation given is kind of a hand calculation, a little odd to be in a paper. However, the authors are trying to give an order of magnitude estimate to the convince the reader that what they are doing is plausible. Go through the calculation term by term and write a symbolic equations for the production rate and the total nuclei produced. Be sure to the relate it to the cross section, which they hide in the probability. (Why would they hide the cross section estimate this way?) The equation in the paper is reproduced below:

$$R = 10^{-26} \times (0.010/241) \times 6 \times 10^{23}\,(\text{atoms of Am in target}) \times 6 \times 10^{12}\,\left(\text{He}^{++}\text{ ions per second}\right) \cong 1.5 \times 10^{6}\,\left(\text{atoms Bk}^{243}\text{formed per second}\right)[2]$$

In order to write this symbolically, we should try to identify groups that correspond to calculateable quantities and match units.

$$R = \underbrace{10^{-26}}_{?} \times \underbrace{0.010}_{m_{Am}} \times \underbrace{\frac{1}{241}}_{\frac{1}{A}} \times \underbrace{6 \times 10^{23}}_{N_A} \times \underbrace{6 \times 10^{12}}_{?}$$

From the given mass, and general knowledge of the atomic number and Avogadro's number, the three above groups can be determined, leaving two groups unidentified. The second group (6×10^{12}) has units similar to current, so a quick calculation of the number of He^{++} ions in a $2\,\mu$A beam shows

$$I_\alpha = \frac{I}{2 \cdot e^-} = \frac{2 \times 10^{-6}\,\text{A}}{2 \cdot 1.6 \times 10^{-19}\,\text{C}} = 6.25 \times 10^{12}\,\frac{\text{He}^{++}}{\text{s}}$$

showing that the second group is in fact, I_α. The final group, then, must be the probability P of interaction. Thus the final equation is

$$R = P\frac{m_{Am}}{A}N_A I_\alpha$$

In order to determine the cross section, we must relate this to a common definition of the reaction rate, namely

$$R = \sigma N_{Am} I_\alpha = \sigma \frac{N_A \Delta x}{A} I_\alpha$$

and equating this with above gives

$$R = \sigma \frac{N_A \Delta x}{A} I_\alpha = P\frac{m_{Am}}{A}N_A I_\alpha$$

$$\sigma = P\frac{m_{Am}}{\Delta x}$$

and assuming the thickness to be a couple microns, we get

$$\sigma = 10^{-26}\frac{0.010}{10.0 \times 10^{-4}} = 1.0 \times 10^{-25}\,\text{cm} = 0.1\,\text{b}$$

They must hide the cross section in this way to stop other countries from knowing whether berkelium can be used to create a bomb or not.

[Q7] What is the technique of ion exchange separation? What is the modern term for this technique? Pull down a picture of a modern system and explain in a few words the basic principle of operation. Ion exchange separation is now commonly referred to as Ion Exchange Chromotography or Ion Chromotography Separation. It involves an analyte (the material of interest), which is moved by the "mobile phase" (usually a buffer solution) into a column containing the "stationary phse" (usually a gel matrix), which has open charges that will allow for the analyte to attach to the stationary solution. After this process, elution can be performed to obtain the analyte, and chromotography or conductivity testing to determine the presence and identity of the analyte [1]. The following figure is a schematic diagram of that process.

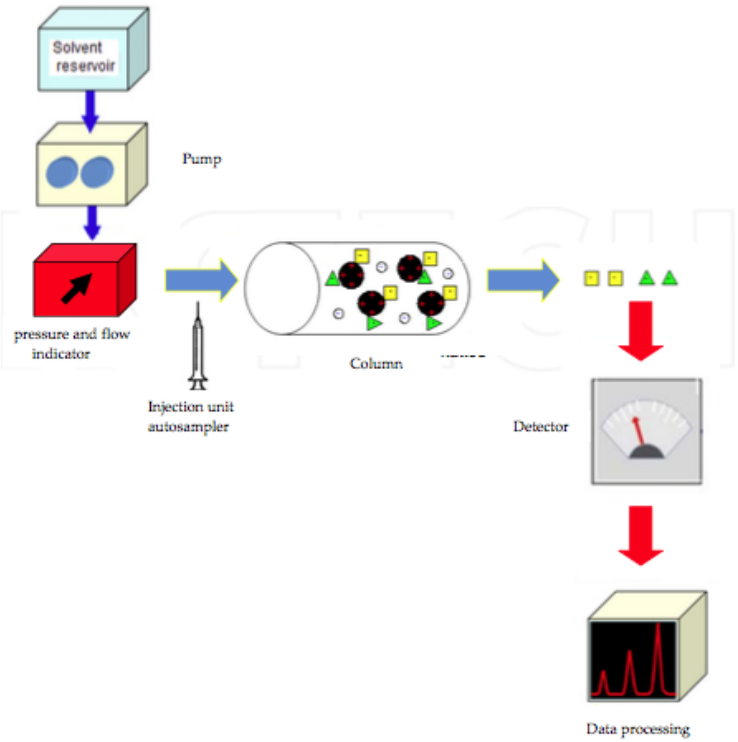

Figure 1: Ion-exchange chromotography system [1]

[Q8] The discovery graph is Figure 2. Explain this graph as if to a friend. Also, why is the top graph of Figure 1 important in your discussion? "The graph in Figure 2 shows the activity in a detector versus the amount of elutriant (a chemical that washes the analyte - or chemical of interest - out of a gel matrix). Peaks appear in graphs like this because the amount of elutriant required to remove isotopes form the matrix is ordered in such a way that less elutriant will wash out isotopes with higher atomic numbers (for an example of this, see the lanthanide chain in Figure 1). Because of this, the peak in Figure 2 that occurs **before** the peak for Cm and Am shows that the atomic number of that element is **higher** than that of Cm, namely it is the discovered isotope $Z = 97$."

[Q9] Write an equation like (1) and (2) for the electron capture process.

$$ {}^{1}_{1}p + e^{-} \rightarrow {}^{1}_{0}n + \nu_e $$

or

$$ {}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X} \rightarrow {}^{\mathrm{A}}_{\mathrm{Z-1}}\mathrm{X} + \nu_e $$

[Q10] What is the problem in preparing microgram amounts of new materials using accelerators? What is the best way to prepare more massive samples? Because of two reasons, accelerators could not be used to create massive samples of high Z elements such as Bk. One is the small beam current in the accelerators, which may have increased since the publication of the Cunningham paper. The other is the thermal stability because of the short half lives (creation of heat from decay would cause the aggregates to be thermally unstable). Instead, Cunningham points out that in high neutron flux regions, several captures can lead to a high Z element, so some high flux isotope creation reactors are a better way to create massive samples [2].

[Q11] How were more massive samples prepared and how long did it take? Is this still the best approach? Using MTR, a 5 yr neutron irradiation of ^{239}Pu was performed, which generated 0.6 μg of Bk [2]. This seems like a miniscule amount to be created in 5 yrs, but MTR does not have as high of a neutron flux as some modern reactors. For instance, creation of Bk targets using irradiation in HFIR takes only just over a year [3].

[Q12] What are the three methods to produce Super Heavy Elements in the discovery phase? The methods to create elements past $Z = 114$ are given anecdotally in the paper as:

Heavy-ion Fusion Using a doubly magic ^{48}Ca projectile onto actinide targets and following their α decay chain afterward [3].

Neutron Bombardment With high flux sources such as HFIR, neutron bombardment can create elements up to $Z = 97$ and perhaps futher. Chemical separation is required after this process [3].

"Hot Fusion" Reactions For the nucelei of elements with $Z = 106,\ 108,\ 110$, a so-called "hot-fusion" technique was used, although not elucidated upon by Oganessian [4].

[Q13] What is the goal of searching in these Super Heavy Elements? The overarching goal is to determine the accuracy of the shell model, especially as it concerns "islands of stability". As Dr. Oganessian states, every advanced model of the nucleus predicts another island of stability after ^{208}Pb near a spherically doubly magic nucleus [3].

[Q14] Write out at least three questions concerning the introduction material in the discovery paper of Livermorium.

1. What are the differences between "hot-" and "cold-" fusion? The paper states that there is 30 MeV excitation in a "cold-fusion" reaction, and $E^* = 45$ MeV for a "hot-fusion" reaction [4], but those don't seem that different.
2. In the synthesis of the ^{249}Bk target from Oak Ridge, neutron bombardment in HFIR was used [3]. How much does the quoted 250 d irradiation in HFIR cost?
3. What type of thermal properties are required for creation of these super heavy elements? Is everything cryogenic? Are any of the decays gaseous?

References

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