PHYS 556 HMWK 2

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[Q1] Calculate the average binding energy of a nucleon in $^{58}_{28}\text{Ni}_{30}$. Start with the value in the Appendix C of Krane. Read pages 59-65 in Krane. Are there corrections to be made to the value found in Appendix C of Krane. To calculate the binding energy per nucleon, first we must calculate the binding energy of the nickel isotope. This can be calculated simply from the mass defect.

$$\begin{aligned} BE_{\text{Ni}} &= 30 \cdot m_n + 28 \cdot m_p - M_{\text{Ni}} \\ &= 30 \cdot (939.573\,\text{MeV}) + 28 \cdot (938.280\,\text{MeV}) - (931.502\,{}^{\text{MeV}}\!/\text{u}) \cdot (57.935346\,\text{u}) \\ &= 492.139\,\text{MeV} \end{aligned}$$

and then averaging this over the amount of nucleons, which are 58.

$$\overline{BE_{\text{Ni}}} = \frac{BE_{\text{Ni}}}{A_{Ni}} = \frac{492.139\,\text{MeV}}{58\,\text{nucelon}} = 8.485\,{}^{MeV}\!/\text{nucleon}$$

Krane describes some corrections that are required to this value, one of which is the inclusion of the electron mass. This is required because the mass spectrographic technqiues used generally use the neutron atom, and not a fully ionized nucleus. The size of the electron mass compared to the neutron and the proton is miniscule though, and so it would not cause any change higher than the uncertainty in the above calculation. The second correction involved the relative size of the binding energy of quarks to that of the neutrons. For example, it is hypothesized that a quark may have a rest mass energy up to 100 GeV, and with three quarks in each nucleon, that means that the differences between 300 GeV in the quarks generates 1 GeV binding energy for the nucelons. Without being able to measure the mass of a free quark (which may be impossible), it is difficult to tell [2, p. 59].

[Q2] Go to the National Nuclear Data Center. Lookup S_n, the 1st neutron, and S_p, the 1st proton separation energy. That is the energy required to take 1 neutron or 1 proton from the nucleus. Using the chart of nucleeides from NNDC for $^{58}_{28}\text{Ni}_{30}$, the values for S_n, S_p, and Δ were found, and are tabulated below.

Table 1: Neutron and proton separation energies for $^{58}_{28}\text{Ni}_{30}$ [1]

Isotope	Q [MeV]	S_n [MeV]	S_p [MeV]
$^{58}_{28}\text{Ni}_{30}$	8.5610	12.2163	8.1722

[Q3] Given the knowledge gained in Q1 and Q2 give an argument why the study of Nuclear Structure can be considered non-relativistic. What sets the energy scale? Using the rule-of-thumb that anything under $0.2c$ can be considered non-relativistic, we can look at the kinetic energies associated with nucleons to make them relativistic. To do this, we use the simple definition of kinetic energy:

$$KE = \frac{1}{2}mv^2$$

and apply this to a nucleon, giving us

$$KE = \frac{1}{2}\left(\sim 940\,\frac{\text{MeV}}{\text{c}^2}\right)(0.2c)^2 = 18.8\,\text{MeV}$$

as the kinetic energy at which a nucleon becomes relativistic. It can be seen in Table 1 that the separation energies for nucleons do not reach this level, and it can also be seen that the binding energy per nucleon in Q1 is not relativistic.

[Q4] In class we found the classical radius of the electron, $r_{classical}$ by assuming that all the mass of the electron, $511\,\mathrm{keV}$ is due to its electric field energy. In a similar way estimate the amount of the proton mass that can be attributed to the electric field energy of the proton. The charge distribution of the neutron and proton is found to have a radius of $0.8\,\mathrm{fm}$. We have an expression for the electric field energy as

$$mc^2 = \frac{\epsilon_0}{2}\int_0^\infty E^2 dV$$

where E denotes the electric field and dV is a volume element in spherical coordinates, given as

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$dV = r^2 \sin(\theta)\, d\theta d\varphi dr$$

which gives us a final expression as

$$\begin{aligned} mc^2 &= \frac{\epsilon_0}{2}\int_0^\infty\int_0^{2\pi}\int_0^\pi \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^4}r^2\sin(\theta)\, d\theta d\varphi dr \\ &= \frac{1}{8\pi}\int_0^\infty\int_0^{2\pi}\int_0^\pi \frac{q^2}{r^2}\sin(\theta)\, d\theta d\varphi dr \\ &= \frac{1}{4\pi}\int_0^\infty\int_0^{2\pi}\frac{q^2}{r^2}d\varphi dr \\ &= \frac{1}{2}\int_0^\infty \frac{q^2}{r^2}dr \\ &= \frac{1}{2}\left[-\frac{q^2}{r}\right]_0^\infty \end{aligned}$$

This would obviously give us an infinite radius to have finite mass, so we can now used the classical radius in the integral as the lower argument, obtaining

$$mc^2 = \frac{q^2}{4\pi\epsilon_0 r_p}$$

and

$$r_p = \frac{q^2}{mc^2} = \frac{1.44\,\mathrm{MeV}\cdot\mathrm{fm}}{938.272046\,\mathrm{MeV}} = 0.00153\,\mathrm{fm}$$

, which is much smaller than the given radius of $0.8\,\mathrm{fm}$. To determine the mass of the proton due to electric charge, we can use that above radius in

$$mc^2 = \frac{q^2}{4\pi\epsilon_0 r_p} = \frac{1.44\,\mathrm{MeV}\cdot\mathrm{fm}}{0.8\,\mathrm{fm}} = 1.8\,\mathrm{MeV}$$

[Q5] The proton and neutron can be considered to be a nucleon. In other words they are same particle, just as the electron can be considered to have two states spin up and spin down. A nucleon has two states isospin up (proton) and isospin down (neutron). Using the results of question [4] at what level would you expect this concept of isospin to break down. Inside the charge radius of $0.8\,\mathrm{fm}$, the isospin assumption would break down.

[Q6] Solve the infinite well potential

[a] List the step required to solve a quantum mechanical potential for the quantum energy spectrum and the normalized wave functions.

[b] Solve for $\psi(x)$ and E_n for the following potential.

$$V(x) = \begin{cases} \infty & x < -\frac{a}{2},\, x > \frac{a}{2} \\ 0 & -\frac{a}{2} \leq x \leq \frac{a}{2} \end{cases}$$

This problem has two regions, one with an infinite potential, and one with no potential (a free particle). In the the free particle region, schrodinger's equation is given as

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi$$

which, when solved is given as

$$\psi(x) = A^{'} \sin kx + B^{'} \cos kx$$

The continuity conditions give

$$\psi\left(-\frac{a}{2}\right) = 0$$

and

$$\psi\left(\frac{a}{2}\right) = 0$$

which gives

$$A^{'} \sin \frac{ka}{2} + B^{'} \cos \frac{ka}{2} = A^{'} \sin -\frac{ka}{2} + B^{'} \cos -\frac{ka}{2}$$

and using the property that sin is an antisymmetric function ($\sin x = -\sin -x$), and cos is a symmetric function ($\cos x = \cos -x$), we get

$$A^{'} \sin \frac{ka}{2} + B^{'} \cos \frac{ka}{2} = -A^{'} \sin \frac{ka}{2} + B^{'} \cos \frac{ka}{2}$$

$$2A^{'} \sin \frac{ka}{2} = 0$$

showing that

$$A^{'} = 0$$

so applying the upper boundary condition, we have

$$B^{'} \cos \frac{ka}{2} = 0$$

, with $B^{'}$ cannot equal 0. This gives

$$\frac{ka}{2} = 0$$

or

$$\frac{ka}{2} = n\pi \quad n = 1, 3, 5, ...$$

Giving

$$\psi(x) = B^{'} \cos \frac{2n\pi}{a} x$$

and

$$E_n = \frac{h^2 k^2}{2m} = \frac{2h^2\pi^2 n^2}{ma^2}$$

[c] Draw the wave function and probability function for the first four energy states within the well. Because the energy is quantized, the corresponding wave functions are

$$\psi_n = B^{'} \cos \frac{2\pi n}{a} x \;\; n = 1, 3, 5, ...$$

which is shown in arbitrary scale in Figure 1.

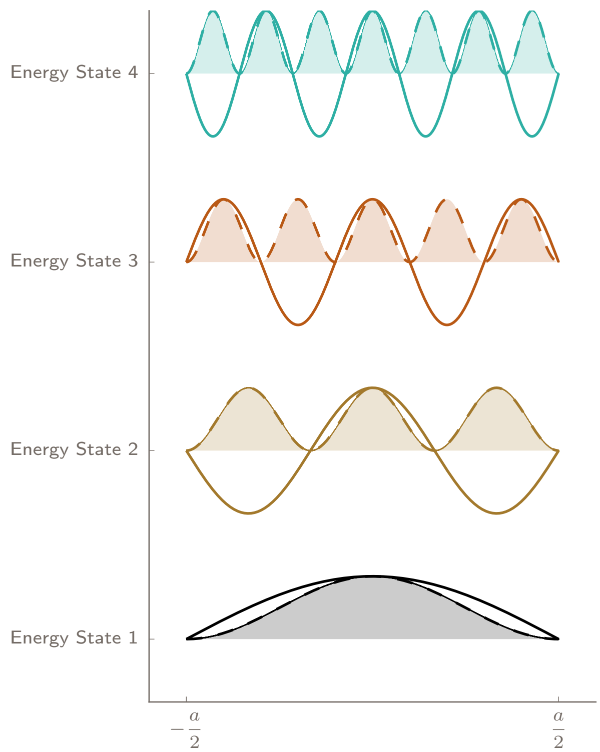

Figure 1: First four energy states within the well of given wave function

[d] Is the potential invariant to parity? This potential is invariant to parity.

[e] Are the wave functions eigenstates of parity? The wave functions are eigenstates of parity.

[f] Assign even and odd parity to each state. What is the pattern? All of the states are even parity.

[g] Fill this potential with 3 protons. What is the parity of the resulting nucleus? Krane states that "the parity of the combined wave function representing any number of even-parity particles or an even number of odd-parity particles; it will be odd if there is an odd number of odd-parity particles." [2, p. 38] Protons are +1 parity, or even parity, so this situation, representing 3 even parity particles, has even parity.

[h] Fill this nucleus with 2 protons and 1 neutron. What is the parity of the resulting nucleus? This again has even parity, because both proton and neutrons are even parity.

[Q7] Fill out the following table. See Table 2 on the following page.

Table 2: Particle Properties

Particle	Symbol	Class/spin	Lepton #	Baryon #	Charge	Mass	Color	Isospin	Lifetime
up quark	u			0	$+2/3$	2.3 MeV	yes	$1/2$	
down quark	d			0	$-2/3$	4.8 MeV	yes	$1/2$	
gluon	g			0	0	0	octet	1	
electron	e^-		+1	0	-1	0.511 MeV		$1/2$	stable
muon	μ^-		+1	0	-1	105.7 MeV	none	$1/2$	2.20×10^{-6} s
neutrino	ν_e, ν_μ, ν_τ		+1	0	0	0.320 eV		$1/2$	stable
photon	γ		0	0	0	0		1	
w-boson	$W^\pm$			0	± 1	80.39 GeV		1	
z-boson	Z			0	0	91.19 GeV		1	
proton	p			+1	+1	938.27 MeV		$1/2$	stable
neutron	n			+1	0	939.57 MeV		$1/2$	614 s
pion	π			0	0, ± 1	134.98 MeV, 139.57 MeV		0	2.20×10^{-8} s

References

[1] M. R. Bhat. Evaluated Nuclear Data Structure File (ENSDF), 1992.

[2] Kenneth S. Krane. *Introductory Nuclear Physics*. John Wiley & Sons, Inc., Hoboken, NJ, 2nd edition, 1988.