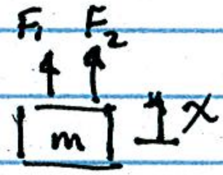
ME 513 Fall 2015 solution PS#2

1.2.1

(a)



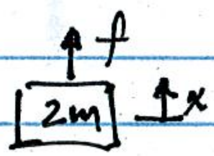
$F_1 = F_2 = -sx$

$$\therefore \quad m\frac{d^2x}{dt^2} = -2sx$$

$$\frac{d^2x}{dt^2} + \frac{2s}{m}x = 0 \quad \rightarrow \quad \omega_0 = \sqrt{\frac{2s}{m}} \ [\text{rad/s}]$$

In this and all cases $f_0 = \frac{\omega_0}{2\pi}$ [Hz]

(b)

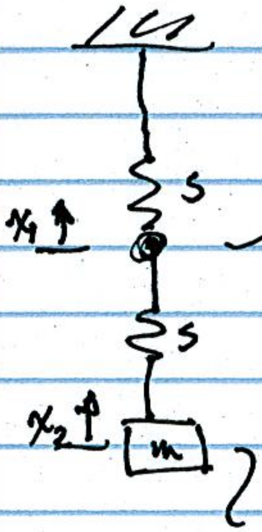


$f = -sx$

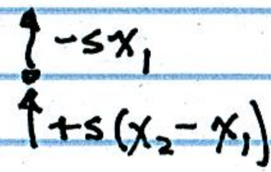
$$\therefore \quad 2m\frac{d^2x}{dt^2} = -sx$$

$$\frac{d^2x}{dt^2} + \frac{s}{2m}x = 0 \quad \rightarrow \quad \omega_0 = \sqrt{\frac{s}{2m}}$$

(c)



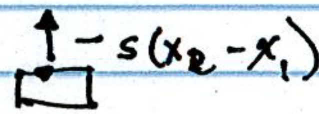
FBD ①



$$\sum f_y = 0$$

$$s(x_2-x_1) - sx_1 = 0 \longrightarrow x_1 = \frac{x_2}{2}$$

FBD ②

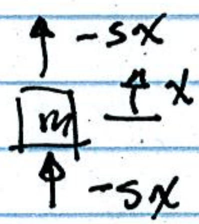


$$\therefore\ m\frac{d^2x_2}{dt^2} = -s(x_2-x_1)$$

$$= -s\frac{x_2}{2}$$

$$\frac{d^2x}{dt^2} + \frac{s}{2m}x = 0 \longrightarrow \omega_0 = \sqrt{\frac{s}{2m}}$$

(d)



$$m\frac{d^2x}{dt^2} + 2sx = 0$$

$$\frac{d^2x}{dt^2} + \frac{2s}{m}x = 0$$

$$\omega_0 = \sqrt{\frac{2s}{m}}$$

1.3.2 $\omega_0 = 5$ rad/s $x_0 = 0.03$ m

$u_0 = 0$

simple oscillator

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0$$

General Solution

$$x(t) = A_1 \cos\omega_0 t + A_2 \sin\omega_0 t$$

$$x(0) = A_1 = 0.03$$

$$\dot{x}(t) = -\omega_0 A_1 \sin\omega_0 t + \omega_0 A_2 \cos\omega_0 t$$

$$\ddot{x}(t) = -\omega_0^2 A_1 \cos\omega_0 t - \omega_0^2 A_2 \sin\omega_0 t$$

(a) $\rightarrow$ $\ddot{x}(0) = -\omega_0^2 A_1 = -25(0.03) = \cancel{\theta} - 0.75 \text{ m/s}^2$

$$\dot{x}(0) = +\omega_0 A_2 = 0 \rightarrow A_2 = 0$$

so $x(t) = A_1 \cos\omega_0 t = 0.03 \cos\omega_0 t$

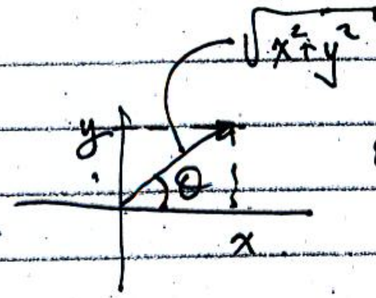
(b) Amplitude $= 0.03$ m

velocity $\dot{x}(t) = -\omega_0 A_1 \sin\omega_0 t$

(c) so max speed $= \omega_0 A_1 = 0.15$ m/s

1.5.2

(a) $\sqrt{x+jy}$



$\theta = \tan^{-1}\frac{y}{x}$

So $\sqrt{x+jy} = \sqrt[4]{x^2+y^2}\, e^{j\theta/2}$

$Re\{\sqrt{x+jy}\} = \sqrt[4]{x^2+y^2}\cos\theta/_2$ where $\theta = \tan^{-1}\frac{y}{x}$

Magnitude $= \sqrt[4]{x^2+y^2}$

Phase $= \theta/2$

(b) $A e^{j(\omega t+\phi)}$

where A is real

$Re\{A e^{j(\omega t+\phi)}\} = A\cos(\omega t+\phi)$

Magnitude $= A$

Phase $= \omega t + \phi$

(c) $\left(1 + e^{-2j\theta}\right) e^{j\theta}$

$= e^{j\theta} + e^{-j\theta}$

$= 2\cos\theta$

$\therefore$ Real part $= 2\cos\theta$

Magnitude $= 2$

Phase $= 0$ (since result is real)

1.6.1 For a damped oscillator

$$x = Ae^{-\beta t}\cos(\omega_d t + \phi)$$

where $\omega_d = \sqrt{\frac{s}{m} - \frac{R_m^2}{4m^2}}$ $\quad \beta = \frac{R_m}{2m}$

Spring stretches 0.04m when a mass of 0.2 kg is attached

$$\Rightarrow s = \frac{0.2 \times 9.8}{0.04} = 49 \text{ N/m}$$

at $t=1$ $\quad \frac{Ae^{-\beta}}{A} = \frac{1}{e} \rightarrow \beta = 1$

$$\therefore R_m = 2m\beta = 1 \quad \text{Ns/m}$$

$$\therefore \omega_d = \sqrt{\frac{s}{m} - \frac{R_m^2}{4m^2}} = 9.85 \text{ rad/s}$$

The initial conditions are:

$t=0 \quad u = 0$

$t=0 \quad x = 0.04\,m$

$$u = \frac{dx}{dt} = -A\beta e^{-\beta t}\cos(\omega_d t + \phi) - A\omega_d e^{-\beta t}\sin(\omega_d t + \phi)$$

$$\therefore u|_{t=0} = -\cancel{A}\cos\phi - \cancel{A}\omega_d \sin\phi = 0$$

$$\cos\phi + \omega_d \sin\phi = 0$$

$$\tan\phi = -\frac{1}{\omega_d}$$

$$\rightarrow \phi = -5.8^\circ$$

$$x\Big|_{t=0} = A\cos\phi = 0.04$$

$$\rightarrow A = 0.0402 \text{ m}$$