Nuclear Physics
Homework 4

[1] Use the extreme independent particle model to predict the ground state, spin and parity, J^p for each of the following nuclei;

$$^{17}F \quad ^{18}F \quad ^{19}F \quad ^{33}S \quad ^{55}Mn \quad ^{91}Zr$$

Compare your prediction to the know value. If they don't agree, look at the level structure in the NNDC. Is there a low lying excited state with the "expected" value? It is found that the level spacing is sensitive to the size of the nucleus. Supplement your original estimate for the ground state Jp with the shift in the energy levels as a function of A using the enclosed figure. Thus your answer should appear as a table with 3 coulombs, extreme independent model, measure value, corrected extreme independent model.

[2] Using simply the extreme independent particle model and the neutron and proton magnetic moments, attempt to predict the following nuclear magnetic moments and compare to the measured values.

Table 3.2 Sample Values of Nuclear Magnetic Dipole Moments

Nuclide	$\mu(\mu_N)$
n	−1.9130418
p	+2.7928456
^{2}H (D)	+0.8574376
^{17}O	−1.89379
^{57}Fe	+0.09062293
^{57}Co	+4.733
^{93}Nb	+6.1705

All values refer to the nuclear ground states; uncertainties are typically a few parts in the last digit. For a complete tabulation, see V. S. Shirley, in *Table of Isotopes* (Wiley: New York, 1978), Appendix VII.

[3] When $j_z = j\hbar$ the magnetic moment operator in the state with maxiumum z projection of angular momentum is

$$\mu = \frac{\mu_N}{\hbar}(g_l l_z + g_s s_z)$$

Compute the expectations of the magnetic momentum for a nucleus by computing the expectation value of $< s_z >$, to find

$$j = l + \frac{1}{2} \quad <\mu> = [g_l(j - \frac{1}{2}) - \frac{g_s}{2}]\mu_N$$

$$j = l - \frac{1}{2} \quad <\mu> = [g_l \frac{j(j + \frac{2}{3})}{(j-1)} - \frac{g_s}{2}\frac{1}{j+1}]\mu_N$$

[4] Using the "Unified Nuclear Model" for hafnium ${}^{177}_{72}Hf_{105}$ answer the following;

(a) Assuming that hafnium-177 is a prolate nucleus with β equal to between +0.2 and +0.3 what is the expected ground state and explain why?

(b) Explain why there is an excited state of Hf-177 at $J^\pi = \frac{9^+}{2}$ state?

(c) Why is the second excited state of Hf-177 a $J^\pi = \frac{11^-}{2}$ state? Is there a sequence of states related to this excited states and why?

(d) What is the 2nd excited state J^π predicted by shell model?

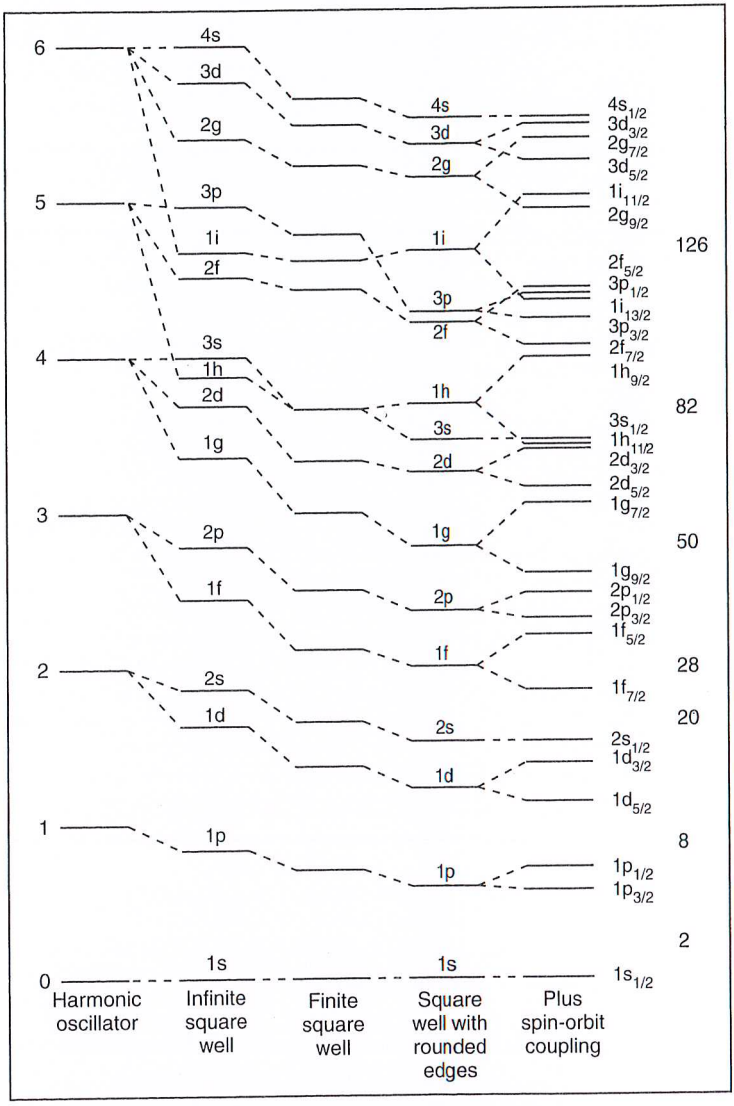
[5] Use the enclosed Clebsch-Gordan Coefficients Table to argue that when adding $2 \oplus 2 \oplus 2$ for the combination of 3 spin 2 nuclear phonons, that the allowed states are;

$$0^+ \quad 2^+ \quad 3^+ \quad 4^+ \quad 6^+$$

[6] Make a complete list of the distinct types of nuclear excitations.

Figure 5.11 | Nuclear Energy Levels for an Infinite Square Well, a Finite Square Well, a Square Well with Rounded Edges, and a Square Well with Rounded Edges Including Spin Orbit Coupling

The numbers on the right hand side of the figure show the cumulative occupancy for the final case. The harmonic oscillator energy levels are shown for comparison.



From B. T. Feld, *Ann. Rev. Nucl. Sci.* 2 (1953), 239. Reprinted with permission from the *Annual Review of Nuclear Science*, Volume 2 © 1953 by Annual Reviews, www.annualreviews.org.

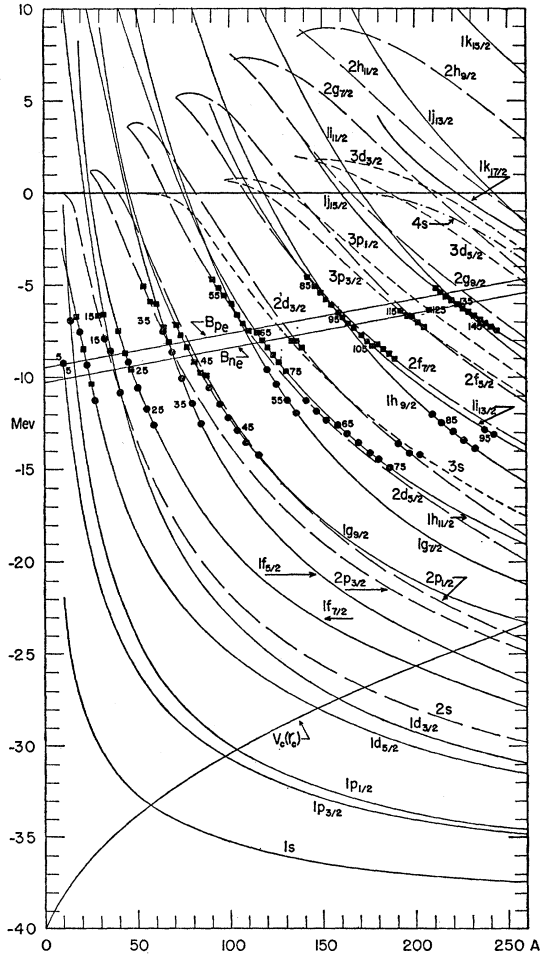


FIG. 6. Nuclear energy levels. The locations of last neutrons in beta-stable nuclei are indicated by squares. The location of the last protons, if no extra force acted upon them, are indicated by circles. The curve labeled $v_c(r_c)$ indicates the major correction associated with classical Coulomb perturbation.

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

J	J	...
M	M	...
m_1	m_2	
m_1	m_2	Coefficients
.	.	
.	.	
.	.	

1/2×1/2

	+1	
+1/2 +1/2	1	1 0 / 0 0
+1/2 −1/2	1/2 1/2	1
−1/2 +1/2	1/2 −1/2	−1
−1/2 −1/2		1

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

2×1/2

	+5/2	
+2 +1/2	1	5/2 3/2 / +3/2 +3/2
+2 −1/2	1/5 4/5	5/2 3/2
+1 +1/2	4/5 −1/5	+1/2 +1/2
+1 −1/2	2/5 3/5	5/2 3/2
0 +1/2	3/5 −2/5	−1/2 −1/2
0 −1/2	3/5 2/5	5/2 3/2
−1 +1/2	2/5 −3/5	−3/2 −3/2
−1 −1/2	4/5 1/5	5/2
−2 +1/2	1/5 −4/5	−5/2
−2 −1/2		1

1×1/2

	+3/2	
+1 +1/2	1	3/2 1/2 / +1/2 +1/2
+1 −1/2	1/3 2/3	3/2 1/2
0 +1/2	2/3 −1/3	−1/2 −1/2
0 −1/2	2/3 1/3	3/2
−1 +1/2	1/3 −2/3	−3/2
−1 −1/2		1

3/2×1/2

	+2		
+3/2 +1/2	1	+1 +1	2 1 / 0 0
+3/2 −1/2	1/4 3/4	2 1	
+1/2 +1/2	3/4 −1/4	0 0	
+1/2 −1/2	1/2 1/2	2 1	
−1/2 +1/2	1/2 −1/2	−1 −1	
−1/2 −1/2	3/4 1/4	2	
−3/2 +1/2	1/4 −3/4	−2	
−3/2 −1/2		1	

2×1

	+3		
+2 +1	1	+2 +2	
+2 0	1/3 2/3	3 2 1	
+1 +1	2/3 −1/3	+1 +1 +1	
+2 −1	1/15 1/3 3/5		
+1 0	8/15 1/6 −3/10	3 2 1	
0 +1	2/5 −1/2 1/10	0 0 0	
+1 −1	1/5 1/2 3/10		
0 0	3/5 0 −2/5	3 2 1	
−1 +1	1/5 −1/2 3/10	−1 −1 −1	
0 −1	2/5 1/2 1/10		
−1 0	8/15 −1/6 −3/10	3 2	
−2 +1	1/15 −1/3 3/5	−2 −2	
−1 −1	2/3 1/3	3	
−2 0	1/3 −2/3	−3	
−2 −1		1	

3/2×1

	+1		
+3/2 +1	1	+3/2 +3/2	
+3/2 0	2/5 3/5	5/2 3/2 1/2	
+1/2 +1	3/5 −2/5	+1/2 +1/2 +1/2	
+3/2 −1	1/10 2/5 1/2		
+1/2 0	3/5 1/15 −1/3	5/2 3/2 1/2	
−1/2 +1	3/10 −8/15 1/6	−1/2 −1/2 −1/2	
+1/2 −1	3/10 8/15 1/6		
−1/2 0	3/5 −1/15 −1/3	5/2 3/2	
−3/2 +1	1/10 −2/5 1/2	−3/2 −3/2	
−1/2 −1	3/5 2/5	5/2	
−3/2 0	2/5 −3/5	−5/2	
−3/2 −1		1	

1×1

	+2		
+1 +1	1	+1 +1	
+1 0	1/2 1/2	2 1 0	
0 +1	1/2 −1/2	0 0 0	
+1 −1	1/6 1/2 1/3		
0 0	2/3 0 −1/3	2 1	
−1 +1	1/6 −1/2 1/3	−1 −1	
0 −1	1/2 1/2	2	
−1 0	1/2 −1/2	−2	
−1 −1		1	

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 JM\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 JM\rangle$$

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

3/2×3/2

	+3		
+3/2 +3/2	1	3 2 / +2 +2	
+3/2 +1/2	1/2 1/2	3 2 1	
+1/2 +3/2	1/2 −1/2	+1 +1 +1	
+3/2 −1/2	1/5 1/2 3/10		
+1/2 +1/2	3/5 0 −2/5	3 2 1 0	
−1/2 +3/2	1/5 −1/2 3/10	0 0 0 0	
+3/2 −3/2	1/20 1/4 9/20 1/4		
+1/2 −1/2	9/20 1/4 −1/20 −1/4		
−1/2 +1/2	9/20 −1/4 −1/20 1/4	3 2 1	
−3/2 +3/2	1/20 −1/4 9/20 −1/4	−1 −1 −1	
+1/2 −3/2	1/5 1/2 3/10		
−1/2 −1/2	3/5 0 −2/5	3 2	
−3/2 +1/2	1/5 −1/2 3/10	−2 −2	
−1/2 −3/2	1/2 1/2	3	
−3/2 −1/2	1/2 −1/2	−3	
−3/2 −3/2		1	

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

	+7/2		
+2 +3/2	1	+7/2 +5/2 / +5/2 +5/2	
+2 +1/2	3/7 4/7	7/2 5/2 3/2	
+1 +3/2	4/7 −3/7	+3/2 +3/2 +3/2	
+2 −1/2	1/7 16/35 2/5		
+1 +1/2	4/7 1/35 −2/5	7/2 5/2 3/2 1/2	
0 +3/2	2/7 −18/35 1/5	+1/2 +1/2 +1/2 +1/2	
+2 −3/2	1/35 6/35 2/5 2/5		
+1 −1/2	12/35 5/14 0 −3/10		
0 +1/2	18/35 −3/35 −1/5 1/5		
−1 +3/2	4/35 −27/70 2/5 −1/10		
+1 −3/2	4/35 27/70 2/5 1/10		
0 −1/2	18/35 3/35 −1/5 −1/5		
−1 +1/2	12/35 −5/14 0 3/10	7/2 5/2 3/2	
−2 +3/2	1/35 −6/35 2/5 −2/5	−3/2 −3/2 −3/2	
0 −3/2	2/7 18/35 1/5		
−1 −1/2	4/7 −1/35 −2/5	7/2 5/2	
−2 +1/2	1/7 −16/35 2/5	−5/2 −5/2	
−1 −3/2	4/7 3/7 7/2		
−2 −1/2	3/7 −4/7 −7/2		
−2 −3/2		1	

2×2

	+4		
+2 +2	1	4 3 / +3 +3	
+2 +1	1/2 1/2	4 3 2	
+1 +2	1/2 −1/2	+2 +2 +2	
+2 0	3/14 1/2 2/7		
+1 +1	4/7 0 −3/7		
0 +2	3/14 −1/2 2/7		
+2 −1	1/14 3/10 3/7 1/5		
+1 0	3/7 1/5 −1/14 −3/10		
0 +1	3/7 −1/5 −1/14 3/10		
−1 +2	1/14 −3/10 3/7 −1/5		
+2 −2	1/70 1/10 2/7 2/5 1/5		
+1 −1	8/35 2/5 1/14 −1/10 −1/5		
0 0	18/35 0 −2/7 0 1/5		
−1 +1	8/35 −2/5 1/14 1/10 −1/5		
−2 +2	1/70 −1/10 2/7 −2/5 1/5		
+1 −2	1/14 3/10 3/7 1/5		
0 −1	3/7 1/5 −1/14 −3/10		
−1 0	3/7 −1/5 −1/14 3/10		
−2 +1	1/14 −3/10 3/7 −1/5		
0 −2	3/14 1/2 2/7		
−1 −1	4/7 0 −3/7		
−2 0	3/14 −1/2 2/7		
−1 −2	1/2 1/2	4	
−2 −1	1/2 −1/2	−4	
−2 −2		1	

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,2} = \left(\frac{1+\cos\theta}{2}\right)^2$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

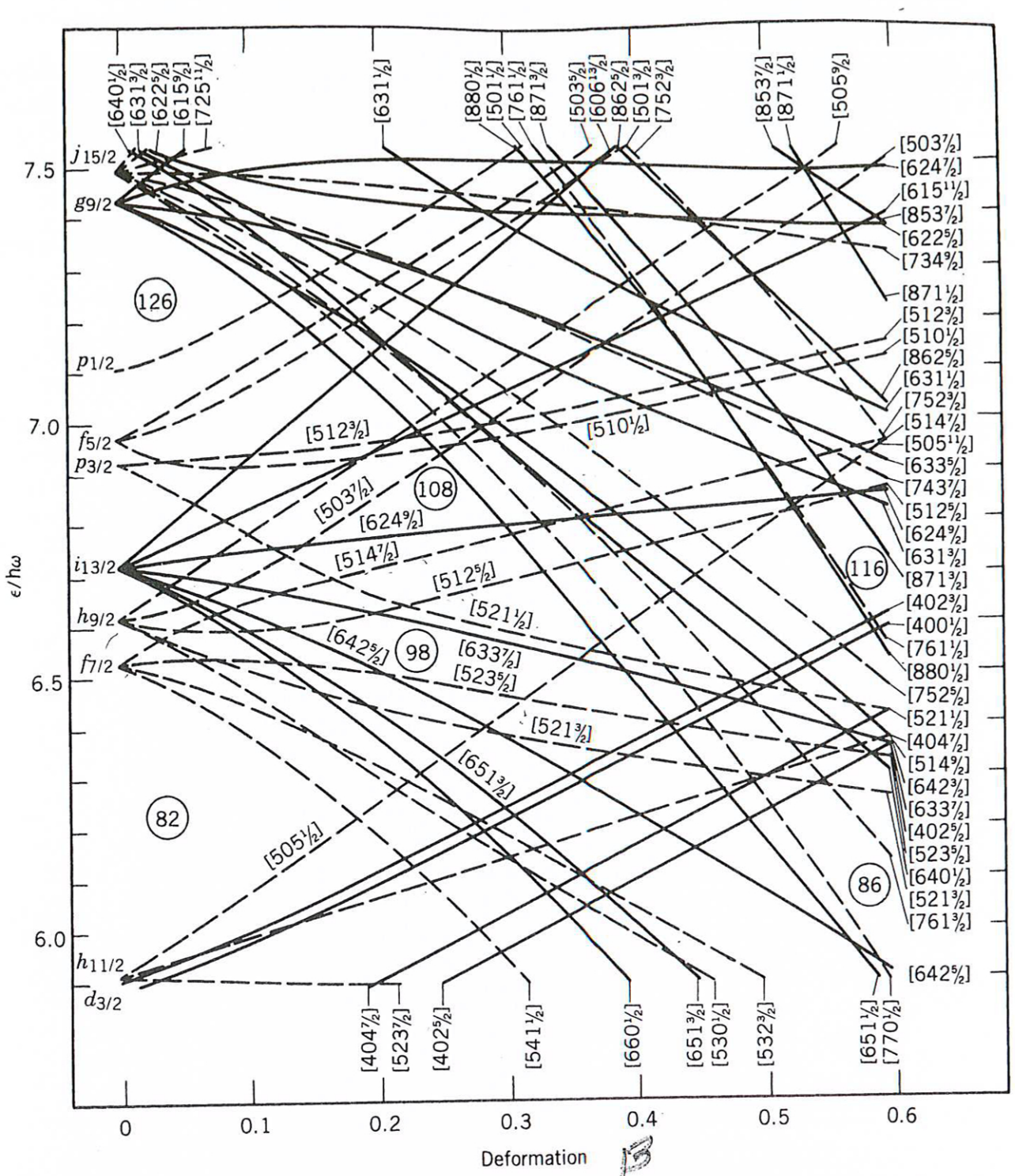
$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta - 1)$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta - 1}{2}\cos\frac{\theta}{2} \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta + 1)$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \left(\frac{1-\cos\theta}{2}\right)^2 \qquad d^2_{0,0} = \left(\frac{3}{2}\cos^2\theta - \frac{1}{2}\right)$$

Figure 35.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients have been [illegible] calculated using computer programs written independently by Cohen and at LRNL.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g9/2
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