ME 513 HMWK 5

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12/4/15

7.1.4 A simple source of sound in air radiates an acoustic power of $10\,\mathrm{mW}$ **at** $400\,\mathrm{Hz}$**. At** $0.5\,\mathrm{m}$ **from the source, compute**

(a) the intensity, The intensity is given by equation (7.2.15)

$$I = \frac{1}{8}\rho_0 c \left({}^{Q}\!/\!{}_{\lambda r}\right)^2$$

We can find Q from the equation for acoustic power (7.2.16)

$$\begin{aligned}\Pi &= \frac{1}{2}\pi\rho_0 c \left({}^{Q}\!/\!{}_{\lambda}\right)^2 \\ &= \frac{1}{2}\pi\rho_0 c \frac{Q^2}{{}^{c^2}\!/\!{}_{f^2}} \\ Q &= \sqrt{\frac{2\Pi c}{\pi\rho_0 f^2}}\end{aligned}$$

which gives

$$Q = \sqrt{\frac{2\left(1.\times 10^{-2}\,\mathrm{W}\right)\left(343\,{}^{m}\!/\!{}_{\mathrm{s}}\right)}{\pi\left(1.275\,{}^{kg}\!/\!{}_{\mathrm{m}^3}\right)\left(400\,\mathrm{Hz}\right)^2}} = 3.27\times 10^{-3}\,{}^{m^3}\!/\!{}_{\mathrm{s}}$$

and so, the intensity is

$$I = \frac{1}{8}\left(1.275\,{}^{kg}\!/\!{}_{\mathrm{m}^3}\right)\left(343\,{}^{m}\!/\!{}_{\mathrm{s}}\right)\left[\frac{\left(3.27\times 10^{-3}\,{}^{m^3}\!/\!{}_{\mathrm{s}}\right)\left(400\,\mathrm{Hz}\right)}{\left(343\,{}^{m}\!/\!{}_{\mathrm{s}}\right)\left(0.5\,\mathrm{m}\right)}\right]^2 = 3.16\,{}^{mW}\!/\!{}_{\mathrm{m}^2}$$

(b) the pressure amplitude, The pressure amplitude is given by equation 7.2.14

$$P = \frac{1}{2}\rho_0 c \left({}^{Q}\!/\!{}_{\lambda r}\right) = \frac{1}{2}\left(1.275\,{}^{kg}\!/\!{}_{\mathrm{m}^3}\right)\left(343\,{}^{m}\!/\!{}_{\mathrm{s}}\right)\frac{\left(3.27\times 10^{-3}\,{}^{m^3}\!/\!{}_{\mathrm{s}}\right)\left(400\,\mathrm{Hz}\right)}{\left(343\,{}^{m}\!/\!{}_{\mathrm{s}}\right)\left(0.5\,\mathrm{m}\right)} = 1.663\,\mathrm{Pa}$$

(c) the particle speed amplitude, Using linear acoustics, we have that

$$I = \frac{1}{2}PU$$

and therefore

$$U = \frac{2I}{P} = \frac{2\cdot 3.16\,{}^{mW}\!/\!{}_{\mathrm{m}^2}}{1.663\,\mathrm{Pa}} = 3.8\,{}^{mm}\!/\!{}_{\mathrm{s}}$$

(d) the particle displacement amplitude, and Finally, the displacement amplitude is another time integral from the particle speed, so

$$\Xi = \frac{U}{f} = \frac{3.8\,{}^{mm}\!/\!{}_{\mathrm{s}}}{400\,\mathrm{Hz}} = 9.5\,\mu\mathrm{m}$$

(e) the condensation amplitude. The condensation is define as the pressure divided by the density times the speed of sound squared

$$S = \frac{P}{\rho_0 c^2} = \frac{1.663\,\mathrm{Pa}}{\left(1.275\,{}^{kg}/_{\mathrm{m}^3}\right)\left(343\,{}^{m}/_{\mathrm{s}}\right)^2} = 1.1 \times 10^{-5}$$

7.4.1 For a baffled piston of radius a driven at angular frequency ω,

(a) find the smallest angle θ_1 for which the pressure is zero in the far field, To find the angle θ_1 desired, we use the smallest node given by

$$ka \sin\theta_m = j_{1m} \qquad m = 1, 2, 3, \ldots$$

so

$$\begin{aligned} ka \sin\theta_1 &= j_{11} \\ \sin\theta_1 &= \frac{j_{11}}{ka} \\ \theta_1 &= \arcsin\left(\frac{j_{11}}{ka}\right) = \arcsin\left(\frac{j_{11}c}{\omega a}\right) \end{aligned}$$

(b) find the greatest finite distance for which the pressure is zero on the acoustic axis, and From the equation (7.4.4), we have

$$\tilde{p}\left(r, 0, t\right) = \rho_0 c u_0 \left\{1 - \exp\left[-jk\left(\sqrt{r^2 + a^2} - r\right)\right]\right\} \exp\left[j\left(\omega t - kr\right)\right]$$

which has the magnitude

$$p\left(r, 0\right) = 2\rho_0 c u_0 \left|\sin\left\{\frac{1}{2}kr\left[\sqrt{1 + \left({}^{a}/_{r}\right)^2} - 1\right]\right\}\right|$$

and the extrema of this is at

$$\frac{1}{2}kr\left[\sqrt{1 + \left({}^{a}/_{r}\right)^2} - 1\right] = \frac{m\pi}{2} \qquad m = 0, 1, 2, \ldots$$

therefore, moving down from large r, we can find the first minima, r_2 at

$$\begin{aligned} \frac{r_2}{a} &= \frac{a}{2\lambda} - \frac{\lambda}{2a} \\ r_2 &= \frac{a^2}{2\lambda} - \frac{\lambda}{2} \end{aligned}$$

and with

$$k = \frac{\omega}{c}$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi c}{\omega}$$

so

$$r_2 = \frac{a^2\omega}{4\pi c} - \frac{\pi c}{\omega}$$

(c) discuss the possibility of obtaining $\theta_1 \ll 1$ and ${}^{r_1}/_{a} \ll 1$ simultaneously. At very large ${}^{r_1}/_{a}$, we approach a monopole, and as such, $\theta_1 \to \frac{\pi}{2}$. This means that $\theta_1 \ll 1$ only when ${}^{r_1}/_{a} \gg 1$ (or when we're in the very dipole like region).

7.4.6C

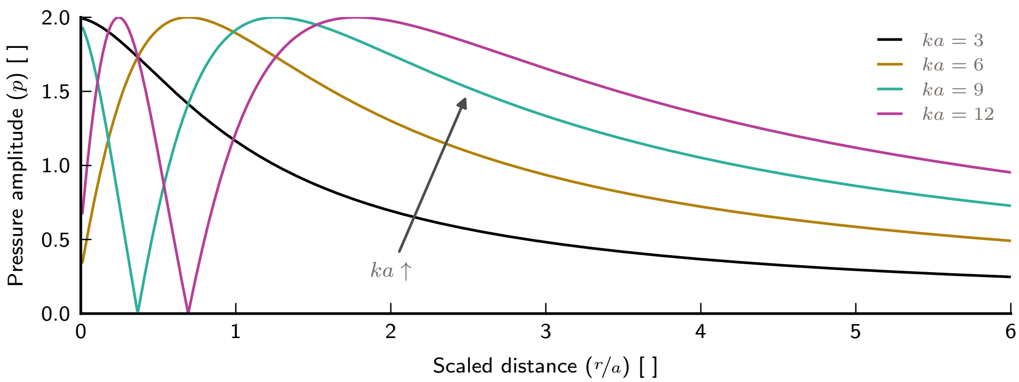

Figure 1: Plot of pressure amplitude with scaled distance for different values of ka

(a) For a ciruclar piston, plot the on-axis pressure amplitude as a function of scaled distance r/a for several values of ka between 3 and 12. Using equation (7.4.5), we have

$$p\,(r,0) = 2\rho_0 c u_0 \left|\sin\left\{\frac{1}{2}kr\left[\sqrt{1+(a/r)^2}-1\right]\right\}\right|$$

and we can convert this into the quantity ka as

$$p\,(r,0) = 2\rho_0 c u_0 \left|\sin\left\{\frac{1}{2}\frac{ka}{a}r\left[\sqrt{1+(a/r)^2}-1\right]\right\}\right|$$

$$p\,(r/a,0) = 2\rho_0 c u_0 \left|\sin\left\{\frac{1}{2}ka\,(r/a)\left[\sqrt{1+(r/a)^{-2}}-1\right]\right\}\right|$$

The solutions to this with $ka = 3, 4, 5, \ldots, 12$ are shown in figure 1.

(b) Plot the range beyond which the pressure amplitude is within 10% of the asymptotic form (7.4.7). With the asymptotic form of

$$p\,(r) = \frac{1}{2}\rho_0 c u_0\,(r/a)^{-1}\,ka$$

we can find where that reaches 90% of the true form by solving the equation

$$2\cancel{\rho_0 c u_0}\left|\sin\left\{\frac{1}{2}ka\,(r/a)\left[\sqrt{1+(r/a)^{-2}}-1\right]\right\}\right| = 0.9\cdot\frac{1}{2}\cancel{\rho_0 c u_0}\,(r/a)^{-1}\,ka$$

$$4.44 = \frac{(r/a)^{-1}\,ka}{\left|\sin\left\{\frac{1}{2}ka\,(r/a)\left[\sqrt{1+(r/a)^{-2}}-1\right]\right\}\right|}$$

which, solved numerically, occurs at $r/a = 4.26$ for a $ka = 12$. Figure 2 shows this region on a plot of scaled distance.

(c) For a piston of 20 cm radius operating at 4 kHz in water, find the distance corresponding to (b). First, we must determine the r/a for asymptotic behvior with the given ka.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{2\pi\cdot 4\times 10^3\,\mathrm{Hz}}{343.2\,\frac{\mathrm{m}}{\mathrm{s}}}$$

so,

$$ka = 366.153$$

and solving numerically, the asymptotic region starts at

$$r/a = 122.23$$

which is

$$r = 24.4\,\mathrm{m}$$

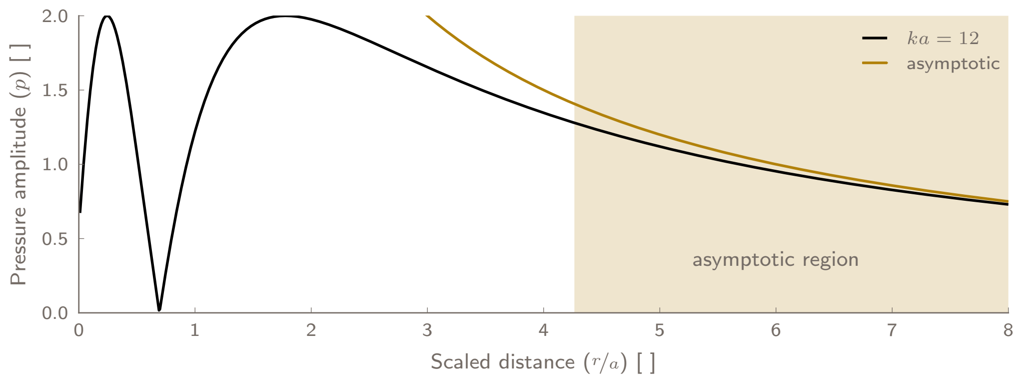

Figure 2: Asymptotic region highlight on plot of pressure amplitude with scaled distance for a single value of ka

7.5.1

(a) Find the resonance frequency of a piston transducer with the mechanical properties m, s, and R_m radiating into a fluid with specific acoustic impedance $\rho_0 c$. Assume $ka \gg 2$. Using equation (7.5.11), the impedance is

$$z_r = \rho_0 c S \left[R_1(2ka) + jX_1(2ka)\right]$$

and well above $ka = 2$, we have the approximations

$$\begin{aligned} kX_1(2ka) &\to \frac{2/\pi}{ka} = \frac{2}{\pi ka} \\ R_1(2ka) &\to 1 \\ z_r &\to R_r \approx S\rho_0 c \end{aligned}$$

therefore

$$z_r = \rho_0 c S \left[1 + j\frac{2}{\pi ka}\right]$$

and comparing that to the form of the mechanical impedance

$$z_m = R_m + j\left(\omega m - \frac{s}{\omega}\right)$$

we can split the radiation impedance into parts

$$\begin{aligned} z_m + z_r &= (R_m + \rho_0 c S) + j\left(\frac{2\omega m}{\pi ka} - \frac{2s}{\pi\omega ka}\right) \\ &= (R_m + \rho_0 c S) + j\left(\frac{2cm}{\pi a} - \frac{2cs}{\pi\omega^2 a}\right) \end{aligned}$$

and the resonance frequency is where

$$\operatorname{Im}\{z_m + z_r\} = 0$$

which is

$$\begin{aligned} \frac{2\omega m}{\pi ka} - \frac{2s}{\pi\omega ka} &= 0 \\ \frac{2\omega m}{\pi ka} &= \frac{2s}{\pi\omega ka} \\ \omega^2 &= \frac{s}{m} \\ \omega &= \sqrt{\frac{s}{m}} \end{aligned}$$

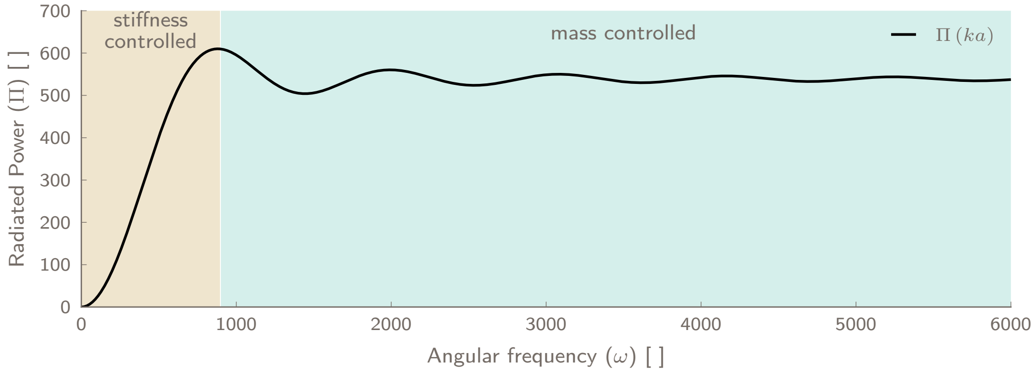

Figure 3: Power radiated as a function of frequency with mass and stiffness controlled regions labeled

(b) Sketch the frequency dependence of the radiatated power if the transducer is driven with a force of constant amplitude. Assume that the resonance frequency occurs well above the lower limit of the approximations implicit in $ka \gg 2$. Indicate where the transducer is mass controlled and where it is stiffness controlled. The radiated power is given by

$$\pi = \frac{1}{2} u_0^2 R_r$$

and assuming along the axis, we have

$$\begin{aligned}\Pi &= \frac{1}{2} u_0^2 R_r \\ &= \frac{1}{2} u_0^2 \rho_0 c S \cdot \mathrm{Re}\left\{\left[1 - \frac{2\mathrm{J}_1\left(2ka\right)}{2ka}\right] + j\left[\frac{2\mathrm{H}_1\left(2ka\right)}{2ka}\right]\right\}\end{aligned}$$

and this is shown in figure 3. The mass controlled region occurs when

$$z_m + z_r \approx j\frac{2cm}{\pi a}$$

or when the frequency is high (on the plateau of the chart). The stiffness controlled region occurs when

$$z_m + z_r \approx -j\frac{2cs}{\pi\omega^2 a}$$

or in the increase part of the chart.