ME 513 HMWK 4

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6.2.2 If a plane wave is reflected from the ocean floor at normal incidence with a level 20 dB below that of the incident wave, what are the possible values of the specific acoustic impedance of the fluid bottom material? The level of 20 dB below gives us

$$20\log_{10}\frac{|p_r|}{|p_i|} = -20$$

Therefore

$$\frac{|p_r|}{|p_i|} = |R| = 0.1 \quad \therefore \quad R = \pm 0.1$$

and determining the reflection coefficient, we have

$$R = \frac{{}^{r_2}/_{r_1} - 1}{{}^{r_2}/_{r_1} + 1} \quad \therefore \quad |R| = \left|\frac{{}^{r_2}/_{r_1} - 1}{{}^{r_2}/_{r_1} + 1}\right|$$

and solving this for ${}^{r_2}/_{r_1}$

$$\frac{r_2}{r_1} = \frac{1-|R|}{1+|R|} = 0.818,\ 1.222$$

To determine specific acoustic impedance, we then must find

$$z = \frac{p}{u} = \rho_2 c_2$$

and using the ratio ${}^{r_2}/_{r_1}$, we can find this

$$\frac{r_2}{r_1} = \frac{\rho_2 c_2}{\rho_1 c_1} = 0.818,\ 1.222$$

and therefore

$$z_n = \rho_2 c_2 = \rho_1 c_1 \frac{r_2}{r_1}$$

and for water

$$z_n = \left(1.5\times10^6\,\text{rayls}\right)\frac{r_2}{r_2} = 1.227\times10^6,\ 1.833\times10^6$$

6.2.3

(a) A plane wave in seawater is normally incident on the water-air interface. Find the pressure and intensity transmission coefficients. Using the definitions

$$R = \frac{{}^{r_2}/_{r_1} - 1}{{}^{r_2}/_{r_1} + 1}$$

$$T = \frac{2\,{}^{r_2}/_{r_1}}{{}^{r_2}/_{r_1} + 1}$$

and

$$R_I = R^2$$

$$T_I = \frac{4\,{}^{r_2}/_{r_1}}{\left({}^{r_2}/_{r_1} + 1\right)^2}$$

where

$$\frac{r_2}{r_1} = \frac{\rho_2 c_2}{\rho_1 c_1} = \frac{415}{1.5 \times 10^6} = 2.77 \times 10^{-4}$$

where r_2 refers to the air, and r_1 refers to the seawater. This gives

$$R = -0.999$$

$$T = 5.54 \times 10^{-4}$$

$$R_I = 0.998$$

$$T_I = 1.1 \times 10^{-3}$$

which shows that a wave in water is practically perfectly reflected at its surface with air.

(b) Repeat (a) for a wave in air normally incident on the air-water interface. Using the reciprocal of the previous analysis, we have

$$\frac{r_2}{r_1} = \frac{\rho_2 c_2}{\rho_1 c_1} = \frac{1.5 \times 10^6}{415} = 3610.1$$

where r_2 refers to the seawater, and r_1 refers to the air. This gives

$$R = 0.999$$

$$T = 1.999$$

$$R_I = 0.998$$

$$T_I = 1.1 \times 10^{-3}$$

This shows that the power transmission and reflection is the same as in the converse case, but the pressure field is different, with the reflection being in phase and the transmission being double.

6.2.6C Plot the pressure reflection and transmission coefficients and the internsity reflection and transmission coefficients for normal incidents of a plane wave on a fluid-fluid boundary for $0 < {}^{r_1}\!/_{r_2} <$ 10. Comment on the results for ${}^{r_1}\!/_{r_2} = 0$, ${}^{r_1}\!/_{r_2} = 1$, and ${}^{r_1}\!/_{r_2} \to \infty$. Using the definitions for the transmission and reflection ratios, we have

$$R = \frac{{}^{r_2}\!/_{r_1} - 1}{{}^{r_2}\!/_{r_1} + 1}$$

$$T = \frac{2{}^{r_2}\!/_{r_1}}{{}^{r_2}\!/_{r_1} + 1}$$

and

$$R_I = R^2$$

$$T_I = \frac{4{}^{r_2}\!/_{r_1}}{\left({}^{r_2}\!/_{r_1} + 1\right)^2}$$

these are plotted through a range of values in figure 1.

The cases requested are:

${}^{r_1}\!/_{r_2} = 0$ With ${}^{r_1}\!/_{r_2} = 0$, ${}^{r_2}\!/_{r_1} = \infty$, which gives $R = 1$, $T = 2$, $R_I = 1$, and $T_I = 0$, which gives a nonphysical case, in which the power is fully reflected, but the pressure on the final side of the boundary is twice that of the initial side.

${}^{r_1}\!/_{r_2} = 1$ This case creates the solution that $R = R_I = 0$ and $T = T_I = 1$, which shows that there is no energy loss in the (full) transmission. The case that this illustrates is when the medium ($\rho_n c_n$) is the same for both the incident medium and the final medium, which emulates no boundary case.

${}^{r_1}\!/_{r_2} \to \infty$ At ${}^{r_1}\!/_{r_2} = \infty$, ${}^{r_2}\!/_{r_1} = 0$. This case pushes $R = -1$, $R_I = 1$, $T = 0$, and $T_I = 0$, which reflects all of the sound energy back into the initial medium, though at a 180^o phase shift.

6.3.3 For a $2\,\mathrm{kHz}$ plane wave in water impinging normally on a steel plate of $1.5\,\mathrm{cm}$ thickness

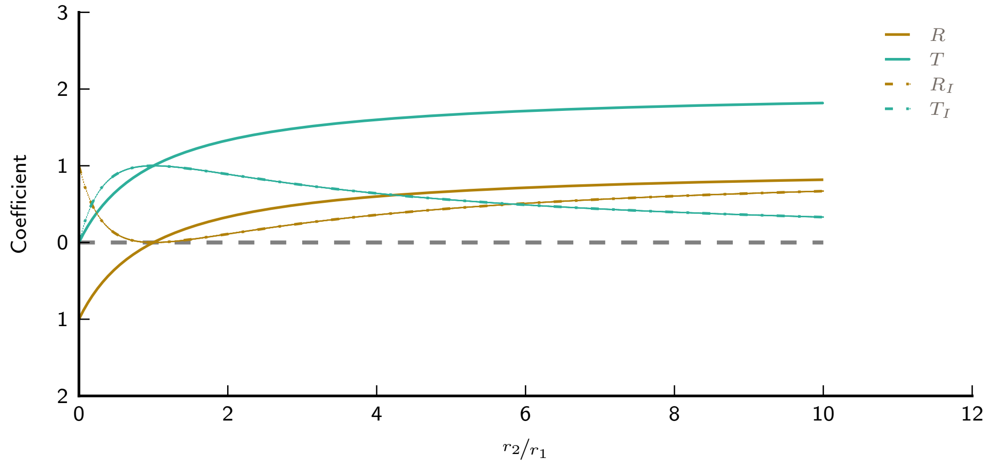

Figure 1: Reflection and Transmission Coefficients and Power Coefficients Plotted through a Range of Values

(a) what is the transmission loss, expressed in dB**, through the steel plate into water on the opposite side?** The transmission loss will be defined as

$$L_T = 20 \log_{10} \frac{|p_t|}{|p_i|}$$

and

$$\frac{|p_t|}{|p_i|} = |T|$$

, so, using the mass law, we have

$$T = \frac{1}{1 + \left[\left(\frac{\omega \rho_s}{2 r_1} \right) \cos \theta \right]^2}$$

and with $\rho_s = 7859\, {}^{kg}/_{\mathrm{m}^3}$, $\omega = 2\,\mathrm{kHz}$, $r_1 = 1.5 \times 10^6$ rayls for water, and $\theta = 0$ for normal, we have

$$T = \frac{1}{1 + \left[\left(\frac{((2000)(7859))}{2(1.5 \times 10^6)} \right) \right]^2} = 0.035$$

and thus, the transmission loss will be

$$L_T = 29.11\,\mathrm{dB}$$

(b) What is the power reflection coefficient of this plate? The power reflection coefficient is

$$R_I = R^2$$

with

$$R = 1 - T = 1 - 0.035 = 0.965$$

therefore

$$R_I = 0.931$$

(c) Repeat (a) and (b) for a 1.5 cm **slab of sponge rubber having a density of** $500\, {}^{kg}/_{\mathrm{m}^3}$ **and a longitudinal wave speed of** $1000\, {}^{m}/_{\mathrm{s}}$

$$T = \frac{1}{1 + \left[\left(\frac{((2000)(500))}{2(1.5 \times 10^6)} \right) \right]^2} = 0.9$$

$$L_T = 2.11\,\mathrm{dB}$$

$$R = 1 - T = 1 - 0.9 = 0.1$$

$$R_I = 0.01$$

6.6.1 An acoustic tile panel is characterized by a normal specific acoustic impedance of $900 - j1200\ {}^{Pa\cdot s}/_{\mathrm{m}}$**.**

(a) For what angle of incidence in air will the poewr reflection coefficient be a minimum? The intensity reflection coefficient for non-normal incidence is

$$R_I = \frac{\left(r_n - {}^{r_1}/_{\cos\theta_i}\right)^2 + \chi_n^2}{\left(r_n + {}^{r_1}/_{\cos\theta_i}\right)^2 + \chi_n^2}$$

and the normal specific acoustic impedance is

$$z_n = r_n + j\chi_n = 900 - j1200 \quad \therefore \quad r_n = 900,\ \chi_n = -1200$$

so

$$R = \frac{\left(900 - {}^{415}/_{\cos\theta_i}\right)^2 + (-1200)^2}{\left(900 + {}^{415}/_{\cos\theta_i}\right)^2 + (-1200)^2}$$

and, solving with wolfram alpha

$$\frac{dR}{d\theta_i} = 0$$

at

$$\theta_i = 73.9^o$$

(b) What is the power reflection coefficient for an angle of incidence of 80^o**?**

$$R = \frac{\left(900 - {}^{415}/_{\cos 80^o}\right)^2 + (-1200)^2}{\left(900 + {}^{415}/_{\cos 80^o}\right)^2 + (-1200)^2} = 0.2984$$

(c) What is the power reflection coefficient for an angle of normal incidence?

$$R = \frac{\left(900 - {}^{415}/_{\cos 0^o}\right)^2 + (-1200)^2}{\left(900 + {}^{415}/_{\cos 0^o}\right)^2 + (-1200)^2} = 0.5286$$

Additional Problem:

(a) Derive an expression for the surface normal impedance (at normal incidence) of a fluid layer of depth L **(having density** ρ_1 **and speed of sound** c_1**) above a perfectly hard backing.** The pressure field and velocity are given by

$$p_1 = \exp(j\omega t)\left[A\exp(-jk_1x) + B\exp(jk_1x)\right]$$

and

$$u_1 = \exp(j\omega t)\left[\frac{A}{\rho_1 c_1}\exp(-jk_1x) - \frac{B}{\rho_1 c_1}\exp(jk_1x)\right]$$

At a depth of L, we have the boundary conditions of no velocity, so

$$u_1|_{x=L} = 0$$

$$\exp(j\omega t)\left[\frac{A}{\rho_1 c_1}\exp(-jk_1L) - \frac{B}{\rho_1 c_1}\exp(jk_1L)\right] = 0$$

so

$$B = A\exp(-2jk_1L)$$

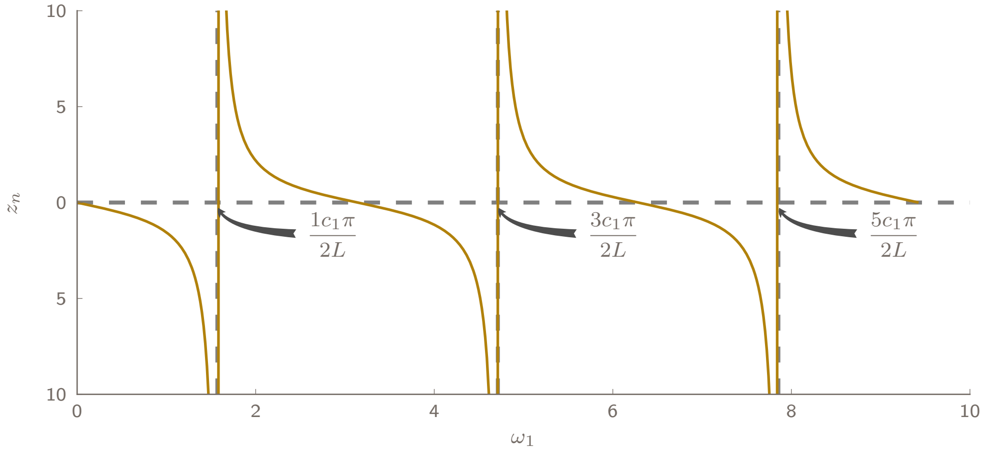

Figure 2: Surface normal impedance as a function of frequency

So solving the surface normal impedance, we have

$$
\begin{aligned}
z = \left.\frac{p_1}{u_1}\right|_{x=0} &= \frac{\cancel{\exp(j\omega t)}\left[A\exp(-jk_1x) + B\exp(jk_1x)\right]}{\cancel{\exp(j\omega t)}\left[\frac{A}{\rho_1 c_1}\exp(-jk_1x) - \frac{B}{\rho_1 c_1}\exp(jk_1x)\right]} \\
&= \frac{\left[\cancel{A}\exp(-jk_1x) + \cancel{A}\exp(-2jk_1L)\exp(jk_1x)\right]}{\left[\frac{\cancel{A}}{\rho_1 c_1}\exp(-jk_1x) - \frac{\cancel{A}\exp(-2jk_1L)}{\rho_1 c_1}\exp(jk_1x)\right]} \\
&= \rho_1 c_1 \frac{\cancelto{1}{\exp(-jk_1x)} + \exp(-2jk_1L)\cancelto{1}{\exp(jk_1x)}}{\cancelto{1}{\exp(-jk_1x)} - \exp(-2jk_1L)\cancelto{1}{\exp(jk_1x)}} \\
&= \rho_1 c_1 \frac{1 + \exp(-2jk_1L)}{1 - \exp(-2jk_1L)} \\
&= -j\rho_1 c_1 \cot(k_1 L)
\end{aligned}
$$

(b) Sketch the impedance. Using the typical wavenumber definition of $k_1 = \frac{\omega_1}{c_1}$, we can plot

$$
z = -j\rho_1 c_1 \cot\left(\frac{\omega_1 L}{c_1}\right)
$$

in figure 2.

(c) Calculate the plane wave pressure reflection coefficient for this layer (the ambient density outside the layer is ρ_0 and the ambient sound speed is c) and show that the magnitude of the reflection coefficient is always equal to unity in this case. The plane wave pressure reflection coefficient is

$$
R = \frac{z_n - \rho_0 c}{z_n + \rho_0 c}
$$

which is easily cast as

$$
R = \frac{\cot(k_1 L) - \frac{\rho_0 c}{\rho_1 c_1}}{\cot(k_1 L) + \frac{\rho_0 c}{\rho_1 c_1}}
$$

but using the periodic nature of cot, we can see that around $n\pi = k_1 L$, $\cot(k_1 L) \gg \frac{\rho_0 c}{\rho_1 c_1}$ and therefore,

$$
R = \frac{\cot(k_1 L) - \cancelto{0}{\frac{\rho_0 c}{\rho_1 c_1}}}{\cot(k_1 L) + \cancelto{0}{\frac{\rho_0 c}{\rho_1 c_1}}} = 1
$$

for $n\pi = k_1 L$.