Physics 556
Nuclear Physics
Homework #3

[1]

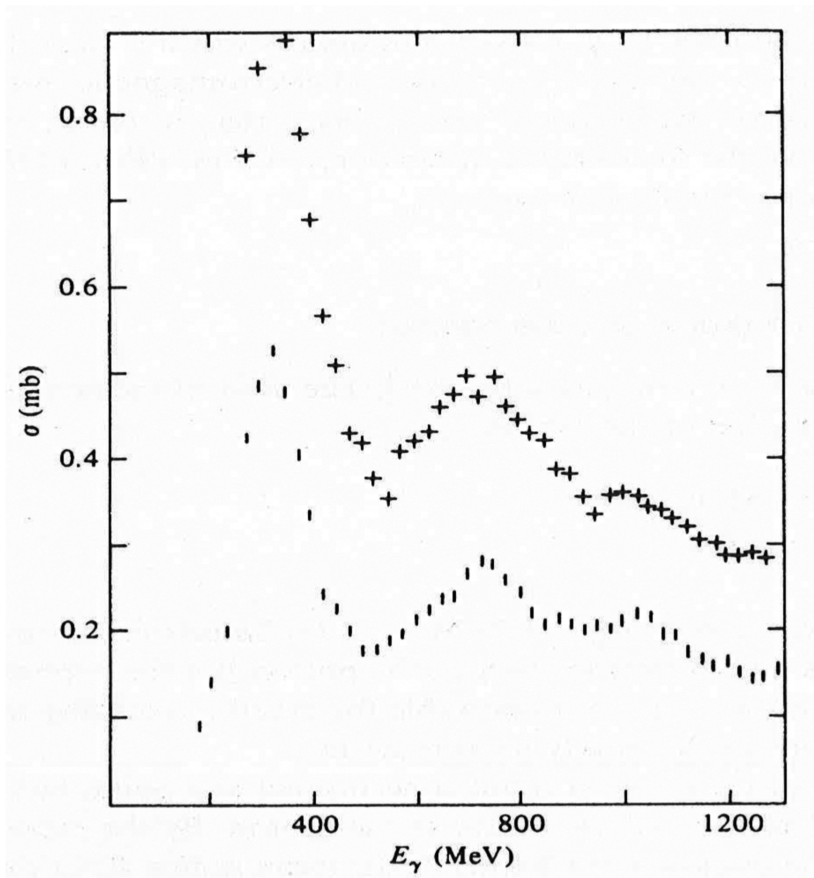

Fig. 3.1 The total photon cross-section for hadron production on protons (dashes) and deuterons (crosses). The difference between these cross-sections is approximately the cross-section on neutrons. (After Armstrong, T. A. *et al.* (1972), *Phys. Rev.* **D5**, 1640; *Nucl. Phys.* **B41**, 445.)

Shown above is the cross-section for absorption of photons by protons and deuterons as a function of photon energy up to 1300 MeV. The peaks are due to photons being absorbed to produce an excited state when the photon energy matches the excitation energy of the state. The subtraction of the proton cross-section from the deuteron cross-section yields the neutron cross section.

[a] Find the energy associated with the 1st and 2nd resonance peaks.

[b] Estimate the energy widths of 1st and 2nd resonance peaks and estimate the life times of these resonances using the uncertainty principle

$$\Delta E\,\Delta t \sim \hbar$$

[c] Since these resonances are the absorption of a gamma ray by a nucleon estimate some of their nuclear quantum numbers. Give your reasoning behind your QM selections.

Resonance	1st resonance	2nd resonance	Comment
Mass			
Baryon #			
Spin			
Isospin			
Parity			
Related Charge states			

[d] To check your ideas go to the Particle Data Group (PDG) web site

http://pdg.lbl.gov/2014/tables/contents_tables_baryons.html

and print out a summary: All Baryons. From the table make your best guess at the resonances being observed. Then look up in detail their measured properties.

[2] Starting from Coulomb's Law show that the potential function for a uniformly charged sphere is given by

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 R}\left(\frac{3}{2} - \frac{1}{2}\left(\frac{r}{R}\right)^2\right)$$

Where R is the charge radius of the sphere.

Find the difference between this potential function and a point like nucleus on the energy of an electron in the ground state electron wave function from the hydrogen atom.

[3] The spherical Harmonics are given by;

$$Y_{lm}(\theta,\phi) = \varepsilon \left\{ \frac{2l+1}{4\pi} \frac{(l-|m|)!}{(l+|m|)!} \right\}^{1/2} P_l^m(\cos\theta) e^{im\phi}$$

such that

$$\begin{aligned} \varepsilon &= (-1)^m \qquad & m \geq 0 \\ \varepsilon &= 1 \qquad & m < 0 \end{aligned}$$

Where

$$P_l^m(x) = (1-x^2)^{m/2} \frac{d^m}{dx^m} P_l(x)$$

$$P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2-1)^l$$

$P_l(x)$ is the Legendre Functions, and $P_l^m(x)$ is the associated Legendre Functions.

(a) Starting with the associated Legendre Functions, write out $Y_{3,\,+1}$.

(b) Check the normalization of your results by using the orthogonality condition.

(c) Show how any $Y_{lm}(\theta,\phi)$ transforms under parity.

(d) How is $P_l^m(x)$ *related to* $P_l^{-m}(x)$?

(e) For the Rodrigues' formula for the Legendre Functions prove orthogonality for any value of *l*. That is

$$\int_{-1}^{1} P_n(x) P_m(x) dx = \delta_{nm}$$

[4] The wave function of the two-nucleon state may be written;

$$\psi_{total} = \Phi_{space} \alpha_{spin} \chi_{isospin}$$

In this case we consider the proton and neutron to be identical particles with just different projections in isospin space. This leads to the extended Pauli Exclusion Principle that two nucleons must have an overall wave function that is anti-symmetric w.r.t. interchange of the two particles.

(a) Discuss the space part of the wave function and symbolically write out the spatial wave function.

(b) Discuss the spin part of the wave function and symbolically write out the spin wave function.

(c) Given our knowledge of the space and spin of the deuteron what must the symmetry of the isospin wave function be? Write explicitly the isospin wave function of the deuteron.

[5] Check if the following reaction violates any of the conservation laws you know including isospin, both **I** and **I_z**.

$$d + d \rightarrow {}^{4}He + \pi^{0}$$

[6] Given the Pauli Spin Matrices

$$\tau_1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \tau_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \tau_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

evaluate

$$(a)\,\tau_3|p\rangle \quad (b)\,\tau_3|n\rangle \quad (c)\,\vec{\tau}^2 = \vec{\tau}\circ\vec{\tau} \quad (d)\,\vec{\tau}^2|p\rangle \quad (e)\vec{\tau}^2|n\rangle$$
$$(f)\,\tau_+ = \tau_1 + i\tau_2 \quad (g)\,\tau_- = \tau_1 - i\tau_2$$

(h) Operate $\tau_\pm$ *on* $|p\rangle$ *and* $|n\rangle$