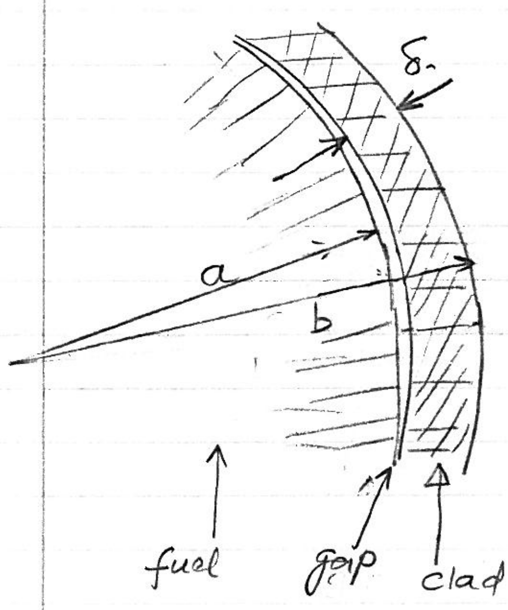
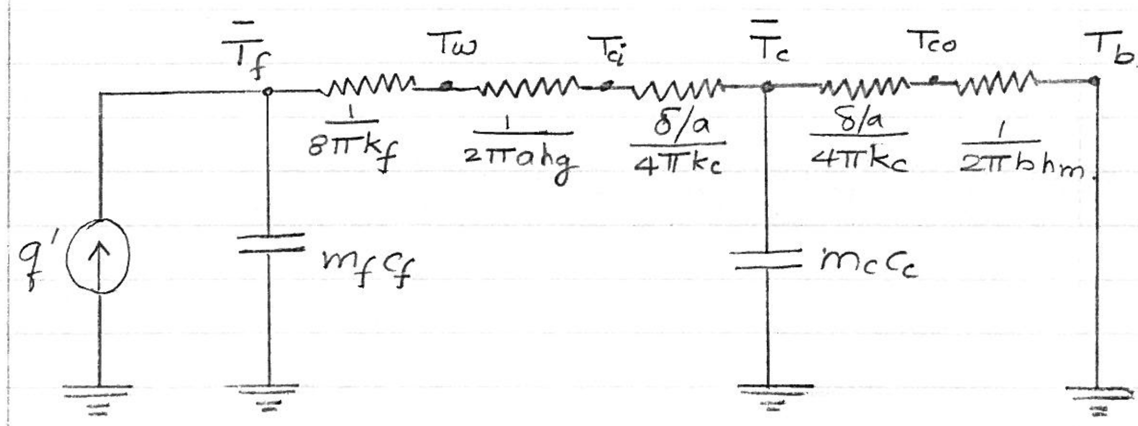
Transient Conduction - Lumped Parameter Model



Consider fuel with clad and gas gap.



Data

fuel pellet radius $a = 5$ mm, clad thickness $\delta = 0.8$ mm.

gap thickness $\delta_{gap} \simeq 0$.

$k_f = 2.0$ W/m °C

$k_c = 20.0$ W/m°C

$c_f = 340$ J/kg °C

$c_c = 340$ J/kg °C

$\rho_f = 10{,}000$ kg/m^3

$\rho_c = 6{,}500$ kg/m^3

$h_g = 5{,}700$ W/m^2·°C

$h_m = 57{,}000$ W/m^2 °C

The time constants for lumped parameter nodes

- fuel $\rightarrow \tau_f = m_f c_f R_{fc}$

$$R_{fc} = \frac{1}{8\pi k_f} + \frac{1}{2\pi a h_g} + \frac{\delta/a}{4\pi k_c} = 0.0216 \ \frac{m\,^\circ C}{W}.$$

$$m_f = \pi a^2 \rho_f = 0.785 \ kg/m$$

$$\tau_f = \underline{5.76\,s}$$

- Clad $\quad \tau_c = m_c c_c R_{cm}$

$$R_{cm} = \frac{\delta/a}{4\pi k_c} + \frac{1}{2\pi b\, h_m} = 0.0011 \ \frac{m\,^\circ C}{W}$$

$$m_c = 2\pi a \delta \rho_c = 0.163. \ kg/m.$$

$$\tau_c = \underline{0.061\,s}$$

The time constant of the clad is about 1% of the fuel time constant. The response of the clad during transient is almost instantaneous.

- If a fuel rod suffers a complete cooling failure at $t=0$, calculate the time taken for the temperature difference between fuel and clad to reach 10% of the initial temperature difference. Assume power drops to zero after cooling failure.

$$m_f c_f \frac{d\bar{T}_f}{dt} = q' - \frac{1}{R_{fc}}\left(\bar{T}_f - \bar{T}_c\right)$$

$$m_c c_c \frac{d\bar{T}_c}{dt} = \frac{1}{R_{fc}}\left(\bar{T}_f - \bar{T}_c\right)$$

$$\frac{d}{dt}(\bar{T}_f - \bar{T}_c) + \left(\frac{1}{m_f c_f R_{fc}} + \frac{1}{m_c c_c R_{fc}}\right)(\bar{T}_f - \bar{T}_c) = \cancel{\frac{q'}{m_c c_f}}\ 0.$$

$$\tau = \left(\frac{1}{m_f c_f R_{fc}} + \frac{1}{m_c c_c R_{fc}}\right)^{-1} \longrightarrow \text{time constant (fuel + clad)}$$

Solution:

$$\frac{\bar{T}_f - \bar{T}_c}{(\bar{T}_f - \bar{T}_c)_0} = e^{-t/\tau}, \longrightarrow 0.1 = e^{-t/\tau}$$

$$\therefore\ t = -\tau \ln 0.1 = 2.3\tau.$$

$$\tau = \underline{\underline{1.2}} \qquad \therefore\ t = \underline{2.8\,s}$$

If average fuel temperature was initialy 1200°C higher than clad temperature, then the time to reach a temperature difference of 120°C, is 2.8 s.