uniform 2 MeV S in inf hydrogen
calculate probability jump over 5eV to 2keV

elastic scattering

probability of arriving below E_a from collisions above E_b directly is

$$\int_0^{E_a} dE \int_{E_b}^{E_0} dE \frac{F(E')}{E'} dE'$$

F is total scattering rate

$$F(E) = \Sigma_s(E)\, \varphi(E)$$

$$\varphi(E) = \frac{S}{E\Sigma_s(E)} \quad \text{in inf hydrogen}$$

then

$$\int_0^{E_a} dE \int_{E_b}^{E_0} dE' \frac{S}{(E')^2} dE' = SE_a\left(\frac{1}{E_b} - \frac{1}{E_0}\right)$$

so probability is

$$P = E_a\left(\frac{1}{E_b} - \frac{1}{E_0}\right)$$

Lab section and COM

$$\cos\theta_s = \frac{1+A\cos\theta_c}{\sqrt{A^2+2A\cos\theta_c+1}}$$

show scattered neutrons always forward directional $(0 \le \theta_s \le \pi/2)$

$$\cos\theta_s = \frac{1+\cos\theta_c}{\sqrt{2+2\cos\theta_c}} = \sqrt{\frac{1+\cos\theta_c}{2}}$$

$$2\cos^2\theta_s = \cos\theta_c + 1 \Rightarrow 2\cos^2\theta_s - 1 = \cos\theta_c$$

$$\cos(2\theta_s) = \cos\theta_c \Rightarrow 2\theta_s = \theta_c \Rightarrow \theta_s = \theta_c/2$$

so $0 \le \theta_s \le \pi/2$ bc. $0 \le \theta_c \le \pi$

find scattering XS as function of μ_c

$$\sigma_s(\theta_c, \varphi_c) = \frac{\sigma_s}{4\pi} \Rightarrow \text{integrate} \quad \sigma_s(\mu_c) = \frac{\sigma_s}{2}$$

find XS in LS as function of μ_c

$$\sigma_s(\mu_s) = \sigma_s(\mu_c)\left|\frac{d\mu_c}{d\mu_s}\right|$$

$$\mu_s = \sqrt{\frac{1+\mu_c}{2}} \Rightarrow d\mu_s = \frac{1}{2\sqrt{2(1+\mu_c)}}\,d\mu_c$$

~~integrate~~

$$\sigma_s(\mu_s) = \sigma_s\sqrt{2(1+\mu_c)} = 2\sigma_s\mu_s$$

Probability of decay bf. absorption

$v = 2200$ m/s , $\Sigma_a = 0.022$ cm^{-1}

$T_{1/2} = 12$ min

$\lambda_a = 1/\Sigma_a = 50$ cm, $\tau_a = \frac{\lambda_a}{v} = 2\times10^{-4}$ s

$\tau_d = \frac{1}{\lambda} = \frac{720\text{ s}}{\ln 2}$

$P = \frac{\tau_a}{\tau_d} = 1.93\times10^{-7}$

Find resonance escape given an expression for the resonance integral.

$$I_{eff} = 3.8\left(\frac{\Sigma_s}{N_u}\right)^{0.42}$$

so find $\frac{\Sigma_s}{N_u}$ first.

$$\frac{\Sigma_s}{N_u} = \frac{\sigma_s^u N_u + \sigma_s^c N_c}{N_u} = \sigma_s^u + \sigma_s^c \frac{N_c}{N_u}$$

$$I_{eff} = 3.8\left(\frac{\Sigma_s}{N_u}\right)^{0.42} \qquad \alpha_c = \left(\frac{A-1}{A+1}\right)^2 \Rightarrow \xi$$

$$\bar{\xi} = \frac{N_c \sigma_c \xi_c + N_u \sigma_u \xi_u}{N_c \sigma_c + N_u \sigma_u} = 0.157$$

$$p = \exp\left(-\frac{N_u}{\bar{\xi}\Sigma_s} I_{eff}\right)$$

that comes from the prob of reaching E_i and being absorbed in i.

$$\pi_i = \frac{1}{\varepsilon_{i-1}} \int_{E_i}^{E_{i-1}} \Sigma_a(E)\,\varphi(E)\,dE \Rightarrow \frac{N_F}{\bar{\xi}\Sigma_s} I_i$$

then the res escp prob.

$$p = \prod_{i=1}^{N} (1-\pi_i) = \prod_{i=1}^{N} \exp(-\pi_i) \approx \exp\left[-\frac{N_F}{\bar{\xi}\Sigma_s} I\right]$$

cylindrical BF_3 counter in neutron flux w/ maxwellian dist of 25°C. BF_3 has pressure at 25 cm Hg (at 20°C), is 6 cm in dia, 25 cm long. Counter has 1% eff., placed in isotropic flux, gives 10000 cpm.

what's total incident flux?

uses ^{10}B rxn for thermal detection, 96% enriched

$P = 25\,\text{cm Hg}/76\,\text{cm Hg} = 0.329\,\text{atm}$

$V = \pi(0.6/2\,\text{cm})^2 \times 25\,\text{cm} = 7.069 \times 10^{-3}\,L.$

$T = 20°C = 293.15\,K, \quad R = 0.08206\ L\,\text{atm}/K\,\text{mol}$

so

$PV = nRT \Rightarrow n = \frac{PV}{RT} = 9.67\times10^{-5}\,\text{mol}$

so

$N_{B10} = 9.67\times10^{-5} \times 96\% \times 6.022\times10^{23}\ \text{atoms/mol} = 5.59\times10^{14}\,\text{atoms}$

σ_a @ thermal is 3843 b

so

$\overline{\sigma_a}^{B10}(T) = \frac{\sqrt{\pi}}{2}\,\sigma_0\sqrt{\frac{T_0}{T}} = \frac{\sqrt{\pi}}{2}\,\sigma_0\sqrt{\frac{293.15}{298.15}} = 3377\ \text{barns}$

$R_a^{B10} = \frac{10000\ \text{cpm}}{\cancel{0.01}\ 1\%} \Rightarrow \text{cps} = 1.667\times10^{4}\ \text{cps}$

$\phi(T) = \frac{R_a^{B10}(T)}{N_{B10}\ \overline{\sigma_a}^{B10}(T)} = 8.83\times10^{4}\ \#/\text{cm}^2\text{s}$

How many elastic scatters to get to a certain energy

find $\alpha = \left(\frac{A-1}{A+1}\right)^2$

$\xi = 1 + \frac{\alpha}{1-\alpha} \ln \alpha$

$\bar{n} = \frac{1}{\xi} \ln \frac{E_0}{E}$

Deriving Compton Scattering

Photon has momentum

$$p = E/c$$

using special relativity for electron

$$T_\gamma = \sqrt{p_e^2 c^2 + m_e^2 c^4} - m_e c^2$$

because elastic

$$E = E' + T$$

so

law of cosines

$$(p')^2 = |\vec{p} - \vec{p}_e|^2 = p^2 + p_e^2 - 2 p p_e \cos\theta_r$$

$$p_e^2 = |\vec{p} - \vec{p}'|^2 = p^2 + (p')^2 - 2 p p' \cos\theta_s$$

rewrite as

$$(E')^2 = E^2 + T(T + 2m_e c^2) - 2E\sqrt{T(T + 2m_e c^2)}\cos\theta_r$$

$$T(T + 2m_e c^2) = E^2 + (E')^2 - 2EE'\cos\theta_s$$

now conserve energy $T = E - E'$

$$E^2 + (E')^2 - 2EE'\cos\theta_s = (E - E')(E - E' + 2m_e c^2)$$

$$[(1 - \cos\theta_s) E + m_e c^2] E' = m_e c^2 E$$

$$\boxed{E' = \frac{E}{1 + (E/m_e c^2)(1 - \cos\theta_s)}, \quad 0 \leq \theta_s \leq \pi}$$

and using momentum eq

$$E^2 + T(T + 2m_e c^2) - 2E\sqrt{T(T + 2m_e c^2)}\cos\theta_r = (E - T)^2$$

$$[(E + m_e c^2)^2 - E^2\cos^2\theta_r] T = 2m_e c^2 E^2 \cos^2\theta_r$$

$$T = \frac{2m_e c^2 E^2 \cos^2\theta_r}{(E + m_e c^2)^2 - E^2\cos^2\theta_r}, \quad 0 \leq \theta_r \leq \frac{\pi}{2}$$

Reflector Savings

for slab geometry

$$s = \frac{1}{B_m}\tan^{-1}\left(\frac{D_c B_m}{D_r \kappa}\tanh(\kappa \tau)\right)$$

and also find the ~~too~~ critical dimension to find the last dimension (w/ savings)

Prove cube is best parallelpiped

minimize volume for

$$V = abc, \qquad B^2 = \left(\frac{\pi}{a}\right)^2 + \left(\frac{\pi}{b}\right)^2 + \left(\frac{\pi}{c}\right)^2$$

turn to Lagrange Multiplier

$$V(a,b,c,\lambda) = abc + \lambda\left[B^2 - \left(\frac{\pi}{a}\right)^2 - \left(\frac{\pi}{b}\right)^2 - \left(\frac{\pi}{c}\right)^2\right]$$

$$\left.\begin{aligned} \frac{\partial V_a}{\partial a} &= bc - \frac{2\pi^2\lambda}{a^3} = 0 \Rightarrow abc = 2\lambda\frac{\pi^2}{a^2} \\ \frac{\partial V_b}{\partial b} &= ac - \frac{2\pi^2\lambda}{b^3} = 0 \Rightarrow abc = 2\lambda\frac{\pi^2}{b^2} \\ \frac{\partial V_c}{\partial c} &= ab - \frac{2\pi^2\lambda}{c^3} = 0 \Rightarrow abc = 2\lambda\frac{\pi^2}{c^2} \end{aligned}\right\} a = b = c$$

$$\frac{\partial V_a}{\partial \lambda} = B^2 - \left(\frac{\pi}{a}\right)^2 - \left(\frac{\pi}{b}\right)^2 - \left(\frac{\pi}{c}\right)^2 = 0$$

$$V = a^3 \Rightarrow B^2 = 3\left(\frac{\pi}{a}\right)^2$$

Consider a H_2O slab four diffusion lengths thick w/ uniform infinite plane neutron source S n/cm² located at one diffusion length from one side and three diffusion lengths from the otherside. What fraction of source neutrons escape? Does the ratio change if source placed at center?

origin at center, plane source at $x = L$

$$\begin{cases} -D\frac{d^2}{dx^2}\phi(x) + \Sigma_a\phi(x) = 0, \quad x \neq L; \quad \underbrace{\phi(\pm 2L) = 0}_{\text{boundaries}} \\ \underbrace{\lim_{x\to L^-} D\frac{d\phi}{dx} = \frac{S}{2}}_{\text{source condition}} \qquad \lim_{x\to L^+}\left[-D\frac{d\phi}{dx}\right] = \frac{S}{2} \end{cases}$$

solve for the geometry

$$\phi(x) = A\cosh\frac{x}{L} + C\sinh\frac{x}{L}$$

solve for left side of plane

$$\phi(x) = C\sinh\left(\frac{x}{L+2}\right) \quad \text{b.c. } \phi(-2L) = 0$$

$$\text{Source at } x = L^- \Rightarrow D\frac{d\phi}{dx}\bigg|_{x=L} = C\frac{D}{L}\cosh 3 = \frac{S}{2}$$

now source is

$$\phi(x) = \frac{SL}{2D\cosh(3)}\sinh(2 - x/L) \qquad \cancel{L \leq x <} \quad -2L \leq x < L$$

and right side

$$\phi(x) = \frac{SL}{2D\cosh(1)}\sinh(2 - x/L) \qquad L < x \leq 2L$$

$$\text{escaping sides} \quad n_L = D\frac{d\phi}{dx}\bigg|_{x=-2L} + D\frac{d\phi}{dx}\bigg|_{x=2L} = \frac{S}{2\cosh(3)} + \frac{S}{2\cosh(1)}$$

for centerline

$$\phi(x) = \frac{SL}{2D\cosh(2)}\sinh\left(2 - |x|/L\right)$$

leakage becomes

$$n_L = \frac{S}{2\cosh(2)} + \frac{S}{2\cosh(2)}$$

Solve for buckling and the critical radius

$$\nabla^2\phi(r) + B^2\phi(r) = 0$$

and

$$[D(E)B^2 + \Sigma_t(E)]\varphi(E) - \int_0^\infty \Sigma_s(E' \to E)\varphi(E')dE' = \lambda\chi(E)\int_0^\infty \nu\Sigma_f(E')\varphi(E')dE'$$

simplify for one energy group and plane geo

$$\frac{\partial^2\phi(x)}{\partial x^2} + B^2\phi(x) = 0$$

and

$$(DB^2 + \Sigma_t)\varphi - \Sigma_s\varphi = \lambda\nu\Sigma_f\varphi$$

find matl buckling

$$B_m^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} = \frac{k_\infty - 1}{L^2}$$

find a_c

$$B_g^2 = B_m^2 \qquad B_g^2 = \left(\frac{\pi}{a_c}\right)^2 =$$

Increase the critical dimension and find change in absorption needed for crit

$$B'^2 = \left(\frac{\pi}{1.05a}\right)^2$$

$$k = \frac{\nu\Sigma_f}{DB_g^2 + \Sigma_a} = 1$$

$$\Sigma_a' = \nu\Sigma_f - DB'^2 \implies \delta\Sigma_a = \Sigma_a' - \Sigma_a$$

source multiplication factor

$$M = \frac{\text{number of neutrons total}}{\text{source neutrons}}$$

$$k = S + Sk + Sk^2 + \dots = S/(1-k)$$

$$M = 1/1-k$$

linearly interpolate last two inverse of multiplication factor points.

~~$m = 3.5 + 0.04$~~

Diffusion Eqn in matrix form and energy dependent in energy vector form

2 group diffusion eqn.

$$-D_1 \frac{d^2}{dx^2}\phi_1(x)+\Sigma_{r1}\phi_1(x)=\frac{1}{k}\left[\nu\Sigma_{f1}\phi_1(x)+\nu\Sigma_{f2}\phi_2(x)\right]$$
$$-D_2 \frac{d^2}{dx^2}\phi_2(x)+\Sigma_{a2}\phi_2(x)=\Sigma_{s1\to2}\phi_1(x)$$

now assume separable

$$\phi_1(x)=\varphi_1\,\phi(x) \quad \text{an} \quad \phi_2(x)=\varphi_2\,\phi(x)$$

substitute

$$-D_1\varphi_1 \frac{d^2}{dx^2}\phi(x)+\Sigma_{r1}\varphi_1\phi(x)=\frac{1}{k}\left[\nu\Sigma_{f1}\varphi_1+\nu\Sigma_{f2}\varphi_2\right]\phi(x)$$
$$-D_2\varphi_2\frac{d^2}{dx^2}\phi(x)+\Sigma_{a2}\varphi_2\phi(x)=\Sigma_{s1\to2}\varphi_1\phi(x)$$

now divide by ϕ

$$-D_1\varphi_1\frac{\frac{d^2}{dx^2}\phi(x)}{\phi(x)}+\Sigma_{r1}\varphi_1=\frac{1}{k}\left[\nu\Sigma_{f1}\varphi_1+\nu\Sigma_{f2}\varphi_2\right]$$
$$-D_2\varphi_2\frac{\frac{d^2}{dx^2}\phi(x)}{\phi(x)}+\Sigma_{a2}\varphi_2=\Sigma_{s1\to2}\varphi_1$$

now assume

$$-B^2=\frac{\frac{d^2}{dx^2}\phi(x)}{\phi(x)} \Rightarrow \frac{d^2}{dx^2}\phi(x)+B^2\phi(x)=0$$

and
$$+\left(D_1B^2+\Sigma_{r1}\right)\varphi_1=\frac{1}{k}\left[\nu\Sigma_{f1}\varphi_1+\nu\Sigma_{f2}\varphi_2\right]$$
$$\left(D_2B^2+\Sigma_{a2}\right)\varphi_2=\Sigma_{s1\to2}\varphi_1$$

Find k when slab thickness

$$B^2=\left(\frac{\pi}{a}\right)^2$$

matrix form

$$\begin{bmatrix} D_1B^2+\Sigma_r-\frac{1}{k}\nu\Sigma_{f1} & -\frac{1}{k}\nu\Sigma_{f2} \\ -\Sigma_{s1\to2} & D_2B^2+\Sigma_{a2}\end{bmatrix}\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix}=0$$

take det.

$$\left(D_1B^2+\Sigma_{r1}-\frac{1}{k}\nu\Sigma_{f1}\right)\left(D_2B^2+\Sigma_{a2}\right)-\frac{1}{k}\nu\Sigma_{f2}\Sigma_{s1\to2}=0$$
$$\frac{1}{k}\left[\nu\Sigma_{f1}\left(D_2B^2+\Sigma_{a2}\right)+\nu\Sigma_{f2}\Sigma_{s1\to2}\right]=\left(D_1B^2+\Sigma_{r1}\right)\left(D_2B^2+\Sigma_{a2}\right)$$

$$k = \frac{\nu\Sigma_{f1}}{D_1B^2+\Sigma_{r1}} + \frac{\Sigma_{s1\to2}}{D_1B^2+\Sigma_{r1}}\ \frac{\nu\Sigma_{f2}}{D_2B^2+\Sigma_{a2}}$$

and when a → infinity

$$k \to k_\infty = \frac{\nu\Sigma_{f1}}{\Sigma_{r1}} + \frac{\Sigma_{s1\to2}}{\Sigma_{r1}}\ \frac{\nu\Sigma_{f2}}{\Sigma_{a2}}$$

and

average n's emitted per thermal n absorbed in fuel η_T

$$\eta_T = \frac{\nu\Sigma_{f2}}{\Sigma_{aF2}}$$

fast fission factor ϵ

$$\epsilon = \frac{\nu\Sigma_{f1}/\Sigma_{s1\to2} + \nu\Sigma_{f2}/\Sigma_{a2}}{\nu\Sigma_{\cancel{f2}}/\Sigma_{a2}}$$

resonance escape probability

$$p = \frac{\Sigma_{s12}}{\Sigma_{r1}} = \frac{\Sigma_{s12}}{\Sigma_{s12}+\Sigma_{a1}}$$

thermal utilization factor

$$f = \frac{\Sigma_{aF2}}{\Sigma_{a2}}$$

infinite multiplication factor

$$k_\infty = \eta\epsilon pf$$

Differential elastic scattering xs of ^4He is given by

$$\sigma_s(\theta_c) = \frac{\sigma_s}{4\pi}(1+\cos\theta_c)$$

find σ_s

$$\sigma_s = \int_{4\pi} \sigma_s(\theta_c)\,d\Omega(\theta_c) = \int_0^{2\pi}\int_0^{\pi} \frac{\sigma_s}{4\pi}(1+\cos\theta_c)\sin\theta_c\,d\theta_c\,d\varphi$$

$$= 2\pi\int_{-1}^{1} \frac{\sigma_s}{4\pi}(1+\mu_c)\,d\mu_c = 2\pi\left[\frac{2\sigma_s}{4\pi}\right] = \sigma_s$$

what fraction of elastic neutrons appear at angles greater than 90° in COM?

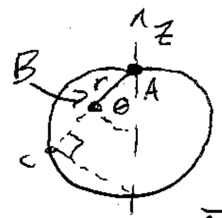
$$\int_0^{2\pi}\int_{\frac{\pi}{2}}^{\pi} \frac{\sigma_s}{4\pi}(1+\cos\theta_c)\sin\theta_c\,d\theta_c\,d\varphi = 2\pi\int_{-1}^{0}\frac{\sigma_s}{4\pi}(1+\mu_c)\,d\mu_c = \frac{\sigma_s}{4}$$

so 1/4

Find ϕ, J in spherical fusion chamber

Neutrons are produced uniformly and isotropically throughout a spherical chamber containing a mixture of ^3H and ^2H gases at 10^8 K, fusion rxns $T(d,n)He$ and $D(d,n)^3He$. $\lambda = \infty$, S, R find n wall flux $\vec{e}_r$ is radial unit vector

make sphere w/ z axis through point A.



$\overline{AC} = 2R\cos\theta$

dV is the volume of the skirt $\overline{AB}$.

so $$d\phi = \underbrace{\frac{S}{4\pi r^2}}_{\text{isotropic source}} \underbrace{dV}_{\text{volume}}$$

$\phi = \int d\phi$, so integrate over 0 to $2R\cos\theta$

$$\phi = \int_0^{\pi/2} \int_0^{2R\cos\theta} \frac{S}{4\pi r^2} \underbrace{r^2 \sin\theta\, dr\, d\theta\, d\varphi}$$ > spherical lagrangian

$$\phi = \frac{S}{2} \int_0^1 \int_0^{2R\mu} dr\, d\mu = SR\int_0^1 \mu\, d\mu = \frac{SR}{2}$$ ← wall flux!

now to find current

$$\Omega = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)$$

$$J = \int_0^{\pi/2} \int_0^{2R\cos\theta} \int_0^{2\pi} \frac{S}{4\pi r^2} \Omega\, r^2 \sin\theta\, d\varphi\, dr\, d\theta$$

w/ $J_x = 0$, $J_y = 0$ b.c. $\int_0^{2\pi} \cos\varphi\, d\varphi = \int_0^{2\pi} \sin\varphi\, d\varphi = 0$

$$J_z = \int_0^{\pi/2} \int_0^{2R\cos\theta} \int_0^{2\pi} \frac{S}{4\pi} \cos\theta \sin\theta\, d\varphi\, dr\, d\theta$$

$$= \frac{S}{2} \int_0^1 \int_0^{2R\mu} \mu\, dr\, d\mu = \frac{SR}{3} \vec{e}_z$$ which is radial at point A, so

$$J_z = \frac{SR}{3} \vec{e}_r$$

Find ϕ, J, J^+ and J^-

$\psi = a + b\cos\theta$

for ϕ:

integrate over Ω

$\phi(\vec{r}) = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, a + b\cos\theta \, d\theta d\varphi$

$= 2\pi \int_{-1}^{1} d\mu (a + b\mu) = 4\pi a$

for J:
integrate $\psi\Omega$ over Ω

$J = \int_0^{2\pi}\int_0^{\pi} \sin\theta \, \Omega (a + b\cos\theta) d\theta d\varphi$

$J_x = \int_0^{2\pi} \cos\varphi \int_0^{\pi} \sin^2\theta (a + b\cos\theta) d\theta d\varphi$
$= 0$

$J_y = \int_0^{2\pi} \sin\varphi \int_0^{\pi} \sin^2\theta (a + b\cos\theta) d\theta d\varphi$
$= 0$

$J_z = \int_0^{2\pi}\int_0^{\pi} \sin\theta\cos\theta (a + b\cos\theta) d\theta d\varphi$
$= 2\pi \int_{-1}^{1} \mu (a + b\mu) d\mu = \frac{4\pi}{3} b$

for partial current
integrate over φ and for
half of θ

$J_z^+ = \int_0^{2\pi}\int_0^{\pi/2} \sin\theta\cos\theta (a + b\cos\theta) d\theta d\varphi$

$= 2\pi \int_0^1 \mu (a + b\mu) d\mu = \frac{1}{4}\phi(r) + \frac{1}{2} J_z(r)$

$J_z^- = \frac{1}{4}\phi(r) + \frac{1}{2} J_z(r)$

$\psi = \phi_0 (\cos Bz + A\mu \sin Bz)$

for ϕ, integrate over μ

$\phi(z) = \int_{-1}^{1} \cos Bz + A\mu \sin Bz \, d\mu = 2\phi_0 \cos Bz$

for J, integrate over μ, $\psi\mu$

$J(z) = \int_{-1}^{1} (\cos Bz + A\mu \sin Bz) \mu d\mu = \frac{2}{3}\phi_0 A \sin Bz$

for partial currents only integrate over half of μ

$J^+ = \phi_0 \int_0^1 (\cos Bz + A\mu \sin Bz) \mu d\mu = \frac{1}{2}\phi_0 \cos Bz + \frac{1}{3}\phi_0 A \sin Bz$

$J^- = \frac{1}{2}\phi_0 \cos Bz - \frac{1}{3}\phi_0 A \sin Bz$

to rewrite ψ as ϕ, J

$\psi = \frac{1}{2}\phi + \frac{3}{2}\mu J$

Given reactivity is given by

$$\rho = \rho_0 + \gamma\, g(t)$$

we know we have a prompt jump component and a feedback component s.t.

$$\rho = \rho_1 + \gamma \int_0^t p(t')\,dt'$$

so we write the form

$$\rho = \rho_1 + \gamma\, g(t) \quad \text{with} \quad g(t) \equiv \int_0^t p(t')\,dt'$$

By taking the ~~prompt~~ point kinetics equations and assuming prompt kinetics (PKA), we know that the delayed source can be completed by the initial condition alone, so

$$\dot{p} = \frac{\rho - \beta}{\Lambda} p \quad \text{with} \quad p(0) = p^0 = \frac{\rho_1}{\rho_1 - \beta} p_0$$

substitute

$$\rho_p = \rho - \beta$$

$$\Lambda \dot{p} = \rho_p\, p$$

and

$$\dot{g} = p, \quad \ddot{g} = \dot{p}$$

$$\Lambda \ddot{g} = \rho_p \dot{g} = (\rho_{p1} + \gamma g(t))\,\dot{g}$$

$$\Lambda \underbrace{\int_0^t \ddot{g}\,dt}_{g cd} = \rho_{p1} \underbrace{\int_0^t \dot{g}\,dt}_{g el} + \gamma \underbrace{\int_0^t g\,\dot{g}\,dt}_{\text{integration by parts}}$$

$u = g \quad v = g$

$du = \dot{g}\,dt \quad dv = \dot{g}\,dt$

$\int_0^t g\dot{g}\,dt = g^2 - \int_0^t g\dot{g}$

$\int_0^t g\dot{g}\,dt = \frac{1}{2} g^2$

$$\Lambda [\dot{g}\,\cancel{}] = \rho_{p1}[g(t) - g(0)] + \frac{\gamma}{2}[g^2(t) - g^2(0)]$$

and $g(0) = 0$

$$\cancel{\Lambda \dot{g} = \rho_{p1} g + \frac{\gamma}{2} g^2} \quad \Lambda[\dot{g}(t) - \dot{g}(0)] = \rho_{p1} g + \frac{\gamma}{2} g^2$$

$$\dot{g}(0) = p(0) = \frac{\rho_1}{\rho_1 - \beta} p_0 = \frac{\rho_1}{\rho_{p1}} p_0$$

$$\Lambda \dot{g} = \frac{\gamma}{2} g^2 + \rho_{p1} g + \Lambda p(0)$$

then use quadratic equation

$$\dot{q} = (q - q_1)(q - q_2)$$

where q_1, q_2 are roots from

$$q_1, q_2 = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

~~so~~ so, with separation of variables

$$\frac{dq}{(q - q_1)(q - q_2)} = dt$$

then from integration tables

$$\frac{\ln(q - q_1) - \ln(q - q_2)}{q_1 - q_2} = t - t_0$$

Xenon Concentration

yield decay

$$\frac{dN_I}{dt} = \gamma_I F(t) - \bar{\lambda}_I N_I(t)$$

$$\bar{\lambda}_I = \lambda_I + \sigma_I \phi(t)$$

yield production decay

$$\frac{dN_{Xe}}{dt} = \gamma_{Xe} F(t) + \lambda_I N_I(t) - \bar{\lambda}_{Xe} N_{Xe}(t)$$

$$\bar{\lambda}_{Xe} = \lambda_{Xe} + \sigma_{Xe} \phi(t)$$

$$\frac{d}{dt}\left[N_I(t) e^{\bar{\lambda}_I t}\right] = \gamma_I F e^{\bar{\lambda}_I t}$$

$$\frac{d}{dt}\left[N_{Xe} e^{\bar{\lambda}_{Xe} t}\right] = \left[\gamma_{Xe} F + \lambda_I N_I(t)\right] e^{\bar{\lambda}_{Xe} t}$$

$$N_I(t) = N_I^0 e^{-\bar{\lambda}_I t} + \frac{\gamma_I F}{\bar{\lambda}_I}\left(1 - e^{-\bar{\lambda}_I t}\right)$$

$$N_{Xe}(t) = N_{Xe}^0 e^{-\bar{\lambda}_{Xe} t} + \frac{\gamma_{Xe} F}{\bar{\lambda}_{Xe}}\left(1 - e^{-\bar{\lambda}_{Xe} t}\right) + \frac{\lambda_I N_I^0}{\bar{\lambda}_{Xe} - \bar{\lambda}_I}\left(e^{-\bar{\lambda}_I t} - e^{-\bar{\lambda}_{Xe} t}\right)$$

$$+ \frac{\lambda_I \gamma_I F}{\bar{\lambda}_I}\left[\frac{1}{\bar{\lambda}_{Xe}}\left(1 - e^{-\bar{\lambda}_{Xe} t}\right) - \frac{1}{\bar{\lambda}_{Xe} - \bar{\lambda}_I}\left(e^{-\bar{\lambda}_I t} - e^{-\bar{\lambda}_{Xe} t}\right)\right]$$

Reactor Dynamics Equations

⟶ Inhour / Reactivity Equation [L2, p. 423–

assume: Infinite Reactor
prompt neutrons slow down instantaneously
Delayed " " " "

then:

$$-\phi_T(t) + \frac{S_T(t)}{\Sigma_a} = t_d \frac{d\phi_T(t)}{dt}$$

becomes

$$[(1-\beta)k_\infty - 1]\phi_T(t) + \frac{p}{\Sigma_a} \sum \lambda_i C_i(t) = t_d \frac{d\phi_T(t)}{dt} \quad (1)$$

and

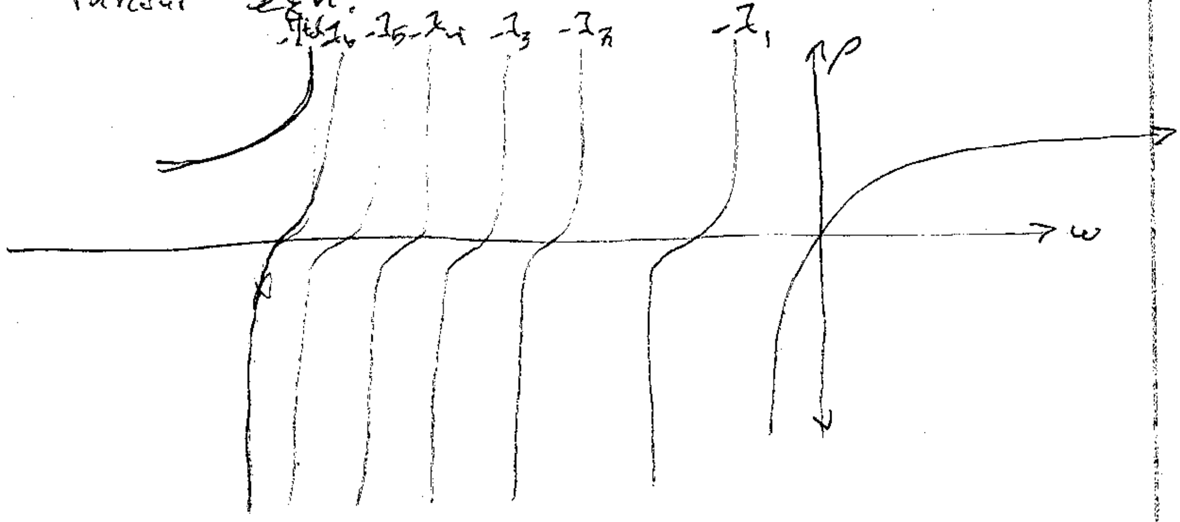
$$\frac{dC_i(t)}{dt} = \beta_i \frac{k_\infty}{p} \Sigma_a \phi_T(t) - \lambda_i C_i(t) \quad (2\text{-}7)$$

using the 7 eqs from above

we get

$$\rho = \frac{k_\infty - 1}{k_\infty} = \frac{\omega t_d}{1+\omega t_d} + \frac{\omega}{1+\omega t_d} \sum_i \frac{\beta_i}{\omega + \lambda_i}$$

inhour eqn!



root for positive $\rho \rightarrow \infty$ ~~[illegible]~~

root for negative ρ

There is a long process to determine constants on [L2, p. 426]

Reactivity Feedbacks and Transient Behaviors

[L2, ch 13]

Temperature feedbacks

$$\alpha = \frac{d\rho}{dT}$$

effect

thermal utilization
resonance escape probability
diffusion length
~~thermr~~ fast fission factor
neutron age
buckling
thermal and fast neu leakage

parameter	sign	parameter	sign
η_T	small except for Pu	L_T^2	+
f	+, small	τ_T	+
p	−, large	B^2	−, small
ϵ	small	P_T	−
k_∞	−	P_F	−
		k	−

Other feedbacks

poisoning

depletion

Transient behavior

poisoning
burnup
after shutdown

Neutronics – Dynamics

References:

[O1] Ott – Nuclear Reactor Statics > THEORY
[O2] Ott – Nuclear Reactor Dynamics
[L2] Lamarsh – Reactor Theory > EXAMPLES

Delayed Neutrons and Reactor Control

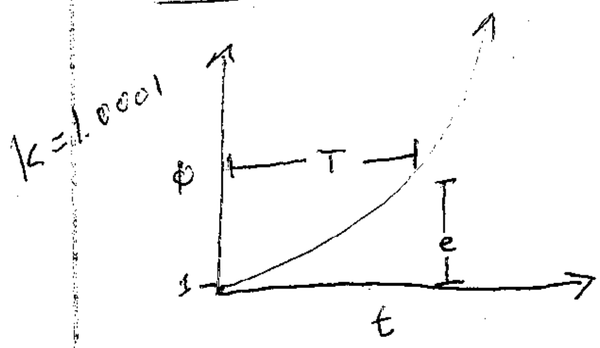
[L2, p. 426 –] Has full Derivation

Delayed neutrons come from Fission products, decay on 6 group basis w/ 6 half lives ranging from ~~0.2 s – 20~~ 0.2 s – 60s (Table 3-6 of [L2])

Reactor Control — also see ch 2 of [O2] for quantitative

Qualitative

w/o delayed | w/ delayed



and T is how long it takes for neutrons to diffuse, very fast! | T is much longer because it is ~~time~~ weighted averaged w/ the λs of delayed neutrons.

Quantitative

RXR behaves as $N_F(t) = N_F(0)\,e^{t/T}$ with $T = \frac{l_p}{k_\infty - 1}$ where l_p is mean generation time.

w/o delayed

$$l_p = t_s + t_d \approx t_d = t_{dM}(1-f)$$

$$l_p \approx 10^{-4}$$

so

$$T = \frac{10^{-4}}{k_\infty - 1} \Rightarrow 0.1\,s \text{ for } k_\infty = 1.001$$

w/ delayed

~~$l_p = t_s + t_d + \sum$~~

$$l = (1-\beta)l_p + \sum_{i=1}^{6} \beta_i l_i$$ mean neutron life

bc $\beta \ll 1$, and $l_p \ll \sum \beta_i l_i$

$$l = \sum_{i=1}^{6} \beta_i l_i = 0.1\,s$$

$$T = \frac{0.1\,s}{k_\infty - 1} \Rightarrow 100\,s \text{ for } k_\infty = 1.001$$

Point Kinetics and Solution Methods

Point Kinetics — assume no spatial dependence so we can separate the PDE

Equations [OZ] p. 24-25

Intuitive

$\Lambda \rightarrow$ generation time

$$\frac{d\hat{\phi}}{dt} = \underbrace{\frac{\rho-\beta}{\Lambda}}\hat{\phi} + \frac{1}{\cancel{\Lambda} v \Sigma_f}\sum_i \lambda_i \hat{C}_i$$

and

$$\frac{d\hat{C}_i}{dt} = \underbrace{-\lambda_i \hat{C}_i}_{\text{decay}} + \underbrace{\nu_{d\ell}\Sigma_f\hat{\phi}}_{\text{production}}$$

One Group [OZ, p. 31]

$$\frac{dp}{dt} = \frac{\rho-\beta}{\Lambda}p + \frac{1}{\Lambda}\sum_i \lambda_i \xi_i + \frac{f}{\Lambda}s(t)$$

$$\frac{d\xi_i}{dt} = -\lambda_i \xi_i + \beta_i p_i(t)$$

but we usually get changes given in reactivity!

so the inhour form is more useful.

Solution Methods

Basically all of ~~ott is~~ [OZ] is different appropriate solution methods. We should know PROMPT JUMP.

PJ [LZ] p 434

tells to what level the flux will rise or drop

we have

$$t_1 \frac{d\phi_T(t)}{dt} = [(1-\beta)k - 1]\phi_T(t) + \frac{\beta k}{\Sigma_1 k_\infty}\sum_i \lambda_i C_i(t)$$

constant by hypothesis

set $\frac{d\phi_T(t)}{dt} = 0$ @ $t=0$, $k=1 \Rightarrow$ find $\frac{\beta k}{\Sigma_1 k_\infty}\sum_i \lambda_i C_i(t)$

plug back in $\rightarrow$ get

$$\phi_T(t) \rightarrow \frac{\beta\phi_T(0)}{1-(1-\beta)k} = \frac{\beta(1-\rho)}{\beta-\rho}\phi_T(0) \quad \text{or generally} \quad \frac{\phi_T^+}{\phi_T^-} = \frac{1-\rho_-'}{1-\rho_+'}$$

$$\bar{t}_i = \frac{1}{\lambda_i} \quad \text{and} \quad T = \frac{1}{\beta}\sum \beta_i \bar{t}_i$$