NUCL 511 Nuclear Reactor Theory and Kinetics

Homework #8

Due April 3

Write a computer program to solve the point kinetics equation with six delayed neutron groups. This program should be able to simulate both reactivity and source transients. Using this program, answer the following questions. Use $\Lambda = 3\times 10^{-6}$s and the delayed neutron data in the table below with $\gamma_k = 1.07$.

Group	β_k	λ_k (s^{-1})
1	0.00025	0.0129
2	0.00150	0.0311
3	0.00144	0.1340
4	0.00258	0.3310
5	0.00074	1.2600
6	0.00029	3.2100

1. Source Drop in Critical Reactor: Analyze the source drop into a critical reactor specified in the homework problem 4 of Chapter 6. Specifically, determine the two reduced source magnitudes to yield the flux amplitude in Fig. 6.22 by running the point kinetics program for various source magnitudes, and show the flux magnitude for the initial 900 seconds. (50 points)

   Answer) The point kinetics equations for the source drop in a critical reactor can be written as

$$\begin{aligned} \Lambda\dot{p}(t) &= -\beta p(t) + \sum_k \lambda_k \zeta_k(t) + [s_1 H(t) + s_2 H(t-t_1)] \\ \dot{\zeta}_k(t) &= \beta_k p(t) - \lambda_k \zeta_k(t) \end{aligned} \tag{1}$$

   where $H(t)$ and $H(t-t_1)$ are the Heaviside unit function. The initial conditions are given by the steady state conditions

$$p(0) = p_0, \quad \zeta_k(0) = \frac{\beta_k}{\lambda_k} p(0) = \frac{\beta_k}{\lambda_k} p_0 \tag{2}$$

   Taking the Laplace transform of Eq. (1) and using Eq. (2), we have

$$\begin{aligned} \Lambda[\alpha\tilde{p}(\alpha) - p_0] &= -\beta\tilde{p}(\alpha) + \sum_k \lambda_k \tilde{\zeta}_k(\alpha) + \frac{1}{\alpha}(s_1 + s_2 e^{\alpha t_1}) \\ \alpha\tilde{\zeta}_k(\alpha) - \frac{\beta_k}{\lambda_k} p_0 &= \beta_k \tilde{p}(\alpha) - \lambda_k \tilde{\zeta}_k(\alpha) \end{aligned} \tag{3}$$

   Eq. (3) can be solved for $\tilde{p}(\alpha)$ as

$$\tilde{p}(\alpha) = \frac{p_0}{\alpha} + \frac{1}{\alpha^2[\Lambda + \sum_k \beta_k / (\alpha + \lambda_k)]}(s_1 + s_2 e^{\alpha t_1}) \tag{4}$$

The denominator of the second term of Eq. (4) is the in-hour equation for a reactivity of zero

$$\alpha\left(\Lambda+\sum_k \frac{\beta_k}{\alpha+\lambda_k}\right)=0 \tag{5}$$

This equation has seven roots, and they can be determined numerically using the program for solving the in-hour equation as

$$\begin{aligned}&\alpha_1=0.0,\quad \alpha_2=-0.01503,\quad \alpha_3=-0.07717,\quad \alpha_4=-0.22375\\ &\alpha_5=-1.13825,\quad \alpha_6=-3.08901,\quad \alpha_7=-2423.77\end{aligned} \tag{6}$$

These roots are the poles of $\tilde{p}(\alpha)$ with α_1 being second order and the others being first order. Thus, by taking the Laplace inverse transform of Eq. (4), $p(t)$ can be determined as

$$p(t)=\frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\tilde{p}(\alpha)e^{\alpha t}d\alpha=\sum_{i=1}^{7}\operatorname*{Res}_{\alpha=\alpha_i}[\tilde{p}(\alpha)e^{\alpha t}] \tag{7}$$

The residues can be determined as

$$\begin{aligned}\operatorname*{Res}_{\alpha=\alpha_1}[\tilde{p}(\alpha)e^{\alpha t}]&=\lim_{\alpha\to\alpha_1}\frac{d}{d\alpha}[(\alpha-\alpha_1)^2\,\tilde{p}(\alpha)e^{\alpha t}]\\ &=p_0+\frac{(\sum_k\beta_k/\lambda_k^2)[s_1+s_2H(t-t_1)]}{[\Lambda+\sum_k\beta_k/\lambda_k]^2}+\frac{s_1t+s_2(t-t_1)H(t-t_1)}{\Lambda+\sum_k\beta_k/\lambda_k}\end{aligned} \tag{8}$$

$$\operatorname*{Res}_{\alpha=\alpha_i}[\tilde{p}(\alpha)e^{\alpha t}]=\lim_{\alpha\to\alpha_i}[(\alpha-\alpha_i)\,\tilde{p}(\alpha)e^{\alpha t}]=\frac{s_1e^{\alpha_i t}+s_2e^{\alpha_i(t-t_1)}H(t-t_1)}{-\alpha_i^2\sum_k\beta_k/(\alpha_i+\lambda_k)^2},\quad i=2,\cdots,7 \tag{9}$$

By evaluating the residues numerically, the power transient for two source drops can be determined as

$$p(t)=\begin{cases}p_0+s_1f(t), & t<t_1\\ p_0+s_1f(t)+s_2f(t-t_1) & t\ge t_1\end{cases} \tag{10}$$

where

$$\begin{aligned}f(t)=&\,391.70+10.80t-68.59e^{-0.0150t}-125.46e^{-0.0772t}-41.52e^{-0.2238t}-12.76e^{-1.1383t}\\ &-4.76e^{-3.0890t}-137.59e^{-2423.8\mathrm{t}}\end{aligned} \tag{11}$$

After a sufficient time elapses from each source drop, all exponential terms die away, and hence the amplitude rises linearly

$$\begin{aligned}p(t)&\approx p_0+\frac{\sum_k\beta_k/\lambda_k^2}{[\Lambda+\sum_k\beta_k/\lambda_k]^2}s_1+\frac{s_1}{\Lambda+\sum_k\beta_k/\lambda_k}t,\quad 0<<t<t_1\\ p(t)&\approx p_0+\frac{\sum_k\beta_k/\lambda_k^2}{[\Lambda+\sum_k\beta_k/\lambda_k]^2}(s_1+s_2)+\frac{1}{\Lambda+\sum_k\beta_k/\lambda_k}[s_1t+s_2(t-t_1)],\quad t>>t_1\end{aligned} \tag{12}$$

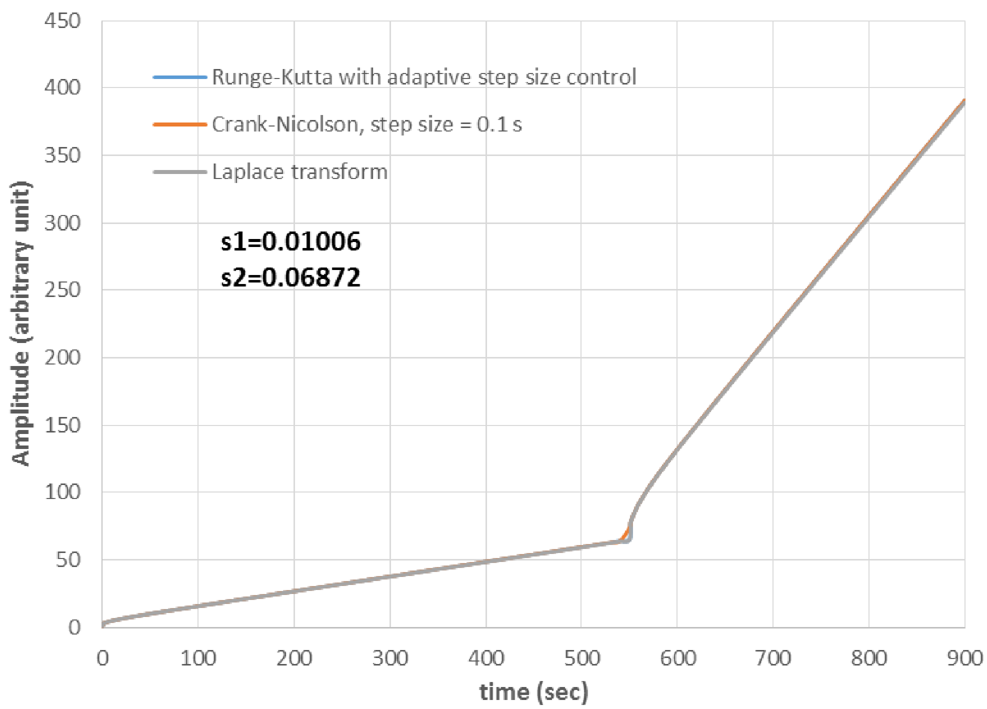
The slope of the amplitude rise thus becomes

$$\begin{aligned}
\frac{dp}{dt} &\approx \frac{s_1}{\Lambda + \sum_k \beta_k / \lambda_k} \approx \frac{s_1}{\sum_k \beta_k / \lambda_k} = \frac{\bar{\lambda}_{in}}{\beta} s_1, \quad 0 << t < t_1 \\
\frac{dp}{dt} &\approx \frac{s_1 + s_2}{\Lambda + \sum_k \beta_k / \lambda_k} \approx \frac{\bar{\lambda}_{in}}{\beta}(s_1 + s_2), \quad t >> t_1
\end{aligned} \tag{13}$$

Therefore the reduced sources can be determined from the asymptotic slopes of the amplitude rise as

$$s_1 \approx 0.0101, \quad s_1 + s_2 \approx 0.0788 \tag{14}$$

The figure below compares the solutions of the point kinetics equations obtained with the Runge-Kutta method with adaptive step size control and the Crank-Nicolson method with the semi-analytic solution obtained by the use of Laplace transform. For the Runge-Kutta method, a relative error criterion of 10^{-5} was used; this criterion resulted in a minimum step of 1.795×10^{-4} s, a maximum size of 1.942×10^{-2} s, and an average step of 1.545×10^{-3} s. A uniform time step size of 0.1s was used for the Crank-Nicolson method. It can be seen that the Runge-Kutta solution agree well with the semi-analytic solution. Although a relatively large time step size was used, the Crank-Nicolson method also reproduces the semi-analytic solution except for the prompt jumps.



2. <u>Asymmetric Saw-tooth Reactivity Insertion</u>: Analyze the reactivity transients specified in the homework problem 2 of Chapter 8, where an asymmetric saw-tooth reactivity insertion means the reactivity transient in Eq. (1).

$$\rho(t) = \begin{cases} at, & t \le t_1 \\ at_1 - a'(t - t_1), & t > t_1 \end{cases} \tag{1}$$

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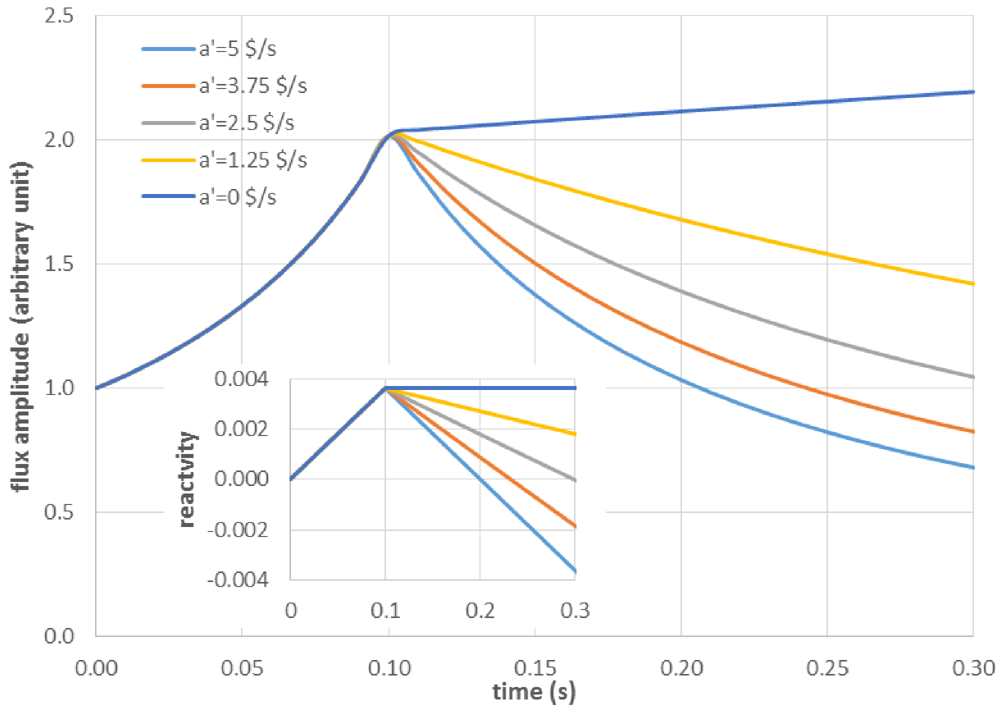
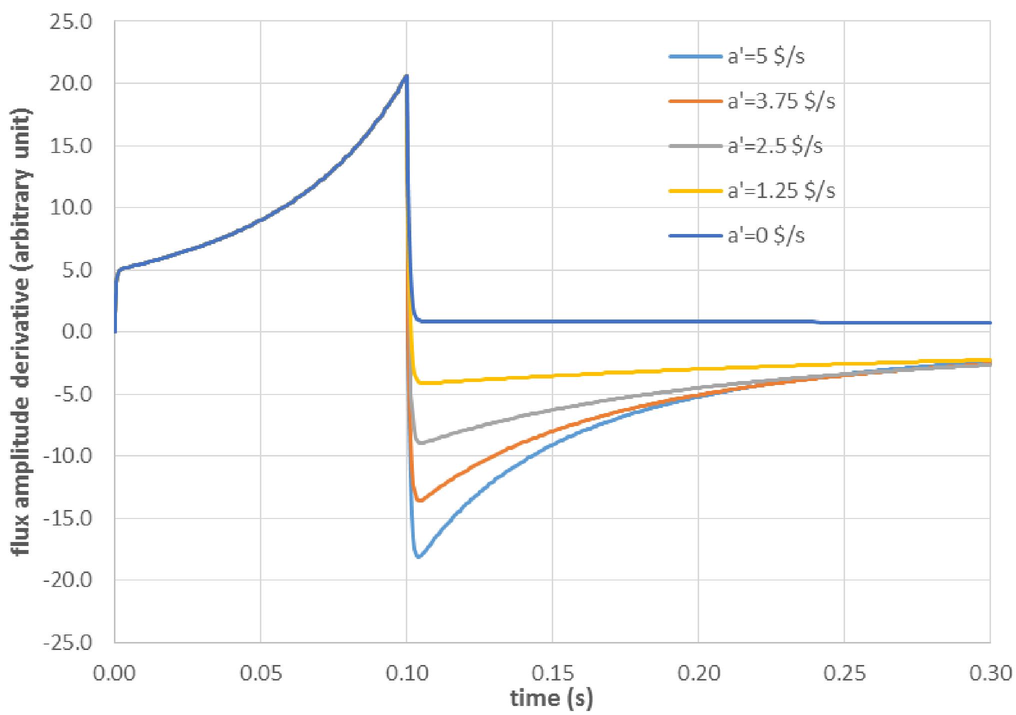
Use $t_1 = 0.1\text{s}$, $a = 5\$/\text{s}$, and five different ramp rate values on the declining part; $a' = a$, $a' = 0.75a$, $a' = 0.5a$, $a' = 0.25a$, and zero. Plot the power amplitude and its derivative for initial 0.3s and estimate the time for which $\dot{p} = 0$ and the reactivity at this time point. (50 points)

Answer) The figures below show the flux and its derivative behaviors during the initial 0.3 s.





For the first four cases, the reactivity decreases after $t = t_1 = 0.1\,\text{s}$ and becomes negative after $t = (1 + a / a')t_1$. However, the flux amplitude still increases slightly after $t = t_1$ due to the precursor buildup during the rising period. This can be seen clearly in the second figure where the amplitude derivative $\dot{p}$ passes the zero value after $t = t_1$. The time where $\dot{p} = 0$ can be calculated from the appoint kinetics code and the corresponding reactivities as

a' (\$/s)	t_0 (s) for $\dot{p}(t_0) = 0$	Reactivity (\$)
5	0.1006	0.4971
3.75	0.1007	0.4972
2.5	0.1010	0.4975
1.25	0.1015	0.4981