NUCL 511
Nuclear Reactor Theory and Kinetics

Lecture Note 1

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Course Description

- This course complements the reactor theory course NUCL 510 and covers the area of static perturbation theory and reactor dynamics.
- The static perturbation theory is a powerful method for analyzing the core reactivity and in calculating the reactivity feedback coefficients that determine the changes in reactivity due to the variations in material and temperature distributions.
- Reactor dynamics deals with the transient behaviors of nuclear reactors, including the coupling of neutronics and thermo-mechanics (i.e., reactivity feedbacks).
- Proper prediction of transient behaviors is crucial in reactor design to ensure the operational stability and the safe termination of anticipated accidents.

Course Goals

- Perturbation theory and application for reactivity change calculations
- Point kinetics equations
- Approximate kinetics and solution methods
  - Prompt jump approximation
  - Prompt kinetics approximation
- Point reactor model with thermal feedback
- Numerical solution methods for the spatial kinetics equations
  - PARCS
- Practical analysis of important transients with feedback
  - Control rod ejection accident
  - Main steam line break accident

Governing Equations of Reactor Physics

- The theory and governing equations for reactor physics analysis are well known;
  - Boltzmann equation for neutron and gamma transports
    - *Boltzmann equation is a linear integro-differential equation with seven independent variables (three in space, two in angle, one in energy and time)*
  - Bateman equation for fuel composition evolution
    - *Bateman equation is a system of ordinary differential equations*
- The coefficients of these equations are determined by nuclear data, geometry, and composition.
- The challenge in neutronics analysis is to determine the solution efficiently by taking into account geometric complexity and complicated energy dependence of nuclear data.

Time-Dependent Neutron Transport Equation

- Time-dependent Boltzmann transport equation

$$\frac{1}{v(E)}\frac{\partial}{\partial t}\psi(\vec{r},E,\vec{\Omega},t) = -\vec{\Omega}\cdot\nabla\psi(\vec{r},E,\vec{\Omega},t) - \Sigma_t(\vec{r},E,t)\psi(\vec{r},E,\vec{\Omega},t) + Q(\vec{r},E,\vec{\Omega},t)$$

$$Q(\vec{r},E,\vec{\Omega},t) = S_s(\vec{r},E,\vec{\Omega},t) + S_p(\vec{r},E,\vec{\Omega},t) + S_d(\vec{r},E,\vec{\Omega},t) + S_{in}(\vec{r},E,\vec{\Omega},t)$$

- Scattering source

$$S_s(\vec{r},E,\vec{\Omega},t) = \sum_i N_i(r,t)\sum_{l=0}^{L}\sum_{k=-l}^{l} Y_{lk}(\vec{\Omega})\int dE'\sigma_{sl}^{i}(\vec{r},E'\rightarrow E)\psi_{lk}(\vec{r},E',t)$$

$$\psi_{lk}(\vec{r},E,t) = \int_{4\pi} d\Omega\,\overline{Y}_{lk}(\Omega)\psi(\vec{r},E,\vec{\Omega},t)$$

- Prompt fission neutron source

$$S_p(\vec{r},E,\vec{\Omega},t) = \frac{1}{4\pi}\sum_i N_i(r,t)\int dE'\chi_{pi}(E'\rightarrow E)\nu_{pi}(E')\sigma_{fi}(r,E')\phi(\vec{r},E',t)$$

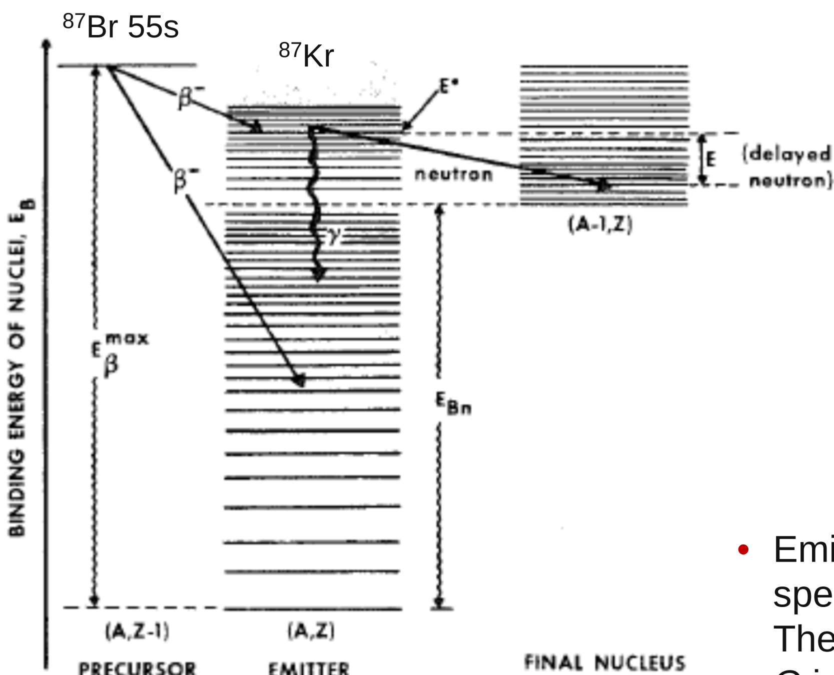
$$\phi(\vec{r},E,t) = \int_{4\pi} d\Omega\,\psi(\vec{r},E,\vec{\Omega},t)$$

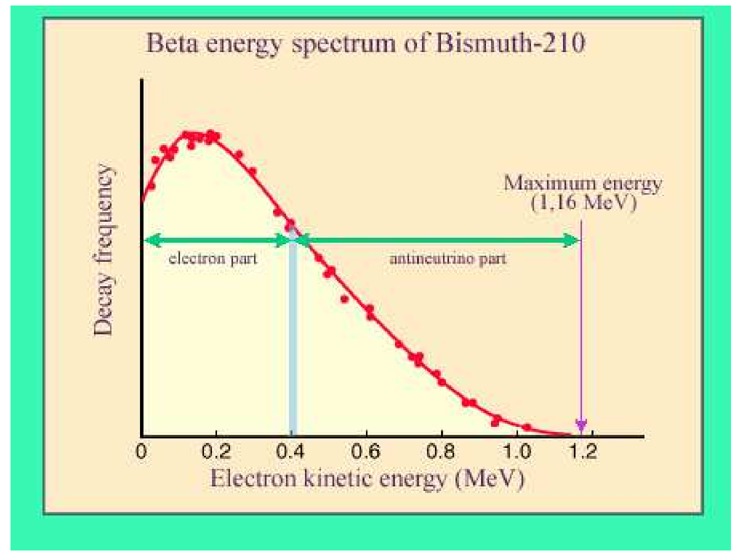
Delayed Neutron Source

- Generated following beta decays of certain fission products
- Grouped into six families depending on decay constants

$$S_d(\vec{r},E,\vec{\Omega},t)=\frac{1}{4\pi}\sum_i\sum_k \chi_{dki}(E)\lambda_{ki}C_{ki}(r,t)$$

$$\frac{\partial}{\partial t}C_{ki}(r,t)=-\lambda_{ki}C_{ki}(\vec{r},t)+N_i(r,t)\int dE'\nu_{dki}(E')\sigma_{fi}(\vec{r},E')\phi(\vec{r},E',t)\quad (k=1,2,\cdots,6)$$





- Emitted beta particles have a continuous kinetic energy spectrum, ranging from 0 to the maximal available energy (Q). The continuous energy spectra of beta particle occurs because *Q* is shared between the beta particle and a neutrino.

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Time Dependent Phenomena in Nuclear Reactors

- Three distinct time-dependent phenomena in nuclear reactors
  - Short time phenomena: typical time intervals of milliseconds to seconds (many minutes in special cases)
    - *Transient analysis*
  - Medium time phenomena: hours to days corresponding to the mean buildup and decay times of certain fission products
    - *Xe and Sm effects*
  - Long time phenomena: several months or years
    - *Fuel depletion analysis*
- Causal relationship between the neutron flux and the physical reactor system
  - Changes in the composition, geometry or temperature of the system may cause a change in the flux
  - Changes in the flux may alter the composition and temperature

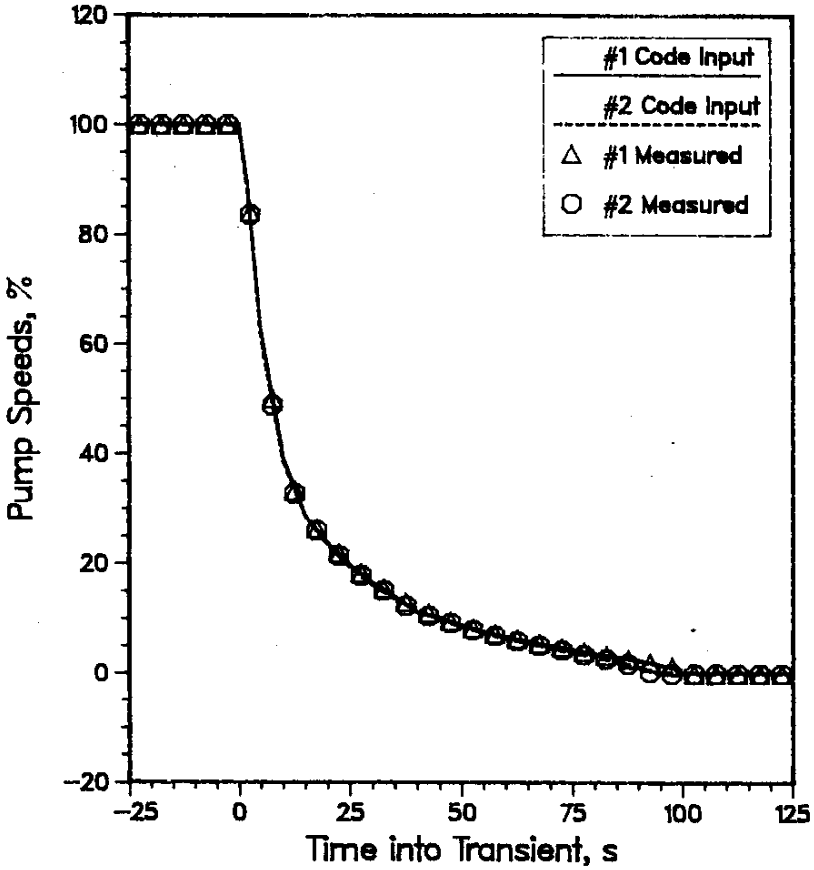
Short Time Phenomena

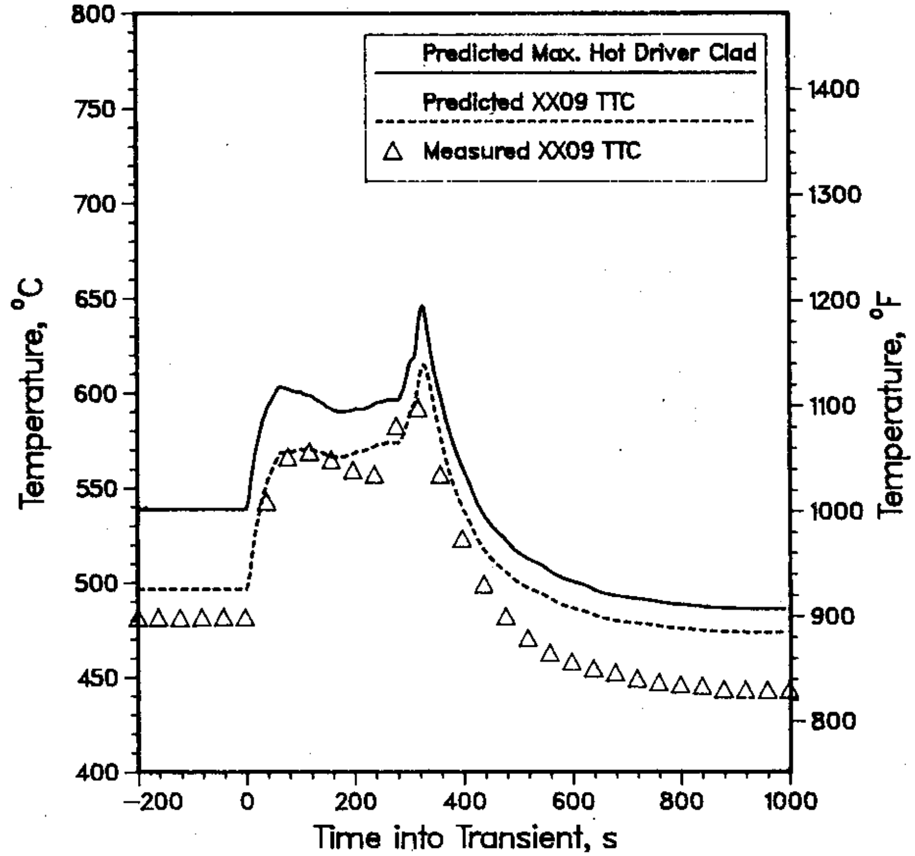
- Relatively rapid changes in the neutron flux due to intended or accidental changes in the system
  - Changes in system influence the flux through feedback
- Accident and safety analyses
  - Explicit treatment of delayed neutrons
  - Reactivity feedbacks due to temperature, composition, and geometry
  - Decay heat removal
- Experiments with time-dependent neutron fluxes
- Reactor operation such as startup, load change and shutdown
  - Some startup procedures may take hours
- Stability analysis with respect to neutron flux changes
  - Typically coupled neutronics and thermal-hydraulics stability

Scope of Reactor Dynamics

- Dynamics vs. Kinetics
  - Dynamics: Prediction of time-dependent reactor behavior with thermal feedback
  - Kinetics: Short time transient behavior at very low power, involving no temperature change
- Subjects to be considered in Reactor Dynamics
  - Neutron Balance
    - *Distinction between prompt and delayed neutrons*
    - *Time dependent behavior of neutron flux shape and amplitude (flux level)*
      - Point kinetics neglects the change in shape, but consider amplitude change only while spatial kinetics deals with both changes
  - Reactivity Feedback
    - *Fuel temperature effect*
    - *Moderator density effect*
  - Measurement of Reactivity
    - *Extent of external perturbation → different reactor behavior*
    - *Control rod worth*
    - *Degree of subcriticality*

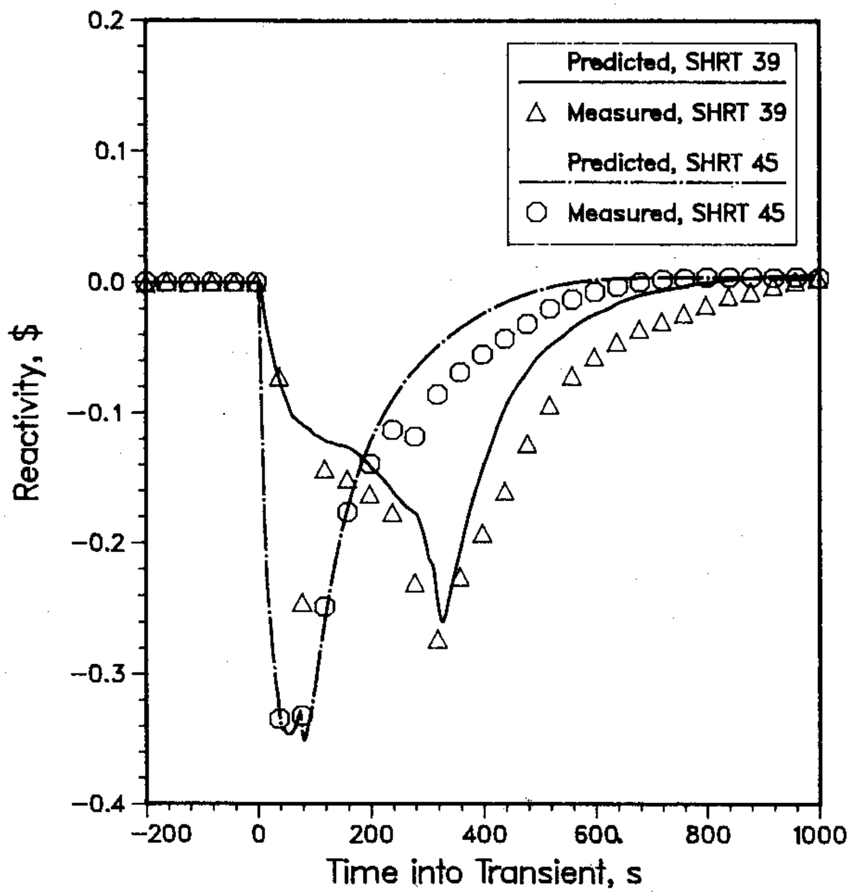
EBR-II Unprotected Loss of Flow Sequence Results

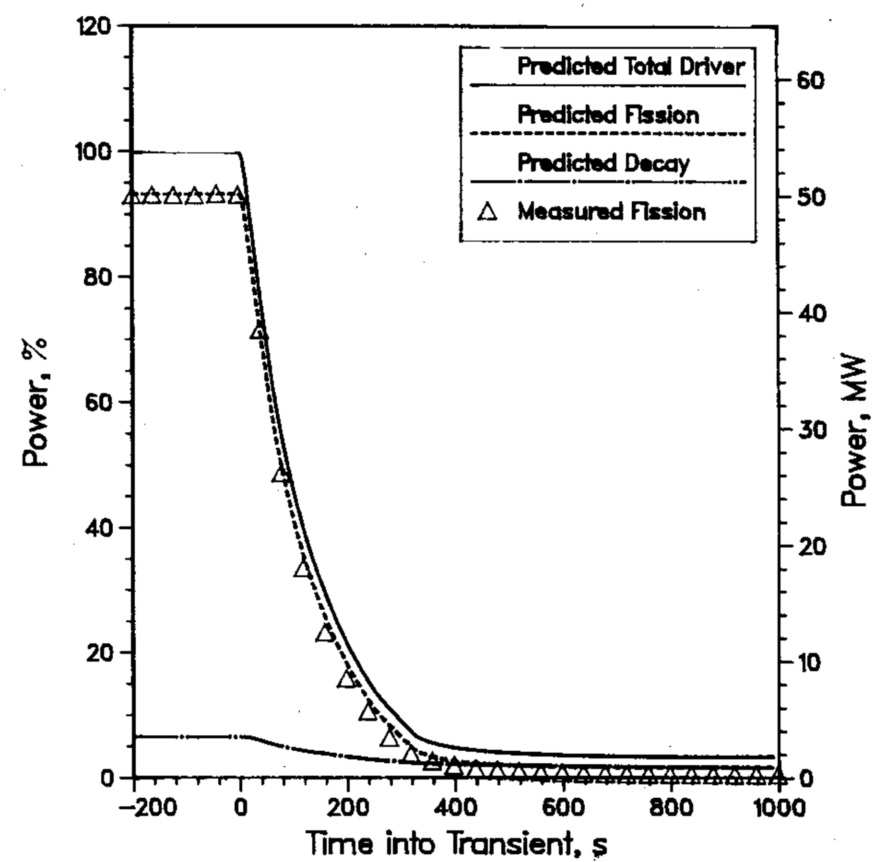




Pump coast down without scram… **causes transient temperature rise…**

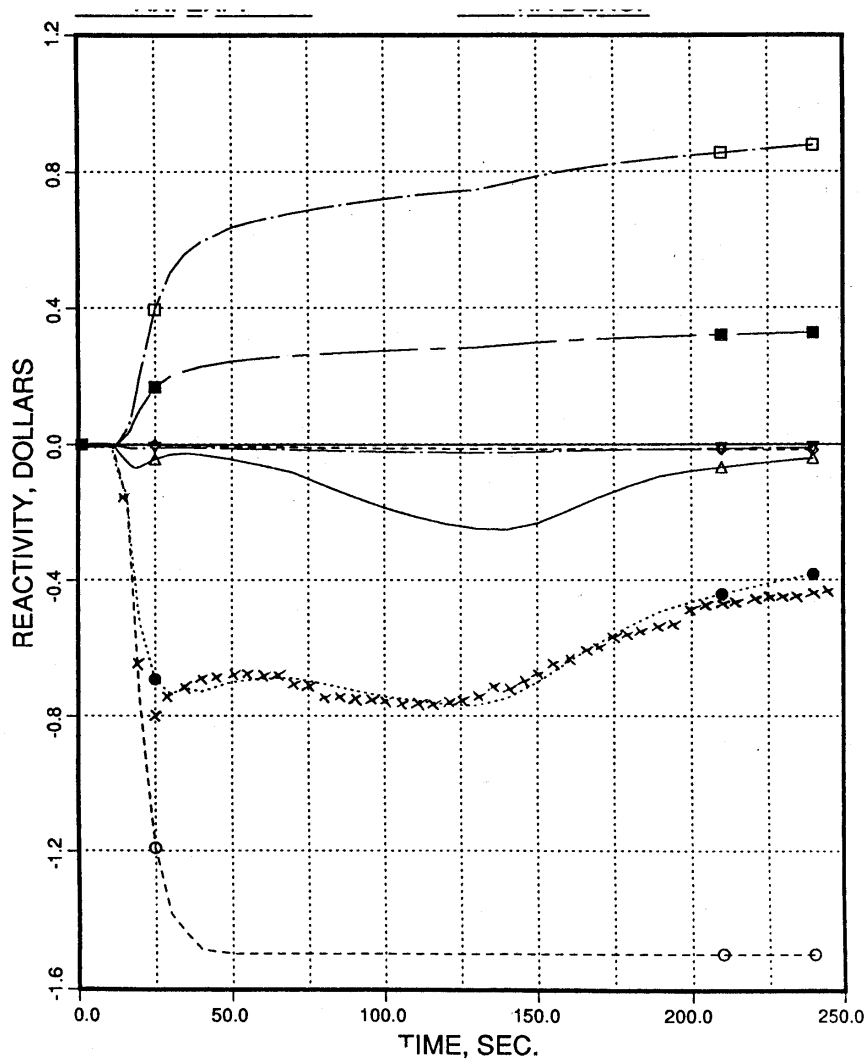
EBR-II Unprotected Loss of Flow Sequence Results





introducing negative reactivity… **reducing reactor power to decay heat.**

FFTF Inherent Safety Test Results



Comparison of Measured and Calculated Reactivity

Loss of forced reactor coolant flow without scram raises temperature, causing negative net reactivity, which reduces reactor power.

FFTF LOFWOS WITH GEMS
50 % POWER

● NET
○ PROGRAMMED
□ DOPPLER
■ AX. EXP.
x DATA
△ RAD. EXP.
◇ CRDL EXP.
▽ NA DENS.

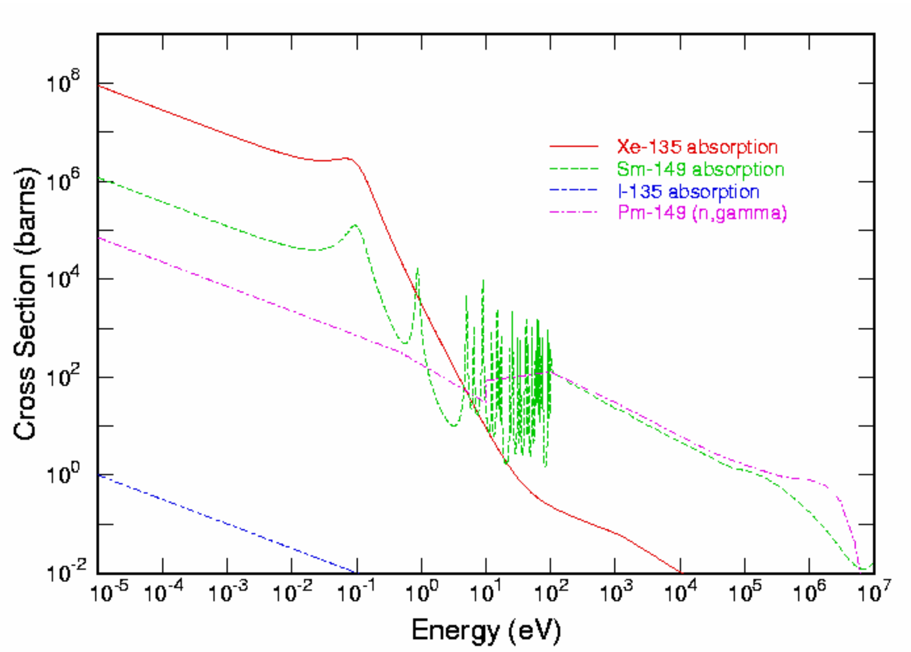
Medium Time Phenomena

- Medium time phenomena are generally associated with the buildup, burnup, and beta decay of fission products ^{135}Xe and ^{149}Sm in thermal reactors

$$ {}^{135}_{52}\mathrm{Te} \xrightarrow[19\ \mathrm{s}]{} {}^{135}_{53}\mathrm{I} \xrightarrow[6.7\ \mathrm{hr}]{} {}^{135}_{54}\mathrm{Xe} \xrightarrow[9.2\ \mathrm{hr}]{} {}^{135}_{55}\mathrm{Cs} \xrightarrow[2.3\times10^{6}\ \mathrm{yr}]{} {}^{135}_{56}\mathrm{Ba}\ (\text{stable}) $$

$$ {}^{149}_{60}\mathrm{Nd} \xrightarrow[1.7\ \mathrm{hr}]{} {}^{149}_{61}\mathrm{Pm} \xrightarrow[53\ \mathrm{hr}]{} {}^{149}_{62}\mathrm{Sm}\ (\text{stable}) $$

  - These two fission products have very large thermal neutron absorption cross sections and thus require special attention in thermal reactors
- Appendix A of textbook



Long Time Phenomena

- Long time phenomena include particularly the burnup and buildup of fissionable isotopes, as well as the buildup, beta decay and burnup of fission products
- Fuel burnup analysis is concerned with long-term reactor behavior which is determined by
  - In-reactor fuel irradiation
  - Various operations performed on fuel between burn cycles
    - *Discharge of burned fuel*
    - *Shuffling of partially burned fuel*
    - *Loading of charged fuel*
- Fuel burnup analysis is part of fuel cycle analysis consists of three main components
  - Front-end cycle: preparation of charged fuel
  - Fuel burnup analysis
  - Back-end cycle: reprocessing of discharged fuel and spent fuel and/or waste treatment

Xe-135 and Sm-149 Poisoning

- Buildup and burnup of ^{135}Xe

$$
{}^{135}_{52}\text{Te} \xrightarrow[19\text{ s}]{} {}^{135}_{53}\text{I} \xrightarrow[6.7\text{ hr}]{} {}^{135}_{54}\text{Xe} \xrightarrow[9.2\text{ hr}]{} {}^{135}_{55}\text{Cs} \xrightarrow[2.3\times10^{6}\text{ yr}]{} {}^{135}_{56}\text{Ba (stable)}
$$

$$
\frac{dN_I}{dt} = \gamma_I F - (\lambda_I + \sigma_I \phi) N_I
$$

$$
\frac{dN_{Xe}}{dt} = \gamma_{Xe} F + \lambda_I N_I - (\lambda_{Xe} + \sigma_{Xe} \phi) N_{Xe}
$$

$$
F(t) = \Sigma_f \phi(t) \quad \text{(Fission rate)}
$$

- Buildup and burnup of ^{149}Sm

$$
{}^{149}_{60}\text{Nd} \xrightarrow[1.7\text{ hr}]{} {}^{149}_{61}\text{Pm} \xrightarrow[53\text{ hr}]{} {}^{149}_{62}\text{Sm (stable)}
$$

$$
\frac{dN_{Pm}}{dt} = \gamma_{Pm} F - \bar{\lambda}_{Pm} N_{Pm}
$$

$$
\frac{dN_{Sm}}{dt} = \lambda_{Pm} N_{Pm} - \sigma_{Sm} \phi N_{Sm}
$$

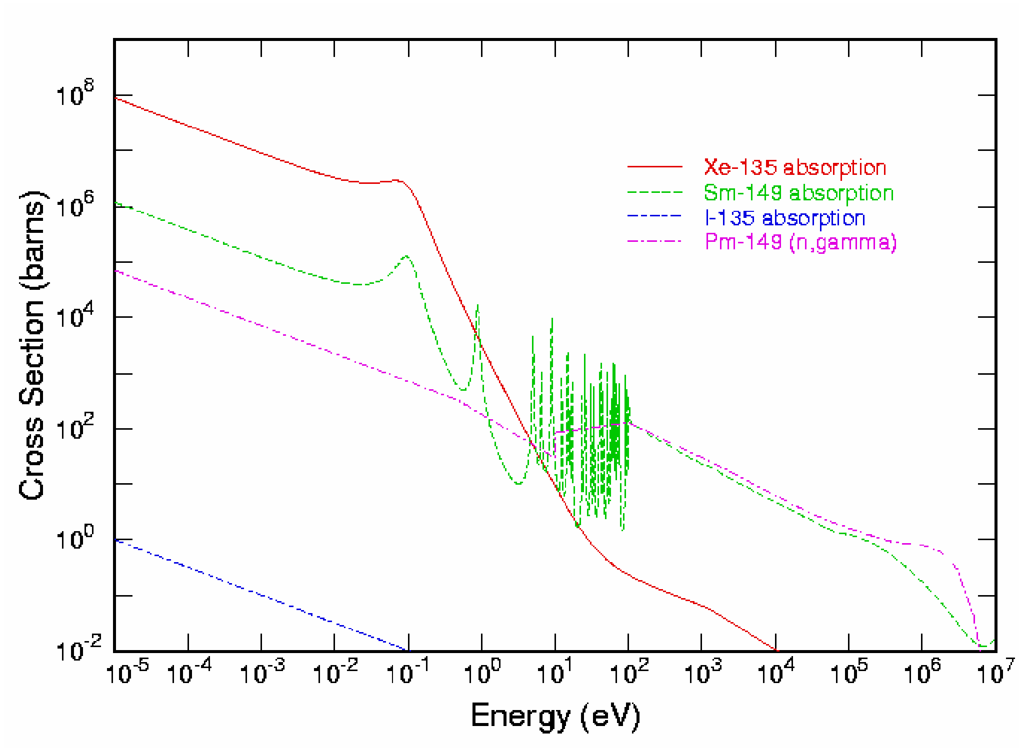
Nuclide	Half-life	XS at 0.0253 eV (b)
Te-135	19.0 s	
I-135	6.57 hr	0.020
Xe-135	9.14 hr	2647580
Cs-135	2.3E+06 yr	8.702
Ba-135	stable	5.796
Nd-149	1.728 hr	
Pm-149	53.08 hr	1400
Sm-149	> 2E+15 yr	40150

Yield Fractions

- Decay data of reaction products is given in the FILE 8 of ENDF/B
  - Fission product yield data (MT=454 and 459) for fissionable materials and spontaneous radioactive decay data (MT=457) are included
  - Independent yields (MT=454) are direct yields per fission prior to delayed neutron, beta, etc., decay. The sum of all independent yields is 2.0 for any particular incident neutron energy
  - Cumulative yields (MT=459) are specified for the same set of fission products. These accounts for all decay branches, including delayed neutrons
- Decay chain models
  - I-135, Xe-135, and Cs-135 for the Xe-135 chain
  - Pm-149 and Sm-149 for the Sm-149 chain.
- Cumulative yield for I-135 and Pm-149, but the independent yields for the other isotopes

Nuclide	Independent	Cumulative
I-135	2.9274E-02	**6.2819E-02**
Xe-135	**2.5663E-03**	6.5385E-02
Cs-135	**4.9096E-06**	6.5390E-02
Pm-149	3.8697E-08	**1.0816E-02**
Sm-149	**1.7099E-12**	1.0816E-02

Thermal XSs and Westcott's Non-1/v Factors



@ 0.0253 eV	Xe-135	Sm-149
capture (b)	2647580	40150

Temp. (°C)	Xe-135	Sm-149
20	1.1581	1.6170
100	1.2103	1.8874
200	1.2360	2.0903
400	1.1864	2.1854
600	1.0914	2.0852
800	0.9887	1.9246
1000	0.8858	1.7568

$$\bar{\sigma}_a = \int_0^{E_{th}} \sigma_a(E)\varphi_M(E)dE \approx \sigma_0\sqrt{\frac{T_0}{T}}\int_0^{\infty}\sqrt{\frac{E}{kT}}e^{-E/kT}\frac{dE}{kT} = \frac{\sqrt{\pi}}{2}\sigma_0\sqrt{\frac{T_0}{T}}$$

$$\bar{\sigma}_a = \frac{\sqrt{\pi}}{2}\sigma_0\sqrt{\frac{T_0}{T}}\,g_a(T)$$

Xe-135 Concentration (1)

- System of differential equations

$$\frac{dN_I}{dt} = \gamma_I F(t) - \bar{\lambda}_I N_I(t)$$

$$\frac{dN_{Xe}}{dt} = \gamma_{Xe} F(t) + \lambda_I N_I(t) - \bar{\lambda}_{Xe} N_{Xe}(t)$$

$$\bar{\lambda}_I = \lambda_I + \sigma_I \phi(t), \quad \bar{\lambda}_{Xe} = \lambda_{Xe} + \sigma_{Xe}\phi(t)$$

(Effective decay constants including burnup)

- I-135 and Xe-135 concentrations for constant thermal flux

$$\frac{d}{dt}[N_I(t)e^{\bar{\lambda}_I t}] = \gamma_I F e^{\bar{\lambda}_I t}$$

$$\frac{d}{dt}[N_{Xe}(t)e^{\bar{\lambda}_{Xe} t}] = [\gamma_{Xe} F + \lambda_I N_I(t)]e^{\bar{\lambda}_{Xe} t}$$

$$N_I(t) = N_I^0 e^{-\bar{\lambda}_I t} + \frac{\gamma_I F}{\bar{\lambda}_I}(1 - e^{-\bar{\lambda}_I t})$$

$$N_{Xe}(t) = N_{Xe}^0 e^{-\bar{\lambda}_{Xe} t} + \frac{\gamma_{Xe} F}{\bar{\lambda}_{Xe}}(1 - e^{-\bar{\lambda}_{Xe} t}) + \frac{\lambda_I N_I^0}{\bar{\lambda}_{Xe} - \bar{\lambda}_I}(e^{-\bar{\lambda}_I t} - e^{-\bar{\lambda}_{Xe} t})$$
$$+ \frac{\lambda_I \gamma_I F}{\bar{\lambda}_I}\left[\frac{1}{\bar{\lambda}_{Xe}}(1 - e^{-\bar{\lambda}_{Xe} t}) - \frac{1}{\bar{\lambda}_{Xe} - \bar{\lambda}_I}(e^{-\bar{\lambda}_I t} - e^{-\bar{\lambda}_{Xe} t})\right]$$

Xe-135 Concentration (2)

- Equilibrium concentrations
  - Because of half-lives of Xe-135 and I-135 are so short and the absorption cross section of Xe-135 is so large, the concentrations of these isotopes quickly arise to their saturation or equilibrium values

$$\bar{\lambda}_I = \lambda_I + \sigma_I \phi(t) \cong \lambda_I$$

$$N_I^\infty = \frac{\gamma_I F}{\bar{\lambda}_I} \cong \frac{\gamma_I F}{\lambda_I}$$

$$N_{Xe}^\infty = \frac{\gamma_{Xe} F + \lambda_I N_I^\infty}{\bar{\lambda}_{Xe}} \cong \frac{(\gamma_{Xe} + \gamma_I) F}{\bar{\lambda}_{Xe}}$$

- Using equilibrium concentrations, the time-dependent concentrations can be rewritten as

$$N_I(t) = N_I^0 e^{-\lambda_I t} + N_I^\infty (1 - e^{-\lambda_I t})$$

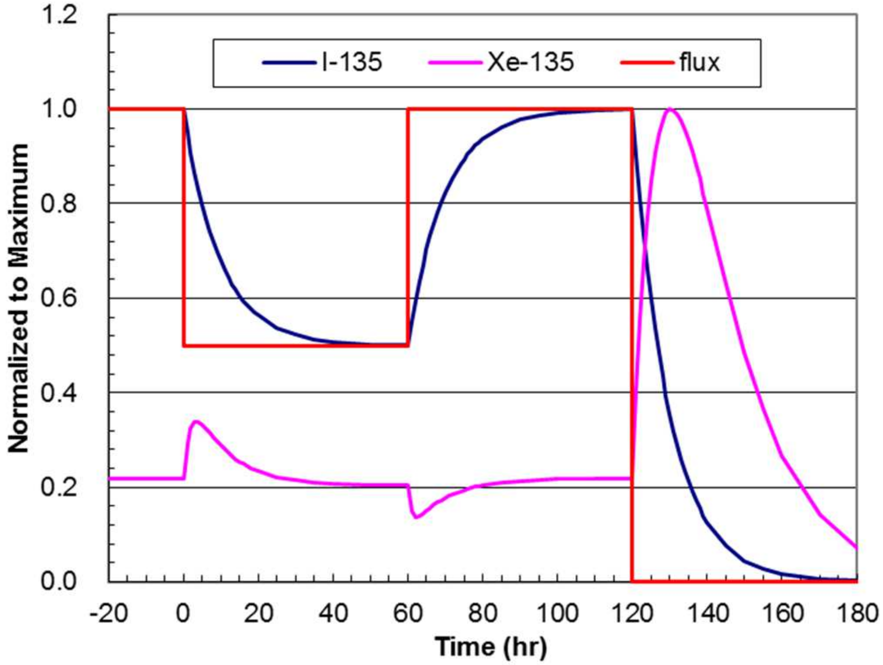
$$N_{Xe}(t) = N_{Xe}^0 e^{-\bar{\lambda}_{Xe} t} + N_{Xe}^\infty (1 - e^{-\bar{\lambda}_{Xe} t}) + \frac{\lambda_I N_I^0 - \gamma_I F}{\bar{\lambda}_{Xe} - \lambda_I} (e^{-\lambda_I t} - e^{-\bar{\lambda}_{Xe} t})$$

Xe-135 Concentration (3)

- Xe-135 buildup after shutdown
  - Because of the decay of I-135, the Xe-135 concentration initially increases after shutdown, although it eventually disappears by its own decay

$$N_I(t) = N_I^0 e^{-\lambda_I t}$$

$$N_{Xe}(t) = N_{Xe}^0 e^{-\lambda_{Xe} t} + \frac{\lambda_I N_I^0}{\lambda_{Xe} - \lambda_I}(e^{-\lambda_I t} - e^{-\lambda_{Xe} t})$$



I-135 and Xe-135 Buildup and Burnout Following Step Changes of Power

Sm-149 Concentration (1)

- System of differential equations

$$\frac{dN_{Pm}}{dt} = \gamma_{Pm} F(t) - \bar{\lambda}_{Pm} N_{Pm}(t)$$

$$\frac{dN_{Sm}}{dt} = \lambda_{Pm} N_{Pm}(t) - \bar{\lambda}_{Sm} N_{Sm}(t)$$

$$\bar{\lambda}_{Pm} = \lambda_{Pm} + \sigma_{Pm}\phi(t), \quad \bar{\lambda}_{Pm} = \sigma_{Pm}\phi(t)$$

(Effective decay constant including burnup)

- Pm-149 and Sm-149 concentrations for constant thermal flux

$$N_{Pm}(t) = N_{Pm}^{0} e^{-\bar{\lambda}_{Pm} t} + \frac{\gamma_{Pm} F}{\bar{\lambda}_{Pm}} (1 - e^{-\bar{\lambda}_{Pm} t})$$

$$N_{Sm}(t) = N_{Sm}^{0} e^{-\bar{\lambda}_{Sm} t} + \frac{\lambda_{Pm} N_{Pm}^{0}}{\bar{\lambda}_{Sm} - \bar{\lambda}_{Pm}} (e^{-\bar{\lambda}_{Pm} t} - e^{-\bar{\lambda}_{Sm} t})$$

$$+ \frac{\lambda_{Pm} \gamma_{Pm} F}{\bar{\lambda}_{Pm}} \left[\frac{1}{\bar{\lambda}_{Sm}} (1 - e^{-\bar{\lambda}_{Sm} t}) - \frac{1}{\bar{\lambda}_{Sm} - \bar{\lambda}_{Pm}} (e^{-\bar{\lambda}_{Pm} t} - e^{-\bar{\lambda}_{Sm} t}) \right]$$

- Equilibrium concentrations

$$N_{Pm}^{\infty} = \frac{\gamma_{Pm} F}{\bar{\lambda}_{Pm}}, \quad N_{Sm}^{\infty} = \frac{\lambda_{Pm} \gamma_{Pm} F}{\bar{\lambda}_{Pm} \bar{\lambda}_{Sm}}$$

Sm-149 Concentration (2)

- Using equilibrium concentrations, the time-dependent concentrations can be rewritten as

$$N_{Pm}(t) = N_{Pm}^{0} e^{-\bar{\lambda}_{Pm} t} + N_{Pm}^{\infty}(1 - e^{-\bar{\lambda}_{Pm} t})$$

$$N_{Sm}(t) = N_{Sm}^{0} e^{-\bar{\lambda}_{Sm} t} + N_{Sm}^{\infty}(1 - e^{-\bar{\lambda}_{Sm} t}) + \frac{\lambda_{Pm}(N_{Pm}^{0} - \gamma_{Pm} F / \bar{\lambda}_{Pm})}{\bar{\lambda}_{Sm} - \bar{\lambda}_{Pm}}(e^{-\bar{\lambda}_{Pm} t} - e^{-\bar{\lambda}_{Sm} t})$$

- Sm-149 buildup after shutdown

$$N_{Pm}(t) = N_{Pm}^{0} e^{-\lambda_{Pm} t}$$

$$N_{Sm}(t) = N_{Sm}^{0} e^{-\lambda_{Sm} t} + \frac{\lambda_{Pm} N_{Pm}^{0}}{\lambda_{Sm} - \lambda_{Pm}}(e^{-\lambda_{Pm} t} - e^{-\lambda_{Sm} t})$$

$$= N_{Sm}^{0} + N_{Pm}^{0}(1 - e^{-\lambda_{Pm} t})$$

Fission Product Poisoning in Thermal Reactors

- In thermal systems, the effect of fission product poisons on the multiplication factor can be approximated by the change of thermal utilization

$$k = k_\infty P_{NL} = \varepsilon \eta_T f p P_{NLF} P_{NLT} \quad \text{(six factor formula)}$$

$$\eta_T = \frac{\nu \Sigma_f}{\Sigma_{aF}} \quad \text{(average fission neutrons per thermal neutron absorbed in fuel)}$$

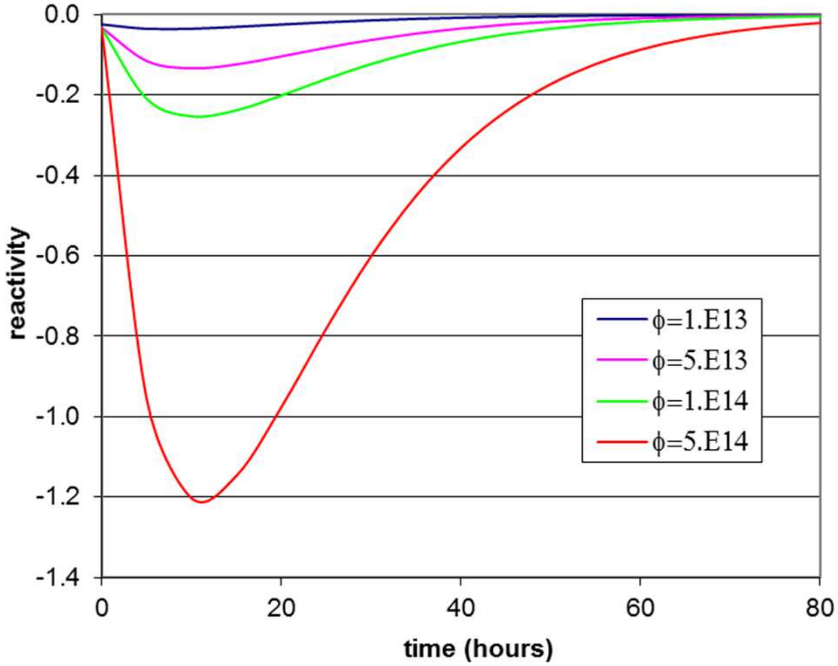
$$f = \frac{\Sigma_{aF}}{\Sigma_a} \quad \text{(thermal utilization factor)}$$

$$\rho = \frac{\delta k}{k} = \frac{\delta k_\infty}{k_\infty} = \frac{\delta f}{f} = -\frac{\delta \Sigma_a}{\Sigma_a}$$

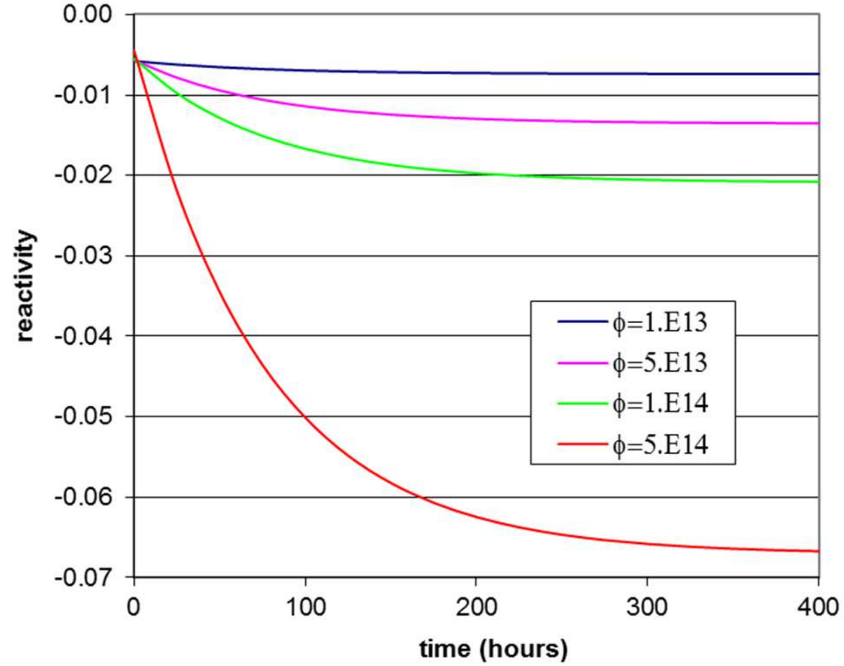
$$\Sigma'_a = \Sigma_a + \Sigma_{aP} \quad \text{(thermal absorption XS with FP poisons)}$$

$$\rho = -\frac{\delta \Sigma_a}{\Sigma_a} = -\frac{\Sigma_{aP}}{\Sigma_a} = -f\frac{\Sigma_{aP}}{\Sigma_{aF}} = -f\eta_T \frac{\Sigma_{aP}}{\nu \Sigma_f} = -\frac{k_\infty}{p\varepsilon}\frac{\Sigma_{aP}}{\nu \Sigma_f}$$

Xe-135 and Sm-149 Poisoning after Shutdown



Reactivity of Xe-135 Buildup after Shutdown



Reactivity of Sm-149 Buildup after Shutdown