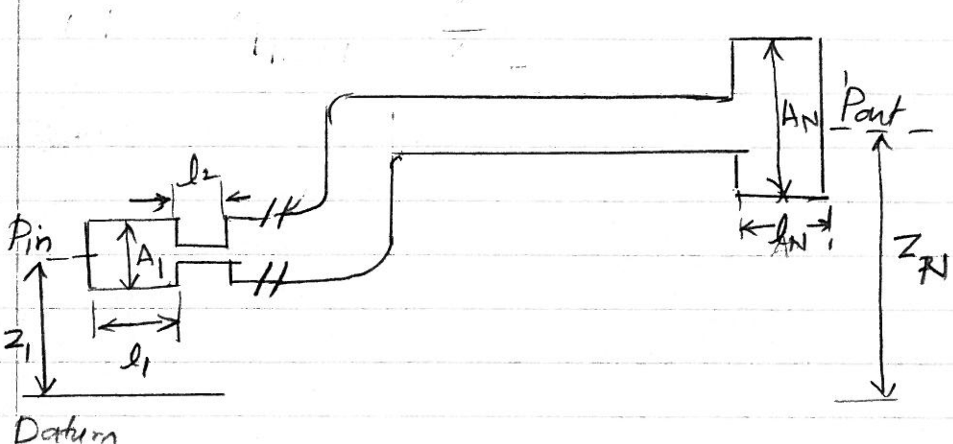
Single Phase Pressure Drop.

Any channel:

$$p_{in} - p_{out} = \Delta p_{inertia} + \Delta p_{acc} + \Delta p_{gravity} + \Delta p_{friction} + \Delta p_{form}$$



where $\Delta p_{inertia} = \left(\frac{l}{A}\right)_T \left(\frac{d\dot{m}}{dt}\right)$

$$\Delta p_{acc} = \frac{\dot{m}^2}{2\rho}\left(\frac{1}{A_N^2} - \frac{1}{A_1^2}\right)$$

$$\Delta p_{gravity} = \rho g \,(z_N - z_1)$$

$$\Delta p_{form} \equiv K\left(\frac{\rho V_{ref}^2}{2}\right)$$

$$\Delta p \equiv f \frac{L}{D}\left(\frac{\rho V_{ref}^2}{2}\right)$$

* Table 9-1. for K- form losses.

friction factor $f = \frac{64}{Re}$ for laminar flow

Fig. 9-15, 9-16, 9-17.

Turbulent flow: $\frac{1}{\sqrt{f}} = -0.8 + 0.87 \ln(Re\sqrt{f})$

Table 9-1 Form loss coefficients for various flow restrictions*

Parameter	$(L/D)_{equiv}$	K	Reference velocity
Pipe entrance from a plenum			
Well rounded entrance to pipe		0.04	In pipe
Slightly rounded entrance to pipe		0.23	In pipe
Sharp-edged entrance		0.50	In pipe
Projecting pipe entrance		0.78	In pipe
Pipe exit to a plenum			
Any pipe exit		1.0	In pipe
Sudden changes in cross-sectional area			
Sudden contraction		$0.5\,(1 - \beta^2)$	Downstream
Sudden expansion		$(1 - \beta)^2$	Upstream
where $\beta \equiv \dfrac{\text{small cross-sectional area}}{\text{large cross-sectional area}}$			
Fittings†			
90° Standard elbow	30	0.35–0.9	
90° Long-radius elbow	20	0.2 –0.6	
45° Standard elbow	16	0.17–0.45	
Standard tee (flow through run)	20	0.2 –0.6	
Standard tee (flow through branch)	60	0.65–1.70	
Valves (various types)			
Fully open		0.15–15.00	
Half-closed		13 –450	

*Approximate values; consult Idelchik [13] for extensive tabulation. Also, section VII gives an accounting for the theoretical basis for obtaining K.

†Values of K depend on the pipe diameter.

and

$$K_T = \sum_i K_i \left(\frac{A_{ref}}{A_i}\right)^2$$

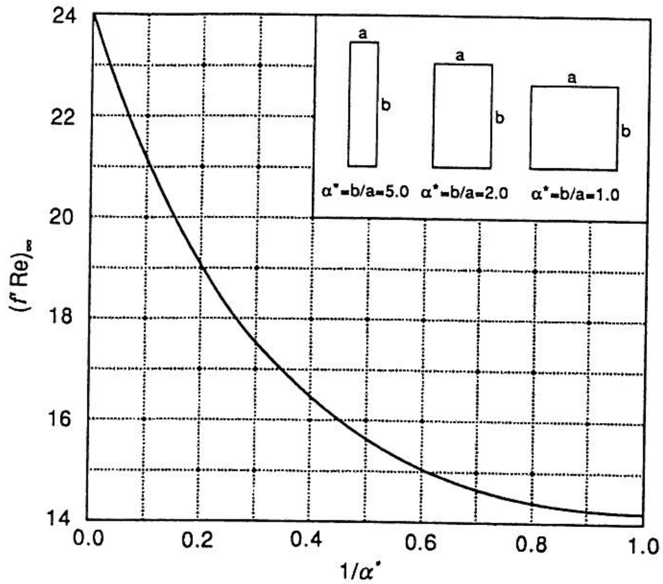


Figure 9-15 Product of laminar friction factor and Reynolds number for fully developed flow with rectangular geometry. *(From Kays [14].)*

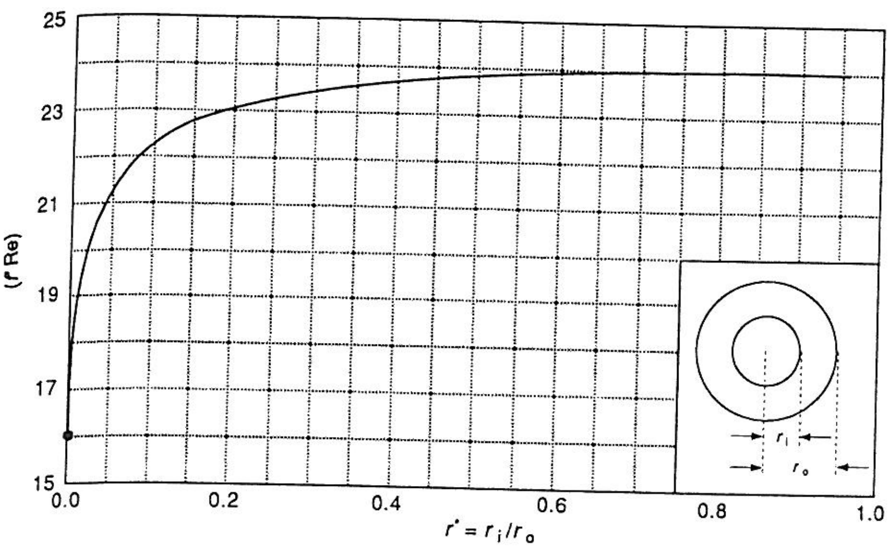


Figure 9-16 Product of laminar friction factor and Reynolds number for fully developed flow in an annular channel. *(From Kays [14].)*

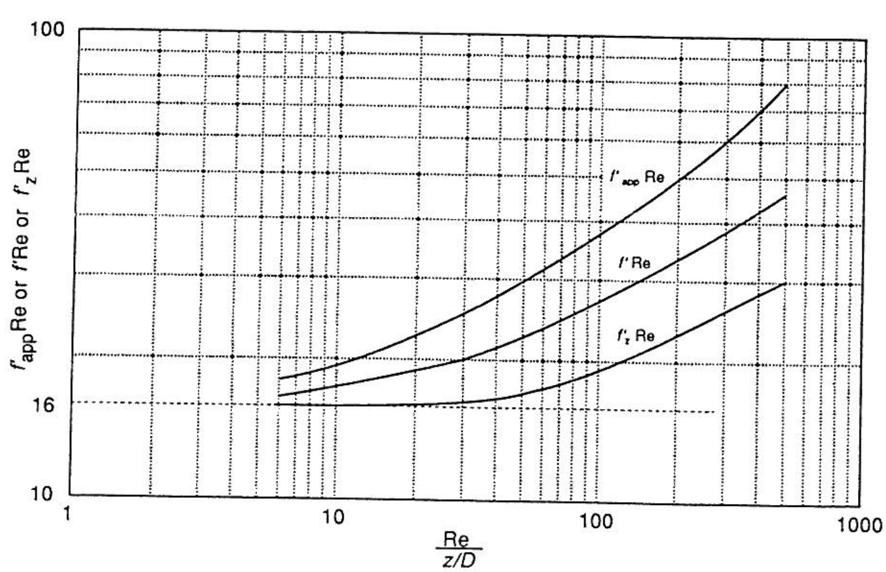


Figure 9-17 Developing laminar flow friction factor. *(From Langhaar and No [17].)*

$f = 0.184\, Re^{-0.2}$ $\quad 30{,}000 < Re < 1 \times 10^{6}$ (McAdam)

$f = 0.316\, Re^{-0.25}$ $\quad Re < 30{,}000$ (Blasius)

Fig. 9-20, Moody's chart

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{\lambda/D}{3.70} + \frac{2.51}{Re\sqrt{f}}\right] \text{ —(Colebrook)}$$

Bare Rod bundle:

Fig 9-22, Table 9-2, 9-3.

Fig 9-23.

$$f_{iL} = \frac{C'_{fiL}}{Re'^{\,n}_{iL}}$$

$$C'_{fiL/T} = a + b_1(P/D - 1) + b_2(P/D - 1)^2$$

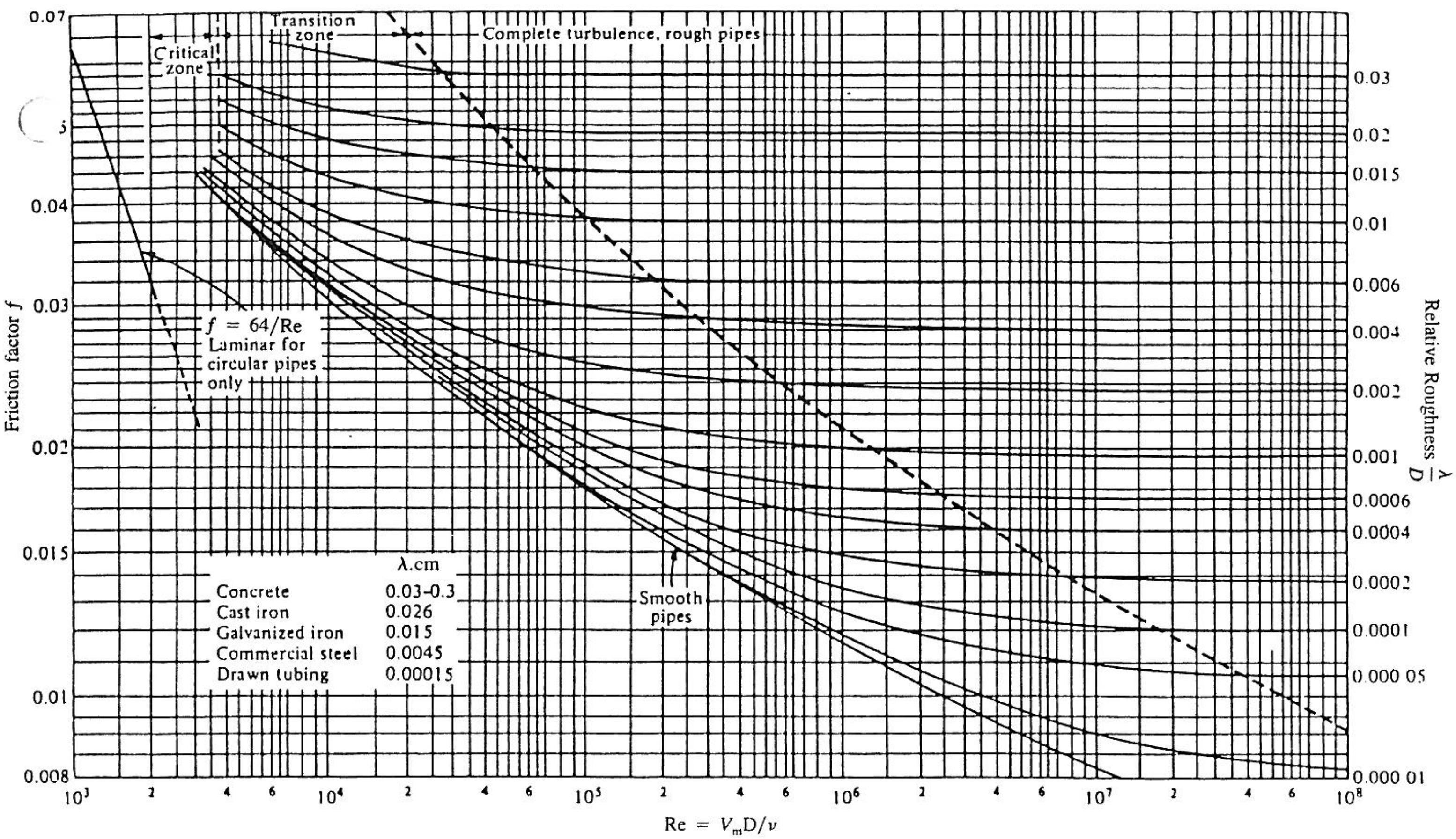


Figure 9-20 Moody's chart for friction factors: Friction factor for use in the relation Δp for pressure drop for flow inside circular pipes. *(From Moody [20].)*

Table 9-2 Coefficients in Eqs. 9-82 and 9-87 for bare rod subchannel friction factor constants C'_{fi} in hexagonal array

	$1.0 \le P/D \le 1.1$			$1.1 < P/D \le 1.5$		
Subchannel	a	b_1	b_2	a	b_1	b_2
Laminar flow						
Interior	26.00	888.2	−3334	62.97	216.9	−190.2
Edge	26.18	554.5	−1480	44.40	256.7	−267.6
Corner	26.98	1636.	−10,050	87.26	38.59	−55.12
Turbulent flow						
Interior	0.09378	1.398	−8.664	0.1458	0.03632	−0.03333
Edge	0.09377	0.8732	−3.341	0.1430	0.04199	−0.04428
Corner	0.1004	1.625	−11.85	0.1499	0.006706	−0.009567

Table 9-3 Coefficients in Eqs. 9-82 and 9-87 for bare rod subchannel friction factor constants C'_{fi} in square array

	$1.0 \le P/D \le 1.1$			$1.1 < P/D \le 1.5$		
Subchannel	a	b_1	b_2	a	b_1	b_2
Laminar flow						
Interior	26.37	374.2	−493.9	35.55	263.7	−190.2
Edge	26.18	554.5	−1480	44.40	256.7	−267.6
Corner	28.62	715.9	−2807	58.83	160.7	−203.5
Turbulent flow						
Interior	0.09423	0.5806	−1.239	0.1339	0.09059	−0.09926
Edge	0.09377	0.8732	−3.341	0.1430	0.04199	−0.04428
Corner	0.09755	1.127	−6.304	0.1452	0.02681	−0.03411

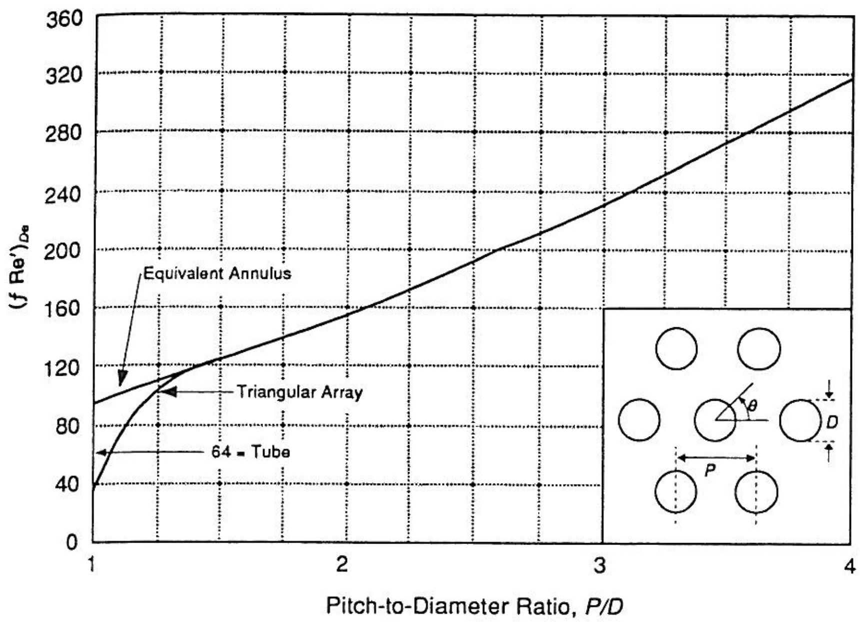


Figure 9-22 Product of laminar friction factors and Reynolds number for parallel flow in a rod bundle. (*From Sparrow and Loeffler* [28].)

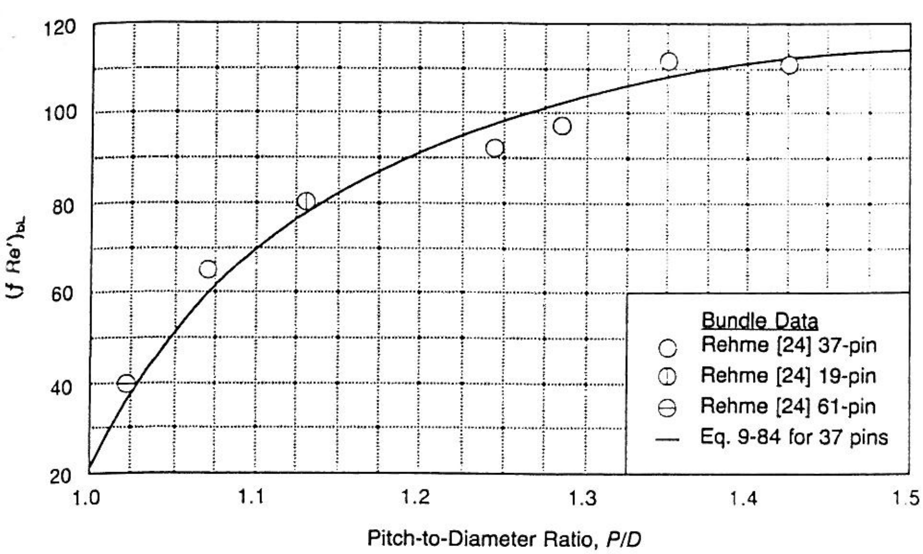


Figure 9-23 Laminar flow results in triangular array bare rod bundles. (*From Cheng and Todreas* [3].)

Two-Phase Pressure Drop

- Flow Regime: Verticle: bubbly
  slug
  Churn
  Annular

  Horizontal: Bubbly
  Plug
  Stratified
  Wavy
  Slug.
  Annular.

- Flow Regime Maps.

$$\Delta p = \Delta p_{acc} + \Delta p_{fric} + \Delta p_{gravity}$$

$$\Delta p_{acc} = \left(\frac{G_m^2}{\rho_m}\right)_{out} - \left(\frac{G_m^2}{\rho_m}\right)_{in}$$

$$\Delta p_{fric} = \int_{z_{in}}^{z_{out}} \frac{\tau_w P_z}{A_z} \cdot dz$$

$$\Delta p_{gravity} = \int_{z_{in}}^{z_{out}} \rho_m g \cos\theta \, dz$$

$$\left(\frac{dp}{dz}\right)_{fric} = \frac{\tau_w P_w}{A_z} \equiv \frac{f_{TP}}{D_e}\left[\frac{G_m^2}{2\rho_m}\right]$$

$$D_e = \frac{4A_z}{P_w.}$$ – equivalent hydraulic diameter

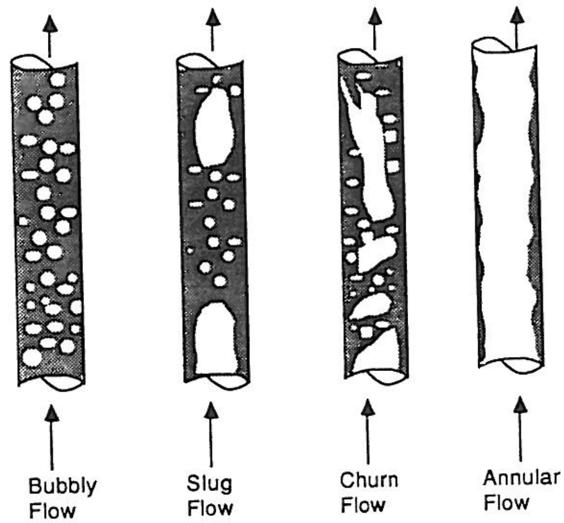


Figure 11-2 Flow patterns in vertical flow.

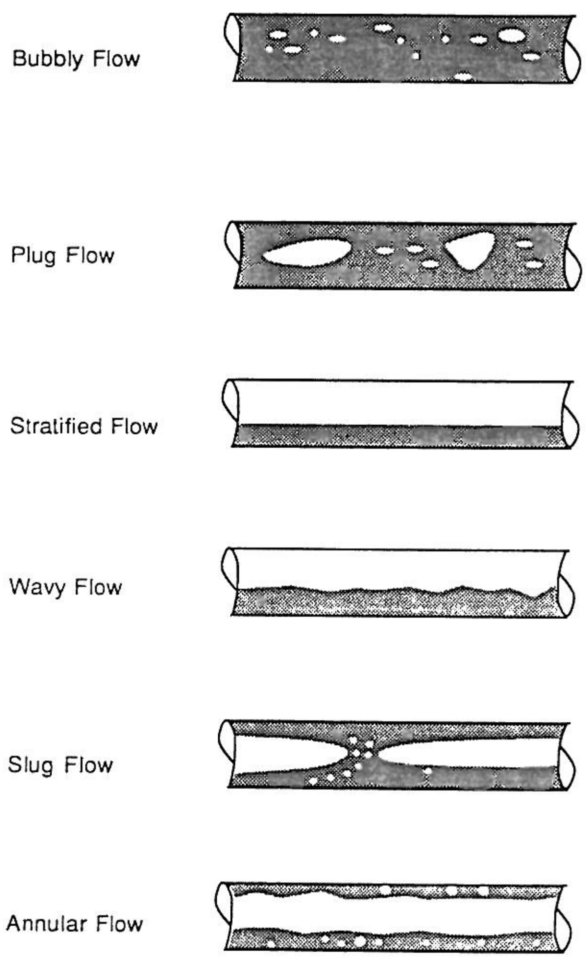


Figure 11-3 Flow patterns in horizontal flow.

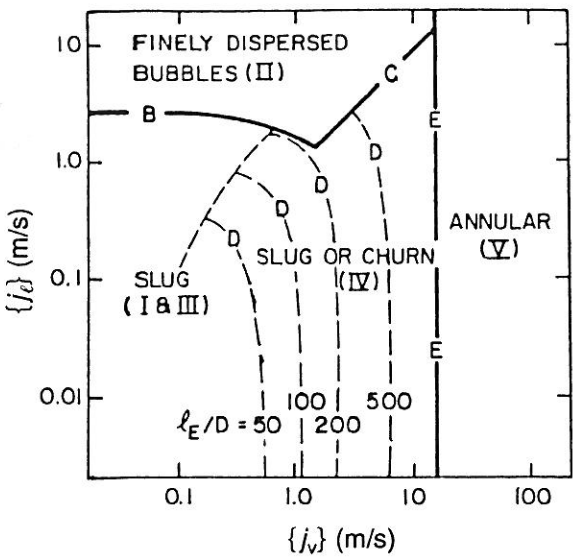


Figure 11-9 Flow regime map of Taitel et al. [36] for air–water at 25°C and 0.1 MPa in 25 mm diameter tube.

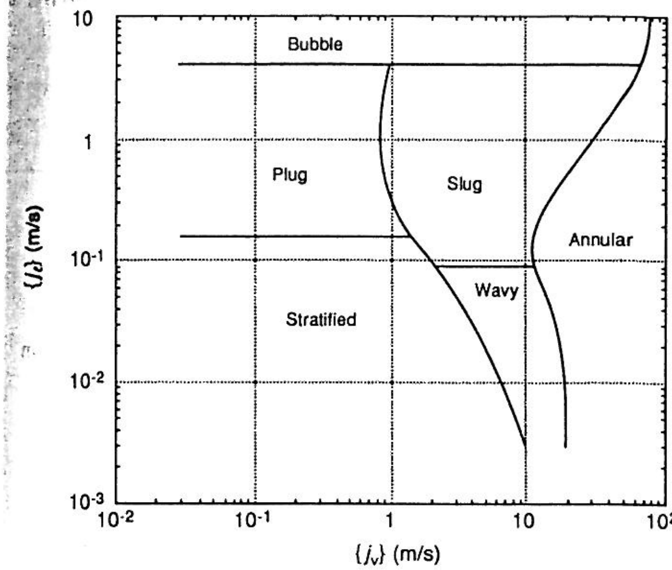


Figure 11-10 Flow map of Mandhane et al. [26] for horizontal flow.

Table 11-3 Two-phase multiplier ($\phi^2_{\ell o}$) of various models

	$\phi^2_{\ell o}$ at various qualities (x)					
p(psia)	x = 0.0	x = 0.1	x = 0.2	x = 0.5	x = 0.8	x = 1.0
1020	1	2.73	4.27	8.30	11.81	13.98
1020	1	2.07	4.14	10.35	16.6	20.7
1020	1	5.4	8.6	17.0	22.9	15.0
738	1	3.9	6.4	12.9	18.5	21.9
738	1	2.98	5.96	14.9	23.8	29.8
738	1	7.1	12.4	25.5	35	22.5
291	1	8.25	14.4	29.7	42.9	51.0
291	1	8.5	17.0	42.5	67.0	85.0
291	1	18.4	36.2	90	132	80.0

• Single phase mass flux ≡ two-phase mass flux

$$\left(\frac{dp}{dz}\right)^{TP}_{fric} = \phi_{lo}^2 \left(\frac{dp}{dz}\right)^{lo}_{fric} = \phi_{vo}^2 \left(\frac{dp}{dz}\right)^{vo}_{fric}$$

single phase multipliers → $\phi_{lo}^2 = \frac{\rho_l}{\rho_m} \frac{f_{TP}}{f_{lo}}$

" $\phi_{vo}^2 = \frac{\rho_v}{\rho_m} \frac{f_{TP}}{f_{vo}}$

• Homogeneous Equilibrium Model
  - Two-phase have equal velocities.
  - " " thermodynamic equilibrium.

Table 11-3.

• Separated Flow Model.
  Lockhart-Martinelli -

$$\left(\frac{dp}{dz}\right)^{TP}_{fric} = \phi_l^2 \left(\frac{dp}{dz}\right)^{l}_{fric} = \phi_v^2 \left(\frac{dp}{dz}\right)^{v}_{fric}$$

• single phase flow separately.

$$\phi_l^2 = 1 + \frac{C}{X} + \frac{1}{X^2}$$

$$\phi_v^2 = 1 + CX + X^2$$

Liquid-Gas	C
T-T	20
V-T	12
T-V	10
V-V	5

T - Turbulent
V - Viscous (laminar)

Fig. 11-14

Martinelli - Nelson. (Steam-Water)

Fig 11-15.

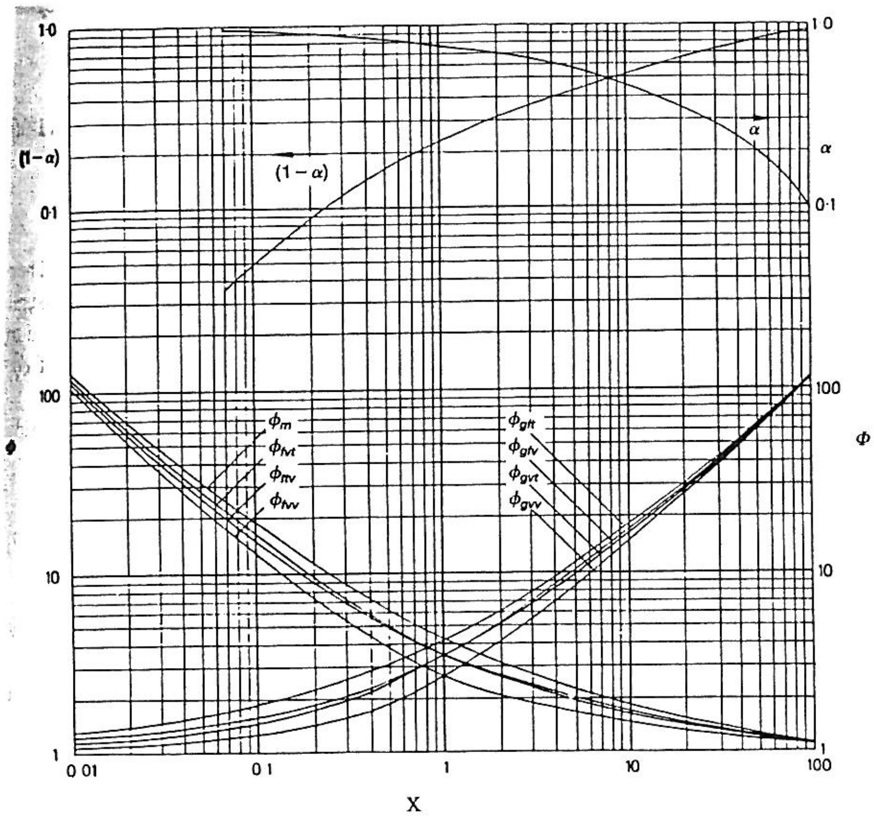


Figure 11-14 Martinelli model for pressure gradient ratios and void fractions *(From Martinelli and Nelson* [27].*)*

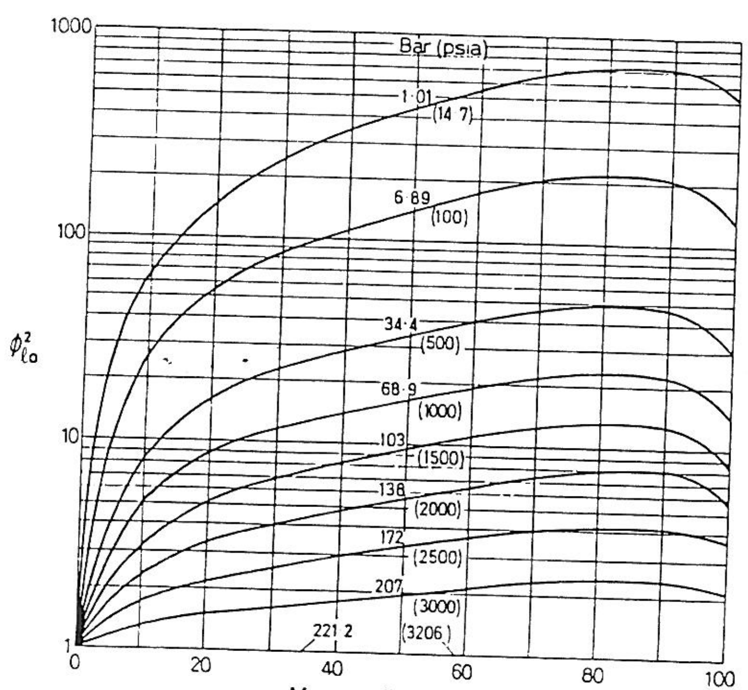


Figure 11-15 Martinelli-Nelson's $\phi^2_{\ell o}$ as a function of quality and pressure. *(From Martinelli and Nelson* [27].*)*

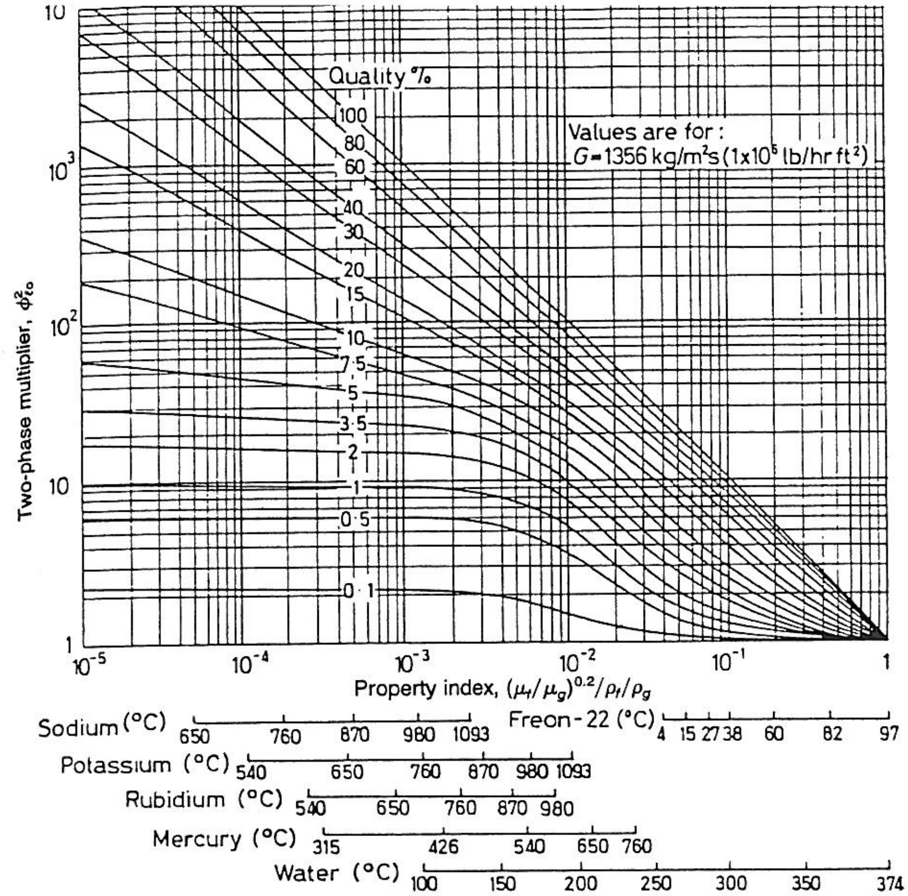


Figure 11-21 Baroczy two-phase friction multiplier ($\overline{\phi^2_{\ell o}}$). *(From Baroczy [4].)*

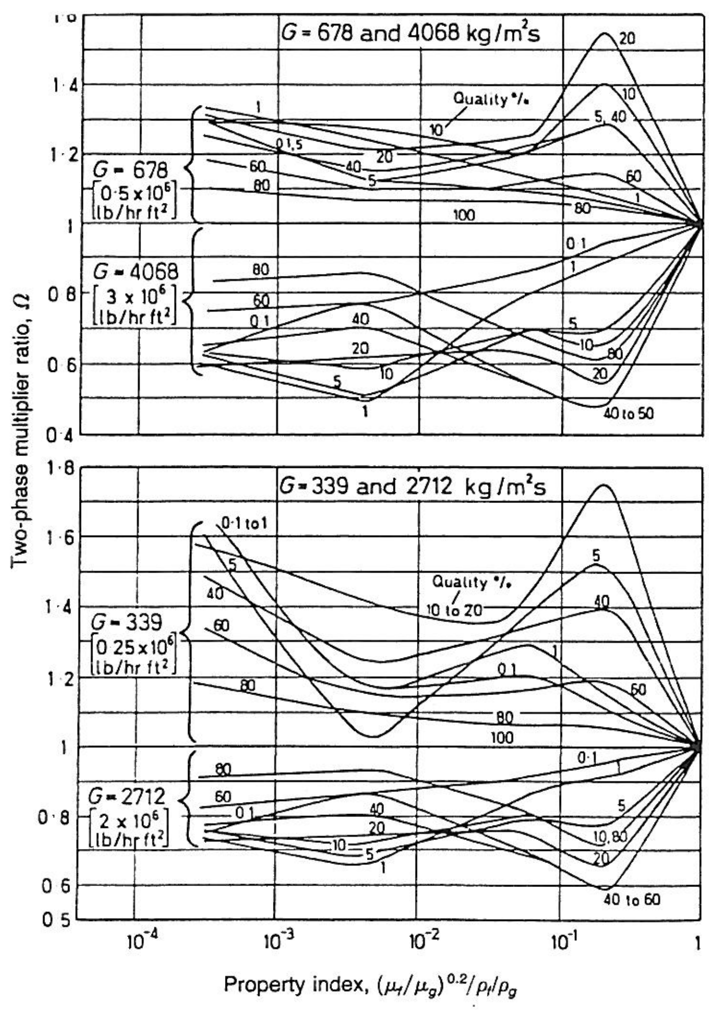


Figure 11-22 Baroczy mass flux correction factor. *(From Baroczy [4].)*