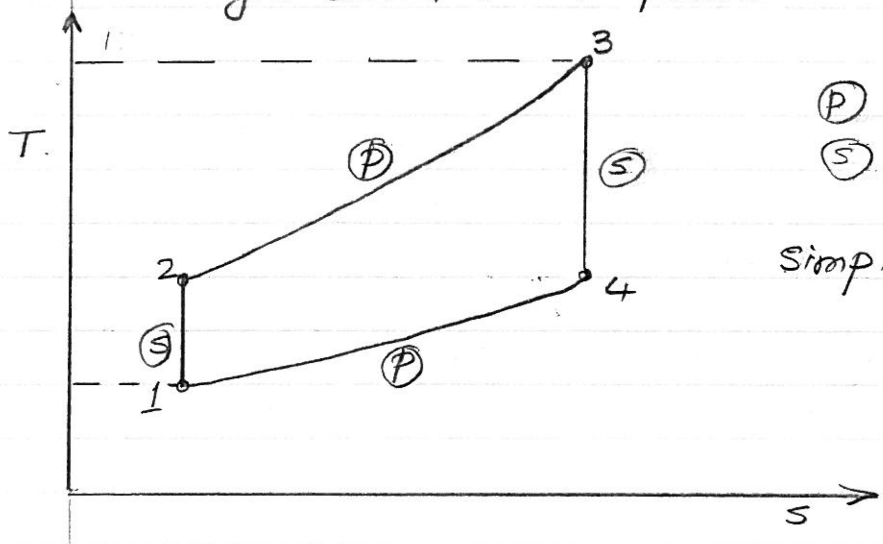
Brayton Cycle

- gas turbine plants
- gas-cooled reactor plants
- single-phase



Process
P - constant pressure
S - constant entropy

Simple Brayton Cycle.

Pressure or compression ratio of the cycle

$$r_p = \frac{p_2}{p_1} \equiv \frac{p_3}{p_4}$$

For a perfect gas with isentropic processes.

$$T V^{\gamma-1} = \text{constant}$$

$$T p^{\frac{1-\gamma}{\gamma}} = \text{Constant} \qquad \because pV^{\gamma} = \text{const.}$$

$$\gamma \equiv c_p/c_v$$

enthalpy $\Delta h = c_p \Delta T$.

c_p - constant.

Turbine and Compressor Work

Turbine: $\dot{W}_T = \dot{m} c_p (T_3 - T_4) = \dot{m} c_p T_3 \left(1 - \frac{T_4}{T_3}\right)$

for isentropic process $\dot{W}_T = \dot{m} c_p T_3 \left[1 - \frac{1}{(r_p)^{(\gamma-1/\gamma)}}\right]$

Compressor: $\dot{W}_{cp} = \dot{m} c_p (T_2 - T_1) = \dot{m} c_p T_1 \left[(r_p)^{\frac{\gamma-1}{\gamma}} - 1 \right]$

Heat input from reactor: $\dot{Q}_R = \dot{m} c_p (T_3 - T_2) = \dot{m} c_p T_1 \left[\frac{T_3}{T_1} - (r_p)^{\frac{\gamma-1}{\gamma}} \right]$

Heat rejected by heat exchanger: $Q_{HX} = \dot{m} c_p (T_4 - T_1) = \dot{m} c_p T_3 \left[\frac{1}{(r)^{\gamma-1/\gamma}} - \frac{T_1}{T_3} \right]$

Maximum useful work

$$\dot{W}_{u,max} \equiv \dot{Q}_R = \dot{m} c_p T_1 \left[\frac{T_3}{T_1} - (r_p)^{\frac{\gamma-1}{\gamma}} \right]$$

Brayton nuclear plant thermodynamic efficiency:

$$\eta = \frac{\dot{W}_T - \dot{W}_{cp}}{\dot{W}_{u,max}} = \frac{\left[T_3 - T_1 (r_p)^{(\gamma-1/\gamma)} \right] \left[1 - \frac{1}{(r_p)^{(\gamma-1/\gamma)}} \right]}{T_1 \left[\frac{T_3}{T_1} - (r_p)^{(\gamma-1/\gamma)} \right]}$$

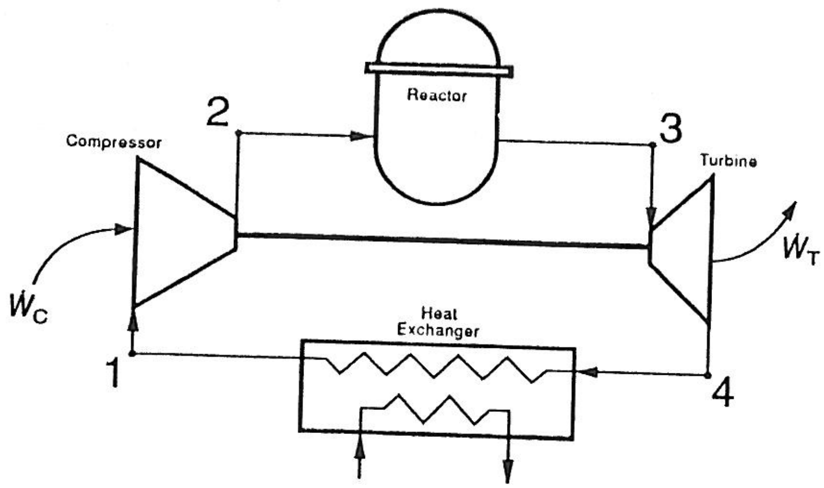
$$\eta = 1 - \frac{1}{(r_p)^{(\gamma-1/\gamma)}}$$

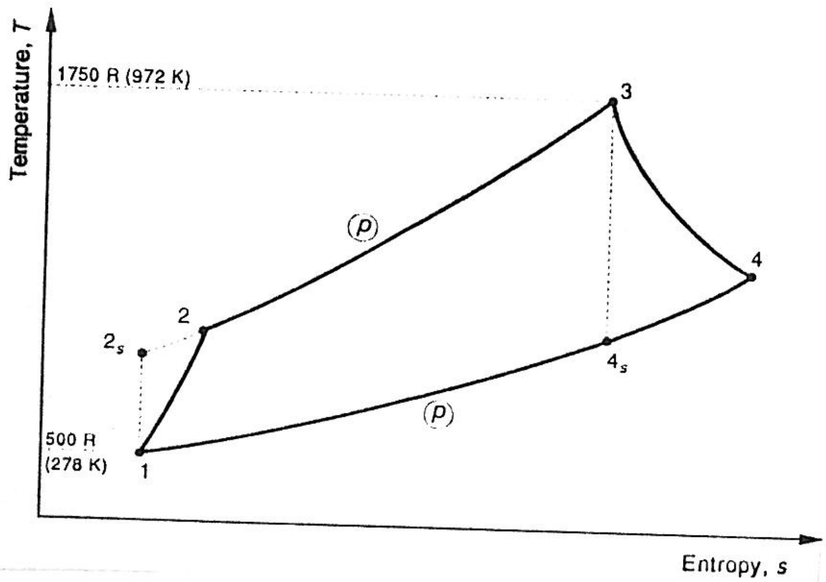
Optimum pressure ratio for maximum net work:

$$\frac{d[\dot{W}_T - \dot{W}_{cp}]}{d r_p} = 0 \quad : \quad (r_p)_{optimum} = \left(\frac{T_3}{T_1} \right)^{\gamma/2(\gamma-1)}$$

Brayton Cycle with Real Components

1. Helium working fluid (assume perfect gas)
   $c_p = 5230$ J/kg°K, $\gamma = 1.658$,
2. $r_p = 4$.
3. $T_3 = 972$°K, $T_1 = 278$°K.
4. isentropic efficiencies of turbine & compressor 90%





$$\eta_T = \frac{\text{Actual work out of turbine}}{\text{ideal turbine work}} = \frac{\dot{W}_T}{\dot{W}_{Ti}} = \frac{\dot{m} c_p (T_3 - T_4)}{\dot{m} c_p (T_3 - T_{4s})}$$

$$\therefore \dot{W}_T = \eta_T \dot{W}_{Ti} = \eta_T \dot{m} c_p T_3 \left(1 - \frac{T_{4s}}{T_3}\right)$$

$$= \eta_T \dot{m} c_p T_3 \left(1 - \frac{1}{(\gamma_p)^{(\gamma-1/\gamma)}}\right) = \underline{1.935\, \dot{m}}\ \mathrm{MJ/s}$$

$$\eta_{cp} = \frac{\text{ideal compresser work}}{\text{actual compressor work}} = \frac{\dot{W}_{cpi}}{\dot{W}_{cp}} = \frac{\dot{m} c_p (T_{2s} - T_1)}{\dot{m} c_p (T_2 - T_1)}$$

$$\therefore \dot{W}_{cp} = \frac{\dot{W}_{cpi}}{\eta_{cp}} = \frac{\dot{m}}{\eta_{cp}} c_p T_1 \left(\frac{T_{2s}}{T_1} - 1\right) = \frac{\dot{m} c_p T_1}{\eta_{cp}}\left(\gamma_p^{(\gamma-1/\gamma)} - 1\right)$$

$$= \underline{1.184\, \dot{m}\ \mathrm{MJ/s}}.$$

$\dot{W}_{NET} = \dot{W}_T - \dot{W}_{CP} = \dot{m}\,(1.935 - 1.184) = \underline{0.752 \cdot \dot{m}}\ MW$

$\dot{Q}_R = \dot{m}\, c_p\, (T_3 - T_2)$

$\dot{W}_{CPi} = \dot{m}\, c_p \left[T_{2s} - T_1\right] = \dot{m}\, c_p\, T_1 \left[r_p^{\frac{\gamma-1}{\gamma}} - 1\right] = 1.066 \cdot \dot{m}\ MJ/s.$

$$T_2 - T_1 = \frac{\dot{W}_{cp}}{\dot{m} c_p} = \frac{\dot{W}_{cpi}}{\dot{m} c_p \cdot \eta_{cp}} = 226.5\,°K$$

$\therefore T_2 = 226.5 + 278 = 504.5°K$

$\dot{Q}_R = \dot{m}\, c_p (T_3 - T_2) = 2.45\,\dot{m}\ MW.$

$$\eta_{th} = \dot{W}_{NET} / \dot{Q}_R = \frac{0.752 \cdot \cancel{\dot{m}}}{2.45 \cdot \cancel{\dot{m}}} = 0.307$$

$\rightarrow \underline{30.7\%}$

Brayton cycle with Reheat and Intercooling

1. Fluid - He perfect gas, $c_p = 5230\ J/kg°K$, $\gamma = 1.658$
2. pressure ratio = 4, Intercooling: $p_1'/p_1 = p_2/p_1' = r_p'$; $T_1'' = T_1$

   Reheat: $\dfrac{p_3'}{p_4} = \dfrac{p_3}{p_3'} = r_p'$ $\quad T_3'' = T_3.$

3. $T_3 = 972°K$, $\quad T_1 = 278°K$
4. Isentropic efficiencies of turbine & compressor 90%

$$\dot{W}_{cp} = \dot{m}\, c_p (T_1' - T_1) + \dot{m}\, c_p\, (T_2' - T_1'')$$

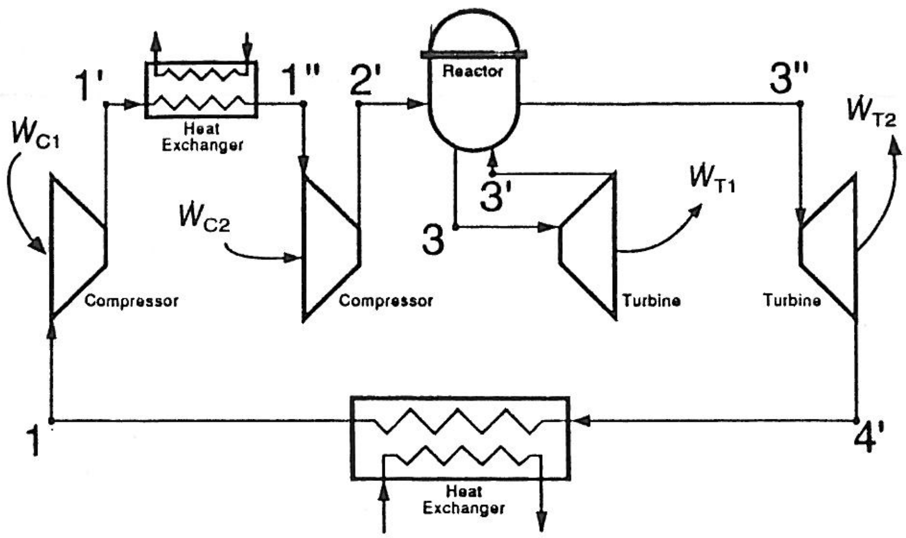
$$= \dot{m}\, c_p T_1 \left[(r_p')^{\frac{\gamma-1}{\gamma}} - 1\right] + \dot{m}\, c_p T_1'' \left[(r_p')^{\gamma-1/\gamma} - 1\right]$$

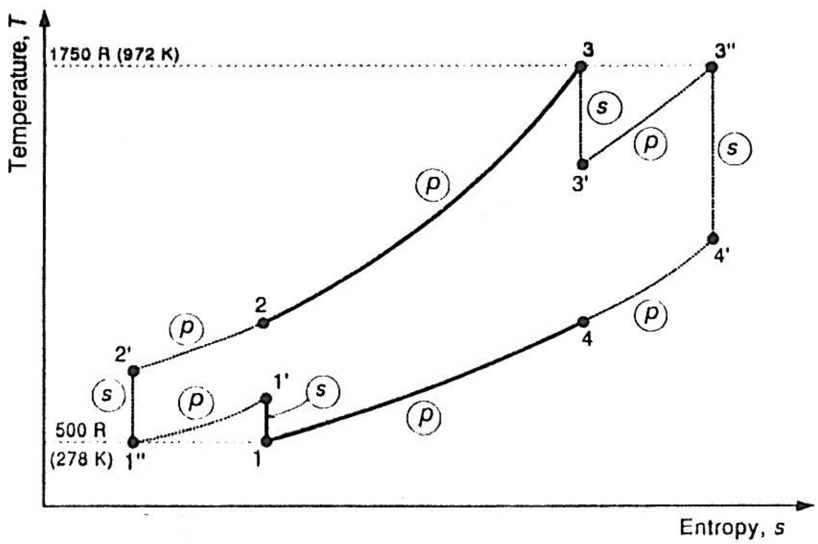
$$= 0.920 \cdot \dot{m}\ MW.$$

$$\dot{W}_T = \dot{m}\, c_p (T_3 - T_3') + \dot{m}\, c_p\, (T_3'' - T_4') = \dot{m}\, c_p T_3 \left[1 - \frac{1}{r_p'^{\frac{\gamma-1}{\gamma}}}\right]$$

$\dot{W}_T =$

$$+ \dot{m}\, c_p T_3'' \left[1 - \frac{1}{r_p'^{\gamma-1/\gamma}}\right]$$





$\dot W_T = 2.44\, \dot m$ MW.

$\frac{T_3''}{T_3} = \frac{1}{\gamma_p^{\gamma-1/\gamma}}$ $\therefore$ $T_3' = 738.3°K$

$T_2' = T_1''(\gamma_p)^{\gamma-1/\gamma}$ and $T_1'' = T_1 = 278°K$

$T_2' = 365.8°K$

$\dot Q_R = \dot m c_p(T_3 - T_2) + \dot m c_p(T_3'' - T_3') = 4.391\, \dot m$ MW.

$\dot W_{NET} = \dot W_T - \dot W_{cp} = 1.524$ MW.

$\eta_{th} = \frac{\dot W_{NET}}{\dot Q_R} = 0.347 \rightarrow 34.7\%$ $\leftarrow$