NUCL 402 Engineering of Nuclear Power Systems

Lecture 6: Reactor control
-Kinetics review

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Fission chain reaction

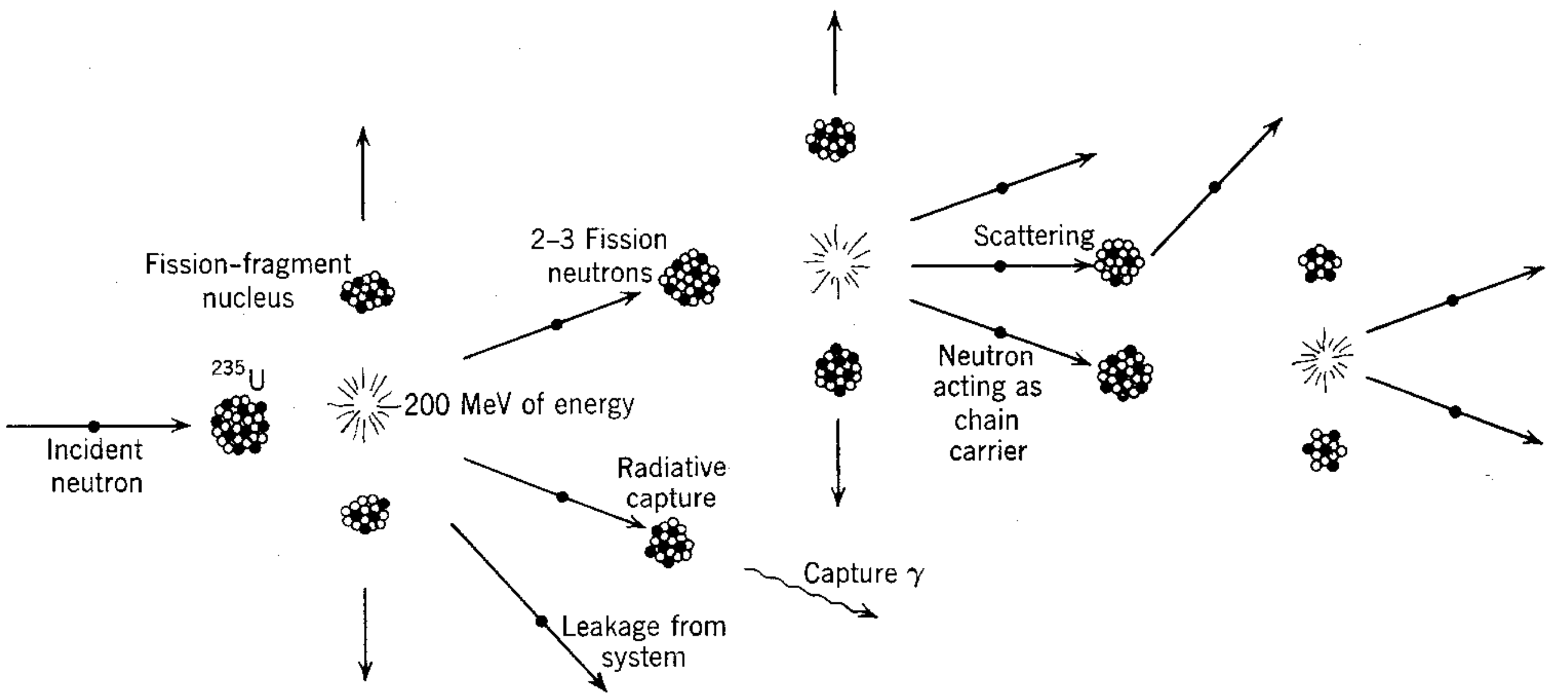


FIGURE 3-1. **A simple schematic of a fission chain reaction.**

$$k \equiv \text{multiplication factor} \equiv \frac{\text{number of neutrons in one generation}}{\text{number of neutrons in preceding generation}}$$

Reactor Kinetics and Control

Steady State Operation of reactor – Critical State

Number of neutron produced = umber of neutrons lost

Reactor power → Neutron density

Why control is necessary?

1) neutron balance-fission burnup

2) neutron poisons –Xe transients

Control Rods, shim rods

Neutron lifetime - 10^{-4}s (slowdown+ subsequent fission)

Power rise in 1000 generation (0.1 sec) with k=1.001 →2.7

Small fraction of delayed neutron provide effective control of power

k –multiplication factor

Four factor formula

$$k_\infty = \eta f \varepsilon p$$

Six factor formula

$$k = \eta f \varepsilon p P_{FNL} P_{TNL} = k_\infty / (1 + L^2 B_g^2)$$

Kinetics

General multigroup neutron diffusion equations:

$$\frac{1}{v_g}\frac{\partial}{\partial t}\phi_g(\underline{r},t) = \underbrace{\nabla\cdot D_g(r)\nabla\phi_g(\underline{r},t)}_{\text{leakage}} - \underbrace{\Sigma_{a\,g}(r)\phi_g(\underline{r},t)}_{\text{loss by absorption}} - \underbrace{\Sigma_{s\,g}(r)\phi_g(\underline{r},t)}_{\text{removal by scattering}} + \underbrace{\sum_{g'=1}^{G}\Sigma_{sg'g}(r)\phi_{g'}(\underline{r},t)}_{\text{scattering into group g}}$$

$$+ \underbrace{\chi_g}_{\text{fraction appearing in group g}} \underbrace{\sum_{g'=1}^{G}\nu_{g'}\Sigma_{f\,g'}(r)\phi_{g'}(\underline{r},t)}_{\text{total fission production}} \underbrace{+S_g^{ext}}_{\text{external source}}$$

One speed form $\quad \frac{1}{v}\frac{\partial}{\partial t}\phi(r,t) = \nabla\cdot D(r)\nabla\phi(r,t) - \Sigma_a(r)\phi(r,t) + \nu\Sigma_f(r)\phi(r,t)$

Source in terms of prompt and delayed neutrons

$$\frac{1}{v}\frac{\partial}{\partial t}\phi(r,t) = \nabla\cdot D(r)\nabla\phi(r,t) - \Sigma_a(r)\phi(r,t) - (1-\beta)\nu\Sigma_f(r)\phi(r,t) + \sum_{i=1}^{6}\lambda_i C_i$$

$$\frac{\partial}{\partial t}C_i(r,t) = -\lambda_i C_i(r,t) + \beta_i\nu\Sigma_f(r)\phi(r,t)$$

Separation of variables, $\quad \nabla^2\phi + B_g^2\phi = 0$

- ✓ Point kinetic equations

$$\frac{\partial n(t)}{\partial t} = \frac{(\rho - \beta)}{\Lambda} n(t) + \sum_{i=1}^{6} \lambda_i C_i(t)$$

$$\frac{\partial}{\partial t} C_i(t) = -\lambda_i C_i(t) + \frac{\beta_i}{\Lambda} n(t), \; i = 1 \ldots 6$$

$$\rho \equiv \frac{k-1}{k} = \text{reactivity} \;\; \text{and} \;\; \Lambda \equiv \frac{\ell}{k} = \text{mean generation time}$$

$$k = \frac{\nu \Sigma_f / \Sigma_a}{1 + L^2 B_g^2}, \qquad \ell = \frac{1}{\nu \Sigma_a (1 + L^2 B_g^2)} = \text{neutron lifetime}$$

- ✓ Solution

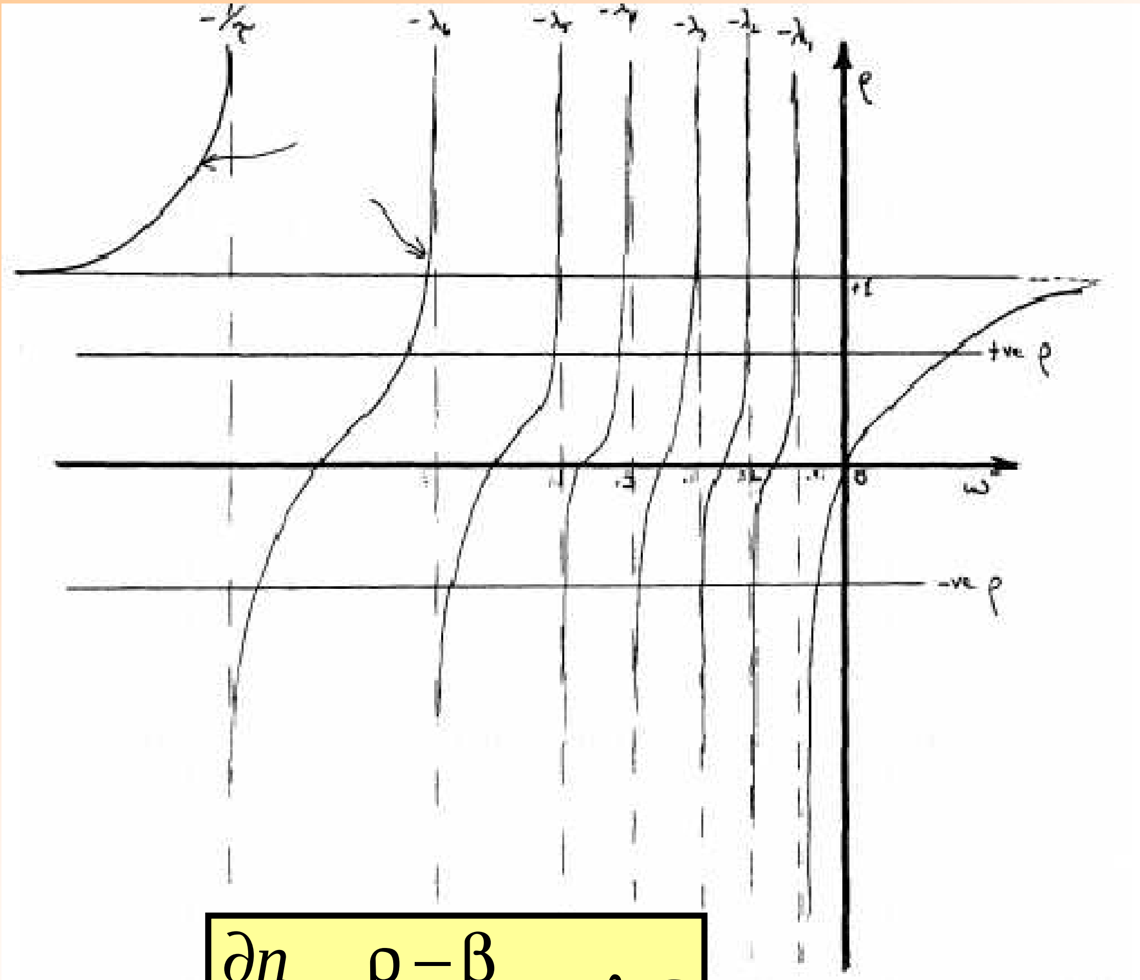
$$n = Ae^{\omega t} \qquad C_i = C_{i0} e^{\omega t}$$

- ✓ Inhour equation

$$\rho = \frac{\omega \ell}{(1 + \omega \ell)} + \frac{1}{(1 + \omega \ell)} \sum_{i=1}^{6} \left(\frac{\omega \beta_i}{(\omega + \lambda_i)} \right)$$

- ✓ Seven roots: Soln

$$n = n_0 \sum_{j=1}^{7} A_j e^{\omega_j t} \qquad C_i = C_{i0} \sum_{j=1}^{7} B_{ij} e^{\omega_j t}$$



$$\frac{\partial n}{\partial t}=\frac{\rho-\beta}{\Lambda}n+\lambda C$$

$$\frac{dC}{d\rho}=-\lambda C+\frac{\beta}{\Lambda}n$$

For $\rho>0$, there is only one $\omega>0$.
All the other ω's are <0, i.e, they represent dying components.

$$n = n_o\exp(t\omega_o)$$
$$= n_o\exp(t/T_p)$$

T_p –reactor period, time required for n to change by factor e.

$T_p = 1/\omega_o$ –stable reactor period

One group delayed neutrons

One group delayed neutrons $\lambda = \dfrac{\beta}{\sum_{i=1}^{6}(\beta_i / \lambda_i)} = \dfrac{0.0065}{0.084} = 0.08\ s^{-1}$

then reactivity is given as $\rho \approx \omega l + \omega\beta/(\omega + \lambda)$

now the roots are $\omega_o \approx \lambda\rho/(\beta - \rho), \quad \omega_1 \approx -(\beta - \rho)/l$

Thus

$$n = n_o\left[\frac{\beta}{\beta-\rho}e^{\lambda\rho t/(\beta-\rho)} - \frac{\rho}{\beta-\rho}e^{-t(\beta-\rho)/l}\right]$$

For +ve ρ=0.0022, $l=10^{-3}s$, $T_p \approx (\beta-\rho)/\lambda\rho = 24.43s$

For -ve ρ=0.0022, $T_p = 49.43s$

If all fission neutrons were prompt, T_p =0.45 s

Stable reactor period for one group of delayed neutrons In reactor shutdown $T_p \approx (\beta-\rho)/\lambda\rho \approx 1/\lambda$ since $|\rho| >> \beta$, *typically* $\rho=-0.1$: $T_p = 80$ s

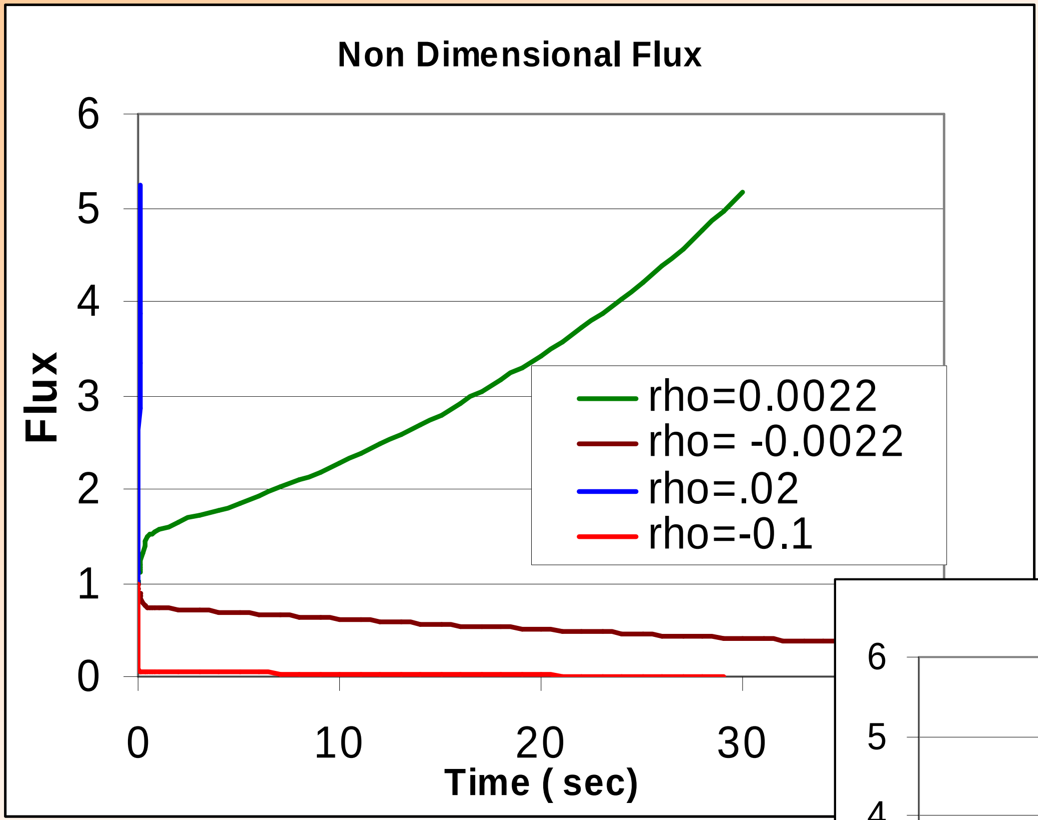
For small ρ, *Then* $T_p \approx \beta/\lambda\rho$

- ✓ Thermal reactor (graphite and $H2O$) reactor period ~ 80s

With rapid insertion of control rod

$$\frac{\phi}{\phi_o} \approx \frac{\beta}{\beta-\rho} e^{\lambda\rho t/(\beta-\rho)} = \frac{0.0065}{0.1} e^{t/80} = 0.06510^{-t/184}$$

Power falls by factor of 10 for every 184 seconds

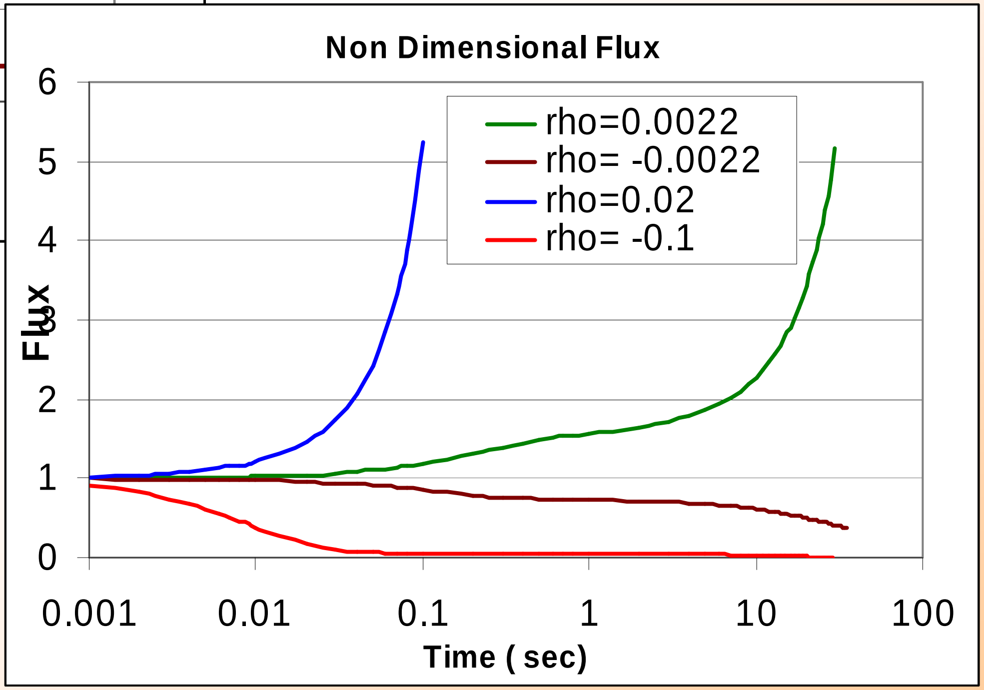


Neutron Flux change with time for

B= 0.0065

$l = 10^{-3}$s

$\lambda = 0.08\text{s}^{-1}$



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If no delayed neutron

$$\frac{dn}{dt} = \frac{\rho - \beta}{l} n$$

For reactor to be critical $dn/dt = 0$, then $\rho = \beta$

Reactor becomes prompt critical when reactivity equal to the fraction of delayed neutrons

Stable reactor period without delayed neutron $T_p = l/\rho$ *(prompt)*

or $T_p = l/(\rho - \beta)$

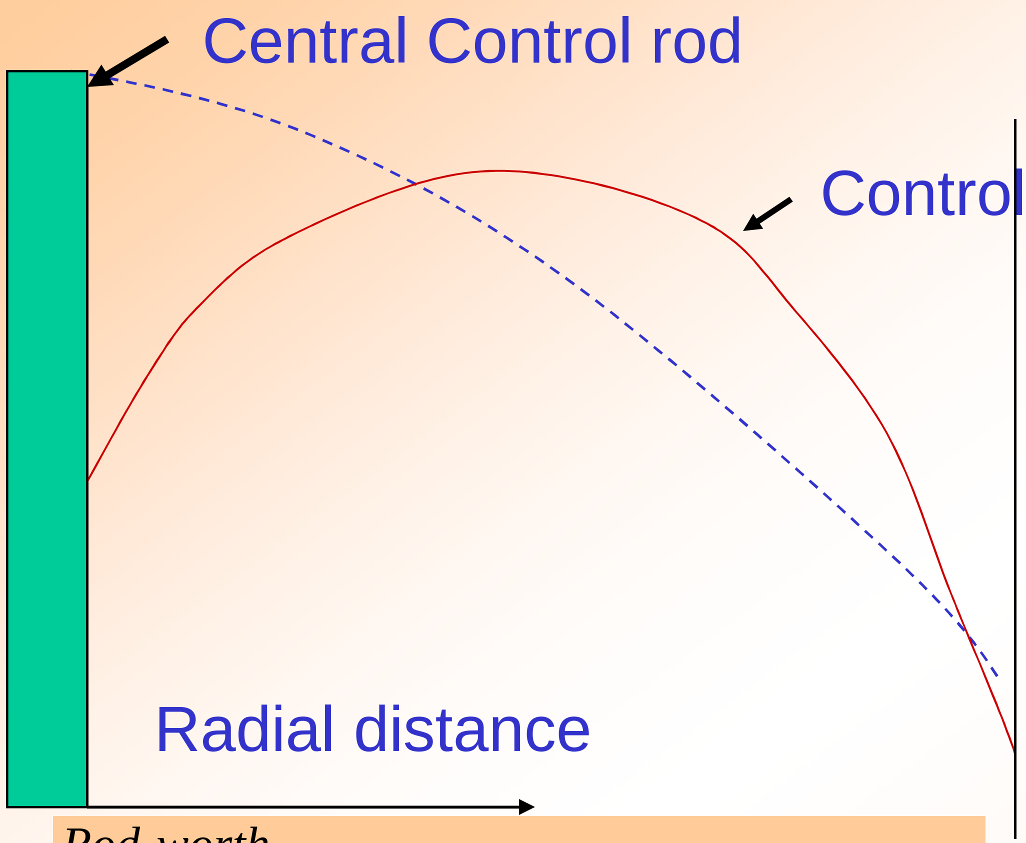
For large ρ *stable reactor period :* $T_p = l/\rho$

Control Rods

1. To control the reactivity –raise or lower power
2. Keep reactor critical for compensating for changes in criticality

Chemical elements with a sufficiently high capture cross section for neutrons

- ✓ silver, indium and cadmium.
- ✓ boron, cobalt, hafnium, dysprosium, gadolinium, samarium, erbium, and europium,
- ✓ alloys and compounds, e.g. high-boron steel, silver-indium-cadmium alloy, boron carbide, zirconium diboride, titanium diboride, hafnium diboride, gadolinium titanate, and dysprosium titanate.



$$\nabla^2\phi_T + B_o^2\phi_T = 0 \quad \textit{rod out}$$

$$\nabla^2\phi_T + B^2\phi_T = 0 \quad \textit{rod in}$$

$$k_0 = \frac{k_\infty}{1+B_o^2M_T^2}, \textit{ rod in}$$

$$k = \frac{k_\infty}{1+B^2M_T^2}, \textit{ rod out}$$

$$M_T^2 = L_T^2 + \tau_T$$

Rod worth

$$\rho_w = \frac{k_0 - k}{k} \approx \frac{2M_T^2B_o(B-B_o)}{1+B_o^2M_T^2}$$

$$= \frac{7.43M_T^2B_o}{(1+B_o^2M_T^2)R^2}\left[0.116 + \ln(r/2.405a) + d/a\right]^{-1}$$

$$d = 2.131\bar{D}\frac{a\sum_t + 0.9354}{a\sum_t + 0.5098},$$

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Rod worth

$$\rho_w = \frac{k_0 - k}{k} \approx \frac{2M_T^2 B_o (B - B_o)}{1 + B_o^2 M_T^2}$$

$$= \frac{7.43 M_T^2 B_o}{(1 + B_o^2 M_T^2) R^2} \left[0.116 + \ln(r / 2.405a) + d / a \right]$$

$$d = 2.131 \bar{D} \frac{a \sum_t + 0.9354}{a \sum_t + 0.5098},$$

System Project Groups	
Group	**Full Name**
	JOHNSON,RANDALL,CRAIG, NUNN,JOSHUA,ALLEN
1	DUC,JOSHUA,BRIAN FRENCH,DOUGLAS,CHARLES JAWORSKI,JASON,MICHAEL SZUMSKI,CATHERINE,ELIZABETH
2	JOSEPH,CAROLYNE,MARIE MEYER,BEN,P RUMSCHLAG, DAVIS TYLER
3	TUBERGEN,JOHN,LOYD WEBSTER,JEFFREY,ALEXANDER
4	BROOKS,CALEB,STEPHEN FULLMER,WILLIAM,DAVID JOSHI,TENZING,HENRY YATISH LIETWILER,CLAYTON,D
5	ALBRECHT,KENNETH,JOSEPH CAMPBELL,CHARLTON,FABIAN EMERICK,AARON,M SANCHEZ,JEFFREY,ANTHONY
6	BLOINK,CARRIE,ADELE CHESTERFIELD,KEVIN,JAMES COLEMAN,JOSHUA,PAUL
8	ANDREW,GREGORY,JAMES EGGERS,RICHARD,LEE CHRISTIAN JABAAY,DANIEL,ROBERT
9	BROWN,MICHAEL,RAY DOWNEY,JASON,ROBERT PALUTSIS,PHILIP,STANLEY

Nuclear criticality

K > 1 supercritical
power increases
K = 1 critical
power constant
K < 1 subcritical
power decreases

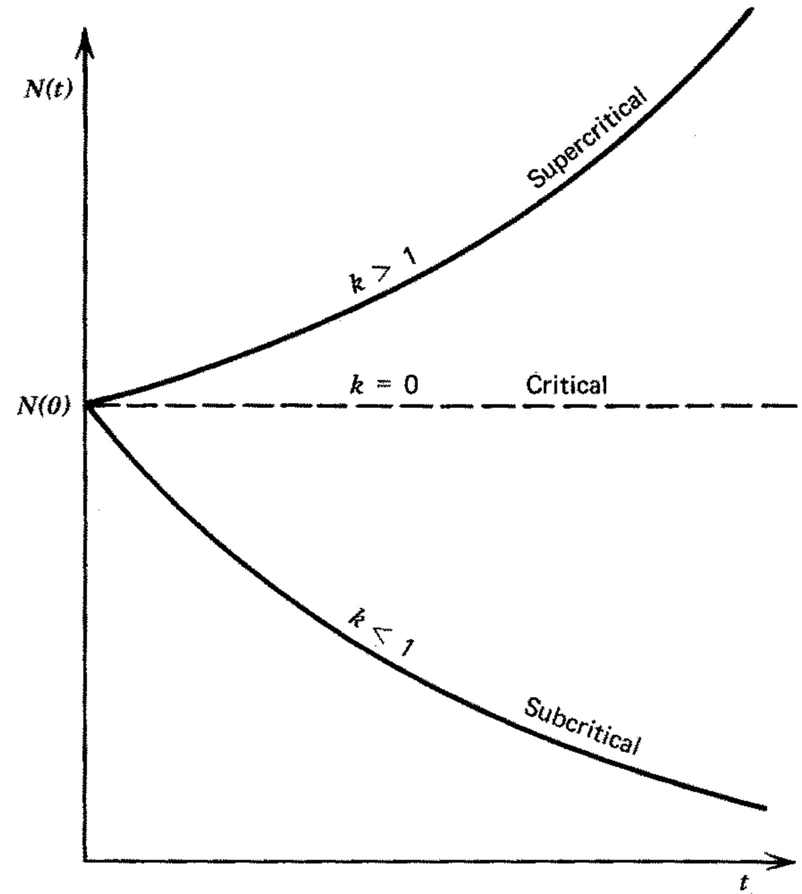


FIGURE 3-2. Time behavior of the number of neutrons in a reactor.

Another definition of *k*

$$k \equiv \frac{\text{rate of neutron production}}{\text{rate of neutron loss}} \equiv \frac{P(t)}{L(t)}$$

$$l \equiv \text{neutron lifetime} \equiv \frac{N(t)}{L(t)}$$

$$N(t) = \text{total neutron population in reactor}$$

Kinetics of fission chain reaction

$$\frac{dN(t)}{dt} = \text{production rate-loss rate} = P(t) - L(t)$$

$$\frac{dN(t)}{dt} = \left[\frac{P(t)}{L(t)} - 1\right] L(t) = (k-1)L(t)$$

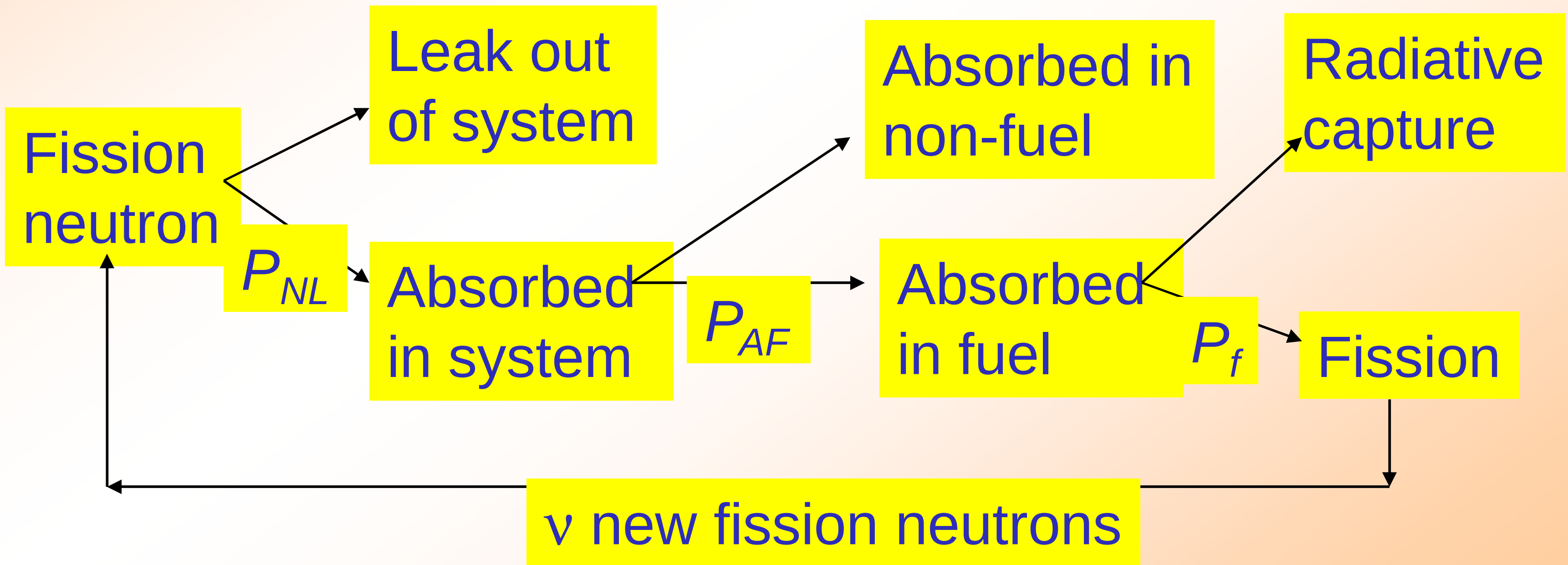
$$\frac{dN(t)}{dt} = \frac{(k-1)}{l} N(t)$$

$$N(t) = N_0 \exp\left[\frac{(k-1)}{l} t\right], \quad \text{let } T = \frac{l}{(k-1)}$$

$$N(t) = N_0 \exp(t/T), \quad T \text{ is the reactor period}$$

k: four factor formula

Follow the fate of a neutron from one generation to the next with conditional probabilities of various loss mechanisms.



k: four factor formula (2)

$P_{NL} \equiv$ probability that neutron will NOT leak out of the reactor before being absorbed. This is called the "non-leakage" probability.

$P_{AF} \equiv$ conditional probability that if a neutron is absorbed, it is absorbed in the fuel. This is called the "thermal utilization" f.

$P_f \equiv$ conditional probability that if a neutron is absorbed in the fuel, it will cause a fission

k: four factor formula (3)

$$P_{AF} = \frac{\Sigma_a^F}{\Sigma_a}, \quad \text{where } \Sigma_a = \Sigma_a^F + \Sigma_a^{other}$$

$$P_f = \frac{\Sigma_f^F}{\Sigma_a^F}, \quad \text{where } \Sigma_a^F = \Sigma_f^F + \Sigma_\gamma^F$$

Follow the fate of the neutron from one generation to the next.

$$N_2 = \nu P_f P_{AF} P_{NL} N_1$$

$$N_2 = \eta f P_{NL} N_1$$

k: four factor formula (4)

$$k = \frac{N_2}{N_1} = \eta f P_{NL}$$

So where are the four factors?
There are only three. Duh…
We are not nearly done yet.

$$k_\infty = \eta f$$

Consider an infinitely large reactor so $P_{NL}=1$. Then the multiplication factor depends only on the composition of the reactor.

k-infinity is a very important concept in reactor physics. It's numerical value is also very important. But this two factor formula for *k*-infinity is not complete.

Neutron energy dependence during lifetime

MeV fission neutrons

Scattering with light nuclei

Resonance absorption

Thermal absorption or leakage

Fission

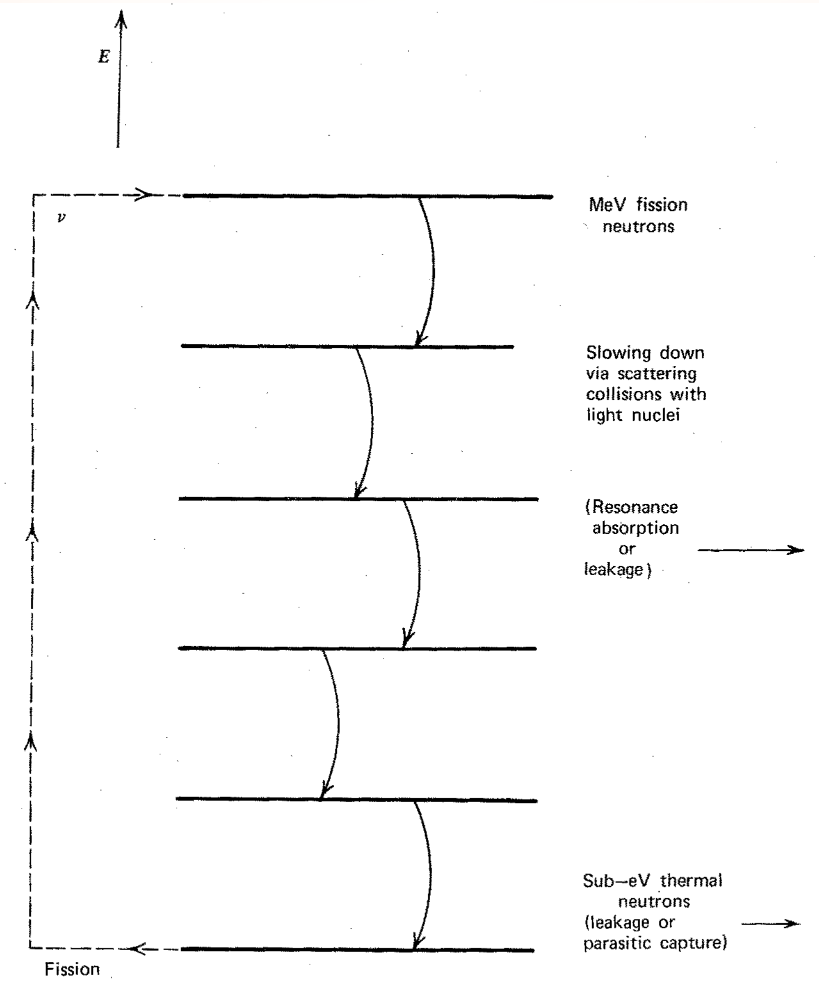


FIGURE 3-3. Processes characterizing a neutron generation in a thermal reactor.

Additional two factors in four factor formula for *k*-infinity

$$\varepsilon \equiv \text{fast fission factor}$$

$$= \frac{\text{number of fast and thermal fission neutrons}}{\text{number of thermal fission neutrons}}$$

$$p \equiv \text{resonance escape probability}$$

$= $ fraction of fission neutrons that slow down from fission energy to thermal energies without being absorbed.

k-infinity: four factor formula

$$k_\infty = \eta f \varepsilon p$$

Recite it in your sleep!

k: six factor formula

$$P_{NL} = P_{FNL} P_{TNL}$$

$P_{FNL} \equiv$ probability that fast neutron will not leak out

$P_{TNL} \equiv$ probability that thermal neutron will not leak out

$$k = \eta f \varepsilon p P_{FNL} P_{TNL}$$

Six factor formula for a typical thermal reactor

$$\eta = 1.65$$
$$f = 0.71$$
$$\varepsilon = 1.02$$
$$p = 0.87$$
$$P_{FNL} = 0.97$$
$$P_{TNL} = 0.99$$

$$k = 1$$

How do we know these values?
Lots of complex calculations.