NUCL 511 HMWK 5

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Calculate β for a fast reactor with 85% fissions in ^{239}Pu and 15% in ^{235}U. Use the ν_{dk}values from Table 2-III and the γ_k values from Table 5-I [1]. Use the delayed neutron data in the lecture note 2 instead of those in Table 2-III. We can find the effective delayed neutron fraction assuming a separable adjoint as $\phi_0^*(r, E) = \phi_0^*(r)\varphi_0^*(E)$, and this is given by

$$\beta_k(t) = \gamma \frac{\nu_{dk}}{\overline{\nu}}$$

where ν_{dk} is the delayed neutron yield for each group, and $\overline{\nu}$ is the energy averaged neutron yield per fission. We must do a weighted average to determine the total effective delayed neutron fraction because there are two different isotopes.

From Lecture Note 2, we can obtain the delayed neutron yield per fission for different isotopes and families. This is given in Table 1. We can retrieve the relative importance of delayed and prompt fission neutrons from Table 5-I [1, p. 72]. This is shown in Table 2.

Finally, it is assumed that the energy average fission neutron yield is approximately the total fission neutron yield $\overline{\nu} \approx \nu$. Thus, the total β can be given by

$$\beta = \sum_{k=1}^{6} \beta_k \tag{1}$$

with

$$\beta_k = \gamma_{dk} \left(0.15 \cdot \frac{\nu_{dk,^{235}U}}{\nu_{^{235}U}} + 0.85 \cdot \frac{\nu_{dk,^{239}Pu}}{\nu_{^{239}U}} \right)$$

To determine the value of ν for each isotope, we use the delayed neutron fractions for fast reactors given in Lecture 2 and the equation

$$\beta_k^{phys} = \frac{\nu_{dk}}{\nu}$$

from this, and Equation 1, it can be seen that

$$\nu = \frac{\sum \nu_{dk}}{\beta^{phys}} \tag{2}$$

Table 3shows the results of these calculations

Calculate a burnup-dependent β for an LWR with 2% of fissions in ^{238}U and initially 98% in ^{235}U. As ^{239}Pu is produced, it takes over part of the fission rate. Let f_{239} be the fraction of the total fission rate that comes from fissioning ^{239}Pu. Use $\gamma_{dk} = 1.08$ [1].

Table 1: Delayed Neutron Yield Per Fission

Family	1	2	3	4	5	6	$\sum$
^{235}U	0.00058	0.00302	0.00288	0.00646	0.00265	0.00111	0.01670
^{238}U	0.0061	0.00496	0.00576	0.01695	0.0118	0.00454	0.04400
^{239}Pu	0.00023	0.00153	0.00115	0.00211	0.00110	0.00033	0.00645

Table 2: Relative Importance of Delayed and Prompt Fission Neutrons (reproduced from [1])

Family	Relative Importance γ_{dk}	Family	Relative Importance γ_{dk}
1	0.802	4	0.825
2	0.831	5	0.825
3	0.818	6	0.825

Table 3: Effective Delayed Neutron Fraction in a Fast Reactor with 85% Fissions in ^{239}Pu and 15% fissions in ^{235}U

Family	1	2	3	4	5	6	$\sum$
β_k ($\times 10^{-4}$)	0.816	5.023	4.071	8.042	3.844	1.306	23.102

Find $\beta(f_{239})$ for $f_{239} \leq 50\%$. As described in the previous problem, the effective delayed neutron fraction can be determined using a weighted average (weighted by the fission rate), and the relative importance of each family's delayed neutrons. In a LWR, this relative importance will be greater than one. It is assumed that each group has the same relative importance of $\gamma_d = 1.08$. The delayed fission neutron yields for different isotopes and families are given in Table 1. It is also assumed that ^{239}Pu takes over equal part of the fission rate that was previously in ^{235}U and ^{238}U. Thus, the final effective delayed neutron fraction can be given in equation 1, with

$$\beta_k = \gamma_{dk}\left((0.02 - 0.02 \cdot f_{239}) \cdot \frac{\nu_{dk,^{238}U}}{\nu_{^{238}U}} + (0.98 - 0.98 \cdot f_{239}) \cdot \frac{\nu_{dk,^{235}U}}{\nu_{^{235}U}} + f_{239} \cdot \frac{\nu_{dk,^{239}Pu}}{\nu_{^{239}Pu}}\right)$$

therefore

$$\beta(f_{239}) = 1.08 \cdot \sum_{k=1}^{6}\left((0.02 - 0.02 \cdot f_{239}) \cdot \frac{\nu_{dk,^{238}U}}{\nu_{^{238}U}} + (0.98 - 0.98 \cdot f_{239}) \cdot \frac{\nu_{dk,^{235}U}}{\nu_{^{235}U}} + f_{239} \cdot \frac{\nu_{dk,^{239}Pu}}{\nu_{^{239}Pu}}\right) \tag{3}$$

to find ν for each isotope, we can use Equation 2 but this time use the thermal reactor β^{phys} given in Lecture 2 ($\beta^{phys}_{^{235}U} = 0.00651$, $\beta^{phys}_{^{238}U} = 0.01766$, and $\beta^{phys}_{^{239}Pu} = 0.00225$).

Plot your results as a function of f_{239}. Plotting the function given in Equation 3 is shown in Figure 1.

An (α, n) point source is moved in a vertical guide tube toward a swimming pool reactor core. Suppose that the adjoint flux varies along the guide tube as $\phi^*(z) = A\cos(z/100)$, where z is the distance from the core mid-plane in cm and A is a constant, and the steady state reactor power is $5\,W$ when the source is located $10\,cm$ above the core mid-plane. Determine the steady state reactor power as a function of source position for the source position from $40\,cm$ to $0\,cm$. It is assumed that there is no reactivity change of the reactor as the source is moved (i.e. it does not scatter or absorb neutrons), thus we must determine the source multiplication factor as a function of position.

We know that the reactor must be subcritical, because an independent source creates a steady state. Using the source multiplication factor not accounting for a delayed neutron source, we have

$$p_0(z) = \frac{s_0(z)}{-\rho_0}$$

and

$$p_0(10\,cm) = 5\,W$$

with

$$s_0(z) = \frac{\left\langle \phi_0^*, S\right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi\right\rangle}$$

$$p_0(z) = \frac{\left\langle \phi_0^*, S\right\rangle}{-\rho_0 \cdot \left\langle \phi_0^*, \mathbf{F}\psi\right\rangle}$$

assuming that the reactor is symmetric, the forward flux shape function can be found as

$$\psi(z) = C\cos(z/100)$$

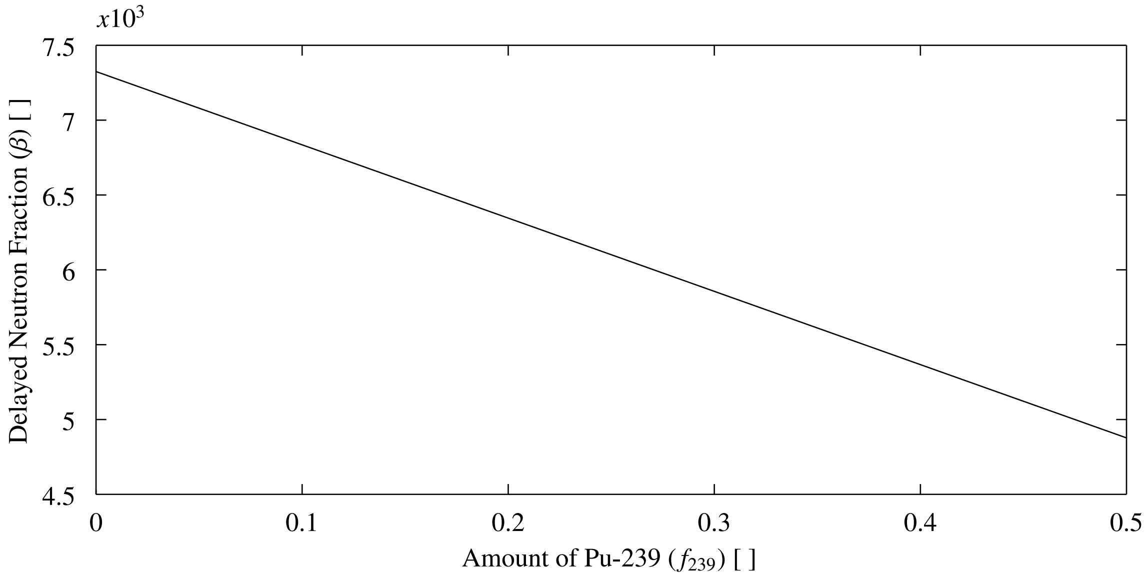

Figure 1: Total Effective Delayed Neutron Fraction versus Fraction of Fissions Occurring in ^{239}Pu

$$p_0(z) = \frac{\int\int S \cdot A\cos(z/100)\, dEdV}{-\rho_0 \int\int \nu\Sigma_f \cdot C\cos(z/100) \cdot A\cos(z/100)\, dEdV}$$

all of the constants will get absorbed into A, therefore

$$p_0(z) = A\frac{\int\int \cos(z/100)\, dEdV}{\int\int \cos^2(z/100)\, dEdV} = A\int\int 1/\cos(z/100)\, dEdV$$

and with

$$p_0(10\,cm) = 5\,W = A\int\int 1/\cos(10/100)\, dEdV$$

$$A = \frac{5\,W}{\int\int 1/\cos(0.1)\, dEdV}$$

and then the function

$$p_0(z) = 5\,W \cdot \int\int \frac{\cos(0.1)}{\cos(z/100)} dEdV$$

describes the power level at any position z of the source. This is shown in Figure 2.

References

[1] K Ott and R Neuhold. *Introductory Nuclear Reactor Dynamics*. American Nuclear Society, La Grange Park, Illinois, 1985.

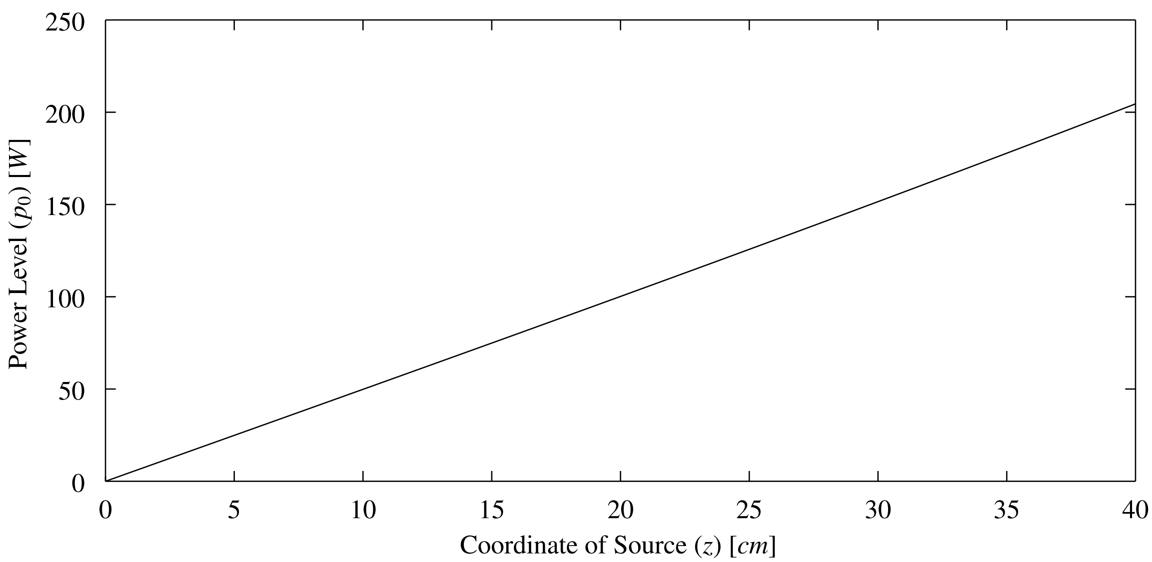

Figure 2: Power Level of Reactor versus Source Location