



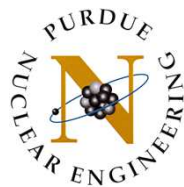
NUCL 511

Nuclear Reactor Theory and Kinetics

Lecture Note 11

Prof. Won Sik Yang

**Purdue University
School of Nuclear Engineering**



PURDUE
UNIVERSITY

Point Reactor Model for T/H Feedback

■ Reactor power with point kinetics model

$$P(t) = P_n p(t)$$

$p(t)$ = power or flux amplitude in PKE

P_n = nominal power level (Watts)

■ Heat balance in fuel

$$C_f \frac{d\bar{T}_f}{dt} = P_n p(t) - Q_{out} = P_n p(t) - U(\bar{T}_f - \bar{T}_c)$$

$C_f = \rho_f c_p V_f$ = total heat capacity of fuel

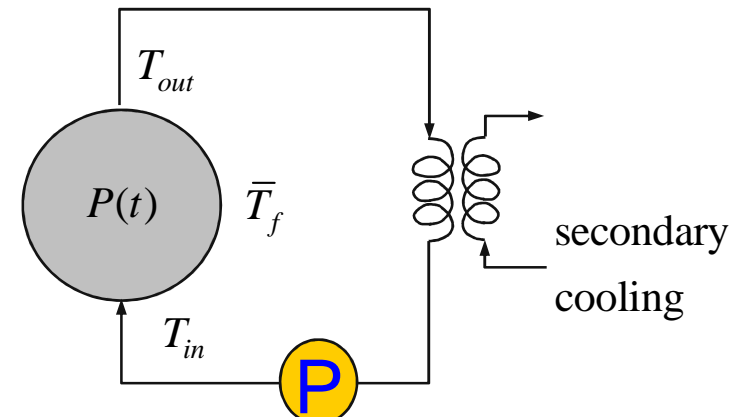
$U = hA$ = overall heat transfer coefficient

– Steady state heat balance

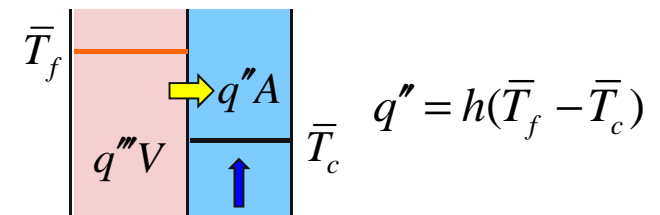
$$0 = P_n p_0 - U(\bar{T}_{f0} - \bar{T}_{c0})$$

■ Change in average fuel temperature

$$C_f \frac{d}{dt} \delta T_f = P_n \delta p - U(\delta T_f - \delta T_c) \quad \delta p = p - p_0, \quad \delta T_f = \bar{T}_f - \bar{T}_{f0}, \quad \delta T_c = \bar{T}_c - \bar{T}_{c0}$$



Average Coolant Channel



Point Reactor Model for T/H Feedback

- Change in average fuel temperature

$$\frac{d}{dt} \delta T_f = \frac{P_n}{C_f} \delta p(t) - \frac{U}{C_f} (\delta T_f - \delta T_c) = C_{IT} \delta p(t) - \lambda_H (\delta T_f - \delta T_c)$$

$C_{IT} = P_n / C_f$ = rate of temperature rise per second (K/s) of nominal power application

$\lambda_H = U / C_f$ = time constant for heat transfer from fuel to coolant (1/s)

- Rapid transients where $\delta T_f \gg \delta T_c$

$$\frac{d}{dt} \delta T_f(t) = -\lambda_H \delta T_f(t) + C_{IT} \delta p(t), \quad \delta T_f(0) = 0$$

$$\frac{d}{dt} [\delta T_f(t) e^{\lambda_H t}] = C_{IT} e^{\lambda_H t} \delta p(t) \Rightarrow \delta T_f(t') e^{\lambda_H t'} \Big|_0^t = \delta T_f(t) e^{\lambda_H t} = C_{IT} \int_0^t e^{\lambda_H t'} \delta p(t') dt'$$

$$\delta T_f(t) = C_{IT} \int_0^t e^{-\lambda_H(t-t')} \delta p(t') dt' = \frac{P_n}{C_f} \int_0^t e^{-\lambda_H(t-t')} \delta p(t') dt' = \frac{P_n}{C_f} \int_0^t e^{-\lambda_H(t-t')} [p(t') - p_0] dt'$$

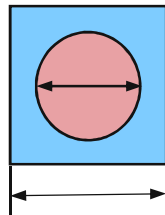
- Adiabatic boundary condition (very rapid transient, $\lambda_H = 0$)

$$\delta T_f(t) = C_{IT} \int_0^t \delta p(t') dt' = \frac{P_n}{C_f} \delta I(t) = \frac{\delta Q(t)}{C_f} = \frac{P_n}{C_f} \int_0^t [p(t') - p_0] dt'$$

Typical PWR Data

■ Power density

- Core average power density: $\sim 100 \text{ kW}/\ell$ (cf. $\sim 50 \text{ kW}/\ell$ for BWR)
- Power density in fuel: $100/0.32 \approx 300 \text{ kW}/\ell$



Pellet dia = 0.8 cm

Cell pitch = 1.25 cm

Fuel Volume Fraction ≈ 0.32

■ Heat capacity of fuel: $0.32 \text{ kJ}/\text{kg}\cdot\text{K}$

- Fuel density: $\sim 10 \text{ g}/\text{cc} = \sim 10 \text{ kg}/\ell$
- Volumetric heat capacity: $\sim 3.2 \text{ kJ}/\ell\cdot\text{K}$ (cf. $4.2 \text{ kJ}/\ell\cdot\text{K}$ for H_2O)

■ Fuel temperature rise per full-power-second

$$\delta T_f = \frac{300 \text{ kW}/\ell \times 1 \text{ sec}}{3.2 \text{ kJ}/\ell\cdot\text{K}} = 94 \text{ K}/\text{fp-s}$$

■ Reactivity change per full-power-second

$$\begin{aligned} \delta \rho / \text{fp-s} &= 94 \text{ K}/\text{fp-s} \times (-3) \text{ pcm}/\text{K} = -280 \text{ pcm}/\text{fp-s} \\ &= -0.4 \$ / \text{fp-s} \equiv \gamma \text{ (full power coefficient, 1/s)} \end{aligned}$$

PKE with Prompt Reactivity Feedback

- Reactivity with feedback (induced by fuel temperature change)

$$\rho(t) = \rho_e(t) + \gamma \int_0^t e^{-\lambda_H(t-t')} [p(t') - p_0] dt'$$

$$\gamma = C_{IT} \alpha_F = \frac{P_n}{C_f} \alpha_F$$

- If γ is constant, then reactivity change is proportional to energy deposit
 - Linear energy feedback model for rapid transients
- Point kinetics equations with reactivity feedback

$$\Lambda \dot{p}(t) = \left[\rho_e(t) - \beta + \gamma \int_0^t e^{-\lambda_H(t-t')} [p(t') - p_0] dt' \right] p(t) + \sum_{k=1}^K \lambda_k \zeta_k(t)$$

$$\dot{\zeta}_k(t) = -\lambda_k \zeta_k(t) + \beta_k p(t)$$

- Sub-prompt critical transients
 - Transient with step reactivity insertion, $\rho_1 < 1\$$
 - Prompt jump approximation with pseudo initial condition

$$p(0^+) = p^0 = \frac{1}{1 - \rho_{1\$}} p_0$$

Short-Time Transient with Step Reactivity Insertion

■ Prompt jump approximation

- Adiabatic approximation: heat confined in fuel for short time
- One group of delayed neutrons

$$0 = \left[\rho_1 + \gamma \int_0^t [p(t') - p_0] dt' - \beta \right] p(t) + \bar{\lambda} \zeta(t)$$

$$\dot{\zeta}(t) = -\bar{\lambda} \zeta(t) + \beta p(t)$$

■ Taylor expansion for relatively slow change after prompt jump

$$p(t) = p^0 + p' t + p'' t^2 + \dots$$

$$\zeta(t) = \zeta_0 + \zeta' t + \zeta'' t^2 + \dots$$

■ Reactivity derivative in terms of power

$$\dot{\rho}(t) = \frac{d}{dt} \left[\rho_1 + \gamma \int_0^t [p(t') - p_0] dt' \right] = \gamma [p(t) - p_0] = \gamma (p^0 - p_0 + p' t + p'' t^2 + \dots)$$

$$\Rightarrow \dot{\rho}(0) = \gamma (p^0 - p_0)$$

■ Precursor equation

$$\zeta' + 2\zeta'' t + \dots = -\bar{\lambda}(\zeta_0 + \zeta' t) + \beta(p^0 + p' t + \dots) \Rightarrow \zeta'(0) = -\bar{\lambda}\zeta_0 + \beta p^0 = \beta(p^0 - p_0)$$

Short-Time Transient with Step Reactivity Insertion

■ Power equation

$$\left[\rho_1 - \beta + \gamma \int_0^t [p^0 - p_0 + p' t'] dt' \right] (p^0 + p' t) + \bar{\lambda} (\zeta_0 + \zeta' t) = 0$$

– Constant term

$$p^0 (\rho_1 - \beta) + \bar{\lambda} \zeta_0 = 0 \Rightarrow p^0 = \frac{\bar{\lambda} \zeta_0}{\beta - \rho_1} = \frac{\beta p_0}{\beta - \rho_1} = \frac{1}{1 - \rho_{1\$}} p_0 \text{ (prompt jump)}$$

– 1st order term

$$(\rho_1 - \beta) p' + \gamma (p^0 - p_0) p^0 + \bar{\lambda} \beta (p^0 - p_0) = 0 \Rightarrow p' = \frac{p^0 - p_0}{\beta - \rho_1} (\gamma p^0 + \bar{\lambda} \beta)$$

$$p^0 = \frac{\beta p_0}{\beta - \rho_1} \Rightarrow \frac{1}{\beta - \rho_1} = \frac{p^0}{\beta p_0} \Rightarrow p' = \frac{p^0}{\beta p_0} (p^0 - p_0) (\gamma p^0 + \bar{\lambda} \beta)$$

■ Condition for $p' = 0$

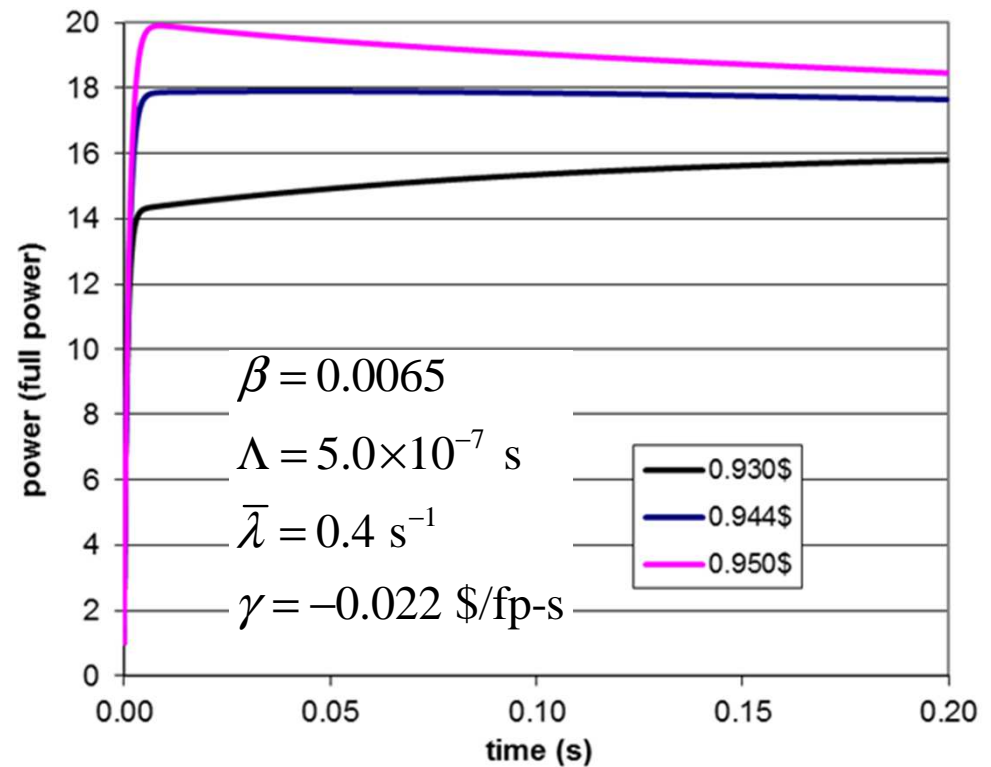
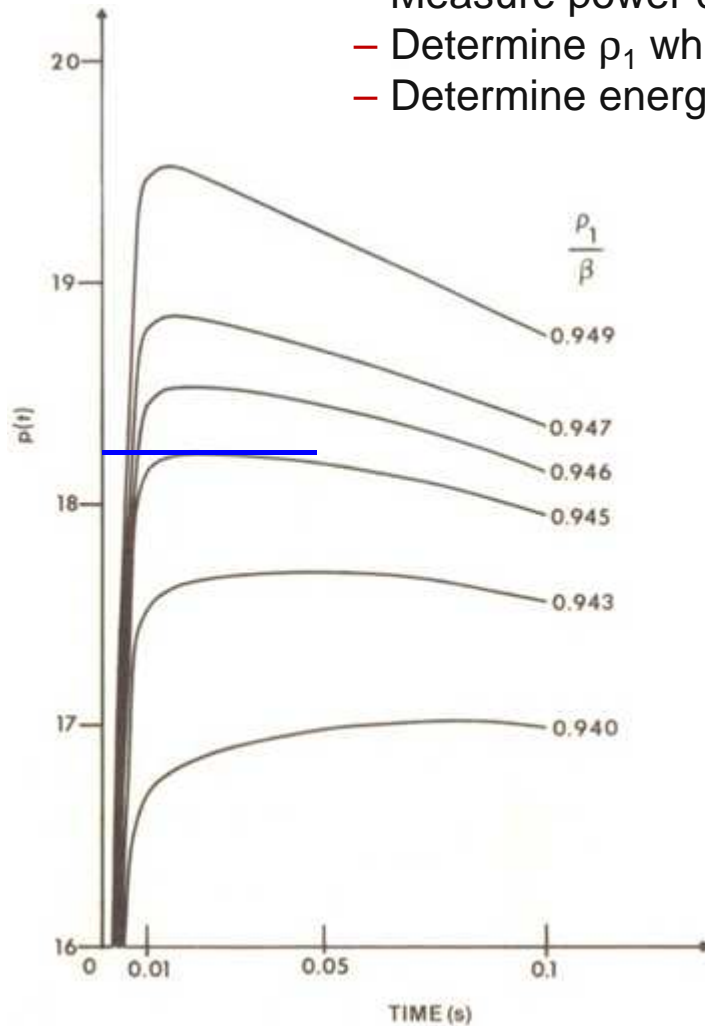
$$\gamma p^0 + \bar{\lambda} \beta = 0 \Rightarrow p^0 = \frac{\bar{\lambda} \beta}{-\gamma} \equiv p^{00}$$

$$\frac{1}{1 - \rho_{1\$}} p_0 = \frac{\bar{\lambda} \beta}{-\gamma} \Rightarrow \rho_{1\$} = 1 + \frac{\gamma p_0}{\bar{\lambda} \beta}$$

- When the prompt jump is p^{00} , the power change after the prompt jump vanishes
- The influence of feedback and delayed neutrons on the power change after the prompt jump is cancelled

Experimental Determination of Energy Coefficient

- Measure power change vs. time for different values of ρ_1
- Determine ρ_1 which gives $p'=0$
- Determine energy coefficient γ



Asymptotic Solution of Sub-Prompt Critical Transient

■ Asymptotic state with adiabatic approximation

$p \uparrow \Rightarrow T \uparrow \Rightarrow \rho \downarrow \Rightarrow \rho = 0$ @ what power?

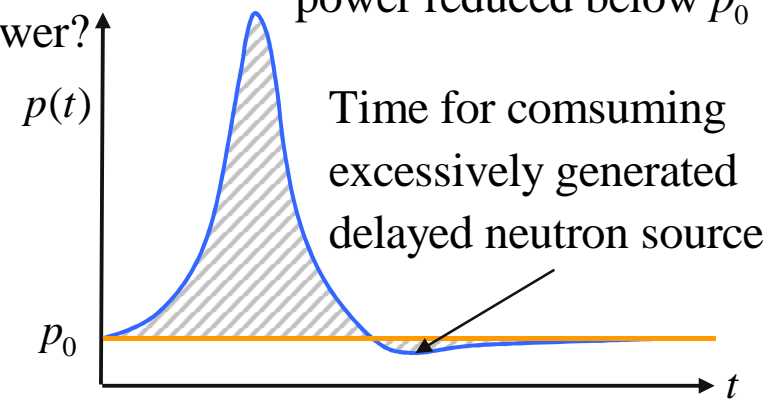
$$\dot{\rho}_{as} = \frac{\lambda \rho_{as}(t) + \dot{\rho}_{as}(t)}{\beta - \rho_{as}(t)} p_{as} = 0 \quad (\Leftarrow \text{PJA})$$

$$\dot{\rho}_{as} = \gamma(p_{as} - p_0) = 0 \Rightarrow p_{as} = p_0 \text{ as } t \rightarrow \infty$$

$$\rho_{as} = \rho_1 + \gamma \int_0^\infty [p(t') - p_0] dt' = 0 \Rightarrow I_{as} = \frac{\rho_1}{-\gamma}$$

$$\delta Q = P_n I_{as} \Rightarrow \text{Heat added} \Rightarrow T_f^{as} > T_{f0}$$

Too strong feedback \rightarrow
power reduced below p_0



– External reactivity insertion is compensated for by increased fuel temperature

■ Asymptotic state with first-order heat transfer ($\lambda_H \neq 0$)

$$0 = \lim_{t \rightarrow \infty} \left\{ \rho_1 + \gamma \int_0^t [p(t') - p_0] \exp[-\lambda_H(t - t')] dt' \right\}$$

$$\approx \rho_1 + \gamma \int_{-\infty}^t (p_{as} - p_0) \exp[-\lambda_H(t - t')] dt' = \rho_1 + \frac{\gamma}{\lambda_H} (p_{as} - p_0)$$

$$p_{as} = p_0 + \frac{\rho_1 \lambda_H}{-\gamma}$$

– Asymptotic power is higher than the initial power to maintain a high temperature to compensate for the external reactivity insertion with heat transfer to coolant

Super-Prompt Critical Excursion by Step Reactivity

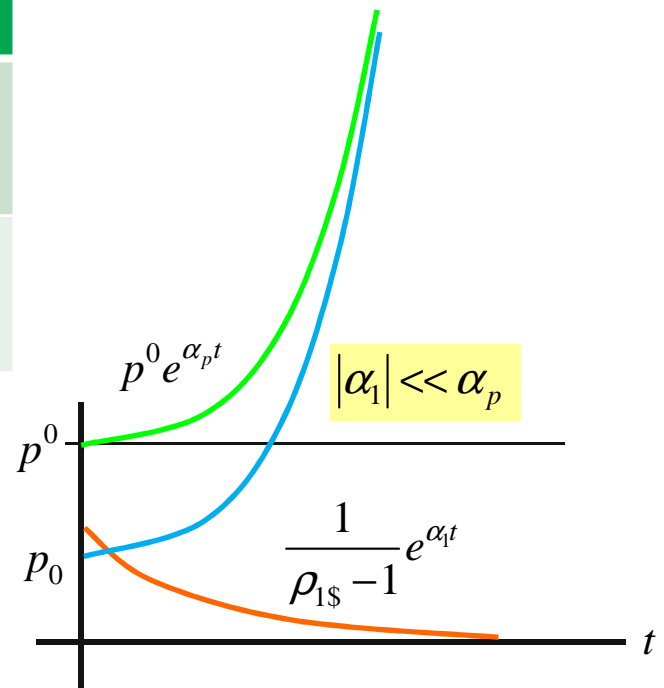
- Step reactivity insertion $\rho > \beta$
- Solution with one delayed neutron group

$$p(t) = p_0 \left(\frac{1}{1 - \rho_{1\$}} e^{\alpha_1 t} - \frac{\rho_{1\$}}{1 - \rho_{1\$}} e^{\alpha_p t} \right), \quad \alpha_p = \frac{\rho_1 - \beta}{\Lambda}, \quad \alpha_1 = \frac{\rho_{1\$}}{1 - \rho_{1\$}} \bar{\lambda}$$

ρ	α_p	α_1	Prompt jump	Asymptotic solution
$< \beta$	< 0	> 0	$p^0 = \frac{p_0}{1 - \rho_{1\$}}$	$p^0 e^{\alpha_1 t}$
$> \beta$	> 0	< 0	$p^0 = \frac{\rho_{1\$} p_0}{\rho_{1\$} - 1}$	$p^0 e^{\alpha_p t}$

- Solution of prompt kinetics equation with pseudo initial condition

$$p(t) \approx \frac{p_0 \rho_{1\$}}{\rho_{1\$} - 1} e^{\alpha_p t} = p^0 e^{\alpha_p t}$$



Super-Prompt Critical Excursion with Prompt Feedback

- Linear energy model with adiabatic boundary condition

$$\rho(t) = \rho_1 + \gamma \int_0^t [p(t') - p_0] dt' \cong \rho_1 + \gamma \int_0^t p(t') dt'$$

$\Leftarrow p(t) \gg p_0$ for prompt critical transient

- At maximum power

$$\Lambda \dot{p} = [\rho(t_m) - \beta] p(t_m) = 0$$

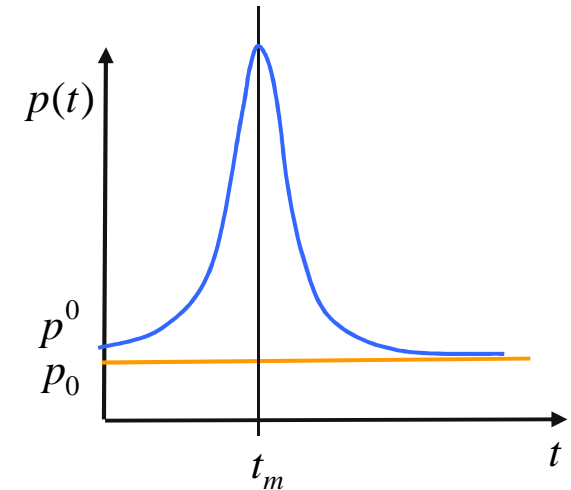
$$\Rightarrow \rho(t_m) = \rho_1 + \gamma \int_0^{t_m} p(t') dt' = \beta$$

$$I(t_m) = \int_0^{t_m} p(t') dt' = \frac{\rho_1 - \beta}{-\gamma}$$

$$\delta Q = P_n I(t_m) = P_n \frac{\rho_1 - \beta}{-\gamma} = \frac{\rho_1 - \beta}{-\gamma / P_n} = \frac{\rho_1 - \beta}{-\gamma_E}$$

- Fuel temperature rise for $\rho_1 = 1.2\$$

$$\frac{0.2\$}{0.4\$ / \text{fp-s}} = 0.5 \text{ fp-s} \Rightarrow \delta T_f \sim 50\text{K} \quad (1 \text{ fp-s} \rightarrow \sim 94\text{K})$$



The energy released until the power reaches its maximum is dependent only on ρ_1 , but independent of Λ

Solution of Prompt Kinetics Equation

■ System of first order ODEs

$$\dot{p}(t) = \frac{\rho(t) - \beta}{\Lambda} p(t) \quad (\text{prompt kinetic equation})$$

$$\rho(t) = \rho_1 + \gamma \int_0^t p(t') dt' \quad (\text{reactivity feedback}) \Rightarrow \dot{\rho}(t) = \gamma p(t)$$

■ Second order ODE for reactivity

$$\ddot{\rho}(t) = \gamma \dot{p}(t) = \gamma \frac{\rho(t) - \beta}{\Lambda} p(t) = \frac{\rho(t) - \beta}{\Lambda} \dot{\rho}(t) \Rightarrow \Lambda \ddot{\rho}_p(t) = \rho_p(t) \dot{\rho}_p(t), \quad \rho_p = \rho(t) - \beta$$

$$\text{Initial conditions: } \rho_p(0) = \rho(0) - \beta = \rho_1 - \beta, \quad \dot{\rho}_p(0) = \gamma p(0) = \gamma p^0$$

■ Maximum power

$$\Lambda [\dot{\rho}_p(t) - \dot{\rho}_p(0)] = \frac{1}{2} [\rho_p^2(t) - \rho_p^2(0)] \Rightarrow \gamma \Lambda [p(t) - p^0] = \frac{1}{2} [\rho_p^2(t) - \rho_{1p}^2]$$

$$- \text{ At maximum power, } p(t_m) = p_m, \quad \rho(t_m) = \beta \Rightarrow \rho_p(t_m) = 0$$

$$\gamma \Lambda (p_m - p^0) = -\frac{1}{2} (\rho_1 - \beta)^2 \Rightarrow p_m = p^0 + \frac{(\rho_1 - \beta)^2}{2\Lambda(-\gamma)}$$

$$p_m = \frac{1.2}{0.2} p_0 + \frac{0.2^2 \times 0.007^2}{2 \times 10^{-5} \times (0.4 \times 0.007)} = 6p_0 + 35$$

PKA Solution beyond the Power Peak

- After the power peak, the reactivity is reduced below β , and the neutron flux of the super-prompt reactor dies away rapidly, and the flux is eventually determined by the source multiplication factor of the delayed neutrons
- When the power is back to the initial jump level

$$p(t) - p^0 = \frac{1}{2}[\rho_p^2(t) - \rho_{p1}^2] = 0$$

$$\rho_p^2(t) = \rho_{p1}^2 \Rightarrow \rho_p(t) = \pm \rho_{p1}$$

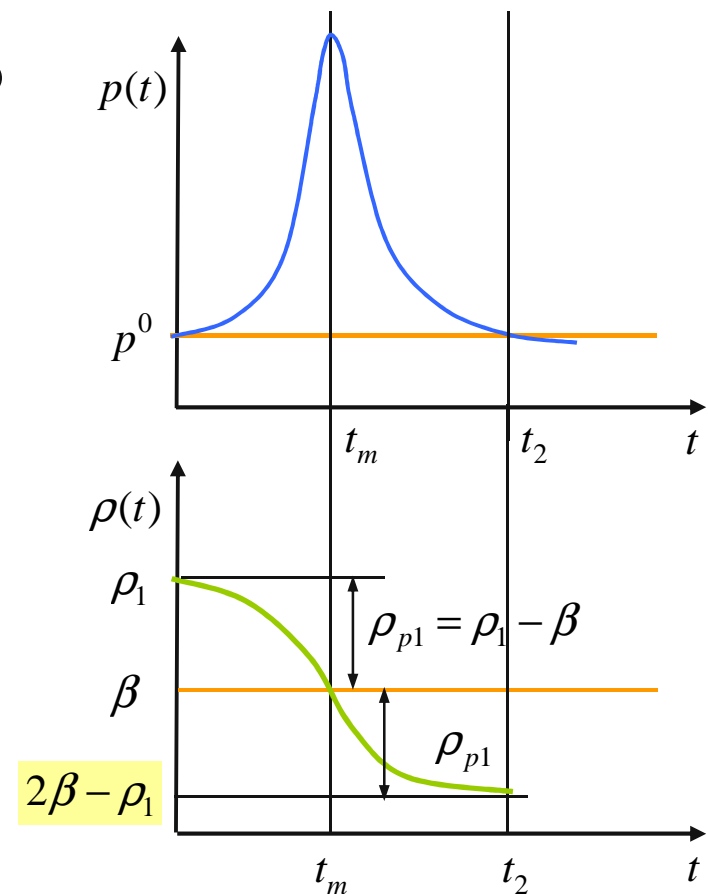
$$\Rightarrow \rho_p(t_2) = -\rho_{p1}$$

$$\Delta\rho(t_2) = \rho(t_2) - \rho(0) = -2\rho_{p1} = -2(\rho_1 - \beta)$$

$$\Delta\rho(t_2) = \gamma I(t_2)$$

$$I(t_2) = 2 \frac{\rho_1 - \beta}{-\gamma} = 2I(t_m)$$

Two times the energy release to the power peak



Solution of Prompt Kinetics Equation

- First integral of second order ODE for reactivity

$$\Lambda[\dot{\rho}_p(t) - \dot{\rho}_p(0)] = \frac{1}{2}[\rho_p^2(t) - \rho_p^2(0)] \quad \rho_{p1} = \rho_p(0) = \rho_1 - \beta, \quad \dot{\rho}_p(0) = \gamma p^0$$

$$2\Lambda\dot{\rho}_p(t) = \rho_p^2(t) - \rho_{p1}^2 + 2\Lambda\gamma p^0 = \rho_p^2(t) - \rho_b^2$$

$$\rho_b^2 = \rho_{p1}^2 + 2\Lambda(-\gamma)p^0 > \rho_{p1}^2$$

$$\frac{\dot{\rho}_p(t)}{[\rho_p(t) + \rho_b][\rho_p(t) - \rho_b]} = \frac{1}{2\Lambda} \Rightarrow \left(\frac{1}{\rho_p(t) + \rho_b} - \frac{1}{\rho_p(t) - \rho_b} \right) \frac{\dot{\rho}_p(t)}{(-2\rho_b)} = \frac{1}{2\Lambda}$$

$$\left(\frac{1}{\rho_p - \rho_b} - \frac{1}{\rho_p + \rho_b} \right) \frac{d\rho_p}{\rho_b} = \frac{dt}{\Lambda}$$

- Integrate over (0,t)

$$\frac{t}{\Lambda} = \frac{1}{\rho_b} \left[\ln \frac{\rho_b + \rho_{p1}}{\rho_b - \rho_{p1}} - \ln \frac{\rho_b + \rho_p(t)}{\rho_b - \rho_p(t)} \right] \quad \rho_p(t) \leq \rho_b$$

Solution of Prompt Kinetics Equation

- At peak power, $\rho_p(t_m) = 0$

$$\frac{t_m}{\Lambda} = \frac{1}{\rho_b} \left[\ln \frac{\rho_b + \rho_{p1}}{\rho_b - \rho_{p1}} - \ln \frac{\rho_b + \rho_p(t_m)}{\rho_b - \rho_p(t_m)} \right] \Rightarrow t_m = \frac{\Lambda}{\rho_b} \ln \frac{\rho_b + \rho_{p1}}{\rho_b - \rho_{p1}}$$

$$\frac{t}{\Lambda} = \frac{t_m}{\Lambda} - \frac{1}{\rho_b} \ln \frac{\rho_b + \rho_p(t)}{\rho_b - \rho_p(t)} \Rightarrow \frac{1}{-\rho_b} \ln \frac{\rho_b + \rho_p(t)}{\rho_b - \rho_p(t)} = \frac{t - t_m}{\Lambda}$$

- Reactivity and power

$$\rho_p(t) = \rho_b \frac{e^{-\rho_b(t-t_m)/\Lambda} - 1}{e^{-\rho_b(t-t_m)/\Lambda} + 1} = \rho_b \tanh \left[-\frac{\rho_b}{2\Lambda} (t - t_m) \right] \Rightarrow \rho_p(\infty) = -\rho_b \quad (\text{background})$$

$$p(t) = \frac{\dot{\rho}_p(t)}{\gamma} = \frac{p_m}{\cosh^2 \left[-\frac{\rho_b}{2\Lambda} (t - t_m) \right]} \quad \text{Symmetric around } t_m!$$

$$p_m = \frac{\rho_b^2}{2\Lambda(-\gamma)} = \frac{\rho_{p1}^2 - 2\Lambda\gamma p^0}{2\Lambda(-\gamma)} = \frac{\rho_{p1}^2}{2\Lambda(-\gamma)} + p^0$$

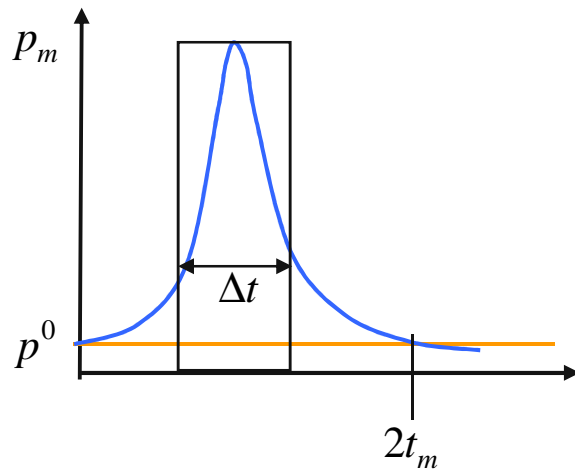
Equivalent Width of Power Burst

- At $t = 2t_m$ (when the power returns to the initial jump)

$$\rho_p(2t_m) = \rho_b \frac{e^{-\rho_b t_m / \Lambda} - 1}{e^{-\rho_b t_m / \Lambda} + 1} \quad e^{-\rho_b t_m / \Lambda} = e^{-\frac{\rho_b}{\Lambda} \frac{\Lambda}{\rho_b} \ln \frac{\rho_b + \rho_{p1}}{\rho_b - \rho_{p1}}} = \frac{\rho_b - \rho_{p1}}{\rho_b + \rho_{p1}}$$

$$\rho_p(2t_m) = \rho_b \frac{\frac{\rho_b - \rho_{p1}}{\rho_b + \rho_{p1}} - 1}{\frac{\rho_b - \rho_{p1}}{\rho_b + \rho_{p1}} + 1} = \rho_b \frac{-2\rho_{p1}}{2\rho_b} = -\rho_{p1} \Rightarrow 2t_m = t_2$$

- Equivalent width (of power burst)



$$p_m \Delta t = \int_0^{2t_m} p(t) dt = \frac{2\rho_{p1}}{-\gamma}$$

$$\Delta t = \frac{2\rho_{p1}}{-\gamma p_m} = \frac{2\rho_{p1}}{-\gamma} \frac{1}{\frac{\rho_{p1}^2}{2\Lambda(-\gamma)} + p^0}$$

$$\Delta t \sim \frac{4\Lambda}{\rho_{p1}} \propto \Lambda \quad \text{for } p_0 \ll 1$$

Example of Burst Estimate

- $\rho_1 = 1.1\$$ in PWR

$$\gamma = -0.4\$ / \text{fp-s}, \quad \beta = 0.007, \quad \Lambda = 2.5 \times 10^{-5} \text{ sec}$$

$$p^0 = \frac{\rho_{1\$}}{\rho_{1\$} - 1} p_0 = \frac{1.1}{0.1} p_0 = 11 p_0$$

$$p_m - p^0 = \frac{(\rho_1 - \beta)^2}{2\Lambda(-\gamma)} = \frac{\beta(\rho_{1\$} - 1)^2}{2\Lambda(-\gamma / \beta)} = \frac{0.007 \times 0.1^2}{2 \times 2.5 \times 10^{-5} \times 0.4} = 3.5 \text{ fp}$$

$$p_m = 11 p_0 + 3.5 \approx 3.5 \text{ fp if } p_0 \ll 1$$

$$\Delta t = \frac{4\Lambda}{0.1\beta} = \frac{4 \times 2.5 \times 10^{-5}}{0.1 \times 0.007} = 14.3 \text{ ms}$$

Precursor Accumulation Solution After Burst

- After the reactivity is reduced below β , PKA is not valid, and the flux is determined by the source multiplication factor of the delayed neutrons

- A first approximation can be obtained by using the initial value of the delayed neutron source and the reactivity after the burst

$$p_{ab} = \frac{s_{d0}}{\beta - \rho(2t_m)} = \frac{\beta p(2t_m)}{\beta - \rho(2t_m)} = \frac{\beta p^0}{\rho_0 - \beta} = \frac{\beta \rho_0 p_0}{(\rho_0 - \beta)^2}$$

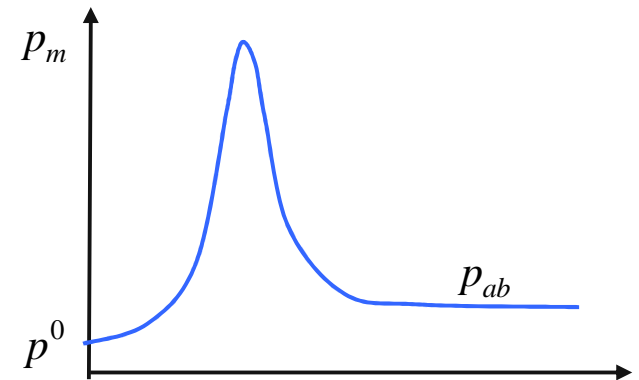
- An improved estimate of the delayed neutron source multiplication requires accounting for the increase of the precursor population during the flux burst

- Precursor accumulation model

$$\zeta(2t_m) \approx \zeta_0 + \beta \int_0^{2t_m} p(t) dt = \zeta_0 + \beta \frac{2\rho_{p1}}{-\gamma}$$

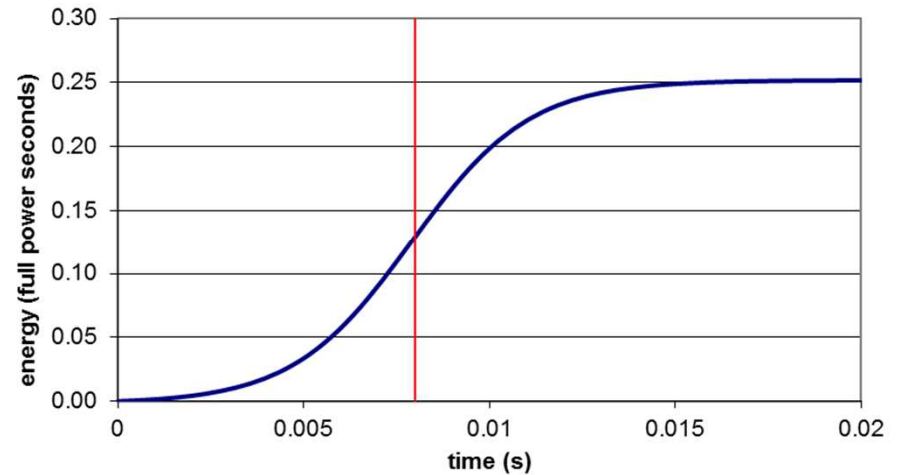
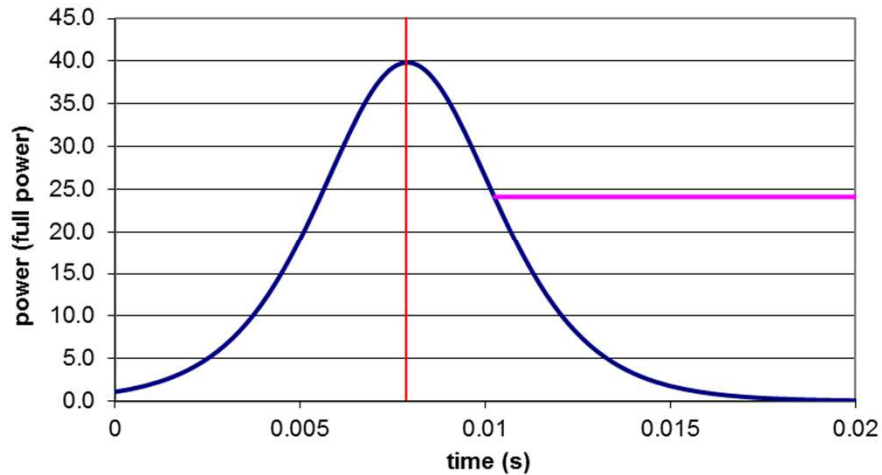
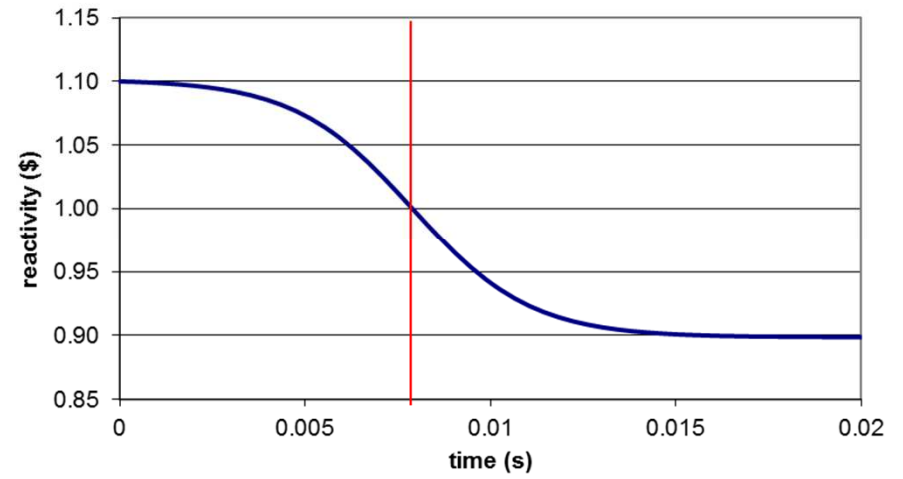
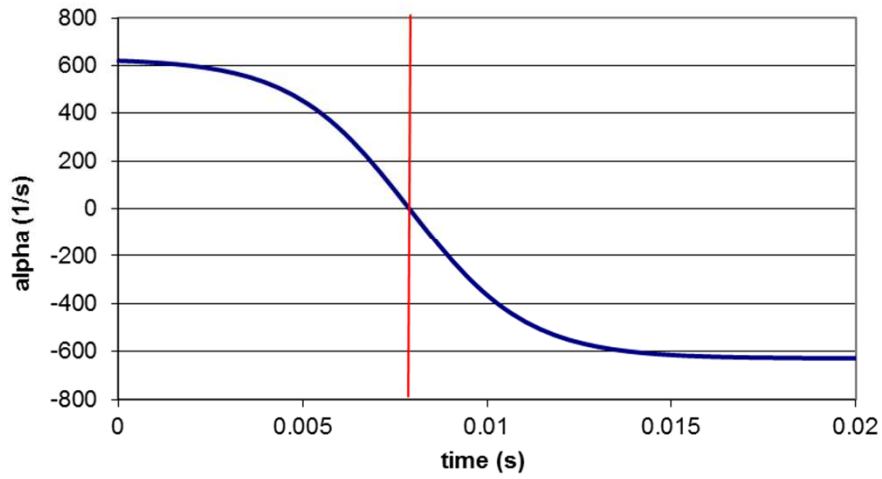
$$\lambda \zeta_2 = \lambda \zeta_0 + \lambda \beta \frac{2\rho_{p1}}{-\gamma} = \beta p_0 \left(1 + 2 \frac{\lambda \rho_{p1}}{-\gamma p_0} \right) = \frac{1}{\rho_{p1\$}} \left(1 + 2 \frac{\lambda \rho_{p1}}{-\gamma p_0} \right) p_0$$

$$p_{ab} = \frac{s_d}{\beta - \rho} = \frac{\beta p_0}{\rho_{p1}} \left[1 + 2 \frac{\lambda \rho_{p1}}{(-\gamma) p_0} \right]$$

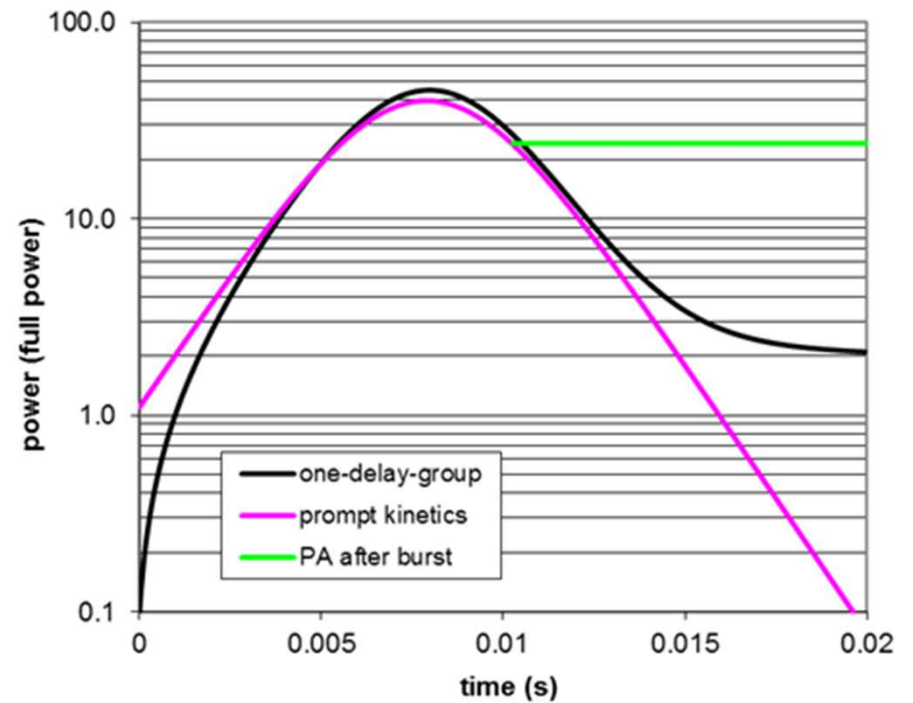
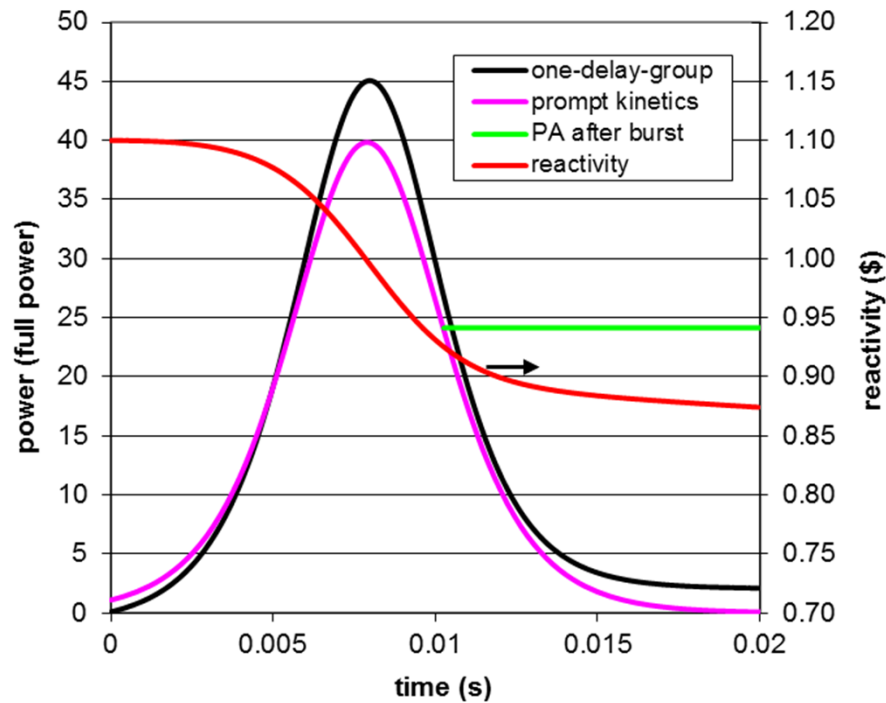


$$\lambda \zeta_2 = \beta p_0 \left(1 + \frac{2 \times 0.4 \times 0.1}{0.4 \times 10^{-4}} \right) = 2000 \beta p_0$$

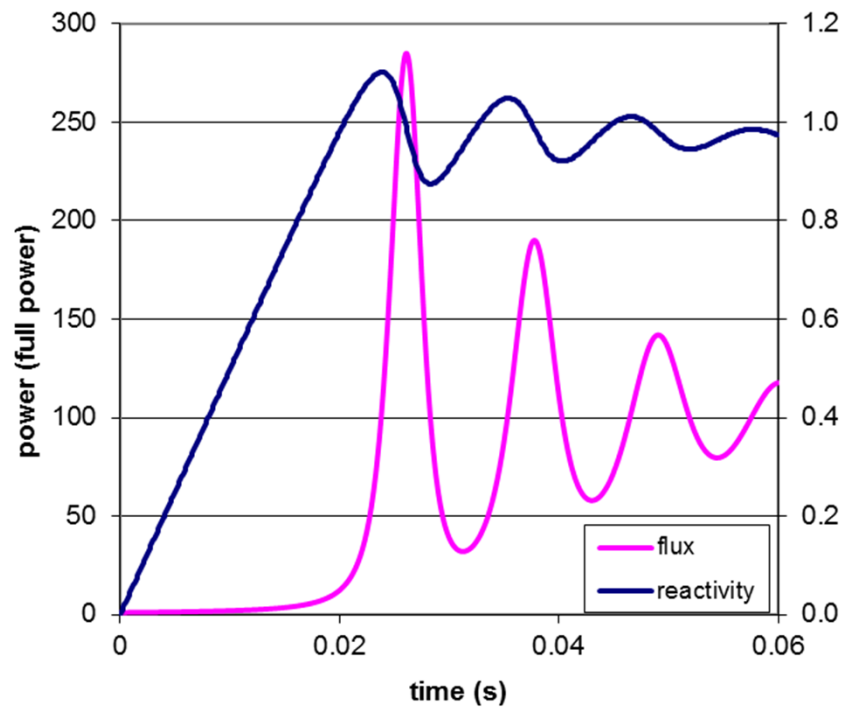
PKA Solutions of Super-prompt Critical Transient



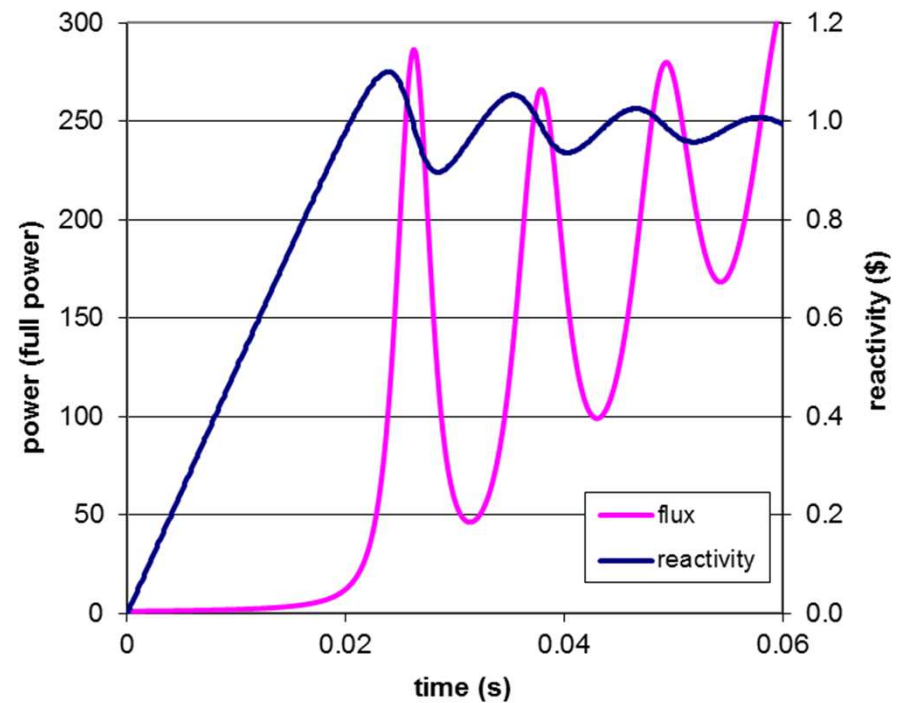
Solutions of Super-prompt Critical Transient



Super-prompt Critical Transient by Reactivity Ramp

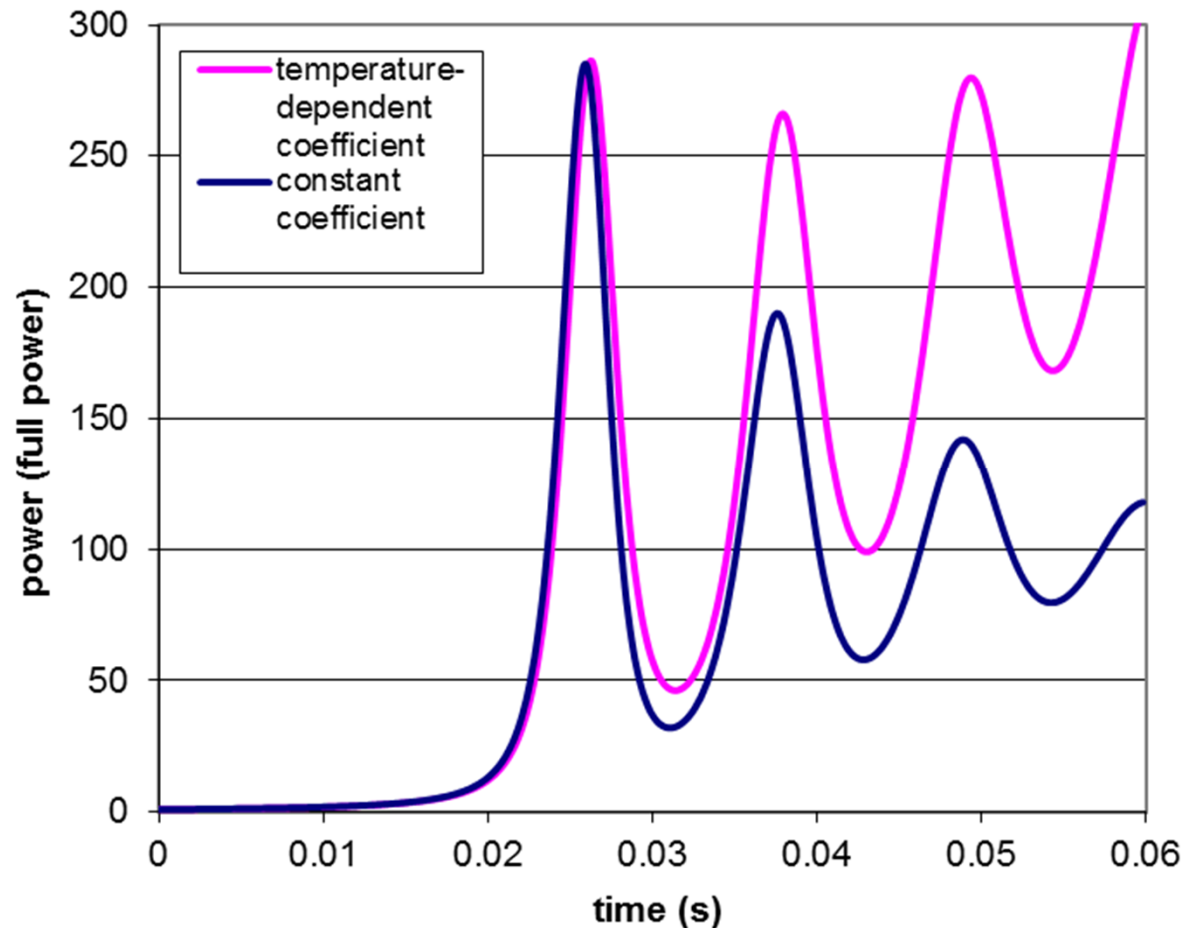


Super-prompt Critical Transient Induced by Reactivity Ramp with Constant Feedback Coefficient



Super-prompt Critical Transient Induced by Reactivity Ramp with Energy Coefficient Proportional to $1/T$

Super-prompt Critical Transient by Reactivity Ramp



Comparison of Flux Transients between Constant and Temperature-Dependent Feedback Coefficients