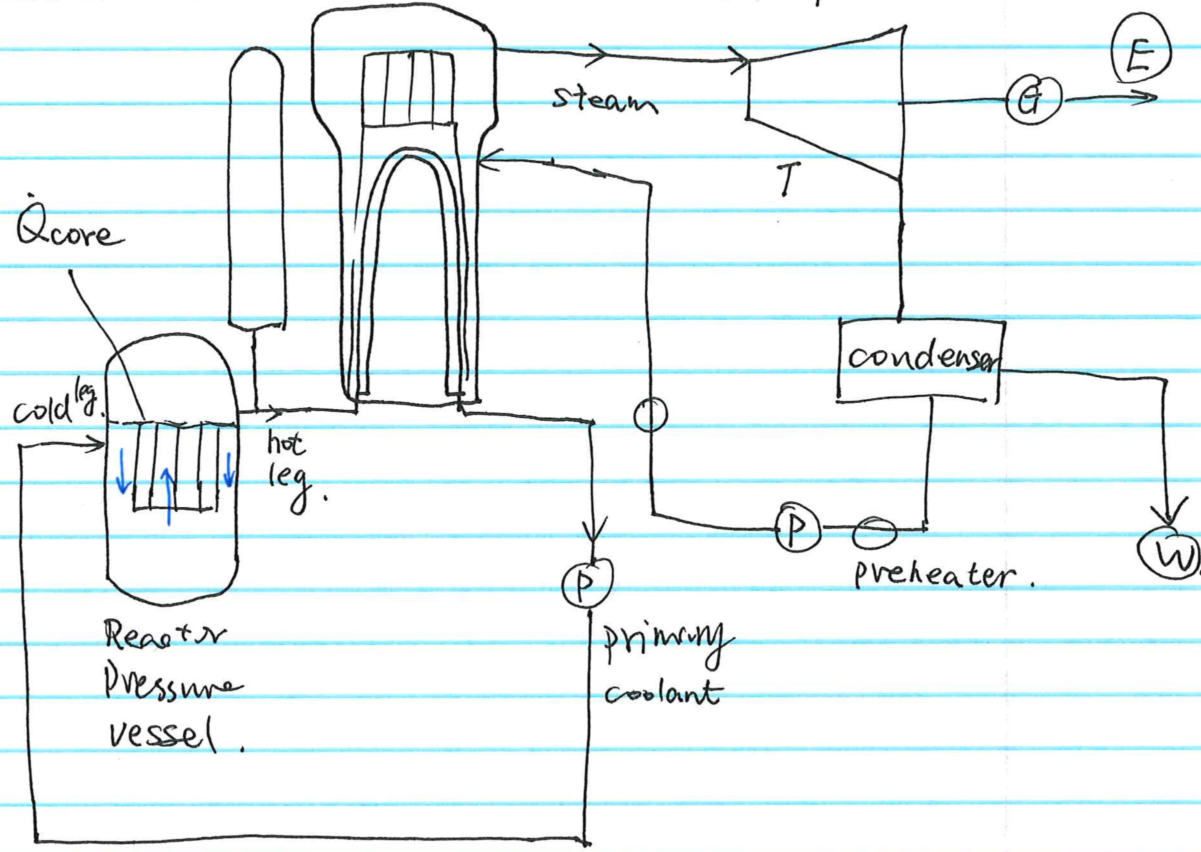
§ Introduction to Nuclear Thermal hydraulic.

Nuclear Power system. (PWR & BWR).

Pressurized Water Reactor. (70%).

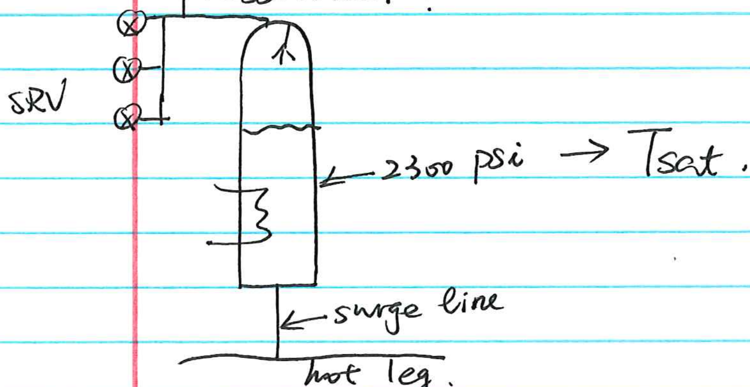


the limit of efficiency. is material.

Steel.

Reactor. : Fuel Rods → subassembly → core.
↳ chain Reaction → heat generation → conduction.
convective cooling, boiling heat transfer.
critical heat flux.

Pressurizer :



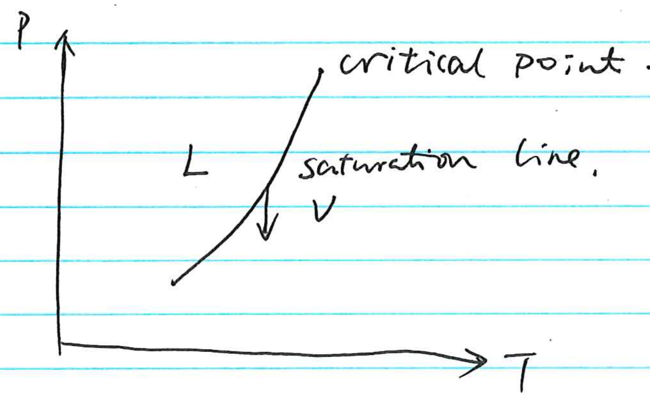
v. Boiling

v. Spray condensation.

v. flushing (Relif Valve)

o droplet T_L

Tsat Steam.



Steam . Generator.

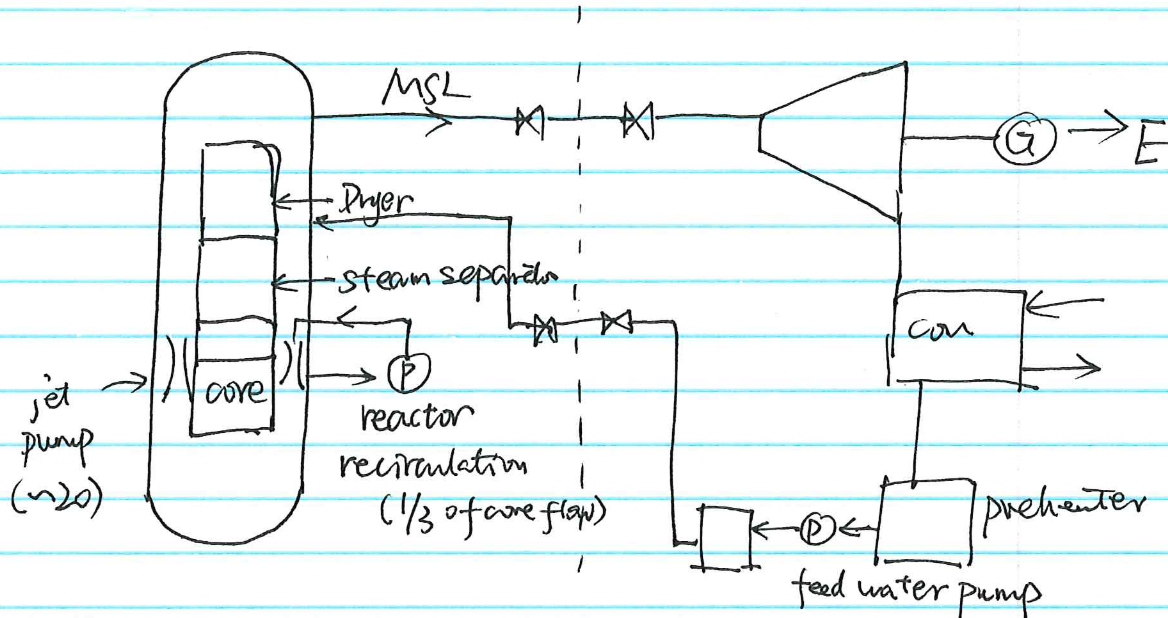
- Inside of tubes. convection.
- outside { Nucleate boiling. CHF. (Dry out).

Turbine

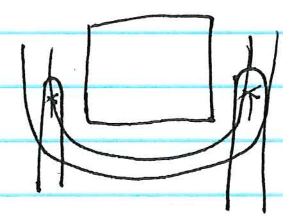
compresive flow (Expansion)

Condensor : conduction.

Boiling Water reactor. (BWR).



BWR
ABWR →



Reactor vessel. { lower Plenum
core (~45000 rods) $x_e \simeq 0.14$
(core inlet flow resriction)
upper plenum
steam Separator. (vortex)
dryer.

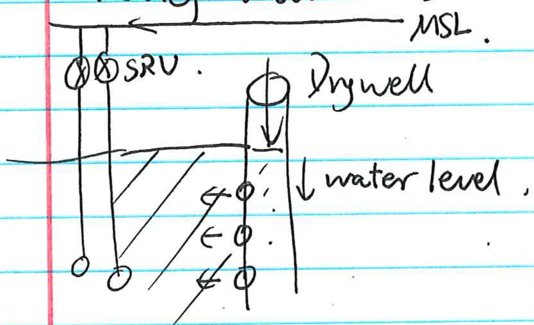
$X \geq 0.996$.

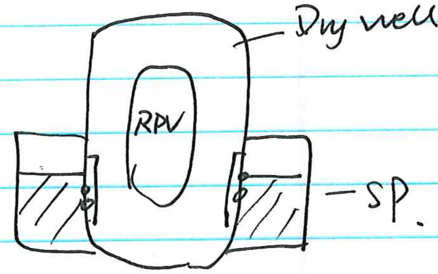
MSL (Isolation Values).
Turbine generator
condenser
Feedwater pump

Safety System

Isolation value

Relief value → MSL ⇒ SP





ECCS

Containment cooling system

Steady state, start up, shutdown
(Normal operation)

Coolant Flow

pump power (pressure relief value ↓)

Pressure regulation (boiling, condensation, Flashing ~~Flooding~~)

Heat removal

Heat loss

Steam generator (2 phase RPV or SG)

compressible flow

condensation

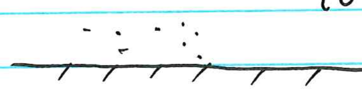
critical heat flux

PWR. Departure from Nucleate Boiling (DNB)
(Bubbly flow) q_c''

BWR Dryout of Liquid Film.
Total power.

DNB.

local



film dryout

Accident and safety analysis.

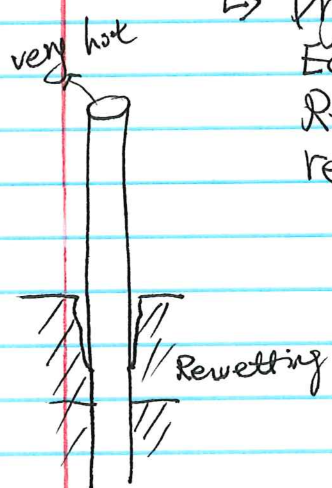
Critical heat flux

- Full power (DNB . dryout)
- LOCA many different Mechanisms.

Transient flow (singlep or two phase)
Thermal shock
Turblent flow
stress corrosion cracking
Two phase flow phenamenon

Critical Heat Flux
Blow Down (Depressurization)
Break → critical . flow. (Boundary condition for LOCA).

→ Dryout of fuel.
ECCS Injection
Reflooding
rewetting



Passive Safe Reactor.
Avoid fuel Dryout.
Natural circulation of Two-phase flow.

Severe accident. → core damage.

Multiphase flow
chemical reaction
Radiation
core melt down.

core phenamenon

Bubble dynamic.

Nuclearation. bubble departure, Phase change
coalensence. break-up.
Drag force
Phase distribution.
DNB

Droplet

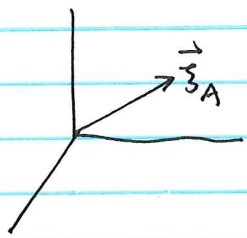
entrainment from liquid film. deposition

Film waves.
Break (LOCA) critical flow.
ECCS. Reflood re wetting
Flooding phenomenon (CCFL)
Counter current flow limitation.

HW #1. Read Appendix A (Transport

4. Particle Path. Velocity and Accerlatation.

· $\vec{\xi}$ material coordinate (Initial fluid particle position)



$t = 0.$ $\quad \vec{x} = \vec{\xi}$

· $\vec{x}$ spatial coordinate

$t = t,$ $\quad \bar{x} = \bar{x}(\vec{\xi}, t)$

Axial of conteinuity.

$\vec{\xi} = \vec{\xi}(\vec{x}, t) \Leftarrow \vec{x} = \vec{x}(\vec{\xi}, t)$

Any property of the fluid may be follaved along a pareicle path.

$$F(\vec{x}, t) = F(\vec{x}(\vec{\xi}, t), t) = F(\vec{\xi}, t)$$

$F(\vec{x}, t)$: Eularian Formulation $\Leftarrow$

$F(\vec{\xi}, t)$: lagiarian Formulation.

· Velocity. $\qquad i: 1, 2, 3$

$\qquad\qquad x, y, z$

$$v_i \equiv \frac{\partial x_i}{\partial t}$$

$$\boxed{\vec{v} = \frac{\partial \vec{x}}{\partial t}.}$$: definition of velocity.

follow the pareicle.

Eularian formulation $F(\vec{x},t)$

$$\frac{\partial}{\partial t} = \left.\frac{\partial}{\partial t}\right)_{\vec{x}}$$ derivative with respect to time. keep $\vec{x}$ constant (observer fixed in space)

$$\frac{D}{Dt} = \left.\frac{\partial}{\partial t}\right)_{\vec{\xi}}$$

keep $\vec{\xi}$ fixed
(observer moving with fluid).

→ Substantial
Material } Derivative.
Total

$\left.\frac{\partial F}{\partial t}\right)_{\vec{x}}$ time rate of change of F at a fixed point

$\left.\frac{DF}{Dt}\right)$ F moving with fluid.

$$\frac{DF}{Dt} = \frac{\partial F(\vec{\xi},t)}{\partial t} = \frac{\partial F[\vec{x}(\vec{\xi},t),t]}{\partial t}$$

$$= \frac{\partial F}{\partial x_i}\left.\frac{\partial x_i}{\partial t}\right)_{\xi} + \left.\frac{\partial F}{\partial t}\right)_x$$

$$= \frac{\partial F}{\partial t} + v_i \frac{\partial F}{\partial x_i}.$$

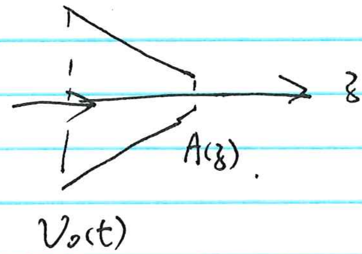
$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + V\cdot\nabla F$$

· Acceleration. change of velocity with respect to time

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{V} = \vec{a}.$$

local accelaration

convective accelration



Incompressible.

$\rho_o V_o A = \rho v A$.

$v = \frac{A_o}{A(z)} V_o(t)$

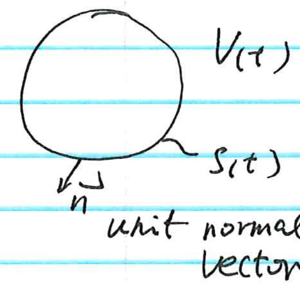
local accel $\frac{A_o}{A} \frac{\partial V_o(t)}{\partial t}$.

convective acce. $\vec{v} \cdot \nabla \vec{v}|_z = \frac{A_o}{A} V_o \frac{\partial}{\partial z} \left(\frac{A_o}{A} V_o \right)$

$= \underbrace{- v^2 \frac{1}{A} \frac{dA}{dz}}$?

Integral Theorey.

Divergency theory.

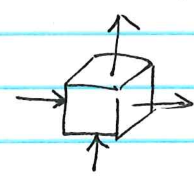


$$\int_V (\nabla \cdot \vec{v}) dV = \oint_S (\vec{n} \cdot \vec{v}) ds$$

$\oint_S$: surface of volume V.

Volume integral $\Rightarrow$ surface integral.

$\nabla \cdot \vec{v} = ?$



green's theory

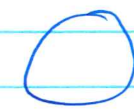
$$\int_V (\nabla \cdot \psi) dV = \oint_S \vec{n} \cdot \psi ds$$

ψ : scalor vector ~ tensor

$$\int_V \nabla a \, dV = \oint_S \vec{n} \cdot a \, ds.$$

$$\int_V (\nabla \cdot \pi) dV = \oint_S (\vec{n} \cdot \pi) dS$$

$$\int_V \nabla p \, dV = \oint_S p \vec{n} \, ds.$$



· leibnitz Formula.

$$\frac{d}{dt} \int_{x=\alpha(t)}^{x=\beta(t)} f(x,t) dx = \int \frac{\partial f}{\partial t} + \frac{d\beta}{dt} f(x,t) \Big|_{x=\beta} - \frac{d\alpha}{dt} f(x,t) \Big|_{x=\alpha}.$$

$$\frac{d}{dt} \int_{V(t)} \psi \, dV = \int \frac{\partial \psi}{\partial t} dV + \underbrace{\oint_S \psi (\vec{v}_S \cdot \vec{n}) ds}_{\text{surface flux}}.$$

V_s : surface velocity.

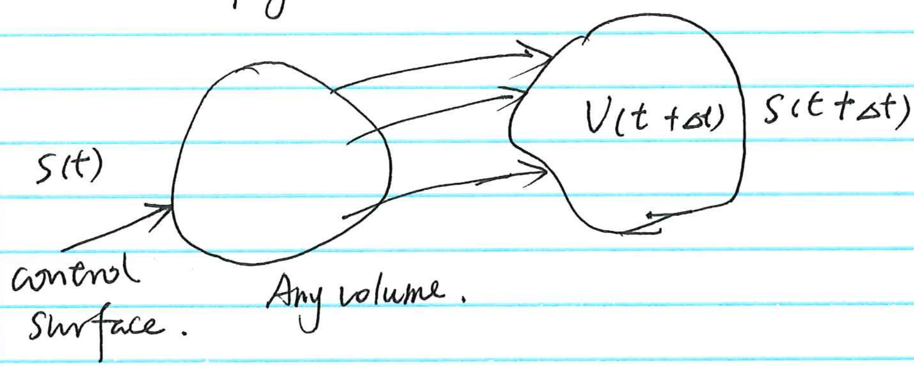
$\boxed{\text{Appendix A}}$

Aris's Book.

$$\frac{d}{dy} \int_{g_1(y)}^{g_2(y)} f(x,y) dx = \int_{g_1(y)}^{g_2(y)} \frac{\partial f(x,y)}{\partial y} dx + g_2'(y) f(g_2(y), y) - g_1'(y) f(g_1(y), y)$$

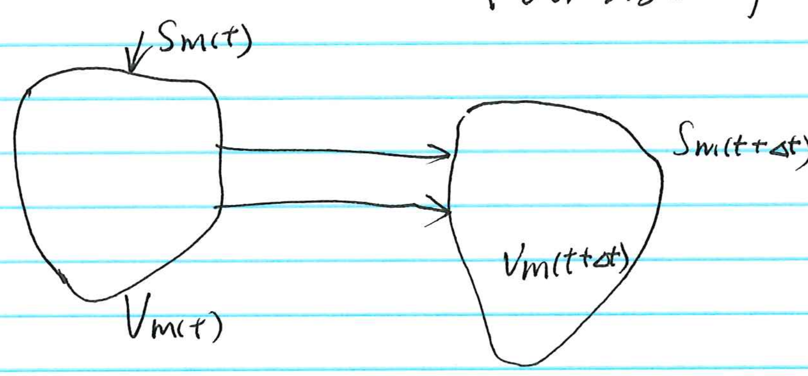
e.g. train

- Control Volume.
  simply connected.



- Material Volume

Boundary of system: { Boundary of Mass.
no mass cross the boundary.
consist of same mass.



$\frac{d}{dt}\int_V \psi \, dV$: change of ψ in V with respect to time

$\frac{d}{dt}\int_{V_m} \psi \, dV$ $\qquad$ V_m .. .. ..

$\updownarrow$

$\frac{D}{Dt}\int_{V_m} \psi \, dV$

Leibnitz Formula

$$\underbrace{\frac{d}{dt}\int_V \psi \, dV}_{\substack{\text{time rate of} \\ \text{change of total} \\ \psi \text{ in } V.}} = \underbrace{\int_V \frac{\partial \psi}{\partial t} dV}_{\substack{\text{Total time} \\ \text{rate of change} \\ \text{of } \psi \text{ in } V.}} + \underbrace{\oint_S \psi (\vec{v}_s \cdot \vec{n}) ds.}_{\substack{\text{change in amount} \\ \text{of total } \psi \text{ due to vol.} \\ \text{change caused by surface motion}}}$$

surface velocity

Special case of Material Volume.

$$\frac{d}{dt}\int_{V_m} \psi \, dV = \int_{V_m} \frac{\partial \psi}{\partial t} dV + \oint_{S_m} \psi (\vec{v}_s \cdot \vec{n}) ds.$$

$\vec{v}_s = \vec{v}$: material velocity

$$\oint_{S_m} \psi (\vec{v}_s \cdot \vec{n}) ds = \oint \psi (\vec{v} \cdot \vec{n}) ds.$$

use divergence theorosm $\oint \rightarrow$ vol. integral.

$$\oint_S (\psi \vec{v}) \cdot \vec{n} \, ds = \int_V \nabla \cdot (\psi \vec{v}) dV.$$

$$\frac{d}{dt}\int_{V_m} \psi \, dV = \int_{V_m} \left[\frac{\partial \psi}{\partial t} + \nabla \cdot \psi \vec{v} \right] dV.$$

Reynolds Transport theorem

$$\boxed{\frac{D}{Dt}\int_{V_m} \psi \, dV \equiv \int_{V_m} \left[\frac{\partial \psi}{\partial t} + \nabla \cdot \psi \vec{v} \right] dV.}$$

Integral Balance $\Rightarrow$ Differential Balance

II general Formulation.

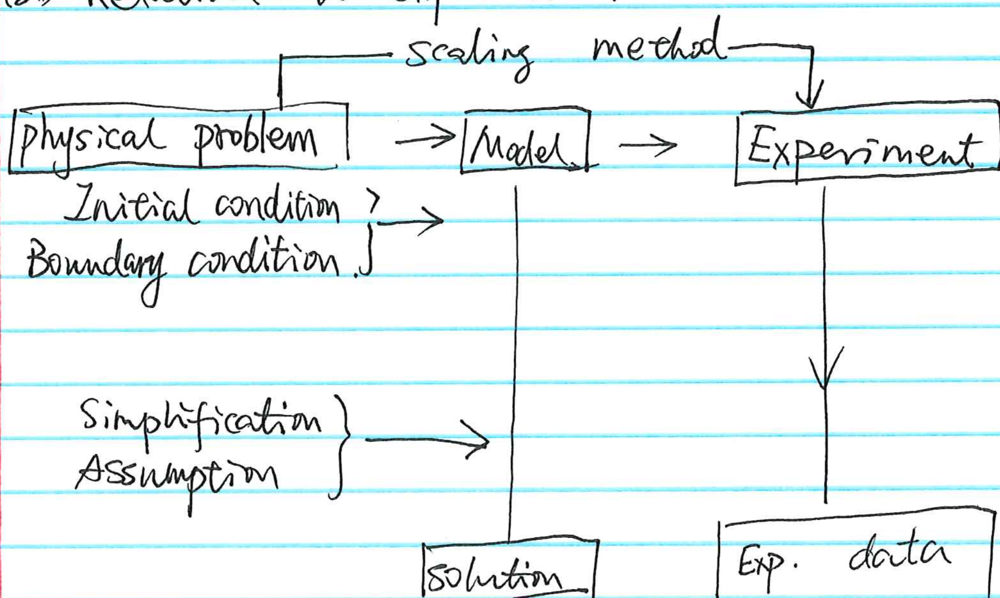
(1) Fundametal approach.

(i) Mathematical Model.

physical system	Mathematical System	Model.
physical concept	Math concept (variable)	→ Variables,
physical law	general Axiom	→ Field Eq.
particular class of Material	Constitute Axiom	→ Constitutive Relation

$\boxed{\text{Appendix A + Chapter II.}}$

(ii) Relation to experiment.



Appendix K — CSAU Methodology ⇐ NRC
Code, Scaling, Applicability
Uncertainty

Nuclear Reactor Thermal-hydraulics.

- Single phase flow
  - Laminar flow Model
  - Turbulent flow Model.

3-D
1-D
(Quasi 1-D)

Control Volume.

- Two phase Flow (Mutiphase flow)
  - Local Instant Formulation with interface
  - ⇒ Two-fluid Model
  - Drift flux model
  - slip Flow model.
  - homogenous Flow model ($T_g = T_f$, $V_g = V_f$)

3-D
⇒ 1-D
Quasi 1-D : subchannel Model.
control volume (0-D)

- Need of experiment data

1) Observe phenomena → scoping experiment

2) fundamental Understanding → Basic experiment
   - Detailed Instrumentation
   - wide ranged of Parameter
   - Simplified geometry

$\downarrow$

$\boxed{\text{model}}$

3) Complicated Thermal hydraulic phenomena (Nuclear Reactor)

<u>Integral Test</u>

- scaled down
- geometry is similar (most complicated)

Seperate - effect test

- component
- Region
- particular. Phenomena

Design Data Base

CHF

2) conservation principle.

Integral Balance ⇐ you can observe

↓

$\boxed{\text{physical laws}}$.

① Mass

$$\underbrace{\frac{D}{Dt}\int_{V_m} \rho \, dV}_{\text{Total Mass}} = 0.$$

② Momentum Equation.

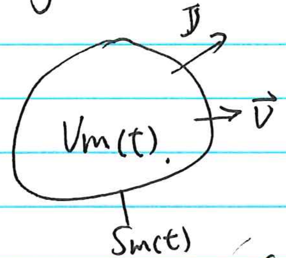
Mass × Acc. = Σ Forces.

$$\underbrace{\frac{D}{Dt}\int_{V_m} \rho \vec{v} \, dV}_{\text{Total momentum}} = \underbrace{-\oint \mathbb{J} \cdot \vec{n} \, ds}_{\text{surface force}} + \underbrace{\int \psi_g \, dV}_{\text{Body force. (重力)}}$$

③ energy Equation

$$\frac{D}{Dt}\int_{V_m} \underbrace{\rho \left(u + \frac{V^2}{2}\right) dV}_{\text{Total Energy}} = \underbrace{-\oint \mathbb{J} \cdot \vec{n} \, ds}_{\substack{\text{Heat Tran. and} \\ \text{work done at surface}}} + \underbrace{\int \psi_g \, dV.}_{\substack{\text{Heat generation} \\ \text{work done by body force}}}$$

3. general balance Equation.



V_m : fixed mass volume

· ψ property enclosed in vol.
(mass . momentum . energy)

$\int \psi \, dv$ = total amount of ψ in V.

· J : flux of property across the surface S_m.

$-\oint J \cdot \vec{n} \, ds$: net flux in cross the surface S_m.

· $\dot{\psi}_g$: generation of ψ in vol. V

$\int \dot{\psi}_g \, dV$: total generation of ψ in V

$$\boxed{\text{Change of } \psi \text{ in Vol} = \text{Influx through surface} + \text{generation in Vol}}$$

$$\boxed{\frac{D}{Dt}\int_{V_m} \psi \, dV = -\oint_{S_m} J \cdot \vec{n} \, ds + \int_{V_m} \dot{\psi}_g \, dv} \qquad ①$$

change of ψ in V. — Influx of ψ through S_m — generation of ψ in V

$\boxed{\text{Integral Balance}} \rightarrow \boxed{\text{Differential Balance}}$

(i) $$\frac{D}{Dt}\int_{V_m} \psi \, dV = \int_V \left[\frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \vec{v})\right] dV \qquad ②$$

(Reynals Transport theory)

(ii) Green's theorm.

$$-\oint \mathbb{J}\cdot\vec{n}\,ds = -\int \nabla\cdot\mathbb{J}\,dV. \quad ③$$

② ③ → ①

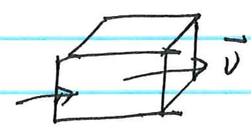
$$\int_{V_m}\left[\frac{\partial\psi}{\partial t} + \nabla\cdot(\psi\cdot\vec{v})\right]dV = -\int_{V_m}\nabla\cdot\mathbb{J}\,dV + \int_{V_m}\dot{\psi}_g\,dV.$$

$$\hookrightarrow \frac{\partial\psi}{\partial t} + \boxed{\nabla\cdot(\psi\cdot\vec{v})} = -\nabla\cdot\mathbb{J} + \dot{\psi}_g$$

Differential Balance Eq.

Physical meaning

- $\frac{\partial\psi}{\partial t}$: Time rate of change of ψ in unit volume
- $\nabla\cdot(\psi\cdot\vec{v})$: convection of ψ by material motion.
- $-\nabla\cdot\mathbb{J}$: in flux of ψ at the surface
- $\dot{\psi}_g$: generation of ψ in Vol.



$\vec{v}$ ← what's the difference between 2.

ψ → Mass
Momentum
energy.

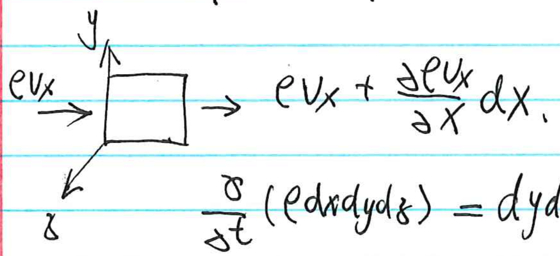
(+) Mass Balance Eq. ⇒ continuity.

$$\begin{cases}\psi = \rho \quad \text{density} \\ \mathbb{J} = 0 \\ \psi_g = 0\end{cases} \Longrightarrow \frac{d\rho}{dt} + \nabla(\rho\cdot\vec{v}) = 0$$

General Balance Equation

$$\frac{\partial \psi}{\partial t} + \nabla \cdot \psi \cdot \vec{v} = -\nabla \cdot J + \dot{\psi}_g$$

$\nabla \rho \vec{v}$: rate of convection of mass due to material motion per unit volume (with respect to fixed frame).



$$\frac{\partial}{\partial t}(\rho\, dx dy dz) = dy dz \left[\rho v_x - \left(\rho v_x + \frac{\partial \rho v_x}{\partial x} dx\right)\right]$$

$$+ \cdots$$

$$+ \cdots$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0.$$

$$\frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho + \rho \nabla \cdot \vec{v} = 0$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho$$

$$\therefore \quad \frac{1}{\rho} \frac{D\rho}{Dt} = -\nabla \cdot \vec{v}.$$

* Incompressible fluid $\leftarrow$ most liquid flow
many gas flow $M \ll 1$

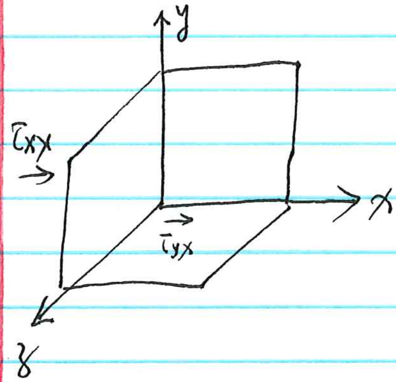
In this case. $\frac{D\rho}{Dt} = 0$: fluid does not change as it flows
(need not have uniform density)

$$\nabla \cdot \vec{v} = 0$$

* constant density flow.

$$\rho = \text{const.} \Rightarrow \nabla \cdot \vec{v} = 0$$

5) Momentum Balance Equation. – Equation of Motion.



τ_{xx} : x comp force on x direction Surface
τ_{yx} : x   y
τ_{zx} : x   z.

Surface direction   force direction

Quantity to Balance : momentum/vol. $\rho\vec{v}$.

$$\psi = \rho\vec{v}.$$

$$J = \Pi = p\cdot\mathbb{I} + \tau$$

(momentum flux – surface force).

Π : total stress
p : pressure
τ : viscous stress
$\mathbb{I}$: unit tensor.

$$\dot{\psi}_g = \rho\vec{F} = \rho\vec{g}$$

(momentum source – body force)

$\vec{F}$. body force
$\vec{g}$: gravity
(EM)

Momentum Equation.

$$\frac{\partial\rho\vec{v}}{\partial t} + \nabla\cdot(\rho\vec{v}\cdot\vec{v}) = -\nabla p - \nabla\cdot\tau + \rho\vec{g}$$

- $\frac{\partial\rho\vec{v}}{\partial t}$: time rate of change of the momentum/vol.
- $\nabla\cdot(\rho\vec{v}\cdot\vec{v})$: rate of momentum change by convection.
- $-\nabla p$: pressure force/vol.
- $-\nabla\cdot\tau$: viscous force/vol.
- $\rho\vec{g}$: gravitation body force/vol.

Cauchy's Equation of Motion.

M.E

$$LHS = \rho\frac{\partial\vec{v}}{\partial t} + \vec{v}\frac{\partial\rho}{\partial t} + \rho\vec{v}\cdot(\nabla\vec{v}) + \vec{v}(\nabla\cdot\rho\vec{v}).$$

$$= \rho\left(\frac{\partial\vec{v}}{\partial t} + \vec{v}\cdot\nabla\cdot\vec{v}\right] + \underbrace{\vec{v}\left[\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\vec{v}\right]}_{\text{C.t.}} \rightarrow 0$$

$$= \rho\frac{D\vec{v}}{Dt}.$$

$$\rho\frac{D\vec{v}}{Dt} = -\nabla\cdot p - \nabla\tau + \rho\vec{g}$$

$\left(\frac{\text{mass}}{\text{vol}}\right) \times \left[\begin{array}{l}\text{total} \\ \text{acceralation}\end{array}\right] = \Sigma\,\text{force} \leftarrow$ Newton's Law

$\left[\begin{array}{ll}\text{local accer.} & \left(\frac{\partial\vec{v}}{\partial x}\right) \\ +\text{converctive accer} & (\vec{v}\cdot\nabla\vec{v})\end{array}\right.$

Σ force → Sum of $\left\{\begin{array}{l}\text{surface force} \\ \text{body force}\end{array}\right.$

(7) Natural Convection

Bossinecq Assumption

- Density change due to thermal expansion
- Density change only important in gravity term

Thermal Expansion Coefficient : β

$$\beta \equiv \frac{1}{V}\frac{\partial V}{\partial T}\Big)_p = -\frac{1}{\rho}\frac{\partial\rho}{\partial T}\Big)_p. \quad (1)$$

V: specific volume

For small density change $\quad d\rho = -\rho\beta dT \quad (2).$

Reference Temp: $\bar{T}$

Density: $\bar{\rho}$ ~~(3)~~

$$\rho - \bar{\rho} \doteq -\bar{\rho}\beta(T-\bar{T}) \qquad (3).$$

(3) $\rightarrow$ Momentum Eq.

$$\bar{\rho}\frac{D\vec{V}}{Dt} = -\nabla P - \nabla\cdot\Pi + [\bar{\rho} - \bar{\rho}\beta(T-\bar{T})]\vec{g}$$

Note: in the inereia term $\rho = \bar{\rho}$

β: thermal physreal properties ($\leftarrow$ Table)

$\rightarrow$ · If the pressure field is approximately hydrostatic ($\vec{V}$ is small in large tank)

$$-\nabla P + \bar{\rho}\vec{g} = 0. \quad \text{(hydrostatic pressure)}$$

· Momentum Eq.

$$\bar{\rho}\frac{D\vec{V}}{Dt} = -\nabla\cdot\Pi - \bar{\rho}\beta(T-\bar{T})\vec{g}$$

$\rightarrow$ Natural convection case

$$\begin{cases} \nabla P = \nabla P_D + \nabla P_s \\ \nabla P_s = \vec{g}\bar{\rho} \end{cases}$$

D: Dynamic

S: Static

M.E

$$\rho\frac{D\vec{V}}{Dt} = \underset{\substack{\uparrow \\ \text{dynamic} \\ \text{pressure} \\ \text{gradient}}}{-\nabla P_D} + \underset{\substack{\uparrow \\ \text{viscous} \\ \text{force}}}{\mu\nabla^2\vec{V}} - \underbrace{\bar{\rho}\beta(T-\bar{T})\vec{g}}_{\text{buoyance force}}$$

(negligible in many cases)

Mechanical Energy Eq. (scalar Eq.)

By scalar product of the local velocity $\vec{v}$ with M.E

$$\vec{v}\cdot\left[\rho\frac{D\vec{v}}{Dt}=-\nabla p-\nabla\cdot\mathbb{T}+\rho\vec{g}\right].$$

$$\rho\frac{D\,v^2/2}{Dt}=-\vec{v}\cdot\nabla p-\vec{v}\cdot(\nabla\cdot\mathbb{T})+\rho\vec{v}\cdot\vec{g}$$

use C.E. ✓

CE

$$\rho\frac{D\,v^2/2}{Dt}=\frac{\partial\rho\frac{v^2}{2}}{\partial t}+\nabla\cdot\left(\rho\frac{v^2}{2}\cdot\vec{v}\right)$$

also.
$$\begin{cases}\vec{v}\cdot\nabla p=\nabla\cdot(p\vec{v})-p\nabla\cdot\vec{v}\\ \vec{v}\cdot\nabla\mathbb{T}=\nabla(\mathbb{T}\cdot\vec{v})-\mathbb{T}:\nabla\vec{v}\end{cases}$$

$$\underbrace{\frac{\partial\rho\frac{v^2}{2}}{\partial t}}_{\text{rate of increase}}+\underbrace{\nabla\left(\tfrac{1}{2}\rho v^2\vec{v}\right)}_{\text{convection}}=\underbrace{-\nabla(p\vec{v})}_{\text{work done by pressure}}+\underbrace{p\nabla\cdot\vec{v}}_{\text{reversible conversion to internal energy}}$$
$$\underbrace{-\nabla(\mathbb{T}\cdot\vec{v})}_{\text{work done by viscous force}}+\underbrace{\mathbb{T}\cdot\nabla\vec{v}}_{\text{irreversible conversion to internal energy}}$$
$$\underbrace{+\rho(\vec{v}\cdot\vec{g})}_{\text{work done by body force}}$$

Energy Equation.

Quantity to be balanced : total energy.

$\psi=\rho(u+\frac{v^2}{2})$ energy / volume

$$\begin{cases}u: \text{internal energy / mass}\\ \frac{v^2}{2}: \text{K.E. / mass}\end{cases}$$

$\mathbb{J} = \vec{q} + \Pi \cdot \vec{V}$. energy flux

heat flux + work done by surface force

$\Pi = p\mathbb{I} + \tau$

$\dot{\psi}_g = \rho \vec{V} \cdot \vec{g} + \dot{q}$ energy source

(work done by body force + internal heat generation ~~from~~

nuclear

E.M

ch. reaction

Energy Eq.

$$\frac{\partial \rho (u + \frac{V^2}{2})}{\partial t} + \nabla \cdot \left[\rho (u + \frac{V^2}{2}) \vec{V}\right] = -\nabla \cdot \vec{q} \quad \text{heat conduction}$$

$$-\nabla(p\vec{V}) \quad \text{work done by pressure}$$

$$-\nabla(\Pi \cdot \vec{V}) \quad \text{work done by viscous force}$$

$$+\rho \vec{V} \cdot \vec{g} \quad \text{work done by gravity.}$$

$$+\dot{q} \rightarrow \text{internal heat generation}$$

time rate of change — energy convection

$$\underbrace{\frac{\partial \rho (u + \frac{V^2}{2})}{\partial t} + \nabla \cdot \left[\rho (u + \frac{V^2}{2}) \vec{V}\right]}_{\rho \frac{D}{Dt}(u + \frac{V^2}{2})}$$

If the body force field has a potential Φ

$\vec{g} = -\nabla \Phi$ $\quad\Phi$: gravitational potential.

then, $\rho \vec{v} \cdot \vec{g} = -\rho (\vec{v} \cdot \nabla \Phi) = -\rho \frac{D\Phi}{Dt} + \cancel{\rho \frac{\partial \Phi}{\partial t}} \to 0$

$$\rho \vec{v} \cdot \vec{g} = -\rho \frac{D\Phi}{Dt}$$

Then total energy may be defined as

$$e = u + \frac{v^2}{2} + \Phi \quad (\text{internal } E. + K.E + \text{potential})$$

E.E.

$$\rho \frac{D}{Dt}\left(u + \frac{v^2}{2} + \Phi\right) = -\nabla \cdot \vec{q} - \nabla \cdot (p\vec{v}) - \nabla \cdot (\Pi \cdot \vec{v}) + \dot{q}$$

Separation of M.E and Thermal Energy.

Internal Energy Eq.

Elimination of K.E terms in view of M.E Eq.

To.E $\rho \dfrac{D(u + v^2/2)}{Dt} = -\nabla \vec{q} - \nabla \cdot (p\vec{v}) - \nabla \cdot (\Pi \cdot \vec{v}) + \rho \vec{v} \cdot \vec{g} + \dot{q}$

$-$

K.E $\rho \dfrac{D\frac{v^2}{2}}{Dt} = -\vec{v} \cdot \nabla p \oplus - \vec{v}(\nabla \cdot \Pi) + \rho \vec{v} \cdot \vec{g}.$

Internal. E.

$= \rho \dfrac{Du}{Dt} = -\nabla \cdot \vec{q} - p \nabla \cdot \vec{v} - \Pi \nabla \vec{v} + \dot{q}$

C.E $\Rightarrow \rho \dfrac{Du}{Dt} = \dfrac{\partial \rho u}{\partial t} + \nabla \rho u \vec{v}.$

$$\frac{\partial \rho u}{\partial t} + \nabla (\rho u \vec{v}) = -\nabla \cdot \vec{q} \quad \leftarrow \text{rate of energy tr. by heat C.}$$

time rate of change of i.e. per vol | rate of convection of i.e.

$-p\nabla \cdot \vec{v}$ ← rate of reversible conversion of work done by pressure

$-\Pi : \nabla \vec{v}$ ← rate of irreversible conversion of work done by viscous force.

$+\dot{q}$

This is not a basic balance equation

the RHS is not in $\nabla(\ )$ form

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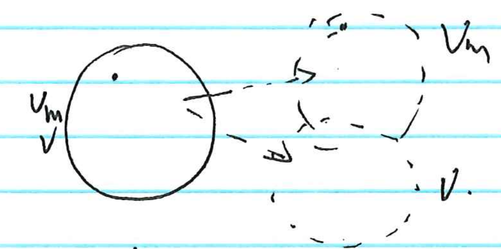
HW.

1) consider fixed mass volume V_m

control vol. V

at $t = t_0$, $V_m = V$



$$\frac{d}{dt}\int_{V_m} \psi \, dv = \frac{d}{dt}\int_V \psi \, dv + \oint_S \psi [\vec{V} - \vec{V}_s] \cdot \vec{n} \, ds.$$

$\vec{V}$: fixed mas. vel.

$\vec{V}_s$: cont vol. surface, velocity.

② obtain general (Integral) balance equation for control vol.

$$\frac{d}{dt}\int_{V_m} \psi \, dv = -\oint \mathbb{J} \cdot \vec{n} \, ds + \int_V \dot{\psi}_g \, dv.$$

$$\frac{d}{dt}\int_V \psi \, dv =$$

③ write Differential Balance Equation (general B.)

(1) Apply this to mass → cont eq

momentum → momentum eq.

Energy equation

$$\rho \frac{D\left(u+\frac{v^2}{2}\right)}{Dt} = -\nabla\cdot\vec{q} - \nabla\cdot(p\vec{v}) - \nabla\cdot(\bar{\bar{\tau}}\cdot\vec{v}) + \rho\vec{v}\cdot\vec{g} + \dot{q}$$

$$\ominus \quad \underbrace{\rho\frac{D\frac{v^2}{2}}{Dt}}_{K.E} = -\vec{v}\cdot\nabla p - \vec{v}\cdot[\nabla\cdot\bar{\bar{\tau}}] + \rho\vec{v}\cdot\vec{g}$$

$$\rho\frac{Du}{Dt} = \underbrace{-\nabla\cdot\vec{q}}_{\text{heat flux}} \underbrace{- p\nabla\cdot\vec{v}}_{\substack{\text{pressure work}\\ \text{converted to energy}\\ \text{(reversible)}}} - \underbrace{\cancel{\bar{\bar{\tau}}:\nabla\vec{v}}}_{\substack{\text{viscous work}\\ \text{converted to energy}\\ \text{viscous dissipation}}} \xrightarrow{} 0 + \underbrace{\cancel{\dot{q}}}_{\text{heat generation.}} \to 0$$

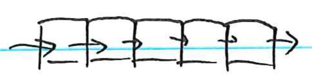
$M = \frac{v}{a} < 0.3.$ K.E is not important.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\nabla$$

$$\rho\frac{Du}{Dt} = \rho\left[\frac{\partial u}{\partial t} + \vec{v}\cdot\nabla u\right] \quad \text{non conservative form}$$

$$\neq \frac{\partial \rho u}{\partial t} + \nabla\cdot\rho u\vec{v} \quad \text{conservative form}$$

error.



$$\boxed{\rho\frac{Du}{Dt} = -\nabla\cdot\vec{q} - p\nabla\cdot\vec{v}.}$$

$$\updownarrow$$

$$dU = dQ - pdV.$$

Enthalphy. Equation.

$$i \equiv u + P/\rho.$$

$$\underbrace{\frac{\partial \rho i}{\partial t} + \nabla\cdot\rho(i\vec{v})} = -\nabla\cdot\vec{q} + \frac{DP}{Dt} - \tau : \cancel{\nabla\cdot\vec{v}} + \cancel{\dot{q}}$$

$$\rho\frac{Di}{Dt} \doteq -\nabla\cdot\vec{q} + \frac{DP}{Dt}$$

(1) Single Mixture (steam + Air)

ρ_k : mass concentration.
(mass of k component/vol.).

w_k : mass fraction $= \frac{\rho_k}{\rho}$.

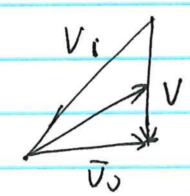
ρ : mixture density $\rho = \sum_{k=1}^{n} \rho_k$.

V_k : velocity of component k.

$\rho_k\vec{v}_k$: momentum of k component.

$\rho\vec{v} = \sum_{k=1}^{n} \rho_k\vec{v}_k$: total momentum

$\vec{v} = \frac{\sum \rho_k\vec{v}_k}{\sum \rho_k}$ = center mass velocity.



Diffusion velocity.

$$\vec{V}_{km} = \vec{V}_k - \vec{v} \qquad \sum \rho_k \vec{V}_{km} = 0.$$

In general balance Eq.

$$\begin{cases} \psi = \rho_k \\ \mathbb{J} = \rho_k(\vec{V}_k - \vec{v}) = \rho_k V_{km} = \vec{J}_k & \text{diffusion mass flux} \\ \dot{\psi}_g = r_k. & \text{reaction rate.} \end{cases}$$

$$\frac{\partial \rho_k}{\partial t} + \nabla\cdot\rho_k\vec{v} = \underbrace{-\nabla\cdot\vec{J}_k}_{\substack{\text{diffusion}\\ \text{mass flux}\\ \text{with respect}\\ \text{to } \vec{v}}} + \underbrace{\dot{r}_k}_{\substack{\text{reaction}\\ \text{rate}}}$$

$$\vec{J}_k = \rho_k(\vec{V}_k - \vec{v})$$

$$\frac{\partial \rho_k}{\partial t} + \nabla\cdot\rho_k\vec{V}_k = 0 + \dot{r}_k$$ ← multifield. two-field.

$$\sum \dot{r}_k = 0$$

$$\sum\left\{\frac{\partial \rho_k}{\partial t} + \nabla\rho_k\vec{V}_k = \dot{r}_k\right\}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\vec{v} = \sum \dot{r}_k = 0.$$

M.M $\psi = \rho\vec{v}$

$$\mathbb{J} = p\,\mathbb{I} + \mathbb{T} + \sum \rho_k \vec{V}_{km}\vec{V}_{km} = \pi$$

$$\dot{\psi}_g = \sum \rho_k \vec{g}.$$

$$\frac{\partial \rho\vec{v}}{\partial t} + \nabla\cdot\rho\vec{v}\vec{v} = -\nabla p - \nabla\cdot\mathbb{T} - \nabla\cdot\sum\rho_k\vec{V}_{km}\vec{V}_{km} + \sum\rho_k\vec{g}$$

solve for $\vec{v}$

Chapter 3.

Single flow constitutive Relation

(1) closure of Equation System.

Balance Eq. (Mass. Momentum. Energy) $\to$ 3.

unknown : Mass ρ $\vec{v}$
Momentum p . τ . $\vec{g}$
Energy u . $\vec{q}$. $\dot{q}$

8 unknowns.

① Balance Equation should be supplemented by various consititutive relations (closure relations)

② Consititutive relations.

. model material response observed in experiment

· violate any physical law

· reasonable simple forms

· fit to certain practical idealization

(2) 2nd law of Thermodynamics

(i) most important physical law outside of. Conservation Equation.

· existance of Entropy . S
Temperature T

· 2nd Law

$$\frac{d}{dt}\int_{V_m} \rho s \, dV + \underbrace{\oint_{S_m} \frac{\vec{q}}{T} \cdot \vec{n} \, ds}_{\text{enterolpy flux}} - \underbrace{\int_{V_m} \frac{\dot{q}}{T} dV}_{\text{entrolpy source}} \geq 0$$

use Reynolds Transport theorum

$$\frac{\partial}{\partial t} \rho s + \nabla \cdot (\rho s \vec{v}) + \nabla \cdot \left(\frac{\vec{q}}{T}\right) - \frac{\dot{q}}{T} \equiv \Delta \geq 0$$

Δ: entrolpy production.
$=$: reversible system (ideal)
$>$: irreversible system

general guidelines.

- Entropy Inequality.
- Determinism
- Frame indifference.
- Local action

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Consistentive Relation.

- Thermodynamic Property
- Forces → Momentum
- heat energy.

~~#~~ (4) Equation of state

· Fundamental Equation of state

$u = u(s, \rho)$ (2)

u: internal energy
s: entrolpy
ρ: density.

$$\begin{cases} T \equiv \left.\frac{\partial u}{\partial s}\right)_\rho & \text{Th. temperature (3).} \\ p = -\frac{\partial u}{\partial (1/\rho)} & \text{Th. Dy. pressure (4)} \end{cases}$$

$du = Tds - p\,d(1/\rho)$

Legendre. Transformation $\Leftarrow$ Book.

Practical Equation of State

$s \rightarrow T, p.$

1. Eq. (Fundamental Eq. of State) $\Rightarrow$ 2 Eqs of state

$\rightarrow \begin{cases} p = p(\rho, T) & \text{Thermal Eq. of State} \\ u = u(\rho, T) & \text{Calorie Eq. of State.} \end{cases}$

Example

(a). Incompressible fluid

$\rho = \text{constant}$

$u = u(T)$. ~~$du = C_v dT$~~ $du = C_v dT$

(b) Ideal gas (steam, air, N_2). (approximation)

$p = RT\rho$

$u = u(T)$

(3) Mechanical Constitutive Relation

$\uparrow$ motion force

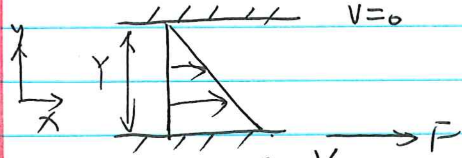
$\tau, \vec{g}$

viscous. force.

(a). Inviscid fluid. $\tau = 0$

$\begin{cases} \text{slip at the wall.} \\ \text{no frictional resistance} \end{cases}$

(b) Linear viscous Fluid (牛 Newton)



$$\frac{F}{A} = \mu \frac{V}{Y}$$

$\downarrow$

$\tau_{yx} = -\mu \, dV_x/dy$ ~ velocity gradient

μ: viscosity

Force ~ velocity gradient (Linear)

Newton's law of viscosity ⇒ Newtonian fluid.

Note: Similarity between

- Momentum transfer — viscous force τ
- Heat (energy) transfer — heat flux $\vec{q}$
- Diffusion Mass flux — $\vec{j}_k$

$\tau_{yx} = -\mu \frac{du_x}{dy}$ (Newtonian Law of viscosity)

$q_y = -k \, dT/dy$ (Fourier's Law of conduction)

$j_{ky} = -\rho D_k \, dw_k/dy$ (Fick's Law of Diffusion)

- w_k : mass
- D : diffusion coefficient
- μ : viscosity
- k : thermal conducity

$$\tau = -\mu\left[\nabla\vec{V} + (\nabla\vec{V})^{t}\right]$$

t : transported Tensor

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

generalized it $\Rightarrow$ N-S Fluid

$$\tau = -\mu\left[\nabla\vec{V} + (\nabla\vec{V})^{t}\right] + \left(\frac{2}{3}\mu - \mu'\right)\nabla\cdot\vec{V}\cdot\mathbb{I}$$

c) Body force

Gravitational force $\vec{g}$ = const.

Electrostatic. } much complicated.

Ele magnatic. }

$\downarrow$

Magnetic Fluid Dynamic

Plasma Physics.

(4) Thermal conservative Relation

① conduction in Isotopic Materal.

$$\vec{q} = -k\nabla T$$

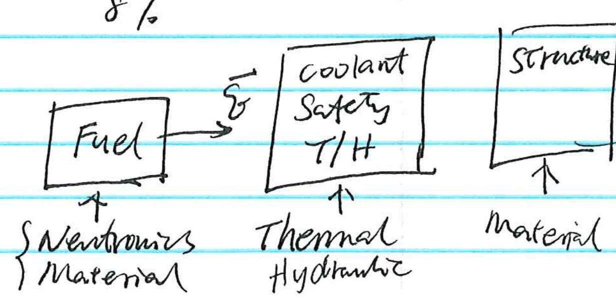
k : conductivity $\qquad k = k(T)$

② Internal heat generation. $\dot{q}$

$\dot{q}$: Fission & decay heat.

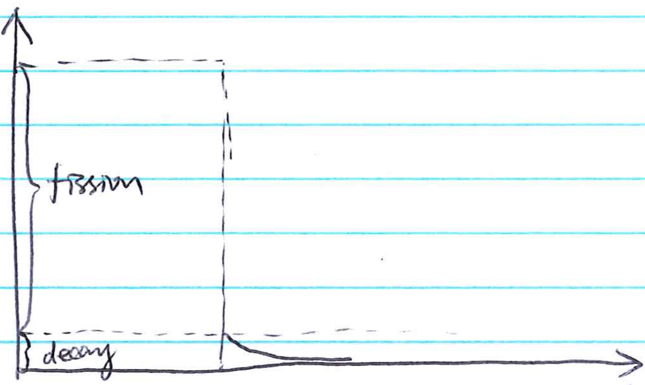
92% $\qquad$ 8%

N reactor



- $\vec{j}_k = -\rho D \nabla w_k$

$\dot{q} \leftarrow$ fission & decay heat.



$\dot{r}_k \neq \cancel{r_k}$ chemical reaction rate

$\dot{r}_k = \dot{r}_k(w_k, T, p)$.

(7). Entropy generation check.

Fundamental Eq. of state

$$T ds = du - \frac{p}{\rho^2} d\rho \qquad s = s(u, \rho)$$

$\downarrow$

internal energy Eq.

$$\rho \frac{Du}{Dt} = -\nabla \cdot \vec{q} - p \nabla \vec{v} - \tau : \nabla \vec{v} + \dot{q}$$

$$\rho \left\{ T \frac{Ds}{Dt} + \frac{p}{\rho^2} \frac{D\rho}{Dt} \right\} \qquad CE \rightarrow \rho \left\{ T \frac{Ds}{Dt} - \frac{p}{\rho} \nabla \cdot \vec{v} \right\}$$

Internal Energy Eq. $\rightarrow$ Entropy Energy Eq.

$$\rho \frac{Ds}{Dt} = -\frac{\nabla \cdot \vec{q}}{T} - \frac{\tau : \nabla \vec{v}}{T} + \frac{\dot{q}}{T}$$

2nd law

$$\rho \frac{Ds}{Dt} + \nabla \cdot \left(\frac{\vec{q}}{T} \right) - \frac{\dot{q}}{T} \equiv \Delta > 0$$

$$\boxed{\Delta = -\frac{\vec{q}\cdot\nabla T}{T^2} - \frac{\tau : \nabla\vec{V}}{T} \geq 0}$$ Requirement

$$\left[\begin{cases} \vec{q} = -k\nabla T \\ \tau = -\mu[\nabla\vec{V} + (\nabla\vec{V})^{+}] \end{cases}\right.$$ Appendix A.

$-\vec{q}\cdot\nabla T \geq 0$

and $-\tau : \nabla\vec{V} \geq 0$.

Equation

C.E	$\rho, \vec{V}$	3. field equation
M.E	$p, \tau, \vec{g}$	$\tau =$
I.E.E.	$u, \vec{q}, \dot{q}$	$g =$
	T	$q =$
	$\underbrace{\hphantom{xxxxxxxx}}_{9.}$	$\dot{q}$ -

Thermal Eq. of state $p = p(T,\rho)$

Caloric Eq. of state $u = u(T,\rho)$

3) constant Trasport property Case.

$\mu = \text{const}$

$k = \text{const}$

$$\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\vec{V} = 0.$$

Newtonian

$$\rho\frac{D\vec{V}}{Dt} = -\nabla p + \mu\nabla^2\vec{V} + \frac{\mu}{3}\nabla\cancel{(\nabla\cdot\vec{V})} + \rho\vec{g}$$

$$\rho C_v\frac{DT}{Dt} = k\nabla^2 T - \cancel{\tau : \nabla\vec{V}} + T\left.\frac{\partial p}{\partial T}\right)_{\rho}\nabla\cdot\vec{V} + \cancel{\dot{q}}$$

4) Incompressible with const. Tr. Pr.

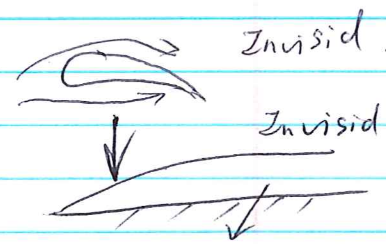
$\rho = \text{const}$

$\mu = \text{const}$

$k = \text{const}$

$$\begin{cases} \nabla\cdot\vec{V} = 0. \\ \rho\, D\vec{V}/Dt = -\nabla p + \mu\nabla^2\vec{V} + \rho\vec{g} \\ \rho c\frac{DT}{Dt} = k\nabla^2 T + \cancel{\dot{q}} \end{cases}$$

$\vec{V}$, p, T } specify c, μ, k, ρ.



5) Invisid Compressible Flow.

$$\frac{\partial \rho}{\partial t} + \nabla \rho \vec{v} = 0$$

$$\rho \frac{D\vec{V}}{Dt} = -\nabla p + \rho \vec{g}$$ ← Eurler 1755

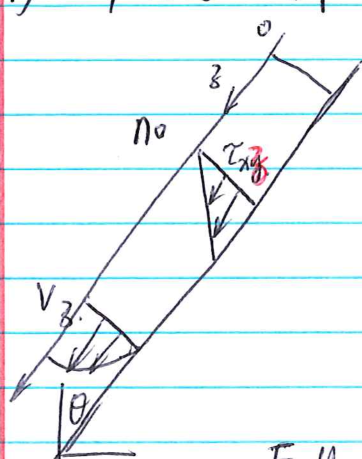
$$\rho C_v \frac{DT}{Dt} = -\nabla \cdot \vec{q} - T \left.\frac{\partial P}{\partial T}\right)_\rho \nabla \cdot \vec{V}$$

$\nabla \cdot \vec{q}$ $\qquad$ $\vec{q} = -k\Delta T$

$\rho = \rho(T, p)$ Ided gas law.

$u = u(T)$.

1) Flow of Falling Liquid film.



Steady state.

Fully developed $\partial/\partial z \to 0$

No shear at Gas-Liquid Int.

Laminar flow

~~Adiabatic~~ Isoth.

Liquid ⇒ Incompressible.

Fully developed. $\quad V_z = V_z(x)$, $V_y = 0$ } 2-D ①

C.E. $\nabla \cdot \vec{v} = 0$

$$\frac{\partial V_x}{\partial x} + \cancel{\frac{\partial v_y}{\partial y}} + \cancel{\frac{\partial V_z}{\partial z}} = 0 \qquad \frac{\partial V_x}{\partial x} = 0. \quad V_x = 0$$

(at wall. $V_x = 0$)

$V_z = V_z x.$

Momentum Eq.

$$z: \rho\left\{\frac{\partial v_z}{\partial t} + v_x\frac{\partial V_z}{\partial x} + V_y\frac{\partial V_z}{\partial y} + V_z\frac{\partial V_z}{\partial z}\right\} = \frac{-\partial p}{\partial z}$$

$$+\mu\left\{\frac{\partial^2 V_z}{\partial x^2} + \cancel{\frac{\partial^2 V_z}{\partial y^2}} + \cancel{\frac{\partial^2 V_z}{\partial z^2}}\right\} + \rho g_z.$$

$$-\frac{\partial p}{\partial z} + \mu\frac{\partial^2 V_z}{\partial x^2} + \rho g_z = 0. \qquad ②$$

$$x: \quad -\frac{\partial p}{\partial x} + \rho g_x = 0. \qquad ③$$

$g_x = \rho \cos\theta$

$g_z = \rho \sin\theta.$

B.C. $x=0$ $\quad p = p_\infty$ free surface.

$p = \rho g_x x + C_1(z).$ $\qquad C_1(z) = p_\infty.$

$$p = \rho g_x x + p_\infty \qquad ④.$$

$\frac{\partial p}{\partial z} = 0 \rightarrow ②$ Integrate.

$$V_z = -\frac{\rho g_z x^2}{2\mu} + C_2 x + C_3.$$

at $x = \delta.$ $\quad V_z = 0$ (B.C.)

$C_3 = \frac{\rho g_z \delta^2}{2\mu}$ $\qquad \delta$: film thickness

1) Velocity profile.

$$V_z = \frac{\rho g \delta^2 \cos\theta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]. \qquad \text{parabolic.}$$

2) Max. Vel.

$$V_{z\,max} = \frac{\rho g \delta^2 \cos\theta}{2\mu}.$$

(3). Average.

$$\langle V_z \rangle = \frac{1}{\delta}\int_0^\delta V_z\,dx = \frac{\rho g \delta^2 \cos\theta}{3\mu}.$$

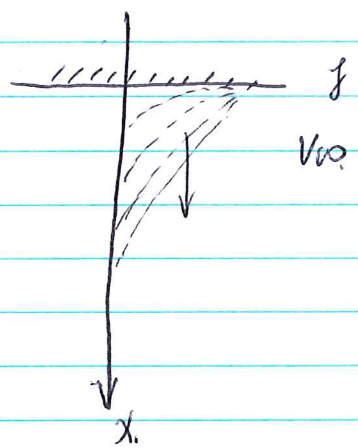
(4) Vol. Flow with width. W

$$Q = \delta W \langle V_z \rangle = \frac{\rho g W \delta^3 \cos\theta}{3\mu}.$$

(5) Film thickness

$$\delta = \sqrt[3]{\frac{3\mu Q}{\rho g W \cos\theta}}.$$

Transient solution. Chara



2-D

$t<0$ $\quad V_x = 0,\ V_y = 0.$

$t = 0^+$ $\quad V_y(0,y) = V_\infty.$

$p =$ uniform at $x = 0.$ $\quad p = p_\infty$

$V_x = 0.$ at $x = 0$ $\quad V_z = 0$ everywhere (2-D)

· Incompressible. $\quad \rho = \text{const}$

const properties. $\quad \mu = \text{const}.$

C.E. $\dfrac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0.$ $\quad \nabla\vec{v} = 0$ $\quad \dfrac{\partial V_x}{\partial x} + \dfrac{\partial V_y}{\partial y} = 0$

$V_y = V_y(t, x).$ (not function of y)

from C.E $\quad \dfrac{\partial V_x}{\partial x} = 0.$ $\quad V_x = 0.$ at everywhere.

B.C. at $x = 0.$ $V_x = 0.$

Momentum Eq.

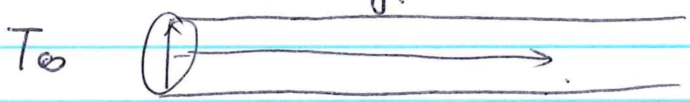
$$\rho\left\{\frac{\partial v_z}{\partial t} + \cancel{v_x \frac{\partial v_z}{\partial x}} + \cancel{v_y \frac{\partial v_z}{\partial y}} + \cancel{v_z \frac{\partial v_z}{\partial z}}\right\} = -\frac{\partial p}{\partial z}$$

$$+ \mu\left\{\frac{\partial^2 v_z}{\partial x^2} + \cancel{\frac{\partial^2 v_z}{\partial y^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}\right\} + \cancel{\rho g_z}$$

$$\rho \frac{\partial v_z}{\partial t} = \mu \frac{\partial^2 v_z}{\partial x^2} \Rightarrow \frac{\partial v_z}{\partial t} = \nu \frac{\partial^2 v_z}{\partial x^2}$$

$\nu = \frac{\mu}{\rho}$: kinematic viscosity.

Sudden heating.



$T = T_0 \qquad t < 0$

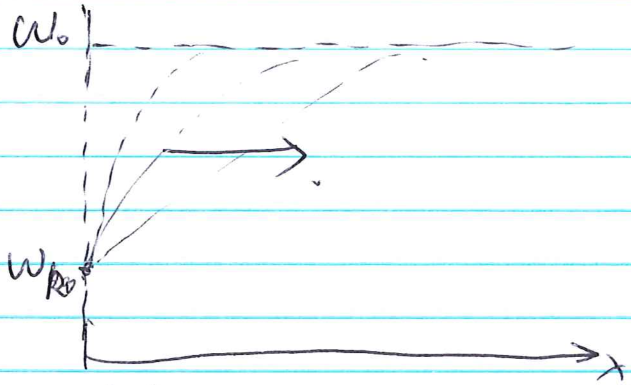
$T = T_\infty \qquad t \geq 0$

at $x = 0$.

$$\rho C_v \left[\frac{\partial T}{\partial t} + \cancel{\vec{v}\,\nabla T}\right] = -k\nabla^2 T + \cancel{q_T}$$

$$\rho C_v \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} \qquad \frac{\partial T}{\partial t} = \frac{k}{\rho C_v}\frac{\partial^2 T}{\partial x^2} \rightarrow \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

$\alpha = \frac{k}{\rho C_v}$ thermal diffusivity.



$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot \rho_k \vec{v} = -\nabla \cdot \vec{J}_{in} + j_k .$$

mass fraction $\quad \frac{\rho_k}{\rho} = W_k .$

$$\begin{cases} \rho = \text{const.} \\ \text{no motion } \vec{v} = 0 \\ r_k = 0 \end{cases}$$

$$\frac{\partial W_k}{\partial t} = D \nabla^2 \cdot W_k .$$

D: Mass diffusion coefficient.

$$\begin{cases} v \\ T \\ W_k \end{cases} \Rightarrow$$

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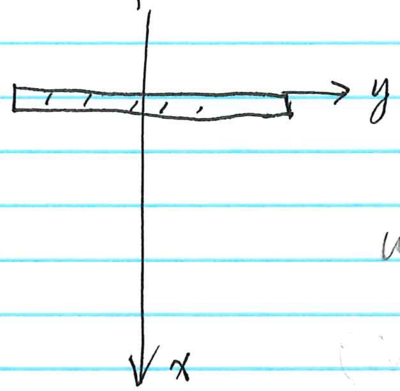
HW #3

Solve for the case of sudden motion.



$V=0 \quad t<0$

$\quad\quad\;\, t=0^+$

$x=0 \quad V=V_\infty$

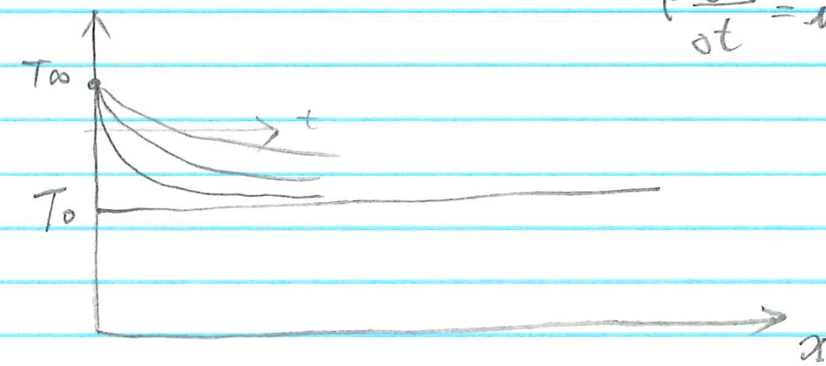
uniform in y direction.

B.C.

$$\frac{dV}{dt} = \mu \frac{\partial^2 V}{\partial x^2} \qquad \frac{\partial V}{\partial t} = \nu \frac{\partial^2 V}{\partial x^2}$$

$$\rho \frac{\partial V}{\partial t} = \mu \frac{\partial^2 V}{\partial x^2}$$

$$\nu = \frac{\mu}{\rho}$$



$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_v}\frac{\partial^2 T}{\partial x^2}$$

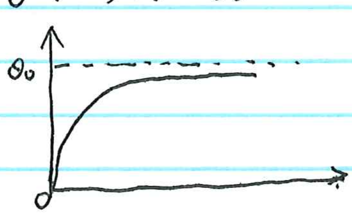
$t \le 0 \quad T(0, x) = T_0$

$t = 0^+ \quad T(0^+, 0) = T_\infty$

$\theta = T - T_\infty \qquad \dfrac{\partial \theta}{\partial t} = a \dfrac{\partial^2 \theta}{\partial x^2} \qquad a = \dfrac{k}{\rho C_v}$

Thermal diffusity.

$$\begin{cases} \theta(x, 0) = T_0 - T_\infty = \theta_0 \\ \theta(0, t) = 0 \\ \theta(\infty, t) = \theta_0 \end{cases}$$



* Suppose the solution has the form

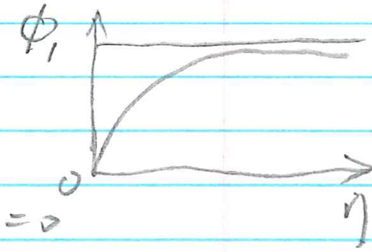
$$\frac{\theta}{\theta_0} = \phi(\eta) \qquad \eta = \frac{x}{\sqrt{4\alpha t}}$$

(similarity transformation)

$$x \to \eta$$

$$\frac{\partial(\theta/\theta_0)}{\partial t} = -\frac{1}{2}\frac{\eta}{t}\phi'$$

$$\frac{\partial^2(\theta/\theta_0)}{\partial x^2} = \frac{\eta^2}{x^2}\phi'' + 0$$



$$\phi'' + 2\eta\phi' = 0 \qquad \begin{cases} \eta = 0, & \phi = 0 \\ \eta = \infty, & \phi = 1 \end{cases}$$

$$\phi' = C_1 e^{-\eta^2}$$

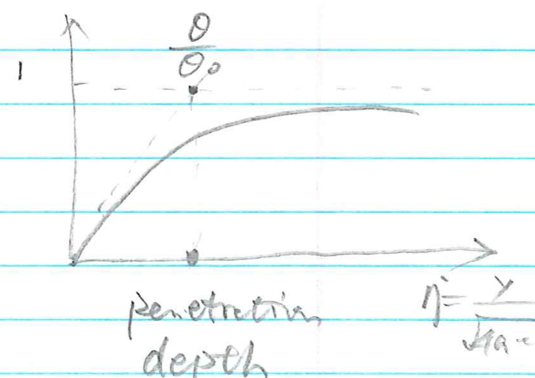
$$\phi = C_1 \int_0^\eta e^{-\xi^2} d\xi + C_2.$$

$$\text{B.C.} \to C_1 = \frac{1}{\int_0^\infty e^{-\xi^2} d\xi}, \quad C_2 = 0.$$

$$\phi = \frac{\int_0^\eta e^{-\xi^2} d\xi}{\int_0^\infty e^{-\xi^2} d\xi} = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\xi^2} d\xi$$

$$= \operatorname{erf} \eta$$

$$\operatorname{erf} \eta \equiv \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\xi^2} d\xi.$$



$$\frac{\theta}{\theta_0} = \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

$$\downarrow$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = \operatorname{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right).$$

$$\frac{\partial T}{\partial x} = \frac{\partial \theta}{\partial x} = \frac{\partial \phi \theta_0}{\partial x} = \theta_0 \frac{\partial \phi}{\partial x} = \theta_0 \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x}.$$

$$\frac{\partial \phi}{\partial \eta} = \frac{\partial}{\partial \eta}\left[\operatorname{erf} \eta\right] = \frac{\partial}{\partial \eta}\left[\frac{2}{\sqrt{\pi}} \int_0^\eta e^{-\xi^2} d\xi\right]$$

$$= \frac{2}{\sqrt{\pi}}\left[\int \frac{\partial}{\partial \eta} e^{-\xi^2} d\xi + \frac{\partial \eta}{\partial \eta} e^{-\xi^2}\Big| - \frac{\partial 0}{\partial \eta} e^{-\xi^2}\Big|_0 = 0.\right.$$

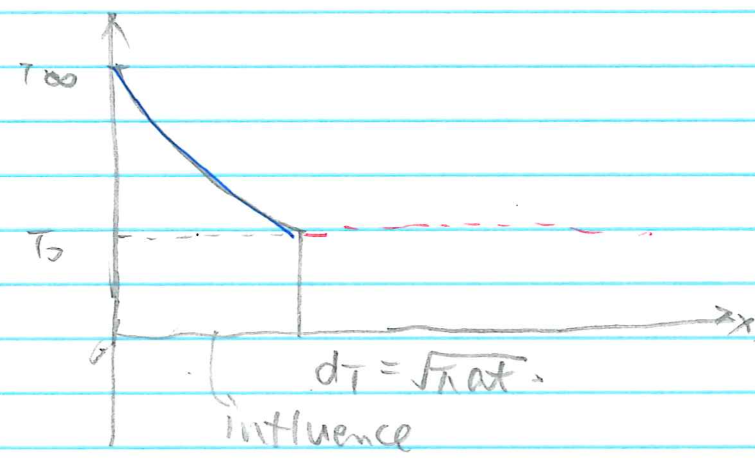
$$= \frac{2}{\sqrt{\pi}} e^{-\eta^2}.$$

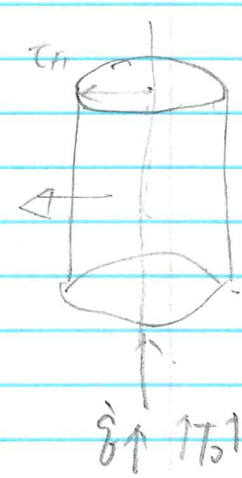
$$\frac{\partial \eta}{\partial x} = 1/\sqrt{4at}$$

$$\left.\frac{\partial T}{\partial x}\right|_0 = \theta_0 \frac{2}{\sqrt{\pi}} e^{-\eta^2} \frac{1}{\sqrt{4at}}\Big|_{x=0} = \frac{\theta_0}{\sqrt{\pi a t}}$$

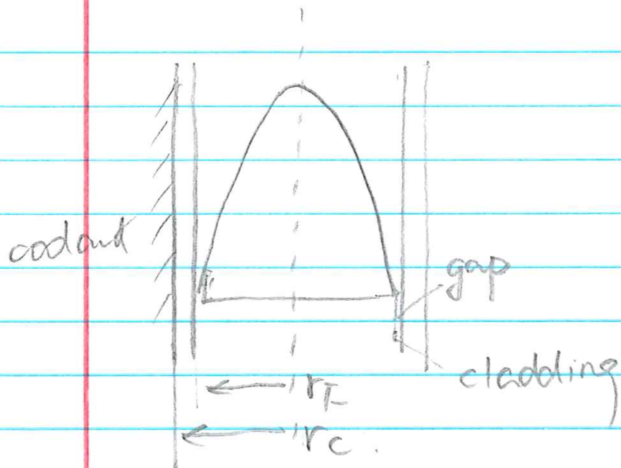
$$\left.\frac{\partial T}{\partial x}\right|_0 = \frac{(T_0 - T_\infty)}{\sqrt{\pi a t}}$$

$\sqrt{\pi a t} = d_T$: Thermal Penetration Depth





* D_F : Diameter of the fil $D_F/3 = r_F$.



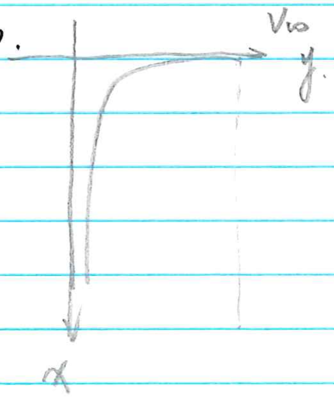
$$d_T = \sqrt{\pi a T} = r_F = D_F/2$$

$$\pi a t = \left(\frac{D_F}{2}\right)^2$$

$$t = \frac{1}{\pi a}\left(\frac{D_F}{2}\right)^2 = \tau_F.$$

* neutronic excursion. → worst accident.

hw.



1. from 3-D. momentum diffusion.
2. solve.
3. the physical meaning.

UO_2 $a = 5.65 \times 10^{-3}$ cm²/sec.

$Na = 6.48 \times 10^{-1}$ cm²/sec,

S.S. $a = 6.65 \times 10^{-2}$ cm²/sec. $H_2O = 1.43 \times 10^{-3}$ cm²/sec

$d_{th} = \sqrt{\pi a t}$	UO_2.	S.S.	H_2O	Na.
1 sec.	1.33mm	4.4mm	0.67mm	14.3mm.
10 sec	4.22mm	13.9mm	2.1mm	45 mm
100 S	13.3mm	44 mm	67 mm	143 mm

$\begin{cases} H_2O & \nu = 1 \times 10^{-2} \text{ cm}^2/\text{sec} \quad (20°C) \\ Na & \nu = 3.3 \times 10^{-3} \text{ cm}^2/\text{sec} \quad (370°C) \end{cases}$

$d_m = \sqrt{\pi \nu t}$ $\quad H_2O \Rightarrow Pr = 7$ $\quad Na : Pr = 5\times10^{-3}$ $\quad Pr = \frac{\nu}{a}$

1 s	1.77 mm	1.0 mm
10 s	5.6 mm	3.2 mm

* Binary diffusion.

H_2O vapor in Air $\quad 16^\circ C \quad D = 2.8\times10^{-1}\ cm^2/sec.$

Schmidt No. $\quad Sc = \frac{\nu}{D}$

$d_c = \sqrt{\pi D t}$.

1 s	9.4 mm.
10 s	29.7 mm.

* 1. velocity profile.

2. if suddenly change.

(7) Note on Turbulent flow.

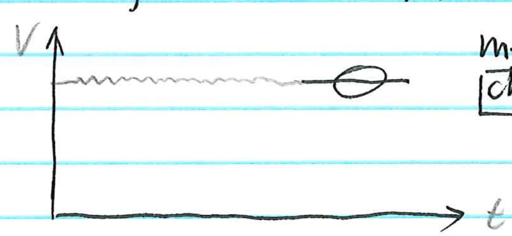
Origin: Navier Stokes. Eq. (Momentum Eq.)

becomes unstable for large Reynold Number.

$$Re = \frac{\rho v D}{\mu} = \frac{\text{inertial force}}{\text{viscous force}}.$$

$\begin{cases} \text{random system} \\ \text{chaotic system} \end{cases}$

Turbulent flow with Fluctuations.



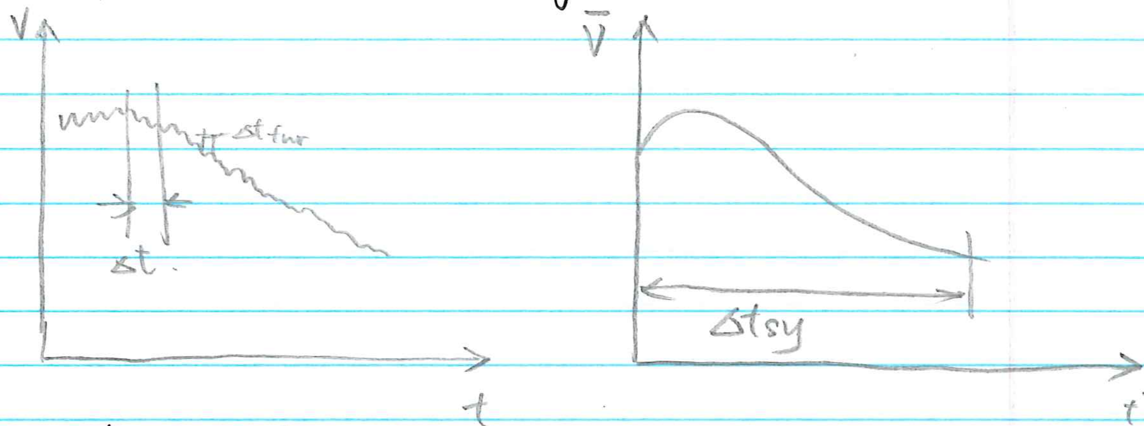
mean value.

$\boxed{\text{chaotic}}$ not random

Time average solution for turbulent flow
$\Rightarrow$ for most engineering problems : sufficient

Chapter 5 Turbulent Flow

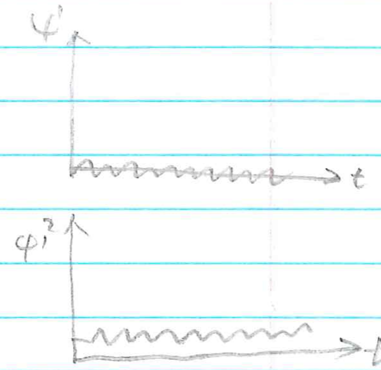
1) Temporal (Time) Average



$\Delta t_{sy} \gg \Delta t$

$$\bar{\varphi} = \frac{1}{\Delta t}\int_{\Delta t} \varphi \, dt \qquad ①$$

$\Delta t_{tur} < \Delta t < \Delta t_{sy}$



$$\varphi = \bar{\varphi} + \varphi' \qquad ②$$

$\bar{\varphi}$: average
φ' : fluctuate component
$\overline{\varphi'} = 0$

$\sqrt{\overline{\varphi'^2}} / \bar{\varphi}$ = Turbulent density $\neq 0$

$I \sim 1$ to 10% (mostly $< 5\%$)

2) Time averaged Field Equation.

$$\psi = \bar{\psi} + \psi'$$

$\bar{\psi} = ?$ $\rightarrow$ Enengeering Problem.

$\left|\frac{\rho'}{\rho}\right| << \left|\frac{v'}{v}\right|$ assumption ~ very good

(i) Continuity Equation.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0.$$

$$\left.\begin{array}{l} \rho = \bar{\rho} + \cancel{\rho'} \\ v = \bar{v} + v' \end{array}\right\} \rightarrow \boxed{\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \bar{\rho} \vec{\bar{v}} = 0.} \leftarrow$$

$\rho = \text{const}.$ $\nabla \cdot \vec{\bar{v}} = 0.$

$\nabla \cdot v' = 0.$

(ii) M.E.

nonlinear

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v} \vec{v} = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g}.$$

$$\left.\begin{array}{l} v = \bar{v} + v' \\ p = \bar{p} + p' \\ \rho = \bar{\rho} \end{array}\right\} - \text{M.E.}$$

$$\overline{v} = \overline{\bar{v} + v'} = \vec{\bar{v}} + \cancel{\overline{v'}}_{0}$$

$$\rho \vec{v} = \bar{\rho}(\bar{v} + v')$$

10.6. Midterm.

$$\frac{\overline{\partial \bar{\rho}(\bar{v} + v')}}{\partial t.} = \frac{\partial \bar{\rho} \bar{v}}{\partial t} + \frac{\partial \bar{\rho}}{\partial t} \cancel{\overline{v'}}$$

$$\overline{\nabla \cdot \rho \vec{v} \vec{v}} = \nabla \cdot \overline{\bar{\rho}(\vec{\bar{v}} + v')(\vec{\bar{v}} + v')}$$

$$= \nabla \cdot \bar{\rho} \vec{\bar{v}} \vec{\bar{v}} + \nabla \cdot \bar{\rho} \overline{v'v'}$$

Turbulent.

$$\frac{\partial \bar{\rho}\bar{V}}{\partial t} + \nabla\cdot\rho\bar{V}\bar{V} = -\nabla\vec{\bar{p}} + \left[\mu\nabla^2\bar{V} + \nabla\cdot\rho\overline{V'V'}\right] + \rho\vec{g}$$

✓ ✓ ✓ ✓ Turbulent flux (Reynolds stress) ✓

$$\tau^t = \rho\overline{V'V'}$$
$$\tau^\mu = -\mu\left[\nabla V + (\nabla V)^t\right]$$
$$\tau^T = \tau^\mu + \tau^t$$
$$\frac{\partial \rho\bar{V}}{\partial t} + \nabla\cdot\rho\bar{V}\bar{V} = -\nabla\bar{p} - \nabla\cdot(\tau^\mu + \tau^t) + \rho\vec{g}$$
$$\frac{\partial \rho\bar{V}}{\partial t} + \nabla\cdot\rho\bar{V}\bar{V} = -\nabla\bar{p} - \nabla\cdot(\tau^T) + \rho\vec{g}$$

$$\tau^t = ?$$

$$\tau^\mu = -\nu\rho\left[\nabla\vec{V} + (\nabla V)^t\right]$$
$$\tau^t = -\varepsilon_M\rho\left[\nabla\bar{V} + (\nabla V)^t\right]$$

ε_M : turbulent diffusivity
ν : viscous "

→ Energy Equation.

$$\rho C_v \frac{DT}{Dt} = k\nabla^2 T - T\left.\frac{\partial p}{\partial T}\right)_p \cancel{\nabla\cdot\vec{V}} - \cancel{\tau:\nabla V^t} + \dot{q}$$

$$C_v\left[\frac{\partial \rho T}{\partial t} + \nabla\cdot\rho T\vec{V}\right]$$

$\rho = \bar{\rho}$ $\quad \nabla\cdot\vec{V} \to 0$ $\quad \tau:\nabla\vec{V} \approx 0$

$$\begin{cases} T = \bar{T} + T' \\ V = \bar{V} + V' \end{cases} \qquad \overline{T'} = 0, \quad \overline{V'} = 0$$

$$\rho C_v \frac{D\bar{T}}{Dt} = \left[k\nabla^2\bar{T} - \nabla\cdot\rho C_v\overline{T'V'}\right] + \dot{q}$$

↑ Turbulent Energy Flux

$$C_v\left[\frac{\partial \rho \bar{T}}{\partial t} + \nabla\cdot\rho\bar{T}\bar{V}\right]$$

$$\begin{cases} \vec{q}^{\,t} = \rho C_v \overline{T'V'} \\ \vec{q}^{\,k} = -k\nabla T \end{cases} \qquad \begin{aligned} &= -\rho\cdot C_v \varepsilon_k \nabla\bar{T} \\ &= -k\nabla\bar{T} \end{aligned}$$

ε_k: Turbulent Energy Diffusity

$$\rho C_v \frac{D\bar{T}}{Dt} = -\nabla\cdot(q^k + q^t) + \dot{q}$$

$$q^T = q^k + q^t.$$

$$\rho C_v \frac{D\bar{T}}{Dt} = -\nabla\cdot q^T + \dot{q}$$

(3) characterisseics of turbulent flow

(i) Transition.

Laminar $\rightarrow$ Turbulent.

$Re \geq 2000$ for pipe flow

(ii) Velocity profile

Laminar $\quad v = U_{max}\left[1-\left(\frac{r}{r_0}\right)^2\right]$

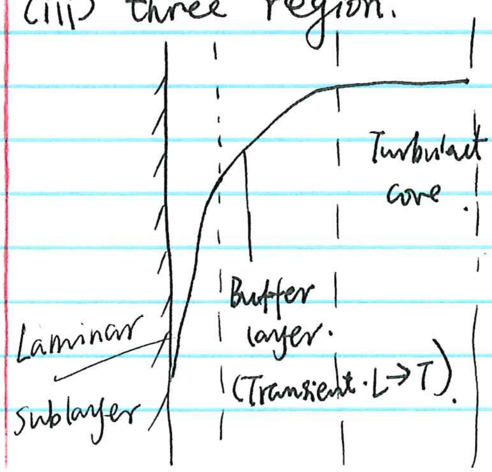
Turbulent $\quad v \sim U_{max}\left(1-\frac{r}{r_0}\right)^{1/n}$

($1/n$ power profile)

$\hookrightarrow$ good for estimate of v

cannot used to get τ

(iii) three region.



flunctuation large.

$\tau^u \sim \tau^t$

iv. Stress in Turbulent flow.

$$\frac{\partial \rho \bar{V}}{\partial t} + \nabla \rho \bar{V}\bar{V} = -\nabla \bar{p} - \nabla\cdot(\tau^{\mu} + \tau^{t}) + \rho \vec{g}$$

In pipe z component.

$$\rho\left(\frac{\partial \bar{V}_z}{\partial t} + \cancel{\bar{V}_r}\frac{\partial \bar{V}_z}{\partial r} + \frac{\bar{V}_\theta}{r}\frac{\partial \bar{V}_\theta}{\partial \theta} + \bar{V}_z\frac{\partial \bar{V}_z}{\cancel{\partial z}}\right)$$

$$= -\frac{\partial \bar{P}}{\partial z} + \cancel{\rho g_z} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r\,\tau_{rz}{}^{T}) + \frac{1}{r}\frac{\partial \tau_{\theta z}{}^{T}}{\partial \theta} + \frac{\partial \tau_{zz}{}^{T}}{\partial z}\right].$$

Steady. S $\frac{\partial}{\partial t} = 0$

no gravity $g_z = 0$

fully developed $\frac{\partial \bar{V}_z}{\partial z} = 0$ $\bar{V}_r = \bar{V}_\theta = 0$

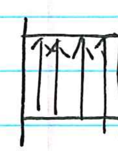
axi symmetric $\frac{\partial}{\partial \theta} = 0$. τ_{zz} independent of z

$$\boxed{\frac{\partial \bar{P}}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\,\tau_{rz}{}^{T}\right)}$$

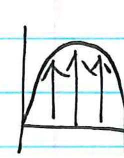
↑ independ of r ↑ independent of z.

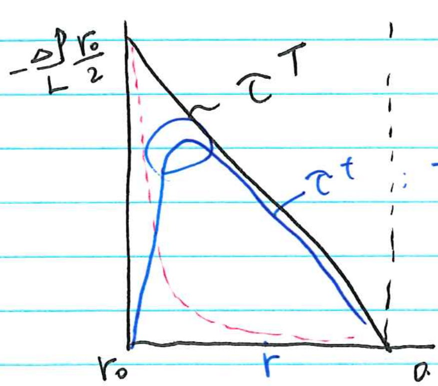
Solve for $\tau_{rz}{}^{T}$; $\tau_{rz}{}^{T} = \tau_{rz}{}^{\mu} + \tau_{rz}{}^{t}$

$$= \left(\frac{dp}{dz}\right)\frac{r}{2}$$


potential flow

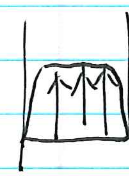
$\frac{dp}{dz} = $ const. $\tau_{rz}{}^{T}$ linear in r.


laminar



near wall region

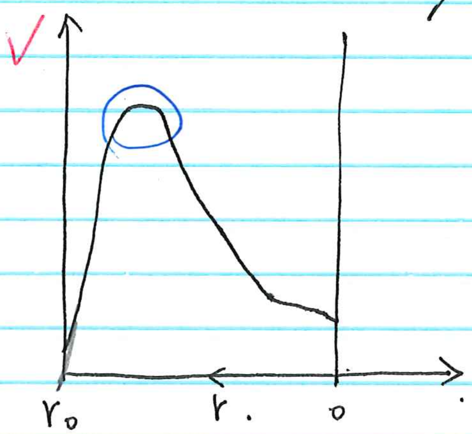
Turbulent component


turbulent

V). Turbulent intensity.

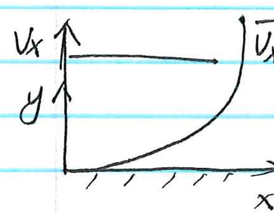
$\cancel{I = \sqrt{\overline{V_z'^2}}}$

$$I = \sqrt{\overline{V_z'^2}} \Big/ \overline{V}_{z\,max}$$



potential flow cant calculate resistance.

(4) Turbulent flow model.



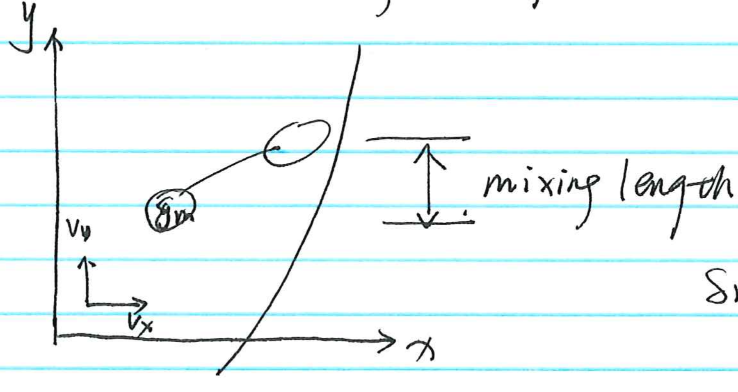
· Bousinesq's Eddy Diffusity (viscosity).

$$\tau_{yx}^{t} = -\mu^t \frac{d\overline{V_x}}{dy}.$$

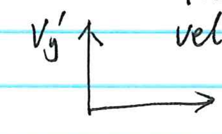
μ^t : Turbulent viscosity.

$\frac{\mu^t}{\rho} = \varepsilon_M.$ Eddy diffusity.

· Prandle's Mixing length Model (1925)



Turbulent vel. Fl



δm : finite mass.

move l :

(1) δm does not loss or gain any x momentum

until ~~max~~ move finite distance l → then complete momentum exchange

(2) → x comp momentum gained by δm
$\delta m \, \delta V_x$

- momentum transfer rate

$$\frac{\delta m \, \delta V_x}{\delta t}$$

- shear force acted on fluid $F = \delta m \delta V_x / \delta t$

$$\tau^t = \frac{F}{A} = \frac{1}{A} \frac{\delta m}{\delta t} \delta V_x$$

- $\delta V_x \approx \frac{d\bar{V}_x}{dy} l$

- From continuity $\frac{1}{A} \frac{\delta m}{\delta t} = \rho |V_y'|$

$$\frac{\tau^t}{\rho} = -l |V_y'| \frac{d\bar{V}_x}{dy}$$

Eddy diffusity ε_M

$$\frac{\tau^t}{\rho} = -\varepsilon_M \frac{d\bar{V}_x}{dy} \qquad \varepsilon_M = l |V_y'|$$

Total $\frac{\tau^T}{\rho} = -(\varepsilon_M + \nu) \frac{d\bar{V}_x}{dy}$

- Prandtle's assumption. physical meaning

$|V_y'| = k_1 V_x'$

$V_x' = k_2 \delta V_x = k_2 l \frac{dV_x}{dy}$

all unknown const $k_1, k_2 \to l$.

$$\boxed{\frac{\tau^t}{\rho} = -l^2 \left|\frac{d\bar{V}_x}{dy}\right| \frac{d\bar{V}_x}{dy}}$$ non linear.

$l = ky$. linear proportional to the position to wall

$$\boxed{\frac{\tau^t}{\rho} = -k^2 y^2 \left|\frac{d\bar{V}_x}{dy}\right| \frac{d\bar{V}_x}{dy}}$$

09/27/11

Prantle's Mixing length Model.

Velocity profile near the wall.

$v^+ = \bar{V}_x / \sqrt{\tau_w/\rho}$ non-dimensional vel.

τ_w $y^+ = y\sqrt{\tau_w/\rho}/\nu$ non-dimensional distance.

$\tau_w = f_i \frac{1}{2}\rho u^2 \Rightarrow \sqrt{\frac{\tau_w}{\rho}} = \sqrt{0.5 f_i}\, u_0$

f_i: Fanning friction factor.

f: 0.001 ~ 0.01

$\sqrt{0.5f}$: 1/45 1/14 $y^+ = Re \frac{\sqrt{\tau_w/\rho}}{\nu} \frac{y}{D} \sim Re \frac{1}{40} \frac{y}{R}$

Three Regions

① laminar sublayer. $\tau^t = 0$ $y^+ < 5$.

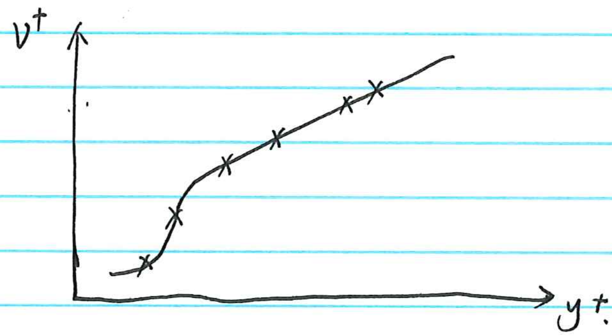
$v^+ = y^+$

② Buffer layer $\quad 5 < y^+ < 30$

$$V^+ = -3.05 + 5 \ln y^+$$

③ Turbulent core $\quad y^+ > 30$

$$V^+ = 5.5 + 2.5 \ln y^+$$



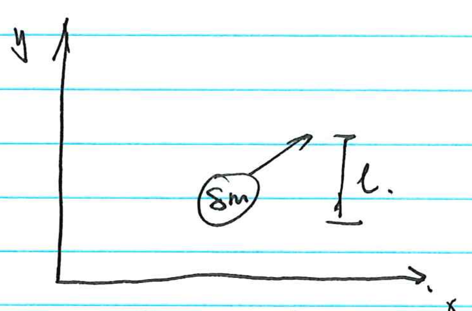
universal velocity profile.

(5). Turbulent heat transfer.

Reynolds Analogy.

$$\frac{\tau}{\rho} = -\varepsilon_M \frac{\partial u}{\partial y}$$

$$\frac{q^+}{\rho c} = -\varepsilon_H \frac{\partial T}{\partial y}.$$



Energy transfer by convection

$$\frac{\delta m\, C \delta T}{\delta t}$$

$$\tau = \frac{F}{A} = \frac{1}{A} \frac{\delta m}{\delta t} \delta V_x = -(\rho |V_y'|)\left(\frac{d\bar{V}_x}{dy} l\right) = -\rho l |V_y'| \frac{d\bar{V}_x}{dy}$$

$$q'' = \frac{Q}{A} = \frac{1}{A} \frac{\delta m}{\delta t} c \delta T = -(\rho |V_y'|)\left(C \frac{d\bar{T}}{dy} l\right) = -\rho C l |V_y'| \frac{d\bar{T}}{dy}$$

$\varepsilon_M = l\, |V_y'| \qquad \varepsilon_H = l\, |V_y'|$

$\varepsilon_M = \varepsilon_H$

$$\frac{\tau}{\rho} = -\varepsilon_M \frac{d\bar{v}_x}{dy} \qquad \frac{q''}{\rho c} = -\varepsilon_H \frac{d\bar{T}}{dy}.$$

(ii) Karman - Boelter - Martinelli Analogy

$$\frac{\tau}{\rho} = -(\varepsilon_M + \nu)\frac{\partial v}{\partial y}$$

$$\frac{q''}{\rho c} = -(\varepsilon_H + a)\frac{\partial T}{\partial y}.$$

· Eddy diffusivity ε_M. Universal Velocity profile $v^+ = f(y^+)$

$$\begin{cases} 0 < y^+ < 5. & \varepsilon_M/\nu = 0. \\ 5 < y^+ < 30 & \dfrac{\varepsilon_M}{\nu} = \dfrac{y^+}{5} - 1 \\ y^+ < 30 & \varepsilon_M/\nu = y^+(1 - y/r_0)/2.5 - 1 \end{cases}$$

For Moderate Prandtl No. $\varepsilon_M = \varepsilon_H$.

Energy Eq. $\begin{cases} \text{fully developed} & V_r = 0 \\ \text{axsymmetric} & \dfrac{\partial}{\partial \theta} = 0 \\ \text{neg. axial. conduction.} & \end{cases}$

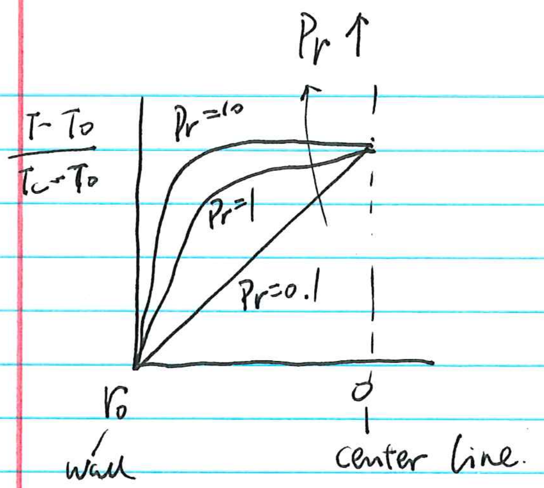
$$\rho c V_z \frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r \rho c (\varepsilon_H + a)\frac{\partial T}{\partial r}\right].$$

$$q'' = \rho c (\varepsilon_H + a)\frac{\partial T}{\partial y}.$$

$V_z = V_z(r)$ $\quad$ 1/7 or universal velocity profile.

$\varepsilon_H = \varepsilon_M \leftarrow$ Eddy Diffusity

Solve for $T(r, Pr)$ $\quad Pr = \dfrac{\nu}{a}.$

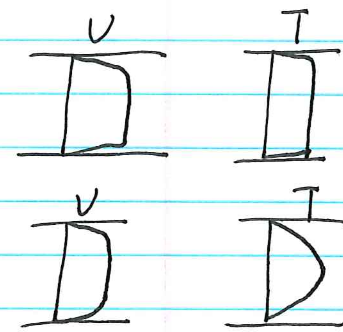


Effect of Prandtl number.

$Pr >> 1$ (oil) $\frac{\nu}{a} >> 1$

large momentum diffusion

$Pr << 1$ (liquid metal) $\frac{\nu}{a} << 1$



iv) Diffusion Equation.

$$\frac{\partial \rho_k}{\partial t} + \nabla\cdot\rho_k \vec{V} = -\nabla\cdot J_k + r_k$$ → chemical reaction rate

mass fraction $w_k = \frac{\rho_k}{\rho}$

$$J_k = -\rho D \nabla w_k$$

$$\frac{\partial \rho w_k}{\partial t} + \nabla\cdot\rho w_k\cdot\vec{v} = \nabla\cdot(\rho D \nabla w_k) + r_k$$

$w_k = \overline{w}_k + w_k'$ $\qquad \overline{\nabla\cdot\rho w_k \vec{v}} = \nabla\cdot\rho\overline{w}_k\cdot\bar{v} + \nabla\cdot\rho\,\overline{w_k' v'}$

$v = \cancel{v_k}\ \bar{v} + v'$

$$\frac{\partial \rho \overline{w}_k}{\partial t} + \nabla\cdot(\rho\overline{w}_k\bar{v}) = \nabla\cdot(\rho D\nabla w_k) - \nabla\cdot(\rho\,\overline{w_k' v'}) + r_k$$

$J_k^T = J_k^l + J_k^t$

$J_k^l = \rho D^l \nabla w_k$

$J_k^t = \rho\,\overline{w_k' v_k'} = -\rho D^t \nabla w_k$

Heat and Mass Transfer Analogy

$$D^t = \varepsilon_H$$

$$\text{or } D^t = \varepsilon_M$$

$$\frac{\partial \rho \overline{w_k}}{\partial t} + \nabla \cdot (\rho \overline{w_k} \bar{v}) = \nabla \cdot [\rho (D^l + D^t) \nabla \overline{w_k}] + r_k.$$

Chapter 6. Single phase dimensional Analysis.

use of Balance Equation for similarity Parameters.

* for non-dimensional parameters.

(1) Scaling Parameters.

- Distance $x^* = \frac{x}{D}$, $y^* = \frac{y}{D}$, $z^* = \frac{z}{D}$, $\bar{x}^* = \frac{\bar{x}}{D}$

D: characteristic length (Hydraulic diameter)

$$D \equiv \frac{4A}{P} = \frac{4 \times \text{Flow area}}{\text{Wetted Perimeter}}.$$

Time scale:

$$\tau = \frac{D}{V} \qquad t^* = \frac{t}{\tau} = \frac{tV}{D}$$

V: characteristic velocity.

- Velocity

$$\bar{v}^* = \frac{\vec{v}}{V}, \quad v_x^* = \frac{v_x}{V}, \quad v_y^* = \frac{v_y}{V}, \quad v_z^* = \frac{v_z}{V}.$$

- Pressure

$$p^* = \frac{P - P_0}{\rho_0 V^2}$$

P_0, ρ_0 } reference P. and density

· Temperature

$$T^* = \frac{T - T_0}{\Delta T} \qquad T_0 \text{ reference} \qquad \Delta T \text{ temperature change scale}$$

· Density

$$\rho^* = \frac{\rho}{\rho_0}$$

· Differential Operators.

$$\nabla^* = D \nabla$$

$$\nabla^{*2} = D^2 \nabla^2$$

$$\frac{D}{Dt^*} = \frac{D}{V} \frac{D}{Dt} \qquad \frac{\partial}{\partial t^*} = \frac{D}{V} \cdot \frac{\partial}{\partial t}$$

2) Force Convection.

· Continuity Equation.

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^* \cdot (\rho^* \vec{V}^*) = 0$$

· Equation of Motion

$$\rho \frac{D\vec{V}}{Dt} = (\rho_0 \frac{V^2}{D}) \rho^* \frac{D\vec{V}^*}{Dt^*}$$

$$\nabla p = \frac{\rho_0 V^2}{D} \nabla^* p^*$$

$$\mu \nabla^2 \vec{V} = \frac{\mu V}{D^2} \nabla^{*2} (\vec{V}^*)$$

$$\frac{1}{3} \mu \nabla(\nabla \cdot \vec{V}) = \frac{1}{3} \mu \frac{V}{D^2} \nabla^* (\nabla^* \cdot \vec{V}^*)$$

$$\vec{g} \rho = \vec{g} \cancel{g} \rho_0 \rho^*$$

Divide E.M. by $\rho_0 V^2 / D$

$$\rho^* \frac{D\vec{V}^*}{Dt^*} = -\nabla^* p^* + \left(\frac{1}{Re}\right) \left[\nabla^{*2} \vec{V}^* + \frac{1}{3} \nabla^* (\nabla^* \cdot \vec{V}^*) \right]$$

$$+\left(\frac{1}{Fr}\right)\rho^*\left(\frac{\vec{g}}{g}\right)$$

$\boxed{\text{Reynolds No.}}$ $Re = \rho_0 VD/\mu = \dfrac{\rho V^2/D}{\mu(V/D)/D} = \dfrac{\text{inertial force/vol}}{\text{viscous force/vol}}$

$\boxed{\text{Froude No.}}$ $Fr = \dfrac{V^2}{gD} = \dfrac{\rho V^2/D}{\rho g} = \dfrac{\text{inertia}}{\text{gravity}}$

Fluid Mechanics Problem: If two systems have
Re, Fr, identical
Dimentionless B.C. I.C. identical } in terms of non-dimensional parameters.
they are identical.

geometrical similarity.
$\Rightarrow$ Dynamically similar.

· Energy Equation

$$V\rho C_p \frac{DT}{Dt} = \left(\rho_0 C_p \frac{V}{D}\Delta T\right)\rho^* \frac{DT^*}{Dt^*}$$

$$Vk\nabla^2 T = \left(\frac{k\Delta T}{D^2}\right)\nabla^{*2} T^*$$

$$\tau : \nabla\vec{V} = \mu\Phi = \mu\frac{V^2}{D}\Phi^*$$

$$\frac{T}{\rho}\left.\frac{\partial\rho}{\partial T}\right)_p \frac{Dp}{Dt} = \frac{\rho_0 V^3}{D}\frac{T^*}{\rho^*}\left.\frac{\partial\rho^*}{\partial T^*}\right)\frac{Dp^*}{Dt^*}$$

$$\rho^*\frac{DT^*}{Dt^*} = \frac{k}{D\rho_0 C_p V}\nabla^{*2}T^* + \frac{\mu V}{D\rho_0 C_p \Delta T}\Phi^* - \frac{V^2}{C_p\Delta T}\frac{T^*}{\rho^*}\frac{\partial\rho^*}{\partial T^*}\frac{Dp^*}{Dt^*} + \frac{\dot{q}D}{\rho C_p \Delta T V}$$

$$\frac{\mu V}{D\rho_0 C_p \Delta T}\Phi^* \qquad \frac{1}{Re}\frac{V^2}{C_p\Delta T}.$$

Eckert No. $\boxed{E_c \equiv \frac{V^2}{C_p\Delta T}} = \frac{\rho V^2/D}{C_p\rho\Delta T/D} = \frac{\text{inertial force}}{\text{thermal driving force}}$

$$\text{or} = \frac{\rho V^2 V/D}{C_p\rho\Delta T V/D} = \frac{\text{K.E convection}}{\text{Enthalpy convection}}$$

for perfect gas

$\gamma = C_p/C_v$. $\quad R = C_p - C_v$.

R: gas constant $= \frac{R}{M_w} = \frac{\text{universal gas constant}}{\text{mol weight}}$.

γ : specific heat ratio.

$$\frac{p}{\rho} = RT = C_p\left(\frac{\gamma-1}{\gamma}\right)T$$

velocity of sound $U_s^2 = \left.\frac{\partial p}{\partial\rho}\right)_s = \gamma RT = C_p T(\gamma-1)$

$$E_c = \frac{V^2}{C_p\Delta T} = \frac{V^2 T_0(\gamma-1)}{U_s^2\Delta T} = (\gamma-1)M^2\left(\frac{T_0}{\Delta T}\right)$$

$M = \frac{V}{U_s}$. Mach. No.

$E_c \sim M^2$. Then for subsonic flow terms with E_c maybe neglected.

Incompressible flow $M \lesssim 0.3$.

Energy Eq.

$$\rho^* \frac{DT^*}{Dt^*} = \frac{k}{D\rho_o C_p V} \nabla^{*2} T^* + \underbrace{\frac{\mu V}{D\rho_o C_p \Delta T}}_{\frac{1}{Re}\frac{V^2}{C_p\Delta T}} \Phi^* - \underbrace{\frac{V^2}{C_p \Delta T}}_{\frac{V^2}{C_p\Delta T}} \frac{T^*}{\rho^*} \frac{\partial \rho^*}{\partial T^*} \frac{Dp^*}{Dt^*} + \left(\frac{\dot{q} D}{\rho C_p \Delta T V}\right)$$

Eckert NO.

$$E_c = \frac{V^2}{C_p \Delta T} = \frac{\text{K.E convection}}{\text{Enthalpy convection}}$$

Peclet NO.

$$\boxed{Pe = \frac{\rho_o C_p D V}{k}} = \frac{\rho_o C_p \Delta T\, V/D}{k \Delta T / D^2} = \frac{\text{H.T. by Convection}}{\text{H.T by conduction}}$$

Prandtl NO.

$$P_r = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} = \frac{\text{Momentum Diffusion}}{\text{Thermal Diffusion}}$$

$$Pe = Re\, P_r$$

Heat generation NO

$$q^* \equiv \frac{\dot{q} D}{\rho C_p \Delta T V} = \frac{\text{Heat generation}}{\text{Heat convection}}$$

$$\rho^* \frac{DT^*}{Dt^*} = \frac{1}{Pe} \nabla^{*2} T^* + \underbrace{\frac{E_c}{Re} \Phi^* - E_c \frac{T^*}{\rho^*} \frac{\partial \rho^*}{\partial T^*} \frac{Dp^*}{Dt^*}}_{\text{for most cases can be neglected } (M \leq 0.3)} + \dot{q}^*$$

Incompressible flow $\rho^* = 1$ $(M \leq 0.3)$

$$\frac{DT^*}{Dt^*} = \frac{1}{Pe}\nabla^{*2}T^* + \frac{Ec}{Re}\Phi^* + \dot{q}^*$$

No heat generation

$$\frac{DT^*}{Dt^*} = \frac{1}{Pe}\nabla^{*2}T^*$$

Thermal similarity governed by Pe.

$$Pe = Pr\,Re$$

Summary.

$$\begin{cases} \dfrac{\partial \rho^*}{\partial t^*} + \nabla^* \cdot (\rho^* \vec{V}^*) = 0 \\ \rho^* \dfrac{DV^*}{Dt^*} = -\nabla^* p^* + \dfrac{1}{Re}\left[\nabla^{*2}\vec{V}^* + \dfrac{1}{3}\nabla^* \cdot (\nabla^* \vec{V}^*)\right] + \dfrac{\rho^*}{Fr}\left(\dfrac{\vec{g}}{g}\right) \\ \rho^* \dfrac{DT^*}{Dt^*} = \dfrac{1}{Pe}\nabla^{*2}T^* + \dfrac{Ec}{Re}\Phi^* - \dfrac{Ec\,T^*}{\rho^*}\left.\dfrac{\partial \rho^*}{\partial T^*}\right)_p \dfrac{Dp^*}{Dt^*} + \dot{q}^* \end{cases}$$

?

$$Re = \frac{\rho_0 V D}{\mu}, \quad Fr = \frac{V^2}{gD}, \quad Pe = \frac{VD}{a} \quad \left(a = \frac{k}{\rho c_p}\right)$$

$$E_c = \frac{V^2}{c_p \Delta T}. \qquad (Pe = Pr\,Re).$$

Incompresible flow No heat generation.

fluid

$$\begin{cases} \nabla^* \vec{V}^* = 0 \\ \dfrac{D\vec{V}^*}{Dt^*} = -\nabla^* p^* + \dfrac{1}{Re}\nabla^{*2}\vec{V}^* + \dfrac{1}{Fr}\left(\dfrac{\vec{g}}{g}\right) \\ \dfrac{DT^*}{Dt^*} = \dfrac{1}{Pe}\nabla^{*2}T^* + \dfrac{Ec}{Re}\Phi^* \end{cases}$$

Dynamics similarity. Re Fr.

Thermal similarity. Pe.

3) Natural Circulation.

- Velocity scale.: self determined.

(1) Creeping flow. $Re \simeq 1$ $\quad \frac{\rho V D}{\mu} = 1$

$$V = \nu / D$$

(2) Acc. due to gravity. $\left.\begin{matrix} \rho \frac{V}{t} = \rho_0 g \beta \Delta T \\ t = \frac{D}{V} \end{matrix}\right\}$

$$V = \sqrt{g \beta \Delta T D}$$

* In general.

$$v^* = \frac{v}{V}, \quad t^* = \frac{t}{\tau}, \quad \tau = \frac{D}{V}.$$

$$\nabla^* = D\nabla, \quad \vec{x}^* = \frac{\vec{x}}{D} \quad \nabla^{*2} = D^2 \nabla^2$$

$$T^* = \frac{T - T_\infty}{T_0 - T_\infty} = (T - T_\infty)/\Delta T$$

$$p^* = \frac{p - p_0}{\rho V^2}. \qquad \rho^* \simeq 1.$$

M.E. $\frac{D\vec{V}^*}{Dt^*} = -\nabla^* p^* - \left(\frac{g \beta \Delta T D}{V^2}\right) T^* + \frac{1}{Re} \nabla^{*2} \vec{v}^*$

E.E. $\frac{DT^*}{Dt^*} = \frac{k}{\rho_0 C_p V D} \nabla^{*2} T^* + \cancel{E_c} \bar{\Phi}^* + \cancel{E_c} \beta T \frac{Dp^*}{Dt^*}$

$E_c \ll 1$

① Creeping flow

very small velocity $Re \simeq 1$

$$V = \frac{\nu}{D}$$

E.M $\frac{D\vec{V}^*}{Dt^*} = -\nabla^* p^* - \frac{g\beta\Delta T D^3}{\nu^2} T^* + \nabla^{*2}\vec{V}^*$

Grashof No.

$$Gr = \frac{g\beta\Delta T D^3}{\nu^2} = \frac{\text{buoyant} \times \text{inertia}}{(\text{viscous force})^2}$$

Note: No Reynolds number ← Creeping flow

E.E.

$$\frac{DT^*}{Dt^*} = \frac{1}{Pr}\nabla^{*2}T^* + Ec\Phi^* + E_c(\beta T)\frac{Dp^*}{Dt^*}$$

Prantl No. $Pr = \frac{\nu}{a} = \frac{\rho_0 C_p \nu}{k}$

only $\left.\begin{matrix} Gr \\ Pr. \end{matrix}\right\}$ control the $\begin{matrix} \text{dynamic} \\ \text{thermal} \end{matrix}$ similarity

2) $V = \sqrt{g\beta\Delta T D}$ $\qquad \tau = \frac{D}{V} = \frac{D}{\sqrt{g\beta\Delta T D}}$

$$\frac{D\vec{V}^*}{Dt^*} = -\nabla^* p^* - T^* + \frac{1}{\sqrt{Gr}}\nabla^{*2}\vec{V}^*$$

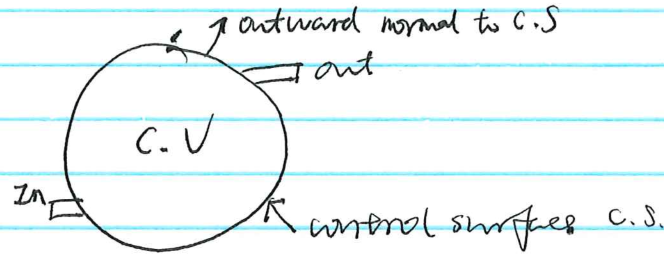
$$\frac{DT^*}{Dt^*} = \frac{1}{Pr\sqrt{Gr}}\nabla^{*2}T^*$$

In either way $T^* = f[Gr, Pr]$

Chapter 7

Control volume analysis of Reactor System

1) conservation Equations of control volume fixed in space
$\vec{V_c} = 0$: such as nuclear reactor



· Mass conservation.

$$\frac{d}{dt}\int_{cv} \rho dV = -\oint_{C.S} \rho \vec{V} \cdot \vec{n} dA.$$

If only one inlet and one outlet.

$$\frac{d}{dt}\int_{cV} \rho dV = (\rho AV)_{in} - (\rho AV)_{out}$$

$$\boxed{\frac{d}{dt}\int_{cV} \rho dV = \sum_i \dot{m}_s}$$

i: area where fluid cross the boundary
$\dot{m}_i$: mass inflow at i section (+)
outflow (−)

· Momentum conservation.

$$\frac{d}{dt}\int_{cV} \rho \vec{V} dv = \overrightarrow{\Sigma F} + \sum_i (\dot{m}_i \vec{V_i})$$

Rate of change of momentum in CV = Total force + Momentum influx

$\begin{cases} \dot{m}_i > 0 & \text{in} \\ \dot{m}_i < 0 & \text{out} \end{cases}$

Force $\begin{cases} \text{gravity} \\ \text{external force (contact force)} \\ \text{pressure} \\ \text{shear.} \end{cases}$

· Energy balance

$$\frac{d}{dt}\int_{cv} \rho e \, dV = \dot{Q} + \dot{W}_s + \Sigma(\dot{m}_i)(e + \frac{p}{\rho})$$

Heat Tr. work + energy influx

$e = u + \frac{V^2}{2} + gz$ (total energy)

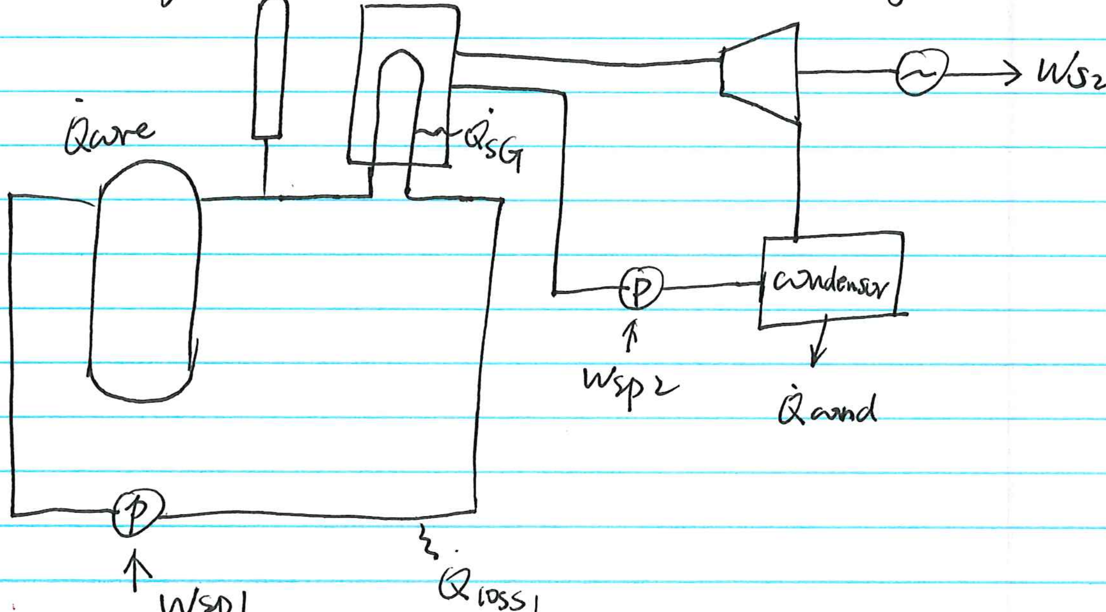
$\dot{Q}$: heat addition

$\dot{W}_s$: work done on system (shaft work). pump. etc.

$u + \frac{p}{\rho} = i$ enthalpy

$\frac{V^2}{2} + gz$. in many heat transfer problems these are negligible.

2). Integral Nuclear Reactor Thermal hydraulics.



Primary system

$\dot{Q}_{core}$ (>0)   Nuclear Heat Generation (~ 3000MWt)

$\dot{Q}_{SG}$ (<0)   Heat Tr. to SG (2nd side)

$\dot{Q}_{loss}$ (<0)   heat loss (small)

$\dot{W}_{sp1}$ (>0)   work done by Pump (~30 MW)
~40,000 hp

Mass balance

$$\frac{d}{dt}\int_{cv} \rho dV + \oint_{cs} \rho \vec{V} \vec{n} dA = 0.$$

total coolant inventory

(i) steady state: both terms are 0

(ii) LOCA

$$\oint_{cs} \rho \vec{V} \vec{n} \cdot dA \longrightarrow (\rho VA)_{break} > 0$$

Initially

$$\frac{d}{dt}\int_{cv} \rho dV = -\oint \rho \vec{V} \cdot \vec{n} dA = -(\rho VA)_{break}.$$

primary coolant inventory decreases as $-\int (\rho VA)_{break} dt$.

· ECCS come in

$$\frac{d}{dt}\int_{cv} \rho dV = (\rho VA)_{ECCS} - (\rho VA)_{break}$$

$= \dot{m}_{ECCS} - \dot{m}_{break}$

If $\dot{m}_{ECCS} > \dot{m}_{break}$.
Primary coolant inventory may recover.

prevention of core unrecovery.

· Momentum balance

· Energy balance

$$\frac{d}{dt}\int_{cv} \rho e dV + \int_{cs} \rho \left(e + \frac{P}{\rho}\right)(\vec{v} \cdot \vec{n}) dA = \dot{Q} + \dot{W}_s$$

10/4/11.

Review

1. General balance equation.
C.E M.E. E.E.

· fixed mass volume.
explain each term. pressure.
conseitutive
{ eq. of state
mechanical
energy. heat.

3 basic
① Laminar flow pabolic
② transiant solution. err function suddenhatg.
physical meaning of solution.
③ fuel rod how long. pipe flow ~~tire teketi~~

Mass balance.

$$\frac{d}{dt}\underbrace{\int_{cv}\rho\, dv}_{\text{total mass}} + \oint_{cs}\rho\vec{v}\cdot\vec{n}\, dA = 0.$$

(i) steady state

$$\oint \rho\vec{v}\cdot\vec{n}\, dA = 0$$

$$\frac{d}{dt}\int_{cv}\rho\, dv = 0. \quad \frac{dM}{dt} = 0. \quad M = \int \rho\, dv.$$

(ii) LOCA.

$$\oint_{cs}\rho\vec{v}\cdot\vec{n}\, dA = (\rho VA)_{break}$$

$$\frac{d}{dt}\int_{cv}\rho\, dv = -\oint \rho\vec{v}\cdot\vec{n}\, .dA = -(\rho VA)_{break}$$

→ Primary coolant inventory.
→ core uncovery.
→ fuel dryout
→ cladding oxidation ⇒ melt down.

- Safety strategy. ECCS

$$\frac{d}{dt}\int_{cv}\rho\, dv = (\rho VA)_{ECCS} - (\rho VA)_{break}$$

$$= \dot{m}_{ECCS} - \dot{m}_{break}.$$

· $\dot{m}_{break}$ is very important → B.C.

$$\begin{cases} \text{Large break flow} \rightarrow \text{Low pressure injection.} \\ \text{small break flow} \rightarrow \text{high pressure injection} \end{cases}$$

Design criteria

$\dot{m}_{ECCS} > \dot{m}_{break}$

Energy Equation.

$$\frac{d}{dt}\int_{cv} \rho e \, dV + \oint \rho\left(e + \frac{P}{\rho}\right)\vec{V}\cdot\vec{n}\, dA = \dot{Q} + \dot{W}_s$$

(i) steady state

No. break $\oint \to 0$

$\frac{d}{dt} \to 0$ } LHS = 0

$$\dot{Q}_{core} - \dot{Q}_{SG} - \dot{Q}_{loss} + \dot{W}_{sp} = 0.$$

$$\dot{Q}_{SG} = \dot{Q}_{core} + \dot{W}_{sp} - \dot{Q}_{loss} \Rightarrow \text{2ndary}$$

(ii) unsteady operation (No. break).

$\oint = 0.$

$$\frac{d}{dt}\int_{cv} \rho e \, dV = (\dot{Q}_{core} + \dot{W}_{sp}) - (\dot{Q}_{SG} + \dot{Q}_{loss})$$

Start up $\dot{Q}_{core} \uparrow$.

$\dot{Q}_{core} + \dot{W}_{sp} > \dot{Q}_{SG} + \dot{Q}_{loss}$

$\frac{d}{dt}\int_{cv} \rho e \, dV \uparrow$ $T \uparrow$

$p \uparrow$.

· Transient overpower.

· LOCA.

at break

$$\oint \rho\left(e+\frac{p}{\rho}\right)\vec{v}\cdot\vec{n}\,dA = \dot{m}_{break}\underbrace{\left(e+\frac{p}{\rho}\right)}_{\text{enthalpy}}{}_{break}$$

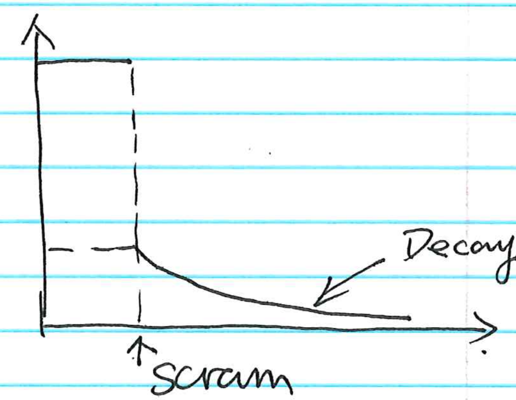
K.E ~ negligible

Initially

$$\frac{d}{dt}\int \rho e\,dv = (\dot{Q}_{core} + \dot{W}_{sp}) - (\dot{Q}_{SG} + \dot{Q}_{loss})$$
$$- \dot{m}_{break}\left(e+\frac{p}{\rho}\right)_{break}$$

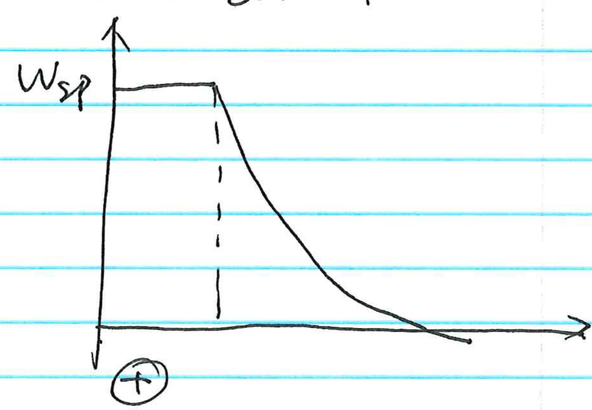
(i) Reactor scram

$Q_{core} \rightarrow \dot{Q}_{decay}$



(ii) $\dot{W}_{sp} \rightarrow 0.$

pump trip



$\frac{d}{dt}\int \rho e\,dv \downarrow$

LOCA $\quad \frac{d}{dt}\int e\,dv \downarrow.$

$p \downarrow$ } good cooling

$t \downarrow.$

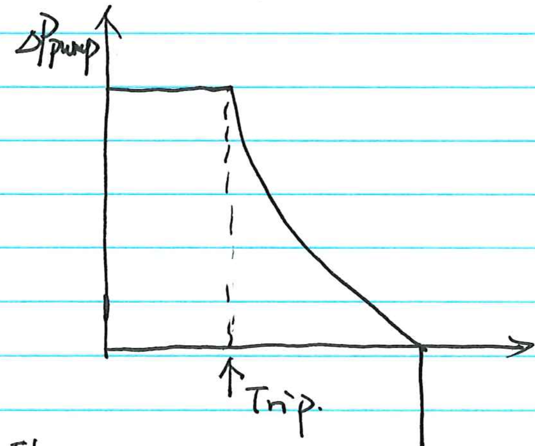
problem is core uncovery

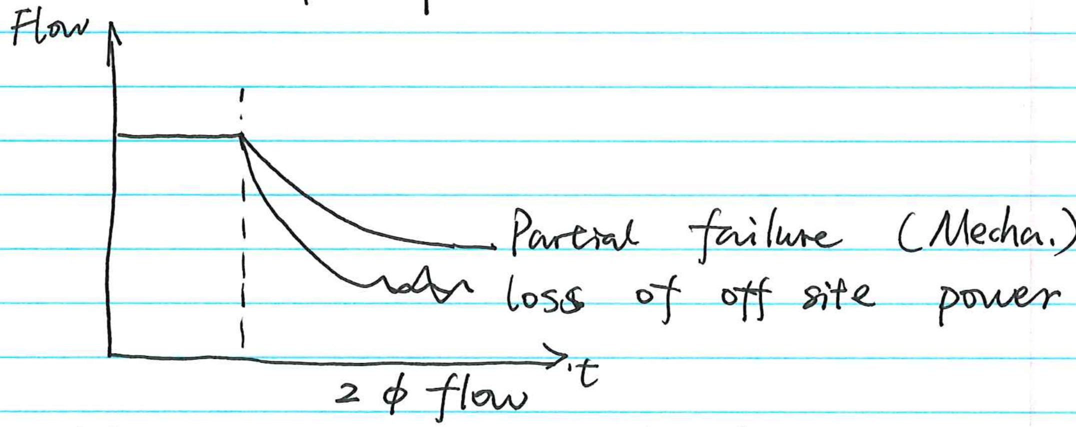
ECCS.

$$\frac{d}{dt}\int_{cv} \rho e\, dv = \dot{Q}_{decay} + \cancel{\dot{W}_{sp}} - (\dot{Q}_{sg} + \dot{Q}_{loss})$$
$$- \left[\dot{m}\left(e + \frac{p}{\rho}\right)\right]_{break} + \left[\dot{m}\left(e + \frac{p}{\rho}\right)\right]_{ECCS.}$$

$\left[\dot{m}\left(e+\frac{p}{\rho}\right)\right]_{ECCS}.$ Temp $= T_0$

2. Loss of flow (pump trip) most important for fast reactor.





(1) $\dot{W}_{sp} \to 0.$ pump coast down $\to 0.$

(2) $\dot{Q}_{SG}$ ↓ decay to natural circulation cooling

↓

(3) $\dot{Q}_{core} \to \dot{Q}_{decay}$ over heating.

Pr. relief valve operate

Small break LOCA

Scram

$\dot{Q}_{core} \rightarrow \dot{Q}_{decay} \downarrow \leftrightarrow \dot{Q}_{SG}$

3. Loss of heat sink accident

turbine trip.

2ndary Loop mulfunction,

- Feed water pump trip.
- 2nd pipe break (overcooling → LHS).
- condensor failure

Real loss of heat sink $\begin{cases} \dot{Q}_{cond} \rightarrow 0 \\ W_{sp} \rightarrow 0. \end{cases}$

$\dot{Q}_{SG} \downarrow \rightarrow \sim 0$

$$\frac{d}{dt}\int \rho e dv = (\dot{Q}_{core} + W_{sp}) - (\underset{\text{become smaller.}}{\underset{\downarrow}{\dot{Q}_{SG}}} + \dot{Q}_{loss}) > 0$$

$\begin{cases} \dot{Q}_{core} \rightarrow \dot{Q}_{decay} \\ W_{sp} \rightarrow 0 \end{cases}$

$$\frac{d}{dt}\int \rho e dv = (\dot{Q}_{decay} + 0) - (\dot{Q}_{SG} + \dot{Q}_{loss}) = ?$$

>0. 2nd can not cooling

heat up pressurize → Pr relief valve

small break LOCA

High pressure Injection system is used.

ECCS → Reactor. → relif valve.

Evaporate →

station black out.

AC power supply lost.

off-site } lost ⇒
Emergency AC power }

pump trip → Loss of Flow

{ primary
{ secondary → Loss of heat sink

reactor may be scrammed (DC power)

$$\frac{d}{dt}\int \rho e dV = (\dot{Q}_{core} + \cancel{W_p}) - (\dot{Q}_{SG} + \dot{Q}_{loss})$$

$\dot{Q}_{core} \downarrow Q_{decay}$, $\dot{Q}_{SG} \downarrow$ down.

10/13/11.

Chapter 7-2.

{ 1-D formulation
Quasi 1-D.

general 3-D formulation.

(1) Too complicated for most application.

(2) Turbulent model not well established
particular for 2 phase flow

(3) often 3-D information we needed.

(4) 3-D Analysis maybe needed in limit number of cases & component.

{ core
SG 2-D or 3-D.
downcomer
plenum

under certain condition

1-D. or Quasi 1-D Formulation.

(1) most important engineering tool for reactor thermal hydraulics & safety analysis. (single or two phase)

(2) 1-D formulation → $\underbrace{\text{finite difference}}_{\text{control volume}}$.

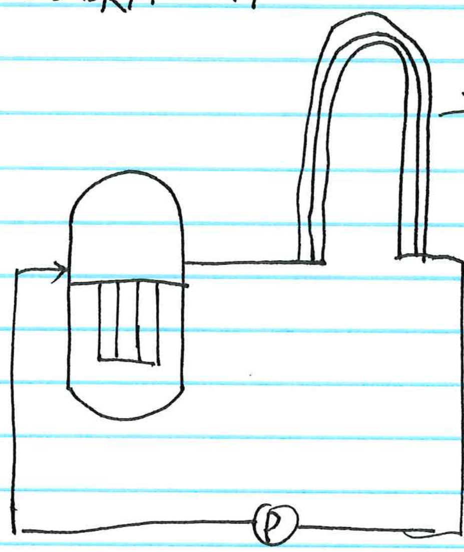
(3) State of the art in two phase flow model.

(4) efficient solution techniques

(5) economical.

(6) Quasi 1-D simulate 2-D or 3-D behaviors (sometimes)

1-D code
Relap 5
TRAC (PP. BD)
RETRAN
(TRACE)
(Two or three - D) subchannel code
(Parallel channel of 1-D)
COBRA
COBRA - TF



→ component.
-RPV
core
downcomer
Lower plenum
upper plenum

-Hot leg. (Pressurizer)
sauge line
pressurize.

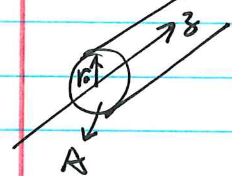
-cold leg.

pump.

-SG I. plenum
up flow
U bend
down flow
exit plenum

1-D Model for single phase flow.

1) Area average.



$$\frac{1}{A}\int_A \psi\, dA = \langle \psi \rangle$$

Axisymmeeric flow $\partial/\partial\theta = 0$. $V_\theta = 0$.

Constant area $A = const$.

$$\frac{1}{\pi r_0^2}\int \psi\, 2\pi r\, dr = \langle\psi\rangle \qquad (1)$$

Time derivative

$$\frac{1}{A}\int_A \frac{\partial \rho\psi}{\partial t}\, dA = \frac{\partial}{\partial t}\frac{1}{A}\int \rho\psi\, dA = \frac{\partial}{\partial t}\langle\rho\psi\rangle.$$

Gradiant Operator in z direction

∇p.

$$\frac{1}{A}\int_A (\nabla S)_z\, dA = \frac{1}{A}\int \frac{\partial S}{\partial z}\, dA = \frac{\partial}{\partial z}\langle S\rangle.$$

Average of vector operator or Sym. tensors.

$$\frac{1}{A}\int_A(\nabla\cdot\tau)_z\, dA = \frac{1}{A}\int\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}) + \frac{\partial\tau_{zz}}{\partial z}\right]\underbrace{dA}_{2\pi r\, dr}$$

$$= \frac{\partial}{\partial z}\langle\tau_{zz}\rangle + \frac{1}{A}\left[(2\pi r_0)\,\tau_{rz}\big|_{r_0} - 0\right]$$

$$\frac{1}{A}\int_A \nabla\cdot w\, dA = \frac{\partial}{\partial z}\langle w_z\rangle + \frac{1}{A}\left[(2\pi r_0)\, w_r\big|_{r_0} - 0\right].$$

Introduce hydraulic diameter.

$$D \equiv \frac{4A}{\xi}. \qquad A: \text{flow area} \quad \xi: \text{wetted perimeter.}$$

$$\frac{2\pi r_0}{A} = \frac{4}{D}$$

D: Important length scale for 1-D formulation.

2) 1-D Balance equation

continuity Eq.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0$$

$$\frac{1}{A} \int \left[\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} \right] = 0.$$

$$\boxed{\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z} \langle \rho V_z \rangle = 0} \qquad \text{area average.}$$

$$\langle\langle V_z \rangle\rangle \equiv \frac{\langle \rho V_z \rangle}{\langle \rho \rangle} \qquad \text{mass weighted mean value}$$

$$\boxed{\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z} \langle \rho \rangle \langle\langle V_z \rangle\rangle = 0}$$

or Assume in A, $\langle \rho \rangle = \rho$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} \rho \langle V_z \rangle = 0$$

$$\langle V_z \rangle = \frac{1}{A} \int V_z \, dA.$$

1-D Momentum Eq.

$$\frac{1}{A}\int\left[\frac{\partial \rho \vec{v}}{\partial t} + \nabla \rho \vec{v}\vec{v} = -\nabla p - \nabla \underline{\underline{\tau}} + \rho \vec{g}\right] dA$$

$$\rho \frac{D\vec{V}}{Dt} = \cdots$$

$$\frac{D\vec{V}}{Dt} = \bar{Q}$$

HW. ρ = uniform in A.

obtain 1-D continuity, momentum, Energy Eq.

$$\frac{\partial}{\partial t}\rho\langle v_z\rangle + \frac{\partial}{\partial z}\rho\langle v_z v_z\rangle = -\frac{\partial\langle p\rangle}{\partial z}$$ pressure gradient

$$-\frac{\partial}{\partial z}\langle \tau_{zz}\rangle$$ normal stress gradient

$$-\frac{4}{D}\tau_{rz}\Big|_{r_0}$$ wall shear

$$+\rho g_z$$ gravity.

1-D Energy Eq.

$$\frac{\partial \rho i}{\partial t} + \nabla\cdot(\rho i \vec{v}) = -\nabla \vec{q} + \frac{Dp}{Dt} - \cancel{\underline{\underline{\tau}} : \nabla v} + \dot{q}$$

$\frac{1}{A}\int dA$ for above eq.

$$\frac{\partial \rho\langle i\rangle}{\partial t} + \frac{\partial}{\partial z}\rho\langle i v_z\rangle = -\frac{\partial}{\partial z}\langle q_z\rangle$$ axial heat conduction

$$+ \xi_h q_0''/A$$ wall heat flux

$$+ D\langle p\rangle/Dt$$ part of pressure work

$$+ \langle \dot{q}\rangle$$ heat generation

Covariance

Note $\langle AB \rangle \neq \langle A \rangle \langle B \rangle$

~~cov(A B) =~~

$$\mathrm{cov}(A\,B) = \langle AB \rangle - \langle A \rangle \langle B \rangle$$

$\langle V_z V_z \rangle$

$\langle i\, V_z \rangle$

(ii) Laminar fully developed flow

$$V_z = 2\langle V_z \rangle \left(1 - \frac{r^2}{r_0^2}\right)$$

$$\mathrm{cov}(V_z, V_z) = \frac{1}{A}\int V_z^2\, dA - \langle V_z \rangle^2 = \frac{1}{3}\langle V_z \rangle^2.$$

$$\mathrm{cov}(i, V_z) = 0.375 \langle i \rangle \langle V_z \rangle$$

Turbulent flow

$$V_z = V_c \left(1 - \frac{r}{r_0}\right)^{1/n}$$

$$V_c = \frac{\langle V_z \rangle}{0.817} = 1.22 \langle V_j \rangle \left(1 - \frac{r}{r_0}\right)^{1/n}.$$

$$\mathrm{cov}(V_z, V_z) = 0.02 \langle V_z \rangle^2.$$

$$\mathrm{cov}(i, V_z) = 0.02 \langle i \rangle \langle V_z \rangle$$

For turbulent flow covariance maybe neglected

$$1 + 0.02 < C_m < 1 + \frac{1}{3}$$

$$1.02 < C_m < 1.33 \qquad C_m \approx 1$$

$$\langle i\, V_z \rangle = C_E \langle i \rangle \langle V_z \rangle$$

$$1.02 < C_E < 1.375. \qquad C_E = 1$$

C_m. C_E. distribution parameter.

Energy eq. $\langle i \rangle$

Alteranate Energy Eq. in terms of T

$$\rho C_p \left[\frac{\partial \langle T \rangle}{\partial t} + \langle V_z \rangle \frac{\langle \partial T \rangle}{\partial z} \right]$$

$$= \frac{\xi_h q_o^+}{A} - \frac{\langle \partial q_z \rangle}{\partial z} - \frac{\langle T \rangle}{\langle \rho \rangle} \left.\frac{\partial \rho}{\partial T}\right)_p \frac{D\langle p \rangle}{D\langle t \rangle} + \dot{q}$$

↓ wall heat flux

↓ axial conduction

Water : axial conduction can be neglected

Na : normally included. ($Pr \ll 1$)

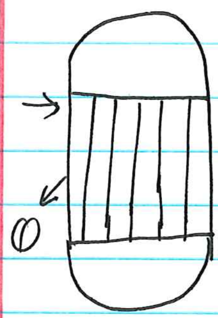
Natural circulation force

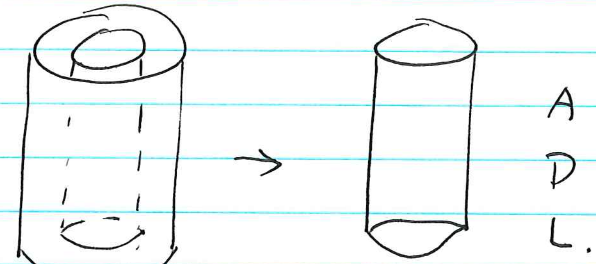
M. E change of ρg_z due to T change.

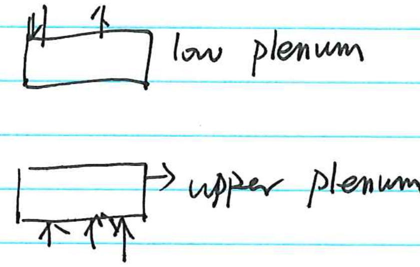
$$\underbrace{\rho g_z = \bar{\rho} g_z - \bar{\rho} \beta \Delta T g_z}.$$

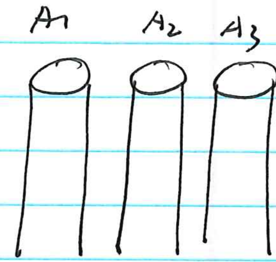
Taylor expansion.

β : Thermal Expansion coefficient.









Summary.

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho V}{\partial z} = 0$$

$C_m = 1.$ $$\frac{\partial \rho V}{\partial t} + \frac{\partial \rho V^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{4\tau_w}{D} + \rho g_z + \cancel{\phi}$$

$$\frac{\partial \rho i}{\partial t} + \frac{\partial}{\partial z}\rho i V = \frac{\xi_i q_w''}{A} + \dot{q} + \left[\frac{\partial P}{\partial z} + V\frac{\partial P}{\partial z}\right] + \cancel{\phi}$$

$$\rho c\left[\frac{\partial T}{\partial t} + V\frac{\partial T}{\partial z}\right] \doteq \frac{\xi_i q_w''}{A} + \dot{q} - \frac{T}{\rho}\frac{\partial \rho}{\partial T}\Big)_p \frac{DP}{Dt}.$$

$$\frac{-4\tau_w}{D} = -\frac{f}{2D}\rho V|V|$$

conservative relation ← f : friction factor. $f = f(Re, \frac{\varepsilon}{D})$ → roughness.

$$q_w'' = h(T_w - T)$$

← h → heat tr. coefficient.

Natural circulation they are not sufficient.

Friction factor.

· smooth pipe. $f = f(Re)$.

Laminar: $Re \le 2300$ $\quad f = 64/Re$. $\quad Re \equiv \dfrac{\rho V D}{\mu}$.

Turbulent: $Re > 2300$

Blasius $\quad f = 0.316/Re^{0.25}$. $\quad Re < 10^5$

McAdams $\quad f = 0.184/Re^{0.2}$ $\quad Re < 2\times10^6$

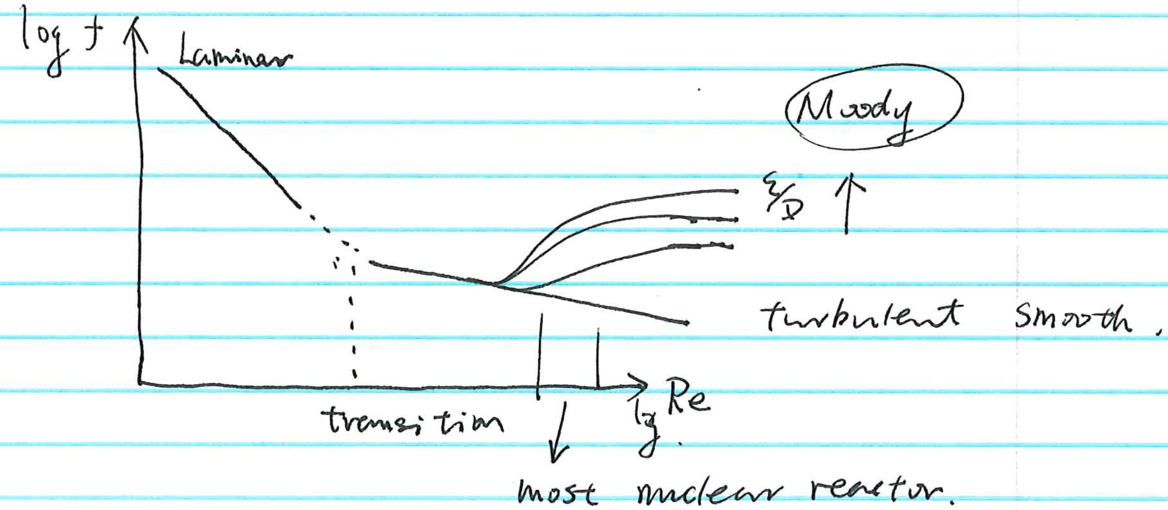
· Rough pipe $\quad f = f(Re, \frac{\varepsilon}{D})$.

↳ completely rough pipe Regime $\quad f = f(\frac{\varepsilon}{D})$.

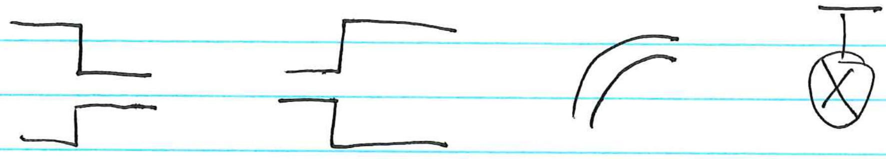
$$f = \frac{1}{(2\log\frac{D}{\varepsilon} + 1.14)^2}.$$ (Von karman & Nik.)

· whole regime

$$\frac{1}{\sqrt{f}} = 1.14 - 2\log\left(\frac{\varepsilon}{D} + \frac{18.7}{Re\sqrt{f}}\right).$$



Minor loss.



Loss coefficient. or k factor.

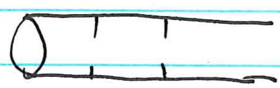
$\Delta p = k \rho V^2/2$. k is given

$k = \sim$ const. $= k(Re)$ weak

$\Delta p \sim -\frac{4\tau_w}{D} = -\frac{f}{2}\rho V^2$. $f \sim Re^{-0.2}$

$Re = \frac{\rho V D}{\mu}$.

$Q = AV. \sim D^2 V$



$\Delta p = \Delta p_{pipe} + \Delta p_c = (\frac{fl}{D} + K)\frac{\rho V^2}{2}$.

$K = k(\text{geometry}, Re)$. ↑ $f = f(Re, \frac{\varepsilon}{D})$

(i) $q_w'' = h(T_w - T_m)$

h: heat tr. coefficient

T_m: mixed mean temperature

$T_m = \frac{1}{A\langle V\rangle}\int T v \, dA$.

Turbulent flow $T_m \simeq \langle T \rangle$.

$h =$ $Nu = \frac{hD}{k}$. non-dim

$\boxed{\text{Nusselt No}}$

☆ $Nu = Nu(Re, Pr)$

$10^{-2} < Pr < 10^{4}$

- Laminar flow $\quad Nu = 4.364$ (q_w'' const)

$= 3.658$ (T_w'' const)

- turbulent flow

$Pr < 0.1$ (liquid metal) $Pr = \nu / a$

$Nu_H = 6.3 + 0.003\, Re Pr \sim$ const

$Nu_T = 4.8 + 0.003\, Re Pr. \sim$ const

$0.5 < Pr < 1.0$ (gases)

$Nu_H = 0.022\, Pr^{0.6}\, Re^{0.8} \sim Re^{0.8} \sim V.$

$Nu_T = 0.021\, Pr^{0.6}\, Re^{0.8}$

H:	q_w const
T:	T const.

☆ difference between liquid metal and water.

$1.0 < Pr < 20$ (water

$Nu = 0.0155\, Pr^{0.3}\, Re^{0.83}$

$Pr > 20$ (heavy oil)

$Nu = 0.0118\, Pr^{0.3}\, Re^{0.9}$

Natural circulation

Boussinesq Assumption

ρ change only important in ρg_z.

all other term $\rho = \bar{\rho}$

$\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} \Big)_P$ $\quad$ Thermal Expansion coefficient.

Equation of state $\quad \beta$ can be obtained

Fundamental Eq of state

$u = u(s,\rho) \to \rho = \rho(p,T)$

$\to u = u(p,T)$

β is a first derivative of thermal eq of state

$$\boxed{\rho g_z \doteq \bar{\rho} g_z - \bar{\rho}\beta \Delta T g_z}$$

$$\downarrow$$

$$-\frac{1}{\rho}\frac{\partial \rho}{\partial T}\Delta T \to \Delta\rho$$

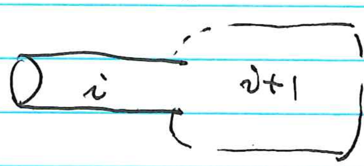
small change .

not small

except ρg_z, → Assume incompressive

$\rho = \bar{\rho}$ = const

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0. \qquad \frac{\partial v}{\partial z} = 0. \text{ for const Area}$$



$V_i = V_i(t)$ only depend on time.
→ i component.

$$\cancel{\rho_i} V_i a_i = \cancel{\rho_{i+1}} V_{i+1} a_{i+1} = V_r a_r.$$

r : reference section.

$v_i a_i = V_r a_r$.

a_r : area of reference section

V_r : vel in reference section

a_i : ith section area.

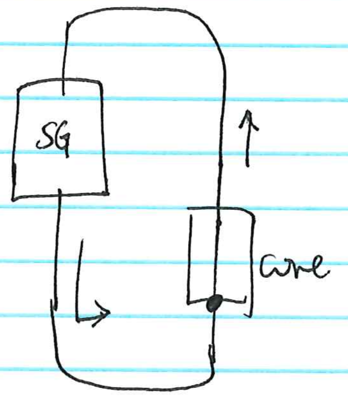
v_i : " velocity

$$V_i = \frac{a_r}{a_i} V_r. \quad ①$$

only 1 velocity in the velocity variable

⇓

there should be only one momentum eq.



⇓

$\boxed{\text{integral Momentum Eq.}}$

$$\frac{\partial \rho_i V_i}{\partial t} + \frac{\partial \rho_i V_i^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{f_i \rho_i V_i |V_i|}{2D_i} + \rho_i g_{zi}$$

$$\rho g_z = \bar{\rho} g_z - \bar{\rho} \beta \Delta T g_z$$

$$\frac{\partial \bar{\rho} V_i}{\partial t} + \frac{\partial \bar{\rho} V_i^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{f_i \bar{\rho} V_i |V_i|}{2D_i} + \underbrace{\bar{\rho} g_z - \bar{\rho} \beta \Delta T g_z}_{\text{B. Assumption}}$$

⇓

$$\oint \Big[\qquad \Big]$$

$$\oint \frac{\partial \bar{\rho} V_i}{\partial t} dz = \bar{\rho} \oint \frac{\partial}{\partial t}\left(\frac{a_r}{a_i} V_r\right) dz = \bar{\rho} \sum \left(\frac{a_r}{a_i}\right) l_i \frac{\partial V_r}{\partial t}$$

$$V_i = \frac{a_r}{a_i} V_r.$$

= Total inertial effect

$$\oint \frac{\partial \bar{\rho} V_i^2}{\partial z} dz = 0 \qquad d \bar{\rho} V_i^2$$

$$\oint -\frac{\partial P}{\partial z} dz = \Delta P_{pump}$$