NUCL 511
Nuclear Reactor Theory and Kinetics

Lecture Note 6

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Solution of Kinetics Problems

- Steady State Solutions
  - Source Multiplication
- Approximation in Short Time Behavior
  - Constant Delayed neutron Source Approximation
  - Precursor Accumulation
- One-group Kinetics
- Asymptotic Behavior
  - In-hour Equation
  - Transition to Asymptotic Behavior

Steady-State Solution

- Point kinetics equation

$$\dot{p}(t) = \frac{\rho-\beta}{\Lambda} p(t) + \frac{1}{\Lambda} \boxed{\sum_{k=1}^{K} \lambda_k \zeta_k(t)} + \frac{1}{\Lambda} s(t)$$

$$\dot{\zeta}_k(t) = \beta_k p(t) - \lambda_k \zeta_k(t)$$

$s_d(t)$: delayed neutron source

- Initial steady state with independent source (i.e., initially subcritical)

$$\beta_k p_0 - \lambda_k \zeta_{k0} = 0 \quad \Rightarrow \quad \lambda_k \zeta_{k0} = \beta_k p_0 \quad \Rightarrow \quad s_{d0} = \sum_{k=1}^{K} \lambda_k \zeta_{k0} = \sum_{k=1}^{K} \beta_k p_0 = \beta p_0$$

$$\Lambda \dot{p}_0 = (\rho_0 - \beta) p_0 + s_{d0} + s_0 = 0 \quad \Rightarrow \quad (\rho_0 - \beta) p_0 + \beta p_0 + s_0 = 0 \quad \Rightarrow \quad s_0 = -\rho_0 p_0 \quad \Rightarrow$$

$$p_0 = \frac{s_0}{-\rho_0}$$

Source multiplication factor

$$\Lambda \dot{p}_0 = (\rho_0 - \beta) p_0 + s_{d0} + s_0 = 0 \quad \Rightarrow$$

$$p_0 = \frac{s_{d0} + s_0}{\beta - \rho_0}$$

Multiplication of delayed neutron source as well as independent source (Generalized source multiplication factor)

Source Multiplication

- Steady-state subcritical reactor with independent source s_0

$$k_0 < 1, \quad \rho_0 = 1 - \frac{1}{k_0} < 0$$

  – Fission neutrons generated by source s_0

$$s_0 \rightarrow k_0 s_0 \rightarrow k_0^2 s_0 \rightarrow k_0^3 s_0 \cdots \quad \Rightarrow \quad n_0 = \frac{k_0}{1-k_0} s_0 = \frac{1}{1/k_0 - 1} s_0 = \frac{1}{-\rho_0} s_0 = M s_0$$

$$M = \frac{1}{-\rho_0}$$ **Source multiplication factor**

  – Continuous source of s_0 neutrons per unit time yields n_0 neutrons per unit time (proportional to power)

$$p_0 = \frac{s_0}{-\rho_0}$$

  – Total neutrons including source itself

$$n_{t0} = n_0 + s_0 = \frac{s_0}{1-k_0}$$

Generalized Source Multiplication

- Generalized source multiplication during transient

$$\Lambda\dot{p}(t) = [\rho(t) - \beta]p(t) + s_d(t) + s(t)$$

$$\Lambda\dot{p}(t) \sim 0 \quad \Rightarrow \quad (\rho - \beta)p(t) + s_d(t) + s(t) = 0$$

$$p(t) = \frac{s_d(t) + s(t)}{\beta - \rho(t)} = M'[s_d(t) + s(t)] \qquad M'(t) = \frac{1}{\beta - \rho(t)}$$

– For a critical reactor ($\rho = 0$ & $s = 0$)

$$p_0 = \frac{s_{d0}}{\beta} \quad \Rightarrow \quad M' = \frac{1}{\beta}$$

$$s_{d0} = \beta p_0 \quad \Rightarrow \quad \frac{\beta p_0}{\beta} = p_0$$

– For a subcritical reactor ($\rho < 0$)

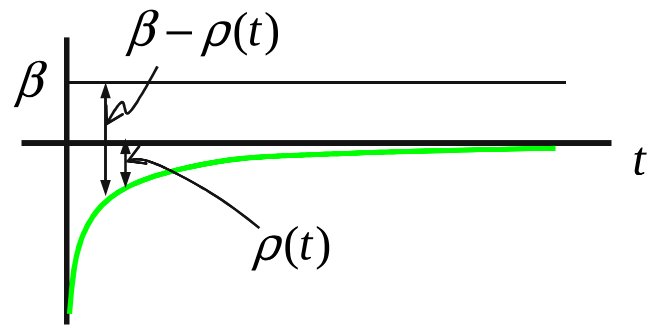
$$\beta - \rho > -\rho$$

$$\Rightarrow \quad M' = \frac{1}{\beta - \rho} < M = \frac{1}{-\rho}$$

Subcritical Source Multiplication & Reactivity Perturbation

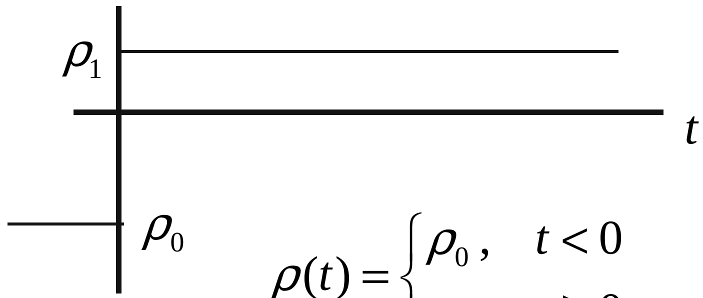
- Generalized source multiplication
  - Denominator decreases with increasing ρ
  - If ρ increases with time, power increases as well even at subcritical state

$$p(t) = \frac{s_d(t) + s(t)}{\beta - \rho(t)}$$



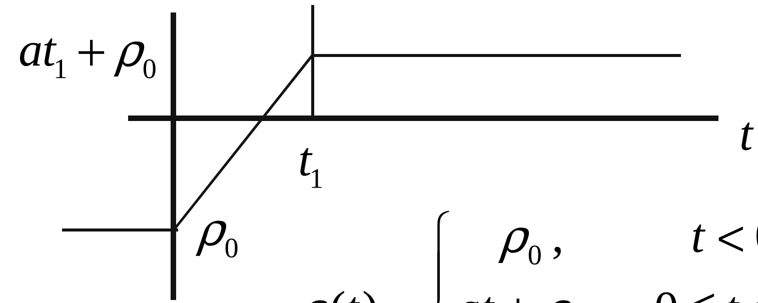
- Two types of reactivity perturbation to consider

Step reactivity change



$$\rho(t) = \begin{cases} \rho_0, & t < 0 \\ \rho_1, & t \geq 0 \end{cases}$$

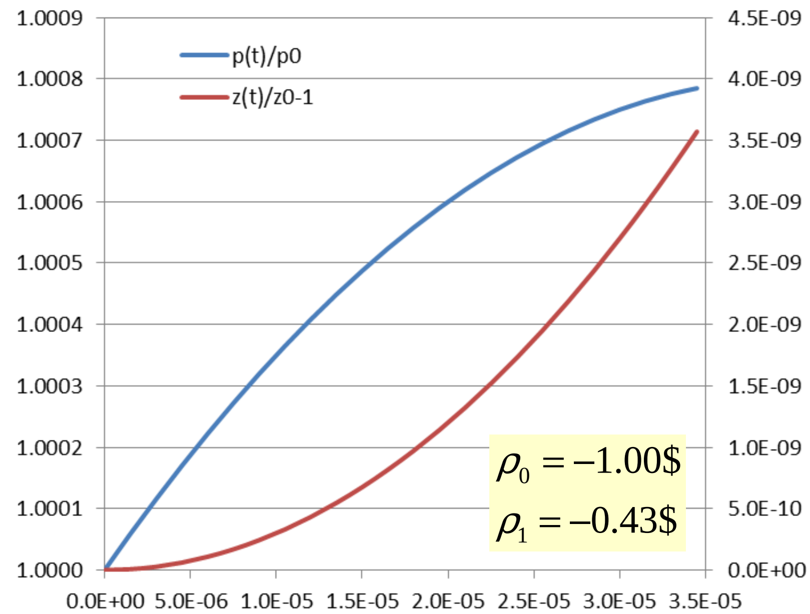
Ramp reactivity change



$$\rho(t) = \begin{cases} \rho_0, & t < 0 \\ at + \rho_0, & 0 \leq t \leq t_1 \\ at_1 + \rho_0, & t \geq t_1 \end{cases}$$

Power Series Solution for Short Time Behavior

- Assume a <u>step change</u> in reactivity with constant source from an <u>initially subcritical</u> state



- Point kinetics equation

$$\Lambda \dot{p}(t) = (\rho_1 - \beta) p(t) + \sum_k \lambda_k \zeta_k(t) + s_0$$

$$\dot{\zeta}_k(t) = \beta_k p(t) - \lambda_k \zeta_k(t)$$

- Power series expansion around *t*=0

$$p(t) = p_0 + p_0^{(1)} t + p_0^{(2)} t^2 + \cdots$$

$$\zeta_k(t) = \zeta_{k0} + \zeta_{k0}^{(1)} t + \zeta_{k0}^{(2)} t^2 + \cdots$$

- Precursor equation

$$\begin{aligned} \zeta_{k0}^{(1)} + 2\zeta_{k0}^{(2)} t + \cdots &= \beta_k (p_0 + p_0^{(1)} t + p_0^{(2)} t^2 + \cdots) - \lambda_k (\zeta_{k0} + \zeta_{k0}^{(1)} t + \zeta_{k0}^{(2)} t^2 + \cdots) \\ &= (\beta_k p_0 - \lambda_k \zeta_{k0}) + (\beta_k p_0^{(1)} - \lambda_k \zeta_{k0}^{(1)}) t + (\beta_k p_0^{(2)} - \lambda_k \zeta_{k0}^{(2)}) t^2 + \cdots \end{aligned}$$

zeroth order: $\zeta_{k0}^{(1)} = \beta_k p_0 - \lambda_k \varsigma_{k0} = 0 \quad \Rightarrow$ slow change in ζ_k as expected (bottom of parabola)

first order: $2\zeta_{k0}^{(2)} = \beta_k p_0^{(1)} - \lambda_k \varsigma_{k0}^{(1)} = \beta_k p_0^{(1)} \quad \Rightarrow \quad \zeta_{k0}^{(2)} = \dfrac{1}{2} \beta_k p_0^{(1)}$

Power Series Solution for Short Time Behavior

- Power amplitude equation

$$\Lambda(p_0^{(1)} + 2p_0^{(2)}t + \cdots) = (\rho_1 - \beta)(p_0 + p_0^{(1)}t + p_0^{(2)}t^2 + \cdots) + \sum_k \lambda_k(\zeta_{k0} + \zeta_{k0}^{(1)}t + \zeta_{k0}^{(2)}t^2 + \cdots) + s_0$$

$$= \left\{(\rho_1 - \beta)p_0 + \boxed{\sum_k \lambda_k \zeta_{k0}} + \boxed{s_0}\right\} + \left\{(\rho_1 - \beta)p_0^{(1)} + \sum_k \lambda_k \zeta_{k0}^{(1)}\right\}t + \cdots$$

$$\beta p_0$$

$$s_0 = (-\rho_0)p_0 \quad \Leftarrow \quad p_0 = s_0 / (-\rho_0)$$

zeroth order: $\Lambda p_0^{(1)} = (\rho_1 - \rho_0)p_0 \quad \Rightarrow \quad p_0^{(1)} = \dfrac{\rho_1 - \rho_0}{\Lambda}p_0 = \dfrac{\Delta\rho}{\Lambda}p_0$

$$\Rightarrow \quad \zeta_{k0}^{(2)} = \frac{1}{2}\beta_k p_0^{(1)} = \frac{\Delta\rho}{2\Lambda}\beta_k p_0 = \frac{\Delta\rho}{2\Lambda}\lambda_k \zeta_{k0}$$

first order: $2\Lambda p_0^{(2)} = (\rho_1 - \beta)p_0^{(1)} \quad \Rightarrow \quad p_0^{(2)} = \dfrac{1}{2\Lambda}(\rho_1 - \beta)p_0^{(1)} = \dfrac{1}{2\Lambda^2}(\rho_1 - \beta)\Delta\rho\, p_0$

- Power series solution in terms of $\Delta\rho$

$$p(t) = p_0\left[1 + \frac{\Delta\rho}{\Lambda}t + \frac{(\rho_1 - \beta)\Delta\rho}{2\Lambda^2}t^2 + \cdots\right] \qquad \zeta_k(t) = \zeta_{k0}\left[1 + \frac{\lambda_k\Delta\rho}{2\Lambda}t^2 + \cdots\right]$$

Approximations for Short Time Behavior

- Approximated delayed neutron source for short time behavior
  - Constant delayed neutron source (CDS) $s_d(t) = s_{d0} = \beta p_0$
    - *Precursor equation is not considered*
  - Precursor accumulation (PA)
    - *Neglect the decay of newly formed precursor in the precursor equation*

$$\dot{\zeta}_k(t) = \beta_k p(t) - \lambda_k \zeta_k(t) = \beta_k [p_0 + \Delta p(t)] - \lambda_k [\zeta_{k0} + \Delta\zeta_k(t)] \quad (\beta_k \Delta p >> \lambda_k \Delta\zeta_k)$$
$$\approx \beta_k [p_0 + \Delta p(t)] - \lambda_k \zeta_{k0} = \beta_k \Delta p(t) = \beta_k [p(t) - p_0]$$

- CDS Approximation

$$\Lambda \dot{p}(t) = (\rho_1 - \beta) p(t) + s_{d0} + s_0 \quad \Rightarrow \quad \dot{p}(t) = \frac{\rho_1 - \beta}{\Lambda} p(t) + \frac{\beta p_0 + s_0}{\Lambda}$$

$$\dot{p}(t) - \alpha_p p(t) = \frac{s_{t0}}{\Lambda}, \qquad \alpha_p = \frac{\rho_1 - \beta}{\Lambda} < 0 \text{ for } \rho < 0$$

$$\frac{d}{dt}[p(t)e^{-a_p t}] = \frac{s_{t0}}{\Lambda} e^{-a_p t} \quad \Rightarrow \quad p(t)e^{-a_p t} - p_0 = \frac{s_{t0}}{\Lambda \alpha_p}(1 - e^{-a_p t})$$

$$\Rightarrow \quad p(t) = p_0 e^{a_p t} + \frac{s_{t0}}{\Lambda \alpha_p}(e^{a_p t} - 1) = p_0 e^{a_p t} + \frac{s_{t0}}{\Lambda(-\alpha_p)}(1 - e^{a_p t})$$

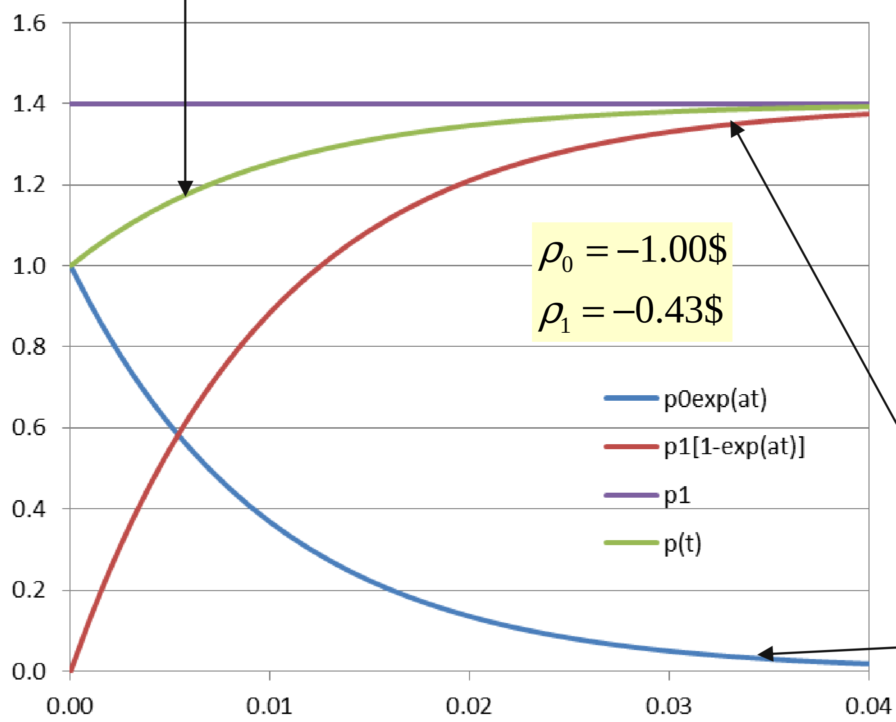
CDS Approximation

- Constant delayed neutron source (CDS)

$$p(t) = p_0 e^{a_p t} + \frac{s_{t0}}{\Lambda(-\alpha_p)}(1-e^{a_p t})$$
$$= p_0 e^{a_p t} + \frac{\beta-\rho_0}{\beta-\rho_1}(1-e^{a_p t}) = p_0 e^{a_p t} + \frac{1-\rho_0(\$)}{1-\rho_1(\$)}(1-e^{a_p t})$$

$$s_{t0} = s_{d0} + s_0 = \beta p_0 - \rho_0 p_0$$

$$\alpha_p = \frac{\rho_1 - \beta}{\Lambda}$$



$$p_1 = \frac{1-\rho_0(\$)}{1-\rho_1(\$)} p_0$$

The power stays at a new power level of $1.4p_0$ because of no change in delayed neutron source

Power buildup due to new prompt neutrons

Decay out of initial source driven power

Precursor Accumulation (PA)

- Precursor accumulation (PA) approximation
  - Neglect the decay of newly formed precursor in the precursor equation
    - *Constant decay rate in the precursor equation*
  - A small source error is amplified by Λ in amplitude equation

$$\Lambda\dot{p}(t) = [\rho(t) - \beta]p(t) + \sum_k \lambda_k \zeta_k(t) + s_0$$

$$\dot{\zeta}_k(t) = \beta_k p(t) - \lambda_k \zeta_{k0} = \beta_k [p(t) - p_0]$$

  - Integration of precursor equation

Integral of power increment, i.e., energy increment

$$\zeta_k(t) - \zeta_{k0} = \beta_k \int_0^t [p(t') - p_0]dt' = \beta_k \boxed{I_{\Delta p}(t)}$$

  - Delayed neutron source and amplitude equation

$$\sum_k \lambda_k \zeta_k(t) = \sum_k \lambda_k \zeta_{k0} + \sum_k \lambda_k \beta_k I_{\Delta p}(t) = \beta p_0 + \beta\bar{\lambda} I_{\Delta p}(t)$$

$$\bar{\lambda} = \frac{1}{\beta}\sum_{k=1}^{K} \lambda_k \beta_k$$

(average decay constant)

$$\Lambda\dot{p}(t) = [\rho(t) - \beta]p(t) + \beta p_0 + \beta\bar{\lambda} I_{\Delta p}(t) + s_0$$
$$= [\rho(t) - \beta]p(t) + \beta\bar{\lambda} I_{\Delta p}(t) + (\beta - \rho_0)p_0$$

Integro-differential equation

Precursor Accumulation (PA)

- PA approximation
  - Integro-differential equation

$$\Lambda\dot{p}(t) = [\rho(t) - \beta]p(t) + \beta\bar{\lambda} I_{\Delta p}(t) + (\beta - \rho_0)p_0$$

  - Second order differential equation and two initial conditions

$$\Lambda\ddot{p}(t) = [\rho(t) - \beta]\dot{p}(t) + \dot{\rho}(t)p(t) + \beta\bar{\lambda}[p(t) - p_0]$$

$$p(0) = p_0, \quad \dot{p}(0) = \frac{\rho(t) - \rho_0}{\Lambda} p_0 = \frac{\Delta\rho}{\Lambda} p_0$$

- For a step reactivity insertion

$$\Lambda\ddot{p}(t) = (\rho_1 - \beta)\dot{p}(t) + \beta\bar{\lambda}[p(t) - p_0]$$

$$\Delta\ddot{p}(t) - \alpha_p \Delta\dot{p}(t) - \frac{\beta\bar{\lambda}}{\Lambda}\Delta p(t) = 0$$

$$\Rightarrow \quad \Delta p(t) = p(t) - p_0 = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t}$$

$$\alpha_{1,2} = \frac{1}{2}\left[\alpha_p \pm \sqrt{\alpha_p^2 + \frac{4\beta\bar{\lambda}}{\Lambda}}\right] = \frac{\alpha_p}{2}\left[1 \pm \sqrt{1 + \frac{4\beta\bar{\lambda}}{\Lambda\alpha_p^2}}\right] \approx \frac{\alpha_p}{2}\left[1 \pm \left(1 + \frac{2\beta\bar{\lambda}}{\Lambda\alpha_p^2}\right)\right]$$

Precursor Accumulation (PA)

- Step reactivity insertion

$$\alpha_1 \approx \frac{\alpha_p}{2}\left[1-\left(1+\frac{2\beta\bar{\lambda}}{\Lambda\alpha_p^2}\right)\right] = \frac{\beta\bar{\lambda}}{\Lambda\alpha_p} = \frac{\beta\bar{\lambda}}{\beta-\rho_1}$$

$$\alpha_2 \approx \frac{\alpha_p}{2}\left[1+\left(1+\frac{2\beta\bar{\lambda}}{\Lambda\alpha_p^2}\right)\right] = \alpha_p + \frac{\beta\bar{\lambda}}{\Lambda\alpha_p} \approx \alpha_p$$

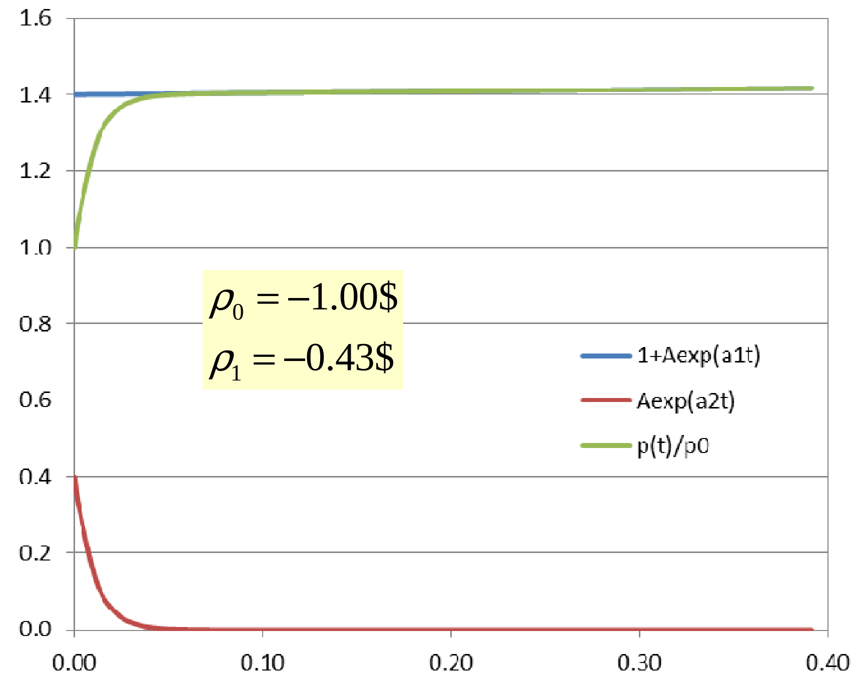
$$\alpha_1 \approx \frac{\beta\bar{\lambda}}{\beta-\rho_1}, \quad \alpha_2 \approx \alpha_p$$

$$A_1 + A_2 = 0, \quad \alpha_1 A_1 + \alpha_2 A_2 = \frac{\Delta\rho}{\Lambda} p_0$$

$$\Rightarrow \quad A_2 \approx \frac{\Delta\rho}{\Lambda\alpha_2} p_0 = -\frac{\rho_1-\rho_0}{\beta-\rho_1} p_0$$

$$A_1 = -A_2 = \frac{\rho_1-\rho_0}{\beta-\rho_1} p_0$$

$$p(t) = p_0\left[1+\frac{\rho_1-\rho_0}{\beta-\rho_1}e^{\alpha_1 t} - \frac{\rho_1-\rho_0}{\beta-\rho_1}e^{\alpha_p t}\right]$$



One Group Kinetics

- Condense six group delayed neutron precursors into one group
  - One-group decay constant needs to be defined properly

$$\Lambda\dot{p}(t) = [\rho(t)-\beta]p(t) + \bar{\lambda}\zeta(t) + s_0$$

$$\dot{\zeta}(t) = \beta p(t) - \bar{\lambda}\zeta(t)$$

$$\bar{\lambda} = \frac{1}{\beta}\sum_{k=1}^{K}\lambda_k\beta_k$$

- How to obtain one-group decay constant
  - For example, average decay constant can be obtained from PA
    - *But initial precursor concentration is NOT conserved*

$$\zeta_0 = \frac{\beta}{\bar{\lambda}}p_0 = \frac{\beta}{\beta^{-1}\sum_k \lambda_k\beta_k}p_0 = \frac{\beta^2}{\sum_k \lambda_k\beta_k}p_0 \neq \zeta(0) = \sum_k \frac{\beta_k}{\lambda_k}p_0 \quad \Leftarrow \quad \zeta_{k0} = \frac{\beta_k}{\lambda_k}p_0$$

  - Alternative one can be obtained by condensing precursor equations

$$\sum_k \dot{\zeta}_k(t) = \sum_k [\beta_k p(t) - \lambda_k\zeta_k(t)] \quad \Rightarrow \quad \dot{\zeta}(t) = \beta p(t) - \sum_k \lambda_k\zeta_k(t) = \beta p(t) - \bar{\lambda}\zeta(t)$$

$$\bar{\lambda}(t) = \frac{1}{\zeta(t)}\sum_k \lambda_k\zeta_k(t) \quad \Rightarrow \quad \bar{\lambda}(0) = \frac{1}{\zeta_0}\sum_k \lambda_k\zeta_{k0} = \frac{\sum_k \beta_k p_0}{\sum_k (\beta_k/\lambda_k)p_0} = \frac{\beta}{\sum_k (\beta_k/\lambda_k)} = \bar{\lambda}_{in}$$

One-Group Decay Constant

- Behavior during initial phase of transient

$$\dot{\zeta}_k(t) = \beta_k p(t) - \lambda_k \zeta_k$$

$$\zeta_k(t) = \zeta_{k0} e^{-\lambda_k t} + \beta_k \int_0^t e^{-\lambda_k (t-t')} p(t') dt'$$

$$s_{dk}(t) = \lambda_k \zeta_{k0} e^{-\lambda_k t} + \lambda_k \beta_k \int_0^t e^{-\lambda_k (t-t')} p(t') dt'$$

$$e^{-\lambda_k t} \approx 1 - \lambda_k t,\ e^{-\lambda_k (t-t')} \approx 1 - \lambda_k (t - t'), \text{ and } \zeta_{k0} = \beta_k p_0 / \lambda_k \quad \Rightarrow$$

$$\zeta_k(t) = p_0(\beta_k / \lambda_k - \beta_k t) + \beta_k \int_0^t p(t') dt' - \beta_k \lambda_k \int_0^t (t - t') p(t') dt'$$

$$s_{dk}(t) = p_0(\beta_k - \lambda_k \beta_k t) + \lambda_k \beta_k \int_0^t p(t') dt' - \beta_k \lambda_k^2 \int_0^t (t - t') p(t') dt'$$

$$\sum_k \zeta_k(t) = p_0(\beta / \bar{\lambda}_{in} - \beta t) + \beta \int_0^t p(t') dt' - \beta \bar{\lambda} \int_0^t (t - t') p(t') dt'$$

$$\sum_k s_{dk}(t) = p_0(\beta - \beta \bar{\lambda} t) + \beta \bar{\lambda} \int_0^t p(t') dt' - \beta \overline{\lambda_k^2} \int_0^t (t - t') p(t') dt'$$

  - $\bar{\lambda}_{in}$ preserves the precursor density during the initial phase of transient
  - $\bar{\lambda}$ preserves the delayed neutron source during the initial phase of transient

One-Group Decay Constant

- Harmonic mean weighted with β_k
  - Arithmetic mean weighted with initial precursor concentration

$$\frac{1}{\bar{\lambda}_{in}} = \frac{1}{\beta}\sum_k \frac{\beta_k}{\lambda_k} \qquad \frac{1}{\zeta_0}\sum_k \lambda_k \zeta_{k0} = \frac{\sum_k \beta_k p_0}{\sum_k (\beta_k/\lambda_k) p_0} = \frac{\beta}{\sum_k (\beta_k/\lambda_k)} = \bar{\lambda}_{in}$$

  - Initial delayed neutron source is preserved with $\bar{\lambda}_{in}$

$$s_{d0} = \bar{\lambda}_{in}\zeta_0 = \sum_k \lambda_k \zeta_{k0}$$

  - But transient delayed neutron source is generally NOT preserved

$$\bar{\lambda}_{in}\zeta(t) \neq \sum_k \lambda_k \zeta_k(t) \qquad \bar{\lambda}_{in}(t) = \frac{1}{\zeta(t)}\sum_k \lambda_k \zeta_k(t)$$

- Arithmetic mean weighted with β_k
  - Arithmetic mean weighted with initial delayed neutron source

$$\bar{\lambda} = \frac{1}{\beta}\sum_k \lambda_k \beta_k$$

$$\frac{1}{s_{d0}}\sum_k \lambda_k s_{dk0} = \frac{1}{\beta p_0}\sum_k \lambda_k \beta_k p_0 = \frac{1}{\beta}\sum_{k=1}^{K} \lambda_k \beta_k = \bar{\lambda}, \quad \bar{\lambda}(t) = \frac{1}{s_d(t)}\sum_k \lambda_k s_{dk}(t)$$

Comparison of One-Group Decay Constants

- Arithmetic mean $\bar{\lambda}$
  - Strongly influenced by short-lifetime precursor groups with large λ_k values
  - Preserves the initial delayed neutron source s_{d0} and the delayed neutron source during the initial transient, but yields different initial precursor density
  - Better to use because a better delayed neutron source yields a better power response
- Harmonic mean $\bar{\lambda}_{in}$
  - Small λ_k values are dominating (Ba-87)
  - Preserve the initial precursor density, the initial delayed neutron source and the precursor density during the initial transient

	$\bar{\lambda}$	$\bar{\lambda}_{in}$
Th-232	0.631	0.105
U-235	0.469	0.090
U-238	0.689	0.139
Np-237	0.441	0.079
Pu-238	0.402	0.075
Pu-239	0.410	0.075
Pu-240	0.433	0.075
Pu-241	0.508	0.088
Pu-242	0.532	0.086
Am-241	0.392	0.073
Am-243	0.402	0.070
Cm-245	0.462	0.096

Asymptotic Behavior and Inhour Equation

- Predict the asymptotic state after perturbation (step change or asymptotic reactivity insertion) for PKE which is a system of 1st order ordinary differential equations
- Homogeneous ODEs with constant coefficients have asymptotic solution

$$p(t) = p_{as} e^{\alpha t}, \ \zeta_k(t) = \zeta_k^{as} e^{\alpha t}$$

$$\Lambda \dot{p}(t) = [\rho(t) - \beta] p(t) + \sum_k \lambda_k \zeta_k(t) \quad \Rightarrow \quad \Lambda \alpha p_{as} e^{\alpha t} = (\rho_1 - \beta) p_{as} e^{\alpha t} + \sum_k \lambda_k \zeta_k^{as} e^{\alpha t}$$

$$\dot{\zeta}_k(t) = \beta_k p(t) - \lambda_k \zeta_k(t) \quad \Rightarrow \quad \alpha \zeta_k^{as} e^{\alpha t} = -\lambda_k \zeta_k^{as} e^{\alpha t} + \beta_k p_{as} e^{\alpha t} \quad \Rightarrow \quad \zeta_k^{as} = \frac{\beta_k}{\alpha + \lambda_k} p_{as}$$

$$\Lambda \alpha = (\rho_1 - \beta) + \sum_k \frac{\lambda_k \beta_k}{\alpha + \lambda_k} \quad \Rightarrow \quad \rho_1 = \Lambda \alpha + \beta - \sum_k \frac{\lambda_k \beta_k}{\alpha + \lambda_k}$$

$$\rho_1 = \Lambda \alpha + \sum_k \frac{\beta_k \alpha}{\alpha + \lambda_k}$$

Inhour equation to find $\alpha = f(\rho_1)$ or $\rho_1 = f(\alpha)$

$$\rho_1 = \frac{l_p}{T + l_p} + \frac{T}{T + l_p} \sum_k \frac{\beta_k}{1 + T\lambda_k}$$

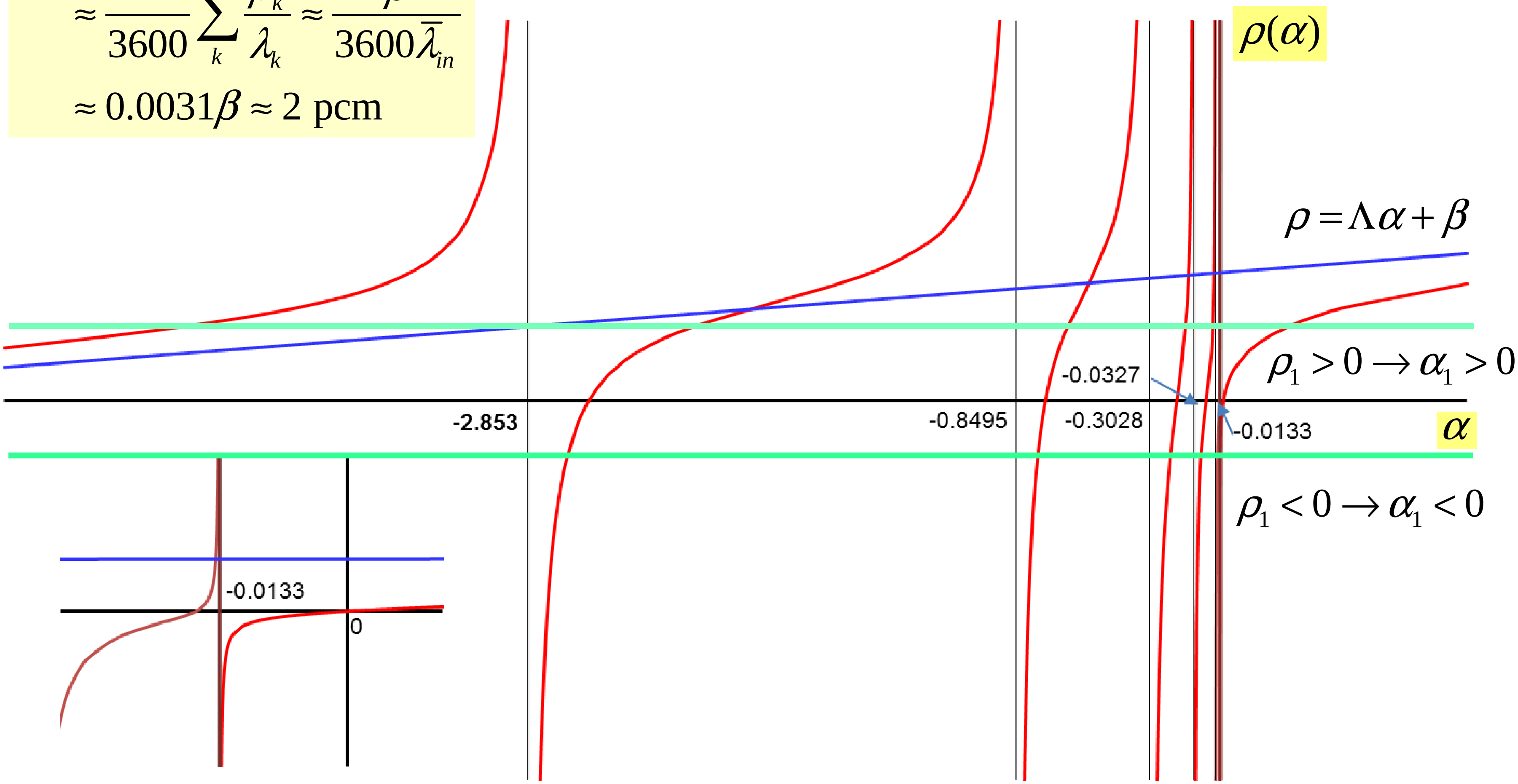
Original inhour equation $T = \alpha^{-1}, \quad \Lambda = (1 - \rho) l_p$

The inverse hour or "in-hour" is the reactivity unit defined by the reactivity that makes the period equal to 1 hour (~0.31¢ for Monju).

Inhour Equation

$$\rho_{1h} = \frac{\Lambda}{3600} + \sum_k \frac{\beta_k / 3600}{1/3600 + \lambda_k}$$
$$\approx \frac{1}{3600} \sum_k \frac{\beta_k}{\lambda_k} \approx \frac{\beta}{3600 \bar{\lambda}_{in}}$$
$$\approx 0.0031\beta \approx 2 \text{ pcm}$$

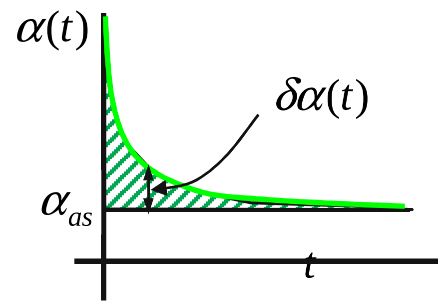
$$\rho_1 = \Lambda\alpha + \sum_k \frac{\beta_k \alpha}{\alpha + \lambda_k} = \Lambda\alpha + \beta - \sum_k \frac{\lambda_k \beta_k}{\alpha + \lambda_k}$$



Asymptotic Transient

- Inverse period during a transient
  - Inverse period generally depends on time
- Transition from initially critical state

$$\alpha(t) = \frac{\dot{p}(t)}{p(t)} \quad \text{(inverse period)}$$



$$\frac{d \ln p}{dt} = \alpha(t) \quad \Rightarrow \quad \ln \frac{p(t)}{p_0} = \int_0^t \alpha(t')dt'$$

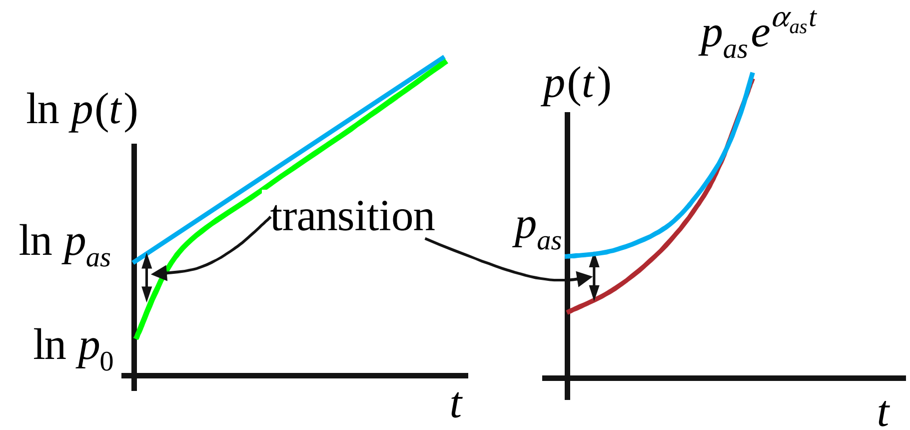
$$\Rightarrow \quad p(t) = p_0 \exp\left[\int_0^t \alpha(t')dt'\right]$$

$$p(t) = p_0 e^{\int_0^t [\alpha_{as} + \delta\alpha(t')[dt'} = p_0 e^{\alpha_{as} t + \int_0^t \delta\alpha(t')dt'}$$

$$= p_0 e^{\int_0^t \delta\alpha(t')dt'} e^{\alpha_{as} t} = \tilde{p}(t) e^{\alpha_{as} t}$$

$$\tilde{p}(t) \to p_{as} \ \text{ as } t \to \infty$$

$$\ln p(t) = \ln \tilde{p}(t) + \alpha_{as} t$$



Asymptotic Behavior of Extreme Cases

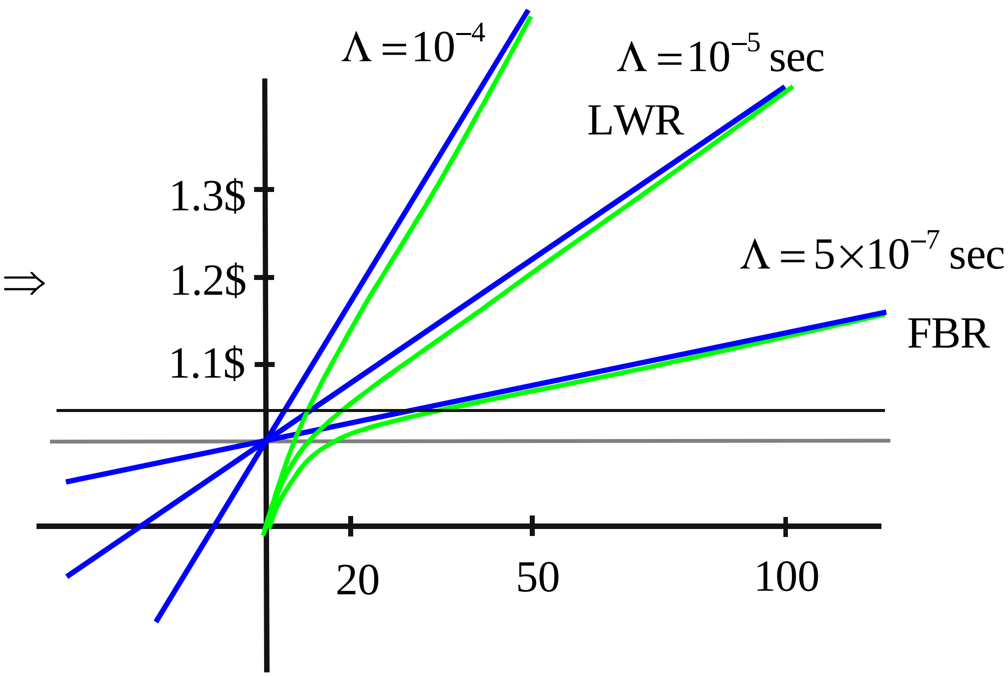
- Extreme cases
  - $\alpha >>> \lambda_k$: Very rapid transient
  - $\alpha >> \lambda_k$: Rapid transient
  - $\alpha << \lambda_k$: Slow transient
- Prompt kinetics
  - Delayed neutron source is completely neglected

$$\dot{p}(t) = \frac{\rho - \beta}{\Lambda} p(t) = \alpha_p p(t)$$

$$\alpha >>> \lambda_k \quad \Rightarrow$$

$$\rho(\alpha) = \Lambda\alpha + \sum_k \frac{\beta_k \alpha}{\alpha + \lambda_k} \simeq \Lambda\alpha + \beta \quad \Rightarrow$$

$$\alpha = \frac{\rho - \beta}{\Lambda} = \alpha_p$$



Asymptotic Behavior of Extreme Cases

- Sub-prompt critical $\alpha >> \lambda_k$

$$\rho = \Lambda\alpha + \sum_k \frac{\beta_k \alpha}{\alpha + \lambda_k} = \Lambda\alpha + \sum_k \frac{\beta_k}{1 + \lambda_k / \alpha}$$

$$\alpha >> \lambda_k \quad \Rightarrow \quad \rho \simeq \Lambda\alpha + \sum_k \beta_k \left(1 - \frac{\lambda_k}{\alpha}\right) = \Lambda\alpha + \beta - \frac{1}{\alpha}\sum_k \beta_k \lambda_k = \Lambda\alpha + \beta - \frac{1}{\alpha}\beta\bar{\lambda}$$

  – Asymptotic solution in one delayed neutron group

$$\zeta(t) = \zeta_{as} e^{\alpha t} \quad \rightarrow \quad \dot{\zeta} = \beta p - \lambda\zeta \quad \Rightarrow \quad \alpha\zeta_{as} = \beta p_{as} - \lambda\zeta_{as} \quad \Rightarrow \quad \zeta_{as} = \frac{p_{as}}{\alpha + \lambda}$$

$$p(t) = p_{as} e^{\alpha t} \quad \rightarrow \quad \Lambda\dot{p} = (\rho - \beta)p + \lambda\zeta \quad \Rightarrow \quad \Lambda\alpha p_{as} = (\rho - \beta)p_{as} + \frac{\lambda\beta}{\alpha + \lambda} p_{as}$$

$$\Rightarrow \quad \rho = \Lambda\alpha + \beta - \frac{\lambda\beta}{\alpha + \lambda} = \Lambda\alpha + \beta\left(1 - \frac{\lambda}{\alpha + \lambda}\right) \approx \Lambda\alpha + \beta\left(1 - \frac{\lambda}{\alpha}\right) = \Lambda\alpha + \beta - \frac{\beta\lambda}{\alpha}$$

$$\Rightarrow \quad \lambda = \bar{\lambda} = \frac{1}{\beta}\sum_k \beta_k \lambda_k$$

For a sub-prompt critical transient analysis with one delayed neutron group (to yield correct asymptotic behavior)

Asymptotic Behavior of Extreme Cases

- Slow transient $\alpha << \lambda_k$

$$\rho = \Lambda\alpha + \beta - \sum_k \frac{\lambda_k \beta_k}{\alpha + \lambda_k} = \Lambda\alpha + \beta - \sum_k \frac{\beta_k}{1 + \alpha / \lambda_k}$$

$$\alpha << \lambda_k \quad \Rightarrow \quad \rho \simeq \Lambda\alpha + \beta - \sum_k \beta_k \left(1 - \frac{\alpha}{\lambda_k}\right) = \Lambda\alpha + \alpha \sum_k \frac{\beta_k}{\lambda_k} = \alpha\Lambda + \alpha \frac{\beta}{\overline{\lambda}_{in}}$$

$$\Lambda << \frac{\beta}{\overline{\lambda}_{in}} \quad \Rightarrow \quad \rho \simeq \alpha \frac{\beta}{\overline{\lambda}_{in}} \quad \Rightarrow \quad \rho_\$ = \frac{\rho}{\beta} \simeq \frac{\alpha}{\overline{\lambda}_{in}}$$

$$\alpha = \overline{\lambda}_{in} \rho_\$ \approx 0.09 \rho_\$ \quad (\alpha \text{ proportional to } \rho)$$

$$\text{For example } \alpha = \frac{1}{3600s} \quad \Rightarrow \quad \rho_\$ = \frac{1}{0.09 \times 3600} = 0.0031 = 0.31 \text{ cents}$$

  – Asymptotic solution in one delayed neutron group

$$\rho = \Lambda\alpha + \beta - \frac{\lambda\beta}{\alpha + \lambda} = \Lambda\alpha + \beta - \frac{\beta}{1 + \alpha / \lambda} \approx \Lambda\alpha + \beta - \beta\left(1 - \frac{\alpha}{\lambda}\right) = \Lambda\alpha + \frac{\alpha\beta}{\lambda}$$

$\lambda = \overline{\lambda}_{in}$ For a slow transient analysis with one delayed neutron group (to yield correct asymptotic behavior)

Summary of Delayed Neutron Source Approximations

Kinetics with no distinction between prompt and delayed neutrons	$\dot{p} = \frac{\rho}{\Lambda} p + \frac{s}{\Lambda}$
Prompt kinetics (neglect delayed neutrons)	$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{s}{\Lambda}$
Constant delayed neutron source (CDS)	$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{\beta p_0}{\Lambda} + \frac{s}{\Lambda}$
Precursor accumulation (PA)	$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{\beta p_0}{\Lambda} + \frac{\lambda\beta}{\Lambda} \int_0^t \left(p(t') - p_0 \right) dt' + \frac{s}{\Lambda}$
One group kinetics	$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{\bar{\lambda}\zeta}{\Lambda} + \frac{s}{\Lambda}$
Six group kinetics	$\dot{p} = \frac{\rho - \beta}{\Lambda} p + \frac{1}{\Lambda} \sum_{k=1}^{K} \lambda_k \zeta_k + \frac{s}{\Lambda}$