NUCL 511 Nuclear Reactor Theory and Kinetics

Homework #9

Due April 15

1. Ch. 10, homework problem 2 (10 points). Find the numerical value of p^{00}, the flux after a prompt jump for which the increase due delayed neutrons is just compensated by Doppler feedback, for an LWR from the typical λ and γ / β values given in the text. Discuss why p^{00} is different from the value for the SEFOR reactor.

   Answer) Using $\gamma / \beta = -0.8$ \$/fp-s in Eq. (10.35) and $\lambda = 0.40\ \text{s}^{-1}$, the prompt jump p^{00} for LWR can be obtained as

   $$p^{00} = \frac{\lambda}{-\gamma / \beta} = \frac{0.4}{0.8} = 0.5 \text{ fp-s}$$

   This value is ~36 times smaller than the value of ~18 for the SEFOR rector. This is due to the large feedback coefficient of LWR relative to the SEFOR rector. The value of -0.8 \$/fp-s for LWR is ~36 times larger in magnitude than the value of -0.022 \$/fp-s for SEFOR given in Eq. (10.50a). Note that the full-power feedback coefficient of LWR given in Eq. (10.35) is ~4 times larger than a typical value for BWR and ~2 times larger than a typical value of PWR.

2. Ch. 10, homework problem 3 (10 points). Consider the reactivity-step-induced transient with an adiabatic boundary condition for the incremental heat, as considered in Sec. 10-2B. List the asymptotic results derived in the text and apply them to estimate the total number of additional delayed neutron source

   $$\int_0^\infty [s_d(t) - s_{d0}]dt$$

   Presents the physical arguments as to why the integral of the additional delayed neutron source must be finite although $s_{das} = s_{d0}$.

   Answer) By solving the precursor equation for a given power amplitude function, the delayed neutron source can be determined as

   $$s_d(t) = \lambda \zeta(t) = \beta \left[p_0 e^{-\lambda t} + \lambda \int_0^t e^{-\lambda(t-t')} p(t') dt' \right] \tag{1}$$

   Therefore the additional delayed neutron sources can be obtained as

   $$\begin{aligned} \int_0^\infty [s_d(t) - s_{d0}]dt &= \beta \int_0^\infty [p_0 e^{-\lambda t} + \lambda \int_0^t e^{-\lambda(t-t')} p(t') dt' - p_0] dt \\ &= \beta p_0 \int_0^\infty e^{-\lambda t} dt + \beta \int_0^\infty \left[e^{\lambda t'} p(t') \int_{t'}^\infty \lambda e^{-\lambda t} dt - p_0 \right] dt' \\ &= \frac{\beta p_0}{\lambda} + \beta \int_0^\infty [p(t') - p_0] dt' \end{aligned} \tag{2}$$

   Since $\rho_{as} = 0$, the inserted reactivity is asymptotically compensated for by the feedback

$$\rho_{as} = \rho_1 + \gamma \int_0^\infty [p(t') - p_0] dt' = 0 \quad \Rightarrow \quad \int_0^\infty [p(t') - p_0] dt' = \frac{\rho_1}{-\gamma} \tag{3}$$

By inserting Eq. (3) into Eq. (1), the additional delayed neutron source can be obtained as

$$\int_0^\infty [s_d(t) - s_{d0}] dt = \beta \left[\frac{p_0}{\lambda} + \int_0^\infty [p(t') - p_0] dt' \right] = \zeta_0 + \frac{\beta \rho_1}{-\gamma} \tag{4}$$

The first term on the rightmost side represents the steady state precursor concentration, which eventually decays and emits a delayed neutron. The second term represents the number of delayed neutrons produced by the additional fissions above the steady state fission rates, which is finite since the prompt feedback forces the power back to the initial value.

3. Ch. 10, homework problem 4 (10 points). Calculate the burst width in a super-prompt-critical transient with the linear energy feedback model from the formulas derived in the text. Use LWR data: $\beta = 0.0065$, $\Lambda = 10^{-5}$ s; let $\rho_1 = 1.1\$$. Also find the energy release

$$Q(t_2) = \int_0^{t_2} P(t) dt$$

Answer) Using $\gamma / \beta = -0.8$ \$/fp-s in Eq. (10.35), $\Lambda = 10^{-5}$ s, Eq. (10.63), and Eq. (10.83), the prompt jump p^0 and the maximum flux amplitude can be determined as

$$p^0 = \frac{\rho_{1\$}}{\rho_{1\$} - 1} p_0 = \frac{1.1}{0.1} p_0 = 11 p_0$$

$$p_m - p^0 = \frac{(\rho_1 - \beta)^2}{2\Lambda(-\gamma)} = \frac{\beta(\rho_{1\$} - 1)^2}{2\Lambda(-\gamma/\beta)} = \frac{0.0065 \times 0.1^2}{2 \times 10^{-5} \times 0.8} = 4.06 \text{ fp}$$

$$p_m \approx \begin{cases} 15.06 \text{ fp} & \text{for } p_0 = 1 \\ 4.06 \text{ fp} & \text{for } p_0 << 1 \end{cases}$$

Therefore, using Eq. (10.96) and Eq. (10.97), the burst width and the energy release can be determined as

$$\Delta t = \frac{2\rho_{p1}}{p_m(-\gamma)} = \begin{cases} \dfrac{2 \times 0.1}{15.06 \times 0.8} = 0.0166 \text{ s} & \text{for } p_0 = 1 \\ \dfrac{2 \times 0.1}{4.06 \times 0.8} = 0.0615 \text{ s} & \text{for } p_0 << 1 \end{cases}$$

$$Q(t_2) = \int_0^{t_2} P(t) dt = \frac{2\rho_{p1}}{(-\gamma)} = \frac{2 \times 0.1}{0.8} = 0.25 \text{ fp-s}$$

Estimate the magnitude of the effects of the following two approximations:

a. The neglect of the heat release (i.e., the assumption of $\lambda_H = 0$). Consider an oxide fuel rod of 0.6 cm in diameter.

NUCL 511 Nuclear Reactor Theory and Kinetics

Answer) With the prompt kinetics approximation and the linear energy feedback model, the governing equations for a super-prompt-critical transient can be written as

$$\dot{p}(t) = \frac{\rho_p(t)}{\Lambda} p(t) \tag{1}$$

$$\rho_p(t) = \rho_{p1} + \gamma \int_0^t e^{-\lambda_H (t-t')} [p(t') - p_0] dt' \tag{2}$$

where $\rho_p(t) = \rho(t) - \beta$ is the prompt reactivity and $\rho_{p1} = \rho_1 - \beta$. The initial condition for Eq. (1) is given by

$$p(0) = p^0 = \frac{\rho_1}{\rho_1 - \beta} p_0 \tag{3}$$

At $t = t_m$ when the power amplitude achieves its maximum, $\dot{p}(t_m) = 0$ and thus the prompt reactivity becomes zero as

$$\rho_p(t_m) = \rho_{p1} + \gamma \int_0^{t_m} e^{-\lambda_H (t_m - t')} [p(t') - p_0] dt' = 0 \tag{4}$$

After passing through zero, ρ_p approaches $-\rho_{p1}$ at $t = t_2$ when the flux amplitude returns to at initial value

$$\rho_p(t_2) = \rho_{p1} + \gamma \int_0^{t_2} e^{-\lambda_H (t_2 - t')} [p(t') - p_0] dt' = -\rho_{p1} \tag{5}$$

Thus the energy release (more precisely, energy deposit in fuel) during the super-prompt transient can be obtained as

$$Q(t_2) = \int_0^{t_2} e^{-\lambda_H (t_2 - t')} [p(t') - p_0] dt' = \frac{2\rho_{p1}}{-\gamma} \tag{6}$$

This is equal to the total energy release in Eq. (10.87) obtained with adiabatic boundary condition. However, the time t_2 when the flux amplitude returns to the initial value increases with heat release. The reactivity feedback is also lagged, and thus the peak power increases with heat release. As a result, if the burst width is simply defined by the flux integral divided by the maximum value, the width decreases with heat release.

<u>Effect of Heat Release</u>: By approximating the flux pulse by a rectangular pulse of height p_m and width Δt , the integral can be approximated as

$$\begin{aligned} Q(t_2) &= \int_0^{t_2} e^{-\lambda_H (t_2 - t')} [p(t') - p_0] dt' \approx \int_0^{\Delta t} e^{-\lambda_H (\Delta t - t')} (p_m - p_0) dt' \\ &= (p_m - p_0) \int_0^{\Delta t} e^{-\lambda_H (\Delta t - t')} dt' = (p_m - p_0) \frac{1 - e^{-\lambda_H \Delta t}}{\lambda_H} \approx (p_m - p_0) \Delta t \left(1 - \frac{\lambda_H \Delta t}{2} \right) \end{aligned} \tag{7}$$

Using $\lambda_H = 0.5\ \mathrm{s}^{-1}$ in Eq. (10.12) for an oxide fuel rod of 0.6 cm in diameter and the burst width obtained without heat release, it can be seen that the burst width is reduced by ~0.41% for $p_0 = 1$ and ~1.5% for $p_0 << 1$. Correspondingly, the maximum power is increased by ~0.41% for $p_0 = 1$ and ~1.5% for $p_0 << 1$.

b. Neglect of P_0 under the feedback integral, i.e., the integral of P_0 over the burst width with $Q(t_2)$.

Answer) The stationary cooling in the feedback integral is equivalent to a positive ramp reactivity as

$$\rho_p(t) = \rho_{p1} + \gamma\int_0^t [p(t') - p_0]dt' = \rho_{p1} + (-\gamma)p_0 t + \gamma\int_0^t p(t')dt' \tag{8}$$

By differentiating Eq. (8), we have

$$\dot{\rho}_p(t) = \gamma[p(t) - p_0] \tag{9}$$

Multiplying Eq. (9) with the prompt kinetics equation in Eq. (1) gives

$$\gamma[p(t) - p_0]\dot{p}(t) = \frac{\rho_p(t)}{\Lambda}\dot{\rho}_p(t)p(t) \tag{10}$$

Dividing Eq. (10) by p yields

$$\left(1 - \frac{p_0}{p(t)}\right)\dot{p}(t) = \frac{\rho_p(t)}{\gamma\Lambda}\dot{\rho}_p(t) \tag{11}$$

Eq. (11) can be readily integrated as

$$p(t) - p^0 - p_0 \ln\frac{p(t)}{p^0} = \frac{1}{2\gamma\Lambda}\left[\rho_p^2(t) - \rho_{p1}^2\right] \tag{12}$$

Since $\rho_p(t_m) = 0$ at the maximum flux amplitude where $\dot{p}(t_m) = 0$, Eq. (12) at $t = t_m$ becomes

$$p_m - p^0 - p_0 \ln\frac{p_m}{p^0} = \frac{\rho_{p1}^2}{2(-\gamma)\Lambda} \tag{13}$$

When $p_0 = 0$, Eq. (13) is reduced to the maximum flux amplitude in Eq. (10.83) obtained without stationary cooling. Eq. (13) can be solved iteratively for the maximum flux as

$$p_m^{(n)} = \frac{\rho_{p1}^2}{2(-\gamma)\Lambda} + p^0 + p_0 \ln\frac{p_m^{(n-1)}}{p^0} \tag{14}$$

It can be seen that the maximum flux amplitude increases with increasing p_0. Eq. (12) shows that $\rho_p(t_2) = -\rho_{p1}$ at $t = t_2$ when the flux amplitude returns to the initial value p^0. Therefore, the heat release during the super-prompt transient can be determined from Eq. (8) as

$$Q(t_2) = \int_0^{t_2}[p(t') - p_0]dt' = \frac{2\rho_{p1}}{-\gamma} \tag{15}$$

Thus the stationary cooling does not change the energy deposit in the fuel during the super-prompt transient. However, the time t_2 increases with stationary cooling. The pulse burst can be determined by dividing the energy release in Eq. (15) by the maximum flux in Eq. (14).

Effect of Stationary Cooling: Using Eq. (14), the maximum flux amplitude for $p_0 = 1$ can be determined to be 15.4. Relative to the value of 15.06 without stationary cooling, it is increased by 2.23%. Accordingly, the burst width is reduced by 2.23%.