NUCL 511 Nuclear Reactor Theory and Kinetics

Homework #3

Due February 6

Consider a three-group representation of a thermal reactor spectrum composed of a fission spectrum, a $1/E$ spectrum and a Maxwell spectrum:

$$\varphi_1(E) = A_1 \frac{2}{\sqrt{\pi}} \frac{\sqrt{E}}{(kT_1)^{3/2}} e^{-E/kT_1}, \quad kT_1 = 1.4 \text{ MeV} \quad \text{for } E_1 = 820.3 \text{ keV} \le E \le E_0 = 20 \text{ MeV}$$

$$\varphi_2(E) = A_2 / E \quad \text{for } E_2 = 0.1 \text{ eV} \le E \le E_1$$

$$\varphi_3(E)dE = A_3 \frac{E}{(kT_3)^2} e^{-E/kT_3}, \quad kT_3 = 0.0253 \text{ eV} \quad \text{for } E_3 = 0 \le E \le E_2$$

1. Find the normalization constants A_a, A_b and A_c such that the spectrum is continuous at the group boundaries and the integration over the whole energy range is unity. Evaluate the group-wise integrals numerically if necessary.

   (Answer) From the continuity conditions at the group boundaries, we have

$$\varphi_1(E_1) = \varphi_2(E_1) \quad \Rightarrow \quad A_1 = A_2 \frac{\sqrt{\pi}}{2} \left(\frac{kT_1}{E_1} \right)^{3/2} e^{E_1/kT_1}$$

$$\varphi_3(E_2) = \varphi_2(E_2) \quad \Rightarrow \quad A_3 = A_2 \left(\frac{kT_3}{E_2} \right)^2 e^{E_2/kT_3}$$

   Thus the integration over the entire energy range becomes

$$\begin{aligned}
1 &= \int_0^{E_0} \phi(E)dE = \int_0^{E_2} \varphi_3(E)dE + \int_{E_2}^{E_1} \varphi_2(E)dE + \int_{E_1}^{E_0} \varphi_1(E)dE \\
&= A_2 \left[\int_0^{E_2} \frac{E}{E_2^2} e^{-(E-E_2)/kT_3} dE + \int_{E_2}^{E_1} \frac{dE}{E} + \int_{E_1}^{E_0} \frac{E^{1/2}}{E_1^{3/2}} e^{-(E-E_1)/kT_1} dE \right] \\
&= A_2 \left[\frac{kT_3}{E_2} \int_{-E_2/kT_3}^{0} \left(1 + \frac{kT_3}{E_2} x \right) e^{-x} dx + \ln \frac{E_1}{E_2} + \frac{kT_1}{E_1} \int_0^{(E_0-E_1)/kT_1} \sqrt{1 + \frac{kT_1}{E_1} x} e^{-x} dx \right] \\
&= A_2 (3.02 + 15.92 + 2.70) = 21.63 A_2
\end{aligned}$$

   Therefore the constants can be determined as

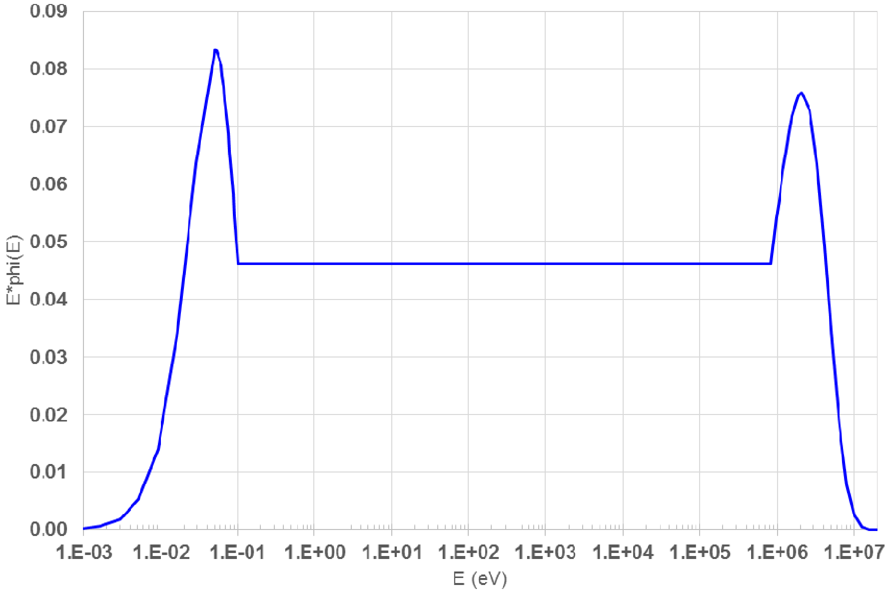
$$A_2 = 0.0462, \quad A_1 = 0.1641, \quad A_3 = 0.1541$$

   Consequently the group-wise spectra become

$$\varphi_1(E) = 1.1178 \times 10^{-10} \sqrt{E} e^{-E/1.4\times10^6} \quad \text{for } E_1 \le E \le E_0$$

$$\varphi_2(E) = 0.0462 / E \quad \text{for } E_2 = 0.1 \text{ eV} \le E \le E_1$$

$$\varphi_3(E)dE = 240.68 E e^{-E/0.0253} \quad \text{for } E_3 \le E \le E_2$$



2. Find the average velocities in individual groups ($\bar{v}_1$, $\bar{v}_2$ and $\bar{v}_3$).

(Answer) For the energy E in eV, the corresponding speed is $v(E) = 1.3831\times10^4\sqrt{E}$ m/s. Therefore, the group fluxes, the velocity integrals over individual groups, and the resulting average group-wise velocities can be determined as

$$\phi_1 = \int_{E_1}^{E_0} \varphi_1(E)dE = 1.1178\times10^{-10}\int_{E_1}^{E_0} \sqrt{E}e^{-E/kT_1}dE = 0.1247$$

$$\int_{E_1}^{E_0} v(E)\varphi_1(E)dE = 1.5461\times10^{-6}\int_{E_1}^{E_0} Ee^{-E/kT_1}dE = 2.6749\times10^6$$

$$\bar{v}_1 = \frac{1}{\phi_1}\int_{E_1}^{E_0} v(E)\varphi_1(E)dE / = 2.1455\times10^7 \text{ m/s}$$

$$\phi_2 = \int_{E_2}^{E_1} \varphi_2(E)dE = 0.0462\int_{E_2}^{E_1} \frac{dE}{E} = 0.7359$$

$$\int_{E_2}^{E_1} v(E)\varphi_2(E)dE = 639.32\int_{E_2}^{E_1} \frac{dE}{\sqrt{E}} = 1.1577\times10^6$$

$$\bar{v}_2 = \frac{1}{\phi_2}\int_{E_2}^{E_1} v(E)\varphi_2(E)dE = 1.5732\times10^6 \text{ m/s}$$

$$\phi_3 = \int_{E_3}^{E_2} \varphi_3(E)dE = 240.68\int_{E_3}^{E_2} Ee^{-E/kT_3}dE = 0.1394$$

$$\int_{E_3}^{E_2} v(E)\varphi_3(E)dE = 3.3289\times10^6\int_{E_3}^{E_2} E^{3/2}e^{-E/kT_3}dE = 377.76$$

$$\bar{v}_3 = \frac{1}{\phi_3}\int_{E_3}^{E_2} v(E)\varphi_3(E)dE = 2709.8 \text{ m/s}$$

3. Find the one-group values of $\bar{v}$ and $\overline{1/v}$ using the three-group values $\bar{v}_1$, $\bar{v}_2$ and $\bar{v}_3$.

   (Answer) The average velocity and the average inverse velocity are defined as

$$\bar{v} = \frac{v_1\phi_1 + v_2\phi_2 + v_3\phi_3}{\phi_1 + \phi_2 + \phi_3} = 3.8329 \times 10^6 \text{ m/s}$$

$$\overline{1/v} = \frac{(1/v_1)\phi_1 + (1/v_2)\phi_2 + (1/v_3)\phi_3}{\phi_1 + \phi_2 + \phi_3} = 5.1917 \times 10^{-5} \text{ s/m}$$

4. Define a one-group $\nu\Sigma_f$ based on the three-group values $\nu\Sigma_{f1} = 0.017 \text{ cm}^{-1}$, $\nu\Sigma_{f2} = 0.015 \text{ cm}^{-1}$, and $\nu\Sigma_{f3} = 0.3 \text{ cm}^{-1}$.

   (Answer) The one-group $\nu\Sigma_f$ is defined as

$$\nu\Sigma_f = \frac{\nu\Sigma_{f1}\phi_1 + \nu\Sigma_{f2}\phi_2 + \nu\Sigma_{f3}\phi_3}{\phi_1 + \phi_2 + \phi_3} = 0.055 \text{ cm}^{-1}$$

5. Calculate the generation time Λ with $\bar{v}$ and $\overline{1/v}$. Discuss the results.

$$\Lambda = \frac{1}{\bar{v}\nu\Sigma_f} = 4.75 \times 10^{-6} \ s$$

$$\Lambda = \overline{\left(\frac{1}{v}\right)}\frac{1}{\nu\Sigma_f} = 9.44 \times 10^{-4} \ s$$

   The average velocity weights more high energy neutrons than low energy neutrons whereas the average inverse velocity weights more low energy neutrons than high energy neutrons. As a result, the generation time is significantly underestimated if is estimated with the average velocity.