NUCL 511
Nuclear Reactor Theory and Kinetics

Lecture Note 10

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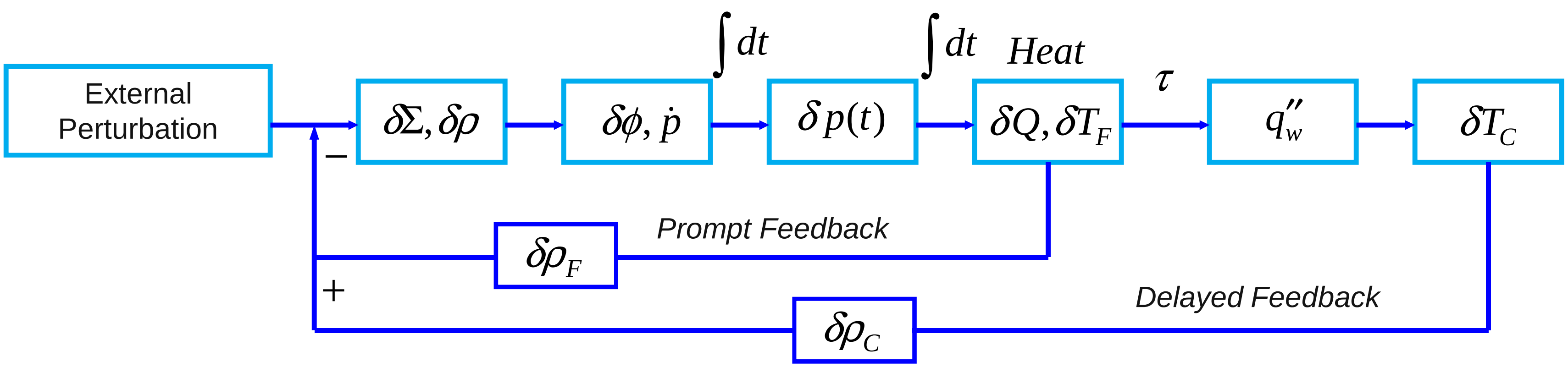
Reactivity Feedback Mechanism

- What is reactivity feedback?
  - Reactivity change induces changes in thermal condition which can affect back to the reactivity → thermal feedback
- Feedback Mechanism



  - Prompt feedback
    - *Occurs immediately due to fuel temperature change*
  - Delayed feedback
    - *Heat conduction to coolant takes time*
    - *Caused by coolant density change (thermal expansion) which changes moderation or leakage*

Doppler Broadening

- The laboratory cross section should be defined to agree with the observed reaction rate.
  - The cross section is determined by the relative speed, as opposed to the laboratory speed of neutron
  - The Doppler broadened cross section will not generally be the same as the cold cross section

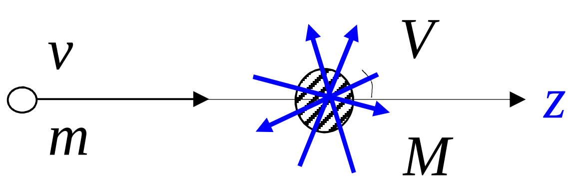
- If $P(\vec{V})d\vec{V}$ is the probability at temperature T that a nucleus (or atom) has velocity $\vec{V}$ within $d\vec{V}$ about $\vec{V}$, the observed reaction rate that a neutron with velocity $\vec{v}$ will collide with a nucleus is

$$R(v,T) = v\sigma(v,T) = \int [v_r\sigma(v_r,0)]P(\vec{V})d\vec{V}$$

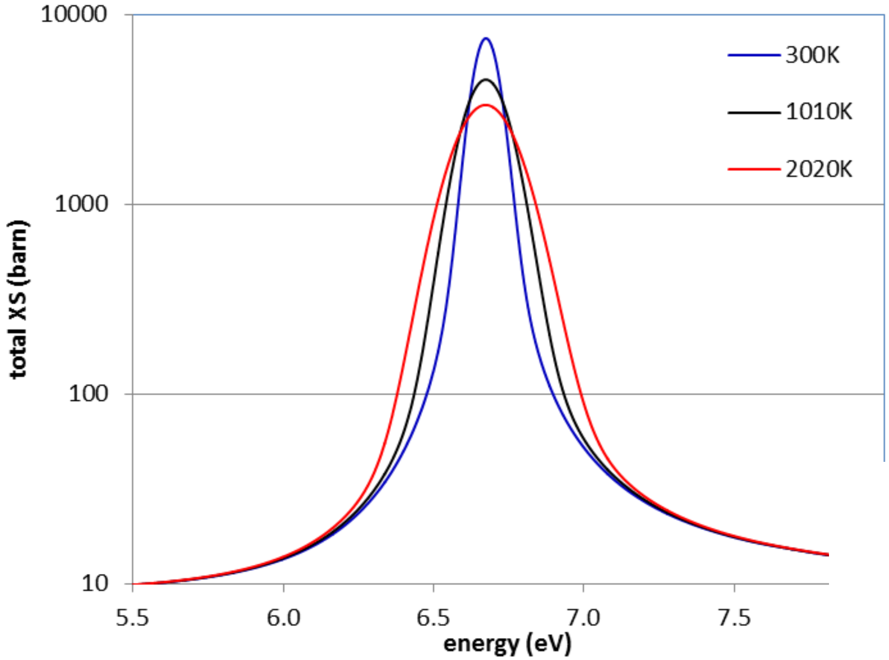
- If the velocity distribution of the target nuclei is isotropic, we have

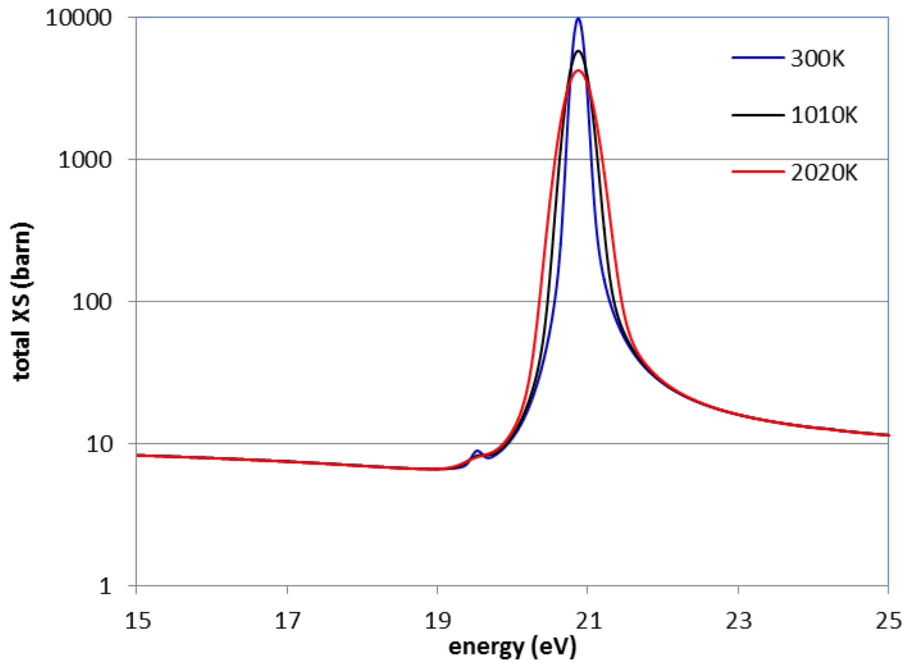
$$P(\vec{V})d\vec{V} = \frac{1}{4\pi}P(V)dVd\mu d\varphi$$

$$\sigma(v,T) = \frac{1}{2v}\int_{-1}^{1}d\mu\int_{0}^{\infty}dV[v_r\sigma(v_r,0)]P(V)$$



Doppler Broadened U-238 Total XS





Free Gas Model

- The relative speed can be written as

$$v_r = |\vec{v} - \vec{V}| = (v^2 + V^2 - 2vV\mu)^{1/2}$$

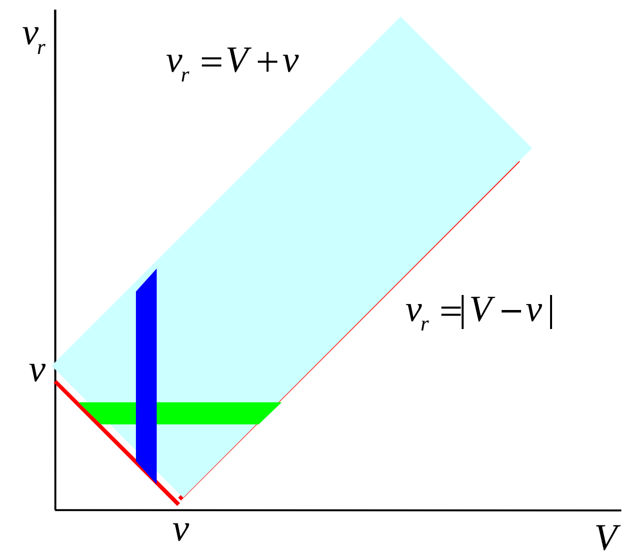
- The Jacobian transformation from the cosine of scattering angle to the relative speed is given by

$$d\mu = v_r dv_r / vV$$

- Changing the integration variable from μ to v_r, we have

$$\sigma(v,T) = \frac{1}{2v^2}\int_{|v-V|}^{v+V}[v_r\sigma(v_r,0)]v_r dv_r \int_0^\infty P(V)\frac{dV}{V}$$

$$= \frac{1}{2v^2}\int_0^\infty [v_r\sigma(v_r,0)]v_r dv_r \int_{|v-v_r|}^{v+v_r} P(V)\frac{dV}{V}$$



- For the Maxwellian monatomic (or free) gas model, the distribution of nuclei speed is given by

$$P(V)dV = 4\pi\left(\frac{a}{\pi}\right)^{3/2} V^2 e^{-aV^2} dV; \quad a = \frac{M}{2kT}$$

Doppler Broadened Cross Section

- The Doppler broadened cross section is obtained as

$$\sigma(v,T) = \frac{2a^{3/2}}{\sqrt{\pi}v^2}\int_0^\infty [v_r\sigma(v_r,0)]v_r dv_r \int_{|v-v_r|}^{v+v_r} Ve^{-aV^2}dV$$

$$= \frac{a^{1/2}}{\sqrt{\pi}v^2}\int_0^\infty [v_r\sigma(v_r,0)]v_r \left[e^{-a(v-v_r)^2} - e^{-a(v+v_r)^2}\right]dv_r$$

- This can be rewritten in terms of energy as

$$\sigma(E,T) = \frac{\alpha^{1/2}}{2\sqrt{\pi E}}\int_0^\infty [\sqrt{E_r}\sigma(E_r,0)]\left[e^{-\alpha(\sqrt{E}-\sqrt{E_r})^2} - e^{-\alpha(\sqrt{E}+\sqrt{E_r})^2}\right]dE_r$$

$$E = \frac{1}{2}mv^2; \quad E_r = \frac{1}{2}mv_r^2; \quad \alpha = \frac{2a}{m} = \frac{M}{mkT} = \frac{A}{kT}$$

- For large $\alpha\sqrt{EE_r}$ $(\sim AE/kT)$, the second exponential can be ignored, compared to the first

$$\sigma(E,T) = \frac{\alpha^{1/2}}{2\sqrt{\pi E}}\int_0^\infty [\sqrt{E_r}\sigma(E_r,0)]e^{-\alpha(\sqrt{E}-\sqrt{E_r})^2}dE_r$$

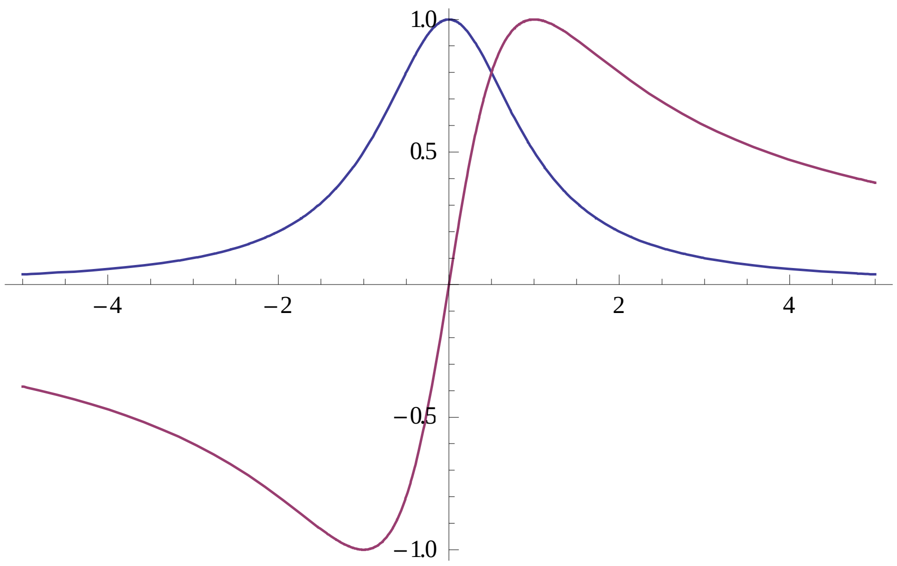
Single Level Breit-Wigner Formula

- Doppler broadened cross section

$$\sigma(E,T) = \frac{1}{\Delta\sqrt{\pi E}} \int_{-\infty}^{\infty} [\sqrt{E_r}\,\sigma(E_r)] e^{-[(E_r - E)/\Delta]^2} dE_r$$

$$\Delta = \left(\frac{4kTE}{A}\right)^{1/2} \quad \text{(Doppler width)}$$

- Single level Breit-Wigner formula



$$\sigma_a(E_r) \approx \sigma_0 \frac{\Gamma_a}{\Gamma} \frac{1}{1+w^2} \quad (a = \gamma, f)$$

$$\sigma_n(E_r) \approx 4\pi a^2 + \sigma_0 \frac{\Gamma_n}{\Gamma} \frac{1}{1+w^2} + \sigma_0 ka \frac{2w}{1+w^2}$$

$$w = \frac{E_r - E_0}{\Gamma / 2}$$

$$\sigma_0 = \frac{4\pi}{k^2} g_J \frac{\Gamma_n}{\Gamma} \text{ (resonance peak)}, \quad k = \frac{2\pi}{\lambda} \text{ (wave number)}$$

Doppler Broadened Line Shape Functions

- Doppler broadened cross sections

$$\sigma_a(E,T)=\sigma_0\frac{\Gamma_a}{\Gamma}\sqrt{\frac{E_0}{E}}\psi(x,\xi),\quad (a=\gamma,f)$$

$$\sigma_n(E,T)=4\pi a^2+\sigma_0\frac{\Gamma_n}{\Gamma}\psi(x,\xi)+2\sigma_0 ka\chi(x,\xi)$$

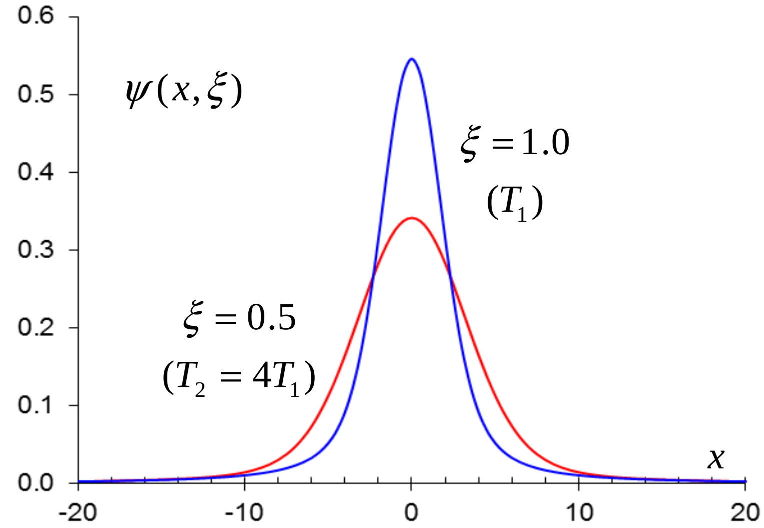
$$x=\frac{E-E_0}{\Gamma/2},\quad \xi=\frac{\Gamma}{\Delta},\quad \sqrt{\frac{E_0}{E}}\simeq 1$$

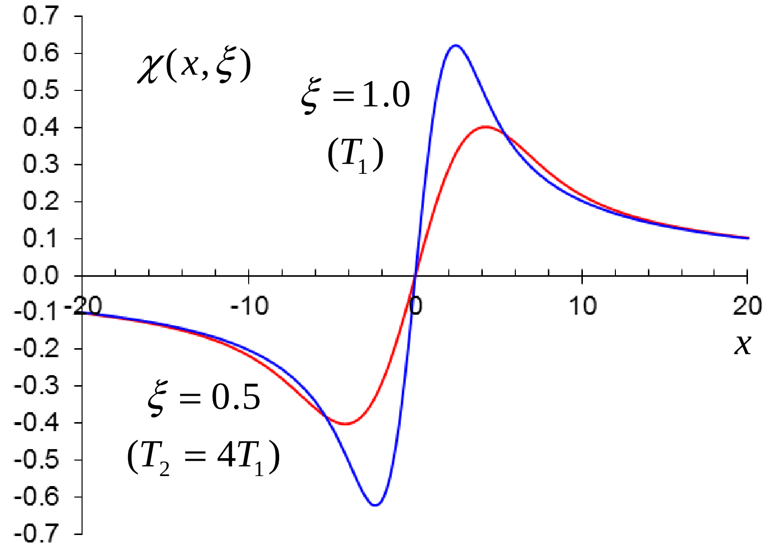
- Symmetric and anti-symmetric Doppler broadened line shape functions

$$\psi(x,\xi)=\frac{\xi}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{1}{1+w^2}\exp\left[-\frac{\xi^2}{4}(x-w)^2\right]dw$$

$$\chi(x,\xi)=\frac{\xi}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{w}{1+w^2}\exp\left[-\frac{\xi^2}{4}(x-w)^2\right]dw$$

$$\sqrt{E_r}\simeq\sqrt{E_0}$$





Psi-Chi Functions

- Doppler broadened line shape functions can be represented as convolution integrals of Lorentzian *L* and Gaussian kernel *G* as

$$\psi(x,\xi)=\int_{-\infty}^{\infty}L(w)G(x-w;\xi)dw=L(x)*G(x,\xi)$$

$$\chi(x,\xi)=\int_{-\infty}^{\infty}2wL(w)G(x-w,\xi)dw=[2xL(x)]*G(x,\xi)$$

$$L(x)=\frac{1}{1+x^2}\ \text{(natural line shape)},\quad G(x,\xi)=\frac{\xi}{2\sqrt{\pi}}\exp\left(-\frac{\xi^2}{4}x^2\right)\ \text{(pure Doppler shape)}$$

- At a very low temperature ($\xi=\Gamma_t/\Delta>>1$), the Doppler broadened line shapes reduce to the natural line shapes

$$\lim_{\xi\to\infty}\psi(x,\xi)=\int_{-\infty}^{\infty}L(w)\delta(x-w)dw=L(x),\quad \lim_{\xi\to\infty}\psi(x,\xi)=\int_{-\infty}^{\infty}2wL(w)\delta(x-w)dw=2xL(x)$$

- At a very high temperature ($\xi=\Gamma_t/\Delta<<1$), the Doppler broadened line shapes reduce to the pure Doppler shapes

$$\lim_{\xi\to 0}\psi(x,\xi)=\frac{\xi\sqrt{\pi}}{2}\int_{-\infty}^{\infty}\delta(t)\exp[-\frac{1}{4}(\xi x-t)^2]dt=\pi G(x,\xi)$$

$$\lim_{\xi\to 0}\chi(x,\xi)=\sqrt{\pi}\int_{-\infty}^{\infty}\delta(t)t\exp[-\frac{1}{4}(\xi x-t)^2]dt=0$$

Self-Shielding Factor

- Group-averaged cross section is computed

$$\sigma_{xg}^{i}(T,\sigma_b)=\frac{\int_g \sigma_x^i(E,T)\phi_i(E,T,\sigma_b)dE}{\int_g \phi_i(E,T,\sigma_b)dE}=\frac{\int_g \sigma_x^i(E,T)C(E)/[\sigma_b+\sigma_t^i(E,T)]dE}{\int_g C(E)/[\sigma_b+\sigma_t^i(E,T)]dE}$$

- Traditional J-integral approach for SLBW resonances

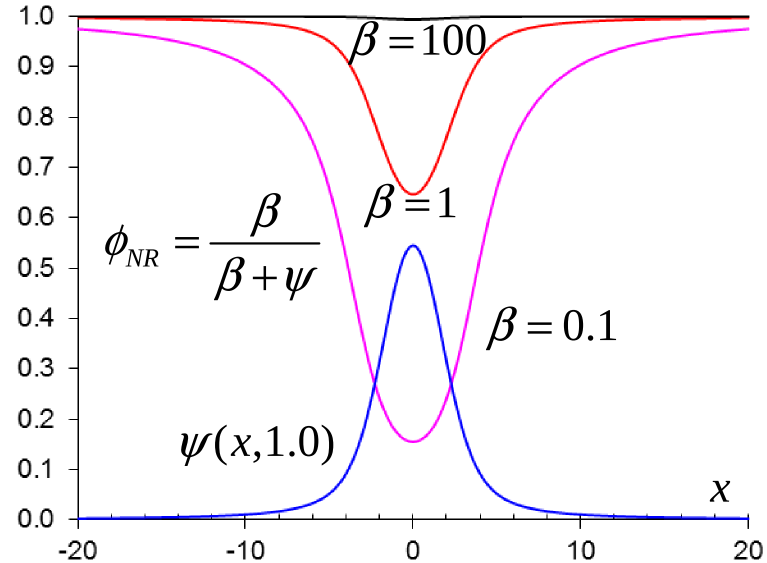
$$I_k^i(T,\sigma_b)=C\int\frac{\sigma_t^i(E,T)}{\sigma_b+\sigma_t^i(E,T)}dE=C\frac{\Gamma}{2}\int\frac{\sigma_0^i\psi(x,\xi)}{\sigma_b+\sigma_0^i\psi(x,\xi)}dx=C\Gamma J(\beta,\xi)$$

$$J(\xi,\beta)=\int_0^\infty\frac{\psi(x,\xi)}{\beta+\psi(x,\xi)}dx,\quad \beta=\frac{\sigma_b}{\sigma_0^i}$$

$$\lim_{\beta\to\infty}J(\xi,\beta)=\frac{1}{2\beta}\int_{-\infty}^{\infty}\psi(x,\xi)dx=\frac{\pi}{2\beta}$$

- Self-shielding factor

$$f(T,\sigma_b)=\frac{\sigma(T,\sigma_b)}{\sigma(0,\infty)}\approx\frac{2\beta}{\pi}J(\xi,\beta)$$

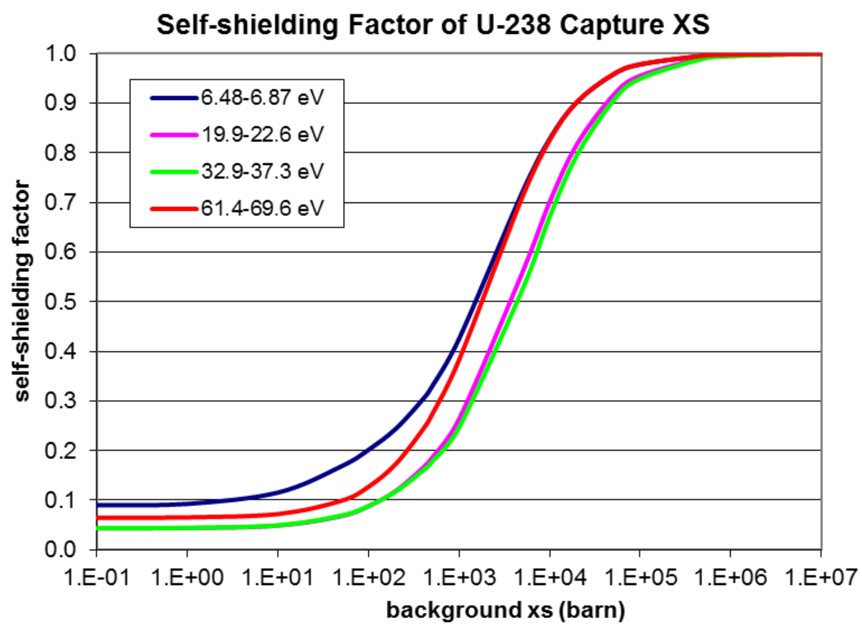


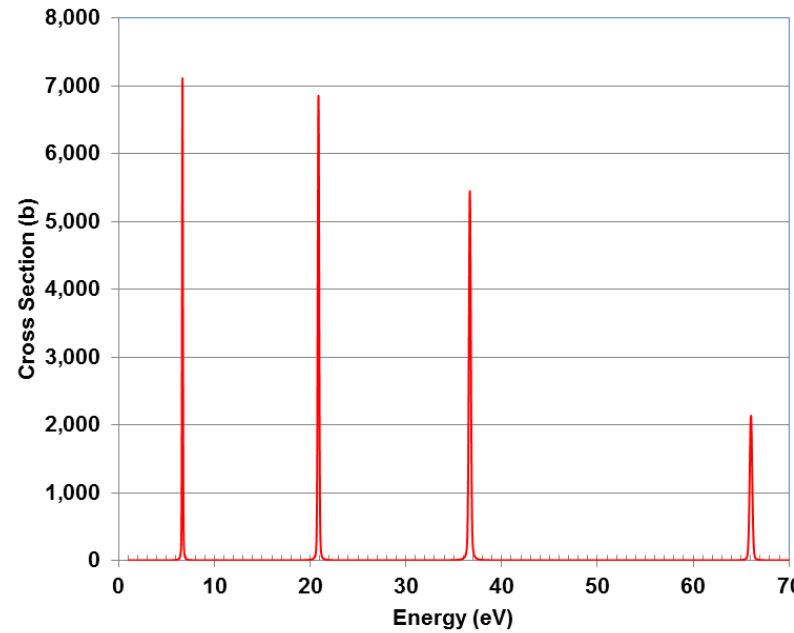
Background Cross Section Dependence

- Absorption decreases with increasing background cross section
  - Self-shielding factor increases

$$\frac{\partial J}{\partial \beta} = -\int_0^\infty \frac{\psi(x,\xi)}{[\beta+\psi(x,\xi)]^2} dx < 0$$

$$\frac{\partial f}{\partial \beta} = \frac{\partial}{\partial \beta}\left[\frac{2\beta}{\pi} J\right] = \frac{2}{\pi}\left[J + \beta \frac{\partial J}{\partial \beta}\right] = \int_0^\infty \left[\frac{\psi(x,\xi)}{\beta+\psi(x,\xi)}\right]^2 dx > 0$$



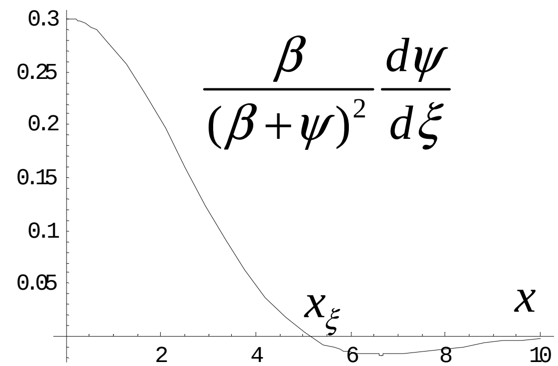


Temperature Dependence

- Absorption increases with increasing temperature
  - Reactivity decreases and thus the Doppler coefficient is negative

$$\frac{\partial J}{\partial \xi}=\int_0^{\infty}\frac{\beta}{[\beta+\psi(x,\xi)]^2}\frac{\partial \psi}{\partial \xi}dx=\int_0^{x_\xi} f(x)\frac{\partial \psi}{\partial \xi}dx-\int_{x_\xi}^{\infty} f(x)\left|\frac{\partial \psi}{\partial \xi}\right|dx$$

$$< f(x_\xi)\int_0^{x_\xi}\frac{\partial \psi}{\partial \xi}dx- f(x_\xi)\int_{x_\xi}^{\infty}\left|\frac{\partial \psi}{\partial \xi}\right|dx = f(x_\xi)\int_0^{\infty}\frac{\partial \psi}{\partial \xi}dx=0$$



$$\frac{\partial}{\partial T}J(\xi,\beta)=\frac{\partial J}{\partial \xi}\frac{d\xi}{dT}=-\frac{\Gamma_t}{2}\left(\frac{A}{4kE}\right)^{1/2}T^{-3/2}\frac{\partial J}{\partial \xi}>0$$

- Self-shielding effect decreases with increasing temperature
  - Self-shielding factor increases

$$\frac{\partial f}{\partial T}=\frac{\partial}{\partial \xi}\left[\frac{2\beta}{\pi}J\right]\frac{d\xi}{dT}=-\frac{\beta\Gamma_t}{\pi}\left(\frac{A}{4kE}\right)^{1/2}T^{-3/2}\frac{\partial J}{\partial \xi}>0$$

Low Temperature

- At a very low temperature, $\xi >> 1$ and the Doppler broadened line shape reduces to the natural line shape.
- In this case, the J function and self-shielding factor become

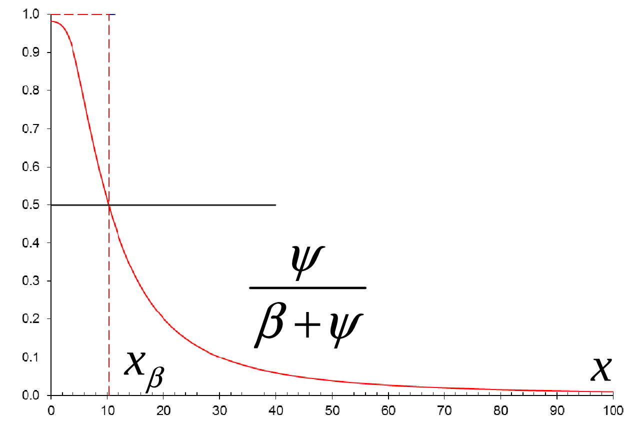
$$\lim_{\xi \to \infty} J(\xi,\beta) = \int_0^\infty \frac{L(x)}{\beta + L(x)} dx = \frac{1}{\beta} \int_0^\infty \frac{dx}{(1+\beta^{-1}) + x^2} = \frac{\pi}{2\beta(1+\beta^{-1})^{1/2}}$$

$$f(\infty,\beta) = \left(\frac{\beta}{1+\beta}\right)^{1/2} \approx \begin{cases} \sqrt{\beta} & \text{for small } \beta \\ 1 - \dfrac{1}{2\beta} & \text{for large } \beta \end{cases}$$

- This eliminates the temperature dependence from the self-shielding factor.
  - The resonance is un-broadened and has its maximum height.
  - This leads to the strongest self-shielding and thus to the smallest value of the self-shielding factor

High Resonances at Low Energies

- For high resonances ($\beta << 1$) at low energies, the integrand of the J function is close to 1 for the high part of the resonance. The integrand decreases on the resonance wings, becomes equal to 1/2 where $\psi(x_\beta, \xi) = \beta$, and vanishes asymptotically.
  - The J integral can be approximated by the area of the rectangle of height 1 and width x_β
  - x_β can be determined from the approximation for dominating Doppler broadening



$$\beta = \psi(x_\beta, \xi) \approx \frac{\sqrt{\pi}\xi}{2} \exp\left(-\frac{\xi^2}{4} x_\beta^2\right)$$

$$J(\xi, \beta) \approx x_\beta \approx \frac{2}{\xi}\sqrt{\ln\frac{\sqrt{\pi}\xi}{2\beta}} \propto \frac{1}{\xi} \propto \sqrt{T}, \quad \frac{\partial}{\partial T} J(\xi, \beta) \propto \frac{1}{\sqrt{T}}$$

- Since high resonances dominate in the 1/E spectrum of thermal reactors, the resonance integral in thermal reactors depends on temperature through a $\sqrt{T}$ term, and the Doppler coefficient has a temperature dependence as $1/\sqrt{T}$

Low Resonances at High Energies

- For resonances which are much lower than the potential cross section, $\beta >> \psi$ and hence the J function can be approximated as

$$J(\xi,\beta)=\int_0^\infty \frac{\psi}{\beta+\psi}dx=\frac{1}{\beta}\int_0^\infty \frac{\psi}{1+(\psi/\beta)}dx \approx \frac{1}{\beta}\int_0^\infty \psi\left(1-\frac{\psi}{\beta}\right)dx=\frac{\pi}{2\beta}-\frac{\pi}{4\beta^2}\psi(0,\sqrt{2}\xi)$$

- For low resonances at high energies, Doppler broadening generally dominates ($\xi >> 1$) and hence

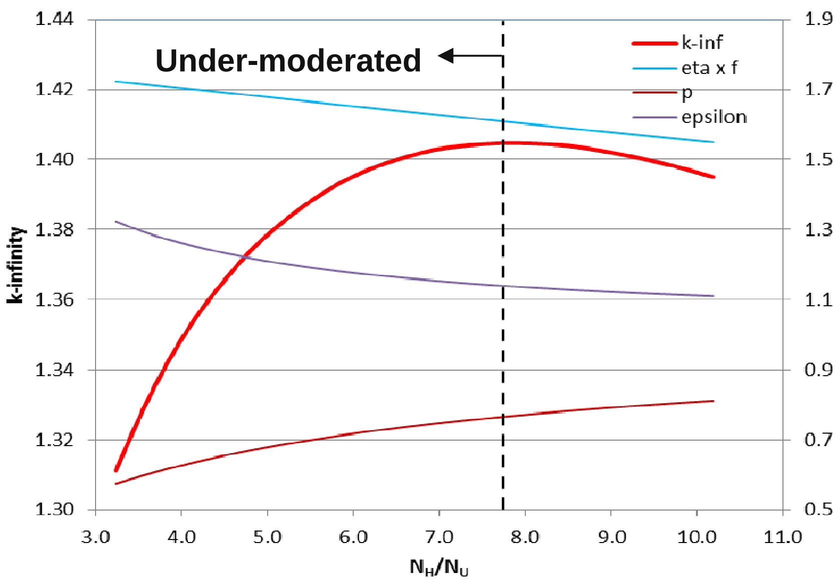
$$\psi(x,\xi)\approx \pi G(x,\xi)=\frac{\xi\sqrt{\pi}}{2}\exp\left(-\frac{1}{4}\xi^2x^2\right) \quad \Rightarrow \quad \psi(0,\sqrt{2}\xi)\approx\sqrt{\frac{\pi}{2}}\xi=\frac{\sqrt{\pi}\Gamma_t}{\sqrt{2}}\left(\frac{A}{4kE}\right)^{1/2}\frac{1}{\sqrt{T}}$$

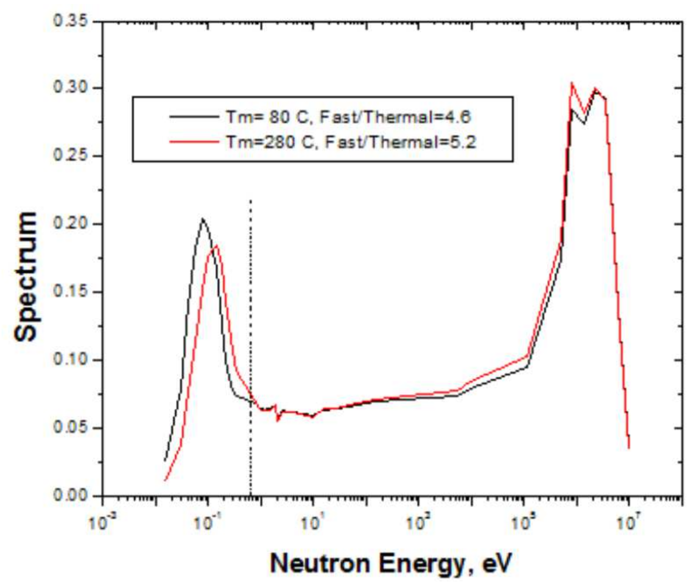
$$\frac{\partial}{\partial T}J(\xi,\beta)\approx\left(\frac{\pi}{2}\right)^{3/2}\frac{\Gamma_t}{4}\left(\frac{A_i}{4kE}\right)^{1/2}\left(\frac{N_i\sigma_m^k}{\Sigma_p}\right)^2 T^{-3/2}$$

  - In the very hard spectrum of small fast reactors, the temperature dependence is determined by the $T^{-3/2}$ contributions of the high energy resonances.
  - In the relatively soft neutron spectra of large ceramic fueled fast reactors, it is between the two limiting cases of high resonances ($T^{-3/2}$) and low resonances ($T^{-1/2}$).
  - A simple analytical evaluation of the medium height resonances is not feasible. A numerical evaluation of the high, medium, and low resonance contributions to the Doppler coefficient in large fast reactors yields an approximate proportionality to T^{-1}

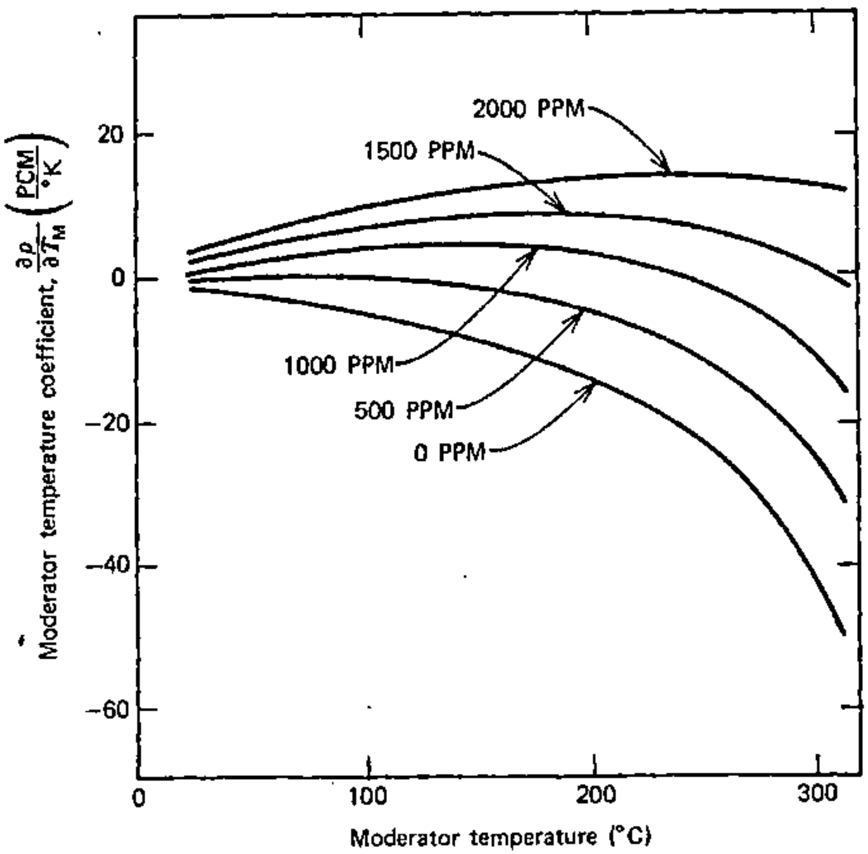
Moderator Effects

- Change in moderator temperature induces changes in moderator density and scattering kernel
- Fuel-to-moderator ratio in under-moderated region for negative MTC
  - $T_m \uparrow \;\Rightarrow\; d_m \downarrow \;\Rightarrow\;$ spectrum hardening $\;\Rightarrow\;$ resonance absorption $\uparrow \;\Rightarrow\; \rho \downarrow$
  - In PWR, $d_m \downarrow \;\Rightarrow\;$ boron concentration $\downarrow \;\Rightarrow\; \rho \uparrow$
    - *MTC becomes less negative as boron concentration increases*
    - *Boron concentration is limited since MTC can be positive at very high concentration (2500 ppm)*

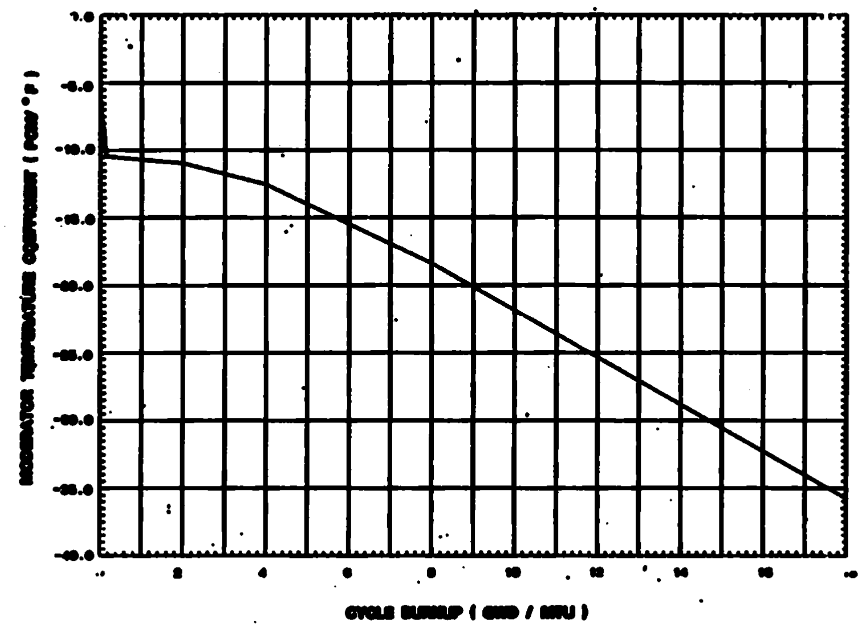




Moderator Temperature Coefficient of PWR

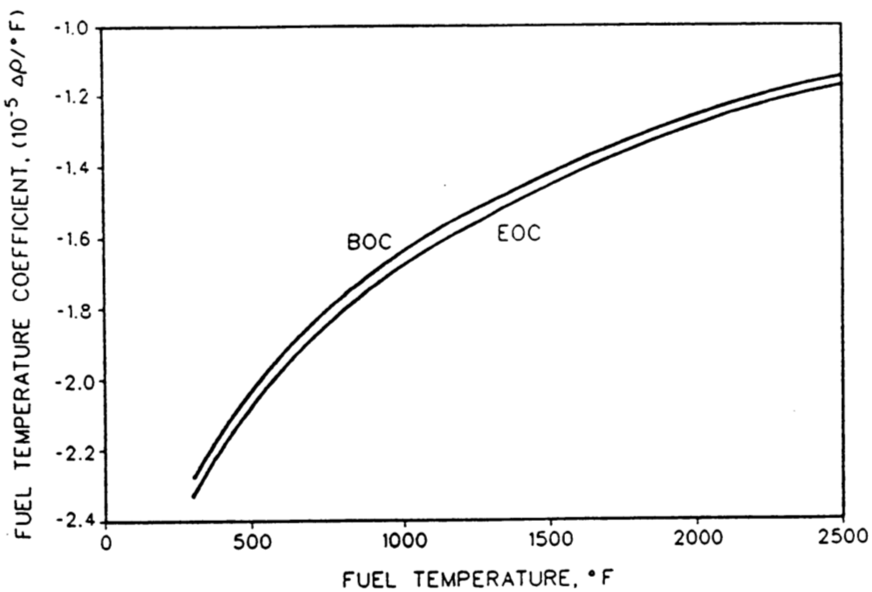


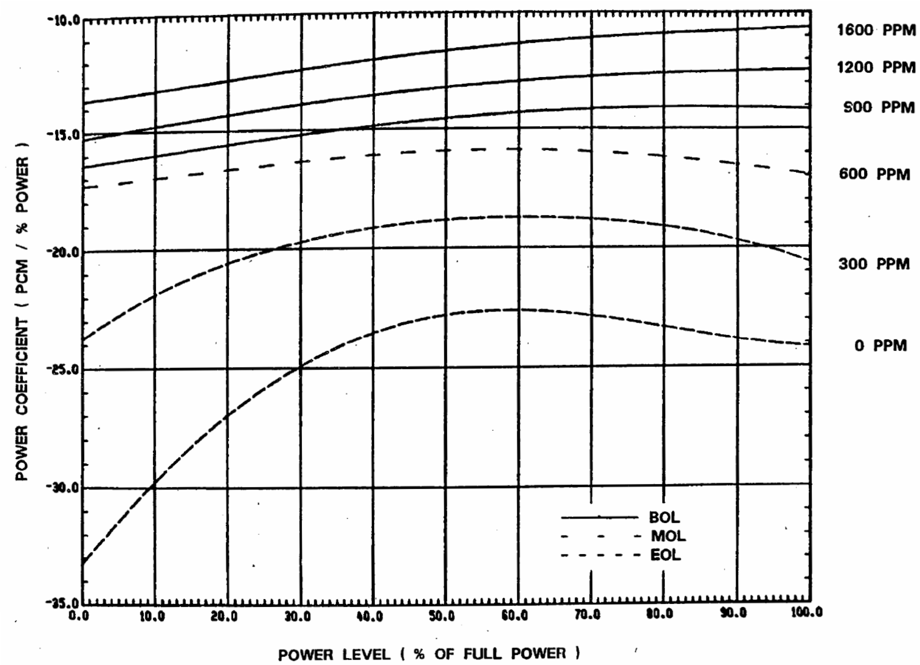
- MTC becomes more positive with boron concentration because of larger reduction in poison content from reduced density (increased temperature)



- MTC becomes more negative with burnup primarily because of the reduction in boron concentration with burnup
- Large negative value would be limiting for a cold water injection incidence
- MTC becomes more negative with control rod insertion because of hardened neutron spectrum and increased leakage

Fuel Temperature and Power Coefficients of PWR

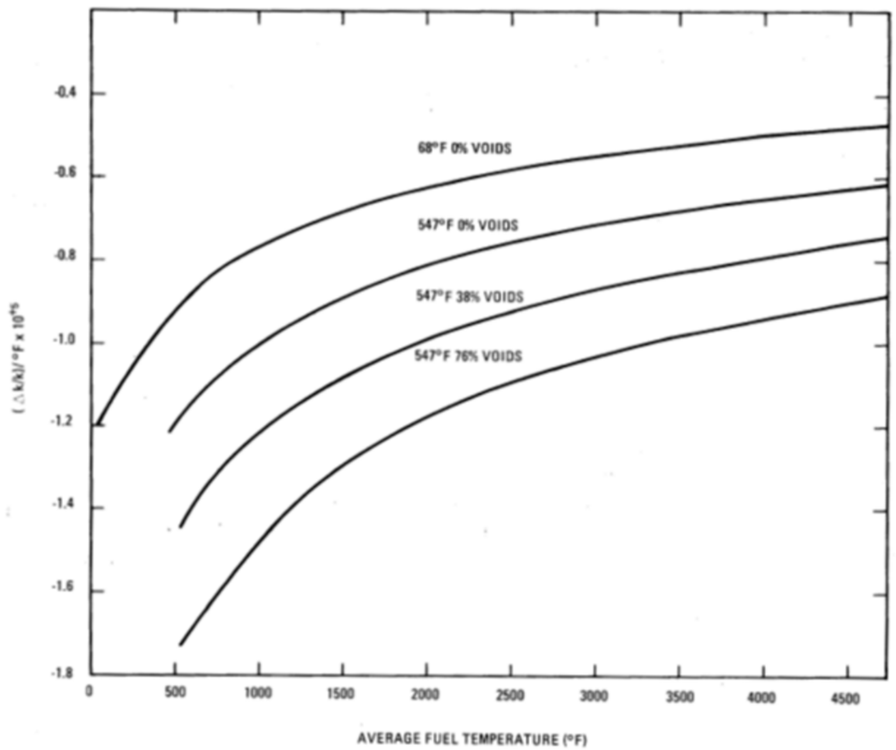


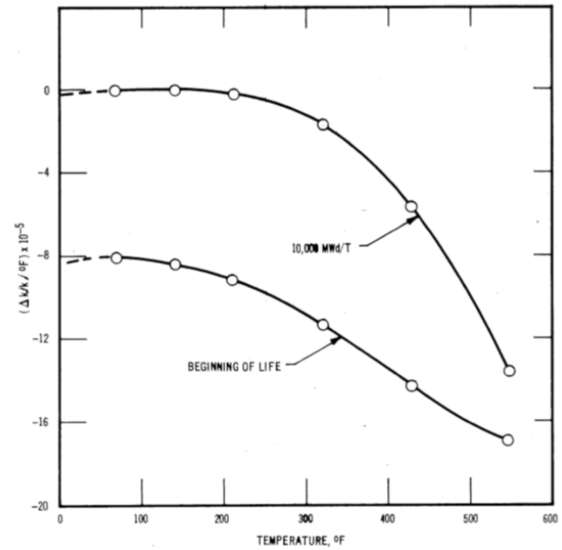


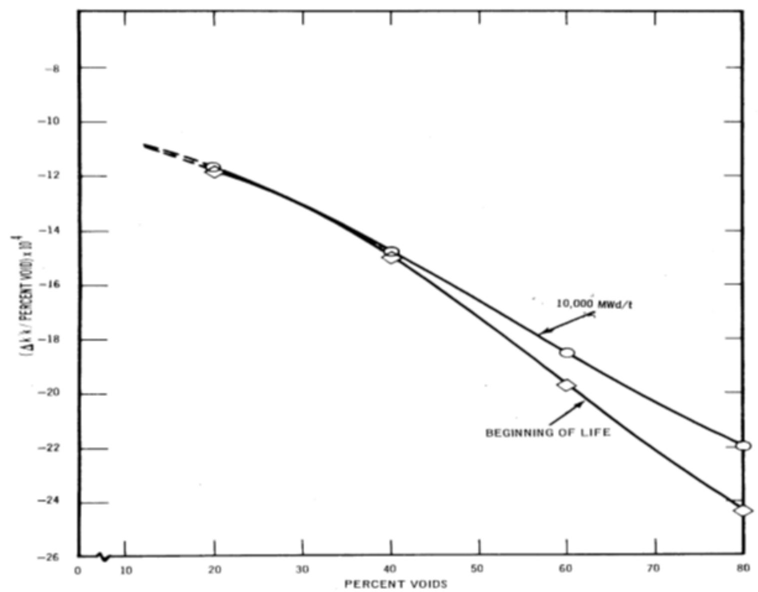
- Magnitude of fuel temperature coefficient decreases with increasing fuel temperature
- Becomes more negative with burnup because of Pu-240 contribution

- Power reactivity coefficient must be negative under all operating conditions
- Coefficient becomes more negative with burnup because of the MTC contribution

Reactivity Feedback Coefficients of BWR







- Coolant temperature coefficient is valid in the core heat-up range
- Void coefficient is valid in the power range of operation

Reactivity Feedback Coefficients of SFR

- The reactivity coefficients and kinetic parameters further define the physics of system
  - Response to a variety of perturbations
- Feedback coefficients are computed for a specific design (geometric and material) configuration
  - Typically evaluated for BOEC and EOEC compositions
- Typical set of whole-core reactivity coefficients
  - Delayed neutron fraction and prompt neutron lifetime
  - Coolant density coefficient and void worth
  - Fuel and structural Doppler coefficient
  - Fuel and structural worth distributions
  - Axial expansion
  - Radial expansion
  - Control rod driveline expansion
- Hummel and Okrent – *Reactivity Coefficients in Large Fast Power Reactors, ANS, 1970* is a good reference for underlying physics

Neutron Balances of Radial and Axial Expansions

	Base Case	Radial Expansion		Axial Expansion	
	balance	balance	Δρ (%)	balance	Δρ (%)
Fission source	100.00	100.00		100.00	
(n,2n) source	0.18	0.18		0.18	
Absorption	68.89	68.93	-0.04	69.06	-0.17
Leakage	31.54	32.16	**-0.63**	31.69	**-0.16**
Radial	17.49	17.72	-0.23	17.65	-0.15
Axial	14.05	14.45	-0.40	14.04	0.01
Sum			-0.67		-0.33

- To first order, radial expansion is an axial leakage effect, and axial expansion is a radial leakage effect
- Because the height is the shorter dimension (more axial than radial leakage), the radial expansion coefficient is more negative
- Axial expansion also give absorption effect from control rod insertion

Coolant Density and Void Coefficients

- Spectral effect
  - Reduced moderation as sodium density decreases
  - In fast regime, this is a positive reactivity effect
    - *From Pu-239 excess neutrons and threshold fission effects*
- Leakage effect
  - Sodium density reduction allows more neutron leakage
  - This is a negative reactivity effect in the peripheral regions
- Capture effect
  - Sodium density decrease results in less sodium capture
  - This is a relatively minor effect
- Coolant density coefficient is computed by first-order perturbation theory to evaluate small density (temperature variation) impacts
- Void worth is evaluated using exact perturbation theory to account for shift in flux distribution for voided condition
  - In general, 10% more positive than the first-order density worth

Sodium Void Worth by Components ($)

		Capture	Spectral	Leakage	Total
1000 MWt ABR (startup metal core)	BOC	0.5	9.1	-5.2	4.4
	EOC	0.5	9.9	-5.5	4.9
250 MWt ABTR (startup metal core)	BOC	0.4	6.4	-5.8	1.0
	EOC	0.4	6.6	-5.8	1.1

- Flowing sodium was voided in active and above-core regions
- Void worth tends to increase with core size
- However, difficult to conceive transient situations that reach boiling
  - Low pressure system
  - More than 300^{o}C margin to boiling
  - Other feedbacks are negative, inhibiting this temperature increase
- Extensive report on void worth reduction – Khalil and Hill, NS&E, 109 (1995)