NUCL 511
Nuclear Reactor Theory and Kinetics

Lecture Note 5

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Exact Point Kinetics Equation (PKE)

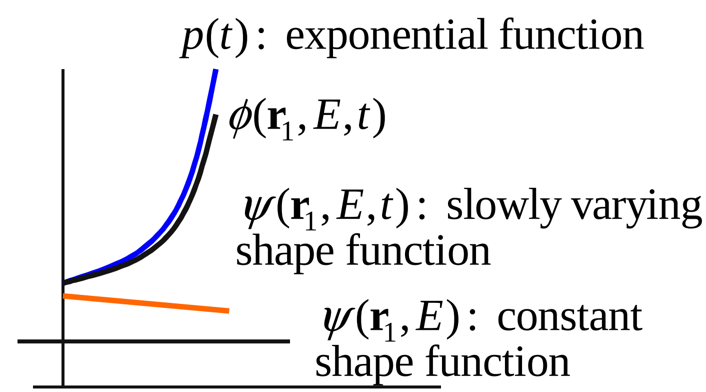
- Point kinetics equation was previously derived by assuming that the time dependency of flux is separable from its space and energy dependence

$$\phi(r,E,t) = p(t)\psi(r,E)$$

- A formally exact point kinetics equation can be derived using the factorization of the flux into a purely time-dependent amplitude function and a slowly varying shape function

$$\phi(r,E,t) = p(t)\psi(r,E,t)$$

  - Factorization is not an approximation such as the separation
  - It splits one function into two
  - One additional equation is needed to make the factorization unique
  - One convenient way is to hold a weighted integral of the shape function in space and energy constant in time



$$\frac{\partial \phi}{\partial t} = \psi \frac{dp}{dt} + p\frac{\partial \psi}{\partial t}$$

$$\int dV \int dE \frac{w(r,E)}{v(E)} \psi(r,E,t) = K_0$$

Time-Dependent Neutron Diffusion Equation

- Time-dependent neutron diffusion equation

$$\frac{1}{v(E)}\frac{\partial \phi(r,E,t)}{\partial t} = (\mathbf{F}_p - \mathbf{M})\phi(r,E,t) + S_d(r,E,t) + S(r,E,t)$$

  - Production rates of prompt fission neutrons

$$\mathbf{F}_p\phi(r,E,t) = \sum_i N_i(r,t)\chi_p^i(E)\int_0^\infty \nu_p^i(E')\sigma_f^i(E')\phi(r,E',t)dE'$$

$$= \chi_p(E)\int_0^\infty \nu_p\Sigma_f(r,E',t)\phi(r,E',t)dE'$$

  - Loss rate by reaction and leakage & scattering source

$$\mathbf{M}\phi(r,E,t) = -\nabla\cdot D(r,E,t)\nabla\phi(r,E,t) + \Sigma_t(r,E,t)\phi(r,E,t)$$

$$-\int_0^\infty \Sigma_s(r,E'\rightarrow E,t)\phi(r,E',t)dE'$$

  - Delayed neutron source

$$S_d(r,E,t) = \sum_k \chi_{dk}(E)\lambda_k C_k(r,t)$$

- Precursor balance equation

$$\frac{\partial C_k(r,t)}{\partial t} = \int_0^\infty \nu_{dk}\Sigma_f(r,E',t)\phi(r,E',t)dE' - \lambda_k C_k(r,t)$$

Time-Dependent Neutron Transport Equation

- Time-dependent neutron transport equation

$$\frac{1}{v(E)}\frac{\partial\psi(r,E,\Omega,t)}{\partial t}=(\mathbf{F}_p-\mathbf{M})\psi(r,E,\Omega,t)+S_d(r,E,\Omega,t)+S(r,E,\Omega,t)$$

  – Production rates of prompt fission neutrons

$$\mathbf{F}_p\psi(r,E,\Omega,t)=(4\pi)^{-1}\sum_i N_i(r,t)\chi_p^i(E)\int_0^\infty \nu_p^i(E')\sigma_f^i(E')\phi(r,E',t)dE'$$

$$=(4\pi)^{-1}\chi_p(E)\int_0^\infty \nu_p\Sigma_f(r,E',t)\phi(r,E',t)dE'$$

  – Loss rate by reaction and leakage & scattering source

$$\mathbf{M}\phi(r,E,t)=\Omega\cdot\nabla\psi(r,E,\Omega,t)+\Sigma_t(r,E,t)\psi(r,E,\Omega,t)$$

$$-\int dE'\int d\Omega'\Sigma_s(r,E'\to E,\Omega'\to\Omega,t)\psi(r,E',\Omega',t)$$

  – Delayed neutron source

$$S_d(r,E,\Omega,t)=(4\pi)^{-1}\sum_k \chi_{dk}(E)\lambda_k C_k(r,t)$$

- Precursor balance equation

$$\frac{\partial C_k(r,t)}{\partial t}=\int_0^\infty \nu_{dk}\Sigma_f(r,E',t)\phi(r,E',t)dE'-\lambda_k C_k(r,t)$$

Fission Operators

- Prompt fission neutron operator

$$\mathbf{F}_p\phi(r,E,t)=\chi_p(E)\int_0^\infty \nu_p\Sigma_f(r,E',t)\phi(r,E',t)dE'$$

$$\nu_p\Sigma_f=\sum_i \nu_p^i\Sigma_f^i,\quad \chi_p=\sum_i \chi_p^i(E)\int_0^\infty \nu_p^i\Sigma_f^i(E')\phi(E')dE'/\int_0^\infty \nu_p\Sigma_f(E')\phi(E')dE'$$

(The space and time variables are omitted for simplicity.)

- Quasi-stationary delayed fission neutron operator
  - This a source of delayed neutrons that would be produced in a stationary reactor with fission cross sections and neutron flux as they exist at time t
  - This is not the actual delayed neutron source since no time delay is included

$$\mathbf{F}_{dk}\phi(r,E,t)=\chi_{dk}(E)\int_0^\infty \nu_{dk}\Sigma_f(r,E',t)\phi(r,E',t)dE'$$

$$\nu_{dk}\Sigma_f=\sum_i \nu_{dk}^i\Sigma_f^i,\quad \chi_{dk}=\sum_i \chi_{dk}^i(E)\int_0^\infty \nu_{dk}^i\Sigma_f^i(E')\phi(E')dE'/\int_0^\infty \nu_{dk}\Sigma_f(E')\phi(E')dE'$$

$$\mathbf{F}_d\phi(r,E,t)=\sum_k \mathbf{F}_{dk}\phi(r,E,t)=\chi_d(E)\int_0^\infty \nu_d\Sigma_f(r,E',t)\phi(r,E',t)dE'$$

$$\nu_d=\sum_k \nu_{dk},\quad \chi_d=\sum_k \chi_{dk}(E)\int_0^\infty \nu_{dk}\Sigma_f(E')\phi(E')dE'/\int_0^\infty \nu_d\Sigma_f(E')\phi(E')dE'$$

Fission Operators

- Total fission neutron operator

$$\mathbf{F}\phi(r,E,t)=\mathbf{F}_p\phi(r,E,t)+\mathbf{F}_d\phi(r,E,t)$$

$$=\chi_p(E)\int_0^\infty \nu_p\Sigma_f(r,E',t)\phi(r,E',t)dE'+\chi_d(E)\int_0^\infty \nu_d\Sigma_f(r,E',t)\phi(r,E',t)dE'$$

$$=\chi(E)\int_0^\infty \nu\Sigma_f(r,E',t)\phi(r,E',t)dE'$$

$$\nu=\nu_p+\nu_d$$

$$\chi=\frac{\chi_p(E)\int_0^\infty \nu_p\Sigma_f(r,E',t)\phi(r,E',t)dE'+\chi_d(E)\int_0^\infty \nu_d\Sigma_f(r,E',t)\phi(r,E',t)dE'}{\int_0^\infty \nu(E')\Sigma_f(E')\phi(r,E',t)dE'}$$

Constraints on Shape Function

- Weighted integration of time-dependent neutron balance equations

$$\frac{1}{v(E)}\frac{\partial\phi(r,E,t)}{\partial t}=(\mathbf{F}-\mathbf{M}-\mathbf{F}_d)\phi(r,E,t)+S_d(r,E,t)+S(r,E,t)$$

$$\left\langle w,\frac{1}{v}\frac{\partial\phi}{\partial t}\right\rangle=\langle w,(\mathbf{F}-\mathbf{M})\phi\rangle-\langle w,\mathbf{F}_d\phi\rangle+\langle w,S_d\rangle+\langle w,S\rangle$$

$$\langle w(r,E),f(r,E,t)\rangle=\int dV\int dE w(r,E)f(r,E,t)$$

- Factorization yields two time derivatives

$$\frac{\partial\phi}{\partial t}=\psi\frac{dp}{dt}+p\frac{\partial\psi}{\partial t}\Rightarrow\left\langle w,\frac{1}{v}\frac{\partial\phi}{\partial t}\right\rangle=\left\langle w,\frac{\psi}{v}\frac{dp}{dt}\right\rangle+\left\langle w,\frac{p}{v}\frac{\partial\psi}{\partial t}\right\rangle=\frac{dp}{dt}\left\langle w,\frac{\psi}{v}\right\rangle+p\frac{\partial}{\partial t}\left\langle w,\frac{\psi}{v}\right\rangle$$

  – Constrain the shape function to yield a unique factorization

$$K(t)=\left\langle w,\frac{\psi}{v}\right\rangle=K_0\ (\text{constant})\quad\Rightarrow\quad\left\langle w,\frac{1}{v}\frac{\partial\phi}{\partial t}\right\rangle=K_0\frac{dp(t)}{dt}$$

    - *The shape function itself is not constant over time*

- Choose the initial adjoint flux as the weighting function to minimize the reactivity error when the shape function is approximated later

$$\left\langle\phi_0^*,\frac{\psi}{v}\right\rangle=\int dV\int dE\frac{\phi_0^*(r,E)}{v(E)}\psi(r,E,t)=K_0$$

- *An un-normalized shape function is normalized to satisfy this constraint*

Exact PKE for Initially Critical Reactor

- Stationary solutions in a critical reactor

$$\frac{\partial C_k}{\partial t} = \int_0^\infty \nu_{dk}\Sigma_f \phi dE' - \lambda_k C_k = 0 \quad \Rightarrow \quad C_{k0} = \frac{1}{\lambda_k}\int_0^\infty \nu_{dk}\Sigma_f \phi dE'$$

$$\Rightarrow \quad S_{d0} = \sum_k \chi_{dk}\lambda_k C_{k0} = \sum_k \chi_{dk}\int_0^\infty \nu_{dk}\Sigma_f \phi_0 dE' = \sum_k \mathbf{F}_{dk0}\phi_0 = \mathbf{F}_{d0}\phi_0$$

$$\frac{1}{v}\frac{\partial \phi}{\partial t} = (\mathbf{F}_p - \mathbf{M})\phi + S_d = 0 \quad \Rightarrow \quad (\mathbf{F}_{p0} - \mathbf{M}_0)\phi_0 + S_{d0} = 0 \quad \Rightarrow \quad (\mathbf{F}_{p0} - \mathbf{M}_0)\phi_0 + \mathbf{F}_{d0}\phi_0 = 0$$

$$(\mathbf{F}_0 - \mathbf{M}_0)\phi_0 = 0, \quad (\mathbf{F}_0^* - \mathbf{M}_0^*)\phi_0^* = 0$$

- Importance-weighted neutron balance equation

$$\left\langle \phi_0^*, \frac{1}{v}\frac{\partial \phi}{\partial t} \right\rangle = \left\langle \phi_0^*, (\mathbf{F} - \mathbf{M})\phi \right\rangle - \left\langle \phi_0^*, \mathbf{F}_d \phi \right\rangle + \left\langle \phi_0^*, S_d \right\rangle$$

$$\phi = p(t)\psi(r,E,t) \quad \Rightarrow \quad K_0 \frac{dp(t)}{dt} = \left\langle \phi_0^*, (\mathbf{F} - \mathbf{M})\psi \right\rangle p(t) - \left\langle \phi_0^*, \mathbf{F}_d \psi \right\rangle p(t) + \left\langle \phi_0^*, S_d \right\rangle$$

$$\left\langle \phi_0^*, (\mathbf{F} - \mathbf{M})\phi \right\rangle = \left\langle \phi_0^*, (\mathbf{F}_0 - \mathbf{M}_0)\phi \right\rangle + \left\langle \phi_0^*, (\Delta\mathbf{F} - \Delta\mathbf{M})\phi \right\rangle = \left\langle \phi_0^*, (\Delta\mathbf{F} - \Delta\mathbf{M})\phi \right\rangle$$

$$\Rightarrow \quad K_0 \frac{dp(t)}{dt} = \left\langle \phi_0^*, (\Delta\mathbf{F} - \Delta\mathbf{M})\psi \right\rangle p(t) - \left\langle \phi_0^*, \mathbf{F}_d \psi \right\rangle p(t) + \left\langle \phi_0^*, S_d \right\rangle$$

Exact PKE for Initially Critical Reactor

- Importance-weighted neutron balance equation

$$K_0 \frac{dp(t)}{dt} = \left\langle \phi_0^*, (\Delta\mathbf{F} - \Delta\mathbf{M})\psi \right\rangle p(t) - \left\langle \phi_0^*, \mathbf{F}_d \psi \right\rangle p(t) + \left\langle \phi_0^*, S_d \right\rangle$$

  – Divide by the importance-weighted quasi-stationary source of fission neutrons, as produced by the flux shape function

$$F(t) = \left\langle \phi_0^*, \mathbf{F}\psi \right\rangle$$

$$\frac{K_0}{F(t)} \frac{dp(t)}{dt} = \frac{\left\langle \phi_0^*, (\Delta\mathbf{F} - \Delta\mathbf{M})\psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle} p(t) - \frac{\left\langle \phi_0^*, \mathbf{F}_d \psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle} p(t) + \frac{\left\langle \phi_0^*, S_d \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle}$$

  – Define kinetics parameters

$$\Lambda(t) = \frac{K_0}{F(t)} = \frac{\left\langle \phi_0^*, (1/v)\psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle}$$ **neutron generation time**

$$\rho(t) = \frac{\left\langle \phi_0^*, (\Delta\mathbf{F} - \Delta\mathbf{M})\psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle}$$ **reactivity**

$$\beta(t) = \frac{\left\langle \phi_0^*, \mathbf{F}_d \psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle} = \sum_k \beta_k(t), \quad \beta_k(t) = \frac{\left\langle \phi_0^*, \mathbf{F}_{dk} \psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle}$$ **delayed neutron fraction**

$$s_d(t) = \frac{\left\langle \phi_0^*, S_d \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle}$$ **delayed neutron source**

$$\frac{dp(t)}{dt} = \frac{\rho(t) - \beta(t)}{\Lambda(t)} p(t) + \frac{s_d(t)}{\Lambda(t)}$$

Exact PKE for Initially Critical Reactor

- Importance-weighted delayed neutron source and precursor concentration

$$s_d(t)=\frac{\langle \phi_0^*, S_d\rangle}{\langle \phi_0^*, \mathbf{F}\psi\rangle}=\sum_k \lambda_k \frac{\langle \phi_0^*, \chi_{dk} C_k\rangle}{F(t)}=\Lambda(t)\sum_k \lambda_k c_k(t) \qquad c_k(t)=\frac{\langle \phi_0^*, \chi_{dk} C_k\rangle}{F(t)\Lambda(t)}=\frac{\langle \phi_0^*, \chi_{dk} C_k\rangle}{K_0}$$

$$\frac{\partial C_k(r,t)}{\partial t}=\int_0^\infty \nu_{dk}\Sigma_f(r,E',t)\phi(r,E',t)dE'-\lambda_k C_k(r,t) \quad \text{(precursor balance equation)}$$

$$\frac{\partial}{\partial t}\langle \phi_0^*, \chi_{dk} C_k\rangle=\langle \phi_0^*, \mathbf{F}_{dk}\psi\rangle p(t)-\lambda_k\langle \phi_0^*, \chi_{dk} C_k\rangle \quad \Rightarrow \quad \frac{dc_k(t)}{dt}=\frac{\langle \phi_0^*, \mathbf{F}_{dk}\psi\rangle}{F(t)\Lambda(t)}p(t)-\lambda_k c_k(t)$$

$$\frac{dc_k(t)}{dt}=\frac{\beta_k(t)}{\Lambda(t)}p(t)-\lambda_k c_k(t)$$

- Importance-weighted reduced precursor concentration

$$\varsigma_k(t)=\Lambda_0 c_k(t)=\frac{F(t)\Lambda(t)}{F_0}c_k(t)=\frac{\langle \phi_0^*, \chi_{dk} C_k\rangle}{F_0}$$

$$\frac{d\varsigma_k(t)}{dt}=\frac{F(t)}{F_0}\beta_k(t)p(t)-\lambda_k\varsigma_k(t)$$

Exact PKE for Initially Critical Reactor

- Exact PKE with precursor concentration

$$\frac{dp(t)}{dt} = \frac{\rho(t)-\beta(t)}{\Lambda(t)} p(t) + \sum_k \lambda_k c_k(t)$$

$$\frac{dc_k(t)}{dt} = \frac{\beta_k(t)}{\Lambda(t)} p(t) - \lambda_k c_k(t), \quad k = 1, 2, \cdots, 6$$

initial conditions

$$p(0) = 1, \quad c_k(0) = \frac{\beta_{k0}}{\lambda_k \Lambda_0}, \quad k = 1, 2, \cdots, 6$$

stationary conditions

$$\rho(0) = 0$$

$$\frac{-\beta_0}{\Lambda_0} p(0) + \sum_k \lambda_k c_k(0) = 0$$

$$\frac{\beta_{k0}}{\Lambda_0} p(0) - \lambda_k c_k(0) = 0$$

- Exact PKE with reduced precursor concentration

$$\frac{dp(t)}{dt} = \frac{\rho(t)-\beta(t)}{\Lambda(t)} p(t) + \frac{1}{\Lambda_0} \sum_k \lambda_k \varsigma_k(t)$$

$$\frac{d\varsigma_k(t)}{dt} = \frac{F(t)}{F_0} \beta_k(t) p(t) - \lambda_k \varsigma_k(t), \quad k = 1, 2, \cdots, 6$$

initial conditions

$$p(0) = 1, \quad \varsigma_k(0) = \frac{\beta_{k0}}{\lambda_k}, \quad k = 1, 2, \cdots, 6$$

stationary conditions

$$\rho(0) = 0$$

$$\frac{-\beta_0}{\Lambda_0} p(0) + \frac{1}{\Lambda_0} \sum_k \lambda_k \varsigma_k(0) = 0$$

$$\beta_{k0} p(0) - \lambda_k \varsigma_k(0) = 0$$

Exact PKE for Initially Subcritical Reactor

- Time-dependent neutron balance equation and initial state

$$\frac{1}{v}\frac{\partial \psi}{\partial t} = (\mathbf{F}_p - \mathbf{M})\psi + S_d + S$$

$$(\mathbf{F}_0 - \mathbf{M}_0)\psi_0 + S_0 = 0$$

$$(\mathbf{F}_0^* - \mathbf{M}_0^*)\phi_0^* = 0$$

- Importance-weighted neutron balance equation

$$\left\langle \phi_0^*, \frac{1}{v}\frac{\partial \phi}{\partial t} \right\rangle = \left\langle \phi_0^*, (\mathbf{F} - \mathbf{M})\phi \right\rangle - \left\langle \phi_0^*, \mathbf{F}_d \phi \right\rangle + \left\langle \phi_0^*, S_d \right\rangle + \left\langle \phi_0^*, S \right\rangle$$

$$\frac{K_0}{F(t)}\frac{dp(t)}{dt} = \frac{\left\langle \phi_0^*, (\Delta\mathbf{F} - \Delta\mathbf{M})\psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle} p(t) - \frac{\left\langle \phi_0^*, \mathbf{F}_d \psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle} p(t) + \frac{\left\langle \phi_0^*, S_d \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle} + \frac{\left\langle \phi_0^*, S \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle}$$

$$\Lambda(t)\frac{dp(t)}{dt} = \rho(t)p(t) - \beta(t)p(t) + \sum_k \lambda_k \frac{\left\langle \phi_0^*, \chi_{dk} C_k \right\rangle}{F(t)} + \frac{\left\langle \phi_0^*, S \right\rangle}{F(t)}$$

$$\frac{dp(t)}{dt} = \frac{\rho(t) - \beta(t)}{\Lambda(t)} p(t) + \sum_k \lambda_k c_k(t) + \frac{1}{\Lambda(t)} s(t)$$

$$s(t) = \frac{\left\langle \phi_0^*, S \right\rangle}{F(t)}$$

Exact PKE for Initially Subcritical Reactor

- Exact PKE with precursor concentration

$$\frac{dp(t)}{dt}=\frac{\rho(t)-\beta(t)}{\Lambda(t)}p(t)+\sum_k \lambda_k c_k(t)+\frac{s(t)}{\Lambda(t)}$$

$$\frac{dc_k(t)}{dt}=\frac{\beta_k(t)}{\Lambda(t)}p(t)-\lambda_k c_k(t),\quad k=1,2,\cdots,6$$

stationary conditions

$$\frac{\rho_0-\beta_0}{\Lambda_0}p(0)+\sum_k \lambda_k c_k(0)+\frac{s_0}{\Lambda_0}=0$$

$$\frac{\beta_{k0}}{\Lambda_0}p(0)-\lambda_k c_k(0)=0$$

initial conditions

$$p(0)=\frac{s_0}{-\rho_0},\quad c_k(0)=\frac{s_0}{-\rho_0}\frac{\beta_{k0}}{\lambda_k\Lambda_0},\quad k=1,2,\cdots,6$$

- Exact PKE with reduced precursor concentration

$$\frac{dp(t)}{dt}=\frac{\rho(t)-\beta(t)}{\Lambda(t)}p(t)+\frac{1}{\Lambda_0}\sum_k \lambda_k \varsigma_k(t)+\frac{s(t)}{\Lambda(t)}$$

$$\frac{d\varsigma_k(t)}{dt}=\frac{F(t)}{F_0}\beta_k(t)p(t)-\lambda_k\varsigma_k(t),\quad k=1,2,\cdots,6$$

stationary conditions

$$\frac{\rho_0-\beta_0}{\Lambda_0}p(0)+\frac{1}{\Lambda_0}\sum_k \lambda_k \varsigma_k(0)+\frac{s_0}{\Lambda_0}=0$$

$$\beta_{k0}p(0)-\lambda_k\varsigma_k(0)=0$$

initial conditions

$$p(0)=\frac{s_0}{-\rho_0},\quad \varsigma_k(0)=\frac{s_0}{-\rho_0}\frac{\beta_{k0}}{\lambda_k},\quad k=1,2,\cdots,6$$

Point Kinetics Equations

- In the conventional point kinetics equation, the shape function, the importance-weighted quasi-stationary source of fission neutrons, and the delayed neutron source operator are approximated by those at t=0

$$\psi(\mathbf{r},E,t)=\phi_0(\mathbf{r},E)$$

$$F(t)=<\phi_0^*,F\psi>\cong<\phi_0^*,F\phi_0>$$

$$\cong<\phi_0^*,F_0\phi_0(\mathbf{r},E)>=F_0$$

$$<\phi_0^*,\mathbf{F}_k\psi>=<\phi_0^*,\mathbf{F}_{k0}\phi_0>$$

$$\Lambda=\Lambda_0=\frac{K_0}{F_0}$$

$$\rho(t)=\frac{1}{F_0}<\phi_0^*,(\Delta\mathbf{F}-\Delta\mathbf{M})\psi_0>$$

$$\beta_k=\beta_{k0}=\frac{1}{F_0}<\phi_0^*,\mathbf{F}_{k0}\psi_0>$$

$$\frac{dp(t)}{dt}=\frac{\rho(t)-\beta}{\Lambda}p(t)+\sum_k\lambda_k c_k(t)$$

$$\frac{dc_k(t)}{dt}=\frac{\beta_k}{\Lambda}p(t)-\lambda_k c_k(t),\quad k=1,2,\cdots,6$$

initial conditions

$$p(0)=1,\quad c_k(0)=\frac{\beta_k}{\lambda_k\Lambda},\quad k=1,2,\cdots,6$$

$$\frac{dp(t)}{dt}=\frac{\rho(t)-\beta}{\Lambda}p(t)+\frac{1}{\Lambda}\sum_k\lambda_k\varsigma_k(t)$$

$$\frac{d\varsigma_k(t)}{dt}=\beta_k p(t)-\lambda_k\varsigma_k(t),\quad k=1,2,\cdots,6$$

initial conditions

$$p(0)=1,\quad \varsigma_k(0)=\frac{\beta_k}{\lambda_k},\quad k=1,2,\cdots,6$$

Effective Delayed Neutron Fraction

- Effective delayed neutron fraction

Weak dependence on energy

$$\beta_k(t) = \frac{\left\langle \phi_0^*, \mathbf{F}_{dk}\psi \right\rangle}{\left\langle \phi_0^*, \mathbf{F}\psi \right\rangle} = \frac{\left\langle \phi_0^*(r,E), \chi_{dk}(E)\int_0^\infty \nu_{dk}\Sigma_f(r,E',t)\psi(r,E',t)dE' \right\rangle}{\left\langle \phi_0^*(r,E), \chi(E)\int_0^\infty \nu\Sigma_f(r,E',t)\psi(r,E',t)dE' \right\rangle}$$

$$\cong \frac{\left\langle \phi_0^*(r,E), \chi_{dk}(E)\nu_{dk}\int_0^\infty \Sigma_f(r,E',t)\psi(r,E',t)dE' \right\rangle}{\left\langle \phi_0^*(r,E), \chi(E)\overline{\nu}(r)\int_0^\infty \nu\Sigma_f(r,E',t)\psi(r,E',t)dE' \right\rangle}$$

Energy average

- If the adjoint separable as $\phi_0^*(r,E) = \phi_0^*(r)\varphi_0^*(E)$

$$\beta_k(t) = \frac{\int_0^\infty \varphi_0^*(E)\chi_{dk}(E)dE}{\int_0^\infty \varphi_0^*(E)\chi(E)dE} \times \frac{\nu_{dk}\int_V \phi_0^*(r)\int_0^\infty \Sigma_f(r,E',t)\psi(r,E',t)dE'dV}{\int_V \phi_0^*(r)\overline{\nu}(r)\int_0^\infty \Sigma_f(r,E',t)\psi(r,E',t)dE'dV} = \gamma\frac{\nu_{dk}}{\overline{\nu}}$$

$$\frac{\nu_{dk}}{\overline{\nu}} \approx \frac{\nu_{dk}}{\nu} = \beta_k^{phy}$$

$$\beta_k = \gamma\beta_k^{phy}, \quad \gamma \simeq \begin{cases} 1.05 \text{ for thermal reactors} \\ 0.85 \text{ for fast reactors} \end{cases}$$

Delayed Neutron Fractions and Decay Constants

- The precursor families are isotope-dependent, and thus the traditional six delay group equations are obtained by combining the contributions of all the fissionable isotopes to each delay neutron group
- The delayed neutron fraction of each of the six delay groups can be obtained by the simple summation

$$\beta_k = \sum_i \beta_{ki}$$

- The isotope-independent decay constants of six delay groups should be determined to accurately represent the stationary precursor concentrations
  - Stationary precursor concentration

$$c_k(t) = \sum_i c_{ki}(t) \quad \Rightarrow \quad \frac{p(t)}{\Lambda}\frac{\beta_k}{\lambda_k} = \frac{p(t)}{\Lambda}\sum_i \frac{\beta_{ki}}{\lambda_{ki}}$$

  - The decay constant of each of six delay groups can be determined as

$$\lambda_k = \frac{\beta_k}{\sum_i (\beta_{ki}/\lambda_{ki})} = \frac{\sum_i \beta_{ki}}{\sum_i (\beta_{ki}/\lambda_{ki})}$$