# NUCL 510 Nuclear Reactor Theory I

**Fall 2011**

## Homework #6

Due October 13

1. Suppose an angular flux in a slab geometry is given by

   $$\psi(z,\mu) = \phi_0(\cos Bz + A\mu \sin Bz).$$

   Here, $\mu$ is the cosine of the polar angle measured from the $z$ axis. (a) Find the flux $\phi(z)$ and the $z$-directional current $J(z)$. (b) Find the partial currents $J^+(z)$ and $J^-(z)$ in the upper and lower half of the solid angle. (c) Rewrite the angular flux in terms of $\phi(z)$ and $J(z)$. (d) Rewrite $\phi(z)$ and $J(z)$ in terms of the partial currents. (Homework problem #5 of Ch. 1)

2. Derive the one-group integral transport equation for the angular flux $\psi(r,\vec{\Omega})$ for a spherical geometry consisting of two media (a center sphere in a spherical shell with a vacuum outside). Assume isotropic scattering and uniformly distributed independent source. (Homework problem #2 of Ch. 6)

3. Investigate the iterative computational procedure for finding the flux $\phi(r)$ in the spherical problem described in problem 2. Assume the inner sphere of radius $R = 2$ cm is black (i.e., its absorption cross section is infinite); take $\Sigma_s = 0.2$ cm$^{-1}$ and $\Sigma_a = 0$ in the outer medium, assumed to be infinitely large, with $\phi(r \to \infty) = \phi_\infty$. (Homework problem #5 of Ch. 6)

   a. Calculate the first iterate $\phi^{(1)}(r)$ from an assumed flux distribution as the initial guess

      $\phi^{(0)}(r) = \phi_\infty$, in the outer medium.

   b. Plot $\phi^{(1)}(r)/\phi_\infty$ and discuss the result.

   c. Discuss qualitatively the changes to be expected in the next iterate $\phi^{(2)}(r)$ as well as in the converged solution.

   d. Solve the one-group diffusion equation for this case; plot, compare, and discuss the results.

4. By applying the separation of energy and spatial variables to the energy-dependent diffusion equation, the following balance equations are obtained:

   $$\nabla^2\phi(r) + B^2\phi(r) = 0$$

   $$[D(E)B^2 + \Sigma_t(E)]\varphi(E) - \int_0^\infty dE'\Sigma_s(E' \to E)\varphi(E') = \lambda\chi(E)\int_0^\infty dE'\nu\Sigma_f(E')\varphi(E')$$

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Simplify the above equations so that they represent the corresponding pair of equations for plane geometry [ $\phi(x)$ ] and one energy group. Assuming $\phi(\pm a)=0$ as boundary conditions and using the one-group values of $D=0.4$ cm, $\Sigma_a=0.0044$ cm$^{-1}$, and $\nu\Sigma_f=0.00584$ cm$^{-1}$, answer the following questions. (Homework problem #5 of Ch. 2)

a. Find the material buckling and describe the general procedure.

b. Find $k=1/\lambda$ for $a=50$ cm and describe the general procedure.

c. Find the critical dimension $a_c$ and describe the general procedure.

d. Increase the critical dimension by 5%, i.e., $a'=1.05a_c$, and find the required increase in absorption (i.e., $\delta\Sigma_a$) to make the system critical. Describe the general procedure.

e. Take the critical dimension and modify the original composition by increasing $\nu\Sigma_f$ such that the resulting $k$ equals 1.05. Describe the general procedure.