NUCL 510 Nuclear Reactor Theory I
Fall 2011

Homework #3

Due September 15

1. Two mono-energetic neutron beams, each of intensity 5×10^8 neutrons/cm^2-sec, cross at an angle of 45°. Compute the neutron flux and current in the region where the beams intersect.

$$\phi = \int \phi(\Omega)d\Omega = 5\times10^8 + 5\times10^8 = 10^9\, n/cm^2 \sec$$

Taking the direction of one beam as the x-axis, the direction of the other beam can be represented as $(1/\sqrt{2}, 1/\sqrt{2})$.

$$\begin{aligned}\vec{J} &= 5\times10^8 \vec{e}_x + 5\times10^8(1/\sqrt{2}\vec{e}_x + 1/\sqrt{2}\vec{e}_y) \\ &= 8.536\times10^8 \vec{e}_x + 3.536\times10^8 \vec{e}_y\ n/cm^2 \sec\end{aligned}$$

2. The density distribution function for mono-energetic neutrons at a point $\vec{r}$ is

$$n(\vec{r},\vec{\Omega}) = ne^{-\chi/\lambda}(1+\cos\theta)$$

where n and λ are constants, and ϑ is the angle between $\vec{\Omega}$ and the z-axis. Compute at $\vec{r}$ (a) the neutron density, (b) the flux, and (c) the current.

$$\begin{aligned}n(\vec{r}) &= \int_{4\pi} n(\vec{r},\vec{\Omega})d\Omega = 2\pi\int_0^\pi n\, e^{-x/\lambda}(1+\cos\theta)\sin\theta\, d\theta \\ &= 2\pi\, n\, e^{-x/\lambda}\int_{-1}^{1}(1+\mu)d\mu = 4\pi\, n\, e^{-x/\lambda}\end{aligned}$$

$$\phi(\vec{r}) = \int \psi(\vec{r},\vec{\Omega})\, d\Omega = \int n(\vec{r},\vec{\Omega})v\, d\Omega = n(\vec{r})v = 4\pi\, n\, e^{-x/\lambda}\, v$$

$$\vec{J}(\vec{r}) = \int \psi(\vec{r},\vec{\Omega})\vec{\Omega}d\Omega = \vec{J}_x + \vec{J}_y + \vec{J}_z$$

$$\vec{\Omega} = \Omega_x\vec{e}_x + \Omega_y\vec{e}_y + \Omega_z\vec{e}_z = \sin\theta\cos\varphi\vec{e}_x + \sin\theta\sin\varphi\vec{e}_y + \cos\theta\vec{e}_z$$

$$\vec{J}_x = \int_0^{2\pi}\int_0^\pi n\, e^{-x/\lambda}\, v(1+\cos\theta)\sin\theta\cos\varphi(\sin\theta\, d\theta\, d\varphi) = 0$$

$$\vec{J}_y = \int_0^{2\pi}\int_0^\pi n\, e^{-x/\lambda}\, v(1+\cos\theta)\sin\theta\sin\phi(\sin\theta\, d\theta\, d\phi) = 0$$

$$\begin{aligned}\vec{J}_z &= \int_0^{2\pi}\int_0^\pi n\, e^{-x/\lambda}\, v(1+\cos\theta)\cos\theta\sin\theta\, d\theta\, d\phi \\ &= 2\pi\, n\, e^{-x/\lambda}\, v\int_{-1}^{1}(1+\mu)\mu\, d\mu = \frac{4\pi}{3} n\, e^{-x/\lambda}\, v\end{aligned}$$

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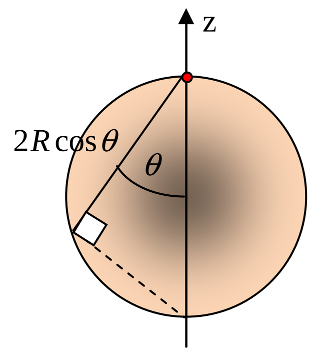
3. Neutrons are produced uniformly and isotropically throughout a spherical chamber containing a mixture of ^{3}H and ^{2}H gases at high temperature (~10^8 °K) and low density. (The neutrons originate in the ^{3}H(d,n)^{4}He and ^{2}H(d,n)^{3}He fusion reactions.) Show that the neutron flux and current at any point on the surface of the chamber are given by $\varphi = SR/2$ and $\vec{J} = SR\vec{e}_r/3$, respectively, where S is the source density (neutrons/cm^3-sec), R is the radius of the chamber, and $\vec{e}_r$ is a unit radial vector. The neutron mean free path in the medium is essentially infinite.

   Since $\lambda_t = \infty$, there is no attenuation ($\Sigma_t = 0$).



$$n(\vec{r},\vec{\omega}) = \frac{S}{4\pi r^2}$$

$$\varphi = \int_0^{\pi/2} d\theta \int_0^{2R\cos\theta} dr \int_0^{2\pi} d\phi\, \frac{S}{4\pi r^2} r^2 \sin\theta$$

$$= \frac{S}{2}\int_0^1 d\mu \int_0^{2R\mu} dr = S\,R\int_0^1 \mu\, d\mu = \frac{SR}{2}$$

$$\vec{J} = \int_0^{\pi/2} d\theta \int_0^{2R\cos\theta} dr \int_0^{2\pi} d\phi\, \frac{S}{4\pi r^2}\vec{\Omega}(r^2\sin\theta)$$

$$\vec{\Omega} = (\sin\theta\cos\phi, \sin\theta\sin\phi, -\cos\theta)$$

$$J_x = J_y = 0 \quad \left(\int_0^{2\pi} \cos\phi\, d\phi = \int_0^{2\pi} \sin\phi\, d\phi = 0\right)$$

$$J_z = -\int_0^{\pi/2} d\theta \int_0^{2R\cos\theta} dr \int_0^{2\pi} d\phi\, \frac{S}{4\pi}\cos\theta\,\sin\theta = -\frac{S}{2}\int_0^1 d\mu \int_0^{2R\mu} \mu\, dr = -\frac{SR}{3}$$

$$\vec{J} = -\frac{SR}{3}\vec{e}_z = \frac{SR}{3}\vec{e}_r$$

4. For an energy-independent s-wave scattering, the energy transfer function is given by

$$f_s(E \to E') = \frac{\sigma_{s0}\eta^2}{2E}\left\{ \mathrm{erf}\left(\eta\sqrt{\frac{E'}{kT}} - \rho\sqrt{\frac{E}{kT}}\right) \mp \mathrm{erf}\left(\eta\sqrt{\frac{E'}{kT}} + \rho\sqrt{\frac{E}{kT}}\right) \right.$$
$$\left. + e^{(E-E')/kT}\left[\mathrm{erf}\left(\eta\sqrt{\frac{E}{kT}} - \rho\sqrt{\frac{E'}{kT}}\right) \pm \mathrm{erf}\left(\eta\sqrt{\frac{E}{kT}} + \rho\sqrt{\frac{E'}{kT}}\right)\right]\right\}$$

   where

$$\eta = \frac{A+1}{2\sqrt{A}}, \quad \rho = \frac{A-1}{2\sqrt{A}}$$

and the upper signs are to be used for $E' > E$ and the lower signs for $E' < E$. Plot the energy transfer functions of incident neutrons of 0.025 eV, 10 eV and 100 eV for the following targets: (a) H-1 at 300 °C, (b) O-16 2000 °C and (c) U-238 at 2000 °C.