

# **NUCL 402 Engineering of Nuclear Power Systems**

## **Lecture 20: Reactor Heat Generation and Decay Heat**

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# Fission energy in reactor

For uranium -235, the average energy released in fission is 200 MeV or 32 pJ . This energy is equivalent to 3 million times as much energy released with combustion of carbon of same mass as one uranium-235 atom.

**Example 1** Calculate the amount of U-235 consumer per day in a thermal reactor operating at power P megawatts.

*Solution:*

Assuming recoverable energy of 200 MeV (32pJ) per fission, the fission rate for a year producing P megawatts power,

$$\begin{aligned}\text{Fission rate} &= P \text{ (MW)} \frac{10^6 \text{ (J)}}{1 \text{ (MW} - \text{sec)}} \times \frac{\text{fission}(\#)}{32 \times 10^{-12} \text{ (J)}} \times \frac{86,400 \text{ (sec)}}{1 \text{ (day)}} \times \frac{365 \text{ (days)}}{1 \text{ (year)}} \\ &= 9.855 \times 10^{23} P \text{ (\#/year)}\end{aligned}$$

$$\text{Burnup rate per year} = 9.855 \times 10^{23} P \text{ (\#/ year)} \times \frac{235 \text{ (g / mole)}}{6.022 \times 10^{23} \text{ (\#/ mole)}} = 384.58 P \text{ (g)}$$

In reality the fissile material is consumed both in fission and in radiative capture.

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The radiative capture rate is given as

Radiative capture rate =  $\alpha \times$  Fission rate

Where  $\alpha$  is the ratio of microscopic absorption cross section to the microscopic fission cross section and for U-235 its value is 0.169. Thus

Consumption rate of U235 =  $384.58 (1+\alpha) P$  (g)

= 449.57 (g)

	Type	Form	Emitted energy (MeV)	Recoverable energy (MeV)
Fission	I Instantaneous energy	Kinetic energy of fission fragments	168	168
		Kinetic energy of newly born fast neutrons	5	5
		Prompt gamma energy	7	7
	II Fission product decay -delayed energy	Beta rays energy	7	7
		Gamma ray energy	6	6
		Neutrinos energy –with beta decay	10	-
		Kinetic energy of delayed neutrons	0.4	0.4
	III Excess neutron gamma capture reaction	Nonfission reactions due to excess neutrons and the proceeding beta, gamma decay energy	-	7

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## Volumetric Heat Generation in Fuel

Fission generates neutrons with large energies ( 2-10 MeV) - these neutrons are slowed down to thermal energies ( $\sim 0.025$  eV). The thermal neutrons have a Maxwellian energy distribution dependent upon the temperature of the medium. In the low energy range the neutron absorption cross section for fissile nuclei is inversely proportional to the squarer of the neutron energy or is inversely proportional to neutron speed.

The average volumetric heat generation rate  $q'''$  (W/m<sup>3</sup>) in the reactor core is given as the product of total reaction rate  $R$  (1/s) and the energy per reaction  $G_f$  (J). The reaction rate  $R$  is calculated as

$$R = N_f \int \sigma_f(E) \phi(E) dE$$

where  $N_f$  is density of the fissionable fuel (nuclei/ cm<sup>3</sup>),  $\sigma_f(E)$  is the microscopic cross section for the fissionable fuel (cm<sup>2</sup>) at energy  $E$  and  $\phi(E)$  is flux of the neutron (neutron/s cm<sup>2</sup>) having energy between  $E$  and  $E + dE$ . The total reaction rate is obtained by taking integral over entire range of neutron energy (0,  $\infty$ ). The fuel volume

By defining flux of all neutrons,  $\phi$ , and effective microscopic cross section  $\sigma_f$  as

$$\phi = \int \phi(E) dE \text{ and } \sigma_f \phi = \int \sigma_f(E) \phi(E) dE$$

## Volumetric Heat Generation in Fuel

The volumetric heat generation rate  $q'''$  is given by

$$\begin{aligned} q''' &= Gf Nf \sigma f \phi \text{ (MeV/s cm}^3\text{)} \\ &= 1.602 \times 10^{-10} Gf Nf \sigma f \phi \text{ (kW/m}^3\text{)} \end{aligned}$$

Since some of the heat is generated in non fuel due to various capture reactions, the overall power generation in the core  $Q$  (W) is related to the  $q'''$  as

$$Q = q''' V_{\text{fuel}} / \gamma,$$

where  $V_{\text{fuel}}$  is the fuel volume and  $\gamma$  is the fraction of the power generated in the fuel.

The core power density  $Q'''$  (W/m<sup>3</sup>) is related to the  $Q$  and  $q'''$  as

$$Q''' = Q/V_{\text{core}} = q''' (V_{\text{fuel}}/ V_{\text{core}} \gamma)$$

**Example:** Calculate the volumetric thermal source strength in a reactor core. The core has neutron flux  $10^{14}$  neutron/s  $\text{cm}^2$  with the effective microscopic cross section for the fissionable fuel of 350 barn and density of the fissionable fuel,  $2 \times 10^{21}$  nuclei/  $\text{cm}^3$ .

**Solution:** Given  $N_f = 2 \times 10^{21}$  nuclei/  $\text{cm}^3$ ,  $\phi = 10^{14}$  neutron/s  $\text{cm}^2$ ,  $\sigma_f = 350 \times 10^{-24}$   $\text{cm}^2$

$$\begin{aligned}\text{Therefore } q''' &= 200 \times 2 \times 10^{22} \times 350 \times 10^{-24} \times 10^{14} \\ &= 14 \times 10^{15} \text{ MeV/s cm}^3 \\ &= 14 \times 10^{15} \times 1.602 \times 10^{-10} = 22.680 \times 10^5 \text{ kW/m}^3 \\ &= 14 \times 10^{15} \times 1.5477 \times 10^{-8} = 21.668 \times 10^7 \text{ Btu/hr ft}^3\end{aligned}$$

## Heat Generation During Transient

- ✓ Up to 50% of the reactor power the heat generation rate can be assumed to follow in proportional to the neutron flux. However if the reactor is shut down the reactor core produces heat at 7% of the reactor power due to delayed neutron fission, fission product decay and activation products from neutron capture.
- ✓ Immediately after the shut down the reactor power decreases exponential from 7% of the core power with a period of 80 seconds. The heat generation from fission product decay is a result of beta and gamma emission from fission product with decay half life ranging from micro seconds to million years.
- ✓ **Decay heat** is the heat produced by the decay of radioactive fission products after a nuclear reactor has been shut down. Decay heat is the principal reason of safety concern in Light Water Reactors (LWR). It is the source of 60% of radioactive release risk worldwide.

## Decay Heat- Shutdown

- ✓ The decay heat power comes mainly from five sources:
- ✓ Unstable fission products, which decay via  $\alpha$ ,  $\beta^-$ ,  $\beta^+$  and  $\gamma$  ray emission to stable isotopes.
- ✓ Unstable actinides that are formed by successive neutron capture reactions in the uranium and plutonium isotopes present in the fuel.
- ✓ Fissions induced by delayed neutrons.
- ✓ Reactions induced by spontaneous fission neutrons.
- ✓ Structural and cladding materials in the reactor that may have become radioactive.
- ✓ Heat production due to delayed neutron induced fission or spontaneous fission is usually neglected. Activation of light elements in structural materials plays a role only in special circumstances, and is usually excluded from decay heat analyses.



# ANS standard for decay heat calculation

- ✓ In 1971, the results were adopted by the American Nuclear Society (ANS) to assemble the first decay heat standard (ANS-5.1/N18.6).
- ✓ The latest version of the standard is ANS-5.1-1994 (Current Standard, Revision of ANSI/ANS-5.1-1979;R1985).
- ✓ The ANS-5.1 standard models the energy release from the fission products of  $^{235}\text{U}$ ,  $^{238}\text{U}$  and  $^{239}\text{Pu}$  using a summation of exponential terms with empirical constants. Corrections are provided to account for energy release from the decay of  $^{239}\text{U}$  and  $^{239}\text{Np}$  and for the neutron activation of stable fission products.
- ✓ Decay heat power from other actinides and activation products in structural materials, and fission power from delayed neutron-induced fission, are not included in this standard.

## Integrated Beta And Gamma Emission Rates

The rates per U-235 fission, and as a function of decay time in days, are

$$\beta(t_d) = 1.50 \cdot 10^{-6} \cdot t_d^{-1.2} \quad \text{MeV/s-f}$$

$$\gamma(t_d) = 1.67 \cdot 10^{-6} \cdot t_d^{-1.2} \quad \text{MeV/s-f}$$

These energy rates are roughly equal for 0.4 MeV mean energy beta particles and 0.7 MeV mean energy gamma-rays.

For a fuel assembly irradiated continuously for  $t_i$  days at a constant fuel assembly power ( $P$ ), the heat ( $H$ ) load power per assembly,  $t_d$  days after irradiation is

$$H = 6.85 \cdot 10^{-3} \cdot P \cdot (t_d^{-0.2} - (t_i + t_d)^{-0.2}) \quad \text{Watts (1)}$$

For a low duty-factor fuel assembly irradiation, the power and irradiation time are replaced by an average power and an elapsed time. With over all irradiation segments, the heat ( $H$ ) load power per assembly is

$$H \cong 6.85 \cdot 10^{-3} \cdot \bar{P} \cdot (t_d^{-0.2} - (t_e + t_d)^{-0.2}) \quad \text{Watts}$$

where  $\bar{P}$  is the average fuel assembly power in watts and  $t_e$  is the elapsed time in days from the initial through the final irradiation segment.

A convenient estimate for the average power is

$$\overline{P} = (G / t_g) / 1.25 \cdot 10^{-6} \quad \text{Watts}$$

where  $G$  is the mass of U-235 burned in the fuel assembly in grams, and the constant is g235U burned per Watt-day.

Another similar heat load expression is

$$H = 6.22 \cdot 10^{-2} \cdot P \cdot (t_d^{-0.2} - (t_i + t_d)^{-0.2}) \quad \text{Watts}$$

with all times in seconds. With all times in days, this heat load expression is

$$H = 6.40 \cdot 10^{-3} \cdot P \cdot (t_d^{-0.2} - (t_i + t_d)^{-0.2}) \quad \text{Watts}$$

Fuel assembly decay heat loads calculated with these expressions are expected to be conservative, and within a factor of two or less of measured heat loads.

This same conservative heat load estimate also has been found to be true for heat load calculations made with the ORIGEN code. The thermal heat load of a fuel assembly is independent of the fuel assembly type.

The constants used in the above equations are based upon empirical data and therefore, are not necessarily exact; it is not uncommon to find several percent variation in a recommended value. The constants considered here, and their range, are:

the beta plus gamma fission product energy rate per fission;

$2.7 - 3.2 \cdot 10^{-6} \text{ MeV/s-f}$ ,

the total energy release per fission;  $190 - 200 \text{ MeV/f}$ ,  
and

the mass of  $^{235}\text{U}$  burned per megawatt-day;  $1.2 - 1.3 \text{ g}^{235}\text{U/MWd}$ .

Depending upon the specific values of the constants that are chosen, the calculated heat load can vary by several percent. In any case, the thermal decay heat is expected to be over predicted.

## Decay Heat Curves

An analytical expression given by El-Wakil , which correlate with the decay heat curves of Ref 11, estimate heat loads about one-half the heat loads calculated above. This heat load expression is

$$H = 4.95 \cdot 10^{-3} \cdot P \cdot t_d^{-0.06} \cdot (t_d^{-0.2} - (t_i + t_d)^{-0.2}) \quad \text{Watts (2)}$$

where all symbols, etc. have the same meaning as above and the times are in days.

The ratio of Eq. (2) to Eq. (1) is

$$4.95 \cdot t_d^{-0.06} / 6.85 = 0.72 \cdot t_d^{-0.06}$$

For decay times  $t_d$  greater than 1 year, the ratio is approximately 0.5.

# Experimental Decay Heat Data

Another analytical expression given by Untermeyer and Weills, has been used to fit experimental decay heat data. This heat load expression is

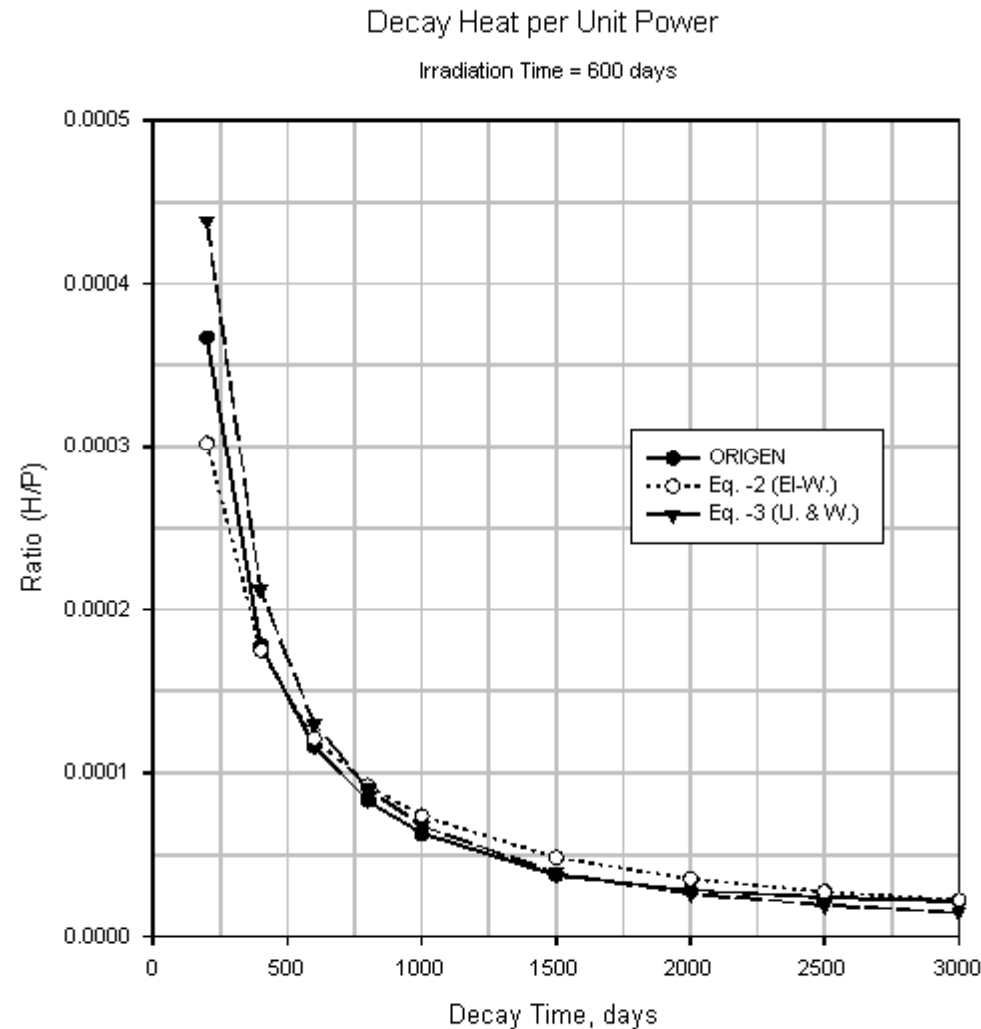
$$H = 0.1 \cdot P \cdot [(t_d + 10)^{-0.2} - (t_i + t_d + 10)^{-0.2}] \text{ Watts (3)}$$

where the irradiation  $t_i$  and decay  $t_d$  times are in seconds.

A plot of the ratio  $H/P$  for Eqs. (2) and (3) are shown in Fig. 1 as a function of decay time and for an irradiation time of 600 days. The ratio calculated with the ORIGIN code is also shown for comparison.

These data clearly show the relative decay heat estimated by the decay heat expressions for a typical irradiation time. The ORIGIN ratio is in good agreement with both Eqs. (2) and (3).

# Experimental Decay Heat Data



**Figure 1.** Comparison of Decay Heat Equations (2) and (3) with ORIGEN