Energy Equation

$\psi = \rho(u + V^2/2)$ → energy/vol

u = internal energy

$\mathbb{J} = \vec{q} + \Pi \cdot \vec{V}$ → energy flux

heat flux + work done by surface force

$\dot{\psi} = \rho \vec{V} \cdot \vec{g} + \dot{q}$ → energy source

General balance eqn

conservative form

$$\frac{\partial \rho(u + v^2/2)}{\partial t} + \nabla \cdot [\rho(u + v^2/2)\vec{v}] = -\nabla \cdot \vec{q}$$ heat flux

$-\nabla \cdot (p\vec{v})$ work done by pressure

$-\nabla \cdot (\tau \cdot \vec{v})$ work done by shear

$+\rho \vec{v} \cdot \vec{g}$ work done by body force

$+ q'''$ internal heat generation

$$\rho \frac{D}{Dt}(u + v^2/2) = \rho\left[\frac{\partial}{\partial t}(u + v^2/2) + \vec{v} \cdot \nabla(u + v^2/2)\right]$$

non conservative form b/c of this term

Total Energy eqn

$E \to u + v^2/2$ ← want to convert to just thermal energy

$$\rho \frac{D}{Dt}(u + v^2/2) = -\nabla \vec{q} - \nabla \cdot (p\vec{v}) - \nabla \cdot (\tau \cdot \vec{v}) + \rho \vec{v} \cdot g + \dot{q}$$

Kinetic Energy Eqn ($V \cdot$ [ME])

subtract to isolate internal energy

$$\rho \frac{D v^2/2}{Dt} = -\vec{v} \cdot \nabla P - \vec{v} \cdot [\nabla \cdot \tau] + \rho \vec{v} \cdot \vec{g}$$

$dE = dQ - PdV$

$$\boxed{\rho \frac{D\vec{u}}{Dt} = -\nabla \cdot \vec{q} - p\nabla \cdot \vec{v} - \tau : \nabla \vec{v} + \dot{q}}$$ Thermal energy eqn

$\frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho u \vec{v})$

heat flux

Reversible conversion of work to internal energy

irreversible conversion of work to internal energy

internal heat generation

Converting IE equation

$u \to T$ Temp (good for single phase)

$u \to i$ Enthalpy (good for 2ϕ)

$$\frac{\partial \rho i}{\partial t} + \nabla\cdot(\rho i \cdot \vec{v}) = -\nabla\cdot\vec{q} + \frac{Dp}{Dt} - \cancel{\overset{\leftrightarrow}{\tau} : \nabla\vec{v}} + \dot{q}$$

$\frac{\partial p}{\partial t} + \vec{v}\cdot\nabla p$

Single phase mixture

ρ_K = mass concentration (component density) (mass of component/volume)

w_K = mass fraction ρ_w/ρ

ρ ; Mixture density $\rho = \sum_{k=1}^{n} \rho_K$ mass/vol

V_K ; velocity of component K

$\rho_K V_K$ momentum of K component

$\rho\vec{v} = \sum \rho_K V_K$ momentum of mixture

$\vec{v} = \frac{\sum \rho_K V_K}{\rho} = \frac{\sum \rho_K V_K}{\sum \rho_K}$

mass weighted mean velocity

Diffusion Velocity

$\vec{V}_{cm} = \vec{V}_K - \vec{v}$

But

$$\sum_{K=1}^{n} \rho_K V_{cm} = 0$$

Diffusion Formulation

Continuity eqn for general component

$$\psi = \rho_K$$

$$\mathbb{J} = \rho_K(\vec{v}_K - \vec{v}) - \rho_K \vec{V}_{cm} = \vec{j}_K$$

$$\dot{\psi}_g = \Gamma_K \quad \text{reaction rate}$$

$$\frac{\partial \rho_K}{\partial t} + \nabla \cdot \rho_K \vec{v} = -\nabla \cdot \vec{j}_K + \Gamma_K$$ — diffusion formulation

$$-\nabla \cdot \rho_K (\vec{v}_K - \vec{v})$$

$$\boxed{\frac{\partial \rho_K}{\partial t} + \nabla \cdot \rho_K \vec{v}_K = \Gamma_K}$$ — multicomponent

$\vec{j}_K = ?$ need to specify

$$\sum_{K=1}^{n} \left[\frac{\partial \rho_K}{\partial t} + \nabla \cdot \rho_K v_K = \Gamma_K \right]$$

$$\Downarrow$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = \sum_{K=1}^{n} \Gamma_K \qquad \sum_{K=1} \Gamma_K = 0 \text{ mass must be conserved}$$

Mixture momentum equation

$$\psi = \rho \vec{v}$$

$$\mathbb{J} = p\mathbb{I} + \tau + \Sigma \rho_K \vec{V}_{cm} \vec{V}_{cm} = \pi$$

$$\dot{\psi}_j = \Sigma \rho_K \vec{g}$$

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v}\vec{v} = -\nabla P - \nabla \cdot \tau - \underline{\nabla \cdot \Sigma \rho_K \vec{V}_{cm} \vec{V}_{cm} + \rho \vec{g}}$$

$$\nabla \cdot \Sigma \rho_K \vec{v}_K \vec{v}_K$$

$\vec{v}_K$ x

$\left\{ \begin{array}{l} \vec{v} \\ \vec{V}_{Km} \leftarrow \end{array} \right.$

Mixture Energy Equation

$\psi = \rho\left(u + v^2/2\right)$

$\dot{\psi}_g = \sum \rho_k \vec{v}_k \cdot \vec{g}$

$\mathbb{J} = \pi \cdot \vec{v} + \vec{q}$

$\vec{q}$ { heat conduction (thermal gradient)

$\left(\frac{1}{2}\rho_k v_k^2 + u_k\right)\vec{V}_{km}$ interdiffusion of energy

diffusion th.. effect

energy transfer due to concentration gradient

$= q_{cond} + q_{diffusion} + q_{concentration}$