Prandtl Mixing Length

Momentum transfers only after moving length ℓ

$\tau = \frac{F}{A}$ and $F = \frac{\delta m \, \delta v_x}{\delta t}$

so $\tau = \frac{\delta m \, \delta v_x}{\delta t} \frac{1}{A}$

now

1) $\delta v_y \sim \ell \frac{d\bar{v}_x}{dy}$ for small ℓ
2) CE mass after $\frac{1}{A}\frac{\delta m}{\delta t} = \rho |v_y'|$

plus in $\tau^T = \underbrace{\frac{1}{A}\frac{\delta m}{\delta t}}\delta v_y = -\ell \rho |v_y'| \frac{d\bar{v}_x}{dy}$ or eddy diff $\frac{\tau^T}{\rho} = -\varepsilon_m \frac{d\bar{v}_x}{dy}$

add molecular

$$\frac{\tau}{\rho} = -(\varepsilon_m + \nu)\frac{d\bar{v}_x}{dy}$$

Prandtl's assumption

$$\boxed{\begin{aligned} |v_y'| &= k_1 v_x' \\ v_x' &= k_2 \delta v_x = k_2 \ell \left|\frac{d\bar{v}_x}{dy}\right| \qquad \ell = ky - \text{wall dist} \end{aligned}}$$

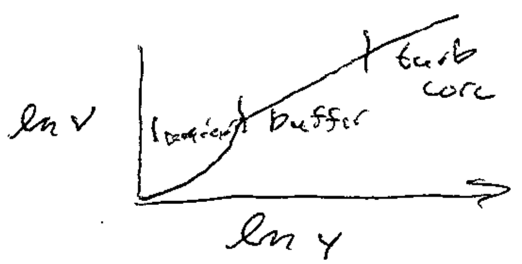
so $|v_y'| = k_1 k_2 \left|\frac{d\bar{v}_x}{dy}\right| \ell$

$$\frac{\tau^t}{\rho} = -|v_y'| \frac{d\bar{v}_x}{dy} \Rightarrow \frac{\tau^t}{\rho} = -\ell\left[k_1 k_2 \left|\frac{d\bar{v}_x}{dt}\right|\right]\frac{d\bar{v}_x}{dy} = -k^2 y^2 \left|\frac{d\bar{v}_x}{dy}\right| \frac{d\bar{v}_x}{dy}$$

at wall, $\tau^t \to \tau^w$, so $\tau^t_{yx} = -\tau_w$

$$\frac{d\bar{v}_x}{dy} = \sqrt{\frac{\tau_w}{\rho}} \frac{1}{ky}$$

$$\bar{v}_x = \frac{1}{k}\sqrt{\frac{\tau_w}{\rho}} \ln|y|$$



Dimensionless Parameters Solution

$$Re = \frac{\text{inertial force}}{\text{viscous force}} = \frac{\rho v D}{\mu}$$

$$Fr = \frac{\text{inertial force}}{\text{gravitational}} = \frac{v^2}{gD}$$

momentum equivalence

$$Ec = \frac{\text{inertial Force}}{\text{Thermal Driving Force}} = \frac{v^2}{c_p \Delta T}$$

energy equivalence

$$Pr = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

$$Pe = Pr\,Re = \frac{\rho v D c_p}{k} = \frac{\text{convection}}{\text{diffusion}}$$

$$Gr = \frac{(\text{buoyancy})(\text{inertial})}{\text{viscous}^2} = \frac{\vec{g}\beta \Delta T D^3}{\nu^2}$$

$$Nu = \frac{\text{convection}}{\text{conduction}} = \frac{hD}{k} \cong C_1 Re^m Pr^n$$

	Forced Circ	Nat Circ
CE	—	—
ME	Re, Fr	Re
EE	Re, Pr	Gr, Pr

And Moody Chart:

for laminar

$$Re < 2400 \Rightarrow f = \frac{64}{Re}$$

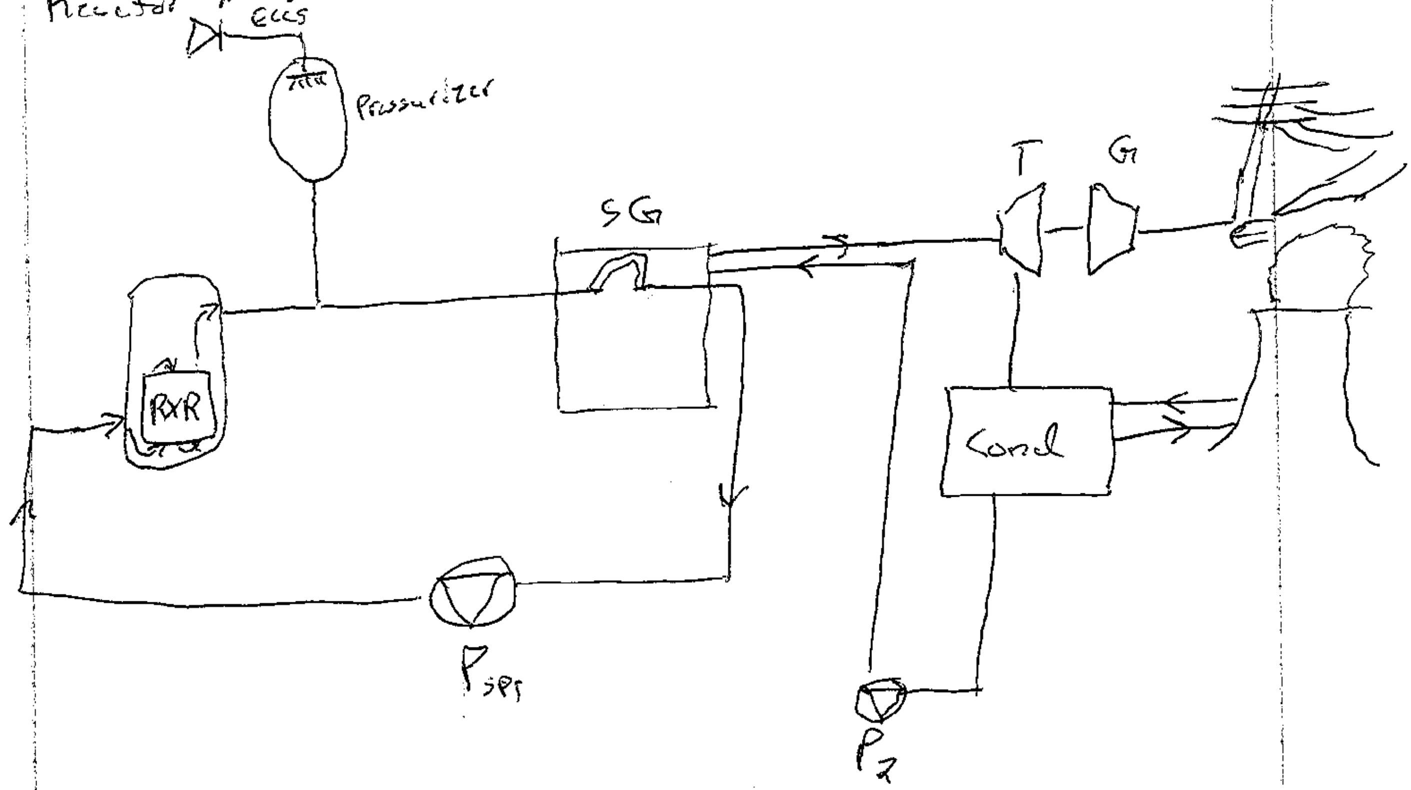
for turbulent

$$Re > 5000$$

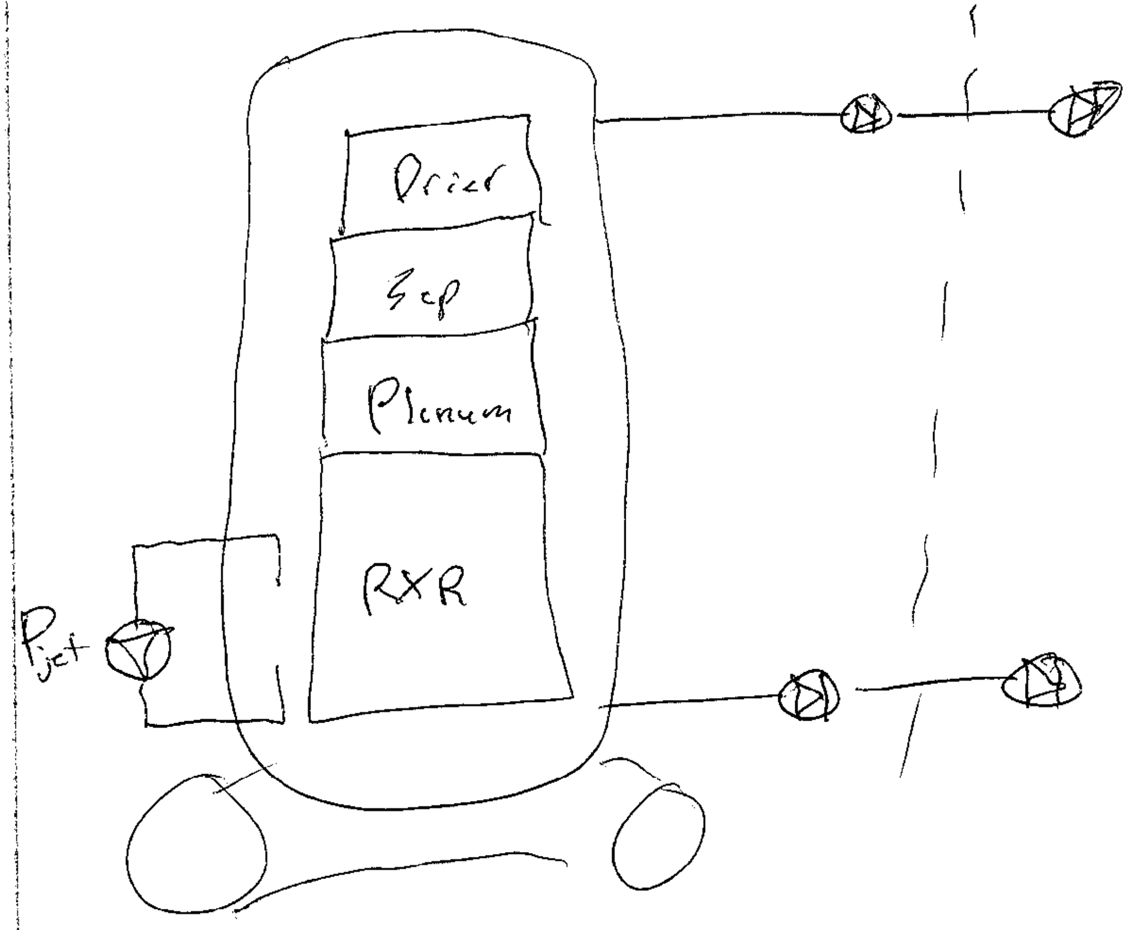
$$f = f\left(Re, \frac{\epsilon}{D}\right)$$

relative roughness

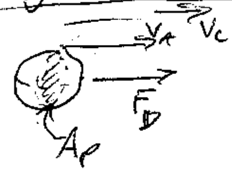
Reactor Diagram



BWR



Drag Force on Particle



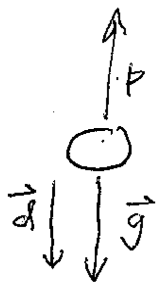
$$F_D = -\frac{C_D}{2}\rho_c v_r |v_r| A_p$$

$$C_D = f(Re), \quad Re \equiv \frac{\rho_c v_r (2r)}{\mu}$$

and regimes $\quad N_\mu \equiv \frac{\mu_c}{(\rho_c \sigma \sqrt{\sigma/g\Delta\rho})^{0.5}}$

Stokes $\left\{ C_D = \frac{24}{Re} \right\}$ Distorted Particle $\left\{ C_D = \frac{\sqrt{2}}{3} N_\mu Re \doteq \left(\frac{4}{3} r_d \sqrt{g\Delta\rho/\sigma}\right) \right\}$

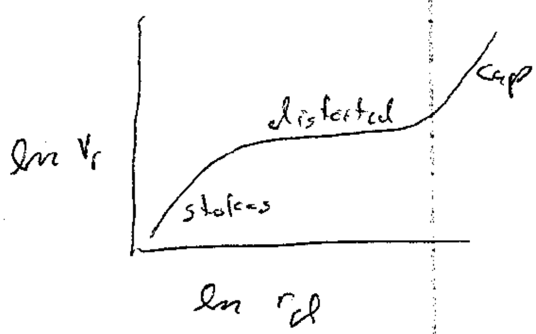
cap bubble $\left\{ C_D = \frac{8}{3}, \quad r_D \geq 2\sqrt{\sigma/g\Delta\rho} \right\}$



$$F_b = \vec{V}(\rho_c - \rho_d)\vec{g}$$

$$F_D = -\frac{1}{2} C_D \rho_c v_r |v_r| A_d$$

Find terminal velocity



Stokes

$$C_D = 24 Re$$

$$\frac{V}{A_d}\left(\frac{\rho_c - \rho_d}{\rho_c}\right) g = \frac{1}{2} C_D$$

$$4/3\, \pi r^3 \frac{(\rho_c - \rho_d)\vec{g}}{\rho_c} = \frac{1}{2}\left[\frac{24\mu}{2r\rho_g |v_r|}\right]$$

$$\frac{8}{3 \cdot 24} \pi r^2 (\rho_c - \rho_d) g = \frac{\mu}{\rho_g v_r}$$

$$v_r = \frac{2}{9} \frac{(\rho_c - \rho_d)\vec{g}\, r^2}{\mu_c}$$

Distorted (typical)

$$C_D = \frac{\sqrt{2}}{3} N_\mu Re \Rightarrow v_r = \sqrt{2}\left(\frac{\sigma \vec{g} \Delta\rho}{\rho_c^2}\right)^{1/4}$$

cap bubble

$$v_r = \sqrt{\frac{\sigma_d \Delta\rho g}{\rho_c}}$$

$$r_d = \left(\frac{3}{4}\frac{V}{A_d}\right)$$

Two Fluid Model

Use interface conditions to relate separate phases, each defined by 3 field equations. Here to do local time averaging to find both edges of each interface.

$$\overline{\nabla F} = \nabla F + \frac{1}{\Delta t}\sum_j \frac{1}{v_{ni}}\left\{ n^+ F^+ + n^- F^- \right\}_j$$

so the field equations are

C.E.

$$\underbrace{\frac{\partial \alpha_k \rho_k}{\partial t}}_{\text{time rate of change of k phase mass}} + \underbrace{\nabla \cdot (\alpha_k \rho_k \vec{v}_k)}_{\text{convection of k phase mass}} = \underbrace{\Gamma_k}_{\text{source of k due to phase change}}$$

M.E.

$$\frac{\partial \alpha_k \rho_k \vec{v}_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k \vec{v}_k) = -\alpha_k \nabla p_k - \nabla \cdot \alpha_k (\tau_k^{\mu} + \tau_k^{t}) + \alpha_k \rho_k \vec{g}$$
$$+ \underbrace{\vec{M}_{ik}}_{\text{drag and lift}} + \underbrace{(\nabla \alpha_k) \cdot \tau_i}_{\text{i'face shear}} + (p_{ki} + p_k)\nabla \alpha_k$$

EE.

$$\frac{\partial \alpha \rho_k i_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k i_k \vec{v}_k) = -\nabla \cdot \alpha_k (q_k^{c} + q_k^{t}) + \alpha_k \frac{D p_k}{Dt} + \underbrace{i_k \Gamma_k}_{\text{phase change enth}} + \underbrace{\alpha_i q_{ki}''}_{\text{i'face heat xfer}} + \phi_k$$

Add some eqs.

$$\underbrace{\sum_{k=1}^{2} \Gamma_k = 0}_{\text{cont.}} \quad \Big\} \quad \underbrace{\sum_{k=1}^{2} \vec{M}_{ik} = 0}_{\text{force}} \quad \Big\} \quad \underbrace{\sum_{k=1}^{2} (\Gamma_k i_{ki} + \alpha_i q_{ki}) \approx 0}_{\text{energy change of i'face}}$$

and

$$\Gamma_g (i_{gi} - i_{fi}) = \Gamma_g (i_{g\,sat} - i_{f\,sat}) = \Gamma_g \Delta i_{fg} \longrightarrow \text{latent heat}$$

u depends on ρ, T (caloric)
p depends on ρ, T (thermal)

Drift Flux Model

For df model, we have 5 eq forms that have 2 CE, 1 ME, and 2 EE eqs, for 4 eq form with 2 CE, 1 ME, 1 EE

we always have mixture CE and ME

Mixture CE

Params:

$$\rho_m = \alpha\rho_g + (1-\alpha)\rho_f$$

$$v_m = \frac{\alpha\rho_g v_g + (1-\alpha)\rho_f v_f}{\rho_m}$$

$$j = \alpha\vec{v}_g + (1-\alpha)\vec{v}_f = j_g + j_f$$

$$v_{gj} = \vec{v}_g - \vec{j} = (1-\alpha)\vec{v}_j$$

$$\sum_{k=1}^{2}\left\{\frac{\partial\,\alpha_k\rho_k}{\partial t} + \nabla(\alpha_k\rho_k\vec{v}_k) = \Gamma_k\right. \Rightarrow \frac{\partial\rho_m}{\partial t} + \nabla\cdot\rho_m\vec{v}_m = 0$$

Vapor

$$\frac{\partial\,\alpha\rho_g}{\partial t} + \nabla\cdot(\alpha\rho_g\vec{v}_g) = \Gamma_g$$

$$\frac{\partial\,\alpha\rho_g}{\partial t} + \nabla\cdot(\alpha\rho_g\vec{v}_m) = \Gamma_g - \nabla\cdot\left(\frac{\alpha\rho_g\rho_f}{\rho_m}\vec{v}_{gj}\right)$$

Mixture Momentum Equation

$\cancel{\frac{\partial\rho_m i_m}{\partial t} + \nabla\cdot(\rho_m i_m}$

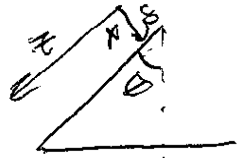
$$\frac{\partial\rho_m\vec{v}_m}{\partial t} + \underbrace{\nabla\cdot(\rho_m\vec{v}_m\vec{v}_m)}_{\text{acc convection}} = \underbrace{-\nabla p_m}_{\text{pres}} + \underbrace{\rho_m\vec{g}}_{\text{grav}} - \underbrace{\nabla\cdot\left\{\tau_m^{\mu} + \tau^{t}\right.}_{\text{wall shear}} + \underbrace{\left.\frac{\alpha\rho_g\rho_f}{(1-\alpha)\rho_m}\vec{V}_{gi}\vec{V}_{gi}\right\}}_{\text{momentum diffusion}}$$

Mixture Energy Equation

$$\underbrace{\frac{\partial\rho_m i_m}{\partial t}}_{\text{time rate of change}} + \underbrace{\nabla\cdot(\rho_m i_m\vec{v}_m)}_{\text{acceleration convection}} = -\nabla\cdot[q_m^{c} + q_m^{t}] - \overbrace{\nabla\cdot\left\{\frac{\alpha\rho_g\rho_f}{\rho_m}(i_{gi} - i_{fi})\vec{V}_g\right\}}^{\text{diffusion}} + \overbrace{\frac{\partial p_m}{\partial t}}^{\text{pressure drift}}$$

$$+\left[\vec{v}_m + \underbrace{\frac{\alpha(\rho_f-\rho_g)}{\rho_m}\vec{V}_{gi}}_{\text{energy diffusion}}\right]\cdot\nabla p_m + \underbrace{\Phi_m}_{\text{dissipation}}$$

→ Laminar Free Flowing Film



<u>Assumptions</u>

→ No Slip
→ Fully Developed, Laminar
→ incompressible
→ isothermal, adiabatic
→ uniform surface pressure
→

$v_x = 0$, ~~$v_y = 0$~~

$v_z(\delta) = 0$

Apply

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$

$$\cancel{\frac{\partial \rho}{\partial t}}^{0} + \rho \left\{ \frac{\partial v_x}{\partial x} + \cancel{\frac{\partial v_y}{\partial y}}^{0} + \cancel{\frac{\partial v_z}{\partial z}}^{0} \right\} = 0$$

$$\frac{\partial v_x}{\partial x} = 0 \implies v_x(\delta) = 0$$

$$\frac{\partial \rho v}{\partial t} + \nabla \cdot \rho \vec{v}\vec{v} = -\nabla p - \nabla \cdot \tau + \rho g \qquad \tau = \mu \nabla \vec{v}$$

z-dir

$$\cancel{\frac{\partial \rho \vec{v}_z}{\partial t}}^{0} + \rho \left\{ \cancel{\vec{v}_x \frac{\partial \vec{v}_z}{\partial x}}^{0} + \cancel{v_y \frac{\partial v_z}{\partial y}}^{0} + \cancel{v_z \frac{\partial v_z}{\partial z}}^{0} \right\} = -\cancel{\frac{\partial p}{\partial z}}^{0}$$ from x-dir

$$+ \mu \left\{ \frac{\partial^2 v_z}{\partial x^2} + \cancel{\frac{\partial^2 v_z}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}^{0} \right\} + \rho g \cos\theta$$

$$\boxed{0 = -\frac{\partial p}{\partial z} + \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g \cos\theta}$$

x-dir

$$\cancel{\frac{\partial \rho v_x}{\partial t}}^{0} + \rho \left\{ \cancel{\vec{v}_x \frac{\partial v_x}{\partial x}}^{0} + \cancel{v_y \frac{\partial v_x}{\partial y}}^{0} + \cancel{v_z \frac{\partial v_x}{\partial z}}^{0} \right\} = -\frac{\partial p}{\partial x}$$

$$+ \mu \left\{ \cancel{\frac{\partial^2 v_x}{\partial x^2}}^{0} + \cancel{\frac{\partial^2 v_x}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}^{0} \right\} + \rho g \sin\theta$$

$$\boxed{\frac{\partial p}{\partial x} = \rho g \sin\theta} \implies p = \rho g \sin\theta\, x + C_1$$

$p(0) = p_0$

$p = \rho g \sin\theta\, x + p_0$

$\therefore \frac{\partial p}{\partial z} = 0$

$$0 = -\cancel{\frac{\partial p}{\partial z}} + \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g \cos\theta$$

$$\frac{\partial^2 v_z}{\partial x^2} = -\frac{\rho g \cos\theta}{\mu}$$

$$\frac{\partial v_z}{\partial x} = -\frac{\rho g \cos\theta\, x}{\mu} + C_1$$

$$@\ x(0) \Rightarrow \frac{\partial v_z}{\partial x} = 0 \Rightarrow C_1 = 0$$

$$v_z = -\frac{1}{2}\frac{\rho g \cos\theta\, x^2}{\mu} + C_2$$

$$v_z(\delta) = 0$$

$$0 = -\frac{\rho g \cos\theta\, \delta^2}{2\mu} + C_2 \Rightarrow C_2 = \frac{\rho g \cos\theta\, \delta^2}{2\mu}$$

$$v_z = \frac{\rho g \cos\theta\, \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

$$\langle v_z \rangle = \frac{1}{\delta}\int_0^\delta v_z\, dx = \frac{1}{\delta}\int_0^\delta \frac{\rho g \cos\theta\, \delta^2}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right] dx$$

$$= \frac{\rho g \cos\theta\, \delta}{2\mu}\int_0^\delta 1 - \frac{x^2}{\delta^2}\, dx$$

$$= \frac{\rho g \cos\theta\, \delta}{2\mu}\left[x - \frac{x^3}{3\delta^2}\right]_0^\delta$$

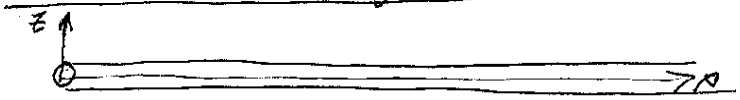
$$= \frac{\rho g \cos\theta\, \delta}{2\mu}\left[\delta - \frac{\delta^3}{3\delta^2}\right]$$

$$= \frac{\rho g \cos\theta\, \delta}{2\mu}\left[\frac{2\delta}{3}\right] = \frac{\rho g \cos\theta\, \delta^2}{3\mu}$$

$$\dot{Q} = \delta w \langle v_z \rangle \quad \left[\frac{m^3}{s}\right]$$

$$\dot{Q} = w\frac{\rho g \cos\theta\, \delta^3}{3\mu}$$

Sudden Heating Problem



before time 0, at T_0
at time 0, $T = T_\infty$

~~Assume~~
Energy Eq

$$\rho C_v\left[\frac{\partial T}{\partial t} + \overset{\text{solid}}{\cancel{\vec{v}\cdot\nabla T}}\right] = k\nabla^2 T + \overset{0}{\cancel{\dot{q}}} + \overset{\text{incomp}}{\cancel{p\nabla\cdot\vec{v}}} - \overset{0\ \text{dissipation}}{\cancel{\tau : \nabla\vec{v}}}$$

$$\rho C_v \frac{\partial T}{\partial t} = k\nabla^2 T \Rightarrow \rho C_v \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial t^2}$$

remember $\alpha = \frac{k}{\rho C_v}$

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial x^2}$$

apply $\theta = T - T_\infty$ and $\theta(x,0) = T_0 - T_\infty = \theta_0$
$\theta(0,t) = 0,\quad \theta(\infty,t) = \theta_0$

$$\frac{\partial\theta}{\partial t} = \alpha\frac{\partial^2\theta}{\partial x^2} \quad \text{and} \quad \theta = \phi(\eta),\ \eta = \frac{x}{\sqrt{4\alpha t}}$$

$$\frac{\partial\theta/\theta_0}{\partial t} = -\frac{1}{2}\frac{\eta}{t}\phi' \Rightarrow \phi'' + 2\eta\phi' = 0$$

integrate

$$\phi' = C_1\exp(-\eta^2),\quad \phi = C_1\int_0^\eta \exp(-\eta^2)\,d\eta + C_2$$

$$C_1 = \frac{1}{\int_0^\infty \exp(-\eta^2)\,d\eta},\quad C_2 = 0$$

$$\phi = \frac{2}{\pi}\int_0^\eta \exp(-\eta^2)\,d\eta = \text{erf}(\eta)$$

$$\theta = \theta_0\,\text{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

Find the gradient!

$$\frac{\partial T}{\partial x} = \frac{\partial\theta}{\partial x} = \frac{\partial\phi\theta}{\partial x} = \theta_0\frac{\partial\phi}{\partial\eta}\frac{\partial\eta}{\partial x}$$

$$\frac{\partial\phi}{\partial\eta} = \frac{2}{\sqrt{\pi}}\exp(-\eta^2),\quad \frac{\partial\eta}{\partial x} = \frac{1}{\sqrt{4\alpha t}}$$

$$\left.\frac{\partial T}{\partial x}\right|_0 = \theta_0\frac{2}{\sqrt{\pi}}\exp(-\eta^2)\left.\frac{1}{\sqrt{4\alpha t}}\right|_0 = \frac{\theta_0}{\sqrt{\pi\alpha t}} = \frac{T_0 - T_\infty}{\sqrt{\pi\alpha t}} \Rightarrow \delta_t = \sqrt{\pi\alpha t}$$

General Balance Equations

General form

$$\frac{d}{dt}\Psi + \nabla\cdot\Psi\vec{v} = -\nabla\cdot J + \dot{\Psi}_g$$

C.E

$$\frac{d}{dt}\rho + \nabla\bar{\rho}\vec{\bar{v}} + \underbrace{\nabla\cdot\overline{\rho' v'}}_{\text{turb}} = 0$$

M.E.

$$\frac{\partial\rho\vec{\bar{v}}}{\partial t} + \nabla\cdot\rho\vec{\bar{v}}\vec{\bar{v}} = -\nabla p - \nabla\cdot\tau + \rho\vec{g} - \underbrace{\nabla\cdot\rho\overline{\vec{v}'\vec{v}'}}_{\text{turbulent convection/shear}}$$

E.E

$$\frac{\partial\rho\left(\vec{u} + \frac{\vec{\bar{v}}^2}{2}\right)}{\partial t} + \nabla\rho\vec{v}\left(\vec{u} + \frac{\vec{v}^2}{2}\right) = -\underbrace{\nabla\bar{q}}_{\text{heat cond}} - \underbrace{\nabla\cdot(pu)}_{\text{pressure}} - \underbrace{\nabla\cdot(\tau\vec{v})}_{\text{shear}} + \underbrace{\rho(\vec{v}\cdot\vec{g})}_{\text{grav}} + \underbrace{\dot{q}}_{\text{gen}}$$

$$-k\nabla^2 T$$

$$\rho C_v\frac{\partial\bar{T}}{\partial t} + \rho C_v\vec{\bar{v}}\nabla\cdot\bar{T} = -k\nabla^2\bar{T} - \bar{T}\left.\frac{\partial p}{\partial T}\right)_\rho\nabla\cdot\vec{\bar{v}} - \tau:\nabla\vec{v} + \dot{q}$$

$$-\nabla\left(\rho C_v\overline{\vec{v}'T'}\right)$$

turbulent heat conduction

Control Volume Analysis

Mass Balance

$$\frac{d}{dt}\int \rho\, dV - \oint_{cs} \rho \vec{V}\cdot\vec{n}\, dA = (\rho \vec{V} A)_{in} - (\rho \vec{V} A)_{out} = \sum_i \dot{m}_i$$

Momentum Balance

momentum entering / leaving

$$\frac{d}{dt}\int \rho \vec{V}\, dV = \sum F + \sum_i \dot{m}\vec{V}$$

body and surf forces

Energy Balance

$$\frac{d}{dt}\int_{cv} \rho e\, dV = \dot{Q} + \dot{W}_s + \sum_i \dot{m}_i \left(e_i + \frac{p_i}{\rho_i}\right)$$

$$e = u + \cancel{\left(\frac{V^2}{2} + gz\right)}^{\,0}$$

Now the cases

LOCA

~~initially~~

$$\frac{d}{dt}\int_{cv} \rho\, dV = -(\rho \vec{V} A)_{break}$$

and

$$\frac{d}{dt}\int_{cv} \rho e\, dV = \dot{Q}_{decay} + \cancel{\dot{W}_{sP_1}}^{\,0} - (\dot{Q}_{SG} + \dot{Q}_{loss}) - \dot{m}_{break}\left(e + \frac{p}{\rho}\right)_{break} \cancel{+ \dot{m}\left(e + \frac{p}{\rho}\right)_{eccs}}$$

ECCS

$$\frac{d}{dt}\int_{cv} \rho\, dV = (\rho \vec{V} A)_{eccs} - (\rho \vec{V} A)_{break}$$

and

$$\frac{d}{dt}\int_{cv} \rho e\, dV = \dot{Q}_{decay} + \dot{W}_{sP_1} - \overbrace{\dot{Q}_{SG} + \dot{Q}_{loss}}^{small} - \dot{m}_{break}\left(e + \frac{p}{\rho}\right)_{break} + \dot{m}\left(e + \frac{p}{\rho}\right)_{eccs}$$

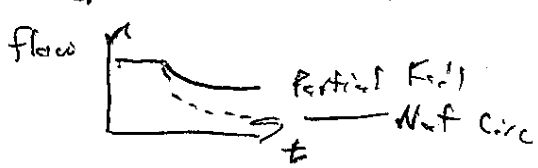
↳ approaches 0, pump coast down

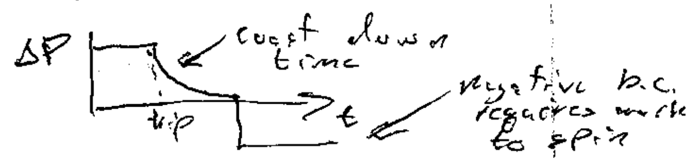
LOFA

$\dot{W}_{P_1} = 0$, $\dot{Q}_{SG} \rightarrow \dot{Q}_{NC}$ (Natural Circulation)

$\dot{Q}_c \rightarrow \dot{Q}_{decay}$

$$\frac{d}{dt}\int_{cv} \rho e\, dV = \dot{Q}_{decay} - (\dot{Q}_{NC} + \dot{Q}_{loss})$$





LOHS

$$\frac{d}{dt}\int_{cv}\rho e\, dV = (\cancel{\dot{Q}_c}^{\dot{Q}_{decay}} + \dot{W}_{SP1}) - (\cancel{\dot{Q}_{SG}}^{0} + \dot{Q}_{loss})$$

and the secondary loop malfunctions

$$\dot{Q}_{cond} = 0,\ \dot{W}_{PZ} = 0,\ \dot{Q}_{SG} = 0$$

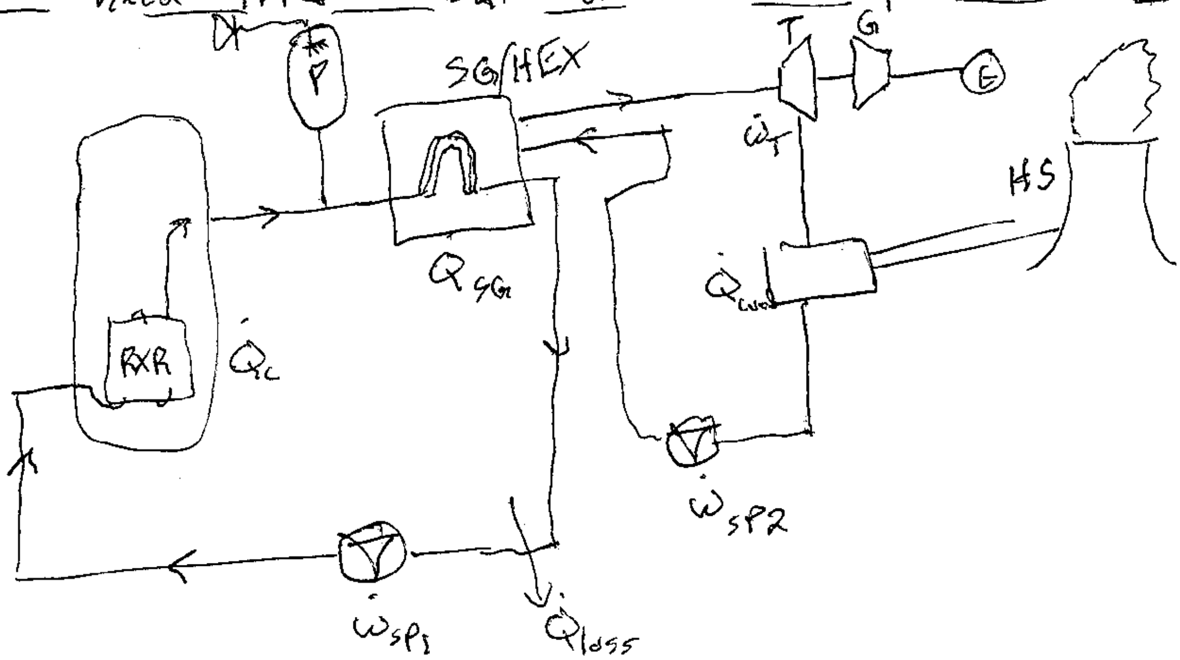
need HPIS to make up for LOHS

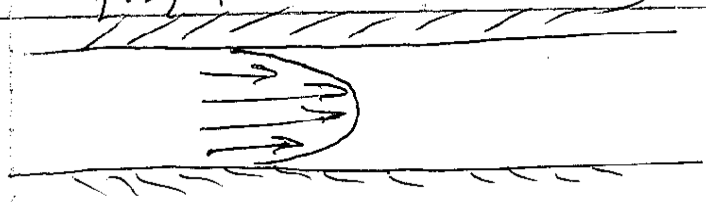
station Blackout

LOFA + LOHS + SCRAM

$$\frac{d}{dt}\int_{cv}\rho e\, dV = (\cancel{\dot{Q}_c}^{\dot{Q}_{decay}} + \cancel{\dot{W}_{SP1}}^{0}) - (\cancel{\dot{Q}_{SG}}^{\dot{Q}_{NC}} + \dot{Q}_{loss})$$

need HPIS but those need pumps!





$$\cancel{F}^{0} = \Delta P \pi r^2 - \tau 2\pi r l$$

$$\tau = -\mu \frac{\partial u}{\partial r}$$

$$\cancel{\frac{\partial u}{\partial r} = \left(\frac{\Delta P}{2\mu l}\right) r}$$

$$0 = \Delta P \pi r^2 - \tau 2\pi r l$$

$$\Delta P \pi r^2 = -\mu \frac{\partial u}{\partial r} 2\pi r l$$

$$\frac{\partial u}{\partial r} = \frac{\Delta P}{2\mu l} r$$

$$u(r) = \left(\frac{\Delta P}{4\mu l}\right) r^2 + C_1$$

$$u(D/2) = 0$$

$$C_1 = -\frac{\Delta P}{4\mu l}\frac{D^2}{4}$$

$$u(r) = \frac{\Delta P}{4\mu l} r^2 - \frac{\Delta P}{4\mu l}\frac{D^2}{4}$$

$$u(r) = \frac{\Delta P D^2}{16\mu l}\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

Natural Circulation

$$\frac{\partial}{\partial t}\rho\langle v_z\rangle - \frac{\partial}{\partial z}\rho\langle v_z v_z\rangle = -\frac{\partial\langle p\rangle}{\partial z} - \cancel{\frac{\partial\langle\tau_{zz}\rangle}{\partial z}}^{\,0}$$

(cancels itself out)

$$-\frac{4}{D}\tau_w + \rho\vec{g_z}$$

$$\oint \frac{\partial \rho v_i}{dt}dz = \sum_i \rho_i \frac{dv}{dt}L_i \qquad v_i = v_r\frac{a_r}{a_i}$$

$$= \rho\frac{dv_r}{dt}\sum_i\left(\frac{a_r}{a_i}\right)l_i$$

Boussinesque use buoyancy to determine forces from heating does change

$$\oint \frac{\partial \rho v_z v_z}{dt} = 0 \qquad \oint \frac{\partial\langle\tau_{zz}\rangle}{\partial z} = 0$$

$$\oint -\frac{\partial p\, dz}{dz} = \Delta p_{pump}$$

$$\oint \frac{-4\tau}{D_i} = \frac{\rho_i f_i v_i v_i}{2D} = \sum_i\left(\frac{fl}{D}+k_i\right)\frac{\rho v_r^2}{2}\left(\frac{a_r}{a_i}\right)^2$$

$$\oint \rho g_z\, dz = \rho g\beta l_h\left(T_{exit}-T_{inlet}\right)_{core}$$

(Integral Momentum Eq)

so

$$\rho_r\frac{dv_r}{dt}\sum_i\left(\frac{a_r}{a_i}\right)l_i = \Delta p_{pump} - \rho_r\vec{g}\beta\Delta T_h l - \frac{\rho_r v_r^2}{2}\sum_i\left(\frac{fl}{D}+k\right)\left(\frac{a_r}{a_i}\right)^2$$

1D forced Convection

Steady state

$$\frac{dv_r}{dt}=0,\quad \Delta p_{pump} = \text{const},\quad \rho_g\vec{g}\beta\Delta T_h l \to 0$$

$$\Delta p_{pump} = \sum_i\left(\frac{fl}{D}+k\right)_i\left(\frac{a_r}{a_i}\right)^2\frac{\rho v_r^2}{2}$$

Transient

$$\rho_r\frac{dv_r}{dt}\sum_i\left(\frac{a_r}{a_i}\right)l_i = \Delta p(t) - \frac{\rho_r v_r^2}{2}\sum_i\left(\frac{fl}{D}+k\right)_i\left(\frac{a_r}{a_i}\right)^2$$

Natural Circulation

Now has to be transient

$$\frac{\partial V_r}{\partial t} = 0, \; \Delta p_{pump} = 0$$

so

$$\rho g \beta \Delta T_n l_h = \frac{\rho_r V_r^2}{2} \sum_i \left(\frac{fl}{D} + k\right)\left(\frac{a_r}{a_i}\right)^2$$

$$-\rho g \beta l_{core} \Delta T_n + \sum \left(\frac{fl}{D} + K\right) \frac{\rho}{2} \left(\frac{a_r}{a_i}\right)^2 V_r^2 = 0$$

$$T_{exit} - T_{inlet} = \Delta T_n$$

$$\rho c_p \left(\frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial z}\right) = \frac{\xi q_0''}{A}$$

$\frac{\partial q_0''}{\partial t} = \text{small} \rightarrow q_0'' = \text{const}$, neglect thermal storage capacity $\cancel{\rho c_p \frac{\partial T_f}{\partial t}}^{0}$

$$\pi R_f^2 \dot{q}_f = 2\pi R_c q_0''$$

$$\partial T = \frac{\xi q_0''}{\rho c_p V_r A} \partial z$$

$$\Delta T_h = \int_{s_0}^{core} \frac{\xi q_0''}{\rho c_p V_r A} dz = \frac{\xi q_0'' l_H}{\rho c_p V_r A}$$

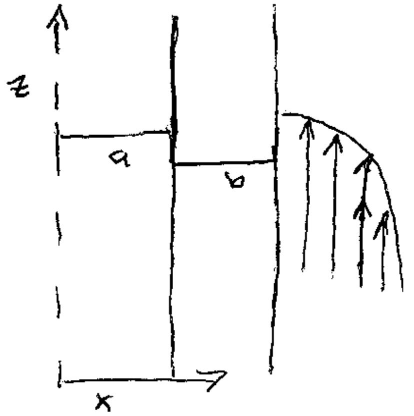
$$-\rho g \beta l_{core} \left(\frac{\xi q_0'' l_H}{\rho c_p V_r A}\right) + \sum \left(\frac{fl}{D} k\right) \frac{\rho}{2} \left(\frac{a_r}{a_i}\right)^2 V_r^2 = 0$$

$$V_r = \left[\frac{g \beta l_H l_{core} \xi q_0''}{1/2 \rho c_p A \cdot \sum \left(\frac{fl}{D} + k\right)\left(\frac{a_r}{a_i}\right)^2}\right]^{1/3}$$

Fuel Plate Heat Conduction/Convection

Refs

[1] - Lamarsh, Intro to Nuclear Engr
[2] - Bird, Transport Phenomena
[3] - Glasstone, Nuclear Reactor Engineering
[4] - Incropera, Fundamentals of Heat and Mass Xfer



Problem statement is 8 10B.3 in [2, p. 322] (for cyl.)

using Lamarsh [1, p.]

start w/ Poisson's eqn

$$\underbrace{\nabla^2 T}_{\text{conduction}} = \underbrace{\frac{q'''}{k}}_{\text{heat generation}}$$

for 1D Plate

$$\frac{\partial^2 T}{dx^2} = \frac{q'''}{k_f} \Rightarrow \frac{\partial T}{dx} = \frac{q'''}{k_f}x + C_1$$

b.c.

$$\left.\frac{dT}{dx}\right]_0 = 0 \Longrightarrow C_1 = 0$$

$$\frac{dT}{dx} = \frac{q'''}{k}x \Longrightarrow T = \frac{q'''}{2k_f}x^2 + C_2$$

$$T]_0 = T_m \Longrightarrow C_2 = T_m$$

$$T - T_m = -\frac{q'''}{2k_f}x^2 \quad \text{and} \quad T_s = T]_a = T_m - \frac{q'''}{2k_f}a^2$$

now find heat flow

$$q = q''' A a = \frac{T_m - T_s}{a/2k_f A}$$

Now the cladding

$$\nabla^2 T = \cancel{\frac{q'''}{k_c}}\ 0$$

for 1D plate

$$\frac{d^2T}{dx^2} = 0 \Rightarrow \frac{dT}{dx} = C_1 \Rightarrow T = C_1 x + C_2$$

b.c.

$$T\Big]_a = T_s \quad , \quad T\Big]_{a+b} = T_c$$

so

$$T_s = C_1 a + C_2 \Rightarrow C_2 = T_s - C_1 a$$

$$T_c = C_1 a + C_1 b + C_2' \Rightarrow T_c = \cancel{C_1 a} + C_1 b + T_s - \cancel{C_1 a}$$

$$C_1 = \frac{T_c - T_s}{b} = -\frac{T_s - T_c}{b}$$

$$C_2 = T_s + \frac{T_s - T_c}{b} a$$

$$T = -\frac{T_s - T_c}{b} x + T_s + \frac{T_s - T_c}{b} a$$

$$T = T_s - \frac{x-a}{b}(T_s - T_c) \quad \text{in cladding}$$

then find heat flow

$$q = q'' A = -k_c \nabla T A = \frac{k_c}{b}(T_s - T_c) A$$

$$T_s - T_c = q \frac{b}{k_c A}$$

so $$T = T_s - \frac{x-a}{b}\left(q \frac{b}{k_c A}\right) = T_s - \frac{x-a}{b}\left[\frac{T_m - T_s}{a/2k_f A} \frac{b}{k_c A}\right]$$

or

$$T_m - T_c = q \underbrace{\left(\frac{a}{2k_f A} + \frac{b}{k_c A}\right)}_{R}$$

Finally in the coolant we use

$$R = \frac{1}{hA}$$

so

$$q = \frac{T_m - T_b}{R}$$ ← bulk coolant temp

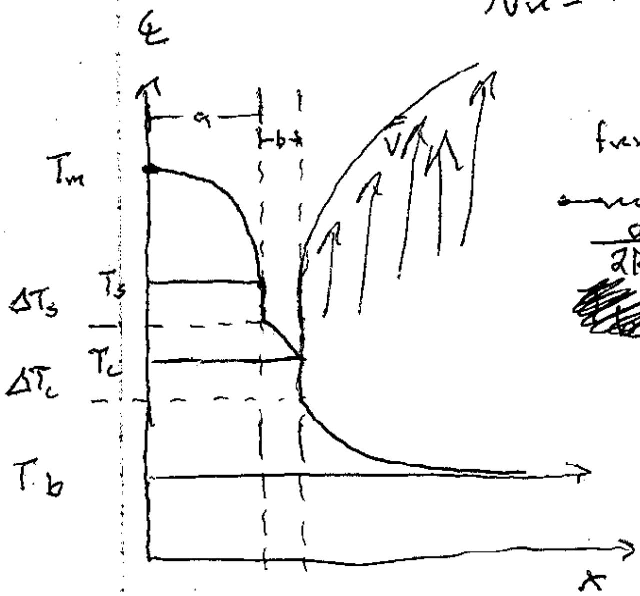
with

$$R = \Big[\underbrace{\frac{a}{2k_f A}}_{\text{fuel}} + \underbrace{\frac{b}{k_c A}}_{\text{cladding}} + \underbrace{\frac{1}{hA}}_{\text{coolant}}\Big]$$

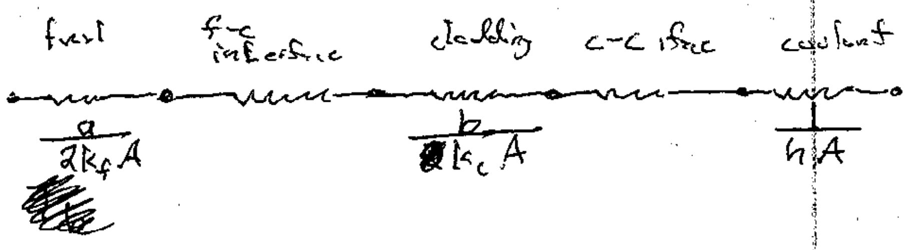
with $h = \bar{h}$ from Nusselt number from [4]

$$\overline{Nu} = C_1 Re^{m} Pr^{n} \quad \text{for laminar}$$

$$Nu = \frac{hD}{k}$$

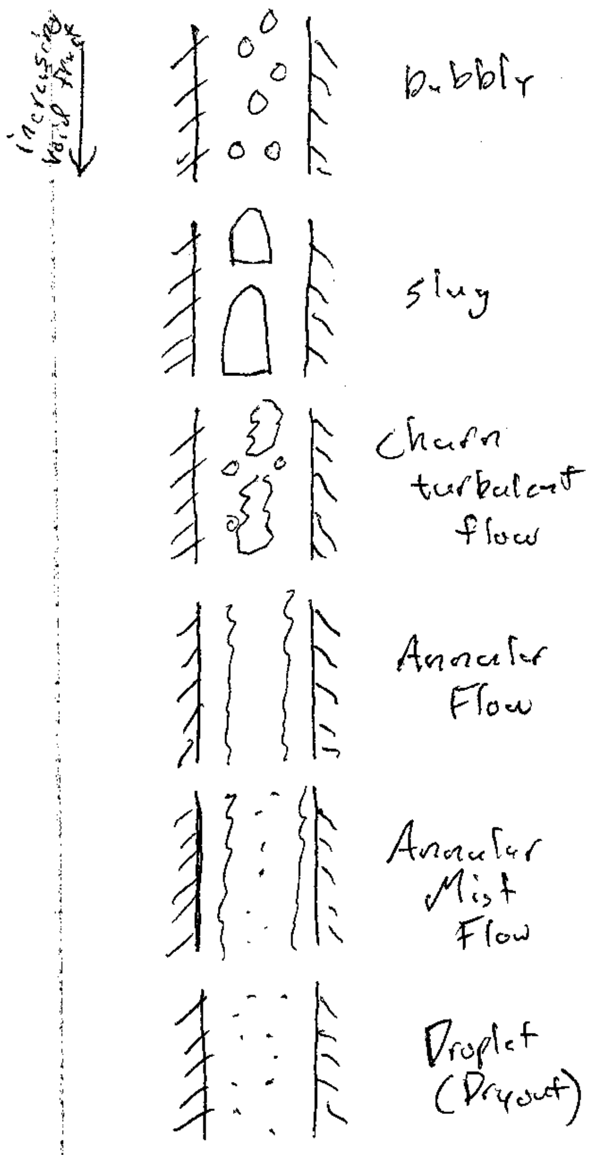


Resistance Diagram

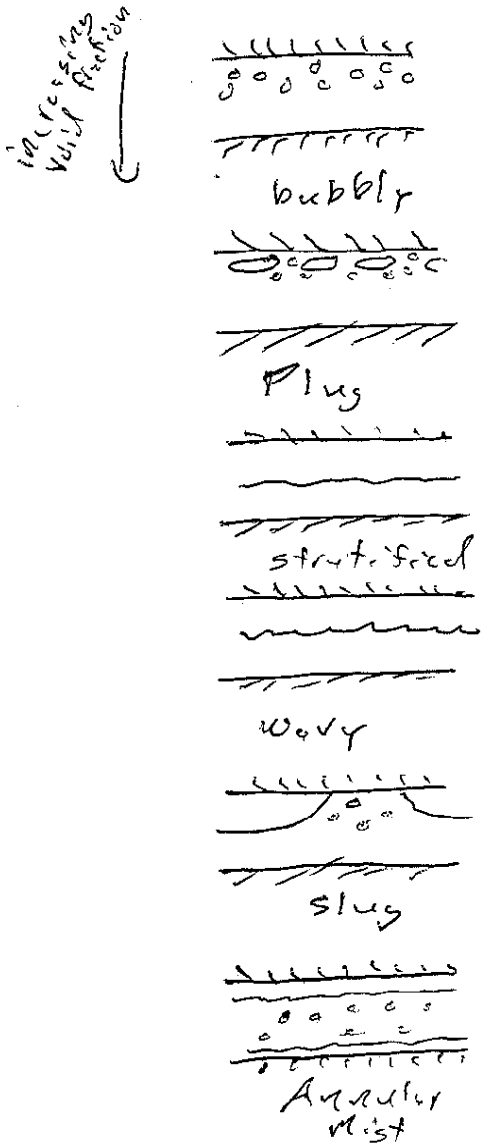


2 Phase Flow Regimes

Vertical Adiabatic



Horizontal Adiabatic



Example CANDU