Steady Heat Conduction

Fourier Law : $q = -kA \cdot \frac{dT}{dx}$: $q'' = q/A$ – heat flux

$q'' = -k\frac{dT}{dx}$

k- thermal conductivity – generally depends on temperature
For UO_2 : $k(T)$ decreases with T.

Combustion Engineering: $k = \frac{38.24}{402.4+T} + 6.1256\times10^{-13}(T+273)^3$.

$k = W/cm\,^\circ C$

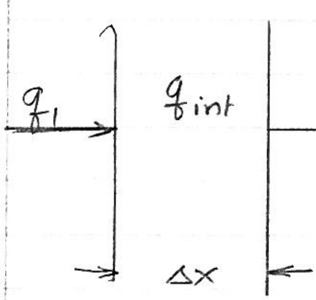
$T = {}^\circ C$.

Westinghouse : $k = \frac{1}{11.8+0.0238T} + 8.775\times10^{-13}T^3$.

k : for Zircaloy 2 (400°C) 13 W/m°C

for S.S 316 (400°C) 23 W/m°C

Conduction with Heat Source



$q_2 = q_1 + q_{int}$: $q_2 - q_1 = q_{in}$

$= q_1 + \frac{dq}{dx}\Big|\Delta x + \frac{d^2q}{dx^2}\Big|\frac{\Delta x^2}{2} + \cdots$

$\lim_{\Delta x \to 0} \frac{dq}{dx}\Big|\Delta x + \frac{d^2q}{dx^2}\Big|\Delta x^2 + \cdots = q_{int}$

$\therefore \frac{dq}{dx} = \frac{q_{int}}{\Delta x}$ $\quad q = -kA\frac{dT}{dx}$

$\therefore \frac{d}{dx}\left(-kA\frac{dT}{dx}\right) = \frac{q_{int}}{\Delta x}$ If A, k – Constant

or $-k\frac{d^2T}{dx^2} = \frac{q_{int}}{A\,\Delta x} = q''' \leftarrow$ Poisson Eqn (1-D)

General Conduction Equation in Cartesian Coordinate

$$\frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + \frac{\partial}{\partial y} k \frac{\partial T}{\partial y} + \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + q''' = 0$$

$$\nabla \cdot (k \nabla T) + q''' = 0$$

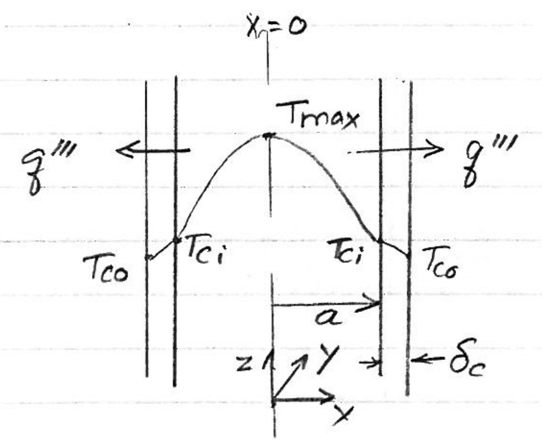
Applications.

Fuel Plate: Symmetric; thin plate $k\frac{\partial T}{\partial y} \simeq k\frac{\partial T}{\partial z} = 0$.

$$\frac{\partial}{\partial x} k \frac{\partial T}{\partial x} + q''' = 0$$

Integrate $k\frac{\partial T}{\partial x} + q'''x = C_1$

At $x=0$ center plane $\quad k\left.\frac{dT}{dx}\right|_{x=0} = 0$



$$k\frac{dT}{dx} + q'''x = 0$$

$$\int_{T_{max}}^{T} k\,dT + \int_0^x q'''x\,dx = 0$$

$$\int_T^{T_{max}} k\,dT = q'''\frac{x^2}{2}.$$

k- constant ; $x=a$, $T=T_{ci}$

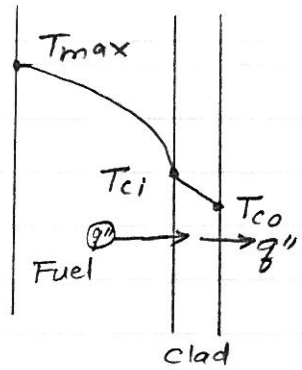
$$T_{max} - T_{ci} = q'''\frac{a^2}{2k} = \frac{q''}{2k/a} \quad : q'' = q'''\cdot a.$$

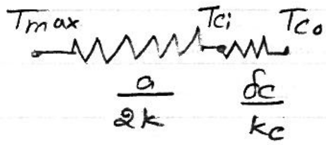
$$V = \frac{q''}{R} \qquad R = \frac{1}{2k/a}$$

Cladding $\quad q''' = 0 \quad : \quad k\frac{dT}{dx} = \text{constant} = \text{heat flux}$

$$-k_c\frac{dT}{dx} = q'' \quad \text{- integrating}$$

$$T_{co} = T_{ci} - \frac{q''\delta_c}{k_c} \quad \therefore \quad R = \frac{1}{k_c/\delta}$$





$$T_{co} = T_{max} - q''\left(\frac{a}{2k} + \frac{\delta_c}{k_c}\right)$$

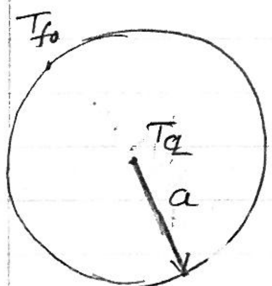
$$q'' = \frac{T_{max} - T_{co}}{\frac{a}{2k} + \frac{\delta_c}{k_c}}$$

$$I = \frac{V}{R}$$

Cylindrical Fuel Pin

Fourier Law $\quad q'' = -kr\frac{dT}{dr}$

Poisson Equation $\quad -\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = q'''$



Integrate once:

$$k_f\frac{dT}{dr} + q'''\frac{r}{2} + \frac{C_1}{r} = 0 \qquad -(1)$$

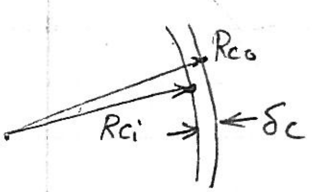
at $r = 0$ $\quad \frac{dT}{dr} = 0; \therefore C_1 = 0$

Integrating again: $r = 0$, to a, $T = T_{\mathcal{L}}$ to T_{fo}

$$(T_{\mathcal{L}} - T_{fo}) = q'''\frac{\pi a^2}{4\pi} = \frac{q'}{4\pi k_f} \qquad q' = q'''\pi a^2$$

Clad

For thin clad $\quad -k_c\frac{dT}{dr} = q'' = \frac{q'}{2\pi R_{ci}}$



$$-k_c\frac{T_{co} - T_{ci}}{\delta_c} = \frac{q'}{2\pi R_{ci}}$$

$$T_{ci} - T_{co} = \frac{q'}{2\pi k_c R_{ci}/\delta_c} \qquad R_{clad} = \frac{\delta_c}{2\pi R_{ci} k_c}$$

Thick clad:

solve cylindrical problem

$$R_{clad} = \frac{\ln\left(\frac{R_{co}}{R_{ci}}\right)}{2\pi k_c}$$

$$q''' \equiv W/m^3$$

$$q'' = \text{heat flux } W/m^2$$

$$q' = \text{linear heat rate } W/m$$

$$q' = q''' \pi a^2$$

$$q_a'' = \frac{q'}{2\pi a}$$