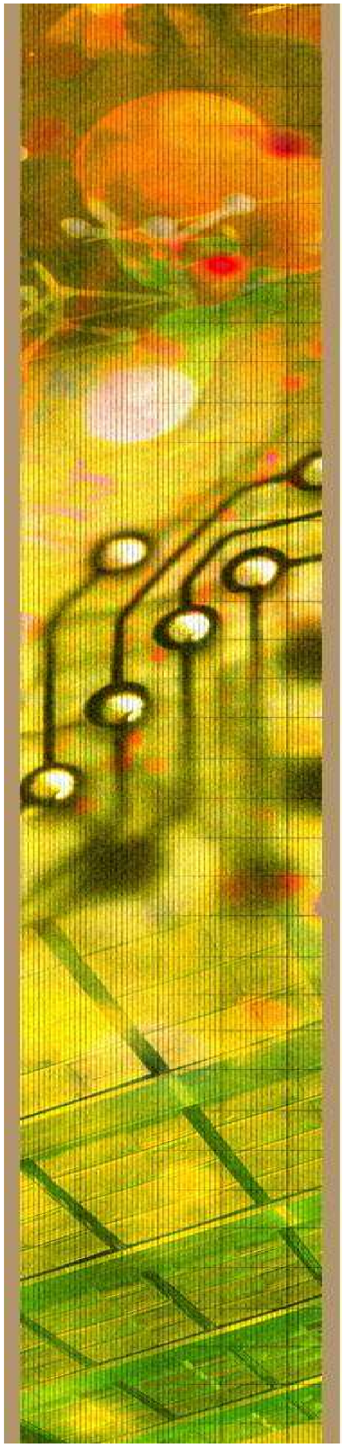

NUCL 510
Nuclear Reactor Theory

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Lecture Note 6

Prof. Won Sik Yang

Purdue University
School of Nuclear Engineering

One-Group One-Region Problem (1)

- One-group diffusion equation with constant cross sections

$$-\nabla \cdot D(\vec{r})\nabla \phi(\vec{r}) + \Sigma_a(\vec{r})\phi(\vec{r}) = \lambda \nu \Sigma_f(\vec{r})\phi(\vec{r})$$

$$-D\nabla^2\phi(\vec{r}) + \Sigma_a\phi(\vec{r}) = \lambda \nu \Sigma_f \phi(\vec{r}) \quad \text{(for constant cross sections)}$$

$$\nabla^2\phi(\vec{r}) + \frac{\lambda\nu\Sigma_f - \Sigma_a}{D}\phi(\vec{r}) = 0 \qquad \frac{\lambda\nu\Sigma_f - \Sigma_a}{D} = B^2$$

$$\nabla^2\phi(\vec{r}) + B^2\phi(\vec{r}) = 0 \quad \text{(Helmholtz equation)}$$

- For λ=1, B^2 is purely a material property called the <u>material buckling</u>

$$B_m^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} = \frac{k_\infty - 1}{L^2}, \quad k_\infty = \frac{\nu\Sigma_f}{\Sigma_a}, \quad L^2 = \frac{D}{\Sigma_a} \text{ (diffusion area)}$$

- Zero flux conditions at extrapolated outer boundaries are often used
  – This yields a homogeneous system of equations, which has a non-trivial solution only for certain values of B^2, and thus <u>an eigenvalue problem</u>; The <u>smallest</u> B^2 is called the <u>geometrical buckling</u>

$$B^2 = -\frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})} \quad \text{(constant curvature)}$$

One-Group One-Region Problem (2)

- Nontrivial solution exists only when B^2 is equal to the geometrical buckling

$$B^2 = \frac{\lambda \nu \Sigma_f - \Sigma_a}{D} = B_g^2$$

- Thus, the off-criticality is given by

$$\frac{1}{\lambda} = k = \frac{\nu \Sigma_f}{D B_g^2 + \Sigma_a} = \frac{\nu \Sigma_f}{\Sigma_a} \frac{1}{1 + L^2 B_g^2} = k_\infty P_{NL} \quad (P_{NL}:\ \text{non-leakage probability})$$

- Using the material buckling, the off-criticality can be written as

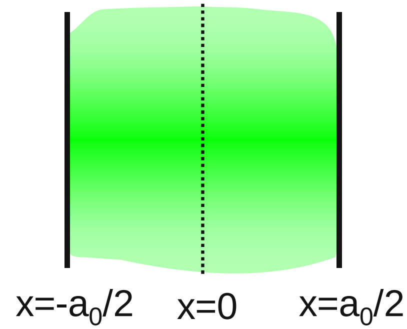
$$\frac{1}{\lambda} = k = \frac{1 + L^2 B_m^2}{1 + L^2 B_g^2}$$

- The material and geometrical buckling determines the off-criticality

$$\begin{aligned}
B_m^2 > B_g^2 &\quad \Rightarrow \quad k > 1 \quad (\lambda < 1) \quad \text{super-critical} \\
B_m^2 = B_g^2 &\quad \Rightarrow \quad k = 1 \quad (\lambda = 1) \quad \text{critical} \\
B_m^2 < B_g^2 &\quad \Rightarrow \quad k < 1 \quad (\lambda > 1) \quad \text{sub-critical}
\end{aligned}$$

1-D Plane Geometry Problem (1)

- Eigenvalues and eigenfunctions



$$\frac{a}{2} = \frac{a_0}{2} + 0.711\lambda_{tr}$$

$$\frac{d^2\phi}{dx^2} + B^2\phi = 0 \quad \Rightarrow \quad \phi(x) = A\cos(Bx) + C\sin(Bx)$$

$$\phi(\pm a/2) = 0 \quad \Rightarrow \quad \phi(\pm a/2) = A\cos(Ba/2) \pm C\sin(Ba/2) = 0$$

$$\begin{bmatrix} \cos(Ba/2) & \sin(Ba/2) \\ \cos(Ba/2) & -\sin(Ba/2) \end{bmatrix} \begin{bmatrix} A \\ C \end{bmatrix} = 0$$

$\exists$ a non-trivial solution iff the determinant $-2\cos(Ba/2)\sin(Ba/2) = 0$

$$1)\ \cos\frac{Ba}{2} = 0 \quad \Rightarrow \quad \frac{Ba}{2} = \frac{n\pi}{2}, \quad n = 1, 3, 5, \cdots \quad \Rightarrow \quad \sin\frac{Ba}{2} \neq 0 \quad \Rightarrow \quad C = 0$$

$$\Rightarrow \quad \phi(x) = A\cos Bx$$

$$2)\ \sin\frac{Ba}{2} = 0 \quad \Rightarrow \quad \frac{Ba}{2} = \frac{n\pi}{2}, \quad n = 2, 4, 6, \cdots \quad \Rightarrow \quad \cos\frac{Ba}{2} \neq 0 \quad \Rightarrow \quad A = 0$$

$$\Rightarrow \quad \phi(x) = C\sin Bx$$

$$B_n = \frac{n\pi}{a}, \quad \phi(x) = \begin{cases} C_n \cos B_n x & \text{for } n = 1, 3, 5, \cdots \\ C_n \sin B_n x & \text{for } n = 2, 4, 6, \cdots \end{cases}$$

1-D Plane Geometry Problem (2)

- Fundamental solution

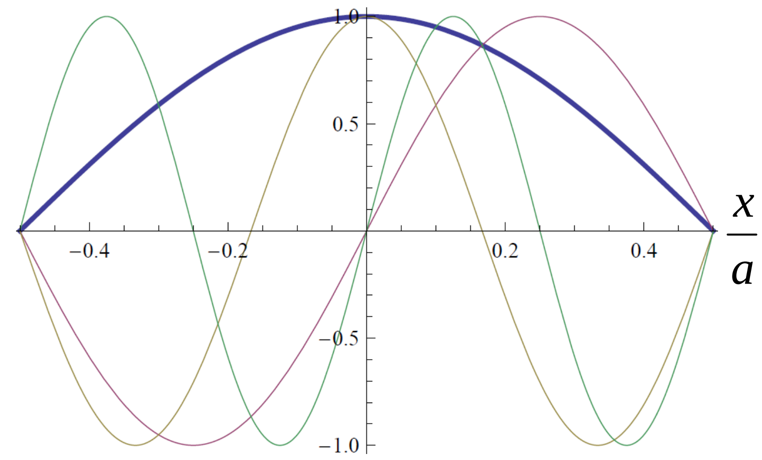
$$\phi(x) \geq 0 \text{ everywhere} \quad \Rightarrow \quad n = 1$$

$$\phi(x) = C_1 \cos\frac{\pi x}{a}$$

  – For time-dependent problems, the flux can be expanded in terms of eigenfunctions as

$$\phi(x,t) = \sum_{n=1,3,5,\cdots} C_n(t)\cos B_n x + \sum_{n=2,4,6,\cdots} C_n(t)\sin B_n x$$

  – For a critical state, the higher order terms decay away and the flux approaches the fundamental solution



- Material and geometrical buckling

$$B_m^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} \qquad B_g^2 = \left(\frac{\pi}{a}\right)^2 = -\frac{1}{\phi}\frac{d^2\phi}{dx^2}$$

- Criticality condition

$$B_m^2 = B_g^2 \quad \Rightarrow \quad \frac{\nu\Sigma_f - \Sigma_a}{D} = \left(\frac{\pi}{a}\right)^2$$

- Off-criticality

$$\frac{1}{\lambda} = k = \frac{\nu\Sigma_f}{DB_g^2 + \Sigma_a} = \frac{k_\infty}{1 + L^2 B_g^2}$$

Eigenvalue and Criticality (1)

- Operator form of multi-group diffusion equation

$$\mathbf{M\varphi} = \mathbf{F\varphi} + \mathbf{s}$$

$$\mathbf{M} = \mathbf{L} - \tilde{\mathbf{S}} = \begin{bmatrix} -\nabla \cdot D_1(\vec{r})\nabla + \Sigma_{r1}(\vec{r}) & -\Sigma_{s21}(\vec{r}) & \cdots & -\Sigma_{sG1}(\vec{r}) \\ -\Sigma_{s12}(\vec{r}) & -\nabla \cdot D_2(\vec{r})\nabla + \Sigma_{r2}(\vec{r}) & \cdots & -\Sigma_{sG2}(\vec{r}) \\ \vdots & \vdots & \ddots & \vdots \\ -\Sigma_{s1G}(\vec{r}) & -\Sigma_{s2G}(\vec{r}) & \cdots & -\nabla \cdot D_G(\vec{r})\nabla + \Sigma_{rG}(\vec{r}) \end{bmatrix}$$

$$\mathbf{F} = \chi \mathbf{f}^T = \begin{bmatrix} \chi_1 \nu\Sigma_{f1} & \chi_1 \nu\Sigma_{f2} & \cdots & \chi_1 \nu\Sigma_{fG} \\ \chi_2 \nu\Sigma_{f1} & \chi_2 \nu\Sigma_{f2} & \cdots & \chi_2 \nu\Sigma_{fG} \\ \vdots & \vdots & \ddots & \vdots \\ \chi_G \nu\Sigma_{f1} & \chi_G \nu\Sigma_{f2} & \cdots & \chi_G \nu\Sigma_{fG} \end{bmatrix}$$

If there is no independent source, a non-trivial solution can be found only when the system is critical

- λ-Eigenvalue problem
  - To obtain a non-trivial solution, the fission source is modified by a factor λ to the degree of off-criticality

$$(\mathbf{M} - \lambda \mathbf{F})\mathbf{\varphi} = 0$$

  - λ is a scalar parameter that makes **(M-λF)** singular

Eigenvalue and Criticality (2)

- Off-criticality

$\lambda=1$: no adjustment of ν $\Rightarrow$ critical ($k=1$)

$\lambda>1$: artificial increase of ν $\Rightarrow$ subcritical ($k<1$)

$\lambda<1$: artificial decrease of ν $\Rightarrow$ supercritical ($k>1$)

- Multiplication factor
  - An inner product of the operator form of diffusion equation with a unit weighting function $\mathbf{w}=(1,1,\ldots,1)^T$ yields

$$(\mathbf{w},\mathbf{M\varphi})=(\mathbf{w},\lambda\mathbf{F\varphi})=\lambda(\mathbf{w},\mathbf{F\varphi})$$

$$(\mathbf{w},\mathbf{F\varphi})=\sum_{g=1}^{G}\chi_g\sum_{g'=1}^{G}\int_V dV\nu\Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r})=\sum_{g'=1}^{G}\int_V dV\nu\Sigma_{fg'}(\vec{r})\phi_{g'}(\vec{r})=\text{ total production}$$

$$(\mathbf{w},\mathbf{M\varphi})=-\sum_{g=1}^{G}\int_V dV\nabla\cdot D_g(\vec{r})\nabla\phi_g(\vec{r})+\sum_{g=1}^{G}\int_V dV\Sigma_{rg}(\vec{r})\phi_g(\vec{r})+\sum_{g=1}^{G}\sum_{g'}^{G}\int_V dV\Sigma_{sg'g}(\vec{r})\phi_{g'}(\vec{r})$$

$$=-\sum_{g=1}^{G}\int_V dV\nabla\cdot D_g(\vec{r})\nabla\phi_g(\vec{r})+\sum_{g=1}^{G}\int_V dV\Sigma_{ag}(\vec{r})\phi_g(\vec{r})\ =\text{total loss}$$

$$k=\frac{\text{total production}}{\text{total loss}}=\frac{(\mathbf{w},\mathbf{F\varphi})}{(\mathbf{w},\mathbf{M\varphi})}=\frac{1}{\lambda}$$

Eigenvalue vs. Source Problem (1)

- Source problem in non-multiplying medium (**F**=0)

$$\mathbf{M\varphi = s} \qquad \Rightarrow -D\frac{d^2\phi(x)}{dx^2} + \Sigma_a\phi(x) = s(x), \quad \phi\left(\pm\frac{a}{2}\right) = 0 \quad \text{(in 1-D plane geometry)}$$

$$\Rightarrow \frac{d^2\phi(x)}{dx^2} - \frac{\Sigma_a}{D}\phi(x) = -\frac{s(x)}{D} \qquad \Rightarrow \frac{d^2\phi(x)}{dx^2} - \frac{1}{L^2}\phi(x) = \tilde{s}(x)$$

  – General solution

$$\phi(x) = \phi_h(x) + \phi_p(x) \quad [\phi_p(x)\text{: particular solution depending on source } \tilde{s}(x)]$$

$$\phi_h(x) = Ae^{x/L} + Ce^{-x/L} = \tilde{A}\cosh(x/L) + \tilde{C}\sinh(x/L)$$

  – Boundary condition

$$\begin{aligned} \phi(a/2) &= Ae^{a/2L} + Ce^{-a/2L} + \phi_p(a/2) = 0 \\ \phi(-a/2) &= Ae^{-a/2L} + Ce^{a/2L} + \phi_p(a/2) = 0 \end{aligned} \quad \Rightarrow \begin{bmatrix} e^{a/2L} & e^{-a/2L} \\ e^{-a/2L} & e^{a/2L} \end{bmatrix} \begin{bmatrix} A \\ C \end{bmatrix} = -\begin{bmatrix} \phi_p(a/2) \\ \phi_p(a/2) \end{bmatrix}$$

    - *The coefficients A and C are uniquely determined for given source distribution*

  – There always exists a physically realizable solution
  – Physically, a stationary flux distribution is sustained by continuously supplied source neutrons

Eigenvalue vs. Source Problem (2)

- Source problem in subcritical multiplying medium (i.e., $\Sigma_a > \nu\Sigma_f$)

$$\mathbf{M\varphi = F\varphi + s} \qquad \Rightarrow -D\frac{d^2\phi(x)}{dx^2} + \Sigma_a\phi(x) = \nu\Sigma_f\phi(x) + s(x), \quad \phi\left(\pm\frac{a}{2}\right) = 0$$

$$\Rightarrow \frac{d^2\phi(x)}{dx^2} + B_m^2\phi(x) = -\frac{s(x)}{D}, \quad B_m^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} < 0 \qquad \Rightarrow \frac{d^2\phi(x)}{dx^2} - |B_m|^2\,\phi(x) = \tilde{s}(x)$$

  – General solution

$$\phi(x) = \phi_h(x) + \phi_p(x) \quad [\phi_p(x)\text{: particular solution depending on source } \tilde{s}(x)]$$

$$\phi_h(x) = Ae^{|B_m|x} + Ce^{-|B_m|x} = \tilde{A}\cosh(|B_m|x) + \tilde{C}\sinh(|B_m|x)$$

  – Boundary condition

$$\begin{bmatrix} e^{|B_m|a/2} & e^{-|B_m|a/2} \\ e^{-|B_m|a/2} & e^{|B_m|a/2} \end{bmatrix} \begin{bmatrix} A \\ C \end{bmatrix} = -\begin{bmatrix} \phi_p(a/2) \\ \phi_p(a/2) \end{bmatrix}$$

    - *The coefficients A and C are uniquely determined for given source distribution*

  – There always exists a physically realizable solution
  – In the limit of $s \to 0$, the only physically realizable solution is the trivial solution

Eigenvalue vs. Source Problem (3)

- Source multiplication
  - In a subcritical multiplying medium, the source neutrons are multiplied in the form of fission neutrons
  - The number of fission neutrons per source neutron is called the source multiplication factor

$$M_s = \int_V dV \int_E dE \nu\Sigma_f(\vec{r},E)\phi(\vec{r},E) / \int_V dV \int_E dE s(\vec{r},E)$$

  - An inner product with a weighting function **w** yields

$$\mathbf{M}\boldsymbol{\varphi} = \mathbf{F}\boldsymbol{\varphi} + \mathbf{s} \quad \Rightarrow (\mathbf{w},\mathbf{M}\boldsymbol{\varphi}) = (\mathbf{w},\mathbf{F}\boldsymbol{\varphi}) + (\mathbf{w},\mathbf{s})$$

  - Using the off-criticality of the system, the left-hand side can be written as

$$\lambda(\mathbf{w},\mathbf{F}\boldsymbol{\varphi}) = (\mathbf{w},\mathbf{F}\boldsymbol{\varphi}) + (\mathbf{w},\mathbf{s}) \;\Rightarrow (\lambda-1)(\mathbf{w},\mathbf{F}\boldsymbol{\varphi}) = (\mathbf{w},\mathbf{s})$$

$$M_s = \frac{(\mathbf{w},\mathbf{F}\boldsymbol{\varphi})}{(\mathbf{w},\mathbf{s})} = \frac{1}{\lambda-1} = \frac{1}{1/k-1} = \frac{1}{-\rho}$$

  - The degree of off-criticality is generally represented by reactivity

$$\rho = 1 - \frac{1}{k} = \frac{k-1}{k} = \frac{\Delta k}{k}$$

$$\begin{aligned} \text{subcritical } (k<1) &\quad \rho<0 \\ \text{critical } (k=1) &\quad \rho=0 \\ \text{supercritical } (k>1) &\quad \rho>0 \end{aligned}$$

Eigenvalue vs. Source Problem (4)

- Source problem in supercritical multiplying medium (i.e., $\Sigma_a < \nu\Sigma_f$)

  $\mathbf{M\varphi} = \mathbf{F\varphi} + \mathbf{s}$ more fission than loss

  $\mathbf{M\varphi} - \mathbf{F\varphi} = \mathbf{s}$ possible only if flux is negative at least in some subdomain

  $$\rho > 0 \quad \Rightarrow \quad (\mathbf{w}, \mathbf{F\varphi}) = \frac{<\mathbf{w}, \mathbf{s}>}{-\rho} < 0 \quad \Rightarrow \quad \text{some components of } \boldsymbol{\varphi} < 0$$

  – There exists only a mathematical solution, which is not physically allowed

- Eigenvalue problem

  $\boxed{(\mathbf{M} - \lambda\mathbf{F})\boldsymbol{\varphi}_\lambda = 0}$

  – To obtain non-trivial solutions, the fission source is modified by an arbitrary factor λ

  – There exists a physically allowed solution (i.e., non-negative), which is called a lambda mode flux

  – But it cannot be physically realized except for a critical problem, since the fission source is arbitrarily modified

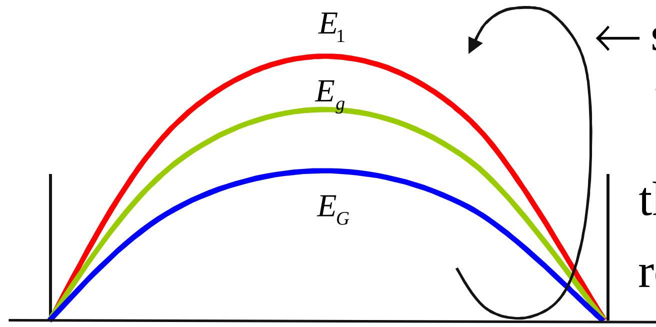
Types of Solutions of the Neutron Balance Equation

- Physically Realizable Non-trivial Solution
  - Source problem in non-multiplying medium
  - Source problem in subcritical medium
  - Eigenvalue problem for critical reactor
- Physically Allowed Solution
  - Eigenvalue problem for non-critical reactor
    - *λ-mode flux, not realized in reality*
- Mathematical Solution
  - Higher mode solution in the eigenvalue problem
  - Source problem in supercritical system
- Trivial Problem
  - Subcritical system with no source
  - Critical system with no initial action

Separation of Space and Energy Dependencies

- Fundamental Theorem of Reactor Theory (by Weinberg and Wigner)
  - In a bare one-region reactor, the neutron flux is separable in its space and energy dependence

$$\phi(\vec{r},E)=\phi(\vec{r})\varphi(E)$$



- Solution of diffusion equation by separation of variables
  - Inserting the separation into the one-region diffusion equation yields

$$-D(E)\nabla^2\phi(\vec{r})\varphi(E)+\Sigma_t(E)\phi(\vec{r})\varphi(E)$$

$$=\int_{E'}dE'\Sigma_s(E'\to E)\phi(\vec{r})\varphi(E)+\lambda\chi(E)\int_{E'}dE'\nu\Sigma_f(E')\phi(\vec{r})\varphi(E)$$

  - Dividing by $D(E)\phi(r)\varphi(E)$ separates out the space and energy dependencies

$$-\frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})}=\frac{1}{D(E)\varphi(E)}\left[-\Sigma_t(E)\varphi(E)+\int_0^\infty dE'\Sigma_s(E'\to E)\varphi(E')\right.$$

$$\left.+\lambda\chi(E)\int_0^\infty dE'\nu\Sigma_f(E')\varphi(E')\right]=B^2 \quad \text{(constant)}$$

Equations for Space and Energy Dependencies

- Equation for space dependency

$$-\frac{\nabla^2\phi(\vec{r})}{\phi(\vec{r})}=B^2 \qquad \Rightarrow \nabla^2\phi(\vec{r})+B^2\phi(\vec{r})=0$$

  - Wave or Helmholtz equation to determine the <u>fundamental mode flux shape</u>
  - B^2 represents the geometrical curvature of the flux, and it is an eigenvalue to be determined from boundary conditions
    - *The <u>smallest eigenvalue</u> B^2 is called the <u>geometrical buckling</u>*

- Equation for energy dependency

$$[D(E)B^2+\Sigma_t(E)]\varphi(E)-\int_0^\infty dE'\Sigma_s(E'\rightarrow E)\varphi(E')=\lambda\chi(E)\int_0^\infty dE'\nu\Sigma_f(E')\varphi(E')$$

  - Integral equation for <u>fundamental or asymptotic spectrum</u> ($\lambda=1$)
    - *$\phi(r,E)$ is not separable around interfaces, but it is separable far away from interfaces (e.g., <u>asymptotic spectrum in a large medium</u>)*
  - The asymptotic spectrum is independent of boundary conditions
    - *It is independent of the size of the core, and it is the same in the reflected core as in an un-reflected core of the same material, provided the regions are large enough*

Types of Solutions of Separated Equations (1)

- Multiplication factor (k) for given geometry and composition
  - Solve the spatial equation with the boundary conditions and determine the geometrical buckling

$$\nabla^2\phi(\vec{r}) + B_g^2\phi(\vec{r}) = 0$$

  - Using the geometrical buckling, solve the energy equation and determine the smallest eigenvalue λ and the λ-mode spectrum $\varphi_\lambda(E) \rightarrow k = 1/\lambda$

$$[D(E)B_g^2 + \Sigma_t(E)]\varphi_\lambda(E) - \int_0^\infty dE'\Sigma_s(E' \rightarrow E)\varphi_\lambda(E') = \lambda\chi(E)\int_0^\infty dE'\nu\Sigma_f(E')\varphi_\lambda(E')$$

- Infinite medium multiplication factor (k) for a given composition
  - In an infinite medium $B^2 = 0$, and thus determine the eigenvalue λ_∞ and the infinite spectrum $\varphi_\infty(E)$ by solving the energy equation

$$\Sigma_t(E)\varphi_\infty(E) - \int_0^\infty dE'\Sigma_s(E' \rightarrow E)\varphi_\infty(E') = \lambda_\infty\chi(E)\int_0^\infty dE'\nu\Sigma_f(E')\varphi_\infty(E')$$

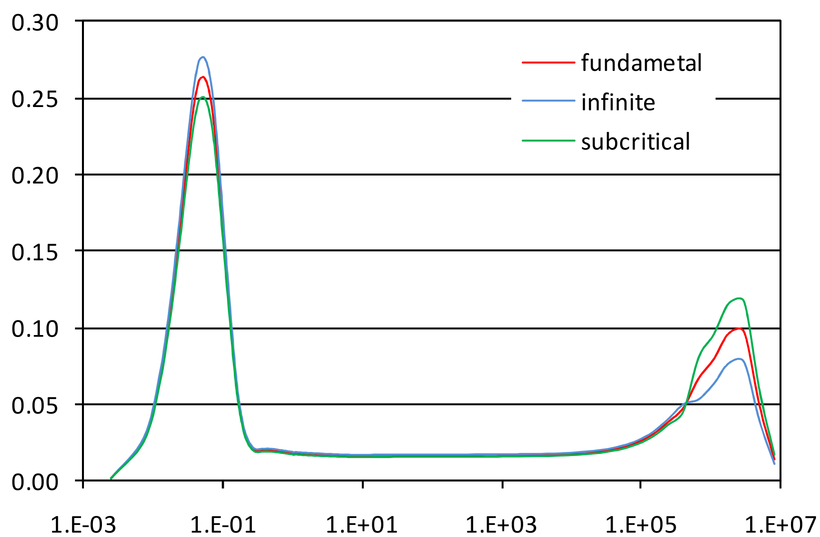
$$k_\infty^0 = \frac{1}{\lambda_\infty} = \frac{\int_0^\infty \chi(E)\int_0^\infty \nu\Sigma_f(E')\varphi_\infty(E')dE'dE}{\int_0^\infty \Sigma_t(E)\varphi_\infty(E)dE - \int_0^\infty\int_0^\infty \Sigma_s(E' \rightarrow E)\varphi_\infty(E')dE'dE} = \frac{\int_0^\infty \nu\Sigma_f(E)\varphi_\infty(E)dE'}{\int_0^\infty \Sigma_a(E)\varphi_\infty(E)dE}$$

Types of Solutions of Separated Equations (2)

- Material buckling and fundamental spectrum for a given composition
  - Solve the buckling eigenvalue problem with $\lambda=1$

$$[D(E)B_m^2+\Sigma_t(E)]\varphi_m(E)-\int_0^\infty dE'\Sigma_s(E'\to E)\varphi_m(E')=\chi(E)\int_0^\infty dE'\nu\Sigma_f(E')\varphi_m(E')$$

  - Starting with an initial guess of B^2, the integral equation is solved for the spectrum and B^2 is adjusted until the spectrum converges
    - *The largest eigenvalue B^2 is the material buckling and the corresponding eigenfunction $\varphi(E)$ is the fundamental mode spectrum*



$\lambda=1$ for fundamental

$\lambda<1$ for infinite medium

$\lambda>1$ for subcritical

$B_g^2=0$ for infinite medium

B_g^2 for subcritical $>B_m^2$

Types of Solutions of Separated Equations (3)

– Infinite multiplication factor obtained with fundamental spectrum

$$\int_0^\infty dE[D(E)B_m^2 + \Sigma_t(E)]\varphi_m(E) - \int_0^\infty dE \int_0^\infty dE' \Sigma_s(E' \to E)\varphi_m(E')$$
$$= \int_0^\infty dE \chi(E) \int_0^\infty dE' \nu\Sigma_f(E')\varphi_m(E')$$

- *One-group cross sections*

$$\Sigma_a = \frac{\int_0^\infty \Sigma_a(E)\varphi_m(E)dE}{\int_0^\infty \varphi_m(E)dE} = \int_0^\infty \Sigma_a(E)\varphi_m(E)dE$$

$$\varphi_m = \int_0^\infty \varphi_m(E)dE = 1$$
normalized spectrum

- *One-group balance equation*

$$DB_m^2\varphi_m + \Sigma_t\varphi_m - \Sigma_s\varphi_m = \nu\Sigma_f\varphi_m \quad \Rightarrow \quad (\nu\Sigma_f - \Sigma_a)\varphi_m = DB_m^2\varphi_m$$

$$B_m^2 = \frac{\nu\Sigma_f - \Sigma_a}{D} = \frac{k_\infty - 1}{L^2}, \quad k_\infty = \frac{\nu\Sigma_f}{\Sigma_a}, \quad L^2 = \frac{D}{\Sigma_a}$$

$$B_m^2 > 0 \quad \text{for } k_\infty > 1$$
$$B_m^2 < 0 \quad \text{for } k_\infty < 1$$

$$k_\infty = 1 + L^2 B_m^2 \neq k_\infty^0 \quad (\text{infinite medium } k)$$

The fundamental spectrum obtained with appropriate leakage that makes the medium critical is different from the infinite medium spectrum.

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Types of Solutions of Separated Equations (4)

- Critical geometry for a bare homogeneous medium
  - Solve the energy equation for the given composition and determine the material buckling
  - Find the geometry of which geometrical buckling is equal to the material buckling

$$B_g^2 = \left(\frac{\pi}{H'}\right)^2 = B_m^2 \quad \text{(1-D slab reactor)}$$

- Critical composition for a bare homogeneous system
  - Solve the spatial eigenvalue problem and determine the geometrical buckling
  - Find the composition iteratively such that the resulting material buckling is equal to the geometrical buckling
- Neutron slowing-down spectrum
  - Important for resonance absorption calculation and group constant generation
  - For given fission source and leakage, solve an inhomogeneous problem

$$[D(E)B^2 + \Sigma_t(E)]\varphi(E) - \int_E^\infty \Sigma_s(E' \to E)\varphi(E')dE' = \chi(E)s_0$$