NON DIMENSIONAL NUMBERS

$$Re = \frac{\rho V D}{\mu} = \frac{\rho V^2/D}{\mu V/D/D} = \frac{\text{INERTIAL}}{\text{VISCOUS}}$$

$$Pr = \frac{\nu}{\alpha} = \frac{C_p \mu}{k} = \frac{\text{VISCOUS DIFFUSION RATE}}{\text{THERMAL DIFFUSION}}$$

$$Re \cdot Pr = Pe = \frac{\rho_o C_p D V}{k} = \frac{\rho_o C_p \Delta T\, V/D}{k \Delta T/D^2} = \frac{\text{HEAT TRANSFER CONVECTION}}{\text{CONDUCTION}}$$

$$Fr = \frac{V^2}{gD} = \frac{\rho V^2/D}{\rho g} = \frac{\text{INERTIAL}}{\text{GRAVITY}}$$

$$E_c = \frac{V^2}{C_p \Delta T} = \frac{\rho V^2/D}{C_p \rho \Delta T/D} = \frac{\text{KINETIC CONVECTION}}{\text{ENTHALPY CONVECTION}}$$

$$Gr = \frac{g \beta \Delta T D^3}{\nu^2} = \frac{\text{BUOYANCY} \times \text{INERTIA}}{\text{VISCOUS}}$$

OIL → $Pr \sim 10^2\ 10^3$

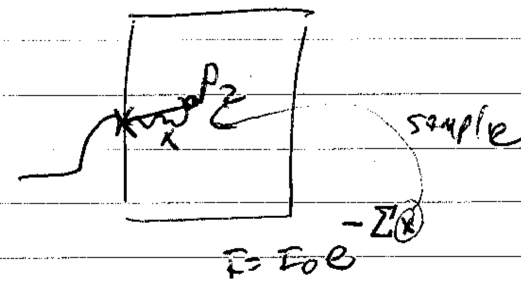
WATER / gas $Pr \sim 1$

Na Metal $Pr = 10^{-2}$

WATER → Low α, but $C_p \uparrow$ $h_{vap} \uparrow$

SMALL MCNP PROGRAM TO FIND THE PROBABILITY

NON-MULTIPLYING MEDIUM



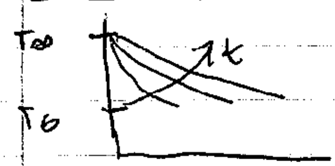
$P_1(E, \vec{\Omega})$ ~ η_2, η_3

η_1 ~ PDF for ENERGY SPECTRUM

DISCRETIZE, CALCULATE REACTION RATE OF Σ_s, Σ_a for each zone,

find surface flux at right + left

TRANSIENT HEAT CONDUCTION



$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_v}\frac{\partial^2 T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

IMPOSE $\theta = T - T_\infty$ TO EXTRACT INITIAL CONDITIONS

$$\theta(x,0) = T - T_\infty = \theta_0$$
$$\theta(0,t) = 0$$
$$\theta(\infty,t) = \theta_0$$

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2}$$

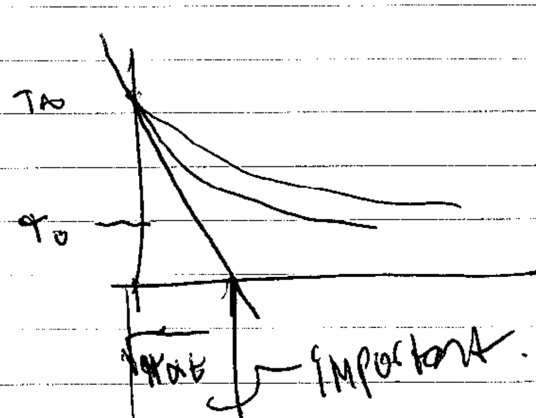
SOLVE USING SIMILARITY SOLUTION WITH $\eta = \frac{x}{\sqrt{4\alpha t}}$

$$\frac{\theta}{\theta_0} = \phi(\eta)$$

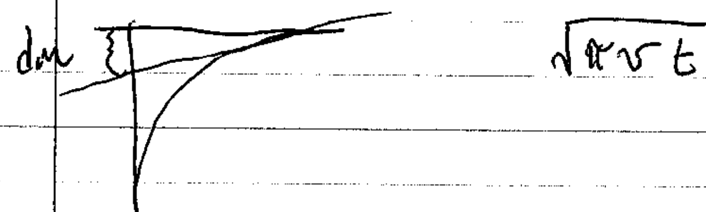
$$\frac{\theta}{\theta_0} = \mathrm{erf}\left(\frac{x}{\sqrt{4\alpha t}}\right)$$

USE TO SOLVE

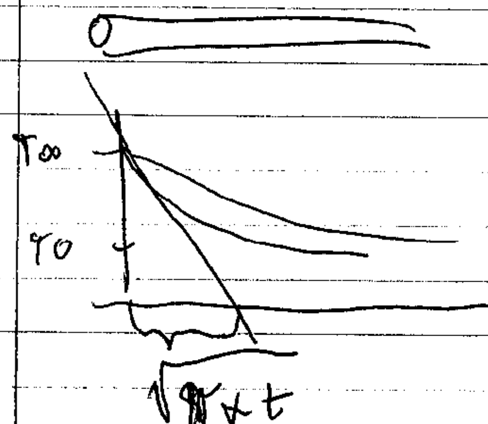
$$\frac{\partial \phi}{\partial \eta} \rightarrow \delta_T = \sqrt{\pi \alpha t}$$



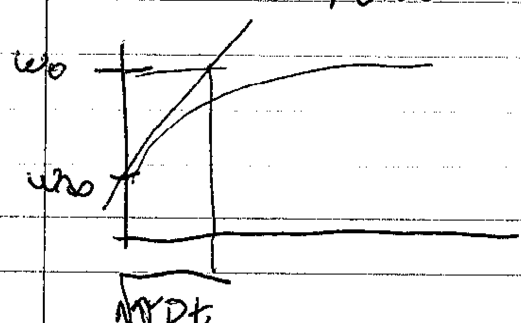
MOMENTUM PENETRATION



THERMAL PENETRATION



DIFFUSION PENETRATION

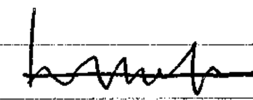


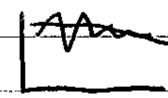
TURBULENT FLOW

TEMPORAL / STATISTICAL

$$\bar{\Psi} = \frac{1}{\Delta t}\int_{\Delta t} \Psi \, dt$$ $\Delta turb < \Delta t < \Delta System$

$$\Psi = \bar{\Psi} + \Psi'$$

If set $\bar{\Psi} = 0$ $\rightarrow$ 

OR $I = \frac{\sqrt{\overline{\Psi'^2}}}{\bar{\Psi}} > 0$ $\rightarrow$ 

TIME AVERAGED, INCOMPRESSIBLE

$$\left|\frac{\rho'}{\rho}\right| \ll \left|\frac{v'}{\rho}\right|$$

$$\frac{\partial \rho}{\partial t} + \nabla \rho \vec{v} = \vec{0} \rightarrow \nabla \cdot \vec{v} = 0$$

$$\vec{v} = \bar{\vec{v}} + v' \qquad \nabla \bar{v} = 0$$

OR

$$\frac{\partial \rho}{\partial t} + \nabla \rho \bar{v} = 0$$

TURBULENT MOMENTUM

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla (\rho \vec{v}\vec{v}) = -\nabla p + \mu \nabla^2 \vec{v} + \rho g$$

$$\vec{v} = \bar{v} + v'$$

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \rho \bar{v}\bar{v} + \nabla \rho (\overline{v'v'}) = -\nabla \bar{p} + \mu \nabla^2 \bar{v} + \rho \vec{g}$$

$$\boxed{\frac{\partial \rho \vec{v}}{\partial t} + \nabla \rho \vec{v}\vec{v} = -\nabla \bar{p} + \left[\mu \nabla^2 \bar{v} - \nabla \rho \overline{v'v'}\right] + \rho g}$$

$$\boxed{\tau^T = \tau^\mu + \tau^t}$$

$$\tau^\mu = -\mu \left(\nabla \bar{v} + (\nabla \bar{v})^t\right)$$
$$\tau^t = \rho \overline{v'v'}$$

$$\frac{\partial \rho \bar{v}}{\partial t} + \nabla \rho \bar{v}\bar{v} = -\nabla p - \nabla \tau^T + \rho g$$

$$\tau^\mu = -\nu \rho \left[\nabla \bar{v} + (\nabla \bar{v})^t\right]$$

$$\tau^t = -\epsilon_m \rho \left[\nabla \bar{v} + (\nabla \bar{v})^t\right]$$

ϵ_m: TURBULENT DIFFUSIVITY.

TURBULENT ENERGY

$$\rho C_v \frac{DT}{Dt} = k\nabla^2 T - \cancel{T\partial u} + \cancel{\tau_{ij}\nabla v} + \dot{q}$$

$$\downarrow$$

$$C_v \frac{\partial \rho T}{\partial t} + \nabla \rho T \vec{v}$$

$$v = \bar{v} + v'$$
$$T = \bar{T} + T'$$

$$\Downarrow$$

$$\rho C_v \frac{D\bar{T}}{Dt} = \left[k\nabla^2 T - \nabla \rho C_v \overline{T'v'}\right] + \dot{q}$$

$q^k = k\nabla^2 T$ - heat conduction

$q^t = \rho C_v \overline{T'v'}$ = TURBULENT HEAT FLUX

$$\rho C_v \frac{D\bar{T}}{Dt} = -\nabla(q^k + q^t) + \dot{q}$$

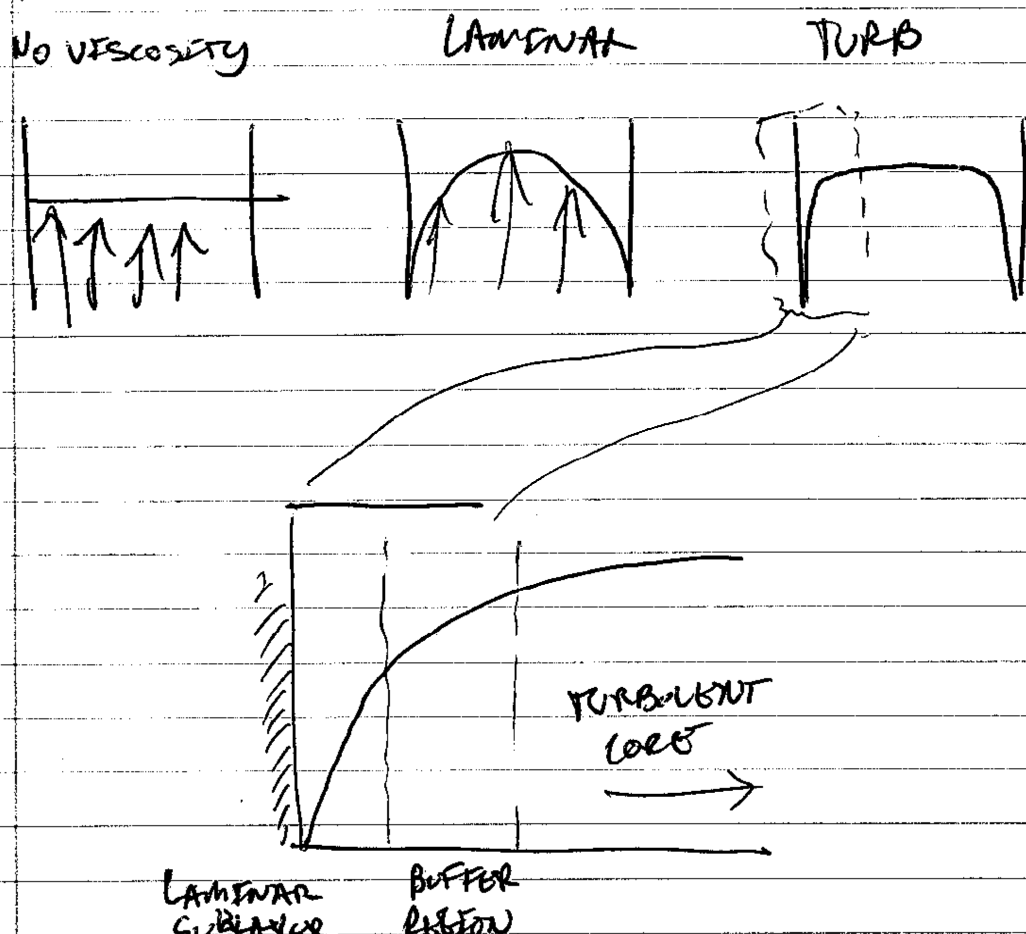
$$q^T = q^k + q^t$$

$$\boxed{\rho C_v \frac{D\bar{T}}{Dt} = -\nabla q^T + \dot{q}}$$

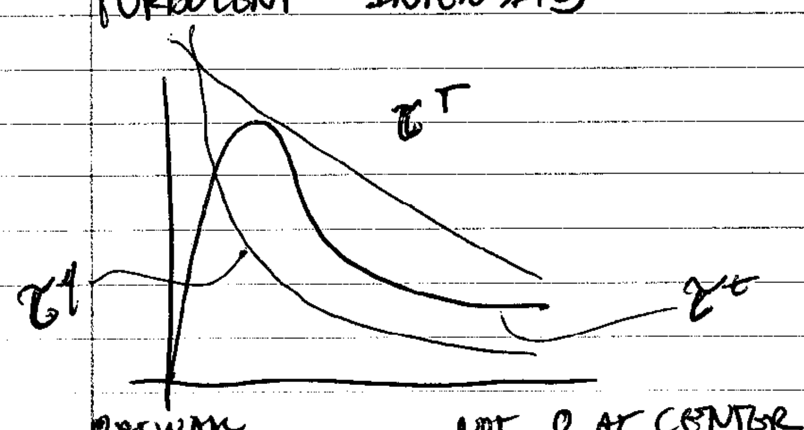
CHARACTERISTICS OF TURB FLOW

$$\text{LAMINAR} = V_{MAX}\left[1-\left(\frac{r}{r_0}\right)^2\right]$$

$$\text{TURB.} = V_{MAX}\left[1-\left(\frac{r}{r_0}\right)\right]^{\frac{1}{7}}$$



TURBULENT INTENSITY



PRANDTYL

1. FLUID ELEMENT MOVES, but no momentum transferred till travelled "L"

2. x component gained by δm = $\delta m \delta v_x$

TRANSFER RATE $\frac{\delta m \delta v}{\partial t}$ = ~~SHEAR FORCE~~

$$\frac{F}{A} = \frac{1}{A}\frac{\delta m\,\delta v_x}{\delta t}$$

$$\delta v_x = \frac{dv_x}{dy} \cdot L$$

$$\frac{1}{A}\frac{\delta m}{\delta t} = \rho\,|v_y'|$$

$$\tau^t = -L\,|v_y'|\,\frac{dv_x}{dy}$$

3. $V_y \propto V_x$

$$|v_y'| = k_1\,|V_x'|$$

$$V_x' = k\,\delta v_y = kL\,\frac{dv_x}{dy}$$

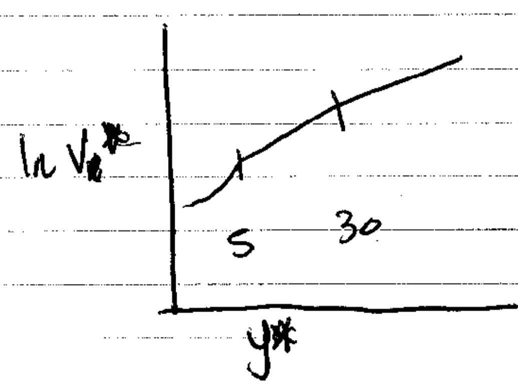
$$\frac{\tau^t}{\rho} = -L^2\left|\frac{dv_x}{dy}\right|\frac{dv_x}{dy}$$

$L = ky$

$$\frac{\tau^t}{\rho} = -k^2 y^2\left|\frac{dv_x}{dy}\right|\frac{dv_x}{dy}$$

EXPO DETERMINED.

$$\tau^t = -\tau^w \Rightarrow \frac{dv_x}{dy} = \frac{1}{k}\sqrt{\frac{\tau_w}{\rho}}\,\frac{1}{y}$$



SINGLE PHASE FLOW DIMENSIONAL SCALING

BALANCE EQUATION

SPACE $x^* = \frac{x}{D}$ $y^* = \frac{y}{D}$ $z^* = \frac{z}{D}$ $D = \frac{4A}{P}$

TIME $\tau = \frac{D}{V}$ $t^* = \frac{t}{\tau}$

↳ CHANNEL INLET VELOCITY

VELOCITY $v^* = \frac{\vec{v}}{V}$

PRESSURE $p^* = \frac{p - p_0}{\rho_0 V^2}$

DENSITY $\rho^* = \frac{\rho}{\rho_P}$

$$\frac{\partial \rho}{\delta t} + \nabla \rho \vec{v} = 0 \Rightarrow \boxed{\frac{\partial \rho^*}{\partial t^*} + \nabla^*(\rho^* v^*) = 0}$$

MOMENTUM

$$\rho \frac{D\vec{v}}{Dt} = \left(\rho_0 \frac{V^2}{D}\right) \rho^* \frac{D\vec{v}^*}{Dt^*}$$

$$\nabla p = \frac{\rho_0 V^2}{D} \nabla^* p$$

$$\mu \nabla^2 \vec{v} = \frac{\mu V}{D^2} \nabla^{*2} \vec{v}^*$$

$$\boxed{\rho^* \frac{Dv^*}{Dt^*} = -\nabla^* p^* + \frac{1}{Re}\left(\nabla^{*2} v^*\right) + \frac{1}{Fr} \rho^* \left(\frac{\vec{g}}{g}\right)}$$

$$Re = \frac{\rho V D}{\mu} \qquad Fr = \frac{V^2}{gD}$$

ENERGY

$$\rho C_p \frac{DT}{Dt} = \left(\rho_0 C_0 \frac{V}{D} \Delta T\right) \rho^* \frac{DT^*}{Dt^*}$$

$$k^2 \nabla T = \left(\frac{k \Delta T}{D^2}\right) \nabla^{*2} T^*$$

$$\tau : \nabla \vec{v} = \left(\mu \frac{V^2}{D^2}\right) \Phi^*$$

$$\frac{T}{\rho} \frac{\partial \rho}{\partial t}\Big|_p \frac{Dp}{Dt} = \frac{\rho_0 V^3 T^*}{D\, \rho^*} \frac{\partial \rho^*}{\partial T^*} \frac{Dp^*}{Dt^*}$$

$$\rho^* \frac{DT^*}{Dt^*} = \left(\frac{k}{D \rho_0 c_p V}\right) \nabla^2 T^* + \left(\frac{\mu V}{D \rho_0 c_p \Delta T}\right) \Phi^* - \left(\frac{V^2}{c_p \Delta T}\right) \frac{T^*}{\rho^*} \frac{\partial \rho^*}{\partial t^*} \frac{Dp^*}{Dt} + \frac{\dot{q} D}{\rho c_p \Delta T V} \dot{q}^*$$

$$Ec = \frac{V^2}{c_p \Delta T}$$

$$Pe = \frac{\rho_0 c_p D V}{k} =$$

$$Pe = Re \cdot Pr$$

$$\boxed{\rho^* \frac{DT^*}{Dt^*} = \frac{1}{Re\, Pr} \nabla^{*2} T^*} + \frac{Ec}{Re} \Phi^* - Ec \frac{T^*}{\rho^*} \frac{Dp^*}{Dt^*} + \dot{q}^*$$

DIMENSIONLESS INCOMPRESSIBLE, NO HEAT GAIN

$$\rho^* \frac{DT^*}{Dt^*} = \frac{1}{Pe} \nabla^{*2} T^* + \cancel{\Phi} \quad \frac{Ec}{1}$$

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^* (\rho^* V^*) = 0$$

$$\rho^* \frac{D\vec{V}^*}{Dt^*} = -\nabla^* P^* + \frac{1}{Re}\left[\nabla^{*2}\vec{V}^* + \frac{1}{3}\vec{V}^*(\vec{V}^* \cdot \vec{V}^*)\right] + \frac{\rho^*}{Fr}\frac{\vec{g}}{g}$$

incompressible $\nabla^* V^* = 0$

CREEP FLOW $(Re \sim 1)$

$$V = \frac{\nu}{D} \qquad \rho \frac{V}{t} = \rho_0 g_c \beta \Delta T \qquad t = \frac{D}{V}$$

$$V = \sqrt{g \beta \Delta T D}$$

$$\frac{Dv^*}{Dt^*} = -\nabla^* p^* - \left(\frac{g \beta \Delta T D}{V^2}\right) T^* + \frac{1}{Re} \nabla^{*2} V^*$$

$$\frac{DT^*}{Dt^*} = \underbrace{\frac{k}{\rho_0 c_p V D}}_{\frac{1}{Pr}} \nabla^{*2} T^*, \qquad \frac{DT^*}{Dt^*} = \frac{1}{Pr} \nabla^{*2} T^*$$

$$\frac{D\vec{V}^*}{Dt} = -\nabla^* p^* - Gr \, T^* + \nabla^{*2} \vec{V}^*$$

$$T^*(Pr, Gr)$$

GENERAL BALANCE

Ψ - $\frac{\text{AMOUNT IN VOLUME}}{\text{UNIT VOLUME}}$

J - FLUX ACROSS FIXED MASS SURFACE

$\dot{\Psi}$ - GENERATION OF Ψ per unit volume

$\int \Psi \, dV$ - TOTAL

$-\oint \vec{J} \cdot \hat{n} \, ds$ - net flow across surface

$\int_V \dot{\Psi}_g \, dV$ - generation in volume

CHANGE OF TOTAL Ψ = SURFACE FLUX + GENERATION

$$\frac{D}{Dt} \int_{V_m} \Psi \, dV = -\oint J \cdot \hat{n} \, ds + \int \dot{\Psi}_g \, dV$$

USING

REYNOLD'S TRANSPORT

$$\frac{D}{Dt} \int_{V_m} \Psi \, dV = \int_V \left[\frac{\partial \Psi}{\partial t} + \nabla \cdot (\Psi \vec{v}) \right] dV$$

GREEN'S THEOREM

$$-\oint J \cdot \hat{n} \, ds = -\int_V \nabla \cdot J \, dV$$

$$\int_V \left[\frac{\partial \Psi}{\partial t} + \nabla (\Psi \cdot \vec{v}) \right] dV = -\int_V \nabla \cdot J \, dV + \int_V \dot{\Psi} \, dV$$

$$\boxed{\frac{\partial \Psi}{\partial t} + \nabla (\vec{\Psi} \cdot \vec{v}) = -\nabla \cdot J + \dot{\Psi}_g}$$

TIME RATE OF CHANGE + CONVECTION OF $\vec{v}$ = FLUX + GENERATION

CYLINDRICAL COORDINATE

CONTINUITY

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}\left(\rho r V_r\right) + \frac{1}{r}\frac{\partial \rho V_\theta}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} = 0$$

MOMENTUM

r:
$$\rho\left(\frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\phi}{r}\frac{\partial V_r}{\partial \phi} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}\right)$$
$$= -\frac{\partial P}{\partial r} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2} + \frac{\partial^2 V_r}{\partial z^2} - \frac{V_r}{r} - \frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}\right] + \rho g_r$$

θ:
$$\rho\left(\frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_r V_\theta}{r}\right) =$$
$$-\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_\theta}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2} + \frac{\partial^2 V_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial V_r}{\partial \theta} - \frac{V_\theta}{r^2}\right] + \rho g_\theta$$

z:
$$\rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right) =$$
$$-\frac{\partial P}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial V_z}{\partial \theta} + \frac{\partial^2 V_z}{\partial z^2}\right] + \rho g_z$$

DIFFERENTIAL MASS BALANCE

$$\frac{\partial \psi}{\partial t} + \nabla(\psi \cdot \vec{v}) = -\nabla \cdot J + \dot{\psi}_g$$

$$\left.\begin{array}{l}\psi = \rho \\ J = 0 \\ \dot{\psi}_g = 0\end{array}\right\} \quad \frac{\partial \rho}{\partial t} + \nabla \rho \cdot \vec{v} = 0$$

CHAIN RULE

$$\frac{\partial \rho}{\partial t} + \vec{v}\nabla\rho + \rho\nabla\vec{v} = 0$$

FIXED MASS

$$\frac{\partial \rho}{\partial t} + \rho\nabla\vec{v} = 0 \Rightarrow \frac{1}{\rho}\frac{\partial \rho}{\partial t} = -\nabla\cdot\vec{v}$$

INCOMPRESSIBLE

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} = 0 \Rightarrow \nabla\cdot\vec{v} = 0$$

MOMENTUM

$$\psi = \rho\vec{v}$$

$$J = T = \underbrace{P\mathbb{I}}_{\text{PRESSURE}} + \underbrace{\tau}_{\text{SHEAR}}$$

$$\dot{\psi} = \rho\vec{g}$$

$$\frac{\partial \psi}{\partial t} + \nabla(\psi \cdot \vec{v}) = -\nabla\cdot J + \dot{\psi}_g$$

$$\underbrace{\frac{\partial(\rho v)}{\partial t} + \nabla(\rho\vec{v}\vec{v})} = -\nabla p - \nabla\tau + \rho\vec{g}$$

$$\boxed{\rho\frac{D\vec{v}}{Dt} = -\nabla p - \nabla\tau + \rho\vec{g}}$$

$$\underbrace{\frac{\partial\rho\vec{v}}{\partial t}}_{\substack{\text{TIME RATE}\\\text{OF CHANGE}\\\text{OF MOMENTUM}\\\text{PER VOLUME}}} + \underbrace{\nabla(\rho\vec{v}\vec{v})}_{\text{CONVECTION}} = \underbrace{-\nabla p - \nabla\tau}_{\text{SURFACE}} + \underbrace{\rho\vec{g}}_{\text{GRAVITY}}$$

mass $\Psi = \rho$

$\bar{J} = 0$

$\dot{\Psi}_g = 0$

momentum $\Psi = \rho \vec{v}$

$J = P + \tau$

$\dot{\Psi}_g = \rho \vec{g}$

kinetic ENERGY $= \vec{v} \cdot$ momentum

$\left(-\nabla p - \nabla \tau \cdot v\right) = \left[-\nabla(\tau \cdot \vec{v}) + \tau : \nabla \vec{v}\right]$

GENERAL ENERGY $\Psi = u + \frac{V^2}{2}$

$\bar{J} = \bar{q} + T\vec{v}$

$\dot{\Psi}_g = \rho \vec{v} g + \dot{q}$

$\rho v_g = -\rho \frac{d\Phi}{dt}$

General $\left[\rho \frac{D}{Dt}\left[u + \frac{V^2}{2} + \Phi\right] = -\nabla \vec{q} - \nabla(\rho \vec{v}) - \nabla(\tau \cdot \vec{v}) + \dot{q}\right]$

GEN. $-$KE $=$ $\rho \frac{Du}{Dt} = -\nabla \cdot \vec{q} - p \nabla \vec{v} - \tau : \nabla \vec{v} + \dot{q}$

[ENTHALPY $i = u + \frac{p}{\rho}$ $\frac{\partial \rho i}{\partial t} + \nabla \rho i \vec{v} = -\nabla \cdot \dot{q} + \frac{Dp}{Dt}$]

NAT CIRC

$\Psi = \rho \vec{v}$

$J = P + \tau$

$\dot{\Psi}_g = \left[\bar{\rho} - \bar{\rho}\beta(T - \bar{T})\right] g$

$d\rho = \rho \beta dT$

[hydrostatic $= -\nabla p + \bar{\rho} g = 0$]

NATURAL CIRCULATION

FROM MOMENTUM $\frac{\partial \rho \vec{v}}{\partial t} + (\nabla \cdot \rho \vec{v}\vec{v}) = -\nabla P - \nabla \tau + \rho \vec{g}$

BOUSSINESQ

IMPOSE β, THERMAL EXPANSION COEFFICIENT

$$\beta = \frac{1}{v}\frac{\partial v}{\partial T}\bigg|_P = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}\bigg|_P \qquad \rho(T,P) \sim \rho(T)$$

USE A FUNCTION OF T BECAUSE CHANGES IN ρ ARE SMALL.

~~COMP~~

$$d\rho = \rho \beta dT$$

USING REFERENCE DENSITY, TEMPERATURE

$$\bar{\rho} = \rho - \bar{\bar{\rho}} = -\rho\beta(T - \bar{T})$$

SUBSTITUTE BACK INTO MOMENTUM

$$\boxed{\bar{\rho}\frac{D\vec{v}}{Dt} = -\nabla P - \nabla \tau + [\bar{\rho} - \bar{\rho}\beta(T-\bar{T})]\vec{g}}$$

FOR LOW $\bar{v}$, APPROX HYDROSTATIC

$$-\nabla P + \bar{\rho} g = 0$$

$$\downarrow$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla \tau - \bar{\rho}\beta(T-\bar{T})\vec{g}$$

THERMAL ENERGY EQUATION

GENERAL $\rho \frac{D}{Dt}\left[u + \frac{v^2}{2} + \Phi\right] = -\nabla \vec{q} - \nabla(p\vec{v}) - \nabla(\tau \cdot \vec{v}) + \dot{q}$

− KINETIC $\frac{\partial(\rho \frac{v^2}{2})}{\partial t} + \nabla(1/2\, \rho \vec{v}^2 \cdot \vec{v}) = -\nabla(\tau \cdot \vec{v}) + \tau : \nabla \vec{v} + \rho \vec{v} g$

$$\boxed{\rho \frac{Du}{Dt} = -\nabla \cdot \vec{q} - p\nabla\vec{v} - \tau : \nabla\vec{v} + \dot{q}}$$

ENTHALPY ENERGY

$$i = u + \frac{p}{\rho}$$

$$\frac{\partial \rho i}{\partial t} + \nabla(\rho i \vec{v}) = -\nabla \cdot \dot{q} + \frac{Dp}{Dt}$$

$$\sum \rho_k \vec{V}_{km} = 0$$

COMPONENT CONTINUITY

$$\Psi = \rho_k$$
$$J = \rho_k (V_k - \vec{V}) = \vec{j}$$
$$\Psi_g = \Gamma_k$$
$$\frac{\partial \rho}{\partial t} + \nabla \rho_k \vec{V} = -\nabla j_k + \Gamma_k \qquad \text{drift flux}$$

MIXTURE MOMENTUM

$$\Psi = \rho \vec{V}$$
$$J = p + \tau + \sum \rho_k V_k V_{km}$$
$$\Psi_g = \sum \rho_k g$$

MIXTURE ENERGY

$$\Psi = \rho \left(u + \frac{V^2}{2}\right)$$
$$J = \pi\, v \cdot \vec{v}$$
$$\Psi_g = \sum \rho_k V_k \vec{g}$$

KINETIC ENERGY

$$\boxed{\rho \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \tau + \rho \vec{g}} \cdot \vec{v}$$

$$\rho \frac{D\left(\frac{\vec{v}^2}{2}\right)}{Dt} = -\vec{v}\nabla p - \vec{v}\nabla\tau + \rho\vec{v}\vec{g}$$

$$\frac{\partial\left(\rho\frac{v^2}{2}\right)}{\partial t} + \nabla\left(\frac{1}{2}\rho\vec{v}^2\cdot\vec{v}\right) = -\nabla(\tau\cdot\vec{v}) + p\cdot\nabla\vec{v}$$

$$\nabla(p\vec{v}) = p\nabla\vec{v} + \vec{v}\nabla p$$

$$\nabla(\tau\vec{v}) = \tau : \nabla\vec{v} + \vec{v}(\nabla\cdot\tau)$$

$$\boxed{\frac{\partial\left(\rho\frac{\vec{v}^2}{2}\right)}{\partial t} + \nabla\cdot\left(\frac{1}{2}\rho\vec{v}^2\cdot\vec{v}\right) = -\nabla(\tau\cdot\vec{v}) + \tau : \nabla\vec{v} + \rho\vec{v}\vec{g}}$$

KINETIC ENERGY TIME RATE OF CHANGE.

VISCOUS FORCE

Irreversible conv. to internal energy

GRAVITY.

GENERAL ENERGY

$$\psi = u + \frac{\vec{v}^2}{2} \quad \text{(Internal + kinetic)}$$

$$J = \vec{q} + T\vec{V} \quad \text{(Heat + Surface energies)}$$

$$\dot{\psi}_g = \dot{q} + \rho\vec{v}g \quad \text{(HEAT GEN + GRAVITY WORK)}$$

$$\frac{\partial\rho}{\partial t}\left(u + \frac{v^2}{2}\right) + \nabla\left(\rho\left(u + \frac{v^2}{2}\right)\vec{v}\right) = \nabla\vec{q} - \nabla p\vec{v} - \nabla(\tau\cdot\vec{v}) + \rho\vec{v}g + \dot{q}$$

$$\rho\vec{v}g = -\rho(v\cdot\nabla\Phi) = -\rho\frac{D\Phi}{Dt}$$

$$\boxed{\rho\frac{D}{Dt}\left[u + \frac{v^2}{2} + \Phi\right] = -\nabla\cdot\vec{q} - \nabla\cdot(p\vec{v}) - \nabla(\tau\cdot v) + \dot{q}}$$

TWO PHASE

ρ_k = MASS CONC. OF K COMPONENT

ω_k = MASS FRACTION $\frac{\rho_k}{\rho}$

ρ = MIXTURE DENSITY.

$$\rho = \sum_i^n \rho_k$$

$\vec{V}_k$ VELOCITY COMPONENT

$\rho_k \vec{V}_k$ MOMENTUM OF K COMPONENT

$$\rho \vec{v} = \sum_i^n \rho_k V_k$$

$$\vec{v} = \frac{\sum_i \rho_k \vec{V}_k}{\rho_k} = \text{CENTER OF MASS VELOCITY.}$$

$$\boxed{\sum \rho_k \vec{V}_{km} = 0}$$

COMPONENT CONTINUITY.

$$\Psi = \rho_k$$

$$\mathbb{J} = \rho_k (\vec{V}_k - \vec{v}) = \vec{J}$$

$$\dot{\Psi} = \Gamma_k$$

$$\frac{\partial \rho}{\partial t} + \nabla \rho_k \vec{v} = -\nabla j_k + \Gamma_k$$

w/V_k $\boxed{\text{DRIFT FLUX}}$

$$\frac{\partial \rho}{\partial t} + \nabla \rho_k \vec{V}_k = \Gamma_k$$

$\boxed{\text{2 FLUID}}$

MIXTURE MOMENTUM

$$\Psi = \rho \vec{v}$$

$$\mathbb{J} = p \mathbb{I} + \tau + \sum_i \rho_k \vec{V}_k \vec{V}_{km} = \Pi$$

$$\dot{\Psi}_g = \sum \rho_k \vec{g}$$

$$\boxed{\frac{\partial \rho \vec{V}}{\partial t} + \nabla \rho \vec{v} \vec{v} = -\nabla p - \nabla \tau - \nabla \sum \rho_k \vec{V}_{km} \vec{V}_k + \sum \rho_k \vec{g}}$$

MIXTURE ENERGY

$$\Psi = \rho \left(u + \frac{v^2}{2}\right)$$

$$\mathbb{J} = \pi \vec{v} \cdot q$$

$$\dot{\Psi}_g = \sum \rho_k v_k \cdot \vec{g}$$

CONSTITUTIVE RELATIONS

BALANCE EQUATION UNKNOWNS

$$\left.\begin{matrix} \rho & \vec{v} & \\ \rho & \tau & \vec{g} \\ u & \vec{q} & \dot{q} \end{matrix}\right\} 8$$

→ SIMPLE MATH

→ PRACTICAL

→ CANT DENY LAWS OF THERMODYNAMICS

EQUATIONS OF STATE

FUNDAMENTAL : $u = u(s, \rho)$ $\quad T = \left.\frac{\partial u}{\partial s}\right|_\rho \quad P = -\left.\frac{\partial u}{\partial(\frac{1}{\rho})}\right|_s$

2ND DERIV : $C_p \; C_v \; \alpha , \beta$

THERMAL

1st deriv : $\rho = \rho(P,T)$ or $P = P(\rho, T)$

CALORIC

A) Incompressible $\quad \rho$ constant

$u = u(T)$ (OFTEN C_V = constant)

B) IDEAL GAS $\quad P = \rho R T$

$u = u(T)$

A) INVISCID FLOW

$\tau = 0$

B LINEARLY VISCOUS = $\tau_{yx} = -\mu \frac{dV_x}{dy}$

NEWTONIAN $\tau_{xy} = -\mu \frac{dV_x}{dy}$

FOURIERS $q = -k \frac{dT}{dy}$

FICKS $J_{xy} = -\rho D_k \frac{d\omega_k}{\partial y}$

HEAT CONDUCTION

$$\dot{q} = \dot{q}(K, T)$$

MASS DIFFUSION

DIFFUSION MASS FLUX

$$J_K = \rho_K(\vec{v}_K - \vec{v}) = -\rho D \nabla \frac{\rho_K}{\rho}$$

$$= -\rho D \nabla w_K$$

$$\gamma_K = \gamma_K(w_1\ w_2 \cdots w_K)$$

$$T ds = du - \frac{p}{\rho^2} d\rho$$

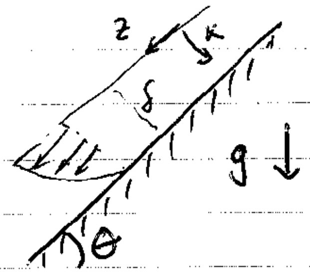
ENTROPY CHANGE

$$\rho \frac{ds}{dt} + \nabla \frac{\vec{q}}{T} - \frac{\dot{q}}{T} > 0$$

$$\Delta = -\frac{\vec{q} \Delta T}{T^2} + \frac{\tau : \nabla \vec{v}}{T} > 0$$

$$\rho C_v \frac{\partial T}{\partial t} = K \nabla^2 T$$

LAMINAR FLOW



ASSUMPTIONS

STEADY STATE $\frac{\partial}{\partial t}=0$

NO SHEAR AT 0:L

FULLY DEVELOPED $\frac{\partial}{\partial z}=0$

NO y DEPENDENCE

INCOMPRESSIBLE

ADIABATIC ISOTHERMAL

CONTINUITY $\cancel{\frac{\partial V}{\partial t}}+\frac{\partial V_x}{\partial x}+\cancel{\frac{\partial V_y}{\partial y}}+\cancel{\frac{\partial V_z}{\partial z}}=0 \qquad \frac{\partial V_x}{\partial x}=0 \Rightarrow V_x=0\Big|_{x=\delta}$

At wall

VELOCITY FIELD $V_z=V_z(x)$

$V_x=V_y=0$

MOMENTUM: $\rho\frac{DV}{Dt}=-\nabla p-\nabla\tau+\rho\vec{g}$

$$z:\quad \rho\left(\cancel{\frac{\partial V_z}{\partial t}}+V_x\cancel{\frac{\partial V_z}{\partial x}}+\cancel{\frac{\partial V_z}{\partial y}}V_y+\cancel{\frac{\partial V_z}{\partial z}}V_z\right)=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 V_z}{\partial x^2}+\cancel{\frac{\partial^2 V_z}{\partial y^2}}+\cancel{\frac{\partial^2 V_z}{\partial z^2}}\right)+\rho g_z$$

$$① \qquad -\frac{\partial p}{\partial z}+\mu\frac{\partial^2 V_z}{\partial x^2}+\rho g_z=0$$

$$x:\quad \rho\left(\frac{\partial V_x}{\partial t}+V_x\frac{\partial V_x}{\partial x}+V_y\frac{\partial V_x}{\partial y}+V_z\frac{\partial V_x}{\partial z}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 V_x}{\partial x^2}+\frac{\partial^2 V_x}{\partial y^2}+\frac{\partial^2 V_z}{\partial z^2}\right)+\rho g_x$$

$$② \qquad -\frac{\partial p}{\partial x}+\rho g\cos\theta=0$$

BOUNDARY CONDITIONS Ⓐ $x=0$, $\frac{\partial V_z}{\partial x}=0$ Ⓑ $x=\delta$, $V_z=0$ Ⓒ $x=0$, $p=p_\infty$

$$② \quad \int\frac{\partial p}{\partial x}=\int\rho g_x \qquad p=\rho g_x x+C_1(x) \qquad C_1(x)=p_\infty$$

$$(2\to1) \quad \frac{\partial(\rho g_x x+p_\infty)}{\partial z}+\mu\frac{\partial^2 V_z}{\partial x^2}+\rho g_z=0$$

$$\mu\frac{\partial^2 V_z}{\partial x^2}+\rho g_z=0$$

$$\frac{\partial V_z}{\partial x} = -\frac{\rho g_z}{\mu} x + C_2(x) \Big|_{x=0} \quad \left(\frac{\partial V_z}{\partial x} = 0 \text{ @ } x=0\right) \quad C_2(x) = 0$$

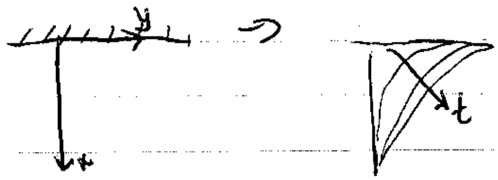
$$V_z = -\frac{\rho g_z}{2\mu} x^2 + C_3(x) \Big|_{x=\delta} \quad (V_z = 0 \text{ @ } x=\delta) \quad C_3 = \frac{\rho g_z \delta^2}{2\mu}$$

$$V_z = -\frac{\rho g_z}{2\mu} x^2 + \frac{\rho g_z}{2\mu} \delta^2$$

$$= \frac{\rho g \delta^2 \cos\theta}{2\mu} \left[1 - \left(\frac{x}{\delta}\right)^2\right]$$

PARABOLIC

DIFFUSION LENGTH



2D

NO DEPENDENCE ON z

INCOMPRESSIBLE

CONSTANT PROPERTIES

$\nabla V = 0$ ISOTHERMAL ADIABATIC

CONTINUITY

$$\frac{\partial \rho}{\partial t} + \nabla \rho \vec{V} = 0$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0 \qquad \frac{\partial V_x}{\partial x} = 0 \qquad V_x \Big|_{x=0} = 0$$

$$\begin{cases} V_y = V_y(t, x) \\ V_x \,\&\, V_z = 0 \end{cases}$$

$$\rho \left[\frac{\partial V_y}{\partial t} + \cancel{\frac{\partial V_y}{\partial x}} + \cancel{\frac{\partial V_y}{\partial y}} + \cancel{\frac{\partial V_y}{\partial z}} \right] = -\frac{\partial P}{\partial y} + \mu \left[\frac{\partial^2 V_y}{\partial x^2} + \cancel{\frac{\partial^2 V_y}{\partial y^2}} + \cancel{\frac{\partial^2 V_z}{\partial z^2}} \right] + \cancel{\rho g_y}$$

$$\rho \left[\frac{\partial V_y}{\partial t} \right] = -\frac{\partial P}{\partial y} + \mu \left[\frac{\partial^2 V_y}{\partial x^2} \right]$$

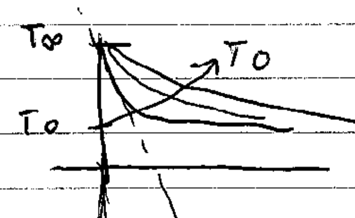
WOULD SOLVE MOMENTUM IN $x \rightarrow P(x) \Rightarrow \frac{\partial P(x)}{\partial y} = 0$

$$\Rightarrow \quad \rho \frac{\partial V_y}{\partial t} = \mu \frac{\partial^2 V_y}{\partial x^2} \qquad \frac{\mu}{\rho} = \nu$$

$$\frac{\partial V_y}{\partial t} = \nu \frac{\partial^2 V_y}{\partial x^2}$$

ν (MOMENTUM DIFFUSIVITY)

THERMAL PENETRATION



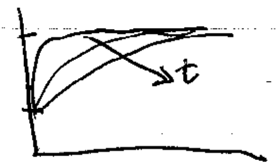
$$\rho C_v \left[\frac{\partial T}{\partial t} + \cancel{\vec{v} \nabla T} \right] = k \nabla^2 T + \cancel{\dot{q}} + \;/\;/$$

$$\rho C_v \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}}$$

$$\rho C_v \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \qquad \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial t^2}$$

α THERMAL DIFFUSIVITY

SUDDEN DIFFUSION



$$\frac{\partial \rho_k}{\partial t} + \nabla \rho_k \vec{V} = -\nabla \cdot \vec{J}_k + r_k$$

ρ = CONSTANT

NO GLOBAL MOTION $\vec{v} = 0$

NO CHEM RXN $\Rightarrow r_k = 0$

$J = \rho D \nabla w_k$

$$\frac{\partial \rho_w}{\partial t} = D \nabla^2 \omega_k$$

$$\frac{\partial \rho_w}{\partial t} = D\left[\frac{\partial^2 w_k}{\partial x^2} + \cancel{\frac{\partial^2 w_k}{\partial y^2}} + \cancel{\frac{\partial^2 w_k}{\partial z^2}}\right]$$

$$\frac{\partial \rho_w}{\partial t} = D \frac{\partial^2 \omega_k}{\partial x^2}$$

ν = KINEMATIC VISCOSITY $\frac{\mu}{\rho}$

α = THERMAL DIFFUSIVITY $\frac{k}{\rho C_p}$

D = MASS DIFFUSIVITY

$$Pr = \frac{\nu}{\alpha}$$