NUCL 511 HMWK 8

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4/2/14

1 ***Source Drop in Critical Reactor.*** **Analyze the source drop into a critical reactor specified in the homework problem 4 of chapter 6 [3]. Specifically, determine the two reduced source magnitudes to yield the flux amplitude in Fig. 6.22 by running the point kinetics program for various source magnitudes, and show the flux magnitude for the initial 900 seconds.** Because this problem involves a source drop, the explicit form of the point kinetics equations numerical method must be used. This form is given from the notes as

$$R_{n-1} = \left[(\rho_{n-1} - \beta)\, p_{n-1} + \sum_{k=1}^{6} \lambda_k \zeta_k^{(n-1)} \right]$$

$$p_n = p_{n-1} + \frac{h}{\Lambda} R_{n-1}$$

$$\zeta_k^{(n)} = (1 - \lambda_k h)\, \zeta_k^{(n-1)} + \beta_k h p_{n-1}$$

To include the independent source, we must modify these equations to include that source. The point kinetics equation including independent source neutrons is

$$\dot{p}(t) = \frac{\rho - \beta}{\Lambda} p(t) + \frac{1}{\Lambda} \sum_{k=1}^{6} \lambda_k \zeta_k(t) + \frac{1}{\Lambda} s(t)$$

To modify the numerical method to include this, we simply add the source neutrons to the power equation, giving us

$$R_{n-1} = \left[(\rho_{n-1} - \beta)\, p_{n-1} + \sum_{k=1}^{6} \lambda_k \zeta_k^{(n-1)} + s_0 \right]$$

To use the explicit form, we also need to be sure to use an appropriate h value, because the explicit method is only stable for

$$h < \frac{1}{\alpha}$$

To determine α, we use the equation $\alpha_P = \frac{\rho - \beta}{\Lambda} = \frac{-\beta}{\Lambda}$, so the h value should be half of that. Notice that the α_P is **negative**, so **it is likely that the explicit method never converges**.

To derive the implicit method for a source drop problem, we start with the following equation

$$\frac{p_n - p_{n-1}}{h} = \frac{1}{\Lambda} \left[(\rho_n - \beta)\, p_n + \sum_{k=1}^{6} \lambda_k \zeta_k^{(n)} + s_n \right]$$

and then using algebra, we get

$$\frac{\Lambda}{h} p_{n-1} + s_n = \left[\frac{\Lambda}{h} + (\beta - \rho_n) \right] p_n - \sum_{k=1}^{6} \lambda_k \zeta_k^{(n)}$$

To do that, we have the coefficient matrix

$$\mathbf{A} = \begin{bmatrix} \frac{1}{h} + \lambda_1 & & & -\beta_1 \\ & \ddots & & \vdots \\ & & \frac{1}{h} + \lambda_6 & -\beta_6 \\ -\lambda_1 & \cdots & -\lambda_6 & \frac{\Lambda}{h} + (\beta - \rho_n) \end{bmatrix}$$

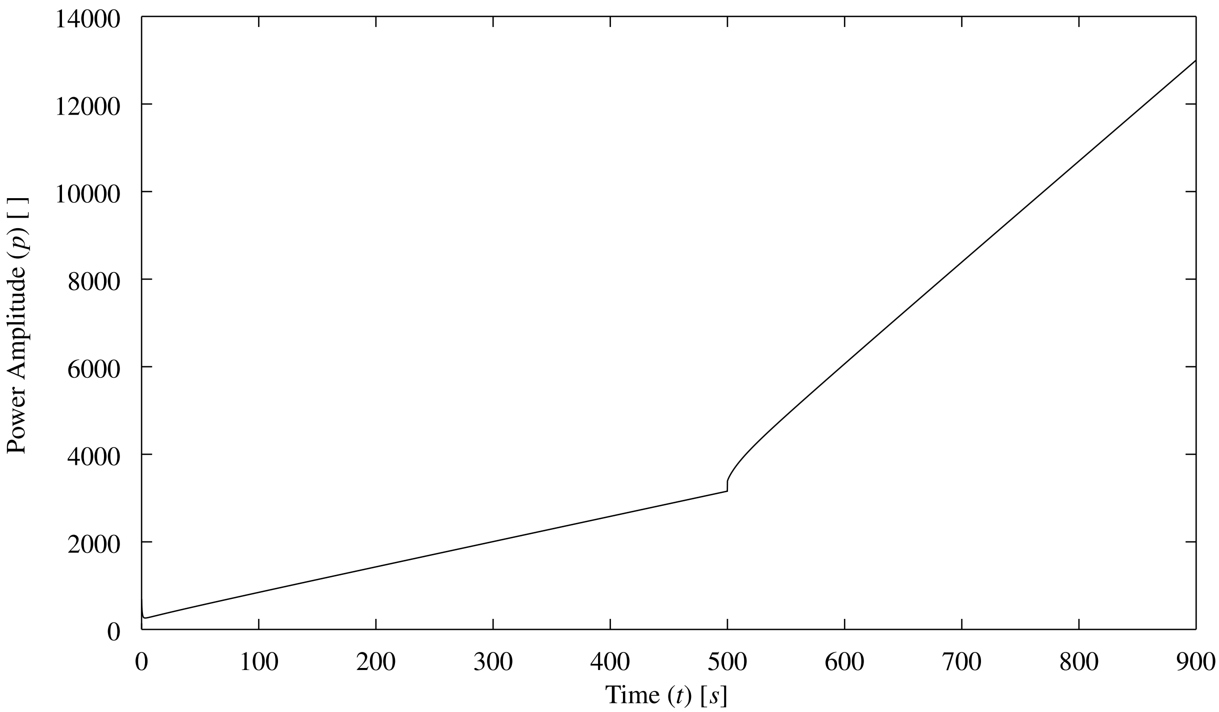


Figure 1: Power Amplitude vs. Time for Two Source Drop Experiment in PUR-1

and the solution vector

$$\vec{b} = \begin{bmatrix} \frac{\zeta_1^{(n-1)}}{h} \\ \vdots \\ \frac{\zeta_6^{(n-1)}}{h} \\ \frac{\Lambda p_{n-1}}{h} + s_n \end{bmatrix}$$

Finally, with those created, the solution of the problem $\mathbf{A}\vec{x} = \vec{b}$ can be found, where $\vec{x}$ is defined as

$$\vec{x} = \begin{bmatrix} \zeta_1^n \\ \vdots \\ \zeta_6^n \\ p_n \end{bmatrix}$$

The figure 1 shows the transient with a source function given by

$$s(t) = \begin{cases} 0.5 & t < 500 \\ 2 & t \geq 500 \end{cases}$$

This figure mimics that of Figure 6.22 from the text, and thus the proportion of the sources as they are dropped in should be 1 : 4.

2 ***Asymmetric Saw-tooth Reactivity Insertion*. Analyze the reactivity transients specified in the homework problem 2 of Chapter 8, where an asymmetric saw-tooth reactivity insertion means the reactivity transient in Eq. (1).**

$$\rho(t) = \begin{cases} at, & t \leq t_1 \\ at_1 - a^{'}(t - t_1), & t > t_1 \end{cases}$$

Table 1: Location of Power Maxima ($\dot{p} = 0$) and Corresponding Reactivity

	Time (t) [s]	Reactivity (ρ) [ ]
$a^{'} = 1.00a$	0.101	-3.4612
$a^{'} = 0.75a$	0.101	-2.5951
$a^{'} = 0.50a$	0.102	-1.7459
$a^{'} = 0.25a$	0.102	-0.8713
$a^{'} = 0.00$	0.103	0.0034

Use $t_1 = 0.1\,s$, $a = 5\,\$/s$, and five different ramp rate values on the declining part; $a^{'} = a$, $a^{'} = 0.75a$, $a^{'} = 0.5a$, $a^{'} = 0.25a$, and $a^{'} = 0$. Plot the power amplitude and its derivative for initial $0.3\,s$ and estimate the time for which $\dot{p} = 0$ and the reactivity at this time point. For a reactivity transient, the implicit numerical method derived in the notes can be applied and solved for each step. This method is unconditionally stable, so the choice of h is trivial, but should still give enough resolution.

This method is simple to program. In general, for each time step, the coefficient matrix is created in arrowhead form, as follows

$$\mathbf{A} = \begin{bmatrix} \frac{1}{h} + \lambda_1 & & & -\beta_1 \\ & \ddots & & \vdots \\ & & \frac{1}{h} + \lambda_6 & -\beta_6 \\ -\lambda_1 & \cdots & -\lambda_6 & \frac{\Lambda}{h} + (\beta - \rho_n) \end{bmatrix}$$

and then the vector can be filled with the previous step solutions

$$\vec{b} = \begin{bmatrix} \frac{\zeta_1^{(n-1)}}{h} \\ \vdots \\ \frac{\zeta_6^{(n-1)}}{h} \\ \frac{\Lambda p_{n-1}}{h} \end{bmatrix}$$

Finally, with those created, the solution of the problem $\mathbf{A}\vec{x} = \vec{b}$ can be found, where $\vec{x}$ is defined as

$$\vec{x} = \begin{bmatrix} \zeta_1^n \\ \vdots \\ \zeta_6^n \\ p_n \end{bmatrix}$$

The solving can be done by gaussian elimination and backsubstitution [1], but in this case the MATLAB backdivision operator [2] was used to save time in programming.

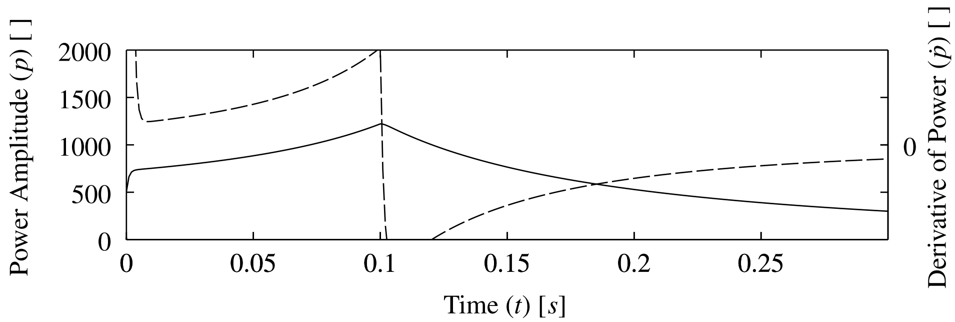
The plots in Figure 2 show the power amplitude and its derivative as a function of time with varying downward slopes. The time for which $\dot{p} = 0$ also corresponds to the maxima of the power, and those are shown in Table 1. **Notice that the reactivity has often already dropped by the time the maxima of power is obtained.**

References

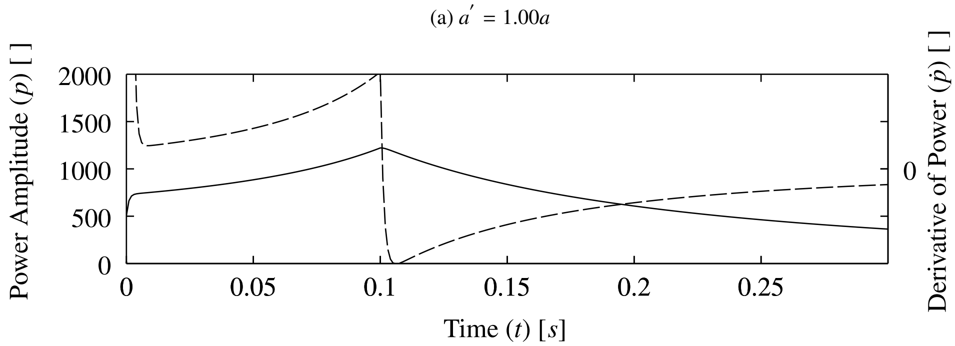
[1] Brian Bradie. *A Friendly Introduction to Numerical Analysis*. Pearson Prentice Hall, Upper Saddle River, New Jersey, 2006.

[2] Mathworks. Symbolic Toolbox: User's Guide (r2013a). Technical report, 2013.

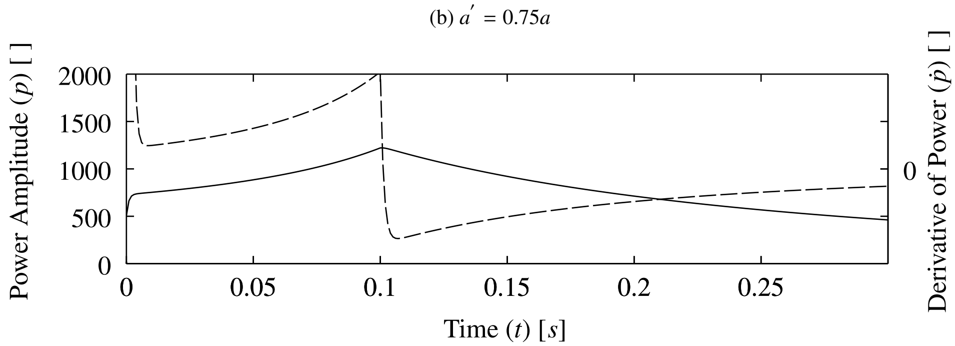
[3] K Ott and R Neuhold. *Introductory Nuclear Reactor Dynamics*. American Nuclear Society, La Grange Park, Illinois, 1985.



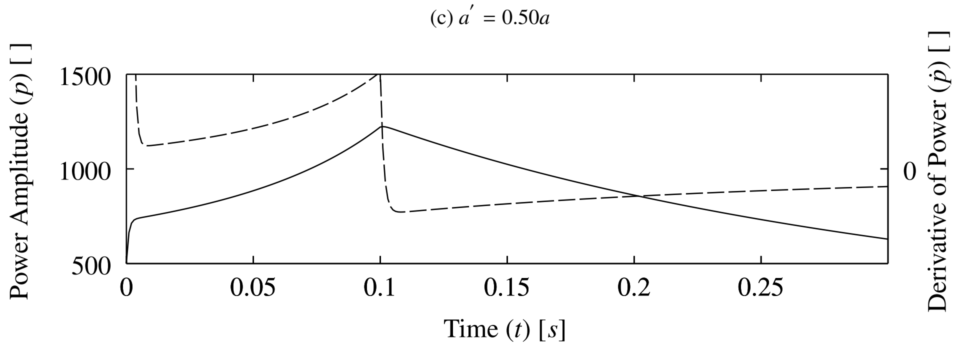
(a) $a' = 1.00a$



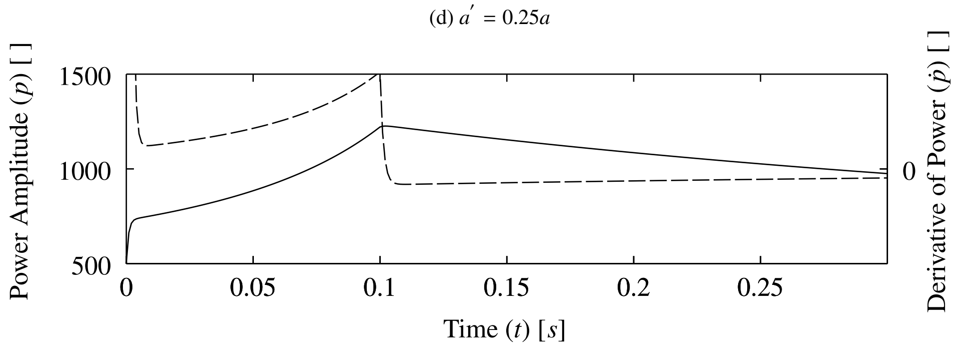
(b) $a' = 0.75a$



(c) $a' = 0.50a$



(d) $a' = 0.25a$



(e) $a' = 0.00$

Figure 2: Power Amplitude and its Derivative for Sawtooth Reactivity Insertion