NUCL 510
Nuclear Reactor Theory

Fall 2011
Lecture Note 1

Prof. Won Sik Yang

Purdue University
School of Nuclear Engineering

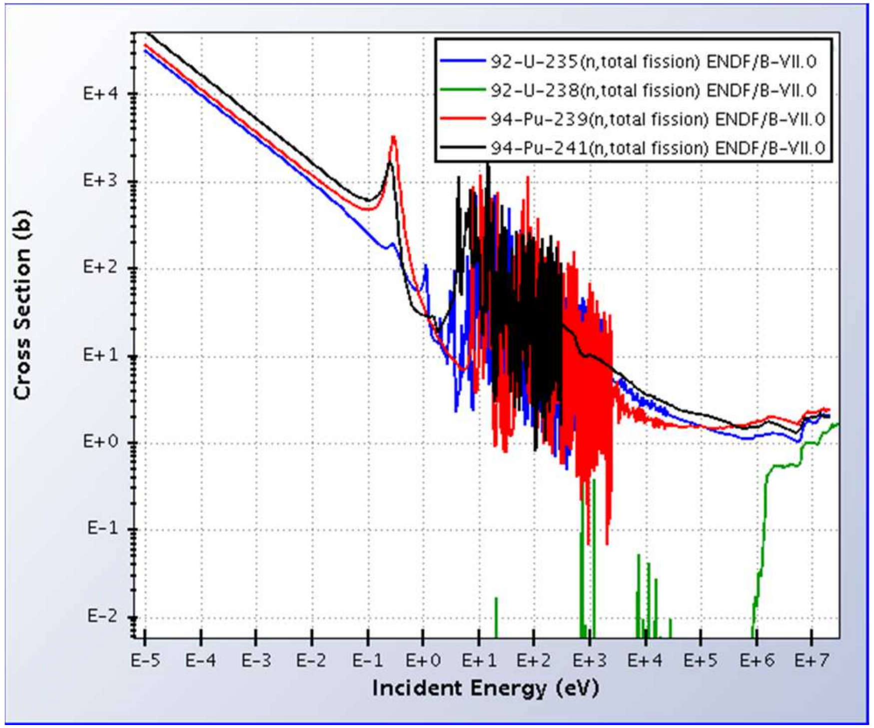
What is Nuclear Reactor Physics?

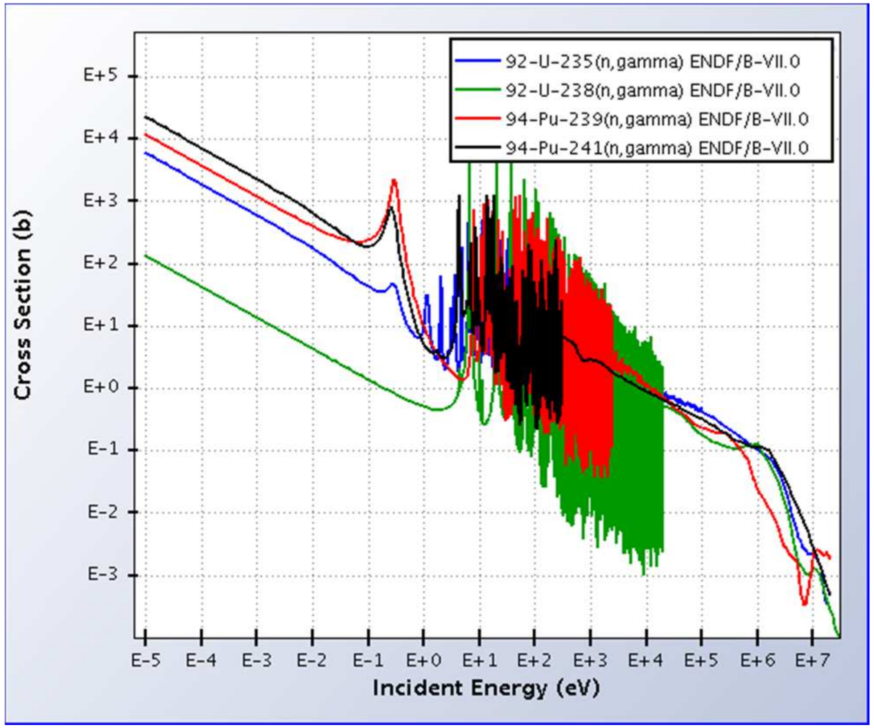
- Nuclear reactor physics deals with the physical processes that determine the behaviors of nuclear chain reaction systems and computational methods for the prediction of these processes.
- Reactor physics or neutronics analysis plays a critical role in reactor design to determine:
  - Fuel dimensions and core configuration
  - Fissile and reactivity control requirements
  - Optimum fuel management scheme
  - Heat generation and deposition
  - Fuel composition evolution with burnup
  - Shielding of ex-core components
- The kinetics parameters and reactivity changes for temperature, core geometry, and material density variations are also determined to assure the favorable safety characteristics.

Governing Equations of Reactor Physics

- The theory and governing equations for reactor physics analysis are well known;
  - Boltzmann equation for neutron and gamma transports
    - *Boltzmann equation is a linear integro-differential equation with seven independent variables (three in space, two in angle, one in energy and time)*
  - Bateman equation for fuel composition evolution
    - *Bateman equation is a system of ordinary differential equations*
- The coefficients of these equations are determined by nuclear data, geometry, and composition.
- The challenge in neutronics analysis is to determine the solution efficiently by taking into account geometric complexity and complicated energy dependence of nuclear data.

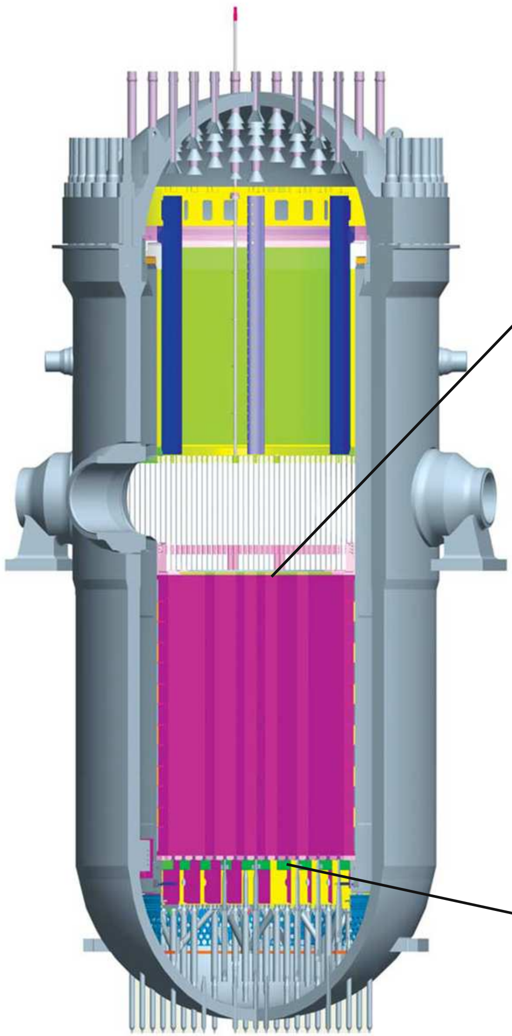
Fission and Capture Cross Sections of Actinides

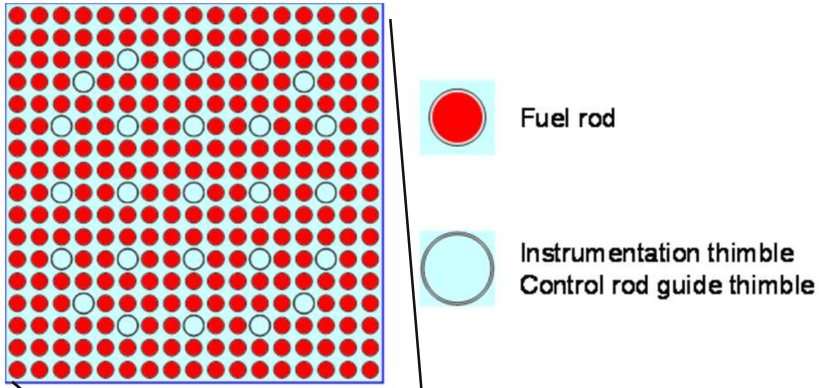


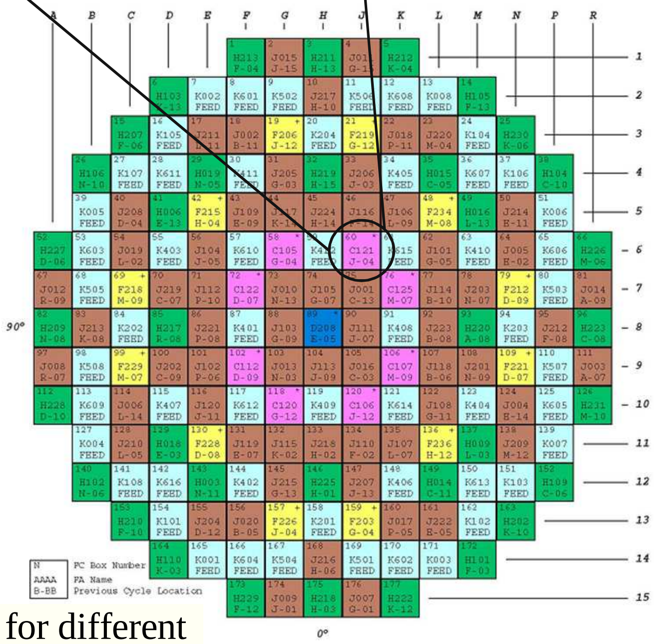


- Complicated resonance structures

PWR Core and Its Major Constituents



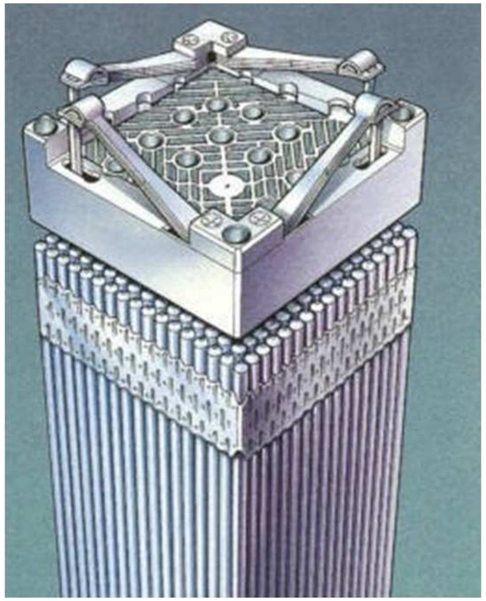




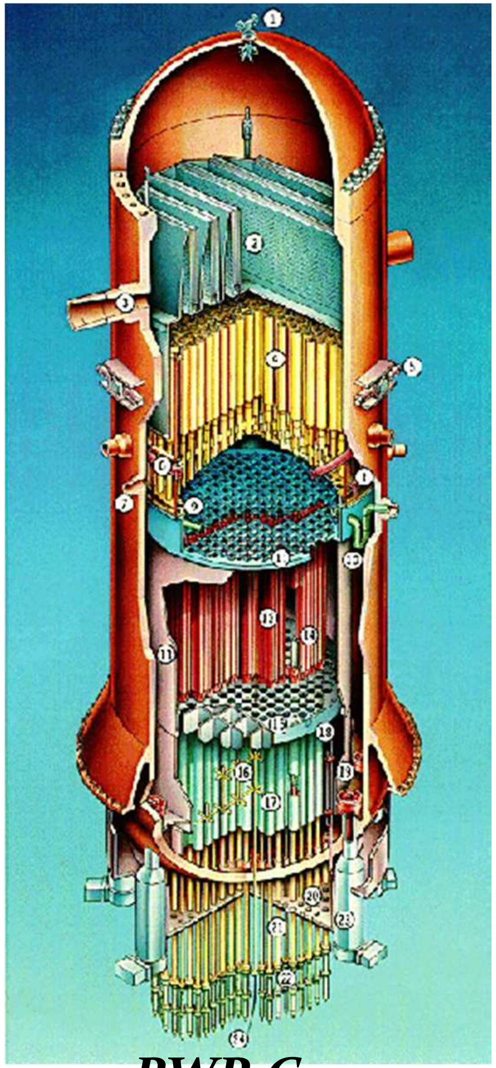
Different color for different fuel enrichment or burnup

LWR Fuel and Core

UO_2 pellet

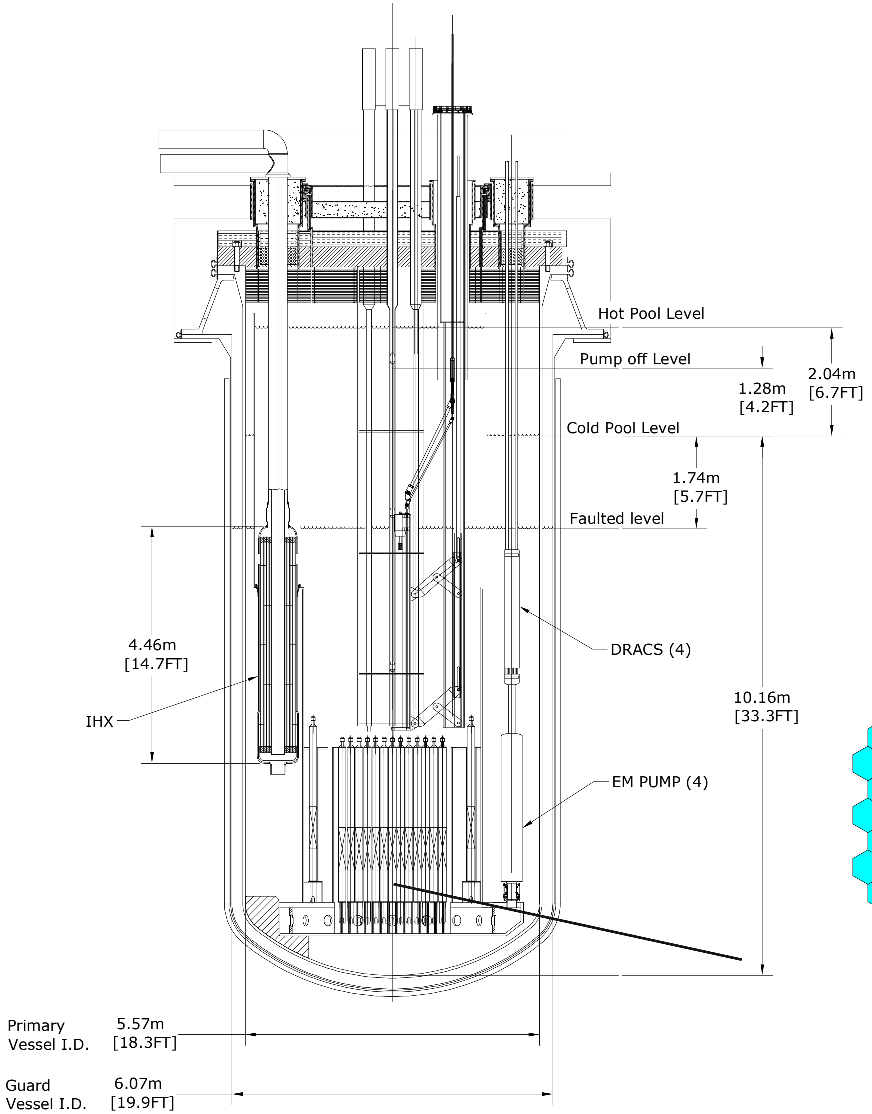


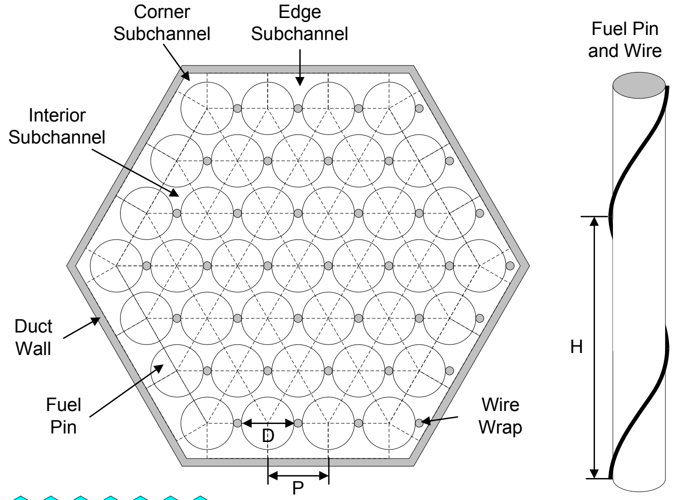
Assembly

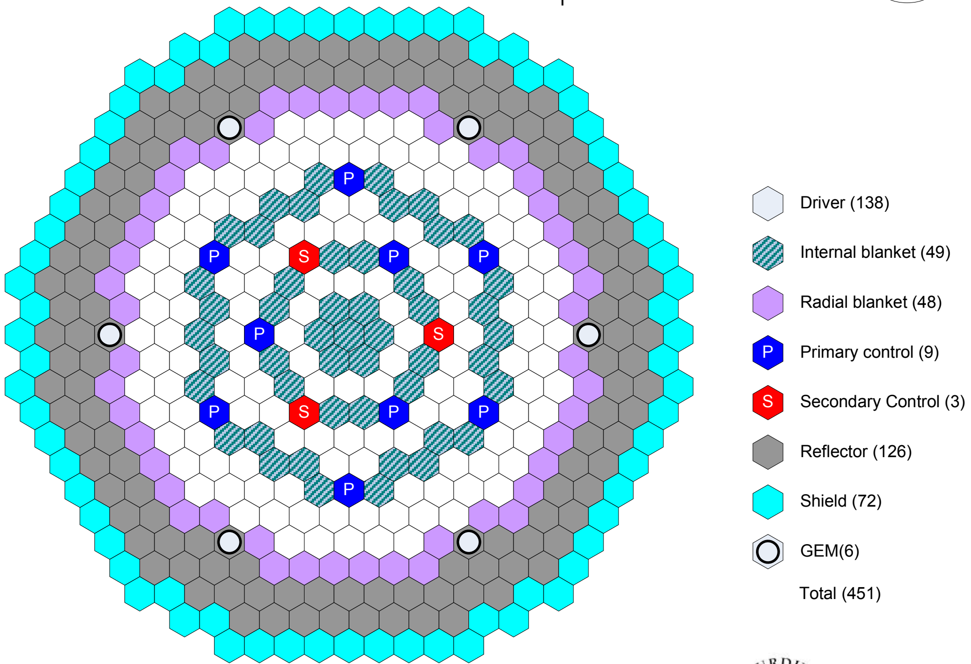


BWR Core

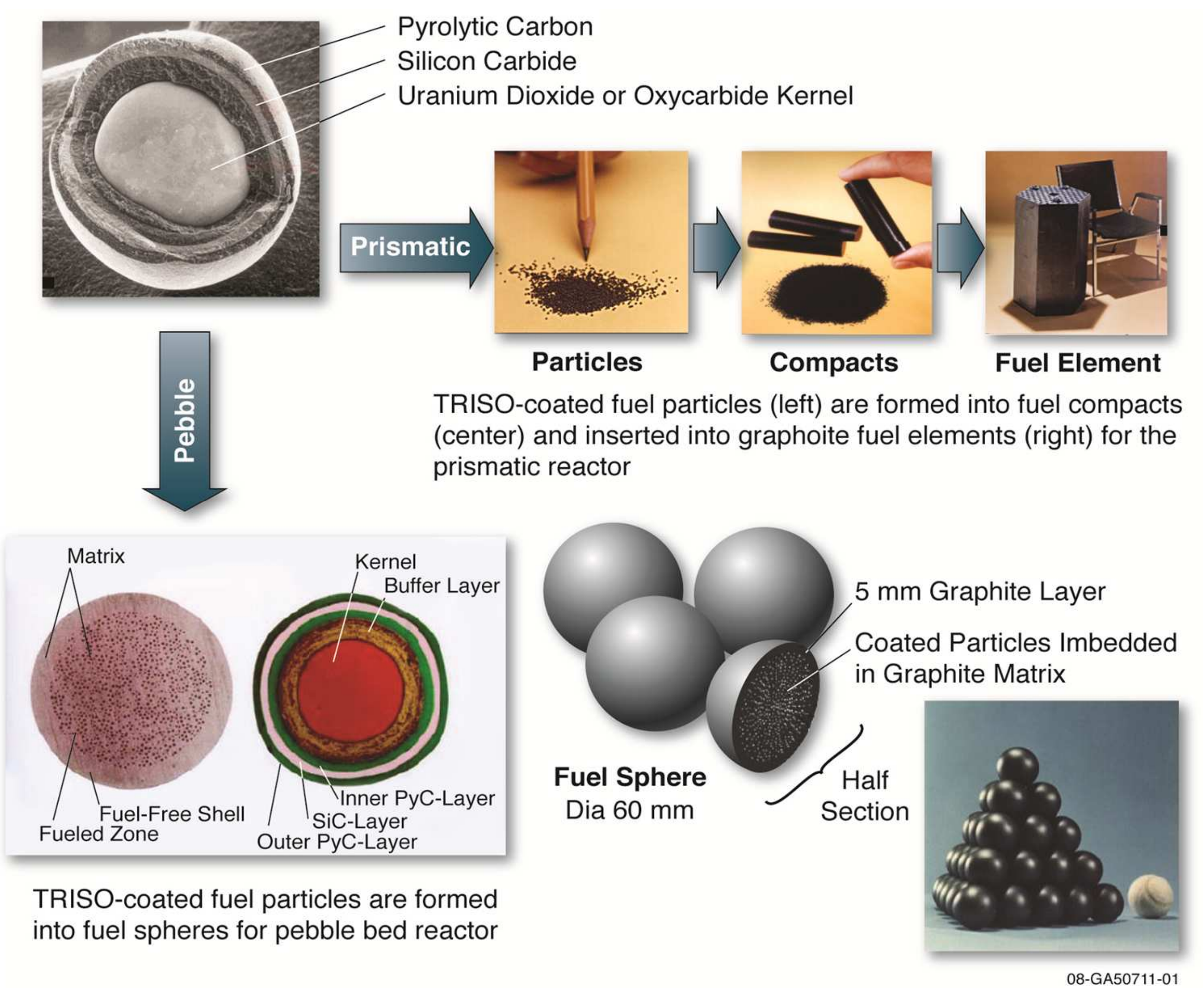
SFR Core and Its Major Constituents

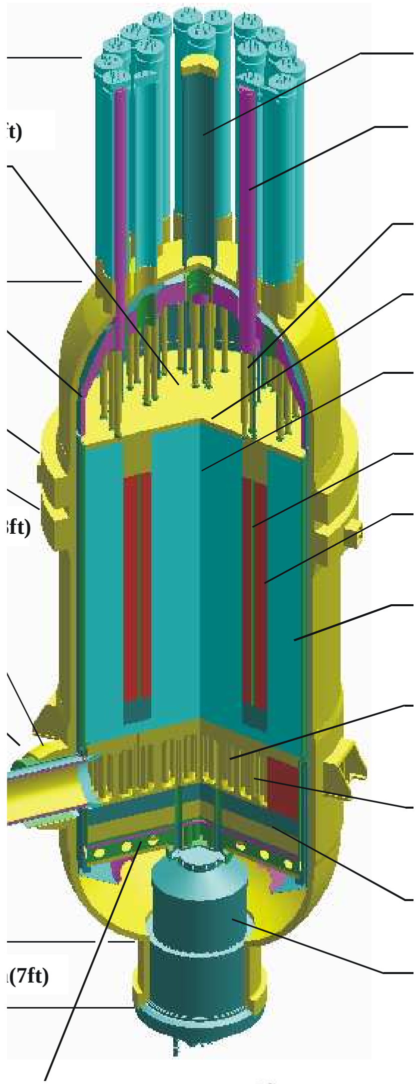






HTR Fuel and Core





Typical Reactor Core Design

- Design Objective
  - Determine fuel composition, local configuration of fuel and coolant, and global arrangement fuel assemblies in a core for specified energy production in a safe and economical manner
- Design Requirements
  - Guarantee specified energy production
    - *Rated power x Duration (e.g. 2775 MW x 15 months x 3 Cycles)*
  - Meet safety criteria
    - *Peak power limit, minimum DNBR limit, negative MTC, discharge burnup, etc.*
- Economic Considerations
  - Minimize generation cost
    - *Higher capacity factor and lower fuel cost*
  - Maximize operational flexibility
    - *Sufficient operating margins*

Design Parameters for Light Water Reactor Cores

- Moderator to Fuel Volume Ratio
  - Optimum fuel pin pitch-to-diameter ratio for maximum k-inf
  - Under-moderation desired for negative temperature coefficients
- Power Density
  - Low power density desired for safety, but higher for economics
- Fuel Enrichment and Arrangement in Fuel Assembly
  - Higher enrichment for more reactivity for longer operation, but higher peaking as well
- Burnable Absorber Contents and Arrangement
  - Effective control of excess reactivity and peaking
- Fuel Assembly Loading Pattern
  - Low leakage fuel loading for economy, but with higher peaking
- Flow Condition
  - Flow rate and inlet temperature, temperature rise

Basic Parameters & Terms in Nuclear Design

- Multiplication Factor

$$k = \frac{\text{\# of neutrons produced}}{\text{\# of neutrons removed}}$$

- Reactivity: Degree of Off-criticality
  - Parameter representing how reactive the core is

$$\rho = 1 - \frac{1}{k_{eff}}$$

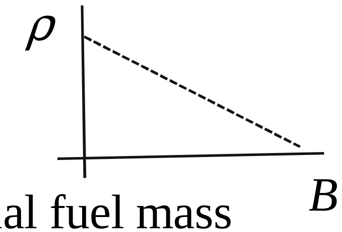
•Unit: % or pcm(per centi milli $= 10^{-5}$)
* 1% reactivity amounts to about 1 month operation in a typical PWR

  - Factors affecting reactivity for a fresh core
    - *Fissile fuel loading amount (fuel enrichment)*
    - *Neutron leakage*
    - *Core thermal condition (Doppler effect, negative temperature feedback)*
    - *Controlling absorber content (control rod, boric acid concentration)*
- Reactivity Coefficient
  - Reactivity change per unit change in specific core parameter $\frac{d\rho}{dT_c}, \frac{d\rho}{dp}$

Basic Parameters & Terms in Nuclear Design

- Burnup: Degree of Fuel Burn-out or Depletion
  - Parameter representing power production per initial fuel loading

$$B = \frac{\text{Energy Produced for an Entity [MWD]}}{\text{Initial Fuel Mass of the Entity [kg]}}$$



* Energy produced is proportial to fuel isotopes fissioned
$\rightarrow$ Relative fration of fuels consumed by dividing by initial fuel mass

- Excess Reactivity
  - Positive reactivity needed to make the core critical throughout the cycle; provided by high fissile loading
  - Factors determining excess reactivity
    - *Temperature defect from cold to hot operating condition*
    - *Fission product buildup*
    - *Fuel depletion (cycle length)*
  - ~20% excess reactivity is required at beginning of cycle (BOC)
  - Can be suppressed by soluble boron(PWR), control rod insertion(BWR), or burnable absorbers

Basic Parameters & Terms in Nuclear Design

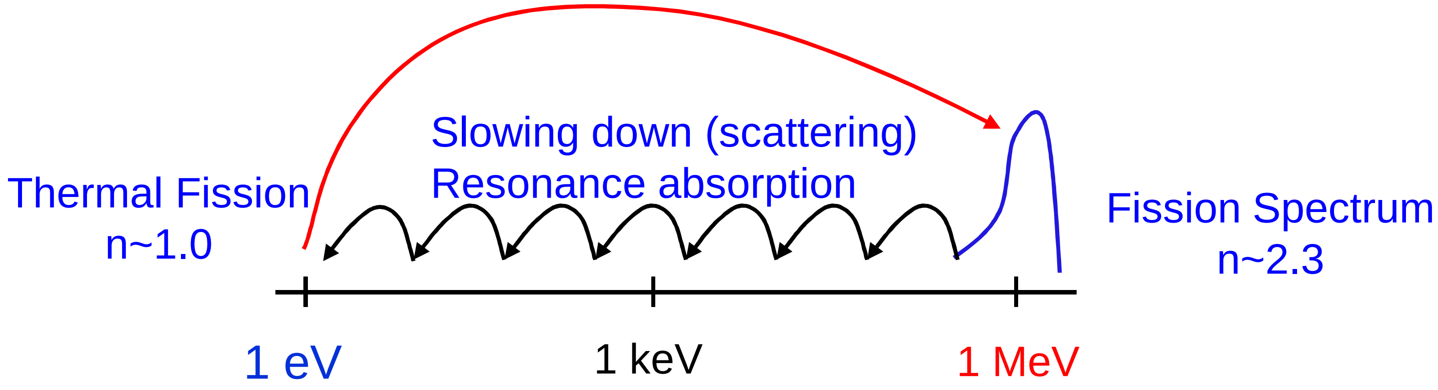
- Burnable Absorber
  - Absorber unit used to control excess reactivity and to control power distribution
  - Typical forms
    - *B_4C or Gadolinia (Gd_2O_3) rods*
- Isotopic Inventory
  - Initial fuel and absorber loading
    - *enough to sustain criticality for a given period (cycle)*
  - Change in nuclide density due to fuel and absorber depletion
    - *fissile consumption and production(from fertile), fission product (Xe, Sm) buildup*
    - *minor actinide production (Am, Np, Cm): long lived waste (long lasting radiation source)*
    - *burnable absorber burnout*

Basic Parameters & Terms in Nuclear Design

- Power Distribution
  - Absolute power density (w/cc) ~100 w/cc for typical PWR
  - Relative value to the average power density ($\bar{p}$)
    - *Power peaking factor of ~2.5*
- Temperature Distribution
  - Peak temperature for safety consideration
    - *Fuel melting, cladding-water interaction*
  - Thermal feedback
    - *Fuel temperature → Doppler effect (intrinsic)*
    - *Coolant temperature → moderator density change*

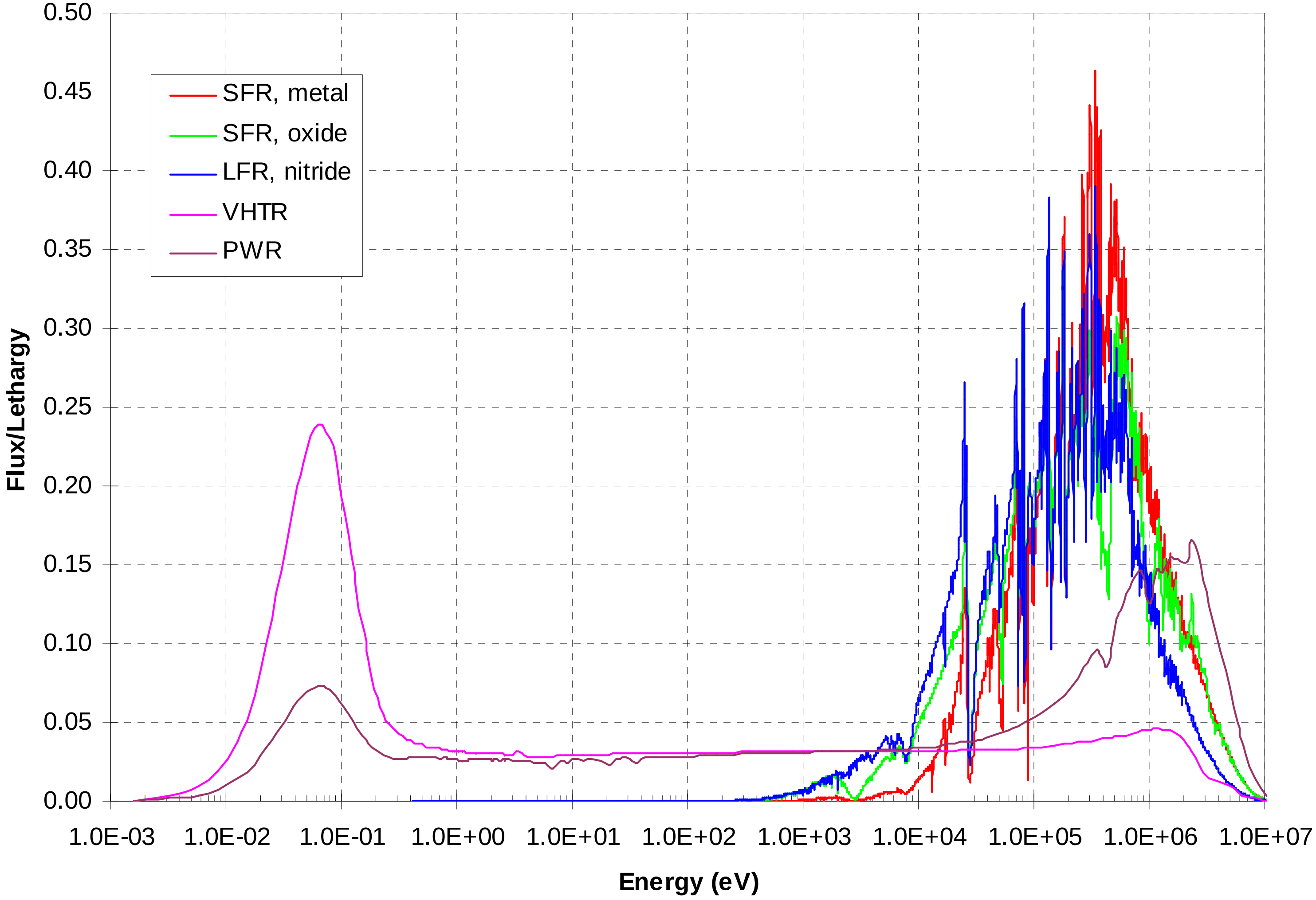
Neutron Life Cycle in Thermal Reactor

- Chain Reaction



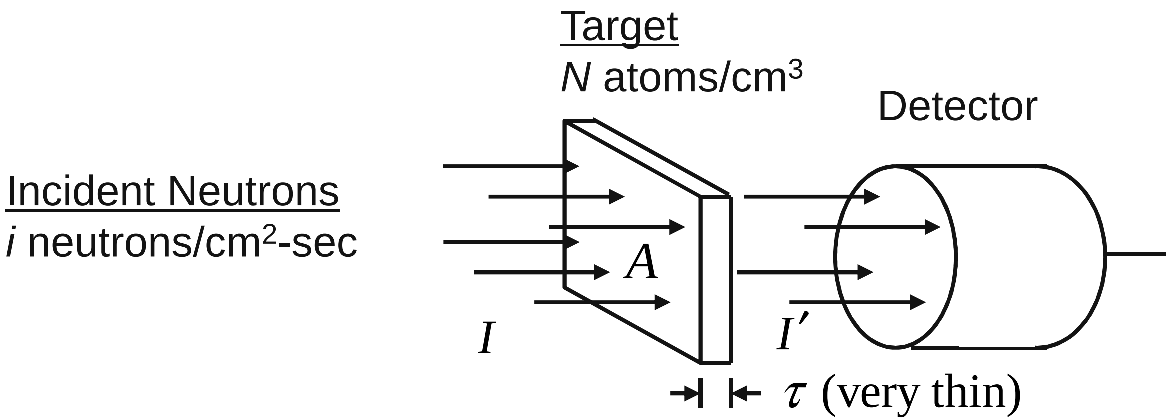
- Loss Mechanisms
  - Leakage: leak out of the system
  - Resonance absorption
  - Nonfuel absorption
  - Capture by fertile
  - Different loss mechanism at different locations and energy→ n(r,E)
    - *neutron population per unit volume and per unit energy interval*
- Sustainable Chain Reaction
  - n_f=1.0 after all those losses

Comparison of Neutron Spectra



Neutron Cross Section, Flux and Other Concepts

- Neutron Beam Attenuation



  – Intensity change observed by experiment

$$\Delta I = I - I' = A(i - i') \propto I \times \tau \times N$$ Loss either by absorption or scattering

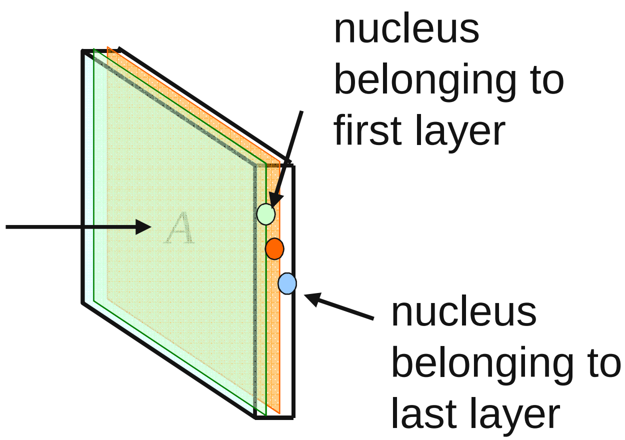
$$\Delta I = \sigma I \tau N$$

$$\frac{\Delta I}{I} = \sigma \tau N = p = \frac{\sigma}{A} A \tau N = \frac{\sigma}{A} N_T$$ Interaction probability per incident neutron

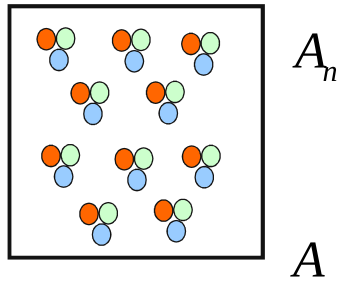
$\rightarrow \sigma$ has a dimension of area because N_T is a number (dimensionless).

Microscopic Cross Section

- What would σ be?
  - Area of a target nucleus



normal through view of the target

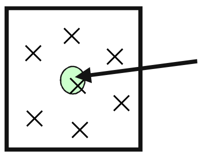


$$A_n = \sigma N_T$$

  - For unit area of impinging neutrons

$$p = \sigma \tau N \rightarrow \boxed{\sigma = \frac{p}{\tau \cdot 1 \cdot N}}$$

Interaction probability per incident neutron per nuclides in unit area



Cross sectional area of the nucleus

$\rightarrow \sigma$ is called microscopic cross section, has the unit of area, specifically,

1 barn $=10^{-24} cm^2$

Macroscopic Cross Section and Reaction Rate

- Macroscopic Cross Section
  - Reaction probability per unit thickness

$$\frac{p}{\tau} = \sigma N \quad \equiv \Sigma$$

Interaction probability for a neutron traveling unit distance

=micro xsec (cm^2) * number density (1/cm^3)→ unit=1/cm

- Reaction Rate
  - Reaction rate per unit volume

$$p = \frac{\Delta I}{I} = \tau\Sigma \quad \rightarrow \Delta I = \tau\Sigma I = \tau\Sigma A i$$

$$\rightarrow \frac{\Delta I}{\tau A} = \Sigma i \qquad \rightarrow \Delta i = \tau\Sigma i$$

Proportional to macro xsec Σ
and neutron beam intensity i

- Infinitesimal change in intensity in dx
  - Change proportional to intensity and macro xsec times dx

$$di = i(x + dx) - i(x) \quad = -i\Sigma dx$$

$$\frac{di}{dx} = -\Sigma i \quad \rightarrow i(x) = i_0 e^{-\Sigma x}$$

Exponential Attenuation

Mean Free Path and Flux

- Mean Free Path
  - Average distance that a neutron travels without collision

$$\frac{i(x)}{i_0} = e^{-\Sigma x} = p(x) : \text{Survival Probablity}$$

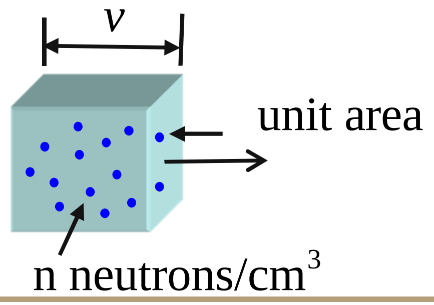
$$e^{-\Sigma x}\Sigma dx \quad : \text{Probability to survive till x then react within dx}$$

$$\lambda = E(x) = \int_0^\infty x e^{-\Sigma x}\Sigma dx = -xe^{-\Sigma x}\Big|_0^\infty + \int_0^\infty e^{-\Sigma x} dx = \frac{1}{\Sigma}$$

$$\int uv' = uv - \int u'v, \quad (u = x, v = -e^{-\Sigma x}) \quad : \text{integration in part}$$

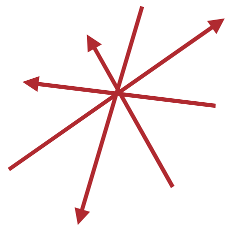
- Neutron Flux for Mono speed and direction
  - What is intensity *i* for mono-directional beam with neutron speed v?

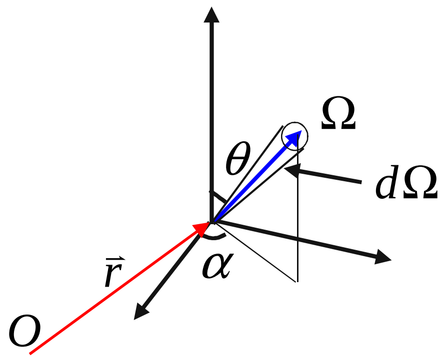
$$i = nv$$



Angular Flux and Scalar Flux

- Angle Dependent Intensity → Angular Flux
  - Neutrons can move in all 4π directions





$$\Omega = \Omega(\theta, \alpha)$$

$n(\Omega)d\Omega$: number density of neutrons moving toward $d\Omega$ about Ω

$i(\Omega)d\Omega = n(\Omega)v(\Omega)d\Omega$: intensity in Ω direction

$\varphi(\Omega) \equiv i(\Omega) = n(\Omega)v(\Omega)$: angular flux [netrons/sec-cm^2-steradian]

- Total Reaction Rate and Scalar Flux
  - Reaction occurs irrespective of direction

$$R = \int_{4\pi} \Sigma\varphi(\Omega)d\Omega = \Sigma\int_{4\pi} \varphi(\Omega)d\Omega = \Sigma\phi$$

$\phi = \int_{4\pi} \varphi(\Omega)d\Omega$ scalar flux (no angle dependence) [n/sec-cm^2]

Energy and Temperature Dependence

- Angular Flux

$n = n(\vec{r}, E, \Omega)$: number density, position, angle + energy dependent

$v = v(E)$: velocity, energy dependent

$$\varphi = n(\vec{r}, E, \Omega) v(E) = \varphi(\vec{r}, E, \Omega)$$
$$= \varphi(x, y, z, E, \alpha, \theta)$$

- Macroscopic Cross Section with Temp. Dependence

$$\Sigma = N(\vec{r}, T)\sigma(E, T) = \Sigma(\vec{r}, E, T)$$

$N = N(\vec{r}, T(\vec{r}))$: nuclide number density, temperature dependent

$\sigma = \sigma(E, T(\vec{r}))$: micro xsec, energy and temperature dependent

Nuclear Data Evaluation

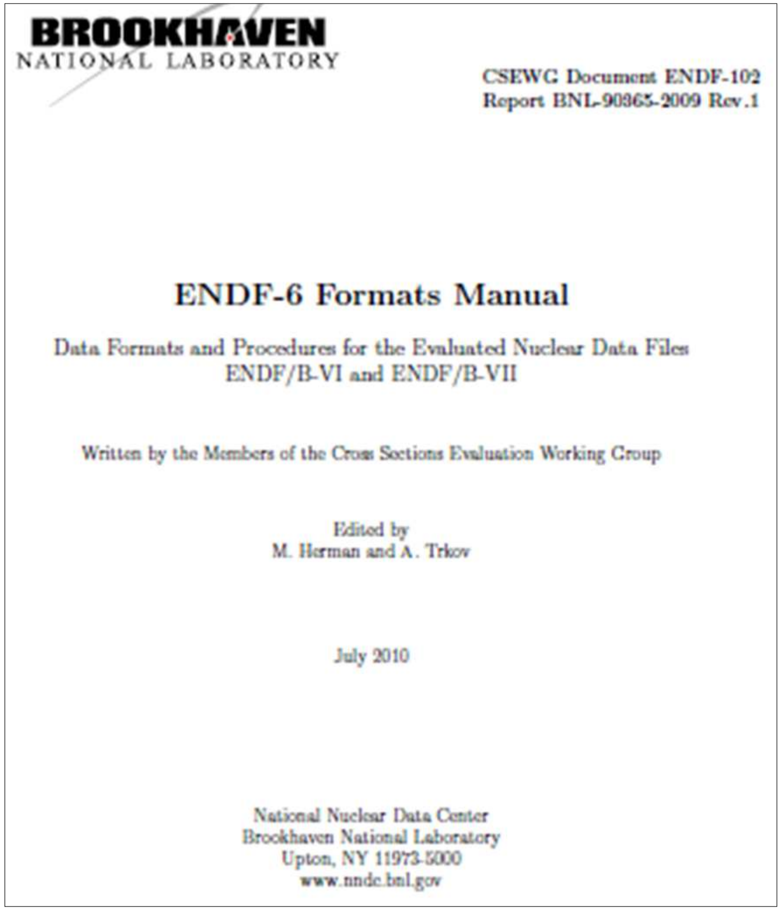
- Nuclear data are usually identified with neutron reaction data, though of importance are other data such as thermal neutron scattering, fission product yields and decay data.
- Evaluation is a set of procedures used to obtain the best data taking into account all available experimental data which are combined with nuclear reaction modeling. Testing and validation is inherent part of evaluation.
- Libraries of evaluated nuclear data typically contain information about few hundred materials (such as moderators, structural materials, fission products and actinides) relevant to nuclear technology applications.
- Historically, systematic nuclear data evaluations started in the US by establishing Cross Section Evaluation Working Group, CSEWG, in 1966.

Basic Ingredients of Evaluation

- Experiment
  - Experimental database EXFOR
    - *EXFOR (**ex**change **for**mat) is international library maintained by Nuclear Reaction Data Center Network coordinated by IAEA.*
  - Atlas of Neutron Resonances (published by Elsevier in 2006)
    - *5th edition of well known **BNL-325**, which contains thermal cross sections and resonance parameters.*
- Theory (also based on experiment!)
  - Nuclear reaction model codes
    - *Used in fast neutron region to guide evaluation and to fill-in gaps in experimental data.*
    - *GNASH (LANL), EMPIRE (BNL and IAEA), TALYS (NRG Petten)*
  - Reference Input Parameter Library, **RIPL**
    - *Developed during 1994-2008 under three IAEA projects.*
    - *Contains all input parameters for nuclear reaction model codes (atomic masses, decay data, level densities, optical model parameters, etc.)*

Formatting, ENDF-6 Formats

- Evaluated data are stored in internationally accepted format ENDF-6
- ENDF (Evaluated Nuclear Data File) formats are maintained by CSEWG (Cross Section Evaluation Working Group) in the US
- Historically, each release of ENDF/B library had its own format, but ENDF/B-VII still uses earlier ENDF-6.
- ENDF-6 organizes data into files, MF and sections, MT.



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CSEWG Document ENDF-102
Report BNL-90365-2009 Rev.1

ENDF-6 Formats Manual

Data Formats and Procedures for the Evaluated Nuclear Data Files
ENDF/B-VI and ENDF/B-VII

Written by the Members of the Cross Sections Evaluation Working Group

Edited by
M. Herman and A. Trkov

July 2010

National Nuclear Data Center
Brookhaven National Laboratory
Upton, NY 11973-5000
www.nndc.bnl.gov

http://www.nndc.bnl.gov/csewg/docs/endf-manual.pdf

Evaluated Nuclear Data Libraries

- There is a number of evaluated nuclear data libraries, of which most important fall into category of general purpose files. Such files are maintained by 5 major countries in accordance with their national interests and priorities.
  - USA: ENDF (Evaluated Nuclear Data File)
    - *ENDF/B-VII.0 (2006); ENDF/B-VII.1 (Expected in December 2011)*
  - Russia: BROND (Biblioteka Rekomendovannych Nejtronnych Dannych - Library of Recommended Neutron Data)
    - *ROSFOND (2008)*
  - Europe: JEFF (Joint European Fission & fusion File)
    - *JEFF-3.1 (2005)*
  - Japan: JENDL (Japan Evaluated Nuclear Data Library)
    - *JENDL-4.0 (2010)*
  - China: CENDL, China Evaluated Nuclear Data Library
    - *CENDL-3.1 (China, 2009)*

National Nuclear Data Center (NNDC)

- http://www.nndc.bnl.gov