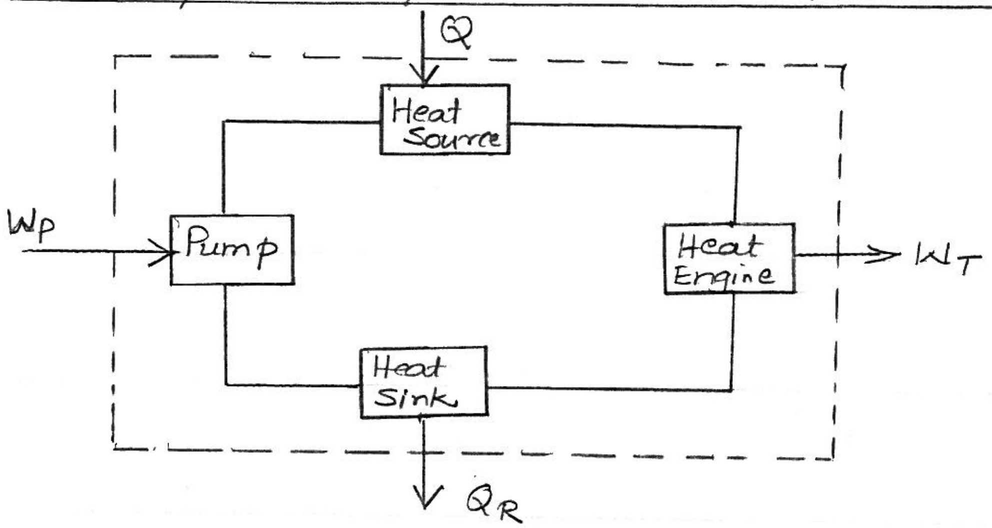
Thermodynamic Cycles and Their Efficiencies.

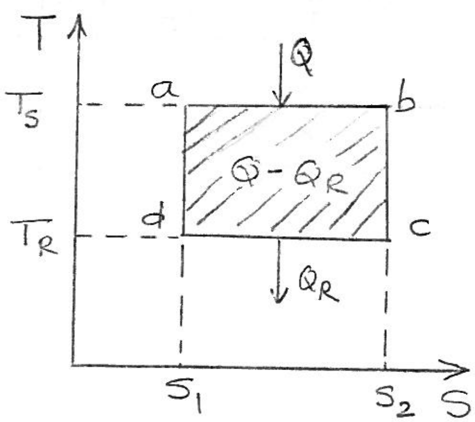


· Reactor Power Cycle

Overall thermal efficiency $\eta = \dfrac{\text{Net output Work}}{\text{Input Heat}} = \dfrac{W_T - W_P}{Q}$

$$= \frac{Q - Q_R}{Q} = 1 - \frac{Q_R}{Q}$$

Carnot Cycle :



Efficiency of this cycle abcd

$$\eta = \frac{Q - Q_R}{Q} = \frac{T_S(S_b - S_a) - T_R(S_c - S_d)}{T_S(S_b - S_a)}$$

$$= 1 - \frac{T_R}{T_S}$$

1. a-b - isothermal heat addition
2. b-c - isentropic expansion
3. c-d - isothermal heat rejection
4. d-a isentropic compression

Shaft work of the turbine

$$W_T = h_1 - h_{2s}$$

pumping Work $W_p = h_{4s} - h_3$.

$$= \frac{v}{J}(p_1 - p_2)$$

Net work per kg of working fluid

$$W_{Net} = (h_1 - h_{2s}) - (h_{4s} - h_3)$$

$$= (h_1 - h_{2s}) - \frac{v}{J}(p_1 - p_2)$$

Heat input

$$Q = (h_1 - h_{4s}) = (h_1 - h_3) - \frac{v}{J}(p_1 - p_2)$$

Overall thermal efficiency

$$\eta = \frac{W_{Net}}{Q} = \frac{(h_1 - h_{2s}) - \frac{v}{J}(p_1 - p_2)}{(h_1 - h_3) - v/J\,(p_1 - p_2)}$$

Actual Ideal Rankine Cycle:

pump (compressor) $\eta_p = \dfrac{h_{4s} - h_3}{h_4 - h_3}$.

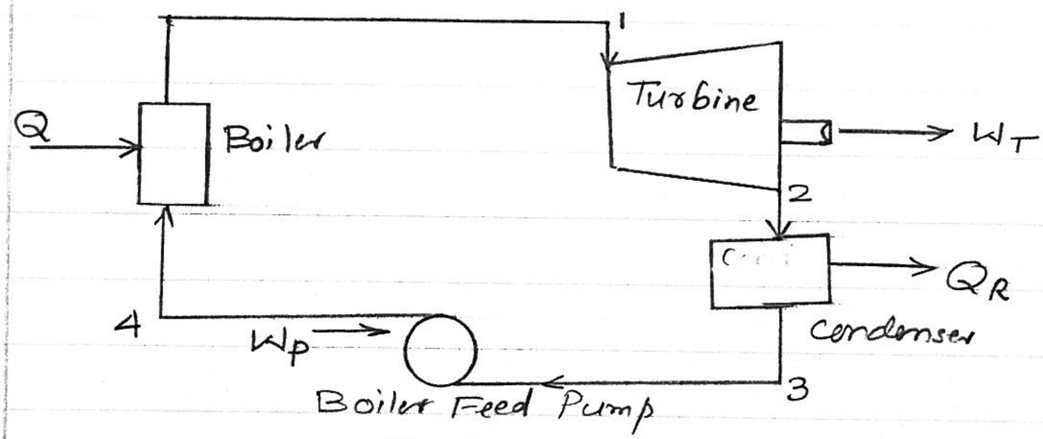
turbine (expansion) $\eta_t = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$.

$$\therefore W_{Net} = \eta_t(h_1 - h_2) - \frac{v\Delta p}{\eta_p J}$$

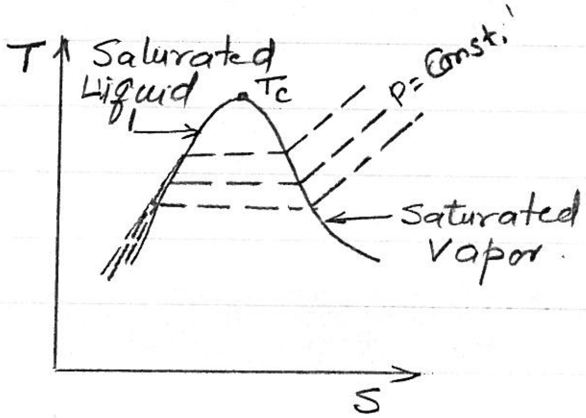
$$Q = (h_1 - h_4) = (h_1 - h_3) - \frac{v\Delta p}{\eta_p J}$$

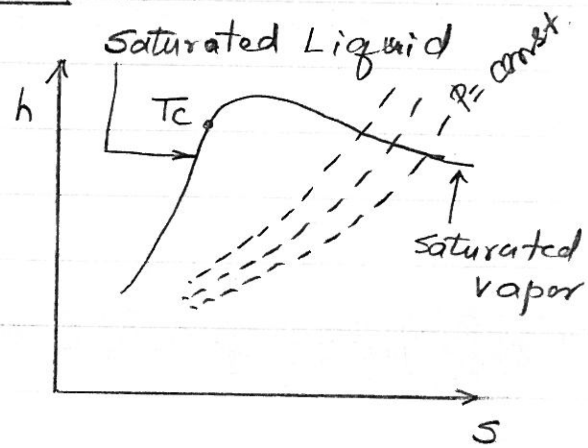
$$\therefore \eta = \frac{\eta_t(h_1 - h_{2s}) - \frac{v\Delta p}{J\eta_p}}{(h_1 - h_3) - \frac{v\Delta p}{J\eta_p}}$$

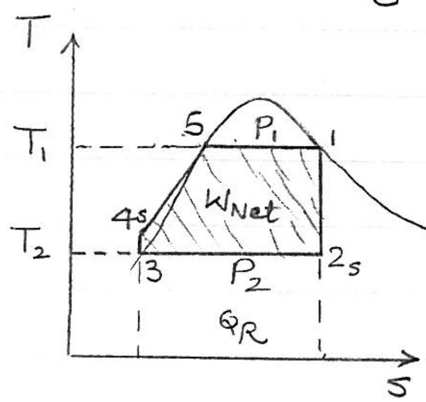
Rankine Cycle (Liquid-vapor cycle)

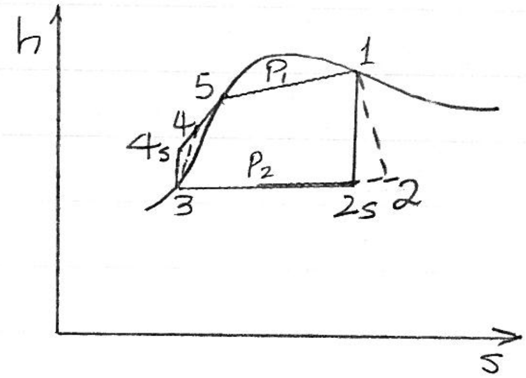


Simple Rankine Cycle.









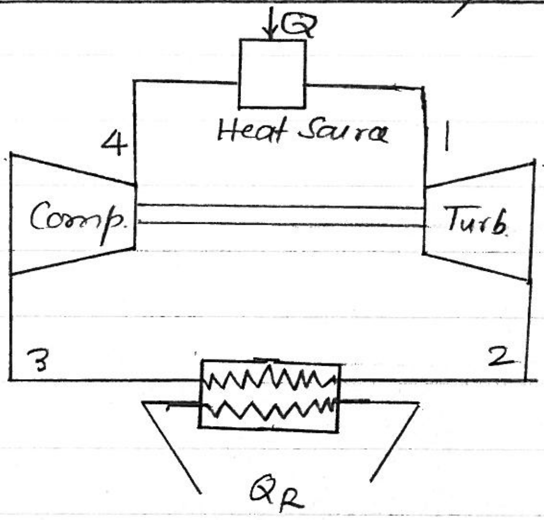
Ideal Rankine Cycle.

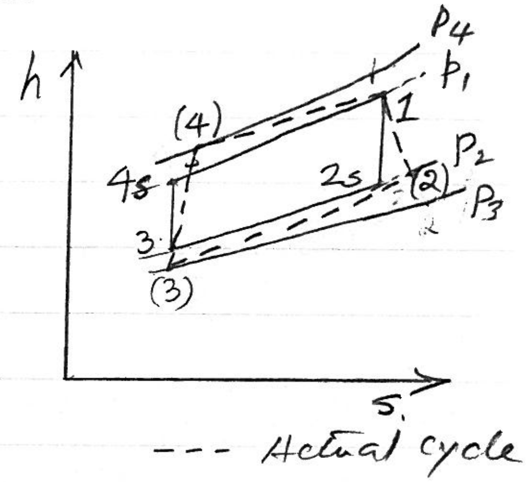
$1 \rightarrow 2_s$ isentropic expansion, $2_s \rightarrow 3$, steam condensation

$3 \rightarrow 4_s$ – isentropic compression $P_2 \rightarrow P_1$ with pump. (boiler pressure)

$4_s \rightarrow 5 - 1$ – high pressure liquid emerges as steam at 1.

Gas Turbine or Brayton Cycles.





1 → 2s − adiabatic & isentropic expansion. (turbine)

2s − 3 − heat removed at constant p.

3 → 4s − adiabatic & isentropic compression (compr.)

4s → 1 − heated at constant p.

Turbine: $W_{Ts} = h_1 - h_{2s}$

Comp. $W_{cs} = h_{4s} - h_3$

heat added $Q = h_1 - h_{4s}$

For ideal Brayton cycle:
$$\eta = \frac{W_{net}}{Q} = \frac{(h_1 - h_{2s}) - (h_{4s} - h_3)}{h_1 - h_{4s}}$$

Actual Brayton Cycle:

compressor efficiency $\eta_c = \dfrac{\text{Ideal Work}}{\text{Actual Work}} = \dfrac{h_{4s} - h_3}{h_4 - h_3}$

turbine " $\eta_t = \dfrac{\text{Actual Work}}{\text{Ideal Work}} = \dfrac{h_1 - h_2}{h_1 - h_{2s}}$.

$$\eta = \frac{(h_1 - h_2) - (h_4 - h_3)}{h_1 - h_4} = \frac{\eta_t (h_1 - h_{2s}) - (h_{4s} - h_3)/\eta_c}{h_1 - h_3 - \dfrac{(h_{4s} - h_3)}{\eta_c}}$$

Thermodynamic Analysis of Nuclear Power Plants

Rankine and Brayton Cycles.
Constant pressure heat addition and rejection for steady flow operation.

Thermodynamic efficiency (or effectiveness) ξ

$$\equiv \frac{\dot{W}_{u,actual}}{\dot{W}_{u,max.}}$$

$$\dot{W}_{u,max} = \frac{\partial (U - T_0 S)}{\partial t} + \sum_{i=1}^{I} \dot{m}_i (h - T_0 s_i) + \left(1 - \frac{T_0}{T_s}\right)\dot{Q}$$

isentropic efficiency

$$\eta_s = \left(\frac{\dot{W}_{u,actual}}{\dot{W}_{u,max}}\right)_{\dot{Q}=0}$$

For adiabatic control volume $\xi = \eta_s$.

$$\dot{W}_{u,max}\big|_{\dot{Q}=0} = -\left[\frac{\partial U}{\partial t}\right]_{c.v.} + \sum_{i=1}^{I} \dot{m}_i h_{is}.$$

steady state

$$\dot{W}_{u,max}\big|_{\dot{Q}=0} = \sum_{i=1}^{I} \dot{m}_i h_{is}.$$

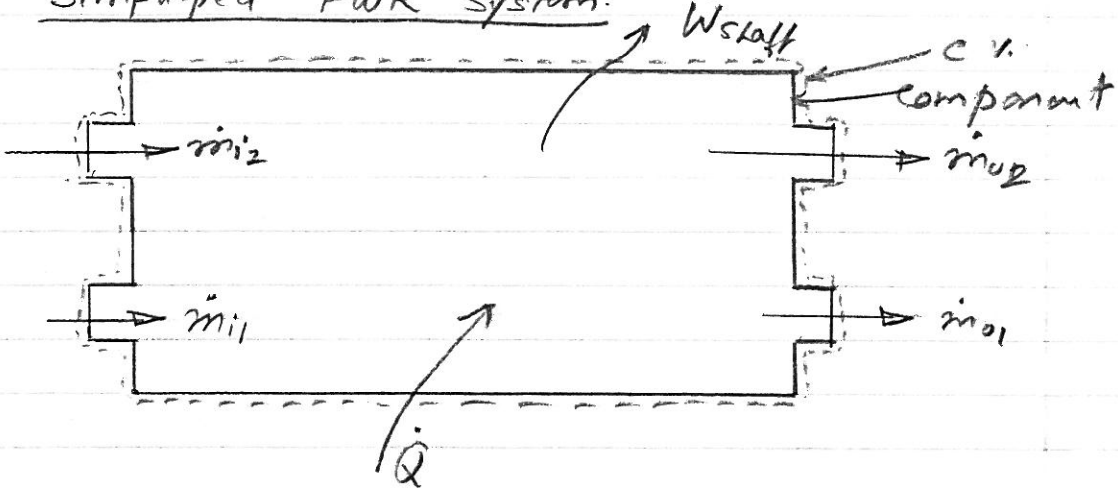
$$\dot{W}_{u,act} = \sum_{i=1}^{I} \dot{m}_i h_i$$

$$\therefore \eta_s = \frac{\sum_{i=1}^{I} \dot{m}_i h_i}{\sum_{i=1}^{I} \dot{m}_i h_{is}}$$

Thermal efficiency.

$$\eta_{th} = \frac{\dot{W}_{u,act}}{\dot{Q}_{in}}$$

Simplified PWR System:



Fixed Volume component

$$\sum_{k=1}^{I} (\dot{m} h)_{in,k} - \sum_{k=1}^{I} (\dot{m} h)_{out,k} = \dot{W}_{shaft} - \dot{Q}$$

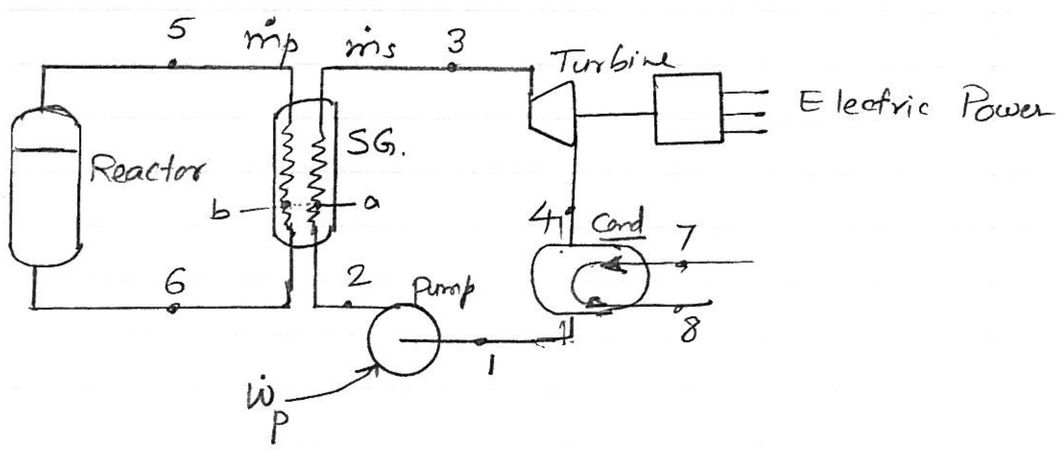
steady state $\quad \dot{m} = \sum_{k=1}^{I} \dot{m}_{in,k} = \sum_{k=1}^{I} \dot{m}_{out,k}$

$$\eta_T = \frac{h_{in} - h_{out}}{h_{in} - h_{out,s}} \quad \text{— turbine}$$

$$\eta_p = \frac{h_{in} - h_{out,s}}{h_{in} - h_{out}} \quad \text{— pump}$$

$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{(h_{out} - h_{in})_s}{(h_{in} - h_{out})_p}$$

p – primary
s – secondary.



$$\frac{\dot{m}_p}{\dot{m}_s} = \frac{h_3 - h_a}{\bar{c}_p[T_5 - (T_a + \Delta T_p)]} = \frac{h_a - h_2}{\bar{c}_p[(T_a + \Delta T_p) - T_6]}$$

$$\dot{W}_{u,actual} = \dot{W}_T + \dot{W}_p = [\dot{m}_s(h_{in} - h_{out})]_T + [\dot{m}_s(h_{in} - h_{out})]_p$$

$$= [\eta_T \dot{m}_s (h_{in} - h_{out,s})]_T + \left[\frac{\dot{m}_s}{\eta_p}(h_{in} - h_{out,s})\right]_p$$

for pump $h_{in} < h_{out,s}$, & for turbine $h_{in} > h_{out,s}$

$$\dot{W}_{u,max} = \dot{m}_p(h_{out} - h_{in})_R = \dot{m}_p(h_5 - h_6)$$

$$= \dot{m}_p(h_5 - h_6) = \dot{m}_s(h_3 - h_2)$$

$$\xi = \frac{[\dot{m}_s(h_{in} - h_{out})]_T + [\dot{m}_s(h_{in} - h_{out})]_p}{[\dot{m}_s(h_{out} - h_{in})]_{SGs.}}$$

$$= \frac{h_3 - h_4 + h_1 - h_2}{h_3 - h_2}$$