1-D Balance Eq.

- mass balance (continuity equation)

$$\underbrace{\frac{\partial \rho}{\partial t}}_{\substack{\text{time rate}\\ \text{of change}\\ \text{in density}\\ \text{(mass)}}} + \underbrace{\nabla \cdot \rho \vec{V}}_{\substack{\text{convetive mass}\\ \text{transfer}}} = 0$$

avg over area $\frac{1}{A}\int dA$ to get 1-D formulation

$$\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z}\langle \rho V_z \rangle = 0$$

not mass weighted mean

note $\langle\langle V_z \rangle\rangle = \frac{\langle\langle \rho V_z \rangle\rangle}{\langle \rho \rangle}$

if $\rho = \langle \rho \rangle$ ρ is uniform in A

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}\rho\langle V_z \rangle = 0}$$

- energy equation

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho i \vec{V}) = -\nabla \cdot \vec{q} + \frac{DP}{Dt} - \tau : \nabla \vec{V} + \dot{q}$$

enthalpy

avg over area, again ρ is uniform in A

$$\frac{\partial \rho \langle i \rangle}{\partial t}$$

$\frac{\partial \rho \langle i \rangle}{\partial t}$ +	$\frac{\partial \rho \langle i V_z \rangle}{\partial z}$ =	$-\frac{\partial \langle q_z \rangle}{\partial z}$	axial conduction
time rate of change of enthalpy	energy transfer by convection	$+\frac{\xi_k q''_w}{A}$	ξ_k heated perimeter $q''_w = h(T_w - T)$ → wall heat flux
		$+\frac{D\langle P \rangle}{Dt}$	energy increase due to pressure
		$+\langle \dot{q} \rangle$	heat generation

• momentum equation

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v}\vec{v} = -\nabla P - \nabla \cdot \underline{\underline{\tau}} + \rho \vec{g}$$

avg over area, again ρ is uniform in A

$\frac{\partial \rho \langle v_z \rangle}{\partial t} + \frac{\partial}{\partial z} \rho \langle v_z v_z \rangle =$	$-\frac{\partial \langle P \rangle}{\partial z}$	pressure
	$-\frac{\partial \langle \tau_{zz} \rangle}{\partial z}$	normal shear
	$-\frac{4}{D} \tau_w$	wall shear $\left(\frac{f}{2D} \rho v \lvert v \rvert\right)$
	$+\rho g_z$	gravity driver

1-D Balance Applied to Reactor System

integrate over primary system

- mass balance $\to 0$ over system

  $\rho_i V_i a_i = \rho_r V_r a_r$

- momentum equation comps.

$$\oint \frac{\partial \rho \langle V_z \rangle}{\partial t} dz = \sum \rho_i \frac{dV_{z,i}}{dt} l_i$$

time rate of change of momentum thru ith component

$$\oint \frac{\partial \rho \langle V_z V_z \rangle}{\partial z} dz = 0$$

convective acceleration

$$\oint -\frac{\partial P}{\partial z} dz = \Delta P_{pump}$$

delta pressure

$$\oint \frac{f_i \rho V_i |V_i|}{2D_i} dz = \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{\rho_i V_i |V_i|}{2}\right)$$

$$\oint \frac{\partial \langle \tau_{zz} \rangle}{\partial z} dz = 0$$

$$\oint \rho g_{z,i} dz = \sum (\rho g l - \rho g \beta \Delta T\, l_h)$$

for whole loop

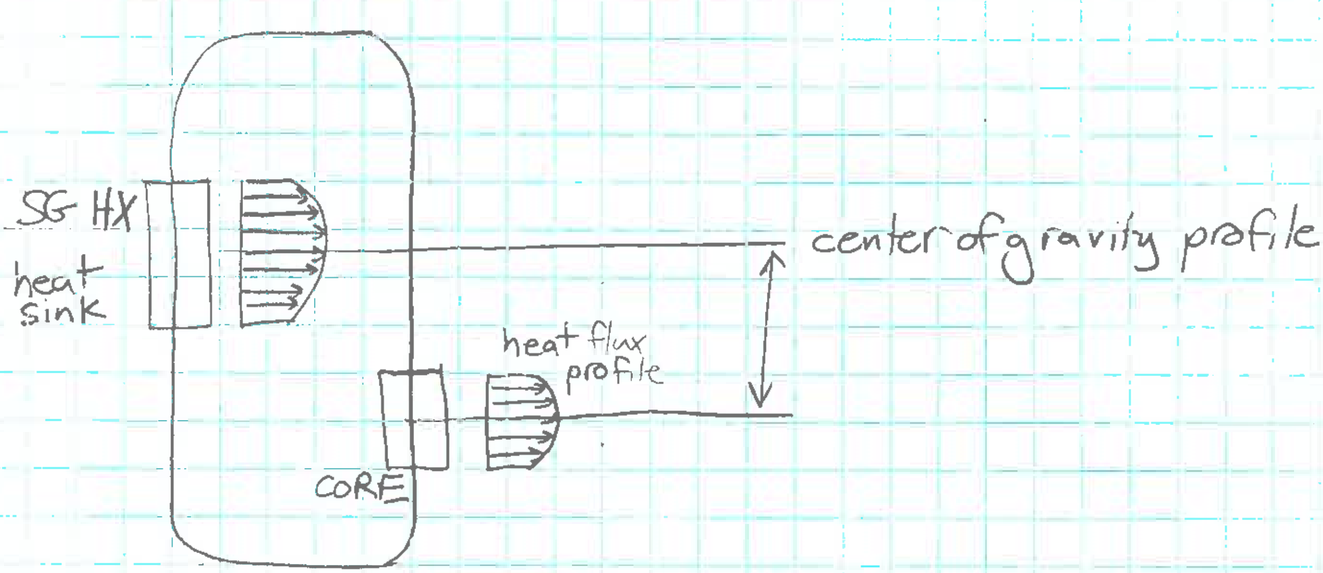
$$= 0 + \rho g \beta \Delta T_h l_h$$

$T_h - T_c$

$$\boxed{\sum \rho_i \frac{dV_i}{dt} l_i = \Delta P_{pump} + \sum (\rho g l - \rho g \beta \Delta T\, l_h)_i - \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{\rho_i V_i^2}{2}\right)}$$

$l_h \rightarrow$ thermal driving head
difference in height of thermal centers

obtained only when heat source is below heat sink

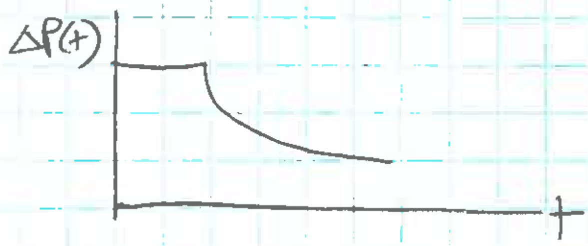


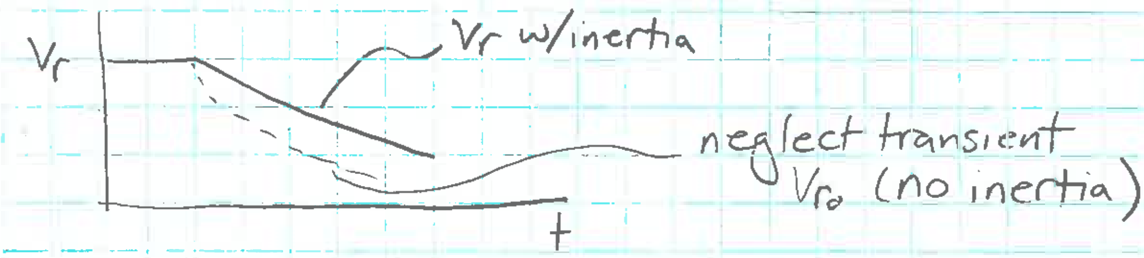
Forced Convection

Steady state
$$\begin{cases} \dfrac{dV_r}{dt} = 0 \\ \Delta P_{pump} = \Delta P_{pump} \text{ const} \\ \text{nat. circ. negligible} \quad \cancel{\rho g \beta \Delta T_h l_h} \to 0 \end{cases}$$

$$\boxed{\Delta P_{pump} - \frac{\rho_r V_r^2}{2} \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2 = 0}$$

Transient $\quad \Delta P = \Delta P(t) \quad$ pump coast down (loss of flow)





$$\boxed{\rho_r \sum \left(\frac{a_r}{a_i} l_i\right) \frac{dV_r}{dt} = \Delta P(t) - \frac{\rho_r V_r^2}{2} \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2}$$

Natural Convection

$$\Delta P_{pump} = 0 \implies \rho g \beta \Delta T_h l_h \neq 0$$

$$\frac{dV_r}{dt} = 0$$

$$\boxed{\rho g \beta \Delta T_h l_h + \frac{\rho V^2}{2} \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2 = 0}$$

General Balance Equation

$$\frac{\partial \psi}{\partial t} + \nabla \cdot \psi \vec{V} = -\nabla \cdot \vec{J} + \dot{\psi}$$

CE
$\psi = \rho$
$J = 0$
$\dot{\psi} = 0$

EE
$\psi = \rho i$
$J = q + \nabla \tau + \nabla P$
$\dot{\psi} = \dot{q}$

ME
$\psi = \rho \vec{V}$
$J = P + \tau$
$\dot{\psi} = \rho \vec{g}$

CE

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0$$

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial \rho \langle V_z \rangle}{\partial z} = 0}$$

for all average over area

$$\frac{1}{A}\int dA$$

$\rho = \langle \rho \rangle$ for uniform ρ dist. in A

not mean weighted mass

$$\langle V_z \rangle \neq \frac{\langle\langle \rho V_z \rangle\rangle}{\langle \rho \rangle}$$

EE

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho i \vec{V}) = -\nabla \vec{q} - \cancel{\tau : \rho \vec{V}} + \frac{DP}{Dt} + \dot{q}$$

$$\frac{\partial \rho \langle i \rangle}{\partial t} + \frac{\partial \rho \langle i V_z \rangle}{\partial z} = -\frac{\partial q}{\partial z}$$ axial conduction

$$- \frac{\xi_k q_w''}{A}$$ heated perimeter, wall heat flux $h(T_w - T)$

$$+ \frac{DP}{Dt}$$ energy increase due to pressure

$$+ \dot{q}$$ heat generation

CE $\rightarrow$ 0

$$\rho_i V_i a_i = \rho_r V_r a_r$$

$$\rho_i = \rho_r = \rho$$ (incompressible flow assumption)

$$\boxed{V_i = V_r \frac{a_r}{a_i}}$$

ME

$$\frac{\partial \rho \vec{V}}{\partial t} + \nabla \cdot \rho \vec{V}\vec{V} = -\nabla P - \nabla \cdot \tau + \rho \vec{g}$$

$$\frac{\partial \rho \langle V_z \rangle}{\partial t} + \frac{\partial \rho \langle V_z V_z \rangle}{\partial z} = -\frac{DP}{Dt}$$ pressure drop

$$-\frac{\partial \langle \tau_{zz} \rangle}{\partial z}$$ normal shear

$$-\frac{4\tau_w}{D}$$ wall shear $= \frac{f}{2D}\rho V_z |V_z|$

$$+\rho \bar{g}_z$$ gravity driver

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integrate over primary system

$$\oint \frac{\partial \rho V_z}{\partial t} dz = \sum \left(\rho_i \frac{dV_i}{dt} l_i \right)$$ change in momentum in comp. i

$$\oint \frac{\partial \rho V_z^2}{\partial z} dz = 0$$ convective acceleration

$$\oint -\frac{DP}{Dt} dz = \Delta P$$

$$\oint \frac{\partial \langle \tau_{zz} \rangle}{\partial z} dz = 0$$ internal fluid momentum losses

$$\oint -\frac{f}{2D}\rho V_z |V_z| dz = -\sum \left(\frac{fl}{D} + k \right)_i \frac{\rho V_i |V_i|}{2}$$

friction loss due to

component friction

$$\oint \rho g_z dz = \sum (\rho g l - \rho g \beta \Delta T l_h)_i = 0 - \rho g \beta \Delta T_h l_h$$

thermal driving head

height

$T_h - T_c$

$$\boxed{\sum \left(\rho_i \frac{dV_i}{dt} l_i \right) = \Delta P - \sum \left(\frac{fl}{D} + k \right)_i \frac{\rho V_i |V_i|}{2} - \rho g \beta \Delta T_h l_h}$$

Void Fraction

phase density function $\chi_k = \begin{cases} 1 & \text{if in phase } k \\ 0 & \text{if not in phase } k \end{cases}$

k is g gas, f liquid; 1 comp 1, 2 comp 2

• Local Void Fraction (time fraction)

$$\alpha_{kt} = \frac{1}{\Delta t}\int_{\Delta t} \chi_k \, dt = \frac{\Delta t_k}{\Delta t}$$

Δt_k ← total time segment related to phase k

Δt ← total sampling time

time fraction of gas + liquid phase = 1

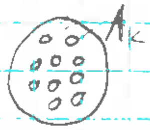
$$\alpha g_t + \alpha f_t = 1$$

• Area Averaged Void Fraction

$$\alpha_{kA} = \frac{1}{A}\int_A \chi_k \, dA = \frac{A_k}{A}$$

A_k ← total cross-sectional area of k phase

A ← $A_g + A_f$ = total cross sectional area



$$\alpha g_A + \alpha f_A = 1$$

• Volume Average Void Fraction

$$\alpha_{kV} = \frac{1}{V}\int_V \chi_k \, dV = \frac{V_k}{V}$$

V_k ~ volume of k phase

V ← $V_g + V_f$ = total volume

$$\alpha g_V + \alpha f_V = 1$$

• Line Average Void Fraction

$$\alpha_{kL} = \frac{1}{L}\int_L \chi_k \, dL = \frac{L_k}{L}$$

L_k ← total line segment for phase k out of L

L ← total sampling line

$$\alpha g_L + \alpha f_L = 1$$

Ergotic Theorem between different void fractions

commutativity of averaging exists

Area average (local void fraction) = Time average (area averaged void fraction)

$$\frac{1}{A}\int_A \frac{1}{\Delta t}\int_{\Delta t} \chi_k \, dt \, dA = \frac{1}{\Delta t}\int_{\Delta t} \frac{1}{A}\int_A \chi_k \, dA \, dt$$

Kelvin-Helmholtz Instability - occurs when sufficient velocity difference exits across interface between two fluids
- waves gen. on surface
- predict onset of turbulence in fluids of different densities moving at different speeds

1) assumptions
- incompressible: $\rho = \text{const.}$
- inviscid flow: $\tau = \text{negligible}$
- irrotational flow: $\nabla \times \vec{V} = 0$

2) local instant formulation
C.E. $\nabla \vec{V} = 0$
M.E. $\rho\left[\frac{\partial \vec{V}}{\partial t} + \nabla \cdot \vec{V}\vec{V}\right] = -\nabla P - \mu \nabla^2 \vec{V} + \rho \vec{g}$

3) define velocity potential
$\vec{V} = -\nabla \phi$

4) apply interfacial boundary conditions

5) assume form for wave interface

6) apply pressure jump condition

7) Substitute into M.E. equation and solve

wave propagation velocity C = mean velocity $\pm$ (capillary wave (gravity wave) - relative velocity)$^{1/2}$

- gravity and surface tension stabilize

- velocity destabilizes

ex. wind blowing over water surface

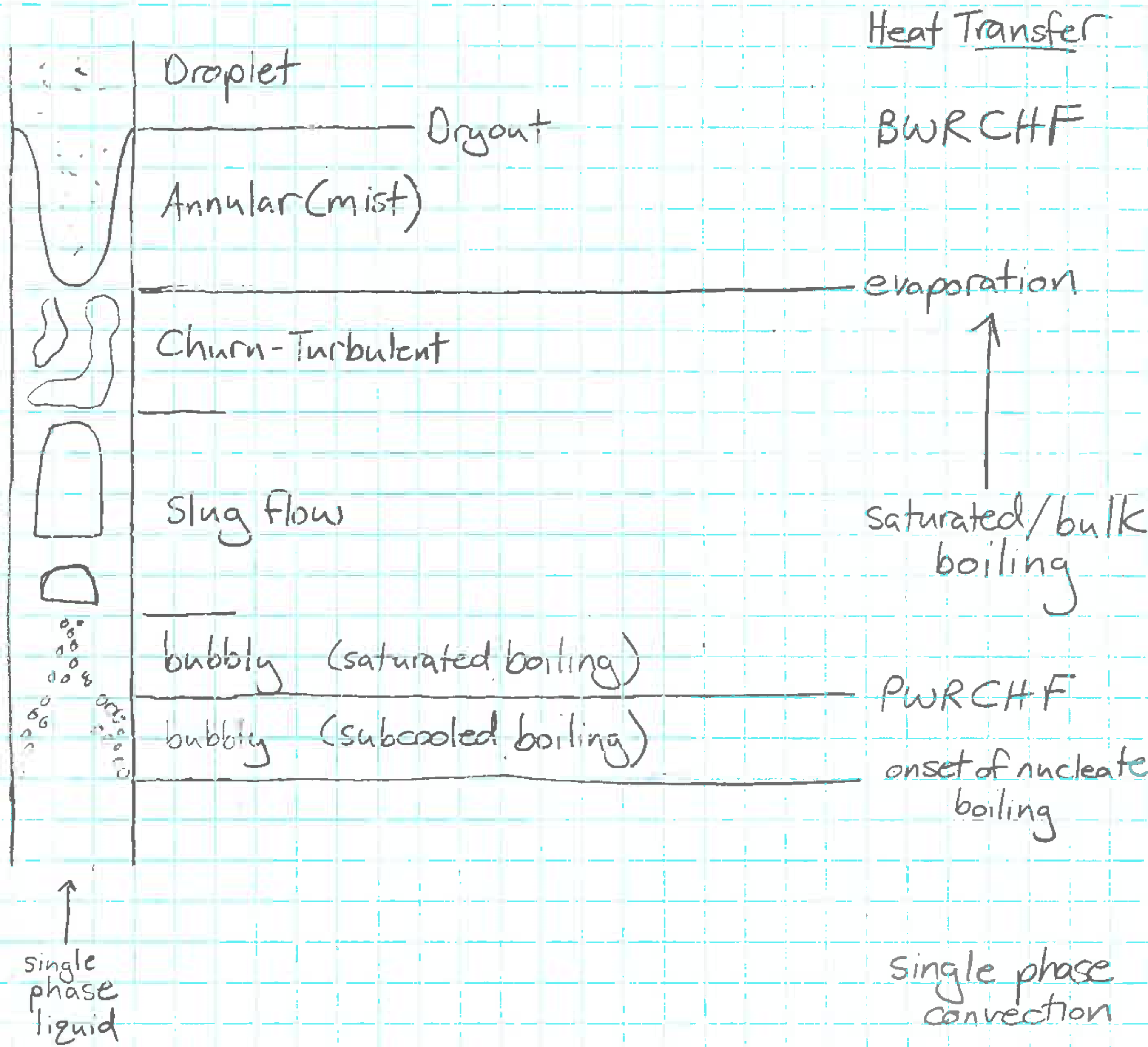
Rayleigh-Taylor Instability - instability of an interface between two fluids of different densities when lighter fluid is pushing heavier fluid

- inverse of K-H Instability

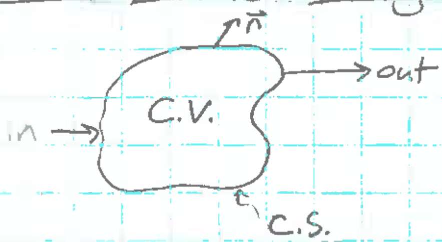
- if surface tension of heavy fluid is large enough system will be stable

$$\lambda_{max} = 2\pi\sqrt{\frac{3\sigma}{g\Delta\rho}} = \sqrt{3}\,\lambda_c$$ critical wavelength

Flow Regime


Heat Transfer
Droplet
Dryout
BWR CHF
Annular (mist)
evaporation
Churn-Turbulent
Slug flow
saturated/bulk boiling
bubbly (saturated boiling)
PWR CHF
bubbly (subcooled boiling)
onset of nucleate boiling
single phase liquid
Single phase convection

Control Volume Analysis



conservation of mass

$$\frac{d}{dt}\int_{CV} \rho \, dV = \sum_i \dot{m}_i$$

[rate of change of mass in the control volume] = [sum of mass flow rate into and out of control volume]

conservation of momentum

$$\frac{d}{dt}\int_{CV} \rho \vec{V} \, dV = \sum \vec{F} + \sum_i (\dot{m}_i \vec{V}_i)$$

[rate of momentum change in the control volume] = [sum of force acting on control volume] + [momentum flux]

- gravity
- external force
- pressure
- shear

conservation of energy

$$\frac{d}{dt}\int_{CV} \rho e \, dV = \dot{Q} + \dot{W}_s + \sum \dot{m}_i \left(e + \frac{p}{\rho}\right)$$

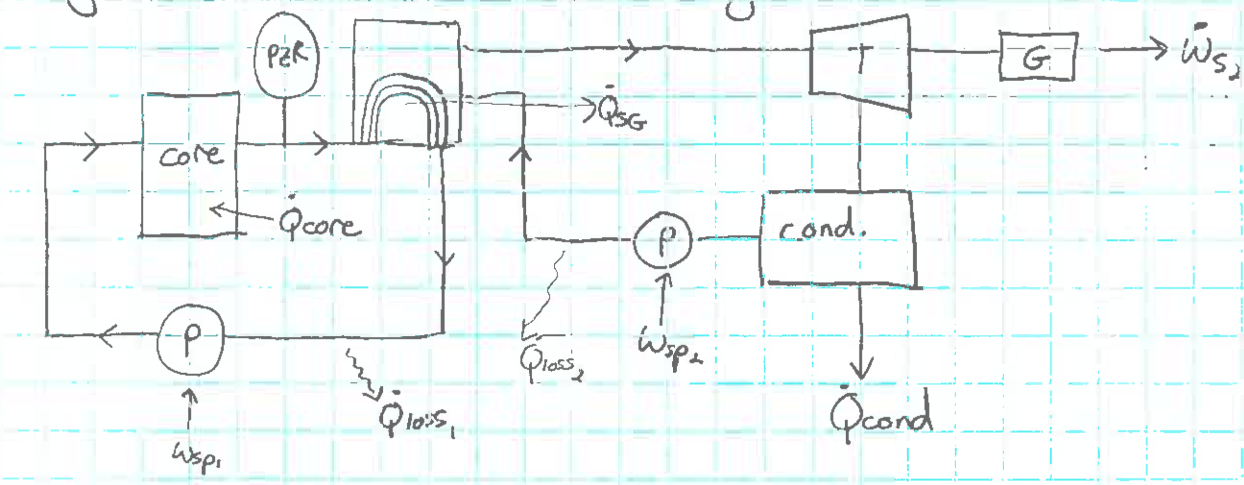
$$e = u + \frac{V^2}{2} + \underbrace{\vec{g} z}_{\text{negligible for } M \ll 1}$$

$\dot{Q}$ heat generation

$\dot{W}_s$ shaft work (work done on system)

$u + \frac{p}{\rho}$ enthalpy

[rate of energy change in the control volume] = [heat generation] + [work done on system] + [energy flux] integral nuclear reactor thermal hydraulic



(1) Steady state (primary loop)

$\dot{m} = 0 \implies \frac{d}{dt}\int_{CV} \rho\, dV = 0$ mass balance

$\underbrace{\frac{d}{dt}\int \rho e\, dV}_{\to 0}$, $\underbrace{\sum \dot{m}_i\left(e + \frac{p}{\rho}\right)}_{\to 0} \implies \dot{Q}_{core} - \dot{Q}_{SG} - \dot{Q}_{loss} + \dot{W}_{sp_1} = 0$ energy balance

(2) LOCA - blowdown (reactor trips)

break flow $\sum \dot{m} = (\rho V A)_{break}$ ← determined by critical flow rate

$\frac{d}{dt}\int_{CV} \rho\, dV = -(\rho V A)_{break}$ mass balance

[mass inventory] [mass loss]

$\frac{d}{dt}\int_{CV} \rho e\, dV = \dot{Q}_{core} - \dot{Q}_{SG} - \dot{Q}_{loss} + \dot{W}_{sp_1} - \dot{m}_{break}\left(e + \frac{p}{\rho}\right)_{break}$

initially $\frac{d}{dt}\int_{CV} \rho e\, dV < 0$

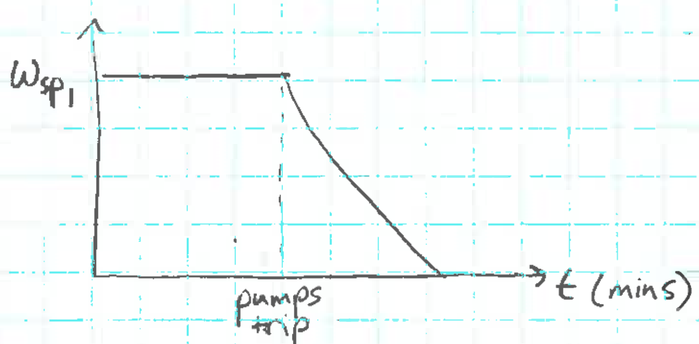
energy balance

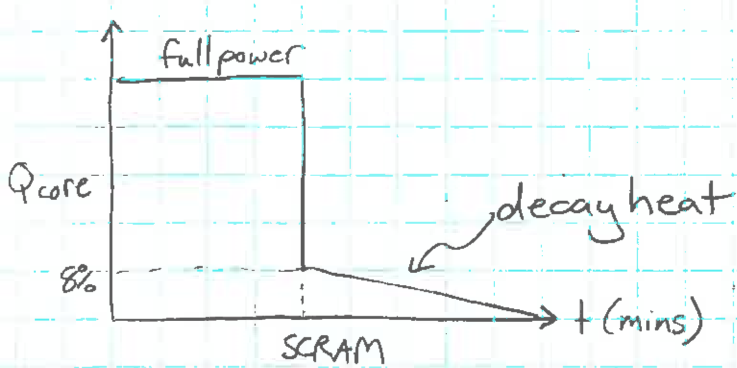
HOWEVER ⇒ due to $-(\rho V A)_{break}$ core will eventually uncover
$T \Uparrow$

LOCA - ECCS injection

$$\frac{d}{dt}\int_{cv}\rho\, dV = \underbrace{-(\rho VA)_{break} + (\rho VA)_{ECCS}}_{\text{must be} > 0!} \quad \Bigg\}\ \text{mass balance}$$

$$\frac{d}{dt}\int_{cv}\rho e\, dV = \dot{Q}_{decay} + \cancel{W_{sp_1}}^{\to 0} - \dot{Q}_{SG} + \dot{Q}_{loss} - \dot{m}_{break}\left(e+\frac{P}{\rho}\right)_{break} + \dot{m}\underbrace{\left(e+\frac{P}{\rho}\right)_{ECCS}}_{\substack{\text{low quality energy} \\ \text{available energy} = 0}} \quad \Bigg\}\ \text{energy balance}$$





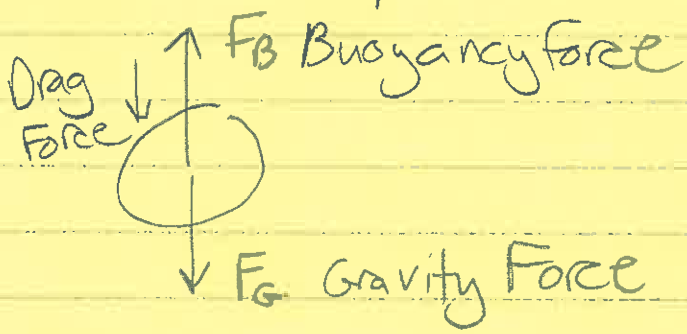
Drag Force

drag is resistance opposed by a medium to anything moving through it

$$F_D = \frac{1}{2} C_D A \rho_m \vec{V_r}^2$$

F_D = Drag Force
C_D = Drag Coefficient
A = cross-sectional area perpendicular to flow
ρ_m = density of medium
V_r = velocity to the body relative to the medium

Drag coefficient expression



$$F_D + F_G = F_B$$

$$\frac{1}{2} C_D A \tilde{n}_m \vec{V_r}^2 + \tilde{n}_{bubble} g V = \tilde{n}_m g V$$

$$C_D = \frac{2(\tilde{n}_m - \tilde{n}_{bubble}) g V}{\tilde{n}_m A V_r^2}$$