NUCL 511 Nuclear Reactor Theory and Kinetics

Homework #7

Due March 13

1. Transients with Constant Reactivity. Find the solution of the point kinetics equations for the following reactivity insertions in a critical reactor and plot the results (in all cases use $\Lambda = 10^{-4}$, 10^{-5} and 10^{-6} s. Use one delayed neutron group with $\beta = 0.007$ and $\bar{\lambda} = 0.1\ \mathrm{s}^{-1}$. (25 points)

   Answer) For a constant reactivity, the general solution of the point kinetics equations with one delayed neutron group is given by

$$p(t) = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} \tag{1}$$

   Inserting Eq. (1) into the point kinetics equations yields the in-hour equation

$$\rho_1 = \Lambda\alpha + \frac{\beta\alpha}{\alpha + \bar{\lambda}} \quad \Rightarrow \quad \alpha^2 - (\alpha_p - \bar{\lambda})\alpha - \frac{\bar{\lambda}\rho_1}{\Lambda} = 0, \quad \alpha_p = \frac{\rho_1 - \beta}{\Lambda} \tag{2}$$

   If $\rho_1 > 0$, Eq. (2) has one positive root (say α_1) and one negative root (say α_2). If $\rho_1 > \beta + \Lambda\bar{\lambda}$, then $\alpha_p > \bar{\lambda}$. Thus we have

$$\alpha_1 = \frac{(\alpha_p - \bar{\lambda})}{2}\left\{1 + \left[1 + 4\frac{\bar{\lambda}\rho_1}{\Lambda(\bar{\lambda} - \alpha_p)^2}\right]^{1/2}\right\}, \quad \alpha_2 = \frac{(\alpha_p - \bar{\lambda})}{2}\left\{1 - \left[1 + 4\frac{\bar{\lambda}\rho_1}{\Lambda(\bar{\lambda} - \alpha_p)^2}\right]^{1/2}\right\} \tag{4}$$

   and $\alpha_1 > |\alpha_2|$. If $\rho_1 = \beta + \Lambda\bar{\lambda}$, $\alpha_1 = |\alpha_2|$ where

$$\alpha_1 = \sqrt{\frac{\bar{\lambda}\rho_1}{\Lambda}}, \quad \alpha_2 = -\sqrt{\frac{\bar{\lambda}\rho_1}{\Lambda}} \tag{5}$$

   If $\rho_1 < \beta + \Lambda\bar{\lambda}$, then $0 < \alpha_1 < |\alpha_2|$ and

$$\alpha_1 = \frac{(\alpha_p - \bar{\lambda})}{2}\left\{1 - \left[1 + 4\frac{\bar{\lambda}\rho_1}{\Lambda(\bar{\lambda} - \alpha_p)^2}\right]^{1/2}\right\}, \quad \alpha_2 = \frac{(\alpha_p - \bar{\lambda})}{2}\left\{1 + \left[1 + 4\frac{\bar{\lambda}\rho_1}{\Lambda(\bar{\lambda} - \alpha_p)^2}\right]^{1/2}\right\} \tag{6}$$

   If $\rho_1 < 0$, both roots become negative. Numbering the two roots such that $\alpha_2 < \alpha_1 < 0$, the two roots are given by Eq. (6). The constants A_1 and A_2 in Eq. (1) can be determined as

$$A_1 = \frac{1}{\alpha_1 - \alpha_2}\left(\frac{\rho_1}{\Lambda} - \alpha_2\right), \quad A_2 = \frac{1}{\alpha_1 - \alpha_2}\left(\alpha_1 - \frac{\rho_1}{\Lambda}\right) \tag{7}$$

   from the initial conditions

$$p(0) = A_1 + A_2 = p_0 = 1, \quad \dot{p}(0) = A_1\alpha_1 + A_2\alpha_2 = \frac{\rho_1}{\Lambda}p_0 = \frac{\rho_1}{\Lambda} \tag{8}$$

   If $|\rho_1 - \beta| >> \Lambda\bar{\lambda}$, the square root in Eq. (4) and Eq. (6) can be approximated as

$$\left[1+4\frac{\bar{\lambda}\rho_1}{\Lambda(\bar{\lambda}-\alpha_p)^2}\right]^{1/2} \approx \left[1+4\frac{\bar{\lambda}\rho_1\Lambda}{(\beta-\rho_1)^2}\right]^{1/2} \approx 1+2\frac{\bar{\lambda}\rho_1\Lambda}{(\beta-\rho_1)^2} \tag{9}$$

As a result, for $\rho_1 > \beta + \Lambda\bar{\lambda}$, the inverse periods can be approximately determined as

$$\alpha_1 \approx \alpha_p + \frac{\bar{\lambda}\rho_1}{\rho_1-\beta} \approx \alpha_p, \quad \alpha_2 \approx -\frac{\bar{\lambda}\rho_1}{\rho_1-\beta} \tag{10}$$

For $\rho_1 < \beta + \Lambda\bar{\lambda}$, the inverse periods can be approximately determined as

$$\alpha_1 \approx \frac{\bar{\lambda}\rho_1}{\beta-\rho_1}, \quad \alpha_2 \approx \alpha_p - \frac{\bar{\lambda}\rho_1}{\beta-\rho_1} \approx \alpha_p \tag{11}$$

a. $\rho_1 = 0.75\$$; $\rho_1 = -1.0\$$; $\rho_1 = -3.0\$$

The exponents and coefficients calculated with Eq. (2) are summarized in the table below.

ρ	0.00525	0.00525	0.00525	-0.00700	-0.00700	-0.00700	-0.02100	-0.02100	-0.02100
Λ	1.00E-04	1.00E-05	1.00E-06	1.00E-04	1.00E-05	1.00E-06	1.00E-04	1.00E-05	1.00E-06
α_p	-17.5	-175.0	-1750.0	-140.0	-1400.0	-14000.0	-280.0	-2800.0	-28000.0
α_1	0.29340	0.29932	0.29993	-0.04998	-0.05000	-0.05000	-0.07499	-0.07500	-0.07500
α_2	-17.9	-175.4	-1750.4	-140.1	-1400.1	-14000.1	-280.0	-2800.0	-28000.0
A_1	3.87057	3.98637	3.99863	0.50036	0.50004	0.50000	0.25013	0.25001	0.25000
A_2	-2.87057	-2.98637	-2.99863	0.49964	0.49996	0.50000	0.74987	0.74999	0.75000

The approximate values of the exponents and coefficients calculated with Eq. (3) are also summarized in the table below. It is noted that the deviations from the exact values increases with increasing reactivity and decreasing generation time.

ρ	0.00525	0.00525	0.00525	-0.00700	-0.00700	-0.00700	-0.02100	-0.02100	-0.02100
Λ	1.00E-04	1.00E-05	1.00E-06	1.00E-04	1.00E-05	1.00E-06	1.00E-04	1.00E-05	1.00E-06
α_p	-17.5	-175.0	-1750.0	-140.0	-1400.0	-14000.0	-280.0	-2800.0	-28000.0
α_1	0.30000	0.30000	0.30000	-0.05000	-0.05000	-0.05000	-0.07500	-0.07500	-0.07500
α_2	-17.5	-175.0	-1750.0	-140.0	-1400.0	-14000.0	-280.0	-2800.0	-28000.0
A_1	3.93258	3.99315	3.99931	0.50018	0.50002	0.50000	0.25007	0.25001	0.25000
A_2	-2.93258	-2.99315	-2.99931	0.49982	0.49998	0.50000	0.74993	0.74999	0.75000

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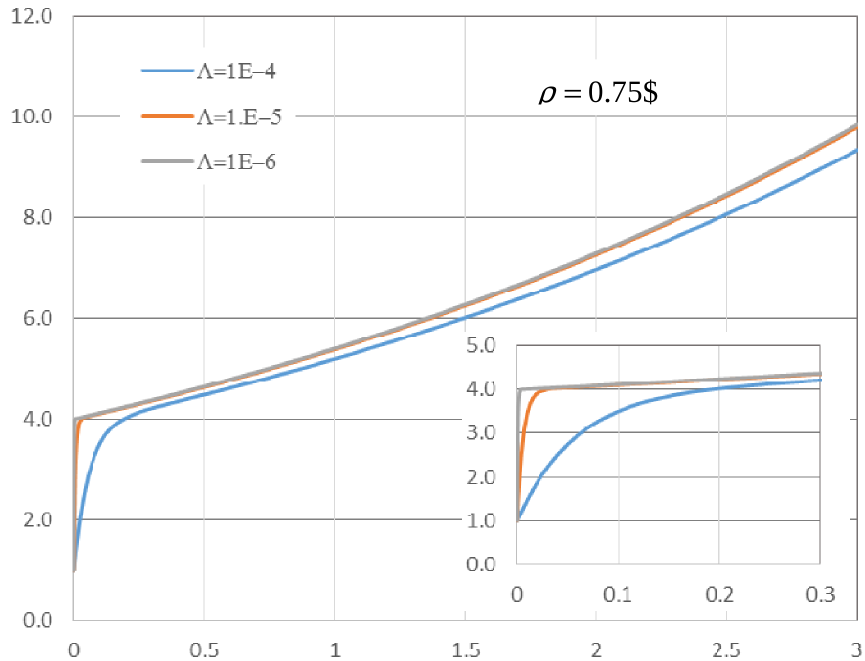
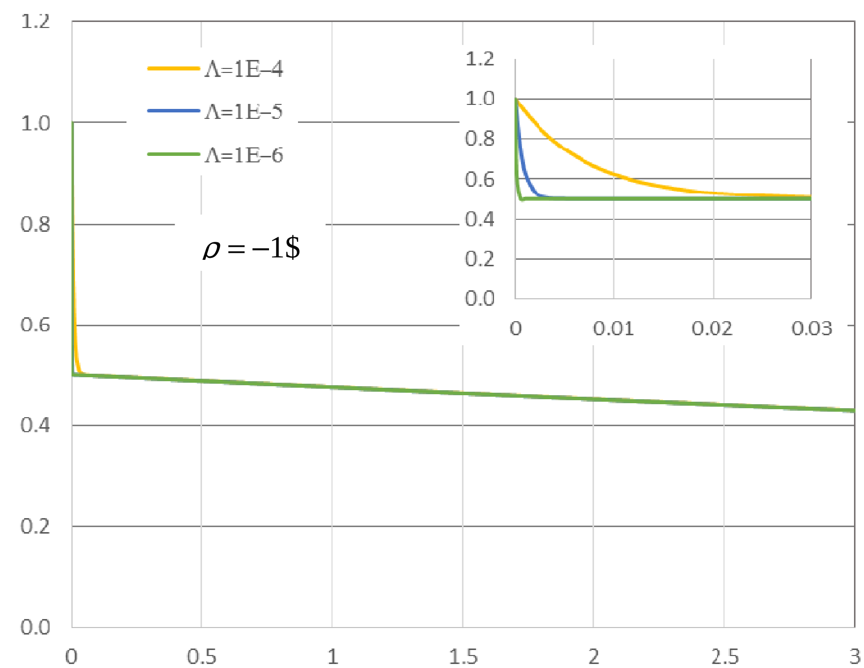
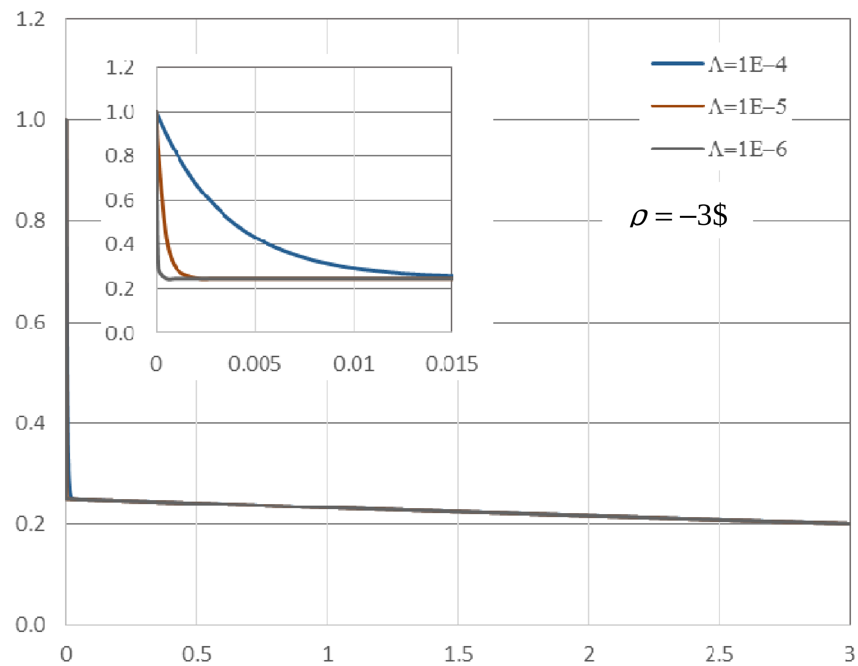
The power transients are shown in the figures below.







b. Estimate first the length of time you want to run the transient; present the rational of your estimate.

Answer) In order to see the asymptotic behavior, it is necessary to the transient at least one stable period (i.e., $1/\alpha_1$).

c. Discuss the transient results, in particular the short-time behavior, asymptotic behavior, and Λ dependence of both.

Answer) Since $\alpha_1 > \alpha_2$, the second term in Eq. (1) will die away quickly, and the first term will dominate. In other words, the power makes a prompt jump or drop, depending on the sign of the reactivity, then shows a transient behavior for a short period, and increases or decreases asymptotically with the stable inverse period α_1. The initial transient period is determined by the transient inverse period α_2 and thus increases as Λ increases. On the other hand, the stable inverse period α_1 shows only very weak dependence on Λ.

2. Find numerically the transient that follows a ρ jump from $\rho_0 = -1\$$ to $\rho_1 = 0$, with a source $s_0 = \beta p_0$ during the transient. Again, use $\Lambda = 10^{-4}$, 10^{-5} and 10^{-6} s and one delayed neutron group with $\beta = 0.007$ and $\bar{\lambda} = 0.1\ \mathrm{s}^{-1}$. (10 points)

Answer) With $\rho_1 = 0$ and $s_0 = \beta p_0$, the point kinetics equations can be written as

$$\begin{aligned} \Lambda \dot{p}(t) &= (\rho_1 - \beta)p(t) + \bar{\lambda}\zeta(t) + s_0 = -\beta p(t) + \bar{\lambda}\zeta(t) + \beta p_0 \\ \dot{\zeta}(t) &= \beta p(t) - \bar{\lambda}\zeta(t) \end{aligned} \quad (1)$$

The initial conditions are given by the steady state conditions

$$p(0) = \frac{s_0}{-\rho_0} = p_0, \quad \zeta(0) = \frac{\beta}{\bar{\lambda}} p(0) = \frac{\beta}{\bar{\lambda}} p_0 \tag{2}$$

By eliminating the reduced precursor concentration, Eq. (1) can be converted into a second order differential equation for the power amplitude as

$$\ddot{p}(t) + \left(\bar{\lambda} + \frac{\beta}{\Lambda} \right) \dot{p}(t) = \frac{\bar{\lambda}\beta p_0}{\Lambda} \tag{3}$$

The corresponding initial conditions are given by

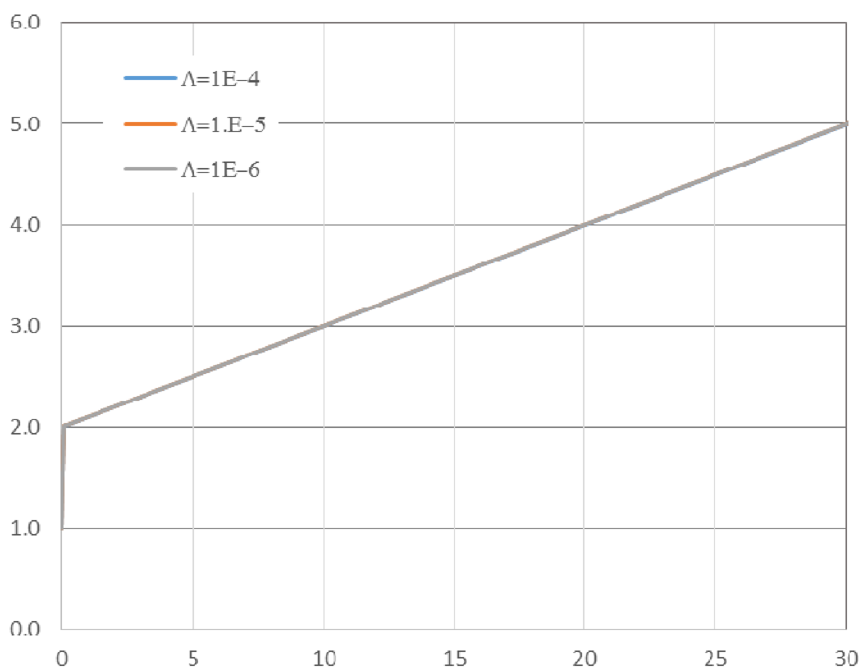
$$p(0) = \frac{s_0}{-\rho_0} = p_0, \quad \dot{p}(0) = \frac{\rho_1}{\Lambda} p_0 + \frac{s_0}{\Lambda} = \frac{\beta}{\Lambda} p_0 \tag{4}$$

Integrating Eq. (3) from time zero to t yields

$$\dot{p}(t) + \left(\bar{\lambda} + \frac{\beta}{\Lambda} \right) p(t) = \frac{\bar{\lambda}\beta p_0}{\Lambda} t + \left(\bar{\lambda} + \frac{2\beta}{\Lambda} \right) p_0 \tag{5}$$

Using the integrating factor, Eq. (5) can be solved as

$$\frac{p(t)}{p_0} = e^{-(\bar{\lambda}+\beta/\Lambda)t} + \frac{\bar{\lambda}\beta}{\bar{\lambda}\Lambda + \beta} t + \left[1 + \left(\frac{\beta}{\bar{\lambda}\Lambda + \beta} \right)^2 \right] (1 - e^{-(\bar{\lambda}+\beta/\Lambda)t}) \tag{6}$$



3. Interpretation of Adjoint Flux. Prove by application of the point kinetics equations the interpretation of the adjoint flux $\phi_0^*(r,E)$, as the asymptotic relative flux rise δp_{as} (aside

from the proportionality constant) that follows a burst insertion of independent source neutrons in a critical reactor at location r with energy E: (15 points)

$$\delta p_{as} = c\phi_0^*(r,E)$$

a. Find the analytical solution for a burst insertion of independent source neutrons in a critical reactor $\delta s(t) = s_0\delta(t)$. Use one group of delayed neutrons, $\lambda = \bar{\lambda}$. Find $p(t)$, and especially δp_{as}.

Answer) With $\rho = 0$ and $\delta s(t) = s_0\delta(t)$, the point kinetics equations can be written as

$$\begin{aligned} \Lambda\dot{p}(t) &= -\beta p(t) + \bar{\lambda}\zeta(t) + s_0\delta(t) \\ \dot{\zeta}(t) &= \beta p(t) - \bar{\lambda}\zeta(t) \end{aligned} \tag{1}$$

The initial conditions are given by the steady state conditions

$$p(0) = p_0, \quad \zeta(0) = \frac{\beta}{\bar{\lambda}} p(0) = \frac{\beta}{\bar{\lambda}} p_0 \tag{2}$$

Taking the Laplace transform of Eq. (1) yields

$$\begin{aligned} \Lambda[\alpha\tilde{p}(\alpha) - p_0] &= -\beta\tilde{p}(\alpha) + \bar{\lambda}\tilde{\zeta}(\alpha) + s_0 \\ \alpha\tilde{\zeta}(\alpha) - \zeta_0 &= \beta\tilde{p}(\alpha) - \bar{\lambda}\tilde{\zeta}(\alpha) \end{aligned} \tag{3}$$

By eliminating $\tilde{\zeta}$, Eq. (3) can be solved for $\tilde{p}$ as

$$\tilde{p}(\alpha) = \frac{p_0}{\alpha} + \frac{\alpha + \bar{\lambda}}{\alpha(s + \bar{\lambda} + \beta/\Lambda)}\frac{s_0}{\Lambda} = \frac{p_0}{\alpha} + \frac{s_0/\Lambda}{\bar{\lambda} + \beta/\Lambda}\left(\frac{\bar{\lambda}}{\alpha} - \frac{\beta/\Lambda}{\alpha + \bar{\lambda} + \beta/\Lambda}\right) \tag{4}$$

By the inverse Laplace transform of Eq. (4), we have

$$p(t) = p_0 + \frac{s_0/\Lambda}{\bar{\lambda} + \beta/\Lambda}\left(\bar{\lambda} - \frac{\beta}{\Lambda}e^{-(\bar{\lambda}+\beta/\Lambda)t}\right) \tag{5}$$

As a result, the asymptotic relative flux increase is given by

$$\delta p_{as} = \frac{\bar{\lambda}/\Lambda}{\bar{\lambda} + \beta/\Lambda} s_0 \tag{6}$$

b. Why is it sufficient to use a single group of delayed neutrons for determining δp_{as} ?

Answer) The asymptotic relative flux increase is governed by the largest inverse period. The largest inverse period of the one delayed neutron group model is zero as that of the six delayed neutron group model is zero. Thus the one group model is sufficient for determining the asymptotic relative flux increase.

c. Relate s_0 to $\phi_0^*(r,E)$ and present the desired proof.

Answer) For a burst insertion of independent source neutrons in a critical reactor at location r with energy E, the reduced source is given by

$$s_0 = <\phi_0^*(r', E'), \delta(r' - r)\delta(E' - E)>_{r', E'} = \phi_0^*(r, E) \tag{7}$$

Using Eq. (6) and (7), the asymptotic relative flux increase can be written as

$$\delta p_{as} = c\phi_0^*(r, E) \tag{8}$$