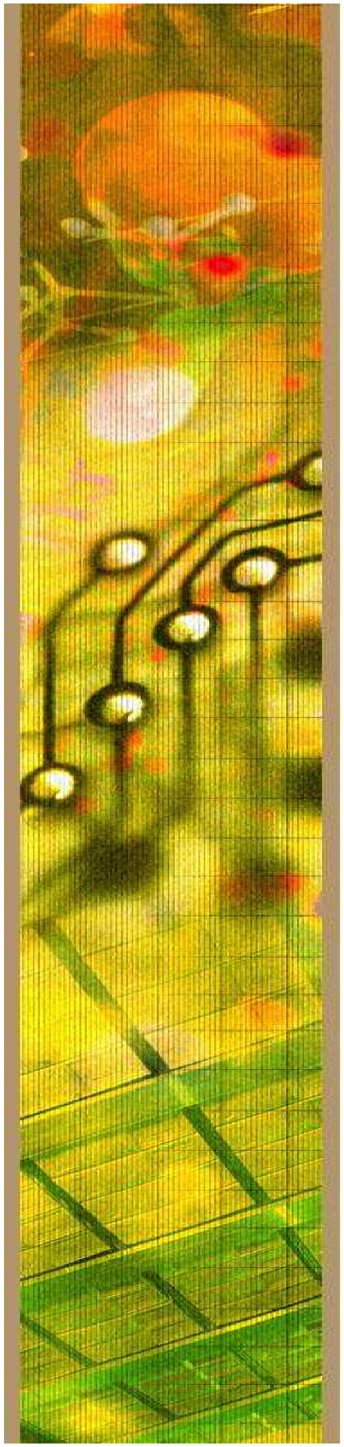

NUCL 510
Nuclear Reactor Theory

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Lecture Note 3

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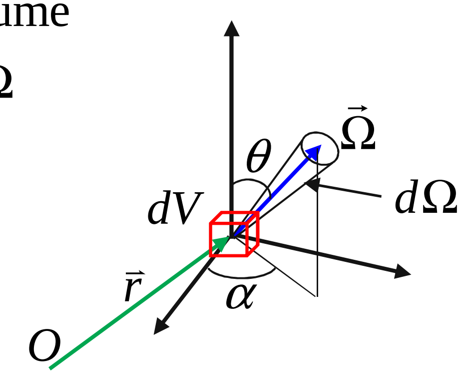
Angular Flux and Reaction Rate

- Neutron density distribution in phase space

  $n(\vec{r},E,\vec{\Omega},t)dVdEd\Omega$ = Expected number of neutrons in a volume element dV about $\vec{r}$ moving in the cone of directions $d\Omega$ about Ω with energies between E and $E+dE$ at time t



- Angular flux

$$\psi(\vec{r},E,\vec{\Omega},t) = v(E)n(\vec{r},E,\vec{\Omega},t)$$

$$\psi(\vec{r},E,\vec{\Omega},t)dVdEd\Omega dt = v(E)n(\vec{r},E,\vec{\Omega},t)dVdEd\Omega dt$$

  Total of the path lengths traveled during dt by all neutrons in the incremental volume $dVdEd\Omega$

- Scalar flux

$$\phi(\vec{r},E,t) = \int_{4\pi} d\Omega\,\psi(\vec{r},E,\vec{\Omega},t)$$

- Reaction rate

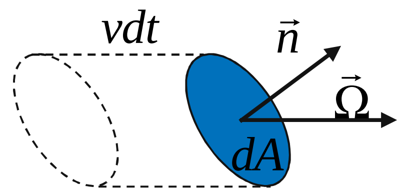
$$\Sigma_x(\vec{r},E,t) = \sum_i N_i(\vec{r},t)\sigma_{ix}(E) = \sum_i \Sigma_{ix}(\vec{r},E,t)$$

  $\Sigma_x(\vec{r},E,t)\psi(\vec{r},E,\vec{\Omega},t)dVdEd\Omega$ = Total number of reactions of type x per unit time in the incremental volume $dVdEd\Omega$

Boundary Crossing and Current

- Neutrons passing trough an incremental surface area dA
  - Number of neutrons in the volume defined by $(vdt)\times(\vec{n}\cdot\vec{\Omega}dA)$

$$n(\vec{r},E,\vec{\Omega},t)[vdt][\vec{n}\cdot\vec{\Omega}dA]dE=\vec{n}\cdot\vec{\Omega}\psi(\vec{r},E,\vec{\Omega},t)dAdtdE$$



Number of neutrons passing through dA with energies between E and $E+dE$ that are going in a particular direction Ω during the time increment from t and $t+dt$

- Net and partial currents
  - Net number of neutrons with energies between E and $E+dE$ crossing dA in the directions of positive $\vec{n}$ regardless of Ω during dt

$$\int_{4\pi}d\Omega\vec{n}\cdot\vec{\Omega}\psi(\vec{r},E,\vec{\Omega},t)dAdtdE=\vec{n}\cdot\vec{J}(\vec{r},E,t)dAdtdE=J_n(\vec{r},E,t)dAdtdE$$

$$\vec{J}(\vec{r},E,t)=\int_{4\pi}d\Omega\vec{\Omega}\psi(\vec{r},E,\vec{\Omega},t)$$

$$J_n(\vec{r},E,t)=\vec{n}\cdot\vec{J}(\vec{r},E,t)=J_n^+(\vec{r},E,t)-J_n^-(\vec{r},E,t)$$

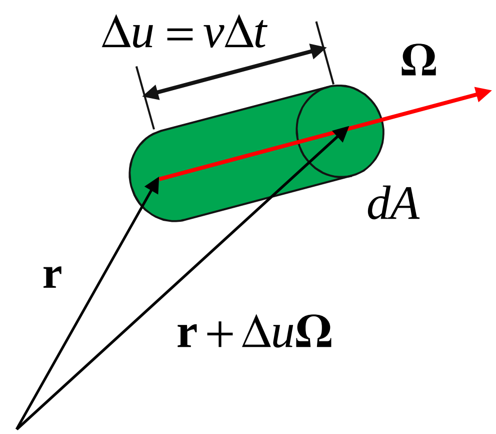
$$\vec{J}_n^+(\vec{r},E,t)=\int_{\vec{n}\cdot\Omega>0}d\Omega\vec{n}\cdot\vec{\Omega}\psi(\vec{r},E,\vec{\Omega},t) \qquad \vec{J}_n^-(\vec{r},E,t)=\int_{\vec{n}\cdot\Omega<0}d\Omega\vec{n}\cdot\Omega\psi(\vec{r},E,\Omega,t)$$

Neutron Balance Equation

- Change in the number of neutrons in $\Delta V d\Omega dE$ between t and $t+\Delta t$

$$\text{(Increase in } n) = -\text{(Losses from streaming)} - \text{(Losses from collision)} + \text{(Neutrons emitted)}$$

$$\begin{aligned}
&[n(\vec{r},E,\vec{\Omega},t+\Delta t) - n(\vec{r},E,\vec{\Omega},t)]\Delta V dE d\Omega \\
&= -[n(\vec{r}+\Delta u\vec{\Omega},E,\vec{\Omega},t) - n(\vec{r},E,\vec{\Omega},t)](v\Delta t\Delta A)dEd\Omega \\
&\quad -\Sigma_t(\vec{r},E,t)\psi(\vec{r},E,\vec{\Omega},t)\Delta V\Delta t dEd\Omega \\
&\quad +Q(\vec{r},E,\vec{\Omega},t)\Delta V\Delta t dEd\Omega
\end{aligned}$$



$$\Delta V = \Delta u \Delta A = v\Delta t\Delta A$$

- Taylor series expansion

$$n(\vec{r},E,\vec{\Omega},t+\Delta t) - n(\vec{r},E,\vec{\Omega},t) \simeq \frac{\partial n}{\partial t}\Delta t = \frac{1}{v(E)}\frac{\partial \psi}{\partial t}\Delta t$$

$$n(\vec{r}+\Delta u\vec{\Omega},E,\vec{\Omega},t) - n(\vec{r},E,\vec{\Omega},t) \simeq \frac{\partial n}{\partial u}\Delta u = (\vec{\Omega}\cdot\nabla n)\Delta u = (\vec{\Omega}\cdot\nabla\psi)\Delta t$$

- Time-dependent Boltzmann transport equation

$$\frac{1}{v(E)}\frac{\partial}{\partial t}\psi(\vec{r},E,\vec{\Omega},t) = -\vec{\Omega}\cdot\nabla\psi(\vec{r},E,\vec{\Omega},t) - \Sigma_t(\vec{r},E,t)\psi(\vec{r},E,\vec{\Omega},t) + Q(\vec{r},E,\vec{\Omega},t)$$

Source Distributions

- Scattering source

$$S_s(\vec{r},E,\vec{\Omega},t)=\int dE'\int d\Omega'\Sigma_s(\vec{r},E'\rightarrow E,\vec{\Omega}'\rightarrow\vec{\Omega},t)\psi(\vec{r},E',\vec{\Omega}',t)$$

- Prompt fission neutrons
  - Fission neutrons are isotropic

$$\mathbf{F}_p\psi(\vec{r},E,\vec{\Omega},t)=\frac{1}{4\pi}\sum_i\int dE'\chi_{pi}(E'\rightarrow E)\nu_{pi}(E')\Sigma_{fi}(r,E',t)\phi(\vec{r},E',t)$$

  - If the fission spectrum is independent of the incident neutron energy,

$$\mathbf{F}_p\psi(\vec{r},E,\vec{\Omega},t)=\frac{1}{4\pi}\sum_i\chi_{pi}(E)\int dE'\nu_{pi}(E')\Sigma_{fi}(r,E',t)\phi(\vec{r},E',t)$$

- Delayed neutron source
  - Generated following beta decays of certain fission products
  - Grouped into six families depending on decay constants

$$S_d(\vec{r},E,\vec{\Omega},t)=\frac{1}{4\pi}\sum_i\sum_k\chi_{dki}(E)\lambda_{ki}C_{ki}(r,t)$$

- Independent source $S(\vec{r},E,\vec{\Omega},t)$

Reactor Kinetics Equations

- Reactor kinetics equations

$$\frac{1}{v(E)}\frac{\partial}{\partial t}\psi(\vec{r},E,\vec{\Omega},t)=(\mathbf{F}_p-\mathbf{M})\psi(\vec{r},E,\vec{\Omega},t)+S_d(\vec{r},E,\vec{\Omega},t)+S(\vec{r},E,\vec{\Omega},t)$$

$$\frac{\partial}{\partial t}C_{ki}(r,t)=-\lambda_{ki}C_{ki}(\vec{r},t)+\int dE'\nu_{dki}(E')\Sigma_{fi}(\vec{r},E',t)\phi(\vec{r},E',t)\quad(k=1,2,\cdots,6)$$

  – Migration and loss operator

$$\begin{aligned}\mathbf{M}\,\psi(\vec{r},E,\vec{\Omega},t)&=\Omega\cdot\nabla\,\psi(\vec{r},E,\vec{\Omega},t)+\Sigma_t(\vec{r},E,t)\psi(\vec{r},E,\vec{\Omega},t)\\&\quad-\int dE'\int d\Omega'\Sigma_s(\vec{r},E'\to E,\vec{\Omega}'\to\vec{\Omega},t)\psi(\vec{r},E',\vec{\Omega}',t)\end{aligned}$$

- Steady-state, total fission operator

$$C_{ki0}(r)=\frac{1}{\lambda_{ki}}\int dE'\nu_{dki}(E')\Sigma_{fi}(\vec{r},E')\phi_0(\vec{r},E')$$

$$S_{d0}(\vec{r},E,\vec{\Omega})=\frac{1}{4\pi}\sum_i\sum_k\chi_{dki}(E)\lambda_{ki}C_{ki0}(r)$$

$$\mathbf{F}\,\psi(\vec{r},E,\vec{\Omega})=\mathbf{F}_p\psi(\vec{r},E,\vec{\Omega})+S_{d0}(\vec{r},E,\vec{\Omega})=\frac{1}{4\pi}\sum_i\int dE'\chi_i(E'\to E)\nu_i(E')\Sigma_{fi}(\vec{r},E')\phi(\vec{r},E')$$

$$\nu_i(E)=\nu_{pi}(E)+\nu_{di}(E),\quad \chi_i(E\to E')=\frac{1}{\nu_i(E)}[\chi_{pi}(E\to E')\nu_{pi}(E)+\chi_{di}(E')\nu_{di}(E)]$$

Time-Independent Transport Equations

- Stationary neutron balance equation

$$\mathbf{M}\,\psi(r,E,\Omega)=\mathbf{F}\,\psi(r,E,\Omega)+S(r,E,\Omega)$$

$$\mathbf{M}\,\psi(\vec{r},E,\vec{\Omega})=\Omega\cdot\nabla\,\psi(\vec{r},E,\vec{\Omega})+\Sigma_t(\vec{r},E)\psi(\vec{r},E,\vec{\Omega}) \\ -\int dE'\int d\Omega'\Sigma_s(\vec{r},E'\rightarrow E,\vec{\Omega}'\rightarrow\vec{\Omega})\psi(\vec{r},E',\vec{\Omega}')$$

$$\mathbf{F}\,\psi(\vec{r},E,\vec{\Omega})=\frac{1}{4\pi}\sum_i\int dE'\chi_i(E'\rightarrow E)\nu_i(E')\Sigma_{fi}(\vec{r},E')\phi(\vec{r},E')$$

  – Non-trivial solution to the source-free transport equation can be found only when the system is critical

- λ-Eigenvalue problem
  – To the degree of off-criticality, the fission source is modified by a factor λ

$$\mathbf{M}\,\psi(\vec{r},E,\vec{\Omega})=\lambda\mathbf{F}\,\psi(\vec{r},E,\vec{\Omega})\quad(\lambda=1/k)$$

  – There exists a non-trivial solution only when (**M**-λ**F**) is singular

$\lambda<1$	$(k>1)$	super-critical
$\lambda=1$	$(k=1)$	critical
$\lambda>1$	$(k<1)$	sub-critical

Operators in Reactor Applications

- Operators express certain mathematical operations or prescriptions to be carried out with a function or a vector
  - Applying an operator to a function (vector) yields another function (vector)

$$\mathbf{D}f(x) = \frac{d}{dx} f(x) \Rightarrow f'(x) \quad \text{(differntial operator)}$$

$$\mathbf{K}f(E) = \int dE' K(E' \to E) f(E') \Rightarrow g(E) \quad \text{(integral operator)}$$

$$\mathbf{A}\vec{u} \Rightarrow \vec{v} \quad \text{(matrix operator)}$$

  - Mapping from a vector (linear) space to another vector (linear) space

- Scalar or inner product

$$(\vec{u},\vec{v}) = \vec{u}^T \vec{v} \quad \text{(real space)}$$

$$(\vec{u},\vec{v}) = \vec{u}^H \vec{v} = (\bar{\vec{u}})^T \vec{v} \quad \text{(complex space)}$$

$$(\vec{u},\vec{v}) = (u_1, u_2)\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = u_1 v_1 + u_2 v_2$$

$$(\vec{u},\vec{v}) = (\bar{u}_1, \bar{u}_2)\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \bar{u}_1 v_1 + \bar{u}_2 v_2$$

$$(f,g) = \int_a^b f(x) g(x) dx \quad \text{(real space)}$$

$$(f,g) = \int_a^b \overline{f(x)} g(x) dx \quad \text{(complex space)}$$

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Adjoint Operators in Reactor Applications

- Adjoint operator

$$(\mathbf{K}^* f, g) = (f, \mathbf{K}g), \quad g \in X, \quad f \in Y, \quad \mathbf{K}: X \to Y, \quad \mathbf{K}^*: Y \to X$$

- Examples

$$(\vec{u}, \mathbf{A}\vec{v}) = \vec{u}^T \mathbf{A}\vec{v} = (\mathbf{A}\vec{v})^T \vec{u} = \vec{v}^T \mathbf{A}^T \vec{u} = (\mathbf{A}^T \vec{u}, \vec{v}) \qquad \Rightarrow \quad \mathbf{A}^* = \mathbf{A}^T \quad \text{(real space)}$$

$$(\vec{u}, \mathbf{A}\vec{v}) = (\bar{\vec{u}})^T \mathbf{A}\vec{v} = (\mathbf{A}\vec{v})^T \bar{\vec{u}} = \vec{v}^T \mathbf{A}^T \bar{\vec{u}} = (\bar{\mathbf{A}}^T \vec{u}, \vec{v}) \qquad \Rightarrow \quad \mathbf{A}^* = \bar{\mathbf{A}}^T = \mathbf{A}^H \quad \text{(complex space)}$$

$$\left(f, \frac{d}{dx} g \right) = \int_a^b f \frac{dg}{dx} dx = fg\Big|_a^b + \int_a^b \left(-\frac{df}{dx} \right) g dx \quad \text{(real space)}$$

$$\Rightarrow \quad \left(\frac{d}{dx} \right)^* = -\frac{d}{dx}, \quad fg\Big|_a^b = f(b)g(b) - f(a)g(a) \quad \text{(bilinear concomitant)}$$

$$\mathbf{K} f(E) = \int dE' K(E' \to E) f(E')$$

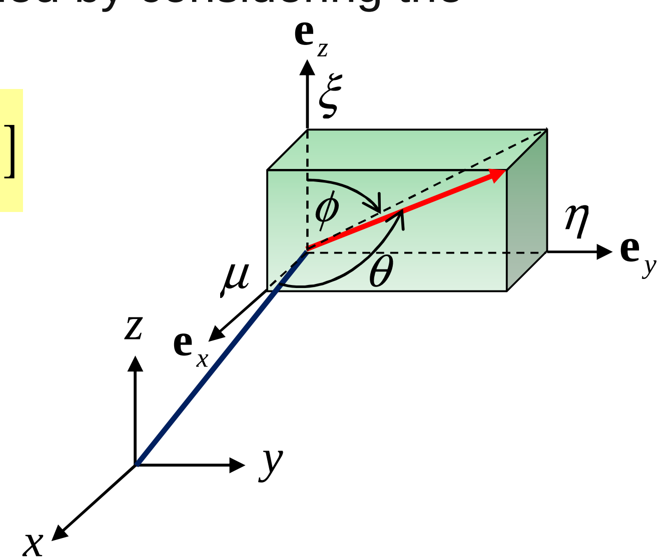
$$\begin{aligned}(f, \mathbf{K}g) &= \int dx f(x) \int dx' K(x \to x') g(x') = \int dx \int dx' f(x) K(x \to x') g(x') \\ &= \int dx' \int dx f(x') K(x' \to x) g(x) = \int dx g(x) \int dx' K(x' \to x) f(x') = (\mathbf{K}^* f, g)\end{aligned}$$

$$\Rightarrow \quad \mathbf{K}^* f(E) = \int dE' K(E \to E') f(E)$$

Streaming Operator

- Streaming operator
  - For curvilinear geometries, the angular coordinates are continuously changing as a particle travels along a straight line
  - The form of the streaming operator is determined by considering the operator in the directional derivative form

$$\mathbf{\Omega}\cdot\nabla\psi(\mathbf{r},\mathbf{\Omega})=\frac{d\psi}{du}=\lim_{du\to 0}\frac{1}{du}\left[\psi(\mathbf{r}+du\mathbf{\Omega},\mathbf{\Omega})-\psi(\mathbf{r},\mathbf{\Omega})\right]$$



- Cartesian space-angle coordinate system

$$\mathbf{\Omega}=\mu\mathbf{e}_x+\eta\mathbf{e}_y+\xi\mathbf{e}_z$$

$$\mu=\mathbf{\Omega}\cdot\mathbf{e}_x \quad \eta=\sqrt{1-\mu^2}\sin\phi \quad \xi=\sqrt{1-\mu^2}\cos\phi$$

$$dx=\mu du \qquad dy=\eta du \qquad dz=\xi du$$

$$\frac{d\psi}{du}=\frac{\partial\psi}{\partial x}\frac{dx}{du}+\frac{\partial\psi}{\partial y}\frac{dy}{du}+\frac{\partial\psi}{\partial z}\frac{dz}{du}=\mu\frac{\partial\psi}{\partial x}+\eta\frac{\partial\psi}{\partial y}+\xi\frac{\partial\psi}{\partial z}$$

$$\mathbf{\Omega}\cdot\nabla\psi(x,y,z,\mathbf{\Omega})=\mu\frac{\partial\psi}{\partial x}+\eta\frac{\partial\psi}{\partial y}+\xi\frac{\partial\psi}{\partial z}$$

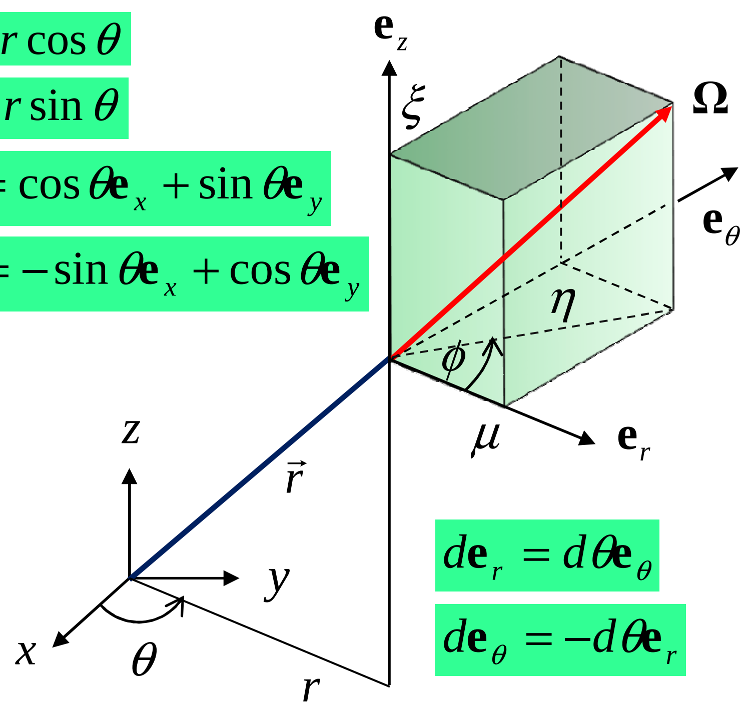
Cylindrical Space-Angle Coordinate System

$x = r\cos\theta$

$y = r\sin\theta$

$\mathbf{e}_r = \cos\theta\mathbf{e}_x + \sin\theta\mathbf{e}_y$

$\mathbf{e}_\theta = -\sin\theta\mathbf{e}_x + \cos\theta\mathbf{e}_y$



$d\mathbf{e}_r = d\theta\mathbf{e}_\theta$

$d\mathbf{e}_\theta = -d\theta\mathbf{e}_r$

$\mathbf{\Omega} = \mu\mathbf{e}_r + \eta\mathbf{e}_\theta + \xi\mathbf{e}_z$

$\mu = \sqrt{1-\xi^2}\cos\phi$

$\eta = \sqrt{1-\xi^2}\sin\phi \quad \xi = \mathbf{\Omega}\cdot\mathbf{e}_z$

$dr = \cos\theta dx + \sin\theta dy = \mu du$

$rd\theta = -\sin\theta dx + \cos\theta dy = \eta du$

$dz = \mathbf{\Omega}\cdot\mathbf{e}_z du = \xi du$

$d\xi = d(\mathbf{\Omega}\cdot\mathbf{e}_z) = 0$

$$d\varphi = -\frac{d\mu}{\eta} = -d\theta = -\eta\frac{du}{r}$$

$$\frac{d\psi}{du} = \frac{\partial\psi}{\partial r}\frac{dr}{du} + \frac{\partial\psi}{\partial\theta}\frac{d\theta}{du} + \frac{\partial\psi}{\partial z}\frac{dz}{du} + \frac{\partial\psi}{\partial\xi}\frac{d\xi}{du} + \frac{\partial\psi}{\partial\varphi}\frac{d\varphi}{du} = \mu\frac{\partial\psi}{\partial r} + \frac{\eta}{r}\frac{\partial\psi}{\partial\theta} + \xi\frac{\partial\psi}{\partial z} - \frac{\eta}{r}\frac{\partial\psi}{\partial\varphi}$$

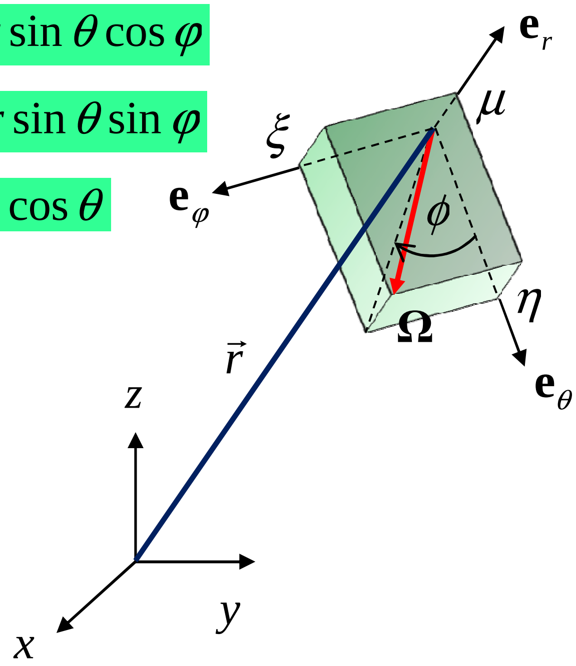
$$\mathbf{\Omega}\cdot\nabla\psi(r,\theta,z,\mathbf{\Omega}) = \frac{\mu}{r}\frac{\partial(r\psi)}{\partial r} + \frac{\eta}{r}\frac{\partial\psi}{\partial\theta} + \xi\frac{\partial\psi}{\partial z} - \frac{1}{r}\frac{\partial(\eta\psi)}{\partial\phi}$$

Spherical Space-Angle Coordinate System (1/2)

$x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

$z = r\cos\theta$



$\mathbf{e}_r = \sin\theta\cos\varphi\mathbf{e}_x + \sin\theta\sin\varphi\mathbf{e}_y + \cos\theta\mathbf{e}_z$

$\mathbf{e}_\theta = \cos\theta\cos\varphi\mathbf{e}_x + \cos\theta\sin\varphi\mathbf{e}_y - \sin\theta\mathbf{e}_z$

$\mathbf{e}_\varphi = -\sin\varphi\mathbf{e}_x + \cos\varphi\mathbf{e}_y$

$\mathbf{\Omega} = \mu\mathbf{e}_r + \eta\mathbf{e}_\theta + \xi\mathbf{e}_\phi$

$\mu = \mathbf{\Omega}\cdot\mathbf{e}_r$

$\eta = \sqrt{1-\mu^2}\cos\varphi$

$\xi = \sqrt{1-\mu^2}\sin\varphi$

$d\mathbf{e}_r = d\theta\mathbf{e}_\theta + \sin\theta d\varphi\mathbf{e}_\varphi$

$d\mathbf{e}_\theta = -d\theta\mathbf{e}_r + \cos\theta d\varphi\mathbf{e}_\varphi$

$d\mathbf{e}_\varphi = -d\varphi(\sin\theta\mathbf{e}_r + \cos\theta\mathbf{e}_\theta)$

$dr = \sin\theta\cos\phi dx + \sin\theta\sin\phi dy + \cos\theta dz = \mu du$

$rd\theta = \cos\theta\cos\phi dx + \cos\theta\sin\phi dy - \sin\theta dz = \eta du$

$r\sin\theta d\phi = -\sin\phi dx + \cos\phi dy = \xi du$

Spherical Space-Angle Coordinate System (2/2)

$$d\mu = \mathbf{\Omega}\cdot d\mathbf{e}_r = \eta d\theta + \xi \sin\theta d\phi = \frac{1}{r}(\eta^2+\xi^2)du = \frac{1}{r}(1-\mu^2)du$$

$$d\varphi = -\frac{1}{\xi}\left(d\eta + \frac{\eta\mu d\mu}{1-\mu^2}\right) = -\frac{1}{\xi}\left(\mathbf{\Omega}\cdot d\mathbf{e}_\theta + \frac{\eta\mu du}{r}\right)$$
$$= -\frac{1}{\xi}\left(-\mu d\theta + \xi\cos\theta d\phi + \frac{\eta\mu du}{r}\right) = -\frac{\xi\cot\theta}{r}du$$

$$\frac{d\psi}{du} = \frac{\partial\psi}{\partial r}\frac{dr}{du} + \frac{\partial\psi}{\partial\theta}\frac{d\theta}{du} + \frac{\partial\psi}{\partial\phi}\frac{d\phi}{du} + \frac{\partial\psi}{\partial\mu}\frac{d\mu}{du} + \frac{\partial\psi}{\partial\varphi}\frac{d\varphi}{du}$$
$$= \mu\frac{\partial\psi}{\partial r} + \frac{\eta}{r}\frac{\partial\psi}{\partial\theta} + \frac{\xi}{r\sin\theta}\frac{\partial\psi}{\partial\phi} + \frac{1-\mu^2}{r}\frac{\partial\psi}{\partial\mu} - \frac{\xi\cot\theta}{r}\frac{\partial\psi}{\partial\varphi}$$

$$\mathbf{\Omega}\cdot\nabla\psi(r,\theta,\phi,\mathbf{\Omega}) = \frac{\mu}{r^2}\frac{\partial(r^2\psi)}{\partial r} + \frac{\eta}{r\sin\theta}\frac{\partial(\sin\theta\psi)}{\partial\theta} + \frac{\xi}{r\sin\theta}\frac{\partial\psi}{\partial\phi}$$
$$+\frac{1}{r}\frac{\partial[(1-\mu^2)\psi]}{\partial\mu} - \frac{\cot\theta}{r}\frac{\partial(\xi\psi)}{\partial\varphi}$$

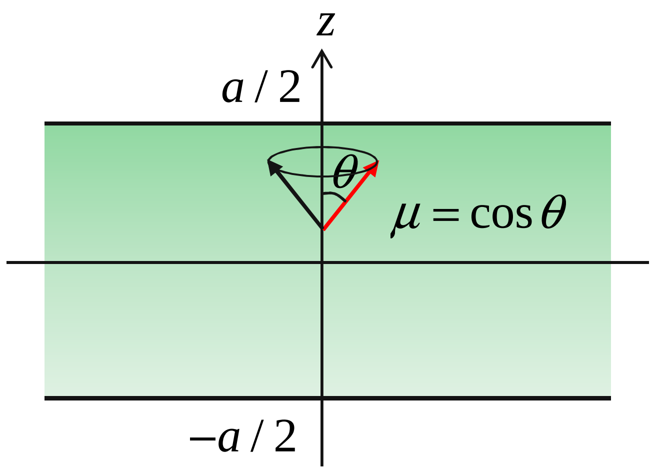
Transport Equation for 1-D Plane Geometry

- 1-D slab geometry
  - In the 1-D plane geometry, the angular flux is independent of the azimuthal angle so that the angular dependence is reduced to the μ interval (-1,1).
  - The angular flux is independent of x and y

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$$

  - Consequently, the angular flux is a function of z and μ only

$$\mathbf{\Omega} \cdot \nabla \psi(z,\mu) = \mu \frac{\partial}{\partial z} \psi(z,\mu)$$

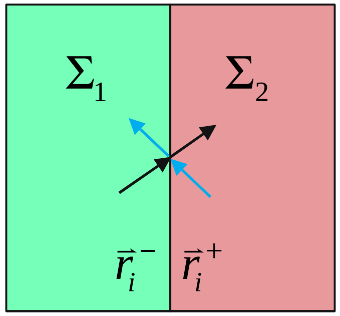


$$\mu \frac{\partial}{\partial z} \psi(z,E,\mu) = \Sigma_t(z,E)\psi(z,E,\Omega) - \int dE' \int d\mu' \Sigma_s(z,E' \to E, \mu' \to \mu)\psi(z,E',\mu')$$
$$+ \frac{1}{4\pi} \sum_i \int dE' \chi_i(E' \to E)\nu_i(E')\Sigma_{fi}(z,E')\phi(z,E') + S(z,E,\mu)$$

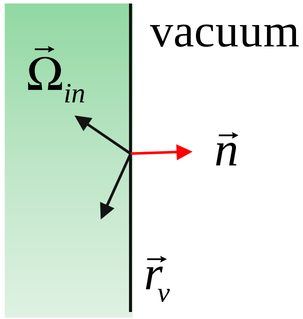
Boundary and Interface Conditions

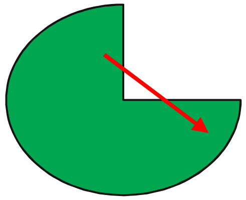
- Boltzmann equation is an integro-differential equation of first order
  - One boundary condition and one condition for each interface are required
- Interface condition
  - Angular flux is continuous at region interfaces, since the interface has neither a finite neutron absorption nor emission capability

$$\psi(\vec{r}_i^{\,-}, E, \vec{\Omega}) = \psi(\vec{r}_i^{\,+}, E, \vec{\Omega})$$



- Boundary condition
  - At the outer boundary with a vacuum outside of a <u>convex</u> medium
  - No neutrons move from the vacuum into the system since a vacuum contains neither a source or scattering material
  - No neutrons emitted into the vacuum from the reactor will ever come back





$$\psi(\vec{r}_v, E, \vec{\Omega}_{in}) = 0 \quad \text{for } \vec{\Omega}_{in} \cdot \vec{n} < 0$$

Simplifications for Practical Core Calculation

- Current design tools solve the transport equations with various approximations and sophisticated multi-step procedures
  - Average parameters for whole-core calculations are determined by a series of sub-domain calculations with increased modeling details and approximate boundary conditions
  - Detailed information is approximately recovered by reconstruction (de-homogenization) method