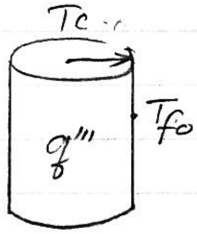
Transient Heat Conduction

Lumped Parameter Model - Cylindrical Fuel



$$\bar{T} = \int_{r=0}^{r_f} T \cdot 2\pi r \cdot dr \Big/ \int_0^{r_f} 2\pi r \cdot dr$$

$$\rho c_p \frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r} k r \frac{\partial T}{\partial r} + q'''$$

multiply by $2\pi r dr$ and integrate w.r.t r

$$\int_0^{r_f} k\, 2\pi r \cdot \frac{1}{r} \cdot \frac{\partial}{\partial r}\left(r \frac{\partial T}{\partial r}\right) dr = 2\pi r_f \cdot k \left(\frac{\partial T}{\partial r}\Big|_{r_f}\right) = -q'_{transfer}$$

$$\int_0^{r_f} \rho c_p \cdot \frac{\partial T}{\partial t} \cdot 2\pi r \cdot dr = \frac{\partial}{\partial t} \rho c_p \int_0^{r_f} 2\pi r \cdot T \cdot dr$$

$$= \rho c_p \cdot \pi r_f^2 \cdot \bar{T} = m' c_p \frac{\partial \bar{T}}{\partial t}$$

$$\therefore\ m' c_p \frac{\partial \bar{T}}{\partial t} = -q'_{transfer} + q'_{source}$$

Here:

$$q'_{transfer} = \frac{\bar{T} - T_{fo}}{R_f}$$

From steady state solution:

$$T_{fo} - T = -q''' \frac{r^2}{4k}\Big|_{r_{fo}}^{r} = -\frac{q'''}{4k}(r_{fo}^2 - r^2)$$

$$T - T_{fo} = \frac{q'}{4\pi k}\left(1 - \frac{r^2}{r_{fo}^2}\right)$$

$$\bar{T} - T_{fo} = \frac{q'}{8\pi k} \qquad \therefore \quad R_f = \frac{1}{8\pi k_f}$$

$$\boxed{m' c_p \frac{\partial \bar{T}}{\partial t} = -\frac{(\bar{T} - T_{f0})}{R_f} + q'}$$

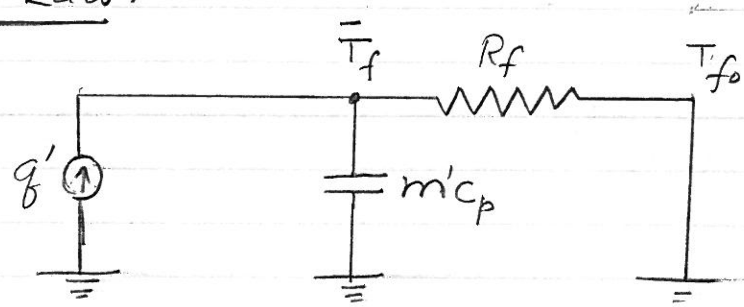
$$\frac{\partial \bar{T}}{\partial t} = -\frac{1}{m' c_p R_f}(\bar{T} - T_{f0}) + q'$$

$$\boxed{\frac{\partial \theta}{\partial t} = -\frac{\theta}{m c_p R_f} + q'} \qquad \theta = \bar{T} - T_{f0}$$

Solution: $\theta = \theta_0 \, e^{-t/\tau} + \frac{\tau q'}{m' c_p}(1 - e^{-t/\tau})$

where: $\tau = \frac{1}{m c_p R_f}$ – Time constant

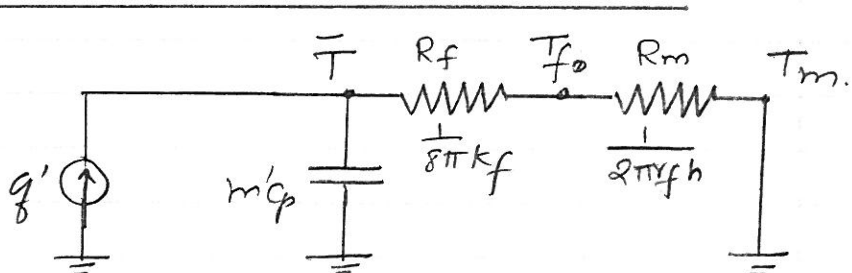
Kirkoff's Law:



$$I = C \frac{dV}{dt} \; ;$$

Cylinder with Convection on the Wall

convection:
$q' = h \, 2\pi r_f \, \Delta T$



$$q' - \frac{1}{(R_f + R_m)}(\bar{T} - T_m) - m' c_p \frac{dT}{dt} = 0$$

$$\tau = m'cR = mc\left(\frac{1}{8\pi k} + \frac{1}{2\pi k\, r_{fo}}\right)$$

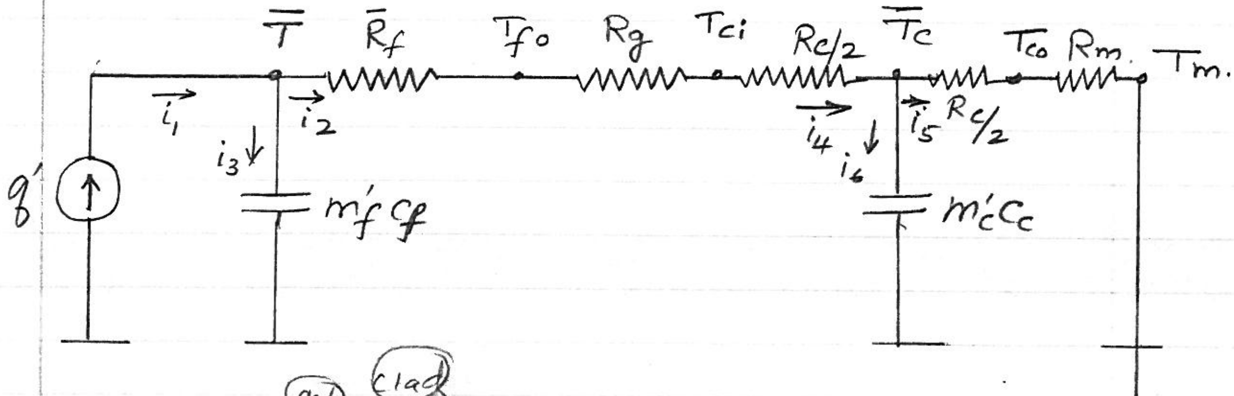
$$= \tau_1 + \tau_2$$

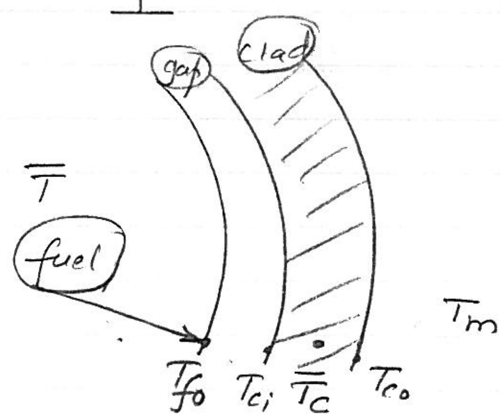
$$\frac{\tau_1}{\tau_2} = \frac{2\pi h a}{8\pi k} = \frac{h\, a/4}{k} = \frac{R_f}{R_m} = Bi \quad \text{– Biot number}$$

$$= \frac{\text{internal resistance}}{\text{surface resistance}}$$

If $Bi \ll 1$ Lumped parameter model is good.

<u>Cylindrical Fuel with Cladding.</u>





$$i_1 - i_2 - i_3 = 0$$

$$q' - \frac{1}{R_{fc}}\left(\bar{T}_f - \bar{T}_c\right) - m_f' c_f \frac{d\bar{T}}{dt} = 0$$

$$R_{fc} = \bar{R}_f + R_g + R_c/2$$

$$i_4 - i_5 - i_6 = 0$$

$$\frac{(\overline{T}_f - \overline{T}_c)}{R_{fc}} - \frac{(\overline{T}_c - T_m)}{R_{cm}} - m_c' c_c \frac{d\overline{T}_c}{dt} = 0$$

$$R_{cm} = R_c/_2 + R_m.$$

Special case : (No heat transfer to the fluid)

$$\frac{d\overline{T}_f}{dt} + \frac{1}{\tau_f}(\overline{T}_f - \overline{T}_c) = q'/m_f c_f$$

$$\frac{d\overline{T}_c}{dt} + \frac{1}{\tau_c}(\overline{T}_f - \overline{T}_c) = 0$$

where time constants $\tau_f = m_f c_f R_{cf}$ - fuel

$\tau_c = m_c c_c R_{fc}$ - clad

$$\frac{d}{dt}(\overline{T}_f - \overline{T}_c) + \frac{1}{\tau}(\overline{T}_f - \overline{T}_c) = \frac{q'}{m_f c_f}$$

$$\tau = \frac{\tau_f + \tau_c}{\tau_f \tau_c}$$

$$\frac{d}{dt}\phi + \frac{1}{\tau}\phi = \frac{q'}{m_f c_f} \qquad \phi = \overline{T}_f - \overline{T}_c$$