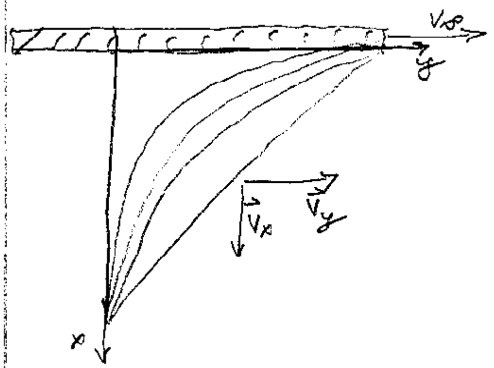
Nucl 551
Exam I
Review

Diffusion Length



<u>Assumptions</u>

2D, No z Dependence $\left(\frac{\partial}{\partial z}=0\right)$
No velocity in z-dir $(v_z=0)$
at $t<0$, $v_x=0$, $v_y=0$, $t=0^+ \Rightarrow v_y(0,y)=\vec{v}_\infty$
$v_x=0$ @ $x=0$
incompressible $(\rho=\text{const}, \nabla\cdot\vec{v}=0)$
const properties $(\mu=\text{const})$
isothermal, adiabatic (no E.E.)
Infinite board $\left(\frac{\partial}{\partial y}=0\right)$

<u>C.E.</u>

$$\cancel{\frac{\partial \rho}{\partial t}}^{0} + \nabla\cdot\rho\vec{v}=0, \qquad \nabla\cdot\vec{v}=0$$

$$\cancel{\rho\nabla\cdot\vec{v}}^{0} + \vec{v}\cdot\nabla\rho=0$$

$$\vec{v}\cdot\nabla\rho=0$$

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0 \qquad \left.\begin{array}{l}\frac{\partial v_x}{\partial x}=0\\ v_x\big|_x=0\end{array}\right\}\Rightarrow v_x=0$$

$$\frac{\partial v_y}{\partial y}=0$$

$$v_y=v_y(t,x)$$

<u>M.E.</u>

$$y\to\ \rho\left\{\frac{\partial v_y}{\partial t}+\cancel{v_x\frac{\partial v_y}{\partial x}}^{0}+v_y\cancel{\frac{\partial v_y}{\partial y}}^{0}+\cancel{v_z\frac{\partial v_y}{\partial z}}^{0}\right\}=-\frac{\partial p}{\partial y}+\mu\left\{\frac{\partial^2 v_y}{\partial x^2}+\cancel{\frac{\partial^2 v_y}{\partial y^2}}^{0}+\cancel{\frac{\partial^2 v_y}{\partial z^2}}^{0}\right\}+\cancel{\rho g_y}^{0}$$

$$\rho\frac{\partial v_y}{\partial t}=-\frac{\partial p}{\partial y}+\mu\frac{\partial^2 v_y}{\partial x^2}$$

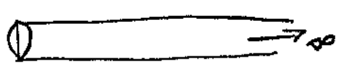
$$p=p(x)\Rightarrow\frac{\partial p}{\partial y}=0$$

$$\rho\frac{\partial v_y}{\partial t}=\mu\frac{\partial^2 v_y}{\partial x^2}$$

use $\nu=\frac{\mu}{\rho}$

$$\frac{\partial v_y}{\partial t}=\nu\frac{\partial^2 v_y}{\partial x^2}$$

becomes similar w/ heat conduction through rod



Energy Eq

$$\rho C_v\left[\frac{\partial T}{\partial t}+\cancel{\vec{v}\cdot\nabla T}^{0\ \text{solid}}\right]=k\nabla^2T+\dot{q}+p\cancel{\nabla\cdot\vec{v}}^{0\ \text{incomp}}-\cancel{\tau:\nabla\vec{v}}^{0\ \text{dissipation}}$$

$$\rho C_v\frac{\partial T}{\partial t}=k\nabla^2T\Rightarrow\rho C_v\frac{\partial T}{\partial t}=k\frac{\partial^2T}{\partial x^2}$$

$$a=\frac{k}{\rho c_v}$$

$$\frac{\partial T}{\partial t}=a\frac{\partial^2T}{\partial x^2}$$

Thermal Penetration Depth

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}$$

$$\Theta = T - T_\infty$$

$$\frac{\partial \Theta}{\partial t} = a \frac{\partial^2 \Theta}{\partial x^2} \quad \text{and} \quad \Theta(x, 0) = T_0 - T_\infty = \Theta_0 \quad \text{and} \quad \Theta(0, t) = 0 \quad \text{and} \quad \Theta(\infty, t) = \Theta_0$$

use similarity

$$\Theta = \phi(\eta)$$
$$\eta = \frac{x}{\sqrt{4at}}$$

$$\frac{\partial \Theta/\Theta_0}{\partial t} = -\frac{1}{2}\frac{\eta}{t}\phi'$$

$$\phi'' + 2\eta\phi' = 0$$

$$\phi' = C_1 \exp(-\eta^2)$$

$$\phi = C_1 \int_0^\eta \exp(-\eta^2) ds + C_2$$

$$C_1 = \frac{1}{\int_0^\infty \exp(-\eta^2) d\eta}, \quad C_2 = 0$$

$$\phi = \frac{\int_0^\eta \exp(-\eta^2) d\eta}{\int_0^\infty \exp(-\eta^2)} = \frac{2}{\pi}\int_0^\eta \exp(-\eta^2) d\eta$$

$$\phi = \operatorname{erf}(\eta) \Rightarrow \frac{\Theta}{\Theta_0} = \operatorname{erf}\left(\frac{x}{\sqrt{4at}}\right)$$

$$t \leq 0 \quad T(0, x) = T_0$$

$$t = 0^+ \quad T(0^+, 0) = T_\infty$$

Temperature Gradient

$$\frac{\partial T}{\partial x} = \frac{\partial \Theta}{\partial x} = \frac{\partial \phi \Theta}{\partial x} = \Theta_0 \frac{\partial \phi}{\partial \eta}\frac{\partial \eta}{\partial x}$$

$$\frac{\partial \phi}{\partial \eta} = \frac{\partial}{\partial \eta}\left[\operatorname{erf}(\eta)\right] = \frac{2}{\pi}\left[\int \frac{\partial}{\partial \eta}\exp(-\eta^2) d\eta + \frac{\partial \eta}{\partial \eta}\exp(-\eta^2)\right] = \frac{2}{\sqrt{\pi}}\exp(-\eta^2)$$

$$\frac{\partial \eta}{\partial x} = \frac{1}{\sqrt{4at}}$$

$$\left.\frac{\partial T}{\partial x}\right|_0 = \Theta_0 \frac{2}{\sqrt{\pi}}\exp(-\eta^2)\left.\frac{1}{\sqrt{4at}}\right|_0 = \frac{\Theta_0}{\sqrt{\pi a t}} = \frac{T_0 - T_\infty}{\sqrt{\pi a t}} \Rightarrow d_T = \sqrt{\pi a t}$$

Hilbert General Balance Equation

$\psi \rightarrow$ property to be balanced

$\int \psi \, dV \rightarrow$ total amount in V

$\mathbb{J} \rightarrow$ Flux property across surface

$-\oint_{S_m} \mathbb{J} \cdot \vec{n} \, ds \rightarrow$ net flow across surface

$\dot{\psi}_g \rightarrow$ generation of ψ per unit volume

$$\left\{ \begin{matrix} \text{change of } \psi \\ \text{in time} \end{matrix} \right\} = \left\{ \begin{matrix} \text{Flux across} \\ \text{surface} \end{matrix} \right\} + \left\{ \text{generation} \right\}$$

$$\frac{D}{Dt} \int_{V_m} \psi \, dV = -\oint_{S_m} \mathbb{J} \cdot \hat{n} \, ds + \int \dot{\psi}_g \, dV$$

using Reynolds

$$\frac{D}{Dt} \int_{V_m} \psi \, dV = \int_V \left[\frac{d\psi}{dt} + \nabla \cdot (\psi \vec{v}) \right] dV$$

and green's

$$-\oint_{S_m} \mathbb{J} \cdot \hat{n} \, ds = -\int_V \nabla \cdot \mathbb{J} \, dV$$

we get

$$\underbrace{\frac{d\psi}{dt}}_{\text{time rate of change}} + \underbrace{\nabla \cdot (\psi \vec{v})}_{\text{convection}} = \underbrace{-\nabla \cdot \mathbb{J}}_{\text{flux}} + \underbrace{\dot{\psi}_g}_{\text{generation}}$$

so mass continuity equation

$$\frac{d\rho}{dt} + \nabla \cdot \rho \vec{v} = \cancelto{0}{-\nabla \cdot \mathbb{J}} + \cancelto{0}{\dot{\rho}_g} \rightarrow \text{most cases in fluids}$$

$$\frac{d\rho}{dt} + \nabla \cdot \rho \vec{v} = \frac{d\rho}{dt} + \vec{v} \cdot \nabla \rho + \rho \nabla \cdot \vec{v} = 0$$

set $\frac{D\rho}{Dt} = \left.\frac{\partial \rho}{\partial t}\right)_{\vec{z}} \rightarrow$ fixed mass

$$\frac{1}{\rho} \frac{D\rho}{Dt} = -\nabla \cdot \vec{v}$$

incompressible $\rightarrow \nabla \cdot \vec{v} = 0$

Cylindrical: $\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (\rho r v_r) + \frac{1}{r} \frac{\partial (\rho v_\theta)}{\partial \theta} + \frac{\partial (\rho v_z)}{\partial z} = 0$

Use Hilbert General Balance Eq

$$\psi = \rho\vec{v}$$

$$\underline{\underline{J}}_i = \underline{\underline{\pi}} = \cancel{p\underline{\underline{I}}} + \underline{\underline{\tau}}$$ (fluid so can't spin)

$$\psi_g = \rho\vec{F} = \rho\vec{g}$$ (only typical volume force unless magnets)

so plugging in:

$$\frac{\partial \rho\vec{v}}{\partial t} + \nabla\cdot(\rho\vec{v}\vec{v}) = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\vec{g}$$

$$\frac{D}{Dt}\psi = \frac{\partial}{\partial t}\psi + \vec{v}\cdot\nabla\psi \Rightarrow \frac{\partial}{\partial t}\psi = \frac{D}{Dt}\psi - \vec{v}\cdot\nabla\psi = \frac{D}{Dt}\psi - \nabla\cdot\psi\vec{v}$$

$$\rho\frac{D\vec{v}}{Dt} = \frac{\partial\rho\vec{v}}{\partial t} + \nabla\cdot(\rho\vec{v}\vec{v})$$

$$\rho\frac{D\vec{v}}{Dt} = -\nabla p - \nabla\cdot\underline{\underline{\tau}} + \rho\vec{g}$$

Cylindrical

r: $$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_z\frac{\partial v_r}{\partial z} - \frac{v_\theta^2}{r}\right) = -\frac{\partial p}{\partial r} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_r}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} + \frac{\partial^2 v_r}{\partial z^2} - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right] + \rho g_r$$

z: $$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

Momentum Eq (Nat. Circ)

Start from momentum Eq (use conservative form: $\nabla \cdot \rho \vec{v}$ not $\rho(\nabla \cdot \vec{v})$)

$$\frac{\partial \rho \vec{v}}{\partial t} + \underbrace{\nabla \cdot (\rho \vec{v}\vec{v})}_{\text{non-linear}} = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + \rho \vec{g}$$

Assumption
Density change due to thermal expansion, but it's only important in gravity terms

Use thermal expansion coefficient β

$$\beta \equiv \frac{1}{v} \left.\frac{\partial v}{\partial t}\right)_p = -\frac{1}{\rho} \left.\frac{\partial \rho}{\partial T}\right)_p$$

$$\rho = \rho(T, p) \sim \rho(T)$$

$$d\rho = -\rho \beta \, dT$$

$$\rho - \bar{\rho} = -\bar{\rho}\beta(T - \bar{T})$$

$$\bar{\rho} \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \cdot \underline{\underline{\tau}} + [\underbrace{\bar{\rho}}_{\text{ref dens.}} - \underbrace{\bar{\rho}\beta(T-\bar{T})}_{\text{density deviation due to thermal expansion}}]\vec{g}$$

then, if $\vec{v}$ is small (hydrostatic)

$$-\nabla p + \bar{\rho}\vec{g} = 0$$

so

$$\bar{\rho} \frac{D\vec{v}}{Dt} = -\nabla \cdot \underline{\underline{\tau}} - \bar{\rho}\beta(T - \bar{T})\vec{g}$$

Kinetic Energy Eq

Use Hilbert General Balance Equation

$$\vec{v}\cdot[M.E] \xrightarrow[\text{from Momentum to KE}]{\text{transform}} [KEE]$$

$$\rho\frac{D\left(\frac{\vec{v}^2}{2}\right)}{Dt} = -\vec{v}\cdot\nabla p - \vec{v}\cdot(\nabla\cdot\tau) + \rho\vec{v}\cdot\vec{g} \Rightarrow \left\{\begin{array}{l}\nabla\cdot(p\vec{v}) = p\nabla\cdot\vec{v} + \vec{v}\cdot\nabla p \\ \nabla\cdot(\tau\cdot\vec{v}) = \tau:\nabla\vec{v} + \vec{v}\cdot(\nabla\cdot\tau)\end{array}\right\}$$

$$\underbrace{\frac{\partial\rho\left(\frac{\vec{v}^2}{2}\right)}{\partial t}}_{\text{change in KE}} + \underbrace{\nabla\cdot\left(\frac{1}{2}\rho\vec{v}^2\vec{v}\right)}_{\text{convection}} = \underbrace{-\nabla\cdot(p\vec{v})}_{\text{work by pressure}} + \underbrace{p\nabla\cdot\vec{v}}_{\text{reversible work converted to internal}} - \underbrace{\nabla\cdot(\tau\cdot\vec{v})}_{\text{work by viscous forces}} + \underbrace{\tau:\nabla\vec{v}}_{\text{irreversible work converted to internal}} + \underbrace{\rho\vec{v}\cdot}_{\text{gravi... work}}$$

$$\Psi = \rho\left(u + \frac{\vec{V}^2}{2}\right)$$

I.E. K.E.

$$\vec{J} = \vec{q} + \tau \cdot \vec{V}$$

heat flux

work done by surface forces

$$\dot{\Psi}_g = \rho \vec{V} \cdot \vec{g} + \dot{q}$$

gravity work

internal heat gen → chem, electrical, nuclear

Hilbert General Balance Equation

$$\frac{\partial \rho(u+\frac{V^2}{2})}{\partial t} + \nabla \cdot \left[\rho\left(u + \frac{\vec{V}^2}{2}\right)\vec{V}\right] = -\nabla \cdot \vec{q} - \nabla \cdot (p\vec{V}) + \nabla \cdot (\tau \cdot \vec{V}) + \rho \vec{V} \cdot \vec{g} + \dot{q}$$

$$\Downarrow$$

Thermal Energy Equation

$$[EE] - [KE]$$

$$\frac{\partial \rho(u+\frac{V^2}{2})}{\partial t} + \nabla \cdot \left[\rho\left(u + \frac{\vec{V}^2}{2}\right)\vec{V}\right] = -\nabla \cdot \vec{q} - \nabla \cdot (p\vec{V}) + \nabla \cdot (\tau \cdot \vec{V}) + \rho \vec{V} \cdot \vec{g} + \dot{q}$$

$$-\left[\rho \frac{\partial \frac{V^2}{2}}{\partial t} = -\vec{V} \cdot \nabla p - \vec{V} \cdot [\nabla \cdot \tau] + \rho \vec{V} \cdot \vec{g}\right]$$

$$\rho \frac{Du}{Dt} = -\nabla \cdot \vec{q} - p \nabla \cdot \vec{v} - \tau : \nabla \vec{V} + \dot{q}$$

heat flux

reversible work

dissipation small, not 0

heat gen

$$\Downarrow$$

$$\rho \frac{Du}{Dt} = -\nabla \cdot q - p \nabla \cdot \vec{v}$$

for most cases (small dissipation, no internal heat gen)

Hilbert General Balance Equation
for k is the component

$\psi = \rho_k$

$\mathbb{J} = \rho_k(\vec{v}_k - \vec{v}) = \rho_k \vec{v}_{km} = \vec{\mathcal{J}}_k$

$\dot{\psi}_g = r_k$

so $\underbrace{\frac{\partial \rho_k}{\partial t}}_{\text{change of mass in time}} + \underbrace{\nabla \cdot \rho_k \vec{v}}_{\text{mass convection}} = \underbrace{-\nabla \cdot \vec{\mathcal{J}}_k}_{\text{diffusion mass flux}} + \underbrace{r_k}_{\text{creation}}$ [Mixture CE]

Hilbert GBE

$\psi = \rho \vec{v}$

$\mathbb{J} = p\,\mathbb{I} + \mathbb{T} + \Sigma \rho_k \vec{v}_k \vec{v}_k = \mathbb{T}\mathbb{T}$

$\dot{\psi}_g = \Sigma \rho_k \vec{g}$

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \rho \vec{v}\vec{v} = -\nabla p - \nabla \cdot \mathbb{T} - \nabla \Sigma \rho_k \vec{v}_{km} \vec{v}_{km} + \Sigma \rho_k \vec{g}$$ [Mixture ME]

Hilbert GBE

$\psi = \rho(u + \frac{\vec{v}^2}{2})$

$\mathbb{J} = \mathbb{T}\mathbb{T} \cdot \vec{v}$

$\dot{\psi}_g = \Sigma \rho_k \vec{v}_k \cdot \vec{g} + \dot{q}_k$

Types

Mechanical $(\tau, \vec{g})$

Thermal $(\vec{q}, \dot{q})$

State Eq Examples → Ideal Gas ($p = RT\rho$, $u = u(T)$), Incomp ($\rho(p,T) = \rho$)

Mechanical

Inviscid (98% correct) → $\tau = 0$

Linearly viscous → $\tau_{yx} = \mu \frac{dV_x}{dy}$

could be viscous force, heat flux, mass flux, diffusion

Thermal

heat conduction → $\vec{q} = -k\nabla T$

internal heat gen → $\dot{q} = \dot{q}(x, T)$ fission, electrical resistance, etc.

Mass Diffusion

Mass diffusion flux $\vec{j}_k = \rho_k(\vec{V}_k - \vec{V})$

Diffusion model specified by constituitive eq

$$j_k \equiv \rho_k(\vec{V}_k - \vec{V}) = -\rho D \nabla\left(\frac{\rho_k}{\rho}\right) = -\rho D \nabla \omega_k$$

Entropy generation check

$T ds = du - \frac{p}{\rho^2} d\rho \qquad s(u, \rho)$

$$\rho\left\{T\frac{Ds}{Dt} + \frac{p}{\rho^2}\frac{D\rho}{Dt}\right\}$$

$$\rho\frac{Ds}{Dt} = -\frac{\nabla \cdot \vec{q}}{T} - \frac{\tau : \nabla\vec{V}}{T} + \frac{\dot{q}}{T}$$

2nd Law

$$\rho\frac{Ds}{Dt} + \nabla\cdot\left(\frac{\vec{q}}{T}\right) - \frac{\dot{q}}{T} \equiv \Delta > 0$$

$$\Delta = -\frac{\vec{q}\cdot\nabla T}{T^2} - \frac{\tau : \nabla\vec{u}}{T} \geq 0$$

$T > 0$, $-\vec{q}\cdot\nabla T \geq 0$; $-\tau : \nabla\vec{V} \geq 0$ so satisfies entropy generation

from high to low

dissipation

1) Scaling parameter

Distance

$$x^* = \frac{x}{D},\ y^* = \frac{y}{D},\ z^* = \frac{z}{D}$$

Time

$$\tau = \frac{D}{V},\quad t^* = \frac{t}{\tau} = \frac{tV}{D}$$

Velocity

$$V^* = \frac{\vec{V}}{V}$$

Pressure

$$p^* = \frac{P - P_0}{\rho_0 V^2}$$

Temperature

$$T^* = \frac{T - T_0}{\Delta T}$$

Density

$$\rho^* = \frac{\rho}{\rho_0}$$

Diff Operator

$$\nabla^* = D\nabla$$

$$\nabla^{*2} = D^2 \nabla^2$$

$$\frac{D}{Dt^*} = \frac{D}{V}\frac{D}{Dt}$$

$$\frac{\delta}{\delta t^*} = \frac{D}{V}\frac{\delta}{\delta t}$$

$$Re = \frac{\rho_0 V D}{\mu} = \frac{\rho V^2/D}{\mu(V/D)/D} = \frac{\text{inertia}}{\text{viscous force}}$$

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_v} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$$

$$Ec = \frac{V^2}{c_p \Delta T} = \frac{\rho V^2/D}{c_p \rho \Delta T/D} = \frac{\text{K.E. convection}}{\text{enthalpy convection}}$$

$$Fr = \frac{V^2}{gD} = \frac{\rho V^2/D}{\rho g} = \frac{\text{inertia}}{\text{gravity}}$$

$$Pe = Re\,Pr$$

Time Averaged Turbulence

$\tau^t = \rho \overline{v'v'}$

$\tau^\mu = -\mu[\nabla\vec{v} + (\nabla\vec{v})^t]$

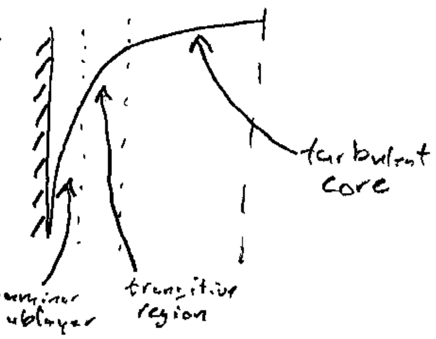
$\tau^T = \tau^\mu + \tau^t$

$\frac{\partial \rho \vec{v}}{\partial t} + \nabla\cdot\rho\vec{v}\vec{v} = -\nabla p - \nabla\cdot\tau^T + \rho\vec{g}$ [Turbulent M.E.]

$\rho C_v \frac{D\overline{T}}{Dt} = [k\nabla^2 T - \nabla\cdot\rho c_v \overline{T'v'}] + g'$ [Turbulent E.E.]

Velocity Profiles

$\Rightarrow$



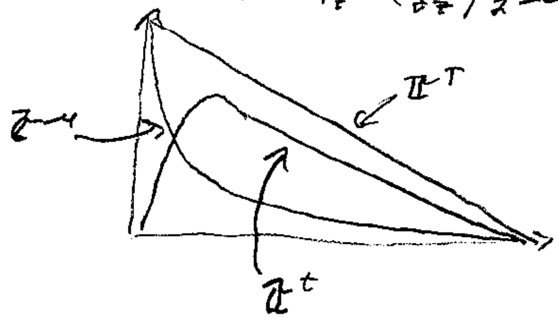
Stress in turbulent pipe flow

steady state
No Gravity
Fully developed
Axisymmetric

$\rho\left(\frac{\partial v}{\partial t} + \overline{v}_r\frac{\partial v_z}{\partial r} + \frac{\overline{v}_\theta}{r}\frac{\partial v_z}{\partial\theta} + \overline{v}_z\frac{\partial\overline{v}_z}{\partial z}\right) = -\frac{\partial\overline{p}}{\partial t} + \rho\vec{g} + \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}^T) + \frac{1}{r}\frac{\partial\tau_{\theta z}^T}{\partial\theta} + \frac{\partial\tau_{zz}^T}{\partial z}\right]$

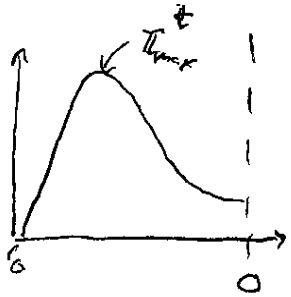
$\frac{\partial\overline{p}}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}^T)$

$\tau_{rz}^T = \tau_{rz}^\mu + \tau_{rz}^t = \left(\frac{\partial p}{\partial z}\right)\frac{r}{2} = -\left(\frac{\Delta p}{L}\right)\frac{r}{2}$



$\Rightarrow$

$I = \frac{\sqrt{\overline{v_z'^2}}}{v_{z,max}}$



τ^t_{max} at $\sim\left(\frac{r}{r_0}\sim 0.9\right)$

Prandtl Turbulence

(1) → Fluid element moves but doesn't transfer any momentum until it has traveled the entire length

(2) momentum x gained by δm

$$\left\{\begin{matrix}\text{x-comp}\\ \text{momentum}\\ \text{gained by}\\ \delta m\end{matrix}\right\} = \delta m\, \delta v_x \;; \quad \left\{\begin{matrix}\text{momentum}\\ \text{transfer}\\ \text{rate}\end{matrix}\right\} = \frac{\delta m\, \delta v_x}{\delta t}$$

$$\left\{\begin{matrix}\text{Shear force}\\ \text{on fluid}\end{matrix}\right\} = \mathbb{F} = \frac{\delta m\ \delta v_x}{\delta t}$$

$$\left\{\begin{matrix}\text{shear}\\ \text{stress}\end{matrix}\right\} = \tau = \frac{\mathbb{F}}{A} = \frac{1}{A}\frac{\delta m}{\delta t}\delta v_x$$

$$\delta v_x = \sim \frac{d\bar{v}_x}{dy}\, l \qquad \Rightarrow \tau^t = -\rho\, |v_y'|\, \frac{d\bar{v}_x}{dy}$$

$$\frac{1}{A}\frac{\delta m}{\delta t} = \rho |v_y'|$$

(3) $v_y \propto v_x$

$$|v_y'| = k_1 v_x'$$

$$v_x' = k_2\, \delta v_y = k_2\, l\, \frac{d\bar{v}_x}{dy} \Rightarrow \frac{\tau^t}{\rho} = -l^2 \left|\frac{d\bar{v}_x}{dy}\right| \frac{d\bar{v}_x}{dy}$$

$l = ky$ → distance from wall,

$$\frac{\tau^t}{\rho} = -k^2 y^2 \left|\frac{d\bar{v}_x}{dy}\right| \frac{d\bar{v}_x}{dy}$$

k → experimentally determined

NEAR wall $\quad \tau^t_{yx} = -\tau_w \Rightarrow \dfrac{d\bar{v}_x}{dy} = \dfrac{1}{k}\sqrt{\dfrac{\tau_w}{\rho}}\,\dfrac{1}{y} \Rightarrow v_x = \log(y)$

Introduce non dimensionalized parameters

$$v^* = \frac{\bar{v}_x}{\sqrt{\tau_w/\rho}}, \quad y^* = \frac{y\sqrt{\tau_w/\rho}}{\nu}$$

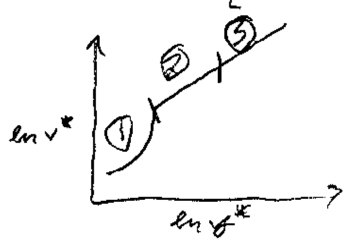
$$\frac{dv^*}{dy^*} = \frac{1}{ky^*} \to v^* = -\frac{1}{k}\ln y^* + C_1$$

so 3 regions

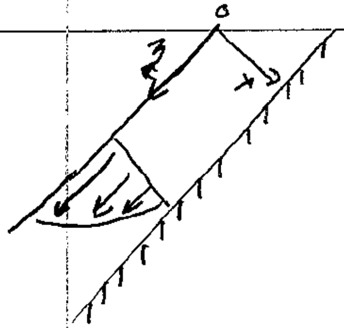
1) Laminar sublayer $\left[\tau^t = 0 \Rightarrow v^* = y^* \quad \text{for } y^* < 5\right]$

2) Buffer layer $\left[v^* = -3.05 + 5\ln y^* \quad \text{for } 5 \le y^* \le 30\right]$

3) Turbulent core $\left[v^* = 5.5 + 2.5\ln y^* \quad \text{for } y^* > 30\right]$



Falling Laminar Film



Assumptions

- $\rightarrow$ steady state $\left(\frac{\partial}{\partial t} \to 0\right)$
- $\rightarrow$ fully developed $\left(\frac{\partial}{\partial z} \to 0\right)$ $\left(\frac{\partial v_z(y)}{\partial y} \to 0\right)$ $\left(v_z = v_z(x),\ v_y = 0\right)$
- $\rightarrow$ Laminar Flow (use navier-stokes)
- $\rightarrow$ Adiabatic, Isothermal (no need for E Equation)
- $\rightarrow$ No y-dependence $\left(2D,\ \frac{\partial}{\partial y} \to 0\right)$
- $\rightarrow$ Incompressible $\left(\rho = \text{const},\ \nabla \cdot \vec{v} = 0\right)$

C.E.

$$\frac{\partial v_x}{\partial x} + \cancel{\frac{\partial v_y}{\partial y}}^{\to 0} + \cancel{\frac{\partial v_z}{\partial z}}^{\to 0} = 0 \implies \frac{\partial v_x}{\partial x} = 0$$

B.C.
$v_x = 0$ at $x = t$ (velocity of wall cannot be finite) $\Rightarrow v_x = 0$ everywhere

M.E.

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \cdot \tau + \rho g$$

$$z \rightarrow \rho\left\{\cancel{\frac{\partial v_z}{\partial t}}^{\to 0} + \cancel{v_x \frac{\partial v_z}{\partial x}}^{\to 0} + \cancel{v_y \frac{\partial v_z}{\partial y}}^{\to 0} + \cancel{v_z \frac{\partial v_z}{\partial z}}^{\to 0}\right\} = -\frac{\partial p}{\partial z} + \mu\left\{\frac{\partial^2 v_z}{\partial x^2} + \cancel{\frac{\partial^2 v_z}{\partial y^2}}^{\to 0} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}^{\to 0}\right\} + \rho g_z$$

$$-\frac{\partial p}{\partial z} + \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g_z = 0$$

$$\rho\left\{\cancel{\frac{D v_x}{Dt}}^{\to 0} + \cancel{v_x \frac{\partial v_x}{\partial x}}^{\to 0} + \cancel{v_y \frac{\partial v_x}{\partial y}}^{\to 0} + \cancel{v_z \frac{\partial v_x}{\partial z}}^{\to 0}\right\} = -\frac{\partial p}{\partial x} + \mu\left\{\cancel{\frac{\partial^2 v_x}{\partial x^2}}^{\to 0} + \cancel{\frac{\partial^2 v_x}{\partial y^2}}^{\to 0} + \cancel{\frac{\partial^2 v_x}{\partial z^2}}^{\to 0}\right\} + \rho \vec{g}_x$$

$$-\frac{\partial p}{\partial x} + \rho \vec{g}_x = 0$$

$$\rho g_x = \rho \cos\theta\, g$$

B.C. at $x = 0$, no shear $\Rightarrow \frac{\partial v_z}{\partial x} = 0$

B.C. at $x = \delta$, $v_z = 0$

$p = p_\infty$

$$p = \rho g_x x + c_1(z)$$
$$p = p_\infty \text{ at } x_0 \implies c_1(z) = p_\infty \implies p = \rho g_x + p_\infty$$

$$-\cancel{\frac{\partial p}{\partial z}}^{\to 0} + \mu \frac{\partial^2 v_z}{\partial x^2} + \rho g_z = 0 \implies \frac{\partial v_z}{\partial x} = \frac{\rho}{\mu} g_z x + C_1 \implies v_z = \frac{\rho g_z}{2\mu} x^2 + C_1 x + C_2$$

From B.C. 2

$$0 = \frac{\partial v_z(0)}{\partial x} = \frac{\rho}{\mu} g_z(0) + C_1 \Rightarrow C_1 = 0$$

From B.C. 1

$$0 = v_z(\delta) = \frac{\rho g_z}{2\mu}\delta^2 + C_2 \Rightarrow C_2 = -\frac{\rho g_z}{2\mu}\delta^2$$

$$v_z = \frac{\rho g \delta^2 \cos\theta}{2\mu}\left[1 - \left(\frac{x}{\delta}\right)^2\right] \Rightarrow$$

NUCL

551

Exam II

Review

Review

CV Analysis

Mass B.
Energy B.

LOCA

Mass B.

1D Formulation → Most Important

Stdy State

Int Momentum Eqn

Pump Coast Down Accident

Energy Eqn. Coast Down

Natl Circ. Tail end of Transient (Balanced by Fric)

$$v = (q'')^{1/3}$$

2 Phase Flow Regimes

Drag Force on Particle (Interfacial Forces)

Drift Flux Models
2 Fluid

Control Volume Analysis
Mass Balance

Average the 3D equation over area

$$\frac{1}{A}\int_A \psi \, dA = \langle \psi \rangle$$

and $\frac{\partial}{\partial \theta} = 0$ for axisymmetric flow. $v_\theta = 0$
using constant area

$$\frac{1}{\pi r_0^2}\int \psi (2\pi r)\, dr = \langle \psi \rangle$$

Time Derivative

$$\frac{1}{A}\int \frac{\partial \rho \psi}{\partial t} dA = \frac{\partial}{\partial t}\frac{1}{A}\int \rho \psi \, dA = \frac{\partial}{\partial t}\langle \rho \psi \rangle$$

Gradient operator in z-dir

$$\frac{1}{A}\int_A (\nabla s)_z \, dA = \frac{1}{A}\int \frac{\partial s}{\partial z} dA = \frac{\partial}{\partial z}\langle s \rangle$$

Average of vector operator (divergence)

$$\frac{1}{A}\int_A (\nabla \cdot \tau)_z \, dA = \frac{\partial}{\partial z}\langle \tau_{zz} \rangle + \frac{1}{A} 2\pi r_0 \, \tau_{rz}\big|_{r_0}$$

Continuity Equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$

$$\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z}\langle \rho v_z \rangle = 0$$

Control Volume Analysis

Energy Balance

start with enthalpy equation

$$\frac{\partial}{\partial t}\rho i + \nabla\cdot(\rho i \vec{V}) = -\nabla\cdot q + \frac{Dp}{Dt} - \tau : \nabla\vec{V} + \dot{q}$$

$$\frac{\partial}{\partial t}\rho\langle i\rangle + \frac{\partial}{\partial z}\rho\langle i V_z\rangle = -\frac{\partial\langle q_z\rangle}{\partial z} + \frac{\xi_h}{A}q'' + \frac{D\langle t\rangle}{Dt} + \langle\dot{q}\rangle$$

what to do with $\langle V_z V_z\rangle$ and $\langle i\, V_z\rangle$?

Covariant : note $\langle AB\rangle \neq \langle A\rangle\langle B\rangle$

difference is covariant

$$\text{cov}\langle AB\rangle = \langle AB\rangle - \langle A\rangle\langle B\rangle$$

i) Laminar Fully developed

$$V_z = 2\langle V_z\rangle\left(1-\frac{r^2}{r_0^2}\right)$$

$$\text{cov}(V_z V_z) = \frac{1}{A}\int V_z^2\, dA - \langle V_0\rangle^2 = \frac{\langle V_z\rangle^2}{3}$$

$$\text{cov}(i\, V_z) \approx 0.375$$

ii) turbulent flow

$$V_z = V_t\left(1-\frac{r}{r_0}\right)^{1/7} \qquad V_t = \frac{\langle V_z\rangle}{0.817}$$

$$= 1.22\langle V_z\rangle\left(1-\frac{r}{r_0}\right)^{1/7}$$

$$\left.\begin{aligned}\text{cov}(V_z V_z) &= 0.02\langle V_z\rangle^2\\ \text{cov}(i\, V_z) &= 0.02\langle i\rangle\langle V_z\rangle\end{aligned}\right\}$$ for turbulent flow, covariant can be ignored

so for turbulent flow

$$\langle V_z V_z\rangle = C_m\langle V_z\rangle\langle V_z\rangle \qquad C_m \approx 1$$

$$\langle i\, V_z\rangle = C_E\langle i\rangle\langle V_z\rangle \qquad C_E \approx 1$$

alternate form

$$\rho C_p\left[\frac{\partial\langle T\rangle}{\partial t} + \langle V\rangle\frac{\partial\langle T\rangle}{\partial z}\right] = \frac{\xi_H q_o'}{A} - \frac{\langle T\rangle}{\rho}\left.\frac{\partial\rho}{\partial T}\right)_p \frac{D\langle p\rangle}{Dt} + \dot{q} - \frac{\partial q_z}{\partial z}$$

wall heat flux

Control Volume Analysis → Mass energy balance
LOCA Mass Balance

Mass B.

$$\dot{m} = 0$$

Energy B.

$$\dot{Q}_{core} - \dot{Q}_{SG} - \dot{Q}_{Loss} + \dot{w}_{SP_1} = 0$$

LOCA

$$\dot{Q}_{core} - \dot{Q}_{SG} - \dot{Q}_{Loss} + w_{SP_1} - \dot{m}_{break}\left(e + \frac{P}{\rho}\right)_{break}$$
$$-(\rho VA)_{break} + (\rho VA)_{ECCS} \gtrless 0$$

must be!

$$\dot{Q}_{decay} + w_{SP_1} - \dot{Q}_{SG} + \dot{Q}_{Loss} - \dot{m}_{break}\left(e + \frac{P}{\rho}\right)_{break} + \dot{m}\left(e + \frac{P}{\rho}\right)_{ECCS}$$

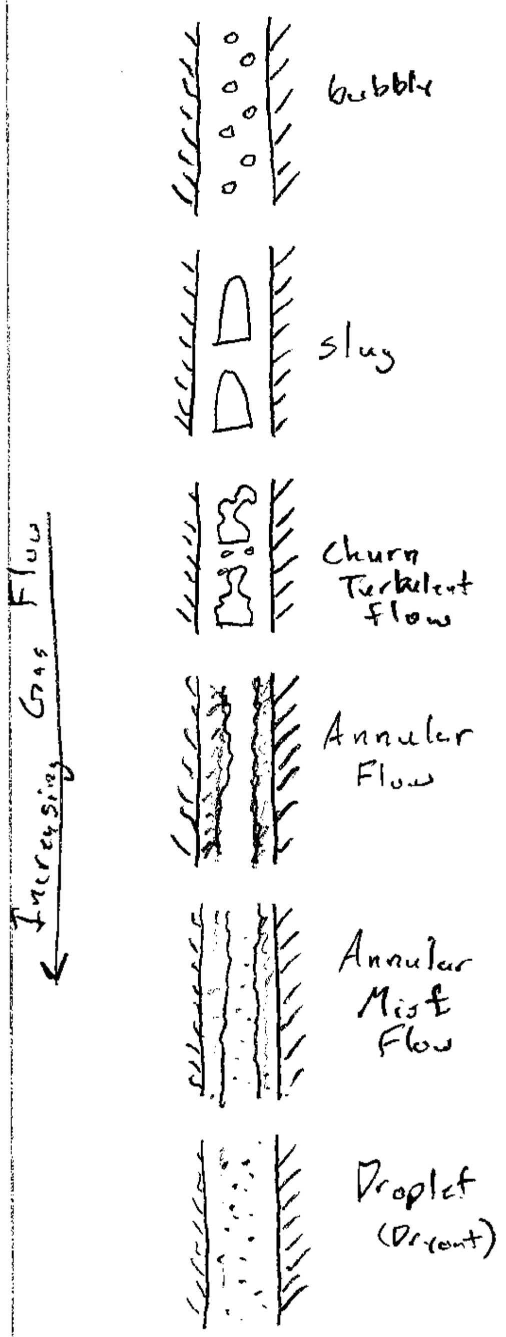
Integral Momentum Eq over Loop

$\oint \frac{\partial \rho \langle v_z \rangle}{\partial t} dz$	$\sum \rho_i \frac{dv_z}{dt} l_i$	time rate of momentum change
$\oint \partial \rho \frac{\langle v_z v_z \rangle}{\partial z} dz$	0	convective acceleration
$\oint - \frac{\partial \langle p \rangle}{\partial z} dz$	Δp_{pump}	pressure change
$\oint \frac{f \rho v \lvert v \rvert}{2D} dz$	$\sum \left(\frac{fl}{D} + k \right)_i \left(\frac{\rho v_i \lvert v_i \rvert}{2} \right)$	Major and minor losses
$\oint \frac{\partial}{\partial z} \langle \tau_{zz} \rangle dz$	0	shear
$\oint \rho g_z \, dz$	$\sum (\overset{\nearrow 0}{\rho g l} - \rho g \beta \Delta T l_n)$	gravity

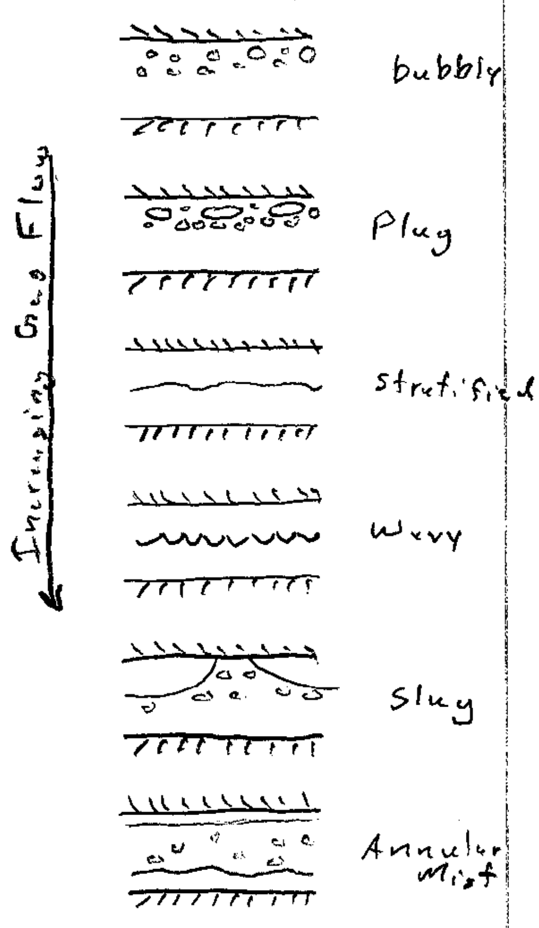
$$\sum \rho_i \frac{dv_i}{dt} l_i = \Delta p_{pump} + \sum_i (\rho g l - \rho g \beta \, \Delta T l_n) - \sum_i \left(\frac{fl}{D} + k \right)_i \left(\frac{\rho v_i \lvert v_i \rvert}{2} \right)$$

2 Phase Flow Regimes

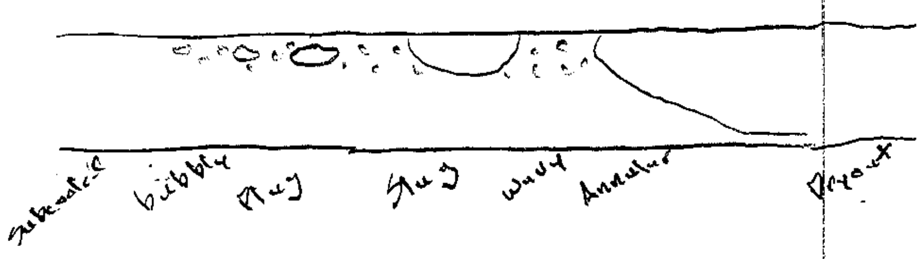
Adiabatic Vertical Flow



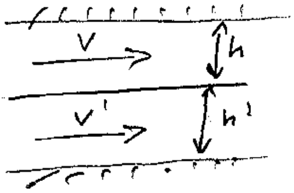
Horizontal Flow



Candu Reactor



K-H



1) assume
   incompressible
   inviscid
   irrotational

2) use local Instant formulation

$$C_g \sim \sqrt{\frac{g\lambda}{2\pi}}$$

3) make a velocity potential field

$$C_c \sim \sqrt{\frac{2\pi\sigma}{\rho_f \lambda}}$$

$$\vec{V} = -\nabla \phi$$

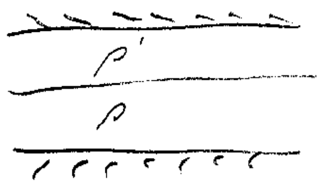
4) apply ~~jump conditions at~~ boundary conditions

5) assume η, interface shape

6) assume pressure jump cond.

7) use field eqs to solve

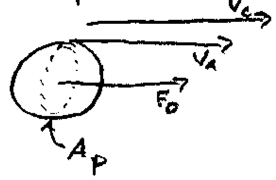
T



$$\lambda_{max} = 2\pi\sqrt{\frac{3\sigma}{g\Delta\rho}} = \sqrt{3}\,\lambda_c$$

Drag Force on Particle (Interfacial Forces)

Solid particle



$$F_D = -\frac{c_D}{2}\rho_c v_r |v_r| A_p$$

$$c_D = f(Re)$$

$$Re \equiv \frac{\rho_c v_r D_p}{\mu} = \frac{\rho_c v_r 2r}{\mu}$$

$$c_D = f(Re, \text{shape})$$

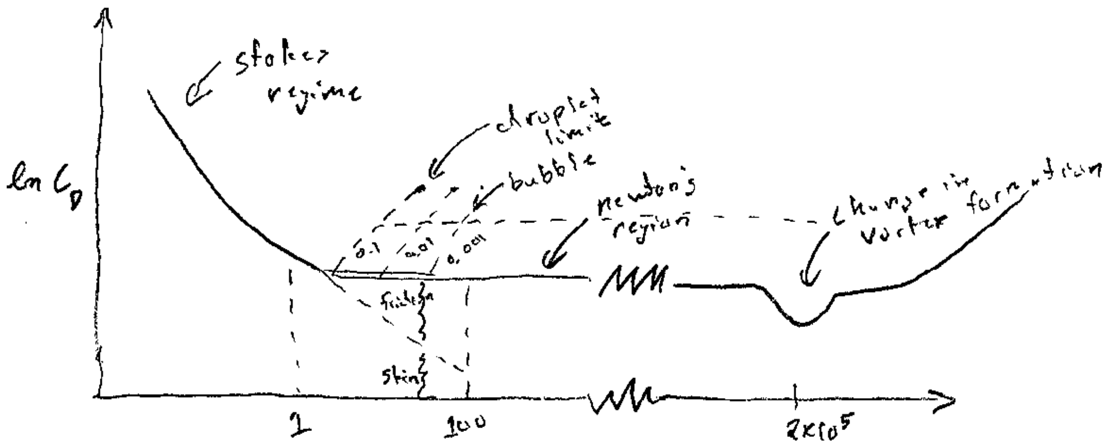
F_D = form drag + skin drag

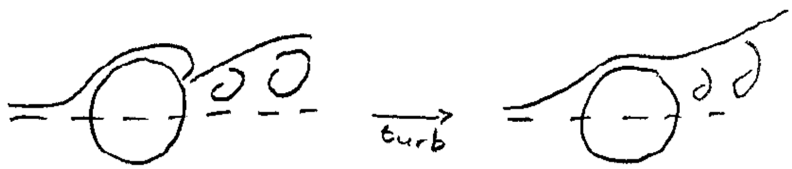
A_p high Re, shear stress

$A_p \rightarrow$ projected area
$v_r \rightarrow$ relative velocity
$c_D \rightarrow$ non-dim drag coefficient

Scaling parameter

$$Re \equiv \frac{\rho_c v_r (r_d * 2)}{\mu}$$





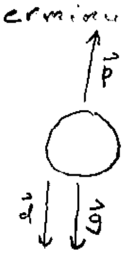
Single Fluid particle

$$N_{Re\infty} = \frac{(2r_p)\rho_c v_c}{\mu_c} \qquad N_\mu \equiv \frac{\mu_c}{(\rho_c \sigma \sqrt{\sigma/g\Delta\rho})^{0.5}}$$

Stokes	Viscous Wake	Distorted Particle
$C_D = \frac{24}{N_{Re\infty}}$	$C_D = \frac{24}{N_{Re\infty}}\left[1 + 0.1 N_{Re\infty}^{0.75}\right]$	$C_D = \frac{\sqrt{2}}{3} N_\mu N_{Re\infty} \cong \left(\frac{4}{3} r_d \sqrt{g\Delta\rho/\sigma}\right)$

Cap Bubble Region	Large Deforming Droplet	Solid Particle
$r_D \geq 2\sqrt{\sigma/g\Delta\rho}$ $C_{D\infty} = \frac{8}{3}$	$C_{D\infty} = 4$ $\quad r_{max} = 3\sqrt{\frac{\sigma}{g\Delta\rho}}$ $\text{taylor} \equiv \sqrt{\frac{\sigma}{g\Delta\rho}}$	$C_{D\infty} = 0.44 \quad N_{Re\infty} \geq 1000$

Terminal Velocity



$$F_b = F_p + F_g = V(\rho_c - \rho_d)g$$

$$F_D = -\frac{1}{2} C_{D\infty} \rho_c v_{r\infty} |v_{r\infty}| A_d$$

Steady-state → terminal velocity

$$v_{r\infty}|v_{r\infty}| = 2\frac{V}{A_d}\left(\frac{\rho_c - \rho_d}{\rho_c}\right)\frac{g}{C_{D\infty}}$$

$C_{D\infty}$ ← $f(Re)$

Stokes Regime

$$\frac{[m^2]}{[s^2]} = \frac{[m^3]}{[m^2]}\frac{[kg]}{[kg]}\frac{[m]}{[s^2]} = \boxed{\frac{m^2}{s^2}}$$

$$C_D = 24\,Re$$

$$\frac{V}{A_d}\left(\frac{\rho_c - \rho_d}{\rho_c}\right)g = \frac{1}{2}C_{D\infty}(Re)$$

$$4/3\,\pi r_d^3(\rho_c - \rho_d)g = \frac{1}{2}\left[\frac{24\mu_c}{2r_d\rho_g|v_{r\infty}|}\right]$$

$$\frac{8}{3*24}\pi r_d^2(\rho_c - \rho_d)g = \frac{\mu_c}{\rho_g v_{r\infty}}$$

$$v_{r\infty} = \frac{2}{9}\frac{(\rho_c - \rho_d)g r_\infty^2}{\mu_c}$$

Distorted Fluid Particle Regime (Typical for 2 phase flow)

$$C_{D\infty} = \frac{\sqrt{2}}{3}N_\mu N_{Re} \Rightarrow v_{r\infty} = \sqrt{2}\left(\frac{\sigma g \Delta\rho}{\rho_c^2}\right)^{1/4} \quad (+,-)$$

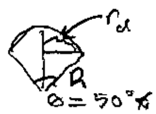
sign convention

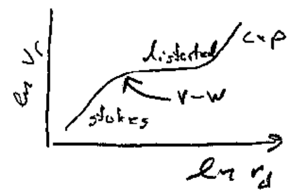
bubble ~ 20 cm/s } important for
droplet ~ 7 cm/s } scrubber

Cap bubble Region

$$v_{r\infty} = \sqrt{\frac{\sigma_d \Delta\rho g}{\rho_c}}$$

$$r_d = \left(\frac{3}{4}\frac{V}{A_d}\right)$$





Two Fluid

$$\overline{\nabla F} = \nabla \overline{F} + \frac{1}{\Delta t}\sum \frac{1}{V_{ni}}\left\{F^{+} n^{+} + F^{-} n^{-}\right\}$$

C.E.

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla\cdot(\alpha_k\rho_k V_k) = \Gamma_k$$

M E

$$\underbrace{\frac{\partial \alpha_k\rho_k V_k}{\partial t}}_{\text{time rate of change of mom.}} + \underbrace{\nabla\cdot(\alpha_k\rho_k V_k\, V_k)}_{\text{acceleration convection}} = \underbrace{-\alpha_k\nabla p_k}_{\text{pressure term}} - \underbrace{\nabla\cdot\alpha_k(\tau_k^{\mu}+\tau_k^{t})}_{\text{wall shear}} + \underbrace{\alpha_k\rho_k\vec{g}}_{\text{gravity}}$$

$$+ \underbrace{\vec{M}_{ik}}_{\text{gen. drag and buoyancy}} + \underbrace{(\nabla\alpha_k)\cdot\tau_i}_{\text{interface shear}} + \underbrace{(p_{ki}-p_k)\nabla\alpha_k}_{\text{from pressure term}} + \underbrace{\vec{V}_k\Gamma_g}_{\text{from mass gen}}$$

EE

$$\underbrace{\frac{\partial \alpha\rho_k i_k}{\partial t}}_{\text{enthalpy change due to time}} + \underbrace{\nabla\cdot(\alpha_k\rho_k i_k\vec{V}_k)}_{\text{enthalpy convection}} = \underbrace{-\nabla\alpha(q_k' + q_k^{t})}_{\text{wall heat transfer}} + \underbrace{\alpha_k\frac{Dp_k}{Dt}}_{\text{resistivity}} + \underbrace{i_k\Gamma}_{\text{phase change}} + \underbrace{\alpha_i q_{ki}''}_{\text{interface heat transfer}} + \underbrace{\phi_k}_{\text{dissip}}$$

Drift Flux

MCE

$$\frac{\partial \alpha \rho_m}{\partial t} + \nabla \cdot \rho_m V_m = 0$$

GCE

use drift flux vel

$$\frac{\partial \alpha \rho_g}{\partial t} + \nabla \cdot \alpha \rho_g V_g = \Gamma_g \implies \frac{\partial \alpha \rho_g}{\partial t} + \nabla \cdot \rho_g \vec{V}_m = \Gamma_g - \nabla \cdot \left\{ \frac{\alpha \rho_g \rho_f}{\rho_m} \vec{V}_{gj} \right\}$$

ME

$$\underbrace{\frac{\partial \rho_m \vec{V}_m}{\partial t}}_{\text{time rate of change}} + \underbrace{\nabla \cdot (\rho_m V_m V_m)}_{\text{acceleration convection}} = \underbrace{-\nabla p_m}_{\text{pressure}} + \underbrace{\rho_m \vec{g}}_{\text{gravity}} - \underbrace{\nabla \cdot \left\{ \tau_m^{\mu} + \tau_m^{t}\right.}_{\text{wall shear}} + \underbrace{\left. \frac{\alpha \rho_g \rho_f}{(1-\alpha)\rho_m} \vec{V}_{gj} \cdot \vec{V}_{gj} \right\}}_{\text{momentum diffusion}}$$

EE

$$\underbrace{\frac{\partial \rho_m i_m}{\partial t}}_{\text{time rate of change}} + \underbrace{\nabla \cdot (\rho_m i_m V_m)}_{\text{acceleration convection}} = \underbrace{-\nabla \cdot \left[q_m^{c} + q_m^{t} \right]} - \underbrace{\nabla \cdot \left\{ \frac{\alpha \rho_g \rho_f}{\rho_m} (i_{gi} - i_{fi}) \vec{V}_g \right\}}_{\text{diffusion}} + \underbrace{\frac{\partial p_m}{\partial t}}_{\text{drift}}$$

$$+ \underbrace{\left[\vec{V}_m + \frac{\alpha(\rho_f - \rho_g)}{\rho_m} \vec{V}_{gj} \right]}_{\text{energy diffusion}} \cdot \nabla p_m + \underbrace{\Phi_m}_{\text{dissipation}}$$

Drift Flux Model

eliminate one momentum equation and replace with mixture velocity equation

Mixture C.E.

$$\sum_{k=1}^{2}\left\{ \frac{\partial \alpha_k \rho_k}{\partial t} + \nabla(\alpha_k \rho_k v_k) = \Gamma_k \right\} \Rightarrow \frac{\partial \rho_m}{\partial t} + \nabla\cdot\rho_m v_m = 0$$

we find mixture parameters by

$$\rho_m = \alpha\rho_g + (1-\alpha)\rho_f$$

$$v_m = \frac{\alpha\rho_g v_g + (1-\alpha)\rho_f v_f}{\rho_m}$$

$$j = \alpha\vec{v}_g + (1-\alpha)v_f = j_g + j_f$$

$$v_{gj} = \vec{v}_g - \vec{j} = (1-\alpha)\vec{v}_j$$

so the vapor C.E. — replace w/ drift velocity

$$\frac{\partial \alpha\rho_g}{\partial t} + \nabla\cdot(\alpha\rho_g\vec{v}_g) = \Gamma_g \qquad v_{gj} \equiv v_g - \underset{v_m}{j}$$

$$\frac{\partial \alpha\rho_g}{\partial t} + \nabla\cdot(\alpha\rho_g\vec{V}_m) = \Gamma_g - \nabla\cdot\left(\frac{\alpha\rho_g\rho_f}{\rho_m}\vec{v}_{gj}\right)$$

Mixture Momentum Equation

$$\frac{\partial\rho_m\vec{v}_m}{\partial t} + \nabla\cdot(\rho_m\vec{v}_m\vec{v}_m) = -\nabla p_m + \rho_m\vec{g} - \nabla\cdot\left\{ \tau_m^{\mu} + \tau_m^{t} + \underbrace{\frac{\alpha\rho_g\rho_f}{(1-\alpha)\rho_m}\vec{v}_{gj}\vec{v}_{gj}}_{\text{momentum diffusion}} \right\}$$

Mixture Energy Equation

$$\frac{\partial\rho_m i_m}{\partial t} + \nabla\cdot(\rho_m i_m\vec{v}_m) = -\nabla\cdot\left[q_m^{c} + q_m^{t}\right] - \nabla\cdot\left\{\frac{\alpha\rho_g\rho_f}{\rho_m}(i_{gi} - i_{fi})\vec{v}_g\right\} + \overbrace{\frac{\partial p_m}{\partial t}}^{\text{pressure drift}}$$

$$+\left[\vec{v}_m + \underbrace{\frac{\alpha(\rho_f-\rho_g)}{\rho_m}\vec{v}_{gj}}_{\text{energy diffusion}}\right]\cdot\nabla p_m + \Phi_m$$

← 4 eq form →

← 5 eq form →

2 C.E.

1 M.E.

2 E.E.

2 Fluid Model

gas field → interfacial transfer ← liquid field

3-D Two Fluid Model

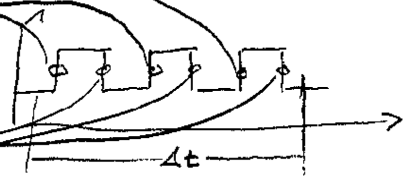
Local time averaging

$\frac{1}{\Delta t}\{\text{B.E. (Single Phase)}\}dt$

local instant

$\frac{\partial F}{\partial t}$ $\quad \nabla\cdot F\vec{v}$ $\quad \nabla\cdot j$ $\quad \phi$

$$\overline{\frac{\partial F}{\partial t}} = \frac{\partial \overline{F}}{\partial t} - \frac{1}{\Delta t}\sum_j \frac{1}{v_{ni}}\{F^+ n^+\cdot v_i + F^- n^-\cdot v_i\}_j$$



j is the jth interface

$-,+$ before and after constitutive

n_j normal vector

v_{ij} interfacial vel.

$$\overline{\nabla F} = \nabla\overline{F} + \frac{1}{\Delta t}\sum\frac{1}{v_{ni}}\{n^+F^+ + n^-F^-\}_j$$

then time balance average equation

C.E.

$$\frac{\partial \alpha_k\rho_k}{\partial t} + \nabla\cdot(\alpha_k\rho_k v_k) = \Gamma_k$$

time rate of change of k phase mass/vol

convection of k phase mass

source of k-values due to phase change (+, -)

M.E.

$-\nabla(\alpha_k p_k)$

$$\frac{\partial \alpha_k\rho_k v_k}{\partial t} + \nabla\cdot(\alpha_k\rho_k\vec{v}_k\vec{v}_k) = -\alpha_k\nabla p_k - \nabla\cdot\alpha_k(\tau_k^{\mu} + \tau_k^{t}) + \alpha_k\rho_k\vec{g} + \vec{M}_{ik} + (\nabla\alpha_k)\cdot\tau_i + (p_{ki} - p_k)\nabla\alpha_k + \vec{v}_{ki}\Gamma_k$$

τ_k^{μ} → average viscous stress

τ_k^{t} → average turbulent stress

v_{ki} → k phase interfacial velocity

M_{ik} → generalized drag and lift force

τ_i : interface shear

Enthalpy energy equation

$$\frac{\partial \alpha\rho_k i_k}{\partial t} + \nabla\cdot(\alpha_k\rho_k i_k\vec{v}_k) = -\nabla\cdot\alpha_k(q_k^c + q_k^t) + \alpha_k\frac{Dp_k}{Dt} + i_k\Gamma_k + \alpha_i q_{ki}'' + \phi_k$$

enthalpy transfer due to phase change

interface heat transfer

$$\frac{D}{Dt}p_k = \frac{\partial p_k}{\partial t} + v_k\cdot\nabla p_k$$

↓ interfacial balance averaging

continuity $\sum_{k=1}^{2}\Gamma_k = 0$ | force balance $\sum_{k=1}^{2}M_{ik} = 0$ | energy change of interface $\sum_{k=1}^{2}(\Gamma_k i_{ki} + \alpha_i q_{ki}) \approx 0$

and

$$\Gamma_g(i_{gi} - i_{fi}) = \Gamma_g(i_{gsat} - i_{fsat}) = \Gamma_g\underbrace{\Delta i_{fg}}_{\text{latent heat}}$$

Constitutive Relation

① interface transfer

$\Gamma_k;\ M_{ki},\ \vec{v}_{ki},\ \tau_i;\ i_{ki}, \alpha_i, q_{ki}$

② Bulk Fluid

$\tau_k^{\mu},\ \tau_k^{t};\ q_k^c,\ q_k^t$

③ Eq. of state for Each Phase

$u_k = u_k(\rho_k, T_k)$ caloric

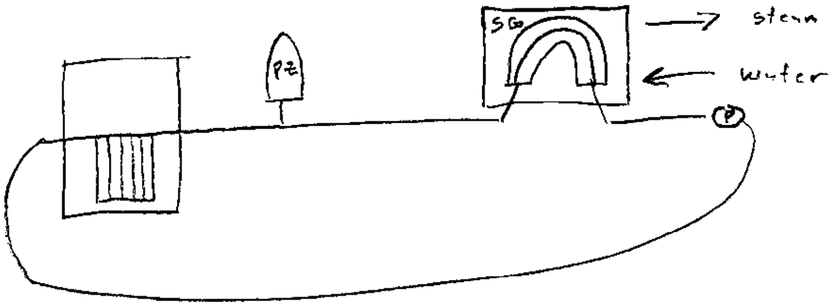
$p_k = p_k(\rho_k, T_k)$ thermal

1-D Formulation

steady state

Int Momentum Equation

Pump Lost Down Accident → Energy Eqn



components

1 core
2 u. plenum
3 hot leg
4 S.G.
5 Cold leg
6 pump
7 cold leg
8 Downcomer
9 lower plenum

Integral Momentum Eq

$$\oint\left\{\frac{\partial \rho v}{\partial t}+\frac{\partial \rho v v}{\partial z}=-\frac{\partial p}{\partial z}-\frac{f}{2D}\rho v|v|+\rho g_z\right\} dl \text{ loop}$$

where $\oint \frac{\partial \rho_i v_i}{\partial z}\,dz=0$, $\oint -\frac{\partial p}{\partial z}\,dz=0+\Delta p_{pump}$

$$\oint \frac{f_i \rho_i v_i |v_i|}{2D_i}\,dz=\sum_i\left(\frac{fl}{D}+k\right)\frac{\rho_i v_i|v_i|}{2}, \quad \oint \rho_i g_{zi}=\cancel{\oint \bar{\rho}\, g_{zi}\,dz}^{\,0}-\sum \rho g_i \beta \Delta T_i l_i$$

$$\oint \frac{\partial \rho_i v_i}{\partial t}\,dz=\rho_i\frac{\partial v_i}{\partial t} l_i$$

Natural Circulation at tail end of transient

(4) General Drag and Lift Forces

The two-fluid model momentum equation

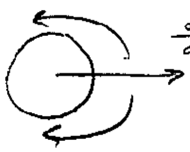
$$\frac{\partial \alpha_k \rho_k \vec{V}_k}{\partial t} + \nabla\cdot(\alpha_k \rho_k \vec{V}_k \vec{V}_k) = -\nabla \alpha_k p_k - \nabla\cdot\alpha_k(\tau_k^{\mu} + \tau_k^{t}) + \alpha_k \rho_k \vec{g}_k$$
$$+ \vec{V}_k \Gamma_k + \vec{M}_{ik} + \nabla\alpha_k \cdot \tau_i + p_{ki} \nabla \alpha_k$$

$$\vec{M}_{ik} = \underbrace{\vec{M}_{ik}^{D}}_{\text{steady state drag force}} + \underbrace{\vec{M}_{ik}^{V}}_{\text{virtual mass force}} + \underbrace{\vec{M}_{ik}^{B}}_{\text{Basset (History) force}} + \underbrace{\vec{M}_{ik}^{L}}_{\text{Lift Force}} + \underbrace{M_{ik}^{C}}_{\text{Collision force}}$$

For most cases, M_{ik}^{D} is sufficient

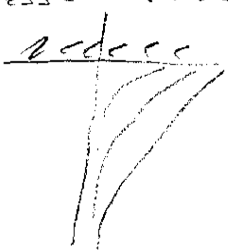
Virtual Mass force

Inviscid Transient



replace mass from front to back

Basset Force



Transient Boundary Layer Drag

For most applications in LWR,
Magnitude is very small b.c. $\frac{dv_r}{dt}$ is small

important for stability of solution method

2 fluid model is only conditionally stable

small 2 → unstable

↳ { cell volume: L is large
numerical viscosity: Donner Cell upwind

Drag Force

$$F_D = -\frac{1}{2} C_D \rho_c V_r |V_r| A_d$$

$$M_{id} = \alpha_r \frac{F_D}{B_d}$$

B_d ; volume of typical particle

Some important length scale

Drag Radius: $r_d \equiv \frac{3B_d}{4A_d}$ A_d; projected area

Volume Eq $r_v = \left(\frac{3}{4\pi} B_d\right)^{1/3}$

Surface Eq $r_s = \left(\frac{A_i}{4\pi}\right)^{1/3}$

Sauter Mean $r_{sm} = \frac{3B_d}{A_i}$

$\frac{r_d}{r_s}$ ----> Shape Factor

$$M_{id}^D = \frac{\alpha_d \vec{u}_D}{B_d} = -\alpha_d \left(\frac{A_d}{B_d}\right) \frac{C_D}{2} \rho_c v_r |v_r|$$

Introduce N_D

$$a_i = N_d A_i$$

$$M_{id}^D = \frac{\alpha_d F_D}{B_d} = -a_i \left[\frac{C_D}{4} \left(\frac{r_{sm}}{r_d}\right) \frac{\rho_c v_r |v_r|}{2} \right]$$

$\left(\frac{r_{sm}}{r_d}\right) \approx 1$

most important parameter
interfacial area concentration $a_i = \frac{A_i}{V}$

ii) Transient Force II: Virtual Mass Force
single Particle

$$F_V = -\frac{B_d}{2} \rho_c \frac{d\vec{v}_r}{dt} \quad \text{virtual mass force}$$

$$F_I = -B_d \rho_d \frac{d\vec{v}_r}{dt} \quad \text{Inertia}$$

$$\underset{\text{Multiparticle}}{\text{Total}} = -B_d \left(\rho_d + \frac{\rho}{2}\right) \frac{d\vec{v}_r}{dt}$$

$$M_{ik}^V = -\frac{\alpha F_V}{B_d} = -\frac{1}{2} \alpha \left(\frac{1+2\alpha_d}{1-\alpha_d}\right) \rho_c \left[\frac{D_d v_r}{Dt} - v_r \cdot \nabla \vec{v}_c\right]$$

$$\frac{D_d}{Dt} = \frac{\partial}{\partial t} + v_\alpha \cdot \nabla$$

$$C_M = \frac{1}{2} \alpha_d \left(\frac{1+2\alpha_d}{1-\alpha_d}\right) \quad \text{induced mass coefficient}$$

Basset Force

$$\vec{M}_{ik}^{B} = \frac{\alpha_d F_B}{B_d}$$

$$= \frac{9}{2}\frac{\alpha_d}{r_d}\sqrt{\frac{\rho_c \mu_c}{\pi}}\underbrace{\int_{-\infty}^{t}\frac{Dl}{D\xi}\frac{d\xi}{\sqrt{t-\xi}}}_{\text{history force}}$$

memory is very short

under development

(5) Relative Velocity between Phases

(1) Drift Flux Model ← V_{gj}

(2) 1-D 2 Fluid Model

i) Local Relative velocity

$$v_r = v_d - v_c$$

$$v_{dj} = v_d - j = (1-\alpha_d)v_r$$

$$v_{gj} = v_g - j = (1-\alpha)v_r$$

$C_D \Rightarrow$ all ~~regions~~ regimes

- Stokes

$$v_{gj} = \frac{2}{9}\frac{g\Delta\rho r_d^2}{\mu_g}(1-\alpha_d)^3$$

- Viscous Wake Regime

$$v_{dj} = \frac{10.8\mu_c}{\rho_c r_d}\left(\frac{\mu_c}{\mu_m}\right)(1-\alpha_d)^2\frac{\psi^{4/3}(1+\psi)}{\left[1+\psi\left\{\frac{\mu_c}{\mu_m}(1-\alpha_d)^{0.5}\right\}^{6/7}\right]}\left(\frac{\rho_c-\rho_d}{\Delta\rho}\right)$$

$$\psi = 0.55\left[(1+0.08r_d^{*3})^{4/7}-1\right]^{0.75}$$

- Disturbed Particle Regime

$$v_{gj} = \sqrt{2}\left(\frac{\sigma g\Delta P}{\rho_f^2}\right)^{1/4}(1-\alpha_d)^{1.75}$$

Chapter 12 → One-D Formulation

$$\langle F \rangle = \frac{1}{A}\int F\,dA$$

$$\langle\langle F_k \rangle\rangle = \frac{\langle \alpha_k F_k \rangle}{\langle \alpha_k \rangle} \quad \text{Void averaging}$$

$$\langle\langle V_k \rangle\rangle = \frac{\langle \alpha_k V_k \rangle}{\langle \alpha_k \rangle} = \frac{\langle j_k \rangle}{\langle \alpha_k \rangle}$$

Continuity

$$\frac{\partial \langle \alpha_k \rangle \rho_k}{\partial t} + \frac{\partial}{\partial z} \langle \alpha_k \rangle \rho_k \langle\langle V_k \rangle\rangle = \langle \Gamma_k \rangle$$

Relative Velocity

$\left\{ \text{Local Relative velocity} \right.$

$$\bar{v}_{gj} = \langle\langle v_g \rangle\rangle - \langle j \rangle \qquad j_k = \frac{Q_k}{A}$$

$$\langle j \rangle = \langle j_g \rangle + \langle j_f \rangle$$

$$v_{gj} = (1 - \langle a_k \rangle)\left(\langle\langle v_g \rangle\rangle - \langle\langle v_f \rangle\rangle\right)$$

$$\langle\langle v_g \rangle\rangle = \frac{\langle \alpha v_g \rangle}{\langle \alpha \rangle}$$

$$\bar{v}_{gj} = \left\langle \frac{\alpha (j + v_{gj})}{\langle \alpha \rangle} - j \right\rangle$$

$$v_{gj} = v_g - j$$

$$\bar{v}_{gj} = \underbrace{\left\{ \frac{\langle \alpha j \rangle}{\langle \alpha \rangle} - \langle j \rangle \right\}}_{\substack{\text{relative} \\ \text{velocity due} \\ \text{profile of} \\ \xi^j_\alpha}} + \underbrace{\frac{\langle \alpha v_{gj} \rangle}{\langle \alpha \rangle}}_{\substack{\text{average} \\ \text{of local} \\ \text{relative} \\ \text{velocity}}}$$

$$C_0 = \frac{\langle \alpha j \rangle}{\langle \alpha \rangle \langle j \rangle}$$

$$v_{gj} = (C_0 - 1)\langle j \rangle + \underbrace{\frac{\langle \alpha v_{gj} \rangle}{\langle \alpha \rangle}}_{\langle\langle v_{gj} \rangle\rangle}$$

$$C_0 = 1.2 - 0.2\sqrt{\rho_g/\rho_f} \qquad \text{No Boiling}$$

$$C_0 = \left(1.2 - 0.2\sqrt{\rho_g/\rho_f}\right)\left(1 - e^{-18\langle \alpha \rangle}\right)$$

(1) C.E. M.E. E.E.

(2) C.E. M.E. E.E.

$\underset{=}{\tau}$
$\vec{q}$
t_g of
Stfc.

η

Jump condition

- Mass $\rightarrow \sum_{k=1}^{2} \dot{m}_k = 0$
- Momentum $\rightarrow \sum_{k=1}^{2} \vec{\tau}_k = 0$; $p_1 - p_2 = -2H_{21}\sigma$
- Energy $\rightarrow \sum_{k=1}^{2} \{ \dot{m}_k i_k + \vec{q}_k \cdot \vec{n} \} \doteq 0$

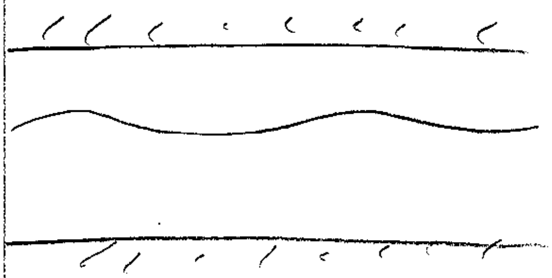
B.C.

$\rightarrow T_{1i} = T_{2i}$

$\rightarrow V_{t1i} = V_{t2i}$

$\rightarrow g_1 \doteq g_2$

$\rightarrow$ Vapor phase is almost saturated at the interface



incompressible, irrotational $\rangle$ —— potential flow

M.E. simplified

$$\boxed{\eta = a e^{i(\omega t - kz)}}$$ a: amplitude

$\phi_1 = c\, f(y)\, \eta$

$c \equiv \frac{\omega}{k} \rightarrow$ wave celerity, phase propagation

solution

$\phi_1' = \frac{-i(u' - c)}{\sinh(kh')} \cosh[k(y - h')]\eta$ $\qquad$ $\phi_1 = \frac{i(u - c)}{\sinh(kh)} \cosh[k(y + h)]\eta$

B.C. pressure boundary condition

$y = \eta$: $p - p' = -\sigma \frac{\partial^2 \eta}{\partial x^2}$ (Momentum Jump)

$p \leftarrow \frac{p'}{\rho'} = \frac{\partial \phi}{\partial t} + u' \frac{\partial \phi_1'}{\partial z} - g\eta$

$\frac{p}{\rho} = \frac{\partial \phi_1}{\partial t} + u \frac{\partial \phi_1}{\partial z} - g\eta$

c is soln

$$\boxed{\rho \coth(kh)(u - c)^2 + \rho' \coth(kh')(u' - c) = \frac{\sigma k^2 + g(\rho - \rho')}{k}}$$

$\rho^{*\prime} \equiv \rho' \coth(kh')$, $\rho^* \equiv \rho \coth(kh)$

$$c^2 - 2c\left\{\frac{\rho^* U + \rho^{*\prime} U'}{\rho^* + \rho^{*\prime}}\right\} + \left\{\frac{\rho^* U^2 + \rho^{*\prime} U'^2}{\rho^* + \rho^{*\prime}}\right\} = \frac{g(\rho - \rho') + \sigma k^2}{k(\rho^* + \rho^{*\prime})}$$

i) Capillary and Gravity wave

- $U = U' = 0$ no flow

$$c^2 = \frac{g(\rho - \rho') + \sigma k^2}{k(\rho^* + \rho^{*\prime})}$$

- $kh \gg 1$ deep water
  $kh' \gg 1$

$\rho = \rho_f$
$\rho' = \rho_g$

ρ_g
ρ_f

$$c^2 = \left(\frac{\omega}{k}\right)^2 = \frac{\sigma k^2 + g(\rho_f - \rho_g)}{k(\rho_f + \rho_g)}$$

$$= \frac{1}{\rho_f + \rho_g}\left\{\frac{\sigma 2\pi}{\lambda} + \frac{g(\rho_f - \rho_g)}{2\pi}\lambda\right\}$$

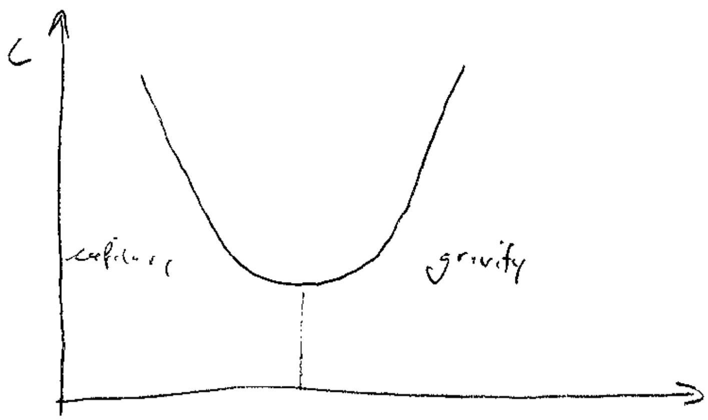
λ very small $\rightarrow$ capillary ~~gravity~~ wave $c^2 = \frac{1}{\rho_f + \rho_g}\frac{\sigma 2\pi}{\lambda}$

$c \approx \sqrt{\frac{2\pi\sigma}{\rho_f \lambda}}$

λ very large $\rightarrow$ gravity wave

$$c^2 = g\frac{\rho_f - \rho_g}{\rho_f + \rho_g}\frac{\lambda}{2\pi}, \quad c \approx \sqrt{\frac{g\lambda}{2\pi}}$$

if water surface just goes up and down, dynamic wave
if water propagates, kinematic wave



$\lambda_{crit} \rightarrow \lambda_c = 2\pi\sqrt{\frac{\sigma}{g\Delta\rho}}$

$\frac{\partial c^2}{\partial \lambda} = 0$ c_{min} at λ_c

$$c_{min} = \sqrt{2}\left(\frac{\sigma g \Delta\rho}{(\rho_f + \rho_g)^2}\right)^{1/4} \approx \sqrt{2}\sqrt[4]{\frac{\sigma g}{\rho_f}}$$

$\lambda_{cap} < \lambda_c = 2\pi\sqrt{\frac{\sigma}{g\Delta\rho}} < \lambda_g$

1.7 cm

Instability

Taylor Instability

Density inversion

ρ_f

ρ_g

$u=0,\ u'=0$ no flow

$kh' \gg 1 \qquad kh \gg 1$

$\rho_f = \rho' \gg \rho_g = \rho$

Q.E. question!

$$c^2 = \left(\frac{\omega}{k}\right)^2 = \underbrace{\frac{\sigma k}{\rho_f+\rho_g}}_{\boxed{\text{stabilizes}}} - \underbrace{\frac{g(\rho_f-\rho_g)}{(\rho_f+\rho_g)k}}_{\boxed{\text{destabilizes}}}$$

$c^2 > 0 \rightarrow$ stable

$c^2 < 0 \rightarrow$ unstable, exp. growing

$= 0 \rightarrow$ stability boundary

$c^2 > 0$ if $\lambda < 2\pi\sqrt{\frac{\sigma}{g\Delta\rho}}$

$c^2 < 0$ if $\lambda > 2\pi\sqrt{\frac{\sigma}{g\Delta\rho}}$

$\lambda_c = 2\pi\sqrt{\frac{\sigma}{g\Delta\rho}}$

↑ taylor wavelength

For instability

$c^2 < 0 \qquad \omega \sim \pm i\beta$

$\eta = a e^{\pm i\beta t - ikz}$, $\quad \omega^2 = \frac{\sigma k^3}{\rho_f+\rho_g} - \frac{g\Delta\rho k}{\rho_f+\rho_g}$

$$\frac{\partial \omega^2}{\partial k} = 0 = \frac{3\sigma k^2 - g\Delta\rho}{\rho_f+\rho_g}$$

$k_{max} = \sqrt{\frac{g\Delta\rho}{3\sigma}}, \quad \lambda_{max} = 2\pi\sqrt{\frac{3\sigma}{g\Delta\rho}} = \sqrt{3}\,\lambda_c$

most dangerous wave

atomic bomb
→ 2 phase flow
→ explosion
→ film boiling

Kelvin Instability

– Helmholtz

Instability due to relative velocity (wind)

$u' \neq 0,\ u \neq 0$

Deep water

$kh \gg 1,\ kh' \gg 1$

$u' = u_g$

$u = u_f$

$\rho^* = \rho = \rho_f$ (water bottom)

$\rho'^* = \rho' = \rho_g$ (gas top)

$$c = \frac{\rho u + \rho' u'}{\rho+\rho'} \pm \left\{ \frac{\sigma k^2 + g(\rho-\rho')}{k(\rho+\rho')} - \frac{\rho\rho'}{(\rho+\rho')^2}(u'-u)^2 \right\}^{1/2}$$

(C.O.M. Vel.)

$\oplus$ $\qquad \ominus$

$\sigma \qquad g$

$\rho-\rho_f > 0$

stab. $\qquad$ stab.

relative vel. destabilizes

unstable

$(u'-u)^2 > \frac{\rho+\rho'}{\rho\rho'}\frac{1}{k}(\sigma k^2 + g\Delta\rho) \rightarrow$ minimum of right hand side

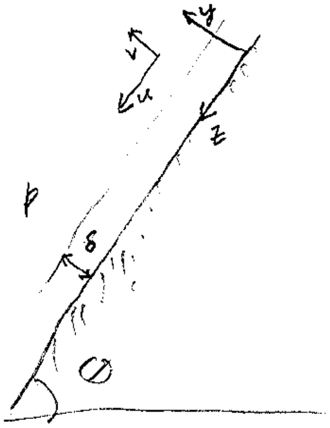
$(u'-u)^2 = \frac{1}{\rho_g}\sigma\sqrt{\frac{g\Delta\rho}{\sigma}} + g\Delta\rho\sqrt{\frac{\sigma}{g\Delta\rho}}$

$\min = \lambda_c = 2\pi\sqrt{\frac{\sigma}{g\Delta\rho}}$

$|(u'-u)| > \sqrt{2}\sqrt[4]{\frac{\sigma g\Delta\rho}{\rho_g^2}} \rightarrow$ unstable

~ 730 cm/s

(2) Laminar Film Flow ($\tau_i = 0$)



Steady state
Fully developed
incompressible
no interfacial shear

$$0 = \mu_f \frac{\partial^2 u}{dy^2} + \rho_f g \sin\theta, \quad \frac{\partial p}{\partial z} = 0$$

$$0 = -\frac{\partial p}{\partial y} + \rho_f g \cos\theta$$

$\left\{ \begin{matrix} CE \\ ME \end{matrix} \right.$

B.C. $y = 0$ @ wall, $u = 0$
$y = \delta$ $\mu \frac{\partial u}{\partial y} = \tau_i = 0$

$$u = \frac{g\rho_f}{\mu}\left(y\delta - \frac{y^2}{2}\right)\sin\theta, \quad u(\delta) = u_c = \frac{g\rho_f}{\mu_f}\frac{\delta^2}{2}\sin\theta$$

$$\langle u \rangle = \frac{1}{\delta}\int_0^\delta u(y)\,dy = \frac{g\rho_f}{\mu_f}\frac{\delta^2}{3}\sin\theta$$

$$\frac{u_c}{\langle u \rangle} = \frac{3}{2}, \quad Q_f = A\langle u \rangle = \xi\delta\langle u \rangle$$

$$\frac{Q_f}{\xi} = \delta\langle u \rangle \quad \text{volumetric flux}$$

Define $\Gamma_f = \rho_f\left(\frac{Q_f}{\xi}\right)$ mean flow

$$\Gamma_f = \rho_f\left(\frac{Q_f}{\xi}\right) = \rho_f\,\delta\langle u \rangle = \frac{\rho_f g}{\nu_f}\sin\theta\,\frac{\delta^3}{3} \propto \delta^3$$

Reynold's number

$$Re \equiv \frac{\delta\langle u \rangle}{\nu_f} = \frac{\Gamma_f}{\mu_f} \Rightarrow \langle u \rangle = \frac{Re\,\nu_f}{\delta} = Re\,\nu_f\left[\frac{g\rho_f\sin\theta}{3\nu_f\Gamma_f}\right]^{1/3}$$

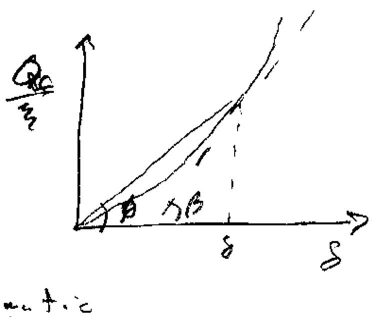
$$\langle u \rangle = \left[\frac{\nu_f g\sin\theta}{3}\right]^{1/3} Re^{2/3}$$

$$\delta = \left[\frac{3\nu_f^2}{g\sin\theta}\right]^{1/3} Re^{1/3}$$

$$\tau_w = \mu_f\frac{du}{dy}\Big|_{y=0} = g\rho_f\,\delta\sin\theta = \rho\{3\nu_f^2 g^2\sin\theta\}^{1/3} Re^{1/3}$$

$$= \frac{f\rho_f\langle u \rangle^2}{2} \quad f \equiv \frac{6}{Re}$$

$$\frac{Q_f}{\xi} = \left(\frac{1}{3}\frac{g\rho_f}{\mu_f}\sin\theta\right)\delta^3$$

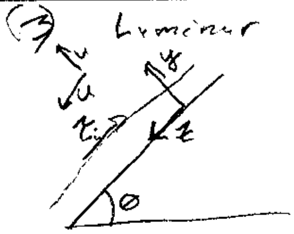


$$\tan\phi = \left(\frac{Q_f}{\xi}\right)\frac{1}{\delta} = \langle u \rangle$$

$$\tan\beta = C_k = \frac{d}{d\delta}\left(\frac{Q_f}{\xi}\right) \quad \text{kinematic velocity}$$

$$= \left(\frac{g\rho_f}{\mu_f}\sin\theta\right)\delta^2 = 3\langle u \rangle = 2u_{max}$$

(3) Laminar Flow Film with interface shear



steady state
Fully developed
2-D
Incompressible

$$0 = -\frac{\partial P}{\partial z} + \mu_f \frac{\partial^2 u}{\partial y^2} + \rho_f g \sin\theta$$

$$0 = -\frac{\partial P}{\partial y} + g\rho_f \cos\theta$$

B.C. $y = 0 \quad u = 0$

$\quad\; y = \delta \quad \mu \frac{\partial u}{\partial y} = -\tau_i$

$\mathbb{P} = g\rho_f \sin\theta - \frac{\partial p}{\partial z}$ ← calculated from gas momentum eq.

$$\frac{du}{dy} = -\frac{\mathbb{P} y}{\mu_f} + C_1$$

$$u(y) = -\frac{\mathbb{P}}{\mu_f}\frac{y^2}{2} + C_1 y + C_2$$

B.C. $C_2 = 0, \quad C_1 = -\frac{\tau_i}{\mu_f} + \frac{\mathbb{P}\delta}{\mu_f}$

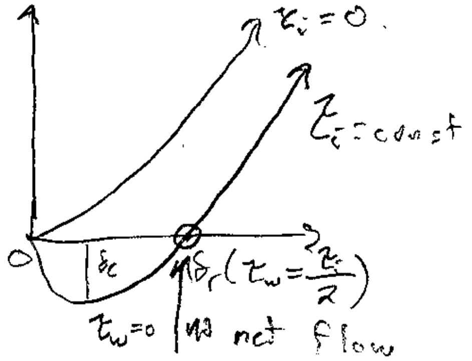
$$u(y) = \frac{\mathbb{P}}{\mu_f} y\left(\delta - \frac{y}{2}\right) - \frac{\tau_i}{\mu_f} y \qquad \text{2nd order}$$

$$u_i = \frac{\mathbb{P}}{\mu_f}\frac{\delta^2}{2} - \frac{\tau_i \delta}{\mu_f}, \qquad \langle u \rangle = \frac{\mathbb{P}}{\mu_f}\frac{\delta^2}{3} - \frac{\tau_i \delta}{\mu_f}$$

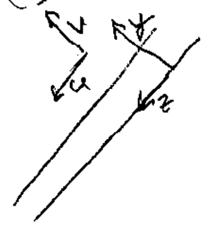
$$\frac{Q_f}{3} = \frac{\mathbb{P}}{\mu_f}\frac{\delta^3}{3} - \frac{\tau_i'}{\mu_f}\frac{\delta^2}{2}$$

$$\Gamma_f = \rho_f\left(\frac{Q_f}{3}\right) = \frac{\mathbb{P}}{\nu_f}\frac{\delta^3}{3} - \frac{\tau_i}{\nu_f}\frac{\delta^2}{2}$$

$$\tau_w = \mu_f \frac{\partial u}{\partial y}\Big|_{y=0} = \mu_f\left[\frac{\mathbb{P}}{\mu_f}\delta - \frac{\tau_i}{\mu_f}\right] = \mathbb{P}\delta - \tau_i$$



(4) Kinematic Waves on Film



continuity Equation

$$\frac{\partial u}{\partial z} + \frac{\partial v}{\partial y} = 0 \qquad (2\text{-}D)$$

(1)

$$\int_0^\delta \frac{\partial u}{\partial z}\,dy + \int_0^\delta \frac{\partial v}{\partial y}\,dy = 0$$

B.C. $y = 0 \quad u = v = 0$

$y = \delta \quad u = u_i$

kinematic interface condition

$y - \delta = 0 \qquad \frac{Dy}{Dt} - \frac{D\delta}{Dt} = 0 \qquad \delta = \delta(z,t)$

$v_y\big|_\delta - \frac{D\delta}{Dt} = 0 \qquad v\big|_\delta = \frac{\partial \delta}{\partial t} + u_i \cdot \frac{\partial \delta}{\partial z}$ (2)

$$\frac{\partial}{\partial z}\int_0^\delta u\,dy = \int_0^\delta \frac{\partial u}{\partial z}\,dy + \frac{\partial \delta}{\partial z} u_i \qquad (3)$$

$$\int_0^\delta \frac{\partial v}{\partial y}\,dy - v(\delta) - 0 = \frac{\partial \delta}{\partial t} + u_i \frac{\partial \delta}{\partial z} \qquad (4)$$

(3) + (4) → (1) $\qquad \frac{\partial \delta}{\partial t} + \frac{\partial}{\partial z}\int_0^\delta u\,dy = 0 \qquad (5)$

$$\langle u \rangle = \frac{1}{\delta}\int_0^\delta u\,dy$$

$$\frac{\partial \delta}{\partial t} + \frac{\partial}{\partial z}(\delta \langle u \rangle) = 0 \qquad (6)$$

$$\frac{Q_f}{\xi} = \delta\langle u \rangle = f(\delta, \mathbb{P}, \tau_i) = \frac{\mathbb{P}}{\mu_f}\frac{\delta^3}{3} - \frac{\tau_i}{\mu_f}\frac{\delta^2}{2} \qquad (7)$$

$$\langle u \rangle = \frac{\mathbb{P}}{\mu_f}\frac{\delta^2}{3} - \frac{\tau_i}{\mu_f}\frac{\delta}{2}$$

suppose $\mathbb{P}$, τ_i given (B.C.)

$\delta\langle u \rangle = f(\delta) \quad \Leftarrow$ steady state

assume correct for transient soln

$$\frac{\partial \delta}{\partial t} + \frac{d}{d\delta}\left(\frac{Q_f}{\xi}\right)\frac{\partial \delta}{\partial z} \qquad (8)$$

$$C_k = \frac{d}{d\delta}\left(\frac{Q_f}{\xi}\right) \qquad (9)$$ kinematic wave velocity

$$\frac{\partial \delta}{\partial t} + C_k\frac{\partial \delta}{\partial z} = 0 \qquad (10)$$

$\Delta\delta \rightarrow$ propagate by C_k, $\quad \frac{d\delta}{dt} + \frac{dz}{dt}) = 0$

$$C_k = \frac{dz}{dt}\Big)_{\delta = \text{const}}$$

Flow = f(concentration)

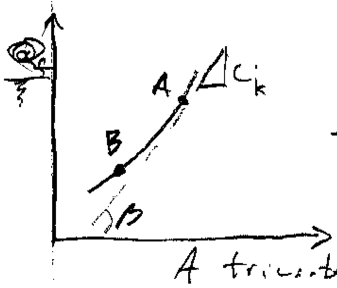
$$\frac{d}{d\delta}\left(\frac{Q_f}{\xi}\right) = C_k = \frac{\mathbb{P}}{\mu_f}\delta^2 - \frac{\tau_i}{\mu_f}\delta$$

$$\langle u \rangle = \frac{\mathbb{P}}{\mu_f}\frac{\delta^2}{3} - \frac{\tau_i}{\mu_f}\delta$$

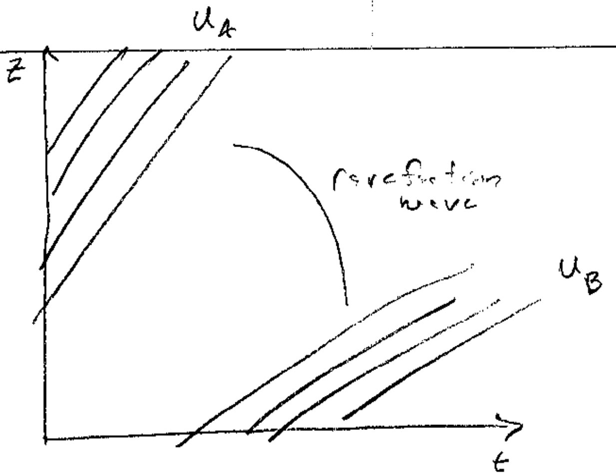
$$u_i = \frac{\mathbb{P}}{\mu_f}\frac{\delta^2}{2} - \frac{\tau_i}{\mu_f}\delta$$

$C_k = \langle u \rangle + \delta\frac{\partial \langle u \rangle}{\partial \delta}$

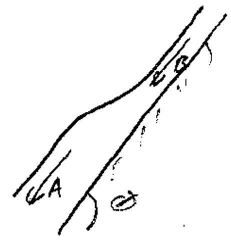
if $C_k = 0$ signal doesn't propagate



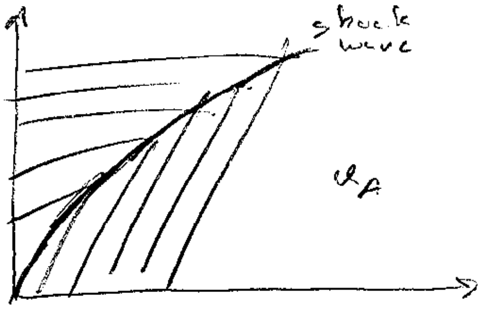
A travels
propagate faster than B



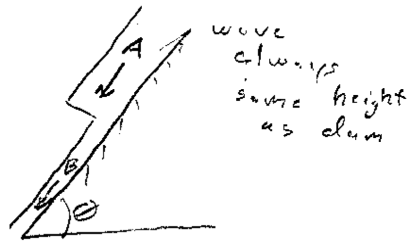
u_A
Z
rarefaction wave
u_B
t



B
A
θ



shock wave
u_A



A
wave always same height as dam
B
θ

Chapter 10 Basic Parameters in Two Phase Flow

(1) ρ_1, v_1, ⋮ (2) ρ_2, v_2, ⋮ Local, Instant Formulation, need Macroscopic

use mathematical average

- 2 Fluid Model (~1975)
- Drift Flux Model (~1965)

only two good models

(1) Eulerian Mean Value

$F' = F'(t, \vec{x})$

- time average $\frac{1}{\Delta t}\int F'(t,\vec{x})\,dt$ (temporal average)
- space average $\frac{1}{\Delta R}\int_{\Delta R} F'(t,\vec{x})\,dR(n)$
  - volume average
  - area average
  - line average
- statistical average $\frac{1}{N}\sum_N F_n(t,\vec{x})$
- mixed average → time & Area

(2) Lagrangian Mean Value

$F = F(t, \vec{\xi})$ $\vec{\xi}$ is name of particle or initial position

$\vec{\xi} = \vec{\xi}(\vec{x}, t)$ or $\vec{x} = \vec{x}(\vec{\xi}, t)$

- time average $\frac{1}{\Delta t}\int_{\Delta t} F(t,\vec{\xi})\,dt$
- statistical $\frac{1}{N}\sum_N F(t,\vec{\xi})$

(3) Boltzmann Statistical Average (Molecular, atoms, particles)

$f = f(\vec{x}, t, \vec{v})$

uses average notation (v_g means now average)

v_g, v_f

v_m

j_g, j_f

j_m $\longrightarrow$ only 2 are important mathematically or you will overspecify the equation

v_{km}

v_{kj}

$v_r = v_g - v_f$

(can solve iterative overspecified eqn... but may be wrong, use only v_g, v_f)

$$V_{gj} \equiv v_g - j = (1-\alpha) v_r$$
$$V_{fj} \equiv v_f - j = -\alpha v_r$$
$$\Rightarrow V_{fj} = -\left(\frac{\alpha}{1-\alpha}\right) V_{gj}$$

$$j = v_m + \alpha(1-\alpha)\frac{(\rho_f - \rho_g)}{\rho_m} v_r$$
$$= v_m + \frac{\alpha(\rho_f - \rho_g)}{\rho_m} V_{gj}$$

Homogeneous flow

$$v_r = 0 \Rightarrow v_g = v_f$$

$$v_{km} = V_{kj} = 0$$

$$v_g = v_f = v_m = j$$

Mixture Pressure

$$p_m = \alpha \langle p_g \rangle + (1-\alpha)\langle p_f \rangle$$

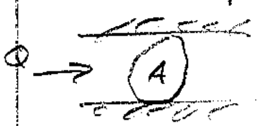
Properties under Area Average

Void fraction $\alpha_A = \frac{A_g}{A} \rightarrow \alpha$

Savannah is weapons creation reactor. Uses CHF not similar to norm. l.

Mixture Density $\rho_m = \alpha \rho_g + (1-\alpha)\rho_f$

Velocity



$\langle v_k \rangle = \frac{Q_k}{A_k}$

need to know void fraction

Volumetric Flux

$$j_g \equiv \frac{Q_g}{A}, \quad j_f \equiv \frac{Q_f}{A} \quad > \quad j = j_g + j_f$$

Mixture Velocity

$$v_m = \frac{\alpha \rho_g \langle v_g \rangle + (1-\alpha)\rho_f \langle v_f \rangle}{\rho_m}$$

Relative Velocity

$$v_r \equiv \langle v_g \rangle - \langle v_f \rangle$$

Drift Velocity

$$\bar{v}_{gj} = \langle v_g \rangle - j = (1-\alpha) v_r$$

Slip Ratio (Depricated)

$$S \equiv \frac{\langle v_g \rangle}{\langle v_f \rangle}$$

Blows up for LOCA

deprecated

Specify Relative Velocity

Quality (x)

$$G_k = \rho_k Q_k = \rho_k j_k A = (\alpha_k \rho_k v_k) A$$ mass flow rate

$$x = \frac{G_g}{G_g + G_f} = \frac{\rho_g j_g}{\rho_g j_g + \rho_f j_f}$$

$$S = \frac{\rho_f}{\rho_g} \frac{x}{1-x} \frac{1-\alpha}{\alpha}$$

$x = x(\alpha)$ void quality correlation

Total volumetric flow rate

$$Q_m = \sum Q_k = Q_g + Q_f$$

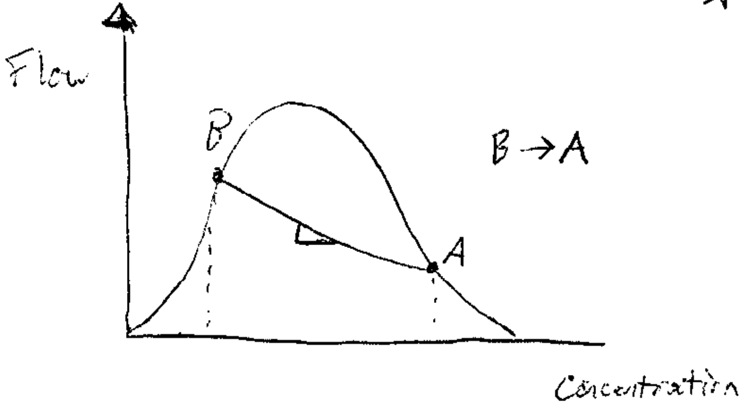
$$Q_m = A(j_g + j_f) = Aj$$

$$j = j_g + j_f$$

Mass flow rate

$$G_k = \rho_k Q_k$$

$$G_m = \sum \rho_k Q_k = A \rho_m v_m = A[\alpha \rho_g$$ [illegible]



Kinematic Shock

$$V_s = \frac{Q_B - Q_A}{j_B - j_A}$$

---

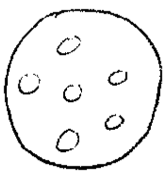
(1) Phase density Function

$$X_K(\vec{x}, t) = \begin{cases} 1 & \text{if } \vec{x} \text{ is in phase } K \\ 0 & \text{" " not " " } \end{cases}$$

K = g or F

i or l

(2) Averaging [Area Average]



$$\langle \psi \rangle_A = \frac{1}{A_K} \int_{A_K} \psi_K \, dA \quad ②$$

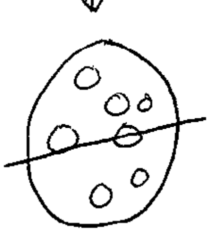
$$\text{total Area average} = A_g + A_F \quad ③$$

$$\langle \psi_K \rangle_V = \frac{1}{V_K} \int \psi_K \, dV \quad ④$$

$$V = V_g + V_F \quad ⑤$$

Line [Average]

$$\langle \psi_k \rangle_L = \frac{1}{L_k}\int \psi_k \, dV$$

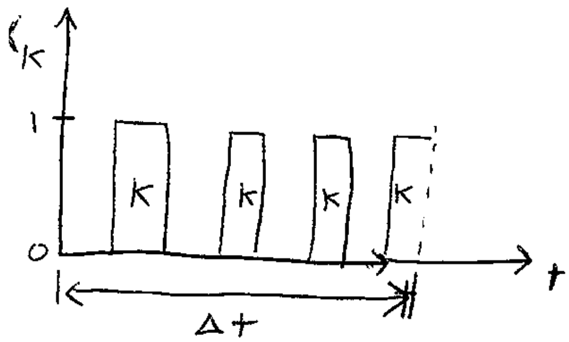


$$L = L_g + L_F$$

[Time Average]

$$\langle \psi_k \rangle_t = \frac{1}{\Delta t_k}\int \psi_k \, dt$$

$$\Delta t = \Delta t_g + \Delta t_F$$



(3) Void Fraction

(i) Local void Fraction (Time Fraction)

$$\alpha_{kt} \equiv \frac{1}{\Delta t}\int_{\Delta t} X_k \, dt = \frac{\Delta t_k}{\Delta t}$$

$$\alpha_{gt} + \alpha_{Ft} = 1$$

(ii) Area Averaged Void Fraction

$$\alpha_{kA} = \frac{1}{A}\int_A X_k \, dA = \frac{A_k}{A}$$

(iii) Volume

$$\alpha_{kV} = \frac{1}{V}\int_V X_k \, dV = \frac{V_k}{V}$$

(iv) Line Averaged

$$\alpha_{kl} = \frac{1}{L}\int_L X_k \, dL = \frac{L_k}{L}$$

Ergodic Theorem exists between various α's

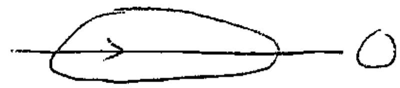
$$\frac{1}{A}\int \frac{1}{\Delta t}\int_{\Delta t} X_k \, dt \, dA = \frac{1}{\Delta t}\int \frac{1}{A}\int X_k \, dA \, dt$$

area average (time avg α_k) = time avg (area avg of α_k)

• Void Fraction Measurement

(1) Photon Attenuation ("γ" or "X" ray)

$$I = I_0 \exp(-\mu x)$$



μ = absorption coeff

ρ = density

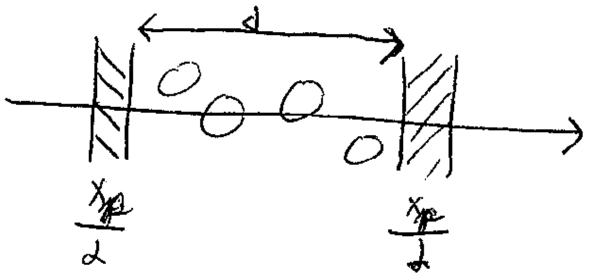
x = distance

I_0 = incident intensity

I = energy intensity

$\frac{\mu}{\rho}$: specific absorption coeff

* Line Void Measurement



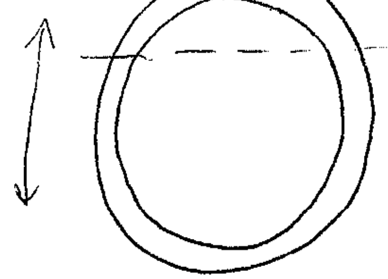
p = pipe

$$I = I_0 \exp(-\mu_p x_p) \underbrace{\exp[-\mu_f (1-\alpha) d]}_{\text{liquid}} \underbrace{\exp[-\mu_g \alpha d]}_{\text{gas}}$$

x_p = total wall thicknesss

d = distance b/w wall

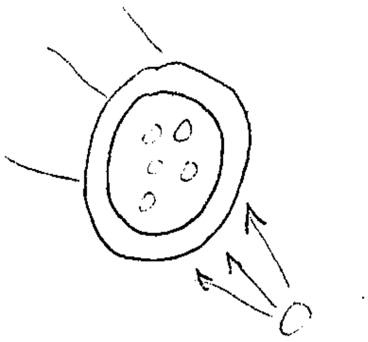
μ_p, μ_f, μ_g = absorption coeff

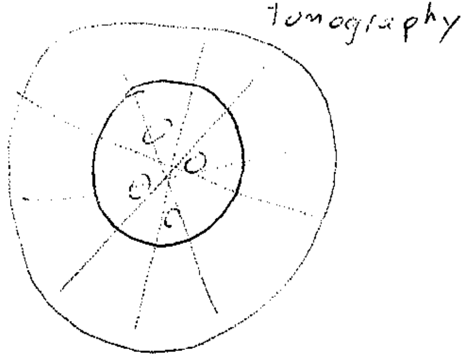


(1) $\alpha = 0$

(2) $\alpha = 1$

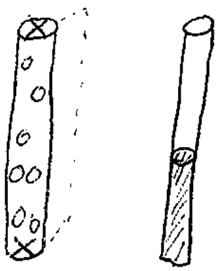
(3) α



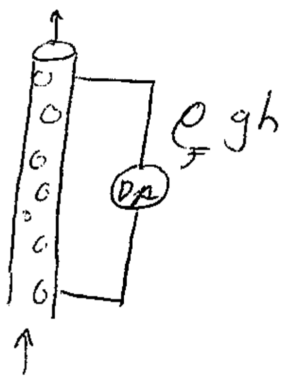


(2) Global Tech

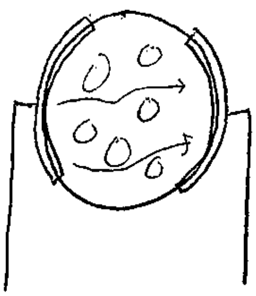
• quick closing Valves

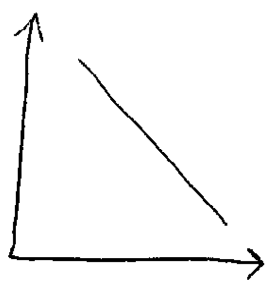


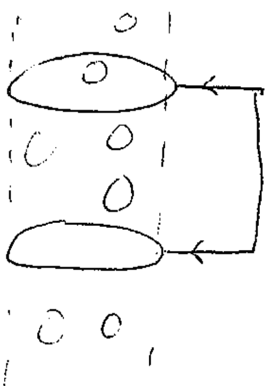
• Δp method (only good for low Flow)



(3) Global Electrical (Impedance)

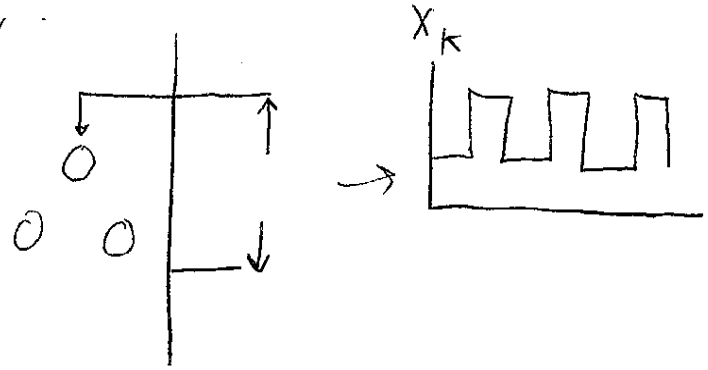


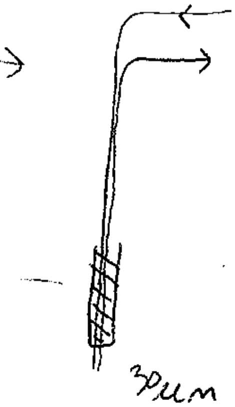


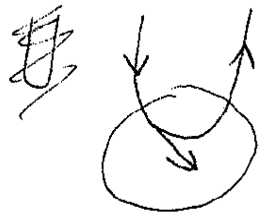


(3) Local Probe

- conductivity Probe
- optical prode







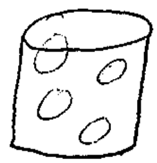
(For high temp)

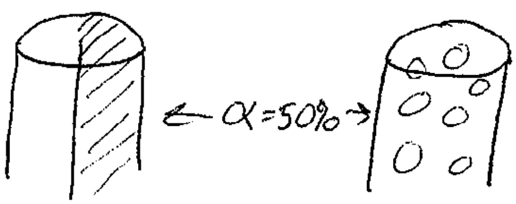
(expensive)
(Fragile)

α : void Fraction of gas α_g
one important geometrical parameter

a_i: interfacial area concentration

$$\boxed{a_i \equiv \frac{\text{Interface Area}}{\text{Mixture Volume}} = \frac{A_i}{V}}$$

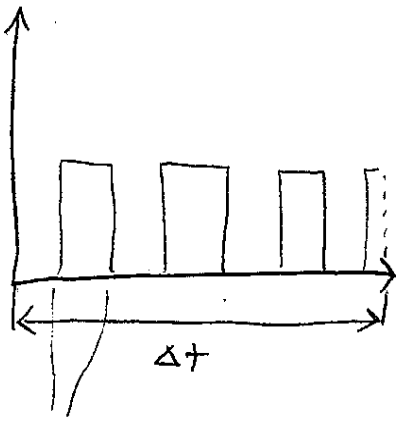




time average a_i (Local point)

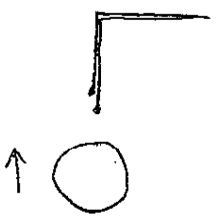
$$a_i = \frac{1}{\Delta t}\sum_j \frac{1}{|\vec{v}_{i*}\cdot\vec{n}|}$$

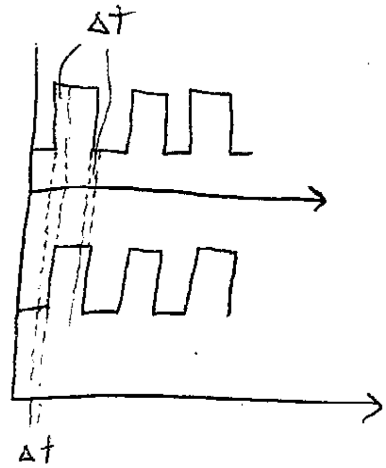
$$a_i = \frac{1}{\Delta t} \sum_j \frac{1}{|\vec{v}_i \cdot \vec{n}|} \longleftrightarrow \text{normal velocity}$$



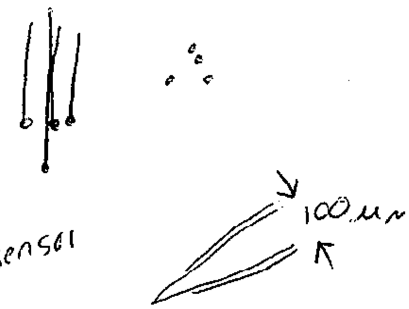
$j = 1, \ldots$

measure velocity of interface $\Rightarrow a_i$





$$V_e = \frac{L}{\Delta \tau}$$



4 sensor

↓

multidimensional velocity of surface

(5) Mixture Properties (time Averaged)

$$\langle \psi \rangle_t = \frac{1}{\Delta t}\int \psi \, dt = \underbrace{\frac{\Delta t_g}{\Delta t}}_{\alpha_{gt}} \langle \psi_g \rangle_t + \underbrace{\frac{\overset{\Delta t_F}{\downarrow}\;\Delta t_F}{\Delta t}}_{\alpha_{Ft}} \langle \psi_F \rangle_t$$

$$\psi = \begin{cases} \psi_g & \text{if } X_g = 1 \\ \psi_F & \text{if } X_F = 1 \end{cases}$$

- Density (mixture)

$$\langle \rho \rangle_t = \frac{\Delta t_g}{\Delta t} \langle \rho_g \rangle_t + \frac{\overset{\Delta t_F}{\cancel{\Delta t_g}}}{\Delta t} \langle \rho_F \rangle_t$$

$$\langle \rho_g \rangle_t = \rho_g \qquad \frac{\Delta t_g}{\Delta t} = \alpha_{gt} = \alpha$$

$$\langle \rho_F \rangle_t = \rho_F$$

$$\boxed{\langle \rho \rangle_t = \rho_m = \alpha \rho_g + (1-\alpha)\rho_F}$$

- Mixture Velocity (Center of Mass)

$$V_m = \frac{\alpha \rho_g \langle v_g \rangle + (1-\alpha)\rho_F \langle v_F \rangle}{\rho_m}$$

$$\rho_m V_m = \sum \underbrace{\alpha_K \rho_K \langle v_K \rangle}_{\text{momentum of "K" phase}}$$

- Volumetric Flux (superficial Velocity)

$j_g = \cancel{\alpha\langle v_g\rangle}\ \alpha\langle v_g\rangle$ Volumetric Flux of gas

$j_F = (1-\alpha)\langle v_F\rangle$ Volumetric Flux of liquid

- Total Volumetric Flux

$$j = j_g + j_F = \alpha\langle v_g\rangle + (1-\alpha)\langle v_F\rangle$$

- Diffusion Velocity

$$V_{km} = \langle v_k\rangle - V_m \qquad \sum \alpha_k \rho_k V_{km} = 0$$

- Drift Velocity

$$V_{kj} = \langle v_k\rangle - j \qquad \sum \alpha_k V_{kj} = 0$$

- Relative Velocity

$$V_r = \langle v_g\rangle - \langle v_F\rangle$$

☆☆ There are two velocities that are important in 2-Fluid model

- 2 Fluid model: $\langle v_g\rangle$, $\langle v_F\rangle$
- Drift Flux model V_m, $\cancel{V_{gF}}$ V_{gj}

$$\rho_m = \alpha\rho_g + (1-\alpha)\rho_f$$

$$v_m = \frac{\alpha \rho_g v_g + (1-\alpha)\rho_f v_f}{\rho_m}$$

$$i_m = \frac{\alpha\rho_g i_g + (1-\alpha)\rho_f i_f}{\rho_m}$$

$$v_m = \alpha\rho_g + (1-\alpha)\rho_f$$

Properties are per unit mass

$$\psi_m = \frac{\sum \alpha_k \rho_k \langle \psi_k \rangle}{\rho_m}$$

~~Two Fluid Model~~
Field Equation

Mixture Model Mixture → Continuum

Two-Phase Flow Model Each Phase → Continuum

Complexity of Model ⟼ AMNT Info in and out

Local Instant Formulation ⟼ Interface + Position

↓

eliminate discontinuity
by averaging (integral operation)

3-D averaged Model → Statistical Aspect
α
turbulence

↓

Eliminate 2-Dimensions

↓

1-D averaged Model ← Wall Transfer
τ_w
q''_w
h_w

Mixture Model

Homogeneous Equilibrium Mode $v_r = 0$, $T_g = T_f$

Slip Flow Model $\left\{ s = \frac{v_g}{v_f} \text{ Specified}, \ \alpha \right.$ → v_m

Drift Flux Model $\left\{ v_m, \ v_{gj} \text{ (given)} \right.$
GE (1966 Zuber)

Two Fluid Model

gas field ⟶ field eq.

$\updownarrow$ interaction

liquid field ⟶ field eq.

⇒ Interfacial Transfer { Mass, Momentum, Energy }

almost expressed by IF

(4) 3D Two Fluid Model (Rudolf)

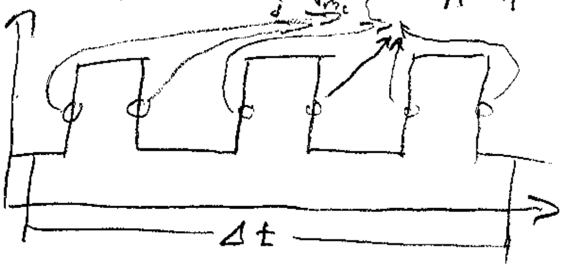
Local Time Averaging

$$\frac{1}{\Delta t}\int \{ B.E. (\text{Single Phase}) \} dt$$

Local Instant

$$\frac{\partial F}{\partial t} \quad \nabla \cdot F \quad \nabla_j \quad \phi$$

$$\overline{\frac{\partial F}{\partial t}} = \frac{\partial F}{\partial t} - \frac{1}{\Delta t}\sum_j \frac{1}{v_{ni}} \{ F^{+} n^{+} \cdot v_i + F^{-} n^{-} \cdot v_i \}_i$$



j is jth interface

$-,+$; function value before and after continuity

n; normal vector

v_{ij} interface velocity

$$\overline{\nabla F} = \nabla F + \frac{1}{\Delta t}\sum_j \frac{1}{v_{ni}} \{ n^{+} F^{+} + n^{-} F^{-} \}_j$$

Time Average Balance Eq.

C.E. $$\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k v_k) = \Gamma_k$$

time rate of change of k phase mass/vol

convection of k phase mass

source of k-values due to phase change (+, −)

$-\nabla(\alpha_k p_k)$

M.E. $$\frac{\partial \alpha_k \rho_k v_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{v}_k \vec{v}_k) = -\alpha_k \nabla p_k - \nabla \cdot \alpha_k (\tau_k^{\mu} + \tau_k^{t}) + \alpha_k \rho_k \vec{g} + \vec{M}_{ik} + (\nabla \alpha_k) \cdot \tau_i + (p_{ki} - p_k) \nabla \alpha_k + \vec{V}_{ki} \Gamma_k$$

τ_k^{μ} average viscous stress

τ_k^{t} turbulent stress

$\vec{V}_{ki}$ k phase interface velocity

M_{ik}: generalized drag and lift force

τ_i: interface shear

Enthalpy Energy Equation

$$\frac{\partial \alpha_k \rho_k i_k}{\partial t} + \nabla\cdot(\alpha_k \rho_k i_k \vec{V}_k) = -\nabla\alpha_k(q_k^c + q_k^t) + \alpha_k \frac{D_k p_k}{Dt} + \underbrace{i_k \Gamma_k}_{\text{enthalpy transfer due to phase change}} + \underbrace{a_i q_{ki}''}_{\text{interface heat transfer}} + \phi_k$$

a_i - interfacial area concentration

$$\frac{D_k}{Dt} p_k = \frac{\partial p_k}{\partial t} + V_k \cdot \nabla p_k$$

---

① ③ interfacial balance averaging ②

{ ① ② interfacial balance

conf. $\sum_{k=1}^{2} \Gamma_k = 0$, $\sum_{k=1}^{2} M_{ik} = 0$, (force balance) $\sum_{k=1}^{2} (\Gamma_k i_{ki} + a_i \underbrace{q_{ki}}_{\text{energy change of interface}}) \approx 0$

$$\Gamma_g(i_{gi} - i_{fi}) = \Gamma_g(i_{gsat} - i_{fsat}) = \Gamma_g \underbrace{\Delta i_{fg}}_{\text{latent heat}}$$

Need Constitutive Relation

① Interface Transfer

Γ_k

$\vec{M}_{ki}$, $\vec{V}_{ki}$, τ_i $\Longrightarrow$?

i_{ki}, $a_i q_{ki}$

② Bulk Fluid

τ_k^{μ}, τ_k^{t} (viscous, turbulent)

q_k^c q_k^t (conduction heat transfer, turbulent heat transfer)

③ Equation of State for Each Phase

$u_k = u_k(p_k, T_k)$ caloric EoS

$p_k = p_k(\rho_k, T_k)$ thermal EoS

(5) 3-D Drift Flux Model (Mixture Model)

eliminate one momentum eq and replace w/ mixture velocity equation

Mixture Continuity Eqns

$$\sum_{k=1}^{2}\left\{\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla(\alpha_k \rho_k \vec{V}_k) = \Gamma_k\right\}$$

$$\boxed{\frac{\partial \rho_m}{\partial t} + \nabla\cdot\rho_m \vec{V}_m = 0}$$

no new mass by phase change

$\rho_m = \alpha\rho_g + (1-\alpha)\rho_f$

$V_m = \frac{\alpha\rho_g V_g + (1-\alpha)\rho_f V_f}{\rho_m}$

$j = \alpha\vec{V}_g + (1-\alpha)V_f \equiv j_g j_f$

$V_{gj} \equiv \vec{V}_g - \vec{j} = (1-\alpha)\vec{V}_j$

Vapor Continuity Eqn

$$\frac{\partial \alpha\rho_g}{\partial t} + \nabla\cdot(\alpha\rho_g\vec{V}_g) = \Gamma_g$$

drift velocity $V_{gj} \equiv V_g - j$

V_m

$$\boxed{\frac{\partial \alpha\rho_g}{\partial t} + \nabla\cdot(\alpha\rho_g\vec{V}_m) = \Gamma_g - \nabla\cdot\left(\frac{\alpha\rho_g\rho_f}{\rho_m}\vec{V}_{gj}\right)}$$

drift vel.

Mixture Momentum Eq

$$\frac{\partial\rho_m\vec{V}_m}{\partial t} + \nabla\cdot(\rho_m\vec{V}_m\vec{V}_m) = -\nabla p_m + \rho_m\vec{g} - \nabla\left\{\tau_m^{\mu} + \tau_m^{t} + \frac{\alpha\rho_g\rho_f}{1-\alpha\;\rho_m}\vec{V}_{gj}\vec{V}_{gj}\right\}$$

momentum diffusion

Mixture Energy Eq

$$\frac{\partial\rho_m i_m}{\partial t} + \nabla\cdot(\rho_m i_m\vec{V}_m) = -\nabla\cdot\left[q_m^c + q_m^t\right] - \nabla\left\{\frac{\alpha\rho_g\rho_f}{\rho_m}(i_{gi} - i_{fi})\vec{V}_{gj}\right\}$$

$$+\frac{\partial p_m}{\partial t} + \left[\vec{V}_m + \frac{\alpha(\rho_f-\rho_g)}{\rho_m}\vec{V}_{gj}\right]\cdot\nabla p_m + \Phi_m$$

energy diffusion

[4 equation Drift Flux Model]

5 equation Drift Flux Model (close to 2 fluid)
2 cont., 1 momentum, ~~+ 1~~ 2 energy eq

Constitutive Relations of D-F Model

$$\vec{V}_{gj} = \vec{V}_{gj}(\alpha, p_m, \vec{g}, \vec{V}_m, \dots)$$

$$\Gamma_g = \Gamma_g(\alpha, p_m, \dots) \quad \text{4 eq case}$$

difficult!

$$\alpha_i \, g_{ki}^{ii} = \alpha_i \, g_{ki}^{ic} \quad \text{(5 eq model)}$$