NUCL 511 Nuclear Reactor Theory and Kinetics

Homework #2

Due January 30

1. The following figure shows a CASMO input data for a PWR pin cell problem composed of fuel, cladding and coolant. The first line is a title, and second line specifies the fuel temperature (TFU), the moderator temperature (TMO), and the boron concentration (BOR) in the coolant. The third line with the key word FUE specifies a fuel density of 10.1 g/cm^3 and a U-235 enrichment of 3.2 wt. %. The fourth line with the key word PIC describes the cylindrical pin cell geometry with a fuel radius of 0.49 cm, a cladding outer radius of 0.55 cm, and a coolant outer radius of 0.81 cm. The PRE line specifies the system pressure in bars, and the PDE line specifies the power density in W/gU. The DEP line with a value of -40 specifies a depletion calculation up to 40 MWd/kgU burnup using the default depletion steps (i.e., 0.1, 0.5, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.5, 15, 17.5, 20, and every 2.5 MWd/kg up to 100 MWd/kg for PWR).

CASMO Input Data for a PWR Pin Cell Problem

```
TTL * Ex11, PIN CELL DEPLETION
TFU=850 TMO=573 BOR=0
FUE 1 10.1/3.2
PIC .49 .55 .81
PRE 155
PDE 30
DEP -40
STA
END
```

Using this CASMO input deck, compute the 1) k-infinity 2) and the nuclide densities of U-235, U-238 and Pu-239 as a function of burnup.

The CASMO manual is available in the directory **/opt/studsvik/1.00.02/share/doc/studsvik** and you can execute the CASMO program using the following command:

casmo4e –v u1.00.02 input

where “input” is your input file name that contains the above input data.

(Answer) The variation of the k-infinity with burnup can be obtained from the “C A S M O – 4E SUMMARY” near the bottom of output, and the nuclide densities can be obtained from “AVERAGED NUMBER DENSITIES OF BURNABLE NUCLIDES” at each burnup state. The k-infinity and nuclide densities are summarized below.

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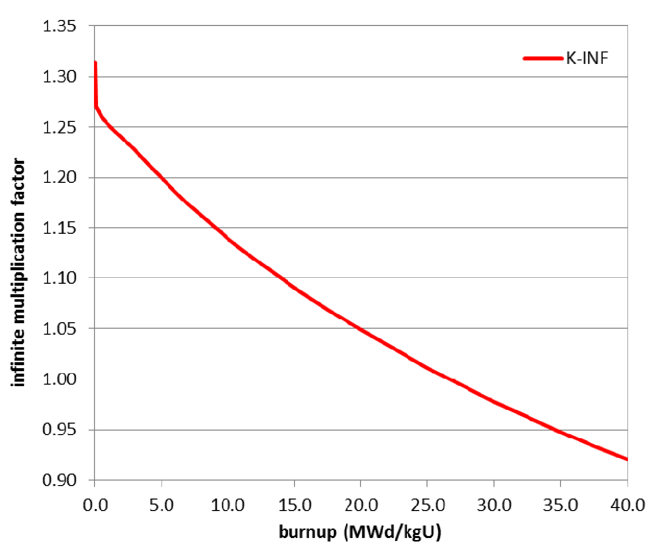
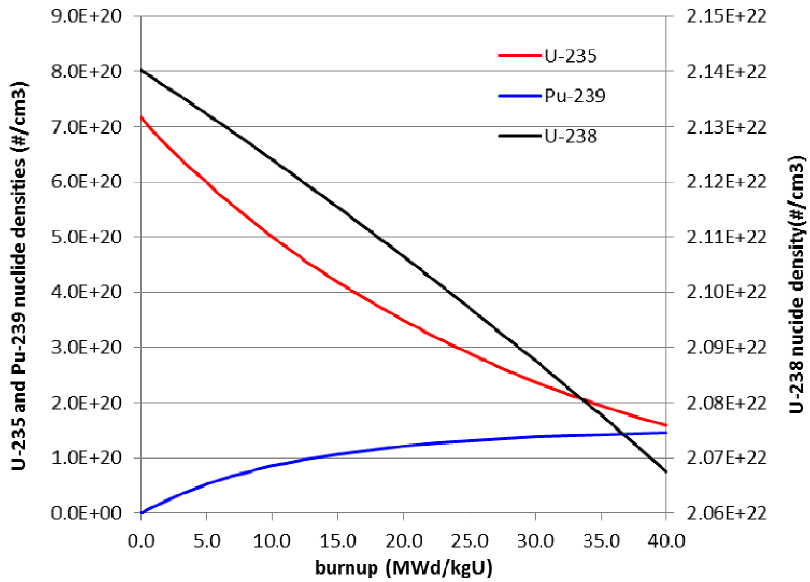
Burnup (MWD/KG)	k-infinity	U-235 density (#/cm^3)	U-238 density (#/cm^3)	Pu-239 density (#/cm^3)
0.0	1.31441	7.1676E+20	2.1403E+22	0.0000E+00
0.1	1.27035	7.1412E+20	2.1401E+22	4.8706E+17
0.5	1.25978	7.0366E+20	2.1395E+22	5.3435E+18
1.0	1.25287	6.9089E+20	2.1387E+22	1.1729E+19
2.0	1.24043	6.6624E+20	2.1371E+22	2.3478E+19
3.0	1.22688	6.4269E+20	2.1355E+22	3.4023E+19
4.0	1.21315	6.2010E+20	2.1339E+22	4.3549E+19
5.0	1.19965	5.9838E+20	2.1323E+22	5.2194E+19
6.0	1.18659	5.7748E+20	2.1307E+22	6.0068E+19
7.0	1.17405	5.5734E+20	2.1290E+22	6.7260E+19
8.0	1.16205	5.3789E+20	2.1274E+22	7.3845E+19
9.0	1.15057	5.1911E+20	2.1257E+22	7.9885E+19
10.0	1.13957	5.0096E+20	2.1240E+22	8.5435E+19
11.0	1.12902	4.8341E+20	2.1223E+22	9.0538E+19
12.5	1.11402	4.5814E+20	2.1197E+22	9.7443E+19
15.0	1.09079	4.1866E+20	2.1154E+22	1.0721E+20
17.5	1.06902	3.8221E+20	2.1110E+22	1.1519E+20
20.0	1.04865	3.4854E+20	2.1064E+22	1.2173E+20
22.5	1.02949	3.1743E+20	2.1018E+22	1.2709E+20
25.0	1.01133	2.8870E+20	2.0972E+22	1.3149E+20
27.5	0.99408	2.6219E+20	2.0924E+22	1.3508E+20
30.0	0.97763	2.3776E+20	2.0876E+22	1.3802E+20
32.5	0.96198	2.1528E+20	2.0827E+22	1.4041E+20
35.0	0.94713	1.9465E+20	2.0777E+22	1.4235E+20
37.5	0.93311	1.7572E+20	2.0727E+22	1.4393E+20
40.0	0.91982	1.5841E+20	2.0676E+22	1.4520E+20





2. Using the delayed and total neutron emission spectra in Table 2-V of the textbook and applying linear interpolation in the group around 1 MeV, estimate the fractions of total and delayed neutrons that can cause fast fission in U-238, assuming a sharp threshold at 1 MeV.

(Answer) The threshold energy of $E_f = 1$ MeV for U-238 fission is in group 5 with the energy boundaries $E_4 = 1.353$ MeV and $E_5 = 0.821$ MeV. Suppose that the fission spectrum varies linearly with lethargy within each group as $\chi_k(u) = a_k u + b_k$.

If $\chi_{k4} = 0$, then $\chi_k(u_4) = 0$. Since its integral over group 5 is χ_{k5}, the spectrum in group 5 can be written as $\chi_k(u) = 2\chi_{k5}(u - u_4)/(u_5 - u_4)^2$. Thus the fraction of neutrons that can cause fast fission of U-238 can be obtained

$$f_k = \frac{2\chi_{k5}}{(u_5 - u_4)^2}\int_{u_4}^{u_f}(u - u_4)du = \frac{(u_f - u_4)^2}{(u_5 - u_4)^2}\chi_{k5} = 0.36623\chi_{k5}$$

Thus, the factions of delayed neutron groups 1, 2, and 3 are 0.33%, 0.81%, and 0.44%, respectively.

If $\chi_{k4} \neq 0$, then $\chi_k(E_4)$ can be approximated as $\chi_k(E_4) = (\chi_{k4} + \chi_{k5})/2$ assuming the spectrum varies linearly with lethargy around the boundary between groups 4 and 5. If the spectrum varies linearly with lethargy, the spectrum in group 5 can be written as

$$\chi_k(u) = \left[\frac{2\chi_{k5}}{(u_5 - u_4)^2} - \frac{\chi_{k4} + \chi_{k5}}{u_5 - u_4}\right](u - u_4) + \frac{\chi_{k4} + \chi_{k5}}{2}$$

Thus the fraction of neutrons that can cause fast fission of U-238 can be obtained

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$$f_k = \sum_{g=1}^{4}\chi_{kg} + \int_{u_4}^{u_f}\chi_{k5}(u)du = \sum_{g=1}^{4}\chi_{kg} + \frac{(u_f-u_4)^2}{(u_5-u_4)^2}\chi_{k5} + \frac{\chi_{k4}+\chi_{k5}}{2}(u_f-u_4)\left[1-\frac{u_f-u_4}{u_5-u_4}\right]$$
$$= \sum_{g=1}^{4}\chi_{kg} + 0.42597\chi_{k5} + 0.05973\chi_{k4}$$

Consequently, the faction of the delayed neutron group 4-6 is 2.02%, and that of the total fission spectrum is 67.5%.