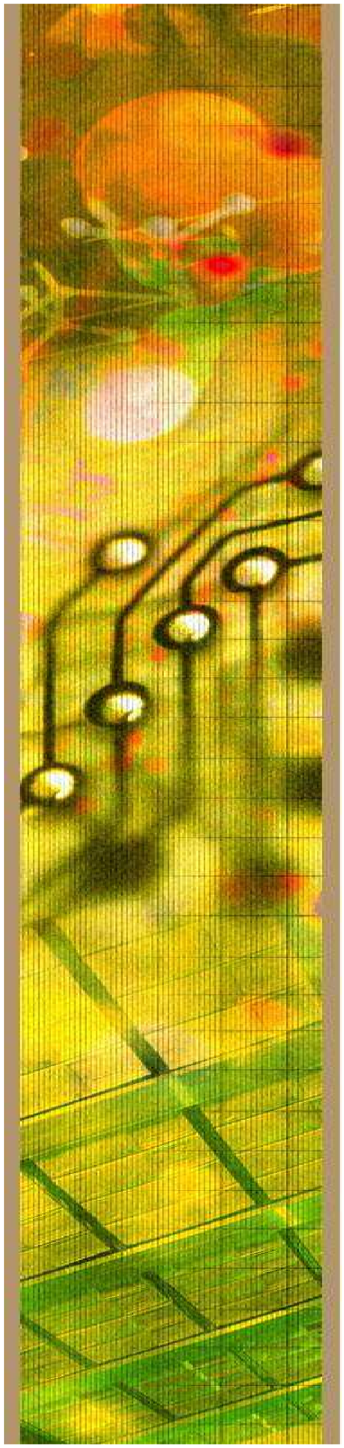

NUCL 511
Nuclear Reactor Theory and Kinetics

Lecture Note 8

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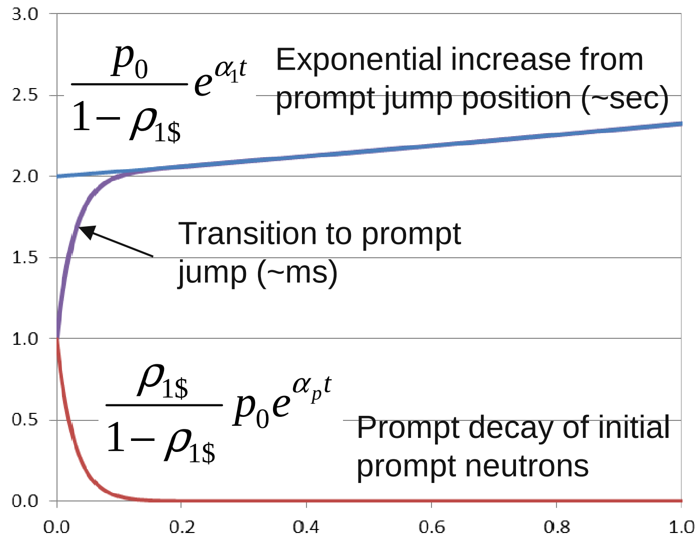
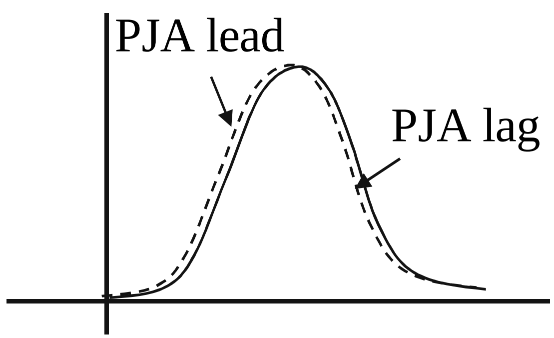
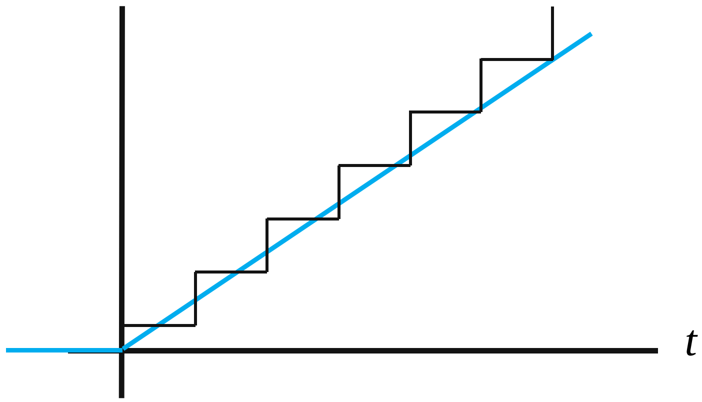
Prompt Jump Approximation

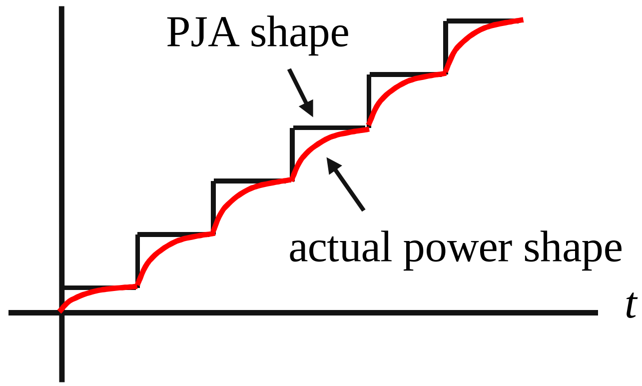
- PJA for modest transients $\Lambda\dot{p} \sim 0$
  - Power response to a step reactivity change





  - Ramp reactivity
    - In reality, no step, but ramp
    - Approximated as a series of steps





Consideration on Prompt Jump Approximation

- Prompt period

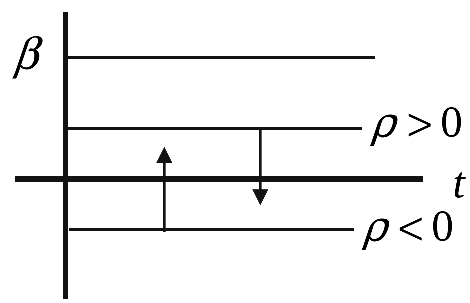
$$T_p = \frac{1}{\alpha_p} = \frac{\Lambda}{\rho - \beta} = \frac{\Lambda}{\beta}\frac{1}{\rho_{1\$} - 1} = \frac{1.4}{\rho_{1\$} - 1} \text{ ms}, \quad T_p =\sim 3 \text{ ms for } \rho = 0.5\$$$

- PJA is more valid as $\Lambda \rightarrow 0$
- PJA yields the time-dependent source multiplication factor
  - Generalized source multiplication factor

$$0 = [\rho(t) - \beta]p(t) + s_d(t) \quad \Rightarrow$$

$$p(t) = \frac{s_d(t)}{\beta - \rho(t)}$$

$$M(t) = \frac{1}{\beta - \rho(t)} \quad \text{under CDS}$$



$$\rho \uparrow \ (\rho \text{ approaching toward } \beta) \quad \Rightarrow \quad p \uparrow \ \text{ even for } \rho < 0$$

$$\rho \downarrow \ (\rho \text{ moving away from } \beta) \quad \Rightarrow \quad p \downarrow \ \text{ even for } \rho > 0$$

One Delayed Neutron Group Kinetics with PJA

- One-group PKE with PJA

$$\begin{cases} \Lambda\dot{p} = [\rho(t)-\beta)]p(t) + \lambda\zeta(t) + s(t) \\ \dot{\zeta}(t) = \beta p(t) - \lambda\zeta(t) \end{cases} \Rightarrow \Lambda\ddot{p} + (\lambda\Lambda + \beta - \rho)\dot{p} - (\lambda\rho + \dot{\rho})p = \lambda s + \dot{s}$$

$$\Lambda \to 0 \quad \Rightarrow \quad (\beta-\rho)\dot{p} - (\lambda\rho + \dot{\rho})p = \lambda s + \dot{s} \quad \Rightarrow \quad \dot{p}(t) = \frac{\lambda\rho(t) + \dot{\rho}(t)}{\beta - \rho(t)} p(t) + \frac{\lambda s(t) + \dot{s}(t)}{\beta - \rho(t)}$$

  - This equation can be integrated from 0^+ to *t* using the integrating factor

$$\exp\left[\int_{0^+}^{t} \frac{\lambda\rho(t') + \dot{\rho}(t')}{\beta - \rho(t')} dt'\right] = \frac{\beta - \rho(0^+)}{\beta - \rho(t)} \exp\left[\int_{0^+}^{t} \frac{\lambda\rho(t')}{\beta - \rho(t')} dt'\right]$$

$$p(t) = p(0^+)\frac{\beta - \rho(0^+)}{\beta - \rho(t)} \exp\left[\int_{0}^{t} \frac{\lambda\rho(t')}{\beta - \rho(t')} dt'\right] + \int_{0^+}^{t} \frac{\lambda s(t') + \dot{s}(t')}{\beta - \rho(t)} \exp\left[\int_{t'}^{t} \frac{\lambda\rho(t'')}{\beta - \rho(t'')} dt''\right] dt'$$

$$p(0^+) = \begin{cases} p_0, & \text{for gradual reactivity insertion } [\rho(0^+) = 0] \\ p^0 = \beta p_0/(\beta - \rho_1), & \text{for initial reactivity step } [\rho(0^+) = \rho_1 \neq 0] \end{cases}$$

$$p(t) = \frac{\beta p_0}{\beta - \rho(t)} \exp\left[\int_{0}^{t} \frac{\lambda\rho(t')}{\beta - \rho(t')} dt'\right] + \frac{1}{\beta - \rho(t)} \int_{0^+}^{t} [\lambda s(t') + \dot{s}(t')] \exp\left[\int_{t'}^{t} \frac{\lambda\rho(t'')}{\beta - \rho(t'')} dt''\right] dt'$$

Step Reactivity Insertion in Critical System

- For an initially critical system

$$p(t) = \boxed{\frac{\beta p_0}{\beta - \rho(t)}} e^{\boxed{\int_0^t \frac{\lambda \rho(t')}{\beta - \rho(t')} dt'}}$$

Delayed neutron adjustment factor

Prompt jump = CDS + instantaneous source multiplication

  – For a step reactivity insertion

$$p^0 = \frac{\beta}{\beta - \rho_1} p_0 = \frac{1}{1 - \rho_{1\$}} p_0$$

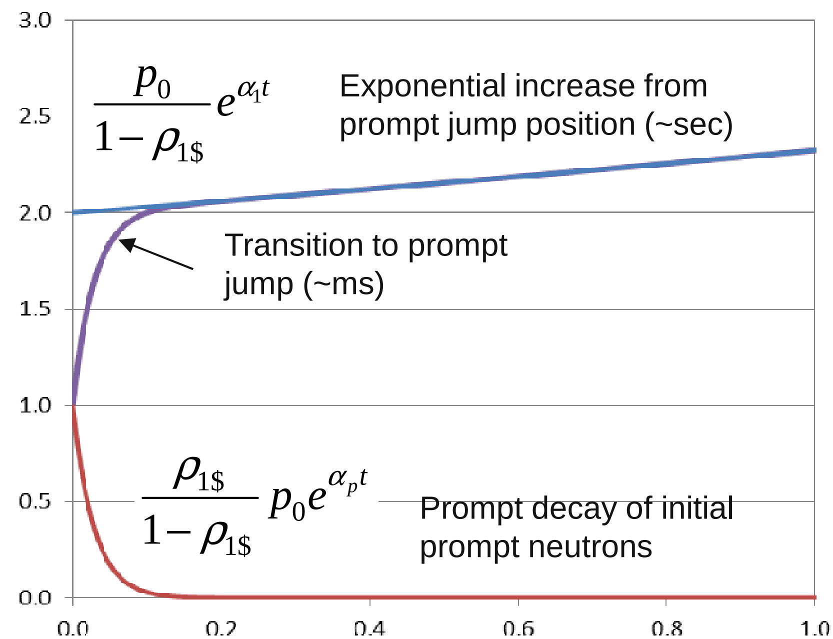
- Time-dependent stable period

$$\alpha_s(t) = \frac{\lambda \rho(t)}{\beta - \rho(t)}$$

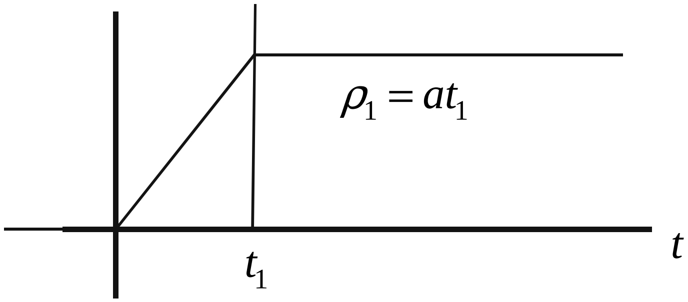
  – For a step reactivity insertion

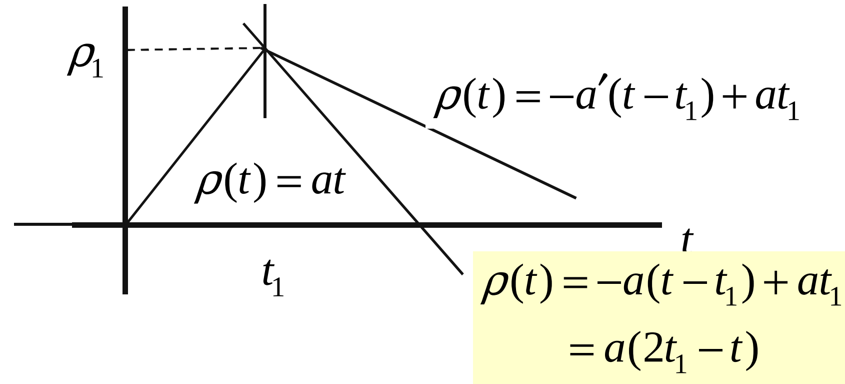
$$\alpha_s = \frac{\lambda \rho_1}{\beta - \rho_1} = \frac{\lambda \rho_{1\$}}{1 - \rho_{1\$}}$$

$$p(t) = p^0 e^{\alpha_s t}$$



Ramp Reactivity Change





$$p(t) = \frac{\beta p_0}{\beta - \rho(t)} e^{\int_0^t \frac{\lambda \rho(t')}{\beta - \rho(t')} dt'}$$

- t < t_1

$$\int_0^t \frac{\lambda \rho(t')}{\beta - \rho(t')} dt' = \lambda \int_0^t \frac{at'}{\beta - at'} dt' = \lambda \int_0^t \left(\frac{\beta}{\beta - at'} - 1 \right) dt' = \lambda \left[-\frac{\beta}{a} \ln\left(1 - \frac{a}{\beta} t\right) - t \right]$$

$$e^{\int_0^t \frac{\lambda \rho(t')}{\beta - \rho(t')} dt'} = e^{-\frac{\lambda \beta}{a} \ln\left(1 - \frac{a}{\beta} t\right) - \lambda t} = \left(1 - \frac{a}{\beta} t\right)^{-\frac{\lambda \beta}{a}} e^{-\lambda t}$$

$$p(t) = \frac{\beta p_0}{\beta - at} \left(1 - \frac{a}{\beta} t\right)^{-\frac{\lambda \beta}{a}} e^{-\lambda t} = \frac{p_0}{(1 - a_\$ t)^{(1 + \lambda / a_\$)}} e^{-\lambda t} \qquad a_\$ = \frac{a}{\beta}$$

Ramp Reactivity Change

- $t = t_1$

$$p(t_1) = \frac{p_0}{(1-\rho_{1\$})^{1+\lambda/a_\$}} e^{-\lambda t_1}$$

- $t > t_1$

$$\int_0^t \frac{\lambda\rho(t')}{\beta-\rho(t')} dt' = \lambda\int_0^{t_1} \frac{at'}{\beta - at'} dt' + \lambda\int_{t_1}^t \frac{a(2t_1-t')}{\beta-a(2t_1-t')} dt'$$

$$\lambda\int_{t_1}^t \frac{a(2t_1-t')}{\beta-a(2t_1-t')} dt' = \lambda\int_{t_1}^t \left[\frac{\beta}{\beta-a(2t_1-t')} - 1\right] dt' = \lambda\left[\frac{\beta}{a}\ln\frac{\beta-a(2t_1-t)}{\beta-at_1} - (t-t_1)\right]$$

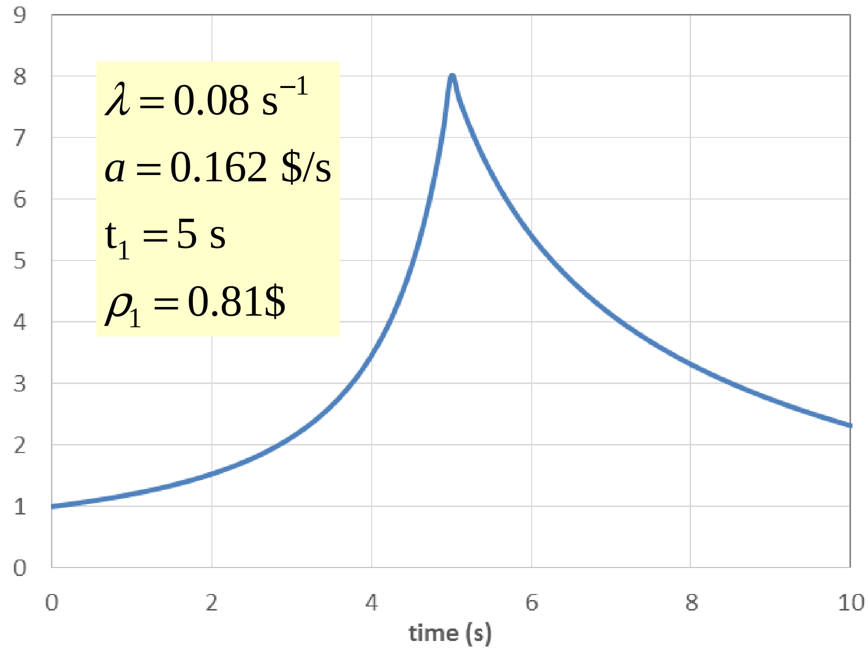
$$p(t) = \frac{p_0}{1-a_\$(2t_1-t)}\left[\frac{1-a_\$(2t_1-t)}{(1-a_\$t_1)^2}\right]^{\frac{\lambda}{a_\$}} e^{-\lambda t} = \frac{p_0}{(1-a_\$t_1)^{2\lambda/a_\$}} \frac{e^{-\lambda t}}{[1-a_\$(2t_1-t)]^{1-\lambda/a_\$}}$$

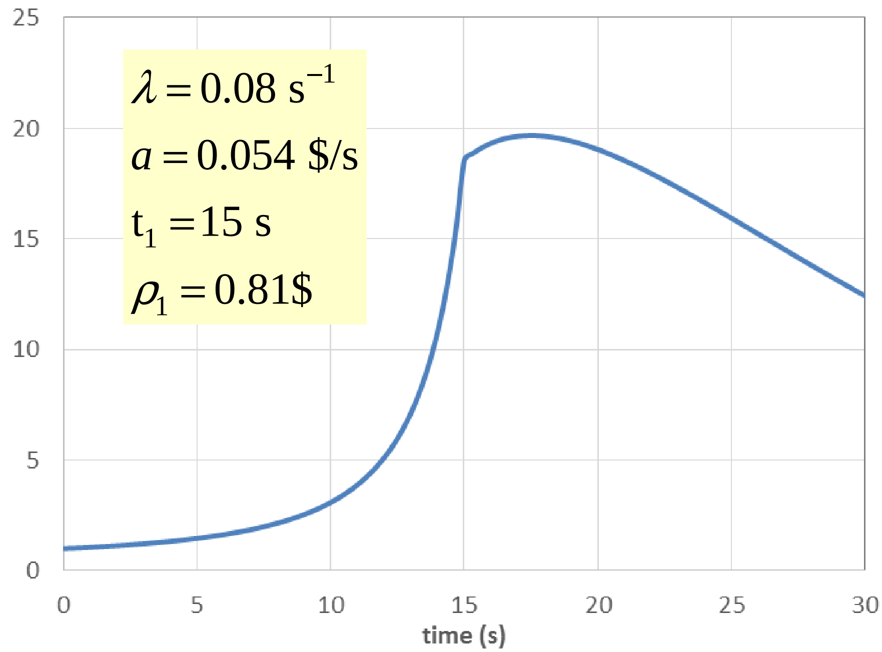
- $t = 2t_1$, $\rho = 0$ again

$$p(2t_1) = p_0 \frac{1}{(1-a_\$t_1)^{2\lambda/a_\$}} e^{-2\lambda t_1} = p_0 \frac{1}{(1-\rho_{1\$})^{2\lambda/a_\$}} e^{-2\lambda t_1} > p_0$$

Due to the precursor buildup during the rising period

Two Cases of Ramp Reactivity Change





Interpretation of PJA Rate of Changes

- Relative change rate of power amplitude

$$\frac{\dot{p}}{p} = \frac{\lambda\rho(t) + \dot{\rho}(t)}{\beta - \rho(t)}$$

- Supercritcal $\rho(t) > 0$

$$\rho(t)\downarrow \quad (\text{away from } \beta) \quad \Rightarrow \quad \dot{p}(t) < 0 \ \text{ if } \ \frac{\dot{\rho}(t)}{\rho(t)} < -\lambda \quad \Rightarrow \quad p\downarrow$$

  – The prompt neutron multiplication decreases more rapidly than the compensation by the delayed neutron generation

- Subcritical $\rho(t) < 0$

$$\rho(t)\uparrow \quad (\text{toward } \beta) \quad \Rightarrow \quad \dot{p}(t) > 0 \ \text{ if } \ \frac{\dot{\rho}(t)}{-\rho(t)} > \lambda \quad \Rightarrow \quad p\uparrow$$

  – The prompt neutron multiplication increases more rapidly than the decrease in the delayed neutron source
  – If there is no external source in a subcritical reactor, flux decreases since the delayed neutron source decreases by $s_d(t) \sim e^{-\lambda t}$

Prompt Kinetics Approximation

- Super-prompt critical

$$\rho > \beta \quad \Rightarrow \quad \alpha_p(t) = \frac{\rho(t) - \beta}{\Lambda} >> \lambda$$

$$\dot{p}(t) = \frac{\rho(t) - \beta}{\Lambda} p(t) + \frac{s_d(t)}{\Lambda} \approx \alpha_p(t) p(t)$$

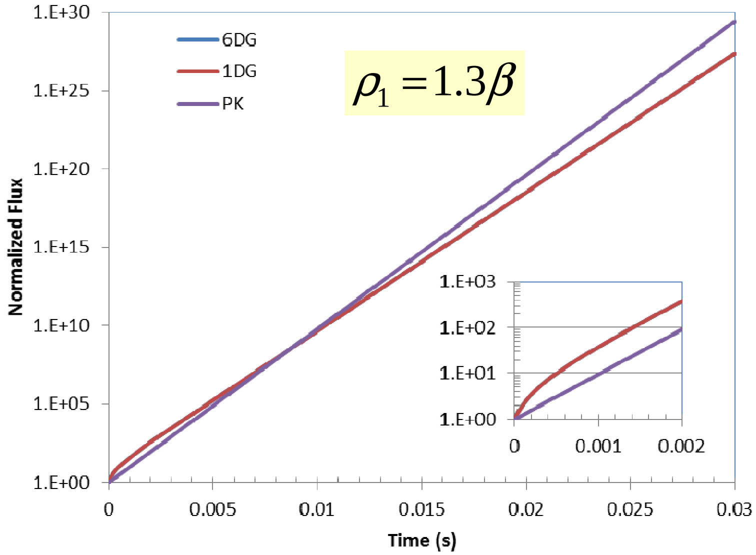
$$p(t) = p^0 \exp\left[\int_0^t \alpha_p(t') dt' \right]$$

- Pseudo-initial flux from 1DG solution for a step reactivity insertion

$$p(t) = p_0 \left[-\frac{\beta}{\rho_1 - \beta} \exp\left(-\frac{\lambda \rho_1}{\rho_1 - \beta} t \right) + \frac{\rho_1}{\rho_1 - \beta} \exp\left(\frac{\rho_1 - \beta}{\Lambda} t \right) \right] \approx \frac{\rho_1 p_0}{\rho_1 - \beta} \exp\left(\frac{\rho_1 - \beta}{\Lambda} t \right)$$

$$p^0 = p_{PK}^0 = \frac{\rho_1 p_0}{\rho_1 - \beta}$$

Comparison of PKE Solutions



$$\Lambda = 4.4\times10^{-7}\ \text{s},\quad \bar{\lambda} = 0.573\ \text{s}^{-1}$$

Group	β_k	λ_k (s^{-1})
1	0.000079	0.012966
2	0.000710	0.031287
3	0.000611	0.134616
4	0.001209	0.344560
5	0.000547	1.383070
6	0.000166	3.763340