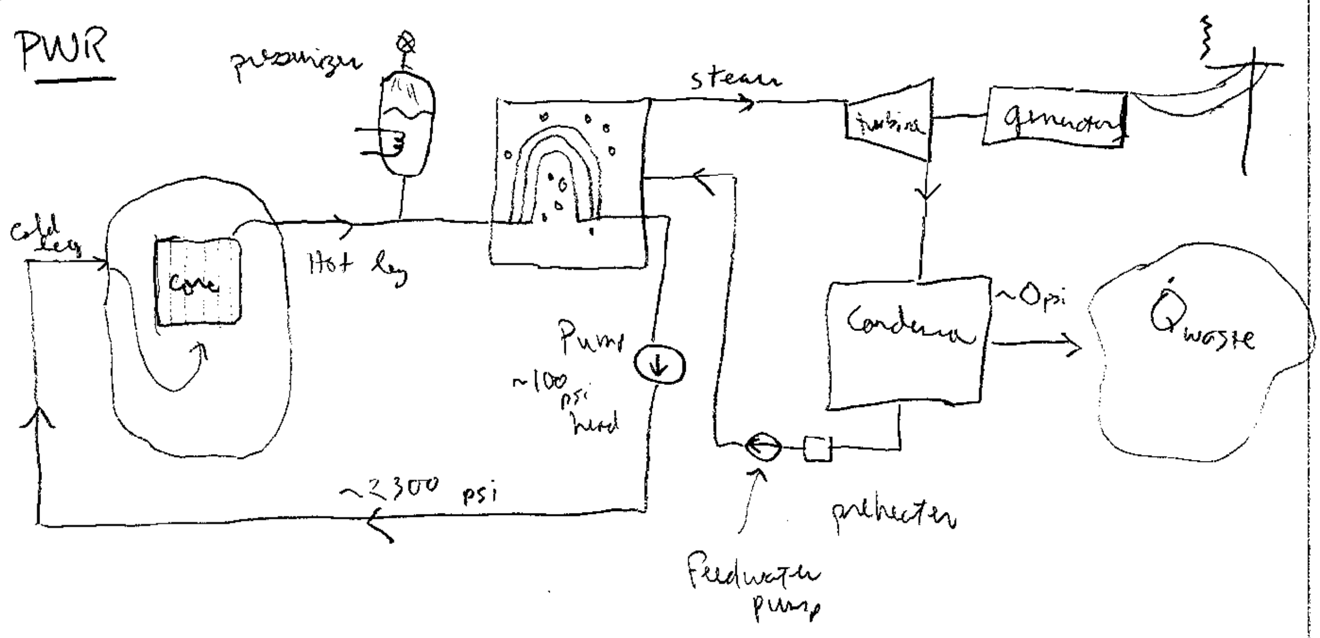
Intro. to Nuclear Thermal-Hydraulics Systems

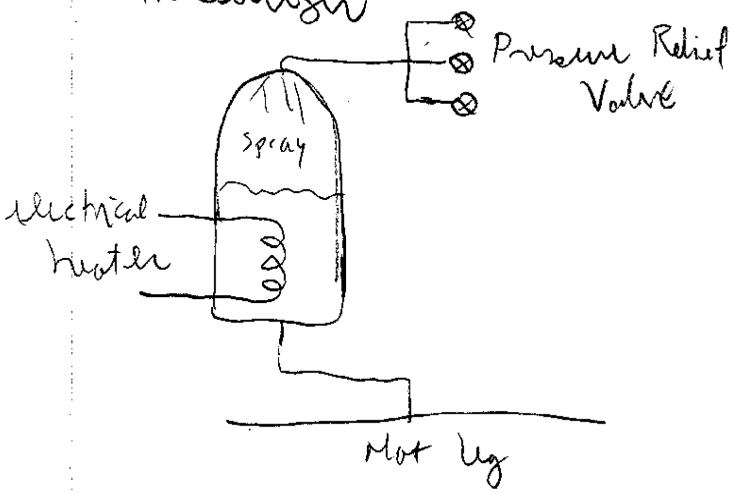
PWR



efficiency $\propto T_{core} \propto T_{cool}$

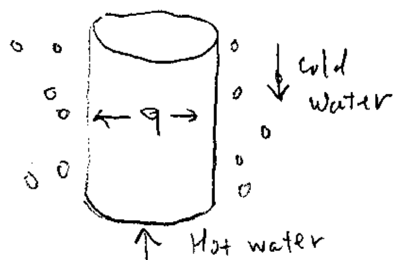
- Reactor
  - fission chain reaction
    - conduction through pellet → convection

- Pressurizer

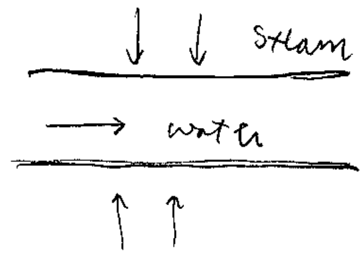


- if P↓ → Heater on ↑ vapor, P↑
- if P↑ → spray condensation, P↓

- Steam Generator
  - convective HT
  - nucleate boiling
  - CHF (Dryout)

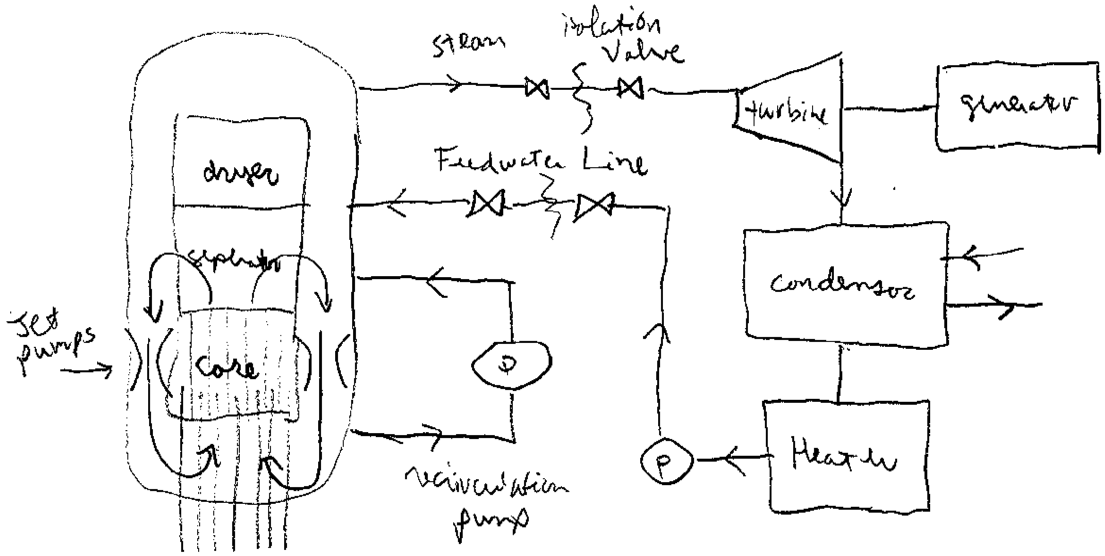


- turbine
  - compressible flow (expansion)
- condensor
  - conduction HT



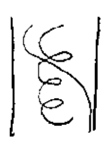
BWR

GE Hitachi, Toshiba (BWR, ABWR) (ESBWR)



CRods (ABWR has internal recirculation pumps)

RPV {
lower plenum
core ~ 45,000 rods in cans
flow restriction at core inlet
upper plenum
steam seperator (vortex flow)
steam dryer
}



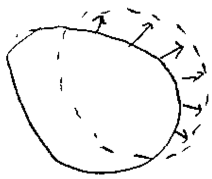
• vortex flow forces water out

Reynolds Transport Theorem Derivation

From Leibnitz Formula

$$\underbrace{\frac{d}{dt}\int_{V(t)} \Psi \, dV}_{\substack{\text{time rate of}\\ \text{change of total}\\ \text{within a CV}}} = \underbrace{\int_V \frac{\partial \Psi}{\partial t} \, dV}_{\substack{\text{total time}\\ \text{rate of}\\ \text{change of}\\ \Psi \text{ in CV}}} + \underbrace{\oint_S \Psi (\vec{v}_s \cdot \vec{n}) \, dS}_{\substack{\text{change in total amount}\\ \text{due to volume change}\\ \text{caused by surface}\\ \text{motion}}}$$

$\vec{v}_s$ = surface velocity



normal velocity
$\vec{v}_s \cdot \vec{n} \, dt$

Special Case of Material Volume

$$V(t) \rightarrow V_m(t)$$

$\vec{v}_s$: surface velocity $\rightarrow$ $\vec{v}$: material velocity

$$S \rightarrow S_m$$

We have

$$\frac{d}{dt}\int_{V_m} \Psi \, dV = \int_{V_m} \frac{\partial \Psi}{\partial t} \, dV + \oint_{S_m} \Psi (\vec{v} \cdot \vec{n}) \, dS_m$$

Using the Divergence theorem

$$\oint_{S_m} \Psi (\vec{v} \cdot \vec{n}) \, dS = \int_{V_m} \nabla \cdot (\Psi \vec{v}) \, dV$$

Yields

$$\underbrace{\boxed{\frac{d}{dt}\int_{V_m} \Psi \, dV}}_{\substack{\text{global}\\ \text{macroscopic}}} \Rightarrow \underbrace{\boxed{\int_{V_m} \left[\frac{\partial \Psi}{\partial t} + \nabla \cdot (\Psi \vec{v}) \right] dV}}_{\substack{\text{local}\\ \text{microscopic}}}$$

Integral Balance Equation

$$\boxed{\underbrace{\frac{D}{Dt}\int_{V_m} \Psi \, dV}_{\substack{\text{change of } \Psi \\ \text{in volume } V_m}} = \underbrace{-\oint_{S_m} \mathbb{J} \cdot \vec{n} \, dS}_{\substack{\text{influx of } \Psi \\ \text{across surface}}} + \underbrace{\int_{V_m} \dot{\Psi}_g \, dV}_{\substack{\text{generation of} \\ \Psi \text{ in } V_m}}}$$

Using Reynolds Transport theorem and Green's theorem

$$\frac{D}{Dt}\int_{V_m} \Psi \, dV = \int_V \left[\frac{\partial \Psi}{\partial t} + \nabla\cdot(\Psi \vec{v})\right] dV$$

and

$$-\oint_{S_m} \mathbb{J} \cdot \vec{n} \, dS = -\int_V \nabla \cdot \mathbb{J} \, dV$$

We have

$$\int_V \left[\frac{\partial \Psi}{\partial t} + \nabla\cdot(\Psi \vec{v})\right] dV = -\int_V \nabla\cdot \mathbb{J} \, dV + \int_{V_m} \dot{\Psi}_g \, dV$$

Integrating yields the

General Balance Equation

$$\boxed{\underbrace{\frac{\partial \Psi}{\partial t}}_{\substack{\text{time rate} \\ \text{of change} \\ \text{per unit} \\ \text{volume}}} + \underbrace{\nabla\cdot(\Psi \vec{v})}_{\substack{\text{convection of} \\ \Psi \text{ by material} \\ \text{motion per} \\ \text{unit volume}}} = \underbrace{-\nabla\cdot \mathbb{J}}_{\substack{\text{surface} \\ \text{influx} \\ \text{across } S_m}} + \underbrace{\dot{\Psi}_g}_{\substack{\text{generation} \\ \text{of } \Psi \text{ in } V_m}}}$$

Mass Balance Equation

$$\Psi = \rho$$
$$\mathbb{J} = 0 \quad \Rightarrow \quad \boxed{\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0}$$
$$\dot{\Psi}_g = 0$$

$$\frac{\partial \rho}{\partial t} + \vec{v}\cdot\nabla\rho + \rho\nabla\cdot\vec{v} = 0$$

Defining the substantial derivative as

$$\frac{D\rho}{Dt} = \frac{\partial\rho}{\partial t} + \vec{v}\cdot\nabla\rho$$

We have

$$\boxed{\frac{1}{\rho}\frac{D\rho}{Dt} = -\nabla\cdot\vec{v}}$$

For incompressible flow,

$$\frac{D\rho}{Dt} = 0$$

$$\boxed{\nabla\cdot\vec{v} = 0}$$

Momentum Balance Equation

$$\Psi = \rho\vec{v}$$
$$\mathbb{J} = \Pi = p\mathbb{I} + \mathbb{T}$$
$$\dot{\Psi}_g = \rho\vec{g}$$

p : pressure

$\mathbb{I}$: unit tensor

τ : viscous shear stress tensor

(i.e. $\tau_{xy} \rightarrow$ y-component force acting on y direction of face)

$\vec{g}$: gravity

$$\boxed{\underbrace{\frac{\partial(\rho\vec{v})}{\partial t}}_{\substack{\text{rate of}\\ \text{change of}\\ \text{momentum}\\ \text{per unit}\\ \text{volume}}} + \underbrace{\nabla\cdot(\rho\vec{v}\vec{v})}_{\substack{\text{rate of}\\ \text{momentum}\\ \text{change by}\\ \text{convection}\\ \text{per unit}\\ \text{volume}}} = \underbrace{-\nabla p}_{\substack{\text{pressure}\\ \text{force}}} \underbrace{-\nabla\cdot\mathbb{T}}_{\substack{\text{viscous}\\ \text{force}}} + \underbrace{\rho\vec{g}}_{\substack{\text{gravitational}\\ \text{force}}}}$$

$$\Downarrow$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \cdot \mathbb{T} + \rho \vec{g}$$

$$\boxed{\rho \vec{a} = \sum \text{forces}}$$

Boussinesq Approximation

- density change due to thermal expansion
- only important in gravity tensor

Thermal Expansion Coefficient, β

$$\beta \equiv \frac{1}{v} \frac{\partial v}{\partial T}\bigg|_p = -\frac{1}{\rho} \frac{\partial \rho}{\partial t}\bigg|_p$$

$$\left(v \equiv \text{specific volume} \equiv \frac{1}{\rho} \right)$$

We have,

$$d\rho = -\rho \beta \, dT$$

$$\boxed{\rho - \bar{\rho} = -\bar{\rho} \beta (T - \bar{T})} \quad \left(\begin{array}{l} \text{reference density } \bar{\rho} \\ \text{at reference temp. } \bar{T} \end{array} \right)$$

$$\therefore \quad \bar{\rho} \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \cdot \mathbb{T} + [\bar{\rho} - \bar{\rho}\beta(T - \bar{T})] \vec{g}$$

Total Energy Balance Eq.

$$\Psi = \rho \left(U + \frac{v^2}{2} \right)$$

$$\mathbb{J} = \vec{q} + \Pi \vec{v}$$

$$\dot{\Psi}_g = \rho \vec{v} \cdot \vec{g} + \dot{q}$$

U = internal energy

$\frac{v^2}{2}$ = kinetic energy

$\vec{q}$ = heat flux

$\Pi \cdot \vec{v}$ = work done by surface force

$\rho \vec{v} \cdot \vec{g}$ = work done by body force

$\dot{q}$ = heat generation

$$\underbrace{\frac{\partial\left(\rho\left(u+\frac{v^2}{2}\right)\right)}{\partial t}}_{\substack{\text{rate of change}\\ \text{of energy over}\\ \text{time}}} + \underbrace{\nabla\cdot\rho\vec{v}\left(u+\frac{v^2}{2}\right)}_{\substack{\text{rate of change}\\ \text{of energy due to convection}}} =$$

$$= \underbrace{-\nabla\cdot\vec{q}}_{\substack{\text{rate of}\\ \text{change}\\ \text{from}\\ \text{conduction}}} \underbrace{-\nabla\cdot(p\vec{v})}_{\substack{\text{work}\\ \text{done}\\ \text{by pressure}}} \underbrace{-\nabla\cdot(\tau\cdot\vec{v})}_{\substack{\text{work done}\\ \text{by viscous}\\ \text{forces}}} + \underbrace{\rho(\vec{v}\cdot\vec{g})}_{\substack{\text{work done}\\ \text{by gravity}}} + \underbrace{\dot{q}}_{\substack{\text{heat}\\ \text{generation}}}$$

<u>separate into mechanical and thermal energy eqs</u>

expand pressure and viscous surface force terms

(i) $-\nabla\cdot(p\vec{v}) = -p\,\nabla\cdot\vec{v}$ reversible pdV work to change T

$\quad - \vec{v}\cdot\nabla p$ kinetic energy

(ii) $-\nabla\cdot(\tau\cdot\vec{v}) = -\tau:\nabla\vec{v}$ irreversible work by viscous forces

$\quad -\vec{v}\cdot\nabla\cdot\tau$ kinetic energy

Separating gives,

<u>Mechanical Energy</u>

$$\boxed{\rho\frac{D\frac{v^2}{2}}{Dt} = -\vec{v}\cdot\nabla p - \vec{v}\cdot\nabla\cdot\tau + \rho\vec{v}\cdot g}$$

<u>Thermal Energy</u>

$$\boxed{\rho\frac{Du}{Dt} = -\nabla\cdot\vec{q} - p\,\nabla\cdot\vec{v} - \tau:\nabla\vec{v} + \dot{q}}$$

Constitutive Relations

Why?

Balance Eq.

$$\left.\begin{array}{ll} \text{Mass} & \rho, \vec{v} \\ \text{Energy} & \rho\vec{v}, \mathbb{T}, \vec{g} \\ \text{Momentum} & u, \vec{q}, \dot{q} \end{array}\right\} \text{3 eqs. 8 unknowns} + T, s$$

What?

- model material response from experiments
- doesn't violate any physical laws
- have reasonably simple mathematical form
- fit practical idealization
- Restrictions:
  - Determinism
  - Frame Independence
  - Local Action
  - Entropy Inequality

$$\frac{\partial}{\partial t} \rho s + \nabla \cdot (\rho s \vec{v}) + \nabla\left(\frac{\vec{q}}{T}\right) - \frac{\dot{q}}{T} \equiv \Delta \geq 0$$

$$\Delta > 0 \rightarrow \text{irreversible (physical)}$$

- Types:
  - Equation of State (relate $\rho, e, u, P, T \ldots$)
  - Mechanistic (gives $\vec{g}, \mathbb{T}$)
  - Thermal (gives $\vec{q}, \dot{q}$)

Equations of state

Fundamental Forms

- $u = u(s, \rho)$
- $T \equiv \left.\frac{\partial u}{\partial s}\right)_\rho$
- $P \equiv -\left.\frac{\partial u}{\partial(1/\rho)}\right)_s$
- $du = T\,ds - P\,d(1/\rho)$

Practical Forms

- $P = P(\rho, T) \Rightarrow$ Thermal EoS
- $u = u(\rho, T) \Rightarrow$ Caloric EoS

Ex: Incompressible Fluids

- $\frac{D\rho}{Dt} = 0$, ρ = constant
- $u = u(T)$, $du = c_v\,dT$

Ideal Gas

- $P = \rho R T$
- $u = u(T)$

Mechanistic Constitutive Relations

- $\vec{g}$ = gravity
- Inviscid $\rightarrow \tau = 0$

  Newtonian $\rightarrow \tau_{yx} = -\mu \frac{dV_x}{dy}$

- Mass diffusion (mixtures)

$$J_k = \rho_k V_{km} = -\rho \nabla\left(\frac{\rho_k}{\rho}\right)$$

- Chemical Reaction

$$\vec{r}_k = r_k(\omega_1, \omega_2 \ldots)$$

Thermal Constitutive Relations

- Conduction

$$\vec{q} = -k \nabla T$$

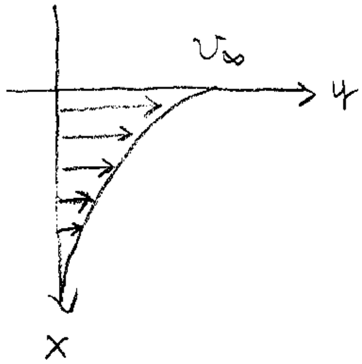
- generation

fission, electrical resistance, magnetic induction
thermal radiation, $\dot{q} = \dot{q}(\vec{x}, t)$ decay heat

Entropy Generation Check

$$s > 0$$

Sudden Motion



- at $t < 0$ $\quad v_x, v_y = 0$
- at $t > 0$ $\quad v_y(0,y) = v_\infty$

  $v_x(0,y) = 0$
- incompressible ρ = constant
- newtonian μ = constant
- uniform pressure $P = P_\infty$

From mass balance equation,

$$\cancel{\frac{\partial \rho}{\partial t}}^{\,0} + \nabla\cdot(\rho v) = 0$$

$$\rho\left(\frac{\partial v_x}{\partial x} + \cancel{\frac{\partial v_y}{\partial y}}^{\,0}\right) = 0$$

$$\frac{\partial v_x}{\partial x} = 0$$

$$v_x = 0$$

From momentum balance equation,

$$\frac{\partial(\rho v)}{\partial t} + \nabla\cdot(\rho v v) = -\nabla p - \nabla\cdot\tau + \rho\vec{g}$$

With newtonian fluid the y-component becomes

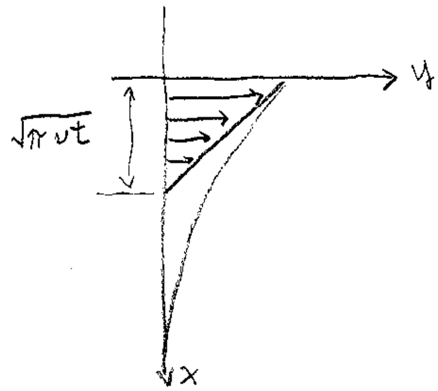
$$\rho\left(\frac{\partial v_y}{\partial t} + \cancel{v_x}^{\,0}\frac{\partial v_y}{\partial x} + v_y \cancel{\frac{\partial v_y}{\partial y}}^{\,0}\right) = -\cancel{\frac{\partial p}{\partial y}}^{\,0} + \mu\left(\frac{\partial^2 v_y}{\partial x^2} + \cancel{\frac{\partial^2 v_y}{\partial y^2}}^{\,0}\right) + \cancel{\rho \vec{g}_y}^{\,0}$$

$$\rho\frac{\partial v_y}{\partial t} = \mu\left(\frac{\partial^2 v_y}{\partial x^2}\right)$$

$$\boxed{\frac{\partial v_y}{\partial t} = \nu \frac{\partial^2 v_y}{\partial x^2}}$$

where $\nu = \frac{\mu}{\rho}$ = kinematic viscosity

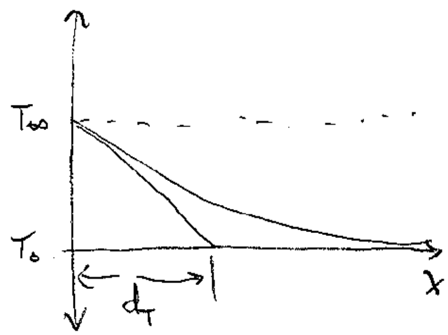
Penetration Depth



momentum penetration depth

$$d_{p\nu} = \sqrt{\pi \nu t}$$

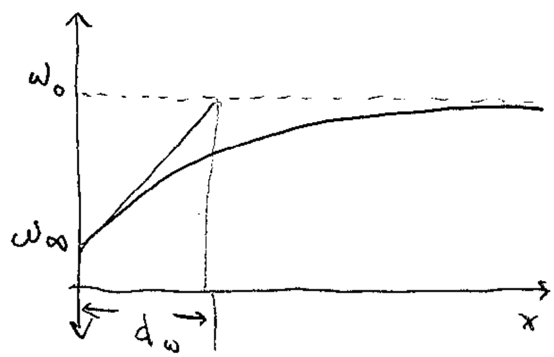
$$\nu = \frac{\mu}{\rho}$$



thermal penetration depth

$$d_T = \sqrt{\pi a t}$$

$$a = \frac{k}{\rho C_v}$$



mass penetration depth

$$d_\omega = \sqrt{\pi D t}$$

D = diffusion coefficient

Heat Conduction in Solids

Fourier's Law of Heat Conduction

$$\boxed{\vec{q}'' = -k\nabla T} \qquad [W/m^2]$$

Newton's Law of Cooling

$$\boxed{q'' = h\left(T_s - T_\infty\right)}$$

Heat equation

$$\boxed{\nabla\cdot(k\nabla T) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}}$$

Cartesian

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Cylindrical

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

Spherical

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t}$$

Boundary Condition

Constant surface temperature $\boxed{T(0,t) = T_s}$

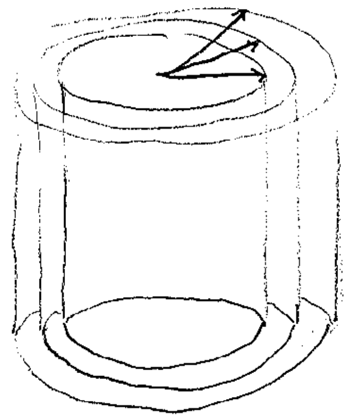
constant surface heat flux

finite $\boxed{-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_s''}$

adiabatic $\boxed{\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0}$

Convection surface condition $\boxed{-k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left(T_\infty - T(0,t)\right)}$

Heat Conduction with Generation



$$\frac{d}{dr}\left(r q_r\right) = S_n r$$

In the fuel fission generation of

$$S_n = S_{n_0}\left[1 - b\left(\frac{r}{R^{(F)}}\right)^2\right]$$

Therefore we have

Fuel: $$\frac{d}{dr}\left(r q_r^F\right) = S_{n_0}\left[1 - b\left(\frac{r}{R^F}\right)^2\right] r$$

Gap: $$\frac{d}{dr}\left(r q_r^G\right) = 0$$

Clad: $$\frac{d}{dr}\left(r q_r^C\right) = 0$$

Integrating gives

$$r q_r^F = \frac{S_{n_0} r^2}{2} - \frac{S_{n_0} b}{R^{F2}} \frac{r^4}{4} + C_1^F$$

$$r q_r^G = C_1^G$$

$$r q_r^C = C_1^C$$

At $r = 0$, q_r^F is finite, therefore $C_1^F = 0$

At $r = R^F$, $q_r^F = q_r^G$

$$\frac{S_{n_0} R^F}{2} - \frac{S_{n_0} b}{R^{F^2}} \frac{R^{F^3}}{4} = \frac{C_1^G}{R^F}$$

$$C_1^G = \frac{S_{n_0} R^{F^2}}{2} - \frac{S_{n_0} b \; R^{F^2}}{4}$$

$$C_1^G = \frac{S_{n_0} R^{F^2} - 2 S_{n_0} b R^{F^2}}{2}$$

$$C_1^G = \frac{S_{n_0} R^{F^2} (1 - 2b)}{2} = S_{n_0} R^{F^2} \left(\frac{1}{2} - b\right)$$

At $r = R^G$, $q_r^G = q_r^C$

$$\frac{S_{n_0} R^{F^2} \left(\frac{1}{2} - b\right)}{R^G} = \frac{C_1^C}{R^G}$$

$$\therefore C_1^C = S_{n_0} R^{F^2} \left(\frac{1}{2} - b\right)$$

therefore

$$q_r^F = S_{n_0} \left(\frac{r}{2} - \frac{b r^3}{4 R^{F^2}}\right)$$

$$q_r^G = \frac{S_{n_0} R^{F^2}}{r} \left(\frac{1}{2} - b\right)$$

$$q_r^C = \frac{S_{n_0} R_F^2}{r} \left(\frac{1}{2} - b\right)$$

Using Fourier's Law

$$-k^F \frac{dT^F}{dr} = \frac{S_{n0}}{2} r - \frac{S_{n0} b}{4 R^{F^2}} r^3$$

$$-k^G \frac{dT^G}{dr} = S_{n0} R^{F^2} \left(\frac{1}{2} - b\right) \frac{1}{r}$$

$$-k^C \frac{dT^C}{dr} = S_{n0} R^{F^2} \left(\frac{1}{2} - b\right) \frac{1}{r}$$

Integrating gives

$$\frac{dT^F}{dr} = -\frac{S_{n0}}{k^F}\left(r - \frac{b}{4R^{F^2}} r^3\right)$$

$$T^F = -\frac{S_{n0}}{k^F}\left(\frac{r^2}{2} - \frac{b}{16 R^{F^2}} r^4\right) + C_2^F$$

$$T^G = \left[-\frac{S_{n0} R^{F^2}}{k^G}\left(\frac{1}{2} - b\right)\right] \ln r + C_2^G$$

$$T^C = \left[-\frac{S_{n0} R^{F^2}}{k^C}\left(\frac{1}{2} - b\right)\right] \ln r + C_2^C$$

At $r = R^F, \quad T^F = T^G$

$r = R^G, \quad T^G = T^C$

$r = R^C, \quad T^C = T_0$

And so on ......

Characteristics of Turbulent Flow

Laminar → Turbulent at $Re \sim 2000$

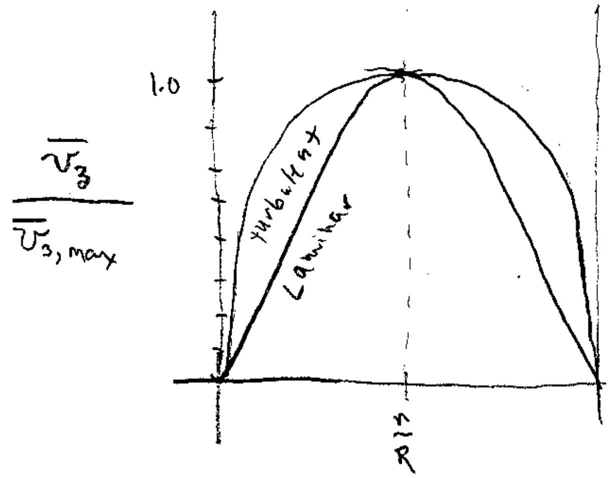
$$Re \equiv \frac{\rho v D}{\mu} = \frac{\text{inertia}}{\text{viscous}}$$

Laminar velocity profile is parabolic

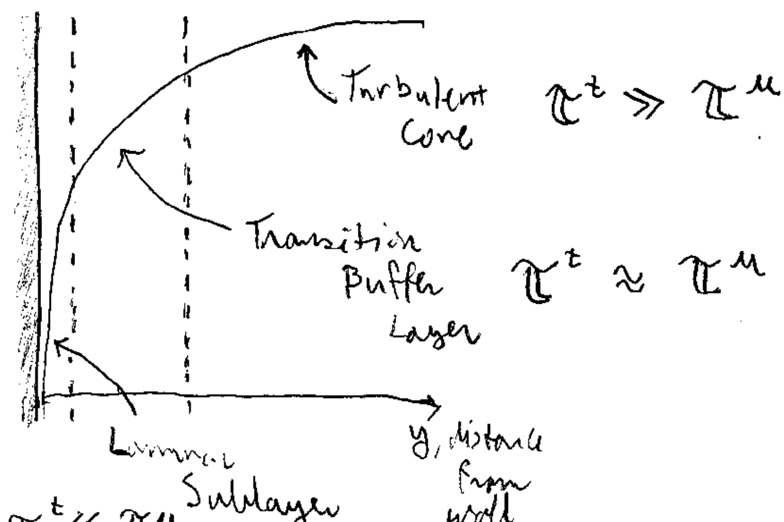
$$v_z = v_{z,max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \frac{\langle v_z \rangle}{v_{z,max}} = \frac{1}{2}$$

Turbulent velocity profile is 1/7 power law

$$v_z = v_{z,max}\left[1 - \left(\frac{r}{R}\right)^{1/7}\right] \qquad \frac{\langle v_z \rangle}{v_{z,max}} = \frac{4}{5}$$



Turbulent Velocity Profiles



Reynold's Stresses Formulation

Using $\left|\frac{\rho'}{\rho}\right| \ll \left|\frac{v'}{v}\right|$ we write

$$\vec{v} = \bar{v} + v'$$
$$\mu = \text{constant}$$
$$\rho = \text{constant}$$

i) Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0 \Rightarrow \nabla \cdot \vec{v} = 0 \Rightarrow \left\{ \begin{array}{l} \nabla \cdot \bar{v} = 0 \\ \nabla \cdot v' = 0 \end{array} \right\}$$

ii) momentum

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v}\vec{v} = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g}$$

averaging gives

$$\frac{\partial \rho \bar{v}}{\partial t} + \nabla \cdot \rho \bar{v}\bar{v} + \nabla \cdot \rho \overline{v'v'} = -\nabla \bar{p} + \mu \nabla^2 \bar{v} + \rho \vec{g}$$

rewriting gives

$$\frac{\partial \rho \bar{v}}{\partial t} + \nabla \cdot \rho \bar{v}\bar{v} = -\nabla \bar{p} + \Big[\underbrace{\mu \nabla^2 \bar{v}}_{\text{viscous flux}} - \underbrace{\nabla \cdot \rho \overline{v'v'}}_{\text{turbulent flux}} \Big] + \rho \vec{g}$$

$\boxed{\text{"Reynolds stresses"}}$

$$\tau^T = \tau^\mu + \tau^t$$

where

$$\tau^\mu = -\nu\rho \left[\nabla \bar{v} + (\nabla \bar{v})^+ \right] \rightarrow \text{newtonian}$$

$$\tau^t = \rho \overline{v'v'} \longrightarrow \text{reynolds stress}$$

Stress in Turbulent Flow

From momentum equation

$$\frac{\partial \rho \bar{v}}{\partial t} + \nabla \cdot \rho \bar{v} \bar{v} = -\nabla \bar{P} - \nabla \left(\tau^{\mu} + \tau^{t} \right) + \rho \vec{g}$$

For pipe flow,

$$\rho \left(\frac{\partial \bar{v}_z}{\partial t} + \bar{v}_r \frac{\partial \bar{v}_z}{\partial r} + \frac{\bar{v}_\theta}{r} \frac{\partial \bar{v}_z}{\partial \theta} + \bar{v}_z \frac{\partial \bar{v}_z}{\partial z} \right) =$$

$$= -\frac{\partial \bar{P}}{\partial z} + \rho g_z + \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \, \tau_{rz}^{T} \right) + \frac{1}{r} \frac{\partial \tau_{\theta z}^{T}}{\partial \theta} + \frac{\partial \tau_{zz}}{\partial z} \right]$$

Assuming .

steady state $\frac{\partial}{\partial t} \to 0$

no gravity $g_z \to 0$

fully developed $\bar{v}_r = \bar{v}_\theta = 0$, $\frac{\partial \bar{v}_z}{\partial z} = 0$

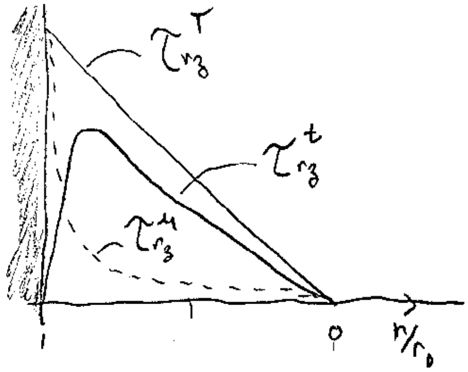
axisymmetric $\frac{\partial}{\partial \theta} = 0$

We have

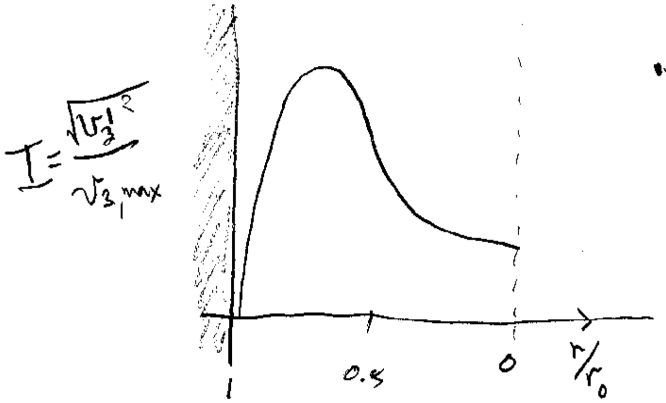
$$\frac{\partial \bar{P}}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} \left(r \, \tau_{rz}^{T} \right)$$

$$\therefore \boxed{\tau_{rz}^{T} = -\frac{\Delta p}{L} \cdot \frac{r}{2}}$$

$$\tau_{rz}^{T} = \tau_{rz}^{\mu} + \tau_{rz}^{t}$$



- max shear stress near wall and 0 at center



- max intensity at $r/r_0 = 0.9$
- $I(r/r_0 = 1) = 0$
- $I(r/r_0 = 0) \neq 1$

- turbulence is generated near the wall
- from instability of navier-stokes equation

Turbulent Flow Models

i) Boussinesq's Eddy Viscosity

analogous to newton

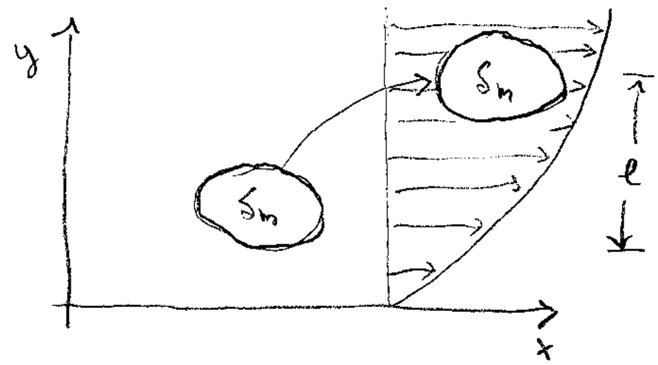
$$\tau^t_{yx} = -\mu^t \frac{d\bar{v}_x}{\partial y}$$

where

μ^t = eddy viscosity

problematic: μ^t is nonlinear and a function of position

ii) Prandtl's Mixing Length Model



l: mixing length
δm: eddy mass

(1) δm does not lose or gain any x-component momentum until it moves l, then complete momentum exchange

(2) x-component gained in δm is given by

$$= \delta m \, \delta v_x$$

At rate defined by

$$= \delta m \frac{\delta v_x}{\delta t}$$

Shear force acting

$$F = \delta m \frac{\delta v_x}{\delta t}$$

Therefore the shear stress is

$$\tau^t = \frac{F}{A} = -\frac{1}{A} \frac{\delta m}{\delta t} \delta v_x$$

For small l,

$$\delta v_x = \frac{d\bar{v}_x}{dy} l$$

Mass transfer given by

$$\frac{1}{A}\frac{\delta m}{\delta t} = \rho |v_y'|$$

Therefore we rewrite the shear stress as

$$\frac{\tau^t}{\rho} = -l |v_y'| \frac{d\bar{v}_x}{dy}$$

Prandtl's assumptions

$$|v_y'| = k_1 |v_x'|$$

$$v_x' = k_2 \delta v_x = k_2 l \frac{d\bar{v}_x}{dy}$$

Therefore

$$\frac{\tau^t}{\rho} = -l^2 \left|\frac{d\bar{v}_x}{dy}\right| \frac{d\bar{v}_x}{dy}$$

Assumed the mixing length l is proportional to the distance from the wall

$$l = ky$$

Therefore

$$\boxed{\frac{\tau^t}{\rho} = -k^2 y^2 \left|\frac{d\bar{v}_x}{dy}\right| \frac{d\bar{v}_x}{dy}}$$

where k is independent of fluid, needed to be determined experimentally

Near the wall

$$\tau^t \approx \tau_w$$

We have

$$\frac{d\bar{v}_x}{dy} = \frac{1}{k}\frac{1}{y}\sqrt{\frac{\tau_w}{\rho}}$$

Taking the non-dimensional forms

$$v^* = \frac{v_x}{\sqrt{\tau_w/\rho}}$$

$$y^* = \frac{y\sqrt{\tau_w/\rho}}{\nu}$$

Gives

$$\frac{dv^*}{dy^*} = \frac{1}{ky^*}$$

With solution

$$\boxed{v^* = \frac{1}{k}\ln(y^*) + C}$$

Experimentally determined constants yield three regions

Laminar Sublayer

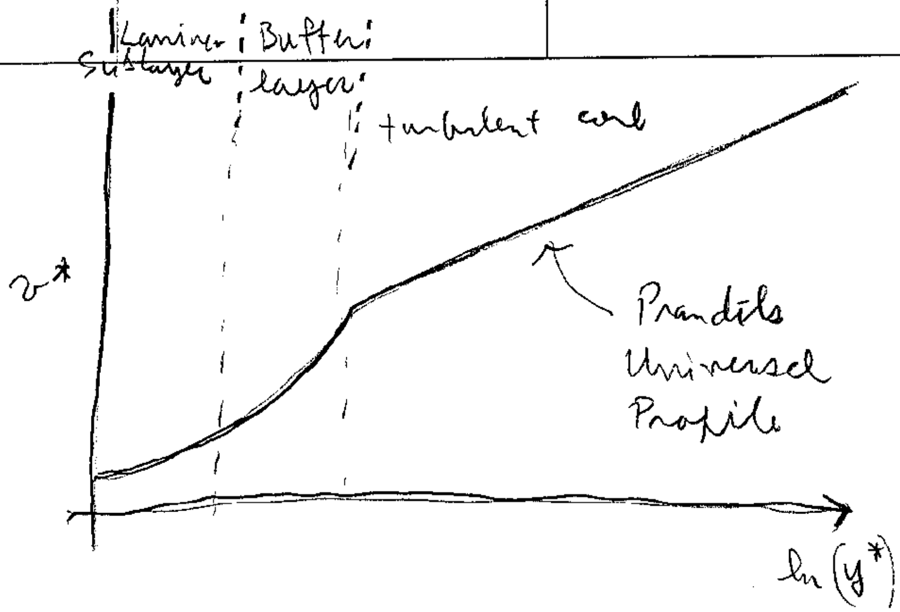
$$\tau^t = 0, \ y^* < 5 \Rightarrow v^* = y^*$$

Buffer Layer

$$5 < y^* < 30 \Rightarrow v^* = -3.05 + 5\ln y^*$$

Turbulent Core

$$y^* > 30 \Rightarrow v^* = 5.5 + 2.5\ln y^*$$


Laminar Sublayer
Buffer layer
turbulent core
v^*
Prandtl's Universal Profile
$\ln(y^*)$

Karman - Martenelli Analogy

$$\frac{\tau^t}{\rho} = -\left(\epsilon_m + \nu\right)\frac{d v_x}{dy}$$

where ϵ_m = eddy diffusivity from Prandtl

Analogous to

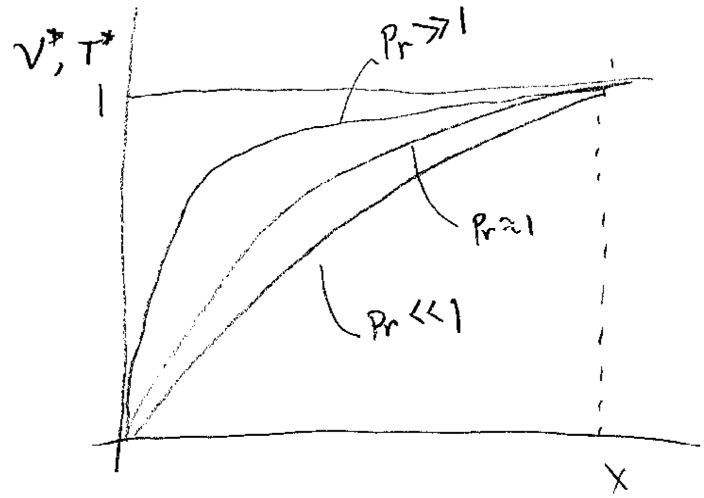
$$\frac{q''}{\rho C} = -\left(\epsilon_H + \alpha\right)\frac{\partial T}{\partial y}$$

Temperature Profile $\Rightarrow T = T(r, Pr)$

where $Pr = 1 \Rightarrow$ T. profile $\approx$ v profile

$$v^* = \frac{v - \bar{v}}{v_c - \bar{v}}$$

$$T^* = \frac{T - \bar{T}}{T_c - \bar{T}}$$



1Φ Non-Dimensional Analysis

D: characteristic length
V: characteristic velocity

Defining the dimensionless variables and operators

$$v^* = \frac{v}{V} \qquad p^* = \frac{p - p_0}{\rho V^2} \qquad t^* = \frac{tV}{D}$$

$$x^* = \frac{x}{D} \qquad y^* = \frac{y}{D} \qquad z^* = \frac{z}{D}$$

$$\nabla^* = D\nabla = \left(\delta_1 \frac{\partial}{\partial x^*} + \delta_2 \frac{\partial}{\partial y^*} + \delta_3 \frac{\partial}{\partial z^*}\right)$$

$$\nabla^{*2} = D^2 \nabla^2 = \frac{\partial^2}{\partial x^{*2}} + \frac{\partial^2}{\partial y^{*2}} + \frac{\partial^2}{\partial z^{*2}}$$

$$\frac{D}{Dt^*} = \left(\frac{D}{V}\right)\frac{D}{Dt}$$

Continuity Eq.

$$\nabla \cdot \vec{v} = 0$$

Eq. of Motion

$$\rho \frac{D\vec{v}}{Dt} = -\nabla P + \mu \nabla^2 \vec{v} + \rho \vec{g}$$

Dimensionless variables yield

$$\nabla^* \cdot \vec{v}^* = 0$$

$$\frac{D\vec{v}^*}{Dt^*} = -\nabla^* p^* + \left[\frac{\mu}{DV\rho}\right]\nabla^{*2}\vec{v}^* + \left[\frac{gD}{V^2}\right]\frac{\vec{g}}{g}$$

Reynolds $\boxed{Re = \frac{DV\rho}{\mu} = \frac{\text{inertial}}{\text{viscosity}}}$

Froude $\boxed{Fr = \frac{V^2}{gD} = \frac{\text{inertial}}{\text{gravity}}}$

$\boxed{\text{Hydrodynamic Similarity} \quad Re,\ Fr}$

Eckert $\boxed{E_c = \frac{v^2}{C_p \Delta T}} = \frac{\text{KE convection}}{\text{enthalpy convection}}$

ideal gas $\boxed{E_c = (\gamma - 1) M^2 \left(\frac{T_0}{\Delta T}\right)}$

Mach $\boxed{M = \frac{V}{u_s}}$

$M > 0.3 \Rightarrow$ compressible flow

Forced Convection

ME $\rightarrow$ Re

EE $\rightarrow$ Re, Pr, (M)

Natural Convection

ME $\rightarrow$ Gr

EE $\rightarrow$ Gr, Pr

Grashof $\boxed{G_r = \frac{g \beta \Delta T D^2}{\nu^2}} = \frac{(\text{bouyancy})(\text{inertial})}{(\text{viscous forces})^2}$

Peclet $\boxed{P_e = \frac{\rho_0 C_p \Delta V}{k}} = \frac{\text{HT by convection}}{\text{HT by conduction}}$

$P_e = Re\, Pr$

Prandtl $\boxed{P_r = \frac{\mu C_p}{k}} = \frac{\text{momentum diffusion}}{\text{thermal diffusion}}$

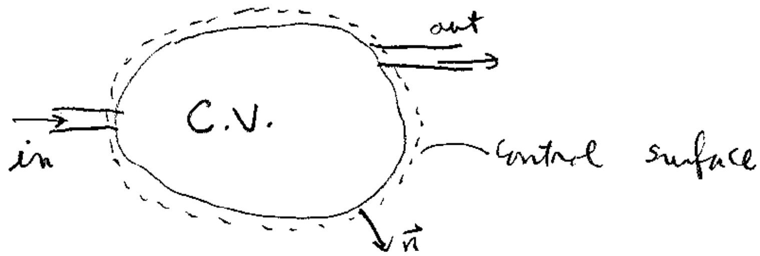
$P_r \ll 1$ for liquid metals

$P_r \gg 1$ for oils

$P_r \approx 1$ for water and gas

$\boxed{\text{Thermal similarity garned by } P_e}$

Control Volume Analysis



· Mass Conservation

$$\frac{d}{dt}\int_{cv} \rho \, d\forall = -\oint_{cs} \rho \vec{U} \cdot \vec{n} \, dA$$

$$\frac{d}{dt}\int_{cv} \rho \, d\forall = (\rho A V)_{in} - (\rho A V)_{out}$$

$$\boxed{\frac{d}{dt}\int_{cv} \rho \, d\forall = \sum \dot{m}_A}$$

· Momentum Conservation

$$\boxed{\frac{d}{dt}\int_{cv} \rho \vec{v} \, d\forall = \sum \vec{F} + \sum_i \dot{m}_i \vec{V}_i}$$

$\vec{F}$, forces: gravity, pressure, shear, external

· Energy Balance

$$\boxed{\frac{d}{dt}\int_{cv} \rho e \, d\forall = \dot{Q} + \dot{W}_s + \sum \dot{m}_i \left(e + \frac{P}{\rho}\right)}$$

where $e = u + \frac{v^2}{2} + gz$

$\dot{Q}$ = heat definition

$\dot{W}_s$ = work done on system

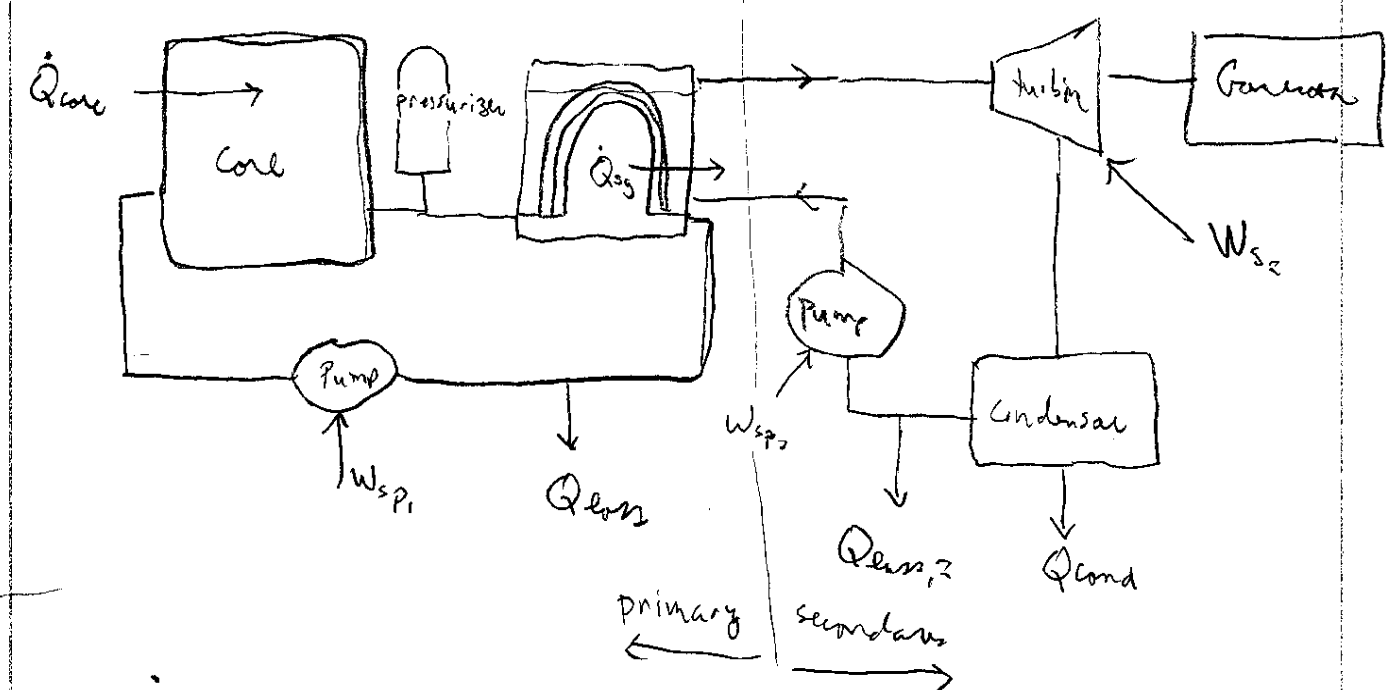
Defining enthalpy

$$u + \frac{P}{\rho} = i$$

Normally $\frac{v^2}{2} + gz \ll u$

$$\boxed{\frac{d}{dt}\int_{cv} \rho u \, dv = \dot{Q} + \dot{W}_s + \sum \dot{m}_i (i)_i}$$

Integral Nuclear Reactor Thermalhydraulics



$\dot{Q}_{core} > 0$ fission, decay heat

$\dot{Q}_{sg} < 0$ $|\dot{Q}_{sg}| > |\dot{Q}_{cond}|$ Transfer to SG

$\dot{Q}_{loss} < 0$ Small

$\dot{W}_{sp_1}$ shaft work by pump, (30 MW)

$\dot{W}_{s_2}$

LOCA

$\dot{m}_i < 0$

$$\frac{d}{dt}\int_{cv} \rho\, dV = -(\rho V A)_{break}$$

- primary coolant inventory decreases
- ECCS comes in

$$\frac{d}{dt}\int_{cv} \rho\, dV = (\rho V A)_{ECCS} - (\rho V A)_{break}$$

- design criteria

$$(\rho V A)_{ECCS} - (\rho V A)_{break} > 0$$

Energy Equation

$$\frac{d}{dt}\int \rho e\, dV = \left(\dot{Q}_{core} + W_{sp}\right) + \sum m_i \left(e + \frac{P}{\rho}\right)$$

steady → $\dot{Q}_{sg} = \dot{W}_{sp} + \dot{Q}_{loss} - \dot{Q}_{core}$

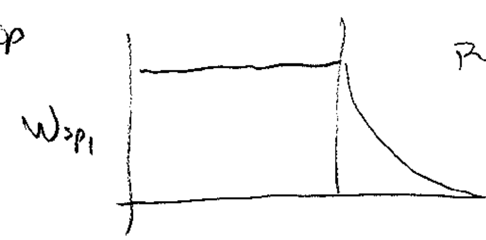
unsteady $\sum m_i \left(e + \frac{P}{\rho}\right) = 0$

LOCA ⇒ $$\frac{d}{dt}(\rho e\, dV) = \left(\dot{Q}_{core} + \dot{W}_{sp_1}\right) - \left(\dot{Q}_{sg} + \dot{Q}_{loss}\right) - \dot{m}_{break}\left(e + \frac{P}{\rho}\right)_{break}$$

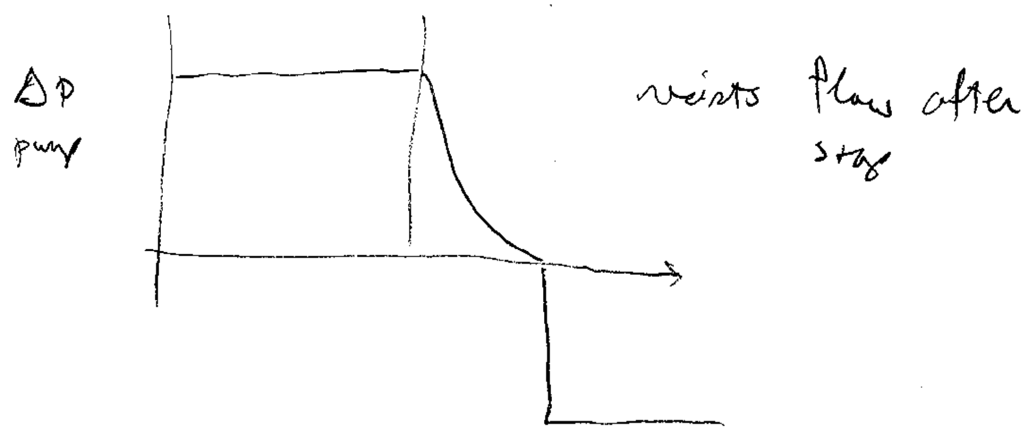
$$\frac{d}{dt}\int \rho e\, dV < 0$$

1) initially $T \downarrow$
2) core uncovered $T \uparrow$

Pump Trip



prevents damage.



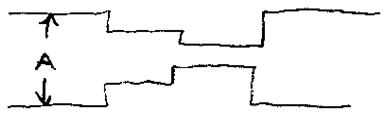
ΔP
pump
resists flow after
stop

Area Averaging

$$\overline{\Psi} = \frac{1}{A}\int_A \Psi \, dA$$

assume axisymmetric $\frac{\partial}{\partial\theta} = 0 \quad V_\theta = 0$

Constant area $\rightarrow A =$ constant



Time derivative

$$\frac{1}{A}\int \frac{\partial \rho\Psi}{\partial t} dA = \frac{\partial}{\partial t}\langle \rho \Psi \rangle$$

Gradient Operation in z

$$\frac{1}{A}\int_A (\nabla s)_z \, dA = \frac{1}{A}\int \frac{\partial s}{\partial z} dA = \frac{\partial}{\partial z}\langle s \rangle$$

Vector Operation

$$\frac{1}{A}\int_A (\nabla \cdot \tau)_z \, dA = \frac{\partial}{\partial z}\langle \tau_{zz} \rangle + \frac{1}{A} 2\pi r_0 \, \tau_{rz}\Big|_{r_0}$$

Hydraulic Diameter, D

$$\frac{2\pi r_0}{A} = \frac{4}{D}$$

1-D formulation

Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0$$

$$\boxed{\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z}\langle \rho v_z \rangle = 0}$$

Momentum Equation

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v}\vec{v} = -\nabla P + \nabla \cdot \tau + \rho \vec{g}$$

$$\boxed{\frac{\partial}{\partial t}\rho\langle v_z\rangle + \frac{\partial}{\partial z}\rho\langle v_z v_z\rangle = \underbrace{-\frac{\partial\langle P\rangle}{\partial z}}_{\text{pressure}} \underbrace{- \frac{\partial}{\partial z}\tau_{zz}}_{\text{normal stress}} \underbrace{- \frac{4}{D}\tau_w}_{\text{wall shear}} \underbrace{+\rho \vec{g}_z}_{\text{gravity}}}$$

Energy Equation

$$\frac{\partial}{\partial t}\rho i + \nabla\cdot\left(\rho i \vec{v}\right) = -\nabla\cdot\vec{q} + \frac{DP}{Dt} - \tau : \nabla\vec{v} + \dot{q}$$

$$\boxed{\frac{\partial}{\partial t}\rho\langle i\rangle + \frac{\partial}{\partial z}\rho\langle i v_z\rangle = \underbrace{-\frac{\partial}{\partial z}\langle q_z\rangle}_{\text{axial conduction}} + \underbrace{\frac{\xi_h}{A}q_o''}_{\text{wall heat flux}} + \frac{D\langle P\rangle}{Dt} + \langle\dot{q}\rangle}$$

where $\frac{D\langle P\rangle}{Dt} = \frac{\partial\langle P\rangle}{\partial t} + \langle v_z\rangle\frac{\partial\langle P\rangle}{\partial z}$

Closure Relations

$-\frac{4\tau_w}{D} = -\frac{f}{2D}\rho v|v|$ f: Darcy friction factor

$q_w'' = h\left(T_w - T\right)$ h: heat transfer coeff.

$f = f\left(Re, \overbrace{\frac{\varepsilon}{D}}^{\text{roughness factor}}\right)$

$Nu = \frac{hD}{k} = Nu\left(Re, Pr\right)$

Natural Circulation

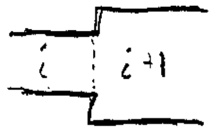
Boussinesq Approximation

$$\rho = \bar{\rho} - \bar{\rho}\beta \Delta T$$

Substituting into Momentum Eq. gives

$$\boxed{\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial z} - \frac{f}{2D}\rho v|v| + \bar{\rho} g_z - \bar{\rho}\beta \Delta T g_z}$$

Integral Momentum Equation



$$\rho_i v_i a_i = \rho_{i+1} v_{i+1} a_{i+1} = \cdots = \rho_r v_r a_r$$

$$\rho_i = \rho_r$$

$$v_i = \frac{a_r}{a_i} v_r$$

Using the momentum equation

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial z} - \frac{f}{2D} \rho v |v| + \bar{\rho} g_z - \bar{\rho} \beta \Delta T g_z$$

We have

$$\frac{\partial \rho_i v_i}{\partial t} + \frac{\partial \rho_i v_i^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{f}{2D} \rho_i v_i |v_i| + \rho_i g_{zi} + \rho_i \beta \Delta T_i g_{zi}$$

Integrating along the entire loop gives

$$\oint_{\text{Pump}}^{\text{Pump}} \left\{ \frac{\partial \rho_i v_i}{\partial t} + \frac{\partial \rho_i v_i^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{f}{2D} \rho_i v_i |v_i| + \rho_i g_{zi} + \rho_i \beta \Delta T_i g_{zi} \right\} dz$$

Term #1 $\quad \oint \frac{\partial \rho_i v_i}{\partial t} dz = \frac{\partial \rho_i v_i}{\partial t} \oint dz = \sum \rho_r \left(\frac{a_r}{a_i}\right) \frac{\partial v_r}{\partial t} l_i$

$$= \left\{ \sum_{i=1}^{n} \rho_r \left(\frac{a_r}{a_i}\right) l_i \right\} \frac{\partial v_r}{\partial t}$$

Term #2 $\quad \oint \frac{\partial \rho_i v_i^2}{\partial z} \partial z \approx 0$

Term #3 $\quad \oint -\frac{\partial P}{\partial z} dz = \Delta P_{pump}$

Term #4 $\quad \oint \frac{f_i \rho_i v_i |v_i|}{2 D_i} dz = \sum \left(\frac{f l}{D} + k\right)_i \frac{\rho_i v_i |v_i|}{2}$

$$= \sum \left(\frac{fl}{D} + k \right)_i \rho_r \left(\frac{a_r}{a_i} \right)^2 \frac{v_r |v_r|}{2}$$

$$= \left\{ \sum_{i=1}^{n} \left(\frac{fl}{D} + k \right)_i \left(\frac{a_r}{a_i} \right)^2 \right\} \frac{\rho_r v_r |v_r|}{2}$$

Term #5 $\oint \rho_i g_{3i} = \sum_{i=1}^{n} \left(\cancel{\rho_r g_i l_i}^{\,0} - \rho_r g_i \beta \Delta T \, l_{Ri} \right)$

$$\boxed{\rho_r \frac{\partial v_r}{\partial t} \sum \left(\frac{a_r}{a_i} \right) l_i = \Delta P + g \beta \rho_r \Delta T_h l_h - \frac{\rho_r v_r^2}{2} \sum \left(\frac{fl}{D} + k \right)_i \left(\frac{a_r}{a_i} \right)^2}$$

1) Forced Convection

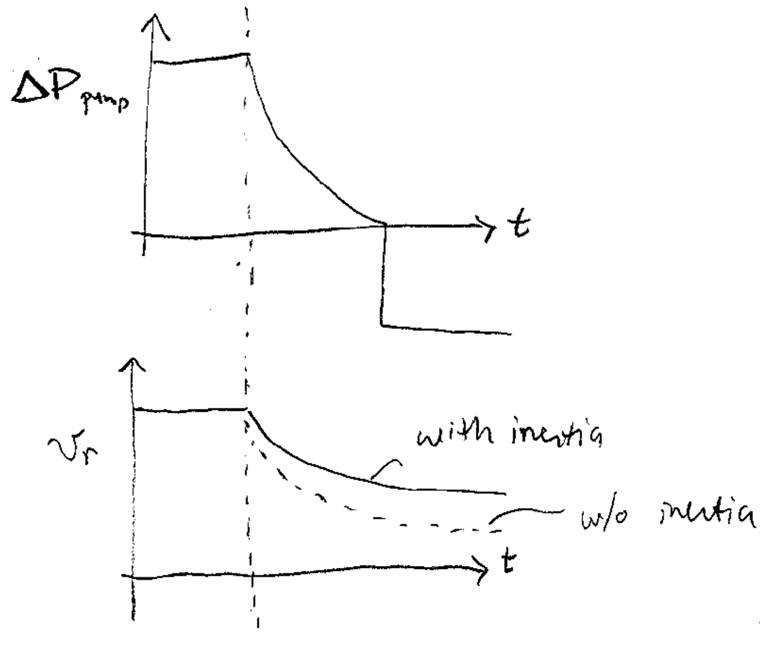
i) steady state $\frac{\partial v_r}{\partial t} = 0$, $\Delta P = \Delta P_{pump}$, $\Delta T_h \approx 0$

$$\boxed{\Delta P_{pump} = \frac{\rho_r v_r^2}{2} \sum \left(\frac{fl}{D} + k \right)_i \left(\frac{a_r}{a_i} \right)^2}$$

ii) transient

$$\Delta P = \Delta P(t)$$

$$\boxed{\rho_r \frac{\partial v_r}{\partial t} \sum \left(\frac{a_r}{a_i} \right) l_i = \Delta P(t) - \frac{\rho_r v_r^2}{2} \sum \left(\frac{fl}{D} + k \right)_i \left(\frac{a_r}{a_i} \right)^2}$$



2.) Natural Circulation

$$\rho C_p \left\{ \frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial z} \right\} = \frac{\xi_h q_0''}{A}$$

At steady state

$$\pi R_F^2 \dot{q}_F = 2\pi R_c q_0''$$

We have

$$\rho C_p v_r \frac{\partial T}{\partial z} = \frac{\xi_h q_0''}{A}$$

Integrating gives

$$\Delta T_h = \frac{\xi_h q_0'' l_{core}}{\rho C_p v_r A}$$

Since $v_r \approx$ constant we have

$$-\frac{\rho_r}{2} \sum \left(\frac{fl}{D} + k \right)_i \left(\frac{a_r}{a_i} \right)^2 v_r^2 + g \beta \rho_r l \left(\frac{\xi_h q_0'' l_{core}}{\rho C_p v_r A} \right) = 0$$

And

$$\boxed{v_r = \left[\frac{g \beta l\, l_{core} \xi_h q_0''}{\frac{\rho C_p A}{2} \sum \left(\frac{fl}{D} + k \right)_i \left(\frac{q_r}{a_i} \right)^2} \right]^{1/3}}$$