NUCL 511
Nuclear Reactor Theory and Kinetics

Lecture Note 2

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Fuel Depletion Equations

- The reactor status for burnup analysis is described by
  - Neutron flux: $\phi(r,E,t)$
  - Nuclide density vector: $\mathbf{N}(r,t)=[N_1(r,t),N_2(r,t),\cdots,N_I(r,t)]^T$
- Neutron flux equation (quasi steady state)

$$\mathbf{M}\phi(r,E,t)=\frac{1}{k}\mathbf{F}\phi(r,E,t)$$

$$\mathbf{M}\phi=-\nabla\cdot D(\vec{r},E,t)\nabla\phi(\vec{r},E,t)+\Sigma_t(\vec{r},E,t)\phi(\vec{r},E,t)-\int_{E'}dE'\Sigma_s(\vec{r},E'\to E,t)\phi(\vec{r},E',t)$$

$$\mathbf{F}\phi=\chi(E)\int_{E'}dE'\nu\Sigma_f(\vec{r},E',t)\phi(\vec{r},E',t)$$

$$\Sigma_x(r,E,t)=\sum_i N_i(r,t)\sigma_x^i(E)$$

  - Flux magnitude is determined by power normalization equation

$$P(t)=\sum_i\int_V dVN_i(r,t)\int_E dE[\kappa\sigma_f^i(E)+\kappa\sigma_c^i(E)]\phi(r,E,t)$$

Energy Release Per Fission

	U235	U238	Pu239	Pu240	Pu241	Pu242	Am241
Kinetic energy of fission fragments	1.69E+08	1.70E+08	1.76E+08	1.74E+08	1.75E+08	1.74E+08	1.76E+08
Kinetic energy of prompt neutrons	4.92E+06	4.80E+06	6.07E+06	6.48E+06	5.99E+06	6.76E+06	6.53E+06
Kinetic energy of delayed neutrons	7.40E+03	1.80E+04	2.80E+03	4.40E+03	5.00E+03	1.00E+04	2.00E+03
Kinetic energy of prompt gammas	6.60E+06	6.68E+06	6.74E+06	6.18E+06	7.64E+06	5.22E+06	7.90E+06
Kinetic energy of delayed gammas	6.33E+06	8.25E+06	5.17E+06	6.49E+06	6.40E+06	7.72E+06	5.51E+06
Total energy released by delayed betas	6.50E+06	8.48E+06	5.31E+06	6.62E+06	6.58E+06	7.87E+06	5.62E+06
Energy carried away by the neutrinos	8.75E+06	1.14E+07	7.14E+06	8.88E+06	8.85E+06	1.06E+07	7.54E+06
Total energy release per fission (sum)	2.02E+08	2.09E+08	2.06E+08	2.08E+08	2.11E+08	2.12E+08	2.10E+08
Total energy less neutrino energy	1.93E+08	1.98E+08	1.99E+08	1.99E+08	2.02E+08	2.02E+08	2.02E+08

Energy Yield Per Capture

H1	2.22	FE54	9.30	ZR90	7.19	PU238	4.81
H2	6.26	FE56	7.65	ZR91	8.64	PU239	6.53
BE9	6.81	FE57	10.04	ZR92	6.73	PU240	5.24
B10	11.46	FE58	6.58	ZR93	8.22	PU241	6.30
C	4.95	CO58	10.45	ZR94	6.47	PU242	5.07
O16	4.14	CO59	7.49	ZR95	7.85	AM241	5.50
NA23	6.96	NI58	9.00	TH232	4.79	AM242	6.40
CR50	9.26	NI59	11.24	U233	6.84	AM42M	6.40
CR52	7.94	NI60	7.82	U235	6.55	AM243	5.40
CR53	9.72	NI61	10.60	U238	4.81	CM242	5.70
CR54	6.25	NI62	6.84	NP237	5.48	CM243	6.80
MN55	7.27	NI64	6.10	NP239	5.17	CM244	5.50

Nuclide Transmutation Equation

- Transmutation equation (Bateman equation)

$$\frac{\partial}{\partial t}\mathbf{N}(r,t) = \mathbf{A}(\varphi,\sigma,\lambda)\mathbf{N}(r,t) \qquad \mathbf{N}(r,t) = [N_1(r,t), N_2(r,t), \cdots, N_I(r,t)]^T$$

$$a_{ij}(r,t) = \sum_x \gamma_{ij}^x \int_0^\infty \sigma_j^x(E)\phi(r,E,t)dE + \gamma_{ij}\lambda_j \quad (i \neq j)$$

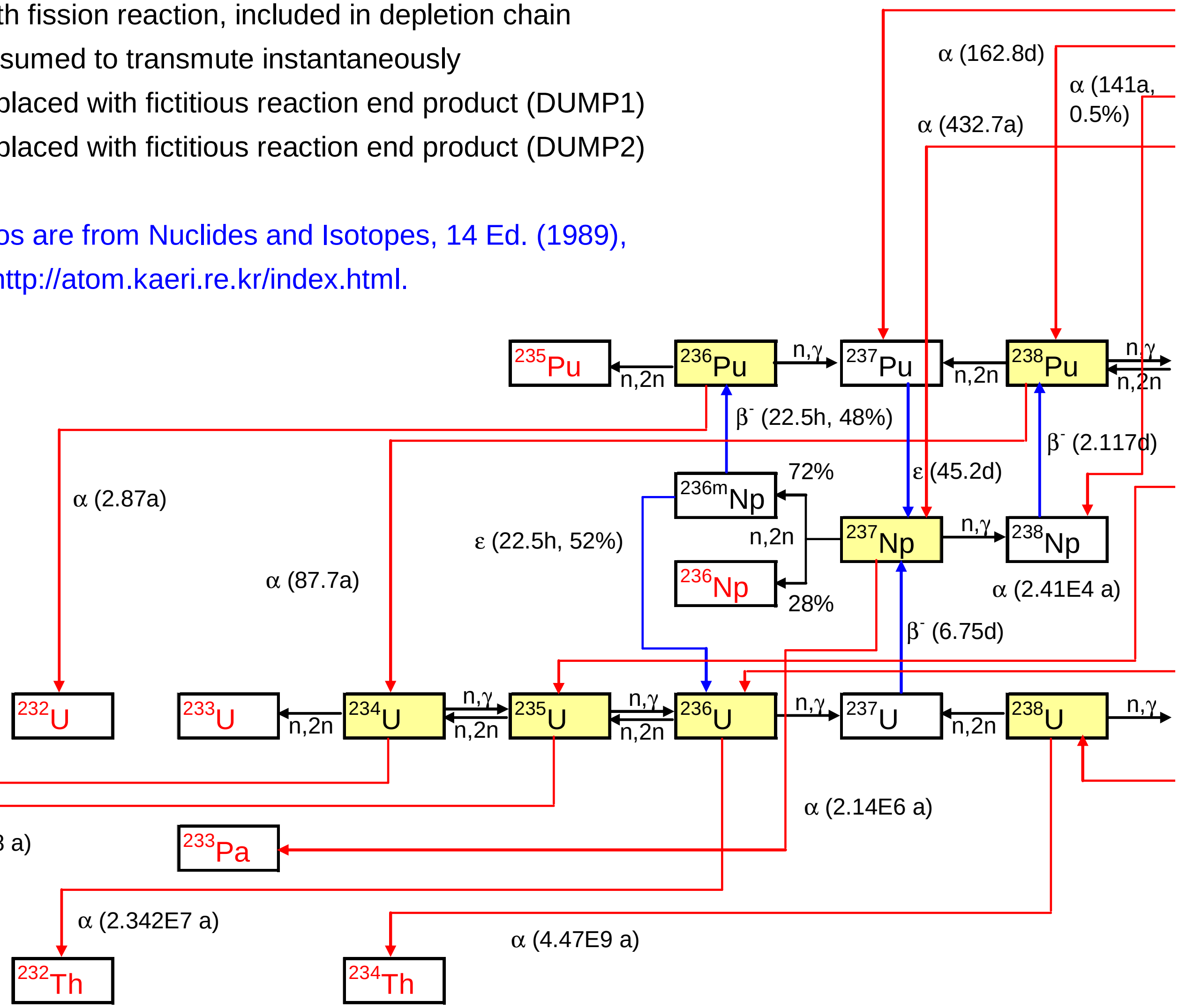
$$a_{ii}(r,t) = -\int_0^\infty \sigma_i^a(E)\phi(r,E,t)dE - \lambda_i$$

$$\frac{d}{dt}\begin{bmatrix} N_{U238} \\ N_{Pu239} \\ N_{Pu240} \\ N_{Pu241} \\ N_{Pu242} \\ N_{Am241} \\ N_{FP1} \\ N_{FP2} \end{bmatrix} = \begin{bmatrix} -\sigma_{U238}^a\phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sigma_{U238}^c\phi & -\sigma_{Pu239}^a\phi & \sigma_{Pu240}^{(n,2n)}\phi & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{Pu239}^c\phi & -\sigma_{Pu240}^a\phi & \sigma_{Pu241}^{(n,2n)}\phi & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{Pu240}^c\phi & -\sigma_{Pu241}^a\phi - \lambda_{Pu241} & \sigma_{Pu242}^{(n,2n)}\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{Pu241}^c\phi & -\sigma_{Pu242}^a\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda_{Pu241} & -\sigma_{Am241}^a\phi & 0 & 0 & 0 \\ \sigma_{U238}^f\phi & \sigma_{Pu239}^f\phi & \sigma_{Pu240}^f\phi & \sigma_{Pu241}^f\phi & \sigma_{Am241}^f\phi & 0 & 0 & 0 \\ \sigma_{U238}^f\phi & \sigma_{Pu239}^f\phi & \sigma_{Pu240}^f\phi & \sigma_{Pu241}^f\phi & \sigma_{Am241}^f\phi & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} N_{U238} \\ N_{Pu249} \\ N_{Pu240} \\ N_{Pu241} \\ N_{Pu242} \\ N_{Am241} \\ N_{FP1} \\ N_{FP2} \end{bmatrix}$$

Burn Chains

^{239}Pu	Isotopes with fission reaction, included in depletion chain
^{237}Pu	Isotopes assumed to transmute instantaneously
^{235}Pu	Isotopes replaced with fictitious reaction end product (DUMP1)
^{247}Cm	Isotopes replaced with fictitious reaction end product (DUMP2)

Halflives and branch ratios are from Nuclides and Isotopes, 14 Ed. (1989), and Table of Nuclide at http://atom.kaeri.re.kr/index.html.



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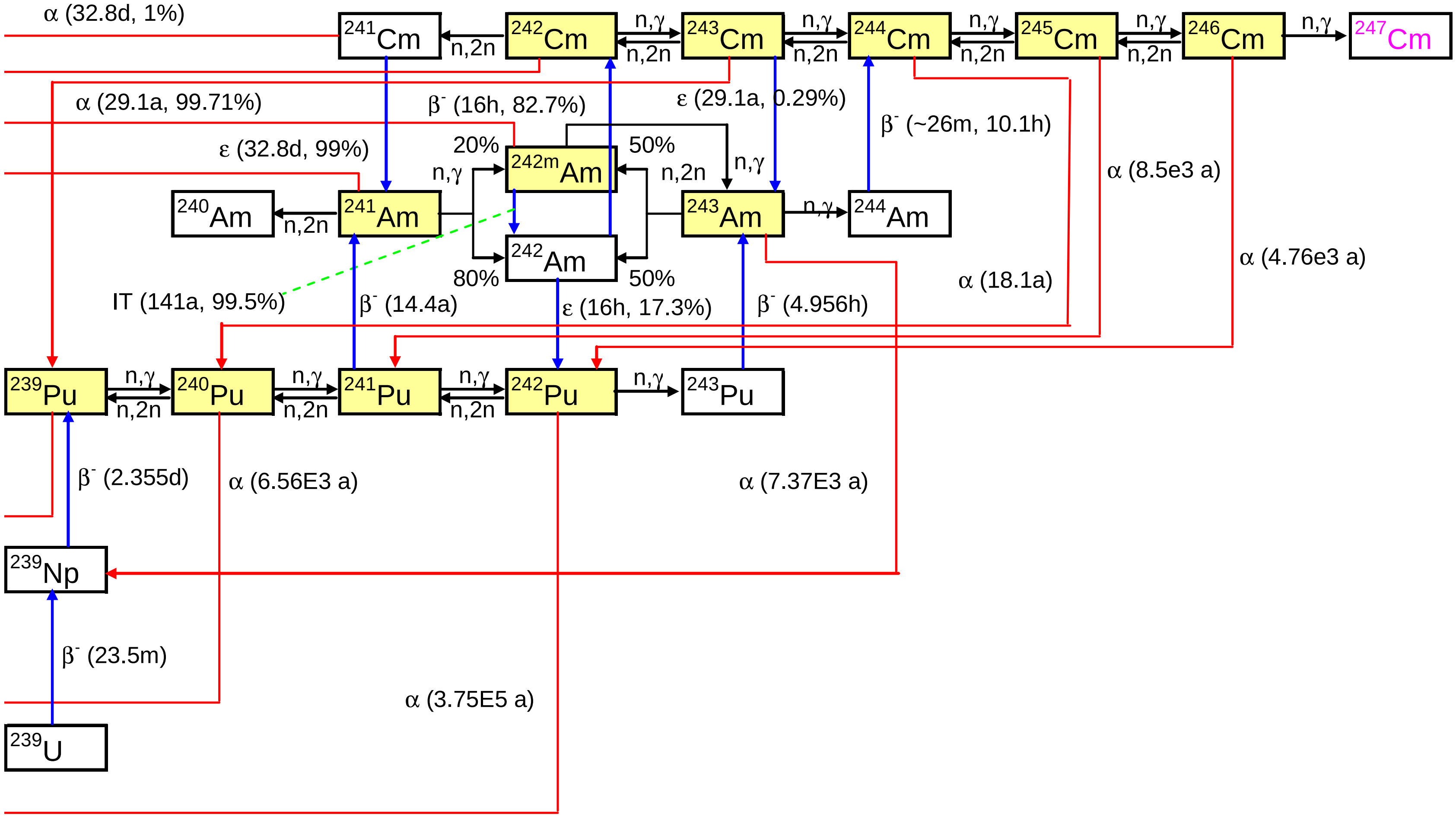
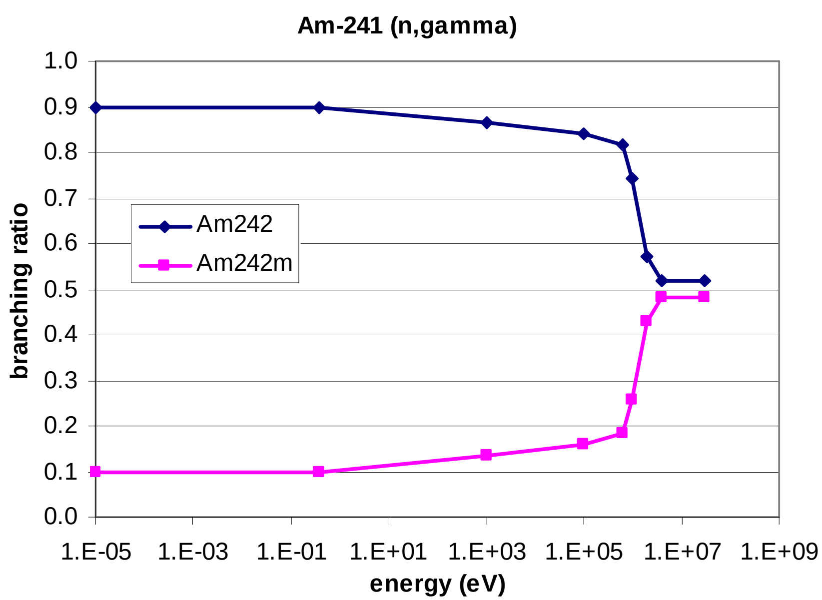
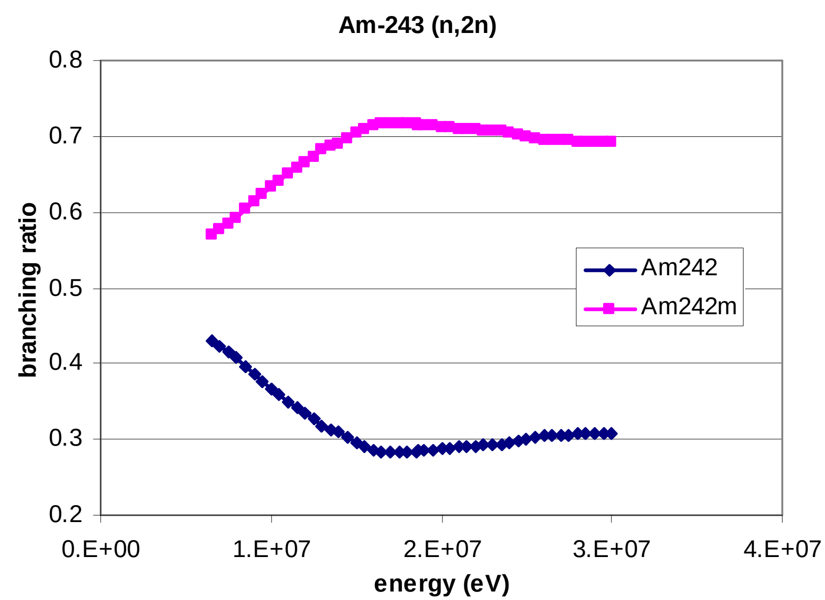
Burn Chains (cont'd)



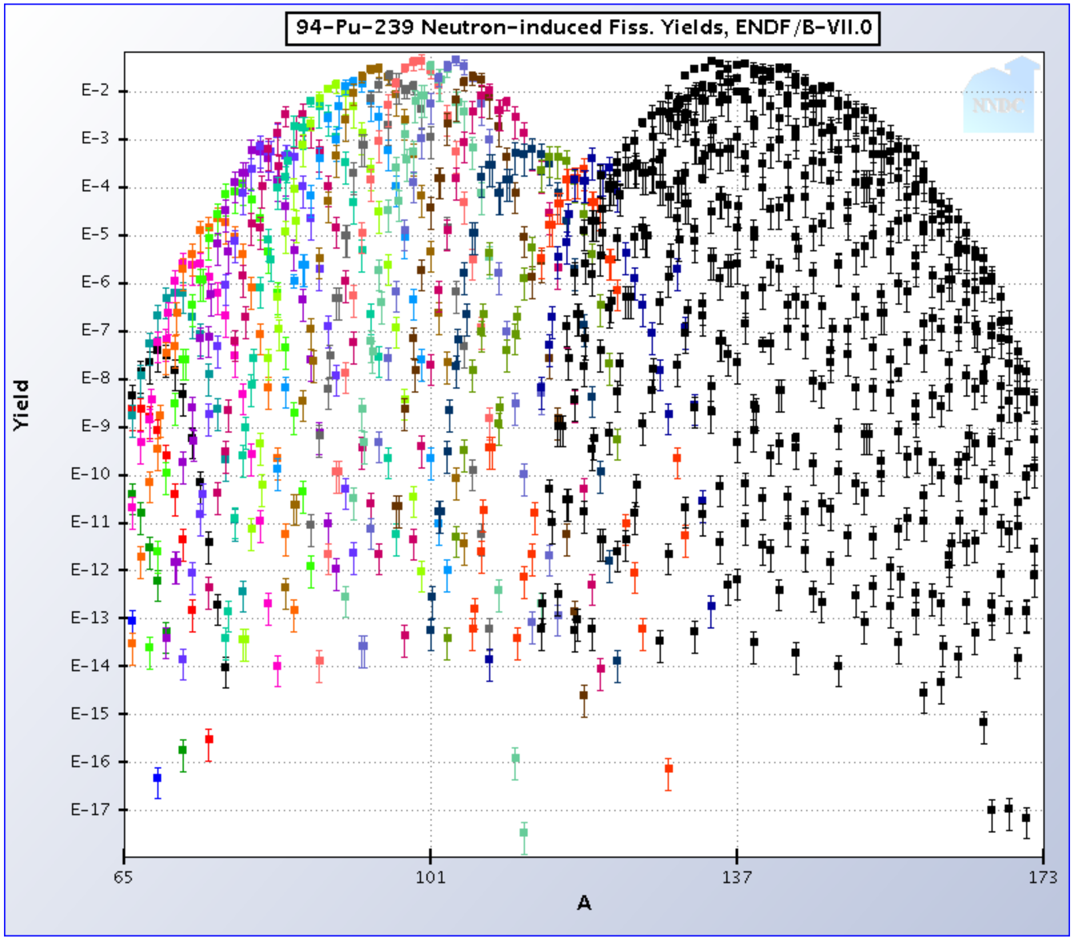
Branching Ratios (ENDF/B-VII.0)

Nuclide	Half life (s)	α	IT	β^-	EC (β^+)
Np236m	8.10E+04			48	52
Am242	5.77E+04			82.7	17.3
Am242m	4.45E+09	0.45	99.55		
Am244m	1.56E+03			99.9639	0.0361
Cm243	9.18E+08	99.71			0.29



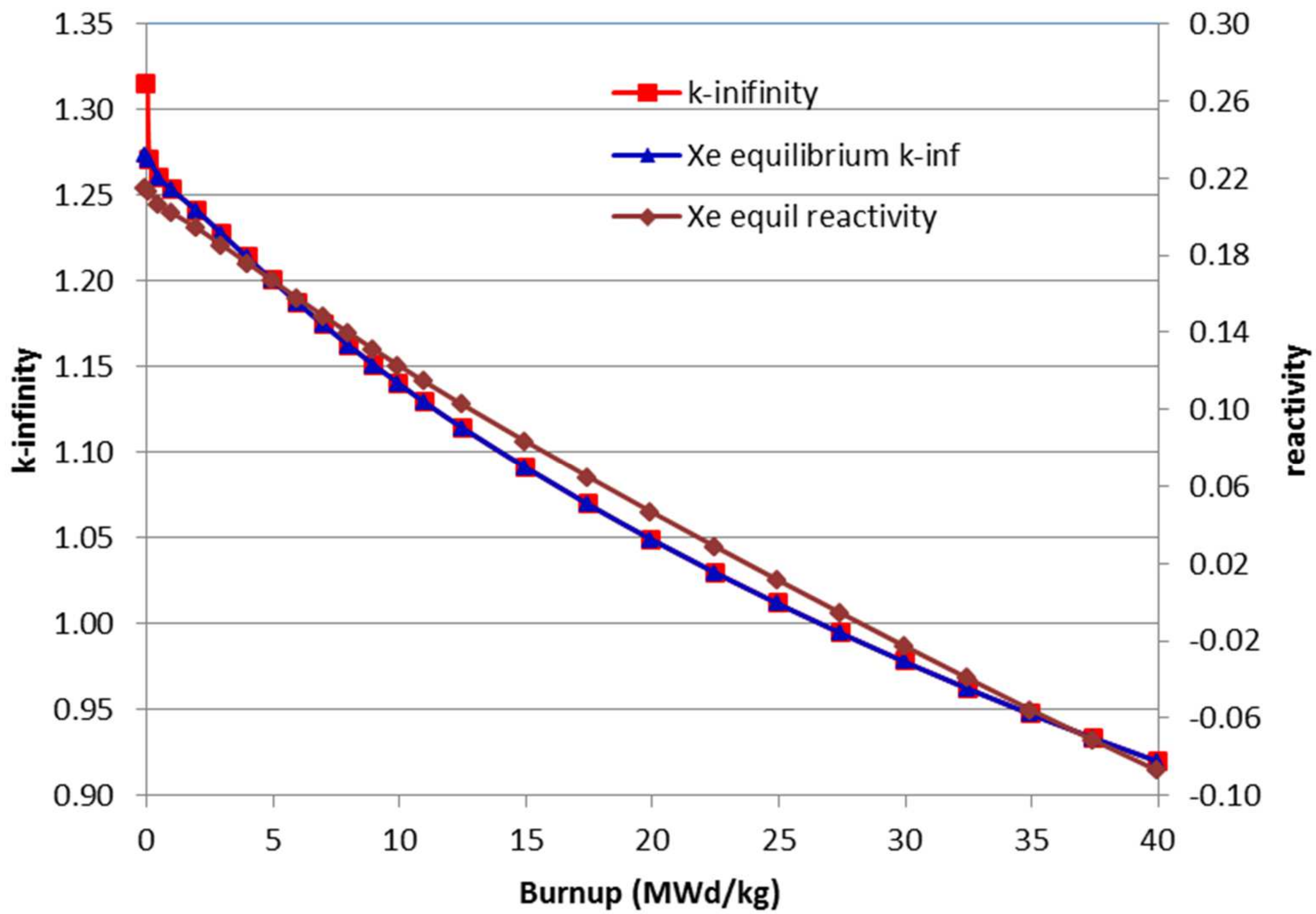


Fission Product Yields

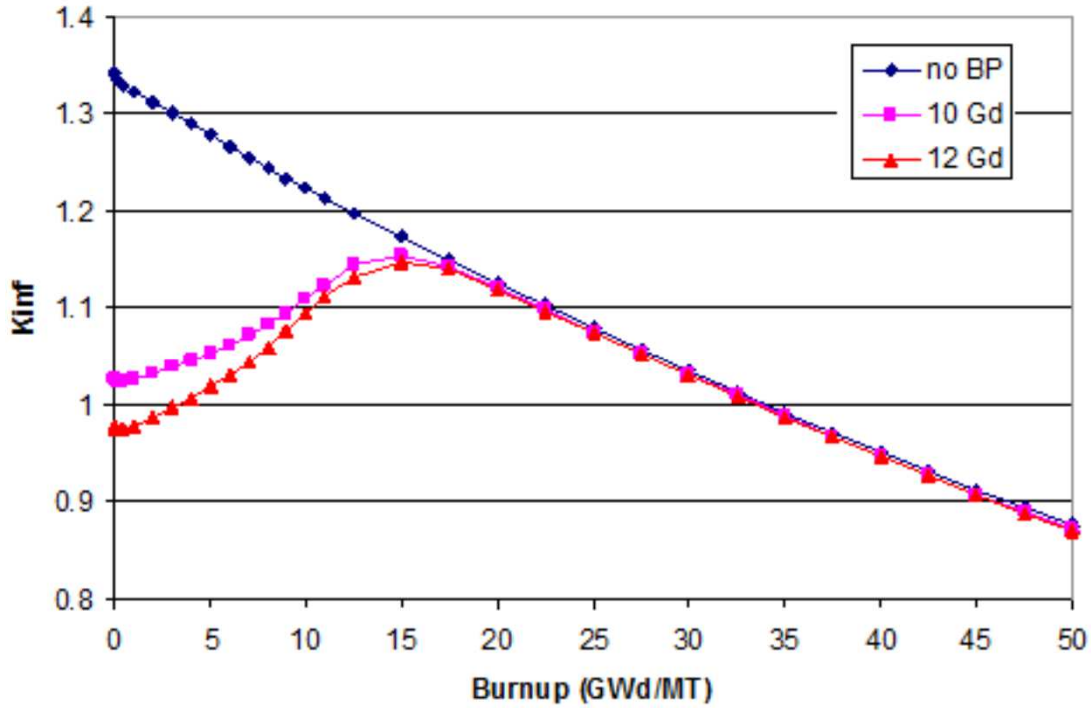


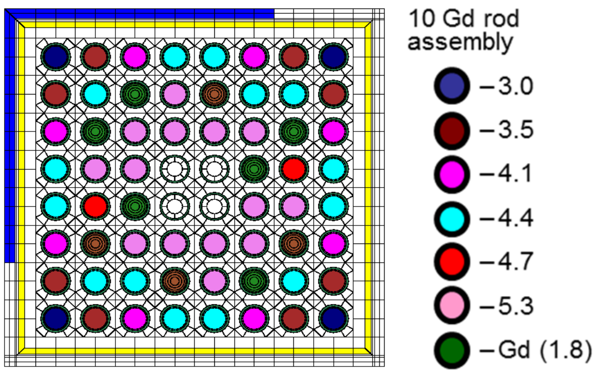
- ENDF/B-VII
  - 1078 fission products
  - Z: 23 – 70
  - A: 66 – 172
- ORIGEN
  - Point depletion model
  - Decay heat, dose rate, neutron source
  - ~850 FPs
- Lattice codes
  - Impacts on reactivity
  - ~100 FPs
- Fast reactor analysis
  - A few lumped fission products

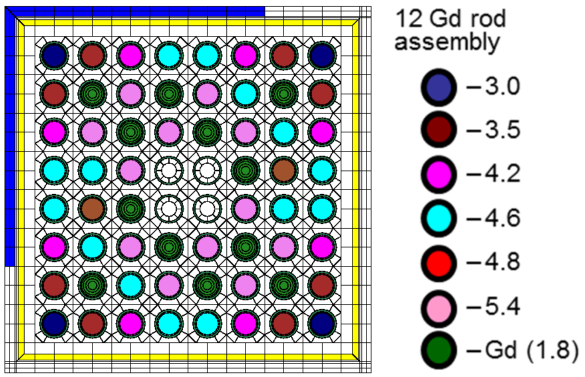
K-Infinity vs. Burnup (PWR)



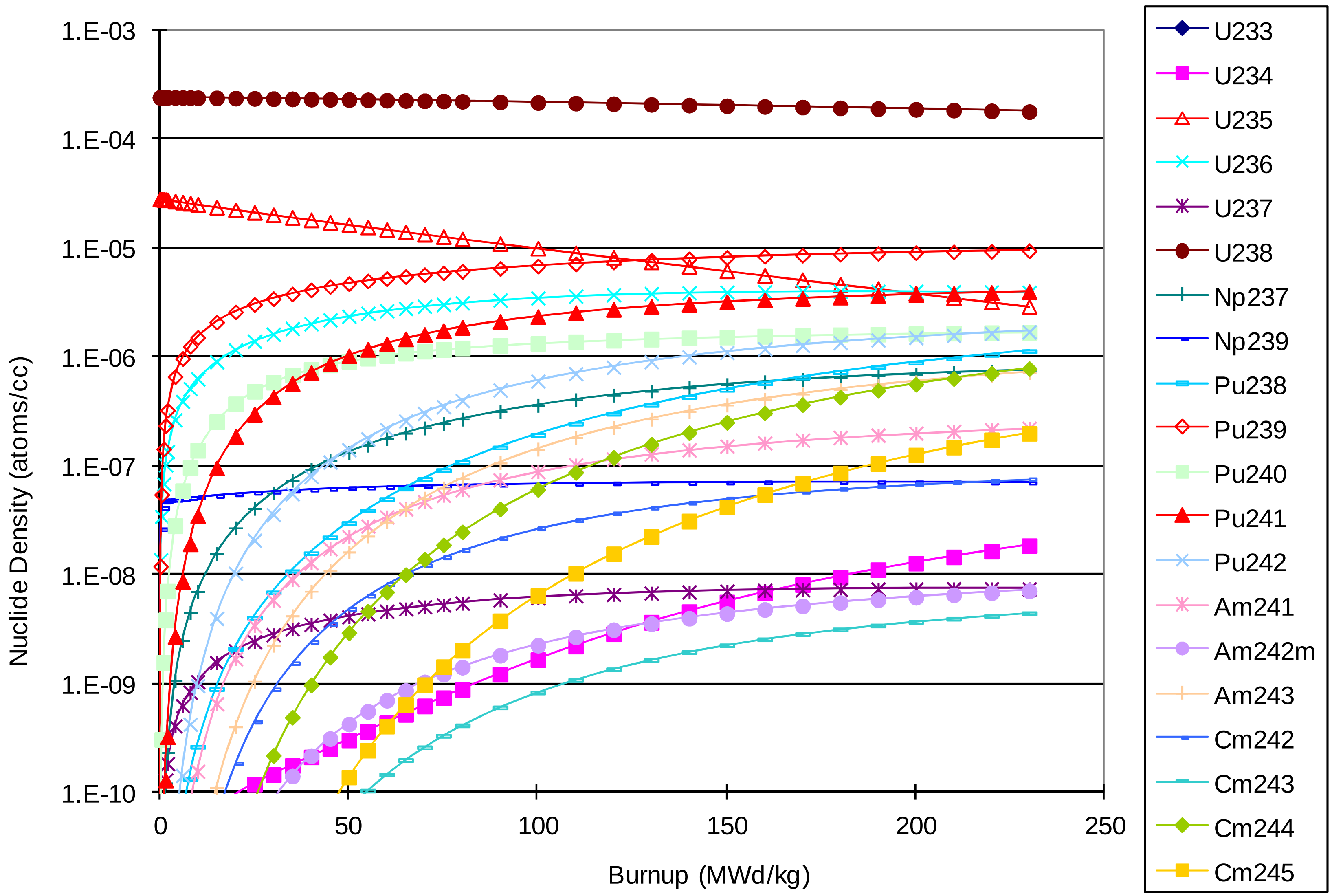
K-Infinity vs. Burnup (BWR)



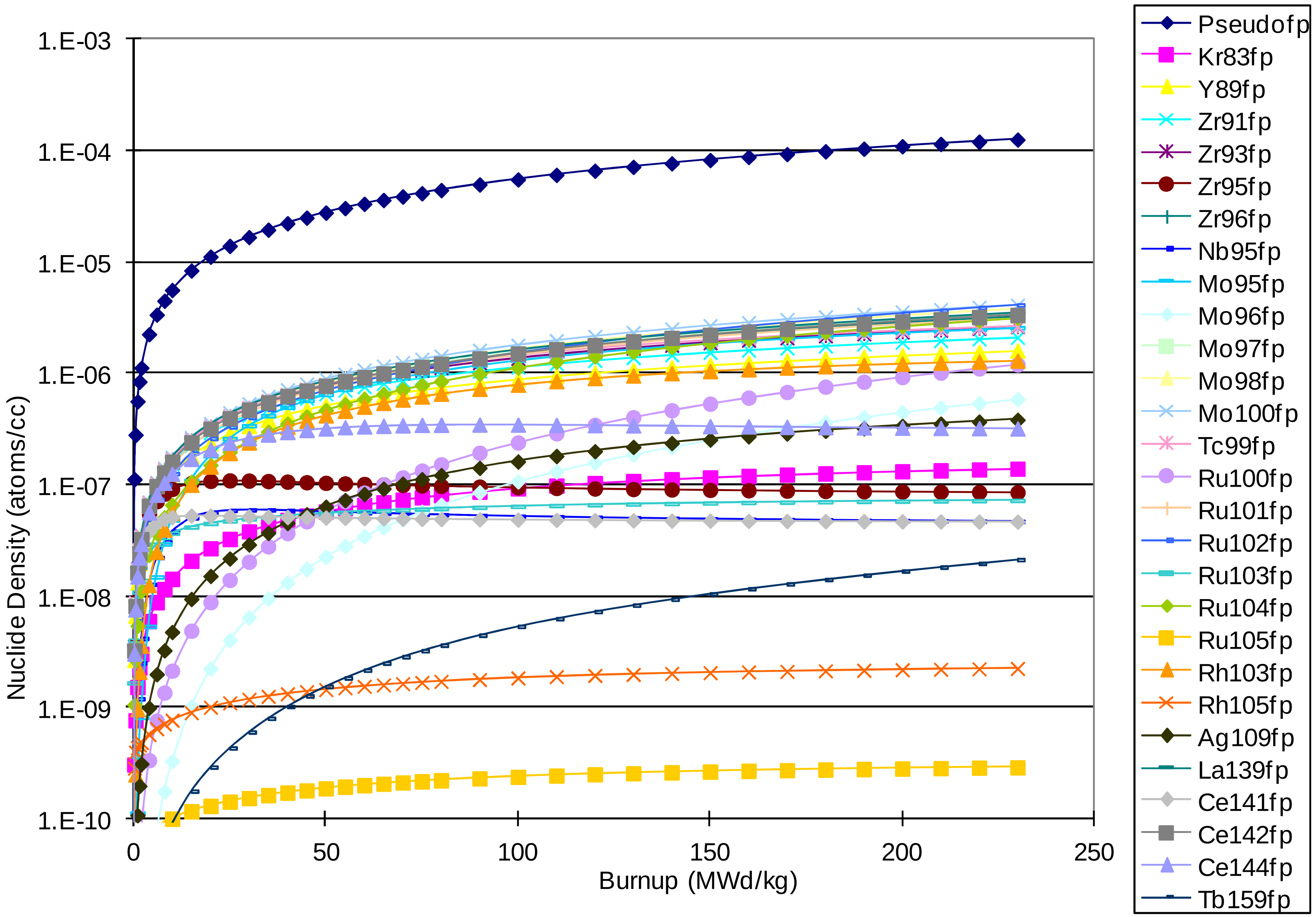




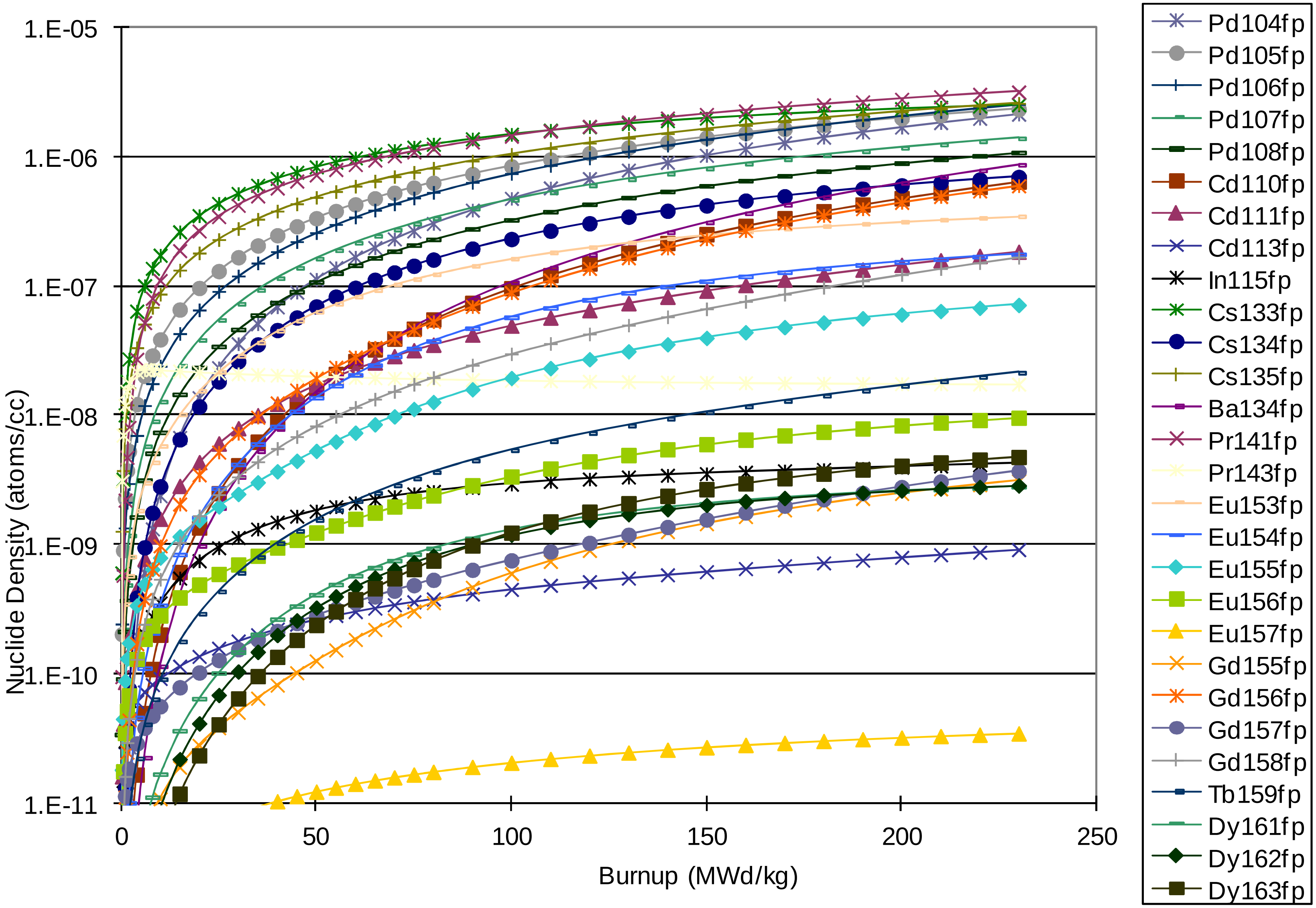
Evolution of Fuel Isotopes in VHTR



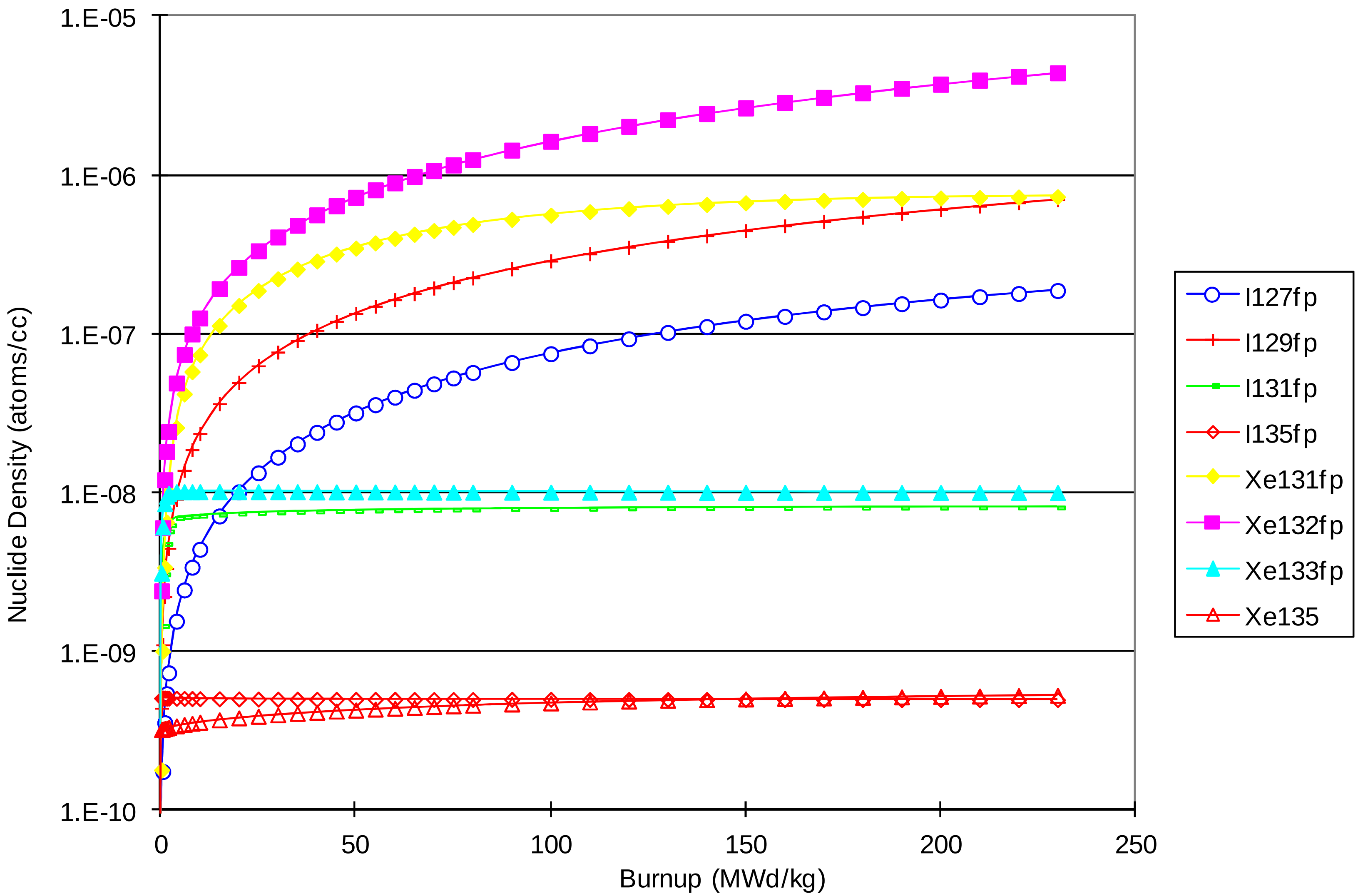
Buildup of Fission Products in VHTR (1)



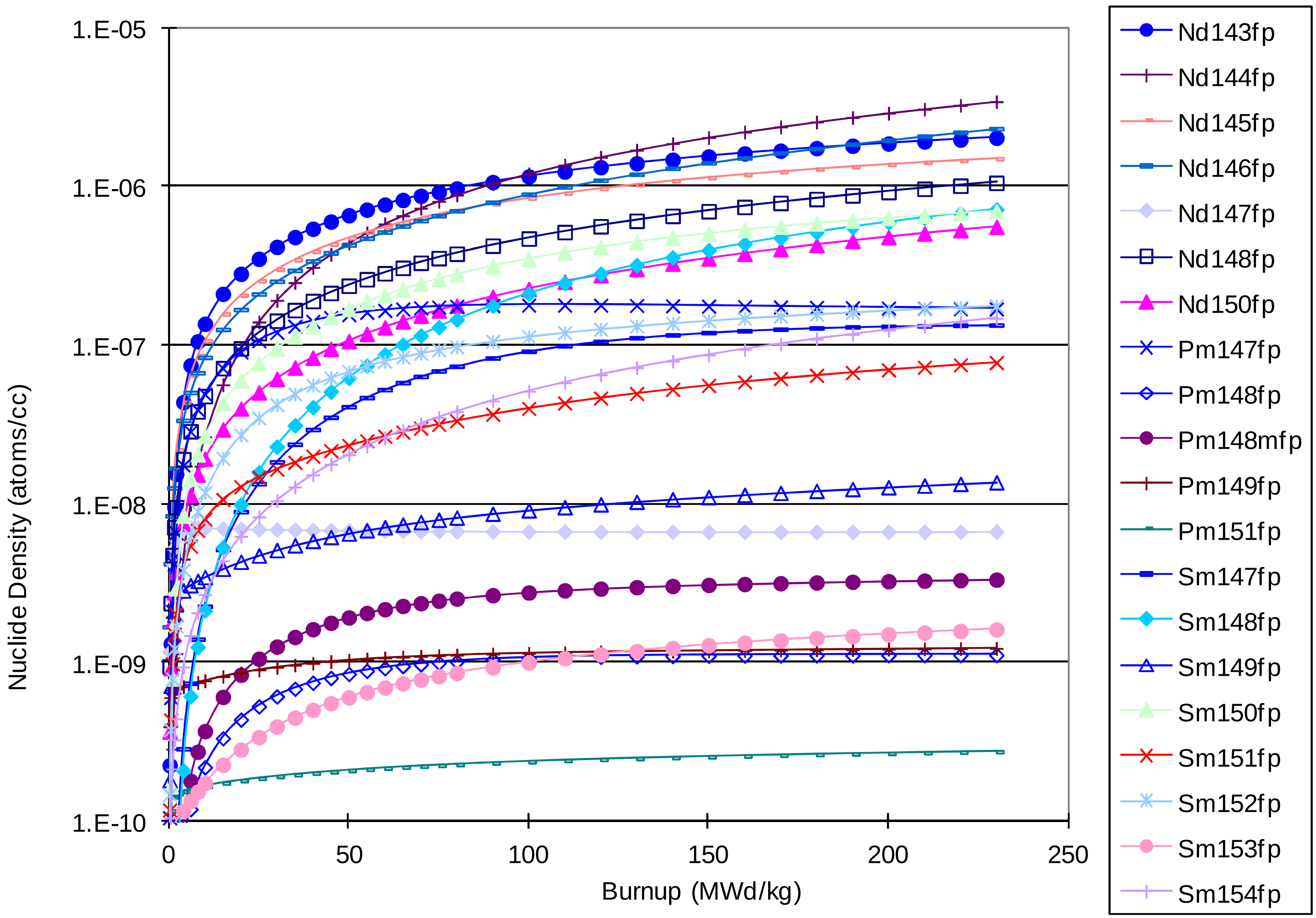
Buildup of Fission Products in VHTR (2)



Buildup of Fission Products in VHTR (3)

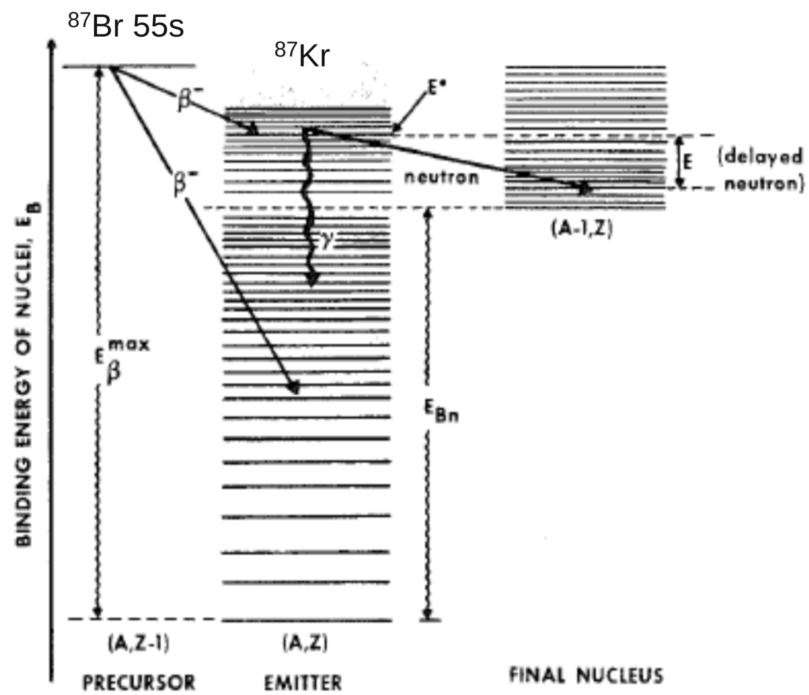


Buildup of Fission Products in VHTR (4)



Prompt and Delayed Neutrons

- Prompt Neutrons
  - Generated simultaneously with fission
- Delayed Neutrons
  - Generated after beta decay of certain fission products
    - *Precursor-emitter pair*
    - *Because of beta decay from precursor to emitter, time delay occurs*
  - Lower energy neutron from emitter
    - *Other energy carried by beta particle*
  - Delayed Neutron Fraction
    - *Strongly isotope dependent, weakly energy dependent*



$$\nu = \nu_p + \nu_d, \quad \beta = \frac{\nu_d}{\nu}$$

- In ENDF/B, the total number of neutrons per fission is given in MF=1, MT=452. Sections may be given for the number of delayed neutrons per fission (MT=455) and the number of prompt neutrons per fission (MT=456), and the components of energy release in fission (MT=458).

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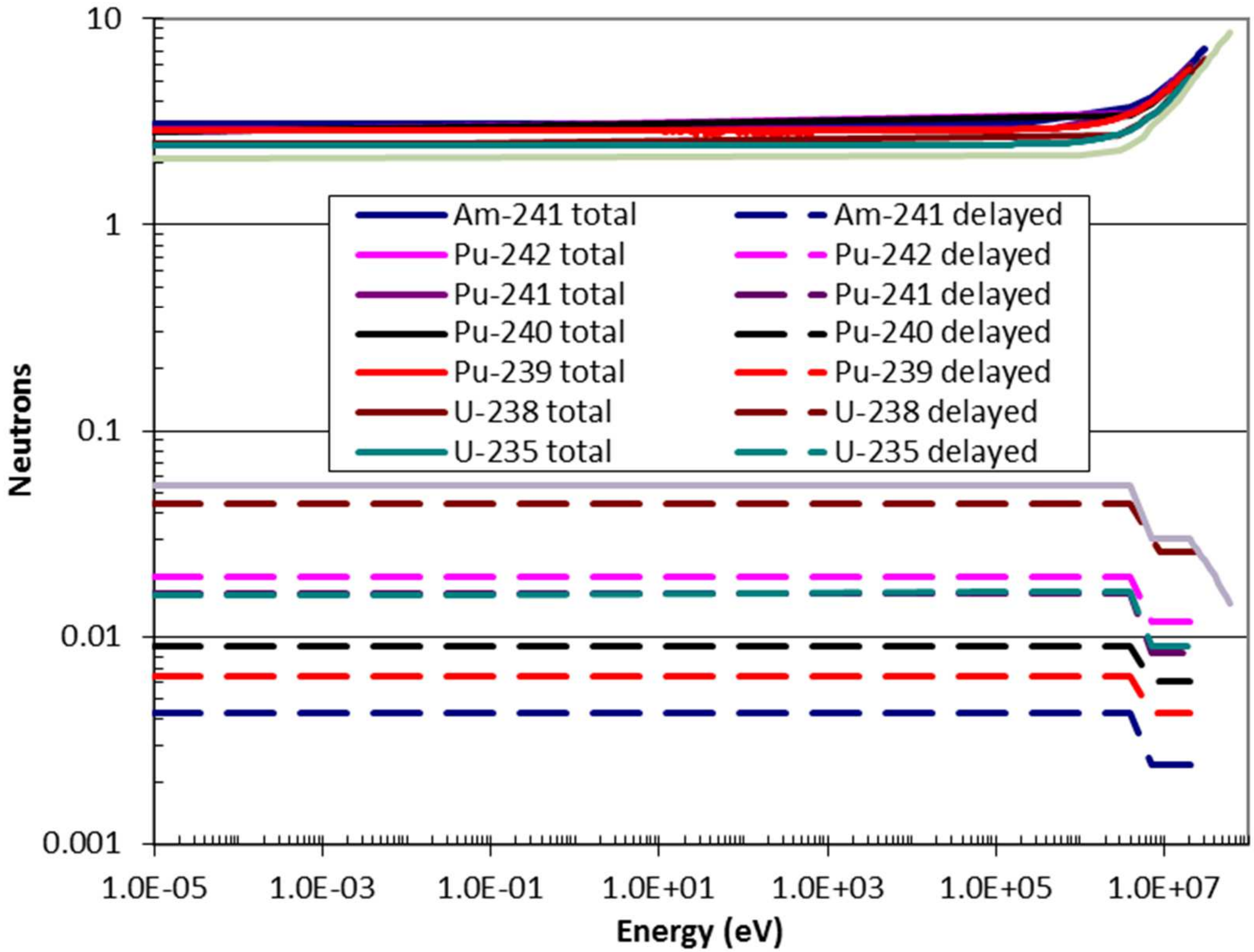
Total and Delayed Neutron Yields Per Fission



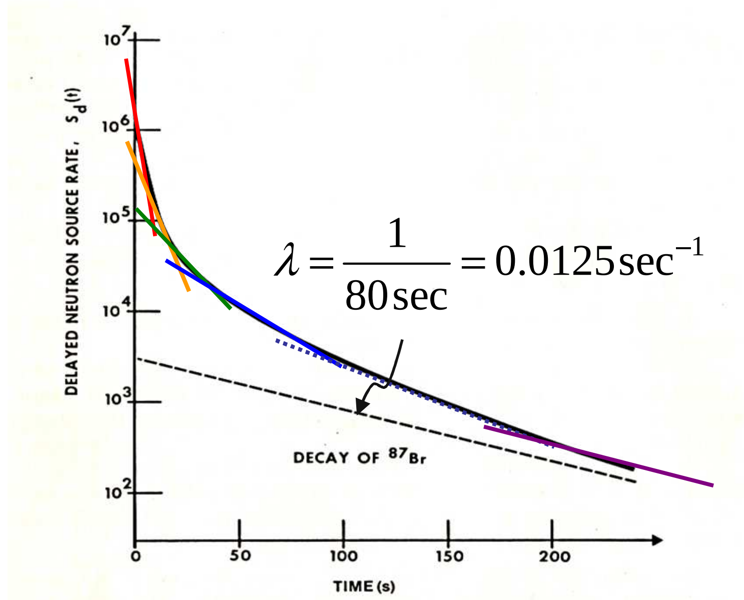
Delayed Neutron Fractions

Isotope	Thermal (0.1 eV)	Fast (350 keV)	Isotope	Thermal (0.1 eV)	Fast (350 keV)
Th-232	0.02599	0.02566	Np-237	0.00410	0.00402
U-235	0.00651	0.00674	Am-241	0.00139	0.00136
U-238	0.01766	0.01748	Am-242m	0.00199	0.00196
Pu-238	0.00144	0.00142	Am-243	0.00243	0.00239
Pu-239	0.00225	0.00220	Cm-243	0.00088	0.00087
Pu-240	0.00321	0.00315	Cm-244	0.00134	0.00132
Pu-241	0.00550	0.00550	Cm-245	0.00178	0.00178
Pu-242	0.00701	0.00688			

Families of Delayed Neutron Precursors

- Delayed neutron precursors are grouped into families
  - ~40 precursors
  - Individual decay constant and yield data are not known
  - Too many for individual representation
  - Typically 6 group representation is used instead
- Determination of decay constants and yield for 6 families
  - Time wise measurement of neutrons after short irradiation of fissioning isotopes
  - Least square fitting in semi-log plot

$$S_d(t) = n_f \sum_{k=1}^{6} \nu_{dk} \lambda_k e^{-\lambda_k t}$$



Group	λ_k	$\nu_{dk}(U^{235})$	β_k / β
1	0.0124	0.00055	0.033
2	0.0305	0.00366	0.219
3	0.111	0.00327	0.196
4	0.301	0.00659	0.396
5	1.14	0.00192	0.115
6	3.01	0.00070	0.042

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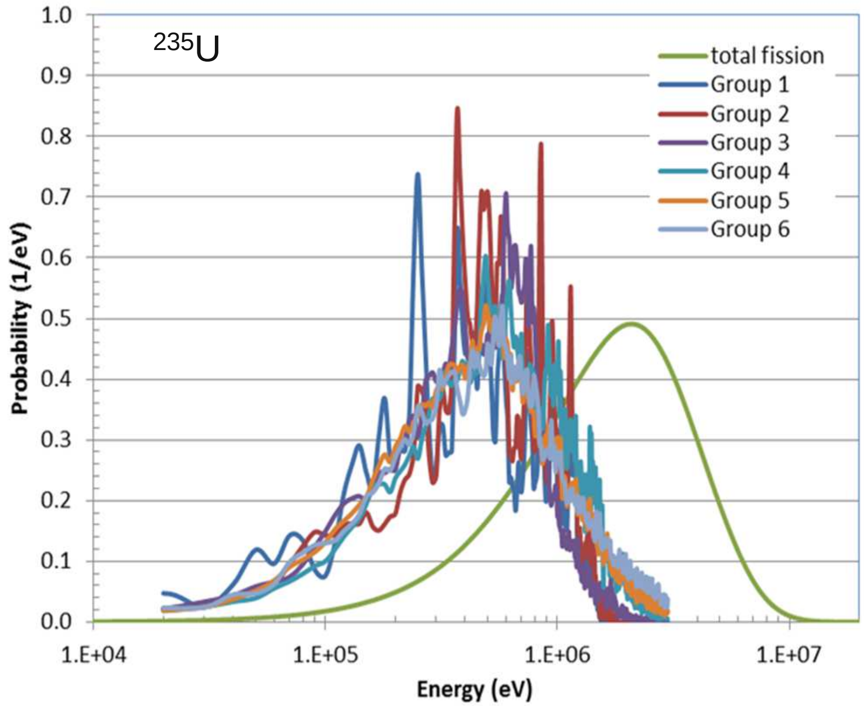
Delayed Neutron Precursor Decay Constants

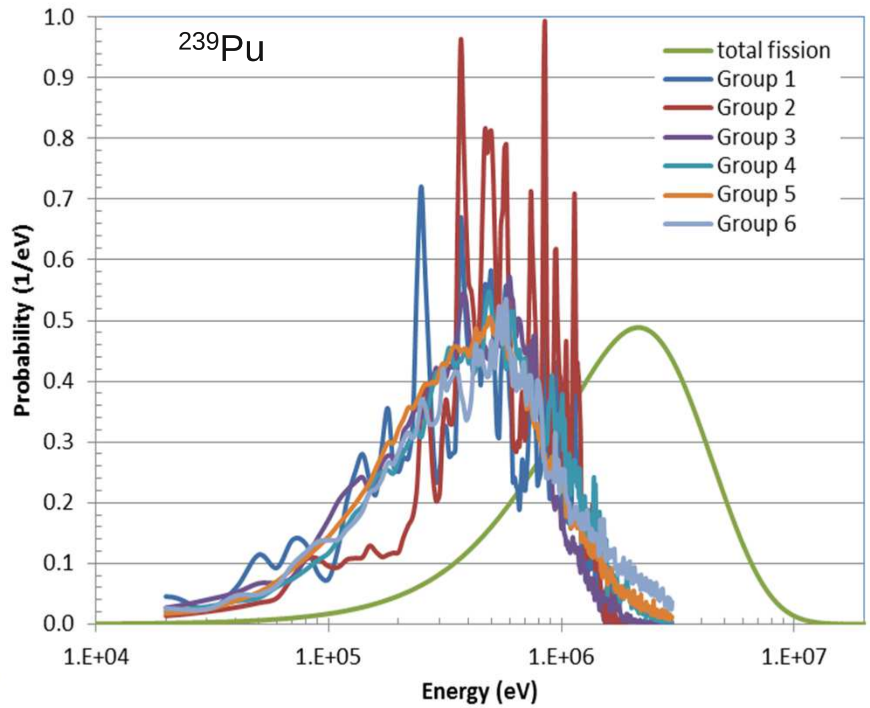
Family	1	2	3	4	5	6
U-234	0.0131	0.0337	0.1210	0.2952	0.8136	2.5721
U-235	0.0133	0.0327	0.1208	0.3028	0.8495	2.8530
U-236	0.0134	0.0322	0.1202	0.3113	0.8794	2.8405
U-238	0.0136	0.0313	0.1233	0.3237	0.9060	3.0487
NP237	0.0133	0.0316	0.1168	0.3007	0.8667	2.7600
PU238	0.0133	0.0312	0.1162	0.2888	0.8561	2.7138
PU239	0.0133	0.0309	0.1134	0.2925	0.8575	2.7297
PU240	0.0133	0.0305	0.1152	0.2974	0.8477	2.8796
PU241	0.0136	0.0300	0.1167	0.3069	0.8701	3.0028
PU242	0.0136	0.0302	0.1154	0.3042	0.8272	3.1372
AM241	0.0133	0.0308	0.1131	0.2868	0.8654	2.6430

Delayed Neutron Yield Per Fission

Family	1	2	3	4	5	6	sum
U-234	0.00071	0.00253	0.00233	0.00500	0.00171	0.00062	0.01290
U-235	0.00058	0.00302	0.00288	0.00646	0.00265	0.00111	0.01670
U-236	0.00070	0.00400	0.00376	0.00891	0.00412	0.00172	0.02320
U-238	0.00061	0.00496	0.00576	0.01695	0.01118	0.00454	0.04400
NP237	0.00043	0.00234	0.00168	0.00393	0.00179	0.00064	0.01081
PU238	0.00016	0.00100	0.00066	0.00149	0.00066	0.00021	0.00418
PU239	0.00023	0.00153	0.00115	0.00211	0.00110	0.00033	0.00645
PU240	0.00029	0.00228	0.00136	0.00297	0.00162	0.00049	0.00900
PU241	0.00029	0.00363	0.00231	0.00566	0.00320	0.00110	0.01620
PU242	0.00039	0.00456	0.00247	0.00643	0.00270	0.00141	0.01796
AM241	0.00015	0.00108	0.00067	0.00144	0.00074	0.00019	0.00427

Prompt and Delayed Neutron Spectra





Prompt and Delayed Neutron Spectra