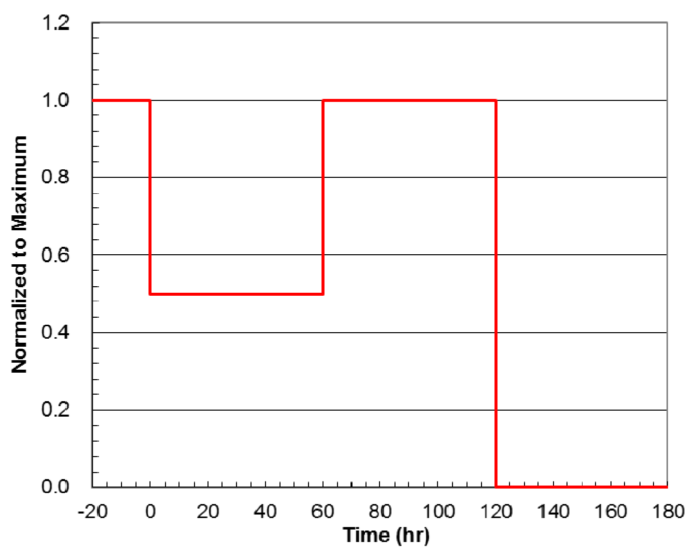
NUCL 511 Nuclear Reactor Theory and Kinetics

Homework #1

Due January 23

1. Using the data given below, determine the I-135 and Xe-135 concentrations and generate their plots for the power history given on slide 20 of the lecture note 1.

One-group flux at full power (#/cm^{2}s)	1.0×10^{14}
Macroscopic fission cross section (Σ_f) (cm^{-1})	0.686
Fission yield of I-135	0.064
Half-life of I-135 (hr)	6.7
Fission yield of Xe-135	0.003
Half-life of Xe-135 (hr)	9.2
Microscopic absorption cross section of Xe-135 (barn)	2.7×10^{6}



(1) For $t<0$ hr, $P(t)=P_0$, where P_0 is the rated full power level. Thus the fission rate and the effective decay constants are given by

$$F(P_0)=\Sigma_f\phi=0.686\times10^{14}\ \text{\#/cm}^3\text{s} = 2.47\times10^{17}\ \text{\#/cm}^3\text{hr}$$

$$\bar{\lambda}_I \cong \lambda_I = \ln 2/6.7 = 0.103\ \text{hr}^{-1}$$

$$\bar{\lambda}_{Xe}(P_0)=\lambda_{Xe}+\sigma_{Xe}\phi(P_0)=\ln 2/9.2+2.7\times10^{-18}\times3.6\times10^{17}=1.05\ \text{hr}^{-1}$$

As a result, the concentrations of I-135 and Xe-135 can be determined as

$$N_I^\infty(P_0)=\frac{\gamma_I F(P_0)}{\lambda_I}=\frac{0.064\times2.47\times10^{17}\ \text{\#/cm}^3\text{hr}}{\ln 2/6.7\ \text{hr}^{-1}}=1.53\times10^{17}\ \text{\#/cm}^3$$

$$N_{Xe}^\infty(P_0)=\frac{(\gamma_{Xe}+\gamma_I)F(P_0)}{\bar{\lambda}_{Xe}}=\frac{(0.003+0.064)\times2.47\times10^{17}\ \text{\#/cm}^3\text{hr}}{1.05\ \text{hr}^{-1}}=1.58\times10^{16}\ \text{\#/cm}^3$$

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(2) For $0 \le t < 60$ hr , $P(t) = 0.5P_0$. In this case, the fission rate, the effective decay constant and the equilibrium concentrations become

$$F(0.5P_0) = 1.23\times10^{17}\ \#/\text{cm}^3\text{hr}$$

$$\bar{\lambda}_{Xe}(0.5P_0) = \lambda_{Xe} + \sigma_{Xe}\phi(0.5P_0) = \ln 2/9.2 + 2.7\times10^{-18}\times1.8\times10^{17} = 0.561\ \text{hr}^{-1}$$

$$N_I^\infty(0.5P_0) = \frac{\gamma_I F(0.5P_0)}{\lambda_I} = 7.64\times10^{16}\ \#/\text{cm}^3$$

$$N_{Xe}^\infty(0.5P_0) = \frac{(\gamma_{Xe}+\gamma_I)F(0.5P_0)}{\bar{\lambda}_{Xe}} = \frac{(0.003+0.064)\times1.23\times10^{17}\ \#/\text{cm}^3\text{hr}}{0.561\ \text{hr}^{-1}} = 1.47\times10^{16}\ \#/\text{cm}^3$$

Thus the time-dependent concentrations can be obtained as

$$\begin{aligned}N_I(t) &= N_I(0)e^{-\lambda_I t} + N_I^\infty(0.5P_0)(1-e^{-\lambda_I t})\\ &= 1.53\times10^{17}e^{-0.103t} + 7.64\times10^{16}(1-e^{-0.103t})\end{aligned}$$

$$\begin{aligned}N_{Xe}(t) &= N_{Xe}(0)e^{-\bar{\lambda}_{Xe}t} + N_{Xe}^\infty(0.5P_0)(1-e^{-\bar{\lambda}_{Xe}t}) + \frac{\lambda_I N_I(0) - \gamma_I F(0.5P_0)}{\bar{\lambda}_{Xe}-\bar{\lambda}_I}(e^{-\lambda_I t} - e^{-\bar{\lambda}_{Xe}t})\\ &= 1.58\times10^{16}e^{-0.561t} + 1.47\times10^{16}(1-e^{-0.561t}) + 1.73\times10^{16}(e^{-0.103t} - e^{-0.561t})\\ &= 1.47\times10^{16} + 1.73\times10^{16}e^{-0.103t} - 1.62\times10^{16}e^{-0.561t}\end{aligned}$$

(3) For $60 \le t < 120$ hr , $P(t) = P_0$. In this case, the fission rate, the effective decay constant and the equilibrium concentrations are the same to those for $t < 0$, and the initial conditions are given by the concentrations at $t = 60$ hr

$$N_I(60) = 7.65\times10^{16}\ \#/\text{cm}^3$$

$$N_{Xe}(60) = 1.48\times10^{16}\ \#/\text{cm}^3$$

Thus the time-dependent concentrations can be obtained as

$$\begin{aligned}N_I(t) &= N_I(60)e^{-\lambda_I(t-60)} + N_I^\infty(P_0)[1-e^{-\lambda_I(t-60)}]\\ &= 7.65\times10^{16}e^{-0.103(t-60)} + 1.53\times10^{17}[1-e^{-0.103(t-60)}]\end{aligned}$$

$$\begin{aligned}N_{Xe}(t) &= N_{Xe}(60)e^{-\bar{\lambda}_{Xe}(t-60)} + N_{Xe}^\infty(P_0)[1-e^{-\bar{\lambda}_{Xe}(t-60)}]\\ &\quad + \frac{\lambda_I N_I(60) - \gamma_I F(P_0)}{\bar{\lambda}_{Xe}-\bar{\lambda}_I}[e^{-\lambda_I(t-60)} - e^{-\bar{\lambda}_{Xe}(t-60)}]\\ &= 1.48\times10^{16}e^{-1.05(t-60)} + 1.58\times10^{16}[1-e^{-1.05(t-60)}] - 8.36\times10^{15}[e^{-0.103(t-60)} - e^{-1.05(t-60)}]\\ &= 1.58\times10^{16} - 8.36\times10^{15}e^{-0.103(t-60)} + 7.36\times10^{15}e^{-1.05(t-60)}\end{aligned}$$

(4) For $t \ge 120$ hr , $P(t) = 0$. In this case, the fission rate and the equilibrium concentrations become zero, and the initial conditions are given by the concentrations at $t = 120$ hr

$$N_I(60) = 7.65\times10^{16}\ \#/\text{cm}^3$$

$$N_{Xe}(60) = 1.48 \times 10^{16} \ \#/\text{cm}^3$$

Thus the time-dependent concentrations can be obtained as

$$N_I(t) = N_I(120)e^{-\lambda_I(t-120)} = 1.53 \times 10^{17} e^{-0.103(t-120)}$$

$$N_{Xe}(t) = N_{Xe}(120)e^{-\lambda_{Xe}(t-120)} + \frac{\lambda_I N_I(120)}{\lambda_{Xe} - \lambda_I}[e^{-\lambda_I(t-120)} - e^{-\lambda_{Xe}(t-120)}]$$
$$= 1.58 \times 10^{16} e^{-0.0753(t-120)} - 5.62 \times 10^{17}[e^{-0.103(t-120)} - e^{-0.0753(t-120)}]$$
$$= 5.46 \times 10^{17} e^{-0.0753(t-120)} - 5.62 \times 10^{17} e^{-0.103(t-120)}$$