NUCL 511 Nuclear Reactor Theory and Kinetics

Homework #5

Due February 27

1. Calculate β for a fast reactor with 85% fissions in ^{239}Pu and 15% in ^{235}U. Use the ν_{dk} values from the lecture note 2 and the γ_k values from Table 5-I. (10 points)

   Answer) Using the separation approximation for the initial adjoint flux, the effective delayed neutron fraction can be obtained as

$$\beta_k = \frac{<\phi^*, F_{dk}\phi>}{<\phi^*, F\phi>} = \frac{\sum_i <\phi^*, F_{i,dk}\phi>}{\sum_i <\phi^*, F_i\phi>} \approx \frac{<\varphi^*(E), \chi_{dk}(E)>_E \sum_i \nu_{dki} <\Sigma_{fi}\phi>_{r,E}}{<\varphi^*(E), \chi(E)>_E \sum_i \nu_i <\Sigma_{fi}\phi>_{r,E}}$$

$$= \gamma_{dk} \frac{\nu_{dk9}R_{f9} + \nu_{dk5}R_{f5}}{\nu_9 R_{f9} + \nu_5 R_{f5}}$$

$$\beta = \sum_k \beta_k$$

   The total fission neutron yields can be obtained using the delayed neutron fractions and yields as

$$\nu_9 R_{f9} + \nu_5 R_{f5} = 0.85(\nu_{d9}/\beta_9) + 0.15(\nu_{d5}/\beta_5)$$
$$= 0.85(0.00645/0.00220) + 0.15(0.01670/0.00674) = 2.86371$$

   Using this value and the delayed neutron yield data, the beta effective value can be obtained as

group		1	2	3	4	5	6	sum
ν_{dk}	U-235	0.00058	0.00302	0.00288	0.00646	0.00265	0.00111	0.01670
	PU239	0.00023	0.00153	0.00115	0.00211	0.00110	0.00033	0.00645
	average	0.00028	0.00175	0.00141	0.00276	0.00133	0.00045	
γ_k		0.802	0.831	0.818	0.825	0.825	0.825	
β_k		0.00008	0.00051	0.00040	0.00080	0.00038	0.00013	0.00230

2. Calculate a burnup-dependent β for an LWR with 2% fissions in ^{238}U and initially 98% in ^{235}U. As ^{239}Pu is produced, it takes over part of the fission rate. Let f_{239} be the fraction of the total fission rate that comes from fissioning ^{239}Pu. Use $\gamma_k = 1.08$ and the delayed neutron data in the lecture note 2. (10 points)

   a. Find $\beta(f_{239})$ for $f_{239} \le 50\%$.

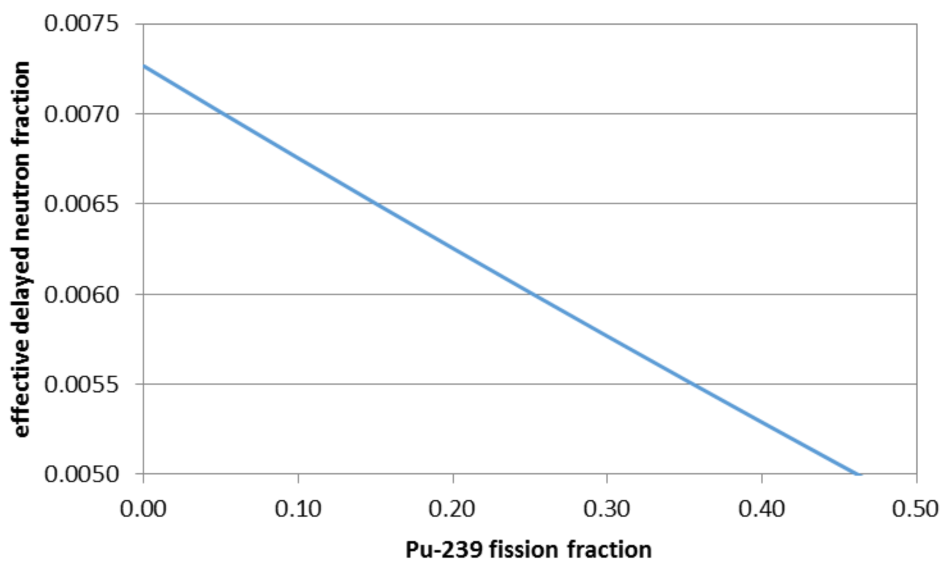
      Answer) Using the formula in Problem 1, the effective delayed neutron fraction can be obtained as

$$\beta(f_{239}) = 1.08 \frac{\sum_k \nu_{dk5}(0.98 - f_{239}) + \nu_{dk9} f_{239} + 0.02\nu_{dk8}}{\nu_5(0.98 - f_{239}) + \nu_9 f_{239} + 0.02\nu_8}$$

NUCL 511 Nuclear Reactor Theory and Kinetics

f239	numerator	denominator	β
0.00	0.01725	2.564	0.00726
0.05	0.01673	2.579	0.00701
0.10	0.01622	2.594	0.00675
0.15	0.01571	2.609	0.00650
0.20	0.01520	2.624	0.00625
0.25	0.01468	2.639	0.00601
0.30	0.01417	2.654	0.00577
0.35	0.01366	2.669	0.00553
0.40	0.01315	2.684	0.00529
0.45	0.01263	2.699	0.00505
0.50	0.01212	2.714	0.00482

b. Plot $\beta(f_{239})$ for $f_{239} \le 50\%$.



3. An (α, n) point source is moved in a vertical guide tube toward a swimming pool reactor core. Suppose that the adjoint flux varies along the guide tube as $\phi^*(z) = A\cos(z/100)$, where z is the distance from the core mid-plane in cm and A is a constant, and the steady state reactor power is 5 watts when the source is located 10 cm above the core mid-plane. Determine the steady state reactor power as a function of source position for the source position from 40 cm to 0 cm. (10 points)

Answer) At steady state, the point kinetics equations yield the source multiplication factor

NUCL 511 Nuclear Reactor Theory and Kinetics

$$\left.\begin{aligned}\Lambda\frac{dp}{dt} &= (\rho-\beta)p+\sum_k \lambda_k\zeta_k + s = 0\\ \frac{d\zeta_k}{dt} &= \beta_k p - \lambda_k\zeta_k = 0\end{aligned}\right\} \Rightarrow \quad \rho p + s = 0 \quad \Rightarrow \quad p = \frac{s}{-\rho}$$

Using the definition, the reduced source $s(z)$ when the point source is located at z cm above the core mid-plane can be obtained as

$$s(z) = <\phi_0^*, S> = A\int_{-c}^{c}\left[\cos\frac{z'}{100}\times S\delta(z'-z)\right]dz' = AS\cos\frac{z}{100}$$

Since the amplitude function is proportional to the reactor power, the reactor power can be determined as a function of the source position as

$$P(s_z) = P(s_{z=10})\,\frac{\cos(z/100)}{\cos(10/100)} = 5.025\cos\frac{z}{10}$$

Source position (cm)	Power (W)
0	5.025
5	5.019
10	5.000
15	4.969
20	4.925
25	4.869
30	4.801
35	4.720
40	4.628