h; heat transfer co-efficient (dimensional)

$q_w'' = h(T_w - T)$

T_w = Twall
T: Bulk fluid temperature

- To nondimensionalized h, the Nusselt Number is introduced:

$$Nu = \frac{hD}{k} \Rightarrow \boxed{h = \frac{k}{D} Nu}$$

In general, $Nu = Nu(Re, Pr)$ for different fluid types.

$Nu_H \rightarrow$ Nusselt No. for constant heat flux
$Nu_T \rightarrow$ Nusselt No. for constant temperature.

- Laminar flow

$Nu_H = 4.364$
$Nu_T = 3.658$

- Turbulent Flow

for $Pr < 0.1$ (Liquid Metal):

$$Nu_H = 6.3 + 0.003\, Re\, Pr$$
$$Nu_T = \underbrace{4.8}_{\text{conduction term}} + \underbrace{0.003\, Re\, Pr}_{\text{convection term}}$$

for $0.5 < Pr < 1.0$ (Gases):

$$Nu_H = 0.022\, Pr^{0.6}\, Re^{0.8}$$

for $1.0 < Pr < 2.0$ (Water)

$$Nu_H = 0.0155\, Pr^{0.5}\, Re^{0.83}$$

1-D Balance Eqns.

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v}{\partial z} = 0 \longrightarrow \text{C.E.}$$

$$\frac{\partial \rho v}{\partial t} + \frac{\partial \rho v^2}{\partial z} = -\frac{\partial p}{\partial z} - \frac{f \rho v |v|}{2D} + \rho g_z \longrightarrow \text{M.E.}$$

$$\rho C_p \left[\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial z}\right] = \frac{\xi_k}{A} h (T_w - T) + q''' \longrightarrow \text{E.E.}$$

- These balance equations cannot be used in accident scenarios

- Not valid for <u>natural circulation</u>.

- <u>Natural Circulation</u> (Bourgeois Assumption)

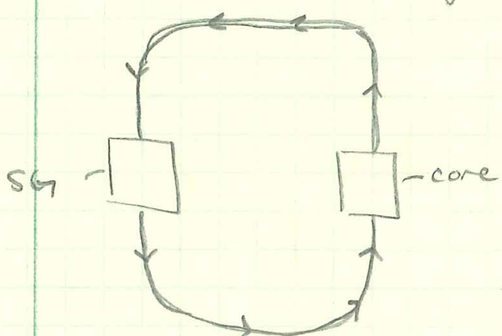
$$\boxed{\rho g z = \bar{\rho} g z \underbrace{- \bar{\rho} \beta \Delta T g z}}$$

$$\beta = \frac{1}{\rho} \left.\frac{\partial \rho}{\partial T}\right)_p \rightarrow \text{(Thermal Ex. Coefficient)}$$

Taylor expansion of density term to account for small changes in density due to temperature change.

<u>Integral Momentum Equation.</u>

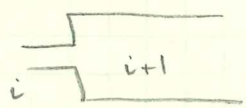
- In most cases it is important to know the solution for the entire reactor system (i.e. accident scenarios - LOCA).



nuclear reactor → i^{th} component → n total components.

$\oint$ 1-D Momentum Eq. ⇒ IME (Integral Momentum Equation)

<u>Continuity Equation</u>



<u>Assumption: Fluid ~ incompressible</u>

$$\rho_i v_i a_i = \rho_{i+1} v_{i+1} a_{i+1} = \rho_r v_r a_r$$ (mass is conserved through the entire system)

$\rho_i = \rho_{i+1} = \rho_r$ ⟶ density can be considered constant.

$$\boxed{v_i = \frac{a_r}{a_i} v_r \quad (1)}$$ ⟶ only one momentum equation

Integrate Momentum Equation along z (1-D → 0-D)

$$\text{1-D: } \boxed{\frac{\partial \rho v}{\partial t} + \frac{\partial}{\partial z}\rho v^2 = \frac{-\partial p}{\partial z} - \frac{f\rho v|v|}{2D} + \bar{\rho} g_z - \bar{\rho}\beta\Delta T g_z}$$

Component-wise Momentum Equation (1-D):

$$\boxed{\frac{\partial \rho_i v_i}{\partial t} + \frac{\partial}{\partial z}\rho_i v_i^2 = \frac{-\partial p}{\partial z} - \frac{f_i \rho_i v_i|v_i|}{2D} + \bar{\rho}_i g_{z,i} - \bar{\rho}_i\beta\Delta T g_{z,i}}$$

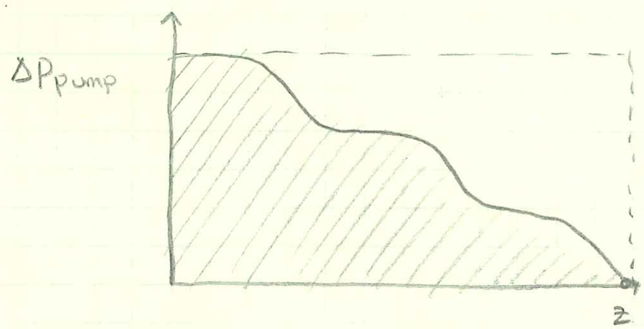
$$\oint \frac{\partial \rho_i v_i}{\partial t} dz = \quad (1)$$

$$\oint \frac{\partial \rho_i v_i^2}{\partial z} dz = \quad (2)$$

$$\oint \frac{-\partial p}{\partial z} dz = \quad (3)$$

$$\oint (\bar{\rho}_i g_{z,i} - \bar{\rho}_i \beta \Delta T g_z)\, dz = \quad (4)$$

$$\oint \frac{f_i \rho_i v_i (v_i)}{2D_i} dz = \quad (5)$$



- Homework → solve for (1) → (5)

answers:

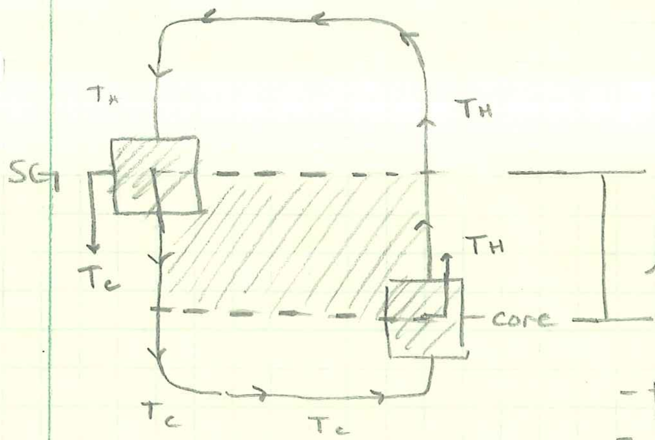
$$(1) = \sum_i \rho_i \frac{dv_i}{dt} l_i = \sum_i \rho_i \left(\frac{a_r}{a_i}\right) \frac{dv_r}{dt} l_i$$

(2) ϕ

(3) ΔP_{pump}

$$(4) \quad \sum_i (\rho_r g_{z,i} l_i \underbrace{- \rho_r g_i \beta \Delta T l_i)}_{\rho_r g_i \beta \Delta T_h l_h}$$

$$(5) \quad \sum_i \left(\frac{fl}{D} + k\right)_i \frac{\rho_i v_i |v_i|}{2} = \sum_i \left(\frac{fl}{D} + k\right)_i \frac{\rho_r v_r^2}{2} \left(\frac{a_r}{a_i}\right)^2$$



l_{H}; difference in thermal centers

- thermal center (hot/cold after the center)
- center of gravity of the heat flux.

- only the thermal $_\wedge$region contributes to natural circulation.
center

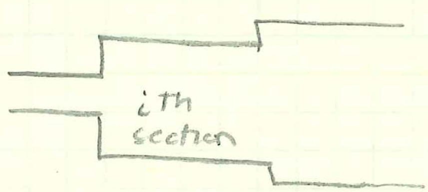
$$\boxed{\rho_r \frac{dv_i}{dt} \sum_i \left(\frac{a_r}{a_i}\right) l_i = \Delta P_{pump} + g \beta \rho_r \Delta T_H l_H - \frac{\rho_r v_r^2}{2} \sum_i \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2} \quad (2)$$

- inertia is driving force.

$(\Delta T_H)_{max} = T_{sat} - T_c \rightarrow$ (limit for single phase natural circulation)

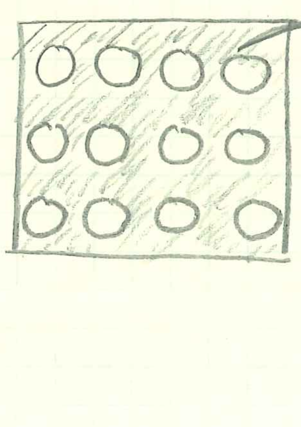
* Derive this equation for homework *

- Define and explain each term.
- Start from 1-D Momentum Eq. with (Bourgeois Assumption).
- Assume single phase / incompressible flow



a_i = constant
l_i = length
f_i = friction factor
D_i = Hydraulic diameter
k_i = minor loss

} B.C. for each node in RELAP/any T-H code.



$$\boxed{D = \frac{4A}{P}}$$

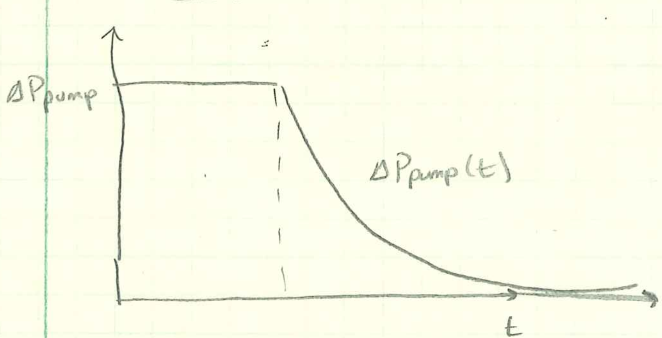
Transition to Natural Circulation

- Assume no change in power (constant heat generation)

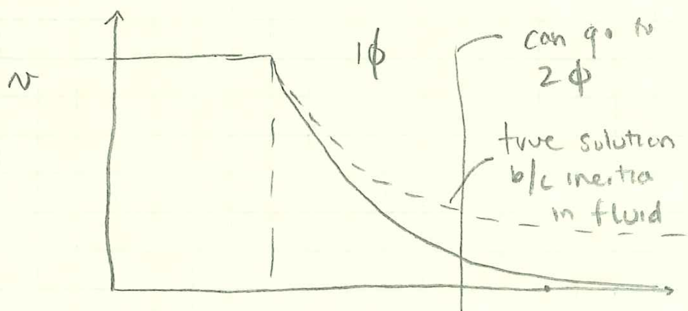
$\dot{q}$ = constant

$$\boxed{\rho_r \sum_i \left(\frac{a_r}{a_i}\right) l_i \frac{\partial v_i}{\partial t} = \Delta p + g \beta \rho_r \Delta T_H l_H - \frac{\rho_r v_r^2}{2} \sum_i \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2}$$

- pump coast down without scram



- Based on the design of the pump



- 1φ initially but 2φ can be reached if $T_{coolant} \geq T_{sat}$. → flow instabilities in nat. circulation.

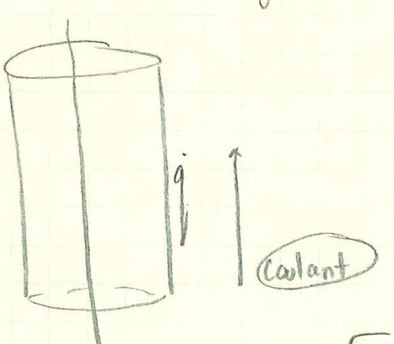
↓

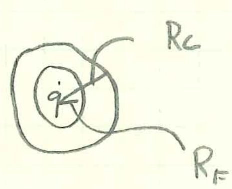
$v_r \sim \sqrt{\Delta P_{pump}}$ (Can be an ok assumption, but the true velocity is larger due to inertial force)

↓

$$\boxed{\rho_r \sum_i \left(\frac{a_r}{a_i}\right) l_i \frac{\partial v_i}{\partial t} = \boxed{\Delta P_{pump}(t)} + \rho_r g \beta \Delta T_H l_H - \frac{\rho_r v_r^2}{2} \sum_i \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2}$$

Consider decay heat only (no pump)





$\pi R_F^2 \dot{q}_F = 2\pi R_c q_c''$

$$\Rightarrow \boxed{q_c'' = \frac{R_F^2}{2R_c} \dot{q}_F} *$$

*[- The q_c'' can be found in simple terms if T_{fuel} is considered constant, otherwise the heat conduction equation inside of the fuel needs to be solved.]