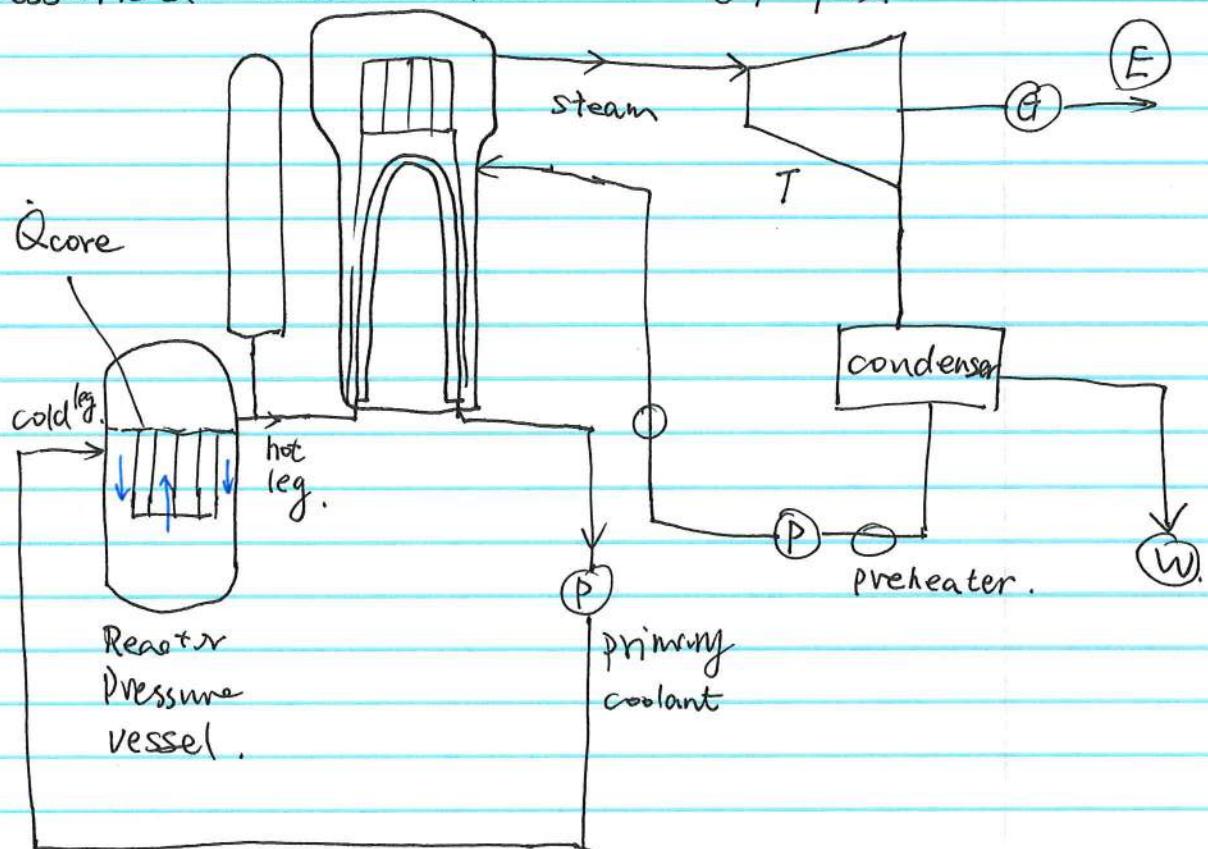


## § Introduction to Nuclear Thermal hydraulic.

Nuclear Power system. (PWR & BWR).

Pressurized Water Reactor. (70%).



the limit of efficiency. is material.

Steel.

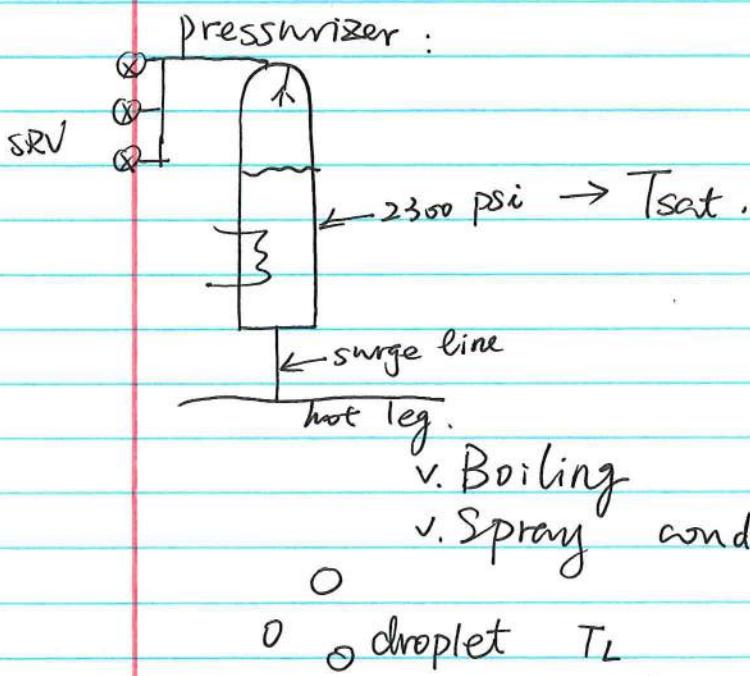
Reactor.: fuel rods  $\rightarrow$  subassembly  $\rightarrow$  core.

$\hookrightarrow$  chain reaction  $\rightarrow$  heat generation  $\rightarrow$  conduction.

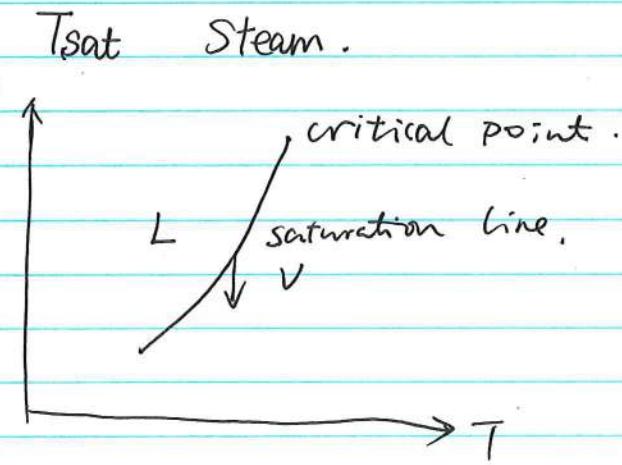
convective cooling, boiling heat transfer.

critical heat flux.

2.



v. flushing (Relief Valve)



Steam . Generator.

• Inside of tubes. convection.

• outside

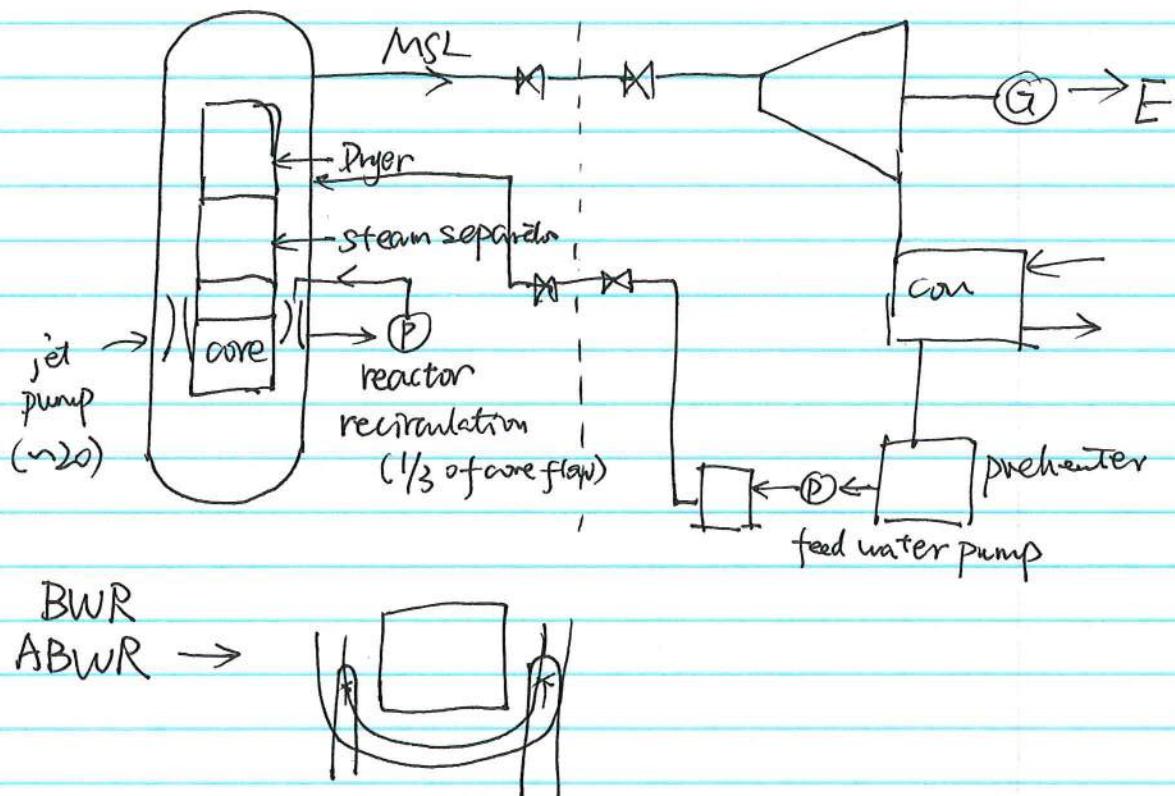
| Moderate boiling.  
| CHF. (Dry out).

Turbine

compressive flow (Expansion)

Condenser : conduction.

## Boiling Water reactor. (BWR).



Reactor vessel. ⌈ lower Plenum

core ( $\sim 4500$  rods)  $X_e \approx 0.14$   
 (core inlet flow restriction)  
 upper plenum  
 Steam Separator. (vortex)  
 dryer.

$$X \geq 0.996.$$

MSL

(Isolation Values).

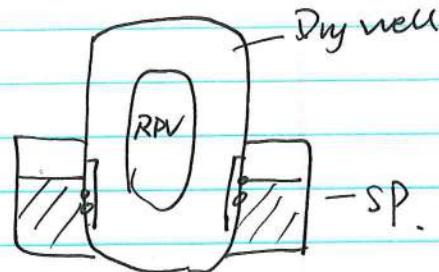
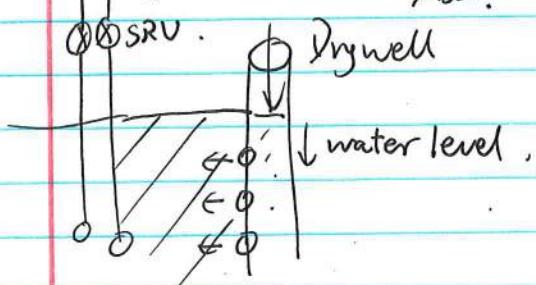
Turbine generator  
condenser

Feedwater pump

## Safety System

Isolation valve

Relief valve  $\rightarrow$  MSL  $\Rightarrow$  SP  
MSL.



ECCS

Containment cooling system

Steady state, start up, shutdown  
(Normal operation)

Coolant Flow

Pump power

(pressure relief valve  $\rightarrow$ )  
Flashing

Pressure regulation (Boiling, Condensation, Flashing)

Heat removal

Heat loss

Steam generator (2 phase RPV or SG)

compressible flow

condensation

critical heat flux

PWR. Departure from Nucleate Boiling (DNB)  
(Bubble flow)

BWR Dryout of liquid film.  
Total power.

~~DNB~~

DNB.

local

~~film dryout~~

film dryout

Accident and safety analysis.

Critical heat flux

- Full power (DNB . dryout)
- LOCA many different mechanisms.

Transient flow (single or two phase)

Thermal shock

Turbulent flow

stress corrosion cracking

Two phase flow phenomenon

critical Heat Flux

Blow Down (Depressurization)

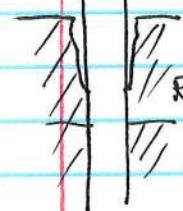
Break  $\rightarrow$  critical. flow. (Boundary condition for LOCA).

$\rightarrow$  Dryout of fuel.

ECCS Injection

Reflooding  
re-wetting

very hot



Passive Safe Reactor.

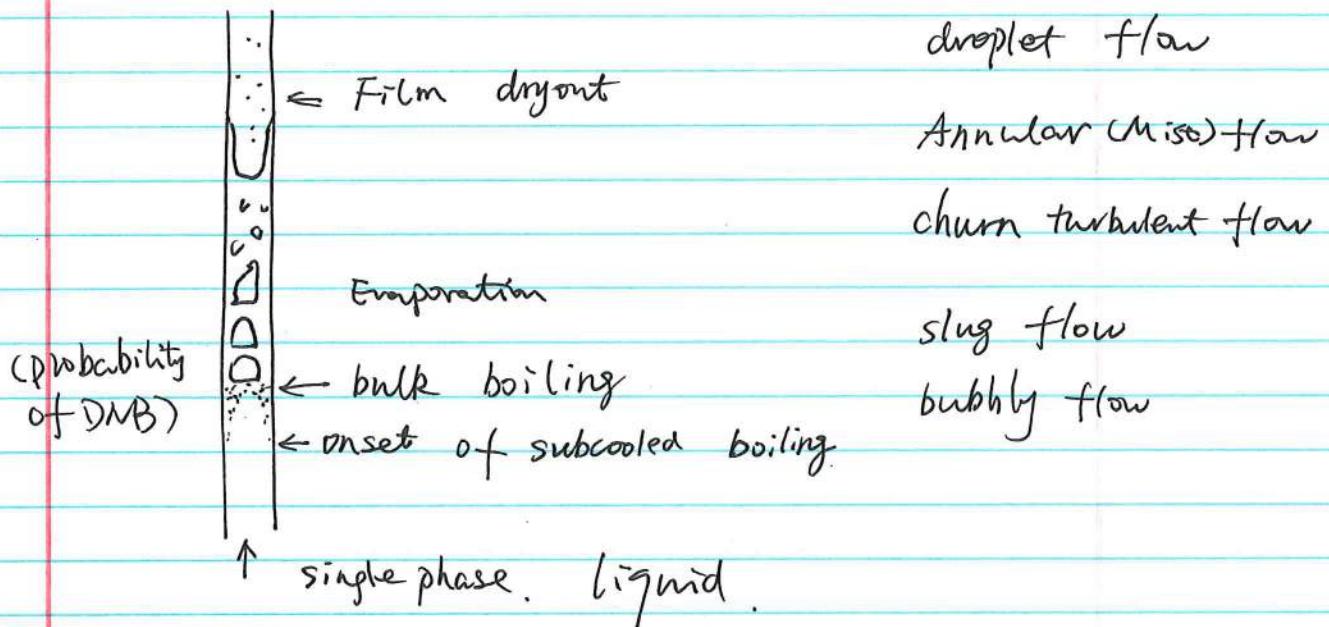
Avoid fuel Dryout.

Natural circulation of Two-phase flow.

Severe accident.  $\rightarrow$  core damage.

Multiphase flow  
chemical reaction  
Radiation  
core melt down.

## Core phenomenon



## Bubble dynamic

Nucleation, bubble departure, Phase change  
coalescence, break-up.

Drag force

Phase distribution.

DNB

## Droplet

entrainment from liquid film, deposition

## Film waves.

Break (LOCA) critical flow.

ECCS. Reflood re wetting

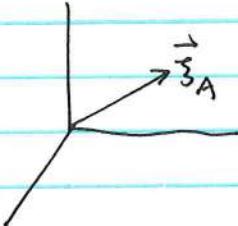
Flooding phenomenon (CCFL)

Counter current flow limitation.

HW #1. Read Appendix A (Transport)

#### 4. Particle Path. Velocity and Acceleration.

- $\vec{s}$  material coordinate (Initial fluid particle position)



$$t=0. \quad \vec{x} = \vec{s}$$

- $\vec{x}$  spatial coordinate  
 $t=t, \quad \vec{x} = \vec{x}(\vec{s}, t)$

Axiom of continuity.

$$\vec{s} = \vec{s}(\vec{x}, t) \Leftarrow \vec{x} = \vec{x}(\vec{s}, t)$$

Any property of the fluid may be followed along a particle path.

$$F(\vec{x}, t) = F(\vec{x}(\vec{s}, t), t) = F(\vec{s}, t)$$

$F(\vec{x}, t)$  : Eulerian formulation  $\Leftarrow$

$F(\vec{s}, t)$  : Lagrangian formulation.

- Velocity.

$i: 1, 2, 3$   
 $x, y, z$

$$v_i \equiv \frac{\partial x_i}{\partial t}$$

$$\boxed{\vec{v} = \frac{\partial \vec{x}}{\partial t}}$$

: definition of velocity.

follow the particle.

Eulerian formulation  $F(\vec{x}, t)$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial t})_{\vec{x}}$$

derivative with respect to time  
keep  $\vec{x}$  constant (observer fixed in space)

$$\frac{D}{Dt} = \frac{\partial}{\partial t})_{\vec{x}}$$

keep  $\vec{x}$  fixed  
(observer moving with fluid).

→ Substantial }  
Material } Derivative.  
Total

$$\left( \frac{\partial F}{\partial t} \right)_{\vec{x}}$$

time rate of change of  $F$  at a fixed point

$$\left( \frac{DF}{Dt} \right)$$

$F$  moving with fluid.

$$\begin{aligned} \frac{DF}{Dt} &= \frac{\partial F(\vec{x}, t)}{\partial t} = \frac{\partial F[\vec{x}(\vec{s}, t), t]}{\partial t} \\ &= \frac{\partial F}{\partial x_i} \frac{\partial x_i}{\partial t} + \frac{\partial F}{\partial t} \\ &= \frac{\partial F}{\partial t} + v_i \frac{\partial F}{\partial x_i}. \end{aligned}$$

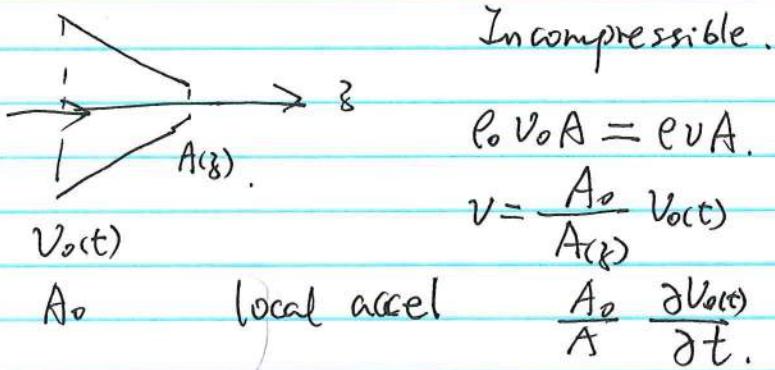
$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \mathbf{V} \cdot \nabla F$$

Acceleration. change of velocity with respect to time

$$\frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = \vec{a}.$$

local acceleration

convective acceleration



$$\text{convective accce. } \vec{v} \cdot \vec{\nabla} v = \frac{A_0}{A} v_0 \frac{\partial}{\partial z} \left( \frac{A_0}{A} v_0 \right)$$

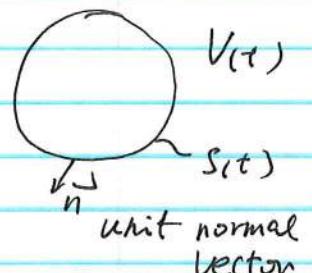
$$= - v^2 \underbrace{\frac{1}{A} \frac{dA}{dz}}_{?}$$

. Integral Theory.

Divergency theory.

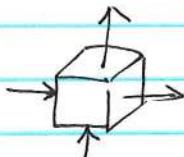
$$\int_V (\nabla \cdot \vec{v}) dV = \oint_S (\vec{n} \cdot \vec{v}) ds$$

$\oint_S$  : surface of volume  $V$ .



Volume integral  $\Rightarrow$  surface integral.

$$\nabla \cdot \vec{v} = ?$$



green's theory

$$\int_V (\nabla \cdot \psi) dV = \oint_S \vec{n} \cdot \psi ds$$

$\psi$  : scalar vector  $\sim$  tensor.

10

$$\int_V \nabla a \, dV = \oint_S \vec{n} \cdot \vec{a} \, ds.$$

$$\int_V (\nabla \cdot \vec{r}) \, dV = \oint_S (\vec{n} \cdot \vec{r}) \, ds$$

$$\int_V \nabla p \, dV = \oint_S p \vec{n} \, ds.$$



• Leibniz Formula.

$$\frac{d}{dt} \int_{x=\alpha(t)}^{x=\beta(t)} f(x,t) \, dx = \left[ \frac{\partial f}{\partial t} + \frac{df}{dx} f(x,t) \right]_{x=\beta} - \left[ \frac{df}{dx} f(x,t) \right]_{x=\alpha}.$$

$$\frac{d}{dt} \int_{V(t)} \psi \, dV = \int \frac{\partial \psi}{\partial t} \, dV + \oint_S \psi (\vec{v}_s \cdot \vec{n}) \, ds.$$

surface flux

$v_s$  : surface velocity.

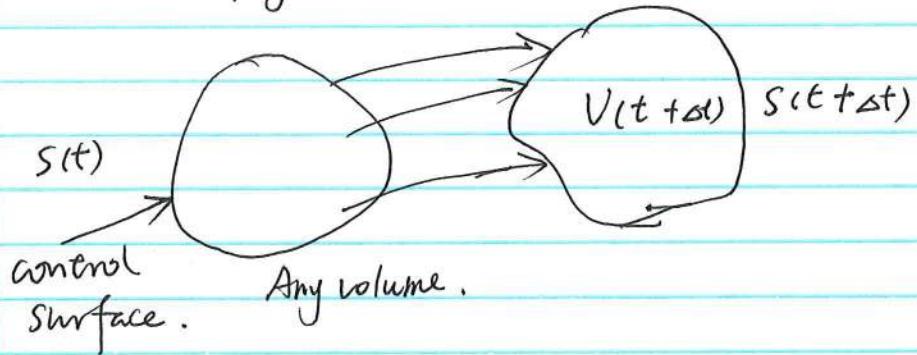
### Appendix A

Anz's Book

$$\frac{d}{dy} \int_{g_1(y)}^{g_2(y)} f(x,y) \, dx = \int_{g_1(y)}^{g_2(y)} \frac{\partial f(x,y)}{\partial y} \, dx + g_2'(y) f(g_2(y), y) - g_1'(y) f(g_1(y), y)$$

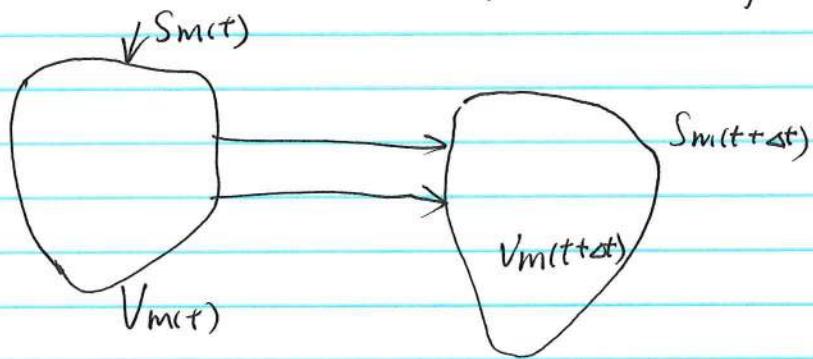
- Control Volume.  
simply connected.

g. 11  
train



- Material Volume

Boundary of system : { Boundary of Mass.  
no mass cross the boundary.  
consist of some mass.



$\frac{d}{dt} \int_V \gamma dV$  : change of  $\gamma$  in  $V$  with respect to time

$$\frac{d}{dt} \int_{V_m} \gamma dV$$

↑

$$\frac{D}{Dt} \int_{V_m} \gamma dV$$

Leibnitz Formula

$$\frac{d}{dt} \int_V \psi dV = \int_V \frac{\partial \psi}{\partial t} dV + \oint_S \psi (\vec{v}_s \cdot \vec{n}) ds.$$

time rate of  
change of total  
 $\psi$  in  $V$ .

Total time  
rate of change  
of  $\psi$  in  $V$ .

surface velocity

change in amount  
of total  $\psi$  due to vol.  
change caused by surface motion

Special case of Material Volume.

$$\frac{d}{dt} \int_{V_m} \psi dV = \int_{V_m} \frac{\partial \psi}{\partial t} dV + \oint_{S_m} \psi (\vec{v}_s \cdot \vec{n}) ds.$$

$\vec{v}_s = \vec{v}$  : material velocity

$$\oint_{S_m} \psi (\vec{v}_s \cdot \vec{n}) ds = \oint \psi (\vec{v} \cdot \vec{n}) ds.$$

use divergence theorem  $\oint \rightarrow$  vol integral.

$$\oint_S (\psi \vec{v}) \cdot \vec{n} ds = \int_V \nabla \cdot (\psi \vec{v}) dV.$$

$$\frac{d}{dt} \int_{V_m} \psi dV = \int_{V_m} \left[ \frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \vec{v}) \right] dV.$$

Reynolds Transport theorem

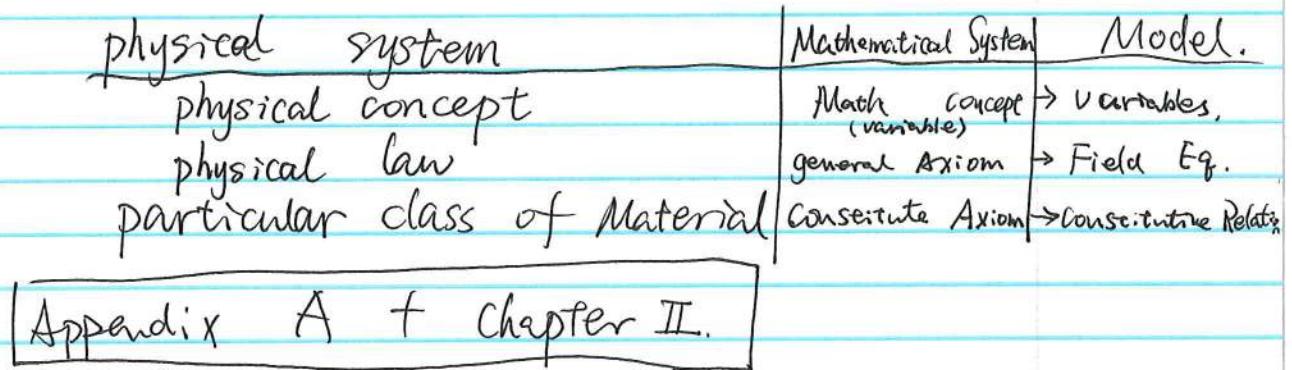
$$\boxed{\frac{D}{Dt} \int_{V_m} \psi dV = \int_{V_m} \left[ \frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \vec{v}) \right] dV.}$$

Integral Balance  $\Rightarrow$  Differential Balance

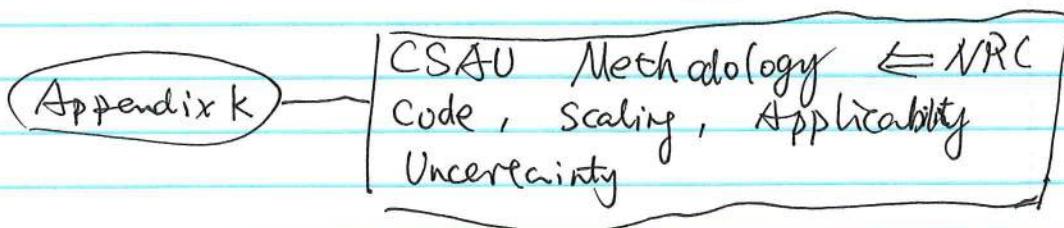
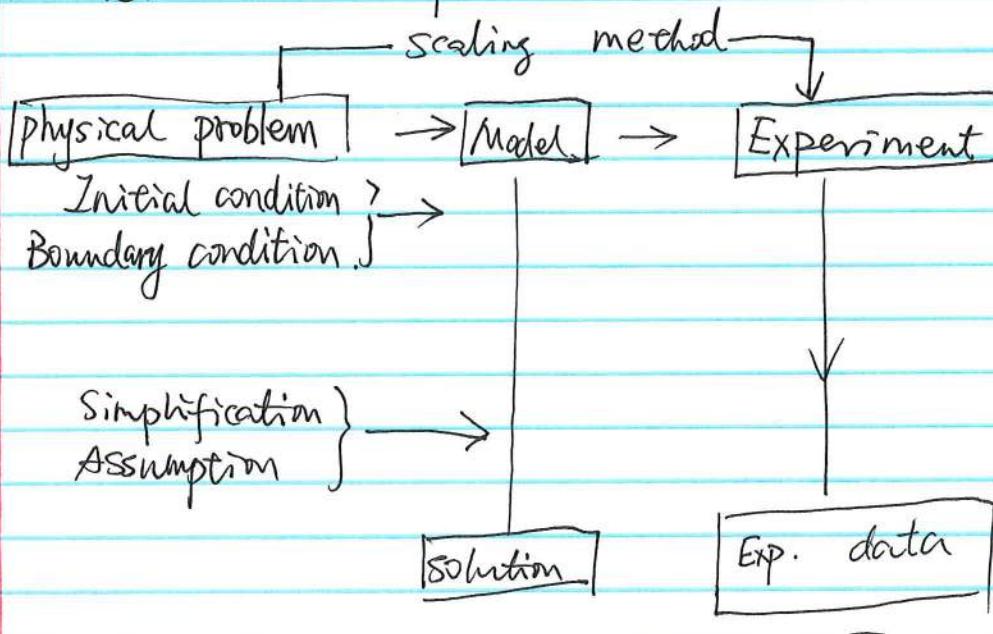
## II general Formulation.

### (1) Fundamental approach.

#### (i) Mathematical Model.



### (2) Relation to experiment.



## Nuclear Reactor Thermal-hydraulics.

- Single phase flow

Laminar flow Model }

3-D

Turbulent flow Model.)

1-D

(Quasi 1-D)

Control Volume.

- Two phase Flow (Multiphase flow)

- Local Instant Formulation with interface

⇒ - Two -fluid Model )

- Drift flux model)

- slip Flow model.

- homogenous Flow model ( $T_g = T_f$ ,  $v_g = v_f$ )

⇒ { 3-D  
1-D

| Quasi 1-D : subchannel Model.

| control volume (0-D)

- Need of experiment data

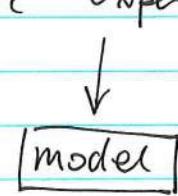
1) Observe phenomena → scoping experiment

2) fundamental Understanding → basic experiment

Detailed Instrumentation )

Wide range of Parameter )

Simplified geometry )



3) Complicated Thermal hydraulic phenomena (Nuclear Reactor)

Integral Test

{ scaled down

{ geometry is similar (most complicated)

## Separate - effect test.

Component

Region

Particular Phenomena

## Design Data Base

CHF

2) conservation principle.

Integral Balance  $\Leftarrow$  you can observe

$\downarrow$   
Physical laws.

① Mass

$$\frac{D}{Dt} \int_{V_{\infty}} e dV = 0.$$

$\underbrace{\phantom{0}}$

Total Mass

② Momentum Equation.

Mass  $\times$  Acc.  $= \sum$  Forces.

$$\frac{D}{Dt} \int_{V_m} e \vec{v} dV = - \oint \vec{J} \cdot \vec{n} ds + \int \vec{f}_g dV$$

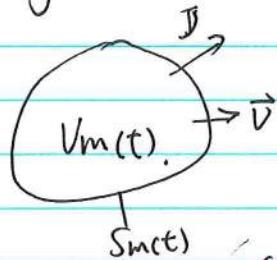
$\underbrace{\phantom{0}}_{\text{Total momentum}}$        $\underbrace{\phantom{0}_{\text{surface force}}}_{\text{Body force. } (\vec{F})}$

③ Energy Equation

$$\frac{D}{Dt} \int_{V_m} e (u + \frac{v^2}{2}) dV = - \underbrace{\oint \vec{J} \cdot \vec{n} ds}_{\text{Heat Trans. and work done at surface}} + \underbrace{\int \vec{f}_g dV}_{\text{Heat generation work done by body force}}$$

$\underbrace{\phantom{0}_{\text{Total Energy}}}_{\text{Heat Trans. and work done at surface}}$

### 3. general balance Equation.



$V_m$ : fixed mass volume

•  $\psi$  property enclosed in vol.

(mass · momentum · energy)

$$\int \psi dV = \text{total amount of } \psi \text{ in } V.$$

•  $J$ : flux of property across the surface  $S_m$ .

$-\phi J \cdot \vec{n} ds$  : net flux in cross the surface  $S_m$ .

•  $\dot{\psi}_g$  : generation of  $\psi$  in vol.  $V$

$$\int \dot{\psi}_g dV : \text{total generation of } \psi \text{ in } V$$

Change of  $\psi$  in Vol = Influx through surface  
+ generation in Vol.

$$\frac{D}{Dt} \int_{V_m} \psi dV = - \oint_{S_m} J \cdot \vec{n} ds + \int_{V_m} \dot{\psi}_g dV \quad \textcircled{P}$$

change of  $\psi$       Influx of  $\psi$       generation of  
in  $V$ .      through  $S_m$        $\psi$  in  $V$

Integral Balance  $\rightarrow$  Differential Balance

$$(i) \frac{D}{Dt} \int_{V_m} \psi dV = \int_V \left[ \frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \vec{v}) \right] dV \quad \textcircled{2}$$

(Reynolds Transport theory)

(ii) Green's theorem.

$$-\oint \mathbf{J} \cdot \hat{\mathbf{n}} dS = - \int \nabla \cdot \mathbf{J} dV. \quad \textcircled{3}$$

$\textcircled{2} \textcircled{3} \rightarrow 0$

$$\int_{V_m} \left[ \frac{\partial \psi}{\partial t} + \nabla \cdot (\psi \vec{v}) \right] dV = - \int_{V_m} \nabla \cdot \mathbf{J} dV + \int_{V_m} \dot{\psi}_g dV.$$

$$\rightarrow \frac{\partial \psi}{\partial t} + \boxed{\nabla \cdot (\psi \vec{v})} = - \nabla \cdot \mathbf{J} + \dot{\psi}_g$$

Physical meaning

Time rate  
of change  
of  $\psi$  in unit  
volume

convection  
of  $\psi$   
by material  
motion.

in flux  
of  $\psi$   
at the  
surface

Differential  
Balance Eq.

generation of  $\psi$   
in  $V_0$ .



what's the difference between 2.

$\psi \rightarrow$  Mass  
momentum  
energy.

(+) Mass Balance Eq.  $\Rightarrow$  continuity,

$$\begin{cases} \psi = \rho \\ \mathbf{J} = \dots \\ \dot{\psi}_g = \dots \end{cases} \xrightarrow{\text{density}} \Rightarrow$$

$$\frac{d\rho}{dt} + \nabla(\rho \cdot \vec{v}) = 0$$

General Balance Equation.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{V} = -\nabla \cdot \vec{J} + \dot{\psi}_g$$

$\nabla \cdot \vec{V}$ : rate of convection of mass due to material motion per unit volume (with respect to fixed frame).

$$\rho v_x + \frac{\partial \rho v_x}{\partial x} dx,$$

$$\frac{\partial}{\partial t} (\rho dy dx ds) = dy ds \left[ \rho v_x - \left( \rho v_x + \frac{\partial v_x}{\partial x} dx \right) \right]$$

+ ...  
+ ...

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{V} = 0.$$

$$\frac{\partial \rho}{\partial t} + \vec{V} \cdot \nabla \rho + \rho \nabla \cdot \vec{V} = 0$$

$$\frac{D\rho}{Dt} = \frac{\partial \rho}{\partial t} + \vec{V} \cdot \nabla \rho$$

$$\therefore \frac{1}{\rho} \frac{D\rho}{Dt} = -\nabla \cdot \vec{V}.$$

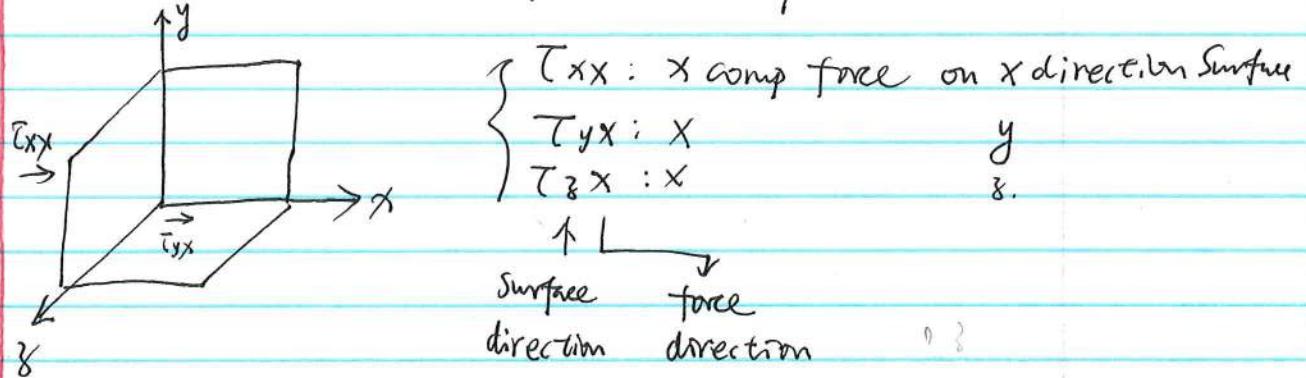
\* Incompressible fluid  $\leftarrow$  most liquid flow  
many gas flow  $M \ll 1$

In this case.  $\frac{D\rho}{Dt} = 0$ : fluid does not change as it flows  
(need not have uniform density)

$$\nabla \cdot \vec{V} = 0$$

\* constant density flow.  
 $\rho = \text{const.} \Rightarrow \nabla \cdot \vec{V} = 0$

## 5) Momentum Balance Equation - Equation of Motion.



Quantity to Balance: momentum/vol.  $\rho \vec{V}$ .

$$\psi = \rho \vec{V}.$$

$$\mathcal{T} = \Pi = p \cdot \mathbb{I} + \tau$$

(momentum flux - surface force).

$\Pi$ : total stress

$p$ : pressure

$\tau$ : viscous stress

$\mathbb{I}$ : unit tensor.

$$\dot{\psi}_g = \rho \vec{F} = \rho \vec{g}$$

(momentum source - body force)

$\vec{F}$ : body force

$\vec{g}$ : gravity  
(EM)

### Momentum Equation.

$$\frac{\partial \vec{V}}{\partial t} + \nabla \cdot (\rho \vec{V} \cdot \vec{V}) = - \nabla p - \nabla \cdot \tau + \rho \vec{g}$$

time rate of change of the momentum/vol.  
rate of momentum change by convection.  
pressure force/vol.  
viscous force/vol.  
gravitation body force/vol.

## Couhy's Equation of Motion.

M.E

$$\begin{aligned} \text{LHS} &= \rho \frac{\partial \vec{v}}{\partial t} + \vec{v} \frac{\partial \rho}{\partial t} + \rho \vec{v} \cdot (\nabla \vec{v}) + \vec{v} (\nabla \cdot \rho \vec{v}) \\ &= \rho \left( \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} \right) + \vec{v} \left[ \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} \right] \\ &= \rho \frac{D\vec{v}}{Dt}. \end{aligned}$$

c.t.

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \tau + \rho \vec{g}$$

↓

$$\left( \frac{\text{mass}}{\text{vol}} \right) \times \left[ \begin{array}{l} \text{total} \\ \text{acceleration} \end{array} \right] = \sum \text{force}$$

← Newton's Law

↓

local acceler.  $\left( \frac{\partial \vec{v}}{\partial x} \right)$

+ convective acceler.  $(\vec{v} \cdot \nabla \vec{v})$

Sum of { surface force  
body force }

### (7) Natural Convection

Bossinesq Assumption

- Density change due to thermal expansion
- Density change only important in gravity term

Thermal Expansion Coefficient :  $\beta$

$$\beta \equiv \frac{1}{V} \frac{\partial V}{\partial T} \Big|_P = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} \Big|_P. \quad (1)$$

$V$ : specific volume

$$\text{For small density change } d\rho = -\rho \beta dT \quad (2).$$

Reference Temp:  $\bar{T}$   
 Density:  $\bar{\rho}$

$$\rho - \bar{\rho} = -\bar{\rho} \beta (T - \bar{T}) \quad (3)$$

(3)  $\rightarrow$  Momentum Eq.

$$\bar{\rho} \frac{D\vec{v}}{Dt} = -\nabla P - \nabla \cdot \pi + [\bar{\rho} - \bar{\rho} \beta (T - \bar{T})] \vec{g}$$

Note: in the inertia term  $\rho = \bar{\rho}$

$\beta$ : thermal physical properties ( $\leftarrow$  Table)

- If the pressure field is approximately hydrostatic  
 $(\vec{v}$  is small in large tank)  
 $-\nabla P + \bar{\rho} \vec{g} = 0$ . (hydrostatic pressure)

- Momentum Eq.

$$\bar{\rho} \frac{D\vec{v}}{Dt} = -\nabla \cdot \pi - \bar{\rho} \beta (T - \bar{T}) \vec{g}$$

- Natural convection case

$$\begin{cases} \nabla P = \nabla P_D + \nabla P_S \\ \nabla P_S = \vec{g} \bar{\rho} \end{cases} \quad \begin{matrix} D: \text{Dynamic} \\ S: \text{static} \end{matrix}$$

M.E

$$\bar{\rho} \frac{D\vec{v}}{Dt} = -\nabla P_D + \underbrace{\mu \nabla^2 \vec{v}}_{\substack{\text{dynamical} \\ \text{pressure} \\ \text{gradient}}} - \bar{\rho} \beta (T - \bar{T}) \vec{g} \underbrace{\vec{g}}_{\substack{\text{viscous} \\ \text{force}}} \underbrace{\bar{\rho} \beta (T - \bar{T}) \vec{g}}_{\substack{\text{buoyance} \\ \text{force}}}$$

(negligible in many cases)

## Mechanical Energy Eq. (scalar Eq.)

By scalar product of the local velocity  $\vec{v}$  with M.E.

$$\vec{v} \cdot [ \rho \frac{D\vec{v}}{Dt} = -\nabla p - \nabla \cdot \tau + \rho \vec{g} ].$$

$$\rho \frac{D \frac{v^2}{2}}{Dt} = -\vec{v} \cdot \nabla p - \vec{v} \cdot (\nabla \cdot \tau) + \rho \vec{v} \cdot \vec{g}$$

use C.E. ✓

$$\rho \frac{D \frac{v^2}{2}}{Dt} = \frac{\partial \rho \frac{v^2}{2}}{\partial t} + \nabla \cdot (\rho \frac{v^2}{2} \cdot \vec{v})$$

(C.E.)

also.  $\begin{cases} \vec{v} \cdot \nabla p = \nabla \cdot (p \vec{v}) - p \nabla \cdot \vec{v} \\ \vec{v} \cdot \nabla \tau = \nabla \cdot (\tau \cdot \vec{v}) - \tau \cdot \nabla \vec{v} \end{cases}$

$$\frac{\partial \rho \frac{v^2}{2}}{\partial t} + \nabla \left( \frac{1}{2} \rho v^2 \vec{v} \right) = -\nabla (p \vec{v}) + p \nabla \cdot \vec{v}$$

↓                          convection                          work done by pressure  
rate of                          reversible conversion  
increase                          to internal energy

-  $\nabla (\tau \cdot \vec{v})$                   +  $\tau \cdot \nabla \vec{v}$   
work done by                          irreversible  
viscous force                          conversion to  
+  $p(\vec{v} \cdot \vec{g})$                           internal energy.  
work done  
by body force.

## Energy Equation.

Quantity to be balanced : total energy.

$$\psi = \rho (u + \frac{v^2}{2}) \quad \text{energy / volume}$$

$$\begin{cases} u: \text{internal energy / mass} \\ \frac{v^2}{2}: \text{k.E. / mass} \end{cases}$$

$$\vec{J} = \vec{g} + \nabla \cdot \vec{v}$$

energy flux

heat flux + work done by surface force

$$\nabla = P \vec{I} + \vec{\tau}$$

$$\dot{q}_g = \rho \vec{v} \cdot \vec{g} + \dot{q}$$

energy source

(work done by + internal heat  
body force generation from

$\uparrow$  nuclear

E.M

ch. reaction

Energy Eq.

$$\frac{\partial \rho(u + \frac{v^2}{2})}{\partial t} + \nabla \cdot [\rho(u + \frac{v^2}{2}) \vec{v}] = -\nabla \cdot \vec{g}$$

heat conduction

time rate of

energy

$$-\nabla(p\vec{v})$$

work done by  
pressure

change

convection

$$-\nabla(\Pi \vec{v})$$

work done by  
viscous force

$$+\rho \vec{v} \cdot \vec{g}$$

work done by  
gravity

$$+\dot{q}$$

$\rightarrow$  internal heat  
generation

$$\rho \frac{D}{Dt} (u + \frac{v^2}{2})$$

If the body force field has a potential  $\Phi$

$$\vec{g} = -\nabla \Phi$$

$\Phi$ : gravitational potential.

$$\text{then, } \rho \vec{v} \cdot \vec{g} = -\rho (\vec{v} \cdot \nabla \Phi) = -\rho \frac{D\Phi}{Dt} + \cancel{\rho \frac{\partial \Phi}{\partial t}} > 0$$

$$\rho \vec{v} \cdot \vec{g} = -\rho \frac{D\Phi}{Dt}$$

Then total energy may be defined as

$$e = u + \frac{v^2}{2} + \phi \quad (\text{internal E.} + \text{k.E} + \text{potential})$$

E.E.

$$\rho \frac{D}{Dt} (u + \frac{v^2}{2} + \phi) = -\nabla \cdot \vec{q} - \nabla \cdot (p \vec{v}) - \nabla \cdot (\tau \cdot \vec{v}) + \dot{g}$$

Separation of M.E and Thermal Energy.

Internal Energy Eq.

Elimination of K.E terms in view of M.E Eq.

$$\text{Ta.E } \rho \frac{D(u + \frac{v^2}{2})}{Dt} = -\nabla \cdot \vec{q} - \nabla \cdot (p \vec{v}) - \nabla \cdot (\tau \cdot \vec{v}) + \dot{e} \vec{v} \cdot \vec{g} + \dot{g}$$

K.E

$$\rho \frac{D \frac{v^2}{2}}{Dt} = -\vec{v} \cdot \nabla p + -\vec{v}(\nabla \cdot \tau) + \dot{e} \vec{v} \cdot \vec{g}$$

Internal E.

$$= \rho \frac{Du}{Dt} = -\nabla \cdot \vec{q} - p \nabla \cdot \vec{v} - \tau \nabla \cdot \vec{v} + \dot{g}$$

$$\text{C.E} \Rightarrow \rho \frac{Du}{Dt} = \frac{\partial \rho u}{\partial t} + \nabla \rho u \cdot \vec{v}$$

$$\frac{\partial \rho u}{\partial t} + \nabla(\rho u \vec{v}) = -\nabla \cdot \vec{q} \quad \leftarrow \text{rate of energy tr. by heat C.}$$

time rate of change of i.e. per vol	rate of conversion of i.e.	$-p \nabla \cdot \vec{v}$	rate of reversible conversion of work done by pressure
		$-\tau \cdot \nabla \vec{v}$	rate of irreversible conversion of work done by viscous force.

This is not a basic balance equation  
the RHS is not in  $\nabla C > 0$  form

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HW.

- 1) consider fixed mass volume  $V_m$   
 control vol.  $v$   
 at  $t = t_0$ ,  $V_m = v$



$$\frac{d}{dt} \int_{V_m} \psi dV = \frac{d}{dt} \int_v \psi dV + \oint_S \psi [\vec{v} - \vec{v}_s] \cdot \vec{n} ds.$$

$\vec{v}$ : fixed mas vel.

$\vec{v}_s$ : cont wl. surface velocity.

- 2) obtain general balance equation for control vol.

$$\frac{d}{dt} \int_{V_m} \psi dV = - \oint_S \vec{J} \cdot \vec{n} ds + \int_v \vec{\psi}_g dV.$$

$$\frac{d}{dt} \int_v \psi dV =$$

- 3) write Differential Balance Equation (general B.)

(1) Apply this to mass  $\rightarrow$  conservation of mass  
 momentum  $\rightarrow$  momentum eq.

## Energy equation

$$\rho \frac{D(u + \frac{v^2}{2})}{Dt} = -\nabla \cdot \vec{g} - \nabla \cdot (p \vec{v}) - \nabla \cdot (\tau \cdot \vec{v}) + \rho \vec{v} \cdot \vec{g} + \dot{q}$$

$$\therefore \rho \frac{D \frac{v^2}{2}}{Dt} = -\vec{v} \cdot \nabla p - \vec{v} \cdot [\nabla \tau] + \rho \vec{v} \cdot \vec{g}$$

K.E

$$\rho \frac{DU}{Dt} = -\nabla \cdot \vec{g} - P \nabla \cdot \vec{v} - \text{IV. } \vec{v} \cdot \vec{v} + \dot{q}$$

heat flux      pressure work  
 converted to energy      converted to energy  
 (reversible)      viscous dissipation

$$M = \frac{V}{a} < 0.3. \quad \text{K.E is not important.}$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla$$

$$\left. \rho \frac{Du}{Dt} = \rho \left[ \frac{\partial u}{\partial t} + \vec{v} \cdot \nabla u \right] \right\} \text{non conservative form}$$

$$A \frac{\partial u}{\partial t} + \nabla \cdot \rho u \vec{v} \quad \text{conservative form}$$

enov.



$$\rightarrow \boxed{\rho \frac{Du}{Dt} = -\nabla \cdot \vec{g} - P \nabla \cdot \vec{v}.}$$



$$dU = dQ - PdV.$$

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Enthalphy. Equation.

$$i = u + p/e.$$

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho(i \vec{v})) = -\nabla \cdot \vec{g} + \frac{\partial p}{\partial t} - \pi; \quad \vec{v} = \vec{u} + \vec{g}$$

$$\rho \frac{\partial i}{\partial t} = -\nabla \cdot \vec{g} + \frac{\partial p}{\partial t}$$

(i) Single Mixture (Steam + Air)

$\rho_k$  : mass concentration,  
(mass of k component/vol.).

$w_k$  : mass fraction =  $\frac{\rho_k}{\rho}$ .

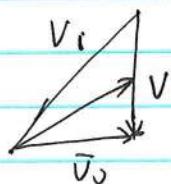
$\rho$  : mixture density  $\rho = \sum_{k=1}^n \rho_k$ .

$v_k$  : velocity of component k.

$\rho_k \vec{v}_k$  : momentum of k component.

$\vec{p}v = \sum_{k=1}^n \rho_k \vec{v}_k$  : total momentum

$\vec{V} = \frac{\sum \rho_k \vec{v}_k}{\sum \rho_k}$  = center mass velocity.



Diffusion velocity.

$$\vec{v}_{km} = \vec{v}_k - \vec{V} \quad \sum \rho_k \vec{v}_{km} = 0$$

In general balance Eq.

$$\left. \begin{array}{l} \psi = \rho_k \\ \overline{J} = \rho_k (\vec{V}_k - \vec{v}) = \rho_k \vec{V}_{km} = \vec{J}_k \quad \text{diffusion mass flux} \\ \dot{r}_k = r_k. \quad \text{reaction rate.} \end{array} \right\}$$

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot \rho_k \vec{V} = - \nabla \cdot \vec{J}_k + \dot{r}_k$$

↓  
 diffusion  
 mass flux  
 with respect  
 to  $\vec{v}$

$$\vec{J}_k = \rho_k (\vec{V}_k - \vec{v})$$

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot \rho_k \vec{V}_k = 0 + \dot{r}_k \quad \left. \begin{array}{l} \text{multi-field.} \\ \text{two-field.} \end{array} \right\}$$

$$\sum \dot{r}_k = 0$$

$$\sum \left\{ \frac{\partial \rho_k}{\partial t} + \nabla \cdot \rho_k \vec{V}_k = \dot{r}_k \right\}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = \sum \dot{r}_k = 0.$$

$$M.N \psi = \rho \vec{v}$$

$$\overline{J} = \rho \overline{I} + \overline{I} + \sum \rho_k \vec{V}_{km} \vec{V}_{km} = \overline{\pi}$$

$$\dot{\psi}_g = \sum \rho_k \vec{J}_k$$

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v} \vec{v} = - \nabla p - \nabla \cdot \overline{I} - \nabla \cdot \sum \rho_k \vec{V}_{km} \vec{V}_{km} + \sum \rho_k \vec{J}_k$$

solve for  $\vec{v}$

## Single flow constitutive Relation

## (1) closure of Equation System.

Balance Eq. (Mass. Momentum. Energy)  $\rightarrow ?$ 

Unknown : Mass  $e \vec{v}$   
 Momentum  $p \cdot \vec{n} \cdot \vec{q}$  } 8 unknowns.  
 Energy  $u \cdot \vec{q} \cdot \vec{q}$  }

① Balance Equation should be supplemented by various constitutive relations (closure relations)

② Constitutive relations

- model material response observed in experiment

- vislate any physical law

- : reasonable simple forms

- fit to certain practical idealization

## (2) 2nd Law of Thermodynamics

(i) most important physical law outside of conservation Equation.

- instance of Entropy  $S$   
Temperature  $T$

- 2nd Law

$$\frac{d}{dt} \int_{V_m} e s dV + \oint \underbrace{\frac{\vec{q}}{S_m T} \cdot \vec{n}}_{\text{entropy flux}} ds - \underbrace{\int_{V_m} \frac{\dot{q}}{T} dV}_{\text{entropy source}} \geq 0$$

use Reynold's Transport theorem

$$\frac{\partial}{\partial t} \rho s + \nabla \cdot (\rho s \vec{v}) + \nabla \cdot \left( \frac{\vec{q}}{T} \right) - \frac{\dot{q}}{T} \equiv \Delta \geq 0$$

$\Delta$ : entropy production.

$\equiv$ : reversible system (ideal)

$>$ : irreversible system

general guidelines.

} Entropy Inequality  
 } Determinism  
 } Frame indifference.  
 Local action

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Conscientive Relation

} Thermodynamic Property  
 } Forces  $\rightarrow$  Momentum  
 heat energy.

# (4) Equation of state

Fundamental Equation of state

$$u = u(s, p)$$

(2)

$u$ : internal energy

$s$ : entropy

$p$ : density

$$\begin{cases} T \equiv \frac{\partial u}{\partial s} \\ p = -\frac{\partial u}{\partial (1/p)} \end{cases}$$

Th. temperature (3)

Th. dy. pressure (4)

$$du = Tds - p d(1/\rho)$$

Legendre Transformation  $\leftarrow$  Book.

Practical Equation of State  
 $S \rightarrow T, P$ .

1. Eq. (Fundamental Eq. of state)  $\Rightarrow$  2 Eqs of state

$$\begin{cases} p = p(p, T) \\ u = u(p, T) \end{cases}$$

Thermal Eq. of state  
 Calorie Eq. of state.

Example

(a). Incompressible fluid

$$\rho = \text{constant}$$

$$u = u(T) . \quad du = C_v dT \quad du = C_v dT$$

(b) Ideal gas (steam, air,  $N_2$ ). (approximation)

$$p = RT$$

$$u = u(T)$$

(3) Mechanical constitutive Relation

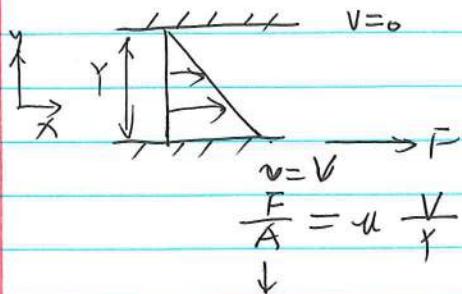
$\uparrow$  motion force

$$\vec{x}, \vec{g}$$

✓ viscous force.

(a). Inviscid fluid.  $\tau = 0$

✗ { slip at the wall.  
 } no frictional resistance

(b) Linear Viscous Fluid (~~Newton~~ Newton)

$$\tau_{xy} = -\mu \frac{dv_x}{dy} \quad \sim \text{velocity gradient}$$

$\mu$ : viscosity

Force  $\sim$  Velocity gradient (Linear)  
 Newton's law of viscosity  $\Rightarrow$  Newtonian fluid.

Note: Similarity between  
 } Momentum transfer

Viscous force  $\propto$

} Heat (energy) transfer  
 Diffusion Mass flux

heat flux  $\frac{\dot{q}}{A}$

$$\tau_{xy} = -\mu \frac{du}{dy} \quad (\text{Newtonian Law of Viscosity})$$

$$\dot{q}_y = -k \frac{dT}{dy} \quad (\text{Fourier's Law of Conduction})$$

$$j_{ky} = -D_k \frac{dw}{dy} \quad (\text{Fick's Law of Diffusion})$$

$\left. \begin{array}{l} w_k: \text{mass} \\ D: \text{diffusion coefficient} \\ \mu: \text{viscosity} \\ k_i: \text{thermal conductivity} \end{array} \right\}$

$$\tau = -\mu [\nabla \vec{v} + (\nabla \vec{v})^t]$$

$\tau$  : transported Tensor

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

generalized it  $\Rightarrow$  n-s Fluid

$$\tau = -\mu [\cancel{\nabla \vec{v}} + (\nabla \vec{v})^t] + \left(\frac{2}{3}\mu - \frac{1}{2}\mu I\right) \nabla \cdot \vec{v} \cdot I$$

c) Body force

Gravitational force  $\vec{g} = \text{const.}$

Electrostatic. } much complicated.

E(e magnetc.)

Magnetic fluid dynamic

Plasma Physics.

(4) Thermal conserutive Relation

① conduction in Isotropic Material.

$$\vec{q} = -k \nabla T$$

$k$ : conductivity  $k = k(T)$

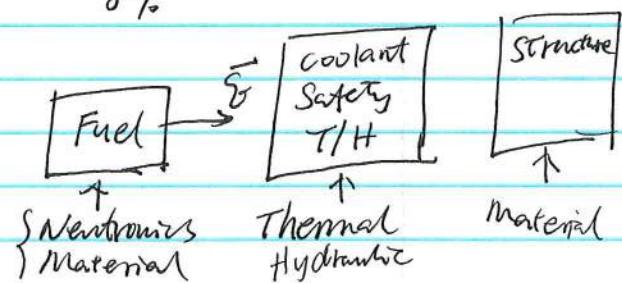
② Internal heat generation.  $\dot{q}$

$\dot{q}$ : Fission & decay heat.

92 %.

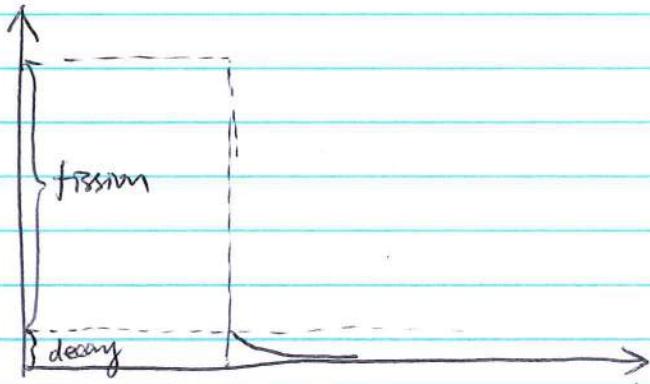
8 %

$N$  reactor



$$\cdot \vec{j}_k = -P D \nabla w_k$$

$\dot{q} \leftarrow$  fission & decay heat.



$$\dot{r}_k = r_k \quad \text{chemical reaction rate}$$

$$\dot{r}_k = r_k(w_k, T, P).$$

### (7). Entropy generation check.

Fundamental Eq. of state  $Tds = du - \frac{P}{P^2} dp \quad s = s(u, P)$

internal energy  $E_f$ .

$$\rho \frac{Du}{Dt} = -\nabla \cdot \vec{q} - P \nabla \cdot \vec{v} - \bar{\tau} : \nabla \vec{v} + \dot{s}$$

$$\rho \left\{ T \frac{Ds}{Dt} + \frac{P}{P^2} \frac{Dp}{Dt} \right\} \quad \text{CE} \rightarrow \rho \left\{ T \frac{Ds}{Dt} - \frac{P}{P} \nabla \cdot \vec{v} \right\}$$

Internal Energy  $E_f \rightarrow$  Entropy Energy  $\dot{s}$ .

$$\rho \frac{Ds}{Dt} = -\frac{\nabla \cdot \vec{E}}{T} - \frac{\bar{\tau} : \nabla \vec{v}}{T} + \frac{\dot{s}}{T}$$

2nd law

$$\rho \frac{Ds}{Dt} + \nabla \cdot \left( \frac{\vec{E}}{T} \right) - \frac{\dot{s}}{T} = \Delta > 0$$

$$\Delta = -\frac{\vec{q} \cdot \nabla T}{T^2} - \frac{\pi : \nabla \vec{v}}{T} \geq 0$$

Requirement

$$\left[ \begin{array}{l} \vec{q} = -k \nabla T \\ \pi = -\mu [\nabla \vec{v} + (\nabla \vec{v})^+] \end{array} \right]$$

Appendix A.

$$-\vec{q} \cdot \nabla T \geq 0$$

$$\text{and } -\pi : \nabla \vec{v} \geq 0.$$

C.E.     $\rho \cdot \vec{v}$   
 M.E.     $\rho, \pi, \vec{g}$ .  
 I.EE.     $u, \vec{q}, \dot{\vec{q}}$   
 $\underbrace{T}_{q}$

3. field equation  
 $\pi =$   
 $\vec{g} =$   
 $\vec{q} =$   
 $\dot{\vec{q}} =$

Thermal Eq. of state  $P = P(T, \rho)$   
 Caloriz Eq. of state  $u = u(T, \rho)$

3) constant Transport property case.

$$u = \text{const}$$

$$k = \text{const}$$

$$\frac{\partial P}{\partial t} + \nabla \cdot P \vec{v} = 0.$$

Newtonian

$$\rho \frac{D \vec{v}}{Dt} = -\nabla p + u \nabla^2 \vec{v} + \frac{u}{3} \nabla (\nabla \cdot \vec{v}) + \rho \vec{g}$$

$$\rho c_v \frac{DT}{Dt} = k \nabla^2 T - \pi \cdot \nabla \vec{v} + T \frac{\partial P}{\partial T} \vec{v} \cdot \nabla \vec{v} + \dot{\vec{q}}.$$

4) Incompressible with const. Tr. Pr.

$$\rho = \text{const}$$

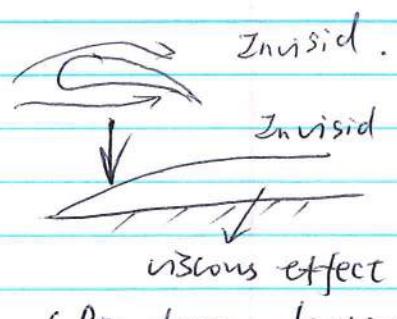
$$u = \text{const}$$

$$k = \text{const}$$

$$\left\{ \begin{array}{l} \nabla \cdot \vec{v} = 0 \\ \rho \frac{D \vec{v}}{Dt} = -\nabla p + u \nabla^2 \vec{v} + \rho \vec{g} \\ \rho c_v \frac{DT}{Dt} = k \nabla^2 T + \dot{\vec{q}} \end{array} \right.$$

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$T$   
 $P$   
 $T$ } specify  $\frac{C}{P}$



5) Inviscid Compressible Flow.

$$\frac{\partial \vec{V}}{\partial t} + \nabla P \vec{V} = 0$$

$$P \frac{D\vec{V}}{Dt} = -\nabla P + \vec{e}g \quad \text{Euler 1755}$$

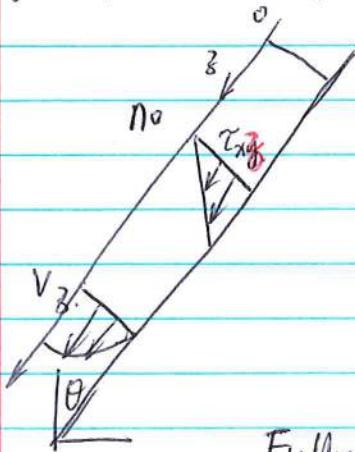
$$P(C_v \frac{DT}{Dt}) = -\nabla \cdot \vec{q} - T \frac{\partial P}{\partial T} \rho \nabla \cdot \vec{V}$$

$$\nabla \cdot \vec{q} \quad \vec{q} = -k \nabla T$$

$$P = P(T, P) \quad \text{Ideal gas law.}$$

$$U = U(T)$$

1) Flow of Falling liquid film.



Steady state.

Fully developed  $\frac{\partial h}{\partial x} \rightarrow 0$

No shear at Gas-Liquid Int.

Laminar flow

Adiabatic Isoth.

Liquid  $\Rightarrow$  Incompressible.

Fully developed.

$$V_x = V_x(x) \quad \left. \begin{array}{l} \text{2-D} \\ \text{0} \end{array} \right\} \quad (1)$$

$$V_y = 0.$$

$$\text{C.E. } \nabla \cdot \vec{V} = 0$$

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$$

$$\frac{\partial V_x}{\partial x} = 0, \quad V_x = 0 \quad (\text{at wall. } V_x = 0)$$

$$V_3 = V_3 x.$$

Momentum Eq.

$$\text{3: } P \left\{ \frac{\partial V_3}{\partial t} + V_x \frac{\partial V_3}{\partial x} + V_y \frac{\partial V_3}{\partial y} + V_3 \frac{\partial V_3}{\partial z} \right\} = - \frac{\partial P}{\partial z} \\ + \mu \left\{ \frac{\partial^2 V_3}{\partial x^2} + \frac{\partial^2 V_3}{\partial y^2} + \frac{\partial^2 V_3}{\partial z^2} \right\} + \rho g_z \\ - \frac{\partial P}{\partial z} + \mu \frac{\partial^2 V_3}{\partial x^2} + \rho g_z = 0. \quad \textcircled{2}$$

$$X_1: - \frac{\partial P}{\partial x} + \rho g_x = 0. \quad \textcircled{3}$$

$$g_x = \rho \cos \theta$$

$$g_y = \rho \sin \theta.$$

B.C.  $x=0 \quad P=P_{\infty}$  free surface.

$$P = \rho g_x X + C_1(z). \quad C_1(z) = P_{\infty}.$$

$$P = \rho g_x X + P_{\infty} \quad \textcircled{4}$$

$$\frac{\partial P}{\partial z} \Rightarrow \rightarrow \textcircled{2} \quad \text{Integrate.}$$

$$V_3 = - \frac{\rho g_x X^2}{2\mu} + C_2 X + C_3.$$

$$\text{at } x=0, V_3=0 \quad (\text{B.C.)}$$

$$C_3 = \frac{\rho g_x \delta^2}{2\mu} \quad \delta: \text{film thickness.}$$

1) Velocity profile.

$$V_3 = \frac{\rho g x^2 \cos \theta}{2\mu} \left[ 1 - \left( \frac{x}{\delta} \right)^2 \right]. \quad \text{parabolic.}$$

2) Max. Vel.

$$V_{3\max} = \frac{\rho g \delta^2 \cos \theta}{2\mu}$$

(3). Average.

$$\langle V_z \rangle = \frac{1}{S} \int_0^S V_z dx = \frac{\rho g S^2 a z \theta}{3 \mu}$$

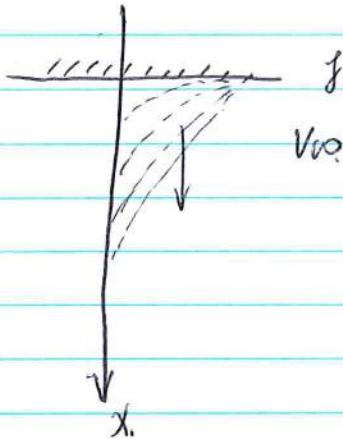
4) Vol. Flow with width,  $W$ 

$$Q = SW \langle V_z \rangle = \frac{\rho g W S^3 a z \theta}{3 \mu}$$

(5) Film thickness.

$$S = \sqrt[3]{\frac{3 \mu Q}{\rho g u / a z \theta}}$$

Transient solution. Chara



2-D

$$t < 0 \quad V_x = 0, V_y = 0$$

$$t = 0^+ \quad V_y(0, y) = V_{in}$$

$$p = \text{uniform at } x=0. \quad p = p_\infty$$

$V_x = 0. \text{ at } x=0 \quad V_z = 0 \text{ everywhere (2-D)}$

Incompressible.  $\rho = \text{const}$   
 viscous properties.  $\mu = \text{const.}$

$$\text{C.E. } \frac{\partial P}{\partial t} + \nabla(PV) = 0. \quad \nabla V = 0 \quad \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = 0$$

$$V_y = V_y(t, x). \quad (\text{not function of } y)$$

$$\text{from C.E. } \frac{\partial V_x}{\partial x} = 0 \dots V_x = 0. \text{ at everywhere.}$$

B.C. at  $x=0. V_x = 0.$

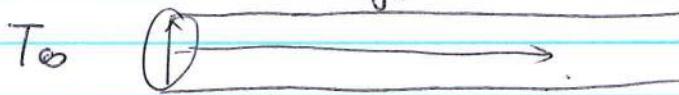
Momentum Eq.

$$\rho \left\{ \frac{\partial V_x}{\partial t} + U_x \frac{\partial V_x}{\partial x} + V_y \frac{\partial V_x}{\partial y} + V_z \frac{\partial V_x}{\partial z} \right\} = - \frac{\partial P}{\partial x} + \left( \mu \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial^2 V_y}{\partial x \partial y} + \frac{\partial^2 V_z}{\partial x \partial z} \right) + \rho g_x$$

$$\rho \frac{\partial V_x}{\partial t} = \mu \frac{\partial^2 V_x}{\partial x^2} \Rightarrow \frac{\partial V_x}{\partial t} = \cancel{\nu \frac{\partial^2 V_x}{\partial x^2}} \nu \frac{\partial^2 V_x}{\partial x^2}$$

$$\nu = \frac{\mu}{\rho} : \text{kinematic viscosity.}$$

Sudden heating.



$$T = T_0 \quad t < 0$$

$$T = T_\infty \quad t \geq 0$$

at  $x=0$ .

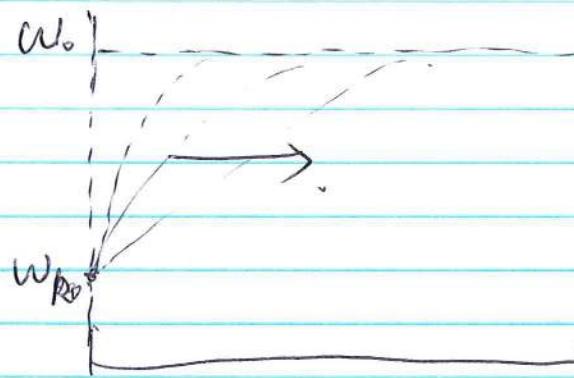
at  $x=0$



$$\rho C_V \left[ \frac{\partial T}{\partial t} + V \frac{\partial T}{\partial x} \right] = -k \frac{\partial^2 T}{\partial x^2} + g T$$

$$\rho C_V \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \quad \frac{\partial T}{\partial t} = \frac{k}{\rho C_V} \frac{\partial^2 T}{\partial x^2}$$

$$\alpha = \frac{k}{\rho C_V} \quad \text{thermal diffusivity.} \quad \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot \rho_k \vec{v} = -\nabla \cdot \vec{j}_{in} + j_k$$

mass fraction  $\frac{\rho_k}{\rho} = w_k$

$$\begin{cases} \rho = \text{const.} \\ \text{no motion } \vec{v} \Rightarrow \\ \vec{r}_k = \vec{r} \end{cases}$$

$$\frac{\partial w_k}{\partial t} = D \nabla^2 w_k$$

D: Mass diffusivity coefficient.

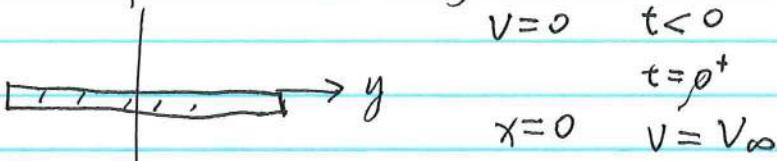
$$\left\{ \begin{array}{l} v \\ T \\ w_k \end{array} \right\} \Rightarrow$$

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HW #3

Solve for the case of sudden motion.



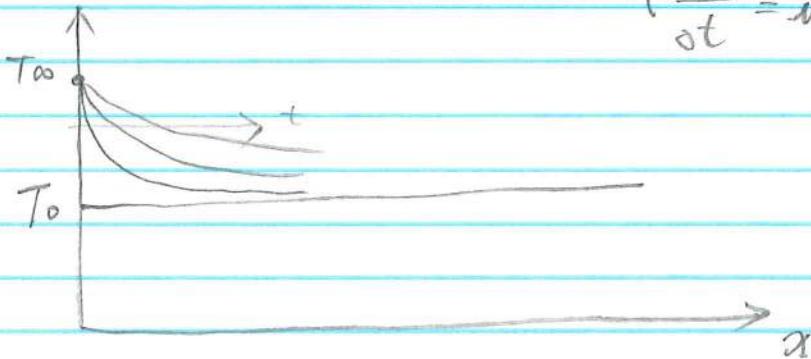
uniform in  $y$  direction.

B.C.

$$\frac{dV}{dt} = \mu \frac{\partial^2 V}{\partial x^2} \quad \frac{\partial V}{\partial t} = \nu \frac{\partial^2 V}{\partial x^2}$$

$$\rho \frac{\partial V}{\partial t} = \mu \frac{\partial^2 V}{\partial x^2} \uparrow$$

$$V = \frac{\eta}{\rho}$$



$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_v} \frac{\partial^2 T}{\partial x^2}$$

$$t \leq 0 \quad T(0, x) = T_0$$

$$t = 0^+ \quad T(0^+, x) = T_\infty$$

$$\theta = T - T_\infty$$

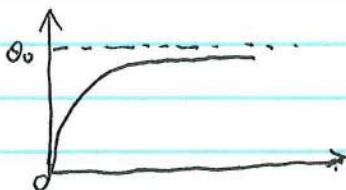
$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial x^2}$$

$$\alpha = \frac{k}{\rho C_v}$$

Thermal diffusivity.

$$\theta(x, 0) = T_0 - T_\infty = \theta_0$$

$$\begin{cases} \theta(0, t) = 0 \\ \theta(\infty, t) = 0 \end{cases}$$



\* Suppose the solution has the form

$$\frac{\theta}{\theta_0} = \phi(\eta) \quad \eta = \frac{x}{\sqrt{4at}}$$

(similarity transformation)

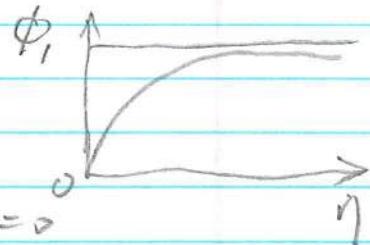
$$x \rightarrow \eta$$

$$\therefore \frac{\partial(\theta/\theta_0)}{\partial t} = -\frac{1}{2} \frac{1}{\eta} \phi'$$

$$\frac{\partial^2(\theta/\theta_0)}{\partial x^2} = -\frac{\eta^2}{x^2} \phi'' + 0$$

$$\phi'' + 2\eta \phi' = 0$$

$$\begin{cases} \eta=0, \phi=0 \\ \eta=\infty, \phi=1 \end{cases}$$



$$\phi' = C_1 e^{-\eta^2}$$

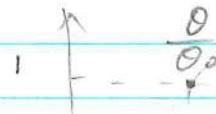
$$\phi = C_1 \int_0^\eta e^{-z^2} dz + C_2.$$

$$\text{B.C. } \rightarrow C_1 = \frac{1}{\int_0^\infty e^{-z^2} dz}, \quad C_2 = 0.$$

$$\phi = \frac{\int_0^\eta e^{-z^2} dz}{\int_0^\infty e^{-z^2} dz} = \frac{z}{\sqrt{\pi}} \int_0^\eta e^{-z^2} dz$$

$$= \operatorname{erf} \eta$$

$$\operatorname{erf} \eta = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-z^2} dz$$



$$\frac{\theta}{\theta_0} = \operatorname{erf} \left( \frac{x}{\sqrt{4at}} \right)$$



$$\frac{T - T_0}{T_b - T_0} = \operatorname{erf} \left( \frac{x}{\sqrt{4at}} \right).$$

penetration depth

$$\eta = \frac{x}{\sqrt{4at}}$$

$$\frac{\partial T}{\partial x} = \frac{\partial \Theta}{\partial x} = \frac{\partial \phi \theta_0}{\partial x} = \theta_0 \frac{\partial \phi}{\partial x} = \theta_0 \frac{\partial \phi}{\partial \eta} \frac{\partial \eta}{\partial x},$$

$$\begin{aligned}\frac{\partial \phi}{\partial \eta} &= \frac{\partial}{\partial \eta} [\operatorname{erf}(\eta)] = \frac{\partial}{\partial \eta} \left[ \frac{2}{\sqrt{\pi}} \int_{-\infty}^{\eta} e^{-s^2} ds \right] \\ &= \frac{2}{\sqrt{\pi}} \left[ \frac{\partial}{\partial \eta} e^{-\eta^2} \Big|_{-\infty} + \frac{\partial \eta}{\partial \eta} e^{-\eta^2} \Big|_{-\infty} - \frac{\partial \eta}{\partial \eta} e^{-\eta^2} \Big|_{-\infty} \right] = 0.\end{aligned}$$

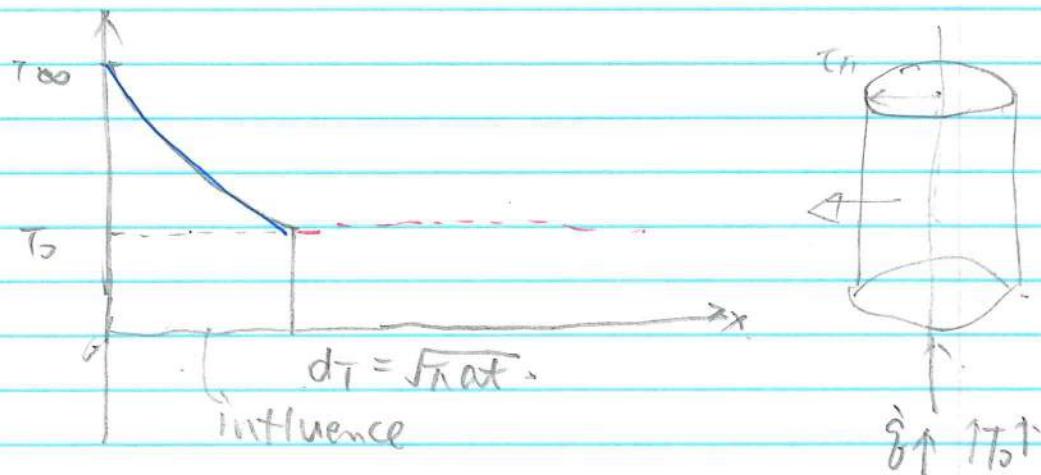
$$= \frac{2}{\sqrt{\pi}} e^{-\eta^2}.$$

$$\frac{\partial \eta}{\partial x} = 1/\sqrt{4at}$$

$$\left. \frac{\partial T}{\partial x} \right|_0 = \theta_0 \frac{2}{\sqrt{\pi}} e^{-\eta^2} \frac{1}{\sqrt{4at}} \Big|_{\eta=0} = \frac{\theta_0}{\sqrt{\pi at}}$$

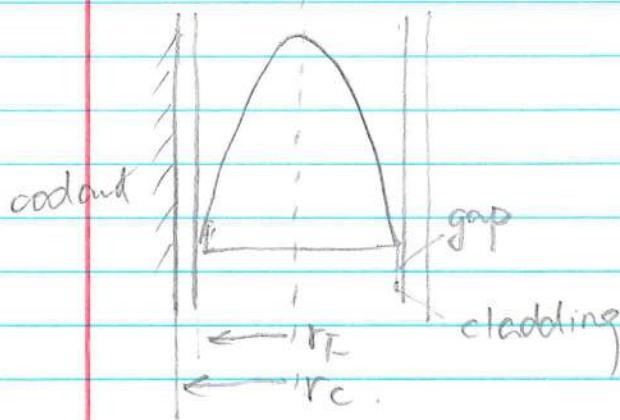
$$\left. \frac{\partial T}{\partial x} \right|_0 = \frac{(T_0 - T_\infty)}{\sqrt{\pi at}}$$

$\sqrt{\pi at} = dr$  : Thermal Penetration Depth



\* D<sub>F</sub>: Diameter of the fil  $D_{fil} = R_F$ .

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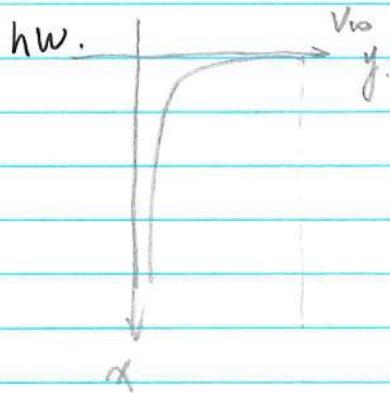


$$d_f = \sqrt{\pi a f} = r_f = D_f / 2$$

$$\pi a f = \left(\frac{D_f}{2}\right)^2$$

$$t = \frac{1}{\pi a} \left(\frac{D_f}{2}\right)^2 = T_f.$$

\* neutronic excursion.  $\rightarrow$  most accident.



1. from 3-D.

momentum diffusiv.

2. solve.

3. the physical meaning.

$$UV_0, \quad a = 5.65 \times 10^{-3} \text{ cm}^2/\text{sec.}$$

$$S.S. \quad Na = 6.48 \times 10^{-1} \text{ cm}^2/\text{sec.}$$

$$H_2O = 1.43 \times 10^{-3} \text{ cm}^2/\text{sec.}$$

$$d_{th} = \sqrt{\pi a f}$$

1 sec.

10 sec

100 s

$$UV_0,$$

$$1.33 \text{ mm}$$

$$4.22 \text{ mm}$$

$$13.3 \text{ mm}$$

$$S.S.$$

$$4.4 \text{ mm}$$

$$13.9 \text{ mm}$$

$$44 \text{ mm}$$

$$H_2O$$

$$0.67 \text{ mm}$$

$$8.1 \text{ mm}$$

$$67 \text{ mm}$$

$$Na$$

$$143 \text{ mm}$$

$$45 \text{ mm}$$

$$143 \text{ mm}$$

$$\begin{cases} H_2O \\ Na \end{cases}$$

$$V = 1 \times 10^{-2} \text{ cm}^2/\text{sec}$$

$$V = 3.3 \times 10^{-3} \text{ cm}^2/\text{sec}$$

$$(20^\circ\text{C})$$

$$(37^\circ\text{C})$$

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$$d_m = \sqrt{\pi D t} \quad H_2O \Rightarrow Pr = 7 \quad Na: Pr = 5 \times 10^{-3} \quad Pr = \frac{V}{\alpha}$$

1 s	1.77 mm	1.0 mm
10 s	5.6 mm	3.2 mm

\* Binary diffusion.

$H_2O$  vapor in Air     $16^\circ C$      $D = 2.8 \times 10^{-1} \text{ cm}^2/\text{sec.}$

Schmidt No.     $Sc = \frac{V}{D}$

$$d_c = \sqrt{\pi D t}$$

1 s	9.4 mm.
10 s	29.7 mm.

\* i. velocity profile .  $\curvearrowright$

ii. if suddenly change.

(7) Note on Turbulent flow.

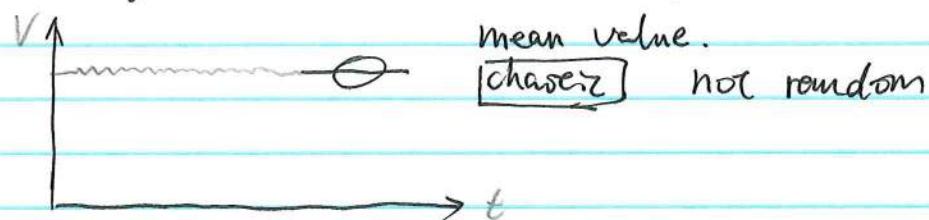
Origin: Navier Stokes. Eq. (Momentum Eq.)

becomes unstable for large Reynold Number.

$$Re = \frac{\rho v D}{\mu} = \frac{\text{inertial force}}{\text{viscous force.}}$$

{ random system  
chaotic system

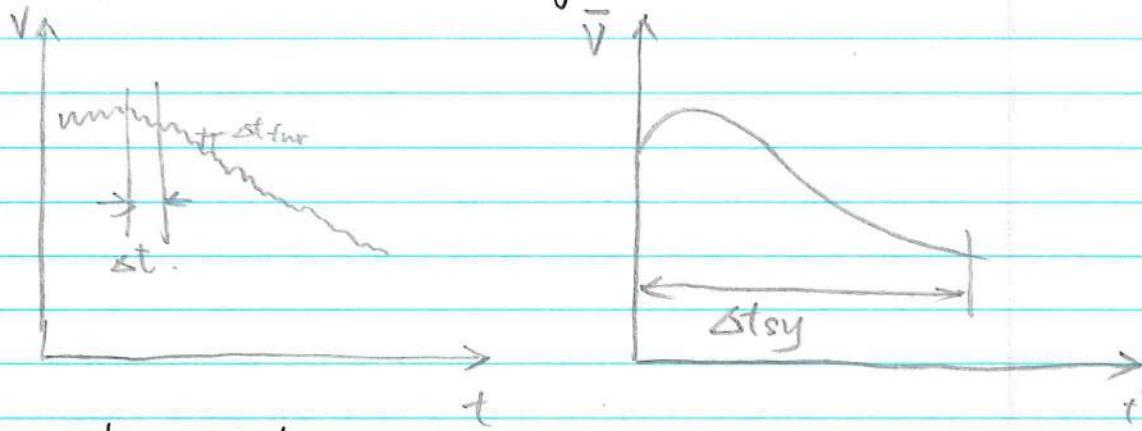
Turbulent flow with Fluctuations.



Time average solution for turbulent flow  
 ⇒ for most engineering problems : sufficient

## Chapter 5 Turbulent Flow

### i) Temporal (Time) Average



$$\Delta t_{sy} \gg \Delta t$$

$$\bar{\varphi} = \frac{1}{\Delta t} \int_{\Delta t} \varphi dt \quad \textcircled{1}$$

$$\Delta t_{\text{obs}} < \Delta t < \Delta t_{sy}$$

$$\varphi = \bar{\varphi} + \varphi' \quad \textcircled{2}$$

$\bar{\varphi}$  : average

$\varphi'$  : fluctuate component

$$\overline{\varphi'} = 0$$



$$\sqrt{\overline{\varphi'^2}} / \bar{\varphi} = \text{Turbulent density} \neq 0$$

$$I \sim 1 \text{ to } 10\% \quad (\text{mostly} < 5\%)$$

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→ Time averaged Field Equation.

$$\psi = \bar{\psi} + \psi'$$

$\bar{\psi} = ? \rightarrow$  Engineering Problem.

$|\frac{\psi'}{\rho}| \ll |\frac{v}{\bar{v}}|$  assumption is very good

(i) continuity Equation.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{v} = 0.$$

$$\left. \begin{array}{l} \rho = \bar{\rho} + \rho' \\ v = \bar{v} + v' \end{array} \right\} \rightarrow \boxed{\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot \bar{\rho} \vec{v} = 0}.$$

$$\rho = \text{const.} \quad \nabla \cdot \vec{v} = 0.$$

$$\nabla \cdot v' = 0.$$

(ii) M.E.

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v} \vec{v} \xrightarrow{\text{nonlinear}} = -\nabla p + \mu \nabla^2 \vec{v} + \rho \vec{g}.$$

$$\left. \begin{array}{l} v = \bar{v} + v' \\ \rho = \bar{\rho} + \rho' \\ \rho = \bar{\rho} \end{array} \right\} \xrightarrow{\text{-M.E.}}$$

$$\bar{v} = \overline{\bar{v} + v'} = \bar{v} + \vec{v}'$$

$$\rho \vec{v} = \bar{\rho} (\bar{v} + v')$$

$$\overline{\frac{\partial \bar{\rho} (\bar{v} + v')}{\partial t}} = \frac{\partial \bar{\rho} \bar{v}}{\partial t} + \frac{\partial \bar{\rho}}{\partial v} \vec{v}'$$

$$\overline{\nabla \cdot \rho \vec{v} \vec{v}} = \nabla \cdot \overline{\rho (\bar{v} + v') (\bar{v} + v')}$$

$$= \nabla \cdot \bar{\rho} \bar{v} \bar{v} + \nabla \cdot \bar{\rho} v' v'$$

↑ Turbulent.

10. b. Midterm.

$$\frac{\partial \bar{P}}{\partial t} + \nabla \cdot \rho \bar{V} \bar{V} = -\nabla \bar{P} + [\mu \nabla^2 \bar{V} - (\nabla \cdot \rho \bar{V} \bar{V})] + \bar{\rho} \bar{g}$$

✓      ✓      ✓      ✓      Turbulent flux  
(Reynolds Stress)

$$\tau^t = \rho \bar{V} \bar{V}$$

$$\tau^u = -\mu [\nabla V + (\nabla V)^T]$$

$$\tau^T = \tau^u + \tau^t$$

$$\frac{\partial \bar{V}}{\partial t} + \nabla \cdot \rho \bar{V} \bar{V} = -\nabla \bar{P} - \nabla \cdot (\tau^u + \tau^t) + \bar{\rho} \bar{g}$$

$$\frac{\partial \bar{V}}{\partial t} + \nabla \cdot \rho \bar{V} \bar{V} = -\nabla \bar{P} - \nabla \cdot (\tau^T) + \bar{\rho} \bar{g}$$

$$\boxed{\tau^t = ?}$$

$$\tau^u = -\nu \rho [\nabla \bar{V} + (\nabla V)^T]$$

$$\tau^t = -(\varepsilon_m) \rho [\nabla \bar{V} + (\nabla V)^T]$$

$\varepsilon_m$ : turbulent diffusivity

$\nu$ : viscous "

→ Energy Equation.

$$\underbrace{\rho C_v \frac{DT}{Dt}}_{C_v \left[ \frac{\partial \bar{T}}{\partial t} + \nabla \cdot \rho \bar{T} \bar{V} \right]} = K \nabla^2 T - T \frac{\partial P}{\partial T} \underbrace{\rho \nabla \cdot \bar{V}}_{\tau: \nabla \bar{V}} - \tau \cdot \nabla V^T + \dot{q}$$

$$C_v \left[ \frac{\partial \bar{T}}{\partial t} + \nabla \cdot \rho \bar{T} \bar{V} \right]$$

$$\rho = \bar{\rho} \quad \nabla \cdot \bar{V} \rightarrow 0$$

$$\tau: \nabla \bar{V} \approx 0$$

$$\begin{cases} T = \bar{T} + T' & \bar{T}' = 0 \\ V = \bar{V} + V' & \bar{V}' = 0 \end{cases}$$

$$\underbrace{\rho C_v \frac{DT}{Dt}}_{\tau: \nabla \bar{V}} = [K \nabla^2 T - \nabla \cdot \rho C_v \bar{T} V^T] + \dot{q}$$

↑ Turbulent Energy Flux

$$\overbrace{C_V \left[ \frac{\partial \bar{T}}{\partial t} + \nabla \cdot \bar{P} \bar{T} \bar{V} \right]}$$

$$\left\{ \begin{array}{l} \vec{g}_b^t = \rho C_V \bar{T}' \bar{V}' \\ \vec{g}_b^k = -k \nabla \bar{T} \end{array} \right. \Leftrightarrow \begin{aligned} &= -\rho C_V (\sum_k) \nabla \bar{T} \\ &= -k \nabla \bar{T}. \end{aligned}$$

$$\rho C_V \frac{D \bar{T}}{D t} = -\nabla \cdot (\vec{g}_b^k + \vec{g}_b^t) + \dot{q}$$

$$\vec{g}_b^T = \vec{g}^k + \vec{g}^t.$$

$$\rho C_V \frac{D \bar{T}}{D t} = -\nabla \cdot \vec{g}^T + \dot{q}$$

Turbulent Energy  
Diffusivity

### (b) characteristics of turbulent flow

#### (i) Transition.

Laminar  $\rightarrow$  Turbulent.

$Re \geq 2000$  for pipe flow

#### (ii) Velocity profile

Laminar

Turbulent

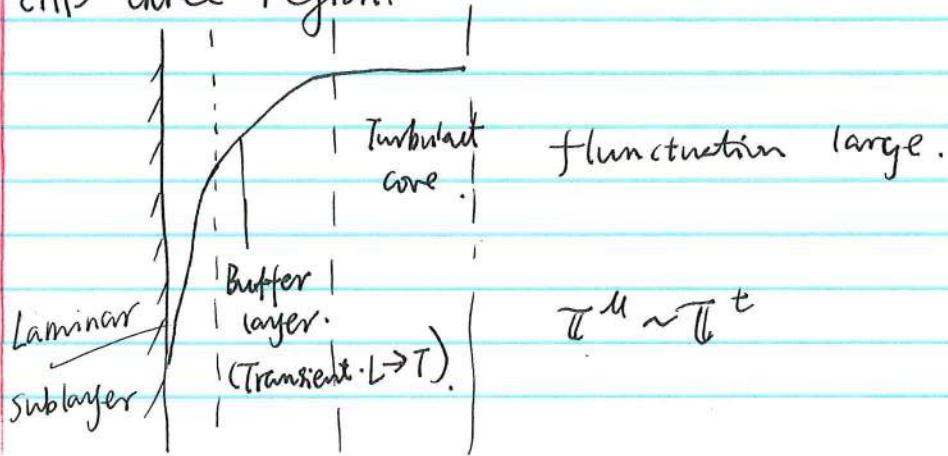
$$\boxed{v = V_{max} \left[ 1 - \left( \frac{r}{r_o} \right)^2 \right]} \quad \cancel{\text{power profile}}$$

$$v \sim V_{max} \left( 1 - \frac{r}{r_o} \right)^{1/\eta}$$

$(1/\eta$  power profile)

$\hookrightarrow$  good for estimate of  $v$   
cannot used to get  $\bar{T}$

#### (iii) three region.



$$\tau^u \sim \bar{U}^t$$

#### IV. Stress in Turbulent flow.

$$\frac{\partial \bar{V}}{\partial t} + \nabla \cdot \bar{V} \bar{V} = -\nabla \bar{P} - \nabla \cdot (\bar{\tau}^u + \bar{\tau}^t) + \rho \bar{g}$$

In pipe  $\bar{z}$  component.

$$\rho \left( \frac{\partial \bar{V}_z}{\partial t} + \bar{V}_r \frac{\partial \bar{V}_z}{\partial r} + \frac{\bar{V}_\theta}{r} \frac{\partial \bar{V}_z}{\partial \theta} + \bar{V}_z \frac{\partial \bar{V}_z}{\partial z} \right)$$

$$= -\frac{\partial \bar{P}}{\partial z} + \rho g_z + \left[ \frac{1}{r} \frac{\partial}{\partial r} (r \bar{\tau}_{rz}^T) + \frac{1}{r} \frac{\partial \bar{\tau}_{\theta z}^T}{\partial \theta} + \frac{\partial \bar{\tau}_{zz}^T}{\partial z} \right]$$

Steady. S  $\frac{\partial}{\partial t} = 0$

no gravity  $g_z = 0$

fully developed  $\frac{\partial \bar{V}_z}{\partial z} = 0$   $\bar{V}_r = \bar{V}_\theta = 0$

axi-symmetric  $\frac{\partial}{\partial \theta} = 0$ .  $\bar{\tau}_{rz}$  independent of  $z$

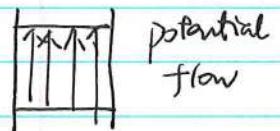
$$\boxed{\frac{\partial \bar{P}}{\partial z} = \frac{1}{r} \frac{\partial}{\partial r} (r \bar{\tau}_{rz}^T)}$$

↑  
independent of  $r$

↑  
independent of  $z$ .

Solve for  $\bar{\tau}_{rz}^T$ ;  $\bar{\tau}_{rz}^T = \bar{\tau}_{rz}^u + \bar{\tau}_{rz}^t$

$$= \left( \frac{dP}{dz} \right) \frac{r}{2}$$



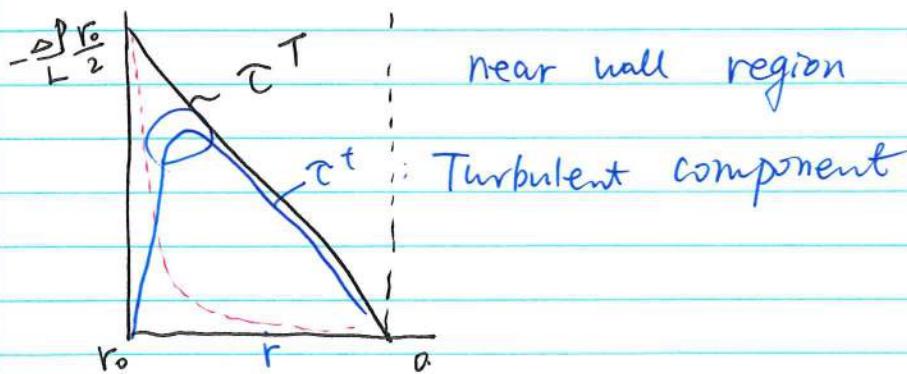
Potential flow

$$\frac{dp}{dz} = \text{const.}$$

$\bar{\tau}_{rz}^T$  linear in  $r$ .



Laminar

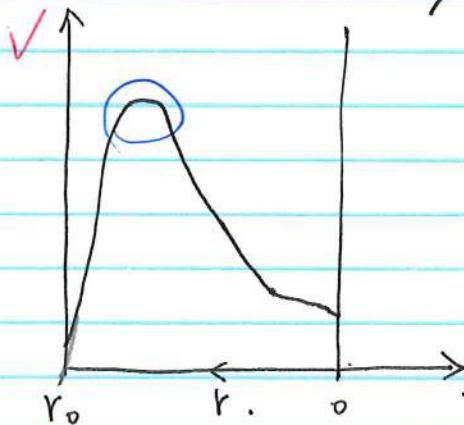


Turbulent

V). Turbulent intensity.

$$I = \sqrt{\overline{V_3^2}}$$

$$I = \frac{\sqrt{\overline{V_3'^2}}}{\overline{V_{3\max}}}$$

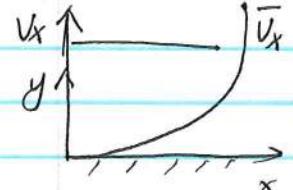


Potential flow can't calculate resistance.

(4) Turbulent flow model.

- Boussinesq's Eddy Diffusivity (viscosity).

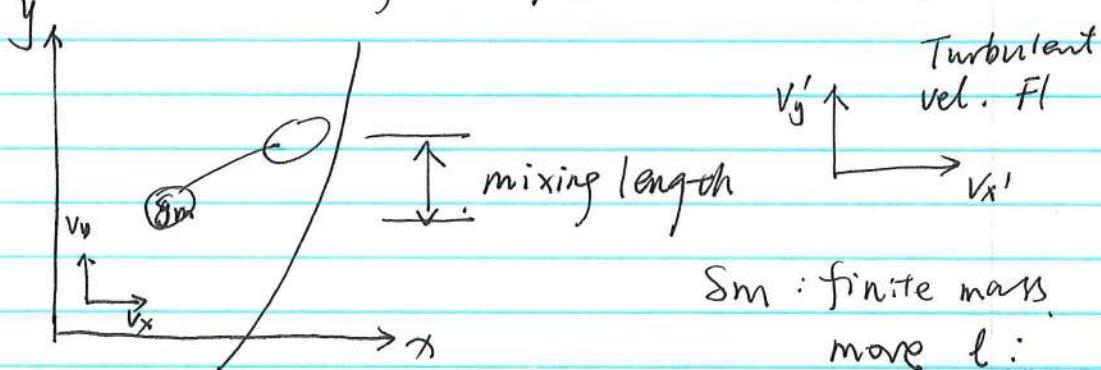
$$\tau_{yx}^t = -\mu^t \frac{d\bar{v}_x}{dy}$$



$\mu^t$ : Turbulent viscosity.

$$\frac{\mu^t}{\rho} = \epsilon_m \quad \text{Eddy diffusivity.}$$

- Prandtl's Mixing length Model (1925)



52.

(1)  $\delta m$  does not loss or gain any  $x$  momentum

until move finite distance  $\ell \rightarrow$  then complete momentum exchange

(2)  $\Rightarrow$  comp momentum gained by  $\delta m$   
 $\delta m \delta V_x$ .

- momentum transfer rate.

$$\frac{\delta m \delta V_x}{\delta t}$$

- shear force acted on fluid  $\bar{F} = \delta m \delta V_x / \delta t$ .

$$\tau^+ = \frac{\bar{F}}{A} = \frac{1}{A} \frac{\delta m}{\delta t} (\delta V_x)$$

$$- \delta V_x \approx \frac{d \bar{V}_x}{dy} \ell$$

$$- \text{From continuity} \quad \frac{1}{A} \frac{\delta m}{\delta t} = \rho |V_y'|$$

$$\frac{\tau^+}{\rho} = - \ell |V_y'| \frac{d \bar{V}_x}{d y}$$

Eddy diffusivity  $\varepsilon_m$

$$\frac{\tau^+}{\rho} = - \varepsilon_m \frac{d \bar{V}_x}{d y} \quad \varepsilon_m = \ell |V_y'|$$

$$\text{Total } \frac{\tau^+}{\rho} = - (\varepsilon_m + \nu) \frac{d \bar{V}_x}{d y}$$

• Prandtl's assumption. physical meaning

$$|V_y'| = k_1 V_x'$$

$$(V_x') = k_2 \delta V_x = k_2 \ell \frac{d V_x}{d y}$$

all unknown const  $k_1, k_2 \rightarrow l.$

$$\boxed{\frac{\tau^t}{\rho} = -l^2 \left| \frac{d\bar{U}_x}{dy} \right| \left| \frac{d\bar{U}_x}{dy} \right|}$$

non linear.

$\ell = k y.$  linear proportional to the position to wall.

$$\boxed{\frac{\tau^t}{\rho} = -k^2 y^2 \left| \frac{d\bar{U}_x}{dy} \right| \left| \frac{d\bar{U}_x}{dy} \right|}$$

09/27/11

Prandtl's Mixing length Model.

Velocity profile near the wall.

$$V^+ = \frac{U_x}{\sqrt{\tau_w/\rho}} \quad \text{non-dimensional vel.}$$

$$\tau_w \quad y^+ = y \sqrt{\tau_w/\rho} / \nu \quad \text{non-dimensional distance.}$$

$$\tau_w = f_i \frac{1}{2} \rho u^2 \Rightarrow \sqrt{\frac{\tau_w}{\rho}} = \sqrt{0.5 f_i} U_o$$

$f_i:$  Fanning friction factor.

$f:$   $0.001 \sim 0.01$

$$\sqrt{0.5f} : \quad 1/45 \quad 1/14 \quad y^+ = Re \frac{\sqrt{\tau_w/\rho}}{\nu} \frac{y}{D} \sim Re \frac{1}{40} \frac{y}{R}$$

Three Regions

① laminar sublayer.  $\tau^t = 0 \quad y^+ < 5.$   
 $v^t = y^+$

(2) Buffer layer

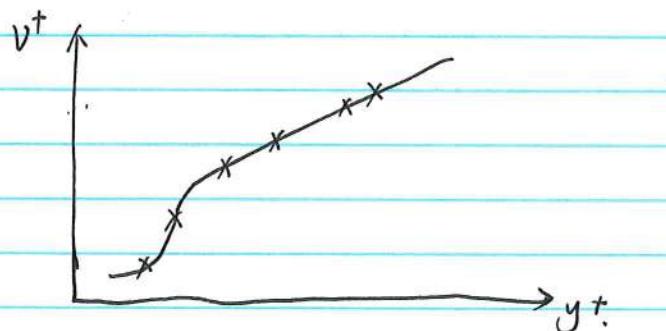
$$V^+ = -3.05 + 5 \ln y^+$$

$$5 < y^+ < 30$$

(3) Turbulent core

$$y^+ > 30$$

$$V^+ = 5.5 + 2.5 \ln y^+$$



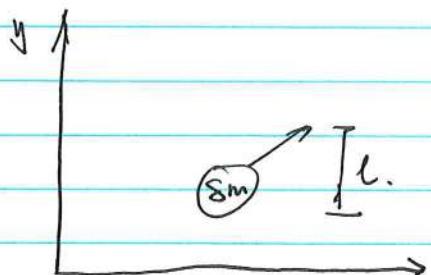
Universal  
velocity  
profile.

(5). Turbulent heat transfer.

Reynolds Analogy.

$$\frac{\tau}{\rho} = - \Sigma_m \frac{\partial u}{\partial y}$$

$$\frac{q''}{\rho c} = - \Sigma_H \frac{\partial T}{\partial y}$$



Energy transfer by convection

$$\frac{\delta m C \delta T}{\delta t}$$

$$\tau = \frac{F}{A} = \frac{1}{A} \frac{\delta m}{\delta t} \delta V_x = -(\rho |V_y|) \left( \frac{d \bar{V}_x}{dy} l \right) = -\rho l |V_y| \frac{d \bar{V}_x}{dy}$$

$$q'' = \frac{Q}{A} = \frac{1}{A} \frac{\delta m}{\delta t} C \delta T = -(\rho |V_y|) \left( C \frac{d \bar{T}}{dy} l \right) = -\rho C l |V_y| \frac{d \bar{T}}{dy}$$

$$\& \Sigma_m = l |V_y| \quad \Sigma_H = l |V_y|$$

$$\Sigma_M = \Sigma_H$$

$$\frac{\tau}{P} = -\Sigma_M \frac{d\bar{v}_x}{dy} \quad \frac{\tau''}{P_C} = -\Sigma_H \frac{d\bar{T}}{dy}.$$

(ii) Karman - Bœlter - Martinelli Analogy

$$\frac{\tau}{P} = -(\Sigma_M + V) \frac{\partial v}{\partial y}$$

$$\frac{\tau''}{P_C} = -(\Sigma_H + a) \frac{\partial T}{\partial y}.$$

Eddy diffusivity  $\Sigma_M$ . Universal Velocity profile  
 $v^+ = f(y^+)$

$$\begin{cases} 0 < y^+ < 5, \quad \Sigma_M/y = 0. \end{cases}$$

$$\begin{cases} 5 < y^+ < 30, \quad \frac{\Sigma_M}{\tau} = \frac{y^+}{5} - 1 \end{cases}$$

$$\begin{cases} y^+ > 30, \quad \Sigma_M/\tau = y^+ (1 - y/r_0)/2.5 - 1 \end{cases}$$

For Moderate Prandtl No.  $\Sigma_M = \Sigma_H$ .

Energy Eq.  $\left\{ \begin{array}{l} \text{fully developed } V_r = 0 \\ \text{axisymmetric } \frac{\partial}{\partial r} = 0 \\ \text{neg. axial. conduction.} \end{array} \right.$

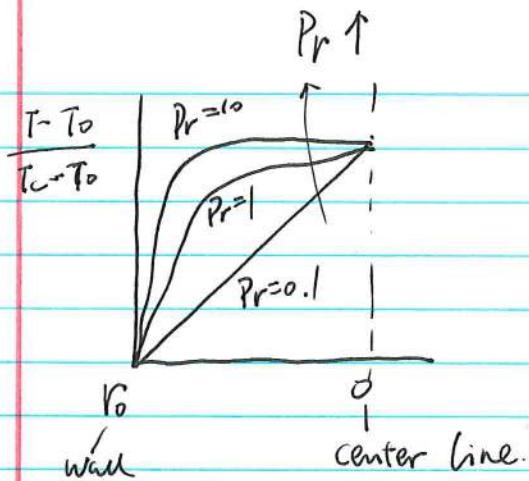
$$P_C V_S \frac{\partial T}{\partial S} = \frac{1}{r} \frac{\partial}{\partial r} [r P_C (\Sigma_H + a) \frac{\partial T}{\partial r}]$$

$$\tau'' = P_C (\Sigma_H + a) \frac{\partial T}{\partial y}.$$

$V_S = V_S(r)$   $\frac{1}{r}$  or universal velocity profile.

$\Sigma_H = \Sigma_M \leftarrow$  Eddy Diffusivity

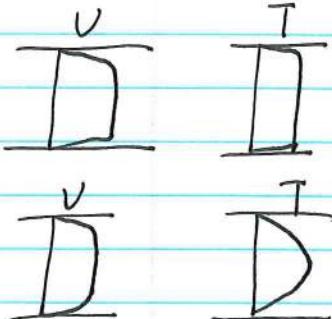
Solve for  $T(r, Pr)$ .  $Pr = \frac{\nu}{\alpha}$ .



Effect of Prandtl number.

$$\text{Pr} \gg 1 \quad (\text{oil}) \quad \frac{V}{a} \gg 1$$

large momentum diffusion



$$\text{Pr} \ll 1 \quad (\text{liquid metal}) \quad \frac{V}{a} \ll 1$$

#### iv) Diffusion Equation.

$$\frac{\partial \rho_k}{\partial t} + \nabla \cdot \rho_k \vec{V} = -\nabla \cdot \vec{J}_k + r_k \quad \xrightarrow{\text{chemical reaction rate}}$$

$$\text{mass fraction. } w_k = \frac{\rho_k}{\rho}$$

$$\vec{J}_k = -\rho D \nabla w_k$$

$$\frac{\partial \rho w_k}{\partial t} + \nabla \cdot \rho w_k \vec{V} = \nabla \cdot (\rho D \nabla w_k) + r_k.$$

$$w_k = \bar{w}_k + w_k' \quad \quad \quad \vec{V} = \cancel{\bar{V}} + v'$$

$$\nabla \cdot \rho w_k \vec{V} = \nabla \cdot \rho \bar{w}_k \vec{V} + \nabla \cdot \rho \bar{w}_k' v'$$

$$\frac{\partial \rho \bar{w}_k}{\partial t} + \nabla \cdot (\rho \bar{w}_k \vec{V}) = \nabla \cdot (\rho D \nabla w_k) - \nabla \cdot (\rho \bar{w}_k' v') + r_k.$$

$$\vec{J}_k^T = \bar{j}_k^T + j_k^T.$$

$$j_k^T = \rho D^T \nabla w_k.$$

$$j_k^T = \rho \frac{\bar{w}_k' v'}{w_k' v_k} = -\rho D^T \nabla w_k$$

## Heat and Mass Transfer Analogy

$$\begin{aligned} D^t &= \Sigma H \\ \text{or } D^t &= \Sigma M \\ \frac{\partial \bar{P}_k}{\partial t} + \nabla \cdot (\rho \bar{w}_k \bar{V}) &= \nabla \cdot [C(D^l + D^t) \bar{w}_k] + R_k. \end{aligned}$$

## Chapter 6. Single phase dimensional Analysis.

use of Balance Equation for similarity Parameters.

\* for non-dimensional parameters.

### (i) Scaling Parameters.

- Distance  $x^* = \frac{x}{D}, y^* = \frac{y}{D}, z^* = \frac{z}{D}, \bar{x}^* = \frac{\bar{x}}{D}$

$D$ : characteristic length (Hydraulic diameter)

$$D = \frac{4A}{P} = \frac{4 \times \text{Flow area}}{\text{Wetted Perimeter}}$$

Time scale:

$$\tau = \frac{D}{V} \quad t^* = \frac{t}{\tau} = \frac{tV}{D}$$

$V$ : characteristic velocity.

Velocity

$$\bar{v}^* = \frac{\bar{v}}{V}, v_x^* = \frac{v_x}{V}, v_y^* = \frac{v_y}{V}, v_z^* = \frac{v_z}{V}$$

Pressure

$$P^* = \frac{P - P_0}{\rho_0 V^2} \quad P_0 \quad \left. \begin{array}{l} \text{reference } P_0 \text{ and density} \\ \rho_0 \end{array} \right.$$

• Temperature

$$T^* = \frac{T - T_0}{\Delta T} \quad T_0 \text{ reference}$$

$\Delta T$  temperature change scale

• Density

$$\rho^* = \frac{\rho}{\rho_0}$$

• Differential Operators.

$$\nabla^* = D \nabla$$

$$\nabla^{*2} = D^2 \nabla^2$$

$$\frac{D}{Dt^*} = \frac{D}{V} \frac{D}{Dt} \quad \frac{\partial}{\partial t^*} = \frac{D}{V} \cdot \frac{\partial}{\partial t}$$

3) Force Convection.

• Continuity Equation.

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^* \cdot (\rho^* \vec{V}^*) = 0$$

• Equation of Motion

$$\rho \frac{D \vec{V}}{Dt} = (\rho_0 \frac{V^2}{D}) \rho^* \frac{D \vec{V}^*}{Dt^*}$$

$$\nabla p = \frac{\rho_0 V^2}{D} \nabla^* p^*$$

$$\mu \nabla^2 \vec{V} = \frac{\mu V}{D^2} \nabla^{*2} (\vec{V}^*)$$

$$\frac{1}{3} \mu \nabla (\nabla \cdot \vec{V}) = \frac{1}{3} \mu \frac{V}{D^2} \nabla^* (\nabla^* \cdot \vec{V}^*)$$

$$\vec{g} \rho = \vec{g} \rho_0 \rho^*$$

Divide E.M. by  $\rho_0 V^2 / D$

$$\rho^* \frac{D \vec{V}^*}{Dt^*} = -\nabla^* p^* + \left( \frac{1}{Re} \right) \left[ \nabla^{*2} \vec{V}^* + \frac{1}{3} \nabla^* (\nabla^* \cdot \vec{V}^*) \right]$$

$$+ \left( \frac{1}{F_r} \right) \rho^* \left( \frac{\vec{g}}{g} \right)$$

Reynolds No.  $R_e = \rho_0 V D / \mu = \frac{\rho V^2 / D}{\mu (V/D) / D} = \frac{\text{inertial force}}{\text{viscous force}}$

Froude No.  $F_r = \frac{V^2}{g D} = \frac{\rho V^2 / D}{\rho g} = \frac{\text{inertia}}{\text{gravity}}$

Fluid Mechanics Problem : If two systems have  
 $R_e$ ,  $F_r$ , identical in terms of  
Dimensions - B.C. I.C. identical non-dimensional  
↑ parameters.  
they are identical.  
geometrical similarity.  
 $\Rightarrow$  Dynamically similar.

### • Energy Equation

$$\nabla P C_p \frac{DT}{Dt} = (\rho_0 C_p \frac{V}{D} \Delta T) \rho^* \frac{DT^*}{Dt^*}$$

$$V k \nabla^2 T = \left( \frac{k \Delta T}{D^2} \right) \nabla^{*2} T^*$$

$$\vec{v} : \nabla \vec{v} = u \vec{\phi} = \mu \frac{V^2}{D} \vec{\phi}^*$$

$$\frac{T}{P} \frac{\partial P}{\partial T} \Big|_P \frac{DP}{Dt} = \frac{\rho_0 V^2}{D} \frac{T^*}{P^*} \frac{\partial P^*}{\partial T^*} \Big|_{T^*} \frac{DP^*}{Dt^*}$$

$$\rho^* \frac{DT^*}{Dt^*} = \frac{k}{D \rho_0 C_p V} \nabla^{*2} T^* + \frac{\mu u}{D \rho_0 C_p \Delta T} \vec{\phi}^* - \frac{V^2}{C_p \Delta T} \frac{T^*}{P^*} \frac{\partial P^*}{\partial T^*} \frac{DP^*}{Dt^*} \\ + \frac{\vec{g} D}{\rho_0 C_p \Delta T V}$$

$$\frac{UV}{DP_0 C_p \Delta T} \Phi^* \quad \frac{1}{Re} \frac{V^2}{C_p \Delta T}$$

Eckert No.

$$Ec = \frac{V^2}{C_p \Delta T} = \frac{\rho V^2 / D}{C_p P \Delta T / D} = \frac{\text{Inertial force}}{\text{Thermal driving force}}$$

$$\text{or } = \frac{\rho V^2 V / D}{C_p P \Delta T V / D} = \frac{\text{K.E conversion}}{\text{Enthalpy convection}}$$

for perfect gas

$$r = C_p / C_v, \quad R = C_p - C_v$$

$$R: \text{gas constant} = \frac{R}{M_w} = \frac{\text{Universal gas constant}}{\text{mol weight.}}$$

$\gamma$  : specific heat ratio.

$$\frac{P}{\rho} = RT = C_p \left( \frac{r-1}{r} \right) T$$

$$\text{velocity of sound } U_s^2 = \frac{\partial P}{\partial \rho}_S = rRT = C_p T(r-1)$$

$$Ec = \frac{V^2}{C_p \Delta T} = \frac{V^2 T_0 (r-1)}{U_s^2 \Delta T} = (r-1) M^2 \left( \frac{T_0}{\Delta T} \right)$$

$$M = \frac{V}{U_s}, \quad \text{Mach. No.}$$

$Ec \sim M^2$ . Then for subsonic flow terms with  $Ec$  maybe neglected.

Incompressible flow  $M \leq 0.3$ .

9/29/11

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Energy Eq.

$$P^* \frac{DT^*}{Dt^*} = \underbrace{\frac{\bar{k}}{DP_0 C_p V} \nabla^{*2} T^*}_{+ \left( \frac{\dot{g} D}{\rho C_p \Delta T V} \right)} + \underbrace{\frac{\mu V}{DP_0 C_p \Delta T} \Phi^* - \frac{V^2}{C_p \Delta T} \frac{T^*}{P^*} \frac{\partial P^*}{\partial T^*} \frac{DP^*}{Dt^*}}_{Re \frac{V^2}{C_p \Delta T}}$$

Eckert No.

$$Ec = \frac{V^2}{C_p \Delta T} = \frac{k \cdot \text{E convection}}{\text{Enthalpy convection}}$$

Pelet No.

$$Pe = \frac{R_o C_p D V}{k} = \frac{\rho_o C_p \Delta T V / D}{k \Delta T / D^2} = \frac{\text{H.T. by convection}}{\text{H.T. by conduction}}$$

Prantl No.

$$Pr = \frac{V}{a} = \frac{\mu C_p}{k} = \frac{\text{Momentum Diffusion}}{\text{Thermal Diffusion}}$$

$$Pe = Re Pr$$

Heat generation NO.

$$g^* = \frac{\dot{g} D}{\rho C_p \Delta T V} = \frac{\text{Heat generation}}{\text{Heat convection}}$$

$$P^* \frac{DT^*}{Dt^*} = \frac{1}{Pe} \nabla^{*2} T^* + \underbrace{\frac{Ec}{Re} \Phi^* - \frac{E}{C} \frac{T^*}{P^*} \frac{\partial P^*}{\partial T^*} \frac{DP^*}{Dt^*} + \dot{g}^*}_{\text{for most cases can be neglected } (M \leq 0.3)}$$

Incompressible flow  $\rho^* = 1$  ( $M \leq 0.3$ )

$$\frac{DT^*}{Dt^*} = \frac{1}{Pe} \nabla^{*2} T^* + \frac{Ec}{Re} \Phi^* + \dot{q}^*$$

No heat generation

$$\frac{DT^*}{Dt^*} = \frac{1}{Pe} \nabla^{*2} T^*$$

Thermal similarity governed by Pe.

$$Pe = Pr Re$$

Summary.

$$\int \frac{\partial \vec{R}^*}{\partial t^*} + \nabla^* \cdot (\rho^* \vec{V}^*) = 0$$

$$\left| \rho^* \frac{D\vec{V}^*}{Dt^*} = -\nabla^* p^* + \frac{1}{Re} \left[ \underbrace{\nabla^{*2} \vec{V}^*}_{\text{Fluid}} + \frac{1}{3} \nabla^* \cdot (\nabla^* \vec{V}^*) \right] + \frac{\rho^*}{Fr} \left( \frac{\vec{g}}{g} \right) \right.$$

$$\left. \rho^* \frac{DT^*}{Dt^*} = \frac{1}{Pe} \nabla^{*2} T^* + \frac{Ec}{Re} \Phi^* - \frac{Ec T^*}{P^*} \frac{\partial e^*}{\partial T^*} \right) \frac{DP^*}{Dt^*} + \dot{q}^*$$

$$Re = \frac{\rho_0 V D}{\mu}, \quad Fr = \frac{V^2}{g D}, \quad Pe = \frac{VD}{a} \quad (a = \frac{k}{\rho c_p})$$

$$Ec = \frac{V^2}{c_p \Delta T} \quad (Pe = Pr Re)$$

Incompressible flow No heat generation.

$$\left\{ \nabla^* \vec{V}^* = 0 \right.$$

*(Fluid)*

$$\left. \frac{D\vec{V}^*}{Dt^*} = -\nabla^* p^* + \frac{1}{Re} \nabla^{*2} \vec{V}^* + \frac{1}{Fr} \left( \frac{\vec{g}}{g} \right) \right.$$

$$\left. \frac{DT^*}{Dt^*} = \frac{1}{Pe} \nabla^{*2} T^* + \frac{Ec}{Re} \Phi^* \right)$$

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Dynamics similarity.  $Re$   $Fr$ .

Thermal similarity.  $Pe$ .

3) Natural Circulation.

- Velocity scale.: self determined.

$$(1) \text{ Creeping flow. } Re \approx 1 \quad \frac{\rho V D}{\mu} = 1$$

$$V = \bar{V}/D$$

$$(2) \text{ Acc. due to gravity. } \rho \frac{\bar{V}}{t} = \rho g \beta \Delta T \quad t = \frac{D}{\bar{V}}$$

$$\bar{V} = \sqrt{g \beta \Delta T D}$$

\* In general.

$$V^* = \frac{V}{\bar{V}}, \quad t^* = \frac{t}{\tau}, \quad \tau = \frac{D}{\bar{V}}$$

$$\nabla^* = D \nabla, \quad \vec{x}^* = \frac{\vec{x}}{D}, \quad \nabla^{*2} = D^2 \nabla^2$$

$$T^* = \frac{T - T_{\infty}}{T_0 - T_{\infty}} = (T - T_{\infty}) / \Delta T$$

$$P^* = \frac{P - P_0}{\rho \bar{V}^2}, \quad \rho^* \approx 1$$

$$\text{M.E. } \frac{D \vec{V}^*}{Dt^*} = -\nabla^* P^* - \left( \frac{g \beta \Delta T D}{\bar{V}^2} \right) T^* + \frac{1}{Re} \nabla^{*2} \vec{V}^*$$

$$\text{E.E. } \frac{DT^*}{Dt^*} = \frac{k}{\rho_0 C_p \bar{V} D} \nabla^{*2} T^* + E_c \dot{\varphi}^* + \frac{E_c \beta T}{\bar{V}} \frac{DP^*}{Dt^*}$$

$E_c \ll 1$

① Creeping flow

Very small velocity  $Re \approx 1$

$$V = \frac{v}{D}$$

$$\text{E.M. } \frac{DV^*}{Dt^*} = -\nabla^* p^* - \frac{g\beta\Delta TD^3}{\nu^2} T^* + \nabla^* \cdot V^*$$

Grashof No.

$$Gr = \frac{g\beta\Delta TD^3}{\nu^2} = \frac{\text{buoyant } \times \text{inertia}}{(\text{viscous force})^2}$$

Note: No Reynolds number  $\leftarrow$  Creeping flow

E.E.

$$\frac{DT^*}{Dt^*} = \frac{1}{Pr} \nabla^* \cdot T^* + E_c \Phi^* + E_c (\beta T) \frac{DP^*}{Dt^*}$$

$$\text{Parametric No. } Pr = \frac{\nu}{\alpha} = \frac{R_o G \nu}{k}$$

only  $\frac{Gr}{Pr}$  control the dynamic similarity thermal

$$2) V = \sqrt{g\beta\Delta TD}$$

$$\bar{T} = \frac{D}{V} = \frac{D}{\sqrt{g\beta\Delta TD}}$$

$$\frac{DV^*}{Dt^*} = -\nabla^* p^* - T^* + \frac{1}{\sqrt{Gr}} \nabla^* \cdot V^*$$

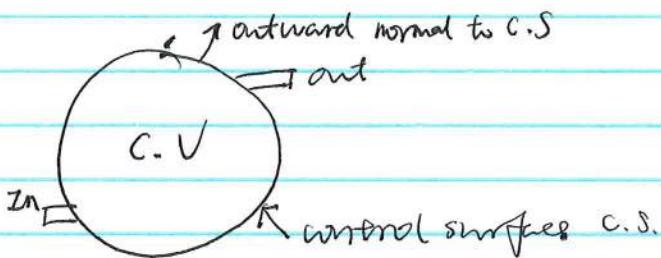
$$\frac{DT^*}{Dt^*} = \frac{1}{Pr\sqrt{Gr}} \nabla^* \cdot T^*$$

In either way  $T^* = f[Gr, Pr]$

## Chapter 7

### Control volume analysis of Reactor System

I) conservation Equations of control volume fixed in space  
 $\vec{V}_c = 0.$  : such as nuclear reactor



• Mass conservation.

$$\frac{d}{dt} \int_{CV} \rho dV = - \oint_{C.S.} \rho \vec{V} \cdot \vec{n} dA.$$

If only one inlet and one outlet.

$$\frac{d}{dt} \int_{CV} \rho dV = (\rho AV)_{in} - (\rho AV)_{out}$$

$$\boxed{\frac{d}{dt} \int_{CV} \rho dV = \sum_i \dot{m}_i}$$

$i$ : area where fluid crosses the boundary

$\dot{m}_i$ : mass inflow at  $i$  section (+)  
 outflow (-)

• Momentum conservation.

$$\frac{d}{dt} \int_{CV} \rho \vec{V} dV = \sum_i (\dot{m}_i \vec{V}_i)$$

Rate of change of momentum in CV =  $\frac{\text{Total force}}{\text{force}} + \frac{\text{Momentum influx}}{\text{influx}}$

$$\left\{ \begin{array}{ll} \dot{m}_i > 0 & \text{in} \\ \dot{m}_i < 0 & \text{out} \end{array} \right.$$

Force - gravity  
 } external force (contact force)  
 pressure  
 shear.

• Energy balance

$$\frac{d}{dt} \int_{CV} p e dV = \dot{Q} + \dot{W}_S + \sum (m_i) (e + \frac{P}{\rho})$$

Heat work + energy influx  
Tr.

$$e = u + \frac{V^2}{2} + gz \quad (\text{total energy})$$

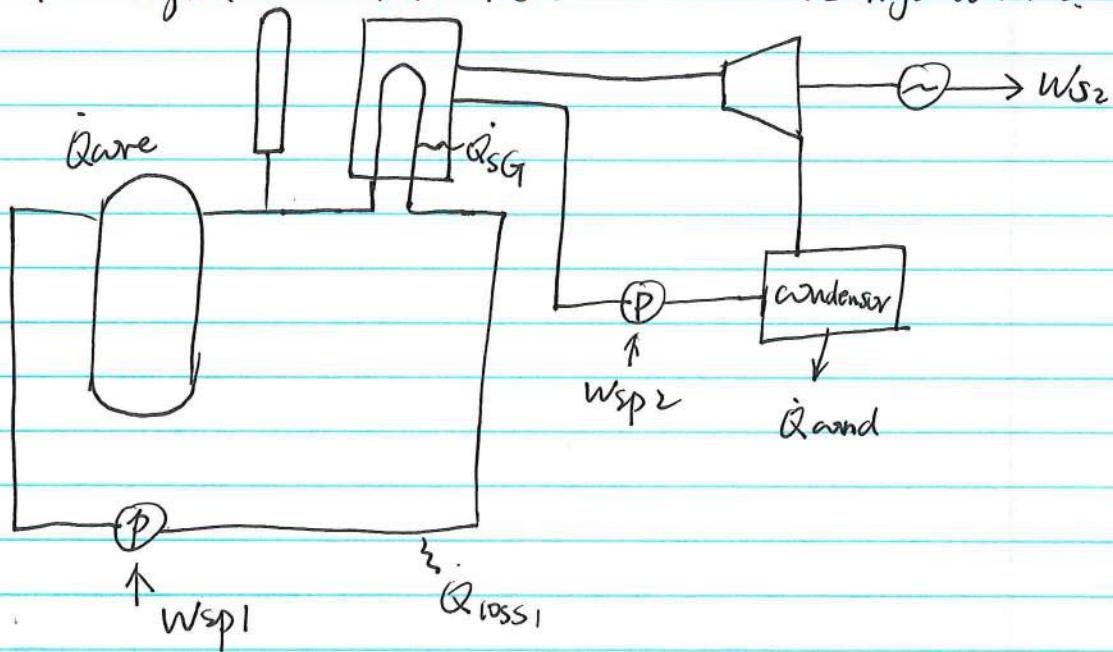
$\dot{Q}$  : heat addition

$\dot{W}_S$  : work done on system (shaft work), pump etc.

$$u + \frac{P}{\rho} = i \quad \text{enthalpy}$$

$\frac{V^2}{2} + gz$ . in many heat transfer problems  
these are negligible.

2). Integral Nuclear Reactor Thermal hydraulics.



## Primary system

$Q_{\text{core}} (>0)$  Nuclear Heat Generation ( $\sim 3000 \text{ MWt}$ )

$Q_{\text{SG}} (<0)$  Heat Tr. to SG (2nd side)

$Q_{\text{loss}} (<0)$  heat loss (small)

$W_{\text{pump}} (>0)$  work done by Pump ( $\sim 30 \text{ MW}$ )  
 $\sim 40,000 \text{ hp}$

## Mass balance

$$\frac{d}{dt} \int_{CV} p dV + \oint_{CS} p \vec{V} \cdot \vec{n} dA = 0.$$

↑                              ↑

total coolant  
inventory

(i) steady state: both terms are 0

(ii) LOCA

$$\oint_{CS} p \vec{V} \cdot \vec{n} \cdot dA \rightarrow (pVA)_{\text{break}} > 0$$

Initially

$$\frac{d}{dt} \int_{CV} p dV = - \oint p \vec{V} \cdot \vec{n} dA = -(pVA)_{\text{break}}$$

primary coolant inventory decreases as  $-\int (pVA)_{\text{break}} dt$ .

• ECR come in

$$\frac{d}{dt} \int_{CV} p dV = (pVA)_{\text{ECR}} - (pVA)_{\text{break}}$$

$$= \dot{m}_{ECCS} - \dot{m}_{break}$$

If  $\dot{m}_{ECCS} > \dot{m}_{break}$ .

Primary coolant inventory may recover.

Prevention of core uncovery.

- Momentum balance

- Energy balance

$$\frac{d}{dt} \int_{CV} \rho e dV + \int_{CS} \rho (e + \frac{P}{\rho}) (\vec{v} \cdot \vec{n}) dA = \dot{Q} + \dot{W}_{IS}$$

10/4/11.

Review

1. General balance equation.

C.E M.E. E.E.

- fixed mass volume.

- explain each term. pressure.

- conservative

- { eq. of state

- } mechanical

- energy. heat.

3 basic

① Laminar flow parabolic

② transient solution. err function suddenly.

physical meaning of solution.

③ fuel rod how long. pipe flow time taken

Mass balance.

$$\frac{d}{dt} \int_{CV} \rho dV + \oint_{CS} \rho \vec{v} \cdot \vec{n} dA = 0.$$

total mass

(i) Steady state

$$\oint \rho \vec{v} \cdot \vec{n} dA = 0$$

$$\frac{d}{dt} \int_{CV} \rho dV = 0. \quad \frac{dM}{dt} = 0. \quad M = \int \rho dV.$$

(ii) LOCA.

$$\oint_{CS} \rho \vec{v} \cdot \vec{n} dA = (\rho VA)_{break}$$

$$\frac{d}{dt} \int_{CV} \rho dV = - \oint \rho \vec{v} \cdot \vec{n} dA = -(\rho VA)_{break}$$

→ Primary coolant inventory.

→ core uncover.

→ fuel dry out

→ cladding oxidation  $\Rightarrow$  melt down.

- Safety strategy. ECCS

$$\frac{d}{dt} \int_{CV} \rho dV = (\rho VA)_{ECCS} - (\rho VA)_{break}$$

$$= \dot{m}_{ECCS} - \dot{m}_{break}.$$

•  $\dot{m}_{break}$  is very important  $\rightarrow$  B.C.

Large break flow  $\rightarrow$  low pressure injection.

Small break flow  $\rightarrow$  high pressure injection

Design criteria

$$m_{ECCS} > m_{break}$$

Energy Equation.

$$\frac{d}{dt} \int_{CV} p e dV + \oint p(e + \frac{P}{\rho}) \vec{v} \cdot \vec{n} dA = \dot{Q} + \dot{W}_S$$

(i) steady state

$$\text{No. break} \quad \begin{cases} \oint \rightarrow 0 \\ \frac{d}{dt} \rightarrow 0 \end{cases} \quad LHS = 0$$

$$\dot{Q}_{core} - \dot{Q}_{SG} - \dot{Q}_{loss} + \dot{W}_{sp} = 0.$$

$$\dot{Q}_{SG} = \dot{Q}_{core} + \dot{W}_{sp} - \dot{Q}_{loss} \Rightarrow \text{secondary}$$

(ii) unsteady operation (No. break).

$$\oint \rightarrow 0.$$

$$\frac{d}{dt} \int_{CV} p e dV = (\dot{Q}_{core} + \dot{W}_{sp}) - (\dot{Q}_{SG} + \dot{Q}_{loss})$$

Start up  $\dot{Q}_{core} \uparrow$

$$\dot{Q}_{core} + \dot{W}_{sp} > \dot{Q}_{SG} + \dot{Q}_{loss}$$

$$\frac{d}{dt} \int_{CV} p e dV \uparrow \quad T \uparrow \quad P \downarrow$$

Transient overpower.

• LOCA.

at break

$$\oint p(e + \frac{P}{\rho}) \vec{V} \cdot \vec{n} dA = \dot{m}_{break} (e + \frac{P}{\rho})_{break} \underbrace{\text{enthalpy.}}_{\text{break}}$$

K.E. ~ negligible

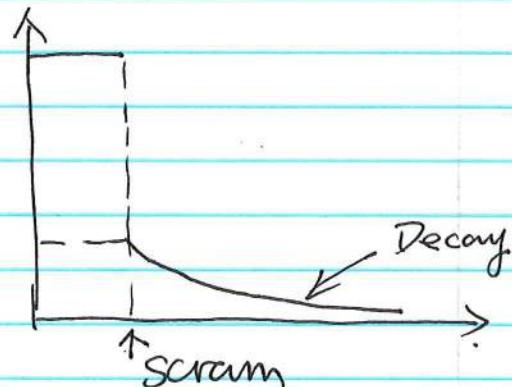
Initially

$$\frac{d}{dt} \int p e dV = (\dot{Q}_{core} + w_{sp}) - (\dot{Q}_{SG} + \dot{Q}_{loss})$$

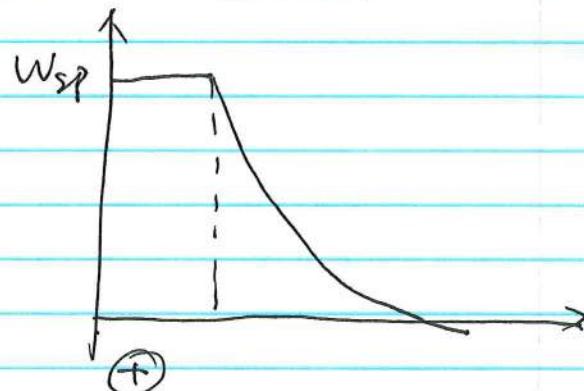
$$\rightarrow \dot{m}_{break} (e + \frac{P}{\rho})_{break}$$

(i) Reactor scram

$\dot{Q}_{core} \rightarrow \dot{Q}_{decay}$



(ii)  $w_{sp} \rightarrow 0$ .  
pump trip



$\frac{d}{dt} \int p e dV \downarrow$

LOCA

$\frac{d}{dt} \int p e dV$

$P \downarrow$

$t \downarrow$

problem is core

$\downarrow$

} good cooling

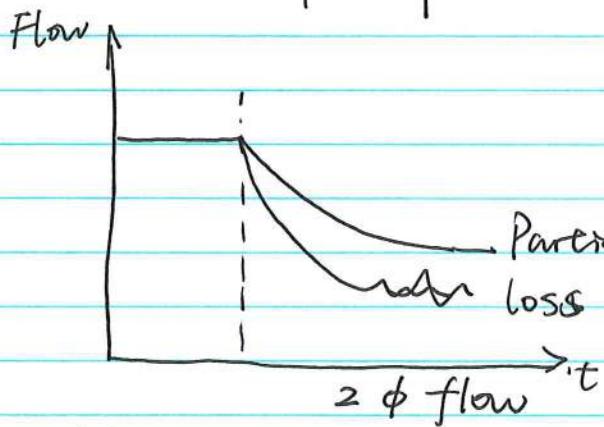
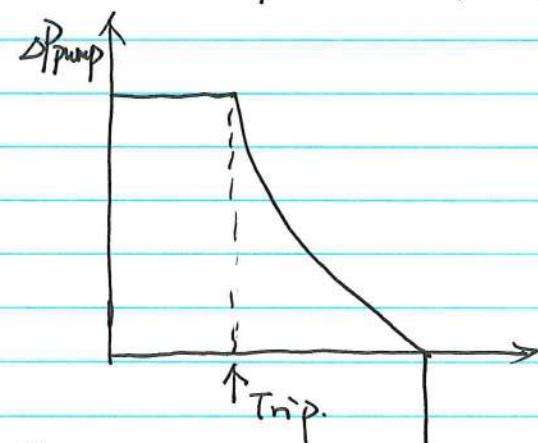
uncovery

ECCS.

$$\frac{d}{dt} \int_{cv} P_e dv = \dot{Q}_{decay} + w_{sp}^{\uparrow} - (Q_{sg} + Q_{loss}) \\ - [m(e + \frac{P}{\rho})]_{break} + [m(e + \frac{P}{\rho})]_{ECCS.}$$

$$[m(e + \frac{P}{\rho})]_{ECCS.} \quad \text{Temp} = T_0$$

2. Loss of flow (pump trip) most important for fast reactor.



Partial failure (Mechan.)

water loss of off site power

$2\phi$  flow

(1)  $w_{sp} \rightarrow 0$ . pump coast down  $\rightarrow 0$ .

(2)  $Q_{SG} \downarrow$  decay to natural circulation cooling

(3)  $Q_{core} \rightarrow Q_{decay}$

over heating.

Pr relief valve operate  
Small break loss

Scram

$$\dot{Q}_{\text{core}} \rightarrow \dot{Q}_{\text{decay}} \downarrow \leftrightarrow \dot{Q}_{\text{SG}}$$

3. Loss of heat sink accident

turbine trip.

secondary loop malfunction,  
 } feed water pump trip.  
 | 2nd pipe break (overcooling  $\rightarrow$  LHS)  
 condenser failure

Real loss of heat sink  $\left\{ \begin{array}{l} \dot{Q}_{\text{cond}} \rightarrow 0 \\ W_{\text{sp}} \rightarrow 0 \end{array} \right.$

$$\dot{Q}_{\text{SG}} \downarrow \rightarrow \sim 0$$

$$\frac{d}{dt} \int p dV = (\dot{Q}_{\text{core}} + W_{\text{sp}}) - (\dot{Q}_{\text{SG}} + \dot{Q}_{\text{loss}}) > 0$$

↓  
become smaller.

$\left\{ \begin{array}{l} \dot{Q}_{\text{core}} \rightarrow \dot{Q}_{\text{decay}} \\ W_{\text{sp}} \rightarrow 0 \end{array} \right.$

$$\frac{d}{dt} \int p dV = (\dot{Q}_{\text{decay}} + 0) - (\dot{Q}_{\text{SG}} + \dot{Q}_{\text{loss}}) = ?$$

> 0. 2nd can not cool if  
 heat up pressurize  $\rightarrow$  pr relief valve  
 small break LOCA

High pressure injection system is used.

ECCS  $\rightarrow$  Reactor.  $\rightarrow$  relief valve.  
 Evaporate  $\rightarrow$

station black out.

AC power supply lost.

off-site

Emergency AC power

} lost  $\Rightarrow$

pump trip  $\rightarrow$  loss of flow

## Sph' many

Secondary  $\rightarrow$  loss of heat sink

reactor may be scanned (DC power)

$$\frac{d}{dt} \int f_e d\omega = (\dot{Q}_{core} + \dot{W}_{p}) - (\dot{Q}_{SG} + \dot{Q}_{loss})$$

↓ ↓  
 Qdecay down

## Chapter 7-2

$\left\{ \begin{array}{l} 1-D \text{ formulation} \\ \text{Quasi } 1-D \end{array} \right.$

general 3-D formulation.

- (1) Too complicated for most application.
  - (2) Turbulent model not well established particularly for 2 phase flow
  - (3) often 3-D information we needed.
  - (4) 3-D Analysis maybe needed in limit number of cases & component
 

$\left\{ \begin{array}{l} \text{core} \\ \text{SG} \quad 2-D \text{ or } 3-D \\ \text{downcomer} \\ \text{plenum} \end{array} \right.$
- under certain condition

1-D. or Quasi 1-D formulation.

- (1) most important engineering tool for reactor thermal hydraulics & safety analysis. (single or two phase)
- (2) 1-D Formulation  $\rightarrow$  finite difference.  
 $\underbrace{\qquad\qquad\qquad}_{\text{control volume}}$
- (3) State of the art in two phase flow model.
- (4) efficient solution techniques
- (5) economical.

(b) Quasi 1-D simulate 2-D or 3-D behaviors (sometimes)

1-D code

Relap 5

TRAC (PP. BD)

RETRAN

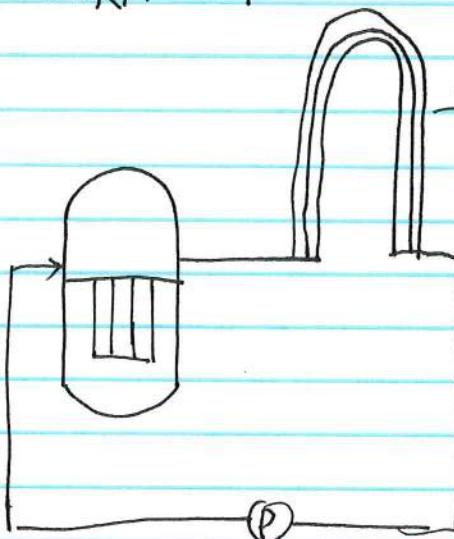
(TRACE)

(Two or three -D) subchannel code

(Parallel channel of 1-D)

COBRA

COBRA-TF



→ component.

-RPV

cove

downcomer

Lower plenum

upper plenum

- Hot leg. (Pressurizer)

↓ sarge line

pressurize.

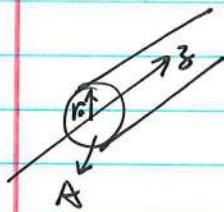
- cold leg.

pump.

- SG I. plenum  
up flow  
U bend  
down flow  
exit plenum

1-D Model for single phase flow.

1) Area average.



$$\frac{1}{A} \int_A \psi dA = \langle \psi \rangle$$

Axially symmetric flow  $\frac{\partial \psi}{\partial \theta} = 0$ ,  $V_\theta = 0$   
constant area  $A = \text{const}$

$$\frac{1}{\pi r_0^2} \int \psi 2\pi r dr = \langle \psi \rangle \quad \textcircled{1}$$

Time derivative

$$\frac{1}{A} \int_A \frac{\partial \rho \psi}{\partial t} dA = \frac{\partial}{\partial t} \frac{1}{A} \int \rho \psi dA = \frac{\partial}{\partial t} \langle \rho \psi \rangle.$$

Gradient Operator in  $\hat{z}$  direction

$\nabla p$ .

$$\frac{1}{A} \int_A (\nabla p)_z dA = \frac{1}{A} \int \frac{\partial p}{\partial z} dA = \frac{\partial}{\partial z} \langle p \rangle.$$

$\nabla \otimes$

Average of vector operator or Sym. tensors.

$$\begin{aligned} \frac{1}{A} \int_A (\nabla \cdot T)_z dA &= \frac{1}{A} \int \left[ \frac{1}{r} \frac{\partial}{\partial r} (r T_{rz}) + \frac{\partial T_{zz}}{\partial z} \right] dA \\ &= \frac{\partial}{\partial z} \langle T_{zz} \rangle + \frac{1}{A} \left[ (2\pi r_0) T_{rz} \Big|_{r_0} - 0 \right] \end{aligned}$$

$2\pi r dr.$

$$\frac{1}{A} \int_A \nabla \cdot w dA = \frac{\partial}{\partial z} \langle w_z \rangle + \frac{1}{A} \left[ (2\pi r_0) W_r \Big|_{r_0} - 0 \right]$$

Introduce hydraulic diameter.

$$D = \frac{4A}{P} \quad A: \text{flow area}$$

$P$ : wetted perimeter.

$$\frac{2\pi r_b}{A} = \frac{4}{D}$$

D: Important length scale for 1-D formulation.

⇒ 1-D Balance equation

continuity Eq.

$$\frac{\partial P}{\partial t} + \nabla \cdot \rho \vec{V} = 0$$

$$\frac{1}{A} \int [ \frac{\partial P}{\partial t} + \nabla \cdot \rho \vec{V} ] = 0$$

$$\boxed{\frac{\partial \langle P \rangle}{\partial t} + \frac{\partial}{\partial z} \langle \rho V_z \rangle = 0}$$

area average.

$$\langle \langle V_z \rangle \rangle = \frac{\langle \rho V_z \rangle}{\langle \rho \rangle} \quad \text{mass weighted mean value}$$

$$\boxed{\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z} \langle \rho \rangle \langle \langle V_z \rangle \rangle = 0}$$

or Assume in  $A, \langle \rho \rangle = \rho$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} \rho \langle V_z \rangle = 0$$

$$\langle V_z \rangle = \frac{1}{A} \int V_z dA$$

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1-D Momentum Eq.

$$\int_A \left[ \frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p - \nabla \bar{C} + \rho \vec{g} \right] dA$$

$$\rho \frac{D\vec{V}}{Dt} = \dots$$

$$\frac{D\vec{V}}{Dt} = \vec{Q}$$

HW.  $\rho$  = uniform. in A.obtain 1-D continuity }  
momentum } Eq.  
Energy }

$$\frac{\partial}{\partial t} \rho \langle V_8 \rangle + \frac{\partial}{\partial z} \rho \langle V_8 V_8 \rangle = - \frac{\Delta p}{\Delta z} \quad \text{pressure gradient}$$

$$- \frac{\partial}{\partial z} \langle T_{88} \rangle \quad \text{normal stress gradient}$$

$$- \frac{4}{D} \tau_{r_8} \Big|_{r_0} \quad \text{wall shear}$$

$$+ \rho g_8 \quad \text{gravity.}$$

1-D Energy Eq.

$$\frac{\partial i}{\partial t} + \nabla \cdot (\rho i \vec{V}) = -\nabla \bar{f} + \frac{DP}{Dt} - \cancel{\pi; \nabla v} + \dot{g}$$

$$\frac{1}{A} \int dA \text{ for above eq.}$$

$$\frac{\partial \langle i \rangle}{\partial t} + \cancel{\pi} \frac{\partial}{\partial z} \rho \langle i V_8 \rangle = - \frac{\partial}{\partial z} \langle f_8 \rangle$$

axial heat conduction

+  $\dot{q}_h \theta_0'' / A$  wall heat flux+  $D(p) / DC$  part of pressure work+  $\langle \dot{g} \rangle$  heat generation

Covariance

$$\text{Note } \langle AB \rangle \neq \langle A \rangle \langle B \rangle$$

$$\text{cov}(A B) =$$

$$\text{cov}(A B) = \langle AB \rangle - \langle A \rangle \langle B \rangle$$

$$\langle V_8 V_8 \rangle$$

$$\langle i V_8 \rangle$$

(ii) Laminar fully developed flow

$$V_8 = 2 \langle V_8 \rangle \left(1 - \frac{r^2}{R_o^2}\right)$$

$$\text{cov}(V_8, V_8) = \frac{1}{\pi} \int V_8^2 dA - \langle V_8^2 \rangle = \frac{1}{3} \langle V_8 \rangle^2.$$

$$\text{cov}(i, V_8) = 0.375 \langle i \rangle \langle V_8 \rangle$$

Turbulent flow

$$V_8 = V_C \left(1 - \frac{r}{R_o}\right)^{1/\eta}$$

$$V_C = \frac{\langle V_8 \rangle}{*0.817} = 1.22 \langle V_j \rangle \left(1 - \frac{r}{R_o}\right)^{1/\eta}$$

$$\text{cov}(V_8, V_8) = 0.02 \langle V_8 \rangle^2.$$

$$\text{cov}(i, V_8) = 0.02 \langle i \rangle \langle V_8 \rangle$$

For turbulent flow covariance maybe neglected

$$1 + 0.02 < C_m < 1 + \frac{1}{3}$$

$$1.02 < C_m < 1.33 \quad C_m \approx 1$$

$$\langle i V_8 \rangle = C_E \langle i \rangle \langle V_8 \rangle$$

$$1.02 < C_E < 1.375. \quad C_E = 1.$$

$C_m, C_E$ . distribution parameter.

Energy eq.  $\langle i \rangle$

B

Alternate Energy Eq. in terms of  $T$

$$\rho C_p \left[ \frac{\partial \langle T \rangle}{\partial t} + \langle v_z \rangle \frac{\partial \langle T \rangle}{\partial z} \right] = \frac{\dot{q}_h g_0^+}{A} - \frac{\langle \partial \tilde{g}_z \rangle}{\partial z} - \frac{\langle T \rangle}{C_p} \frac{\partial p}{\partial T} \Big|_p \frac{Dp}{Dt} + \dot{q}$$

$\downarrow$                        $\downarrow$   
wall heat      axial  
flux            conduction.

water : axial conduction can be neglected  
air : normally included. ( $Pr \ll 1$ )

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Natural circulation force.

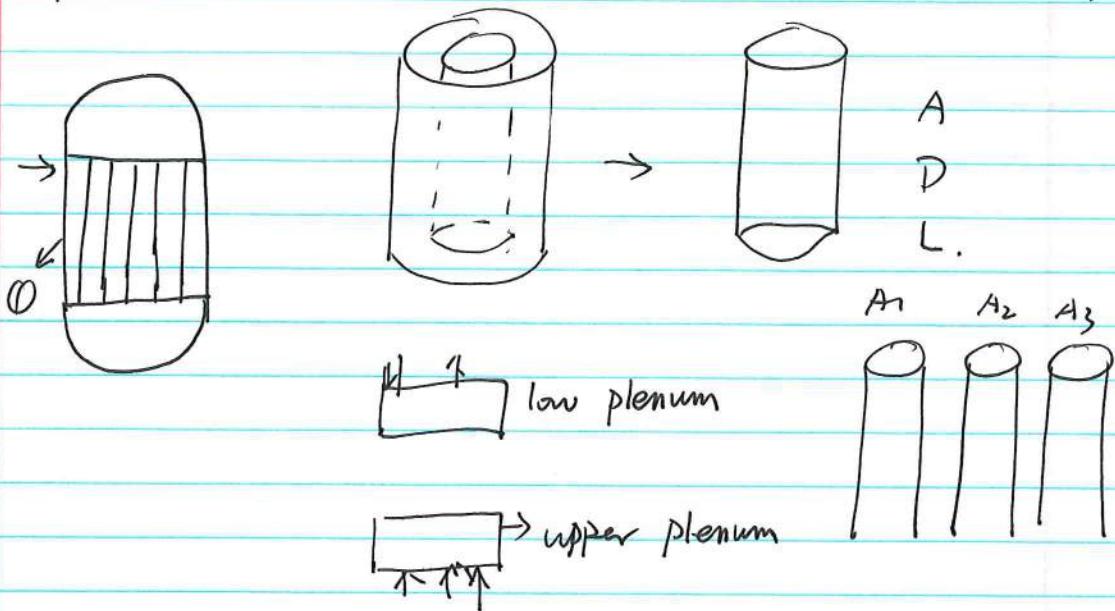
M.E change of  $\rho g_z$  due to  $T$  change.

$$\underline{\rho g_z = \bar{\rho} g_z - \bar{\rho} \beta \Delta T g_z}. \quad \text{Taylor expansion.}$$

$\beta$ : Thermal Expansion coefficient.

(10/18/11)

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Summary.

$$\frac{\partial P}{\partial t} + \frac{\partial PV}{\partial z} = 0.$$

$$C_M = 1. \quad \frac{\partial PV}{\partial t} + \frac{\partial PV^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{4T_w}{D} + \rho g z + \textcircled{J}.$$

$$\left[ \frac{\partial Pi}{\partial t} + \frac{\partial}{\partial z} \rho_i V = \frac{3i g'' w}{A} + \dot{q} + \left[ \frac{\partial P}{\partial z} + V \frac{\partial P}{\partial z} \right] + \textcircled{J} \right]$$

$$\rightarrow \rho C \left[ \frac{\partial T}{\partial t} + V \frac{\partial T}{\partial z} \right] = \frac{3i g'' w}{A} + \dot{q} - \frac{I}{P} \frac{\partial P}{\partial T} \frac{DP}{Dt}.$$

$$-\frac{4T_w}{D} = -\frac{f}{2D} \rho V |V|.$$

*inertial*  $\leftarrow$   $\textcircled{J}$ : friction factor.  $f = f(Re, \frac{\epsilon}{D})$   $\rightarrow$  roughness.

$$g'' w = h(T_w - T).$$

$\leftarrow h \rightarrow$  heat tr. coefficient.

Natural circulation they are not sufficient.

friction factor.

- smooth pipe.  $f = f(Re)$ .

Laminar:  $Re \leq 2300$      $f = 64/Re$ .     $Re \equiv \frac{\rho v D}{\mu}$

Turbulent:  $Re > 2300$

Blaauw's  $f = 0.316/Re^{0.25}$ .     $Re < 10^5$

McAdams  $f = 0.189/Re^{0.2}$      $Re < 2 \times 10^6$

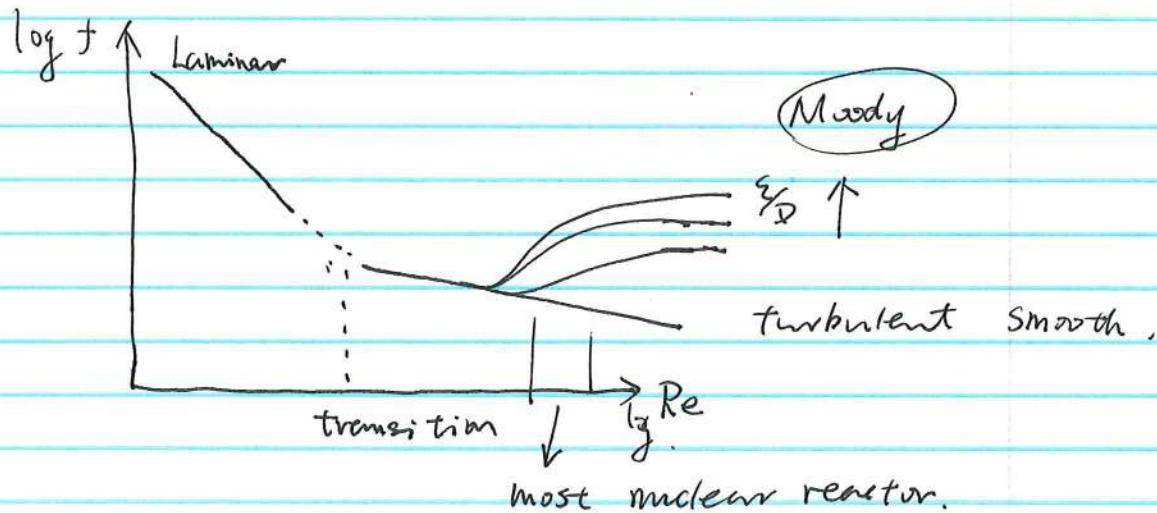
- Rough pipe  $f = f(Re, \frac{\epsilon}{D})$ .

→ completely rough pipe regime  $f = f(\frac{\epsilon}{D})$ .

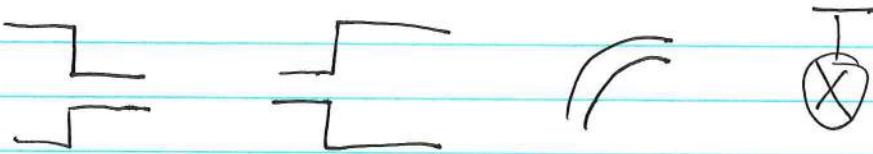
$$f = \frac{1}{(2 \log \frac{D}{\epsilon} + 1.14)^2} \quad (\text{Von Karman \& Nik.})$$

- whole regime

$$\frac{1}{\sqrt{f}} = 1.14 - 2 \log \left( \frac{\epsilon}{D} + \frac{18.7}{Re \sqrt{f}} \right)$$



Minor loss.



loss coefficient. or  $k$  factor.

$$\Delta P = k \rho V^2 / 2. \quad k \text{ is given}$$

$$k = \sim \text{const.} = k(Re) \quad (\text{weak})$$

$$\cancel{\Delta P_n - \frac{4T_w}{D}} = -\frac{f}{2} \rho V^2. \quad \text{for } Re^{-0.2}$$

$$Q = A V. \sim D^2 V$$

$$Re = \frac{\rho V D}{\mu}$$



$$\Delta P = \Delta P_{\text{pipe}} + \Delta P_c = \left(\frac{f L}{D} + K\right) \frac{\rho V^2}{2}$$

$$K = k \text{ (geometry, } Re). \quad \uparrow \quad f = f(Re, \frac{S}{D})$$

$$(i) \quad \dot{q}_w'' = h(T_w - T_m)$$

$h$ : heat tr. coefficient

$T_m$ : mixed mean temperature

$$T_m = \frac{1}{A_{\text{cv}}} \int T v dA.$$

Turbulent flow  $T_m \approx \langle T \rangle$

$$h = \quad Nu = \frac{h D}{k} \quad . \text{non-dim}$$

Nusselt No.

★

$$Nu = Nu(Re, Pr)$$

$$10^{-2} < Pr < 10^4$$

• Laminar flow

$$Nu = 4.364 \quad (\theta_w \text{ const})$$

$$= 3.658 \quad (T_w \text{ const})$$

• turbulent flow

$$Pr < 0.1 \quad (\text{liquid metal}) \quad Pr = \frac{\nu}{\alpha}$$

$$Nu_T = 6.3 + 0.023 Re Pr \sim \text{const}$$

$$Nu_T = 4.8 + 0.023 Re Pr. \sim \text{const}$$

 $0.5 < Pr < 1.0 \quad (\text{gases})$ 

$$Nu_T = 0.022 Pr^{0.6} Re^{0.8} \sim Re^{0.8} \sim V.$$

$$Nu_T = 0.021 Pr^{0.6} Re^{0.8}$$

H:	$\theta_w \text{ const}$
T:	$T_w \text{ const.}$

 $1.0 < Pr < 20 \quad (\text{water})$ 

$$Nu = 0.0155 Pr^{0.3} Re^{0.83}$$

difference between

liquid metal and

water.

 $Pr > 20 \quad (\text{heavy oil})$ 

$$Nu = 0.0118 Pr^{0.3} Re^{0.9}$$

### Natural circulation

Boussinesq Assumption

$\rho$  change only important in  $Pg_z$ .  
 all other term  $\rho = \bar{\rho}$

$$\beta = -\frac{1}{\bar{\rho}} \frac{\partial \rho}{\partial T} \Big|_P \quad \boxed{\text{Thermal Expansion coefficient.}}$$

Equation of state  $\beta$  can be obtained

Fundamental Eq. of state

$$u = u(s, p) \rightarrow p = p(p, T)$$

$$u = u(p, T)$$

$\beta$  is a first derivative of thermal eq. of state

$$\boxed{p g_3 = \bar{p} g_3 - \bar{p} \beta \Delta T g_3}$$

$\downarrow$

$$-\frac{1}{\bar{p}} \frac{\partial p}{\partial T} \Delta T \Rightarrow \Delta p$$

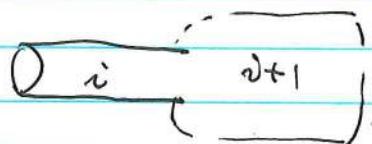
Small change.

not small

except  $\bar{p} g_3$ ,  $\rightarrow$  Assume incompressive

$$p = \bar{p} = \text{const}$$

$$\frac{\partial p}{\partial t} + \frac{\partial p v}{\partial z} = 0, \quad \frac{\partial v}{\partial z} = 0, \quad \text{for const Area}$$



$v_i = v_i(t)$  only depend on time.  
 $\Rightarrow$   $i$  component.

$$\cancel{f_i} v_i A_i = \cancel{f_{i+1}} v_{i+1} A_{i+1} = v_r A_r.$$

r: reference section.

$$v_i A_i = v_r A_r.$$

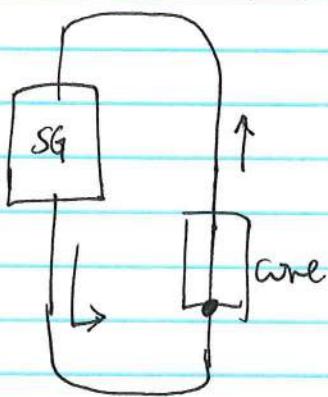
$A_r$ : area of reference section

$v_r$ : vel in reference section

$A_i$ :  $i$ th section area.

$$v_i : \text{velocity} \quad v_i = \frac{A_r}{A_i} v_r, \quad \text{①}$$

only 1 velocity in the velocity variable  
 ↓  
 there should be only one momentum eq.



↓  
 Integrated Momentum Eq.

$$\frac{\partial \rho_i v_i}{\partial t} + \frac{\partial \rho_i v_i^2}{\partial z} = - \frac{\partial P}{\partial z} - \frac{f_i (\rho_i v_i | v_i|)}{2D_i} + \rho_i g_{g_i}$$

$$\rho g_s = \bar{\rho} g_s - \bar{\rho} \beta \Delta T g_s.$$

$$\frac{\partial \bar{\rho} v_i}{\partial t} + \frac{\partial \bar{\rho} v_i^2}{\partial z} = - \frac{\partial P}{\partial z} - \frac{f_i \bar{\rho} v_i |v_i|}{(2D_i)} + \bar{\rho} g_s - \bar{\rho} \beta \Delta T g_s$$

↓

B. Assumption

f [

$$\oint \frac{\partial \bar{\rho} v_i}{\partial t} dz = \bar{\rho} \oint \frac{\partial}{\partial t} \left( \frac{a_r}{a_i} v_r \right) dz = \bar{\rho} \sum_i \left( \frac{a_r}{a_i} l_i \right) \frac{\partial v_r}{\partial t}$$

$$v_i = \frac{a_r}{a_i} v_r.$$

= Total inertial effect

$$\oint \frac{\partial \bar{\rho} v_i^2}{\partial z} dz = 0$$

$$d \bar{\rho} v_i^2$$

$$\oint - \frac{\partial P}{\partial z} dz = \Delta P_{pump}$$