# NUCL 511 HMWK 9

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**1 Find the numerical value of $p^{00}$, the flux after a prompt jump for which the increase due to delayed neutrons is just compensated by Doppler feedback, for an LWR from the typical $\lambda$ and $\gamma/\beta$ values given in the text. Discuss why $p^{00}$ is different from the value for the SEFOR reactor.** Using the condition for which $p' = 0$, or, physically, that the increase from prompt jump is just compenstated for by Doppler feedback (i.e. the power change is zero), we can find that this point is

$$p^0 = \frac{\bar{\lambda}\beta}{-\gamma} \equiv p^{00}$$

*For LWR:*

The literature gives a typical $\bar{\lambda}$ value of $0.43\,\mathrm{s}^{-1}$ [Lecture 2], $\gamma$ of $-0.8\,\$/\mathrm{fp}\cdot\mathrm{s}$ [1, p. 240] and $\beta$ of 0.0065 which is equivalent to 1 \$.

$$p^{00} = \frac{\bar{\lambda}\beta}{-\gamma} = -\frac{\bar{\lambda}\beta}{\gamma} = -\frac{\left(0.43\,\mathrm{s}^{-1}\right)(1\$)}{(-0.8\,\$/\mathrm{fp}\cdot\mathrm{s})} = 0.54\,\mathrm{fp}$$

*For SEFOR:*

The literature gives a typical $\bar{\lambda}$ value of $0.40\,\mathrm{s}^{-1}$ [1, p. 244], $\gamma$ of $-0.022\,\$/\mathrm{fp}\cdot\mathrm{s}$ [1, p. 245] and $\beta$ of 0.0065 which is equivalent to 1 \$.

$$p^{00} = \frac{\bar{\lambda}\beta}{-\gamma} = -\frac{\bar{\lambda}\beta}{\gamma} = -\frac{\left(0.40\,\mathrm{s}^{-1}\right)(1\$)}{(-0.022\,\$/\mathrm{fp}\cdot\mathrm{s})} = 18.8\,\mathrm{fp}$$

While SEFOR was designed to mimic the properties of a large power reactor, the power excursion of a typical LWR should not be the same as SEFOR. It used BeO rods to soften the spectrum (since SEFOR was very small compared to a LWR), and the fuel rods were also fabricated almost 3 times smaller than that of an LWR to mimic the high fuel temperature. While the kinetics of SEFOR may be similar, the power inertia of the LWR system means that it will only increase a small amount compared to what SEFOR will increase [1, p. 246].

**2 Consider the reactivity-step-induced transient with an adiabatic boundary condition for the incremental heat, as considered in Sec. 10-2B. List the asymptotic results derived in the text and apply them to estimate**

$$\int_0^\infty \left[s_d\left(t'\right) - s_{d0}\right] dt = ?$$

**Present the physical arguments as to why the integral of the additional delayed neutron source must be finite although $s_{das} = s_{d0}$.** In the text, the solution for the asymptotic flux is provided as [1, eq. 10.60]

$$p_{as} = p_0 - \rho_1 \frac{\lambda_H}{\gamma} > p_0$$

This result, along with

$$p_0 = \frac{s_{d0}}{\beta}$$

and

$$p = \frac{s_d}{\beta}$$

means we can put the integral in the following form

$$\int_0^\infty \left[s_d\left(t'\right) - s_{d0}\right] dt = \beta \int_0^\infty \left[p\left(t'\right) - p_0\right] dt$$

and understanding that [1, eq. 10.56]

$$\int_0^\infty \left[p\left(t'\right) - p_0\right] dt = -\frac{\rho_1}{\gamma}$$

we conclude that

$$\int_0^\infty \left[s_d\left(t'\right) - s_{d0}\right] dt = -\frac{\beta\rho_1}{\gamma}$$

As stated by Ott, "the asymptotic state is a stationary state and therefore the flux level is independent of the precursor decay constants altogether" [1, pp. 248-249]. This provides the mathematical argument to the contradiction of a constantly "loaded" delayed source (which would result in an infitie delayed source). This mathematical argument is compounded by the physical arguments that the PJA is non physical, so the delayed source would really not being at it's asymptotic level. Also, the kinetics are only taking into account one energy group. If more energy groups were considered, the delayed neutron source would emit a soft spectrum, and not all of the neutrons would compound (so it would not compound infinitely). Finally, there is no consideration of heat transfer in the current model, and heat transfer would change cross sections and adjust the delayed neutron source, keeping it finite.

**3 Calculate the burst width in a superprompt-critical transient with the linear energy feedback model from the formula derived in the text. Use LWR data: $\beta = 0.0075$, $\Lambda = 10^{-5}$ s; let $\rho_1 = 1.1\$$. Also find the energy release**

$$Q\left(t_2\right) = \int_0^{t_2} P\left(t\right) dt$$

**Estimate the magnitude of the effects of the following two approximations:**

**(1) The neglect of the heat release (i.e., the assumption of $\lambda_H = 0$). Consider an oxide fuel rod of $0.6\,\text{cm}$ in diameter.** To determine the burst width, the Lecture notes give the following equation

$$\Delta t = \frac{2\rho_{p1}}{-\gamma} \frac{1}{\frac{\rho_{p1}^2}{2\Lambda(-\gamma)} + p^0}$$

So, with $\Lambda$ and $\rho_1$ given, we must determine $\rho_{p1}$, $\gamma$, and $p^0$. The pseudoinitial power is found by

$$p^0 = \frac{\rho_{1\$}}{\rho_{1\$} - 1} p_0 = \frac{1.1}{0.1} p_0 = 11 p_0$$

and we can assume that $p_0$ is much less than 1, making the following approximation accurate [1, eq. 10.99].

$$\Delta t \approx \frac{4\Lambda}{\rho_{p1}} = \frac{4\Lambda}{0.1\beta} = 0.0533\,\text{s}$$

To find the energy release, we can first find the energy release up to the flux maxima, given by [1, eq. 10.71]

$$\Delta Q_m = \int_0^{t_m} P\left(t'\right) dt' = -\frac{\rho_{p1}}{\gamma_e}$$

The total energy release is just twice this (since it is a symmetric burst) [1, eq. 10.87].

$$\Delta Q\left(t_2\right) = 2\Delta Q_m = -2\frac{\rho_{p1}}{\gamma_e}$$

We now need to find the energy release coefficient for an oxide fuel rod and with $\lambda_H = 0$. By neglecting first order heat transfer, our feedback becomes

$$\delta\rho\left(t\right) = \gamma \int_0^t \left[p\left(t'\right) - p_0\right] \exp\left[-\lambda_H\left(t' - t\right)\right] dt' = \gamma \int_0^t \left[p\left(t'\right) - p_0\right] dt'$$

With that adiabatic neglect of first order heat transfer, we get

$$\delta T_f = 94\,\mathrm{K/fp \cdot s}$$

and therefore

$$\gamma = (94\,\mathrm{K/fp \cdot s})\,(-3\,\mathrm{pcm/K}) = -0.4\,\$/\mathrm{fp \cdot s}$$

To find $\gamma_e$, we need to multiply this with a conversion from $MW$ to $fp$, which exists in the typical average power (2500 MW).

$$\gamma_e = \frac{(-0.4\,\$/\mathrm{fp \cdot s})}{2500\,\frac{\mathrm{MW}}{\mathrm{fp}}} = -1.6 \times 10^{-4}\,\$/\mathrm{MW \cdot s}$$

and therefore the energy release is

$$\Delta Q\,(t_2) = -2\frac{0.1\,(0.0075)}{(-1.6 \times 10^{-4}\,\$/\mathrm{MW \cdot s})} = 9.36\,\mathrm{MW}$$

**(2) Neglect of $P_0$ under the feedback integral, i.e., the integral of $P_0$ over the burst width with $Q\,(t_2)$.** To determine this, we also assume that the heat transfer is adiabatic, so the feedback becomes

$$\delta\rho\,(t) = \gamma \int_0^t \left[p\left(t^{'}\right) - p_0\right] \exp\left[-\lambda_H \left(t^{'} - t\right)\right] dt^{'} = \gamma \int_0^t \left[p\left(t^{'}\right)\right] dt^{'}$$

With the neglect of the baseline power and also first order heat transfer. The solution above still holds, since the power at maxima was assumed to be much higher than that at baseline $p_m \gg p_0$.

# References


[1] K Ott and R Neuhold. *Introductory Nuclear Reactor Dynamics*. American Nuclear Society, La Grange Park, Illinois, 1985.