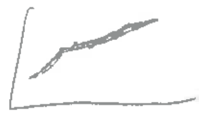


10/5 Exam Review

3-D formulation — energy, momentum, continuity

physical meaning

constitutive relations for single phase

thermo - eq of state

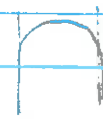
hydrodynamic - stress

heat flux

turbulent flow Prandlt mixing length —

incompressible

inviscid

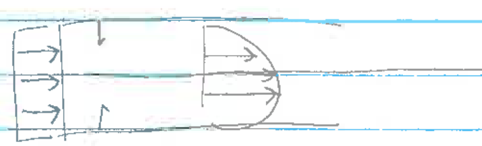


laminar flow parabola steady state

1-D conduction → temp profile

independent, dependent

pen depth



10sec

turbulent (Reynold's Stress)

origin of turbulent stress

approx. velocity profile

mixing length

shear stress of turbulent flow

internal turbulence

turbulent intensity

eddy diffusivity
molecular diffusivity

Prandlt Number

Peclet
Reynolds Number scale mom. eq
Prandlt Number
Frand Number
Mach
Eckart

} non dimension group ✓

Control Volume analysis
mass
momentum
energy

} LOCA
breakflow
(critical flow)

loss of flow
loss of heat sink

Steady Flow

(a) General Balance Eq

$$\begin{bmatrix}\text{change of } \psi \\ \text{in } V\end{bmatrix} = \begin{bmatrix}\text{flux of } \psi \\ \text{across Surface}\end{bmatrix} + \begin{bmatrix}\text{generation of } \psi \\ \text{within } V(t)\end{bmatrix}$$

$$\frac{D}{Dt}\int_{V_m} \psi \, dV = -\oint_S \vec{J}\cdot\vec{n}\,dS + \int_{V_m} \dot{\psi}\, dV$$

Reynolds transport theorem

$$\frac{D}{Dt}\int_{V_m} \psi \, dV = \int_{V_m}\left[\frac{\partial \psi}{\partial t} + \nabla\cdot(\psi\vec{v})\right]dV$$

convective transport

Green's Theorem

$$-\oint_S \vec{J}\cdot\vec{n}\,dS = -\int_{V_m} \nabla\cdot\vec{J}\,dV$$

substitute into general balance eq.

$$\int_{V_m}\left[\frac{\partial \psi}{\partial t} + \nabla\cdot(\psi\vec{v})\right]dV = -\int_{V_m}\nabla\cdot\vec{J}\,dV + \int_{V_m}\dot{\psi}\,dV$$

all over same V_m

$$\frac{\partial \psi}{\partial t} + \nabla\cdot(\psi\vec{v}) = -\nabla\cdot\vec{J} + \dot{\psi}$$

$$\begin{bmatrix}\text{time rate of} \\ \text{change of } \psi \\ \text{per unit vol.}\end{bmatrix} + \begin{bmatrix}\text{convection of } \psi \\ \text{by material motion} \\ \text{per unit vol}\end{bmatrix} = -\begin{bmatrix}\text{influx of } \psi \\ \text{across surface} \\ \text{per unit vol.}\end{bmatrix} + \begin{bmatrix}\text{generation of} \\ \psi \text{ per unit Vol.}\end{bmatrix}$$

(b) Field Equations / (C) meaning

continuity (mass)

$\psi = \rho$

$J = 0$

$\dot{\psi} = 0$ (except for nuclear reactor)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0 ; \quad \begin{bmatrix}\text{rate of} \\ \text{mass change} \\ \text{per unit vol.}\end{bmatrix} + \begin{bmatrix}\text{rate of mass} \\ \text{convected per} \\ \text{unit vol.}\end{bmatrix} = 0$$

(for control volume)

momentum

$\psi = \rho \vec{V}$

$J = P \delta + \tau \Rightarrow$

$$\frac{\partial \rho \vec{V}}{\partial t} + \nabla \cdot \rho \vec{V}\vec{V} = -\nabla p - \nabla \cdot \tau + \rho \vec{g}$$

$$\begin{bmatrix}\text{rate of moment.} \\ \text{change per} \\ \text{unit vol.}\end{bmatrix} + \begin{bmatrix}\text{rate of convected} \\ \text{moment. per} \\ \text{unit vol.}\end{bmatrix} = -\begin{bmatrix}\text{pressure} \\ \text{force}\end{bmatrix} - \begin{bmatrix}\text{viscous} \\ \text{force}\end{bmatrix} + \begin{bmatrix}\text{gravity} \\ \text{force}\end{bmatrix}$$

per unit vol.

$\dot{\psi} = \rho \vec{g}$

energy

$\psi = \rho \left(u + \frac{V^2}{2}\right)$

$J = q + p\vec{V} + \tau \vec{V}$

$\dot{\psi} = \rho \vec{V} \vec{g} + \dot{q}$

$$\partial \rho\left(u + \frac{\vec{V}^2}{2}\right) + \nabla \cdot \left[\rho\left(u + \frac{\vec{V}^2}{2}\right)\vec{V}\right] = -\nabla \cdot \vec{q}$$ heat conduct.

$-\nabla \cdot (p\vec{V})$ work by press.

$-\nabla \cdot (\tau \cdot \vec{V})$ work by shear

$+\rho \vec{V} \cdot \vec{g}$ work by gravity

$+\dot{q}$ internal heat gen

(d) constitutive relations (single phase / laminar)

equations of state (s, u, p, T, ρ)

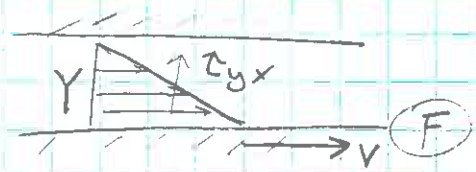
fund $\rightarrow$ $du = Tds - pd\left(\frac{1}{\rho}\right) = C_v dT$ replace w/ 2 thermal caloric

mechanistic relation $(\tau, \vec{q})$

$\tau = 0$ invicid fluid

$\tau_{yx} = -\mu \frac{dV_x}{\partial y}$ $\Longleftarrow$ Newtonian Viscous Fluid

$\frac{F}{A} = \mu \frac{V}{Y}$



$$\tau = -\mu \underbrace{\left[\nabla\vec{v} + (\nabla\vec{v})^T\right]}_{\text{newtonian}} + \frac{2}{3}(\mu - \mu')(\nabla \cdot \vec{v})\overset{\to 0}{\mathbb{I}}$$

compressibility effect

thermal $(\vec{q}, \dot{q})$

$\vec{q} = -k\nabla T$ thermal conduction

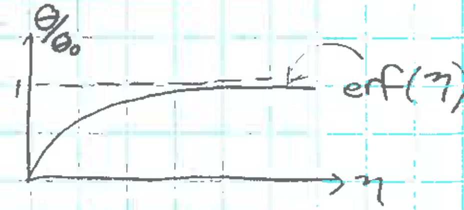
$\dot{q} = \dot{q}(x,t)$ internal heat gen.

Sudden Heating

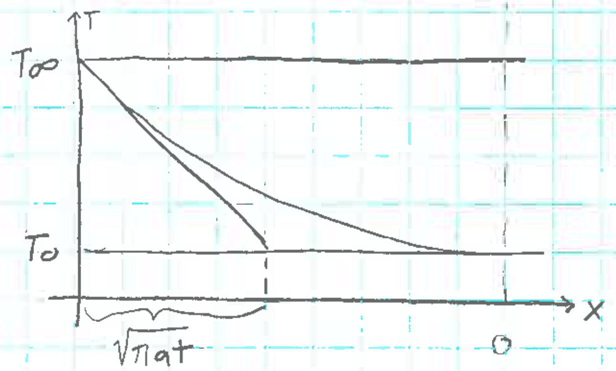
$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

where α is thermal diffusivity

$$\alpha = \frac{k}{\rho C_v}$$



solution $\dfrac{T - T_\infty}{T_0 - T_\infty} = \mathrm{erf}\left(\dfrac{x}{\sqrt{4\alpha t}}\right)$



thermal penetration depth

$\sqrt{\pi \alpha t} = d_T$

distance thermal energy can propagate into a medium

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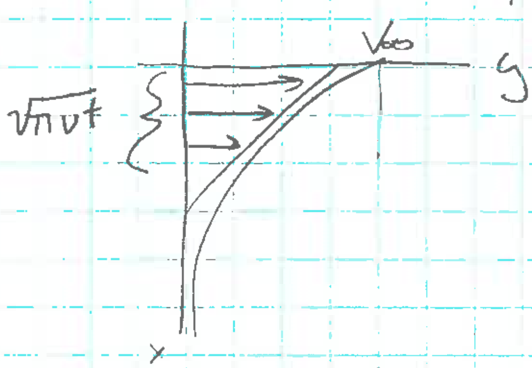
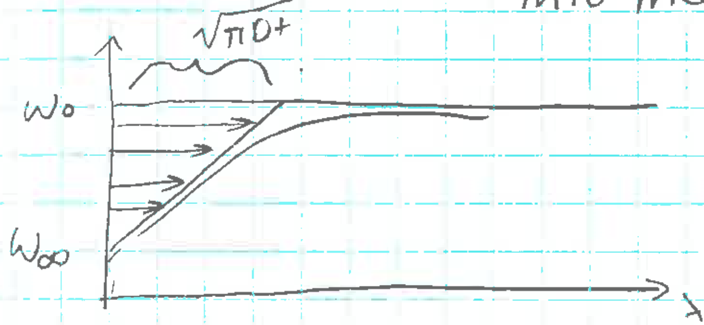
momentum penetration depth

$d_m = \sqrt{\pi \nu t}$ distance momentum / kinetic energy propagates into a medium



diffusion length

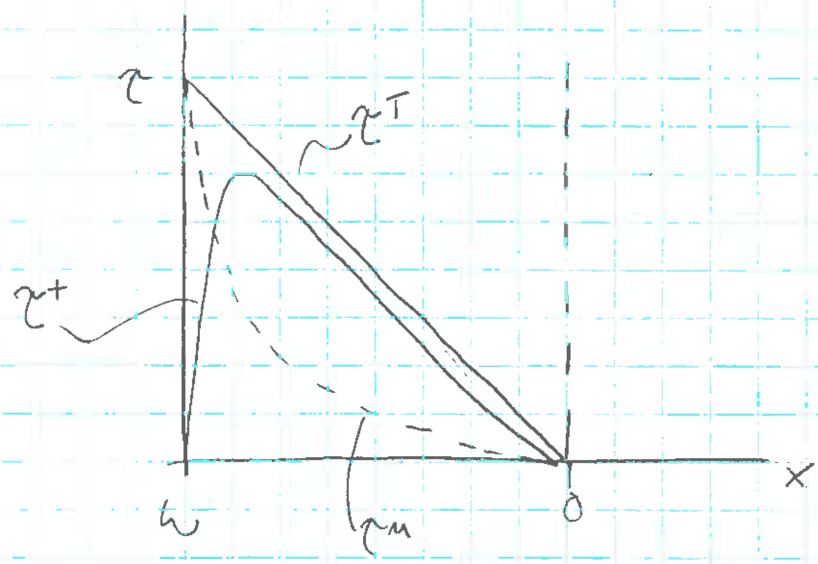
$d_L = \sqrt{\pi D t}$ distance diffusing material propagates into medium

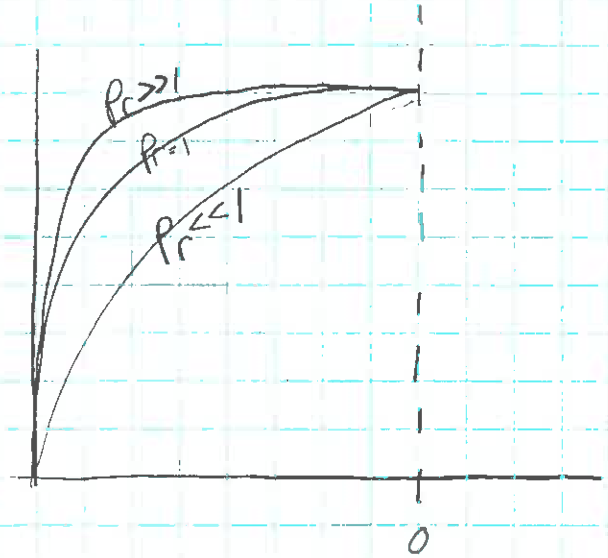


Reynold's (turbulent) stress

$\tau^t = \rho \overline{V'V'}$

substitute $\vec{V} = \overline{V} + v'$ into Navier-Stokes eq.

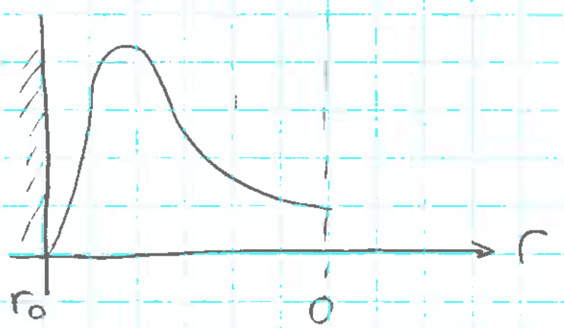




Pr >> 1 mean velocity diffision dominates
Pr << 1 means thermal diffusion dominates
↳ more laminar

Turbulent Intensity

$$I = \frac{\sqrt{\overline{v_z'^2}}}{\overline{v}_{z\,max}}$$



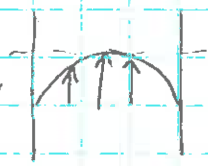
intensity max ~ $\frac{r}{r_0} = 0.9$ (near wall)

zero at wall

NOT zero at center

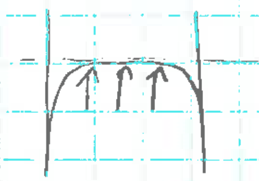
laminar flow in a pipe

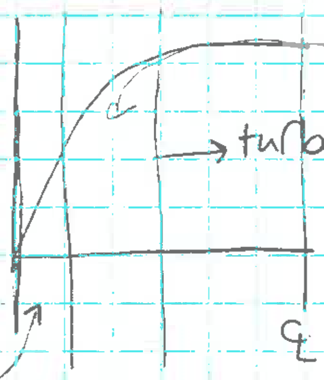
$V = V_{max}\left[1-\left(\frac{r}{r_0}\right)^2\right]$ ← velocity profile $Re < 2000$



turbulent flow in a pipe $Re \geq 2000$

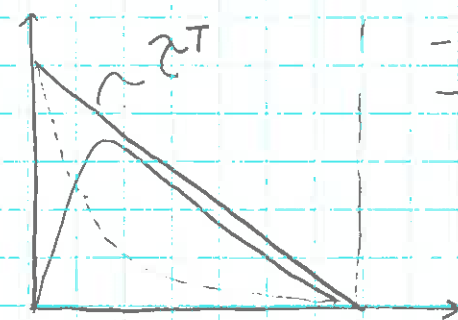
$V = V_{max}\left(1-\frac{r}{r_0}\right)^{1/7}$





total stress in flow

$\tau_{rz}^T = \tau_{rz}^\mu + \tau_{rz}^t$



Origin of turbulence

wall ← basis for mixing length model

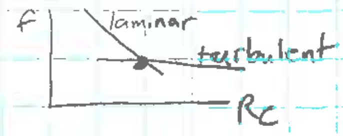
math → instability of the momentum eq for large Re
(Navier-Stokes)

turbulent stress (Reynold's Stress)

$\tau^{t} = \rho \overline{v'v'}$

Scaling

Reynolds Number (Re)



$$Re = \frac{\rho_o V D}{\mu} \qquad \frac{\text{inertia force/vol}}{\text{viscous force/vol}}$$

laminar or turbulent
<2000 , ≥ 2000

Froude Number (Fr)

$$Fr = \frac{V^2}{gD} = \frac{\text{inertia}}{\text{gravity}}$$

Eckert Number (Ec)

$$Ec = \frac{V^2}{C_p \Delta T} = \frac{\text{inertia Force}}{\text{thermal driving Force}} \left(\frac{\text{kE convect.}}{\text{enthalpy convect.}}\right)$$

Mach Number (M)

$$M = \frac{V}{V_o} \qquad \frac{\text{speed}}{\text{speed of sound}}$$

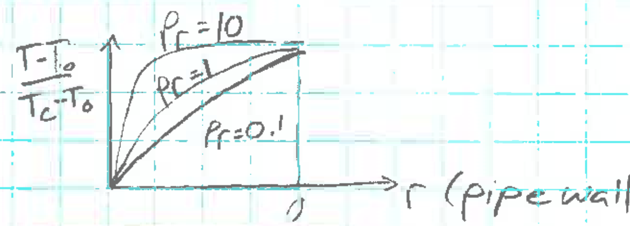
Peclet Number (Pe)

$$Pe = \frac{\rho_o C_p D V}{k} - \frac{\text{heat by convection}}{\text{heat by conduction}}$$

$$= Pr\,Re$$

Prandlt Number (Pr)

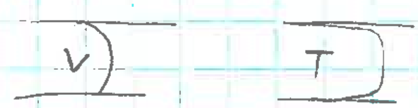
(controls turbulent profile)



$$Pr = \frac{\nu}{\alpha} = \frac{\text{kinematic viscosity}}{\text{thermal diffusivity}}$$

$\alpha = \frac{k}{\rho c_p}$

$Pr >> 1 \longrightarrow$ larger momentum diffusion



$Pr << 1 \longrightarrow$ larger thermal diffusion

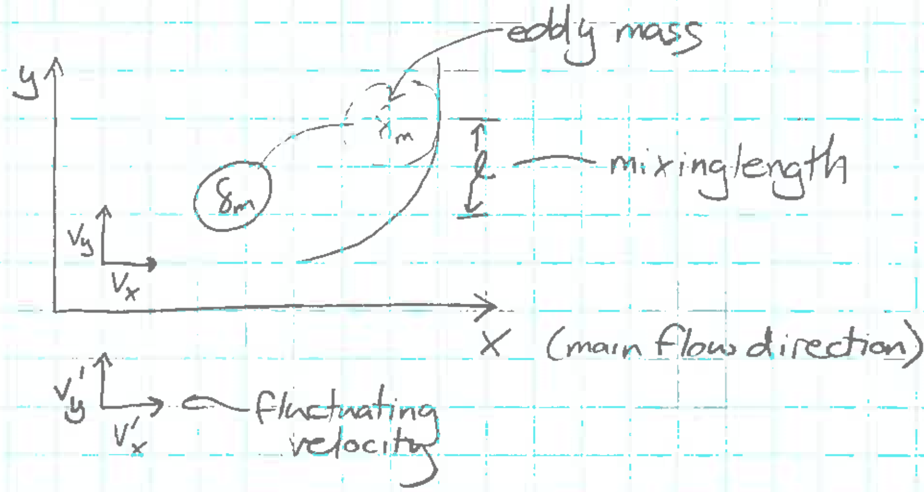
Grashof Number (Gr)

$$Gr = \frac{g \beta \Delta T D^3}{\nu^2} = \frac{(\text{bouyancy force})(\text{inertia})}{(\text{viscous force})^2}$$

	Forced Circ.	Natural Circ
C.E.	none	none
M.E.	Re, Fr (for free surface)	Re
E.E.	Re, Pr	Gr, Pr

Prandlt Mixing Length

model
momentum
transfer



(1) δ_m does not lose or gain any x comp. momentum until it moves ℓ (y-direct)
the complete momentum exchange

(2) x-component momentum gained by δ_m after moving ℓ is $\delta_m \delta V_x$

---

velocity profile near wall with τ_w → wall shear

momentum transfer $\frac{\delta_m \delta V_x}{\delta t}$ by turbulence

shear $\frac{F}{A} = \frac{1}{A}\frac{\delta_m \delta V_x}{\delta t}$

$\delta V_x \approx \frac{d\bar{V}_x}{dy}\ell$ (for small ℓ) velocity transfer by turbulence

$\frac{1}{A}\frac{\delta m}{\delta t} = \rho |V_y'|$ mass transfer by turbulence

$\frac{\tau^t}{\rho} = -|V_y'| \frac{d\bar{V}_x}{dy}\ell$, $\frac{\tau^m}{\rho} = -\nu \frac{dV_x}{dy}$

eddy diffusivity $\varepsilon_m = |V_y'|\ell$

$\frac{\tau^T}{\rho} = -(\varepsilon_m + \nu)\frac{d\bar{V}_x}{dy}$

Prandlt's Assumption

$$\left.\begin{aligned} |V_y'| &= k_1 V_x' \\ V_x' &= k_2 \delta V_x = k_2 \frac{d\bar{V}_x}{dy} l \end{aligned}\right\} \quad |V_y'| = k_1 k_2 \frac{d\bar{V}_x}{dy} l$$

$$\frac{\tau^+}{\rho} = l^2 \left|\frac{d\bar{V}_x}{dy}\right| \frac{d\bar{V}_x}{dy}$$

$l = ky$ ← distance from wall

$$\frac{\tau^+}{\rho} = k^2 y^2 \left|\frac{d\bar{V}_x}{dy}\right| \frac{d\bar{V}_x}{dy}$$

$\tau^+ \rightarrow \tau_w$ near wall

$$\frac{d\bar{V}_x}{dy} = \sqrt{\frac{\tau_w}{\rho}} \frac{1}{ky} \implies \bar{V}_x = \frac{1}{k}\sqrt{\frac{\tau_w}{\rho}} \ln|y|$$

---

thermal diffusivity

energy transfer by turbulence $\frac{\delta_m C \delta T}{\delta t}$

$$q'' = \frac{Q}{A} = \frac{1}{A}\frac{\delta_m C \delta T}{\delta t}$$

$$\delta T = l \frac{d\bar{T}}{dy}$$

$$q'' = \rho |V_y'| C l \frac{d\bar{T}}{dy}$$

$\boxed{\varepsilon_H = l |V_y'|}$ due to Pr mixing length

1-D Balance Eq.

- mass balance (continuity equation)

$$\underbrace{\frac{\partial \rho}{\partial t}}_{\text{time rate of change in density (mass)}} + \underbrace{\nabla \cdot \rho \vec{V}}_{\text{convective mass transfer}} = 0$$

avg over area $\frac{1}{A}\int dA$ to get 1-D formulation

$$\frac{\partial \langle \rho \rangle}{\partial t} + \frac{\partial}{\partial z}\langle \rho V_z \rangle = 0$$

not mass weighted mean

note $\langle\langle V_z \rangle\rangle = \frac{\langle\langle \rho V_z \rangle\rangle}{\langle \rho \rangle}$

if $\rho = \langle \rho \rangle$ ρ is uniform in A

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}\rho \langle V_z \rangle = 0}$$

- energy equation

$$\frac{\partial \rho i}{\partial t} + \nabla \cdot (\rho i \vec{V}) = -\nabla \cdot \vec{q} + \frac{DP}{Dt} - \tau : \nabla \vec{V} + \dot{q}$$

enthalpy

avg over area, again ρ is uniform in A

$$\frac{\partial \rho \langle i \rangle}{\partial t}$$

$\frac{\partial \rho \langle i \rangle}{\partial t}$ +	$\frac{\partial \rho \langle i V_z \rangle}{\partial z}$ =	$-\frac{\partial \langle q_z \rangle}{\partial z}$	axial conduction
time rate of change of enthalpy	energy transfer by convection	$+ \frac{\xi_k q''_w}{A}$	ξ_k heated perimeter $q''_w = h(T_w - T)$ → wall heat flux
		$+ \frac{D\langle P \rangle}{Dt}$	energy increase due to pressure
		$+ \langle \dot{q} \rangle$	heat generation

- momentum equation

$$\frac{\partial \rho \vec{v}}{\partial t} + \nabla \cdot \rho \vec{v}\vec{v} = -\nabla P - \nabla \cdot \underline{\underline{\tau}} + \rho \vec{g}$$

avg over area, again ρ is uniform in A

$$\boxed{\begin{aligned} \frac{\partial \rho \langle v_z \rangle}{\partial t} + \frac{\partial}{\partial z} \rho \langle v_z v_z \rangle = & -\frac{\partial \langle P \rangle}{\partial z} && \text{pressure} \\ & -\frac{\partial \langle \tau_{zz} \rangle}{\partial z} && \text{normal shear} \\ & -\frac{4}{D}\tau_w && \text{wall shear} \quad \left(\frac{f}{2D}\rho v|v|\right) \\ & + \rho g_z && \text{gravity driver} \end{aligned}}$$

1-D Balance Applied to Reactor System

integrate over primary system

- mass balance $\rightarrow 0$ over system
  $\rho_i V_i a_i = \rho_r V_r a_r$
- momentum equation comps.

$$\oint \frac{\partial \rho \langle V_z \rangle}{\partial t} dz = \sum \rho_i \frac{dV_{z,i}}{dt} l_i$$

time rate of change of momentum thru ith component

$$\oint \frac{\partial \rho \langle V_z V_z \rangle}{\partial z} dz = 0$$

convective acceleration

$$\oint -\frac{\partial \langle P \rangle}{\partial z} dz = \Delta P_{pump}$$

delta pressure

$$\oint \frac{f_i \rho V_i |V_i|}{2 D_i} dz = \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{\rho_i V_i |V_i|}{2}\right)$$

$$\oint \frac{\partial \langle \tau_{zz} \rangle}{\partial z} dz = 0$$

$$\oint \rho g_{z,i} dz = \sum (\rho g l - \rho g \beta \Delta T\, l_h)$$

for whole loop

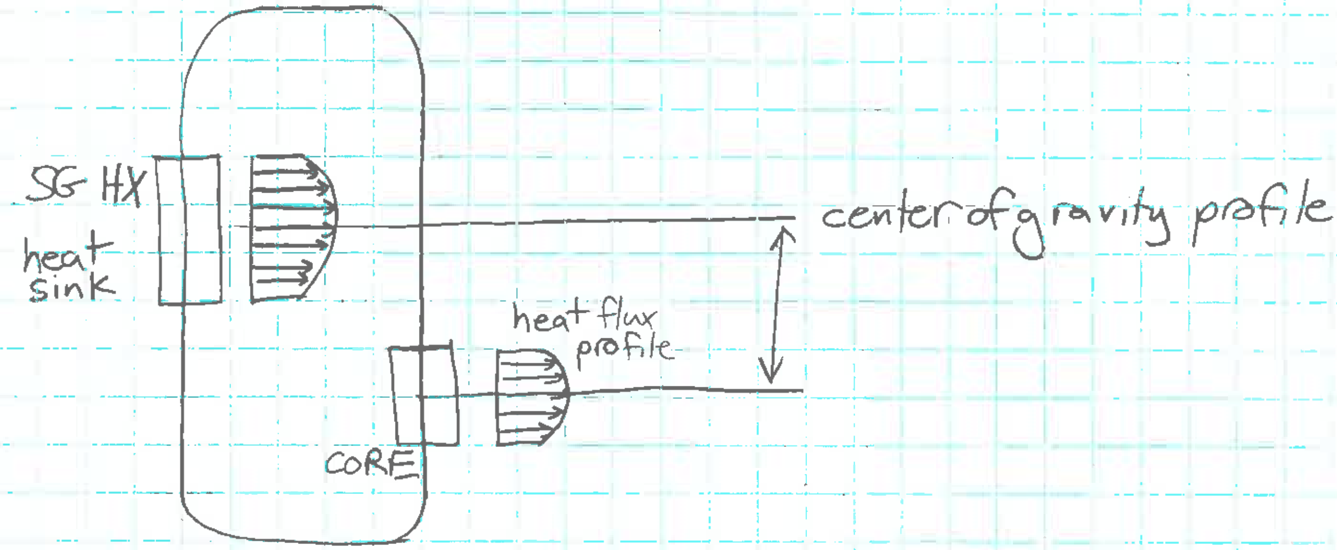
$$= 0 + \rho g \beta \Delta T_h l_h$$

$T_h - T_c$

$$\boxed{\sum \rho_i \frac{dV_i}{dt} l_i = \Delta P_{pump} + \sum (\rho g l - \rho g \beta \Delta T\, l_h)_i - \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{\rho_i V_i^2}{2}\right)}$$

$l_h \rightarrow$ thermal driving head
difference in height of thermal centers

obtained only when heat source is below heat sink

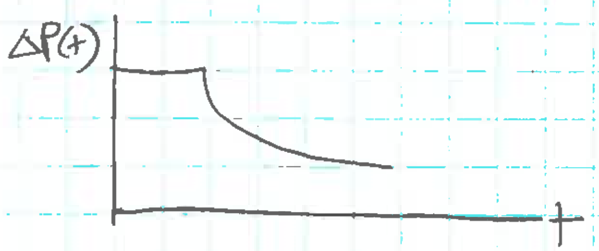


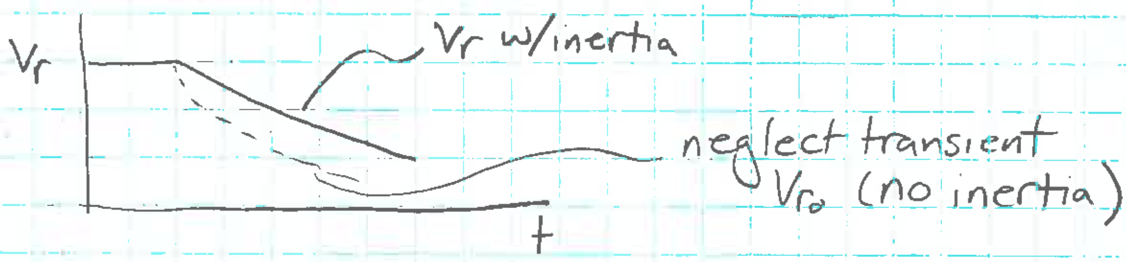
Forced Convection

Steady state
$$\begin{cases} \dfrac{dV_r}{dt} = 0 \\ \Delta P_{pump} = \Delta P_{pump} \text{ const} \\ \text{nat. circ. negligible} \end{cases} \qquad \cancel{\rho g \beta \Delta T_h l_h} \rightarrow 0$$

$$\boxed{\Delta P_{pump} - \frac{\rho_r V_r^2}{2} \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2 = 0}$$

Transient $\quad \Delta P = \Delta P(t) \quad$ pump coast down (loss of flow)





$$\boxed{\rho_r \sum \left(\frac{a_r}{a_i} l_i\right) \frac{dV_r}{dt} = \Delta P(t) - \frac{\rho_r V_r^2}{2} \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2}$$

Natural Convection

$$\Delta P_{pump} = 0 \quad \Longrightarrow \quad \rho g \beta \Delta T_h l_h \neq 0$$

$$\frac{dV_r}{dt} = 0$$

$$\boxed{\rho g \beta \Delta T_h l_h + \frac{\rho V^2}{2} \sum \left(\frac{fl}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2 = 0}$$