Internal Conversion vs. Electron Capture

EC → orbital electron captured by nucleus

has to accelerate, so γ ray emitted

γ ray has energy depending on value of EC

$${}^{A}_{Z}X^{*} \rightarrow {}^{A}_{Z-1}Y + \nu$$ also γ but not included in balance

IC → excited nucleus, transfers energy to orbital electron w/

$$E_{e^-} = E_{ex} - E_b$$

binding energy in shell

practically monoenergetic b.c. shell energy

Decay Situations

given

$^{A}_{Z}X_{N}$ w/ ~~$M'(Z,A)$ is a nucleus mass~~

$M(Z,A)$ is neutral atomic mass

$m_0 \rightarrow$ mass e^-

$931.5 \ \frac{MeV}{amu}$

positron decay

$$^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + \beta^+ + \nu$$

alpha decay

$$^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$$

Beta Decay

$$^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + \beta^- + \bar{\nu}$$

Given

$$\frac{DE}{Dx} = c\left(\frac{Z}{v}\right)^2 \qquad \text{(bethe-bloch)}$$

for a d, p, α

if mass ion increased

→ ~~more~~ less penetrative

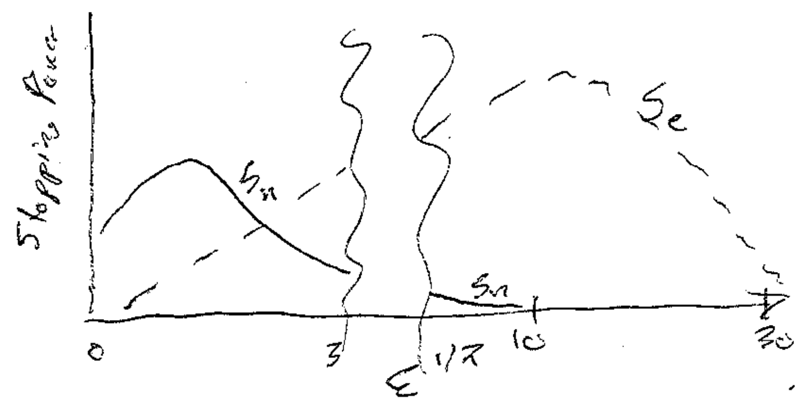
~~→ charge~~

charge increased

→ less penetrative

look at (Bethe-bloch eqn)

$$-\frac{dE}{dx}\Big|_e = \frac{4\pi Z_1^2 e^4 n_e}{m_e v^2} \ln \frac{b_{max}}{b_{min}}$$



$$-\frac{dE}{dx} = \frac{4\pi z^2 e^4}{m_0 v^2} N B$$

$$B \equiv Z\left[\ln \frac{2 m_0 v^2}{I} - \ln\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2}\right]$$

Binding Energy Problem

Calculate binding energy of last neutron in ^{13}C

so ^{13}C is equal to

$$^{12}C + n \rightarrow {}^{13}C$$

use Q value eqn

$$\cancel{Q = \left([BE(^{13}C)] - [BE(^{12}C) + BE(n)] \right) 931.5}$$

$$Q = \left([M(^{12}C) + M(n)] - [M(^{13}C)] \right) 931.5 \text{ MeV}$$

$$Q = (12 + 1.00866 - 13.00335) 931.5 \text{ MeV}$$

$$Q = \underline{4.95 \text{ MeV}}$$

Calculate binding energy per nucleon in ^{13}C

$$\cancel{BE = \frac{13.00335}{13} 931.5 \text{ MeV} =}$$

$$BE_{nucleon} = \frac{[7M_n + 6M_p - M_{C13}] \times 931.5 \text{ MeV}}{13}$$

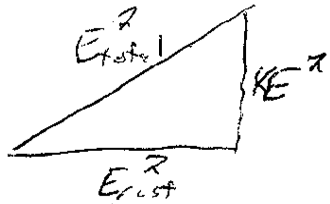
Particle wavelengths (relativistic)

mass bearing

$h = 4.135 \times 10^{-5}$ eV·s

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\sqrt{E_{total}^2 - E_{rest}^2}}$$



$$E_{total} = KE + E_{rest}$$

$$p = \frac{1}{c}\sqrt{T(T + 2mc^2)}$$

zero rest mass

$$\lambda = \frac{hc}{E}$$

Beam of 100 keV γ is compton scattered. $\theta = 30°$ calc E', E_{recoil}, θ_{recoil}, $T_{max, e}$

solve for E'

$$E' = \frac{E E_e}{E(1-\cos\theta) + E_e}$$

solve for E_{recoil}

also rest masses cancel

assume at rest

$$\cancel{E_e} + E \Longrightarrow E_r + E'$$

$$E_r = E' - E$$

$$\cancel{P_e} + P_\gamma = P_{\gamma'} + P_{er}$$

x dir $\rightarrow$ $\frac{E}{c} = \frac{E' \cos(30)}{c} + \cos(\theta)\sqrt{T_r(T_r + 2m_e c^2)}$

y dir $\rightarrow$ $0 = \frac{E'}{c}\sin(30) + \sin(\theta)\sqrt{T_r(T_r + 2m_e c^2)}$

solve for θ

T_{max} occurs when the γ recoils at $\theta = 180°$, so $\theta_e = 0°$ $\longrightarrow$

$$E' = \frac{E E_e}{E(1-\cos\theta) + E_e}$$

$$\cancel{E_e} + E = E_r + E'$$

$$E_r = E' - E$$

Consider

$${}^{9}_{4}Be(p, \alpha) \quad rxn$$

T_p, T_α given

α emerges at angle of $90°$. find T_R

Energy is conserved

$$\cancel{T_{Be}}^{0} + m_{Be}c^2 + T_p + m_p c^2 = T_R + m_R c^2 + T_\alpha + m_\alpha c^2$$

$$T_R = T_\alpha - T_p + m_R c^2 + m_\alpha c^2 - m_p c^2 - m_{Be} c^2$$

momentum is conserved

$$p_p = p_R + p_\alpha$$

$$\frac{1}{c}\sqrt{T_\alpha(T_\alpha + 2m_\alpha c^2)} = \frac{1}{c}\sqrt{T_R(T_R + 2m_R c^2)} + \frac{1}{c}\sqrt{T_\alpha(T_\alpha + 2m_\alpha c^2)}$$

$$\sqrt{T_\alpha(T_\alpha + 2m_\alpha c^2)} = \sqrt{T_R(T_R + 2m_R c^2)} + \sqrt{T_\alpha(T_\alpha + 2m_\alpha c^2)}$$

solve

$\Delta t = 24$ hr

$N_0 = 1683$ counts/10 min

$N_1 = 1683$ counts/20 min

$N_b = 51$ cpm

$$\frac{N_0}{N_1} = \exp(-\lambda \Delta t)$$

$$\lambda = \frac{1}{\Delta t} \ln\left(\frac{N_1}{N_0}\right)$$

$$t_{1/2} = +\ln(2)\, \Delta t \cancel{\ln\left(\frac{N_1}{N_0}\right)} \frac{1}{\ln(N_1) - \ln(N_0)}$$

$$\sigma_{t_{1/2}} = \sqrt{\cancel{\left(\frac{\partial t_{1/2}}{\partial \Delta t}\right)^2 \sigma_{\Delta t}^2}^{\,0} + \left(\frac{\partial t_{1/2}}{\partial N_1}\right)^2 \sigma_{N_1}^2 + \left(\frac{\partial t_{1/2}}{\partial N_0}\right)^2 \sigma_{N_0}^2}$$

careful w/ derivatives

Standard source problem

given efficiency ϵ_{ip}, counts N (full energy)

isotropic, no attenuation

so detector gets $S = N \frac{4\pi}{\epsilon_{ip}\Omega}$

where

then omega is

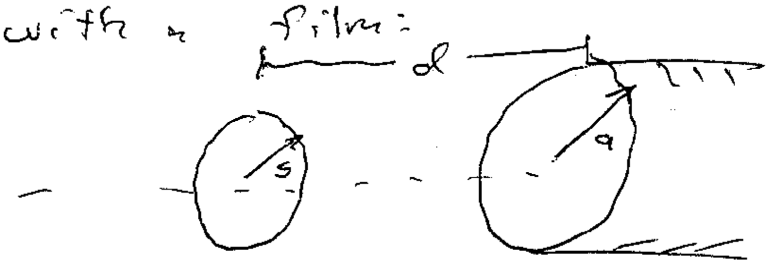
angle between source dir and normal of det.

$$\Omega = \int_A \frac{\cos\alpha}{r^2} dA$$

for geo above $\Omega = 2\pi\left(1 - \frac{d}{\sqrt{d^2+a^2}}\right)$

if $d \gg a \Rightarrow \Omega = \frac{A}{d^2} = \frac{\pi a^2}{d^2}$

Or with a film:



$$\Omega = \frac{4\pi a}{s}\int_0^\infty \frac{\exp(-dk) J_1(sk) J_1(ak)}{k} dk$$

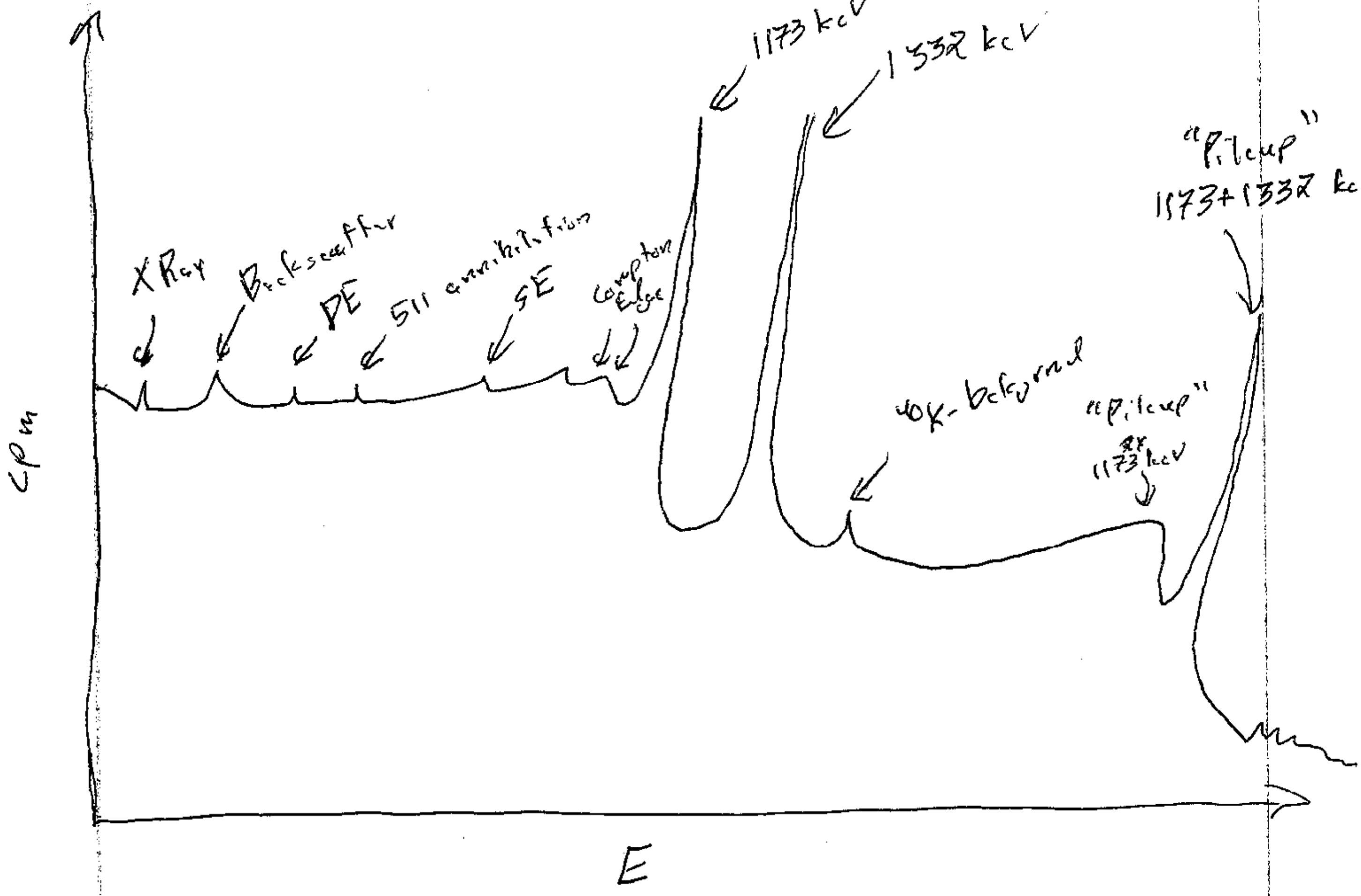
$$\Omega \simeq 2\pi\left\{1 - \frac{1}{(1+\beta)^{1/2}} - \frac{3}{8}\frac{\alpha\beta}{(1+\beta)^{5/2}} + \alpha^2[F1] - \alpha^3[F2]\right.$$

$$F1 = \frac{5}{16}\frac{\beta}{(1+\beta)^{7/2}} - \frac{35}{64}\frac{\beta^2}{(1+\beta)^{9/2}}$$

$$F2 = \frac{35}{128}\frac{\beta}{(1+\beta)^{9/2}} - \frac{315}{256}\frac{\beta^2}{(1+\beta)^{11/2}} + \frac{1155}{1024}\frac{\beta^3}{(1+\beta)^{13/2}}$$

$$\alpha = \left(\frac{s}{d}\right)^2 \quad \beta = \left(\frac{a}{d}\right)^2$$

Peaks in a Pulse height spectrum



Compton Continuum → backscatter and C-Edge

Double Escape → Pair Production both gammas escape (1.022 MeV less than peak)

single Escape → Pair Production, only one gamma escapes (.511 MeV less than peak)

annihilation → in β^+ decay, always annihilates and makes two 0.511 MeV photons.

Backscatter → 0.2 – 0.25 MeV, anything above $\theta=110°$ is essentially same energy so we get peak.

X-Ray → electrons falling shells

5) Radioactive Decay, decay rate, activity

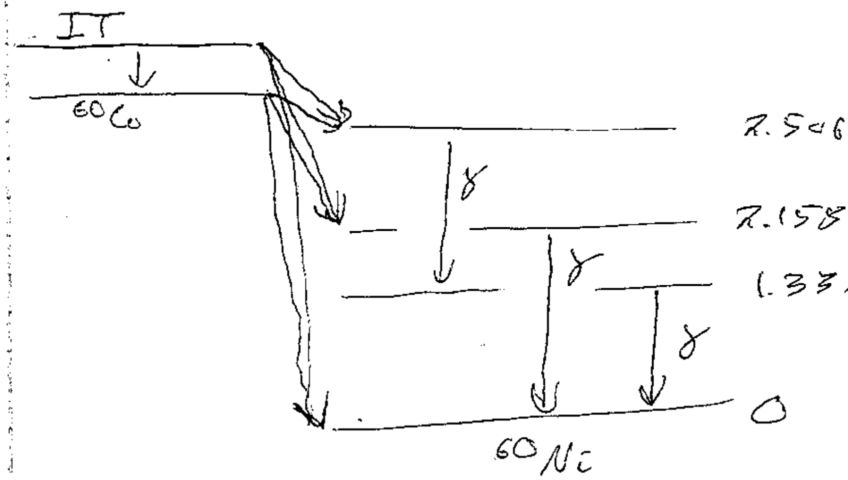
Reference - Lamarsh (5, p. 18-26)

n poor — $Z>N$ — β^+ decay — ${}^{A}X \xrightarrow{\beta^+} {}^{A}X{-1} + \nu$

n rich — $Z<N$ — β^- decay — ${}^{A}X \xrightarrow{\beta^-} {}^{Z}X{+1} + \bar{\nu}$

competes with electron capture

n rich - heavy, very unstable — α decay ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}X{-2} + \alpha$



Decay calcs

$\lambda = \frac{\ln(2)}{T_{1/2}}$ $\qquad N(t)=N(0)e^{-\lambda t}$

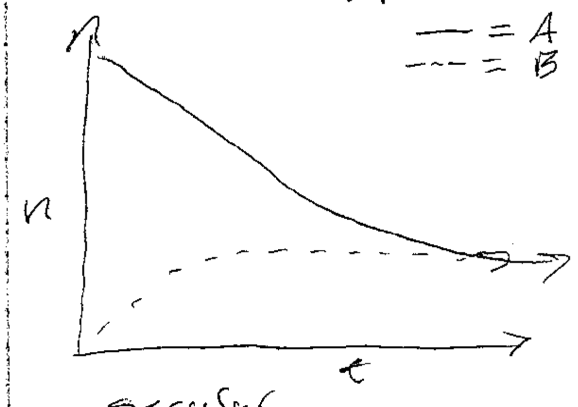
single decay: $\alpha(t)=\alpha_0 e^{-\lambda t}$

~~Binary system~~ Single decay with production: $dn/dt = -\lambda n + R$

$n = n_0 e^{-\lambda t} + \frac{R}{\lambda}(1-e^{-\lambda t})$

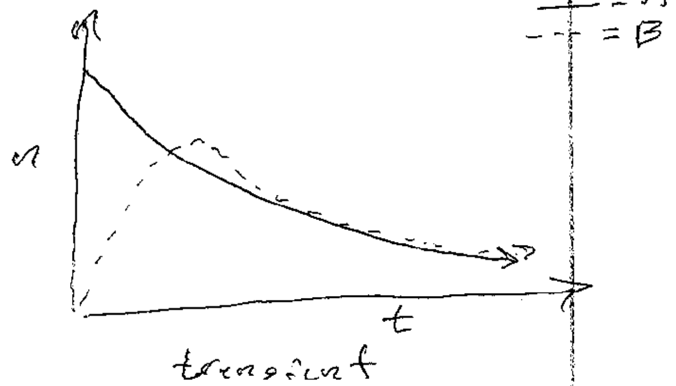
Binary System: $\frac{dn_B}{dt} = -\lambda_B n_B + \lambda_A n_A(t)$

$t^* = \frac{T_p T_d}{\ln(2)(T_p - T_d)} \ln(T_p/T_D)$ $\qquad n_B = n_{B0} e^{-\lambda_B t} + \frac{n_{A0}\lambda_A}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t})$



secular

$\lambda_B > \lambda_A$



transient

$\lambda_B < \lambda_A$

3) Binding Energy for atoms and nuclei, mass defect

Reference - Lamarsh (I, p. 11)

mass → energy by $E_{rest} = m_0 c^2$

then remember that KE is difference between total and rest mass energy

$$E = mc^2 - m_0 c^2 = m_0 c^2 \left[\frac{1}{\sqrt{1-\beta^2}} - 1\right]$$

leads to $\frac{1}{2}mv^2$ for $v \ll c$

Binding Energy:
remember

$$Q = \left[(M_a + M_b) - (M_c + M_d)\right] \underbrace{931\ MeV/amu}_{c^2}$$

so the mass defect is difference between constituents and the actual atom

$$\Delta = ZM_p + NM_n - M_A$$

(Z, N are reactants, atom is product)

so binding energy

$$BE(a) = Z_a M(^1H) + N_a(M_n) - M_a$$

in MeV

LAST neutron is the least bound neutron

$$E_s = \left[M_n + M(^{A-1}Z) - M(^{A}Z)\right] 931\ MeV/amu$$

nuclei

Possible problems:

- Calculate binding energy of the last neutron in ^{13}C. (Lamarsh, p. 33)

2) Nuclear structure and Nuclear Radii Estimate

Reference - Lamarsh (I), p 11, 33-36

$$R = 1.25 \text{ fm} \times A^{1/3}$$

$\Rightarrow$ indicates uniform density

for all atoms w/ several shells, atomic radius is close to 2×10^{-10} m

Reference - Lamber (I) p. 60

$r = 1.2 \times 10^{-15} \times A^{1/3}$

Nuclear Models

Shell - Pauli exclusion principle
there are $2j+1$ possible states for substates w/ total momentum j.
- orbits. Helps solve binding energy problems
- magic numbers correspond to closed shells in both ~~M~~ n and p

Liquid Drop

$$M = NM_n + ZM_p - \alpha A + \beta A^{2/3} + \gamma Z^2/A^{1/3} + \xi (A-2Z)/A + \delta$$

δ: pairing term the bond between n+n or p+p is stronger than n+p

ξ: symmetry term, stronger for Z and N the same

γ: coloumbic term $\rightarrow$ from potential energy

β: surface tension ($R^2 \propto A^{2/3}$)

α: total volumetric correction, total energy of one bond

Possible problems:

probably qualitative or a small part of other problems