NUCL 510
Nuclear Reactor Theory

Fall 2011
Lecture Note 10

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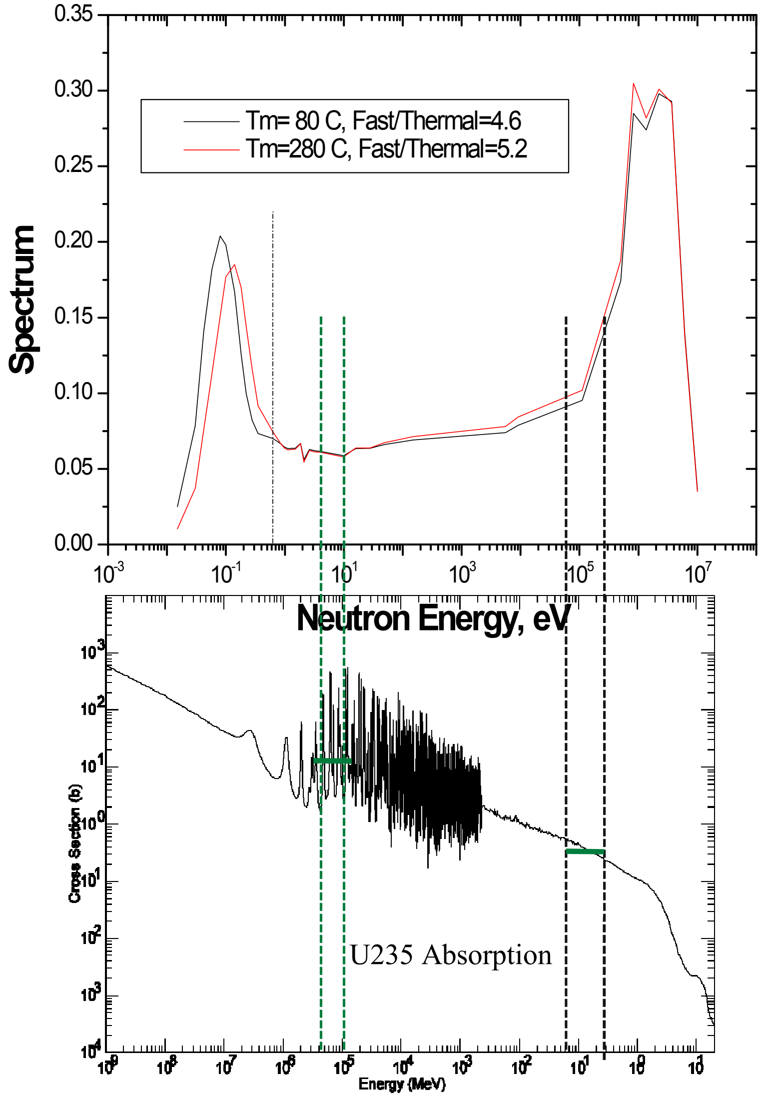
Group Constants in Resonance Region

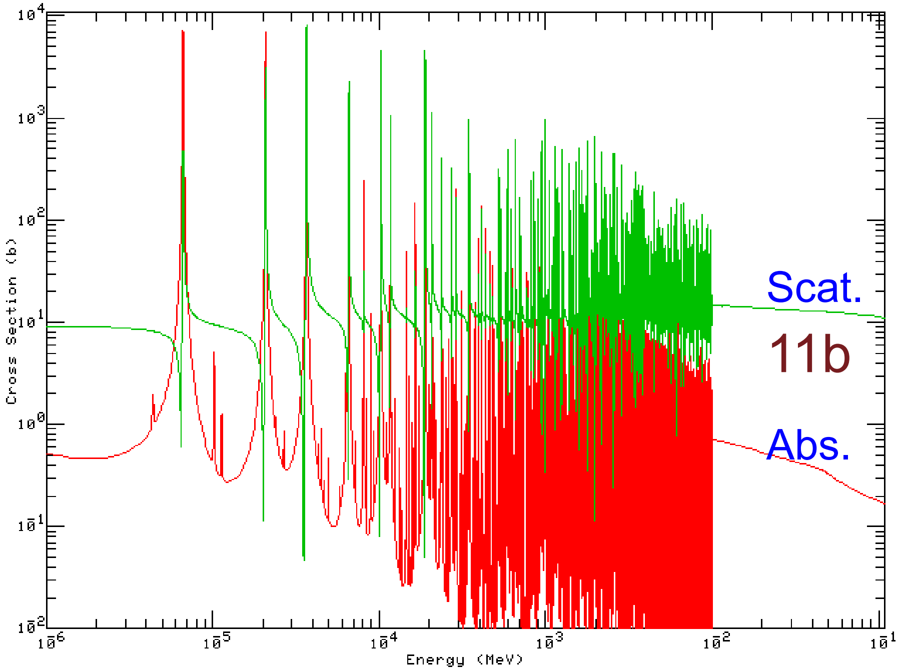
- Group constants
  - Average cross sections; Diffusion constants; Resonance integrals; Fission spectrum

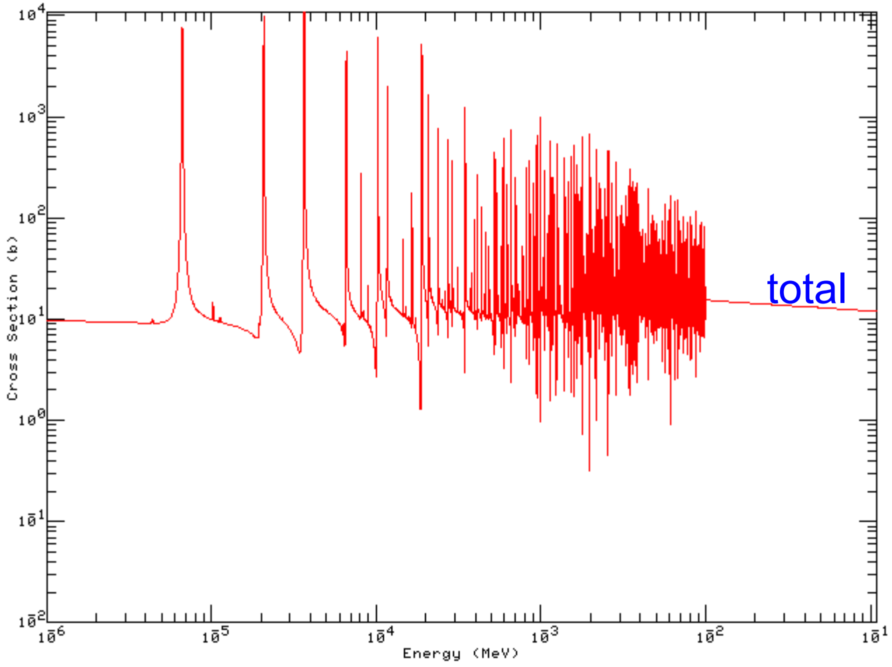
$$\sigma_{xg} = \int_{E_g}^{E_{g-1}} \sigma(E)\phi(E)dE \Big/ \int_{E_g}^{E_{g-1}} \phi(E)dE$$

- Smooth cross sections
  - Fine group cross sections can be obtained by ultrafine group slowing down calculations for typical composition as a function of composition
  - In fine group level, the change in within group spectrum due to deviation from reference condition is not large
- Resonance cross sections
  - Numerous resonances within a fine group and self-shielding changes depending on composition
  - Need to be reevaluated for given composition and temperature

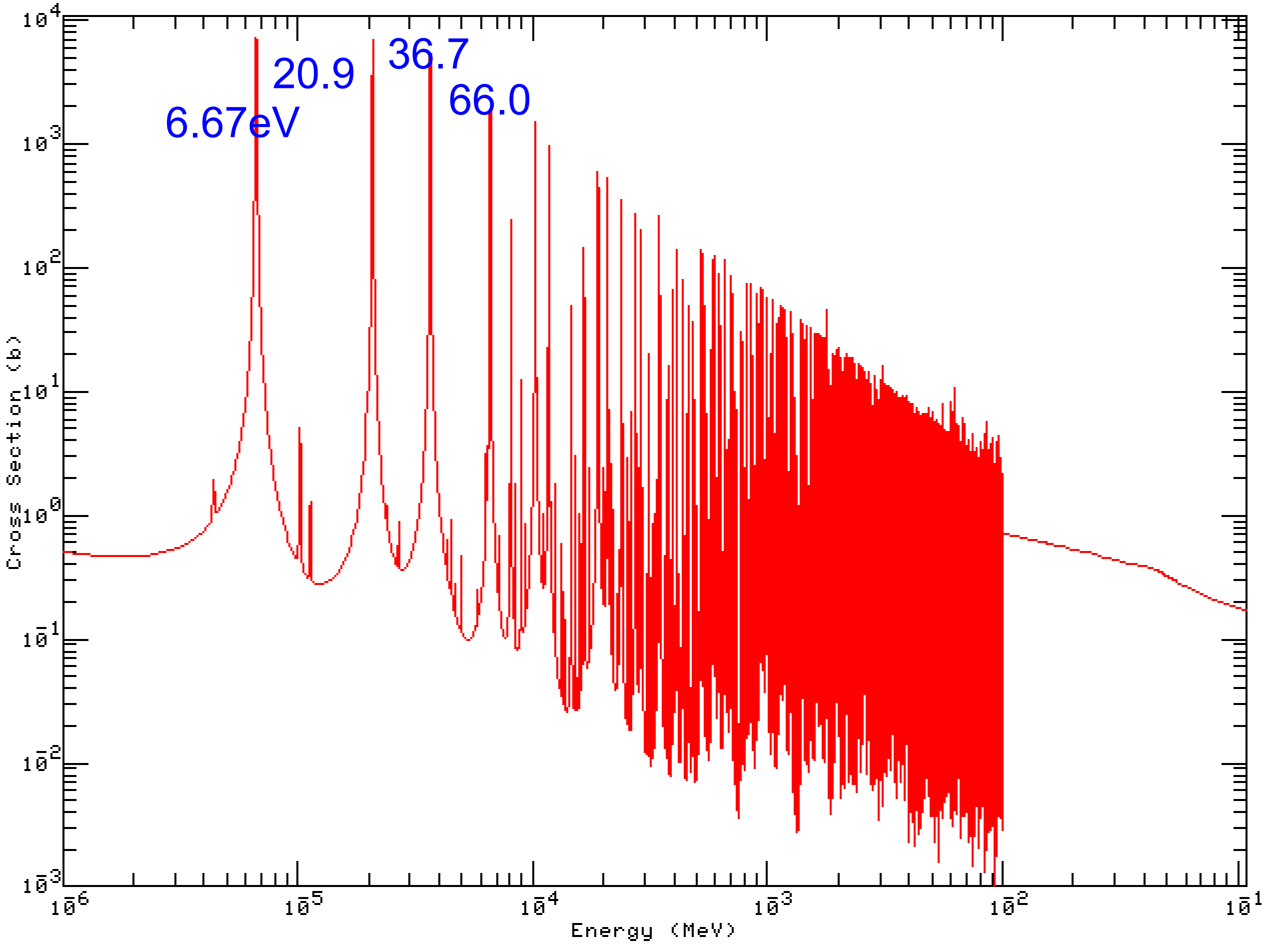


Resonance Behavior of U-238





Resonance Absorption of U-238



Slowing Down in Hydrogen with Absorption (1)

- Homogeneous mixture of resonance absorber and hydrogen moderator

$$\Sigma_t(E) = \Sigma_a^R(E) + \Sigma_s^R(E) + \Sigma_p, \quad \Sigma_p = N_R \sigma_p^R + N_M \sigma_p^M$$

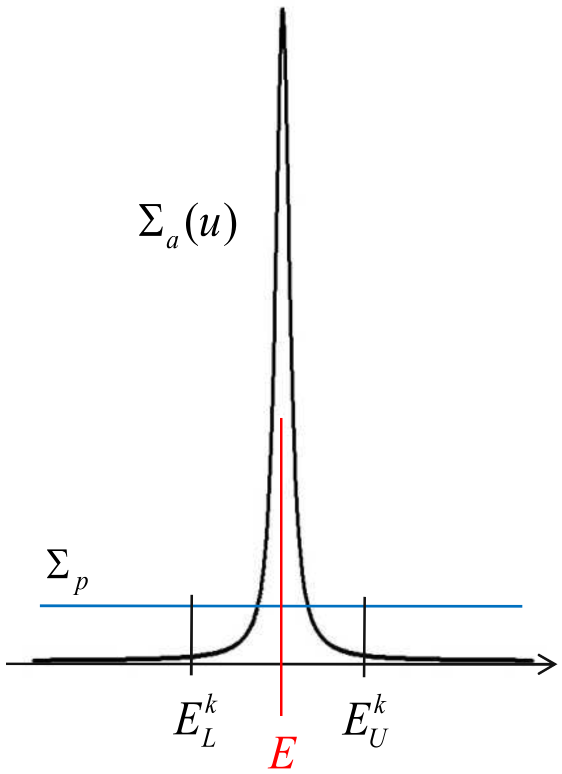
- Balance equation below fission source region

$$\Sigma_t(E)\varphi(E) = \int_E^\infty \frac{\Sigma_s(E)\varphi(E')}{E'} dE'$$

– Differentiate

$$\frac{d}{dE}\Sigma_t(E)\varphi(E) = -\frac{\Sigma_s \varphi(E)}{E} \quad = -\frac{(\Sigma_t(E) - \Sigma_a(E))\varphi(E)}{E}$$

$$= -\left(1 - \frac{\Sigma_a(E)}{\Sigma_t(E)}\right)\frac{\Sigma_t(E)\varphi(E)}{E}$$



$$F(E) = \Sigma_t(E)\phi(E) \quad \text{(collision density)}$$

$$E\frac{d}{dE}F(E) + F(E) - a(E)F(E) = 0 \quad \Rightarrow \quad \frac{d}{dE}[EF(E)] - \frac{a(E)}{E}[EF(E)] = 0$$

Slowing Down in Hydrogen with Absorption (2)

- Solve differential equation using the integrating factor

$$\frac{d}{dE}[EF(E)]-\frac{a(E)}{E}[EF(E)]=0$$

$$e^{h(E)}=\exp\left[\int_E^{E_0}\frac{a(E')}{E'}dE'\right]=\exp\left[\int_E^{E_0}\frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'}\right]$$

$$\frac{d}{dE}\left[e^{h(E)}EF(E)\right]=0$$

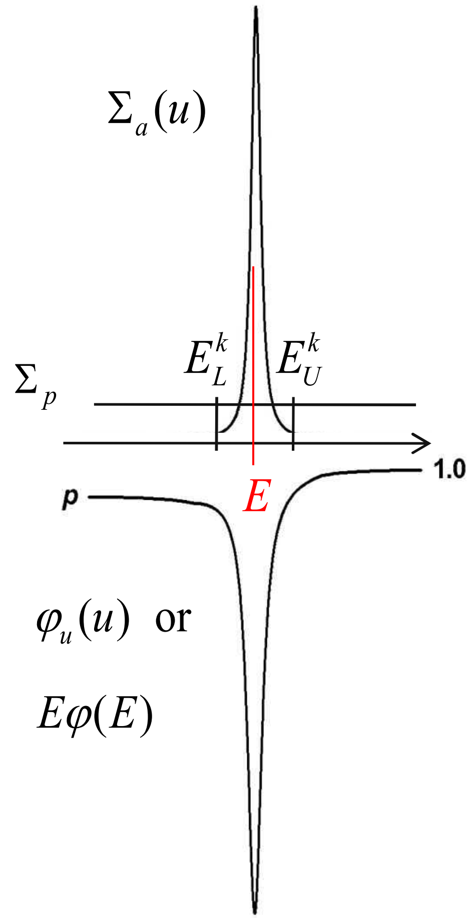
– Integrate from E to E_U

$$e^{h(E)}EF(E)=e^{h(E_u)}E_uF(E_u)=q_{sd}(E_u)$$

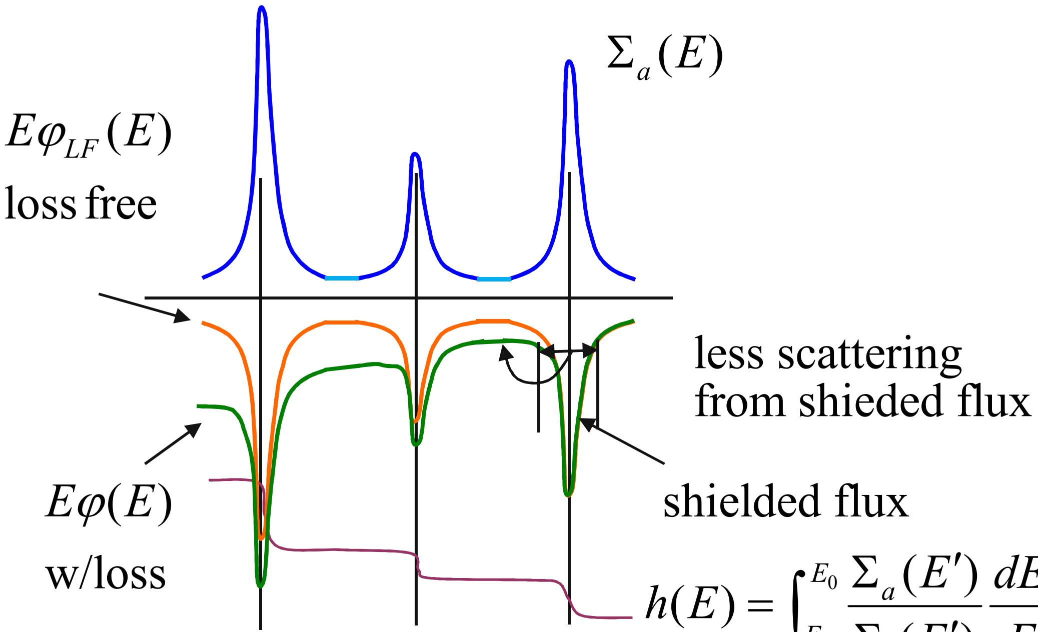
$$E\varphi(E)=\frac{q_{sd}(E_u)e^{-h(E)}}{\Sigma_t(E)}=\frac{q_{sd}(E_u)e^{-h(E)}}{\Sigma_R(E)+\Sigma_p}$$

- Resonance self-shielding
  - Flux depression due to resonance absorption reduces the absorption per fuel nuclide

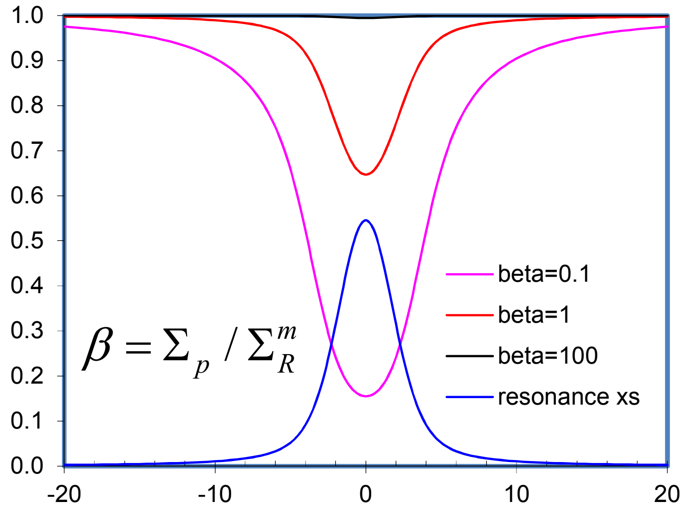
$$\int_{E_L}^{E_U}\frac{\sigma_a(E)}{\Sigma_R(E)+\Sigma_p}\frac{dE}{E}<\int_{E_L}^{E_U}\frac{\sigma_a(E)}{\Sigma_p}\frac{dE}{E}$$



Resonance Self Shielding



$$h(E) = \int_E^{E_0} \frac{\Sigma_a(E')}{\Sigma_t(E')} \frac{dE'}{E'}$$



$$E\varphi(E) = \frac{q_{sd} e^{-h(E)}}{\Sigma_t(E)} \qquad \Sigma_t(E) = \Sigma_R(E) + \Sigma_p = N_F \sigma_R^F(E) + N_F \sigma_p^F + N_M \sigma_s^M$$

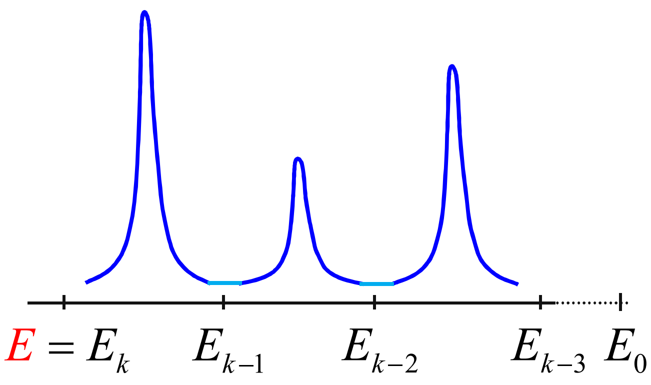
- Degree of self-shielding is determined by relative abundance of fuel to moderator

$$N_F / N_M \downarrow \;\Rightarrow\; \Sigma_R^F(E) / \Sigma_p^M \downarrow \;\Rightarrow\; \text{flux depression} \downarrow$$

$\Rightarrow$ more absorption per fuel nuclide $\quad (N_F / N_M \to 0;$ no depression, infinite dilution)

Resonance Escape Probability

$$h(E)=\int_E^{E_0}\frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'}=\sum_{i=1}^{k}\int_{E_i}^{E_{i-1}}\frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'}$$



- Non-absorption probability

$$p(E)=e^{-h(E)}=\exp\left[-\sum_{i=1}^{k}\int_{E_i}^{E_{i-1}}\frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'}\right]$$

$$=\prod_{i=1}^{k}\exp\left[-\int_{E_i}^{E_{i-1}}\frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'}\right]=\prod_{i=1}^{k}e^{-\pi_i}$$

$$p_i=e^{-\pi_i}\approx 1-\pi_i$$

resonance escape probability for the i-th resonance

- Absorption probability

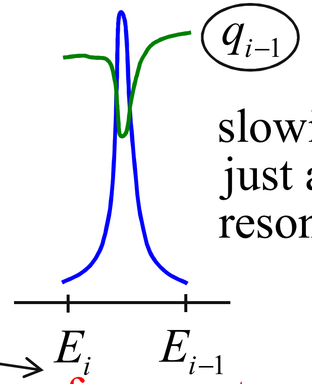
$$\overline{p}(E)=\frac{1}{q_{sd}}\int_E^{E_0}\Sigma_a(E')\varphi(E')dE'=\frac{1}{q_{sd}}\int_E^{E_0}\Sigma_a(E')\frac{q_{sd}e^{-h(E')}}{E'\Sigma_t(E')}dE'=\int_E^{E_0}\frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{e^{-h(E')}}{E'}dE'$$

$$\frac{dh(E)}{dE}=-\frac{\Sigma_a(E)}{E\Sigma_t(E)}\quad\Rightarrow\quad\frac{\Sigma_a(E)}{\Sigma_t(E)}\frac{dE}{E}=-dh(E)\qquad E\to E_0\quad\Rightarrow\quad h(E)\to 0$$

$$\overline{p}(E)=\int_0^{h(E)}e^{-h}dh=1-e^{-h(E)}=1-p(E)$$

Absorption Probability and Resonance Integral

$$h(E) = \int_{E}^{E_0} \frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'} = \sum_{i=1}^{k}\int_{E_i}^{E_{i-1}} \frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'} = \sum_{i=1}^{k}\pi_i$$



slowing down density just above the i-th resonance

$$\pi_i = \int_{E_i}^{E_{i-1}} \frac{\Sigma_a(E')}{\Sigma_t(E')}\frac{dE'}{E'} = \frac{1}{q_{i-1}}\int_{E_i}^{E_{i-1}} \Sigma_a(E')\boxed{\frac{q_{i-1}}{\Sigma_t(E')E'}}dE'$$

Absorption probability by the i-th resonance with loss free spectrum

– Flux outside the resonance

$$E\varphi = \frac{q_{k-1}}{\Sigma_s} = \frac{q_{k-1}}{\Sigma_p}$$

$$\Sigma_t(E) = \underbrace{\Sigma_a^{res}(E) + \Sigma_s^{res}(E)}_{\Sigma_t^{res}(E)} + \underbrace{\Sigma_p^F + \Sigma_p^M}_{\Sigma_p}$$

– Choose q_{i-1} such that $E\varphi = 1$ above the resonance; $q_{i-1} = \Sigma_p E\varphi = \Sigma_p$

$$R_i = \int_{E_i}^{E_{i-1}} \Sigma_a(E')\frac{q_{k-1}}{\Sigma_t(E')E'}dE' \quad \Rightarrow \quad \int_{E_i}^{E_{i-1}} \Sigma_a(E')\frac{\Sigma_p}{\Sigma_t^{res}(E') + \Sigma_p}\frac{dE'}{E'}$$

$$\pi_i = \frac{R_i}{q_{i-1}} \quad \Rightarrow \quad \pi_i = \int_{E_i}^{E_{i-1}} \frac{\Sigma_a(E')}{\Sigma_t^{res}(E') + \Sigma_p}\frac{dE'}{E'}$$

Resonance Integral

$$R_i = \int_{E_i}^{E_{i-1}} \Sigma_a(E') \frac{\Sigma_p}{\Sigma_t^{res}(E') + \Sigma_p} \frac{dE'}{E'} = \int_{E_i}^{E_{i-1}} N_F \sigma_a(E') \frac{\Sigma_p}{N_F \sigma_t^{res}(E') + \Sigma_p} \frac{dE'}{E'}$$

$$= N_F \int_{E_k}^{E_{k-1}} \sigma_a(E') \frac{\Sigma_p / N_F}{\sigma_t^{res}(E') + \Sigma_p / N_F} \frac{dE'}{E'} \quad = N_F \int_{E_k}^{E_{k-1}} \sigma_a(E') \frac{\sigma_b}{\sigma_t^{res}(E') + \sigma_b} \frac{dE'}{E'}$$

$$\sigma_b = \frac{\Sigma_p}{N_F}: \text{ background cross section, } f(N_F, N_M, \sigma_p^F, \sigma_p^M)$$

$$I_i = \int_{E_i}^{E_{i-1}} \sigma_a(E') \frac{\sigma_b}{\sigma_t^{res}(E') + \sigma_b} \frac{dE'}{E'}: \text{ resonance integral of i-th resonance}$$

$$I_i^{\infty} = \int_{E_i}^{E_{i-1}} \sigma_a(E') \frac{dE'}{E'}: \text{ Infinte dilution RI for } \sigma_b = \infty \ (N_F \to 0)$$

$$\pi_i = \frac{R_i}{\Sigma_p} = \frac{N_F}{\Sigma_p} I_i \qquad \Rightarrow \quad e^{-\pi_i}: \text{ resonance escape probability with}$$

loss free absorption probability as the exponent

$$h(E) = \sum_{i=1}^{k(E)} \pi_i, \quad E_{k+1} < E < E_k \quad : \text{total absorption probability with loss free spectrum}$$

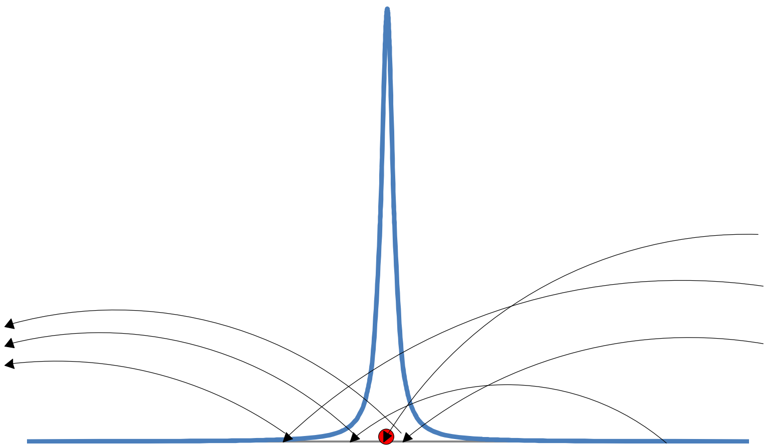
$$p(E) = e^{-h(E)} \quad : \text{resonance escape probability, a function of dilution } (N_M / N_F)$$

$$= \prod_{i=1}^{k(E)} e^{-\pi_i} \qquad I_i = f(\sigma_b): \text{ RI generated as a function of background XS}$$

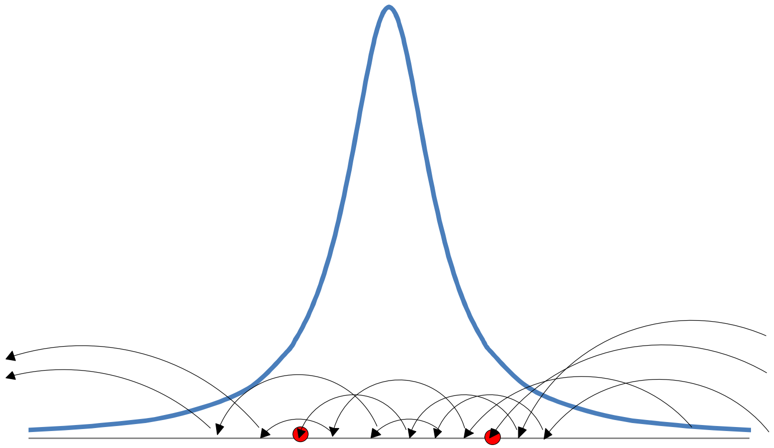
Basic Approximations for Resonance Treatment

- No analytic solution for resonance absorption during slowing down in non-hydrogeneous material

Narrow Resonance



Wide Resonance



- Resonance width is narrow compared with energy loss in scattering on light and heavy nuclei
- Practically all neutrons scattered into the resonance are coming from energies far above that particular resonance
- Scattering inside the resonance brings neutrons out of the resonance
- Valid for resonances at high energy

- Resonance is still narrow compared with scattering energy loss on light nuclei, but wide compared to the energy loss on heavy nuclei
- Neutron resides inside the resonance after scattering
- Regard in-scattering same as out-scattering
- Valid for scattering with heavy nuclides at low energy

Narrow Resonance Approximation

- Assumptions of NR approximation
  - Isolated resonances (well separated between resonances)
  - Resonance is so narrow that the energy density of the scattering source within the resonance is not influenced by the resonance itself

$$\Sigma_t(E) = \Sigma_a^{res}(E) + \Sigma_s^{res}(E) + \Sigma_p^F + \Sigma_p^M; \quad \Sigma_t(E) = \Sigma_p^F + \Sigma_p^M = \Sigma_p \quad \text{(outside resonance)}$$

- Scattering source with flux for no absorption $E\varphi(E) = q_r / \xi\Sigma_p$

$$R_{ss}(E) = \int_E^{E/\alpha_M} \frac{\Sigma_p^M \varphi(E')}{(1-\alpha_M)E'} dE' + \int_E^{E/\alpha_F} \frac{\Sigma_s^F(E')\varphi(E')}{(1-\alpha_F)E'} dE'$$

$$\approx \int_E^{E/\alpha_M} \frac{\Sigma_p^M [q_r / \xi\Sigma_p E']}{(1-\alpha_M)E'} dE' + \int_E^{E/\alpha_F} \frac{\Sigma_p^F [q_r / \xi\Sigma_p E']}{(1-\alpha_F)E'} dE'$$

$$= \frac{q_r \Sigma_p^M}{\xi(1-\alpha_M)\Sigma_p} \int_E^{E/\alpha_M} \frac{dE'}{(E')^2} + \frac{q_r \Sigma_p^M}{\xi(1-\alpha_M)\Sigma_p} \int_E^{E/\alpha_F} \frac{dE'}{(E')^2} = \frac{q_r(\Sigma_p^M + \Sigma_p^M)}{\xi\Sigma_p} = \frac{q_r}{\xi E}$$

- Neutron balance within resonance

$$\Sigma_t(E)\varphi(E) = \int_E^{E/\alpha} \frac{\Sigma_s \varphi(E')}{(1-\alpha)E'} dE' = \frac{q_r}{\xi E} \quad \Rightarrow \quad \varphi(E) = \frac{q_r}{\xi\Sigma_t(E)E}$$

Narrow Resonance Infinite Mass Approximation

- Assumptions of NRIM approximation
  - Resonance is narrow compared to scattering energy loss on light moderator (NR)
  - Scattering energy loss is small compared with the resonance width for fuel nuclides (α=1, IM)

$$\Sigma_t(E) = \Sigma_a^{res}(E) + \Sigma_s^{res}(E) + \Sigma_p^F + \Sigma_p^M; \quad \Sigma_s^F(E) = \Sigma_s^{res}(E) + \Sigma_p^F$$

- Scattering source

$$R_{ss}(E) = \int_E^{E/\alpha_M} \frac{\Sigma_p^M \varphi(E')}{(1-\alpha_M)E'} dE' + \int_E^{E/\alpha_F} \frac{\Sigma_s^F(E')\varphi(E')}{(1-\alpha_F)E'} dE'$$

$$\int_E^{E/\alpha_M} \frac{\Sigma_p^M \varphi(E')}{(1-\alpha_M)E'} dE' = \frac{q_r}{\xi E} \quad \text{(NR approximation)}$$

– L'Hospital's rule

– Leibniz's rule for differentiation under the integral sign

$$\lim_{\alpha_F \to 1} \frac{1}{1-\alpha_F} \int_E^{E/\alpha_F} \Sigma_s^F(E')\varphi(E') \frac{dE'}{E'} = \lim_{\alpha_F \to 1} \left[-\phi\left(\frac{E}{\alpha_F}\right) \Sigma_s^F\left(\frac{E}{\alpha_F}\right) \frac{\alpha_F}{E} \right] \left[-\frac{E}{\alpha_F^2} \right] = \phi(E)\Sigma_s^F(E)$$

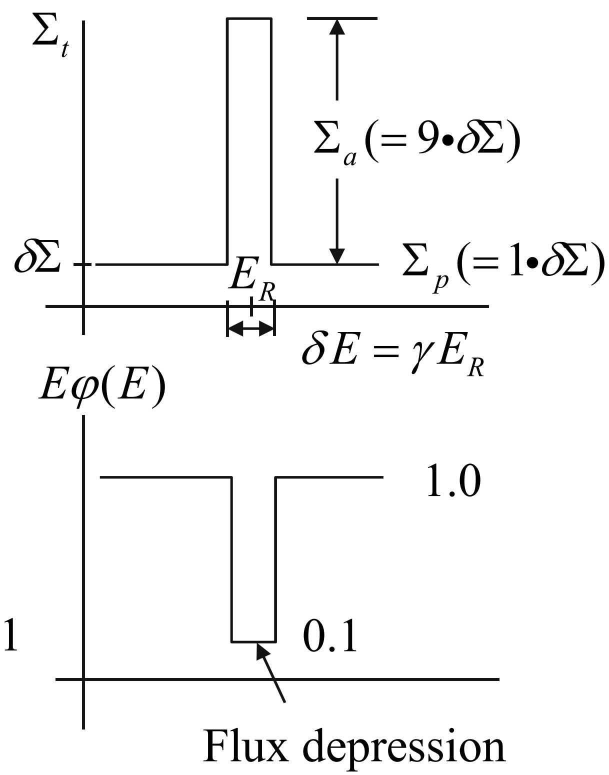
- Neutron balance within resonance

fuel scattering contributions are omitted

$$\Sigma_t(E)\varphi(E) = \frac{q_r}{\xi E} + \Sigma_s^F(E)\varphi(E) \quad \Rightarrow \quad \varphi(E) = \frac{q_r}{\xi E[\Sigma_a^F(E) + \Sigma_p^M]}$$

Shielded Absorption

$$E\varphi(E)=\frac{q_r e^{-h(E)}}{\xi\Sigma_t(E)},\quad \Sigma_t(E)=\Sigma_p+\Sigma_a(E)$$



- Normalization
  - choose q_r such that $E\varphi=1.0$ above resonance

$$E_u\varphi(E_u)=\frac{q_r}{\xi\Sigma_t(E_u)}=\frac{q_r}{\xi\Sigma_p}=1\quad\Rightarrow\quad q_r=\xi\Sigma_p$$

- Flux within resonance

$$E\varphi(E)=\frac{\Sigma_p}{\Sigma_a(E)+\Sigma_p}e^{-h(E)}\quad\Rightarrow\quad E\varphi(E)=\frac{1}{9+1}e^{-h(E)}\approx 0.1$$

- Shielded absorption

$$R_a=\int_{E_R-\frac{\gamma}{2}E_R}^{E_R+\frac{\gamma}{2}E_R}\Sigma_a(E')\varphi(E')dE'=\int_{E_R-\frac{\gamma}{2}E_R}^{E_R+\frac{\gamma}{2}E_R}9\delta\Sigma 0.1\frac{dE'}{E'}=0.9\delta\Sigma\int_{E_R-\frac{\gamma}{2}E_R}^{E_R+\frac{\gamma}{2}E_R}\frac{1}{E}dE \longrightarrow$$

unshielded spectrum

$$\approx 0.9\delta\Sigma\frac{1}{E_R}\int_{E_R-\frac{\gamma}{2}E_R}^{E_R+\frac{\gamma}{2}E_R}dE={\color{red}0.9\delta\Sigma\gamma}\ll 9\delta\Sigma\gamma$$

significantly reduced absorption

Resonance Self-Shielding and Effective XS

- Self-shielded flux

$$E\varphi(E)=\frac{s_0}{\xi\Sigma_t(E)}=\frac{\Sigma_p}{\Sigma_t(E)}=\frac{\Sigma_p}{\Sigma_t^R(E)+\Sigma_p}=\frac{N_R\sigma_p^R+N_M\sigma_p^M}{N_R\sigma_t^R(E)+N_R\sigma_p^R+N_M\sigma_p^M}=\frac{\sigma_b}{\sigma_t^R(E)+\sigma_b}$$

$$\sigma_b=\frac{\Sigma_p}{N_R}=\sigma_p^R+\frac{N_M}{N_R}\sigma_p^M=f\left(\sigma_p^R,\sigma_p^M,\frac{N_M}{N_R}\right)\quad \text{(background cross section)}$$

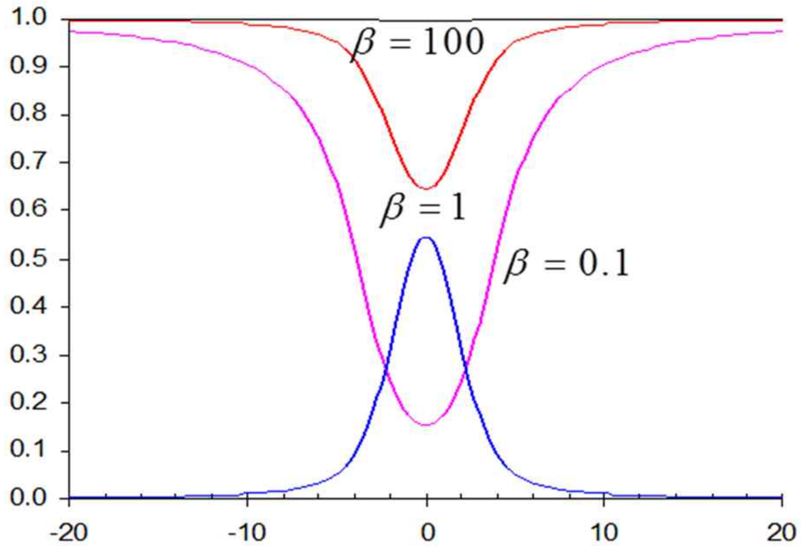
– Unshielded flux with infinite dilution

$$N_R\to 0\quad\Rightarrow\quad\frac{N_M}{N_R}\to\infty\quad\Rightarrow\quad\sigma_b\to\infty\quad\Rightarrow\quad E\varphi(E)=1$$

- Effective cross section

$$\bar{\sigma}_a^R=\frac{\int\sigma_a^R(E)\varphi(E)dE}{\int\varphi(E)dE}=\frac{\int\sigma_a^R(E)\frac{\sigma_b}{\sigma_t^R(E)+\sigma_b}\frac{dE}{E}}{\int\frac{\sigma_b}{\sigma_t^R(E)+\sigma_b}\frac{dE}{E}}$$

$$\bar{\sigma}_a^R=\frac{\int\sigma_a^R(u)\frac{\sigma_b}{\sigma_t^R(u)+\sigma_b}du}{\int\frac{\sigma_b}{\sigma_t^R(u)+\sigma_b}du},\quad\Leftarrow\quad du=-\frac{dE}{E}$$



Resonance Integral

- Reaction rate per nuclide with normalized flux such that $E\varphi(E)$=1 above resonance
  - Provided in a form of 2-dimensional table in XS library

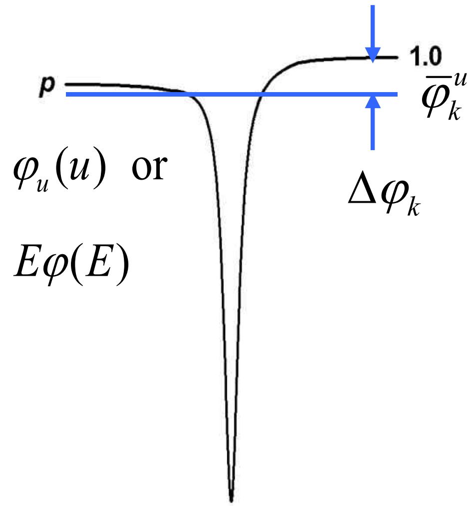
$$I_k = \int_{E_L^k}^{E_U^k} \sigma_a^R(E)\varphi(E)dE = \int_{E_L^k}^{E_U^k} \frac{\sigma_a^R(E)\sigma_b}{\sigma_a^R(E)+\sigma_b}\frac{dE}{E} = \int_{u_L^k}^{u_U^k} \frac{\sigma_a^R(u)\sigma_b}{\sigma_a^R(u)+\sigma_b}du = f(\sigma_b, T)$$

- Effective cross section in terms of RI

$$\bar{\sigma}_a^R = \frac{I_k}{\varphi_k} = \frac{I_k/\Delta u}{\varphi_k/\Delta u} = \frac{\bar{I}_k}{\bar{\varphi}_k^u} = \frac{\bar{I}_k}{1-\Delta\varphi_k}$$

$$\bar{\varphi}_k^u = \frac{1}{\Delta u}\int_{u_L^k}^{u_U^k} \frac{\sigma_b}{\sigma_a^R(u)+\sigma_b}du = \frac{1}{\Delta u}\int_{u_L^k}^{u_U^k}\left(1-\frac{\sigma_a^R(u)}{\sigma_a^R(u)+\sigma_b}\right)du$$

$$= 1-\frac{1}{\sigma_b \Delta u}\int_{u_L^k}^{u_U^k}\frac{\sigma_a^R(u)\sigma_b}{\sigma_a^R(u)+\sigma_b}du = 1-\frac{\bar{I}_k}{\sigma_b} \quad \Rightarrow \quad \Delta\varphi_k = \frac{\bar{I}_k}{\sigma_b}$$



$$\bar{\sigma}_a^R(\sigma_b) = \frac{\bar{I}_k(\sigma_b)}{1-\dfrac{\bar{I}_k(\sigma_b)}{\sigma_b}}\cdots(*)$$

In reality, NRIM does not hold always, then $I_k(\sigma_b)$ is adjusted so that (*) gives the correct average cross section

Intermediate Resonance Approximation

- Scattering source for narrow resonance approximation

$$R_{SS}^{NR}(E) = \sum_i \int_E^{E/\alpha_i} \frac{\Sigma_{si}(E')\varphi(E')}{(1-\alpha_i)E'} dE' \simeq \sum_i \Sigma_{pi} \int_E^{E/\alpha_i} \frac{1}{(1-\alpha_i)E'^2} dE' \simeq \sum_i \Sigma_{pi} \frac{1}{E}$$

$$\varphi(E') = \frac{1}{E'} : \text{Normalized flux above resonance}$$

- Scattering source for wide resonance approximation

$$R_{SS}^{WR}(E) = \sum_i \int_E^{E/\alpha_i} \frac{\Sigma_{si}(E')\varphi(E')}{(1-\alpha_i)E'} dE'$$

$$\approx \sum_i \lim_{\alpha_i \to 1} \frac{1}{(1-\alpha_i)} \int_E^{E/\alpha_i} \frac{\Sigma_{si}(E')\varphi(E')}{E'} dE' \approx \sum_i \Sigma_{si}(E)\varphi(E)$$

- Scattering source for intermediate resonance (IR) approximation

$$R_{SS}(E) = \sum_i \left[\lambda_i \frac{\Sigma_{pi}}{E} + (1-\lambda_i)\Sigma_{si}(E)\varphi(E) \right]$$

$$R_{SS}(u) = \sum_i [\lambda_i \Sigma_{pi} + (1-\lambda_i)\Sigma_{si}(u)\varphi(u)]$$

Slowing Down Equation with IR Source

- Neutron balance equation for a mixture of fuel and moderator

$$\Sigma_t(E)\varphi(E) = \sum_i \left[\lambda_i \frac{\Sigma_{pi}}{E} + (1-\lambda_i)\Sigma_{si}(E)\varphi(E) \right] = \lambda \frac{\Sigma_p}{E} + (1-\lambda_F)\Sigma_s^F(E)\varphi(E) \quad (\lambda_M = 1)$$

$$\Sigma_t(E) = \Sigma_a^F(E) + \Sigma_s^F(E) + \Sigma_p^M; \quad \Sigma_s^F(E) = \Sigma_s^{F,res}(E) + \Sigma_p^F$$

$$\lambda = \frac{\sum_i \lambda_i \Sigma_{pi}}{\Sigma_p} = \frac{\Sigma_p^M + \lambda_F \Sigma_p^F}{\Sigma_p^M + \Sigma_p^F}$$

– Move flux dependent term of RHS to LHS

$$[\Sigma_a^F(E) + \lambda_F \Sigma_s^F(E) + \Sigma_p^M]\varphi(E) = [\Sigma_a^F(E) + \lambda_F \Sigma_s^{res}(E) + \lambda\Sigma_p]\varphi(E) = \frac{\lambda\Sigma_p}{E}$$

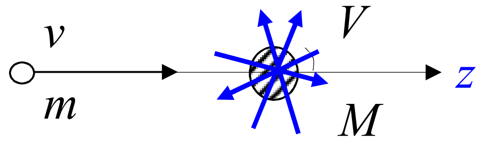
- Flux with IR approximation

$$E\varphi(E) = \frac{\lambda\Sigma_p}{\Sigma_a^F(E) + \lambda_F \Sigma_s^{res}(E) + \lambda\Sigma_p} \Rightarrow E\varphi(E) = \frac{\sigma_b}{\sigma_a^F(E) + \lambda_F \sigma_s^{res}(E) + \sigma_b}; \quad \sigma_b = \frac{\lambda\Sigma_p}{N_F}$$

For $\lambda_F << 1$, $\lambda_F \sigma_s^{res}(u)$ is neglected $\Rightarrow \varphi(u) = \dfrac{\sigma_b}{\sigma_a^F(u) + \sigma_b}$

Doppler Broadening (1)

- Thermal motion of target nuclei affects the observed reaction rate since the cross section is determined by the relative speed



$$v_r = |\vec{v} - \vec{V}| = (v^2 + V^2 - 2vV\mu)^{1/2} \qquad E_r = \frac{1}{2} m v_r^2$$

$$P(V,T)dV = 4\pi \left(\frac{M}{2\pi kT} \right)^{3/2} V^2 e^{-MV^2/2kT} dV \quad \text{(Maxwellian distribution)}$$

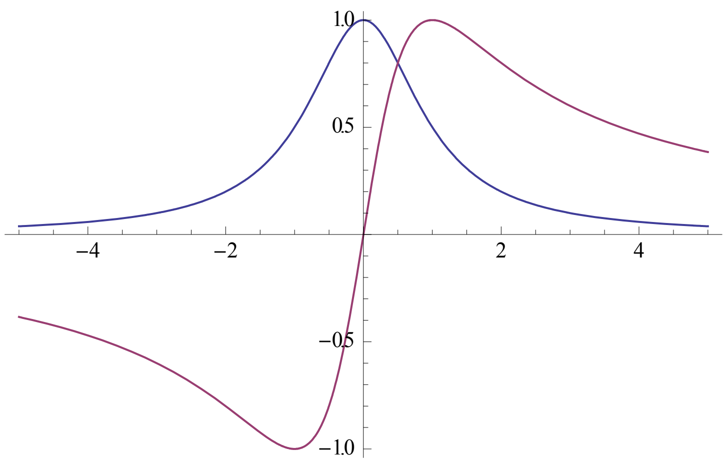
$$\sigma(E,T) = \frac{1}{\Delta\sqrt{\pi E}} \int_{-\infty}^{\infty} [\sqrt{E_r}\,\sigma(E_r)] e^{-[(E_r - E)/\Delta]^2} dE_r \qquad \Delta = \left(\frac{4kTE}{A} \right)^{1/2} \quad \text{(Doppler width)}$$

- Single level Breit-Wigner formula

$$\sigma_a(E_r) \approx \sigma_0 \frac{\Gamma_a}{\Gamma} \frac{1}{1+x^2} \quad (a = \gamma, f)$$

$$\sigma_n(E_r) \approx 4\pi a^2 + \sigma_0 \frac{\Gamma_n}{\Gamma} \frac{1}{1+x^2} + \sigma_0 ka \frac{2x}{1+x^2}$$

$$\sigma_0 = \frac{4\pi}{k^2} g_J \frac{\Gamma_n}{\Gamma}; \quad x = \frac{E_r - E_0}{\Gamma / 2}$$



Doppler Broadening (2)

- Symmetric and anti-symmetric Doppler broadened line shape functions

$$\psi(x,\xi)=\frac{\xi}{2\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{1}{1+w^2}\exp\left[-\frac{\xi^2}{4}(x-w)^2\right]dw$$

$$\chi(x,\xi)=\frac{\xi}{\sqrt{\pi}}\int_{-\infty}^{\infty}\frac{w}{1+w^2}\exp\left[-\frac{\xi^2}{4}(x-w)^2\right]dw$$

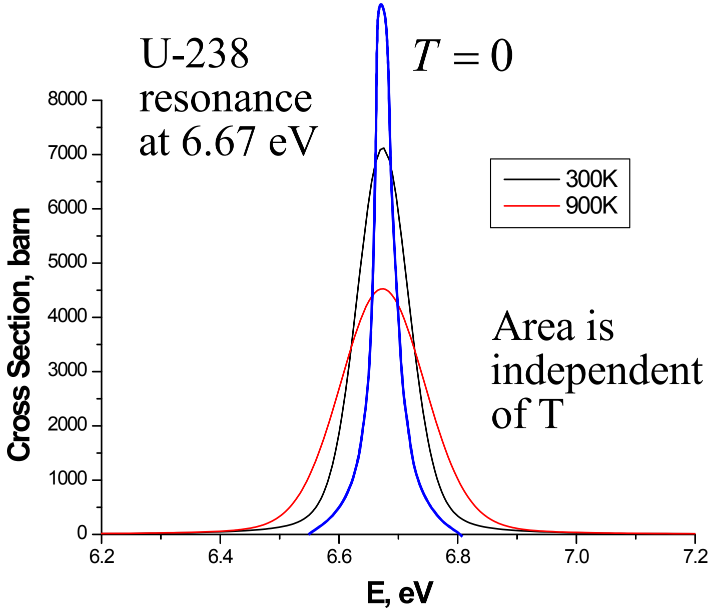
- Self-shielding factor

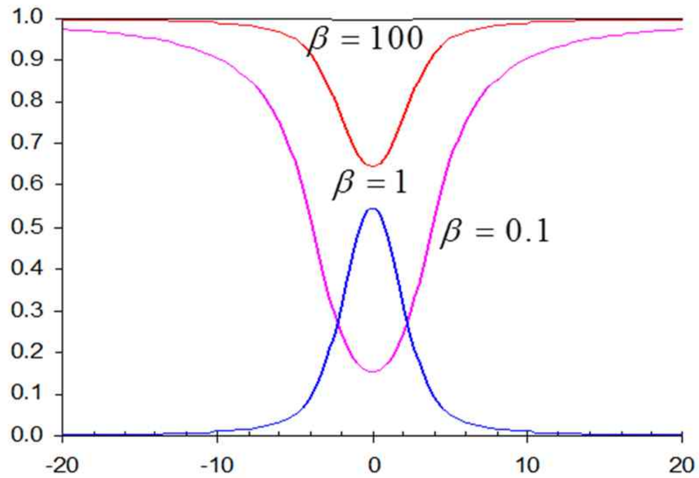
$$f(T,\sigma_b)=\frac{\sigma(T,\sigma_b)}{\sigma(0,\infty)}\approx\frac{2\beta}{\pi}J(\xi,\beta)$$

$$J(\xi,\beta)=\int_0^{\infty}\frac{\psi(x,\xi)}{\beta+\psi(x,\xi)}dx$$

$$\xi=\frac{\Gamma}{\Delta},\quad \Delta=\left(\frac{4kTE_0}{A}\right)^{1/2}$$

$$\beta=\frac{\Sigma_p}{\Sigma_m}=\frac{\Sigma_p}{N_i\sigma_m}=\frac{\sigma_b}{\sigma_m};\quad \sigma_b=\frac{\Sigma_p}{N_i}$$





Doppler Broadening (3)

- Doppler broadening does not change the area under the curve, but reduced self-shielding

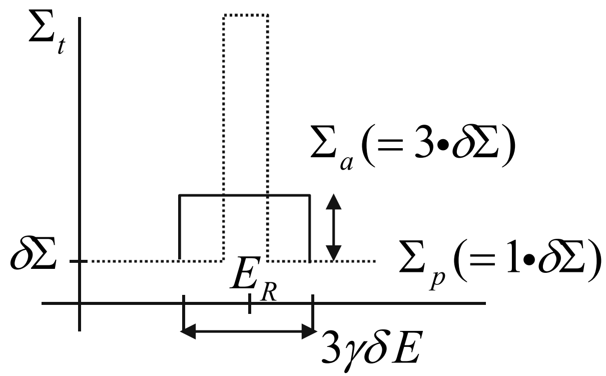
$$E\varphi(E) = \frac{1}{3+1} = 0.25$$

- Reduced self-shielding increases total absorption

$$R_a = \int_{E_R-\frac{3\gamma}{2}E_{Rl}}^{E_R+\frac{3\gamma}{2}E_R} \Sigma_a(E')\varphi(E')dE' = \int_{E_R-\frac{3\gamma}{2}E_R}^{E_R+\frac{3\gamma}{2}E_R} 3\delta\Sigma \frac{1}{4}\frac{dE'}{E'}$$

$$\approx \frac{3}{4}\frac{\delta\Sigma}{E_R}\int_{E_R-\frac{3\gamma}{2}E_R}^{E_R+\frac{3\gamma}{2}E_R} dE' = \frac{9}{4}\delta\Sigma\gamma = \frac{10}{4}0.9\delta\Sigma\gamma$$

$\Rightarrow$ leading to an 2.5 times increase relative to the un-broadened case

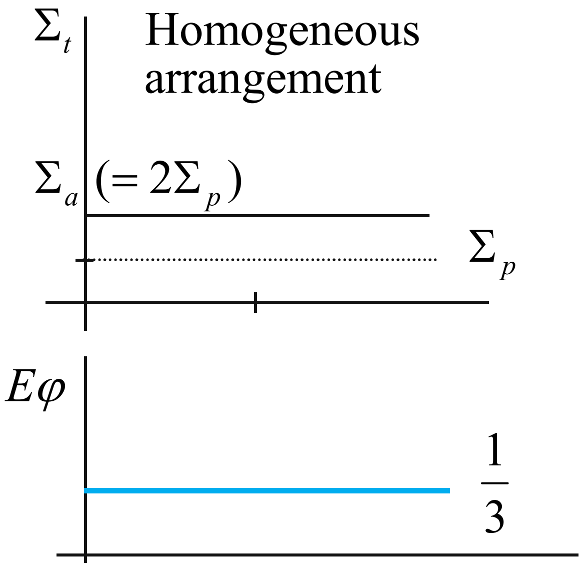


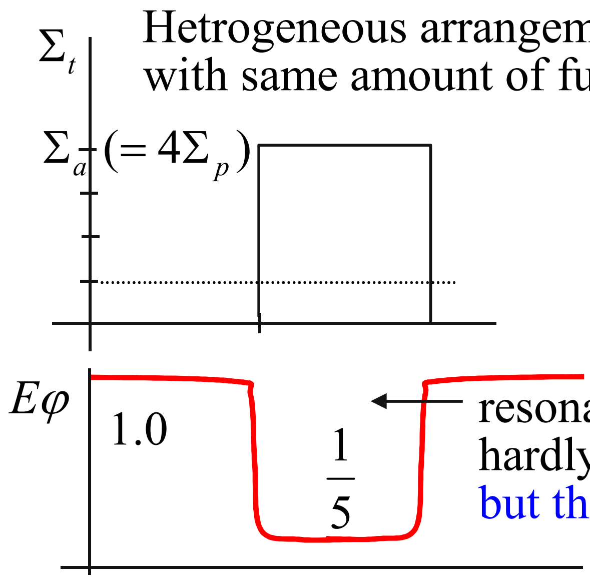
Illustrative example on Slide 14

- Effect of Doppler broadening
  - Increased resonance capture in U-238 due to reduced self-shielding reduces the core reactivity
  - For U-235, reduced self-shielding due to Doppler broadening increases fission as well as capture reactions, which compensate each other and result in a small reactivity change

Spatial Self Shielding

- Heterogeneous arrangement of fuel and moderator





resonance energy neutrons hardly penerate the fuel, but thermal neutrons can ...

– Within resonance

$$E\varphi = \frac{\Sigma_p}{\Sigma_a + \Sigma_p} = \frac{1}{3}$$

– Total absorption

$$\Sigma_a \varphi \gamma V = \frac{1}{3} N_F \sigma_a \gamma V$$

$$E\varphi = \frac{\Sigma_p}{\Sigma_a + \Sigma_p} = \frac{1}{5} \text{ only @ fuel region}$$

$$\Sigma_a' \varphi' \gamma \frac{V}{2} = 2 N_F \sigma_a \frac{1}{5} \gamma \frac{V}{2} = \frac{1}{5} N_F \sigma_a \gamma V$$

less absorption → higher reactivity