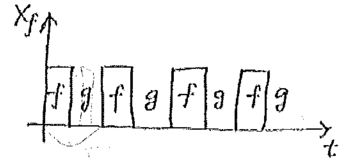
551 Final

1. (a) definition of void fraction



Phase density function (local)

$X_k(\vec{x}, t) = \begin{cases} 1 & \text{if } x \text{ is } k \text{ phase} \\ 0 & \text{if not} \end{cases}$

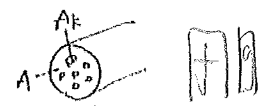
(i) Local void fraction (time average)

$\alpha_{kt} \equiv \frac{1}{\Delta t}\int_{\Delta t} X_k \, dt = \frac{\Delta t_k}{\Delta t}$ probability

$\alpha_{gt} + \alpha_{ft} = 1$

(ii) Area averaged void fraction

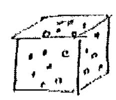
$\alpha_{kA} = \frac{1}{A}\int_A X_k \, dA = \frac{A_k}{A}$



$\alpha_{gA} + \alpha_{fA} = 1$

(iii) Volume averaged void fraction

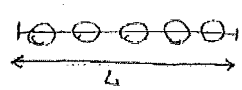
$\alpha_{kV} = \frac{1}{V}\int_V X_k \, dV = \frac{V_k}{V}$



$\alpha_{gV} + \alpha_{fV} = 1$

(iv) line averaged void fraction

$\alpha_{kL} = \frac{1}{L}\int_L X_k \, dL = \frac{L_k}{L}$



$\alpha_{gL} + \alpha_{fL} = 1$

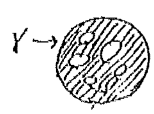
- Ergodic theorem

Area average (Local void fraction) = Time average (Area averaged void fraction)

$\frac{1}{A}\int_A \frac{1}{\Delta t}\int_{\Delta t} X_k \, dt \, dA = \frac{1}{\Delta t}\int_{\Delta t} \frac{1}{A}\int_A X_k \, dA \, dt$

(b) Void fraction measurement

(i) ① photon attenuation technique.



generally attenuation in liquid more than that of gas

$I = I_0 \cdot \exp(-\mu x)$

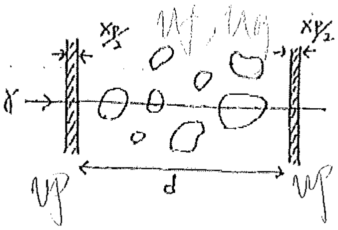
I : ~~emerging~~ attenuated intensity

I_0 : incident intensity

μ : absorption coeff

x : distance

② line measurement



$$I = I_0 \exp(-\mu_p x_p)\exp(-\mu_f(1-\alpha)d)\exp(-\mu_g \alpha d)$$

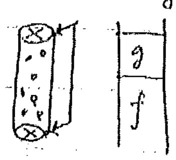
where x_p : total wall thickness
d : distance between walls
μ_p, μ_f, μ_g : absorption coeff.

$\alpha = 1$: $I = I_0 \exp(\underbrace{-\mu_p}x_p)\exp(\underbrace{-\mu_g}d)$

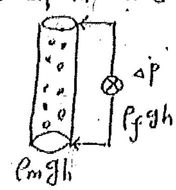
$\alpha = 0$: $I = I_0 \exp(\underbrace{-\mu_p}x_p)\exp(\underbrace{-\mu_f}d)$

(ii) global technique

① quick closing valves



② Δp method



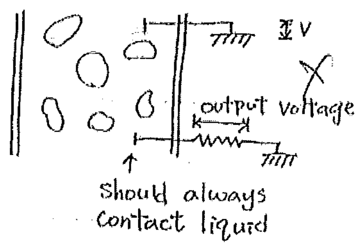
Low flow (friction small) < 1 m/sec

$\underline{\rho_f g h} - \underline{\rho_m g h} = \Delta P$

$\rho_m = \alpha\rho_g + (1-\alpha)\rho_f$

(iii) Local probe technique

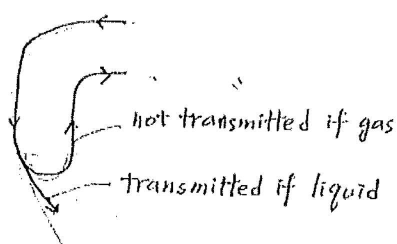
① conductivity probe



discrete resistance when the upper probe is in contact with bubble

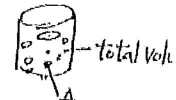
This method will not work in electrically non-conducting fluid.

② optical probe



(iv) Interfacial area concentration

$a_i = \frac{\text{interfacial area}}{\text{mixture volume}} = \frac{A_i}{V}$; $a_i = \frac{\text{Interfacial perimeter}}{\text{cross sectional area}} = \frac{C_i}{A}$



· local interfacial area (time averaged)

$$\boxed{a_i = \frac{1}{\Delta t}\sum_j \frac{1}{|\vec{V_i}\cdot\vec{n}|}} = \frac{1}{\Delta t}\sum_j \frac{1}{(V_{in})_j}$$

$(V_{in})_j$ = jth interfacial normal velocity of interface

2 Natural circulation

balance equation :

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial \rho V}{\partial z} = 0 \\ \frac{\partial \rho V}{\partial t} + \frac{\partial \rho V^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{f}{2D}\rho V_z |V_z| + \rho g_z - \rho\beta\Delta T g_z \\ \frac{\partial \rho_i}{\partial t} + \frac{\partial \rho_i V}{\partial z} = \frac{\xi_h q_0''}{A} \end{cases}$$

assumption : incompressible $\rho = const$
constant : μ, k, C_p

continuity : $\rho_i V_i a_i = \rho_r V_r a_r \Rightarrow V_i = \left(\frac{a_r}{a_i}\right) V_r$

momentum : $\rho \oint \frac{\partial \rho_i V_i}{\partial t} dz = \rho_r \frac{dV_r}{dt} \sum \left(\frac{a_r}{a_i}\right) l_i$

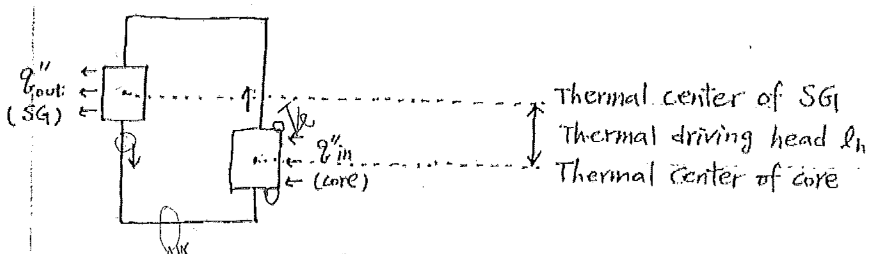
$$\oint \frac{\partial \rho_i V_i^2}{\partial z} dz = 0$$

$$\oint -\frac{\partial P}{\partial z} dz = \Delta P_{pump}$$

$$\oint \frac{f_i \rho_i V_i |V_i|}{2D_i} dz = \sum \left(\frac{fl}{D} + k\right)\left(\frac{a_r}{a_i}\right)^2 \frac{\rho_r V_r^2}{2}$$

$$\oint \rho_i g_i dz = \sum \rho_r g_i l_i = 0$$

$$\oint \rho_i \beta_i \Delta T_i g_i dz = \sum \rho_r \beta g_i \Delta T l_{hi} = \rho_r \beta g \Delta T l_h$$



$$\rho_r \frac{dV_r}{dt} \sum \left(\frac{a_r}{a_i}\right) l_i = \Delta P_{pump} - \sum \left(\frac{fl}{D} + k\right)\left(\frac{a_r}{a_i}\right)^2 \frac{\rho_r V_r^2}{2} + \rho_r \beta g \Delta T l_h$$

$$\Delta T = T_h - T_c = T(\text{core exit}) - T(\text{SG exit})$$

If pump is on, natural circulation force ~ negligible
momentum equation can be solved without energy Eq.

(i) Forced convection

① Steady state $\Rightarrow \frac{\partial V_r}{\partial t} = 0$ negligible natural circulation $\rho_r g \beta \Delta T \ell_h \sim$ small

$$\Rightarrow \Delta P_{pump} - \frac{\rho_r V_r^2}{2} \Sigma \left(\frac{f\ell}{D} + k\right)\left(\frac{a_r}{a_i}\right)^2 = 0 \quad \Rightarrow \; \therefore V_r \sim \sqrt{\Delta P_{pump}}$$

② transient $\Delta P = \Delta P(t)$ pump coast down characteristic

$$\rho_r \frac{dV_r}{dt} \Sigma \left(\frac{a_r}{a_i}\right) \ell_i = \Delta p(t) - \frac{\rho_r V_r^2}{2} \Sigma \left(\frac{f\ell}{D} + k\right)_i \left(\frac{a_r}{a_i}\right)^2$$

③ Energy equation

$$di = C_p \, dT$$

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p V_r \frac{\partial T}{\partial z} = \frac{z_h q_0''}{A}$$

$\frac{D}{Dt}$

at $t = t_1$, and $z = 0$, $T = T_{in}$: $T - T_{in} = \int_{t_1}^{t} \frac{z_h q_0''}{\rho C_p A} dt$

(ii) Transient to natural circulation

$$\rho_r \frac{dV_r}{dt} \Sigma \left(\frac{a_r}{a_i}\right) \ell_i = \Delta P - \Sigma \left(\frac{f\ell}{D} + k\right)\left(\frac{a_r}{a_i}\right)^2 \frac{\rho_r V_r^2}{2} + g\beta \rho_r \Delta T \ell_h$$

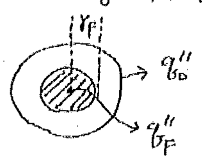
$$\Delta T = (T_{exit} - T_{in})_{core}$$

Energy equation for core : $\rho C_p \left\{ \frac{\partial T}{\partial t} + V_r \frac{\partial T}{\partial z} \right\} = \frac{z_h q_0''}{A}$ ✓

q_0'' : neutronics, decay heat or solid conduction

Fuel : $\rho_F C_F \frac{\partial T_F}{\partial t} = k_F \nabla^2 T_F + \dot{q}_F$, $\dot{q}_F$: neutronics

Cladding : $\rho_c C_c \frac{\partial T_c}{\partial t} = k_c \nabla^2 T_c$



$\sqrt{\pi \alpha t} \sim r_F$

fuel & cladding

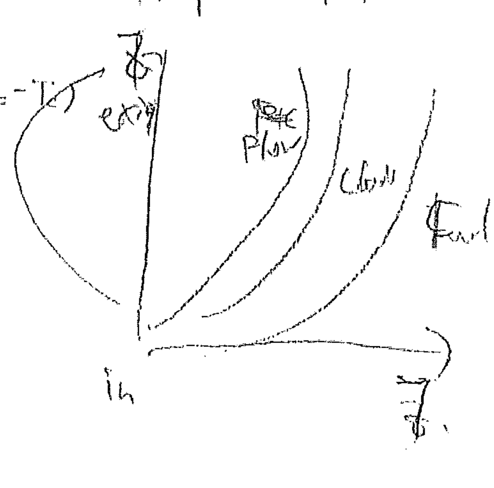
$$-k_F \frac{\partial T}{\partial r}\Big|_{r_F} = -k_c \frac{\partial T}{\partial r}\Big|_{r_F} = h_{gap}(T_F - T_c)$$

$$T_F\big|_{r_F} \neq T_c\big|_{r_F}$$

h_{gap} : gap conductance

$$q_0'' = -k_c \frac{\partial T}{\partial r}\Big|_{r_c} = h(T_c - T)$$

+ Temperature profile



3 Two fluid Model

contribution of interface (mass, momentum, energy transfer at interface)

$$\frac{\partial \rho \psi}{\partial t} + \nabla \cdot \rho \psi v = -\nabla \cdot J + \dot{\phi}_g + \sum_j \{\ \}_j$$

- Continuity equation for phase k

$\psi_k = 1$, $J_k = 0$, $\dot{\phi}_{kg} = 0$

$$\frac{\partial \alpha_k \rho_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{V}_k) = \Gamma_k$$

time rate of change of phase k mass/Vol

convection of k-phase mass/vol

Σ_j { mass transfer }$_j$ source of k-phase due to phase change of interface /Vol

- momentum equation of phase k

$\psi_k = \vec{V}_k$, $\underline{J_k = \Pi_k = P_k \mathbb{I} + \mathbb{T}_k}$, $\dot{\phi}_g = \rho_k \vec{g}$

$$\frac{\partial \alpha_k \rho_k \vec{V}_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k \vec{V}_k \vec{V}_k) = -\nabla \alpha_k P_k - \nabla \cdot \alpha_k (\mathbb{T}_k'' + \mathbb{T}_k^t) + \alpha_k \rho_k \vec{g}$$

local time rate of change of momentum/Vol

convection of k-phase momentum/Vol

pressure force

viscous stress + turbulent stress

body force

$$\underbrace{+ \vec{V}_{ki} \Gamma_k + \vec{M}_{ik} + \mathbb{T}_i \cdot \nabla \alpha_k + P_{ki} \nabla \alpha_k}_{\text{interfacial transfer of momentum}}$$

momentum transfer due to phase change

generalized drag

interfacial shear force

force due to interfacial pressure

- Enthalpy energy equation

$$\frac{\partial \alpha_k \rho_k i_k}{\partial t} + \nabla \cdot (\alpha_k \rho_k i_k \vec{V}_k) = -\nabla \cdot \alpha_k (\vec{q}_k^{\,c} + \vec{q}_k^{\,t}) + \alpha_k \frac{DP_k}{Dt}$$

local time rate of change of Energy/Vol

convection of k-phase Energy/vol

k-phase conduction heat transfer

turbulent heat transfer (conduction)

$$+ \underbrace{(i_{ki} \Gamma_k + a_i q_{ki}'')}_{\text{interfacial transfer of energy}} + \phi_k$$

energy transfer due to phase change

heat flux at interface

dissipation

in two fluid model

Continuity × 2
momentum × 2
Energy × 2
+
interfacial jump condition

$\sum_{k=1}^{2} \Gamma_k = 0$: mass jump

$\sum_{k=1}^{2} \vec{M}_{ik} = 0$: momentum jump

$\sum_{k=1}^{2} (\Gamma_i i_{ki} + a_i q''_{ki}) = 0$: energy jump

- General needs of constitutive equation

① interface transfer

Γ_k

$\vec{M}_{ki}$, $\vec{V}_{ki}$, τ_i

i_{ki} , $a_i q''_{ki}$

② Bulk transfer

τ_k^{μ} , τ_k^{t}

q_k^{c} , q_k^{t}

③ Equation of state for each phase

$i_k = i_k(\rho_k, T_k)$ caloric equation of state

$\rho_k = \rho_k(P_k, T_k)$ thermal equation of state

And we need to specify

$\sum \Gamma_k = 0$

$\sum M_{ik} = 0$

$\sum (\Gamma_k i_{ki} + a_i q''_{ki}) = 0$

a_i

P_{ki}

τ_i

Drift 比較易解?

4 Drift Flux Model

- mixture continuity equation

$$\frac{\partial \rho_m}{\partial t} + \nabla \cdot \rho_m \vec{V_m} = 0 \quad \cdots\cdots ①$$

$$\to \frac{\partial}{\partial t}\Big[\underbrace{\rho_g \alpha + (1-\alpha)\rho_f}_{\rho_m}\Big] + \nabla \cdot \Big[\underbrace{\alpha \rho_g V_g + (1-\alpha)\rho_f V_f}_{\rho_m \vec{V_m}}\Big] = \sum_{k=1}^{2} \Gamma_k$$

$$\frac{\partial \alpha \rho_g}{\partial t} + \nabla \cdot (\alpha \rho_g V_g) = \Gamma_g$$

$$\to \frac{\partial \alpha \rho_g}{\partial t} + \nabla \cdot (\alpha \rho_g V_m) = \Gamma_g - \nabla \cdot \Big(\underbrace{\frac{\alpha \rho_g \rho_f}{\rho_m}}_{\text{mass diffusion}} \vec{V_{gj}}\Big) \quad \cdots\cdots ②$$

expressed by constitutive relation (drift velocity)

- mixture momentum equation

$$\frac{\partial \rho_m \vec{V_m}}{\partial t} + \nabla \cdot (\rho_m \vec{V_m}\,\vec{V_m}) = -\nabla P_m + \rho_m \vec{g} - \nabla \cdot \Big(\tau^{\mu} + \tau^{t} + \underbrace{\frac{\alpha}{1-\alpha}\frac{\rho_g \rho_f}{\rho_m} \vec{V_{gj}}\,\vec{V_{gj}}}_{\text{momentum diffusion}}\Big) \quad \cdots\cdots ③$$

- mixture energy equation

$$\frac{\partial \rho_m i_m}{\partial t} + \nabla \cdot (\rho_m i_m \vec{V_m}) = -\nabla \cdot \Big\{ \vec{q^c} + \vec{q^t} + \frac{\alpha \rho_g \rho_f}{\rho_m} \Delta i_{fg} \vec{V_{gj}} \Big\} + \frac{\partial P_m}{\partial t} + \Big[\vec{V_m} + \frac{\alpha(\rho_f - \rho_g)}{\rho_m}\vec{V_{gj}}\Big] \cdot \nabla P_m + \Phi_m \quad \cdots\cdots ④$$

dissipation ≈ 0
small

⇒ we have 4 field equations [3 mixture field eq / 1 mass diffusion eq.

$\checkmark$

$$\rho_m = \alpha \rho_g + (1-\alpha)\rho_f$$

$$V_m = \frac{\alpha \rho_g V_g + (1-\alpha)\rho_f V_f}{\rho_m}$$

$$\vec{j} = \alpha V_g + (1-\alpha) V_f = j_g + j_f$$

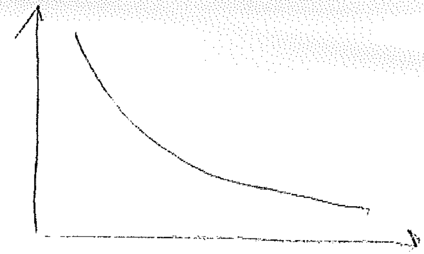
$$\vec{V_{gj}} = V_g - j = V_g - [\alpha V_g + (1-\alpha) V_f] = (1-\alpha)(V_g - V_f) = (1-\alpha) V_r$$

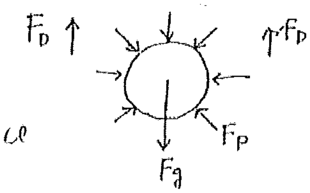
$$j = V_m + \alpha(1-\alpha)\frac{\Delta\rho}{\rho_m} V_r = V_m + \alpha \frac{\Delta\rho}{\rho_m} V_{gj}$$

thus. velocity field $\left.\begin{matrix} V_m \\ V_{gj} \end{matrix}\right\}$ can be expressed in terms of j, V_g, V_f, V_r

$\therefore \begin{cases} \vec{V_{gj}} = \vec{V_{gj}}(\alpha, \rho_m, \vec{g}, \vec{V_m} \cdots) \text{ Drift velocity} \\ \Gamma_g = \Gamma_g(\alpha, \rho_m, \cdots) \end{cases}$

5 terminal velocity ($V_{r\infty}$)





drag force

V: volume of the fluid particle

we have $F_P + F_g = \underbrace{V(\rho_c - \rho_d)g}_{\text{buoyancy force}}$

and $F_D = -\frac{1}{2} C_{D\infty} \rho_c \cancel{V_{r\infty} |V_{r\infty}| A_d}$ ✓

since $\Sigma F = M \frac{dV}{dt} = 0$

$\underline{\text{Drag force}} + (\underline{\text{pressure force}} + \underline{\text{gravity force}}) = 0$

$-\frac{1}{2} C_{D\infty} \rho_c V_{r\infty} |V_{r\infty}| A_d + V(\rho_c - \rho_d) g = 0$

$V_{r\infty} |V_{r\infty}| = 2 \left(\frac{V}{A_d}\right)\left(\frac{\rho_c - \rho_d}{\rho_c}\right) \frac{g}{C_{D\infty}}$

(i) when sphere

$2 \frac{V}{A_d} = 2 \cdot \frac{\frac{4}{3}\pi r_d^3}{\pi r_d^2} = \frac{8}{3} r_d$

$\Rightarrow r_d = \frac{3}{4}\left(\frac{V}{A_d}\right)$: equivalent radius

$\Rightarrow V_{r\infty} |V_{r\infty}| = \frac{8}{3} \frac{r_d}{C_{D\infty}} \left(\frac{\rho_c - \rho_d}{\rho_c}\right) g$ for sphere

(ii) Stokes regime ($C_{D\infty} = \frac{24}{Re_\infty}$)

$Re_\infty = \frac{2\rho_c V_{r\infty} r_d}{\mu_c} \Rightarrow C_{D\infty} = \frac{12\mu_c}{\rho_c V_{r\infty} r_d}$

$V_{r\infty} |V_{r\infty}| = \frac{8}{3} \frac{\rho_c V_{r\infty} r_d^2}{12\mu_c} \left(\frac{\rho_c - \rho_d}{\rho_c}\right) g.$

$\Rightarrow \cancel{V_{r\infty} = \frac{r_d}{3}\sqrt{\frac{2(\rho_c - \rho_d) g}{\mu_c}}}$ $|V_{r\infty}| = \frac{2}{9} \frac{\rho_c \ldots}{\rho}$

(iii) Wake regime : since $C_{D\infty}$ is not linear, we need computer to solve

(iv) Distorted regime : $C_{D\infty} = \frac{\sqrt{2}}{3} N_{\mu} N_{Re\infty}$

$$V_{r\infty} = \sqrt{2} \left(\frac{6 g \Delta \rho}{\rho_c^2}\right)^{1/4}$$

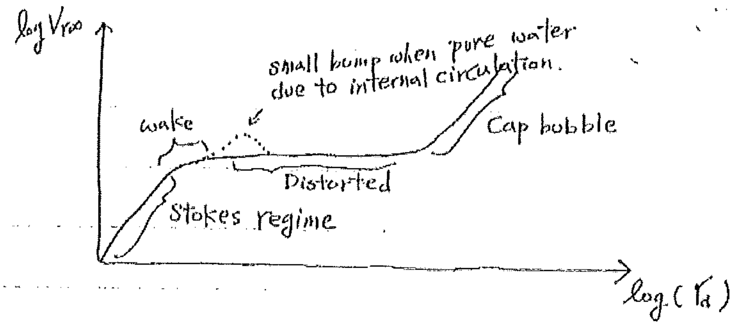
(v) Cap bubble flow : $C_{D\infty} = \frac{8}{3}$

$$V_{r\infty} \left| V_{r\infty} \right| = \frac{r_d (\rho_c - \rho_d) g}{\rho_c}$$

$$\Rightarrow V_{r\infty} = \sqrt{\frac{r_d(\rho_c - \rho_d) g}{\rho_c}}$$

$\therefore \rho_c - \rho_d > 0$: $V_{r\infty} > 0$: rise

$\rho_c - \rho_d < 0$: $V_{r\infty} < 0$: fall



6 Loss of power happening

- balance equation

$$\begin{cases} \frac{\partial \rho}{\partial t} + \frac{\partial \rho V}{\partial z} = 0 \\ \frac{\partial \rho V}{\partial t} + \frac{\partial \rho V^2}{\partial z} = -\frac{\partial P}{\partial z} - \frac{f}{2D} \rho V_z |V_z| + \rho g_z - \rho \beta \Delta T g_z \\ \frac{\partial \rho i}{\partial t} + \frac{\partial \rho i V}{\partial z} = \frac{\xi_h q_o''}{A} \end{cases}$$

assumption : (incompressible $\rho = const$
constant : μ, K, C_p

Continuity : $\rho_i V_i a_i = \rho_r V_r a_r \Rightarrow V_i = \left(\frac{a_r}{a_i}\right) V_r$

momentum: $\oint \frac{\partial \rho_i V_i}{\partial t} dz = \rho_r \frac{dV_r}{dt} \Sigma \left(\frac{a_r}{a_i}\right) l_i$

$$\oint \frac{\partial \rho_i V_i^2}{\partial z} dz = 0$$

$$\oint -\frac{\partial P}{\partial z} dz = \Delta P_{pump}$$

$$\oint \frac{f_i \rho_i V_i |V_i|}{2D_i} dz = \Sigma \left(\frac{fl}{D} + K\right) \left(\frac{a_r}{a_i}\right)^2 \frac{\rho_r V_r^2}{2}$$

$$\oint \rho_i g_i dz = \Sigma \rho_r g_i l_i = 0$$

$$\oint -\rho_i \beta_i \Delta T_i g_i dz = \Sigma \rho_r \beta g_i \Delta T l_{hi} = \rho_r \beta g \Delta T l_h$$

$$\Rightarrow \rho_r \frac{dV_r}{dt} \Sigma \left(\frac{a_r}{a_i}\right) l_i = \Delta P_{pump} - \Sigma \left(\frac{fl}{D} + K\right) \left(\frac{a_r}{a_i}\right)^2 \frac{\rho_r V_r^2}{2} + \rho_r \beta g \Delta T l_h$$

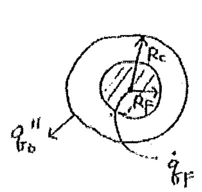
- Energy : In case of power loss

$q_o'' = const$, $\frac{d q_o''}{dt} \sim small$

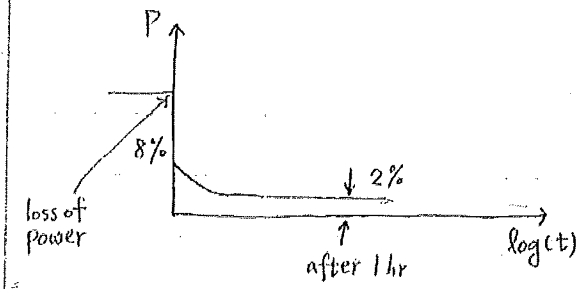
thermal storage change in fuel neglect

$\pi R_F^2 \dot{q}_F = 2\pi R_c q_o''$

$\dot{q}_F = f(t_{operation}, t_{scram})$



LOCA)
ECCS



We are interested after 1 hr, i.e. $\dot{q}_F = f(t) \approx const$.

$\Rightarrow V_r = const$, for steady state, natural circulation

$$\rho C_p V_r \frac{\partial T}{\partial z} = \frac{\xi_h q_0''}{A}$$

$$(T_{exit} - T_{in}) = \Delta T_h = \frac{\xi_h q_0'' l_{core}}{\rho C_p V_r A}$$

Substituting in momentum equation:

$$\cancel{\Delta P_{pump} = -K_{pump}\left(\frac{\rho V_r^2}{2}\right)}$$

$$\Rightarrow 0 = -K_{pump}\left(\frac{\rho_r V_r^2}{2}\right) - \frac{\rho_r V_r^2}{2}\sum\left(\frac{fl}{D}+k\right)\left(\frac{a_r}{a_i}\right)^2 + \rho_r \beta g \Delta T_h l_h$$

$$= -\frac{\rho_r V_r^2}{2}\sum\left(\frac{fl}{D}+k\right)\left(\frac{a_r}{a_i}\right)^2 + \rho_r \beta g \Delta T_h l_h$$

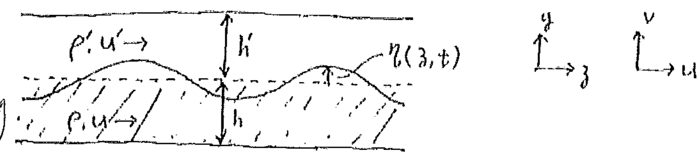
$$= -\frac{\rho_r V_r^2}{2}\sum\left(\frac{fl}{D}+k\right)\left(\frac{a_r}{a_i}\right)^2 + \rho_r \beta g \left(\frac{\xi_h q_0'' l_{core}}{\rho_r C_p (V_r) A}\right) l_h$$

$$\therefore V_r^3 = \frac{1}{\frac{1}{2}\sum\left(\frac{fl}{D}+k\right)\left(\frac{a_r}{a_i}\right)^2} \times \frac{\beta g \xi_h q_0'' l_{core} l_h}{\rho_r C_p A}$$

$$\therefore V_r = \left[\frac{\beta g \xi_h q_0'' l_{core} l_h}{\frac{\rho_r C_p A}{2}\sum\left(\frac{fl}{D}+k\right)\left(\frac{a_r}{a_i}\right)^2}\right]^{1/3}$$

✓

7 Instability of Kelvin-Helmholtz



g - stabilizing

γ - dis-

S - sta

Assume ① no heat transfer (→ drop energy equation)

② Inviscid (→ no shear stress term in momentum equation)

③ Incompressible ($\rho \to$ const)

i) $y > 0$, (upper fluid)

$$\begin{cases} \dfrac{\partial^2 \phi'}{\partial y^2} + \dfrac{\partial^2 \phi'}{\partial z^2} = 0 \\ \dfrac{P'}{\rho'} = \dfrac{\partial \phi'}{\partial t} - \dfrac{V^2}{2} - \Omega \end{cases}$$

ii) $y < 0$ (lower fluid)

$$\begin{cases} \dfrac{\partial^2 \phi}{\partial y^2} + \dfrac{\partial^2 \phi}{\partial z^2} = 0 \\ \dfrac{P}{\rho} = \dfrac{\partial \phi}{\partial t} - \dfrac{V^2}{2} - \Omega \end{cases}$$

introduce perturbation potential

$$\phi' = \underbrace{-u'z}_{\text{steady}} + \underbrace{\phi_1'}_{\text{small perturbation}} \rightarrow \text{disturbance potential}$$

and let $\rho'^* = \rho' \coth(kh')$

$\rho^* = \rho \coth(kh)$

$c = \dfrac{\omega}{k}$: wave velocity.

$$\rightarrow c^2 - 2c\left\{\frac{\rho^* u + \rho'^* u'}{\rho^* + \rho'^*}\right\} + \left\{\frac{\rho^* u^2 + \rho'^* u'^2}{\rho^* + \rho'^*}\right\} = \frac{g(\rho - \rho') + \sigma k^2}{k(\rho^* + \rho'^*)}$$

① Taylor instability

$\vec{g}\downarrow$ ρ_f heavy

ρ_g light

g — distablizing
γ — dis...
S — sta... → stretch

assume $u=0$, $u'=0$

$kh' \gg 1$, $kh \gg 1$ ⟶ deep water assomption

$\rho_f \gg \rho_g$

$$\therefore\ c^2 - 2c\left\{\frac{\rho^* u + \cancel{\rho'^* u'}}{\cancel{\rho^* + \rho'^*}}\right\} + \left\{\frac{\rho^* u^2 + \cancel{\rho'^* u'^2}}{\cancel{\rho^* + \rho'^*}}\right\} = \frac{g(\rho-\rho') + \sigma k^2}{k(\rho^* + \rho'^*)}$$

$$c^2 = \frac{g(\rho-\rho') + \sigma k^2}{k(\rho^* + \rho'^*)}$$

$\rho' = \rho'^* = \rho_f$, $\rho = \rho^* = \rho_g$

$$\therefore\ c^2 = \frac{g(\rho_g - \rho_f) + \sigma k^2}{k(\rho_f + \rho_g)} = \frac{\sigma k^2}{k(\rho_f+\rho_g)} - \frac{g(\rho_f - \rho_g)}{k(\rho_f+\rho_g)}$$

①
$$c^2 = \left(\frac{\omega}{k}\right)^2 = \begin{cases} >0 : \text{stable } (\lambda < \lambda_c) \\ =0 : \text{neutral stability} \\ <0 : \text{unstable } (\lambda > \lambda_c) \end{cases}$$

② $k = \frac{2\pi}{\lambda} \Rightarrow c^2 = \frac{\sigma}{\rho_f+\rho_g}\left(\frac{2\pi}{\lambda}\right) - \frac{\lambda g}{2\pi}\frac{\rho_f-\rho_g}{\rho_f+\rho_g} \Rightarrow \frac{\partial c^2}{\partial \lambda} = 0 \Rightarrow \lambda = 2\pi\sqrt{\frac{\sigma}{g\Delta\rho}} = \lambda_c$

③ $c^2 = \left(\frac{\omega}{k}\right)^2 = \frac{\sigma k^2}{k(\rho_f+\rho_g)} - \frac{g\Delta\rho}{k(\rho_f+\rho_g)}$

$\Delta\rho = \rho_f - \rho_g$

$$\Rightarrow \omega^2 = \frac{\sigma k^3}{\rho_f+\rho_g} - \frac{g\Delta\rho k}{\rho_f+\rho_g}$$

$c^2 = 0$ $k = \frac{2\pi}{\lambda}$

ω^2 ?

$\frac{\partial c^2}{\partial \lambda} = 0$

⇒ k_{max}

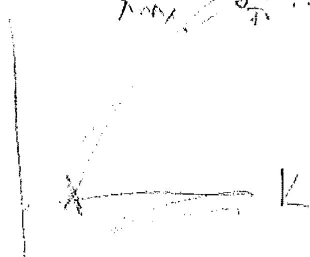
λ_{max} $\frac{\partial \omega^2}{\partial k}$ ..?

$$\rightarrow \frac{\partial \omega^2}{\partial k} = 0 \Rightarrow \frac{3\sigma k^2}{\rho_f+\rho_g} - \frac{g\Delta\rho}{\rho_f+\rho_g} = 0$$

$$k_{max} = \sqrt{\frac{g\Delta\rho}{3\sigma}}$$

$3\sigma k^2 - g\Delta\rho$

$$\lambda_{max} = 2\pi\sqrt{\frac{3\sigma}{g\Delta\rho}} = \sqrt{3}\,\lambda_c$$



② Kelvin - Helmholtz Instability

ρ_g $u' \rightarrow$	gas
ρ_f $u \rightarrow$	liquid

assume $u \neq 0$, $u' \neq 0$

$kh' \gg 1$, $kh \gg 1$ → deep water assumption

from $c^2 - 2c\left\{\frac{\rho^* u + \rho'^* u'}{\rho^* + \rho'^*}\right\} + \left\{\frac{\rho^* u^2 + \rho'^* u'^2}{\rho^* + \rho'^*}\right\} = \frac{g(\rho - \rho') + \sigma k^2}{k(\rho^* + \rho'^*)}$

(use $\cancel{c = \frac{2a \pm \sqrt{b^2 - 2ac}}{2}}$)

let $\rho' = \rho'^* = \rho_g$, $\rho = \rho^* = \rho_f$

$$c = \left(\frac{\rho_f u + \rho_g u'}{\rho_f + \rho_g}\right) \pm \left\{\frac{\sigma k^2 + g(\rho_f - \rho_g)}{k(\rho_f + \rho_g)} - \frac{\rho_f \rho_g}{(\rho_f + \rho_g)^2}(u' - u)^2\right\}^{1/2}$$

above equation (wave velocity) is unstable when

$$\frac{\sigma k^2 + g(\rho_f - \rho_g)}{k(\rho_f + \rho_g)} < \frac{\rho_f \rho_g}{(\rho_f + \rho_g)^2}(u' - u)^2$$

b^2 - 4ac < 0 ⇒ unstable

$$\Rightarrow (u' - u)^2 > \frac{\rho_f + \rho_g}{\rho_f \rho_g} \frac{1}{k}(\sigma k^2 + g\Delta\rho) \qquad \frac{\rho_f + \rho_g}{\rho_f \rho_g}$$

$\sigma k + \frac{g\Delta\rho}{k}$

where $k = \frac{2\pi}{\lambda_c} = \sqrt{\frac{g\Delta\rho}{\sigma}}$

$\sigma\sqrt{\frac{g\Delta\rho}{\sigma}}$

assume $\rho' = \rho_g \ll \rho = \rho_f$

$\sqrt{\sigma g\Delta\rho} + \frac{g\Delta\rho}{\sqrt{\frac{g\Delta\rho}{\sigma}}}$

$$(u' - u)^2 \approx 2\sqrt{\frac{\sigma g\Delta\rho}{\rho_g^2}} \Rightarrow u_r = \sqrt{2}\left(\frac{\sigma g\Delta\rho}{\rho_g^2}\right)^{1/4}$$

$2\sqrt{\sigma g\Delta\rho}$

$\frac{1}{\rho_f} + \frac{1}{\rho_g}$

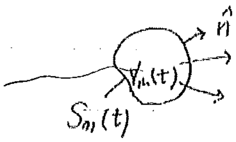
Thus, kelvin - Helmholtz instability is due to relative motion between different phase

① Relative Velocity between phases
② Surface Tension
③ Gravity

551 Mid-term

meaning of various terms in equations

1. General balance equation, each field equation, constitutive relations



J = flux < + : out flow, − : in flow

$\vec{V}$ = material velocity

change of property in volumn V_m = In-flux through the surface + Generation in V_m

$$\boxed{\frac{D}{Dt}\int_{V_m} \psi \, dV = -\oint_{S_m} \hat{n}\cdot J \, ds + \int_{V_m} \dot{\psi}_g \, dV}$$

Reynolds Transport theorem

$$\frac{D}{Dt}\int_{V_m} \psi \, dV = \int_{V_m} \left[\frac{\partial \psi}{\partial t} + \nabla\cdot(\psi \vec{V})\right] dV$$

Green's theorem $\int_{S_m} \hat{n}\cdot \vec{J} \, ds = \int_{V_m} \nabla\cdot\vec{J} \, dV$

$$\because \int_{V_m}\left[\frac{\partial \psi}{\partial t} + \nabla\cdot(\psi\vec{V})\right] dV = -\int_{V_m} \nabla\cdot J \, ds + \int_{V_m} \dot{\psi}_g \, dV$$

$$\Rightarrow \frac{\partial \psi}{\partial t} + \nabla\cdot(\psi\vec{V}) = -\nabla\cdot J + \dot{\psi}_g$$ ⇐ General balance equation

- $\frac{\partial \psi}{\partial t}$: time rate of change of property ψ per unit volume
- $\nabla\cdot(\psi\vec{V})$: convection of ψ by material motion per unit volume.
- $-\nabla\cdot J$: In flux of ψ at the surface
- $\dot{\psi}_g$: generation rate of ψ in volume.

① mass balance equation

$\psi \to \rho$ density

$\vec{V} \to$ material velocity

$J \to 0$

$\dot{\psi}_g \to 0$

then $\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\vec{V} = 0$: continuity equation

- $\frac{\partial \rho}{\partial t}$: time rate of change of mass per unit volume
- $\nabla\cdot\rho\vec{V}$: Convection of mass due to material motion per unit volume.

② momentum balance equation

$\psi \to \rho \vec{V}$

$\mathbb{J} \to \mathbb{T} = p\mathbb{I} + \tau$

$\underset{\text{unit tensor}}{\mathbb{I}}$ $\underset{\text{viscous tensor}}{\tau}$

$-\nabla p \quad \nabla \tau$

$\dot{\psi}_g \Rightarrow \rho \vec{g}$ ← body force

$$\Rightarrow \underbrace{\frac{\partial \rho \vec{V}}{\partial t}}_{\substack{\text{time rate of}\\\text{change of momentum}\\\text{per unit volume}}} + \underbrace{\nabla \cdot (\rho \vec{V}\vec{V})}_{\substack{\text{rate of momentum}\\\text{change by convection}\\\text{per unit volume due}\\\text{to material motion}}} = \underbrace{-\nabla p}_{\substack{\text{pressure force}\\\text{per unit volume}}} - \underbrace{\nabla \cdot \tau}_{\substack{\text{viscous force}\\\text{per unit volume}}} + \underbrace{\rho \vec{g}}_{\substack{\text{body force}\\\text{per unit volume}}}$$

③ energy balance equation

$\psi \to \rho(\underset{\text{internal energy}}{u} + \underset{\text{kinetic energy}}{\tfrac{1}{2}V^2})$

$\mathbb{J} \to \underset{\text{heat flux}}{\vec{q}} + \underset{\substack{\text{work done by}\\\text{surface force}}}{\mathbb{T}\cdot\vec{V}}$ $\qquad \mathbb{T} = p\mathbb{I} + \tau$

$\dot{\psi}_g \to \rho \vec{V}\cdot\vec{g} + \dot{q}$ ← heat generation

work done by gravity
(or work done by body force)

$$\underbrace{\frac{\partial \rho(u+\frac{1}{2}V^2)}{\partial t}}_{\substack{\text{time rate of change}\\\text{of energy per unit volume}}} + \underbrace{\nabla\cdot\rho(u+\tfrac{1}{2}V^2)\vec{V}}_{\substack{\text{rate of energy}\\\text{change by convection}\\\text{per unit volume}\\\text{due to material motion}}} = \underbrace{-\nabla\cdot\vec{q}}_{\text{heat conduction}} \underbrace{-\nabla\cdot p\vec{V}}_{\substack{\text{work done}\\\text{by pressure}\\\text{(reversible)}}} \underbrace{-\nabla\cdot(\tau\cdot\vec{V})}_{\substack{\text{work done by}\\\text{viscous force}\\\text{(irreversible)}}} + \underbrace{\rho\vec{V}\cdot\vec{g}}_{\substack{\text{work done}\\\text{by gravity}}} + \underbrace{\dot{q}}_{\substack{\text{internal}\\\text{heat}\\\text{generation}}}$$

we have 8 unknowns
- mass : $\rho, \vec{V}$
- momentum : $p, \tau, \vec{g}$
- energy : $u, \vec{q}, \dot{q}$

and 3 field equations

So we need 5 constitutive equations to solve them

constitutive equations

i) Equation of state

$$\rho = \rho(p, T) \qquad p = p(\rho, T)$$

$$u = u(p, T) \qquad u = u(\rho, T)$$

ii) Stress tensor

$$\tau = -\mu \left[\nabla \vec{v} + (\nabla \vec{v})^{t} \right]$$

iii) Heat flux

$$\vec{q} = -k \nabla T$$

iv) Heat generation

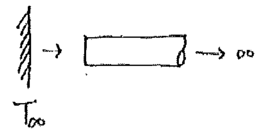
$$\dot{q} = q(x, t) \begin{cases} \text{nuclear fission} \\ \text{Joule heating (Electric)} \\ \text{magnetic induction} \\ \text{thermal radiation} \end{cases}$$

v) body force

$$\vec{g} = \text{const} \begin{cases} \text{gravity} \\ \text{Electrostatic} \\ \text{Electromagnetic.} \end{cases}$$

thermal penetration depth (derive and describe)

2 Transient heat conduction, applicable equation



put T_∞ in contact at $t = 0^+$

consider energy equation

$$\rho C_p \left[\frac{\partial T}{\partial t} + \vec{V}\cdot\nabla T \right] = k \nabla^2 T + \dot{q}$$

$\vec{V} = 0$ in solid no heat generation

$$\rho C_v \left[\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T \right] = k\nabla^2 T + \dot{q}$$

x-component: $\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$

$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2} \quad \left(a = \frac{k}{\rho C_p} : \text{thermal diffusivity} \right)$$

B.C. $t < 0$, $T(0^-, x) = T_0$

$t \geq 0$, $T(0^+, 0) = T_\infty$

$a = \frac{k}{\rho c_p}$

let $\theta = T - T_\infty$

then $\frac{\partial \theta}{\partial t} = a \frac{\partial^2 \theta}{\partial x^2}$

$\theta(x, 0) = \theta_0$

$\theta(0, t) = 0$

$\theta(\infty, t) = \theta_0$

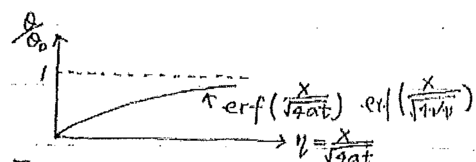
try similarity transformation with

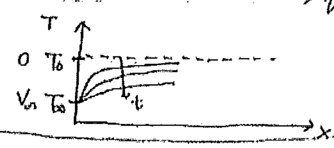
$\frac{\theta}{\theta_0} = \phi(\eta)$, where $\eta = \frac{x}{\sqrt{4at}}$

⋮

get $\frac{\theta}{\theta_0} = \mathrm{erf}(\eta)$

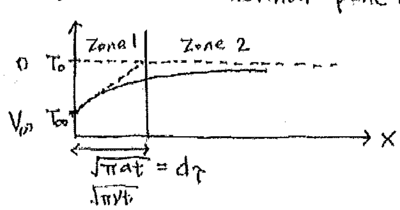
$$\frac{T - T_\infty}{T_0 - T_\infty} = \mathrm{erf}\left(\frac{x}{\sqrt{4at}} \right)$$





⋮

$d_T = \sqrt{\pi a t}$: thermal penetration depth

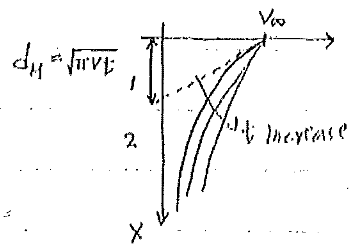


$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = \frac{T_0 - T_\infty}{d_T}$$

Zone 1: zone where temperature (momentum) propagates at a given t

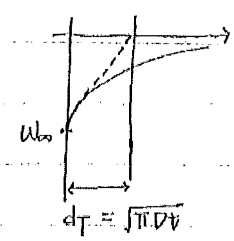
Zone 2: zone where nothing propagates at a give t

Similarity



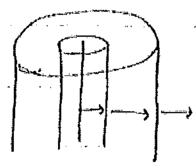
$d_M = \sqrt{\pi \nu t}$

momentum penetration depth



$d_T = \sqrt{\pi D t}$

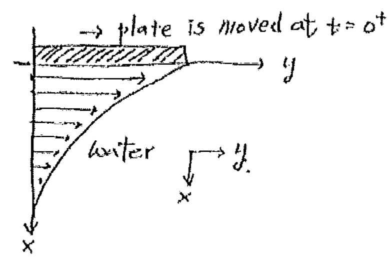
diffusion penetration depth



$\dot{q} \rightarrow T_{fuel} \rightarrow T_{cladding} \rightarrow T_{coolant}$

$d_T = \sqrt{\pi \alpha t}$

4 Transient problem



consider momentum equation

$$\rho \left\{ \frac{\partial \vec{V}}{\partial t} + \vec{V}\cdot\nabla\vec{V} \right\} = -\nabla p + \mu \nabla^2 \vec{V} + \rho \vec{g}$$

y-direction :

$$\rho \left\{ \frac{\partial V_y}{\partial t} + \underbrace{\cancel{V_x \frac{\partial V_y}{\partial x}}}_{\text{continuity \& B.C.}} + \underbrace{\cancel{V_y \frac{\partial V_y}{\partial y}}}_{\text{2D problem}} + \cancel{V_z \frac{\partial V_z}{\partial z}} \right\} = \underbrace{-\cancel{\frac{\partial P}{\partial y}}}_{P \text{ is uniform}} + \mu \left(\frac{\partial^2 V_y}{\partial x^2} + \underbrace{\cancel{\frac{\partial^2 V_y}{\partial y^2}}}_{\text{Continuity}} + \underbrace{\cancel{\frac{\partial^2 V_z}{\partial z^2}}}_{\text{2D problem}} \right)$$

$$\rho \frac{\partial V_y}{\partial t} = \mu \frac{\partial^2 V_y}{\partial x^2}$$

$\frac{\partial V_y}{\partial t} = \nu \frac{\partial^2 V_y}{\partial x^2}$, $\quad \nu = \frac{\mu}{\rho}$: kinetic viscosity

let $\theta = V - V_\infty \Rightarrow \frac{\partial \theta}{\partial t} = \nu \frac{\partial^2 \theta}{\partial x^2}$, $\Rightarrow \frac{\theta}{\theta_0} = \phi(\eta)$, where $\eta = \frac{x}{\sqrt{4\nu t}}$

$$\frac{\theta}{\theta_0} = erf\left(\frac{x}{\sqrt{4\nu t}}\right) = \frac{V - V_\infty}{-V_\infty}$$

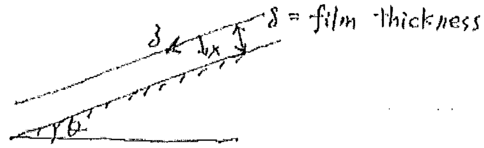
⋮

$$\left.\frac{\partial V_y}{\partial x}\right|_{x=0} = \frac{-V_\infty}{d_M} = \frac{-V_\infty}{\sqrt{\pi \nu t}}$$

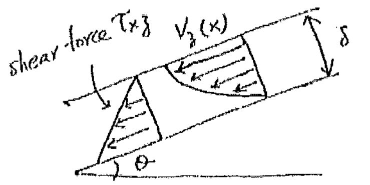
$\frac{V - V_\infty}{-V_\infty}$ $\quad \frac{\theta}{\theta_0} = erf\left(\frac{x}{\sqrt{4\nu t}}\right)$

$d_M = \sqrt{\pi \nu t}$

3 Laminar pipe, uniform entrance velocity



velocity profile $V_z(x) = \frac{\rho}{2\mu} g\sin\theta\cdot\delta^2\left(1-\frac{x^2}{\delta^2}\right)$



$$V_z = \frac{\rho}{2\mu} g\delta^2\cos\theta$$

$$V_z = \frac{\rho g\sin\theta\,\delta^2}{2\mu}\left(1-\frac{x^2}{\delta^2}\right)$$

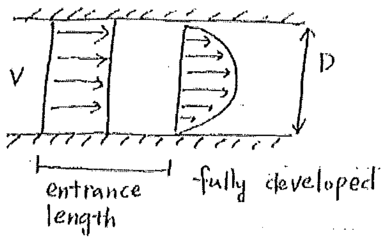
$$V_{max} = \frac{\rho}{2\mu} g\sin\theta\,\delta^2$$

$$\langle V_z\rangle = \frac{1}{\delta}\int_0^\delta V_z\,dx = \frac{\rho}{2\mu} g\sin\theta\cdot\delta\left[x-\frac{x^3}{3\delta^2}\right]_0^\delta = \frac{\rho g\sin\theta\,\delta^2}{3\mu}$$

$$Q = \delta\langle V_z\rangle \underset{\substack{\uparrow\\\text{film width}}}{W} = \frac{\rho g\sin\theta\,\delta^3 W}{3\mu}$$

film thickness $\delta^3 = \frac{3\mu Q}{\rho g w\sin\theta} \Rightarrow \delta = \sqrt[3]{\frac{3\mu Q}{\rho g w\sin\theta}}$

5. Entrance length



entrance length can be calculated from d_M:

$$d_M = \sqrt{\pi \nu t} = \frac{D}{2}$$

$$t = \frac{D^2}{4} \frac{1}{\pi \nu}$$

$$L_{entrance} = t \cdot V = \left(\frac{D}{2}\right)^2 \frac{V}{\pi \nu} \quad , \quad \nu = \frac{\mu}{\rho}$$

$$= \left(\frac{D}{2}\right)^2 \frac{\rho V}{\pi \mu} = \left(\frac{\rho V D}{\mu}\right) \frac{D}{4\pi} = Re \frac{D}{4\pi}$$

$$\therefore \quad \frac{L_e}{D} = Re \frac{1}{4\pi}$$

Same for temperature profile

$$d_T = \sqrt{\pi a t} = \frac{D}{2}$$

$$t = \left(\frac{D}{2}\right)^2 \frac{1}{\pi a}$$

$$L_{entrance} = t \cdot V = \left(\frac{D}{2}\right)^2 \frac{V}{\pi a} \quad , \quad a = \frac{k}{\rho C_p}$$

6 Turbulent flow (a) Reynold's stress

momentum equation

$$\frac{\partial \rho \vec{V}}{\partial t} + \nabla \cdot \rho \vec{V}\vec{V} = -\nabla p + \mu \nabla^2 \vec{V} + \rho \vec{g}$$

assume $\vec{V} = \bar{V} + V'$, $\rho' = 0$ then take time average

$$\Rightarrow \frac{\partial \rho \bar{V}}{\partial t} + \underbrace{\nabla \cdot \rho \, \overline{(\bar{V}+V')(\bar{V}+V')}} = -\nabla \bar{p} + \mu \nabla^2 \bar{V} + \rho \bar{g}$$

$$\nabla \cdot \rho \bar{V}\bar{V} + \cancel{\nabla \cdot \rho \overline{\bar{V} V'}} + \cancel{\nabla \cdot \rho \overline{V' \bar{V}}} + \nabla \cdot \rho \overline{V'V'}$$

$\bar{V}\overline{V'} = 0$ due to $\overline{V'} = 0$, but $\overline{V'V'}$ is not zero due to nonlinear

$$\Rightarrow \frac{\partial \rho \bar{V}}{\partial t} + \nabla \cdot \rho \bar{V}\bar{V} = -\nabla \bar{p} + [\mu \nabla^2 \bar{V} - \nabla \cdot \underbrace{\rho \overline{V'V'}}] + \rho \bar{g}$$

- Reynolds stress
- turbulent stress
- turbulent momentum flux

$$\Rightarrow \frac{\partial \rho \bar{V}}{\partial t} + \nabla \cdot \rho \bar{V}\bar{V} = -\nabla \bar{p} - \nabla \cdot \underbrace{[\pi^{\mu} + \pi^{t}]}_{\pi^{T}} + \rho \bar{g}$$

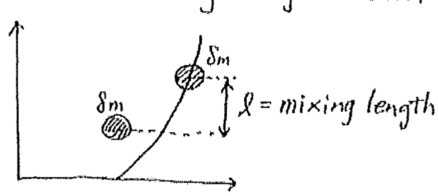
$$\pi^{T} = \pi^{\mu} + \pi^{t}$$

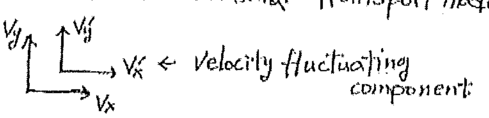
$$\pi^{\mu} = -\nu \rho [\nabla \bar{V} + (\nabla \bar{V})^{+}] \qquad \nu : \text{kinematic viscosity}$$

$$\pi^{t} = -\varepsilon_m \rho [\nabla \bar{V} + (\nabla \bar{V})^{+}] \qquad \varepsilon_m : \text{eddy diffusivity}$$

(b) Prandtl's mixing length model

Prandtl's mixing length model is based on molecular transport model





Prandtl's assumption

i) δm does not lose or gain any x-component momentum until it moves l. Then completely exchange momentum

(ii) x-momentum gained by $\delta m = \delta m\, \delta V_x$

→ momentum transfer rate $= \dfrac{\delta m\, \delta V_x}{\delta t}$

→ shear force acting on fluid $= F = \dfrac{\delta m\, \delta V_x}{\delta t}$

→ shear stress : $\tau^t = \dfrac{F}{A} = \dfrac{1}{A}\dfrac{\delta m\, \delta V_x}{\delta t}$

$V_z = V_z(r) \leftarrow \varepsilon_M$

Here $\delta V_x = \underline{\ell \dfrac{d\bar{V}_x}{dy}}$, $\dfrac{1}{A}\dfrac{\delta m}{\delta t} = \rho |V_y'|$

$\Rightarrow \dfrac{\tau^t}{\rho} = (-)\ell |V_y'| \dfrac{d\bar{V}_x}{dy}$

$\varepsilon_H = \varepsilon_M$

$\left(\dfrac{d\bar{V}_x}{dy}\right.$

$\Rightarrow \dfrac{\tau^t}{\rho} = -\varepsilon_M \dfrac{d\bar{V}_x}{dy}$, $\varepsilon_M = \ell |V_y'|$ eddy diffusivity.

Prandtl assumed $|V_y'| = k_1 V_x'$, $V_x' = k_2\, \delta V_x = k_2 \ell \dfrac{d\bar{V}_x}{dy}$

$\dfrac{\tau^t}{\rho} = -\underline{\ell} k_1 k_2 \ell \left|\dfrac{d\bar{V}_x}{dy}\right| \dfrac{d\bar{V}_x}{dy} = -\ell^2 \left|\dfrac{d\bar{V}_x}{dy}\right| \dfrac{d\bar{V}_x}{dy}$

$\ell = ky$: mixing length depends on how far from the wall

$\Rightarrow \dfrac{\tau^t}{\rho} = -k^2 y^2 \left|\dfrac{d\bar{V}_x}{dy}\right| \dfrac{d\bar{V}_x}{dy}$

(c) use eddy diffusivity model to derive velocity profile

again

$\dfrac{\tau^t}{\rho} = -\ell |V_y'| \dfrac{d\bar{V}_x}{dy} = -\varepsilon_M \dfrac{d\bar{V}_x}{dy}$

$\tau^t = -\tau_w$ is valid near the wall

$\Rightarrow \dfrac{\tau_w}{\rho} = k^2 y^2 \dfrac{d\bar{V}_x}{dy} \cdot \dfrac{d\bar{V}_x}{dy}$

$\Rightarrow \dfrac{d\bar{V}_x}{dy} = \dfrac{1}{ky}\sqrt{\dfrac{\tau_w}{\rho}}$

$V^+ = \dfrac{\bar{V}_x}{\sqrt{\tau_w/\rho}}$: non-dimensional velocity

$v^+ = \dfrac{\bar{V}_x}{\sqrt{\tau_w/\rho}}$

$y^+ = \dfrac{y\sqrt{\tau_w/\rho}}{\nu}$: non-dimensional distance

$y^+ = \dfrac{y\sqrt{\tau_w/\rho}}{\nu}$

$\Rightarrow \dfrac{dV^+}{dy^+} = \dfrac{1}{ky^+}$

$\dfrac{dv^+}{dy^+} = \dfrac{1}{ky^+}$

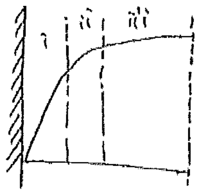
$\dfrac{d\bar{V}_x}{dy} = \dfrac{1}{ky}\sqrt{\dfrac{\tau_w}{\rho}}$

7 Turbulent flow

(a) Viscosity of laminar, turbulent and total

$$\frac{\tau^T}{\rho} = \frac{\tau^\mu}{\rho} + \frac{\tau^t}{\rho} = -(\nu + \varepsilon_M)\frac{d\bar{V}_x}{dy}$$

There are three regions



i) Laminar sublayer ($\tau^\mu > \tau^t$)

ii) Buffer layer ($\tau^\mu \sim \tau^t$)

iii) turbulent core ($\tau^t > \tau^\mu$)

- stress in turbulent flow

from momentum equation: in pipe

$$\frac{\partial \rho \bar{V}}{\partial t} + \nabla\cdot(\rho \bar{V}\bar{V}) = -\nabla \bar{P} - \nabla\cdot(\tau^\mu + \tau^t) + \rho \bar{g}$$

z-direction: $\rho\left(\cancel{\frac{\partial \bar{V}_z}{\partial t}} + \cancel{\bar{V}_r \frac{\partial \bar{V}_z}{\partial r}} + \cancel{\frac{\bar{V}_\theta}{r}\frac{\partial \bar{V}_z}{\partial \theta}} + \cancel{\bar{V}_z \frac{\partial \bar{V}_z}{\partial z}}\right)$

steady state fully developed axisymmetric from continuity equation

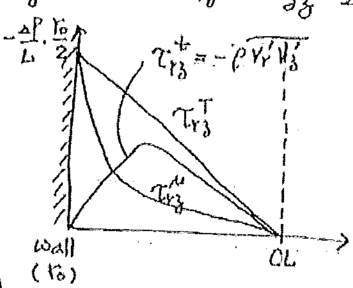
$$= -\frac{\partial \bar{P}}{\partial z} - \left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}^T) + \cancel{\frac{1}{r}\frac{\partial \tau_{\theta z}^T}{\partial \theta}} + \cancel{\frac{\partial \tau_{zz}^T}{\partial z}}\right] + \cancel{\rho g_z}$$

axisymmetric τ_{zz} small g_z neglect

$$\Rightarrow \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz}^T) = -\frac{\partial \bar{P}}{\partial z}$$

$$\frac{\partial}{\partial r}(r\tau_{rz}^T) = -\frac{\partial \bar{P}}{\partial z} r \Rightarrow r\tau_{rz}^T = -\frac{\partial \bar{P}}{\partial z}\cdot\frac{1}{2}r^2 \Rightarrow \tau_{rz}^T = -\frac{\partial \bar{P}}{\partial z}\cdot\frac{r}{2}$$

$$\therefore \tau_{rz}^T = -\frac{r}{2}\left(\frac{\Delta P}{L}\right) = \tau_{rz}^\mu + \tau_{rz}^t$$

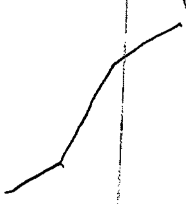


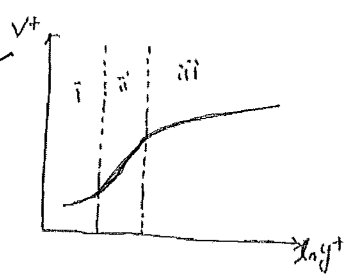
$$\therefore \frac{dV^+}{dy^+} = \frac{1}{k}\frac{1}{y^+} \Rightarrow k\,dV^+ = \frac{1}{y^+}dy^+$$

$$\therefore kV^+ = \ln y^+ + C_1$$

$$\Rightarrow V^+ = \frac{1}{k}\ln y^+ + C$$

$V^+ = \frac{1}{k}\ln y^+ + C$



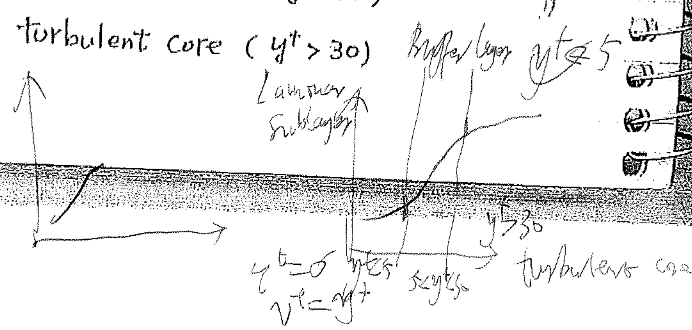


(i) laminar sublayer ($y^+ < 5$)

$\tau^t \simeq 0$, $V^+ = y^+$

(ii) Buffer layer ($5 < y^+ < 30$)

(iii) turbulent core ($y^+ > 30$)



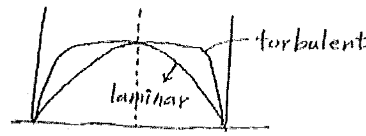
$$-\left(\xi_{ut}+\xi_M\right)\frac{d\bar{v}_x}{dy} \qquad \frac{\cancel{\rho} C_p}{k}\cdot\frac{\mu}{\cancel{\rho}}$$

$$Re = \frac{\rho V D}{\mu}$$

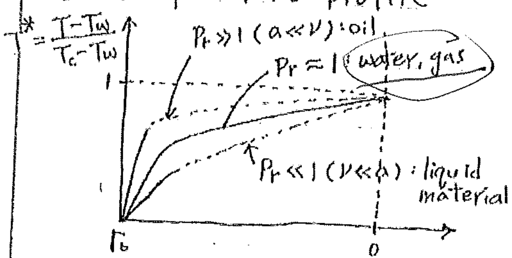
(b) velocity profile

laminar $V = V_{max}\left[1-\left(\frac{r}{r_0}\right)^2\right]$

turbulent $V = V_{max}\left[1-\left(\frac{r}{r_0}\right)\right]^{1/7}$



(c) temperature profile



$$Pr = \frac{\nu}{\alpha} = \frac{\rho C_p}{K}\cdot\frac{\mu}{\rho} = \frac{\mu C_p}{K} = \frac{\text{momentum diffusi}}{\text{thermal diffusion}}$$

thermal diffusivity (α), kinematic (ν)

Effect of Prandtl number

$Pr \gg 1$, $\frac{\nu}{\alpha} \gg 1$, larger momentum diffusion (oil)

$Pr \ll 1$, $\frac{\nu}{\alpha} \ll 1$, larger thermal diffusion (liquid metal)

8 Dimensionless number

$$Re = \frac{\rho V D}{\mu} = \frac{\text{inertia force/volume}}{\text{viscous force/volume}} \qquad \frac{\rho V^2/D}{\mu(V/D)/D}$$

$$Fr = \frac{V^2}{gD} = \frac{\text{inertia force/volume}}{\text{gravitational force/volume}}$$

$$Pe = \frac{VD}{\alpha} = Re\cdot Pr = \frac{\text{heat transfer by convection}}{\text{heat transfer by conduction}}$$

$$E_c = \frac{V^2}{C_p\Delta T} = \frac{\cancel{\text{kinetic energy}}\ \text{K.E. convection}}{\cancel{\text{thermal energy}}\ \text{Enthalpy convection}} = (\gamma-1)M^2\left(\frac{T_0}{\Delta T}\right)$$

$$\frac{V^2}{C_p\Delta T} = \frac{\text{inertia force}}{\text{thermal driving force}}$$

$$Gr = \frac{g\beta\Delta T D^3}{\nu^2} = \frac{\text{bouyant force}\times\text{inertia force}}{(\text{viscous force})^2} \qquad \frac{g\beta\Delta T D^3}{\nu^2}$$

$$Pr = \frac{\nu}{\alpha} = \frac{\text{momentum diffusion}}{\text{thermal diffusion}} \qquad = \frac{\nu}{\alpha}$$