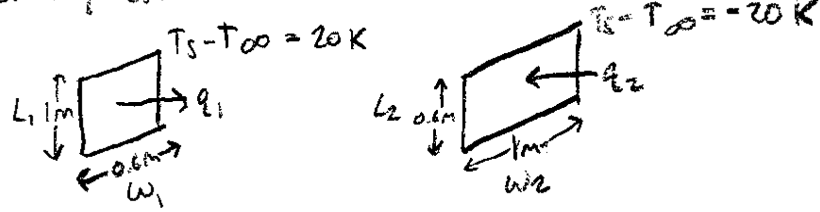
Nickolas Upole | NUCL 351 | 3/30/2014

Problem 9.5: Heat transfer rate by convection from a verticle surface, 1m high by 0.6m wide, to quiescent air that is 20 K cooler. Find the ratio of the heat transfer in the above case to that for a vertical surface that is 0.6m high by 1m wide with quiescent air that is 20 K warmer.



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For air at 300 K and 1atm, $\nu = 15.89\times10^{-6}\ m^2/s$, $\alpha = 22.5\times10^{-6}\ m^2/s$

$$\frac{q_1}{q_2} = \frac{\bar{h}_{L_1} A_{s_1} \Delta T_1}{\bar{h}_{L_2} A_{s_2} \Delta T_2} = \frac{\bar{h}_{L_1}}{\bar{h}_{L_2}}$$

The Rayleigh number is given by $Ra_L = g\beta \Delta T L^3/\nu\alpha$.

$$Ra_{L_1} = (9.8\ m/s^2)\left(\frac{1}{300K}\right)(20K)(1m^3) / \left((15.89\times10^{-6}\ m^2/s)(22.5\times10^{-6}\ m^2/s)\right) = 1.82\times10^9$$

$$Ra_{L_2} = 1.82\times10^9 \left(\frac{0.6}{1}\right)^3 = 3.94\times10^8$$

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = C(Ra_L)^n \quad \text{so} \quad \bar{h}_L = \frac{k}{L} C Ra_L^n$$

where $C_1 = 0.10$, $n_1 = 1/3$ for case 1
$C_2 = 0.59$, $n_2 = 1/4$ for case 2

$$\boxed{\frac{q_1}{q_2} = \frac{(C_1/L_1) Ra_{L_1}^n}{(C_2/L_2) Ra_{L_2}^n} = \frac{(0.10/1m)(1.82\times10^9)^{1/3}}{(0.59/0.6m)(3.94\times10^8)^{1/4}} = 0.881}$$

Problem 9.6: Large vertical plate with uniform temperature of 130°C suspended in quiescent air at 25°C and atmospheric pressure. Find the boundary layer thickness at 0.25 m from lower edge, maximum velocity in boundary layer at this location and position of maximum, heat transfer coefficient at this location, and location where boundary layer becomes turbulent.

For air at 350 K and 1atm, $\nu = 20.92\times10^{-6}\ m^2/s$, $k = 0.030\ W/mK$, and $Pr = 0.700$

$y = \eta x (Gr_x/4)^{-1/4}$ where $Gr_x = g\beta(T_s - T_\infty)x^3/\nu^2 = 9.81\frac{m}{s^2}\cdot\frac{1}{350K}(130-25)K\,x^3/(20.92\times10^{-6}\ m^2/s)^2$

$Gr_x = 6.718\times10^9 x^3$ so $y \approx 5(0.25m)(6.718\times10^9(0.25)^3/4)^{-1/4} = 1.746\times10^{-2}\ m$

$\boxed{y = 17.5\ mm}$

$$u = \frac{2\nu}{x} Gr_x^{1/2} f'(\eta) = \frac{2\times20.92\times10^{-6}\ m^2/s}{0.25\ m}(6.718\times10^9(0.25)^3)^{1/2}\times0.275 = 0.47\ m/s$$

$\boxed{u = 0.47\ m/s}$ and $\boxed{y_{max} = 1/5(17.5mm) = 3.5mm}$

$$Nu_x = h_x x/k = (Gr_x/4)^{1/4} g(Pr) = (6.718\times10^9(0.25)^3/4)^{1/4}\cdot 0.50 = 35.7$$

$$\boxed{h_x = \frac{Nu_x k}{x} = 35.7\times0.030\ W/mK / 0.25m = 4.3\ W/m^2\cdot K}$$

$Ra_{x,c} = Gr_{x,c} Pr \approx 10^9 \quad x_c = [10^9/6.718\times10^9(0.700)]^{1/3} = 0.60m$

$\boxed{x_c = 0.60m}$

Problem 9.28: Coolant flow rate and inlet and outlet temperatures. Dimensions and emissivity of channel side walls. Temperature of surroundings. Power dissipation. Find the sidewall temperatures for $\varepsilon_s=0.15$, Temperature of sidewalls for $\varepsilon_s=0.90$, sidewall temperature with loss of coolant for $\varepsilon_s=0.15$ and $\varepsilon_s=0.90$

For air at 298 K, $c_p = 1007$ J/kg·K

$q = q_c + q_{conv} + q_{rad} = 200$ W

$q_c = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.015$ kg/s $\cdot 1007$ J/kg·K $\times 10$ K $= 151$ W

$q_{conv} = 2\bar{h}A_s(T_s - T_\infty)$ where $A_s = H \times L = 0.32\,m^2$.

$$\bar{h} = \frac{k}{H}\left(0.825 + \frac{0.387\,Ra_H^{1/6}}{\left[1+(0.492/Pr)^{9/16}\right]^{8/27}}\right)^2$$ where $Ra_H = g\beta(T_s - T_\infty)H^3/\alpha\nu$

$q_{rad} = 2A_s\varepsilon_s\sigma(T_s^4 - T_{sur}^4)$

$\boxed{T_s = 35.8°C \text{ for } \varepsilon_s = 0.15}$ $\boxed{T_s = 28.8°C \text{ for } \varepsilon_s = 0.90}$

$\boxed{T_s = 68.8°C \text{ for } \varepsilon_s = 0.15 \text{ and loss of coolant}}$

$\boxed{T_s = 49.5°C \text{ for } \varepsilon_s = 0.90 \text{ and loss of coolant}}$

$$q_1 = h_1 A_1 (T_{s,1} - T_\infty)$$

$$q'' = -k \frac{dT}{dx}$$

$$\Delta x \, K \frac{(T_{m,n-1} - T_{m,n})}{\Delta x}$$

$$\frac{Nu_L}{Nu_L} = \frac{Re_L}{Re_L}$$

$$\Delta E_{st} = -\Delta E_{conv}$$

$$\Delta E_{st} = -hA\left((T_i - T_\infty)e^{-\frac{hA}{\rho V c}t}\right)$$

$$\Delta E_{st} = \frac{+\cancel{hA}(T_i - T_\infty)}{\cancel{hA}} \rho V c t e^{-\frac{hA}{\rho V c}t}$$

$$O = \sqrt{2}\,\Delta x$$

$$\rho V c \frac{dT}{dt} = q_s'' A - hA(T - T_\infty) + \dot{E}_g$$

$$\theta = q_s'' A - hA(T - T_\infty) + \dot{E}_g$$

$$\frac{hL}{K} = A\, Re_L Pr^{1/3}$$

$$\frac{\frac{h_1 \cancel{L_1}}{\cancel{K_1}}}{\frac{h_2 \cancel{L_2}}{\cancel{K_2}}} = \frac{\frac{\cancel{\rho_1} V_1 \cancel{L_1}}{\cancel{\mu_1}}}{\frac{\cancel{\rho_2} V_2 \cancel{L_2}}{\cancel{\mu_2}}}$$

$$q_1 = h_1 A_1 (T_{s,1} - T_\infty)$$

$$h_1 = \frac{q_1}{A_1 (T_{s,1} - T_\infty)}$$

$$\frac{h_1}{h_2} = \frac{V_1}{V_2}$$

$$q_2 = h_2 A_2 (T_{s,2} - T_\infty)$$

$$h_2 = h_1 \left(\frac{V_2}{V_1}\right)$$

$$h_2 =$$

$$0 = k\frac{(T_{m,n-1} - T_{m,n})}{\Delta x} + \Delta x k \frac{(T_{m+1,n} - T_{m,n})}{\Delta x} + h\sqrt{2}(T_\infty - T_{m,n})$$

$$0 = \frac{kT_{m,n-1}}{\Delta x} + k\frac{T_{m+1,n}}{\Delta x} + h\sqrt{2}T_\infty$$

$$T_{m,n}\left(\frac{k}{\Delta x} + \frac{k}{\Delta x} + h\sqrt{2}\right) = \frac{kT_{m,n-1}}{\Delta x} + \frac{kT_{m+1,n}}{\Delta x} + h\sqrt{2}T_\infty$$

$$0 = q_0''\frac{\Delta x}{2} + k\frac{\Delta x}{2}\frac{(T_{m+1,n} - T_{m,n})}{\Delta x} + h\frac{\sqrt{2}\Delta x}{2}(T_\infty - T_{m,n})$$

$$0 = q_0'' \Delta x + k T_{m+1,n} + \frac{h\sqrt{2}\Delta x}{2} T_\infty$$

$$T_{m,n}(k + h\sqrt{2}\Delta x) = q_0''\Delta x + kT_{m+1,n} + h\sqrt{2}\Delta x T_\infty$$

$t=0 \quad T=T_i$

$$\theta = \frac{q_s''A + \dot{E}_g}{T_i - T_\infty}\frac{\rho VC}{hA} + Be^{-\frac{hAt}{\rho VC}}$$

$$e^{\frac{hAt}{\rho VC}}\theta = \frac{C\rho VC}{hA}e^{\frac{hA}{\rho VC}t} + Be^{\frac{-hAt}{\rho VC}}$$

$$1 = \frac{q_s''A + \dot{E}_g}{T_i - T_\infty}\frac{\rho VC}{hA} + B$$

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$$T = q_s''A + \dot{E}_g + T_\infty$$

$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{q_s''A + \dot{E}_g}{T_i - T_\infty}$$

$$\rho VC\frac{dT}{dt} = q_s''A + hA(T - T_\infty) + \dot{E}_g$$

$$\theta = q_s''A - hA(T - T_\infty) + \dot{E}_g$$

$$\frac{d\theta}{dt} = 0 + 0 - hA\frac{dT}{dt}$$

$$\theta = T - T_\infty$$

$$\frac{d\theta}{dt} = \frac{dT}{dt}$$

$$\theta = e^{-\frac{hA}{\rho VC}t}$$

$$\rho VC\frac{dT}{dt} \quad T_i - T_\infty$$

$$\theta = Be^{-\frac{hA}{\rho VC}t}$$

$$\frac{d\theta}{dt} = B\frac{-hA}{\rho VC}\theta$$

$$\rho VC\frac{d\theta}{dt} + hA\theta = \frac{q_s''A + \dot{E}_g}{T_i - T_\infty}$$

$$\frac{d\theta}{dt} + \frac{hA}{\rho VC}\theta = \frac{q_s''A + \dot{E}_g}{T_i - T_\infty}$$

$$B\rho VC\left(\frac{-hA}{\rho VC}\right)\theta + hABe^{\frac{-hA}{\rho VC}t} =$$

$$-hAB\theta + hAB\theta =$$

$$e^{\frac{hA}{\rho VC}}\frac{d\theta}{dt}$$

$$\frac{d}{dt}\left(e^{\frac{hAt}{\rho VC}}\theta\right) = Ce^{\frac{hA}{\rho VC}t}$$

$$e^{\frac{hAt}{\rho VC}}\frac{d\theta}{dt} + \frac{hA}{\rho VC}\theta e^{\frac{hAt}{\rho VC}} = \frac{q_s''A + \dot{E}_g}{T_i - T_\infty}e^{\frac{hA}{\rho VC}t}$$

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HW 7

Problem 6.1: Known: $u(y) = Ay + By^2 - Cy^3$ and $T(y) = D + Ey + Fy^2 - Gy^3$

Find: C_f and h

$$\tau_s = \mu \frac{\partial u}{\partial y}\Big]_{y=0} = \mu\left[A + 2By - 3Cy^2\right]_{y=0} = A\mu$$

$$\boxed{C_f = \frac{\tau_s}{\rho u_\infty^2/2} = \frac{2A\mu}{\rho u_\infty^2} = \frac{2A\nu}{u_\infty^2}}$$

$$h = \frac{-k_f(\partial T/\partial y)_{y=0}}{T_s - T_\infty} = \frac{-k_f\left[E + 2Fy - 3Gy^2\right]_{y=0}}{D - T_\infty} \quad \text{so} \quad \boxed{h = \frac{-k_f E}{D - T_\infty}}$$

Problem 6.4: Known: h_x varies as $x^{-1/2}$ Find: Ratio of $\bar{h}_x/h_x$

$$\bar{h}_x = \frac{1}{x}\int_0^x h_x\,dx = \frac{C}{x}\int_0^x x^{-1/2}\,dx = \frac{C}{x}2x^{1/2} = 2Cx^{-1/2} = 2h_x$$

$$\boxed{\frac{\bar{h}_x}{h_x} = \frac{2h_x}{h_x} = 2}$$

Problem 6.19: Known: $L = 1\,m$, $T_s = 400\,K$, $T_\infty = 300\,K$, $V = 100\,m/s$, $q_1'' = 20{,}000\,W/m^2$

$L = 5\,m$, $T_s = 400\,K$, $T_\infty = 300\,K$, $V = 20\,m/s$

Find: Average convection coefficient

$$\overline{Nu}_L = f(Re_L, Pr) \qquad Re_{L,1} = \frac{V_1 L_1}{\nu_1} = \frac{100\,m/s \cdot 1\,m}{\nu_1} = \frac{100\,m^2/s}{\nu_1}$$

$$Re_{L,2} = \frac{V_2 L_2}{\nu_2} = \frac{20\,m/s \cdot 5\,m}{\nu_2} = \frac{100\,m^2/s}{\nu_2}$$

So $\overline{Nu}_{L,2} = \overline{Nu}_{L,1}$ giving $\bar{h}_2 L_2/k_2 = \bar{h}_1 L_1/k_1$ so $\bar{h}_2 = \bar{h}_1 \frac{L_1}{L_2} = 0.2\,\bar{h}_1$

$$q_1 = \bar{h}_1 A_1 (T_s - T_\infty)_1 \quad \text{so} \quad \bar{h}_1 = \frac{(q_1/A_1)}{(T_s - T_\infty)} = \frac{20{,}000\,W/m^2}{(400-300)\,K} = 200\,W/m^2K$$

hence... $\boxed{\bar{h}_2 = 0.2 \times 200\,W/m^2K = 40\,W/m^2 \cdot K}$

Problem 7.15: Known: Temperature, pressure, and Reynolds number for air flow

Find: a) rate of heat transfer, b) rate of heat transfer at air velocity doubled and pressure increased to 10 atm.

a) $q = \bar{h}_L (w \times L)(T_s - T_\infty)$ and $\overline{Nu}_L = \frac{\bar{h}_L L}{k} = 0.664\,Re_L^{1/2} Pr^{1/3} = 0.664 \cdot 40000^{1/2}\, 0.70^{1/3} = 117.9$

So $\bar{h}_L = 117.9\frac{k}{L} = 117.9 \frac{0.0299\,W/mK}{0.2\,m} = 17.6\,W/m^2K$

$q = 17.6\,W/m^2K\,(0.1\,m \times 0.2\,m)(100-50)K = 17.6\,W$

b) $Re_{L,2}\left(\frac{u_\infty L}{\nu}\right)_2 = 2 \times 10\left(\frac{u_\infty L}{\nu}\right)_1 = 20\,Re_{L,1} = 8 \times 10^5$ so $\overline{Nu}_L = \frac{\bar{h}_L L}{k} = (0.037\,Re_L^{4/5} - 871)\,Pr^{1/3}$

$\overline{Nu}_L = 961$

$\bar{h}_L = 961 \frac{0.0299\,W/mK}{0.2\,m} = 143.6\,W/m^2 \cdot K \quad q = 143.6\,W/m^2 \cdot K\,(0.1\,m \times 0.2\,m)(100-50)K = 143.6\,W$

Problem 7.42: Known: Conditions of air in cross flow over a pipe

Find: a) Drag force per unit length b) heat transfer per unit length of pipe

$F_D = C_D A_f \frac{\rho V^2}{2} \quad F_D' = C_D D \frac{\rho V^2}{2} \qquad Re_D = \frac{VD}{\nu} = \frac{15\,m/s \cdot 0.025\,m}{19.31 \times 10^{-6}\,m^2/s} = 1.942 \times 10^4 \quad C_D \approx 1.1$

$\boxed{F_D' = 1.1 \cdot 0.025\,m \cdot 1.048\,kg/m^3\,(15\,m/s)^2/2 = 3.24\,N/m}$

b) Using Hilpert's relation, with $C = 0.193$ and $m = 0.618$

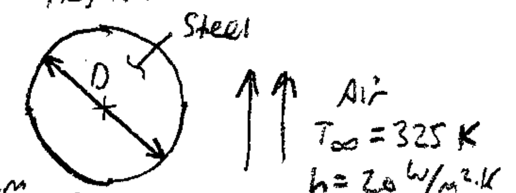
$$\bar{h} = \frac{k}{D} C Re_D^m Pr^{1/3} = \frac{0.0288 \text{ W/m K}}{0.025\text{m}} \times 0.193 \left(1.942 \times 10^4\right)^{0.618} (0.702)^{1/3}$$

$\bar{h} = 88 \text{ W/m}^2\text{K}$ so $q' = \bar{h}(\pi D)(T_s - T_\infty) = 88 \frac{W}{m^2 K}(\pi \times 0.025m)(100-25)K = 520 \text{ W/m}$

$$\boxed{q' = 520 \text{ W/m}}$$

Problem 5.5: Known: $D = 12\,mm$, $T_i = 1150\,K$, $T_f = 400\,K$, $T_\infty = 325\,K$, $h = 20\,W/m^2K$, $k_{st} = 40\,W/m\cdot K$, $\rho_{st} = 7800\,kg/m^3$, and $c_{st} = 600\,J/kg\cdot K$.

Find: Time required to cool to T_f



Equation 5.10 for a sphere is $L_c = r_o/3$,

$$Bi = \frac{hL_c}{k_{st}} = \frac{h(r_o/3)}{k_{st}} = \frac{hD}{6k_{st}} = \frac{20\,W/m^2K \cdot 0.012\,m}{6 \cdot 40\,W/m\,K} = 0.001$$

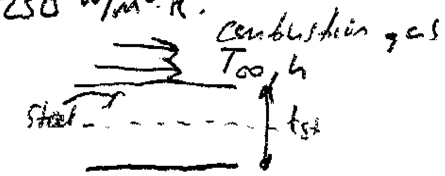
The temperature of steel is near constant during cooling. The lumped capacitance is used.

$$t = \frac{\rho_{st} V c_{st}}{hA} \ln\frac{T_i - T_\infty}{T_f - T_\infty} = \frac{\rho_{st}(\pi D^3/6)c_{st}}{h\pi D^2} \ln\frac{T_i - T_\infty}{T_f - T_\infty}$$

$$\boxed{t = \frac{7800\,kg/m^3 (0.012\,m)\, 600\,J/kg\cdot K}{6 \times 20\,W/m^2\cdot K} \ln\left(\frac{1150-325}{400-325}\right) = 1122\,s}$$

Problem 5.37: Known: $t_{st} = 100\,mm$, $\rho_{st} = 7830\,kg/m^3$, $c_{st} = 550\,J/kg\cdot K$, $k_{st} = 48\,W/m\,K$, $T_i = 200°C$, $T_f = 550°C$, $T_\infty = 800°C$, $h = 250\,W/m^2\cdot K$.

Find: Time required to heat to T_f.



The Biot number is $Bi = \frac{hL}{k_{st}} = \frac{h t_{st}}{2k_{st}}$

$$Bi = \frac{250\,W/m^2K \cdot 0.1\,m}{48\,W/m\,K \cdot 2} = 0.260$$ The lumped capacitance analysis should not be performed

$$\theta_o^* = \frac{T_f - T_\infty}{T_i - T_\infty} = \frac{550-800}{200-800} = 0.417 = C_1 e^{-\zeta_1^2 Fo}$$

where $\zeta_1 \approx 0.488$ rad, $C_1 \approx 1.0396$ (Table 5.1)

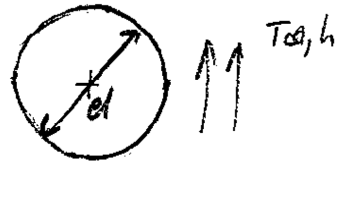
$$\alpha = \frac{k_{st}}{\rho_{st} c_{st}} = 1.115 \times 10^{-5}\,m^2/s$$

$$-\zeta_1^2(\alpha t/L^2) = \ln(0.401) = -0.914$$

$$\boxed{t = \frac{0.914\, t_{st}^2}{4\zeta_1^2 \alpha} = \frac{0.914 \cdot (0.1\,m)^2}{4(0.488^2) \cdot 1.115 \times 10^{-5}\,m^2/s} = 861\,s}$$

Problem 5.51: Known: $d = 40\,mm$, $T_i = 800\,K$, $T_\infty = 300\,K$, $h = 1600\,W/m^2K$, $t = 35$

Find: Temperature of rod after a long period of time.



$$Bi = \frac{hL_c}{k} = \frac{hd}{4k} = \frac{1600\,W/m^2\cdot K\,(0.04\,m)}{4 \cdot 22.3\,W/m\cdot K} = 0.72$$

*cannot use the lumped capacitance model

@ $t = 35\,s$ $$Bi = \frac{hD}{2k} = \frac{1600\,W/m^2\cdot K \cdot 0.040\,m}{2 \cdot 22.3\,W/m\,K} = 1.43$$

use one term approximation

$$Fo = 4\alpha t/d^2 = 4 \cdot 5.259 \times 10^{-6}\,m^2/s \cdot 35\,s/(0.04\,m)^2 = 0.46$$

$$\theta_o^* = C_1 e^{-\zeta_1^2 Fo} = 1.2636\, e^{-1.4036^2 \cdot 0.46} = 0.766.$$

$$\frac{Q}{Q_o} = 1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1) = 1 - \frac{2 \times 0.766}{1.4036} 0.5425 = 0.408$$

$$-Q = \rho c V(T(\infty) - T_\infty) - Q_o$$

$$\boxed{T(\infty) = T_\infty + (T_i - T_\infty)(1 - Q/Q_o) = 300\,K + (800-300)\,K\,(1-0.408) = 596\,K}$$

$$\rho V c \frac{dT}{dt} = q_s''A - h(T-T_\infty) + \dot{E}_g$$

$$\frac{\rho V c \frac{dT}{dt}}{T_i - T_\infty} = \frac{q_s''A}{T_i - T_\infty} - h\frac{(T-T_\infty)}{T_i - T_\infty} + \frac{\dot{E}_g}{T_i - T_\infty} \qquad A = \left(\frac{q_s''A + \dot{E}_g}{T_i - T_\infty}\right)$$

$$\rho V c \frac{dT}{dt} = q_s''A - h(T-T_\infty) + \dot{E}_g$$

$$\rho V c \frac{d\theta}{dt} = \frac{q_s''A}{T_i - T_\infty} - h\theta + \dot{E}_g$$

$$\rho V c \dot{\theta} + h\theta = A$$

$$e^{-\int \frac{h}{\rho V c} dt}$$

$$\dot{\theta} + \frac{h}{\rho V c}\theta = A$$

$$e^{-h/\rho V c t}\dot{\theta} + \frac{h}{\rho V c}\theta e^{\cancel{-}\frac{h}{\rho V c}t} = A e^{-h/\rho V c t}$$

$$\frac{d}{dt}$$

$$\Delta E_{st} = \rho V c \frac{dT}{dt}$$

$$\rho V c \frac{d\theta}{dt} + h\theta = \frac{q_s''A}{T_i - T_\infty} + \frac{\dot{E}_g}{T_i - T_\infty}$$

$$\rho V c \frac{d^2\theta}{dt^2} + h\dot{\theta} = 0$$

$$T = (T_i - T_\infty)e^{-h/\rho V c t} + T_\infty$$

$$e^{\lambda t}$$

$$\frac{dT}{dt} = \frac{-h(T_i - T_\infty)}{\cancel{\rho V c}} e^{-h/\rho V c t}$$

$$\rho V c \lambda^2 \cancel{\theta} + h\lambda\cancel{\theta} = 0$$

$$\cancel{\lambda t}$$

$$\frac{d}{dt}\left(e^{h/\rho V c t}\theta\right) = \frac{A}{\rho V c} e^{h/\rho V c t}$$

$$\lambda(\rho V c \lambda + h) = 0$$

$$\lambda = 0 \text{ and } \lambda = \frac{-h}{\rho V c}$$

$$\cancel{e^{h/\rho V c t}}\theta = \frac{A}{h}\cancel{e^{h/\rho V c t}} + C$$

$$\cancel{C}e$$

$$\theta = C_1 + C_2 e^{-h/\rho V c t}$$

$$\theta = \frac{A}{h} + C$$

$$1 = C_1 + C_2$$

$$\frac{\rho V c \frac{dT}{dt}}{(T_i - T_\infty)} = \frac{q_s'' A}{(T_i - T_\infty)} - h\frac{(T - T_\infty)}{(T_i - T_\infty)} \; \varepsilon\sigma(T$$

$$\rho V c \frac{d\theta}{dt} = \frac{q_s'' A}{(T_i - T_\infty)} - h\theta$$

$$\theta = e^{\lambda t}$$

$$\frac{d\theta}{dt} = \lambda\theta$$

$$\lambda\theta \quad \rho V c \frac{d\theta}{dt} = \frac{q_s'' A}{(T_i - T_\infty)} - h\theta + \dot{E}_g$$

$$\lambda(T - T_\infty)\rho V c$$

T_t $\qquad$ T_j

$$\rho V c \frac{dT}{}$$

$$q_f = R\,\Delta T$$

$$q_f = \frac{\Delta T}{R_f + R_{t,j}}$$

$$q_f(R_f + R_{t,j}) = \Delta T$$

$$\frac{\Delta x^2}{4} + \frac{\Delta x^2}{4}$$

$$0 = E_{in} - E_{out} + \cancel{E_g}$$

$$q_o''\frac{\Delta x}{2} \quad + h(T_\infty - T_{m,n})\frac{\Delta x}{\sqrt{2}} + k\frac{T_{m+1,n} - T_{m,n}}{\Delta x}\frac{\Delta x}{2} = 0$$

$$q_o''\frac{\Delta x}{2} + hT_\infty\frac{\Delta x}{\sqrt{2}} + k\frac{T_{m+1,n}}{2} = hT_{m,n}\frac{\Delta x}{\sqrt{2}} + \frac{T_{m,n}k}{2}$$

Problem 8.1: Known: Fully developed conditions for water, pipe diameter, mass flow rate, temperature

Find: Max velocity and pressure gradient

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4 \times 0.01\ kg/s}{\pi (0.025m)\ 855\times10^{-6}\ kg/m/s} = 596$$

$$\frac{u(r)}{u_m} = 2\left[1-\left(\frac{r}{r_0}\right)^2\right] \quad \text{where} \quad u_m = \frac{\dot{m}}{\rho\pi D^2/4} = \frac{4\cdot 0.01\ kg/s}{998\ kg/m^3 \times \pi (0.025m)^2} = 0.0204\ m/s$$

$$\boxed{u(0) = 2\cdot 0.0204\ m/s = 0.041\ m/s}$$

$$\frac{dp}{dx} = -\frac{64}{Re_D}\frac{\rho u_m^2}{2D} = \frac{-64}{596} \times \frac{998\ kg/m^3\,(0.0204\ m/s)^2}{2\times 0.025 m} = -0.86\ kg/m^2 s^2$$

$$\boxed{\frac{dp}{dx} = -0.86\ Pa/m}$$

Problem 8.7: Known: Velocity and temperature profiles for laminar flow of tube radius $r_0 = 10\ mm$

Find: Bulk temperature

$$T_m = \frac{2}{u_m r_0^2}\int_0^{r_0} u(r)T(r)\,r\,dr \qquad u_m = \frac{2}{r_0^2}\int_0^{r_0} u(r)\,r\,dr$$

$$u_m = \frac{2}{r_0^2} r_0^2 \int_0^1 0.1\left[1-(r/r_0)^2\right](r/r_0)\,d(r/r_0) = 2\left(0.1\left((r/r_0)^2 - \frac{1}{4}(r/r_0)^4\right)\right)\Big|_0^1 = 0.05\ m/s$$

$$T_m = \frac{2}{(0.05)m/s\ r_0^2} r_0^2 \int_0^1 \left(0.1\left(1-(r/r_0)^2\right)\right)\left(344.8 + 75.0(r/r_0)^2 - 18.8(r/r_0)^4\right)(r/r_0)\,d(r/r_0)$$

$$T_m = 4\left((172.40+18.75-3.13)-(86.20+12.50-2.35)\right) = 367\ K$$

$$\boxed{T_m = 367\ K}$$

Problem 8.26: Known: Ethylene glycol flows through a coiled thin walled tube submerged in a well-stirred water bath at constant temperature.

Find: Heat rate and length of tube

$$q_{conv} = \dot{m}c_p(T_{m,o}-T_{m,i}) = 0.01\ kg/s \times 2562\ J/kg\,(35-85) = -1281\ W.$$

$$A_s = q_{conv}/\bar{h}\Delta T_{lm}$$

$$\Delta T_{lm} = (\Delta T_o - \Delta T_i)/\ln\frac{\Delta T_o}{\Delta T_i} = \left((35-25)°C - (85-25)°C\right)/\ln\left(\frac{35-25}{85-25}\right) = 27.9°C$$

$$Re_D = \frac{4\dot{m}}{\pi D\mu} = \frac{4\cdot 0.01\ kg/s}{\pi(0.003m)(0.522\times10^{-2}\ N\cdot s/m^2)} = 813 \qquad \overline{Nu_D} = \frac{\bar{h}D}{k} = 3.66$$

$$\bar{h} = 3.66 \times 0.260\ \frac{W}{mK}/0.003m = 317\ \frac{W}{m^2K}$$

$$A_s = \frac{1281\ W}{317\ W/m^2K \times 27.9°C} = 0.1448\ m^2$$

$$L = \frac{A_s}{\pi D} = 0.1448\ m^2/\pi(0.003\ m) = 15.4\ m \qquad \boxed{L = 15.4\ m}$$

$\frac{h_1 x}{k} = \frac{h_2 x}{k}$

$\frac{\partial u^*}{\partial y^*} = \frac{\partial u(y)}{\partial y}\frac{\partial y}{\partial y^*}\frac{1}{u_\infty}$ $\quad \delta_T \quad \frac{T_s - T(y)}{T_s - T_\infty} = 0.99$

$Pr = \frac{\rho c_p}{k}$ $\quad u^* = \frac{u(y)}{u_\infty} \quad y^* = \frac{y}{\delta}$

$y = \delta$

$\frac{hL}{k} = Nu_L = 0.04 Re_L^{0.85} Pr^{1/2}$

$\tau = \mu \frac{\partial u^*}{\partial y^*}$

$h_L = k$

$\cancel{\dot{E}_{in}}^0 - E_{out} + \dot{E}_g = \cancel{\dot{E}_{st}}^0$

$\frac{\partial u^*}{\partial y^*} = \frac{\partial u(y)}{\partial y}\frac{\delta}{u_\infty}$

$+h\ell^2(T_s - T_\infty) = \dot{E}_g$

$T_s = \frac{\dot{E}_g}{h\ell^2} + T_\infty$

$\frac{\mu\, \partial u(y)\, \delta}{\partial y\, u_\infty}$

$\frac{Nu_2}{Nu_1} = \frac{h_2 L_2 \cancel{k}}{\cancel{k} h_1 L_1} = \frac{h_2 \cancel{x_2}}{h_1 \cancel{x}} = \frac{\cancel{A}\frac{\cancel{\rho} V_2 \cancel{L_2}}{\cancel{\mu}}}{\cancel{A}\frac{\cancel{\rho} V_1 \cancel{L}}{\cancel{\mu}}}$

$h_2 = h_1 \frac{V_2}{V_1}$

$\frac{\rho V d}{\mu}$

$q_1 = h_1 A_1 (T_{s,1} - T_\infty)$

$h_1 = \frac{q_1}{A_1(T_{s,1} - T_\infty)}$

$C_f = \frac{\quad}{Re}$

$q_2 = h_2 A_2 (T_{s,2} - T_\infty)$

$q_2 = \left(\frac{V_2}{V_1}\right)\left(\frac{A_2}{A_1}\right)\frac{(T_{s,2} - T_\infty)}{(T_{s,1} - T_\infty)} q$

$$\dot{q}_{kr} + \dot{q}_{c} - \dot{q}_{k\,r+\Delta r} - \dot{q}_{c\,r+\Delta r} = 0$$

$$\cancel{-k\frac{\partial T}{\partial r}\Big|_{r+\Delta r} 4\pi r^2 + h 4\pi r^2 (T_r - T_\infty)} + k\frac{\partial T}{\partial r}\Big|_{y=r+\Delta r} \cancel{4\pi}(r+\Delta r^2) - h\cancel{4\pi}\cancel{(r+\Delta r)^2 (T_{r+\Delta r} - T_\infty)}$$

$$\cancel{kr^2\left(\frac{\partial T}{\partial r}\Big|_{y=r+\Delta r} - \frac{\partial T}{\partial r}\Big|_{y=r}\right)} + k\frac{\partial T}{\partial r}\Big|_{y=r+\Delta r} 2r\Delta r + \cancel{k\frac{\partial T}{\partial r}\Big|_{y=r+\Delta r}}\Delta r^2$$

$$-hr^2(T_{r+\Delta r} - \cancel{T_\infty} - T_r - \cancel{T_\infty}) - 2r\Delta r h(T_{r+\Delta r} - T_\infty) + \Delta r^2 h(T_{r+\Delta r} - T_\infty) = 0$$

$$kr^2\frac{\partial^2 T}{\partial r^2} + k\frac{\partial T}{\partial r}\Big|_{y=r+\Delta r} 2r - hr^2\left(\frac{\partial T}{\partial r}\right) - 2rh(T_{r+\Delta r} - T_\infty) = 0$$

$$\cancel{k}r^2\frac{\partial^2 T}{\partial r^2} + \cancel{k}\frac{\partial T}{\partial r}2r - hr^2\left(\frac{\partial T}{\partial r}\right) - 2rh(T_r - T_\infty) = 0$$

kr

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} - \frac{h}{k}\left(\frac{\partial T}{\partial r}\right) - \frac{2h}{r}(T_r - T_\infty) = 0$$

$$\theta = (T - T_\infty)$$

$$\frac{\partial \theta}{\partial r} = \frac{\partial T}{\partial r}$$

$$\frac{\partial^2 \theta}{\partial r^2} = \frac{\partial^2 T}{\partial r^2}$$

$$\frac{\frac{h}{k} - \frac{2}{r} \pm \sqrt{\left(\frac{2}{r} - \frac{h}{k}\right)^2 + \frac{4 \cdot 2h}{r}}}{2}$$

$$\frac{h}{2k} - \frac{1}{r} \pm \sqrt{\frac{4}{r^2}}$$

$$\frac{\partial^2 \theta}{\partial r^2} + \left(\frac{2}{r} - \frac{h}{k}\right)\frac{\partial \theta}{\partial r} - \frac{2h}{r}\theta = 0$$

$$\lambda^2 + \left(\frac{2}{r} - \frac{h}{k}\right)\lambda - \frac{2h}{r} = 0$$

Problem 1.1: The thermal conductivity of a sheet of rigid, extruded insulation is reported to be $k = 0.029\ W/m\cdot K$. The measured temperature difference across a 20-mm-thick sheet of material is $T_1 - T_2 = 10°C$.

a.) What is the heat flux through a 2m x 2m sheet of the insulation?

$$q''_t = -k\frac{dT}{dx} = k\frac{(T_1 - T_2)}{t} = 0.029\frac{W}{m\cdot K}\cdot\frac{10\,K}{20\,mm}\times\frac{1000\,mm}{1\,m} = 14.5\frac{W}{m^2}$$

b.) What is the rate of heat transfer through the sheet of insulation?

$$q_t = A\,q''_t = L^2 q''_t = (2m)^2\cdot 14.5\,\frac{W}{m^2} = 58\,W$$

Problem 1.24: Under conditions for which the same room temperature is maintained by a heating or cooling system, it is not uncommon for a person to feel chilled in the winter but comfortable in the summer. Provide a plausible explanation (with supporting calculations) by considering a room whose air temperature is maintained at 20°C throughout the year, while the walls of the room are nominally at 27°C and 14°C in the summer and winter respectively. The exposed surface of a person in the room may be assumed to be at a temperature of 32°C throughout the year and have an emissivity of 0.90. The coefficient associated with heat transfer by natural convection between the person and the room air is approximately $2\ W/m^2\cdot K$.

$T_{room} = 20°C$ $T_{wall,win} = 14°C$ $T_{wall,sum} = 27°C$ $T_{person} = 32°C$ $\epsilon = 0.90$

$h_c = 2\ W/m^2\cdot K$

A chilled feeling is linked to excessive heat loss. Due to the temperature of the room remaining fixed, the chilled feeling cannot be linked to natural convection. In both cases the heat flux is.

sum and win $q''_{conv} = h(T_{person} - T_{room}) = 2\ W/m^2K\,(12K) = 24\ \frac{W}{m^2}$

The heat flux due to radiation must cause this chilled affect then..

Sum $q''_{rad} = \epsilon\sigma(T_{person}^4 - T_{wall,sum}^4) = 0.9\times 5.67\times 10^{-8}\frac{W}{m^2K^4}((305.15K)^4 - (300.15)^4)$

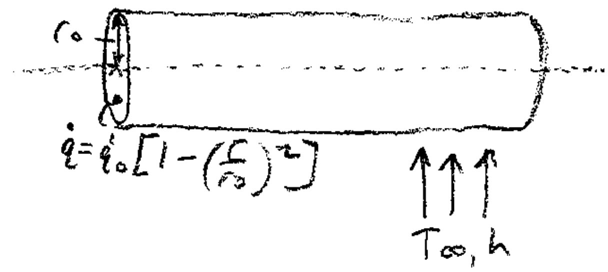
$q''_{rad} = 28.3\ W/m^2$

Win $q''_{rad} = \epsilon\sigma(T_{person}^4 - T_{wall,win}^4) = 0.9\times 5.67\times 10^{-8}\frac{W}{m^2K^4}((305.15K)^4 - (287.15K)^4)$

$q''_{rad} = 95.5\ W/m^2$

The colder walls have a greater effect on the heat transfer due to radiation, causing a chilling effect.

Problem 1.44: Radioactive wastes are packed in a long, thin-walled cylindrical container. The wastes generate thermal energy nonuniformly according to the relationship $\dot{q} = \dot{q}_o\left[1-\left(\frac{r}{r_o}\right)^2\right]$, where $\dot{q}$ is the local rate of energy generation per unit volume, $\dot{q}_o$ is a constant, and r_o is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at T_∞ and provides a uniform convection coefficient h.



Obtain an expression for the total rate at which energy is generated in a unit length of the container. Use this result to obtain an expression for the temperature T_s of the container wall.

$$\dot{E}_g = \int \dot{q}\, dV = \dot{q}_o \int_0^{r_o} \left(1-\left(\frac{r}{r_o}\right)^2\right) 2\pi r L\, dr$$

$$\frac{\dot{E}_g}{L} = 2\pi \dot{q}_o \int_0^{r_o} \left(1-\left(\frac{r}{r_o}\right)^2\right) r\, dr \qquad u = 1-\frac{r^2}{r_o^2}$$

$$du = -\frac{2r}{r_o^2}dr$$

$$\frac{\dot{E}_g}{L} = \cancel{2}\pi \dot{q}_o \int_1^0 u\, \cancel{r}\left(-\frac{r_o^2\, du}{\cancel{2}\cancel{r}}\right) \qquad dr = -\frac{r_o^2\, du}{2r}$$

$$\frac{\dot{E}_g}{L} = -\pi r_o^2 \dot{q}_o \left(\frac{1}{2}u^2\right)\Big|_1^0 = -\pi r_o^2 \dot{q}_o \left(0-\frac{1}{2}\right) = \frac{\pi r_o^2 \dot{q}_o}{2}$$

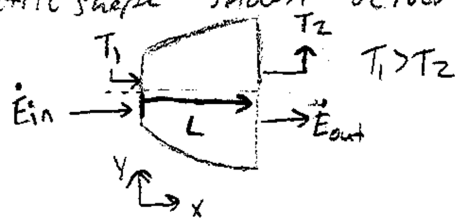
$$\boxed{\frac{\dot{E}_g}{L} = \frac{\pi r_o^2 \dot{q}_o}{2}}$$

$$\frac{\dot{E}_g}{L} - \dot{E}'_{out} = 0$$

$$\frac{\pi r_o^2 \dot{q}_o}{2} - h 2\pi r_o (T_s - T_\infty)$$

$$\boxed{T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}}$$

Problem 2.1: Assume steady-state, one-dimensional heat conduction through the axisymmetric shape shown below.



Assuming constant properties and no internal heat generation, sketch the temp. distribution on T-x coordinates. Briefly explain the shape of your curve.

Performing an energy balance on the object shows $\dot{E}_{in} - \dot{E}_{out} = 0$, it follows that

$$\dot{E}_{in} - \dot{E}_{out} = q_x$$

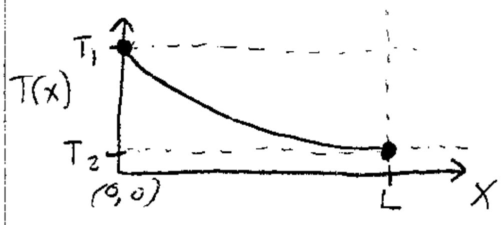
and that $q_x \neq q_x(x)$. The heat rate within the object must be constant. From Fourier's law,

$$q_x = -k A_x \frac{dT}{dx}$$

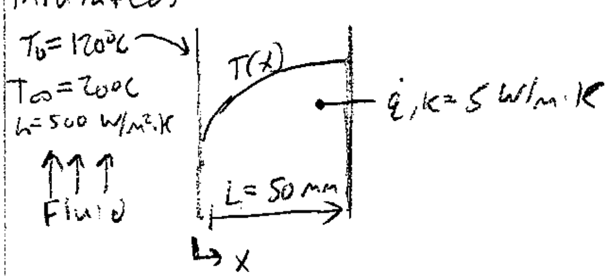
Since q_x and $-k$ are both constants, it follows that

$$A_x \frac{dT}{dx} = \text{constant}.$$

The cross sectional area of the shape increases with x so $\frac{dT}{dx}$ must decrease with x. The temperature distribution is drawn below.



Problem 2.26: One-dimensional steady state conduction with uniform internal energy generation occurs in a plane wall with a thickness of 50 mm and a constant thermal conductivity of 5 W/m·K. For these conditions, the temperature distribution has the form $T(x) = a + bx + cx^2$. The surface at $x=0$ has a temperature of $T(0) = T_0 = 120°C$ and experiences convection with a fluid for which $T_\infty = 20°C$ and $h = 500\ W/m^2 K$. The surface at $x=L$ is well insulated.



a.) applying an overall energy balance to the wall, calculate the internal energy generation rate, $\dot{q}$.

$$\dot{E}''_{in} - \dot{E}''_{out} + \dot{E}''_{gen} = 0 \quad \text{where } \dot{E}''_{in} = q''_{conv}$$

$$q''_{conv} + \dot{E}''_{gen} = h(T_\infty - T_0) + \dot{q} L = 0$$

$$\dot{q} = \frac{h(T_0 - T_\infty)}{L} = \frac{500 \frac{W}{m^2 K}(120-20)K}{\frac{50}{1000} m}$$

$$\boxed{\dot{q} = 1.0 \times 10^6 \frac{W}{m^3}}$$

Problem 2.26 cont.: b.) Determine the coefficients a, b, and c by applying the boundary conditions to the prescribed temperature distribution.

B.C. at $x=0$, conv. surface condition

$$\dot{E}''_{in} - \dot{E}''_{out} = q''_{conv} - q''_x(0) = 0 \quad \text{where} \quad q''_x(0) = -k \left.\frac{dT}{dx}\right|_{x=0}$$

$$h(T_\infty - T_0) - \left[-k(0+b+2cx)_{x=0}\right] = 0$$

$$b = \frac{h(T_1 - T_\infty)}{k} = \frac{500 \frac{W}{m^2 K}(120-20)K}{5 \frac{W}{m \cdot K}} = 1.0 \times 10^4 \ \frac{K}{m}$$

B.C. at $x=L$, adiabatic or insulated surface

$$\dot{E}_{in} - \dot{E}_{out} = -q''_x(L) = 0 \quad \text{where} \quad q''_x(L) = -k \left.\frac{dT}{dx}\right|_{x=L}$$

$$k(0+b+2cx)_{x=L} = 0$$

$$c = \frac{-b}{2L} = \frac{-1.0 \times 10^4 \frac{K}{m}}{2 \cdot \frac{50}{1000} m} = -1.0 \times 10^5 \ \frac{K}{m^2}$$

B.C. at $x=0$, $T(0) = T_0 = 120°C$

$$T(0) = 120°C = a + b \cdot 0 + c \cdot 0$$

$a = 120°C$ (Plot at end)

c.) Consider conditions for which the convection coefficient is halved, but the internal energy generation rate is the same. Determine the new values of a, b, and c.

Overall energy balance

$$a = T_0 = \frac{\dot{q}L}{h} + T_\infty = \frac{1.0 \times 10^6 \frac{W}{m^3} \times \frac{50}{1000} m}{250 \ W/m^2 \cdot K} + 20°C = 220°C$$

Surface Energy balance at $x=0$

$$b = \frac{h(T_0 - T_\infty)}{k} = \frac{250 \frac{W}{m^2 K}(220-20)K}{5 \frac{W}{m K}} = 1.0 \times 10^4 \ \frac{K}{m}$$

Surface Energy balance at $x=L$

$$c = \frac{-b}{2L} = \frac{-1.0 \times 10^4 \frac{K}{m}}{2 \cdot \frac{50}{1000} m} = -1.0 \times 10^5 \ \frac{K}{m^2}$$

d.) The internal energy generation rate is doubled, and the convection coefficient remains unchanged. Determine temperature distribution. Discuss the effects of h and $\dot{q}$ on the distributions.

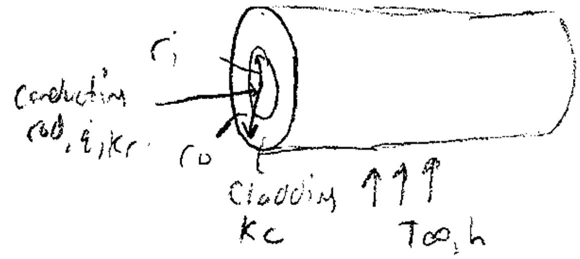
$a = 220°C$ $\quad b = 2 \times 10^4 \frac{K}{m}$ $\quad c = -2 \times 10^5 \ K/m^2$

depends on h $\quad$ depends on $\dot{q}$ $\quad$ depends on $\dot{q}$

Decreasing h increased T_0

Increasing $\dot{q}$ increased T_0

Problem 2.39: Passage of an electric current through a long conducting rod of radius r_i and thermal conductivity k, results in uniform volumetric heating at a rate of $\dot{q}$. The conducting rod is wrapped in an electrically non conducting cladding material of outer radius r_o and thermal conductivity k_c, and convection cooling is provided by an adjoining fluid.



The conducting rod and cladding heat equations are shown below with boundary conditions.

Conducting Rod

$$\frac{k_r}{r}\frac{d}{dr}\left(r\frac{dT_r}{dr}\right)+\dot{q}=0$$

Cladding

$$\frac{d}{dr}\left(r\frac{dT_c}{dr}\right)=0$$

BC

$$\left.\frac{dT_r}{dr}\right|_{r=0}=0$$

$$T_r(r_i)=T_c(r_i)$$

$$k_r\left.\frac{dT_r}{dr}\right|_{r_i}=k_c\left.\frac{dT_c}{dr}\right|_{r_i}$$

$$-k_c\left.\frac{dT_c}{dr}\right|_{r_o}=h\left(T_c(r_o)-T_\infty\right)$$

$$E_{in} - E_{out} = 0$$

$$q''_{cond}\cancel{A} - q''_{rad}\cancel{A} - q''_{conv}\cancel{A} = 0$$

$$q''_{cond} = -k\frac{dT}{dx}$$

$$= -k\frac{(T_s - T_i)}{L}$$

$$= k\frac{(T_i - T_s)}{L}$$

$$conv = h(T_s - T_\infty)$$

$$k\frac{(T_i - T_s)}{L} - 2h(T_s - T_\infty) = 0$$

$$\frac{kT_i}{L} - \frac{kT_s}{L} - 2hT_s + 2hT_\infty = 0$$

$$\frac{kT_i}{L} + 2hT_\infty = \frac{kT_s}{L} + 2hT_s$$

$$T_s = \frac{kT_i + 2hLT_\infty}{L\left(\frac{k}{L} + 2h\right)}$$

$$T_s = \frac{kT_i + 2hLT_\infty}{(k + 2h)}$$

$$\frac{k(T_i - T_s)}{L} - h(T_s - T_\infty) - h_r(T_s - T_{sur}) = 0$$

$$\frac{kT_i}{L} + hT_\infty + h_r T_{sur} = \frac{k}{L}T_s + hT_s + h_r T_s$$

$$kT_i + hLT_\infty + h_r L T_{sur} = T_s(k + hL + h_r L)$$

$$E''_{in} = -k\frac{dT}{dx}$$

$$E''_{in} = k\frac{(T_1 - T_f)}{L}$$

$$E_{in} = \frac{6kw^2(T_1 - T_f)}{L}$$

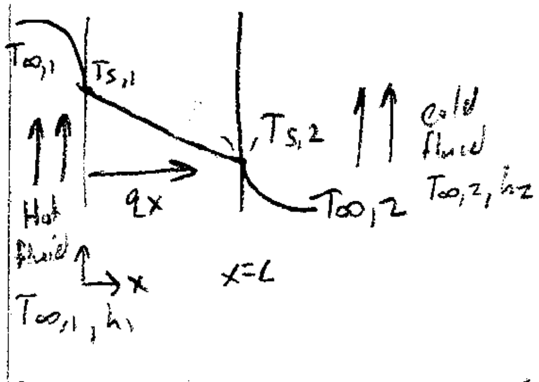
$$Mh_{sf} = \frac{6kW^2(T_1 - T_f)}{L}\Delta t$$

$$\frac{MH_{sf}L}{6kW^2(T_1 - T_f)} = \Delta t$$

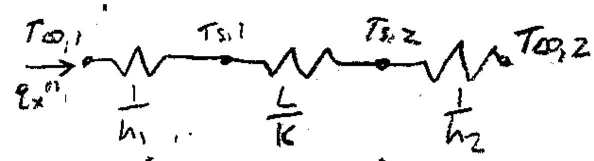
$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st} = \rho V C_p \frac{dT}{dt}$$

$$q_i'' A_i -$$

Problem 3.1: Consider the plane wall of Figure 3.1, seperating hot and cold fluids at temperatures $T_{\infty,1}$ and $T_{\infty,2}$, respectively. Using surface energy balances as boundary condition at $x=0$ and $x=L$ (see equation 2.32), obtain the temperature distribution within the wall and the heat flux in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k, and L.



Set up a thermal circuit



$$q_x''=\frac{(T_{\infty,1}-T_{\infty,2})}{\left(\frac{1}{h_1}+\frac{L}{k}+\frac{1}{h_2}\right)}=-k\frac{dT}{dx}$$

$$\int -k\frac{dT}{dx}\,dx=-k\,T(x)=\frac{(T_{\infty,1}-T_{\infty,2})}{\left(\frac{1}{h_1}+\frac{L}{k}+\frac{1}{h_2}\right)}x+C$$

1

$$T(x)=-\frac{(T_{\infty,1}-T_{\infty,2})}{\left(\frac{1}{h_1}+\frac{L}{k}+\frac{1}{h_2}\right)}\frac{x_1}{k}+C \qquad @\ x=0 \quad T(0)=T_{s,1}$$

$$T(0)=T_{s,1}=C$$

$$-k\frac{dT}{dx}\Big|_{x=0}=h_1(T_{s,1}-T_{\infty,1})$$

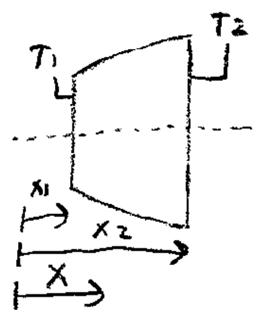
$$-\frac{(T_{\infty,1}-T_{\infty,2})}{\left(\frac{1}{h_1}+\frac{L}{k}+\frac{1}{h_2}\right)}=h_1(T_{s,1}-T_{\infty,1}) \qquad T_{s,1}=\frac{-1}{h_1}\frac{(T_{\infty,1}-T_{\infty,2})}{\left(\frac{1}{h_1}+\frac{L}{k}+\frac{1}{h_2}\right)}+T_{\infty,1}$$

$$\boxed{T(x)=-\frac{(T_{\infty,1}-T_{\infty,2})}{\left(\frac{1}{h_1}+\frac{L}{k}+\frac{1}{h_2}\right)}\left(\frac{x}{k}+\frac{1}{h_1}\right)+T_{\infty,1}} \qquad \boxed{q_x''=-k\frac{dT}{dx}=\frac{(T_{\infty,1}-T_{\infty,2})}{\left[\frac{1}{h_1}+\frac{1}{h_2}+\frac{L}{k}\right]}}$$

Problem 3.29: The diagram shows a conical section fabricated from pure aluminum. It is of circular cross section having diameter $D=ax^{1/2}$, where $a=0.5\,m^{1/2}$. The small end is located at $x_1=25$ mm and large end at $x_2=125$ mm. The end temperatures are $T_1=600$ K and $T_2=400$ K, while the lateral surface is well insulted.

a.) Derive $T(x)$

$$q_x=-kA\frac{dT}{dx}=-k\frac{\pi}{4}D^2\frac{dT}{dx}$$



$$\int_{T_1}^{T}-k\,dT=\int_{x_1}^{x}\frac{4q_x\,dx}{\pi a^2 x}$$

$$-k(T-T_1)=\frac{4q_x}{\pi a^2}\ln(x/x_1)$$

$$T(x) = \frac{-4q_x}{K\pi a^2}\ln(x/x_1) + T_1 \qquad @\ x = x_2 \quad T = T_2$$

$$T(x_2) = T_2 = \frac{-4q_x}{K\pi a^2}\ln(x_2/x_1) + T_1$$

$$q_x = k\,\frac{\pi a^2}{4}\frac{(T_2 - T_1)}{\ln(x_1/x_2)}$$

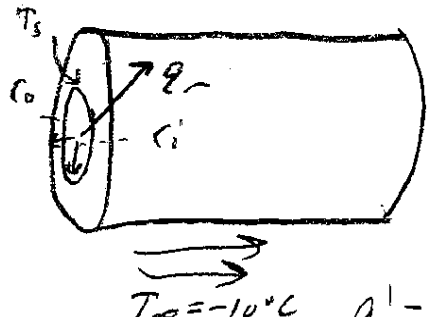
$$T(x) = \frac{(T_1 - T_2)}{\ln(x_1/x_2)}\ln(x/x_1) + T_1$$

b) Calculate the heat rate q_x.

$$q_x = -k\,\frac{\pi}{4}a^2 d\,\frac{(T_1 - T_2)}{\ln(x_1/x_2)}\,\frac{1}{x_1}\,\frac{x_1}{x} + T_1$$

$$q_x = -k\,\frac{\pi}{4}a^2 d\,\frac{(T_2 - T_1)}{\ln(x_1/x_2)}\,\frac{1}{x} + T_1 = 5.76\ kW$$

Problem 3.37: A thin electric heater is wrapped around the outer surface of a long cylindrical tube whose inner surface is maintained at a temp. of 5°C. The tube wall has inner and outer radii of 25 and 75 mm, respectively, and a thermal conductivity of 10 W/m·K. The thermal contact resistance between the heater and the outer surface of the tube (per unit length of the tube) is $R'_{t,c} = 0.01\ m\cdot K/W$. The outer surface of the heater is exposed to a fluid with $T_\infty = -10°C$ and a convection coefficient of $h = 100\ W/m^2\cdot K$. Determine the heater power per unit length of tube required to maintain the heater at $T_o = 25°C$.



$q' \rightarrow$ $T_o = 5°C$ — R_{CR} — T_i — R_c — T_∞

$$q' = \frac{(T_1 - T_0)}{R_{CR}} + (T_\infty + T_1)h2\pi r_o$$

$$q' = \frac{(25°C - 5°C)}{0.01 m\cdot K/W} + (-10°C + 25°C)100\tfrac{W}{m^2K}\cdot\pi\cdot 2\cdot 0.0\ m$$

$$\boxed{q' = 2471\ W/m}$$

$$k\frac{d^2T}{dx^2}+\dot{q}=0$$

$$\frac{d^2T}{dx^2}=-\frac{\dot{q}}{k}$$

$$\frac{d^2T}{dx^2}=\frac{\dot{q}_0\left(\frac{x}{L}-1\right)}{k}$$

$$\frac{dT}{dx}=\frac{\dot{q}_0}{2Lk}x^2-\frac{\dot{q}_0}{k}x+C_1$$

$$\frac{\dot{q}_0 L}{2k}-\frac{\dot{q}_0 L}{k}+C_1=0$$

$$\frac{dT}{dx}=\left(\frac{\dot{q}_0}{2L\cancel{k}}x^2-\frac{\dot{q}_0}{\cancel{k}}x+\frac{\dot{q}_0 L}{2\cancel{k}}\right)$$

$$\frac{dT}{dx}=\frac{\dot{q}_0}{6Lk}x^3-\frac{\dot{q}_0}{2k}x^2+\frac{\dot{q}_0 L}{2k}x+C_2$$

$T(0) = T_0$

$-k \frac{dT}{dx}\Big|_{x=L} = 0$

$-k \frac{dT}{dx} =$

$q_w'' = h_i (T_{s,i} - T_{\infty,i})$

$\frac{q_w''}{h_i} + T_{\infty,i}$

$$\frac{d^2T}{dx} = \frac{-\dot{q}}{k} = \frac{\dot{q}_0\left(\frac{x}{L} - 1\right)}{k}$$

$\frac{dT}{dx} =$

$$\frac{dT}{dx} = \frac{\dot{q}_0}{2kL} x^2 - \frac{\dot{q}_0}{k} x + C_1$$

$$\frac{\dot{q}_0}{2k} L - \frac{\dot{q}_0 L}{k} + C_1 = 0$$

$$\frac{\dot{q}_0 L}{2k} - \frac{2\dot{q}_0 L}{2k} + C_1 = 0$$

$$C_1 = \frac{\dot{q}_0 L}{2k}$$

$$T(x) = \frac{\dot{q}_0}{6kL} x^3 - \frac{\dot{q}_0}{2k} x^2 + C_1 x + C_2$$

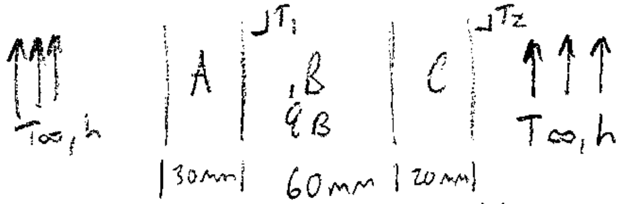
$$C_2 = T_0$$

$$q''(x) = -k \frac{dT}{dx}$$

$$q''(x) = -\cancel{k}\left(\frac{\dot{q}_0 x^2}{2\cancel{k}L} - \frac{\dot{q}_0}{\cancel{k}} x + \frac{\dot{q}_0 L}{2\cancel{k}}\right)$$

$$q''(x) = \frac{-\dot{q}_0 x^2}{2L} + \dot{q}_0 x - \frac{\dot{q}_0 L}{2}$$

Problem 3.73: Consider one-dimensional conduction in a plane composite wall. The outer surfaces are exposed to a fluid at 25°C and a convection heat transfer coefficient of 1000 $W/m^2 \cdot K$. The middle wall B experiences uniform heat generation $\dot{q}_B$, while there is no generation in walls A and C. The temperatures at the interfaces are $T_1 = 261°C$ and $T_2 = 211°C$.



$k_A = 25 \frac{W}{mK}$ $\quad k_C = 50 \frac{W}{mK}$

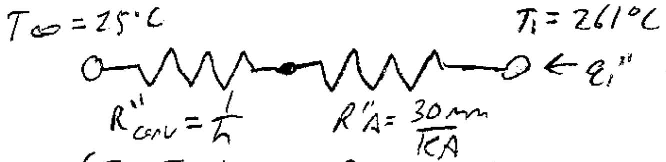
a) Assuming negligible contact resistance at the interfaces, determine the volumetric heat generation $\dot{q}_B$ and the thermal conductivity k_B.

Use wall B energy balance

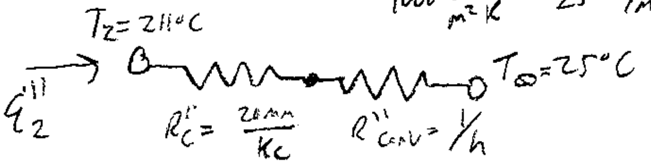
$$\cancel{\dot{E}''_{in}} - \dot{E}''_{out} + \dot{E}''_g = \cancel{\dot{E}''_{st}}$$

$$\dot{E}''_g = \dot{q}''(60mm) = q_1'' + q_2'' \Rightarrow \dot{q}'' = \frac{q_1'' + q_2''}{60mm}$$

Construct Thermal Circuits



$$q_1'' = \frac{(T_1 - T_\infty)}{\left(\frac{1}{h} + \frac{30mm}{k_A}\right)} = \frac{(261-25)\ K}{\left(\frac{1}{1000 \frac{W}{m^2K}} + \frac{0.030m}{25\ W/m \cdot K}\right)} = 107273 \ \frac{W}{m^2}$$



$$q_2'' = \frac{(T_2 - T_\infty)}{\left(\frac{1}{h} + \frac{20mm}{k_C}\right)} = \frac{(211-25)\ K}{\left(\frac{1}{1000 \frac{W}{m^2K}} + \frac{0.020m}{50\ W/mK}\right)} = 132,857 \ W/m^2$$

$$\boxed{\dot{q}_B = \frac{107273 \frac{W}{m^2} + 132,857\ W/m^2}{0.06\ m} = 4.00 \times 10^6 \ \frac{W}{m^3}}$$

To determine k_B we know...

$$T(x) = \frac{-\dot{q}_B}{2k_B} x^2 + C_1 x + C_2 \qquad q_x''(x) = -k_B\left(-\frac{\dot{q}_B}{k_B} x + C_1\right)$$

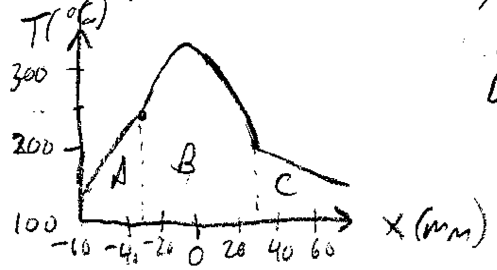
$$T(-L_B) = T_1 = -\frac{\dot{q}_B}{2k_B}(-L_B)^2 - C_1 L_B + C_2 \quad \text{where } T_1 = 261°C$$

$$T(L_B) = T_2 = \frac{-\dot{q}_B}{2k_B}(L_B)^2 + C_1 L_B + C_2 \quad \text{where } T_2 = 211°C$$

$$q_x''(-L_B) = -q_1'' = -k_B\left(-\frac{\dot{q}_B}{k_B}(-L_B) + C_1\right) \quad \text{where } q_1'' = 107,273 \ W/m^2$$

Problem 3.73 cont.: Solving the three equations yields $k_B = 15.3$ W/m·K

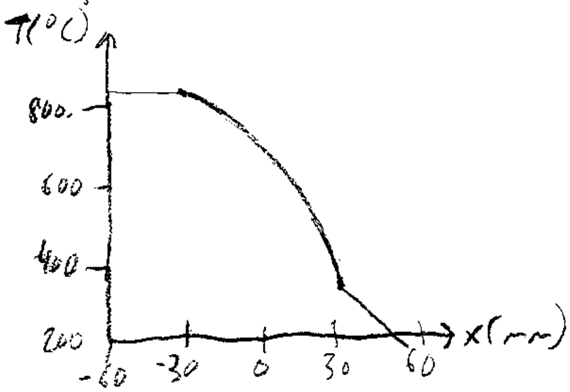
b) Plot the temperature distribution, showing its important features.



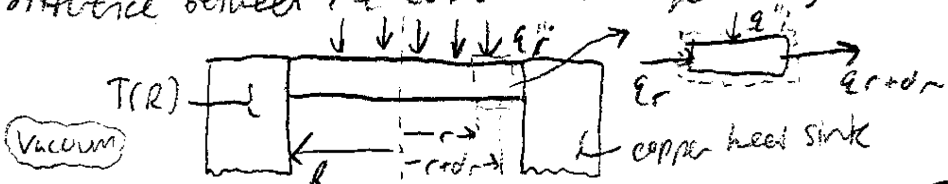
Quadratic in material B, but not symmetric
Linear in A and C.

c) Consider the conditions corresponding to a loss of coolant at the exposed surface of material A ($h=0$). Determine T_1 and T_2 and plot the temp. distribution.

Using the same analysis as part A, $T_1 = 835°C$ and $T_2 = 360°C$



Problem 3.98: Knowing the radius, thickness, and incident flux for a radiation heat gauge, find the expression relating flux to temperature difference between the center and edge of gauge.



$$q_r + q_i''(2\pi r dr) = q_{r+dr} \qquad q_r = -k(2\pi r t)\frac{dT}{dx} \qquad q_{r+dr} = q_r + \frac{dq_r}{dr}dr$$

$$q_i''(2\pi dr) = \frac{d}{dr}\left[(-k 2\pi r t)\frac{dT}{dr}\right]dr$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = -\frac{q_i''}{kt}r$$

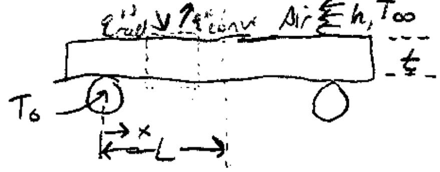
Integrating

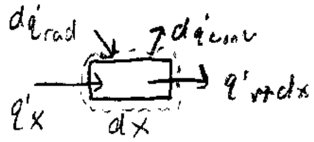
$$r\frac{dT}{dr} = -\frac{q_i'' r^2}{2kt} + C_1 \quad \text{and} \quad T(r) = \frac{-q_i'' r^2}{4kt} + C_1 \ln r + C_2$$

$$T(R) = -\frac{q_i'' R^2}{4kt} + C_2 \qquad C_2 = T(R) + \frac{q_i'' R^2}{4kt}$$

$$T(r) = \frac{q_i''}{4kt}(R^2 - r^2) + T(R) \qquad \boxed{q_i'' = \frac{4kt}{R^2}[T(0) - T(R)] = \frac{4kt}{R^2}\Delta t}$$

Problem 3.100. Knowing the surface conditions and thickness of a solar collection absorber plate and the temperature of the working fluid, determine the differential equation which governs plate temperature distribution and the form of the temperature distribution.





a.) $q'_x + dq'_{rad} = q'_{x+dx} + dq'_{conv}$

$q'_{x+dx} = q'_x + \left(\frac{dq'_x}{dx}\right)dx$

$dq'_{rad} = q'_{rad}\, dx$

$dq'_{conv} = h(T - T_\infty)\, dx$

So... $q'_{rad}\, dx = \left(\frac{dq'_x}{dx}\right)dx + h(T - T_\infty)\, dx$

$q'_x = -kt\, dT/dx$ $\boxed{\frac{d^2T}{dx^2} - \frac{h}{kt}(T - T_\infty) - \frac{q'_{rad}}{kt} = 0}$

b) choosing $y = T - T_\infty$, $\frac{d^2T}{dx^2} = \frac{d^2y}{dx^2}$, the differential equation becomes

$$\frac{d^2y}{dx^2} - \frac{h}{kt}y + \frac{q'_{rad}}{kt} = 0$$

y is of the form, $y(x) = C_1 e^{\lambda x} + C_2 e^{-\lambda x} + S/\lambda^2$

where $\lambda = \left(\frac{h}{kt}\right)^{1/2}$, $S = q'_{rad}/kt$

The boundary conditions are

$y(0) = T_0 - T_\infty = y_0$ $\frac{dy}{dx}\Big|_{x=L} = 0$

$y_0 = C_1 + C_2 + S/\lambda^2$

$\frac{dy}{dx}\Big|_{x=L} = C_1 \lambda e^{\lambda L} - C_2 \lambda e^{-\lambda L} = 0$ $C_2 = C_1 e^{2\lambda L}$

$C_1 = \frac{(y_0 - S/\lambda^2)}{(1 + e^{2\lambda L})}$ $C_2 = \frac{(y_0 - S/\lambda^2)}{(1 + e^{-2\lambda L})}$

So

$$y(x) = (y_0 - S/\lambda^2)\left(\frac{e^{\lambda x}}{1 + e^{2\lambda L}} + \frac{e^{-\lambda x}}{1 + e^{-2\lambda L}}\right) + \frac{S}{\lambda^2}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{-\dot{q}}{k} = \frac{-\dot{q}_0}{k}\left(1-\frac{x}{L}\right) = \frac{-\dot{q}_0}{k} + \frac{\dot{q}_0}{k}\frac{x}{L}$$

$$\frac{\partial T}{\partial x} = \frac{-\dot{q}_0 x}{k} + \frac{\dot{q}_0}{2kL}x^2 + C$$

$$\frac{-\dot{q}_0 L}{k} + \frac{\dot{q}_0 L}{2k} + C = 0 \qquad C = \frac{\dot{q}_0 L}{k} - \frac{\dot{q}_0 L}{2k} = \frac{\dot{q}_0 L}{2k}$$

$$T(x) = \frac{-\dot{q}_0 x^2}{2k} + \frac{\dot{q}_0 x^3}{6kL} + \frac{\dot{q}_0 L x}{2k} + C$$

$$-k\left(\frac{-\dot{q}_0 x}{k} + \frac{\dot{q}_0}{2kL}x^2 + \frac{\dot{q}_0 L}{2k}\right)$$

$$\dot{q}_0 x - \frac{\dot{q}_0 x^2}{2L} - \frac{\dot{q}_0 L}{2}$$

$$\dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{q_i'' \cancel{4\pi} r_i^2 - h(T-T_\infty)\cancel{4\pi} r_0^2}{\rho c_p \frac{4}{3}\cancel{\pi}(r_0^3 - r_i^3)}$$

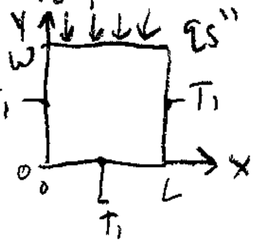
$$\frac{dT}{dt} = 0 = \frac{\cancel{3}(q_i'' r_i^2 - h(T-T_\infty) r_0^2)}{\cancel{\rho c_p (r_0^3 - r_i^3)}}$$

$$q_i'' r_i^2 - hTr_0^2 + hT_\infty r_0^2$$

$$hTr_0^2 = q_i'' r_i^2 + hT_\infty r_0^2$$

$$T = \frac{q_i'' r_i^2}{h r_0^2} + T_\infty$$

Problem 4.5: Know the boundary conditions on four sides of the rectangular plate. Find the temperature distribution.



Defining $\theta = T - T_\infty$, the differential equation is

$$\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$$

$\theta(0,y)=0 \quad \theta(L,y)=0 \quad \theta(x,0)=0 \quad k\frac{\partial\theta}{\partial y}\Big|_{y=W} = q_s''$

The solution is $\theta = \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{L}\sinh\frac{n\pi y}{L}$

To determine C_n, we apply the top surface boundary condition:

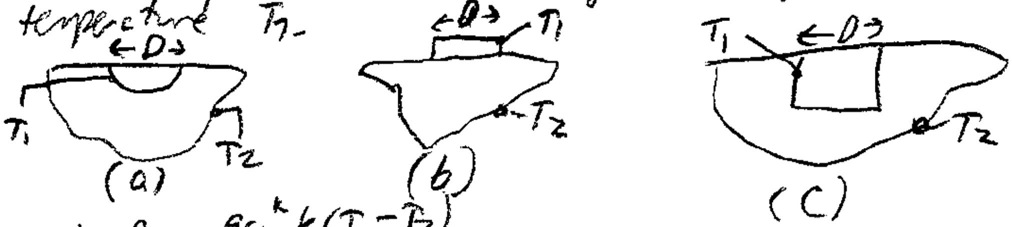
$$\frac{\partial\theta}{\partial y}\Big|_{y=W} = \sum_{n=1}^{\infty} C_n \frac{n\pi}{L}\sin\frac{n\pi x}{L}\cosh\frac{n\pi W}{L}.$$

The equation becomes $\frac{q_s''}{k} = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{L}$ where $A_n = \frac{\frac{q_s''}{k}\int_0^L \sin\frac{n\pi x}{L}dx}{\int_0^L \sin^2\frac{n\pi x}{L}dx}$

$$A_n = \frac{q_s''}{k}\frac{2}{\pi}\frac{(-1)^{n+1}+1}{n} \quad \text{so} \quad C_n = 2\frac{q_s''L}{k}\frac{(-1)^{n+1}+1}{n^2\pi^2\cosh(n\pi W/L)}$$

Problem 4.8: know the shape of objects at surface of semi-infinite medium

Find the shape factors between object at temperature T_1 and semi infinite temperature T_2.



$$q'' = \frac{q}{A_s} = \frac{q_{ss}^{*} k(T_1 - T_2)}{L_c} \quad \text{where } L_c = (A_s/4\pi)^{1/2} \text{ and } A_s \text{ is the area.}$$

$$S = \frac{q}{k(T_1-T_2)} = \frac{q'' A_s/2}{k(T_1-T_2)} = \frac{q_{ss}^{*} A_s}{2L_c} = \frac{q_{ss}^{*}(4\pi A_s)^{1/2}}{2} \quad \text{so} \quad S = q_{ss}^{*}(\pi A_s)^{1/2}$$

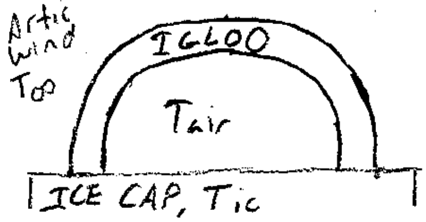
a) $S = 1(\pi \cdot \pi D^2)^{1/2} = \pi D$

b) $S = \frac{2\sqrt{2}}{\pi}\left(\pi\,\frac{\pi D^2}{2}\right)^{1/2} = 2D$

c) $S = 0.932(\pi 2D^2)^{1/2} = \sqrt{2\pi}(0.932)D = 2.34D$

d) $S = 0.961(\pi(2D^2 + 4D2D))^{1/2} = \sqrt{10\pi}(0.961)D = 5.39D$

Problem 4.28 An igloo is built in the shape of a hemisphere, with an inner radius of 1.8 m and walls of compacted snow that are 0.5 m thick. On the inside of the igloo, the surface heat transfer coefficient is 6 W/m²·K; on the outside, under normal wind conditions, it is 15 W/m²·K. The thermal conductivity of compacted snow is 0.15 W/m·K. The ice cap temperature is −20°C and has the same thermal conductivity as the compacted snow.



a.) Assuming that the occupants' body heat provides a continuous source of 320 W within the igloo, calculate the inside air temperature when the outside air temperature is $T_\infty = -40°C$. Be sure to consider heat losses through the floor of the igloo.

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{cv,c} + R_{wall} + R_{cv,o}} + \frac{T_{\infty,i} - T_{ic}}{R_{cv,f} + R_{cap}}$$

Ceiling Convection $R_{cv,c} = \dfrac{2}{h_i(4\pi r_i^2)} = \dfrac{2}{6\frac{W}{m^2\cdot K}\cdot 4\pi(1.8\,m)^2} = 8.19\times10^{-3}\,\dfrac{K}{W}$

Outside Convection $R_{cv,o} = \dfrac{2}{h_o(4\pi r_o^2)} = \dfrac{2}{15\frac{W}{m^2\cdot K}\cdot 4\pi(2.3\,m)^2} = 2.01\times10^{-3}\,\dfrac{K}{W}$

Floor Convection $R_{cv,f} = \dfrac{1}{h_i(\pi r_i^2)} = \dfrac{1}{6\,W/m^2\cdot K\cdot\pi(1.8\,m)^2} = 1.64\times10^{-2}\,\dfrac{K}{W}$

Wall conduction $R_{wall} = 2\left[\dfrac{1}{4\pi k}\left(\dfrac{1}{r_i} - \dfrac{1}{r_o}\right)\right] = 2\left[\dfrac{1}{4\pi\cdot 0.15\frac{W}{mK}}\left(\dfrac{1}{1.8\,m} - \dfrac{1}{2.3\,m}\right)\right]$
$= 1.28\times10^{-1}\,K/W$

Ice Cap Conduction $R_{cap} = \dfrac{1}{kS} = \dfrac{1}{4kr_i} = \dfrac{1}{4\cdot 0.15\,W/m\cdot K\cdot 1.8\,m} = 9.26\times10^{-1}\,\dfrac{K}{W}$

$$q = 320\,W = \frac{T_{\infty,i} + 40°C}{(8.19\times10^{-3} + 2.01\times10^{-3} + 1.28\times10^{-1})\frac{K}{W}} + \frac{T_{\infty,i} + 20°C}{(1.64\times10^{-2} + 9.26\times10^{-1})\frac{K}{W}}$$

$$\boxed{T_{\infty,i} = 1.2°C}$$

Problem 10.7: Known: $h = C(\Delta T_e)^n \left(\frac{p}{p_a}\right)^{0.4}$ where $C=5.56$ and $n=3$ for $15 < q_s'' < 235 \frac{kW}{m^2}$

Find: Compare predictions using this expression with the Rohsenow effect @ $\Delta T_e = 10°C$ and $p_1 = 2$ bar and $p_2 = 5$ bar

Solution: $p = 2$ bar $\quad h = 5.56(10)^3 (2\,bar/1.0133\,bar)^{0.4} = 7298\ W/m^2K$ $\boxed{q_s'' = 73\ kW/m^2}$

$p = 5$ bar $\quad h = 5.56(10)^3 (5\,bar/1.0133\,bar)^{0.4} = 10529\ W/m^2K$ $\boxed{q_s'' = 105\ kW/m^2}$

Using the Rohsenow effect equation.

$p = 2$ bar $\quad q_s'' = 230.7\times10^{-6}\frac{Ns}{m^2} \cdot 2203\times10^3 \frac{J}{kg}\left[\frac{9.8\,m/s^2(942.7-1.1082)\frac{kg}{m^3}}{54.97\times10^{-3}\,N/m}\right]^{1/2}\left[\frac{4244.3\,J/kg K \cdot 10K}{0.013\times2203\times10^3\frac{J}{kg}\,1.43}\right]^3$

$\boxed{q_s'' = 232\ kW/m^2}$

$p = 5$ bar $\quad \boxed{q_s'' = 439\ kW/m^2}$

Problem 10.37: Known: Saturated water at 1 atm and velocity 2 m/s in cross flow over a heater element of 5 mm diameter

Find: Max heating rate, q'

Solution: The Lienhard-Eichhorn correlation is used to estimate q''_{max}. The equation simplifies to the following equation by assuming high-velocity region flow.

$$q''_{max} = \frac{\rho_v h_{fg} V}{\pi}\left[\frac{1}{169}\left(\frac{\rho_l}{\rho_v}\right)^{3/4} + \frac{1}{19.2}\left(\frac{\rho_l}{\rho_v}\right)^{1/2}\left(\frac{\sigma}{\rho_v V^2 D}\right)^{1/3}\right]$$

So $q''_{max} = \frac{1}{\pi} 0.5955\,kg/m^3 \times 2257\times10^3\,J/kg \times 2\,m/s\left[\frac{1}{169}\left(\frac{957.9}{0.5955}\right)^{3/4} + \frac{1}{19.2}\left(\frac{957.9}{0.5955}\right)^{1/2}\left(\frac{58.9\times10^{-3}\,N/m}{0.5955\,kg/m^3\,2^2\,m^2/s^2\,0.005}\right)^{1/3}\right]$

$q''_{max} = 4.331\ MW/m^2$

The high-velocity region assumption is satisfied if

$$\frac{q''_{max}}{\rho_v h_{fg} V} < \frac{0.275}{\pi}\left(\frac{\rho_l}{\rho_v}\right)^{1/2} + 1$$

$$\frac{4.331\times10^6\,W/m^2}{0.5955\,kg/m^3\times2257\times10^3\,J/kg\times2\,m/s} = 1.61 < \frac{0.275}{\pi}\left(\frac{957.9}{0.5955}\right)^{1/2} + 1 = 4.51$$

The inequality is satisfied. The maximum heat rate is then..

$$q_{max} = q''_{max}\pi D = 4.331\,MW/m^2 \cdot \pi \cdot 0.005\,m = 68.0\ kW/m$$

Problem 10.41: Known: Saturated steam @ 0.1 bar condenses on the outside of a brass tube with water flowing on the inside of the tube, convection coefficient prescribed

Find: Steam condensation rate per unit length of the tube.

Solution: The condensation rate is defined as follows.

$\dot m' = q'/h'_{fg}$ where $q' = U_o \pi D_o (T_{sat} - T_m)$

$$U_o = \left[\frac{1}{h_o} + \frac{D_o/2}{k}\ln\frac{D_o}{D_i} + \frac{D_o}{D_i}\frac{1}{h_i}\right]^{-1}$$

$$U_o = \left[\frac{1}{6800\,W/m^2K} + \frac{0.0095\,m}{110\,W/mK}\ln\frac{19}{16.5} + \frac{19}{16.5}\frac{1}{5200\,W/m^2K}\right]^{-1} = 2627\ W/m^2K$$

So $\dot m' = U_o \pi D_o (T_{sat} - T_m)/h'_{fg}$

So $\dot m' = 2627\,W/m^2K\,\pi(0.019\,m)(320-303)\,K/2410\times10^3\,J/kg = 1.11\times10^{-3}\,kg/s$

$\boxed{\dot m' = 1.11\times10^{-3}\,kg/s}$

NUCL 351 Nuclear Thermal Hydraulics – II (Heat Transfer)

Class schedule:	Monday, Wednesday and Friday 11:30 am – 12:20 pm
Class room:	GRIS280
Instructor:	T. Hibiki Office hours: Tuesday and Thursday 10:00 am -11:00 am, Wednesday 12:20 pm – 13:20 pm (**<u>No email policy</u>**, Please visit my office, No appointment required), The office hour is subject to change due to unexpected commitments.
Textbook:	F. P. Incropera, D. P. DeWitt, T. H. Bergman & A. S. Lavine "Fundamentals of Heat and Mass Transfer (6th edition, any edition is OK)," John Wiley & Sons (2007)
References:	R. B. Bird, W. E. Stewart & E. N. Lightfoot "Transport Phenomena (Revised 2nd edition)," John Wiley & Sons (2007) N. E. Todreas & M. S. Kazimi "Nuclear Systems I," Hemisphere Publishing (1990)
Attendance:	Since class discussion is a major course ingredient, regular attendance is mandatory.
Homework:	Homework sets will be assigned as weekly sets in the Friday class and are due at 11:30 in the next Friday class unless otherwise noted. Late homework will not be accepted for grade, however all homeworks should be submitted before April 30. **<u>If all homeworks are not submitted by April 30, it will affect the final grade</u>**. HW grade will be Full (reasonable efforts), Half (not sufficient efforts), Zero (late submission) or Negative Points (no submission). HW can be submitted only in the class room. Please do not put your HW in my office or mail box.
Course grading:	Exams (6) 72 %, Homework 28 %
Final grade scale:	A=85-100; B=75-84; C=65-74; D=55-64; F<54
Course description:	This course concerns the energy exchange processes due to temperature differences (heat transfer) that are relevant to nuclear energy systems. The relevant empirical laws of energy in motion due to temperature gradients, heat, will be studied with applications to nuclear systems as well as many other engineering processes. In particular, the Fourier's law of heat conduction, the Newton cooling law of convection, and the Stefan-Boltzmann law of radiation will be studied. Particular emphasis will be placed on the special processes of convective heat transfer of importance to nuclear systems, in particular, free convection, natural circulation, boiling and condensation.
NUCL351 topics:	Introduction, Introduction to Conduction, One-Dimensional, Steady-State Condition, Two-Dimensional, Steady-State Conduction, Transient Conduction, Introduction to

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Class	Date	Text sections	Read pages	Remarks	HW set	Due date
			(Six Edition)			
1	1/13			Introduction		
2	1/15	1.1-1.7	2-39			
3	1/17	2.1-2.5	58-82		Set 1	1/24
4	1/20			Martin Luther King Holiday		
5	1/22	3.1-3.2	96-116			
6	1/24	3.3-3.5	116-137		Set 2	1/31
	1/27	3.6-3.8	137-168			
7	1/29	4.1-4.3	202-212			
8	1/31	4.4	212-222		Set 3	2/7
9	2/3	4.5-4.6	222-235			
10	2/5	5.1-5.2	256-263			
11	2/7	5.3-5.5	263-276	Taught by TA	Set 4	2/14
12	2/10	5.6	276-283	Taught by TA		
13	2/12			Exam 1		
14	2/14			Exam 2		
15	2/17	5.7-5.9	283-301			
16	2/19	5.10.1	302-310			
17	2/21	5.10.2-5.11	310-318		Set 5	2/28
18	2/24	6.1-6.2	348-358			
19	2/26	6.3-6.4	359-367			
20	2/28	6.5-6.6	367-377		Set 6	3/7
21	3/3	6.7-6.9	377-386			
22	3/5	7.1-7.3	402-422			
23	3/7	7.4	423-433		Set 7	3/14
24	3/10	7.5-7.9	433-456	Taught by TA		
25	3/12	8.1-8.3	486-504	Taught by TA		
26	3/14	8.4	505-513	Taught by TA	Set 8	3/28
	3/17			SPRING BREAK		
	3/19			SPRING BREAK		
	3/21			SPRING BREAK		
27	3/24	8.5-8.10	514-533			
28	3/26			Exam 3		
29	3/28			Exam 4		
30	3/31	9.1-9.6	560-584			
31	4/2	9.7-9.11	584-596			
32	4/4	10.1-10.3	620-626		Set 9	4/11
33	4/7	10.4-10.5	627-640			
34	4/9	10.6-10.8	641-651			
35	4/11	10.9-10.12	651-656		Set 10	4/18
36	4/14	11.1-11.2	670-675			
37	4/16	11.3	675-686			
38	4/18	11.4-11.7	686-706		Set 11	4/25
39	4/21	12.1-12.3	724-744			
40	4/23			Exam 5		
41	4/25			Exam 6		
42	4/28			Spcial lecture (Two-phase flow)		
43	4/30			Spcial lecture (Two-phase flow)		
44	5/2			Spcial lecture (Two-phase flow)		

$$\frac{\partial^2 u}{\partial y^2} = \frac{-g}{\mu_\ell}(\rho_\ell - \rho_v)$$

$$\frac{\partial u}{\partial y} = \frac{-g}{\mu_\ell}(\rho_\ell - \rho_v)y + C \qquad u = \frac{-g}{\mu_\ell}(\rho_\ell - \rho_v)\frac{y^2}{2} + Cy \cancel{+C}$$

$$C = \frac{g}{\mu_\ell}(\rho_\ell - \rho_v)\delta$$

$$U(y) = \frac{g}{\mu_\ell}(\rho_\ell - \rho_v)\delta^2\left(\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right)$$

ρ_ℓ

$$\frac{y^2}{2\delta} - \frac{1}{6}\frac{y^3}{\delta^2}$$

$$\frac{3}{3\cdot 2} - \frac{1}{6}$$

$$d\dot{m}\, h_{fg} = k\frac{(T_{sat} - T_s)}{\delta}\, b\, dy$$

$$\frac{d\dot{m}}{b\,dy} = \frac{\cancel{k_\ell}(\cancel{T_{sat}} - \cancel{T_s})}{\cancel{h_{fg}}\,\delta} = \frac{\cancel{g}}{\cancel{\mu_\ell}}\cancel{\rho_\ell}(\cancel{\rho_\ell} - \cancel{\rho_v})\delta^2\frac{d\delta}{\cancel{dx}}$$

$$\frac{k_\ell(T_{sat} - T_s)\mu_\ell}{h_{fg}\, g\rho_L(\rho_L - \rho_v)}dx = \delta^3 d\delta$$

$$\cancel{\frac{1}{4}\delta^4 = \frac{k_\ell(T_{sat} - T_s)\mu_\ell x}{h_{fg}\, g\rho_L(\rho_L - \rho_v)}}$$

$$q_s'' = A(T_s - T_{sat})^3$$

$$\left(\frac{q_s''}{A}\right)^{1/3} = T_s - T_{sat}$$

$$\dot{m}' = \frac{q'}{h_{fg}'} = \frac{U_0 \pi D_0 (T_{sat} - T_m)}{h_{fg}'}$$

$$q_0'' = K_s \frac{(T_s - T_0)}{L}$$

$$\frac{q_0'' L}{K_s}$$