center: $[X_c, Y_c]$

point: $[X_p, Y_p]$

$X = X_p - X_c$

$Y = Y_p - Y_c$

$$atan2 = \begin{cases} 2\arctan\left(\frac{\sqrt{x^2+y^2} - x}{y}\right) & \text{if } Y \neq 0 \\ 0 & \text{if } x > 0 \ \& \ Y = 0 \\ \pi & \text{if } x < 0 \ \& \ Y = 0 \\ \text{undefined} & \text{if } x = 0 \ \& \ Y = 0 \end{cases}$$

$$\frac{\partial atan2(Y,X)}{\partial X} = \frac{-Y}{X^2+Y^2}$$

$$\frac{\partial atan2(Y,x)}{\partial Y} = \frac{X}{x^2+Y^2}$$

JACOBIAN MATRIX #1

$$\left[\frac{\partial ellipseOrientation}{\partial X_c}, \frac{\partial ellipseOrientation}{\partial Y_c}\right]$$

JACOBIAN MATRIX #2

$$\left[\frac{\partial ellipseOrientation}{\partial X_p}, \frac{\partial ellipseOrientation}{\partial Y_p}\right]$$

JACOBIAN MATRIX CALCULATION:

$ellipseOrientation(center, point) = atan2(X_p - X_c, Y_p - Y_c) = atan2(Y, X)$

$$\frac{\partial atan2(\overbrace{Y_p - Y_c}^{Y}, \overbrace{X_p - X_c}^{X})}{\partial X_c} = \frac{\partial atan2(Y,X)}{\partial X} \cdot \frac{\partial X}{\partial X_c} = \left(\frac{-Y}{X^2+Y^2}\right)\cdot(-1) \quad (1)$$

$$\frac{\partial atan2(Y,X)}{\partial Y_c} = \frac{\partial atan2(Y,X)}{\partial Y} \cdot \frac{\partial Y}{\partial Y_c} = \left(\frac{X}{x^2+Y^2}\right)\cdot(-1) \quad (2)$$

$$\frac{\partial atan2(Y,X)}{\partial X_p} = \frac{\partial atan2}{\partial X} \cdot \frac{\partial X}{\partial X_p} = \left(\frac{-Y}{X^2+Y^2}\right)(1) \quad (3)$$

$$\frac{\partial atan2(Y,X)}{\partial Y_p} = \frac{\partial atan2}{\partial Y} \cdot \frac{\partial Y}{\partial Y_p} = \left(\frac{X}{X^2+Y^2}\right)(1) \quad (4)$$

JACOBIAN MATRIX FOR CENTER POINT 2

$$\left[\frac{Y}{x^2+Y^2}, \frac{-X}{x^2+Y^2}\right]$$

JACOBIAN FOR AXIS POINT 2

$$\left[\frac{-Y}{x^2+Y^2}, \frac{X}{x^2+Y^2}\right]$$

UNIT TEST HAND CALCULATION

center: [5, 5]

point: [10, 10]

$X = 10 - 5 = 5$

$Y = 10 - 5 = 5$

$orientation = 2 \cdot \arctan\left(\frac{\sqrt{50} - 5}{5}\right) = 0.7854$ radians

JACOBIAN 1 (for center) $= \left[\frac{5}{50}, \frac{-5}{50}\right] = [0.1, -0.1]$

JACOBIAN 2 (for axis point) $= \left[\frac{-5}{50}, \frac{5}{50}\right] = [-0.1, 0.1]$