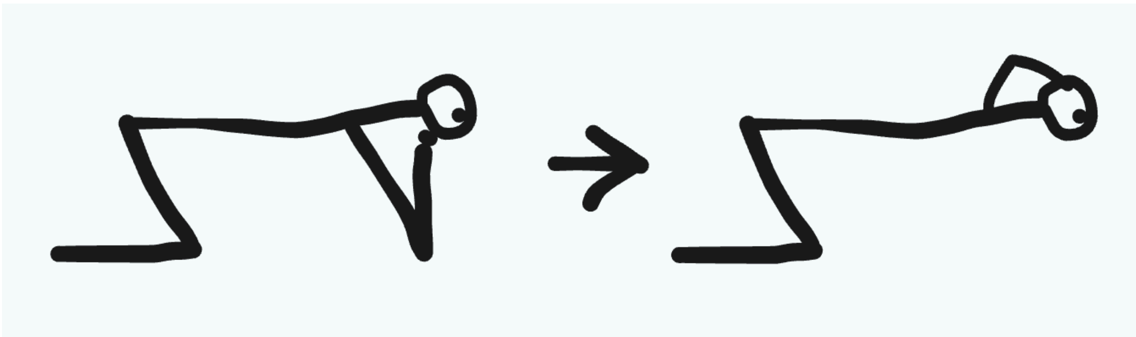
## Background

There is a common social media trend (is social media redundant) where two people of the opposite sex kneel and lean forward with their hands on the backs of their heads like this: (is "like this" redundant? if omitted, it is implied that the proceding image will refer to the text )



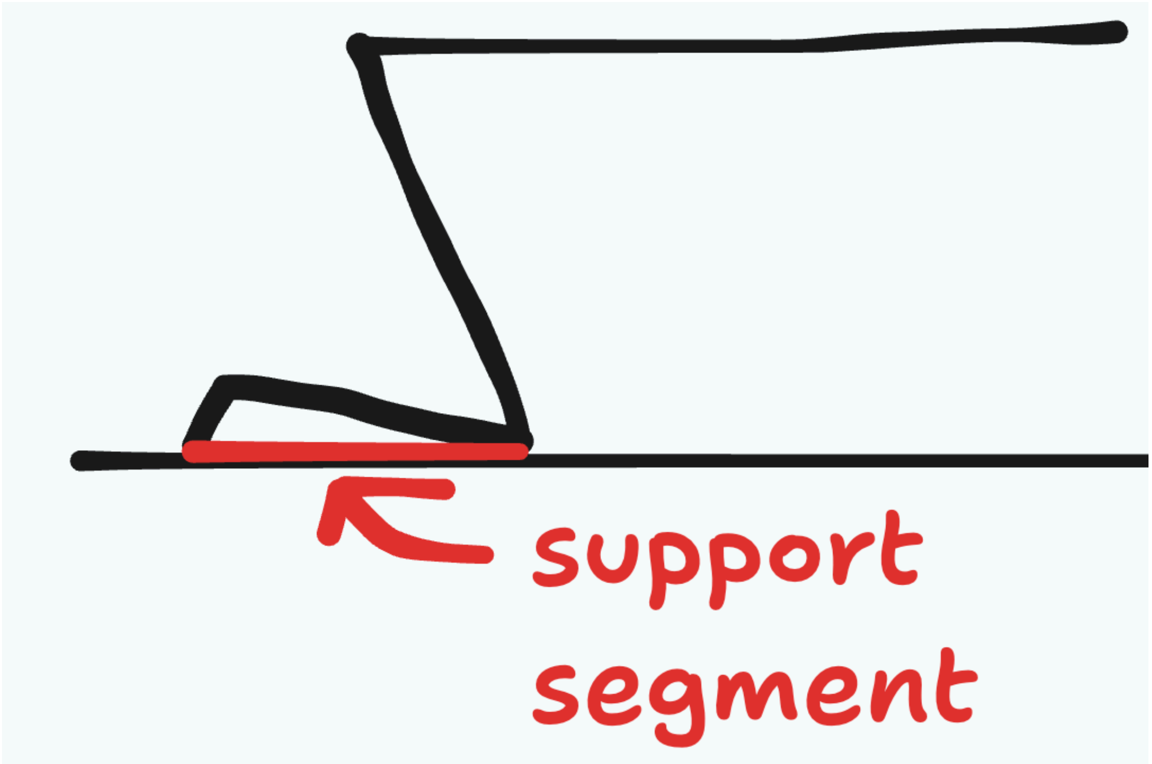
The woman is able to maintain the position, while the man instantly falls forwards on his face.

I assumed the videos were fake and the man was falling intentionally (have there been cases of this? why does this happen) until I tried and failed to keep myself upright. I then asked:

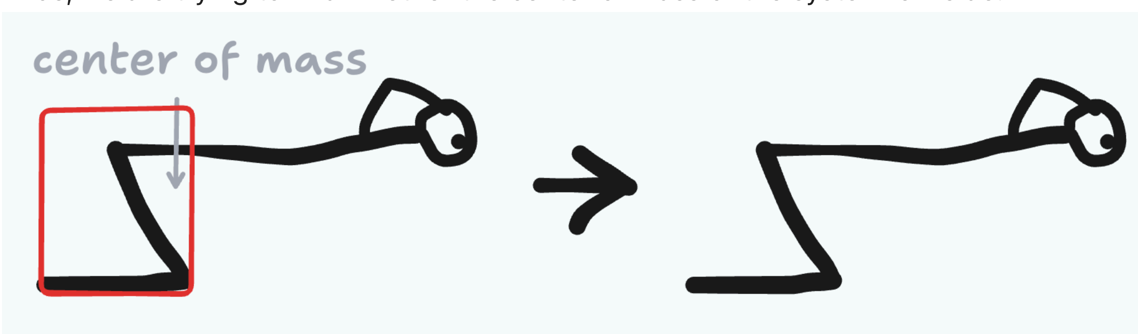
## Why do men fall and women don't?

We can model our system in 2d since we are only concerned with tipping in a single direction (forward)

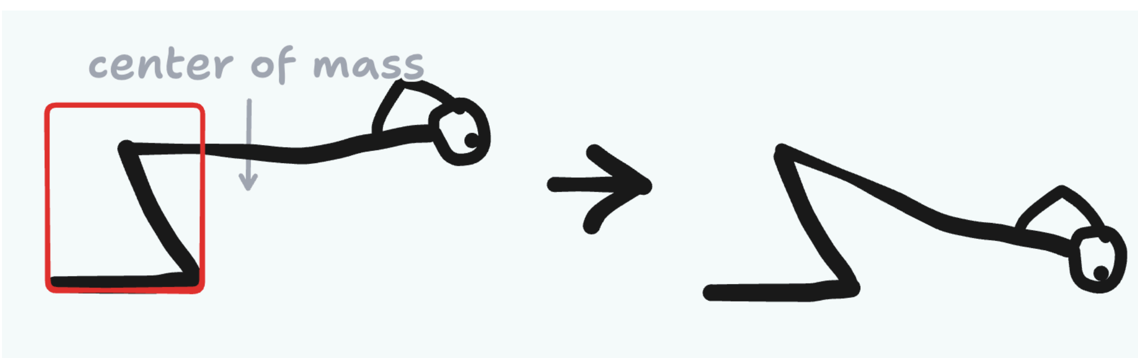
in 2d, objects tip if and only if their center of mass lies outside their support segment (base). In our system, the support segment is the part of the leg between the toe and the knee:




Thus, we are trying to find whether the center of mass of the system is inside:

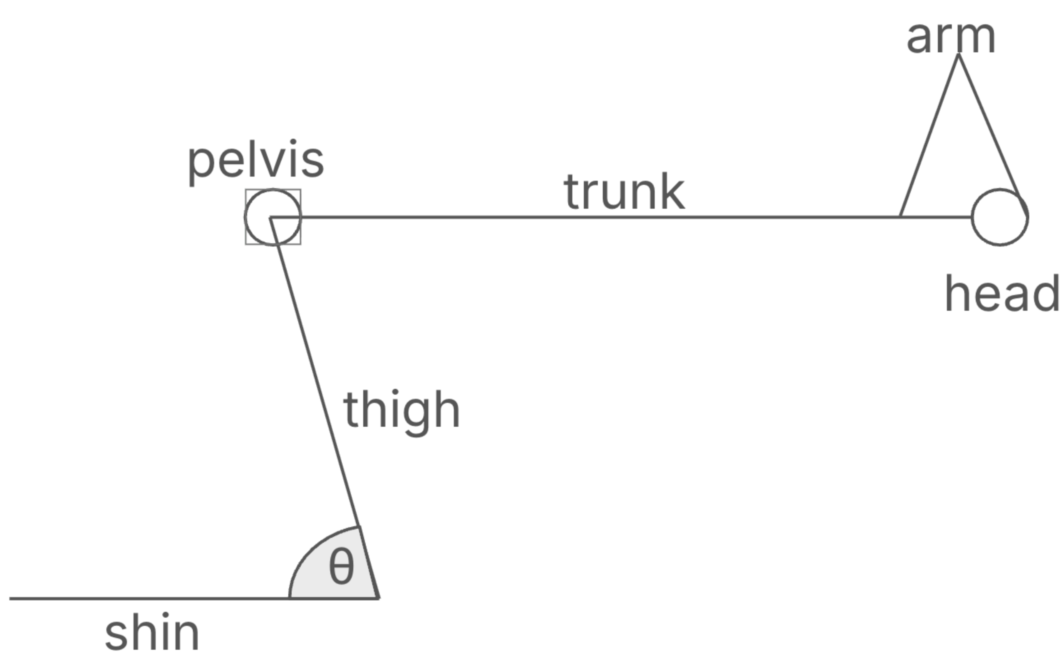



or outside:




of the area between foot and knee.

I found all the data I needed in this NASA paper, which records the length (as a % of height), mass (as a % of body weight), and center of mass of each of all relevant body segments. I then created a more detailed model of the person in the position:




I used the diagram to derive the following equations.

**Notation:** The parameter $p$ represents the fraction of a segment's length from the proximal joint to its center of mass. For example, if $p_{\text{shin}} = 0.4$, the center of mass of the lower leg is located 40% of the way from the knee to the ankle.

I took the angle between the calf and the back of the knee to be $\theta = 60^\circ$ since that was the average angle I saw in the videos.

I define $d = \text{length}_{\text{thigh}} \cdot \cos(\theta)$ as the horizontal projection of the thigh.

The horizontal position of each body part's center of mass is:

$$x_{\text{com,shin}} = \text{length}_{\text{shin}} \cdot (1 - p_{\text{shin}})$$

$$x_{\text{com,thigh}} = \text{length}_{\text{shin}} - d \cdot (1 - p_{\text{thigh}})$$

$$x_{\text{com,pelvis}} = \text{length}_{\text{shin}} - d + \text{length}_{\text{pelvis}} \cdot p_{\text{pelvis}}$$

$$x_{\text{com,upper body}} = \text{length}_{\text{shin}} - d + \text{length}_{\text{upper body}} \cdot p_{\text{upper body}}$$

$$x_{\text{com,arm}} = \text{length}_{\text{shin}} - d + \text{length}_{\text{upper body}} \cdot P_{\text{AS}} + (\text{length}_{\text{upper body}} + \text{length}_{\text{head}}) \cdot p_{\text{ar}}$$

$$x_{\text{com,head}} = \text{length}_{\text{shin}} + \text{length}_{\text{upper body}} - d + \text{length}_{\text{head}} \cdot p_{\text{head}}$$

Here $P_{\text{AS}}$ is the attachment point of the arm along the upper body (as a fraction from the pelvis).

The total center of mass is:

$$x_{\text{com}} = \sum_{\text{part} \in \text{parts}} x_{\text{com, part}} \cdot w_{\text{part}}$$

where $w_{\text{part}}$ is the mass fraction of each body part.

translated to code:

In [1]:
```
from math import cos as c, pi
def cos(T):
    return c((T * pi) / 180)

# pictures show ~ 70 degree angle
T = 60

# arm starts
P_AS = 0.9

# other definitions
parts = {"s", "t", "p", "u", "a", "h"}

l = {
    "m": { "t": 0.245, "s": 0.246, "p": 0.130, "u": 0.320, "a": 0.440, "h":
    "f": { "t": 0.240, "s": 0.245, "p": 0.130, "u": 0.320, "a": 0.436, "h":
}

p = {
    "m": { "t": 0.44, "s": 0.43, "p": 0.50, "u": 0.53, "a": 0.47, "h": 0.50
    "f": { "t": 0.44, "s": 0.43, "p": 0.50, "u": 0.53, "a": 0.47, "h": 0.50
}

w = {
    "m": { "t": 0.210, "s": 0.096, "p": 0.140, "u": 0.300, "a": 0.109, "h":
    "f": { "t": 0.2125, "s": 0.0811, "p": 0.1800, "u": 0.2560, "a": 0.0956,
}


# primary function
def x_com(l, p, part):
    d = l['t'] * cos(T)
    if part == "s": # shin (lower leg)
        return l['s'] * (1 - p['s'])
    elif part == "t": # thigh (upper leg)
        return l['s'] - d * (1 - p['t'])
    elif part == "p": # pelvis (butt)
        return l['s'] - d
    elif part == "u": # upper body/trunk
        return l['s'] - d + l['u'] * p['u']
    elif part == "a": # entire arm
        # bent on elbow; hands on back of head
        # treat flexion as compression preserving p
        return l['s'] - d + l['u'] * P_AS  + (l['u'] + l['h']) * p['a']
    elif part == "h": # head
        return l['s'] + l['u'] - d + l['h'] * p['h']
```

```
for g in ["m", "f"]:
    print("--------------------------------")
    curr_x_com = round(sum(x_com(l[g], p[g], part) * w[g][part] for part in
    curr_shank = l[g]['s']
    print(f"for gender {g}:")
    print(f"center of mass: {curr_x_com}")
    print(f"shank length: {curr_shank}")
    if curr_x_com < curr_shank:
        print(f"since this gender's center of mass is, on average, within th
    else:
        print(f"since this gender's center of mass is, on average, outside t
```

```
--------------------------------
for gender m:
center of mass: 0.2646
shank length: 0.246
since this gender's center of mass is, on average, outside their base, they
are likely to fall forward
--------------------------------
for gender f:
center of mass: 0.2407
shank length: 0.245
since this gender's center of mass is, on average, within their base (feet a
nd lower legs), they are not likely to fall forward
```

The output shows that the average male's center of mass lies outside the supporting segment, where the average female's center of mass lies within the supporting segment. this explains why men fall more often than women.

how much more often? we could model $x_{\text{com}}$ as a probability distribution by applying the same operations on normally distributed measurements, since elementary functions applied to normal distributions yield normal distributions. Taking $\int_0^{\text{length}_{\text{shank}}} x_{com}(x)dx$ would result in the percentage of individuals for that gender who avoid falling in the challenge.

I did not do this because:

1. only the scalar means of the measurements were present in the paper I used for data
2. the resulting distribution still wouldn't be entirely accurate because of the inherent imprecision of contorting and segmenting the body

Thank you for reading. Investigations like these are intellectually trivial and seldom have practical benefits, but it is satisfying to understand why things behave the way they do.