This is a more rigorous derivation of the Newton-Raphson style solution of the inverse kinematics problem of Chapter 6 of Lynch and Park's Modern Robotics[1].

Since the essence of the Newton-Raphson method is to compute the increment through derivative (Jacobian in the $R^n$ case), so my aim is to obtain the Jacobian of pose $T$ (Lie group SE(3)) with respect to angle $\theta$ (joint space).

We have the following chain rule for computing Jacobian:

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \xi}\frac{\partial \xi}{\partial \theta}$$

where $\xi$ is the screw (Lie algebra se(3))

According to [2] 7.115a,

$$\frac{\partial T}{\partial \xi} = J_r$$

We use right Jacobian in this derivation.

According to [2] 7.219, we have the relation between the time derivative of the screw $\xi$ (Lie algebra se(3)) and the twist:

$$V^b = J_r \dot{\xi}$$

According to [1] 5.15 we have:

$$V^b = J_b(\theta)\dot{\theta}$$

Thus we get the Jacobian between screw and joint angle:

$$\frac{\partial \xi}{\partial \theta} = J_r^{-1} J_b(\theta)$$

So finally we get:

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \xi}\frac{\partial \xi}{\partial \theta} = J_r J_r^{-1} J_b(\theta) = J_b(\theta)$$

A similar definition of Jacobian of Lie group with respect to Lie algebra could be found in [3].

# References


[1] Kevin M. Lynch and Frank C. Park. *Modern Robotics*. Cambridge University Press, 2017.

[2] Timothy D. Barfoot. *State Estimation for Robotics*. Cambridge University Press, 2017.

[3] Joan Sola, Jeremie Deray, and Dinesh Atchuthan. A micro Lie theory for state estimation in robotics. `https://arxiv.org/abs/1812.01537`.