This is a more rigorous derivation of the Newton-Raphson style solution of the inverse kinematics problem of Chapter 6 of Lynch and Park's Modern Robotics[1].

Since the essence of the Newton-Raphson method is to compute the increment through derivative (Jacobian in the R^n case), so my aim is to obtain the Jacobian of pose T (Lie group SE(3)) with respect to angle θ (joint space).

We have the following chain rule for computing Jacobian:

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \xi}\frac{\partial \xi}{\partial \theta}$$

where ξ is the screw (Lie algebra se(3))

According to [2] 7.115a,

$$\frac{\partial T}{\partial \xi} = J_r$$

We use right Jacobian in this derivation.

According to [2] 7.219, we have the relation between the time derivative of the screw ξ (Lie algebra se(3)) and the twist:

$$V^b = J_r\dot{\xi}$$

According to [1] 5.15 we have:

$$V^b = J_b(\theta)\dot{\theta}$$

Thus we get the Jacobian between screw and joint angle:

$$\frac{\partial \xi}{\partial \theta} = J_r^{-1}J_b(\theta)$$

So finally we get:

$$\frac{\partial T}{\partial \theta} = \frac{\partial T}{\partial \xi}\frac{\partial \xi}{\partial \theta} = J_r J_r^{-1}J_b(\theta) = J_b(\theta)$$

A similar definition of Jacobian of Lie group with respect to Lie algebra could be found in [3].

[2] 7.219 depends on 7.217. 7.217 depends on 7.200. The derivation of 7.200 can be found in 4.5.2 of [4].

Note, the definition of Jacobian of pose T with respect to screw ξ requires moving from SE(3) to se(3). This naturally leads to that when solving the equation, we need to do a log operation on $T_{sb}^{-1}T_{sd}$. Another way to look at the equation is that the equation is:

$$ln(T_{sb}(\xi)) = ln(T_{sd})$$

The minus operation corresponds to left multiplication to matrix inverse.

$$x - y$$

corresponds to

$$Y^{-1}X$$

The definition of Jacobian on SE(3):

$$ln(T^{-1}(\xi)T(\xi + \delta\xi))^{\vee} = J_r \delta\xi$$

类比于一维实数域 R 上的牛顿法。最重要的对应是：减法对应到左乘矩阵的逆然后取对数。对数从 SE(3) 还原到 se(3) 上，因为自变量 ξ 在 se(3) 上，因变量 T 在 SE(3) 上。有了这个对应之后就可以定义 Jacobian 了。

在牛顿法里的目标差 $c - f(x)$ 也就对应到 $ln(T_{sb}^{-1}(\theta)T_{sd})^{\vee}$

参考文献

[1] Kevin M. Lynch and Frank C. Park. *Modern Robotics.* Cambridge University Press, 2017.

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[4] J. M. Selig. *Geometric Fundamentals of Robotics.* Springer, 2005.