Unification: Exercises

1. Let $\sigma = [g(Y)/X]$ and $\tau = [a/Y, g(X)/Z]$ be two substitutions and let $t = f(X, h(X, Y), Z)$ be a term. Calculate the following:

   (a) $t\sigma$;

   (b) $t\tau$;

   (c) $(\sigma \circ \tau)$ and $(\tau \circ \sigma)$;

   (d) $t(\sigma \circ \tau)$ and $t(\tau \circ \sigma)$.

2. Write down the steps taken by the unification algorithm for each of the following pairs of terms. If the algorithm succeeds then write down the MGU and the corresponding common instance.

   (a) $g(h(X)) \stackrel{?}{=} g(Y)$

   (b) $g(h(X)) \stackrel{?}{=} g(a)$

   (c) $f(X, a) \stackrel{?}{=} f(b, Y)$

   (d) $f(X, X) \stackrel{?}{=} f(a, g(Y))$

   (e) $f(X, a) \stackrel{?}{=} f(g(X), a)$

   (f) $f(f(h(X), a), h(Z)) \stackrel{?}{=} f(f(Y, Z), X)$

   (g) $l(V, k(a, V), W) \stackrel{?}{=} l(g(Y, a), k(Y, X), V)$

   (h) $k(W, k(h(V), W)) \stackrel{?}{=} k(f(X), k(Y, Y))$