

ESPACE: Orbit Mechanics, Exercise 1

Keplerian Orbits in Space-fixed, Earth-fixed and Topocentric systems

Orbits of several satellites are given in an inertial, geocentric reference system (space-fixed) by the Keplerian orbital elements: semimajor axis a , eccentricity e , inclination i , right ascension of ascending node Ω , argument of perigee ω , and perigee passing time T_0 on Nov. 06, 2023.

Satellite	a [km]	e	i [deg]	Ω [deg]	ω [deg]	T_0 [h]
GOCE	6629	0.004	96.6	210	144	01:00
GPS	26560	0.01	55	30	30	11:00
Molniya	26554	0.7	63	200	270	05:00
GEO	geostationary	0	0	0	50	00:00
Michibiki	geosynchronous	0.075	41	200	270	19:00

For the following computations precession, nutation, polar motion and variations in the length of day are neglected. The Earth fixed reference system then rotates with an angular rate of $\omega_{\text{Earth}} = 2\pi/86164\text{s}$ about the \mathbf{e}_3 -axis of the inertial space-fixed reference system. At the time $t_0 = \text{Nov. 06, 2023, 00:00 UT}$ the sidereal angle is 03h 00m.

Part A:

- 1) Create a MATLAB-function `kep2orb.m` that computes polar coordinates r (radius) and v (true anomaly) based on input orbital elements. Formulate your program in a way that the time t can be used as input parameter.
- 2) Plot the orbit for the five satellites in the orbital plane for one orbital revolution.
- 3) Plot the mean anomaly M , the eccentric anomaly E , and the true anomaly v as well as the difference $v - M$ for one orbital revolution for the GPS satellite and the Molniya satellite.
- 4) Create a MATLAB-function `kep2cart.m` that uses `kep2orb.m`, which transforms Keplerian elements to position and velocity in an inertial (space-fixed) system.
- 5) Compute position and velocity vectors of the five satellites for a period of one day. Visualize your results. Plot the trajectory in 3D and 2D (projection to x - y , x - z and y - z planes) as well as a time series of the magnitude of velocity.

Part B:

- 6) Create a MATLAB-function `cart2efix.m` that transforms position and velocity in a spacefixed system into position and velocity in an Earth-fixed system.
- 7) Plot the trajectory of the satellites in 3D for the first two orbital revolutions.
- 8) Calculate and draw the satellite ground-tracks on the Earth surface.
- 9) Create a MATLAB-function `efix2topo.m` that transforms position and velocity in an earthfixed system into position and velocity in a topocentric system centered at the station Wettzell which position vector in an Earth-fixed system is given by: $\mathbf{r}_w = (4075.53022, 931.78130, 4801.61819)^T$ km.
- 10) Plot the trajectory of the satellites as observed by Wettzell using the MATLAB-function `skyplot.m`.
- 11) Calculate visibility (time intervals) for the satellites at the station Wettzell and visualize them graphically.

Use the following values for your computations:

Geocentric gravitational constant $GM = 398.6005 \cdot 10^{12} \text{m}^3/\text{s}^2$

Earth's radius $R_E = 6371 \cdot 10^3 \text{m}$

Prepare a written report with a short description of the way how to perform the computations and comment your results. Include the MATLAB-functions `kep2orb.m`, `kep2cart.m`, `cart2efix.m` and `efix2topo.m`.

Due date for delivery of written report: 08. January 2024

Please send your written report (as .pdf) to:

Zhang Ke (ge85taz@mytum.de)

Urs Hugentobler (urs.hugentobler@tum.de)

or upload to Moodle.