1 Introduction

In this exercise, we will study the Keplerian orbits in different coordinate systems. The Keplerian obital elements for 5 satellites are given in the table below. The orbital elements are given in the following order: semi-major axis a, eccentricity e, inclination i, right ascension of the ascending node Ω, argument of perigee ω, and perigee passing time T_0 on Nov.06,2023.

Satellite	**a[km]**	**e**	**i[deg]**	**Ω[deg]**	**ω[deg]**	**T_0[s]**
GOCE	6629	0.004	96.6	210	144	01:00
GPS	26560	0.001	55	30	30	11:00
Molniya	26554	0.7	63	200	270	05:00
GEO	42164	0	0	0	50	00:00
Michibiki	42164	0.075	41	200	270	19:00

Orbit in 2D plane

In this first task we will calculate the polar coordinates of stellite postion. The Keplerian elements of the semi-major axis and eccentricity are given. These two parameters determine the shape of the orbit. Then we have to calculate the current position of the satellite on this orbit plane with the time of perigee passage, where $\nu = E = M = 0$. This allows us to calculate the orbit evolution over time.

$$\boldsymbol{r}_f(r,\nu) = \begin{bmatrix} r\cos\nu \\ r\sin\nu \\ 0 \end{bmatrix} = \begin{bmatrix} a\cos E - e \\ a\sqrt{1-e^2}\sin E \\ 0 \end{bmatrix}$$

r is the distance from the center of the ellipse to the satellite, and ν is the true anomaly. The true anomaly is the angle between the perigee and the satellite position. The eccentric anomaly E is the angle between the perigee and the position of the satellite on the ellipse. The eccentric anomaly can be calculated by solving the Kepler equation.

$$M = E - e\sin E$$

The mean anomaly M is the angle between the perigee and the position of the satellite on the circle. This is a fictitious angle and it evolves linearly in time. The mean anomaly can be calculated by the following equation.

$$M = n(t - T_0)$$

The mean motion n is the angular velocity of the satellite on the orbit. The mean motion can be calculated by the following equation.

$$n = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{GM}{a^3}}$$

The five satellite orbit in 2D plane are shown in Figure 1. The orbit of GOCE is a circular orbit, and the orbit of GPS is a nearly circular orbit. The orbit of Molniya is a highly eccentric orbit. The orbit of GEO is a circular orbit, and the orbit of Michibiki is a nearly circular orbit.

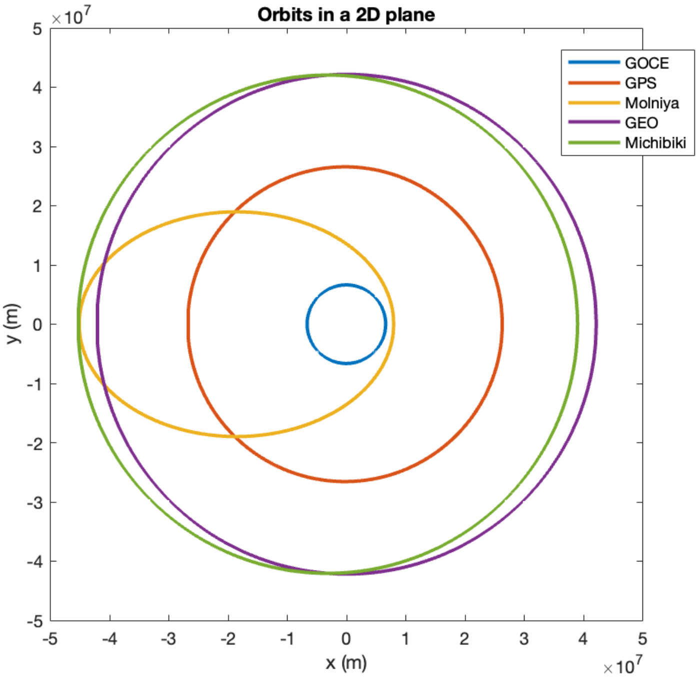
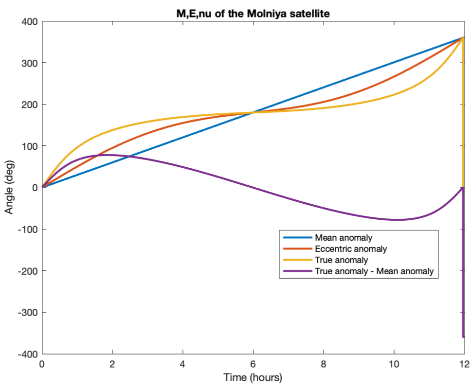
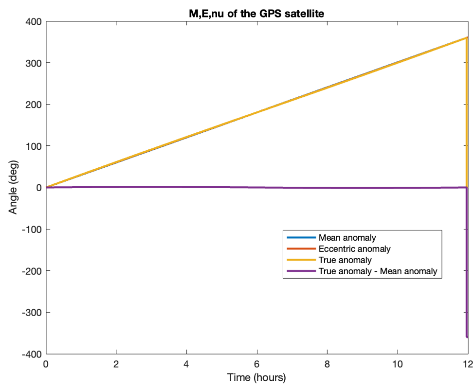


Figure 1: 2D orbit

We also have to plot the mean anomaly M, the eccentric anomaly E and the true anomaly ν over 12 hours for the GPS and Molniya satellite.



a) Mean anomaly



b) Eccentric anomaly

Space-fixed inertial system

In this second task we have to transform the satellite postion and velocity from the perifocal f-frame of 2D plane to the cartesian coordinates of 3D space in an inertial space fixed system.

$$\boldsymbol{r}_i = \boldsymbol{R}_3(-\Omega)\boldsymbol{R}_1(-i)\boldsymbol{R}_3(-\omega)\boldsymbol{r}_f$$
$$\dot{\boldsymbol{r}}_i = \boldsymbol{R}_3(-\Omega)\boldsymbol{R}_1(-i)\boldsymbol{R}_3(-\omega)\dot{\boldsymbol{r}}_f$$

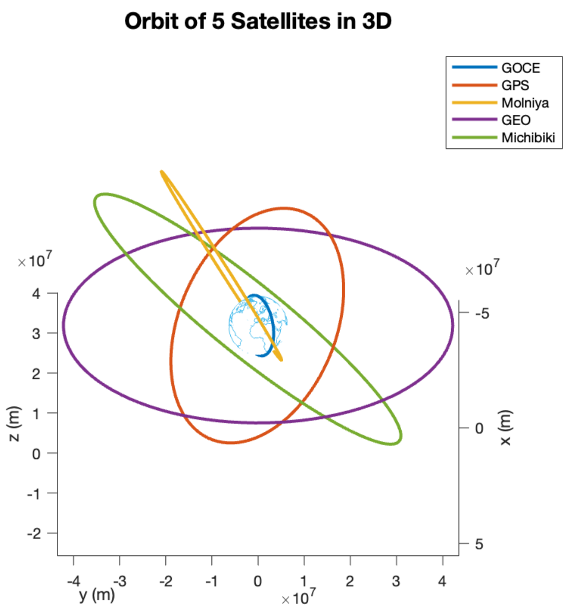
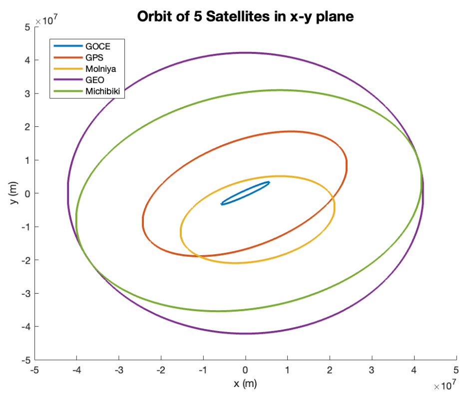
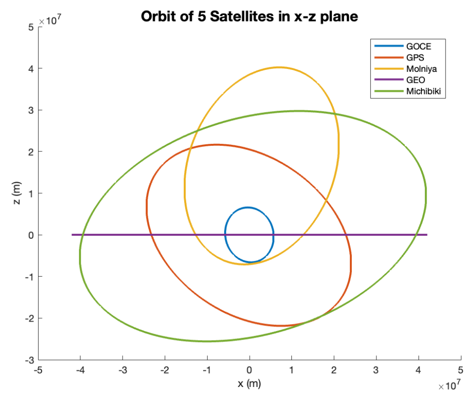


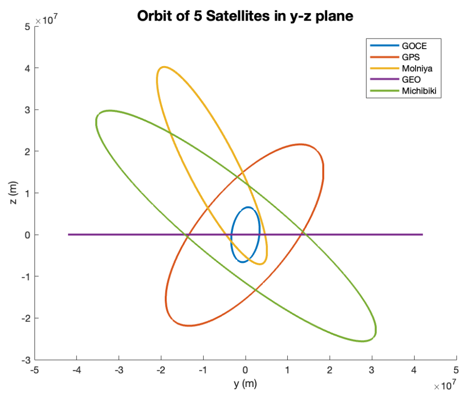
Figure 3: 3D orbit



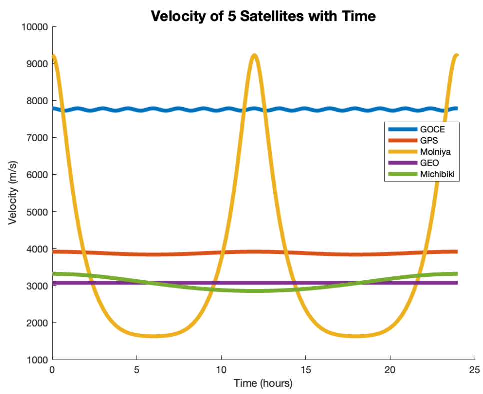
a) Caption for image 1



b) Caption for image 2



c) Caption for image 3



d) Caption for image 4

Figure 4: Caption for the whole figure

Earth-fixed system

In this task we have to transform the satellite postion and velocity from the cartesian coordinates of 3D space in an inertial space fixed system to the cartesian coordinates of 3D space in an Earth-fixed system. The Earth-fixed system is a rotating system with respect to Earth rotation rate. The transformation can be calculated by the following equation.

$$\theta = \dot{\Omega}_E t + \text{sidereal angle}$$
$$\boldsymbol{r}_e = \boldsymbol{R}_3(-\theta)\boldsymbol{r}_i$$

which $\dot{\Omega}_E$ is the Earth rotation rate. The Earth rotation rate is calculated by $\frac{2\pi}{86164}$. The sidereal angle is the angle between the Greenwich meridian and the equinox. The satellite position in Earth-fixed system is shown in Figure 5.

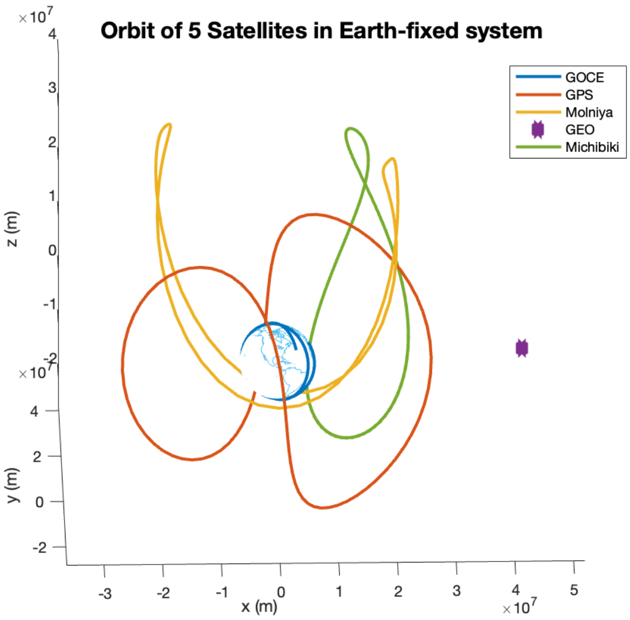


Figure 5: Earth-fixed system

The satellite ground-tracks on the Earth surface will also be calculated and shown in Figure 6.

$$\lambda = \arctan\left(\frac{y}{x}\right)$$
$$\phi = \arctan\left(\frac{z}{\sqrt{x^2+y^2}}\right)$$

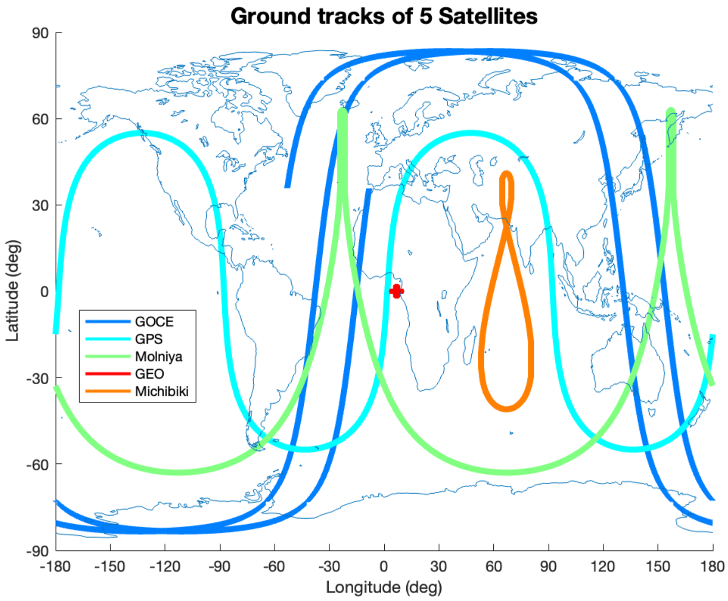


Figure 6: Ground-track

Topocentric system

The position vector of the station Wettzell is given in the Earth-fixed system: $\boldsymbol{r}_w = (4075.53022, 931.78130, 4801.61819)^T$ km. The position vector of the satellite in the topocentric system is calculated by the following equation.

$$\boldsymbol{r}_t = \boldsymbol{r}_e - \boldsymbol{r}_w$$
$$\boldsymbol{r}_t = \boldsymbol{Q}_1 \boldsymbol{R}_2(90 - \Phi_w) \boldsymbol{R}_3(\lambda_w) \boldsymbol{r}_t$$
$$Q_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The Q_1 matrix is used to transfrom from right- to left-handed system.

It's also useful to compute the azimuth and elevation of the satellites in the topocentric system. The azimuth and elevation are calculated by the following equation.

$$\text{azimuth} = \arctan\left(\frac{y_t}{x_t}\right)$$
$$\text{elevation} = \arctan\left(\frac{z_t}{\sqrt{x_t^2 + y_t^2}}\right)$$

And now we can plot the trajectory of the satellites that can be observed by Wettzell. The trajectory of the satellites that can be observed by Wettzell is shown in Figure 7.

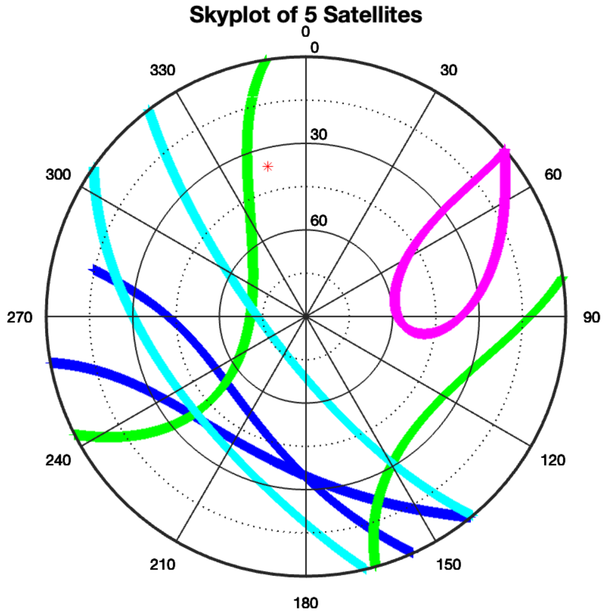


Figure 7: Topocentric system

Finally we can calculate the visibility of the satellites at the station Wettzell. The visibility of the satellites at the station Wettzell is shown in Figure 8.

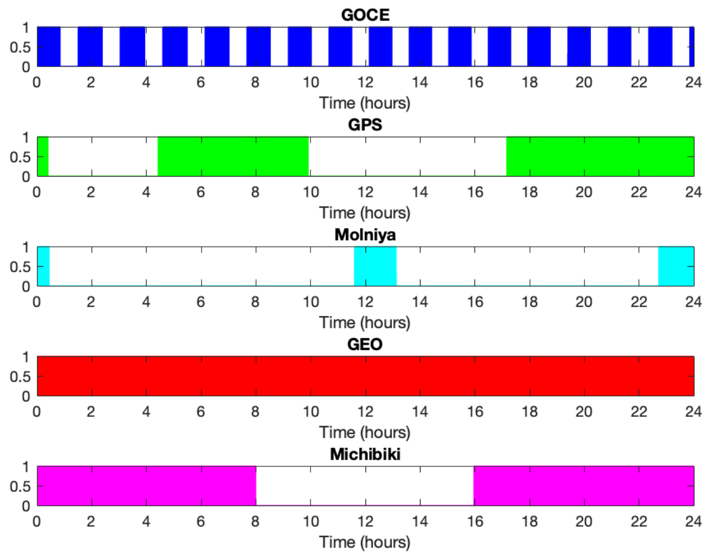


Figure 8: Visibility