Institute for Astronomical and Physical Geodesy
Technische Universität München

Winter Semester 2023/2024

ESPACE: Orbit Mechanics, Exercise 2
Numerical Integration of Satellite Orbits

For numerical integration MATLAB provides the functions **ode23**, **ode45** and **ode113** (see **help**). As input parameters the integration time steps, the initial conditions (position, velocity) of the integration $\mathbf{y_0}$ and the program **yprime,** which contains the differential equation are required.

Compute numerically the trajectory of the Sentinel-3 satellite for 3 revolutions. Use the following Kepler elements for the Sentinel-3 satellite:

Satellit	**a [km]**	**e**	**i [◦]**	**Ω [◦]**	**ω [◦]**	**T_p[sec]**
Sentinel-3	7192	0.004	98.3	257.7	144.2	00:00

Compare the numerically integrated trajectories for the un-disturbed and for the disturbed (due to flattening of the Earth) cases against each other as well as against the analytical derived trajectory (from the Kepler elements). In particular the following sub tasks have to be solved:

1. Analytically compute the undisturbed trajectory (positions & velocities) for 3 revolutions from the Kepler elements (program should be available from previous exercise in orbit mechanics).

2. Write a program **yprime**, providing the differential equation for the un-disturbed Kepler problem. Compute the trajectory (positions & velocities) for 3 revolutions using two different MATLAB integrators and two different step sizes.

3. Compare the analytical and numerical derived results in terms of plots for all three components.

4. Plot the differences between the analytical and numerical case also in radial (R-direction), along-track (S-direction), and cross-track (W-direction) components. Which components are mainly affected by integration errors?

5. Take into account the Earth's flattening and update your program **yprime** accordingly. Apply the formulas given below.

   The disturbed equation of motion is given by:

$$\ddot{\mathbf{r}} = -\frac{GM}{r^3}\left[\mathbf{r} - \frac{3}{2}J_2\frac{a_e^2}{r^2}\begin{pmatrix} x\left(5\sin^2\beta - 1\right) \\ y\left(5\sin^2\beta - 1\right) \\ z\left(5\sin^2\beta - 3\right) \end{pmatrix}\right] \qquad \text{with} \quad \mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad \text{and} \quad \sin\beta = \frac{z}{r}$$

with $J_2 = 0.00108263$; $GM = 398.6005 \cdot 10^{12}$ m^3/s^2; $a_e = 6378$ km

Display the difference between numerical integration in perturbed case to unperturbed case (e.g. from analytical solution) in inertial x-, y-, z-components as well as in R-, S-, W-components.

6. Develop your own simple integrator (e.g. Euler method) for the undisturbed case and investigate the impact of the step size on your result. Optionally you may also develop a simple Runge Kutta algorithm. Compare the results with the analytical solution.

Due date for delivery of written report: 12. February 2024

Please send your written report (as .pdf) to:

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or upload to Moodle.