Institute for Astronomical and Physical Geodesy
Technical University of Munich

Winter Semester 2023/2024

ESPACE: Orbit Mechanics, Exercise 1

Keplerian Orbits in Space-fixed, Earth-fixed and Topocentric systems

Orbits of several satellites are given in an inertial, geocentric reference system (space-fixed) by the Keplerian orbital elements: semimajor axis a, eccentricity e, inclination i, right ascension of ascending node Ω, argument of perigee ω, and perigee passing time T_0 on Nov. 06, 2023.

Satellite	**a [km]**	**e**	**i [deg]**	**Ω [deg]**	**ω [deg]**	**T_0 [h]**
GOCE	6629	0.004	96.6	210	144	01:00
GPS	26560	0.01	55	30	30	11:00
Molniya	26554	0.7	63	200	270	05:00
GEO	geostationary	0	0	0	50	00:00
Michibiki	geosynchronous	0.075	41	200	270	19:00

For the following computations precession, nutation, polar motion and variations in the length of day are neglected. The Earth fixed reference system then rotates with an angular rate of $\omega_{Earth} = 2\pi/86164s$ about the $\mathbf{e}_3$-axis of the inertial space-fixed reference system. At the time t_0 = Nov. 06, 2023, 00:00 UT the sidereal angle is 03h 00m.

Part A:

1) Create a MATLAB-function kep2orb.m that computes polar coordinates r (radius) and v (true anomaly) based on input orbital elements. Formulate your program in a way that the time t can be used as input parameter.

2) Plot the orbit for the five satellites in the orbital plane for one orbital revolution.

3) Plot the mean anomaly M, the eccentric anomaly E, and the true anomaly v as well as the difference v – M for one orbital revolution for the GPS satellite and the Molniya satellite.

4) Create a MATLAB-function kep2cart.m that uses kep2orb.m, which transforms Keplerian elements to position and velocity in an inertial (space-fixed) system.

5) Compute position and velocity vectors of the five satellites for a period of one day. Visualize your results. Plot the trajectory in 3D and 2D (projection to x-y, x-z and y-z planes) as well as a time series of the magnitude of velocity.

Part B:

6) Create a MATLAB-function cart2efix.m that transforms position and velocity in a spacefixed system into position and velocity in an Earth-fixed system.

7) Plot the trajectory of the satellites in 3D for the first two orbital revolutions.

8) Calculate and draw the satellite ground-tracks on the Earth surface.

9) Create a MATLAB-function efix2topo.m that transforms position and velocity in an earthfixed system into position and velocity in a topocentric system centered at the station Wettzell which position vector in an Earth-fixed system is given by: $\mathbf{r}_w = (4075.53022, 931.78130, 4801.61819)^T$ km.

10) Plot the trajectory of the satellites as observed by Wettzell using the MATLAB-function skyplot.m.

11) Calculate visibility (time intervals) for the satellites at the station Wettzell and visualize them graphically.

Use the following values for your computations:
Geocentric gravitational constant $GM = 398.6005 \cdot 10^{12} m^3/s^2$
Earth‘s radius $R_E = 6371 \cdot 10^3$ m

Prepare a written report with a short description of the way how to perform the computations and comment your results. Include the MATLAB-functions kep2orb.m, kep2cart.m, cart2efix.m and efix2topo.m.

Due date for delivery of written report: 08. January 2024

Please send your written report (as .pdf) to:
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Urs Hugentobler (urs.hugentobler@tum.de)
or upload to Moodle.