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Alex H. Room

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I | Introduction

The computation of spectra can be boldly considered the 'fundamental problem of operator theory' [1];

II Spectra

We must first discuss our quantity of interest: the spectrum of an operator.

Definition. ***(Resolvent and spectrum)*** *(Adapted from [2]) Let T be a linear operator on a Banach space. The resolvent of T is the set $\rho(T) := \{\eta \in \mathbb{C} : (T - \eta I) \text{ is bijective}\}$, where I is the identity operator.*

The spectrum of T, denoted Spec(T), is $\mathbb{C} \setminus \rho(T)$, i.e. the set of all complex numbers λ such that the operator $(T - \lambda I)$ does not have a bounded inverse.

Indeed, if the Banach space is finite-dimensional, this definition coincides with that of the eigenvalues of a matrix. An infinite-dimensional operator also has eigenvalues, but these are just a subset of the whole spectrum (and sometimes can even be none of the spectrum), as we will see below. One can very simply check that if a non-zero vector x satisfies $Tx = \lambda x$, then λ must be in $\mathrm{Spec}(T)$.

Example 1. *Let M_f denote the multiplication operator by a function f on $L^2(0,1)$; this operator has action $M_f u(x) = f(x)u(x)$. If f is continuous, then Spec(M_f) is equal to the range of f.*

Proof. Let the range of f be denoted $\mathrm{Ran}(f)$, and consider the function $g = \frac{1}{f(x)-\lambda}$. If $\lambda \notin \mathrm{Ran}(f)$, g is bounded, and so one can see that M_g is an inverse to $M_f - \lambda I$:

$$M_g(M_f - \lambda I)u(x) = M_g(f(x) - \lambda)u(x) = \frac{f(x) - \lambda}{f(x) - \lambda}u(x) = u(x).$$

Now take $\lambda \in \mathrm{Ran}(f)$. Then □

References

[1] William Arveson, FW Gehring, and KA Ribet. *A short course on spectral theory*. Vol. 209. Springer, 2002.

[2] Lawrence C Evans. *Partial differential equations*. 2nd ed. Vol. 19. American Mathematical Society, 2010.