

PROBABILITY

9.30.2020

PROBLEM SET 2

- * Due Friday by 10:59am
- * TA Ria Paul will be holding office hours immediately after class today (12pm CST)
- * see Canvas announcement for Zoom link

RECAP

- * choosing the right way to plot something!
- * colormaps!
 - * sequential, diverging, and qualitative

WHAT IS DATA ANALYSIS?

- * Moving data around (*munging*)
- * Visualizing data
- * **Statistics**



BERNOULLI



- * The Bernoulli distribution is like flipping a coin: the outcome is either heads (1) or tails (0)
- * (but it doesn't have to be a fair coin)
- * It has one parameter: q = the probability that the outcome is 1
- * $X \sim \text{Bernoulli}(q)$ # means that X is a bernoulli random variable with parameter q

BERNOULLI

- * To simulate a Bernoulli random variable (RV) we can use `np.random.rand()` and `>`
- * For example, to get a random True or False with 50% probability:
- * `np.random.rand() > 0.5`

BERNOULLI

- * The **expected value (EV)** is (more or less) what the average value should be for an infinitely large sample
- * It's defined as a sum of the possible values an RV could take, weighted by the probabilities of those values
- * If $X \sim \text{Bernoulli}(q)$, then its expected value $E[X] = q$

WEIGHTED COIN?

- * suppose we have a coin and we want to know if it's fair or not (i.e. does $q=0.5$?)
- * we flip it 100 times, and it lands on heads 63 times
- * is it fair? how do we know?



WEIGHTED COIN?

- * we can test this by simulation!
- * simulate 10,000 experiments where a fair coin ($q=0.5$) is flipped 100 times
- * we know most values should be around 50
- * but how often does a value as extreme as 63 come up?



P-VALUE

- * the **p-value** is the probability of seeing an outcome *at least this extreme* if the “null hypothesis” (i.e. the coin is fair) is true

BINOMIAL DISTRIBUTION



- * it turns out you don't need to simulate lots of experiments to test things about bernoulli RVs
- * because you can compute it exactly using the **binomial distribution!**

BINOMIAL DISTRIBUTION

- * $Y \sim \text{Binomial}(n, q)$ # Y is a Binomial RV with n trials and probability q on each trial
- * Y is number of 1's that came up in n samples from a bernoulli distribution with parameter q
- * (this is exactly what we've been doing—generating binomial RVs!)



BINOMIAL DISTRIBUTION



- * the exact shape of a binomial distribution is given by this formula:

$$\binom{n}{k} q^k (1 - q)^{n-k}$$

|
n-choose-k: the number
of ways to pick k
things out of n things

BINOMIAL DISTRIBUTION



- * the exact shape of a binomial distribution is given by this formula:

$$\binom{n}{k} q^k (1 - q)^{n-k}$$

q^k : the probability
of getting exactly k
1's

BINOMIAL DISTRIBUTION

- * the exact shape of a binomial distribution is given by this formula:

$$\binom{n}{k} q^k (1 - q)^{n-k}$$

$(1-q)^{(n-k)}$: the probability of getting exactly $(n-k)$ 0's



END