LINEAR REGRESSION II

11.6.2020

RECAP

* receptive fields

  * what kind of stuff in the world does a neuron (or voxel, or whatever) respond to?

  * we can estimate receptive fields by measuring responses in different conditions, and then fitting a model

RECAP

* how do we fit a stimulus—>response model?

* spike-triggered average?

  * kinda bad, especially when stimulus features (x’s) are correlated

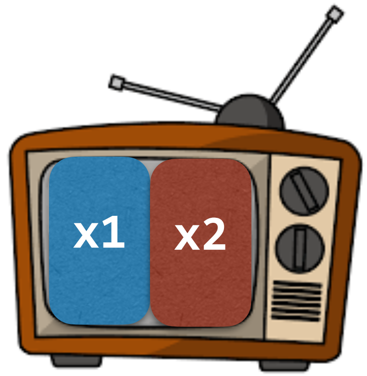
* regression!

  * solves problems with STA!

2D EXAMPLE

subject

stimulus



* **y** = output of a neuron that you are measuring

* **x1** = how many times **left** side of screen flashes per second

* **x2** = how many times **right** side of screen flashes per second

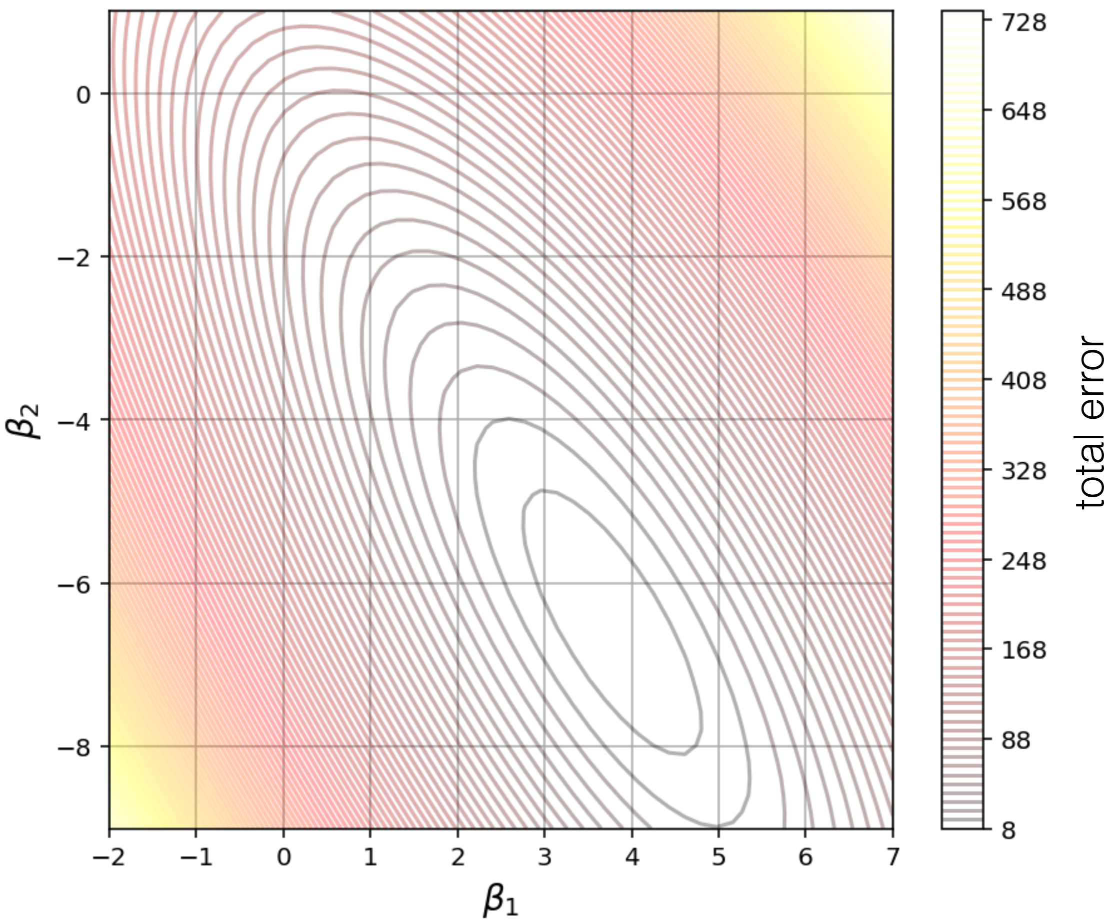
SQUARED ERROR

* the **squared error** is defined as the sum of squared differences between actual and predicted data

$$Err(\beta) = \sum_{t=1}^{T} (y_t - x_t\beta)^2$$

2D EXAMPLE

* here the error function takes two variables, *E(b1, b2)*



LINEAR REGRESSION

* so how do we find the values of b1 and b2 that minimize the loss function?

* one way is to take the derivative (aka the **gradient**) of the error with respect to b, then move in that direction

* this is called **gradient descent**

LINEAR REGRESSION

Squared Error: $$Err(\beta) = (Y - X\beta)^2$$

Gradient: $$\frac{\partial Err(\beta)}{\partial \beta} = -X^\top (Y - X\beta)$$

LINEAR REGRESSION

* but there’s another way to find the optimal b1 and b2

* what’s the shape of the error function?

  * a parabola!

LINEAR REGRESSION

* there is an analytic solution to the minimum of a parabola!

$$\frac{\partial Err(\beta)}{\partial \beta} = -X^\top (Y - X\beta) = 0$$

$$-X^\top Y + X^\top X\beta = 0$$

$$X^\top X\beta = X^\top Y$$

$$\beta = (X^\top X)^{-1} X^\top Y$$

ANALYTIC REGRESSION

* **np.linalg.lstsq** solves least squares regression

* it returns 4 things:

  * the regression weights (beta)

  * the residuals (final squared error)

  * the rank (we’ll talk about this later)

  * the singular values (ditto)

END