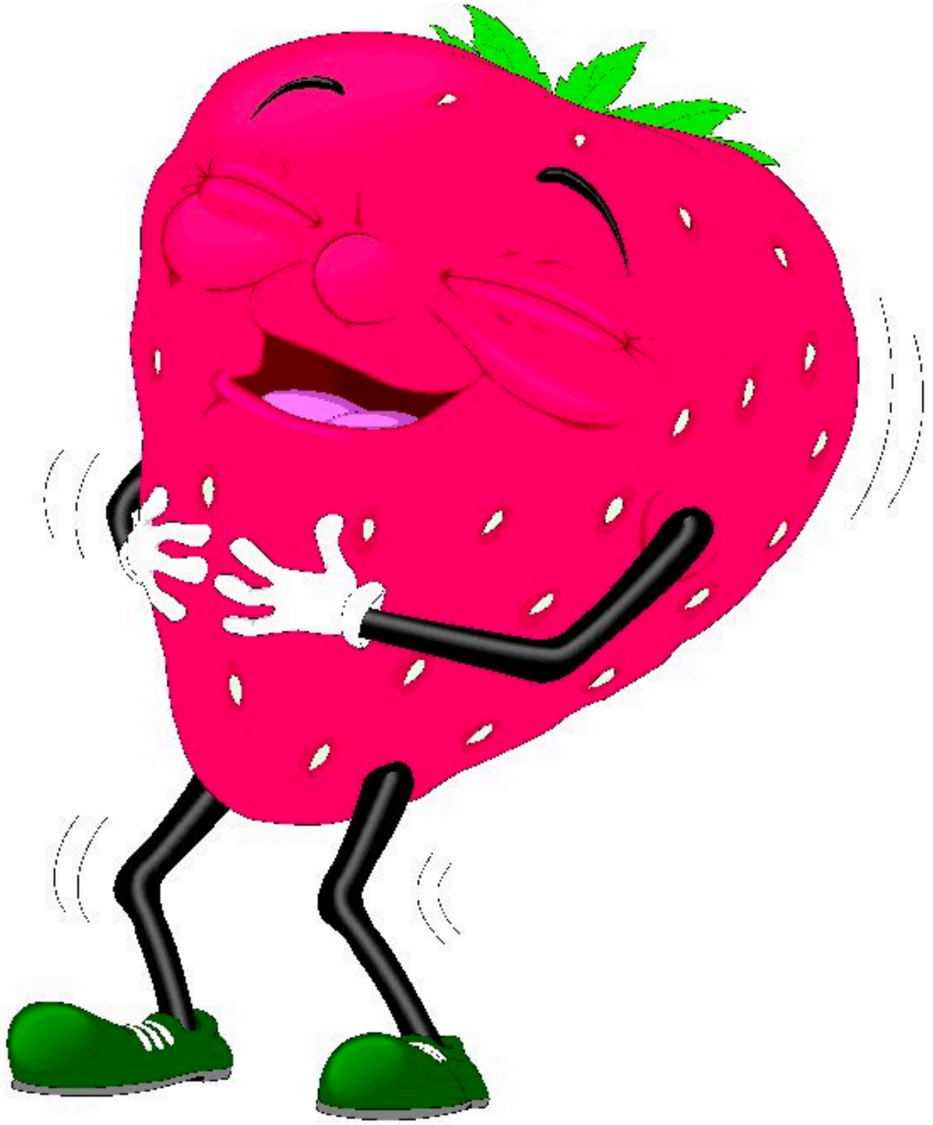
TIMESERIES 4

10.28.2020

PROBLEM SET 4

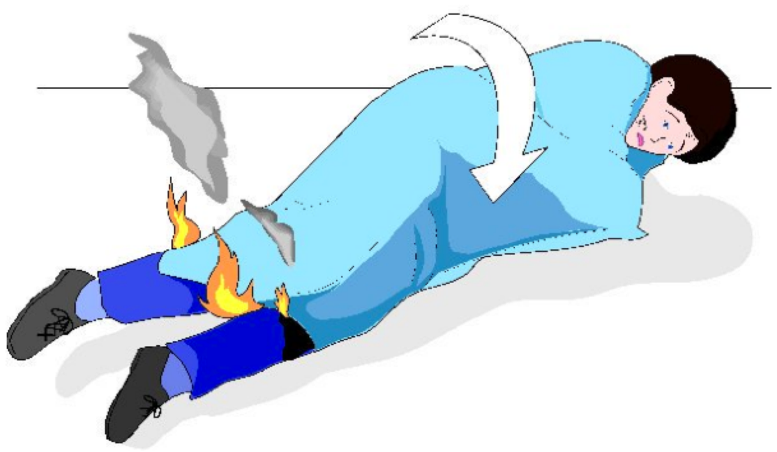
* is due Friday!



PROBLEM SET 5

* will be posted Friday

* but you will have **3 weeks** to finish it



RECAP: SPECTROGRAM

* if we compute the fourier transform for small snippets of time and then stack them together into an array

  * this is the **spectrogram**

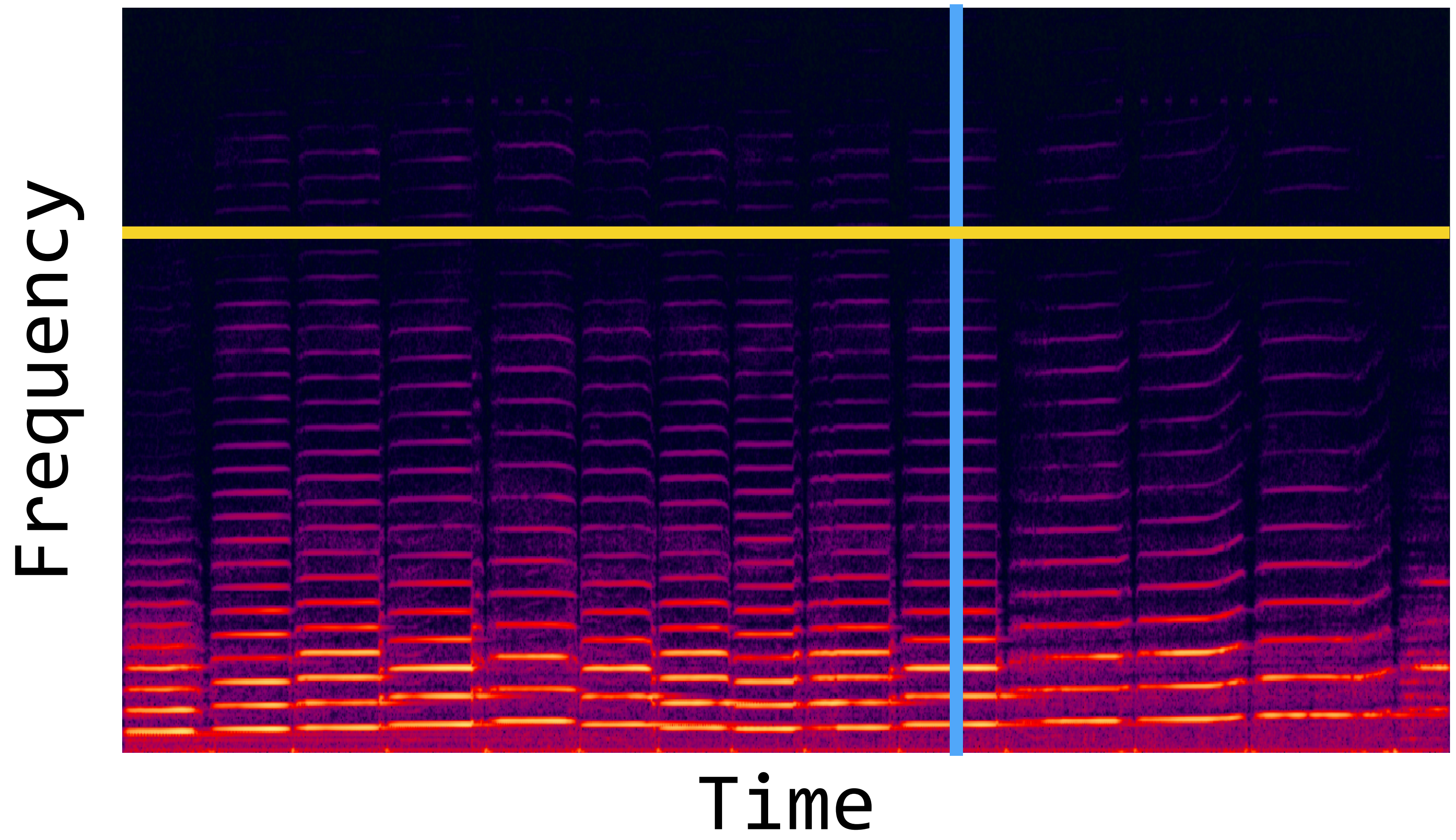
  * it shows which frequencies are present in a timeseries at each point in time

  * *you should know how to read a spectrogram*

THE SPECTROGRAM

* each **column** is the fourier transform of a short snippet

* what about each **row**? what does one row mean?



FILTERING

* **filtering** is a process that removes some frequencies from a timeseries and lets others remain (or even amplifies them)

* this is accomplished by convolving your timeseries with a **filter**, a small array that is designed to have a specific effect

FILTERING

* **low-pass filter:** removes high frequencies, allows low frequencies through

* **high-pass filter:** removes low frequencies, allows high frequencies through

* **band-pass filter:** removes all frequencies except for a specific band (the “pass band”)

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FILTERING

* back to the spectrogram:

  * one row of a spectrogram is a lot like a **band-pass filtered** version of a timeseries

FILTERING

* suppose we have some EEG data from a human subject and we want to filter it so that only alpha-band oscillations remain

  * (this is a band-pass filter)

* how do you make a filter that has the properties you want?

FILTERING

* **scipy.signal** is a module in scipy that contains lots of useful functions for filter design

* **scipy.signal.firwin** creates "finite impulse response" filters with desired properties

ANALYZING A FILTER

* **scipy.signal.freqz** is a great function that tells you what the *frequency response* of your filter looks like

    * i.e. it tells you what the filter is going to do to your signal

RECALL:
FOURIER ANALYSIS

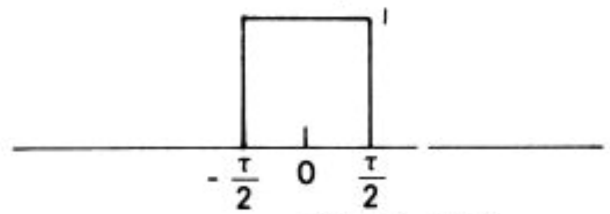
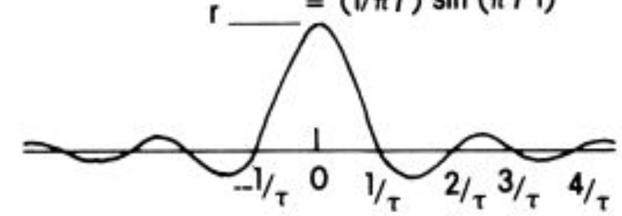
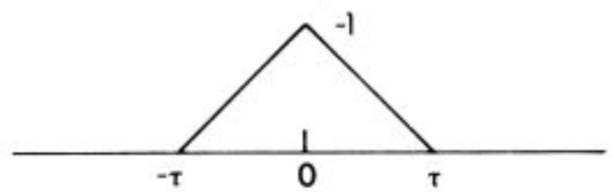
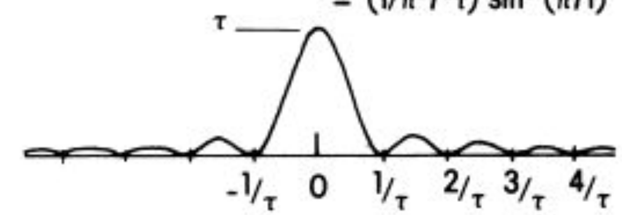
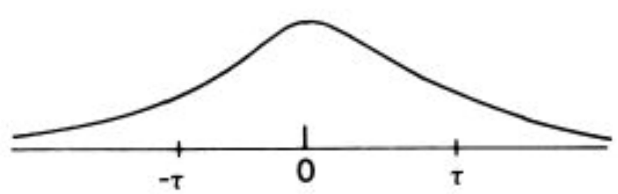
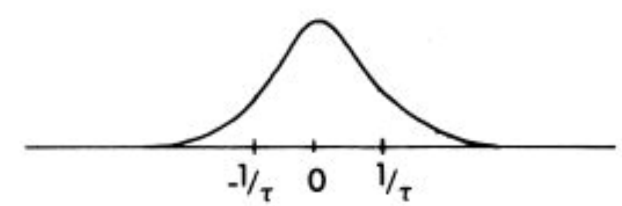
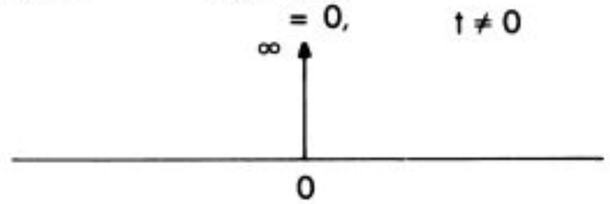
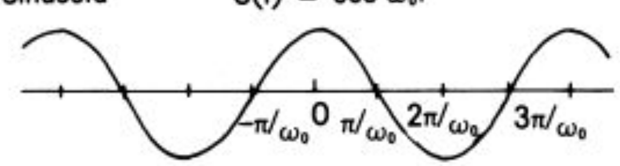
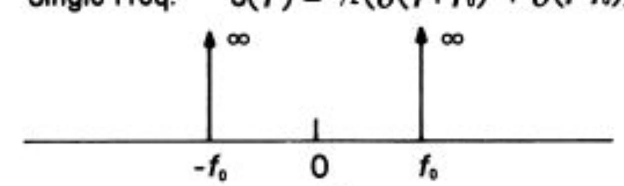
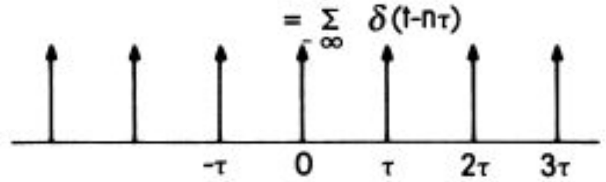
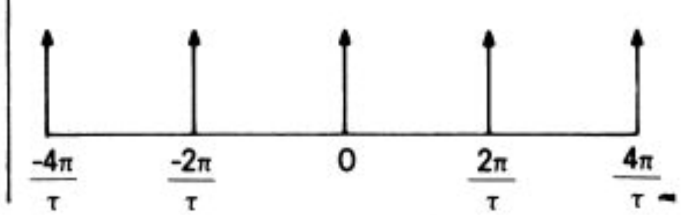
* fourier transforms have an interesting property related to convolution:

* given two timeseries, f and g, the fourier transform of their convolution = the element-wise product of their fourier transforms

$$\mathrm{FT}(f \star g) = F \cdot G$$

* the reverse is also true:

$$F \star G = \mathrm{FT}(f \cdot g)$$

Time Function		Frequency Function	
Boxcar	$G(t) = \begin{cases} 1, & \lvert t \rvert < \tau/2 \\ 0, & \lvert t \rvert > \tau/2 \end{cases}$	Sinc	$S(f) = \tau \,\mathrm{sinc}\,(f\tau) = (1/\pi f) \sin(\pi f t)$
			
Triangle	$G(t) = \begin{cases} 1-\lvert t \rvert/\tau, & \lvert t \rvert < \tau \\ 0, & \lvert t \rvert > \tau \end{cases}$	Sinc²	$S(f) = \tau \,\mathrm{sinc}^2\,(ft) = (1/\pi^2 f^2 \tau) \sin^2(\pi f t)$
			
Gaussian	$G(t) = e^{-\frac{1}{2}t^2}$	Gaussian	$S(f) = \tau(2\pi)^{\frac{1}{2}} e^{-(\pi f \tau)^2}$
			
Impulse	$G(t) = \delta(t)$ $= 0, \quad t \neq 0$	DC Shift	$S(f) = 1$
			
Sinusoid	$G(t) = \cos \omega_0 t$	Single Freq.	$S(f) = \frac{1}{2}(\delta(f+f_0) + \delta(f-f_0))$
			
Comb.	$G(t) = \mathrm{comb}\,(t) = \sum_{-\infty}^{\infty} \delta(t-n\tau)$	Comb.	$S(f) = \sum_{-\infty}^{\infty} \delta(f-n/\tau)$
			

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