LONG SHORT-TERM MEMORY NETWORKS

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RECAP

* Recurrent neural networks (RNNs)
  * Appropriate for problems where data have **sequential dependence**
  * Trainable using **backpropagation through time (BPTT)**
    * This is like "**unrolling**" the recurrent network into a (very) deep feedforward network, and then applying backpropagation
  * RNNs are great for problems like **language**, which requires more sequential processing than vision

RECAP

* Apply the **system construction** approach to the neuroscience of language processing
  * Train an RNN **language model** that predicts future words from past
  * Use its internal states to predict fMRI data collected while subjects listen to natural language
  * This works! But **hierarchy** is difficult to interpret relative to visual CNN models

THE TROUBLE WITH RNNS

* RNNs suffer from the problems of **vanishing** and **exploding gradients**

* This problem is worse when backpropagating over a **long** sequence

  * Thus limiting the temporal dependence of functions that simple RNNs can learn

THE TROUBLE WITH RNNS

* A language model example:

  When she tried to print her tickets, she found that the printer was out of toner. She went to the stationery store to buy more toner. It was very overpriced. After installing the toner into the printer, she finally printed her _____

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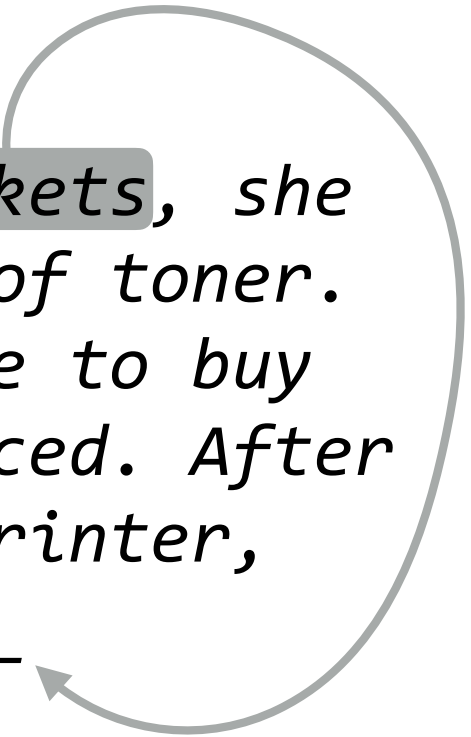

~37 steps back!

THE TROUBLE WITH RNNS

* Simple 1-D example:

$$h_{t+1} = \sigma(wh_t) \qquad \frac{\partial h_{t+1}}{\partial h_t} = w\sigma'(wh_t)$$

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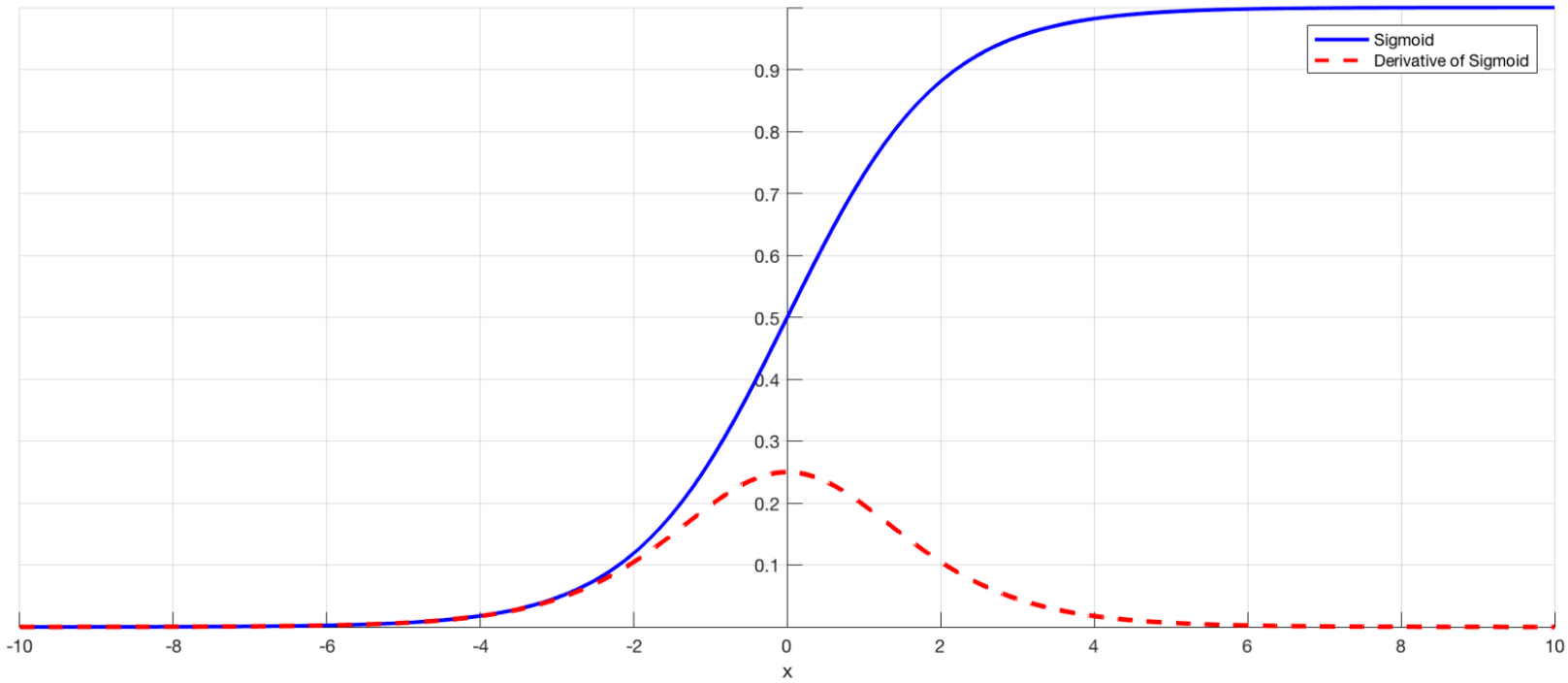
really bad

also bad

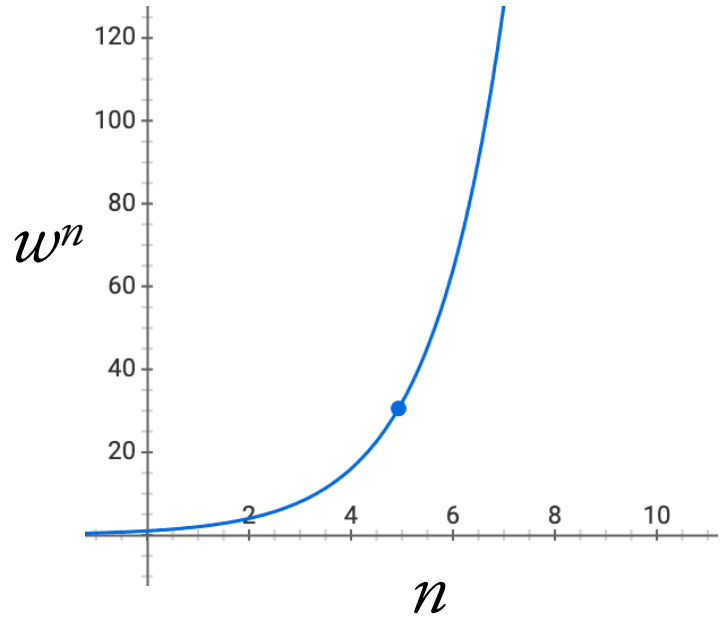
THE TROUBLE WITH RNNS

* **Vanishing gradients** occur if
  * n large, $|w| < 1$, or activations h_t far from 0



THE TROUBLE WITH RNNS

* **Exploding gradients** occur if
  * n large, |w| > 1



THE TROUBLE WITH RNNS

* How can these things be solved?
  * By breaking the chains of (1) weight multiplications, and (2) nonlinearities
* Enter: **Long short-term memory networks (LSTMs)**

LSTM

Jürgen Schmidhuber

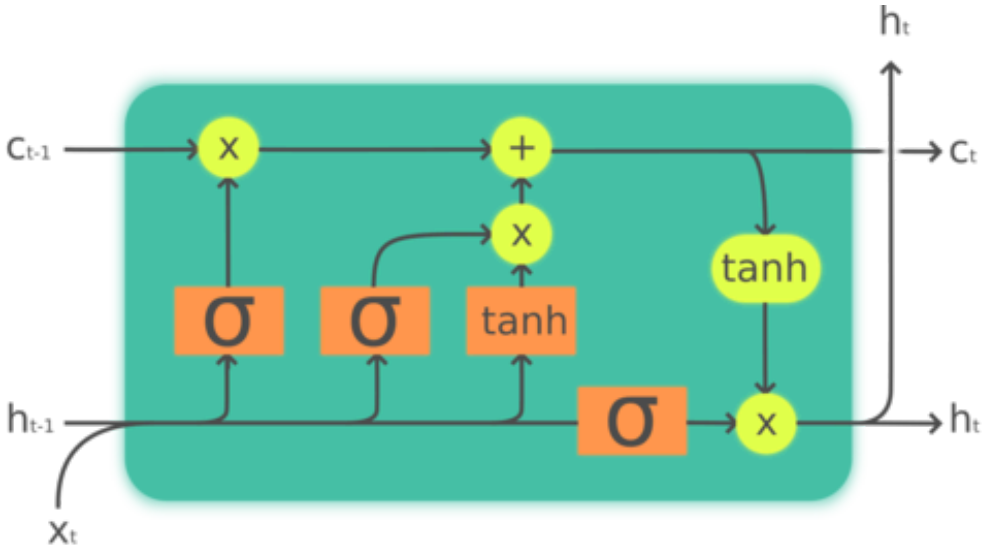
* The LSTM was introduced by Hochreiter & Schmidhuber in 1997

* One example of a **gated recurrent network**

LSTM

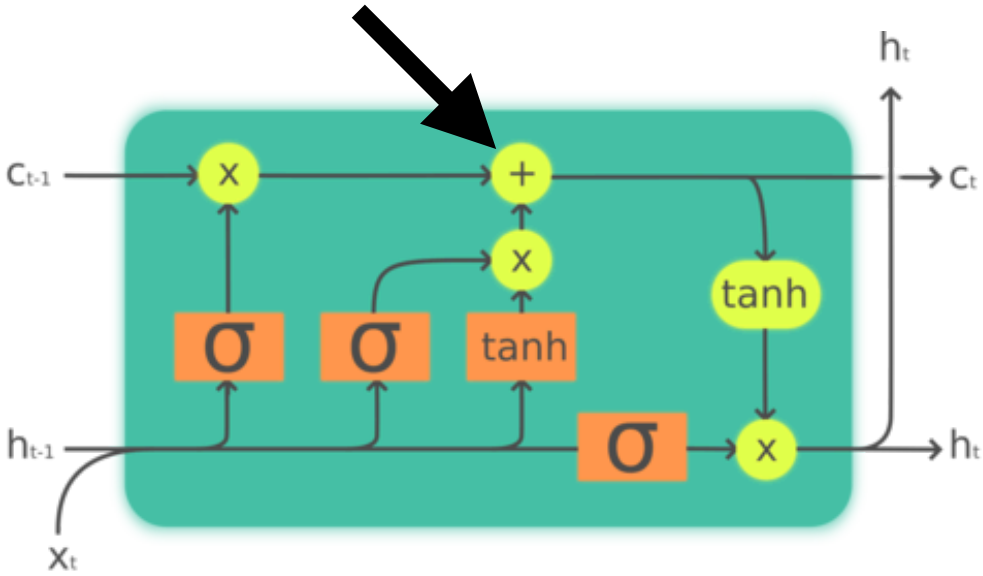
* Basic idea:

* In addition to the **hidden state**, maintain a separate **cell state** (c_t) that does not pass through a non-linearity at each timestep



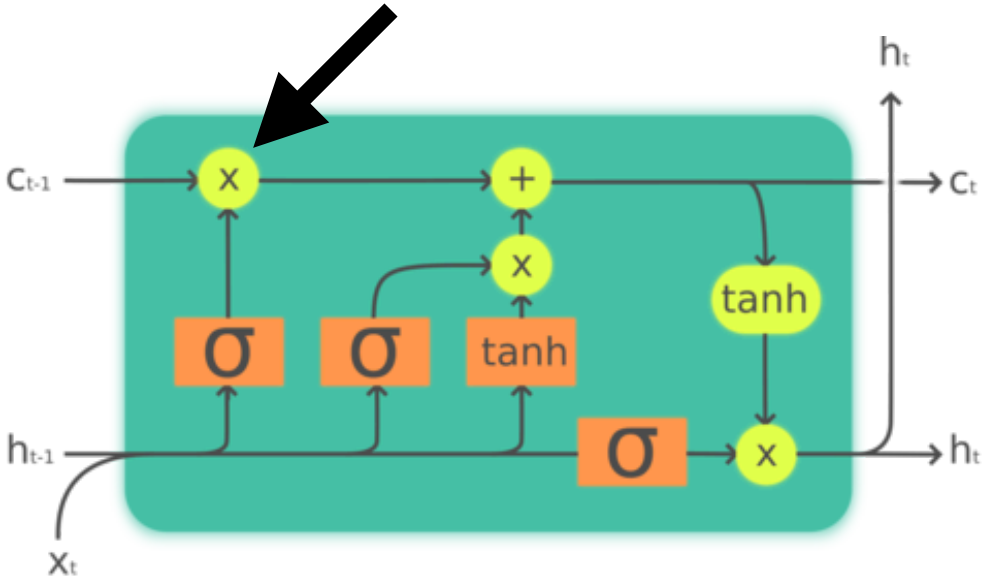
LSTM

* Information can be **added to** the cell state at each timestep



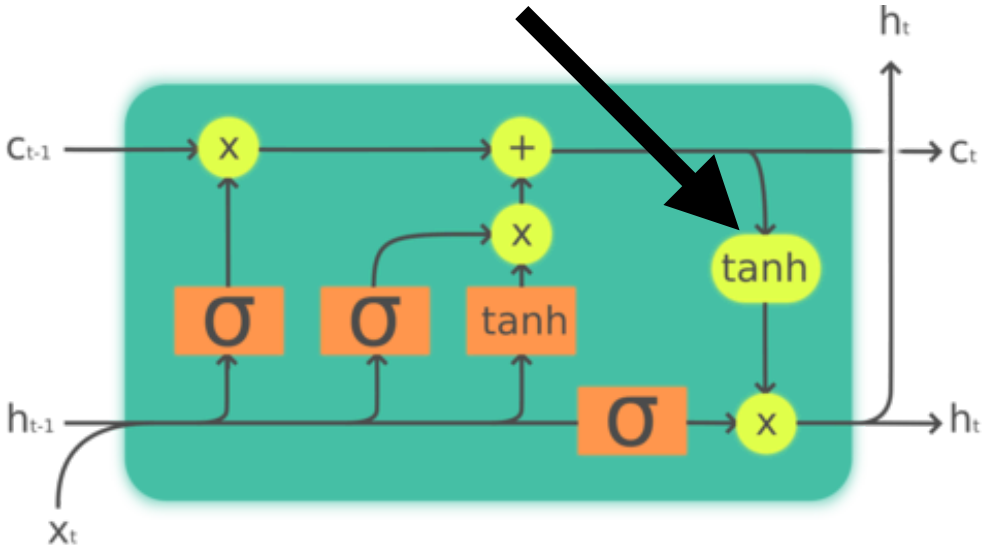
LSTM

* Information can also be **removed from** the cell state (or "forgotten") at each timestep



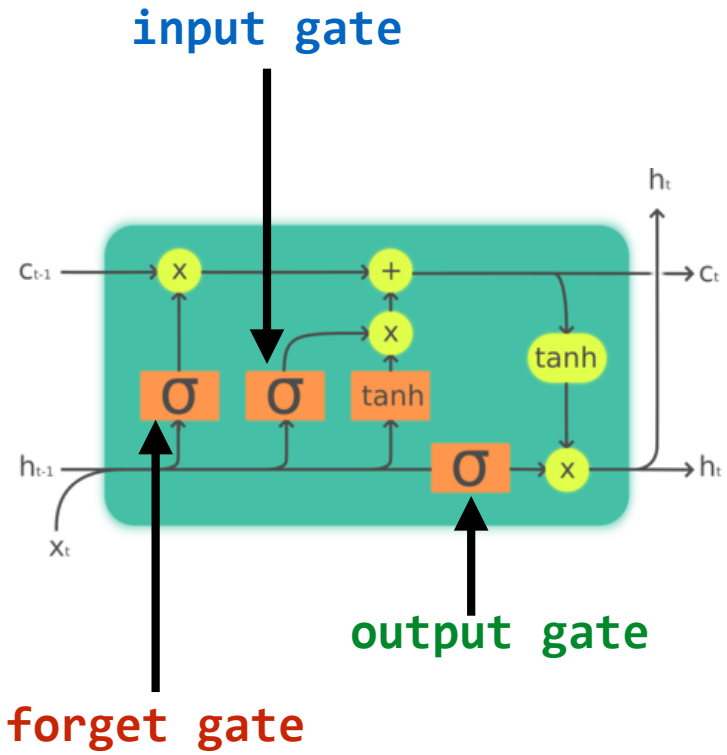
LSTM

* The new cell state is then used to construct the **output** "hidden" state



LSTM

* The LSTM uses a set of **gates** that can open or close to allow information to flow
  * The **input gate** controls how information is added to the cell state
  * The **forget gate** controls how information is removed from the cell state
  * The **output gate** controls how information from the cell state becomes output



LSTM

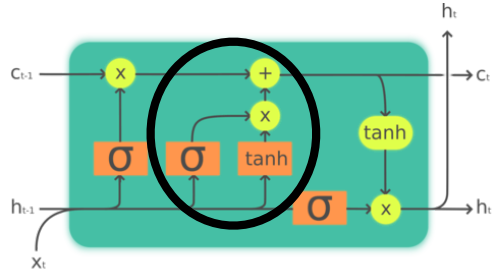
* Each gate is computed as a learnable function of:
  current input and **previous hidden state**

Example:
forget gate

$$f_t = \sigma_g(W_f x_t + U_f h_{t-1} + b_f)$$

sigmoid nonlinearity (0…1)

LSTM - INPUT GATE



* The **input gate** is multiplied by a **proposed cell state** and then added to the **previous cell state**

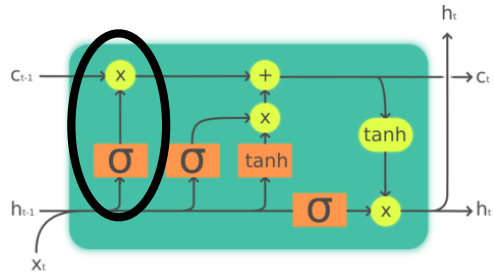
input gate: $i_t = \sigma_g(W_i x_t + U_i h_{t-1} + b_i)$

proposed cell state: $\tilde{c}_t = \tanh(W_c x_t + U_c h_{t-1} + b_c)$

new cell state: $c_t = \cdots + i_t \circ \tilde{c}_t$

element-wise or "Hadamard" product

LSTM - FORGET GATE

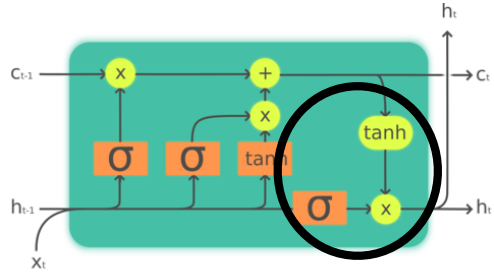


* The **forget gate** is multiplied by the **previous cell state** and then added to the **proposed cell state** times the **input gate**

forget gate: $f_t = \sigma_g(W_f x_t + U_f h_{t-1} + b_f)$

new cell state: $c_t = f_t \circ c_{t-1} + i_t \circ \tilde{c}_t$

LSTM - OUTPUT GATE

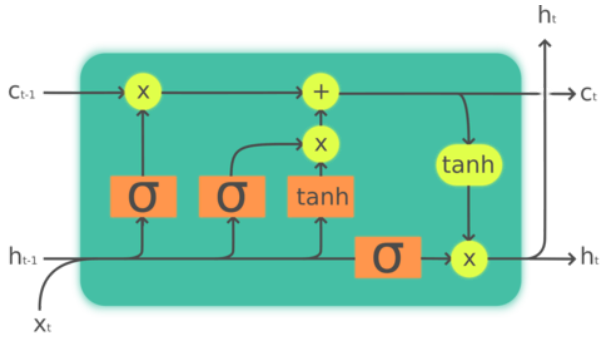


* The **output gate** is multiplied by the **current cell state** (with non-linearity applied) to form the **new hidden state**

output gate: $o_t = \sigma_g(W_o x_t + U_o h_{t-1} + b_o)$

new hidden state: $h_t = o_t \circ \tanh(c_t)$

LSTM



* All together now!
  * Input gate: $i_t = \sigma_g(W_i x_t + U_i h_{t-1} + b_i)$
  * Forget gate: $f_t = \sigma_g(W_f x_t + U_f h_{t-1} + b_f)$
  * Output gate: $o_t = \sigma_g(W_o x_t + U_o h_{t-1} + b_o)$
  * Proposed cell: $\tilde{c}_t = \tanh(W_c x_t + U_c h_{t-1} + b_c)$
  * New cell state: $c_t = f_t \circ c_{t-1} + i_t \circ \tilde{c}_t$
  * New hidden state: $h_t = o_t \circ \tanh(c_t)$

LSTM

* How does the LSTM solve the problems of exploding & vanishing gradients?

* Similar to our earlier 1-D example:

$$c_{t+1} = f_{t+1} \circ c_t + \dots \qquad \frac{\partial c_{t+1}}{\partial c_t} \approx f_{t+1}$$

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$$\frac{\partial c_{t+n}}{\partial c_t} \approx \prod_{j=1}^{n} f_{t+j}$$

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$$c_{t+1} = f_{t+1} \circ c_t + \dots \qquad \frac{\partial c_{t+1}}{\partial c_t} \approx f_{t+1}$$

can shrink (a bit) but not explode! $\longrightarrow$ $\frac{\partial c_{t+n}}{\partial c_t} \approx \prod_{j=1}^{n} f_{t+j}$

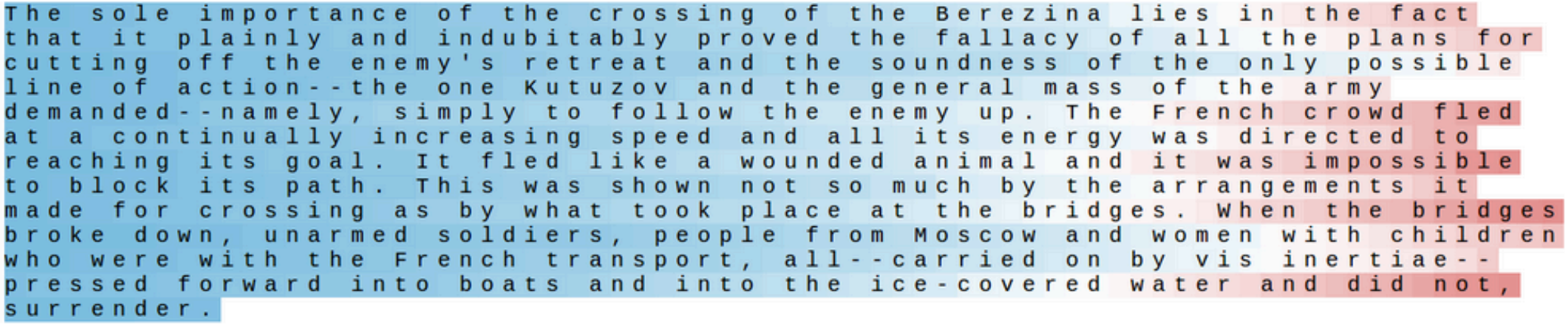
LSTM VISUALIZATOIN

* Suppose we train an LSTM to predicts the **next character** in a sequence from the previous characters
  * This network is trained either on text (e.g. *War and Peace*) or C++ code (e.g. the Linux kernel)
* Visualizing the **cell state values** (for a few units) at each character can show a bit of what the network has learned!

From: http://karpathy.github.io/2015/05/21/rnn-effectiveness/

LSTM VISUALIZATOIN

Cell sensitive to position in line:



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LSTM VISUALIZATOIN

Cell that robustly activates inside if statements:

```
static int __dequeue_signal(struct sigpending *pending, sigset_t *mask,
        siginfo_t *info)
{
  int sig = next_signal(pending, mask);
  if (sig) {
    if (current->notifier) {
      if (sigismember(current->notifier_mask, sig)) {
        if (!(current->notifier)(current->notifier_data)) {
          clear_thread_flag(TIF_SIGPENDING);
          return 0;
        }
      }
    }
    collect_signal(sig, pending, info);
  }
  return sig;
}
```

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LSTMS IN SYSTEM CONSTRUCTION

* Jain & Huth (2018) used (word-level) LSTM language models to extract features for modeling fMRI responses to natural language