SYSTEM IDENTIFICATION

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LAST TIME

* Experimental design
* Deductive
  * Contrast- and hypothesis-driven
* Inductive
  * Natural stimuli
  * Data-driven

TODAY

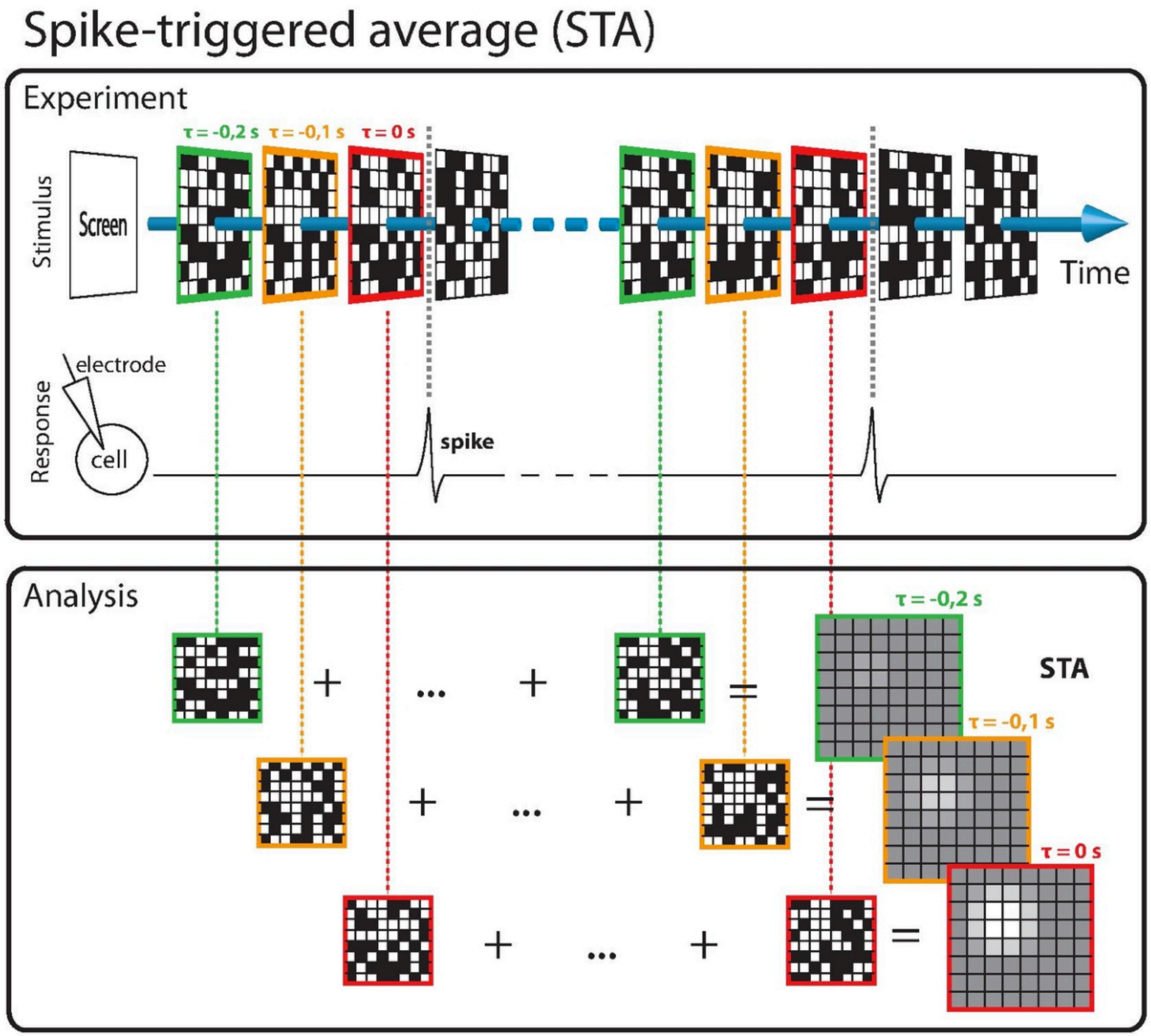
- Spike triggered average (👎)
- Correcting for confounding variables
- System identification

SPIKE-TRIGGERED AVERAGE

* Suppose we are doing an experiment where we record from one neuron in primary visual cortex (V1) while we show images

* How do we characterize the **receptive field** of this neuron?

SPIKE-TRIGGERED AVERAGE

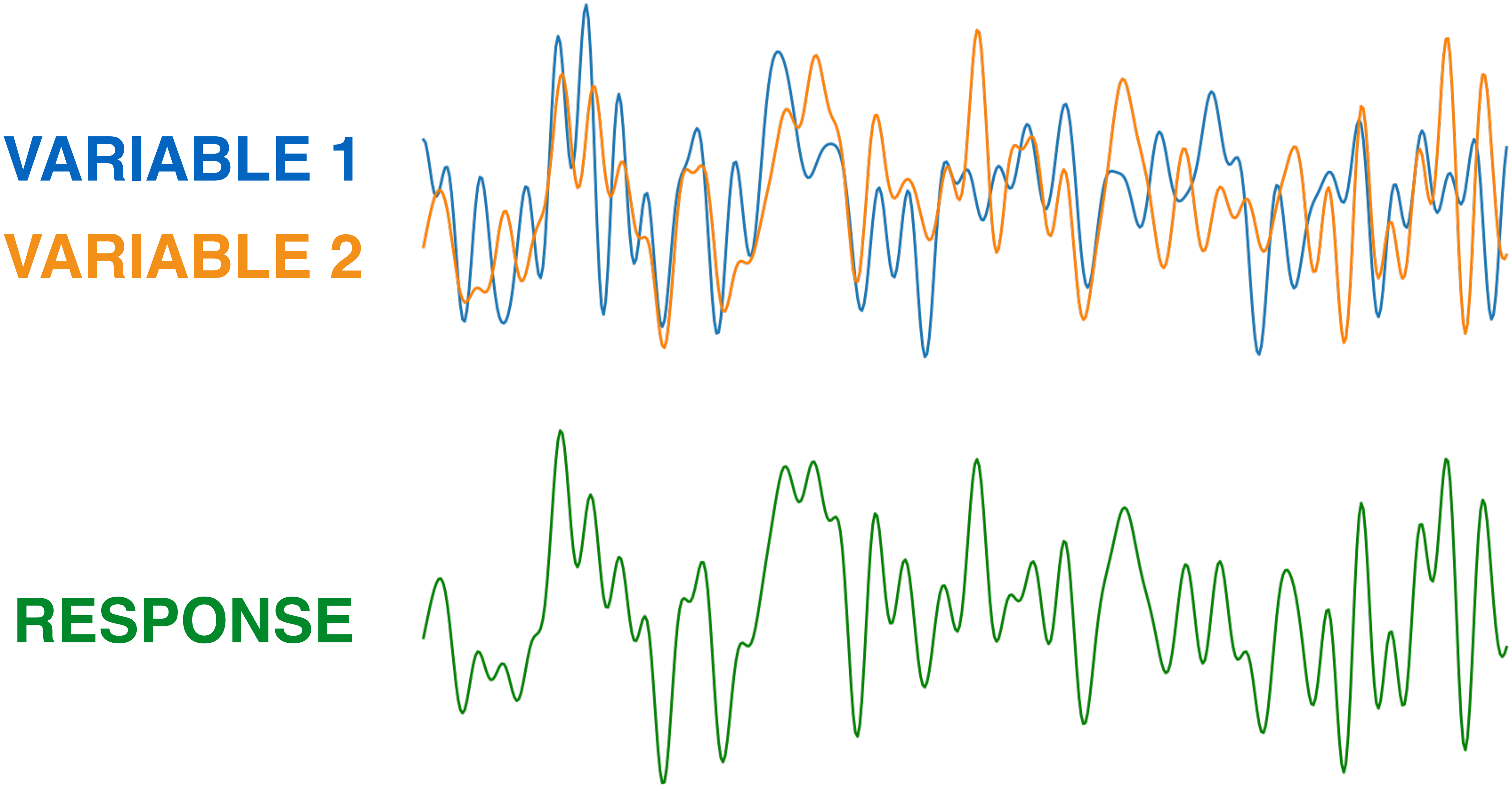


SPIKE-TRIGGERED AVERAGE

* What can go **wrong** with the spike-triggered average (STA)?

* Correctly using STA puts a strong requirement on our **experimental design.** What is it?

VARIABLES ARE CORRELATED?



VARIABLES ARE CORRELATED?

$$\text{RESPONSE} = A \cdot (\text{VARIABLE 1}) + B \cdot (\text{VARIABLE 2}) + E$$

$$A = ? \qquad B = ?$$

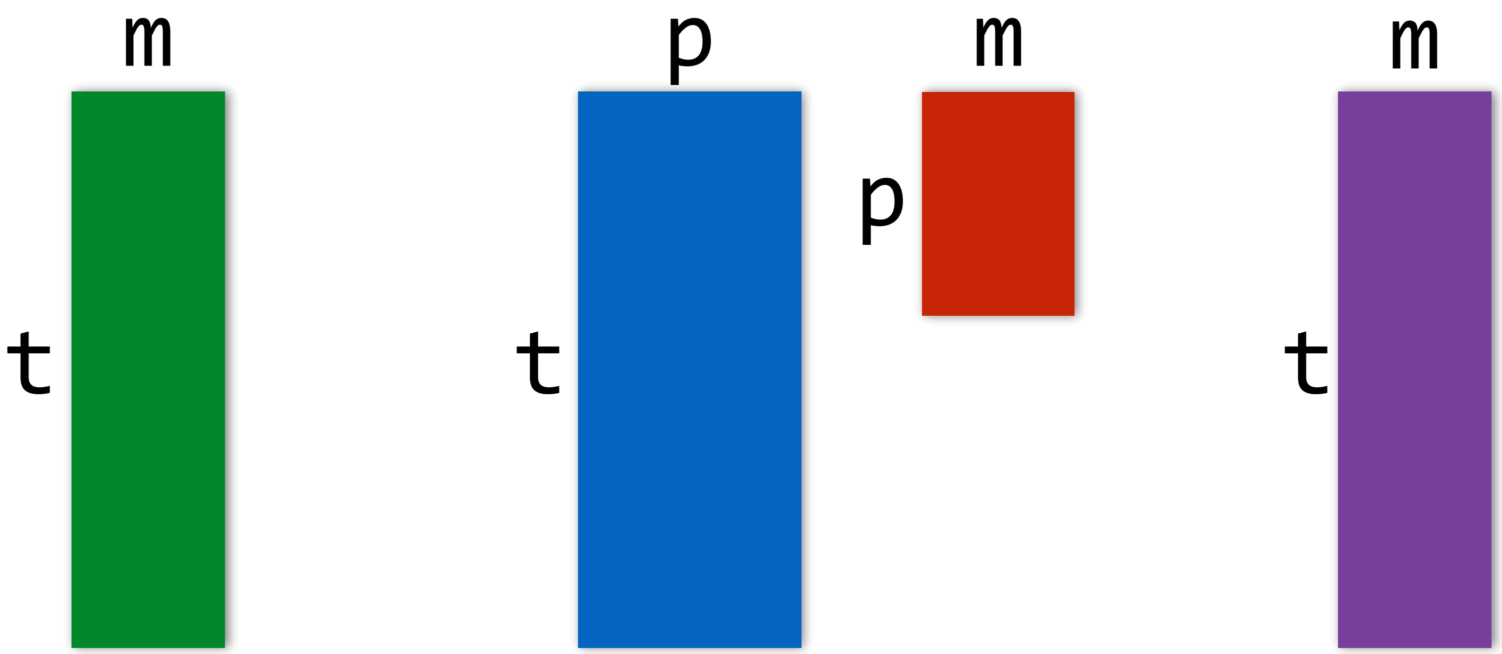
REGRESSION

$$\overset{\textbf{RESPONSE}}{Y} = \overset{\textbf{VARIABLES}}{X}\overset{\textbf{WEIGHTS}}{\beta} + \overset{\textbf{NOISE}}{\epsilon}$$

REGRESSION

RESPONSE **VARIABLES** **WEIGHTS** **NOISE**

$$Y = X\beta + \epsilon$$



REGRESSION

$$\hat{\beta} = \underbrace{(X^\top X)^{-1} X^\top}_{\text{Moore-Penrose pseudoinverse}} Y$$

\hat\beta = (X^\top X)^{-1} X^\top Y

REGRESSION

$$\hat{\beta} = (X^\top X)^{-1} X^\top Y$$

~precision matrix

un-mixes the
variables

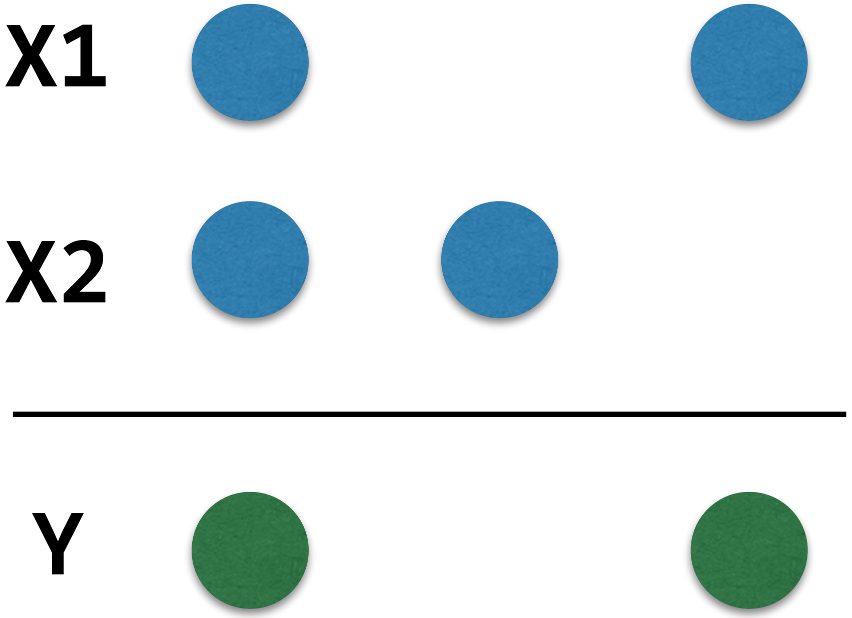
**spike-triggered
average**

correlation
between X & Y

REGRESSION

- *Regression* is the process of **correcting for correlations** between variables (as much as possible)

SIMPLE EXAMPLE



B1 = ?

B2 = ?

SYSTEM IDENTIFICATION

$$Y = f(X)$$

* What kind of a function is f?

SYSTEM IDENTIFICATION

READ THIS PAPER for next Tuesday (2/18):

Complete Functional Characterization of Sensory Neurons by System Identification

Michael C.-K. Wu,[1] Stephen V. David,[2] and Jack L. Gallant[3,4]

https://github.com/alexhuth/neuralcomputation-sp2020/

SYSTEM IDENTIFICATION

* Linear model

$$Y = X\beta$$

* Linearized model

$$Y = \mathbb{L}(X)\beta$$

* Nonlinear model

$$Y = \Theta(X)$$

LINEAR MODELS

$$Y = X\beta$$

image pixels

X1, Y=0.7

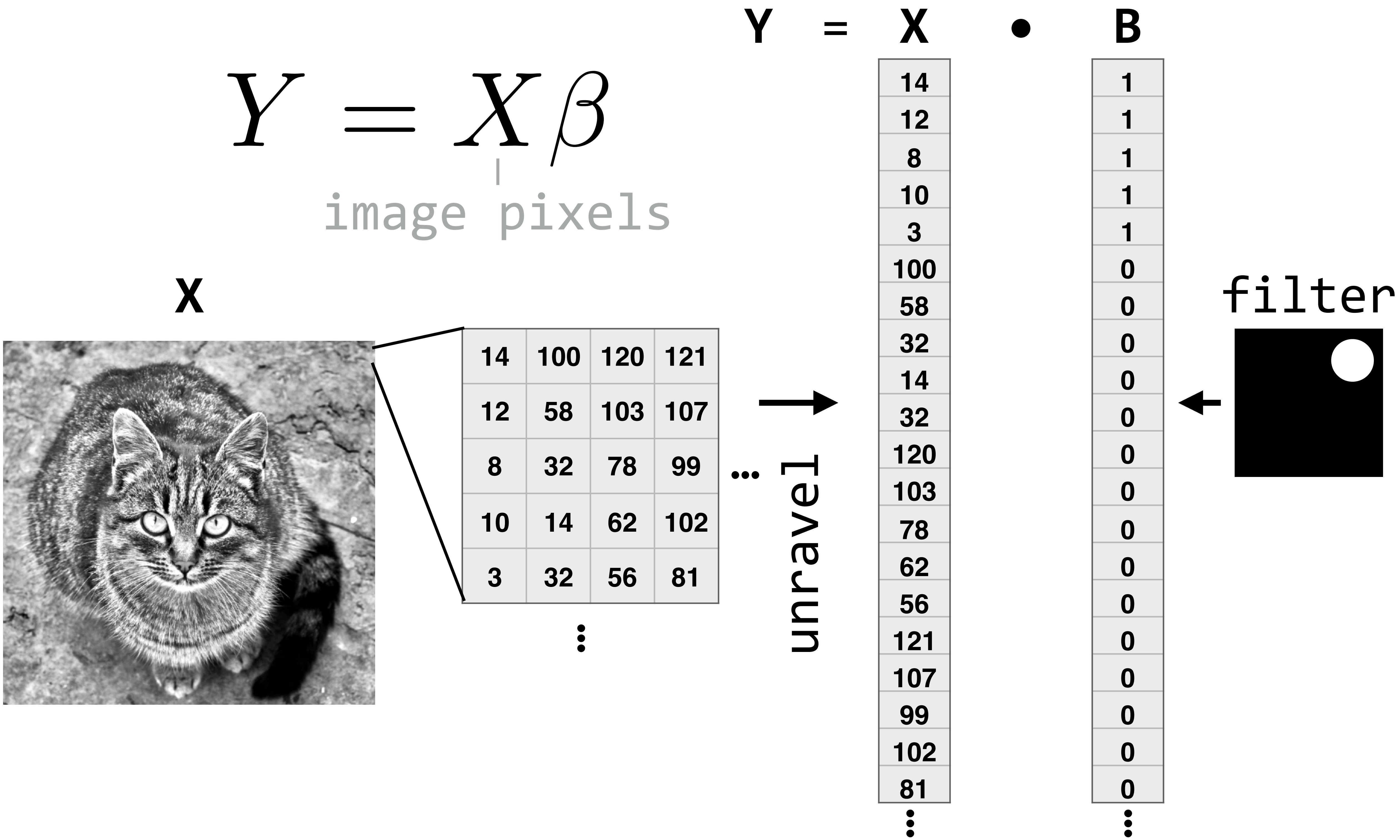
X2, Y=0.3

X3, Y=0.0

LINEAR MODELS

$$Y = X\beta$$

image pixels



X

14	100	120	121
12	58	103	107
8	32	78	99
10	14	62	102
3	32	56	81

⋮ …

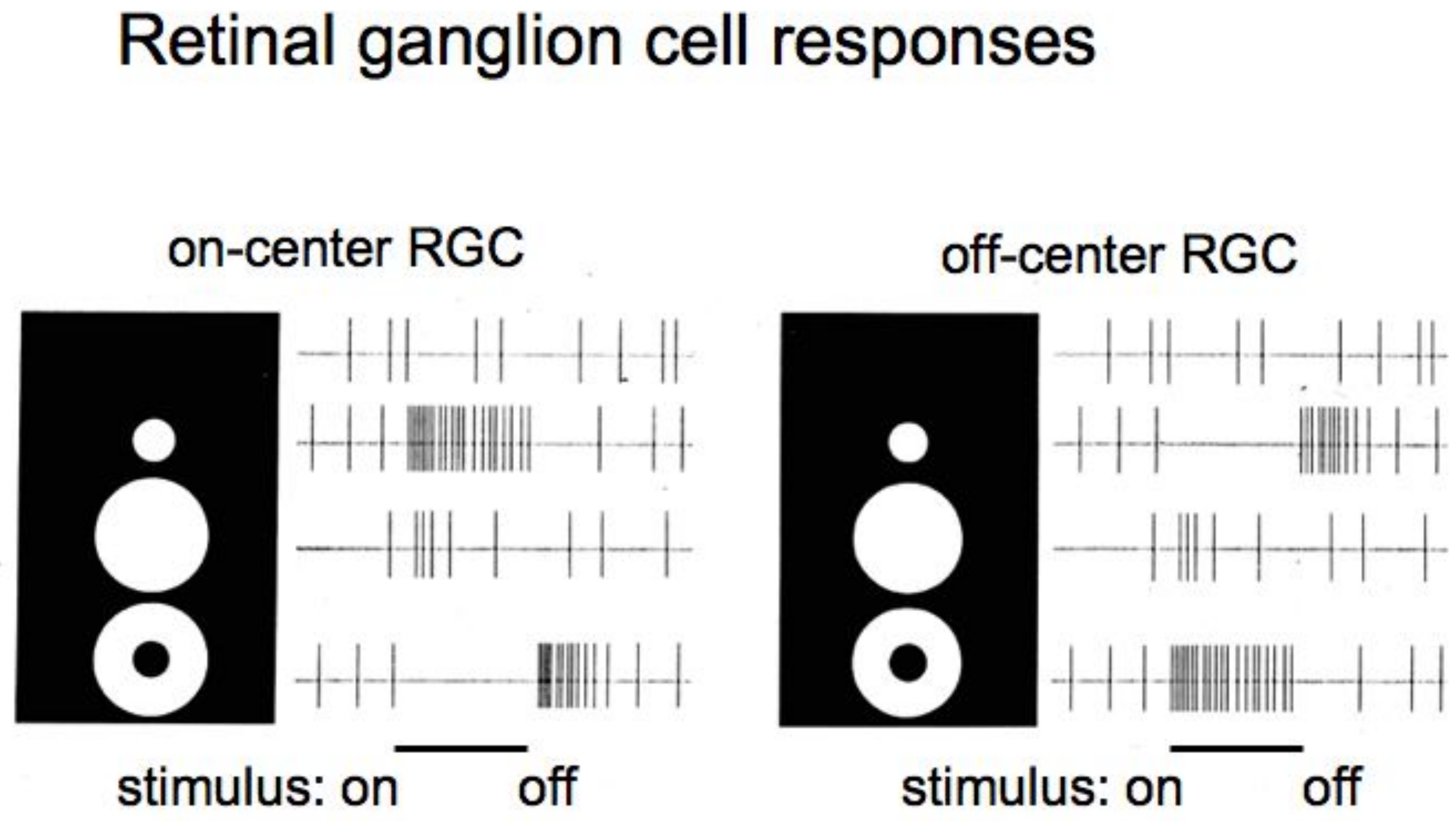
unravel

Y = X • B

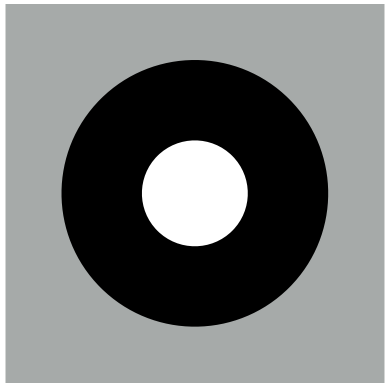
14	1
12	1
8	1
10	1
3	1
100	0
58	0
32	0
14	0
32	0
120	0
103	0
78	0
62	0
56	0
121	0
107	0
99	0
102	0
81	0
⋮	⋮

filter

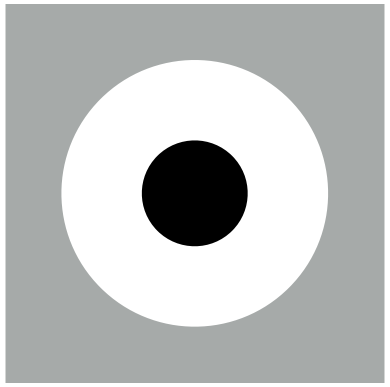
LINEAR MODELS



Beta
(on-center)



Beta
(off-center)



http://www.cns.nyu.edu/~david/courses/perception/lecturenotes/ganglion/ganglion.html

LINEARIZED MODELS

$$Y = \mathbb{L}(X)\beta$$

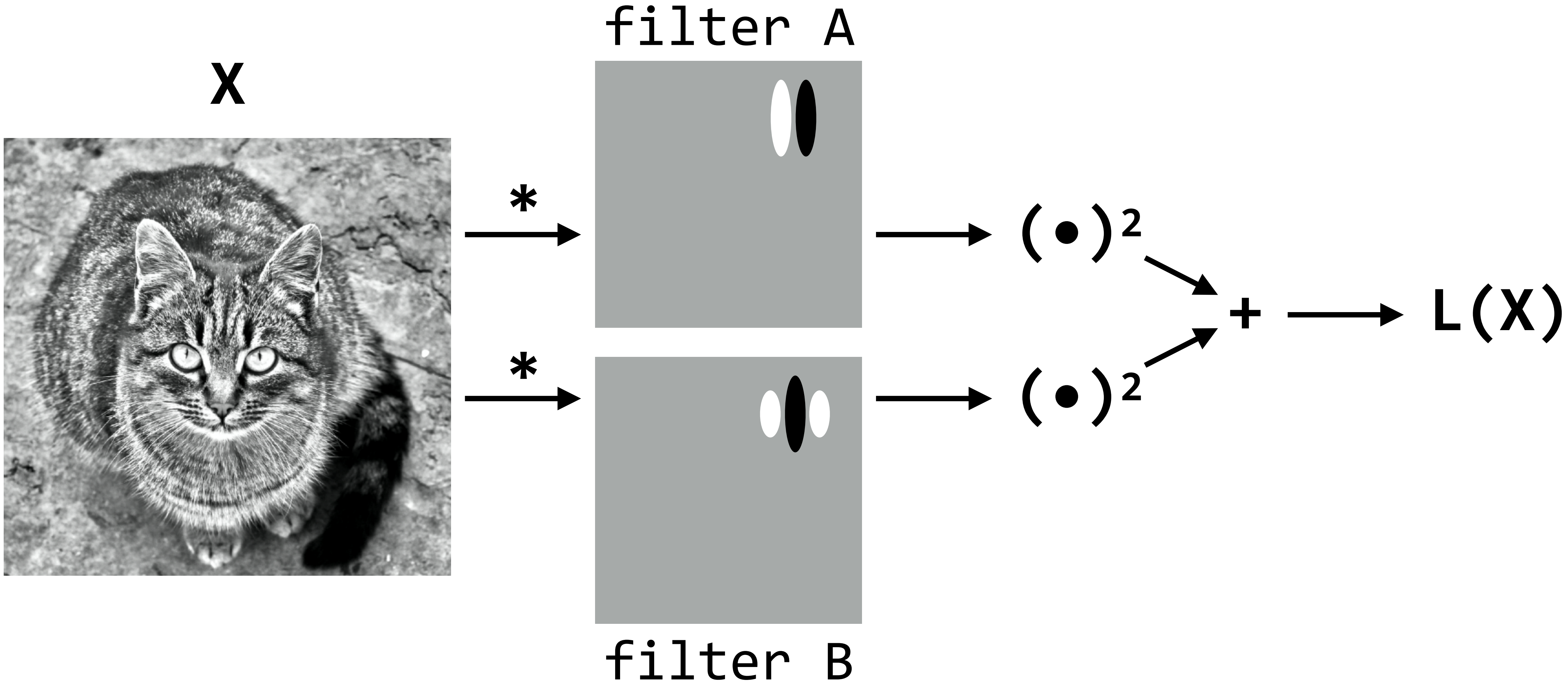
* **L** is some **non-linear** function of the stimulus ***X*** that gives us *features*

  * We call **L** a **linearizing transform**

* ***Beta*** is a linear weighting of the *features* that gives us the response ***Y***

LINEARIZED MODELS

$$Y = \mathbb{L}(X)\beta$$



LINEARIZED MODELS

$$Y = \mathbb{L}(X)\beta$$

story

→

word matrix

word	time			
now	1	0	0	
this	1	0	0	
is	0	1	0	…
a	0	1	0	
story	0	0	1	
⋮				

NONLINEAR MODELS

$$Y = \Theta(X)$$

X1, Y="cat"

X2, Y="dog"

X3, Y="owl"

NONLINEAR MODELS

$$Y = \Theta(X)$$

X1, Y=[1,0,0]

X2, Y=[0,1,0]

X3, Y=[0,0,1]

NONLINEAR MODELS

$$Y = \Theta(X)$$

image pixels

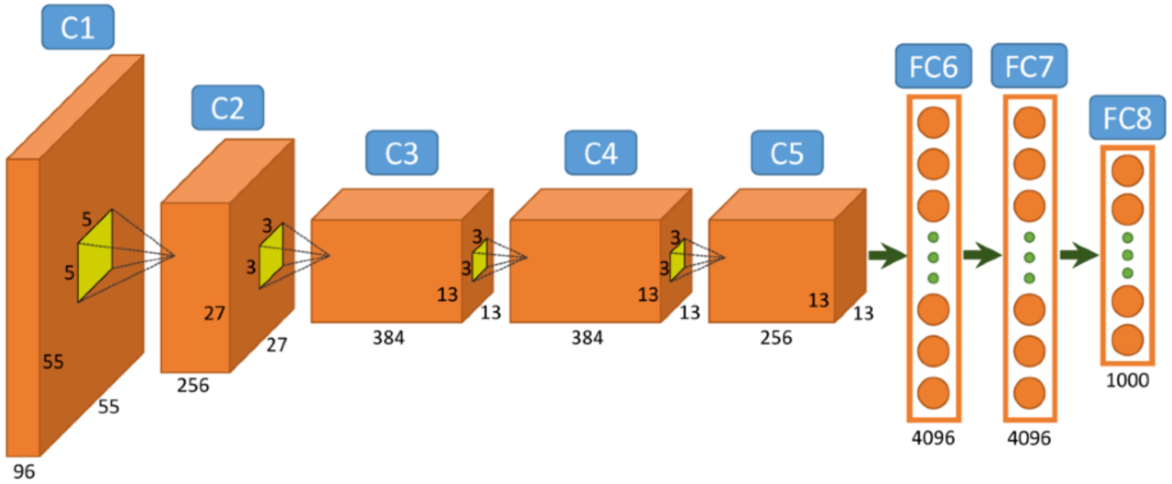
X

Theta

(deep neural network)

Y

→



→ “cat”

https://www.saagie.com/blog/object-detection-part1

SYSTEM IDENTIFICATION

- **Linear model**
  - easy, usually pointless
- **Linearized model**
  - sweet spot, but requires **hypothesis**!
- **Nonlinear model**
  - very expensive, need lots of data

LINEARIZING TRANSFORMATION

=

FEATURE SPACE

=

HYPOTHESIS

RECAP

- Spike-triggered average
- Regression
- System identification
  - Linear
  - Linearized
  - Non-linear

NEXT TIME

- spatiotemporal models
- model fitting