SPATIOTEMPORAL MODELS

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2.25.2020

LAST TIME

* Analytic solutions to L2-regularized regression problems:

* Ridge regression

* Tikhonov regression

TODAY

* Spatiotemporal models
  * Space-time separable
  * Space-time inseparable
* Capturing real dynamics vs. output filtering

SPATIOTEMPORAL MODEL

* We often think of responses as instantaneous & time-invariant (i.e. like a feedforward CNN)

* Real neural systems *always* function across time

* i.e. neural responses are not just a function of the current stimulus, but also of previous stimuli

SPATIOTEMPORAL MODEL

* Temporal dependence can be “real” / *computationally useful*

  * e.g. change detection

  * e.g. computing temporal derivative (chemotaxis/thermotaxis)

SPATIOTEMPORAL MODEL

* Temporal dependence can also be "artifactual" / *imposed by the measurement system*

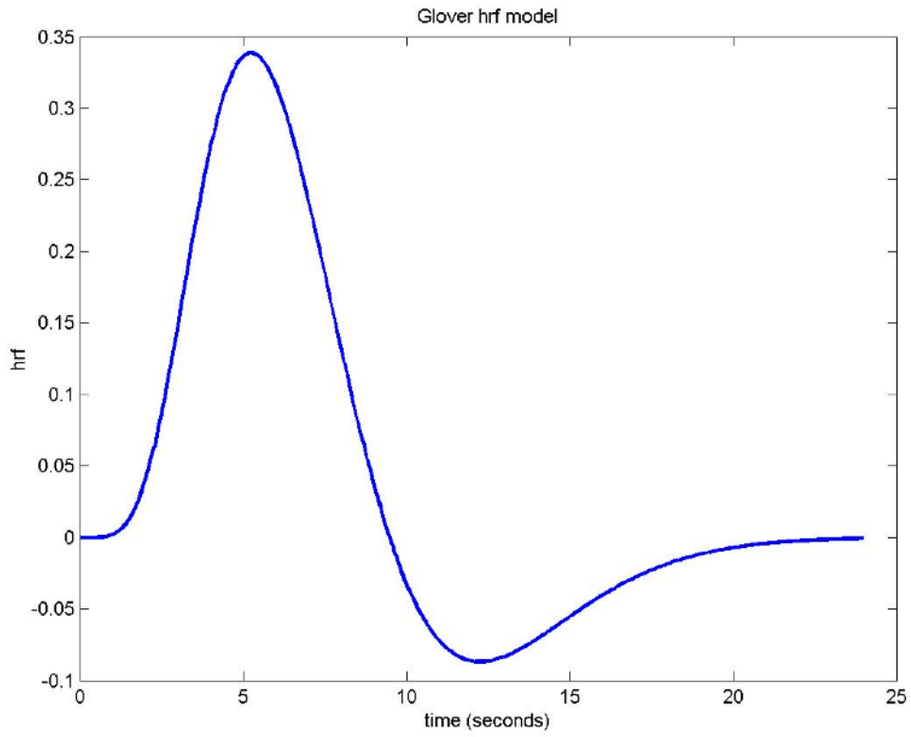
    * e.g. the hemodynamic response in BOLD fMRI

    * e.g. fluorescence decay in GCaMP-based calcium imaging

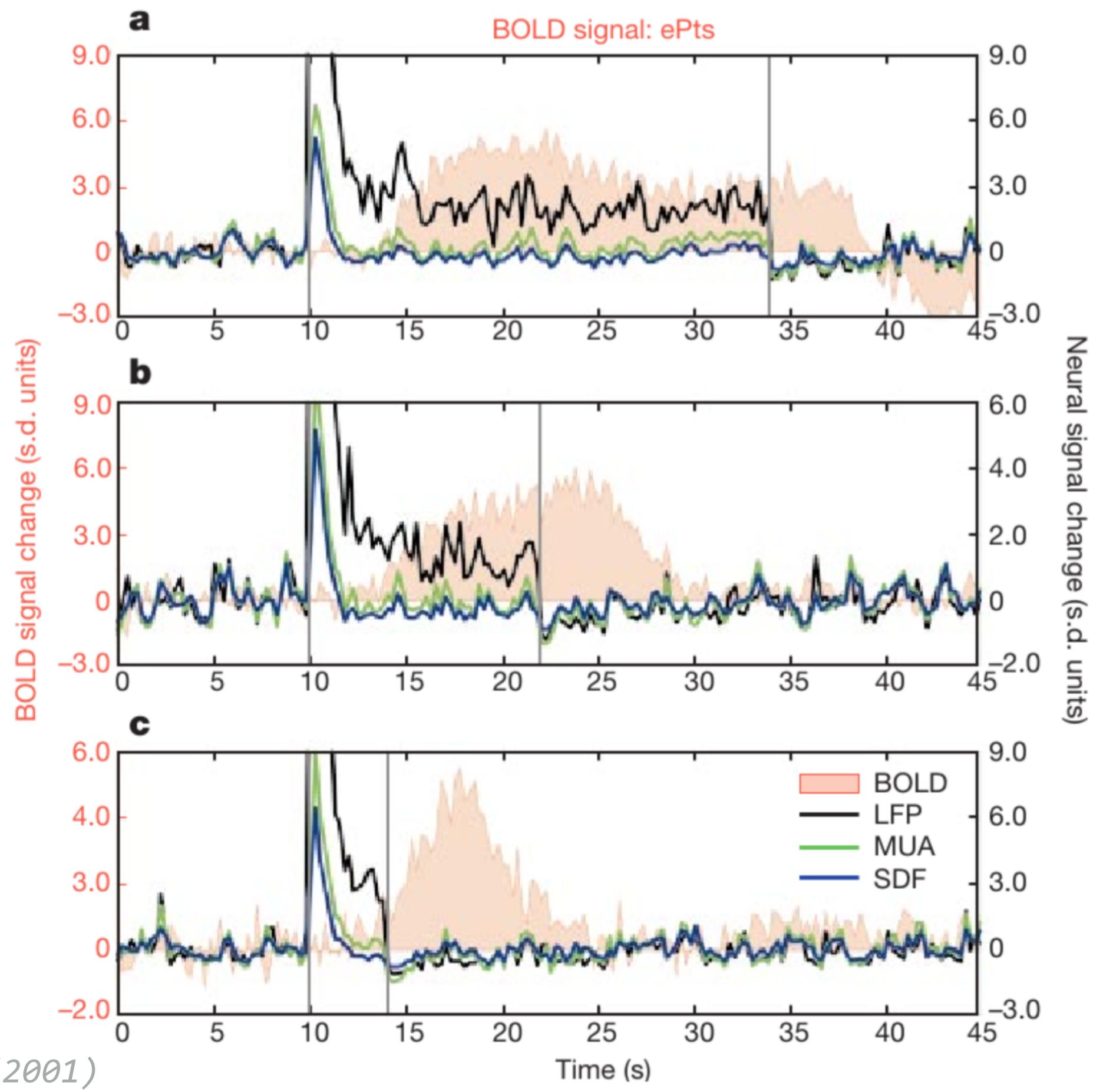
BOLD & HRF

* BOLD = blood-oxygen level dependent

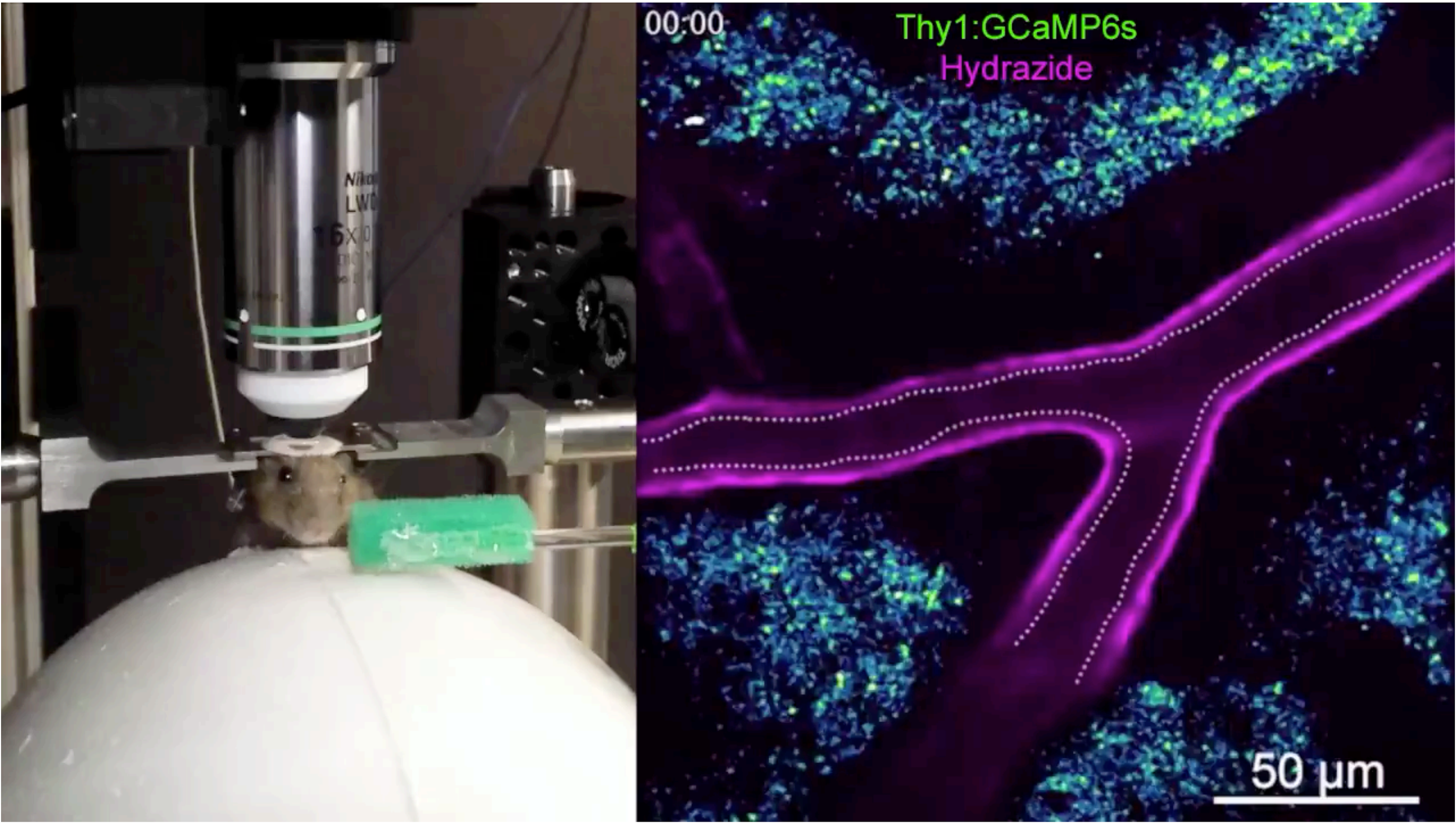
* HRF = hemodynamic response function



typical model HRF



Logothetis et al. (2001)



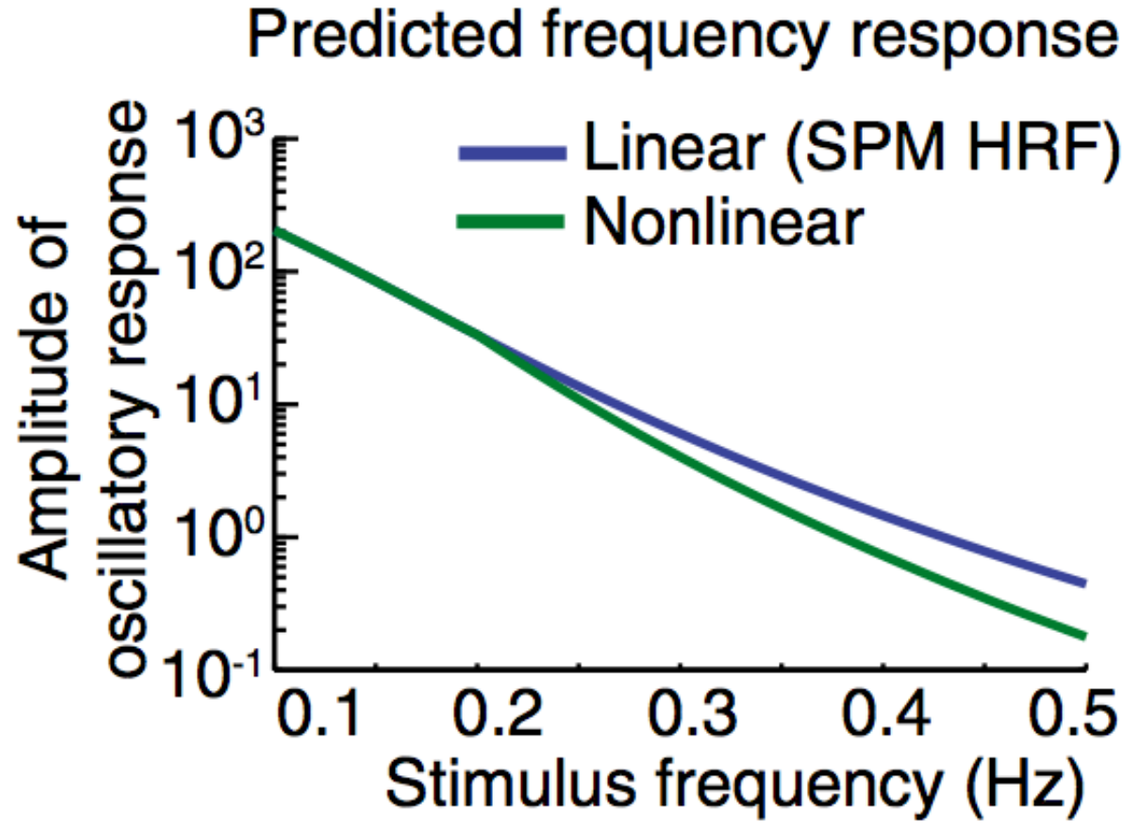
https://twitter.com/Dr_Alex_Crimi/status/1230633683675222016

BOLD & HRF

* HRF is approximately linear

* …but not perfectly

HRF frequency response



Lewis, Setsompop, Rosen, & Polimeni (2016)

SPATIOTEMPORAL MODEL

* HRF needs to be accounted for in model

* Three approaches:

  * assume HRF 👎 👎 👎 🤦‍♀️

  * fit **space-time separable** model

  * fit **space-time inseparable** model

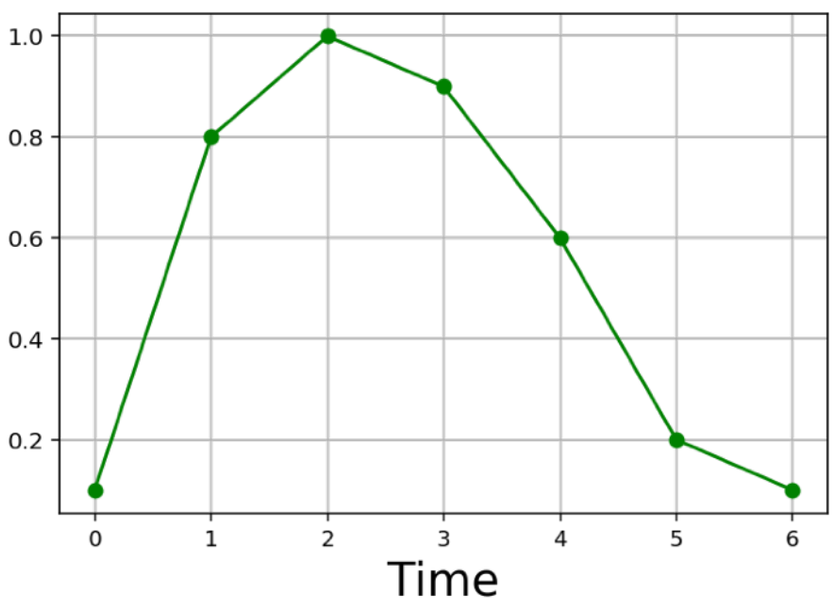
SPACE-TIME SEPARABLE

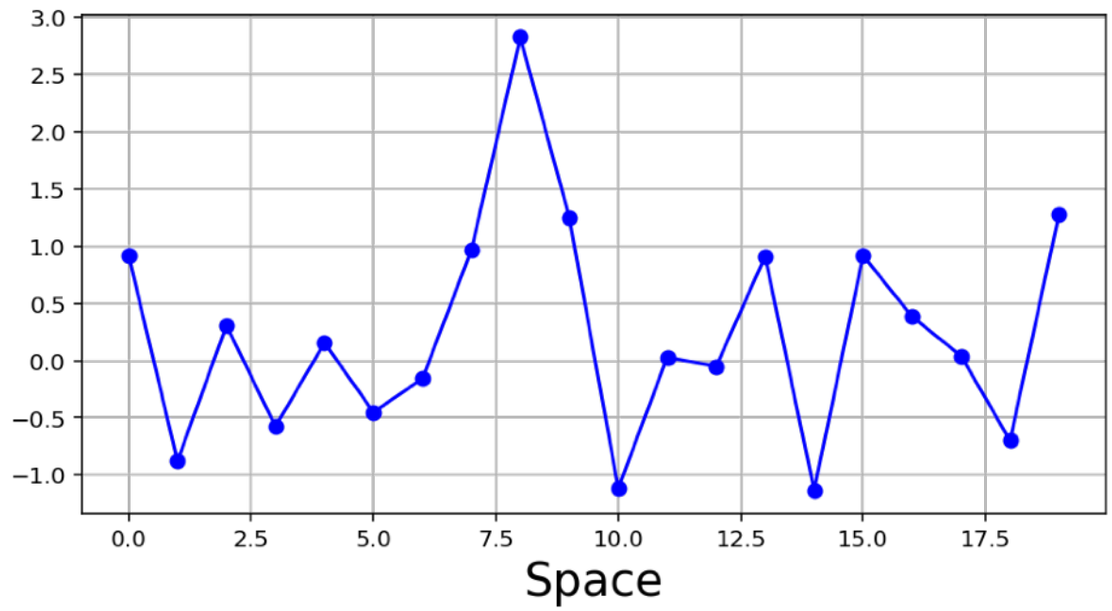
* Model is outer product of one spatial kernel & one temporal kernel

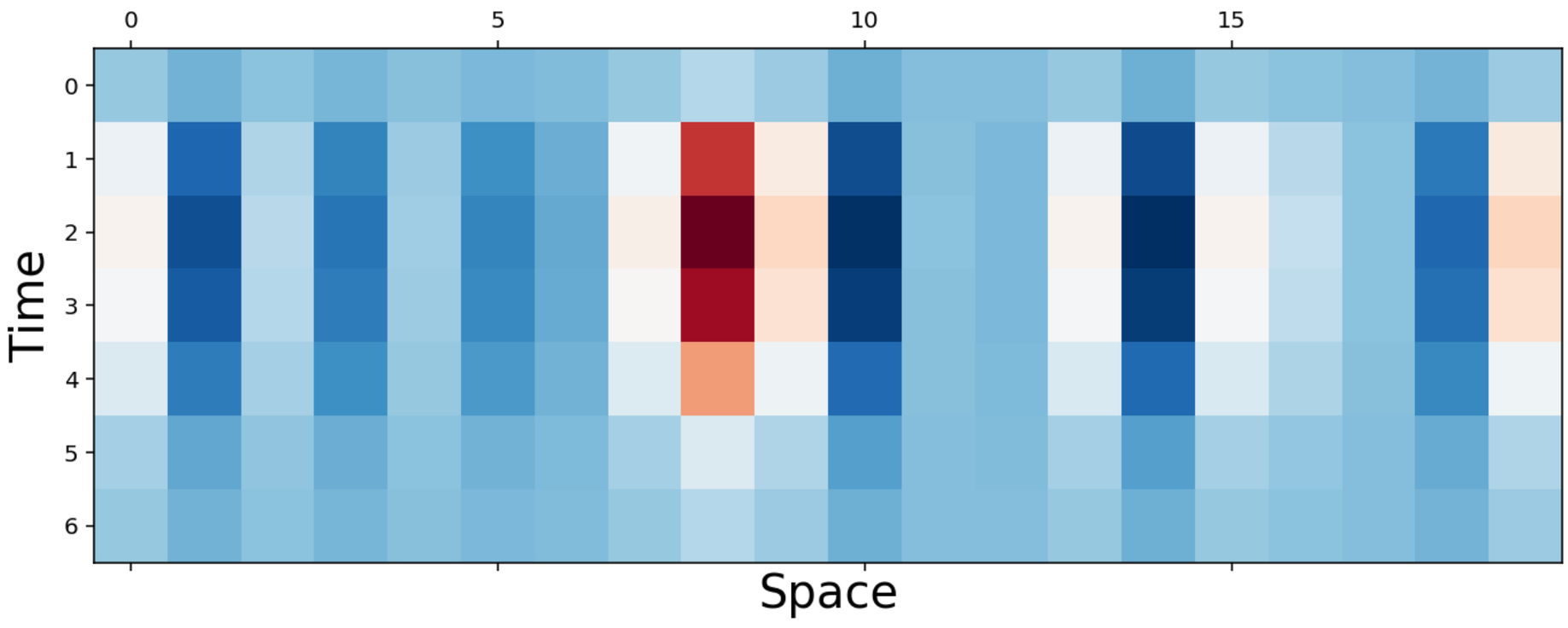
* Fit using e.g. generalized least squares (GLS)

  * Each model (i.e. each voxel) must be fit separately

SPACE-TIME SEPARABLE







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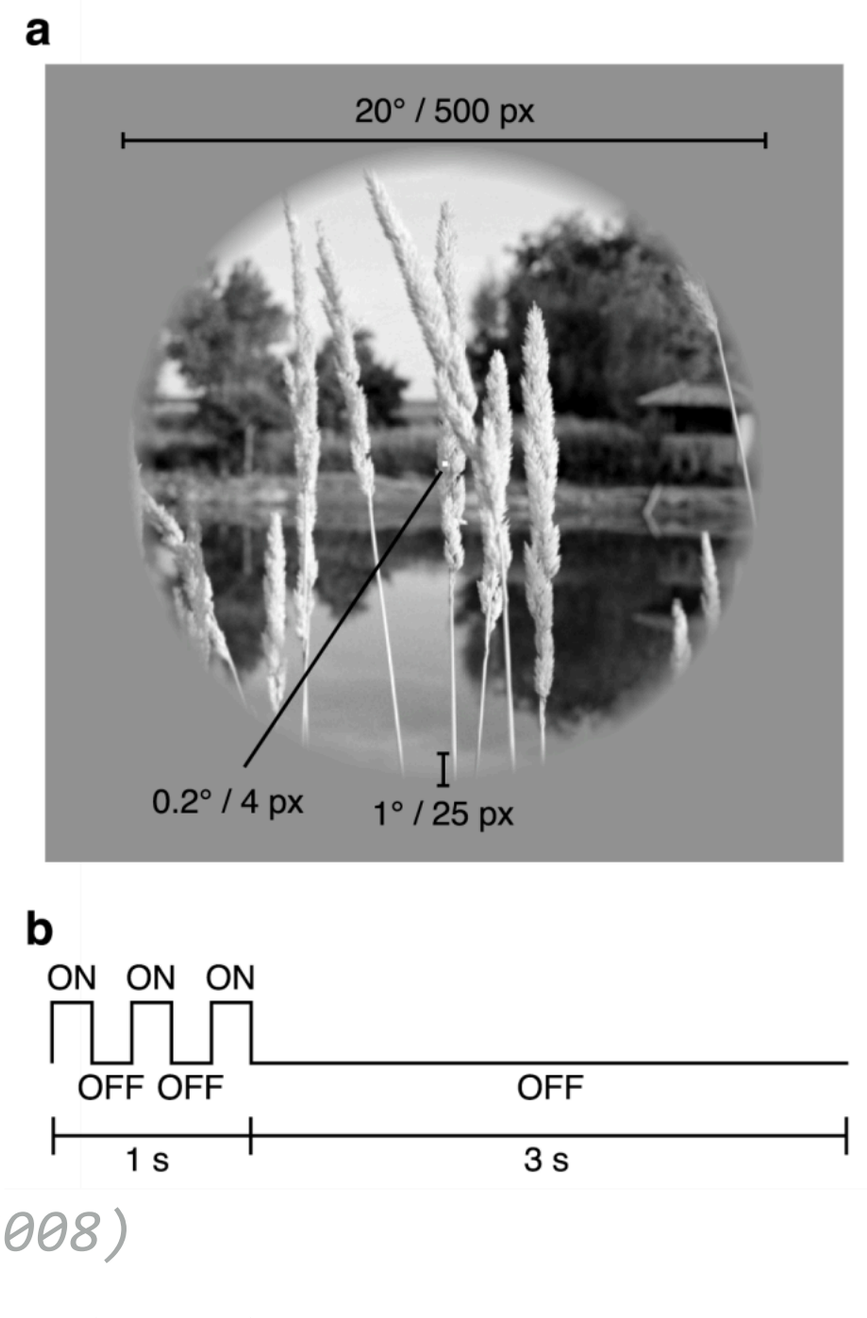
SPACE-TIME SEPARABLE EXAMPLE

* Record BOLD signal using fMRI from visual cortex

* Flash images in random order w/ ~4s spacing



Kay, Naselaris, Prenger, & Gallant (2008)

Kay, David, Prenger, Hansen, & Gallant (2007)

SPACE-TIME SEPARABLE EXAMPLE

* Build 1st stage model: what is response to each image in each voxel?

$$y = (X * k)h + \epsilon$$

design matrix

kernel

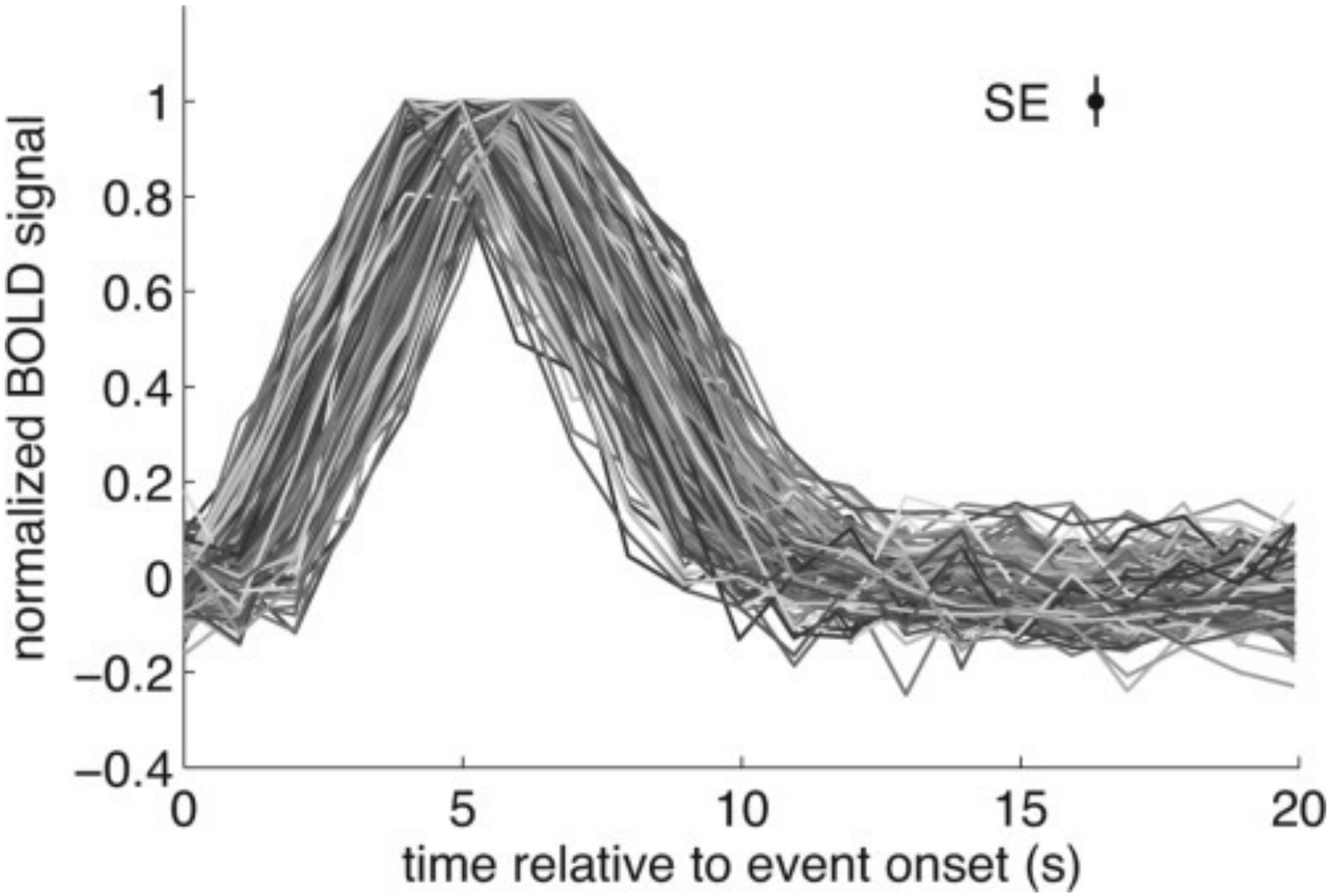
response to each image

Kay, Naselaris, Prenger, & Gallant (2008)

Kay, David, Prenger, Hansen, & Gallant (2007)

SPACE-TIME SEPARABLE EXAMPLE

k for several images & voxels



Kay, Naselaris, Prenger, & Gallant (2008)

Kay, David, Prenger, Hansen, & Gallant (2007)

SPACE-TIME SEPARABLE EXAMPLE

* Build 2nd stage model: what is response to each feature in each voxel?

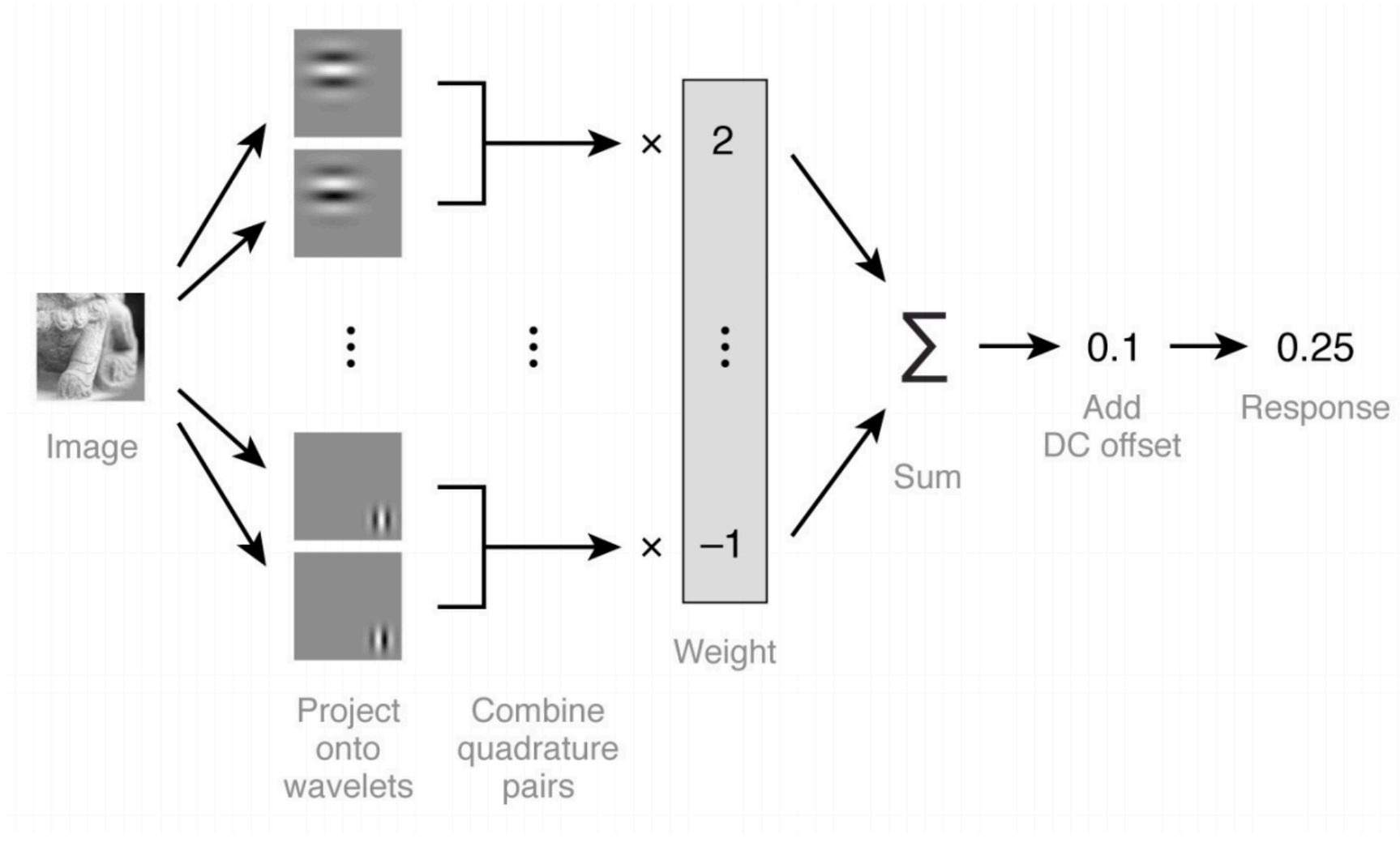
$$h = Z\beta + \epsilon_2$$

response to each image | design matrix | feature weights

Kay, Naselaris, Prenger, & Gallant (2008)

Kay, David, Prenger, Hansen, & Gallant (2007)

SPACE-TIME SEPARABLE EXAMPLE



Kay, Naselaris, Prenger, & Gallant (2008)

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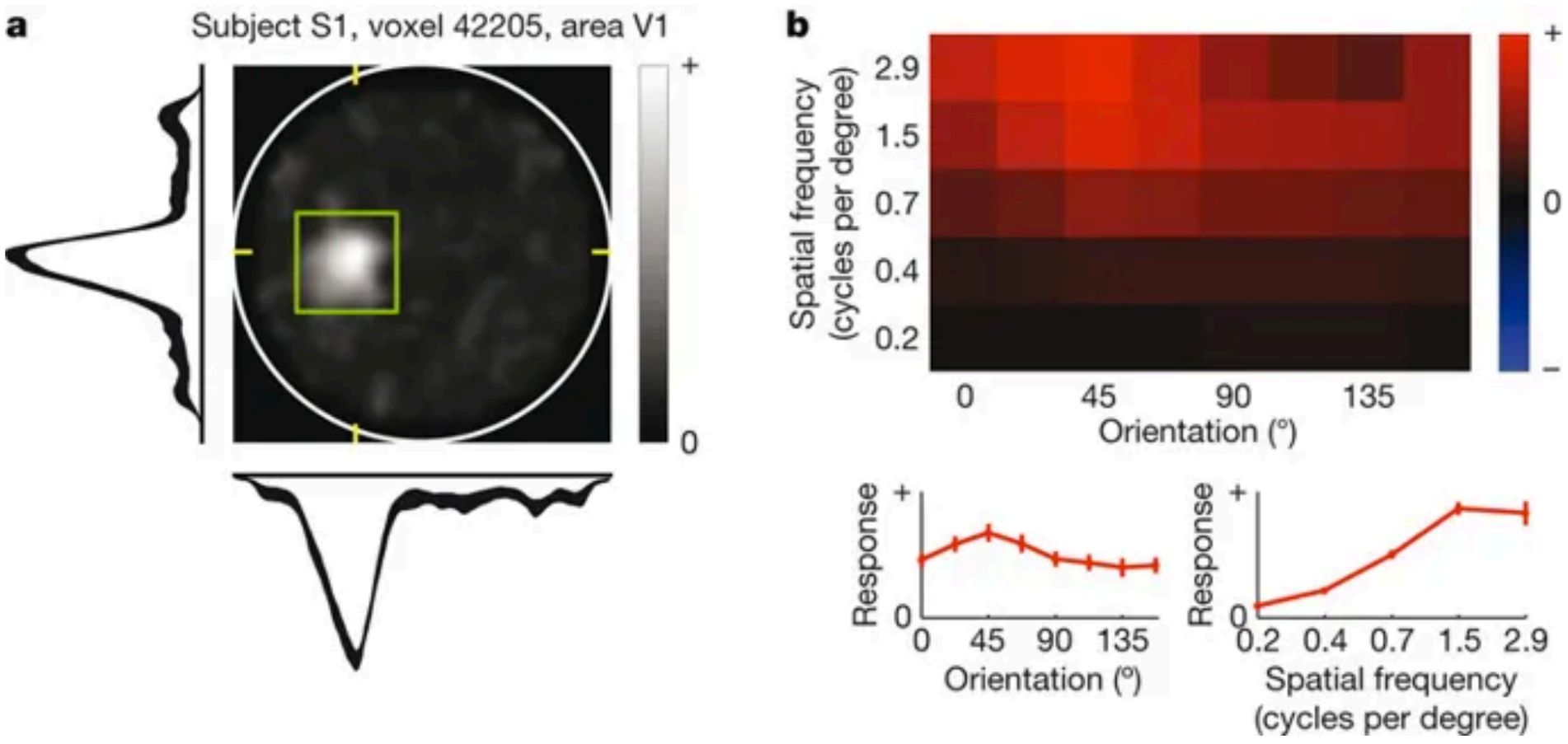
SPACE-TIME SEPARABLE EXAMPLE



Kay, Naselaris, Prenger, & Gallant (2008)

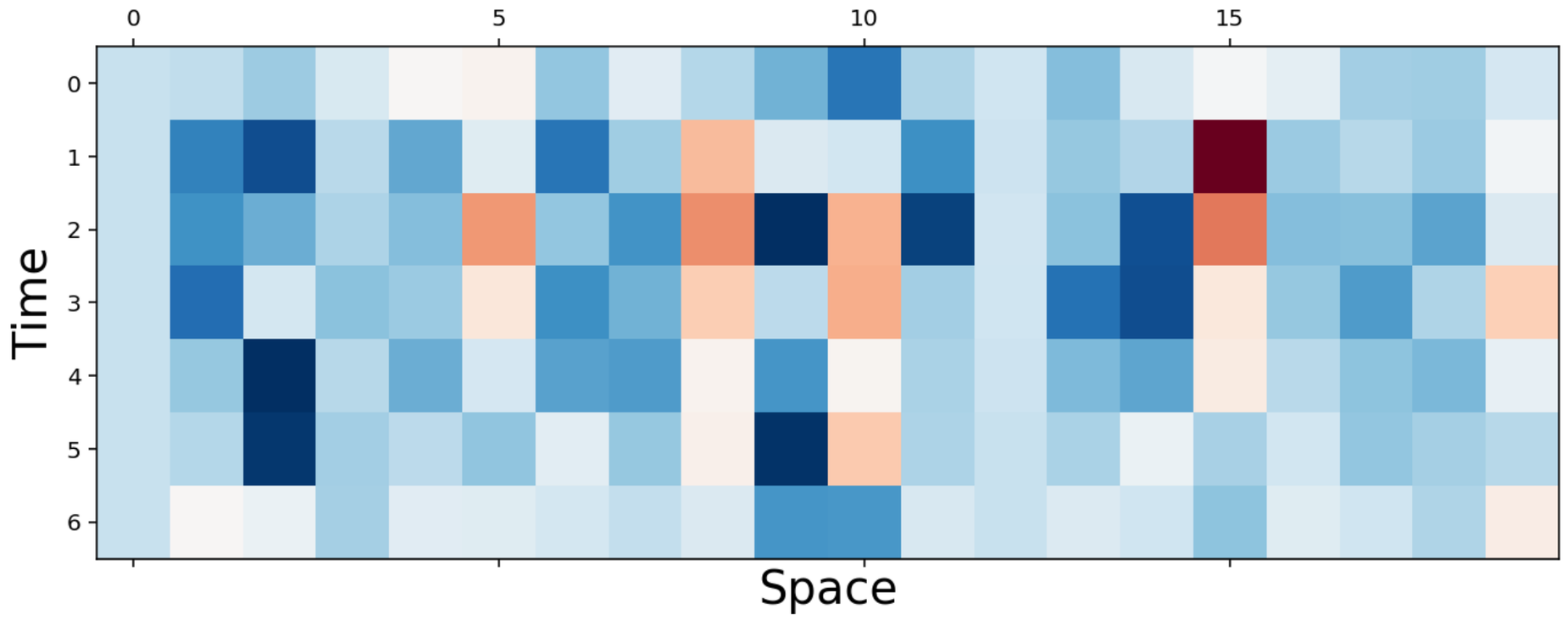
Kay, David, Prenger, Hansen, & Gallant (2007)

SPACE-TIME INSEPARABLE

* Different temporal kernel for each spatial dimensions (i.e. each feature)

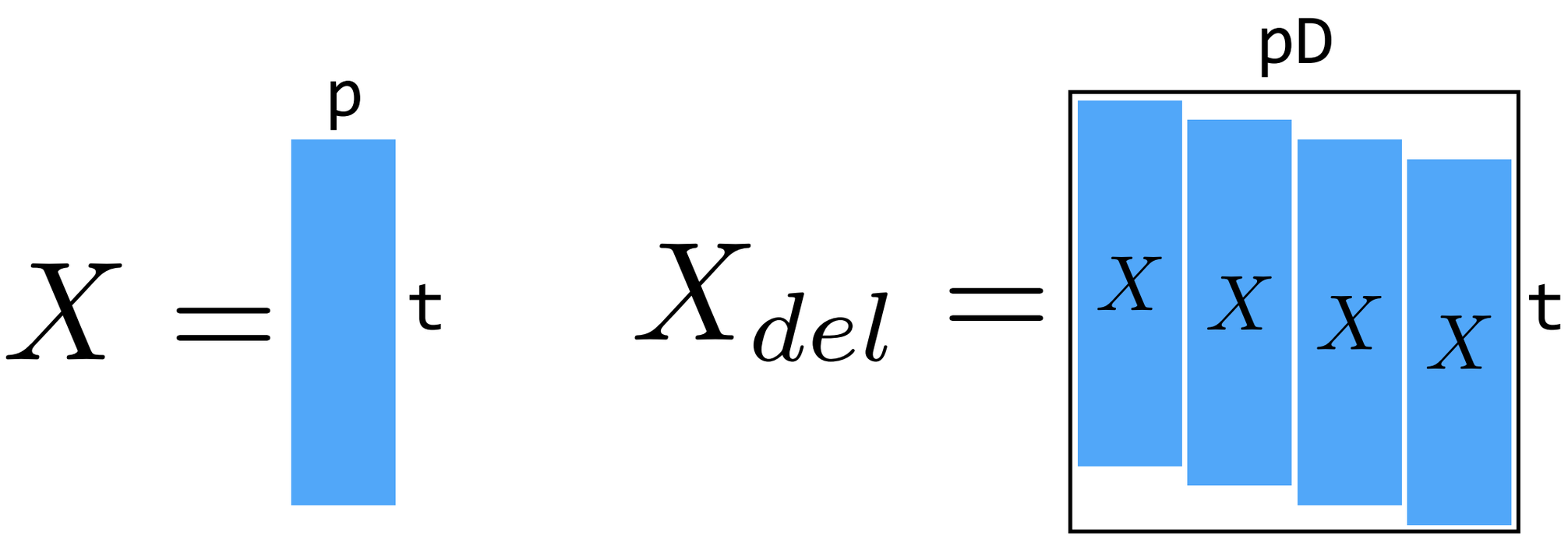
* Fit using ordinary methods

SPACE-TIME INSEPARABLE



FINITE IMPULSE RESPONSE (FIR)

* Concatenate delayed copies of the stimulus matrix



FINITE IMPULSE RESPONSE (FIR)

$$X_{del} = \begin{matrix} \text{Dp} \\ \boxed{\begin{matrix} X & X & X & X \\ X & X & X & X \\ X & X & X & X \\ X & X & X & X \end{matrix}} \end{matrix} \text{t} \qquad Y = X_{del} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{bmatrix}$$

$$Y(t) = \beta_1 X(t) + \beta_2 X(t-1) + \beta_3 X(t-2) + \beta_4 X(t-3)$$

Y(t) = \beta_1 X(t) + \beta_2 X(t-1) + \beta_3 X(t-2) + \beta_4 X(t-3)

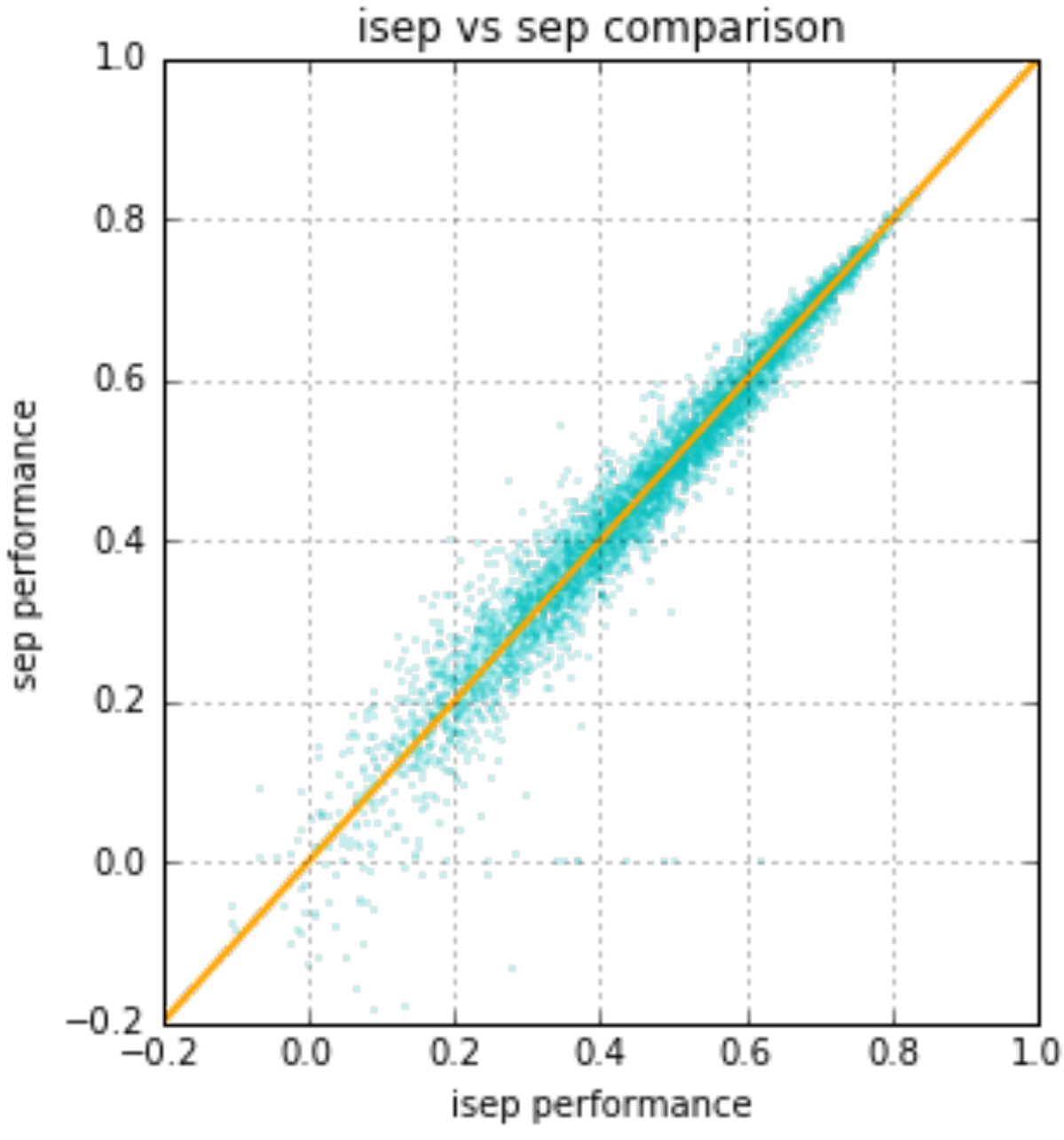
Y = X_{del} \begin{bmatrix}\beta_1\\

SPATIOTEMPORAL MODEL

* **Separable**
    * Expensive
    * Highly constrained
* **Inseparable**
    * Cheap
    * Unconstrained

SPATIOTEMPORAL MODEL

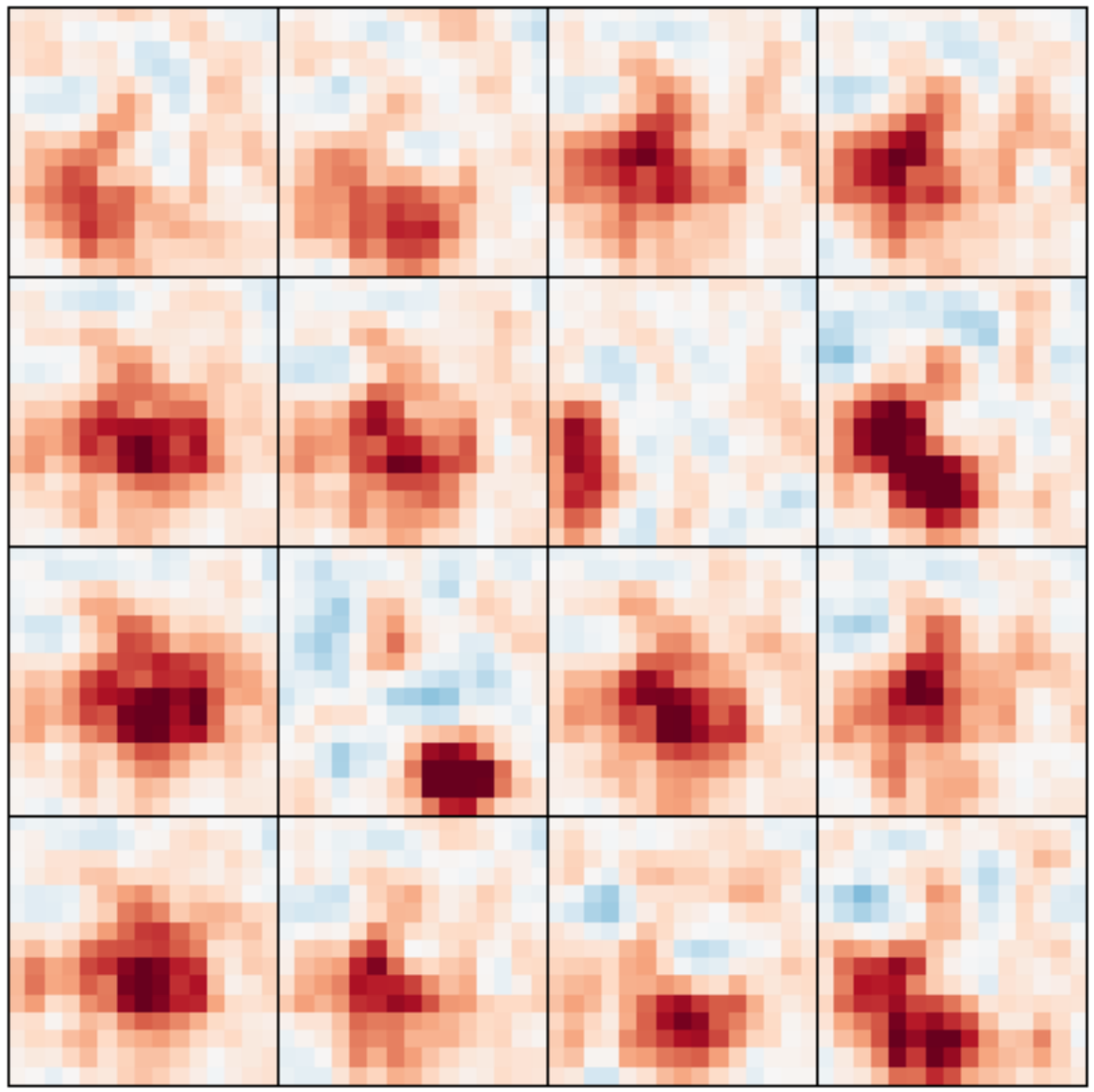
* In practice, both perform very similarly, so I prefer inseparable models



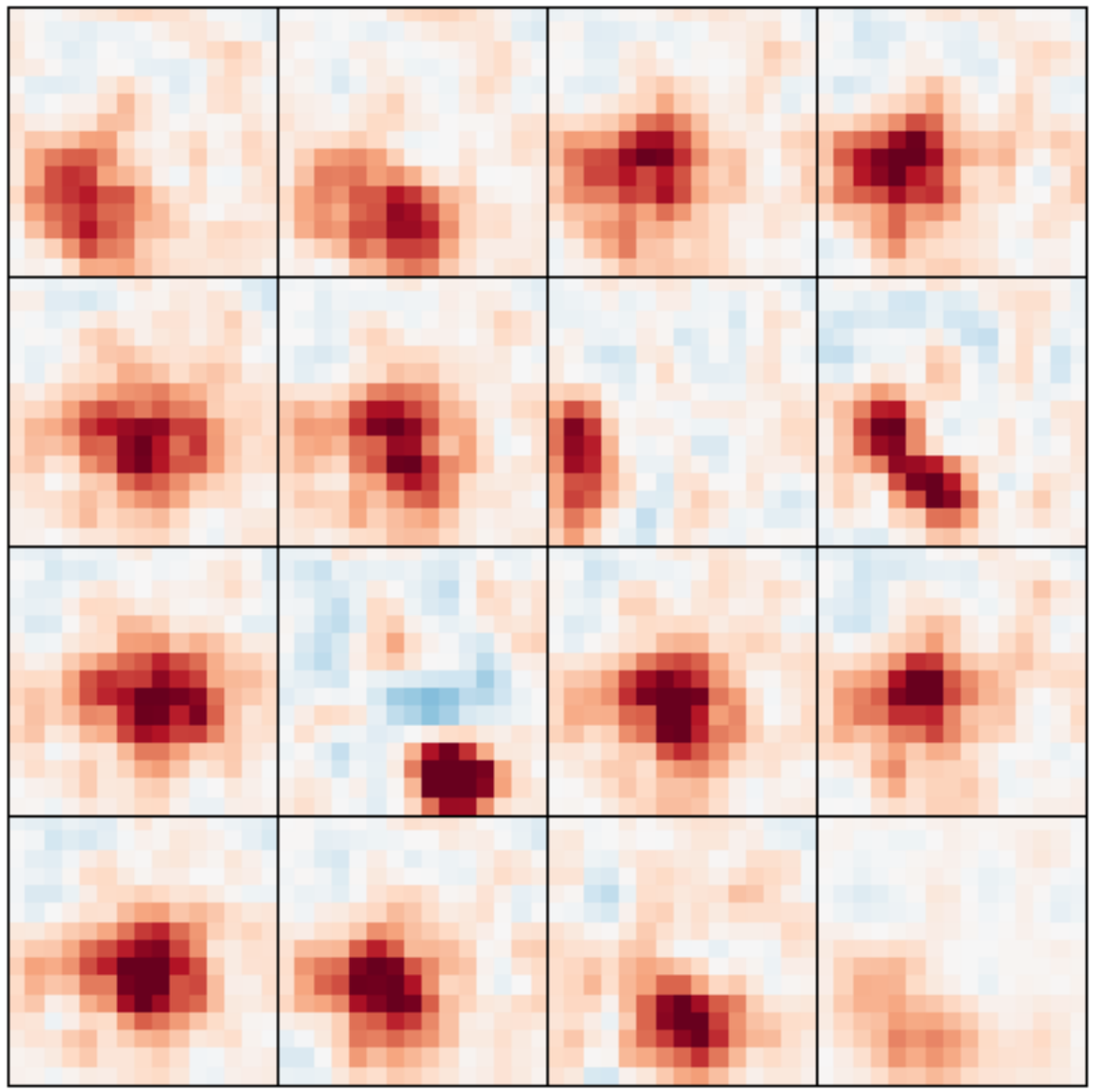
h/t Anwar Nuñez

SPATIOTEMPORAL MODEL

inseparable model weights



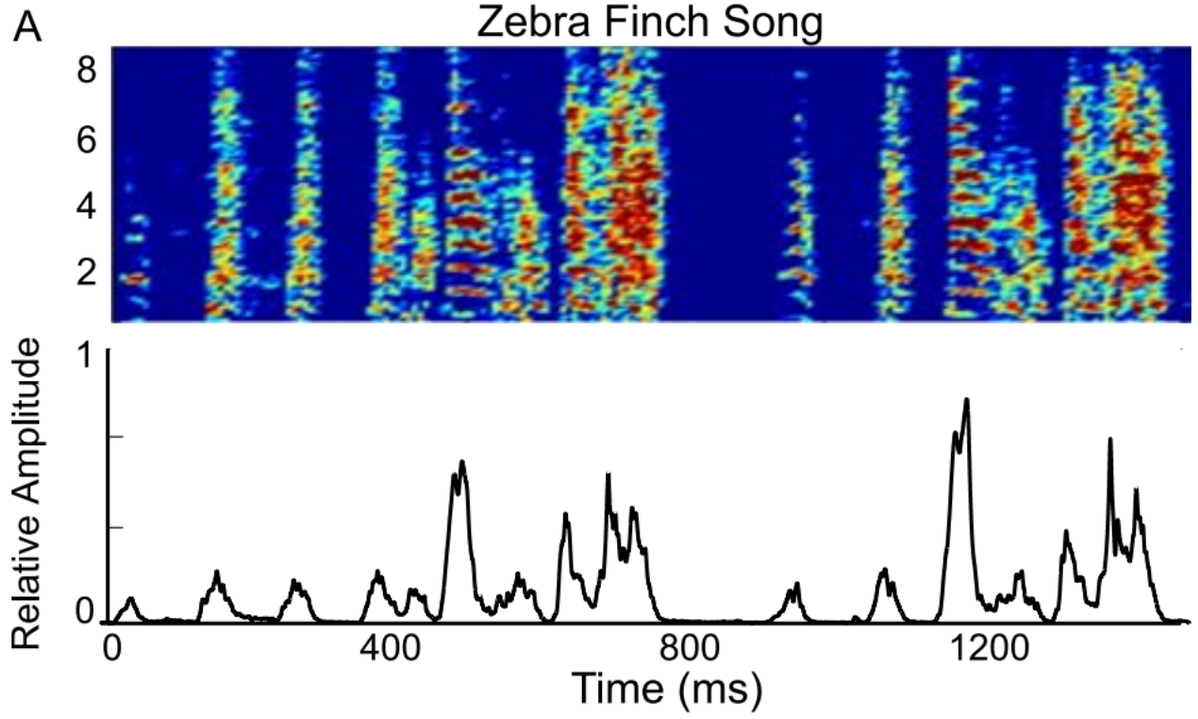
separable model weights



h/t Anwar Nuñez

INSEPARABLE EXAMPLE

* Measure responses in zebra finch neurons while playing them birdsongs

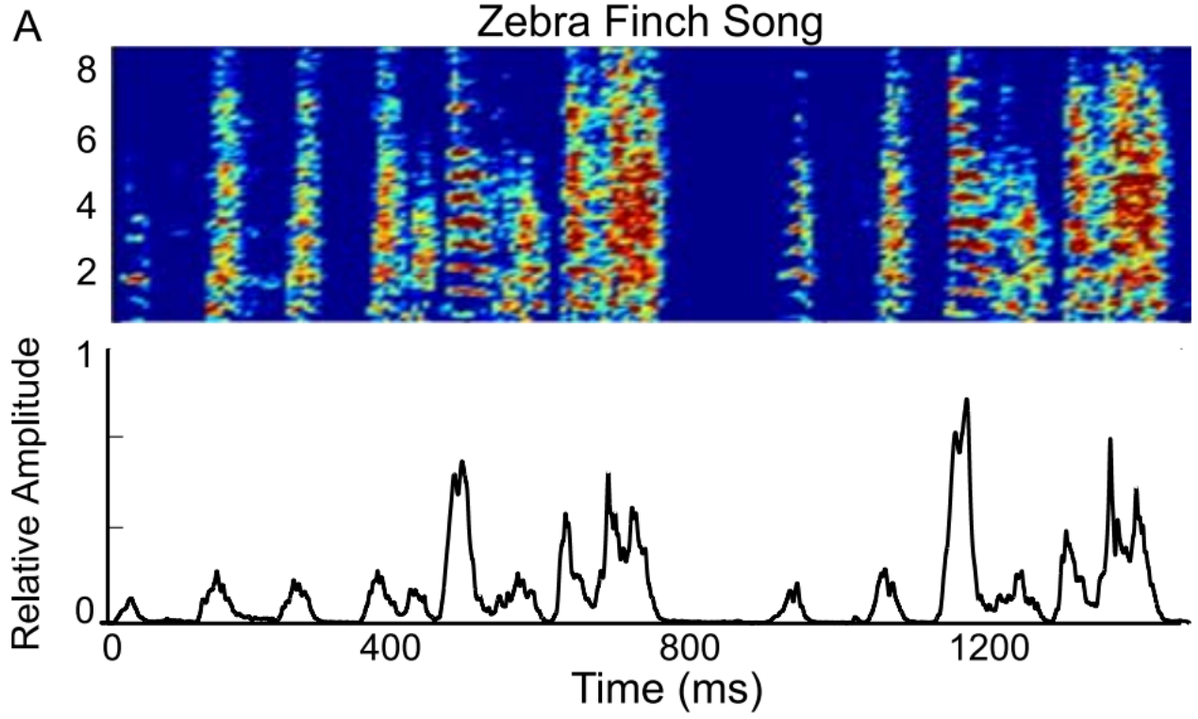


Woolley, Gill, & Theunissen (2006)

INSEPARABLE EXAMPLE

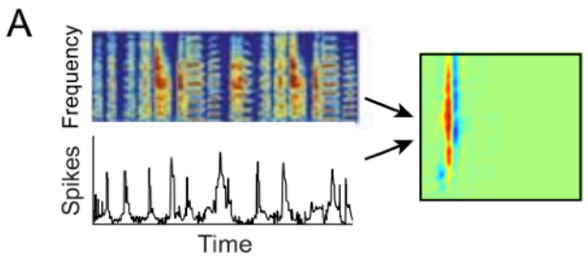
* Fit model that relates spectrogram of birdsong to neural response

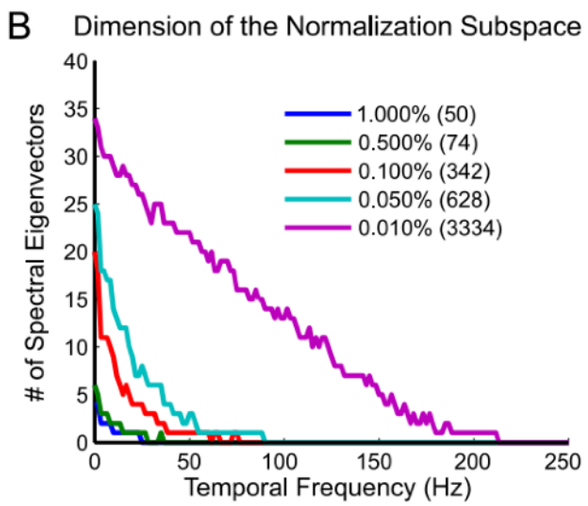
* Regularize using method similar to elastic net (L1 + L2)

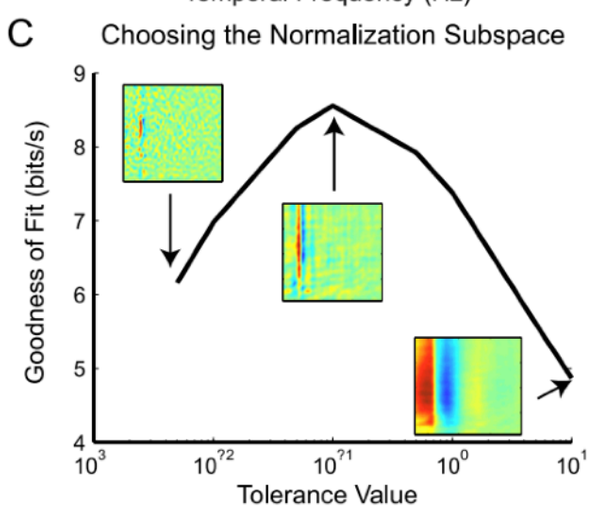


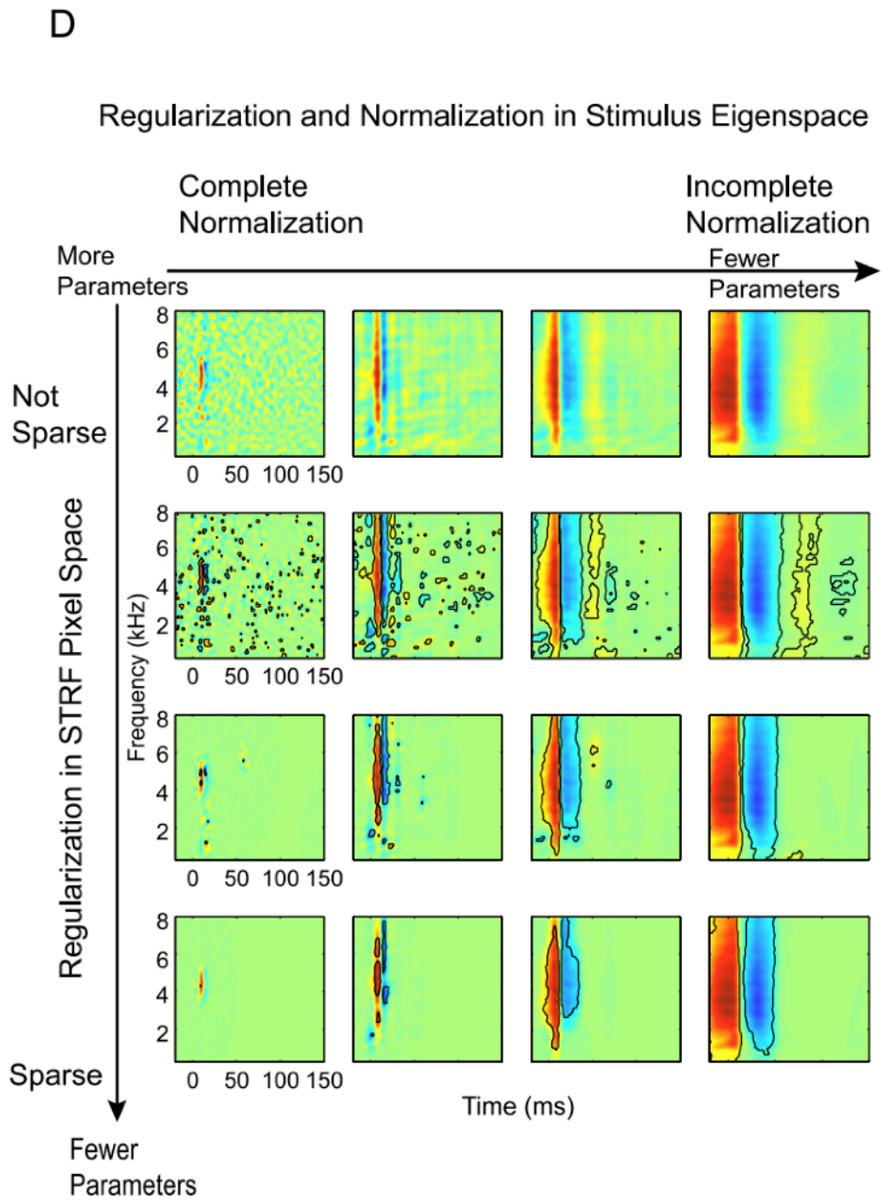
Woolley, Gill, & Theunissen (2006)

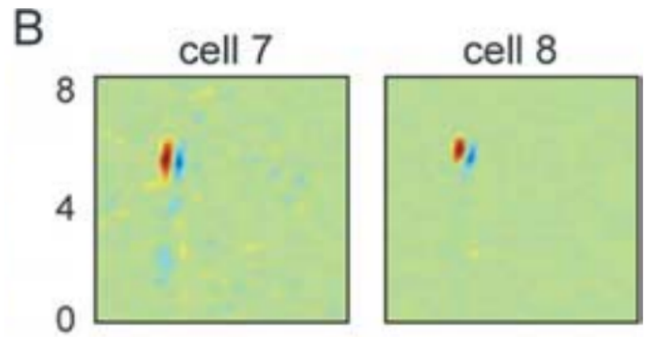
INSEPARABLE EXAMPLE











Inseparable-like response properties!

Woolley, Gill, & Theunissen (2006)

RECAP

* Need to model temporal dependence in system identification

* Temporal dependence can be “real” or “artifactual” (BOLD)

* Spatiotemporal models can be “space-time separable” or “space-time inseparable”

NEXT TIME

* Data quality!