

1 Introduction

Inferring glacier thickness has been a problem for a long time for glaciologists. This data is necessary for determining the total volume of ice for a given glacier outline, but also for modelling glacier flow. Acquiring such data is hard and rather complicated given the wild nature of the fieldwork for glaciologists. Drilling a borehole through a glacier is long and necessitate cumbersome machinery.

Hence the development of indirect methods such as geophysical and remote-sensing methods.

As part of an Undergraduate scholarship research award project, I will be helping two master's students, Tryggvi Unnsteinsson and Andrew Nolan, who are modelling ice flow of two glaciers in western Canada. The goal of this project is to review different ice thickness models and ultimately to infer bed topography for the two studied glaciers.

2 The glaciers studied

The glaciers under study are Job glacier, which is located on Mt. Meager in southwestern British Columbia and an unnamed surging glacier part of the bigger Kluane glacier, in southern Yukon. Various data is available for each one. The next section will go over what is available for each glacier.

To note: further writing will be made here.

2.1 Job glacier

Job glacier is located upon Mt. Meager, a volcano in southwestern British Columbia. The hypsometric curve of Job glacier is shown at figure 1.

The data available for Job glacier consists of a LIDAR survey from (?) and a glacier outline taken from Randolph Glacier Inventory. Also available is ice thickness data from a ground penetrating radar campaign held by Dr. G. Flowers and her team in September 2018.

2.2 Little Kluane

The hypsometric curve of Little Kluane is shown at figure 2. The data available consists of a digital elevation model (?) and a glacier outline from Nolan(2020).

3 The Ice Thickness Models Intercomparison eXperiment, ITMIX

The ever growing supply of different ice thickness models called for the need of a unified intercomparison experiment. The Ice Thickness Models Intercomparison eXperiment, ITMIX, is a project launched by the working group on glacier ice thickness estimation, part of the International Association of Cryospheric sciences.

The experiment consists of 17 different models tested over 21 cases, providing at least, depending on data availability:

- a glacier outline
- a digital elevation model

And a combination of:

- the surface mass balance
- the velocity field
- the rate of ice thickness change

There were 4 main types of models trough this experiment:

- Minimization approaches
 - Those models defines ice thickness inversion as a minimization problem. They use a cost function consisting of minimizing the difference between observed and modelled data. The models presented in ITMIX all need the glacier outline, a digital elevation model and distributed surface mass balance.
- Mass conserving approaches

- These methods are based on the principle of mass conservation (Farinotti, 2020). The ice flux divergence has to be compensated by the rate of ice thickness change and the climatic mass balance:

$$\nabla \cdot q = \frac{\partial h}{\partial t} - \dot{b}$$

To note is that most of the models in this category use the least amount data, only needing the glacier outline and the digital elevation model.

- Shear-stress based approaches

- Those approaches rely on some estimation of the basal shear stress τ . They then solve for ice thickness using the shallow ice approximation (Fowler and Larson, 1978)

$$h = \frac{\tau}{f \rho g \sin \alpha}$$

An important thing to note is that those models don't take into account mass conservation. They then don't need much data to be able to produce a working model, only needing the glacier outline and a digital elevation model.

- Velocity-based approaches

- The models described in this category are based on a form of Glen's flow law (Glen, 1955) and an approximation of either the basal velocity u_b or the depth-averaged profile velocity \bar{u} from the surface velocity u_s . Those models all need the surface velocity field, the glacier outline and the digital elevation model.

- Other approaches

- GCneuralnet (Clarke et al., 2009) is a model based on artificial neural networks. It is based on the assumption that the bedrock topography resembles nearby unglaciated valleys. This model only needs the glacier outline and the digital elevation model.
- Brinkerhoff (Brinkerhoff et al., 2016) is based on the Bayesian inference. The idea is that the bed elevation and the ice flux

divergence can be described as Gaussian random fields with known covariance but unknown mean. This model can only work for the glaciers where all the data offered in ITMIX is available.

As there is little data available for the glaciers under study, Job and Little Kluane, it is important to choose a model corresponding to potentially accessible data. ITMIX concluded that the models using the most data weren't always the best representation of actual bedrock topography. This is probably caused by the high variation in the methodology used by various scientists to capture such data. Thus the lack of surface velocity field or rate of ice thickness change over time data isn't too dramatic.

ITMIX ranked the models from best to worst according to their performance over two rankings:

- Their performance over the glaciers they tested.
- Their performance over all the glaciers, considering the amount of test-cases.

It is however important to note the individual performance of the models when inferring ice thickness of similar glaciers to our study sites, considering location and topography, thereby the need of quantifying similarity in glacier geometry.

4 Comparing glacier geometry

The hypsometric curve is defined by the elevation in relation to relative surface area from a defined location. In such, the hypsometric curve of a definier glacier outline relates the percentage of area over a given elevation. By using relative elevation over absolute elevation, we can quantify similarity between different hypsometric curves. Those curves can be seen in figures 1 and 2.

The topography of ITMIX's glaciers are compared to Job's and Little Kluane's by computing their hypsometric curve and using the values of percentile area corresponding to percentile elevation such that:

$$\%Area = f(\%Elevation) \quad (1)$$

If two glaciers were identical, plotting their $\%Area$ in function of the other would give us points lying on the $x = y$ line. Thus, because the $\%Area$ values

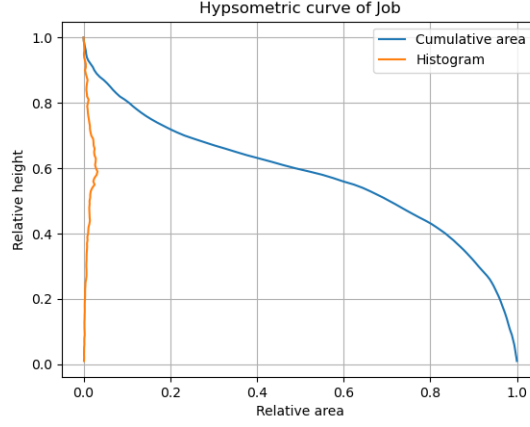


Figure 1: Job glacier's hypsometric curve

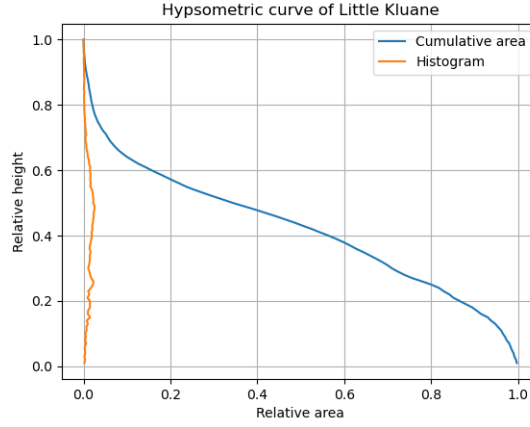


Figure 2: Little Kluane's hypsometric curve

are distributed equally on $\%Elevation$ between $[1, 100]$, we can compare two glaciers by computing a statistical coefficient

$$R^2 = 1 - \frac{\sum (\%Area_x - \%Area_y)^2}{\sum (\%Area_x - \%Area_x)^2} \quad (2)$$

A higher value of R^2 would mean a higher resemblance between the overall topography of the glaciers. The top correlations of each glaciers were

compiled and ranked in Table 1

Table 1: Glacier correlation

Job		Little Kluane	
Glacier name	R^2 value	Glacier name	R^2 value
Kesselwandferner	0.98682	Urumqi	0.99252
Synthetic2	0.98252	NorthGlacier	0.99232
Elbrus	0.97972	Tasman	0.97692
Freya	0.97662	Starbuck	0.96772
Aqutikitsoq	0.96662	Unteraar	0.96532

Those glaciers are the better correlated to our study cases. Their hypso-metric curves can be seen in the appendix. More data would be interesting to look at for better comparison:

- The actual geometry of the glacier has been mostly ignored. It would be great to be able to quantify similarity in the shapes of the glacier.
- Other attributes such as the length, the slope or the equilibrium line altitude were ignored.
- The histogram was computed. It seem to not work completely though, the histogram values are very small. It would be great to see if the similarity between the histograms and the hypsometric curve are the same.

5 Comparing Itmix models

In order to see what models fared better for those glaciers, the error of every model was computed. The error ϵ is computed like this:

$$\epsilon = h - h'$$

Since the h data are points, the error is local and no heat maps were produced. No interpolation between the points for distributed ice thickness was made in order to obtain the "truest" error. The relative error $\epsilon_{\%}$ was computed like this:

$$\epsilon_{\%} = 100 \frac{\epsilon}{h}$$

The mean, the standard deviation, the minima, maxima and the absolute maxima of the error was computed and used as a metrics. Other metrics could and will be used for further investigation of the better models:

- Mean square error $MSE = \frac{1}{n} \sum \epsilon^2$
- R^2 coefficient

The ice thickness of every computed model for each glacier was plotted along with its error. Such a figure is seen at figure 3. A few problems remain

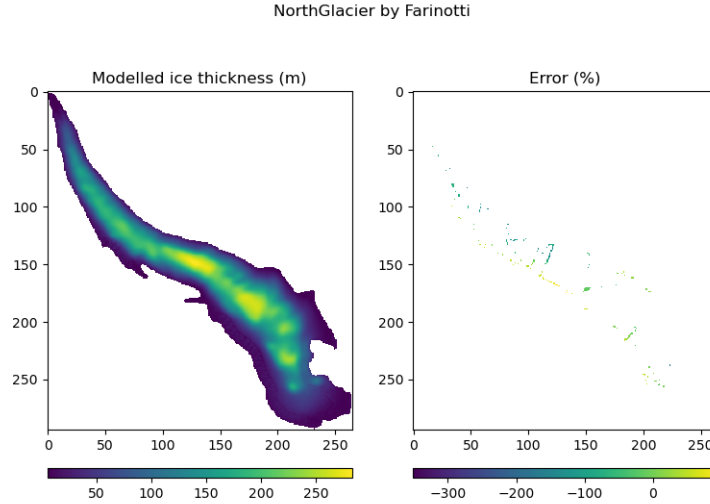


Figure 3: NorthGlacier's modelled ice thickness by Farinotti and its error

with the picture:

- The shapefile hence the geometry of the glacier cannot be seen on the error map. This is a problem being investigated.
- The error cannot be seen well on every map because of the nature of the data, which are points. This problem is also being looked at.
- The x and y axis needs to be the coordinates of the glaciers.

- The error computed seems quite big, verification of the computing needs to be made.

6 What's next

Further investigation will be made to quantify similarity in glacier geometry mainly with:

- The shape of the glaciers. Some interesting ressources can be found here:
 - <http://agri.ckcest.cn/ass/NK005-20160725003.pdf>
 - <https://www.student.cs.uwaterloo.ca/~cs763/Projects/phil.pdf>
- Their more basic geometry, such as length, width, mean slope and ELA
- Making sure the histogram computed is right and comparing their shape the same way it was made with the hypsometric curve

Verification of the error computed for every model and glacier will be made. The error maps will also be adjusted to add the contour of the glaciers and correctly see the error distribution.

When the error is correctly computed and the similarity in glaciers will be better quantified, the best performing model from the chosen metric, either:

- R^2 coefficient
- MSE
- Mean error
- and more!!

will be selected and the inferring work will finally begin...