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Results and Data Analysis

Question 0: The Rotational Wave Approximation (RWA)

Rotational wave approximation or RWA is one of the most techniques in the field of quantum optics where it applies for the case that $\omega_0 \sim \omega$. RWA allows us to transform the corresponding Floquet eigenvalue problem to the following equation,

$$\begin{bmatrix} -\frac{1}{2}\hbar\omega_0 + \hbar\omega & \frac{E}{2} \\ \frac{E}{2} & \frac{1}{2}\hbar\omega_0 \end{bmatrix} \begin{pmatrix} b_{1,i} \\ a_{0,i} \end{pmatrix} = \epsilon_i \begin{pmatrix} b_{1,i} \\ a_{0,i} \end{pmatrix} \tag{12}$$

where for this case, we set $\hbar = 1$, $\hbar\omega$ = 10, $\hbar\omega_0$ = [$\frac{\hbar\omega}{2}$, $\frac{3\hbar\omega}{2}$], and $a_{0,1}$ and $b_{1,i}$ are constants elements of the i-th eigenvector with the eigenvalue of ϵ_i.

The objective is then to solve the above equation using RWA and plot the possible ϵ_i values as a function of $\hbar\omega_0$ with difference values of the coupling strength E in which we have 6 different cases as the following would unpack it.

1. **Case 1: $E < \hbar\omega$**

   The following shows the plot obtained for the first case in which $E = \frac{\hbar\omega}{2} = 5$

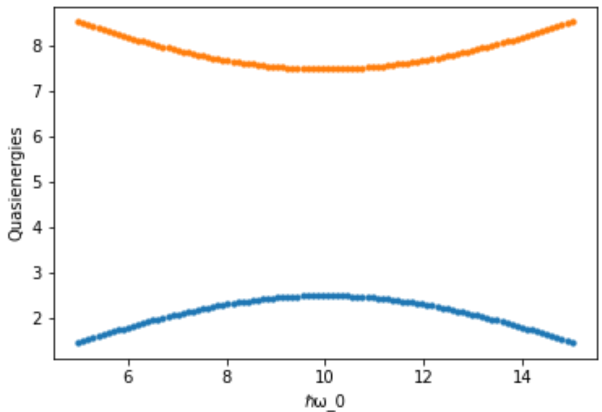


Figure 1. The values of ϵ_1 (blue) and ϵ_2 (orange) plotted versus the values of $\hbar\omega_0$ with $E = \frac{\hbar\omega}{2} = 5$

In addition, below is plotted for the value of $E = \frac{\hbar\omega}{10} = 1$,

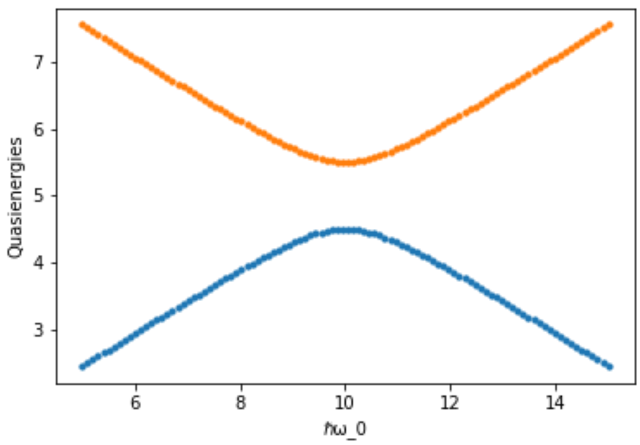


Figure 2. The values of ϵ_1 (blue) and ϵ_2 (orange) plotted versus the values of $\hbar\omega_0$ with $E = \frac{\hbar\omega}{10} = 1$

2. **Case 2: $\boldsymbol{E > \hbar\omega}$**
   The following shows the plot obtained for the second case in which $E = 2\hbar\omega = 20$

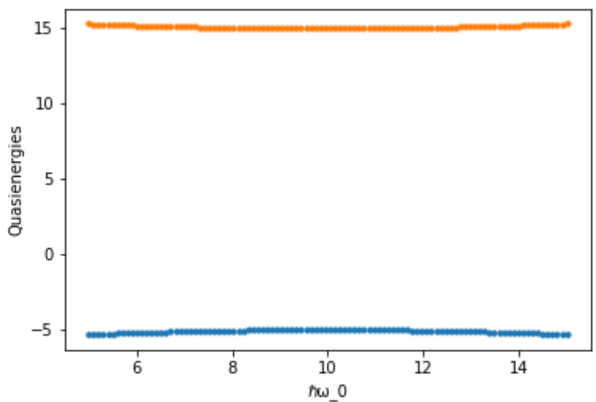


Figure 3. The values of ϵ_1 (blue) and ϵ_2 (orange) plotted versus the values of $\hbar\omega_0$ with $E = 2\hbar\omega = 20$

Additionally, following shows the plot obtained for the second case in which $E = 10\hbar\omega = 100$,

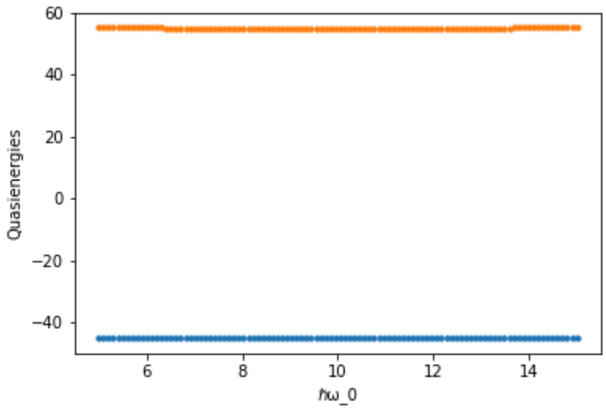


Figure 4. The values of ϵ_1 *(blue) and* ϵ_2 *(orange) plotted versus the values of* $\hbar\omega_0$ *with* $E = 10\hbar\omega = 100$

3. **Case 3: $E = \hbar\omega$**

   The following shows the plot obtained for the third case in which $E = \hbar\omega = 10$

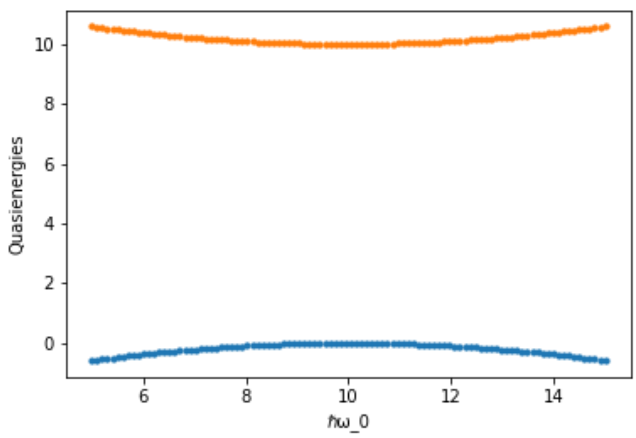


Figure 5. The values of ϵ_1 *(blue) and* ϵ_2 *(orange) plotted versus the values of* $\hbar\omega_0$ *with* $E = \hbar\omega = 10$

From the 3 cases in the above, it could be observed that as the values of E increases, the two quasi-energies values approach a certain constant value even with different $\hbar\omega_0$. It could also be seen that the values of ϵ_1 and ϵ_2 has a converging behaviour to one another when $\hbar\omega_0$ (the energy gap) amounts to the same value as the $\hbar\omega$ which is the energy of the incoming light itself. This converging behaviour becomes stronger with the decrease of the coupling strength E as seen from the plots above until ϵ_1 and ϵ_2 finally meets with small enough E. It could also be seen that that this behaviour of ϵ_1 and ϵ_2 only applies to values of ω_0 comparable to ω as seen from the distinct behaviour of the quasi-energies in the following questions.

Question 1 (Calculating the quasienergies)

Before trying to find the corresponding quasi-energies, we first need to find the appropriate cut-off value (N) for different coupling strength E, which the following shows for the case that will be used in finding the wavefunction.

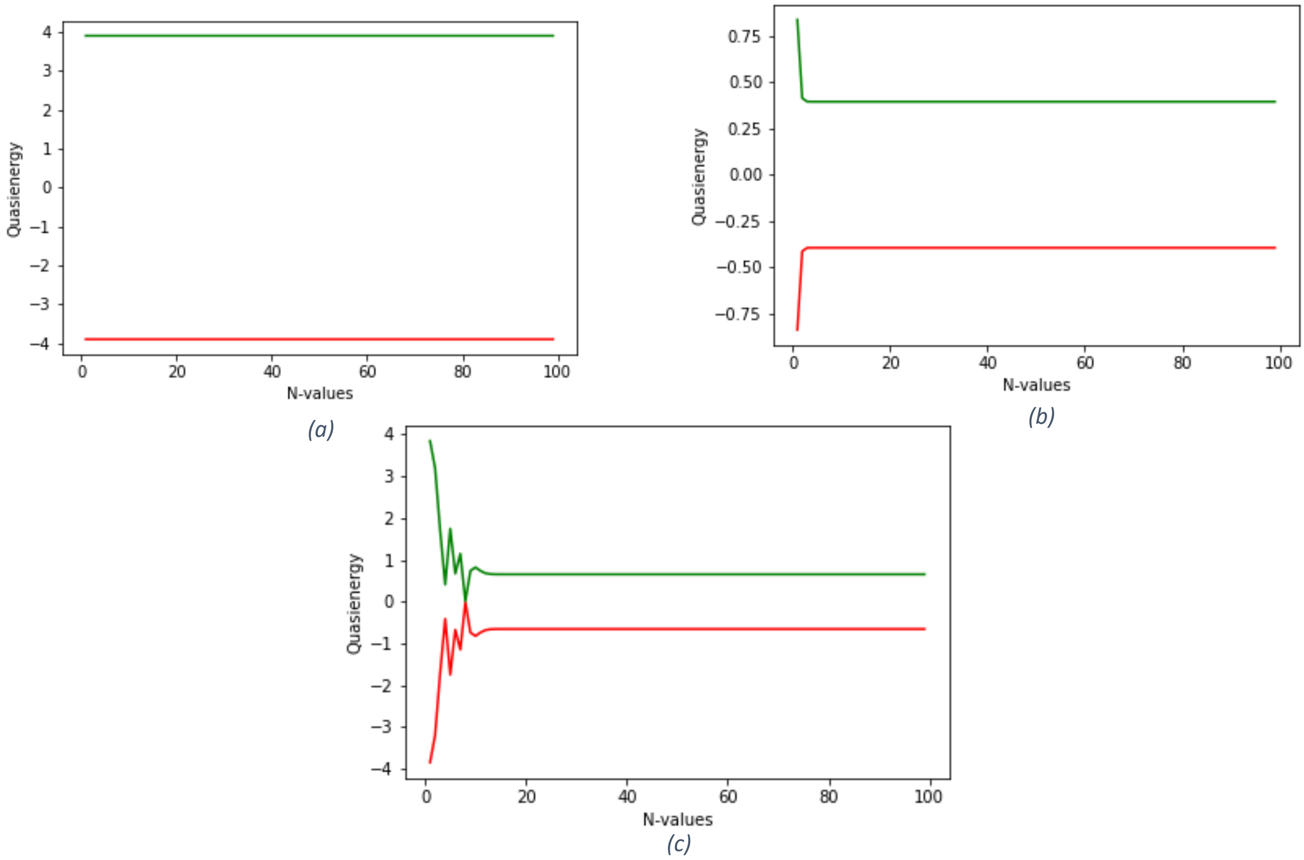


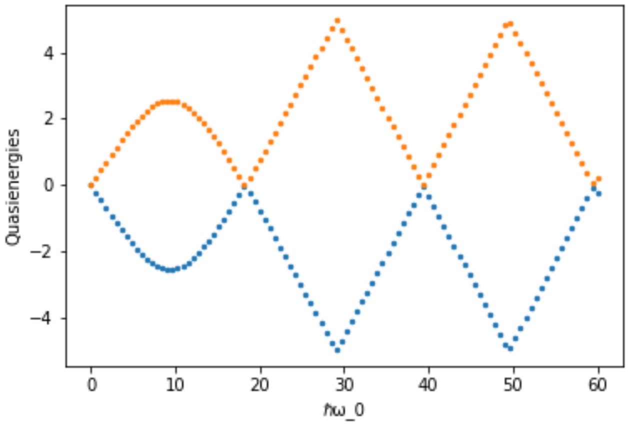
Figure 6. Plot of the quasi-energies (ϵ_1 and ϵ_2) VS the cut-off values (N) with (a) E = $\frac{\hbar\omega}{10}$; (b) E = $\hbar\omega$; and (c) $E = 10\hbar\omega$

It could be observed from the above plots that the quasi-energies, which is ϵ_1 and ϵ_2, stabilize in value with higher cut-off value (N) as E increases. In the above case, when $E = \frac{\hbar\omega}{10}$, the quasi-energies stabilized in value with only $N = 1$, while for $E = \hbar\omega$ and $E = 10\hbar\omega$, ϵ_1 and ϵ_2 would stabilize in value when N equals to approximately 5 and 15, respectively. Using this result, for the rest of the report, N would be taken to be equal to 60 for safety, accuracy, and precision of the results to be used for the rest of the questions.

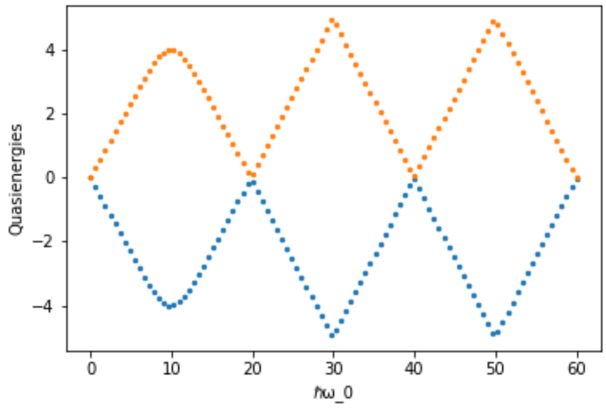
Obtaining the value for the cut-off N, we would now be able to calculate the quasi-energies for different values of E and observe how it changes with the energy gap value $\hbar\omega_0$, where we use the values of $\hbar\omega$ = 10. Similar as before, we would have 3 cases as the following shows,

1. **Case 1: $\boldsymbol{E < \hbar\omega}$**

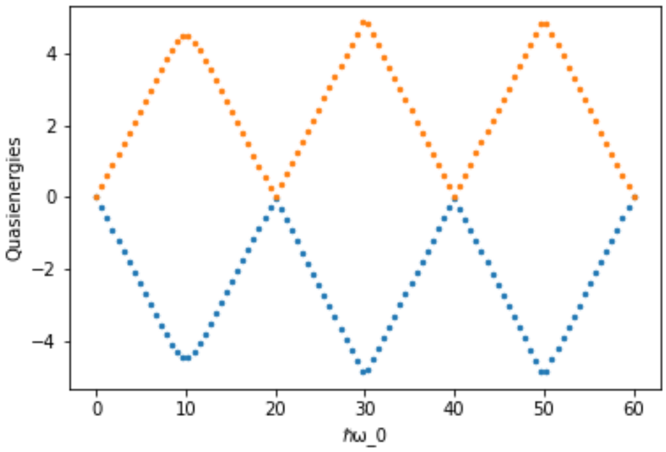
   The following shows the plot obtained for the third case in which $E = \frac{\hbar\omega}{2} = 5$, $N = 60$, and $\hbar\omega_0$ goes from 0 to $6\hbar\omega$,



(a)



(b)

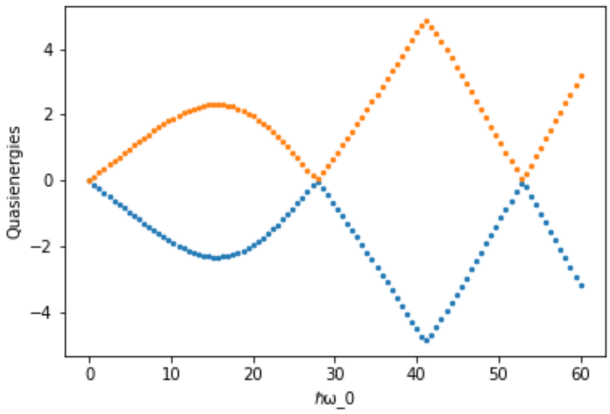


(c)

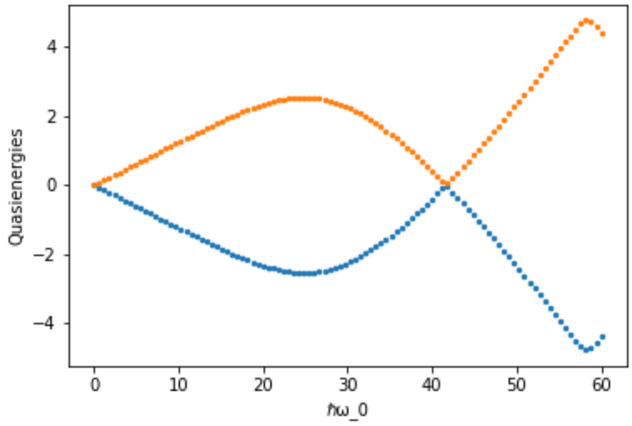
Figure 7. Different plots of the quasi-energies (ϵ_1 (blue color) and ϵ_2 (orange color)) VS $\hbar\omega_0$ with values of $\hbar\omega = 10$, $\hbar\omega_0 = [0{,}6\hbar\omega]$, and (a) $E = \frac{\hbar\omega}{2}$; (b) $E = \frac{\hbar\omega}{5}$; (c) $E = \frac{\hbar\omega}{10}$

2. **Case 2: $\boldsymbol{E > \hbar\omega}$**

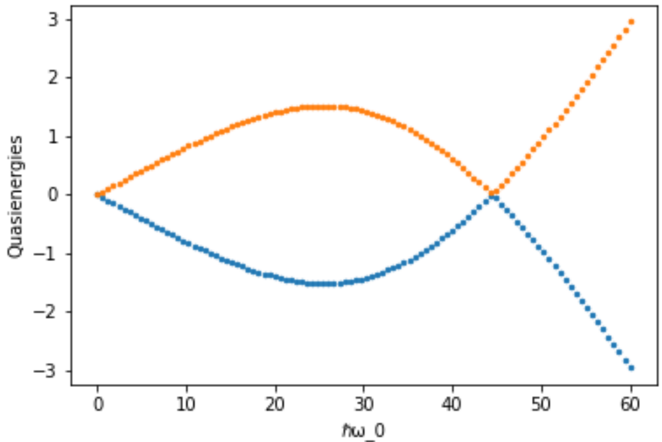
   The following shows the plot obtained for the third case in which $E = \frac{\hbar\omega}{2} = 5$, $N = 60$, and $\hbar\omega_0$ goes from 0 to $6\hbar\omega$,



(a)



(b)



(c)

Figure 8. Different plots of the quasi-energies (ϵ_1 (blue color) and ϵ_2 (orange color)) VS $\hbar\omega_0$ with values of $\hbar\omega = 10$, $\hbar\omega_0 = [0{,}6\hbar\omega]$, and (a) $E = 2\hbar\omega$; (b) $E = 5\hbar\omega$; (c) $E = 10\hbar\omega$

3. **Case 3: $\boldsymbol{E = \hbar\omega}$**
   The following shows the plot obtained for the third case in which $E = \hbar\omega = 10$, $N = 60$, and $\hbar\omega_0$ goes from 0 to $6\hbar\omega$,

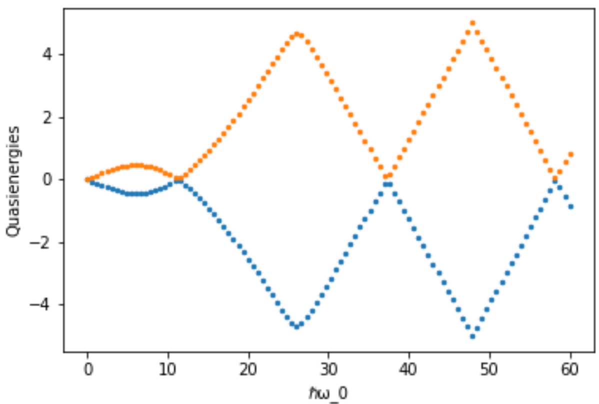


Figure 9. The plot of the quasi-energies (ϵ_1 (blue color) and ϵ_2 (orange color)) VS $\hbar\omega_0$ with values of $\hbar\omega = 10$, $\hbar\omega_0 = [0{,}6\hbar\omega]$, and $E = \hbar\omega$

It could be observed from the above plots of the three cases that the values of ϵ_1 and ϵ_2 oscillates in value as we increase $\hbar\omega_0$. However, this oscillation does not have the same period in $\hbar\omega_0$ for every range of value of $\hbar\omega_0$. It could be seen that the first "diamond shape" changes in shape, amplitude, and period as we change the value of E with respect to $\hbar\omega$, in which as the values of E becomes larger, the peak of the first diamond shape becomes less sharp and , the width of the diamond shape, which signifies the period of getting the same quasi-energy with different $\hbar\omega_0$, would also become larger. On the other hand, the smaller the value of E, the oscillation of ϵ_1 and ϵ_2 values with respect to $\hbar\omega_0$ becomes more uniform with one another and tend to have the same period with different ranges of $\hbar\omega_0$ values. In addition, from the value of $E = \hbar\omega$ (case 3) to $E = 2\hbar\omega$, the last diamond shape of case 3 in Fig. (9) seemingly disappeared and merged with one of the other diamonds and thus producing the kind of figure in Fig. (8a). This result could signify us the relationship between E which has the physical meaning of the overall interference from the applied light and the quasi-energies which physically determine the relative phases of the two eigenstates and hence its interference pattern.

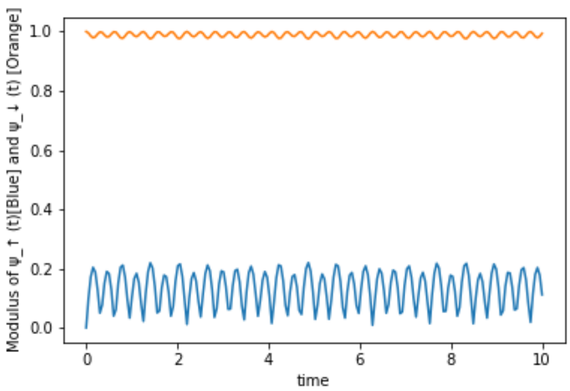
Question 2

Being able to obtain the two quasi-energies and the eigenvectors of the Floquet Hamiltonian ($\hat{H}^{Floquet}_{m,l}$) using the code and the results in question 1, our next step is now therefore to obtain the wavefunction $|\psi(t)\rangle$. Thus, assuming that we have the initial state at $t = 0$ of,

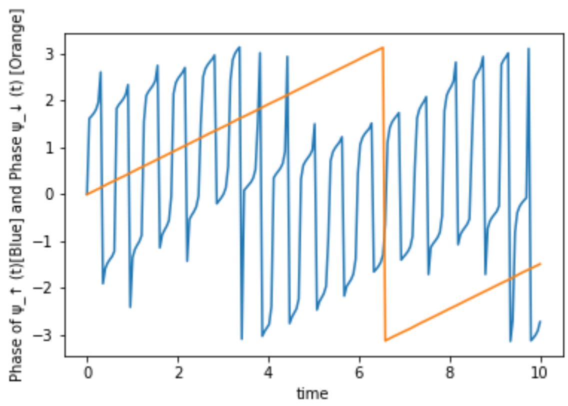
$$|\psi(t=0)\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

then, using Eq. (11), we would then obtain the following plot for $\psi_\uparrow(t)$ and $\psi_\downarrow(t)$ and test it for different values of $\hbar\omega_0$ with a fixed value of E to be $\frac{\hbar\omega}{5}$ and value of $\hbar\omega$ to be equal to 10 as the following shows for each different case.

1. **Case 1: $\hbar\omega_0 < \hbar\omega$**

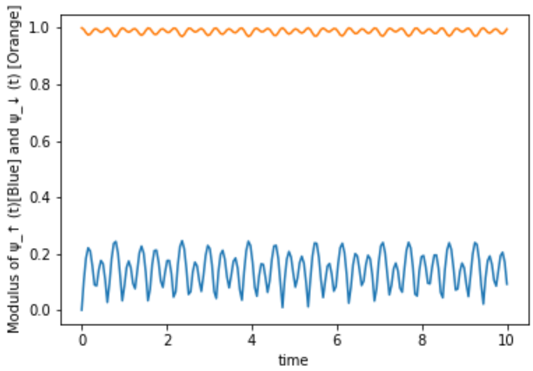


(a)

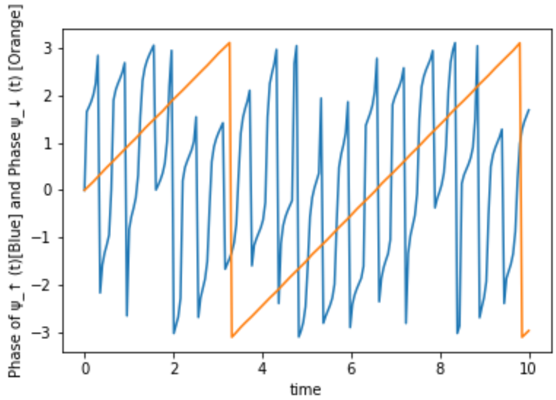


(b)

Figure 10. The plot of the ground and excited state wavefunction ($\psi_\downarrow[orange]$ and $\psi_\uparrow[blue]$) (a) Modulus and (b) Phase VS time (t) with $\hbar\omega = 10$, $\hbar\omega_0 = \frac{\hbar\omega}{10}$, and $E = \frac{\hbar\omega}{5}$

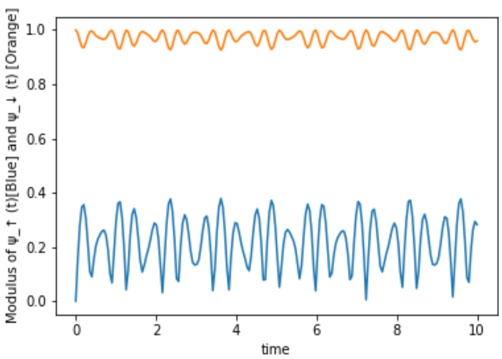


(a)

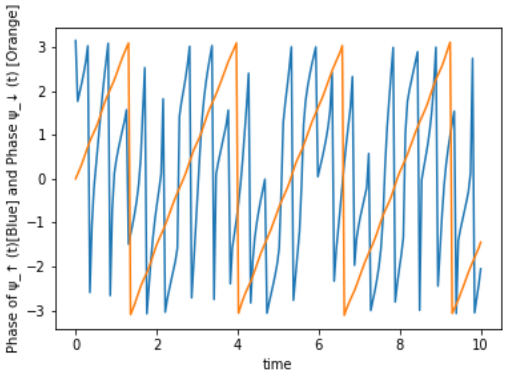


(b)

Figure 12. The plot of the ground and excited state wavefunction ($\psi_{\downarrow}[orange]$ and $\psi_{\uparrow}[blue]$) (a) Modulus and (b) Phase VS time (t) with $\hbar\omega = 10$, $\hbar\omega_0 = \frac{\hbar\omega}{5}$, and $E = \frac{\hbar\omega}{5}$



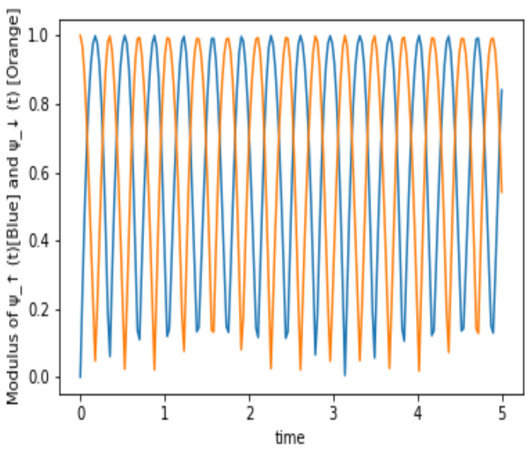
(a)



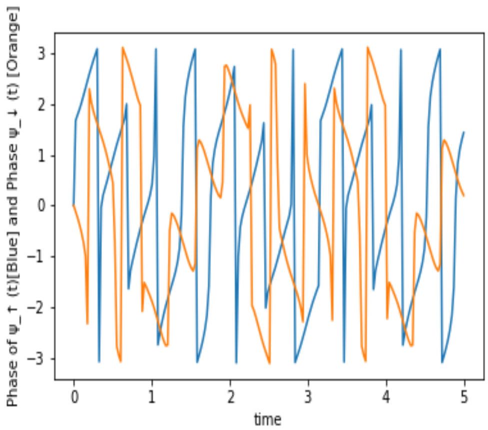
(b)

Figure 11. The plot of the ground and excited state wavefunction ($\psi_{\downarrow}[orange]$ and $\psi_{\uparrow}[blue]$) (a) Modulus and (b) Phase VS time (t) with $\hbar\omega = 10$, $\hbar\omega_0 = \frac{\hbar\omega}{2}$, and $E = \frac{\hbar\omega}{5}$

2. **Case 2: $\hbar\omega_0 = \hbar\omega$**



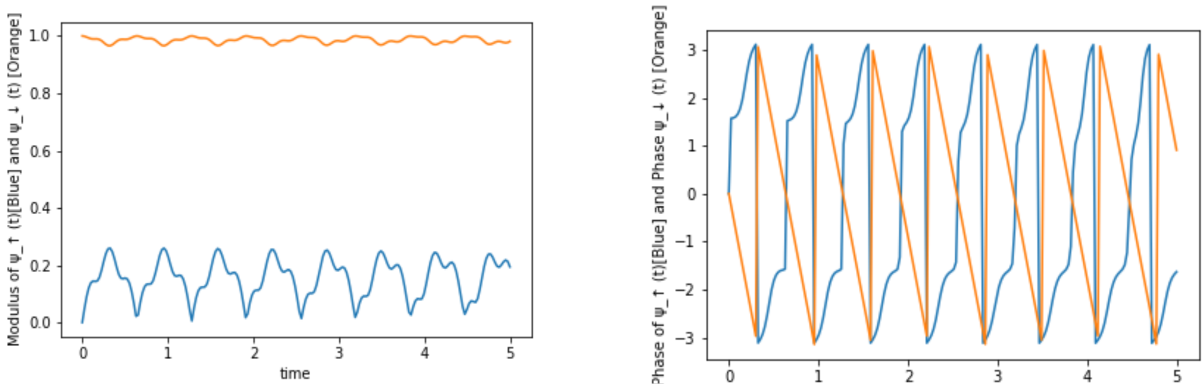
(a)



(b)

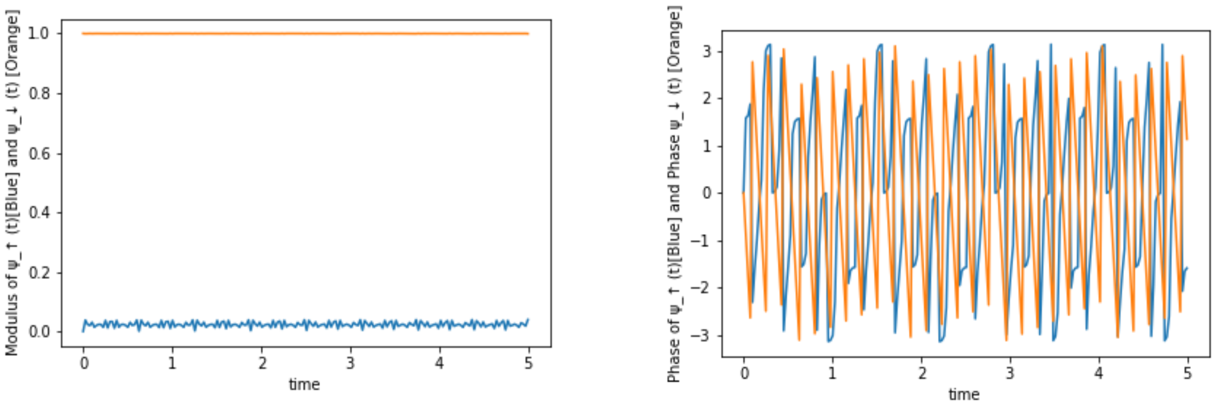
Figure 13. The plot of the ground and excited state wavefunction ($\psi_{\downarrow}[orange]$ and $\psi_{\uparrow}[blue]$) (a) Modulus and (b) Phase VS time (t) with $\hbar\omega = 10$, $\hbar\omega_0 = \hbar\omega$, and $E = \frac{\hbar\omega}{5}$

3. **Case 3: $\hbar\omega_0 > \hbar\omega$**



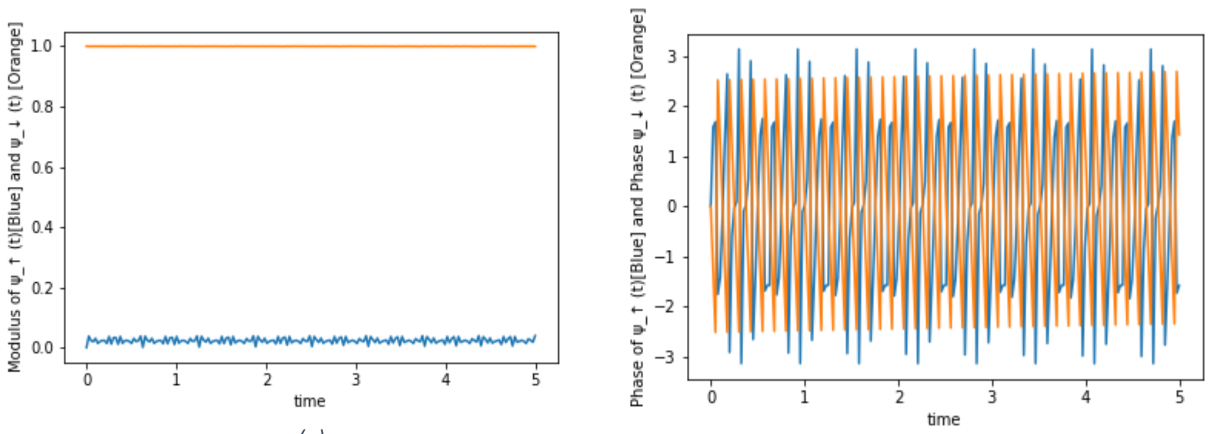
(a) *(b)*

Figure 14. The plot of the ground and excited state wavefunction ($\psi_{\downarrow}[orange]$ and $\psi_{\uparrow}[blue]$) (a) Modulus and (b) Phase VS $time$ (t) with $\hbar\omega = 10$, $\hbar\omega_0 = 2\hbar\omega$, and $E = \frac{\hbar\omega}{5}$



(a) *(b)*

Figure 15. The plot of the ground and excited state wavefunction ($\psi_{\downarrow}[orange]$ and $\psi_{\uparrow}[blue]$) (a) Modulus and (b) Phase VS $time$ (t) with $\hbar\omega = 10$, $\hbar\omega_0 = 5\hbar\omega$, and $E = \frac{\hbar\omega}{5}$



(a) *(b)*

Figure 16. The plot of the ground and excited state wavefunction ($\psi_{\downarrow}[orange]$ and $\psi_{\uparrow}[blue]$) (a) Modulus and (b) Phase VS $time$ (t) with $\hbar\omega = 10$, $\hbar\omega_0 = 10\hbar\omega$, and $E = \frac{\hbar\omega}{5}$

It could be seen from the above three cases that as the values of $\hbar\omega_0$ approaches to the same value as $\hbar\omega$, the wavefunction, which signifies the probability to find the electron in a certain state, of the excited state ($\psi_\uparrow$) increases in amount. The wavefunction of the excited state then has its largest values if $\hbar\omega_0 = \hbar\omega$ (as observed in Fig. (13a)), which is when the energy gap equals to the energy of the applied light (case 2). This result agrees with the theory and what is expected because electrons in the ground state could only be excited to the first excited state when the energy of the applied light has the same energy as the energy gap. Additionally, it could be noted as well that the ground state wavefunction tends and amounts to approximately 1 as $\hbar\omega_0$ becomes too small or too large compared to $\hbar\omega$ which could be seen in Fig. (11a), Fig. (15a), and Fig. (16a). This a further signifies that the applied light did not able to excite the electron to the excited state and thus the electron will always reside in the ground state.

Furthermore, the oscillations of both $\psi_\uparrow$ and $\psi_\downarrow$ appeared in all 3 cases are due to the probability of the process of falling to the ground state and then being excited again to the first excited state and redoing the process all over again. This is occurrence should be due to the *Rabi Oscillation* mentioned in the introduction. Notice also that when the wavefunction on the ground state is at its highest value, the wavefunction of the excited state would at its lowest. This also agrees with the theory since the probability to find the excited electron in the ground state would certainly be lower than founding it in the excited state and vice versa. In addition, it is observed that the oscillation of the phase becomes quicker with higher frequency as the value of $\hbar\omega_0$ gets larger with respect to the value of $\hbar\omega$.

Question 3

Consider now another case, where we add another EM wave with an angular frequency of 5ω, hence the Hamiltonian now also have a new additional component compared to before as the following shows,

$$\widehat{H}(t) = \begin{bmatrix} \frac{1}{2}\hbar\omega_0 & E\cos(\omega t) + E'\cos(5\omega t) \\ E\cos(\omega t) + E'\cos(5\omega t) & -\frac{1}{2}\hbar\omega_0 \end{bmatrix}$$

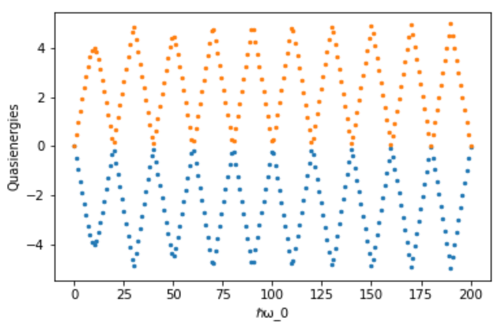
Similarly, using Eq. (8), the Hamiltonian matrix could be Fourier transformed and yields the following new Floquet Hamiltonian matrix $\widehat{H}_{new}^{Floquet}$,

$$
\begin{pmatrix}
\dots & 0 & 0 & E/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E'/2 & 0 & 0 \\
0 & -\frac{1}{2}\hbar\omega_0+3\hbar\omega & E/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E'/2 & 0 & 0 & 0 \\
0 & E/2 & \frac{1}{2}\hbar\omega_0+2\hbar\omega & 0 & 0 & E/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E'/2 \\
E/2 & 0 & 0 & -\frac{1}{2}\hbar\omega_0+2\hbar\omega & E/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E'/2 & 0 \\
0 & 0 & 0 & E/2 & \frac{1}{2}\hbar\omega_0+\hbar\omega & 0 & 0 & E/2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & E/2 & 0 & 0 & -\frac{1}{2}\hbar\omega_0+\hbar\omega & E/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & E/2 & \frac{1}{2}\hbar\omega_0 & 0 & 0 & E/2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & E/2 & 0 & 0 & -\frac{1}{2}\hbar\omega_0 & E/2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & E/2 & \frac{1}{2}\hbar\omega_0-\hbar\omega & 0 & 0 & E/2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & E/2 & 0 & 0 & -\frac{1}{2}\hbar\omega_0-\hbar\omega & E/2 & 0 & 0 & 0 \\
0 & E'/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E/2 & \frac{1}{2}\hbar\omega_0-2\hbar\omega & 0 & 0 & E/2 \\
E'/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E/2 & 0 & 0 & -\frac{1}{2}\hbar\omega_0-2\hbar\omega & E/2 & 0 \\
0 & 0 & 0 & E'/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E/2 & \frac{1}{2}\hbar\omega_0-3\hbar\omega & 0 \\
0 & 0 & E'/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E/2 & 0 & 0 & \dots
\end{pmatrix}
$$

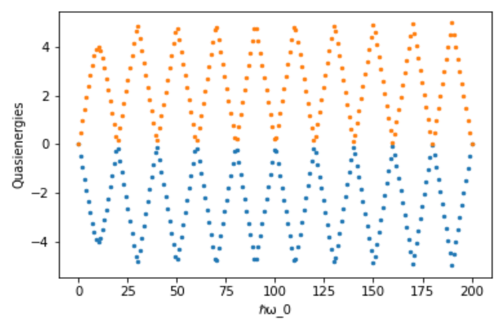
where it also has the size of $2(2N+1) \times 2(2N+1)$ and E' is the new coupling strength term.

Using the above obtained $\widehat{H}_{new}^{Floquet}$ and proceeding with similar process as in question 1, we would obtain the possible values of the two quasi-energies with different values of $\hbar\omega_0$ for 3 different cases values of E', in which we set $\hbar\omega$ = 10, $E = \frac{\hbar\omega}{5}$, and let $\hbar\omega' = 5\hbar\omega$.

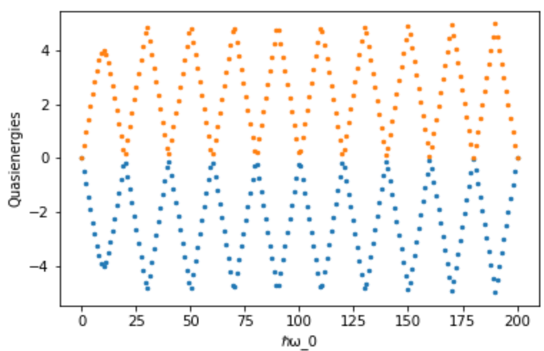
- **Case 1: $E' < E$**



(a)



(b)



(c)

Figure 17. Different plots of the quasi-energies (ϵ_1 (blue color) and ϵ_2 (orange color))VS $\hbar\omega_0$ with values of $\hbar\omega = 10$, $\hbar\omega_0 = [0,200]$, $E = \frac{\hbar\omega}{5}$ and (a) $E' = \frac{E}{2} = \frac{\hbar\omega'}{50}$; (b) $E' = \frac{E}{5} = \frac{\hbar\omega'}{125}$; (c) $E' = \frac{E}{10} = \frac{\hbar\omega'}{250}$

- **Case 2: $E' = E$**

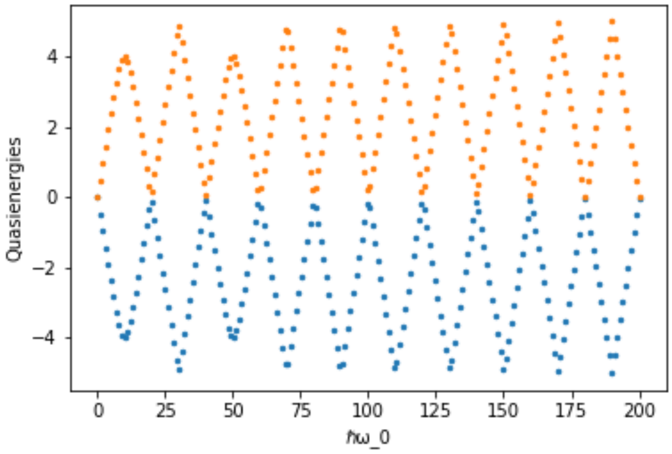
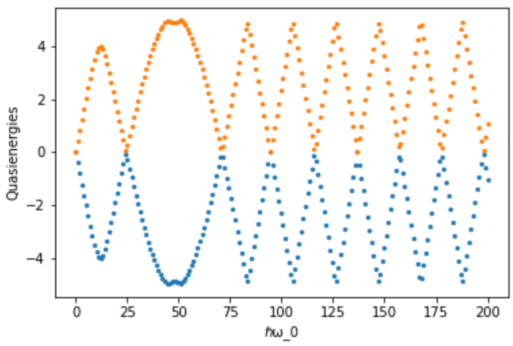
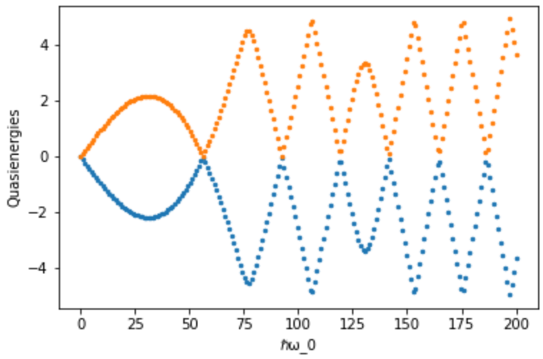


Figure 18. Different plots of the quasi-energies (ϵ_1 (blue color) and ϵ_2 (orange color))) VS $\hbar\omega_0$ with values of $\hbar\omega = 10$, $\hbar\omega_0 = [0,200]$, $E = \frac{\hbar\omega}{5}$, and $E' = E = \frac{\hbar\omega'}{25}$

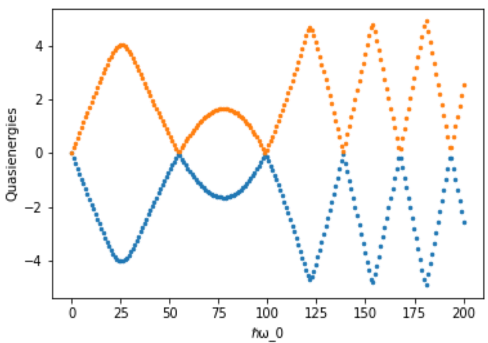
- **Case 3: $E' > E$**



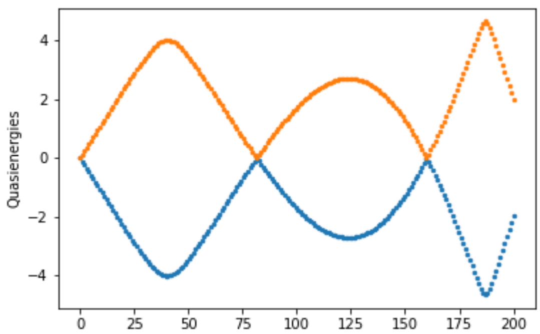
(a)



(b)



(c)



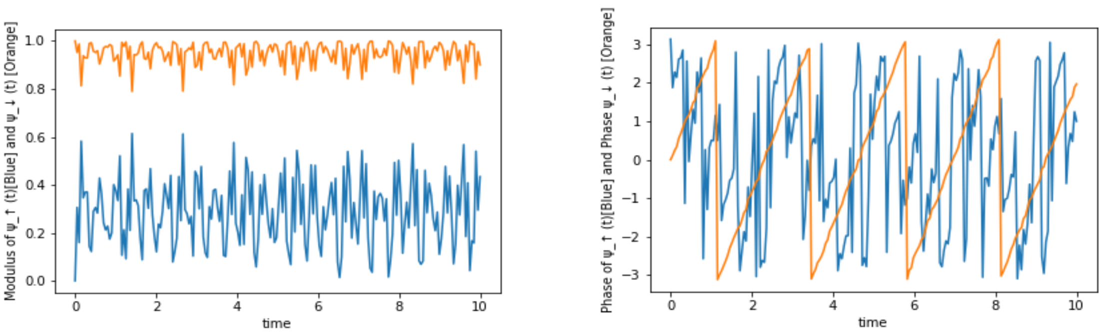
(d)

Figure 19. Different plots of the quasi-energies (ϵ_1 (blue color) and ϵ_2 (orange color))VS $\hbar\omega_0$ with values of $\hbar\omega = 10$, $\hbar\omega_0 = [0,200]$, $E = \frac{\hbar\omega}{5}$ and (a) $E' = 10E = \frac{2\hbar\omega'}{5}$; (b) $E' = 25E = \hbar\omega'$; (c) $E' = 50E = 2\hbar\omega'$; (d) $E' = 125E = 5\hbar\omega'$

From the above plots of quasi-energies with three different cases, we see that for case 1 and case 2, where the values of E' are much smaller than $5\hbar\omega$, the quasi-energy plot is similar to the quasi-energy plot of question 1, particularly Fig (7b) and Fig (7c). This is due to the new EM wave component having small coupling strength value and thus little significance to the overall EM wave. Most significantly, however, by adding the new component of EM wave to our system, we now see two diamond shaped plot that are smoother and smaller than the other diamond shaped, rather than one as in question 1. This implies that the addition of the new EM wave component would realize a new hybridization in our system. In addition, for case 3, as E' becomes bigger and larger than $\hbar\omega'$, we see also the widening of the diamond shape and also the merging of them in the process of increase E which is also a similar behaviour as observed in question 1. This merging could be seen between Fig (19b) to Fig (19c).

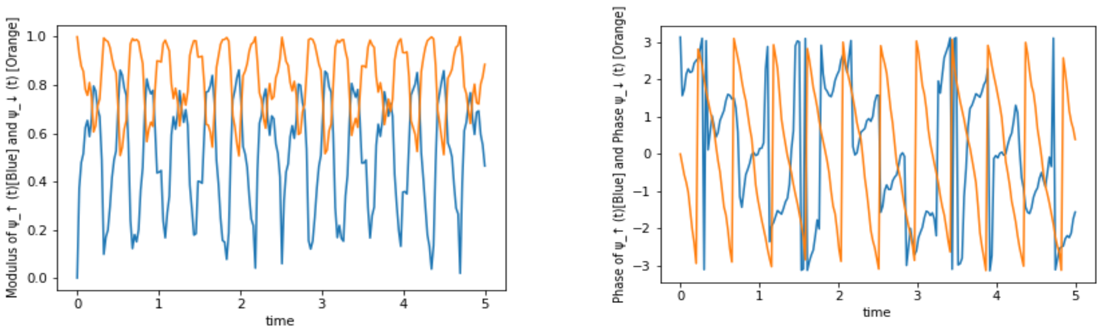
Furthermore, we would also like to explore and study how the wavefunction for both the ground state and the excited state for the new Hamiltonian and doing the similar procedure as in question 2 would give us three different cases where we denote $\hbar\omega_{new} = \hbar(\omega + 5\omega)$ as the new total energy of the incoming light. Here, we set as usual $\hbar\omega = 10$ and $E = \frac{\hbar\omega}{5}$ whereas the new component EM wave have the energy of $5\hbar\omega = 50$ with $E' = \frac{5\hbar\omega}{5}$. The following are the plots of $\psi_\uparrow(t)$ and $\psi_\downarrow(t)$ for the three different cases.

- **Case 1:** $\boldsymbol{\hbar\omega_0 < \hbar\omega_{new}}$



(a) *(b)*

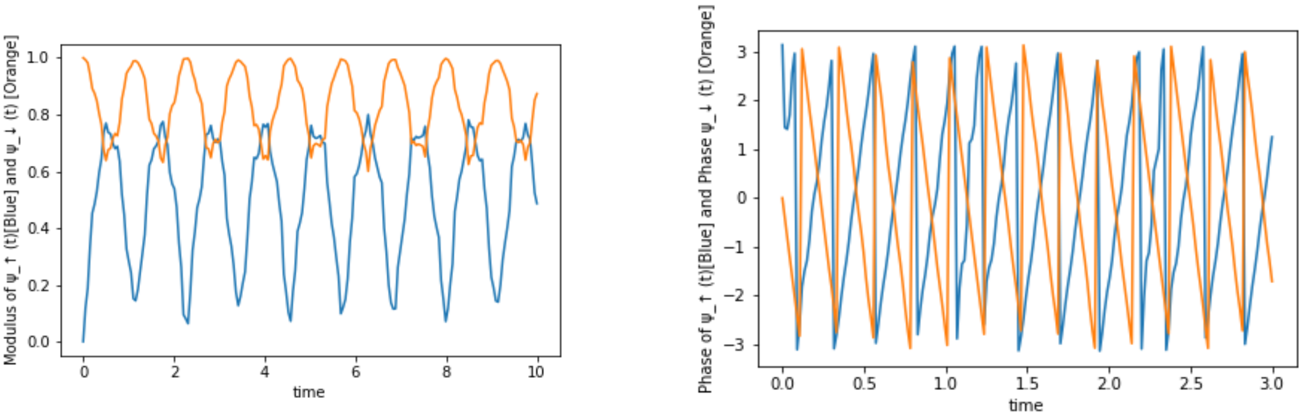
Figure 20. The plot of the ground and excited state wavefunction ($\psi_\downarrow[orange]$ and $\psi_\uparrow[blue]$) (a) Modulus and (b) Phase VS time (t) with $\hbar\omega = 10$, $\hbar\omega' = 5\hbar\omega = 50$, $\hbar\omega_0 = \frac{\hbar\omega_{new}}{10}$, $E = \frac{\hbar\omega}{5}$, and $E' = \frac{\hbar\omega'}{5}$



(a) *(b)*

Figure 21. The plot of the ground and excited state wavefunction ($\psi_\downarrow[orange]$ and $\psi_\uparrow[blue]$) (a) Modulus and (b) Phase VS time (t) with $\hbar\omega = 10$, $\hbar\omega' = 5\hbar\omega = 50$, $\hbar\omega_0 = \frac{\hbar\omega_{new}}{5}$, $E = \frac{\hbar\omega}{5}$, and $E' = \frac{\hbar\omega'}{5}$

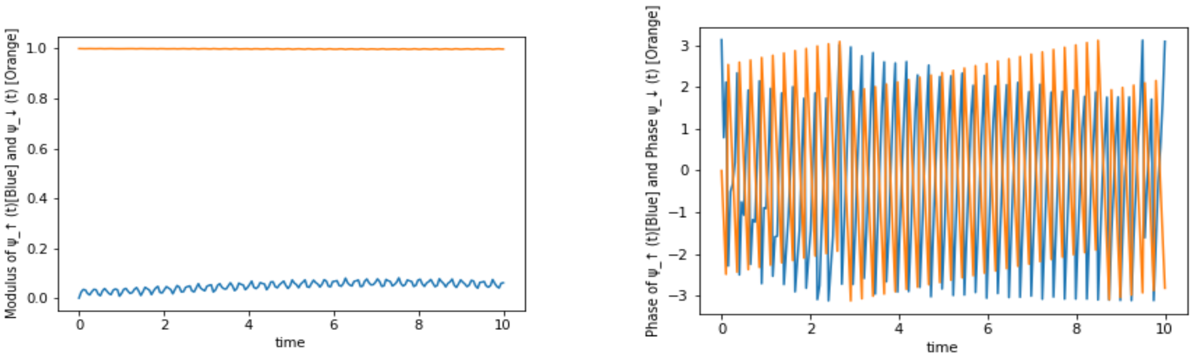
- **Case 2:** $\hbar\boldsymbol{\omega_0} = \hbar\boldsymbol{\omega_{new}}$



(a) *(b)*

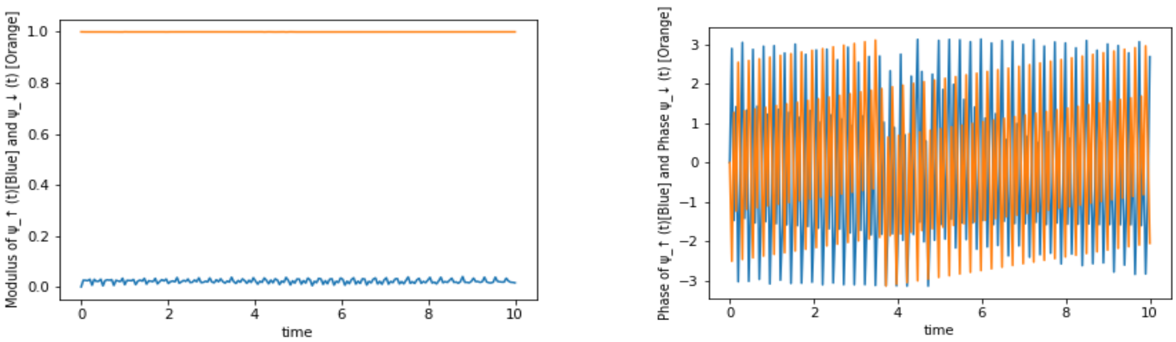
Figure 22. The plot of the ground and excited state wavefunction ($\psi_\downarrow[orange]$ and $\psi_\uparrow[blue]$) (a) Modulus and (b) Phase VS $time\ (t)$ with $\hbar\omega = 10$, $\hbar\omega' = 5\hbar\omega = 50$, $\hbar\omega_0 = \hbar\omega_{new}$, $E = \frac{\hbar\omega}{5}$, and $E' = \frac{\hbar\omega'}{5}$

- **Case 3:** $\hbar\boldsymbol{\omega_0} > \hbar\boldsymbol{\omega_{new}}$



(a) *(b)*

Figure 23. The plot of the ground and excited state wavefunction ($\psi_\downarrow[orange]$ and $\psi_\uparrow[blue]$) (a) Modulus and (b) Phase VS $time\ (t)$ with $\hbar\omega = 10$, $\hbar\omega' = 5\hbar\omega = 50$, $\hbar\omega_0 = 5\hbar\omega_{new}$, $E = \frac{\hbar\omega}{5}$, and $E' = \frac{\hbar\omega'}{5}$



(a) *(b)*

Figure 24. The plot of the ground and excited state wavefunction ($\psi_\downarrow[orange]$ and $\psi_\uparrow[blue]$) (a) Modulus and (b) Phase VS $time\ (t)$ with $\hbar\omega = 10$, $\hbar\omega' = 5\hbar\omega = 50$, $\hbar\omega_0 = 10\hbar\omega_{new}$, $E = \frac{\hbar\omega}{5}$, and $E' = \frac{\hbar\omega'}{5}$

From the above plots, it could be seen that the physical significance of the results are similar to question 2. We see that in cases where the values of $\hbar\omega_0$ are either smaller or larger than $\hbar\omega_{new}$ (case 1 and case 3) has lower values for the excited state wavefunction copmared to the ground state wavefunction which indicates lower probability of the electrons being in the excited state and higher probability of it residing in the ground state. Similar behaviour are also observed for the phase plot with time, where as $\hbar\omega_0$ gets larger the phase oscillates much faster as well. However, we see that when $\hbar\omega_0 = \frac{\hbar\omega_{new}}{5} = \frac{6\hbar\omega}{5}$, the wavefunction of excited state has similar values with the case where $\hbar\omega_0 = \hbar\omega_{new}$ and with a faster oscillation. This might be because the incoming EM wave consists of two components, namely one with energy of $\hbar\omega$ and the other with energy of $5\hbar\omega$, thus when $\hbar\omega_0 = \frac{\hbar\omega_{new}}{5}$ the energy gap has similar value of energy to the first component of the EM wave ($\hbar\omega$) which then would cause the higher probability of the electron to be in the excited state. For the other cases, it could be observed that the wave function of the ground state dominates in value compared to the excited state which is as expected as it is in the analysis of question 2 which could be seen as an example in Fig. (23a) and Fig. (24b).

Conclusion

Overall, this project devised a code to simulate the possible scenarios and cases of Rabi Oscillation phenomenon using techniques coming from, systems of ODE, Fourier series and transformation to solve the general wavefunction. The quasi-energies and wavefunctions are then plotted to expose how they behave with changes in energy gap values and also with time. However, there could be still many cases uncovered from this report which could be studies further in future projects using the developed code.

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References

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