Data Science for Economists

Lecture 10: Econ Applications of Numerical Methods

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Introduction

Agenda

Today will be an easy lecture that shows y'all some applications of the numerical methods we just learned.

The actual lecture slides will be short as we will spend most of the day programming.

Linear Regression

Linear Regression

- As you may have already seen, many times in economics, we have a dependent variable $(y_i)_{i=1}^n$ and indepedent variables $(x_{1i}, x_{2i}, \ldots, x_{ki})_{i=1}^n$ that we would like to relate.
- Specifically, we want to estimate $E[y_i | X_i]$ where $X_i = (x_{1i}, x_{2i}, \ldots, x_{Ki})_{i=1}^n$.
- We do this by assuming a "linear model" and estimating the equation of a line

$$y_i = \alpha + \beta_1 x_{1i} + \ldots + \beta_K x_{1k} + \varepsilon_i$$

- y_i and X_i are data we observe; α and $(\beta_k)_{k=1}^K$ are parameters we would like to estimate.

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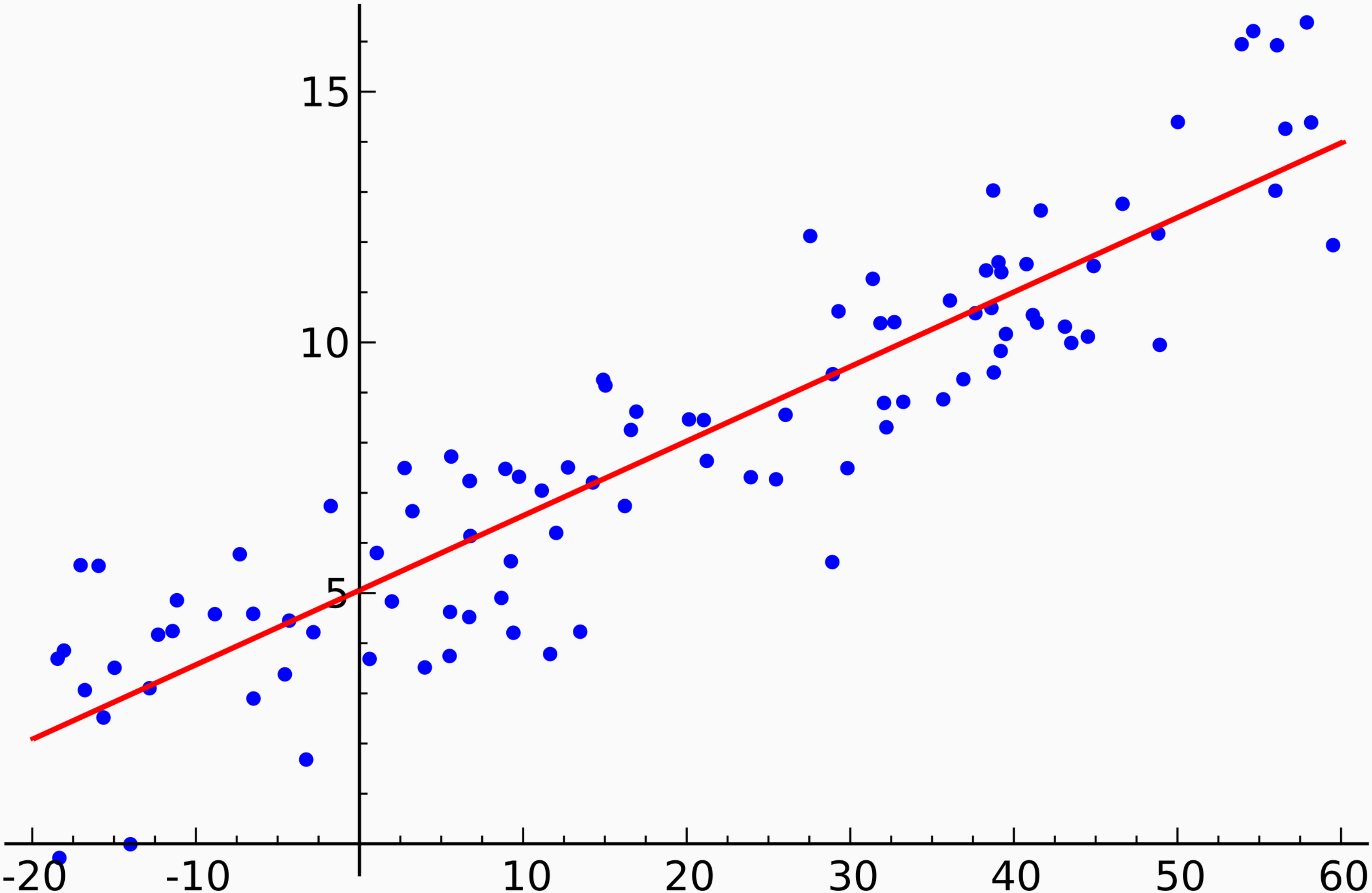
- y_i and X_i are data we observe; α and $(\beta_k)_{k=1}^K$ are parameters we would like to estimate.
- What's ε_i?
- Note: $\varepsilon_i = y_i - \beta_1 x_{1i} - \ldots - \beta_K x_{1k}$
- Idea: we can estimate the parameters by choosing $\theta = (\alpha, \beta_1, \ldots, \beta_K)$ such that we minimize

$$\sum_{i=1}^{N} \varepsilon_i^2$$

Linear Regression



Linear Regression

- Notation:
  - For the true values of $\boldsymbol{\theta}$, we say ε_i is the unobserved error term.
  - When we estimate the values of $\boldsymbol{\theta}$, we typically put "hats" on them to indicate estimates $\hat{\boldsymbol{\theta}}$. Likewise with the error term.
  - We call $\hat{\varepsilon}_i$ the "residual"
  - Error term is population; residual is sample.

- So to estimate $\boldsymbol{\theta}$, we would like to solve the problem

$$\min_{\theta} \sum_{i=1}^{N} \hat{\varepsilon}_i(\theta)^2$$

- Note: $\hat{\varepsilon}_i(\theta) = y_i - \hat{\alpha} - \hat{\beta}_1 x_{1i} - \ldots - \beta_K x_{1K}$
- While this problem has a "closed-form" solution, we can use the numerical methods we talked about in last class.
- Let's go to R!

Supply and Demand Oligopoly

Demand: Oligopoly

- I will be leaving a lot of details out.
- Suppose we have K products all produced by a different firm.
- Suppose consumers get the following utility from product k:

$$u_{ik} = \beta_0 + \beta_x x_k - \alpha p_k + \xi_k + \varepsilon_{ik}$$

- We say consumers will select one of the K products that maximizes their utility (with the option of not buying anything at all).
- Without getting into details, we say that the market share for each product will have the following expression

$$s_k = \frac{e^{u_k}}{1 + \sum_{j=1}^{K} e^{u_j}}$$

  where $u_k = \beta_0 + \beta_x x_k - \alpha p_k + \xi_k$
- We say the market share is the "demand."

Supply: Oligopoly

- On the supply side, we say firms set price to compete with each other to maximize profits:

$$\max_p \pi_k(p) = s_k(p)p - s_k(p)c_k$$

  where c_k is a constant marginal cost of production.
- From a game theory model, we can derive an optimal price setting formula

$$p_k = \frac{1}{\alpha(1 - s_k(p_k))} + c_k$$

  - Reminder: in perfect competition $p_k = c_k$.
- Note that the market share (demand) shows up in the pricing formula.
- If we wanted to simulate this model, we would need to solve for equilibrium prices as demand depends on prices $s_k(p)$ and prices depend on demand $p_k(s)$.
- We need to solve a fixed point where $p_k(s_k(p^*)) = p^*$
- To R for an example!

Up Next: Data Wrangling