Revenue Management with Reallocation*

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Abstract

We develop a model to study the trade-offs associated with introducing re-allocative mechanisms into dynamic-pricing environments with heterogeneous goods and strategic consumers. Our focus is on airlines that sell seats in vertically-differentiated cabins and provide upgrade opportunities after an initial purchase via auctions and fixed-price sales. If consumers anticipate opportunities for an improved reallocation and reduce outright purchases of premium seats, the screening intention of dynamically-set prices can be undermined to create circumstances with a greater probability of upgrades and an ambiguous impact on profits. To study ways to adapt these mechanisms to better complement dynamic-pricing practices, we estimate the model's structural parameters using proprietary data from an airline that includes the price for each itinerary, daily cabin-specific seat inventories for each flight, bids and purchases of upgrades, and information on visits and purchases on the airlines' website. We find that the mechanisms, as implemented, transfer a modest amount of surplus from the airline to consumers. In counterfactual calculations, we explore two ways to improve integration and performance. We find that profits and total welfare increase by introducing state-specific reserve values to provide commitment for the airline to make the auction less accommodating to strategic consumers and making pricing policies dependent on submitted bids to internalize the option value of the auction while setting prices.

Keywords: Dynamic Pricing, Auctions, Price Discrimination, Upgrades, Airline Industry

JEL Codes: C57, D44, D61, L10, L50, L93

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1 Introduction

In many settings, firms use dynamic pricing to allocate a fixed amount of goods or services to a randomly arriving sequence of customers with privately-known valuations. If the items to be allocated are heterogeneous, optimal prices must balance current sales against future demand and effectively screen customers between the differentiated offerings. This is challenging in practice, and demand uncertainty can result in substantial allocative inefficiencies. As such, firms often complement dynamic pricing with other strategies intended to reduce these inefficiencies and increase profits.

In this paper, we develop and estimate a model to study the trade-offs associated with integration of re-allocative strategies into dynamic pricing environments using data from the airline industry. Specifically, we examine auctions that allow consumers to bid for upgrades between vertically-differentiated cabins after an initial purchase.[1] Ideally, bids can be used by the firm to allocate available upgrades to customers with the greatest valuation while freeing up units of the lesser product for sale, increasing allocative efficiency and profits. However, strategic consumers that anticipate opportunities for an improved reallocation may reduce outright purchases of premium seats. This can undermine the screening intention of dynamically-set prices to create circumstances associated with a greater probability of winning an upgrade through these alternative channels and negatively impact profits. Despite this trade-off, most airlines use separate vendors for pricing and upgrade decisions, and the algorithms are not adapted to work together. In particular, the auction is greedy in the sense that it maximizes revenue given the state encountered by the airline at the time of the auction, which creates opportunities for strategic consumers to alter the distribution of states. This type of failure to internalize decision externalities across teams with siloed objectives is common among firms facing complex problems, as highlighted by Hortaçsu et al. [2023]. Our focus is to identify effective ways to improve the performance and integration of these re-allocative mechanisms in settings that rely on dynamic-pricing practices.

Our analysis relies on unique data from a North American airline that implemented auctions and fixed-price upgrade processes. The data include the price paid and timing of purchase for each itinerary, daily seat inventories for every flight in the airline's network, and information on upgrade bidding and purchases. The auction allows consumers that purchase an economy fare to bid among a discrete set of values for an upgrade, and any remaining premium seats at departure are offered at a fixed price on a first-come basis during check-in. We complement the revenue management and upgrade data with information on visits to

[1] This practice is particularly common in travel and leisure markets. For example, see Amtrak's BidUp program: https://www.amtrak.com/bidup.

the airline's website that includes which markets are queried, whether a purchase was made, and tier status of customers. Together, these data provide a complete picture of the timing of consumers' arrivals to the market and their decision making, which permits our analysis of the impact of the auction on market outcomes.

The supply side of our model features a monopolistic airline with a fixed number of seats for sale in two vertically-differentiated cabins. The airline sets prices for both cabins in each period before departure to maximize profits, while offering customers that purchase an economy seat the opportunity to bid among a discrete set of values for a chance to be upgraded to the premium cabin. At a fixed date before departure that is known to consumers, the airline decides how many, if any, bids to accept. If any seats in the premium cabin remain unsold at departure, the airline offers upgrades at a fixed price during check-in. The auction optimizes revenue by balancing the benefit (i.e., bid revenue and one additional economy seat) and cost (i.e., one less premium seat) associated with each bid. This makes the auction forward looking because it accounts for the benefits of reallocation given the state faced by the airline at the time of the auction (i.e., seats remaining), but not backward looking because it fails to internalize the influence of bid-acceptance policies on sales prior to the auction that determine the state.

On the demand side, the presence of the auction and check-in upgrades creates a more complex decision process for customers. Like previous studies of the airline industry, we assume that a random number of short-lived consumers of different types with heterogeneous preferences arrive each period and make a purchase decision: economy, premium, or no-purchase. Those purchasing economy seats can choose to bid for the possibility of an upgrade or wait until departure for a chance to be upgraded at a fixed price. This requires consumers to form beliefs about the possibility of being awarded an upgrade, which must account for the bidding behavior of other customers and the airline's bid-acceptance and pricing strategies. This is particularly complicated because selection into the auction varies by the prices faced by customers at each point in time, which is a non-stationary stochastic process.

Solving the model requires identifying optimal pricing and bid-acceptance policies for the airline and the equilibrium beliefs for consumers that are consistent with the probability of winning an upgrade. The airline's problem is a straightforward, albeit computationally intensive, dynamic program that yields state-dependent (i.e., time and cabin capacities) pricing policies for economy and premium cabins and a threshold value required to accept a given number of bids at a fixed date before departure (i.e., opportunity cost of premium seats). Solving for equilibrium beliefs is complicated by customer selection into bidding, which is driven by the realized path of prices. Specifically, a customer at a particular state must integrate over all possible past, current, and future bidding by others to form an expectation about the probability of winning an upgrade with a particular bid value or check-in purchase. To solve for such state-dependent beliefs, we use an iterative forward simulation procedure. This entails simulating customer choices given an initial guess of beliefs (i.e., probability of winning with a particular bid value at each state), updating those beliefs based on the airline's optimal bid-acceptance policy, and then iterating until convergence between beliefs and the probability of winning. The process typically converges quickly despite the high-dimensional nature of the problem (i.e., many discrete bid values at thousands of states).

To estimate the model, we apply the method-of-moments approach of Fox et al. [2016] and Nevo et al. [2016] to flexibly capture both within- and across-market heterogeneity in preferences for air travel. This two-step procedure also has the advantage of limiting the number of times the model is solved during estimation, which is important for our application.[2] In the first step, we solve and simulate the model for a range of candidate preference parameters that characterize demand for a flight (i.e., a data-generating process) to calculate outcomes like transacted prices, seat occupancy rates, and results from the auction and check-in sales. In the second step, for each market, we match moments from the data to a convex combination of analogous moments for each of the candidate preference parameters. This yields a flexible discrete distribution of consumer preferences for each market that captures heterogeneity in preferences across flights within a market.

We find that the model estimates can reproduce features of the data well. The variation in prices leading up to departure, and between the economy and premium cabins, is rationalized by market-specific arrival patterns, valuations for travel, and fraction of late-arriving frequent fliers with greater valuations for travel and quality. Importantly, predictions regarding participation in the upgrade processes are also consistent with those in the data. We match the roughly 25% of the premium seats that are occupied due to the auction and check-in purchases, and the overall odds of winning the auction closely matches the 0.45 probability observed in the data.

We perform two simple counterfactual calculations to offer comparable results to the extant literature. First, as a benchmark to the reduced-form analysis of Marsh et al. [2024b], we examine the impact on profits and consumer surplus from the introduction of the auction. Consistent with those results, and the influence of strategic consumers predicted by our model, we find that the introduction of the auction results in a small transfer on average from the airline to consumers. Next, like Aryal et al. [2023], we calculate a first-best allocation by solving a linear program that allocates seating capacity to passengers assuming the airline has

[2] In a preliminary step, we directly recover the customer arrival process from the web search data for each market.

perfect foresight about demand (i.e., arrivals and valuations). This characterizes an efficient frontier for any distribution of surplus between the airline and consumers and the opportunity for improvement measured as the gap between current and first-best outcomes. We similarly find that on average, dynamic pricing alone achieves approximately 86.5% of the first-best welfare with revenue and consumer surplus contributing 76.0% and 10.5%, respectively. Consistent with the first counterfactual, dynamic pricing and the auction together reduce total welfare slightly.

Given the failure of the auction to close the gap with first-best efficiency or improve profits, we consider two ways to improve the complementarity of the auction and revenue-management system. First, we consider one way to limit manipulation of the auction by strategic consumers. Specifically, we let the airline commit to optimal state-specific reservation values that increase the cost of accepting bids to internalize the strategic response of consumers to accommodating bid-acceptance polices. So, the auction is now backward- and forward-looking by balancing the benefits of accepting bids and reallocating passengers against the cost of consumers manipulating the path of prices and states prior to the auction. Next, we explore one way to alter the revenue-management system to account for the existing optimization policies of the auction. Currently, no bid information is shared by the auction, which prevents pricing policies from internalizing the option value associated with bids (i.e., raises the opportunity cost of a premium sale). Our counterfactual calculation permits the airline to directly condition pricing policies on collected bids. Calculating the airline's optimal pricing policies in this setting is much more complex. Most directly, incorporating bids expands the state space substantially. We overcome this by considering summary statistics of the distribution of bids collected. Second, and more challenging, a solution to the model now entails solving a game between the airline and consumers to identify optimal pricing policies and equilibrium bidding, because bids directly impact pricing policies. To solve this game, we use an iterative solution method between between the airline and consumers' best responses. These last two counterfactuals will be complete soon.

Related Literature. This paper contributes to the growing literature in industrial organization that empirically studies price discrimination. There are numerous studies that examine a range of discriminatory strategies in different settings: [e.g., Ivaldi and Martimort, 1994, Leslie, 2004, Busse and Rysman, 2005, Crawford and Shum, 2006, Mortimer, 2007, McManus, 2007, Aryal and Gabrielli, 2019, Aryal et al., 2023]. Further, there are many that focus specifically on inter-temporal price discrimination: [e.g., Nevo and Wolfram, 2002, Nair, 2007, Escobari, 2012, Jian, 2012, Hendel and Nevo, 2013, Lazarev, 2013, Cho et al., 2018, Williams, 2022a]. We contribute to this literature by studying complex hybrid approaches used by firms that combine different strategies to price discriminate.

Our model and findings also contribute to the rich literature on dynamic pricing.[3] Foundational theoretical studies include Stokey [1979], Gale and Holmes [1993], Dana [1999], Courty and Li [2000], Armstrong [2006]. Recent empirical work examines a variety of industries where firms sell an expiring asset with dynamically adjusted prices[e.g. Graddy and Hall, 2011, Sweeting, 2010, Cho et al., 2018, Waisman, 2021]. Williams [2022b] and Aryal et al. [2023] are two closely related studies that also study dynamic pricing in the airline industry. Williams [2022b] contrasts market outcomes and welfare under uniform and dynamic pricing for a monopolistic airline with a fixed amount of capacity. Aryal et al. [2023] enrich this framework by modeling a vertically-differentiated aircraft to study both intra-temporal and inter-temporal price-discrimination incentives. We build upon these studies by considering the welfare consequences of strategies like auctions intended to complement dynamic pricing of differentiated goods.

There are a number of theoretical studies on the use of auctions for allocating multiple units of a homogeneous good. Vulcano et al. [2002] characterize optimal auctions for allocating multiple units of a homogeneous good, Talluri and van Ryzin [1998] examine bid-prices as a way to price seats on a flight segment, and Ely et al. [2017] consider the use of auctions to resolve overbooking of a flight. Our setting is similar, but we allow for multiple qualities of seats and demand is non-stationary. We also contribute to the literature on endogenous entry into auctions [e.g. Samuelson, 1985, Levin and Smith, 1994, Marmer et al., 2013, Roberts and Sweeting, 2013, Gentry and Li, 2014, Gentry et al., 2017], which is known to alter classic results from auction theory. In our application, variation in prices determines entry into the auction, such that consumers prefer economy with the possibility of an upgrade to either premium or economy with certainty. This creates a complex form of selection, which we resolve through an iterative forward simulation procedure to solve for equilibrium beliefs.

We also contribute to the expansive literature on pricing and consumer choice in the airline industry. A number of studies examine the relationship between price dispersion, in lieu of a direct measure of price discrimination, and market power like Borenstein and Rose [1994], Puller et al. [2012], and Chandra and Lederman [2018]. A more closely related literature examines strategic aspects of consumer decision making in the industry. Li et al. [2014] measure the fraction of consumers that are strategic in the timing the purchases of their itinerary.[4] Lazarev [2013] estimates a model with forward-looking consumers and a monopoly airline that uses inter-temporal price discrimination to examine the effects of policies prohibiting the resale of airline tickets. Scott [2024] and Li et al. [2024] explore the

[3] See van Ryzin and Talluri [2005] for a general introduction to revenue-management practices.

[4] Related, Dilmé and Li [2018] provide a general theoretical framework to examine how the strategic timing of purchases by consumers alters the optimal pricing of the firm.

effects of uncertainty and cancellation fees in dynamic-pricing environments with forward-looking consumers.

There are a number of recent studies examining upgrades specifically. Cui et al. [2018] theoretically study fixed-price upgrades with strategic consumers, while Cui et al. [2019] empirically examine the effect from the introduction of fixed-price upgrades on price dispersion. [Favrizi, 2024] considers the welfare consequences of fixed-prices upgrades, but assumes that consumers do not anticipate the possibility of an upgrade. Our analysis combines aspects from each of these studies by allowing for strategic consumers and fixed-price upgrades, while also introducing the auction mechanism. Marsh et al. [2024a] and Marsh et al. [2024b] provide additional details on the web-traffic data that we use to estimate the arrival process and a detailed analysis of the auction implementation, respectively.

2 Data and Institutional Details

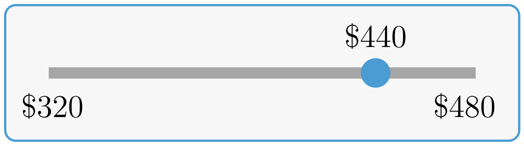
The data for this study come from a major North American airline that uses dynamic pricing strategies and upgrade mechanisms to allocate seats in vertically-differentiated cabins. The primary data describe the revenue management and upgrade processes. We supplement these data with consumer search queries made on the airline's website, providing additional information regarding the consumer arrival and search process. In this section, we detail the upgrade process and describe our data sample in order to motivate a model of revenue management with reallocation.

2.1 Upgrade Process

As is common in the industry, the airline segments each aircraft into a high-quality premium cabin and an economy cabin. Each prospective consumer is able to purchase a ticket for either cabin depending on availability. The airline presents all economy ticket holders with two opportunities to receive an upgrade to the premium cabin: via a fixed fee when checking into their flight or through an auction prior to check-in. The opportunity to upgrade at check-in requires an excess of premium seats at the time of check-in. In contrast, each passenger who books an economy ticket before the auction has the opportunity to place a bid regardless of the remaining premium capacity.

Economy passengers place bids using a slider feature characterized by a minimum bid, a maximum bid, and an initial position. An example slider can be seen in Figure 1. Bids can be placed in discrete increments between the minimum and the maximum. No other information about the auction is communicated to passengers when placing their bid. The airline collects bids and decides which bids to accept (if any) on a fixed day approximately one week before departure. The timing of the bid clearance ahead of departure is uniform across flights. If a bid is accepted, the passenger must pay the bid amount for the upgrade.[5]

Figure 1: Example Slider



Notes: The figure shows an example of a slider seen by a passenger participating in an upgrade auction. The slider features a minimum, maximum, and starting position. Bids can be made by finalizing the position and submitting the bid.

The existing revenue management system is not adjusted after the introduction of either upgrade mechanism. The upgrade fees are set ex-ante and are not updated as demand is realized. Furthermore, the airline does not adjust prices in response to the bids placed by consumers. In fact, the airline uses the services of a separate vendor to manage both upgrade systems, meaning the upgrade process is not integrated into the revenue management system that dynamically sets prices. We exploit this fact in our model by assuming the airline does not account for the option value of an economy ticket that is created by both upgrade mechanisms when setting prices.

2.2 Data Sources

A flight in the data is identified by a unique combination of departure date, directional segment, and flight number. Inventory data track daily changes in seat availability for every flight that the airline scheduled for departure during the sample period. These data reveal the number of seats sold and remaining in each cabin at each day leading up to departure. Booking data contain information from all ticket sales for the corresponding flights, including the itinerary details, the booking date, the fare paid, the original aircraft-cabin class, and a passenger identification number.

In addition to the above revenue management data, we use data collected from the two passenger upgrade mechanisms. First, we observe all upgrades made at passenger check-in via the fee. These data include the passenger's itinerary details and identification number, as well as the segment-specific upgrade fee. We also observe all bids placed by passengers for the upgrade auction held prior to passenger check-in. Observations in these data include

[5]If ties need to be broken, the airline uses the passenger's loyalty status as a tie breaker.

the passenger's itinerary details and identification number, as well as the bid amount, the minimum and maximum bid the passenger could submit, and whether the bid was accepted.

An itinerary captures a passenger's departure date, directional segment, and flight number. This information allows observations to be matched across the aforementioned data sets by the uniquely identified flight in the inventory data. Likewise, passengers in the booking data can be linked to both upgrade data sets via the passenger identification number and the flight information.

We supplement the revenue management data with flight queries made by consumers on the airline's website. Each observation in the search data represents a time-stamped web page visit and includes the search inputs that led the consumer to the web page, such as the trip origin and destination, departure date, and even the consumer's loyalty status if they logged in during their search.

2.3 Sample Selection and Market Definition

The revenue management and upgrade data feature flights drawn from a fifteen month period, during which the airline introduced the auction. We narrow our analysis to the final three months of flights due in large part to the gradual consumer uptake in the auction documented in Marsh et al. [2024b]. For each flight, inventories and bookings span almost a full year leading up to departure. The supplemental search data span nine months in a nearly adjacent, more recent period.

The revenue management data provide information on flight inventories and consumer booking counts. Although the search data do not overlap these data, they allow us to observe both booking counts and search counts for trips in the airline's network in a nearby time period. For a given flight segment and day before departure, we find the booking-to-search ratio in the search data and proportionally match these to the booking counts observed in the revenue management data.[6]

In the revenue management data, there are approximately 400 directional flight segments with an upgrade auction during our three-month sample period. However, because we only observe trip origin and destination pairs in the search data (as opposed to the flight segments observed in the revenue management data), we limit our sample to directional segments that account for all passenger flow to or from one of the airports. This ensures that all searches with an origin or destination that include that airport must travel along the relevant directional segment. This cut leaves 94 directional segments.

[6]We verify that the distribution of bookings in the revenue management data is reasonably summarized by the distribution of bookings in the search data.

We also exclude segments that were present in the three-month sample but not present (or rarely present) in the search data. These include segments that are flown infrequently or only flown seasonally. This cut reduces the number of directional segments to 60. For every remaining directional segment, the returning segment also survives these cuts. We therefore group directional segments together and define a market to be a non-directional segment. Appendix A shows the number of flights and flight network resulting from this sample selection.

2.4 Descriptives

Sample Statistics. Our final sample includes 5,066 flights. Summary statistics from these flights are presented in Table 1. The average flight travels approximately 1,350 miles, and nearly half of the flights travel at least 1,600 miles. These flights have an average economy-to-premium seat ratio of nearly 11 to 1, but the airline configures almost every plane with at least 10 economy seats for every 1 premium seat. Before even selling a ticket, the airline creates a scarcity effect that increases the opportunity cost of allocating a premium seat.

The load factor refers to the share of seats within a specified cabin that are allocated at the reference time. On average, 80% of the seats in both the economy and premium cabins are allocated at the time of departure, but the median flight has a load factor of approximately 90% in both cabins. It is worth noting that only 58% of the seats in the premium cabin are sold at full price. On the day before the auction, 53% of the premium seats are allocated on average, but an interquartile range of 63% highlights the demand uncertainty the airline faces. For nearly one-half of the flights in the sample, less than half of premium seats are allocated a week before departure.

The airline receives 1.54 bids per flight and allocates 0.69 and 1.86 premium seats through the auction and check-in mechanisms, respectively. Although one might consider participation in the auction to be low, more than half the flights receive at least one bid. Further, the relatively cheaper (though less certain) check-in mechanism yields nearly two upgrades per flight. On the median flight, the premium cabin load factor is 90% at departure, with 30% of the premium seats allocated via one of the two upgrade mechanisms.

Summary statistics for consumer transactions are presented in Table 2. Our sample contains more than 550,000 consumer bookings. The average fares paid for economy and premium tickets are \$199.49 and \$457.97, respectively, but there is quite a bit of variation across markets and across time leading up to departure within markets. However, it is not uncommon to see a premium ticket priced at 100% more than an economy ticket.

As referenced above, there is nearly a \$100 difference between the average submitted bid

Table 1: Flight Summary Statistics

Variable	**Mean**	**St. Dev.**	**25th**	**50th**	**75th**	**N**
Flight Details						
Flight Distance	1355.7	908.8	397	1591	2036	5,066
Capacity (Econ./Prem.)	10.89	2.03	10.17	10.17	13.50	5,066
Load Factor (Departure)						
Economy	0.808	0.198	0.722	0.885	0.951	5,066
Premium	0.796	0.266	0.667	0.917	1.000	5,066
Premium (Full Price)	0.581	0.326	0.333	0.600	0.917	5,066
Upgrades						
Check-In Upgrades	1.864	2.116	0	1	3	5,066
Number of Bids	1.541	2.320	0	1	2	5,066
Auction Upgrades	0.688	1.208	0	0	1	5,066

Notes: The table shows descriptive statistics from flights in the sample. Details include the flight distances, seating capacities, allocation of seats, and allocation of upgrades.

($220.71) and the average check-in fee ($129.91). The sign of the difference between the two is intuitive because an empty premium seat has far more value to the airline further from departure. Forgoing the opportunity to sell the seat at full price is much more costly at the time of the auction. The magnitude of both is also reasonable given that the difference in average fares is $258.48. If bids were much higher, we would question why consumers do not purchase a premium ticket at full price.

To measure bid intensity, we normalize each bid relative to the slider presented to the passenger.[7] The average normalized bid is a little less than a quarter (0.228) of the slider, with the 25th percentile being zero (the slider minimum). Conditional on submitting a bid, the probability of being upgraded through the auction is 0.446. In general, the distribution of bids tends heavily towards the lower half of the slider.

Realloaction Mechanisms. The goal of the airline's revenue management system is to maximize expected profits given the number of seats remaining as departure approaches. Due to demand uncertainty, the airline may find itself in an unfavorable state of the world where demand realizations lead to too many or too few seats being allocated. When these unfavorable states are reached, the airline may benefit from reallocating passengers between cabins and reshuffling the number of remaining seats in each cabin.

Figure 2a demonstrates the effect of both upgrade mechanisms on capacity allocation leading up to departure. The airline evaluates the submitted bids in the auction on a

[7] Specifically, the normalized bid is $b_i^{\text{norm}} = \frac{b_i - \underline{b}_i}{\overline{b}_i - \underline{b}_i}$ where b_i^{norm} is the normalized bid of passenger i, b_i is i's bid, $\underline{b}_i$ is i's slider minimum, and $\overline{b}_i$ is i's slider maximum.

Table 2: Customer Transaction Summary Statistics

Variable	Mean	St. Dev.	25th	50th	75th	N
Bookings						
Economy Fare	199.49	144.68	100.86	171.00	254.90	518,411
Premium Fare	457.97	258.94	288.00	412.90	585.54	31,677
Upgrades						
Check-In Fee	129.91	119.48	49.00	59.00	159.00	6,191
Submitted Bid	220.71	145.11	90.00	210.00	311.60	7,806
Slider Minimum	178.91	113.40	80.00	185.00	270.00	7,806
Slider Maximum	368.73	192.95	210.00	420.00	570.00	7,806
Normalized Bid	0.2281	0.2724	0.0000	0.1111	0.3500	7,806
Bid Accepted	0.4461	0.4971	0.0000	0.0000	1.0000	7,806

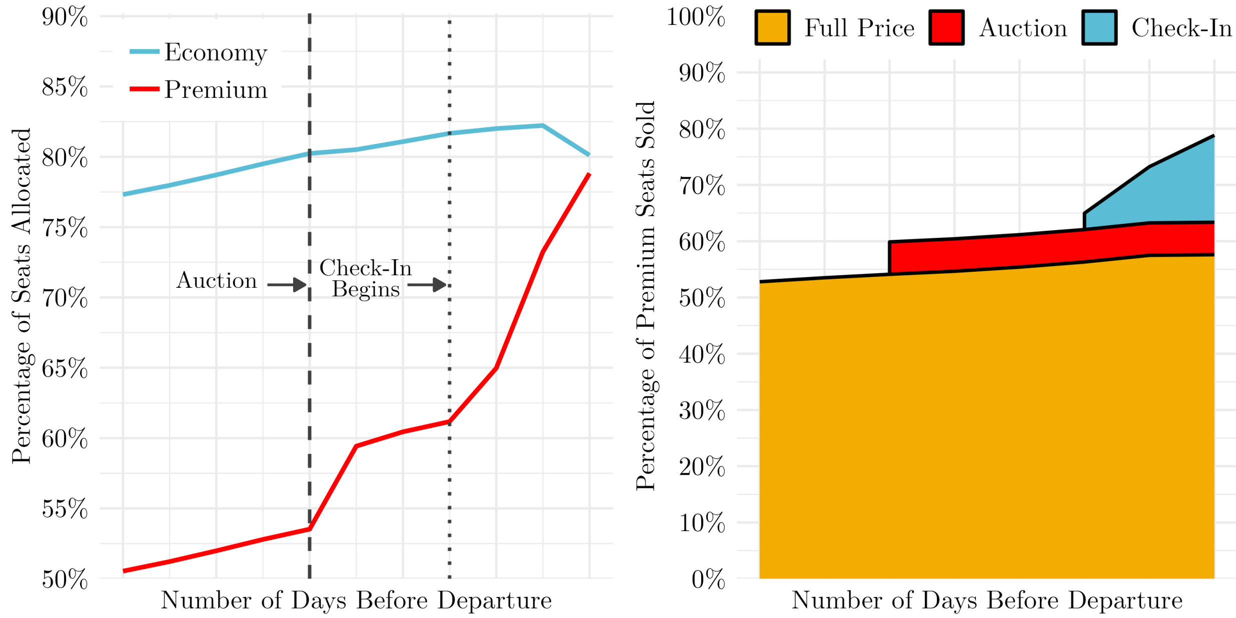
Notes: The table shows descriptive statistics from customer transactions in the sample. These statistics come from bookings, as well as the check-in upgrade mechanism and the auction upgrade mechanism. Details include prices, fees, bids, and auction slider characteristics.

fixed day around a week before departure. At this point, a small dip in the percentage of economy seats allocated can be observed with a corresponding increase in the percentage of premium seats allocated. Likewise, the check-in process begins a few days before departure, and we observe a similar dip in the economy percentage and the corresponding increase in the premium percentage.[8] The percentage of premium seats allocated by each mechanism and day leading up to departure is shown in Figure 2b. The airline allocates 57.6% of all premium seats via full price premium ticket sales. The check-in and auction mechanisms account for 15.5% and 5.8% of premium seats, respectively. This means that roughly one-fifth of all premium seats are allocated using an upgrade mechanism, and nearly one-quarter of all passengers in the premium cabin are allocated their seat via an economy ticket and premium upgrade.

The probability of receiving an upgrade through the auction is highly dependent on the number of seats remaining at the time of the auction. Figure 3a explores the upgrade probability while conditioning on the percentage of seats allocated in each cabin at the time of the auction. As the premium load factor before the auction increases, the probability of being upgraded decreases. This is because premium seats become more value as fewer remain, which increases the opportunity cost associated with an upgrade. As the economy load factor before the auction increases, we observe a slight decrease in the upgrade probability. We believe this is the result of two compounding effects: a competition effect and a demand effect. Flights that have sold more economy seats before the auction will have more bidders

[8] The large decrease in the economy percentage after one day out is associated with check-in upgrades and cancellations.

Figure 2: Upgrades as a Reallocation Mechanism



(a) Share of Seats Allocated Approaching Departure by Cabin

(b) Share of Premium Seats Allocated Approaching Departure by Mechanism

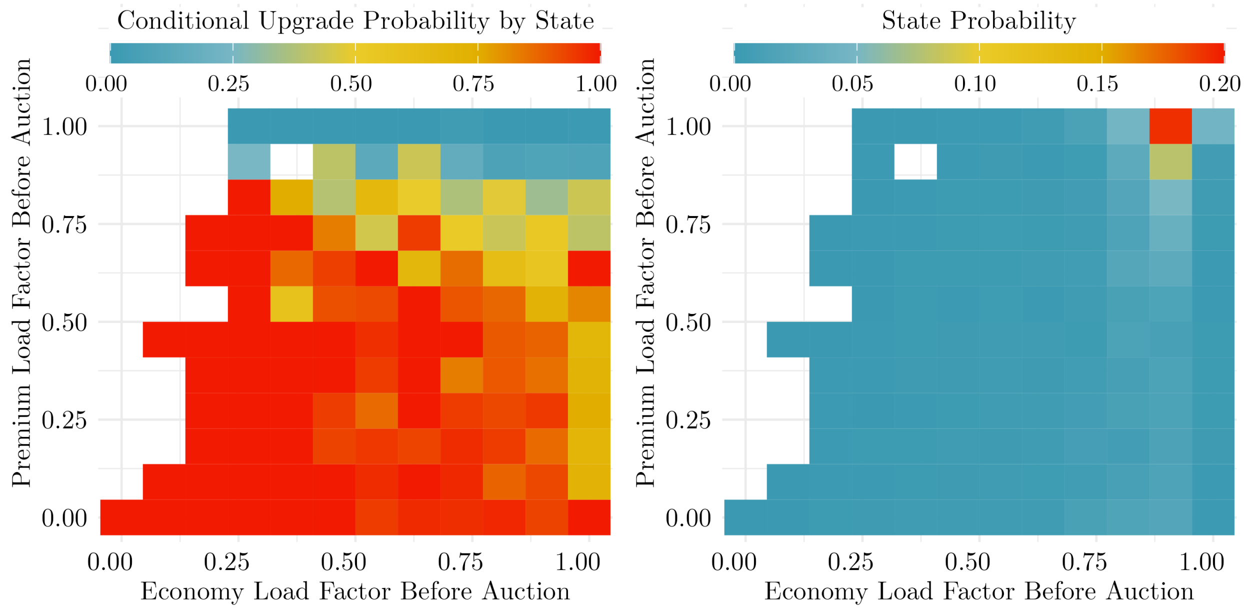
Notes: The airline chooses which bids to upgrade at five days out and the check-in process begins two days out. Almost a quarter (24.6%) of all premium seats are allocated in the last five days. Of that quarter, 13.9% are allocated with posted prices, 62.7% are allocated with the check-in mechanism, and 23.3% are allocated with the auction.

on average. This is the competition effect. However, flights that have sold more economy seats before the auction are likely to signal higher overall demand for the flight, increasing the opportunity cost of each premium seat and crowding out potential upgrades. This is the demand effect.

As the probabilities in Figure 3a are conditional on a state, Figure 3b plots the corresponding probabilities for reaching each state. The modal state corresponds to a full premium cabin and an almost full economy cabin. However, this is only about 20% of the total mass and the probabilities of the surrounding states drop off quickly with most of the remaining states having a relatively flat probability. As most of the variation in the state probabilities is in the region where the economy load factor before the auction is greater than 0.5, this highlights the high variability in the premium load factor.

The different behavior influenced by the number of remaining premium seats can create aggregate upgrade probabilities that are nonmonontic in the strength of the bid, as seen in Figure 4a. The upgrade probability peaks in our sample at a normalized bid of about 0.25 before decreasing. These nonmonotonic upgrade probabilities are the result of averaging across flights, which aggregates two different effects, which we name the capacity effect and bidding effect. Figure 3a has already highlighted that the upgrade probability is heavily

Figure 3: Upgrade Probabilities by Remaining Capacity State



(a) Conditional Probability Heat-Map

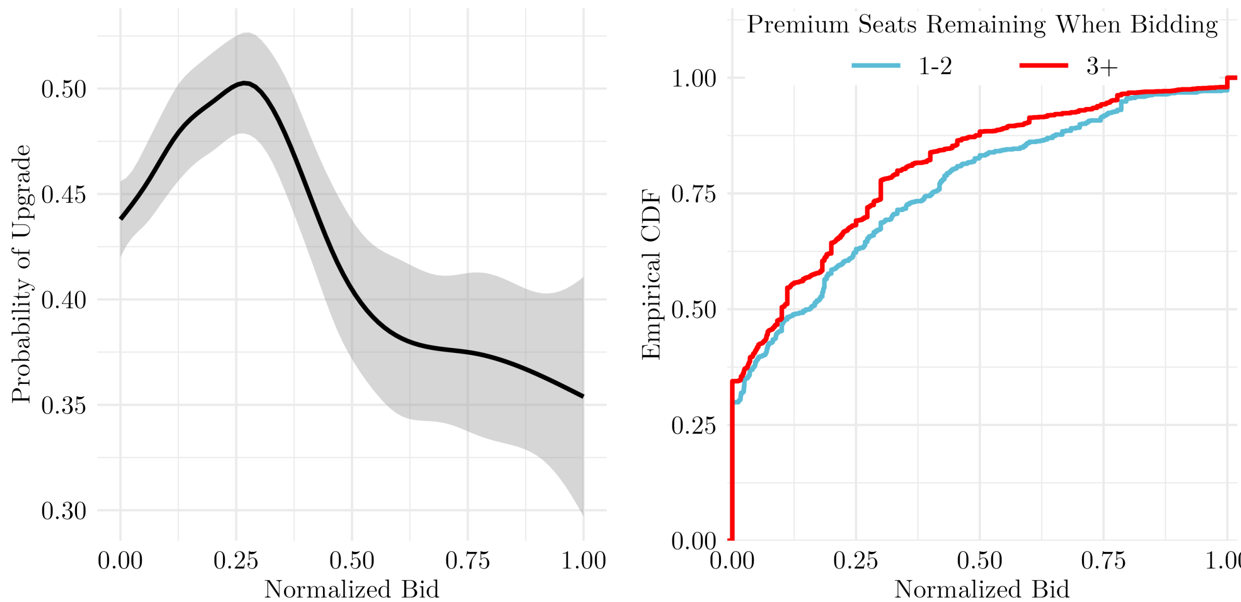
(b) Joint Distribution Heat-Map

Notes: A "state" is an ordered pair of the share of remaining seats (capacity) in each cabin the day before the auction. A tile on the heat-map represents one possible state. The economy and premium load factors before the auction are binned into 12 tiles rounding down. The sum of the product of all tiles in each subfigure equals the unconditional upgrade probability from the auction.

dependent upon the number of premium seats remaining before the auction. This is the capacity effect. However, as fewer premium seats remain, the premium price increases, also increasing the price difference between premium and economy. Because consumers should never bid more than the difference in prices (as they could just buy premium outright instead of bidding), this leads to the largest bids being submitted more often when fewer premium seats remain. This is observed in Figure 4b as the distribution of normalized bids when only one or two premium seats remain when a consumer bidding first-order stochastically dominates the distribution of normalized bids when three or more premium seats remain. The interaction of these two effects produces the nonmonotonic upgrade probabilities because these larger bids from the bidding effect are less likely to be accepted because of the capacity effect. This highlights how the unique two-sided entry in upgrade auctions produces economic behavior different from auctions with traditional one-sided endogenous entry [e.g. Roberts and Sweeting, 2013, Gentry and Li, 2014, Gentry et al., 2017].

Search and Arrivals. The search data allow us to directly observe the number of daily searches on the airline's website, as well as the searches that result in a purchase. Figure 5a shows the densities of all searches, economy purchases, and premium purchases across all markets in our sample. The densities of the searches and economy purchases increase over

Figure 4: Evidence of Nonmonotonic Upgrade Probabilities



(a) Average Upgrade Probability

(b) Submitted Bids Empirical CDF

Notes: The upgrade probability curve in a is calculated using a generalized additive model from ggplot2 in R. The shaded region represents the 95% confidence region. The empirical CDF in b is calculated using the empirical frequency estimator.

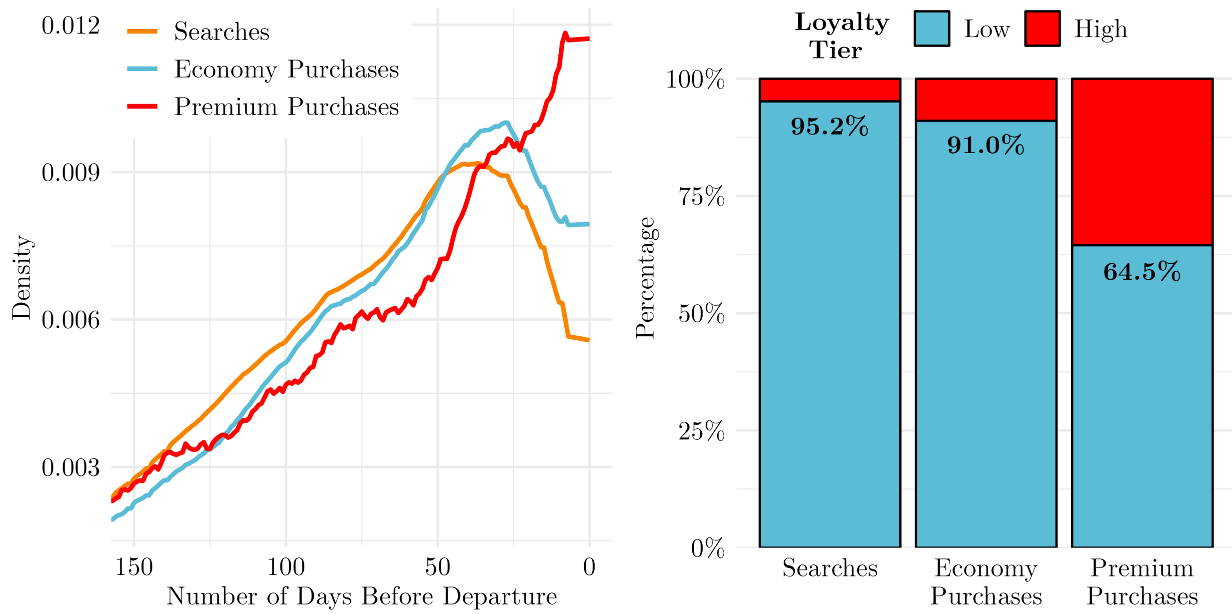
the time horizon until peaking about one month before departure. In contrast, the density of premium purchases increases throughout the time horizon. This highlights the opportunity associated with upgrade passengers through the auction, as many premium purchases are made in the final week before the flight.

If the consumer logs into their customer account during the browsing session, their loyalty status with the airline is also observed.[9] Figure 5b decomposes the searches, economy purchases, and premium purchases by tier status. The vast majority of searches and economy purchases are made by low tier customers. However, more than 35% of premium purchases are made by high tier customers, even though these customers consist of only 5% of searches. This highlights how the airline must balance keeping enough seats to sell to these high valuation consumers with collecting upgrade revenue from economy passengers.

The descriptive analysis highlights various important features in the data that we capture in our model. Around one-third of all premium seats allocated are in the last week before departure with the vast majority through an upgrade mechanism. The upgrade probability in the auction is highly dependent upon the number of premium seats that remain before the auction. Consumers appear to respond by submitting larger bids when fewer premium

[9] To preserve anonymity, we do not provide details regarding the benefits or the requirements of each tier status. We refer to low tier customers as those without loyalty status or the lowest possible status and high tier customers as those with a status above the low tier.

Figure 5: Patterns in Searches and Purchases



(a) Search And Purchase Densities

(b) Searches and Purchases by Tier Status

Notes: The densities in a are calculated by taking the count of each smoothed variable on each scaled by the total count across the entire time horizon. The variables are smoothed using a Gaussian kernel and the rule-of-thumb bandwidth. In b, the raw data is used to calculate the share of total searches and cabin purchases by loyalty tier. We classify customers into the high tier loyalty category if they are tier 2 or higher as this is the first tier that must be earned through frequent flying.

seats remain, which can create aggregate upgrade probabilities that are nonmonotonic in the relative bid. Premium purchases are more likely closer to departure with high loyalty status consumers are responsible for a disproportionate number these purchases. These facts motivate a model of of non-stationary and stochastic consumer demand where consumers behave strategically when presented with upgrade mechanisms alongside dynamic pricing in order to maximize expected utility. The airline will set prices optimally ignoring the upgrade mechanisms and will behave sequentially rational in each period given the remaining seats and realized consumer demand.

3 Model

In this section, we present an equilibrium model featuring a profit-maximizing monopoly airline with a fixed number of vertically-differentiated premium and economy seats on an aircraft to allocate before departure to strategic consumers. The airline uses dynamically-adjusted prices to sell seats to a random number of arriving consumers each period prior to departure. Consumers choose between premium, economy, and no purchase, and economy purchases include the option to secure an upgrade by bidding in an auction or purchasing at check-in for a fixed fee if premium seats remain unsold. Beliefs regarding the possibility of an upgrade are consistent with consumer and airline behavior.

The airline that supplied the data has a pricing team that sets prices for the initial allocation using a revenue management algorithm and an upgrade team that relies on a separate algorithm to design the upgrade mechanisms for a reallocation. Consistent with Hortaçsu et al. [2023], each team does not internalize how their behavior influences the other when making decisions. Therefore, we assume that the pricing team does not consider the upgrade mechanisms, nor any strategic behavior by consumers induced by the upgrade mechanisms, when making pricing decisions. Similarly, we assume prices are not influenced by submitted bids, and the upgrade fee is set in period $t = 1$ and does not change as demand is realized. These modeling assumptions are consistent with the setting and our data.

The pricing part of our model resembles Aryal et al. [2023], but the addition of the upgrade mechanisms substantially increases the complexity. Because consumers are forward-looking and strategically select into cabins, they must have beliefs about the probability of being upgraded when making purchasing decisions. As beliefs affect choices, the evolution of remaining capacities and the upgrade beliefs affect one another. This means that a solution to our model requires solving state-dependent upgrade beliefs in a dynamic, non-stationary environment that form a fixed point between consumer and airline behavior.

3.1 Environment

Time is discrete and indexed with $t \in \mathcal{T} = \{1, ..., T, T+1\}$. Tickets are first made available in period $t = 1$, and the airline is able to sell tickets through period $t = T$. Consumers arrive to the airport in period $T + 1$ to check-in, and the plane departs at the end of this final period. The airline is endowed with an initial flight capacity $\boldsymbol{k}_1 = (k_1^f, k_1^e)$, where k_1^f and k_1^e represent the total number of premium and economy seats, respectively, on the flight. We denote $\boldsymbol{k}_t$ to be the remaining capacities at period t. The relevant state vector in the model is therefore $\boldsymbol{k}_t = (k^f, k^e, t) \in \{0, 1, ..., k_1^f\} \times \{0, 1, ..., k_1^e\} \times \{1, ..., T, T+1\}$.

In each period $t < T + 1$, $N_t \sim \text{Poisson}(\lambda_t)$ consumers arrive and choose between the premium cabin, economy cabin, and outside option, which is represented by the choice set $\{f, e, o\}$. Consumers must make a decision in the period they arrive, which means they cannot strategically delay purchase. Consistent with the airline pricing literature [e.g. Lazarev, 2013, Williams, 2022b, Aryal et al., 2023], each consumer receives a type $\omega \in \{B, L\}$ with probabilities γ_t and $1 - \gamma_t$, respectively. ω represents a consumer's reason for travel:

business (B) or leisure (L).[10]

The airline has two upgrade mechanisms for moving passengers from the economy cabin to the premium cabin: an auction mechanism and a fixed fee mechanism. Consumers are aware of each mechanism and are completely informed about how they work. The fixed fee mechanism allows economy passengers to move to the premium cabin by paying a fee r as long as premium seats remain when they arrive to the airport and check-in.[11] The auction mechanism allows economy passengers the opportunity to submit a bid b immediately following purchase. Bids are from a discrete bid space $\boldsymbol{B} = \{b^1, ..., b^J\}$ with $b^1 > 0$. Consumers do not have to participate.[12] Bids are collected in a portfolio $\boldsymbol{\mathscr{b}}_t = (\mathscr{b}^1, ..., \mathscr{b}^j)$ which summarizes the total number of bids submitted of each type from periods 1 to t. A first price auction is held at the very beginning of period $\tilde{t}$ before the airline sets prices, and the airline sequentially upgrades passengers by choosing the most desirable bids in $\boldsymbol{\mathscr{b}}_{\tilde{t}}$. Consumers who arrive after $\tilde{t}$ cannot submit bids.

3.2 Consumer Demand

Demand is modeled using a pure characteristics approach [e.g. Mussa and Rosen, 1978, Berry and Pakes, 2007] with two vertically differentiated cabins and an outside option normalized to zero [e.g. Bresnahan, 1987, Berry, 1994]. Preferences over the three choices are parameterized as $u^f(p) = \nu\xi - p$, $u^e(p) = \nu - p$, and $u^0(p) = 0$. This implies the quality of the economy cabin is normalized to one ($\xi^e = 1$). Preferences are distributed $(\nu, \xi) \sim F_\nu^\omega \times F_\xi^\omega$ where $\nu \geq 0$, $\xi \geq 1$, and $\nu \perp\!\!\!\perp \xi$. We use i subscripts to indicate that (ν_i, ξ_i) is a realization for consumer i of the random variable (ν_i, ξ_i). This demand structure implies that all consumers agree f is higher quality than e; however, they differ in their willingness-to-pay for the quality. We assume that business travelers have a higher willingness-to-pay for both travel and quality by requiring that $F_\nu^B \geq F_\nu^L$ and $F_\xi^B \geq F_\xi^L$ in first order stochastic dominance.

Consumers have state dependent beliefs for the probability of an upgrade from each bid value as well as being able to purchase an upgrade at check-in which are denoted $\boldsymbol{\varrho}_t(\boldsymbol{k}) = (\varrho_t^1(\boldsymbol{k}), ..., \varrho_t^J(\boldsymbol{k}))$ and $\varphi_t(\boldsymbol{k})$ respectively. These beliefs are assumed rational in that they are consistent with consumer and airline behavior, and beliefs are updated via Bayes' Rule where possible. Define $\boldsymbol{b}_{\tilde{t}}^a$ to be the set of bids accepted by the airline. Then for a bid of type

[10] We label types "business" and "leisure" to be consistent with the literature. However, we use information on loyalty status to estimate γ_t as we do not have information on business/leisure markets. Ultimately, the labels do not matter and one should interpret consumers with $\omega = B$ as having a larger valuation distribution than consumers with $\omega = L$.

[11] While the airline could set this optimally by maximizing $\mathbb{E}[V_1(\boldsymbol{k}_1)|r]$, we do not take a stance on how the airline sets r and rely on the observed values of r in the data.

[12] Not bidding can be interpreted as submitted a bid of zero or $b^0 \equiv 0$.

$b^j \in \boldsymbol{B}$, $\varrho_t^j(\boldsymbol{k}) = \Pr(b^j \in \boldsymbol{b}_t^a | \boldsymbol{k}_t)$. The conditioning on state $\boldsymbol{k}_t$ is from consumers arriving at time t and observe remaining capacities $\boldsymbol{k}$. Similarly, let $T_i^a \in [T+1, T+2)$ be i's check-in time, which is independent across all passengers and unknown to i at the time of purchase. T_i^a determines the order in which consumers check-in for their flight. If $k^f_{T_i^a}$ is how many premium seats remain when i arrives to the airport, then $\varphi_t(\boldsymbol{k}) = \Pr(k^f_{T_i^a} > 0 | \boldsymbol{k}_t)$.

Within time period t, consumers arrive at slightly different times $t_i \in [t, t+1)$ and get to make decisions sequentially based on their continuous arrival time.[13] Consumers choose from $\{f, e, o\}$ by selecting the option with the highest expected utility, $\{\mathcal{U}_{it}^f, \mathcal{U}_{it}^e, \mathcal{U}_{it}^o\}$, that is still available at t_i. As the utility of f and o is certain for i, $\mathcal{U}_{it}^f = u_{it}^f$ and $\mathcal{U}_{it}^o = u_{it}^o$. However, consumers form $\mathcal{U}_{it}^e$ by expected over u_{it}^e and u_{it}^f using their beliefs.

Because the gross utility of the premium cabin is $\nu_i \xi_i$ and the gross utility of the economy cabin is ν_i, the gross utility of an upgrade is the difference $\nu_i \xi_i - \nu_i = \nu_i(\xi_i - 1)$. Consumer i will only purchase an upgrade at check-in if $\nu_i(\xi_i - 1) \geq r$. Because the utility $\nu_i - p_t^e$ is sunk after purchasing an economy ticket and the check-in system should only affect bidding if consumers are willing to pay the upgrade fee, consumer i's optimal bid, denoted b_{it}^*, solves,

$$b_{it}^* = \arg\max_{b^j \in \boldsymbol{B}} \varrho_t^j(\boldsymbol{k})(\nu_i(\xi_i - 1) - b^j) + \varphi_t(\boldsymbol{k})\big(1 - \varrho_t^j(\boldsymbol{k})\big) \max\big\{0, \nu_i(\xi_i - 1) - r\big\}. \quad (1)$$

Let $\varrho_{it}^*(\boldsymbol{k})$ be the belief corresponding to b_{it}^*. Each consumer will have an optimal bid whether they purchase economy or not, and this bid determines entry into the auction. Putting this all together, consumer i's expected utility of purchasing economy $\mathcal{U}_{it}^e$ is,

$$\mathcal{U}_{it}^e = \nu_i - p_t^e + \varrho_{it}^*(\boldsymbol{k})\big(\nu_i(\xi_i - 1) - b_{it}^*\big) + \varphi_t(\boldsymbol{k})\big(1 - \varrho_{it}^*(\boldsymbol{k})\big) \max\big\{0, \nu_i(\xi_i - 1) - r\big\}. \quad (2)$$

Consumers make optimal choice $d_{it}^* \in \{f, e, o\}$ by comparing u_{it}^f, $\mathcal{U}_{it}^e$, and u_{it}^o and selecting the alternative with the highest expected utility still available at purchase time t_i.

3.3 Pricing Team's Dynamic Program

Prices are set for each period by a pricing team who wants to maximize expected profits ignoring upgrades. Let $Q^f(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})$ and $Q^e(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})$ be the quantity demanded for the premium and economy cabins respectively given demand realizations $\boldsymbol{y} = (t_i, \nu_i, \xi_i)_{i=1}^n$, prices $\boldsymbol{p}$, and remaining seats $\boldsymbol{k}$ where n is a realization of N_t.[14] Define $R(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k}) =$

[13] One way to think of this is that consumer arrive via some (potentially non-homogeneous) Poisson process and all consumers who arrived in $[t, t+1)$ see prices $\boldsymbol{p}_t$ which were set by the airline at the beginning of the period. If $\lambda(t)$ is the arrival rate, then $N_t \sim \text{Poisson}(\lambda_t)$ where $\lambda_t = \int_t^{t+1} \lambda(s) ds$.

[14] Note that the quantity demanded for a cabin can never be larger than its remaining capacity due to the sequential purchasing process.

$p^f Q^f(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k}) + p^e Q^e(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})$ to be the revenue from demand realizations $\boldsymbol{y}$, prices $\boldsymbol{p}$, and remaining capacities $\boldsymbol{k}$. Then the expected revenue at t from prices $\boldsymbol{p}$ with remaining capacities $\boldsymbol{k}$ is $\mathbb{E}_t[R(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})] = p^f \mathbb{E}_t[Q^f(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})] + p^e \mathbb{E}_t[Q^e(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})]$ where the expectation is taken over demand realizations $\boldsymbol{y}$.

The pricing team must set prices at the beginning of the period before consumers arrive and never observes ω_i, ν_i, nor ξ_i. Tickets sold in each cabin have a constant marginal "peanut" cost of servicing a passenger in the respective cabin: $\boldsymbol{c} = (c^f, c^e)$ with $c^e \leq c^f$.[15] Costs are realized at the end of period $T + 1$. Then the total cost function for a flight is $C(\boldsymbol{k}) = \boldsymbol{c} \cdot (\boldsymbol{k}_1 - \boldsymbol{k}) = c^f(k_1^f - k^f) + c^e(k_1^e - k^e)$. The airline's discount rate, δ, is set to 1.

Because we assume the pricing team ignores the upgrade mechanisms when setting prices, the terminal condition for unsold capacity at departure is $V_{T+1}(\boldsymbol{k}) = 0, \forall \boldsymbol{k} \in \mathcal{K}$. This sets up a dynamic program for the pricing team to solve to obtain a pricing policy. Using backwards induction, optimal prices in the last period solve

$$V_T(\boldsymbol{k}) = \max_{\boldsymbol{p} \in \mathbb{R}_+^2} \mathbb{E}_T[R(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})] - \int_{\boldsymbol{k}' \in \mathcal{K}} C(\boldsymbol{k}') dH_T(\boldsymbol{k}' | \boldsymbol{k}, \boldsymbol{p}), \tag{3}$$

and in all preceding periods solve

$$V_t(\boldsymbol{k}) = \max_{\boldsymbol{p} \in \mathbb{R}_+^2} \mathbb{E}_t[R(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k})] + \int_{\boldsymbol{k}' \in \mathcal{K}} V_{t+1}(\boldsymbol{k}') dH_t(\boldsymbol{k}' | \boldsymbol{k}, \boldsymbol{p}), \tag{4}$$

where $\mathcal{K}$ is the state space and $H_t(\boldsymbol{k}' | \boldsymbol{k}, \boldsymbol{p}) = \Pr(k^{f'} \leq x, k^{e'} \leq y | \boldsymbol{k}, \boldsymbol{p})$ is the distribution of next period's state conditional upon this period's state and prices. The remaining capacities for each cabin evolve with the laws of motion

$$k_{t+1}^f = k_t^f - Q^f(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k}) \quad \text{and} \quad k_{t+1}^e = k_t^e - Q^e(\boldsymbol{y}, \boldsymbol{p}, \boldsymbol{k}). \tag{5}$$

Define the solution to Equation 3 and 4 with the laws of motion defined by Equation 5 and terminal condition $V_{T+1}(\boldsymbol{k}) = 0, \forall \boldsymbol{k} \in \mathcal{K}$ to be the policy function $\boldsymbol{p}_t(\boldsymbol{k})$.

[15] These marginal costs are referred to as "peanut" costs as they represent the cost of serving a passenger in cabin m, which sometimes include complementary items such as snacks or sleep masks. The items given to premium passengers are often higher quality than those given to economy passengers which is reflected in the higher peanuts costs. One can imagine these costs might also arise from cleaning the seats if it takes longer to clean a premium seat. Ultimately, the source of these costs are outside the scope of this paper and simply reflect the fact that there are costs associated with servicing a passenger on a flight which are higher for premium passengers.

3.4 Upgrade Team's Choice of Auction Upgrades

At the beginning of the auction period, the upgrade team selects the most desirable offers from the collection of submitted bids. $\boldsymbol{b}_{\tilde{t}}$, and upgrades those passengers. Let $\boldsymbol{i^u} = (-1, 1)$ be the upgrade vector. The interpretation is that an upgrade changes the remaining capacity vector $\boldsymbol{k}$ by removing one premium seat (the negative 1) and adding one economy seat (the positive 1). Accepting n bids at $\tilde{t}$ means the upgrade team is changing the remaining capacity state for the flight from $\boldsymbol{k}_{\tilde{t}}$ to $\boldsymbol{k}_{\tilde{t}} + n\boldsymbol{i^u}$ by reallocating n passengers from the economy cabin to the premium cabin. Although the airline collects auction revenue and an additional economy seat to sell in the future with each auction upgrade, it also imposes an opportunity cost on the airline because the premium seat allocated with the auction can no longer be sold in the future through dynamic pricing and, if it remains unsold at $T + 1$, at check-in. We model the upgrade team's optimal number of auction upgrades by assuming that it continues to accept bids until the marginal revenue of the next bid accepted is less than the marginal opportunity cost of the upgrade.

To define the marginal opportunity cost of an upgrade, consider the value of holding $\boldsymbol{k} = (k^f, k^e)$ remaining seats in each cabin at the start of the period t. Each seat can be sold through prices in remaining periods $\{t, t+1, ..., T\}$ and the remaining premium seats can be allocated through check-in upgrades in period $T + 1$ for price r. As $V_t(\boldsymbol{k})$ is the value at t to the airline by selling seats $\boldsymbol{k}$ through dynamic pricing only, define $U_t(\boldsymbol{k})$ to be the value of $\boldsymbol{k}$ at t and selling them with check-in upgrades.[16] Because optimal prices are assumed to be set ignoring the upgrades, the total value of $\boldsymbol{k}$ at t, denoted $TV_t(\boldsymbol{k})$, is simply the sum of $V_t(\boldsymbol{k})$ and $U_t(\boldsymbol{k})$ i.e. $TV_t(\boldsymbol{k}) = V_t(\boldsymbol{k}) + U_t(\boldsymbol{k})$. Therefore, the opportunity cost of n upgrades through the auction is $TV_t(\boldsymbol{k} + n\boldsymbol{i^u}) - TV_t(\boldsymbol{k})$.[17] Define the marginal opportunity cost of the n^{th} upgrade, denoted $\Delta TV_t(n, \boldsymbol{k})$, to be

$$\Delta TV_t(n, \boldsymbol{k}) = \begin{cases} 0 & \text{if } n = 0 \\ TV_t(\boldsymbol{k} + n\boldsymbol{i^u}) - TV_t(\boldsymbol{k} + (n-1)\boldsymbol{i^u}) & \text{if } n \in \{1, 2, ..., k^f\} \\ \infty & \text{otherwise.} \end{cases} \tag{6}$$

If $\Delta TV_t(n, \boldsymbol{k})$ is increasing in n, then the upgrade team will continue accepting the highest bids at time $\tilde{t}$ until the marginal cost of an upgrade exceeds the marginal revenue, or

$$\Delta TV_{\tilde{t}}(n + 1, \boldsymbol{k}) > b_{\tilde{t}}^{(n+1)}, \tag{7}$$

[16] See Appendix B for the formal definition of $U_t(\boldsymbol{k})$,.

[17] Because costs are realized at the end of $T+1$, the change in the passenger's peanut cost from the upgrade is already taken into account through $\Delta V_t(n, \boldsymbol{k})$ as defined in Equation 3.

where $b^{(n)}$ is the n^{th} highest bid.[18] Let $n^u_{\tilde{t}}(\boldsymbol{k}, \boldsymbol{\mathscr{b}})$ be the smallest n that satisfies Equation 7 given remaining capacities $\boldsymbol{k}$ and bid portfolio $\boldsymbol{\mathscr{b}}$ i.e. $n^u_{\tilde{t}}(\boldsymbol{k}, \boldsymbol{\mathscr{b}})$ is the upgrade team's optimal number of auction upgrades given $\boldsymbol{k}$ and $\boldsymbol{\mathscr{b}}$ at $\tilde{t}$.

3.5 Timing of Entire Model

Putting this all together, the timing of the model is:

1. The airline is endowed with a fixed initial capacity vector $\boldsymbol{k}_1$ of premium and economy seats to sell over time and sets a fixed fee r for upgrades if any premium seats remain in period $T+1$.

2. For time periods $t \in \mathcal{T}_T$,

   i. If $t = \tilde{t}$, upgrade team selects $n^u_t(\boldsymbol{k}, \boldsymbol{\mathscr{b}})$ passengers to upgrade and resets capacity state to $\boldsymbol{k} + n^u_t(\boldsymbol{k}, \boldsymbol{\mathscr{b}})\boldsymbol{i^u}$.

   ii. Pricing team sets prices $\boldsymbol{p}_t(\boldsymbol{k})$ given state $\boldsymbol{k}_t$.

   iii. $N_t \sim \text{Poisson}(\lambda_t)$ consumers arrive in $[t, t+1)$ and consumer i has travel type $\omega_i \in \{B, L\}$ with probabilities γ_t and $1-\gamma_t$, respectively.

      a) Each consumer i with travel type ω_i realizes their preferences (ν_i, ξ_i) from distributions F^ω_ν and F^ω_ξ where $\omega = \omega_i$.

      b) Consumers form upgrade beliefs $\varphi_t(\boldsymbol{k})$, $\boldsymbol{\varrho}_t(\boldsymbol{k})$ and choose their most preferred option in $\{f, e, o\}$ sequentially based on their arrival time $t_i \in [t, t+1)$.

      c) If purchasing economy and $t < \tilde{t}$, consumers submits bid b^*_{it} where the win beliefs are calculated via Bayes' Rule, and the airline adds b^*_{it} to $\boldsymbol{\mathscr{b}}_t$.

3. At period $T+1$, consumers receive independent, random check-in times $T^a_i \in [T+1, T+2)$.

   i. If consumer i has an economy ticket, $k^f_{T^a_i} > 0$, and $\nu_i(\xi_i - 1) > r$, i pays the airline r to be upgraded to the premium cabin, and the airline updates $\boldsymbol{k}$ to $\boldsymbol{k} + \boldsymbol{i^u}$.

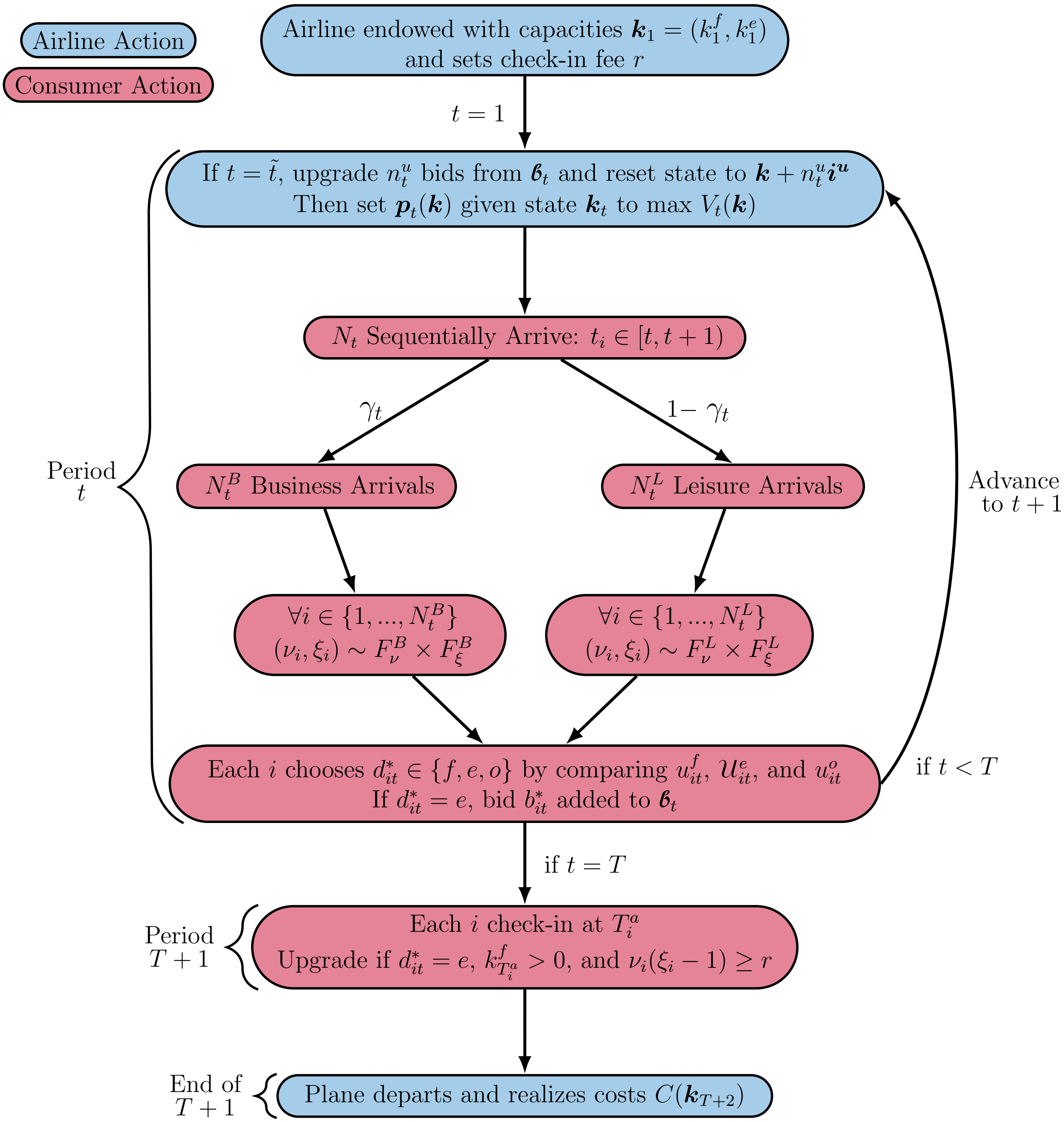
4. After all consumers arrive to the airport and check-in, the plane departs with final state $\boldsymbol{k}_{T+2}$ and realizes costs $C(\boldsymbol{k}_{T+2})$.[19]

In the next section discusses how we solve and estimate this model.

[18] If $\Delta TV_t(n, \boldsymbol{k})$ is not increasing in n, the optimal number of auction upgrades is the number that maximizes the total auction revenue minus the total opportunity cost. See Appendix B for a formal treatment.

[19] $T+2$ is used for the remaining capacity vector at departure to emphasize that it is the remaining capacity after all check-in upgrades since check-in times can be anywhere in $[T+1, T+2)$.

Figure 6: Diagram of Timing for Entire Model



Notes: Flow chart of the entire model.

4 Model Solution and Estimation

Our approach to estimation seeks to limit the computational burden associated with repeatedly solving the model while allowing for a flexible distribution of heterogeneity across markets and between flights within a market. In this section, we first discuss the parameterization of the model and our process for solving the airline's pricing problem and equilibrium beliefs for a given value of the parameter vector. Next, we discuss our adaptation of the moment-based methodology from Fox et al. [2016] and Nevo et al. [2016] to recover consumers' preferences for each market.

4.1 Model Parameterization

The primitives of our model are $\boldsymbol{\psi} = \big((F_\nu^L, F_\nu^B, F_\xi^L, F_\xi^B), (\boldsymbol{k}_1, \boldsymbol{c}), (\tilde{t}, \boldsymbol{B}, r), (\lambda_t, \gamma_t)_{t=1}^T\big)$ where parameters are grouped into sets for the preference distributions, airline capacity and cost parameters, upgrade parameters, and arrival process. The distributions (F_ν^L, F_ν^B) are assumed to be Exponential truncated at the 95$^{\text{th}}$ percentile with means (μ_ν^L, μ_ν^B) where $\mu_\nu^L \leq \mu_\nu^B$.[20] Similarly, the distributions (F_ξ^L, F_ξ^B) are assumed to be one added to an Exponential random variable truncated at the 95$^{\text{th}}$ percentile with means (μ_ξ^L, μ_ξ^B) where $\mu_\xi^L \leq \mu_\xi^B$. To ensure that $\mu_\nu^L \leq \mu_\nu^B$ and $\mu_\xi^L \leq \mu_\xi^B$, we let $\mu_\nu^B = \mu_\nu^L(1+\delta_\nu^B)$ and $\mu_\xi^B = \mu_\xi^L + \delta_\xi^B$ where $\delta_\nu^B \geq 0$, $\delta_\xi^B \geq 0$ and estimate $(\delta_\nu^B, \delta_\xi^B)$ rather than (μ_ν^B, μ_ξ^B). Lastly, we set $\mu_\nu^L = 1 + \delta_\nu^L$ and estimate δ_ν^L to impose that all parameters have the same lower bound i.e. $\delta_\nu^L \geq 0$.

The initial capacities $\boldsymbol{k}_1$ are observed for every flight in the inventory data. The peanut costs are set to be $c^f = 10$ and $c^e = 5$. We take the check-in fee r as given as it is observed in the data. For the bid space, $b^1 \in \boldsymbol{B}$ is set to be 0.8 of r and $b^J \in \boldsymbol{B}$ to be 1.75 of r. These multipliers were calibrated from the data.[21] The remaining points in $\boldsymbol{B}$ are set by forming an equally spaced grid of J points from b^1 to b^J where $J = 15$. Tickets are sold starting 330 days before departure. To reduce the computational burden, we group this time horizon into 15 time periods that each begin at the following number of days before departure: $\{330, 180, 150, 120, 90, 60, 45, 40, 35, 30, 25, 20, 15, 10, 5\}$. That is, the interval $[181, 331)$ represents $t = 1$ and the interval $[0, 6)$ represents $t = T = 15$. We set $\tilde{t} = 15$ which means that the auction is run at the very beginning of period 15 before the airline sets prices and consumers arrive. The search data is used to directly estimate the

[20] Truncation is to prevent "fishing" by the airline when solving prices with simulations. Without it, the exponential distribution can result in a small number of simulations with extraordinarily high utility draws. The airline will cater prices to these outlier consumers even though they are actually very rare.

[21] While there is some variation in the data for the check-in fee within a market, the variation is minimal and we take the modal observed value of the check-in fee for that market. See Appendix D.1 for descriptive evidence that motivates why these multipliers were chosen.

arrival process $\boldsymbol{\Lambda} = (\lambda_t, \gamma_t)_{t=1}^T$ for each market, which we treat as an observed parameter during the rest of estimation.

For this parameterization of the model, a market can be described by an arrival process $\boldsymbol{\Lambda}_m$ and parameter vector $\boldsymbol{\psi} = (\mu_\nu^L, \delta_\nu^B, \delta_\xi^L, \delta_\xi^B) \in [\mathbf{0}, \overline{\boldsymbol{\Psi}}] \subseteq \mathbb{R}_+^4$. We can solve the model as described in Section 4.2 for any given $\boldsymbol{\Lambda}_m$ and $\boldsymbol{\psi}$. This results in the endogenous objects $\boldsymbol{p}_t(\boldsymbol{k})$, $\boldsymbol{\varrho}_t(\boldsymbol{k})$, and $\varphi_t(\boldsymbol{k})$ that we can use to obtain simulated moments from the model to use in estimation.

4.2 Solving the Model

A solution to our model includes the following endogenous objects: a pricing policy $\boldsymbol{p}_t(\boldsymbol{k})$ that solves the airline's dynamic program from Section 3.3 and equilibrium beliefs $\boldsymbol{\varrho}_t(\boldsymbol{k})$ and $\varphi_t(\boldsymbol{k})$. To understand our equilibrium notion, it is important to remember that we are modeling the airline as having two separate teams: a pricing team and an upgrade team. The pricing team sets prices as if there will never be any upgrades and obtains a pricing policy $\boldsymbol{p}_t(\boldsymbol{k})$. The upgrade team then takes $\boldsymbol{p}_t(\boldsymbol{k})$ as given and decides who to upgrade. So consumers are playing a game against each other and the upgrade team. The equilibrium notion we use for the upgrade game is Bayesian Nash Equilibrium, which means equilibrium beliefs must form a fixed point for behavior from consumers and the upgrade team.

Because it is very tedious to derive the exact distribution of the number of seats demanded in each cabin conditional on $(\boldsymbol{p}, \boldsymbol{k}_t)$, and it is very difficult, if not impossible, to solve the equilibrium beliefs analytically, we use simulation methods to solve for the endogenous objects.[22] For a parameterization of the model, solving $\boldsymbol{p}_t(\boldsymbol{k})$ is standard and can be done using backwards induction of the airline's dynamic program, and $\boldsymbol{p}_t(\boldsymbol{k})$ is held fixed while solving equilibrium beliefs.

We use simulation methods to solve for the endogenous objects. For a parameterization of the model, solving $\boldsymbol{p}_t(\boldsymbol{k})$ can be done with backwards induction of the pricing dynamic program. To reduce the computational burden, we solve the dynamic program for a subset of the states in the state-space and interpolate the value and policy functions. The points where we explicitly solve the dynamic program at all combinations of the following sets of remaining premium and economy seats, respectively: $\{0, 1, 2, 3, 6, 9, ..., k_1^f\}$ and $\{0, 1, 2, 3, 6, 9, 12, 22, 32, ..., k_1^e\}$. Once $\boldsymbol{p}_t(\boldsymbol{k})$ is solved for, it is held fixed while solving equilibrium beliefs.

For the equilibrium beliefs, we use an iterative procedure to find $\boldsymbol{\varrho}_t(\boldsymbol{k})$ and $\varphi_t(\boldsymbol{k})$ that form a fixed point between consumer and airline behavior in the model. To do this, we initialize

[22] Quantity demanded in each cabin should follow a generalized Poisson distribution, but the truncation from either cabin selling out makes deriving the exact distribution very tedious.

$\boldsymbol{\varrho}_t(\boldsymbol{k})$ and $\varphi_t(\boldsymbol{k})$ with an initial guess and forward simulate the model. One simulation of the model is an entire path of realized demands in every time period, including submitted bids and check-in upgrades. We use these simulated paths to update the current guess of beliefs until convergence.

To understand our procedure for solving equilibrium bidding beliefs, first note that the belief for bid type j, $\varrho_t^j(\boldsymbol{k}) = \Pr(b^j \in \boldsymbol{b}_{\tilde{t}}^a|\boldsymbol{k}_t)$, can be decomposed using the Law of Total Probability by expected over all possible auction states $\boldsymbol{k}_{\tilde{t}}$ and $\boldsymbol{\mathscr{b}}_{\tilde{t}}$,

$$\varrho_t^j(\boldsymbol{k}) = \Pr(b^j \in \boldsymbol{b}_{\tilde{t}}^a|\boldsymbol{k}_t) = \sum_{(\boldsymbol{k}_{\tilde{t}},\, \boldsymbol{\mathscr{b}}_{\tilde{t}}) \in \mathcal{K}\times\mathcal{B}} \Pr(b^j \in \boldsymbol{b}_{\tilde{t}}^a|\boldsymbol{k}_t, \boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}}) \Pr(\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}}|\boldsymbol{k}_t) \tag{8}$$

$$= \sum_{(\boldsymbol{k}_{\tilde{t}},\, \boldsymbol{\mathscr{b}}_{\tilde{t}}) \in \mathcal{K}\times\mathcal{B}} \Pr(b^j \in \boldsymbol{b}_{\tilde{t}}^a|\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}}) \Pr(\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}}|\boldsymbol{k}_t). \tag{9}$$

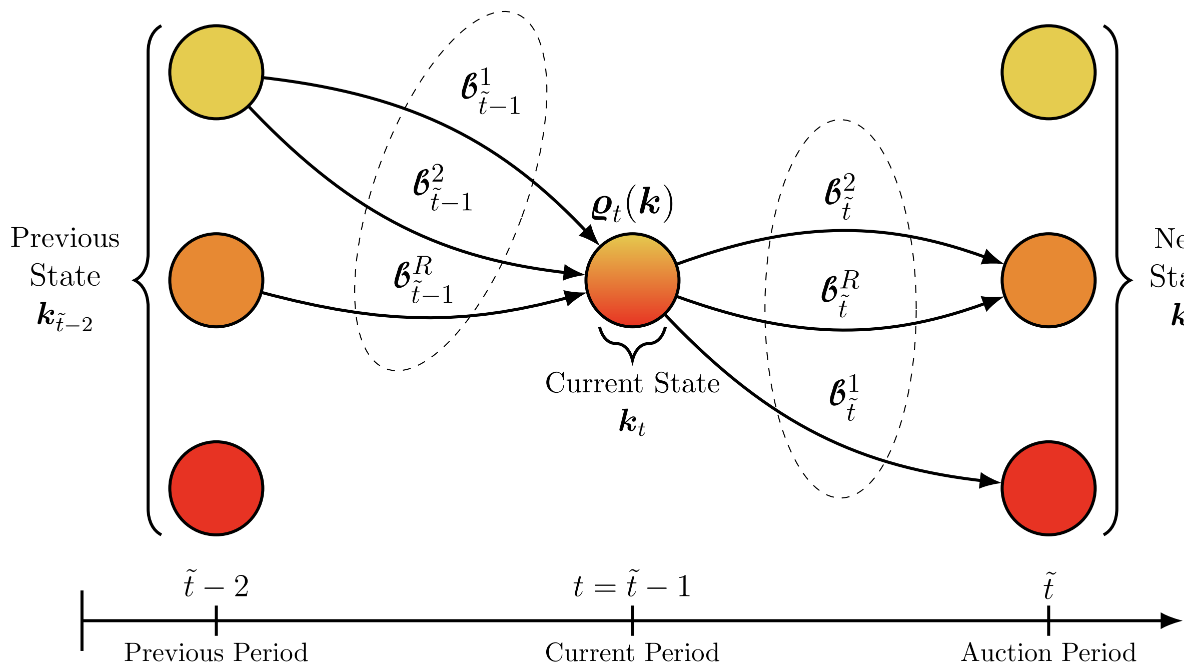
The step between 8 to 9 is from the fact that the optimal bids for the airline to accept are not affected by the path for how the airline found itself in state $\boldsymbol{k}_{\tilde{t}}$ with bid portfolio $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ before the auction. That is, only $\boldsymbol{k}_{\tilde{t}}$ and $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ affect the airline's upgrade decisions.

$\Pr(b^j \in \boldsymbol{b}_{\tilde{t}}^a|\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}})$ is the probability the airline upgrades a bid of size j given a realized capacity state $\boldsymbol{k}_{\tilde{t}}$ and bid portfolio $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ at the auction. Barring pivotal ties which are broken randomly, this probability will be either 0 or 1 as the upgrade process is deterministic once $\boldsymbol{k}_{\tilde{t}}$ and $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ are fixed. If there are more bids of type j than the airline is willing to accept, the probability will be the number of bids of type j the airline accepts divided by the total number of bids of type j. $\Pr(\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}}|\boldsymbol{k}_t)$ is the probability of observing state $\boldsymbol{k}_{\tilde{t}}$ and bid portfolio $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ when the auction occurs given that a consumer arrived in state $\boldsymbol{k}_t$. Put simply, it is the joint distribution of bid portfolios and remaining capacities at the time of the auction given arriving in state $\boldsymbol{k}_t$.

To better understand Equation 9, consider how $\boldsymbol{k}_{\tilde{t}}$ and $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ affect the probability of an upgrade from the auction. $\boldsymbol{k}_{\tilde{t}}$ determines the marginal upgrade opportunity cost curve $\Delta TV_{\tilde{t}}(n, \boldsymbol{k})$ from Equation 6 which the airline uses to select which bids to upgrade. If there are relatively more premium seats remaining than expected, the marginal opportunity cost of an upgrade will be lower and the airline is more likely to accept smaller bids. However, smaller bids may be rejected if $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ contains larger bids. That is, $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ affects the competitiveness of the auction. Figure 7 shows how both $\boldsymbol{k}$ and $\boldsymbol{\mathscr{b}}$ evolve over time and influence the beliefs of a consumer in state $\boldsymbol{k}_t$. Consumers arriving in period $t = \tilde{t} - 1$ (the period before the auction) observe remaining capacities $\boldsymbol{k}$ but do not observe $\boldsymbol{\mathscr{b}}$. Because consumers do not observe $\boldsymbol{\mathscr{b}}$, they must consider the distribution of $\boldsymbol{\mathscr{b}}_t$ conditional on observing the state $\boldsymbol{k}_t$. Furthermore, consumers must consider the purchase decisions and submitted bids in period $\tilde{t} - 1$ because they affect how $\boldsymbol{k}$ and $\boldsymbol{\mathscr{b}}$ evolve into the next period when auction occurs.

Therefore, in order for consumers to form $\boldsymbol{\varrho}_t(\boldsymbol{k})$, they must integrate over the distribution of $\boldsymbol{\mathscr{b}}_t$ as well as the distributions of $\boldsymbol{k}_{\tilde{t}}$ and $\boldsymbol{\mathscr{b}}_{\tilde{t}}$ conditional on observing state $\boldsymbol{k}_t$

Figure 7: Visualizing of Simulated Demand Paths



Notes: Nodes represent remaining capacity states where the more red nodes have less remaining capacity, and arrows represent possible demand paths with a corresponding bid portfolio $\boldsymbol{\mathscr{b}}$ that is not observed by consumers. The color gradient of the node for state $\boldsymbol{k}_t$ represents a general state. The superscripts on the bid portfolios are indices for possible paths of realized demand e.g. the bid portfolio at time t along path r is $\boldsymbol{\mathscr{b}}^r_t$. The dotted ellipses indicate consumers' information sets i.e. they observe $\boldsymbol{k}$ but not $\boldsymbol{\mathscr{b}}$.

We use simulations to approximate each probability in Equation 9. $\Pr(\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}}|\boldsymbol{k}_t)$ can be easily approximated using the frequency of simulated outcomes. $\Pr(b^j \in \boldsymbol{b}^a_{\tilde{t}}|\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}})$ is a bit trickier. For bid types placed within a simulation, the corresponding conditional probability is simply the probability that bid type is upgraded in a given simulation. However, not every bid type is guaranteed to be placed within a simulation. We compute $\Pr(b^j \in \boldsymbol{b}^a_{\tilde{t}}|\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}})$ for bid types not placed within a simulation by using the airline's value function and allowing bidders to decrease their bids as long doing so does not risk altering the allocation. The details of this procedure are outlined in Appendix C.1. Even though $\Pr(b^j \in \boldsymbol{b}^a_{\tilde{t}}|\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}})$ and $\Pr(\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathscr{b}}_{\tilde{t}}|\boldsymbol{k}_t)$ are equilibrium objects that depend upon current beliefs, this procedure guides consumers towards their optimal bids even when the current beliefs are far from their equilibrium values.

The process for updating the check-in beliefs $\varphi_t(\boldsymbol{k})$ is similar but easier than that of $\boldsymbol{\varrho}_t(\boldsymbol{k})$. We also use a frequency simulator to compute $\Pr(k^f_{T^a_i} > 0|\boldsymbol{k}_t)$. For each simulated path through $\boldsymbol{k}_t$, divide the number of remaining premium seats at the beginning of period

$T+1$ by the number of the economy passengers willing to pay that upgrade fee to get the check-in probability for that simulation. Because airport arrival times T_i^a are independent across passengers, the probability of receiving an upgrade conditional on $\boldsymbol{k}_t$ is simply the number of seats remaining at the beginning of periods $T+1$ divided by the number of consumers willing to pay the check-in fee. Consumers awarded an upgrade through the auction are removed from the count if they were willing to pay the check-in fee. The average of check-in probabilities across all simulations through $\boldsymbol{k}_t$ is the updated value of $\varphi_t(\boldsymbol{k})$.

Given the procedures outlined above, we solve for equilibrium beliefs by finding a fixed point between consumer and airline behavior using the following steps.

1. Initialize $\varphi_t^0(\boldsymbol{k})$ and $\boldsymbol{\varrho}_t^0(\boldsymbol{k})$.

2. Given $\varphi_t^0(\boldsymbol{k})$, $\boldsymbol{\varrho}_t^0(\boldsymbol{k})$, and the fixed pricing policy $\boldsymbol{p}_t(\boldsymbol{k})$, simulate R demand paths.

3. $\forall\, \boldsymbol{k}_t \in \mathcal{K} \times \mathcal{T}_T$, compute $\Pr(b^j \in \boldsymbol{b}_{\tilde{t}}^a | \boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathcal{b}}_{\tilde{t}})$, $\Pr(\boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathcal{b}}_{\tilde{t}} | \boldsymbol{k}_t)$, and $\Pr(k^f_{T_i^a} > 0 | \boldsymbol{k}_t)$ using all simulated paths through $\boldsymbol{k}_t$ and update beliefs to $\varphi_t^1(\boldsymbol{k})$ and $\boldsymbol{\varrho}_t^1(\boldsymbol{k})$.

   (a) If no paths go through $\boldsymbol{k}_t$, beliefs are not updated.

   (b) If $t > \tilde{t}$, $\boldsymbol{\varrho}_t^1(\boldsymbol{k}) = \mathbf{0}$ because the auction has already been run.

4. Continue alternating between steps 2 and 3 while updating beliefs from $\varphi_t^l(\boldsymbol{k})$ to $\varphi_t^{l+1}(\boldsymbol{k})$ and $\boldsymbol{\varrho}_t^l(\boldsymbol{k})$ to $\boldsymbol{\varrho}_t^{l+1}(\boldsymbol{k})$ until convergence.

The details of each step and our implementation can be found in Appendix C.2.

4.3 Estimation

Our estimation procedure balances computational burden with capturing flight heterogeneity within and across markets. If the market arrival process $\boldsymbol{\Lambda}_m$ is known, any given parameter vector $\boldsymbol{\psi} = (\mu_\nu^L, \delta_\nu^B, \delta_\xi^L, \delta_\xi^B)$ fully parameterizes the model and would represent a single type of flight in a market. However, our market definition allows for many types of flights in the same market e.g. flights departing on Fridays or in December may be higher average willingness-to-pay flights than those departing on Tuesdays or in early October. Our estimation procedure must allow for this heterogeneity in order to properly capture the uncertainty faced by the airline within each market.

One approach would be to identify flight heterogeneity through functional form assumptions about the demand primitives in $\boldsymbol{\psi}$ and estimate the parameters in the functional form. This requires assumptions about the structure of heterogeneity within and across markets, which is difficult to do *a priori* without adding too many parameters. An extremum estimator could be used to estimate the parameters added by the functional forms. However, this would present further issues because solving our model is very computationally intensive. Instead, we exploit the richness of our data and flexibly estimate the distribution of flights within a market.

To do this, we use a random coefficient model where each flight in a market receives demand primitives $\boldsymbol{\psi}$ drawn from a discrete, market-specific distribution G_m defined by a vector of probability weights $\boldsymbol{\theta}_m$ so that $G_m(\boldsymbol{\psi}) = G(\boldsymbol{\psi}; \boldsymbol{\theta}_m)$. Because our counterfactuals of interest only require identification of the distribution of flights, we do not care why flights within or across markets have different underlying demand. As long as our approach to estimation captures flight heterogeneity, we can perform our counterfactuals using the entire distribution of flights. This also reduces the number of parameters to estimate, which allows us to use richer estimation methods.

We estimate the model using a flexible method of moments approach that combines methodologies from Ackerberg [2009], Fox et al. [2016], and Nevo et al. [2016] to estimate a discrete distribution of candidate types of flights. Estimation is performed in four steps market by market. The first is to estimate the market-specific arrival process $\hat{\boldsymbol{\Lambda}}_m = (\hat{\lambda}_{mt}, \hat{\gamma}_{mt})_{t=1}^{T}$, which is held fixed during the remaining three steps. The details of how this is done can be found in Appendix D.2. Next, we draw $H = 2,500$ candidate types for the parameter vector $(\mu_\nu^L, \delta_\nu^B, \delta_\xi^L, \delta_\xi^B)$ using Halton sets, and solve the model once for all candidate types. When creating the grid of candidate types, we set the lower bound and upper bound for μ_ν^L to be 0 and 350, and for δ_ξ^L, δ_ν^B, and δ_ξ^B, they were set to 0 and 1.5.[23] In the third step, we calculate moments from the data and the corresponding moments from the model for every solution, which can easily be done with simulation using the model solutions for each candidate type. The $N_g \times 1$ column vector of data moments is denoted $\hat{\boldsymbol{g}}_m^{\text{dat}}$ and the $N_g \times H$ matrix of model moments is denoted $\hat{\boldsymbol{g}}_m^{\text{mod}}$ where each column are the equivalent moments for a candidate type. The fourth and final step involves finding weights $\boldsymbol{\theta}$ that minimize the difference between the empirical and simulated moments. Formally, estimated weights $\hat{\boldsymbol{\theta}}_m$ satisfy

$$\begin{aligned} \hat{\boldsymbol{\theta}}_m \;&=\; \arg\min_{\boldsymbol{\theta}} \; \boldsymbol{g}_m(\boldsymbol{\theta})' \boldsymbol{g}_m(\boldsymbol{\theta}) \\ &\text{subject to } \sum_{h=1}^{H} \theta_h = 1 \;\text{ with }\; \theta_h \geq 0. \end{aligned} \tag{10}$$

The vector $\boldsymbol{g}_m(\boldsymbol{\theta}) = \hat{\boldsymbol{g}}_m^{\text{dat}} - \hat{\boldsymbol{g}}_m^{\text{mod}} \boldsymbol{\theta}$ is the difference between the moments from the data

[23] Appendix D.3 shows the resulting parameter grids. Figure D.2 shows the grid used for estimation by travel type and Figure D.3 shows the implied parameters by travel type when points in the estimation grid are transformed back into $(\mu_\nu^L, \mu_\nu^B, \mu_\xi^L, \mu_\xi^B)$.

in market m and the weighted simulated moments from the model for all H types. It is helpful to think of the minimand in Equation 10 as analogously to the minimand of the ordinary least squares estimator where $\hat{\boldsymbol{g}}_m^{\text{dat}}$ is the dependent variable $\boldsymbol{y}$, $\hat{\boldsymbol{g}}_m^{\text{mod}}$ is the matrix of independent variables $\boldsymbol{X}$, $\boldsymbol{\theta}$ is the vector of parameters to estimate $\boldsymbol{\beta}$, and $\boldsymbol{g}_m(\boldsymbol{\theta})$ is the residual function $\boldsymbol{\varepsilon}(\boldsymbol{\beta}) = \boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}$. The solution to Equation 10, $\hat{\boldsymbol{\theta}}_m$, is the estimated vector of probability weights which define a discrete distribution of random coefficients $G(\boldsymbol{\psi}; \hat{\boldsymbol{\theta}}_m)$ for types of flights in market m.

Lastly, it will often be useful to aggregate results across markets. We do by forming probabilities that a consumer arrives to purchase a seat for a flight in each market. Because the arrival process is exogenous, the probability of a market is simply the total number of expected arrivals across all flights in that market divided by the total number of expected arrivals across flights in all markets. If λ_{mt} is the expected number of arrivals in market m at time t, the probability of a market can be expressed as

$$\Pr(i \in m) = \frac{N_m^f \times \sum_{t=1}^{T} \lambda_{mt}}{\sum_{m'=1}^{M} N_{m'}^f \times \sum_{t=1}^{T} \lambda_{m't}}, \tag{11}$$

where $\Pr(i \in m)$ is the probability a consumer arrives to market m to purchase a flight and N_m^f is the number of flights in market m. The estimated market arrival process $\boldsymbol{\Lambda}_m$ and the number of flights in the data can be used to estimate the above probabilities.

4.4 Identification

The objects that need to be identified in this model are the arrival process $\boldsymbol{\Lambda}_m$ and the distribution of random coefficients, G_m, for flights in each market, $m \in \{1, ..., 30\}$. Identification of the arrival process requires a few assumptions because the search data covers a different time period than the rest of the data. For the arrival rates λ_t, the identifying assumption is that the booking rate (i.e. the probability a consumer purchases a ticket after arriving) remains the same in each period between the two different sample periods. Because we observe the number of daily bookings in the revenue management data, the arrival process is identified by taking the ratio of of the average number of booking for each period and market from the revenue management data and dividing by the average booking rate in the corresponding period and market from the search data. For the mixing probability γ_t, we assume it remains constant across the two sample periods. While this might seem like a strict assumption, because we estimate a market average probability, flight heterogeneity is averaged out and what remains is a mixing process that is likely influenced by the inherent features in that market which are unlikely to change. For example, it is unlikely that the popularity of various cities for business travel would change over the time horizon between the two samples. Similarly, it is known that all consumers typically purchase tickets for long-hull international flights further from departure. As long as high loyalty status consumers do not fundamentally change when they arrive to purchase tickets, our estimates are unlikely to be affected.

Identification of the distribution G_m comes down to uniquely identifying the probability weights $\boldsymbol{\theta}_m$. This requires choosing moments that can distinguish between different types of candidate flights. The columns of matrix $\hat{\boldsymbol{g}}_m^{\text{mod}}$ are the moments for each candidate type, and identification of the weights $\boldsymbol{\theta}_m$ requires only the familiar rank condition from OLS: $\text{rank}(\hat{\boldsymbol{g}}_m^{\text{mod}\prime}\hat{\boldsymbol{g}}_m^{\text{mod}}) = |\boldsymbol{\theta}_m| = H$. This means that the moments used for estimation must distinguish between the candidate parameter vectors so that the columns of $\hat{\boldsymbol{g}}_m^{\text{mod}}$ are linearly independent and $\hat{\boldsymbol{g}}_m^{\text{mod}\prime}\hat{\boldsymbol{g}}_m^{\text{mod}}$ is invertible.

Because the model is nonlinear, intuition for how parameters influence moments is not always clear as most moments are influenced by multiple parameters. We present the moments used for estimation below along with a discussion of which the moments are informative for which parameters.

1. The distributions of economy and premium fares for each period;
2. The joint distribution of economy and premium fares for each period;
3. The distributions of economy and premium load factors for each period;
4. The joint distribution of economy and premium load factors for each period;
5. The joint distribution of cabin prices and load factors for both cabins and each period;
6. The distribution of the minimum and maximum difference between premium and economy fares i.e. $\min_{t\in\{1,\dots,T\}}\{p_t^f - p_t^e\}$ and $\max_{t\in\{1,\dots,T\}}\{p_t^f - p_t^e\}$;
7. The distribution of the total dollar value of bids submitted for each period;
8. The distribution of the total dollar value of the bid portfolio in each time period;
9. The distribution of the number of check-in upgrades purchased;
10. The distribution of upgrade load factors i.e. the share of premium seats sold through an upgrade.

Moments (1) and (2) identify all four parameters: μ_ν^L, δ_ν^B, δ_ξ^L, and δ_ξ^B. Each parameter (as well as the arrival process) influences the optimal prices set by the airline. Higher parameter values for μ_ν^L and δ_ν^B should result in higher economy prices in each time period. Likewise, larger values of δ_ξ^L and δ_ξ^B should result in higher premium prices. Furthermore, (2) helps pin down the parameters for F_ν^ω relative to F_ξ^ω as the joint distribution is informative of the fare gap between the two cabins and pins down the parameters of valuation distribution relative to the quality distribution. Moments (3) and (4) also identify all four parameters using similar reasoning; however, they are particularly useful for separating the leisure and business parameters as business travelers are more likely to purchase premium tickets at full price rather than economy tickets. Moments (5) help identify the leisure parameters from the business parameters for similar reasons.

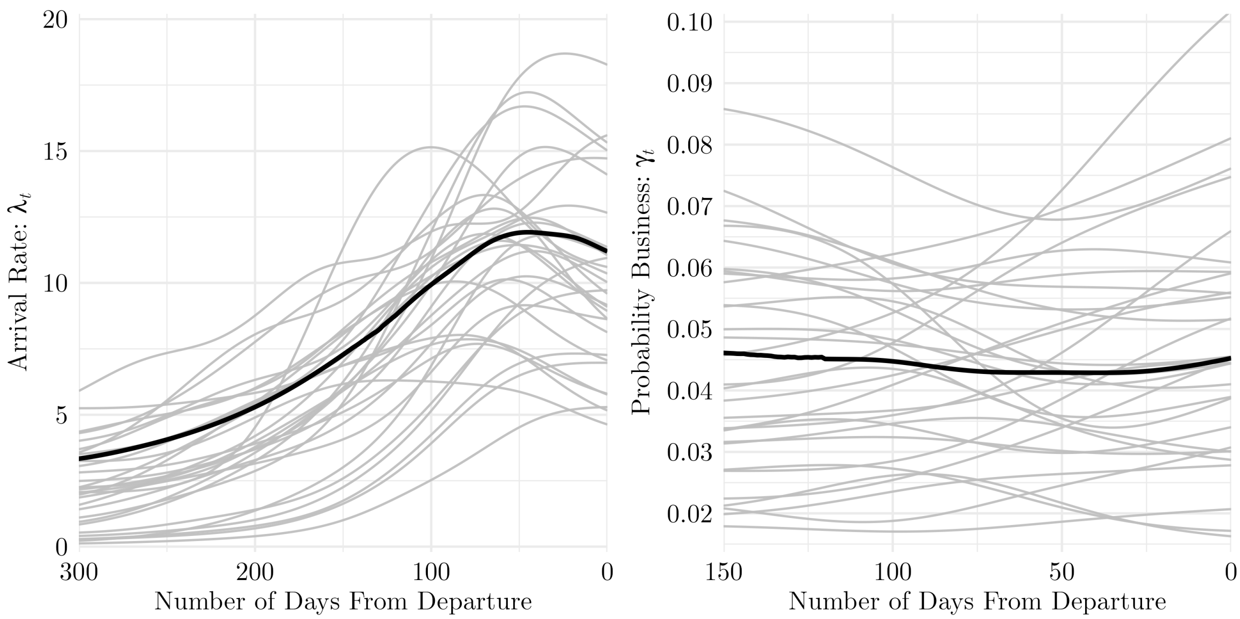
The moments in (6) are informative of μ_ξ^L and δ_ξ^B. For those who purchase premium, their gross utility of the upgrade must be larger than the price difference. Without upgrades, the maximum and minimum price differences across all periods bound the distribution of ξ. However, the presence upgrades introduces time-varying preferences which affects who buys premium outright. Because of this, moments about the upgrade process are needed to separate δ_ξ^L from δ_ξ^B. Moments (7), (8), and (9) help separately identify μ_ν^L from δ_ξ^L. Because business travelers are more likely to purchase full price premium tickets, upgrades are more likely to be purchased by leisure travelers. As the gross utility of an upgrade is $\nu_i(\xi_i - 1)$, the distribution of bids and check-in upgrades purchased helps identify both μ_ν^L and δ_ξ^L. While both parameters will affect the upgrade moments, larger values of δ_ξ^L will result in more and higher bids as well as more check-ins purchased. Whereas, higher values of μ_ν^L alone will result in higher prices all around.

To form moments from the distributions listed above, we create a grid of 100 equally spaced points between the 2nd and 98th percentiles in the data for each distribution. The grids vary by market and, when applicable, time period. For joint distributions of two variables, 10 equally spaced points for each variable are created in the same manner and all combinations of those 10 points forms a grid of 100 points. Each grid is held fixed during estimation and the quantile is calculated at each grid point in the data and equivalent simulated objects from the model. With the list above, 100 moments of 154 different distributions are used to identify the distribution of random coefficients in each market. This means that 15,400 moments are used to identify 2,500 weights.

5 Results

This section lays out the results of estimation and various counterfactuals. We first explore the estimated distribution of demand parameters and how they vary across markets. After that, we explore various counterfactuals aimed at the impact of the upgrade mechanisms on profit and consumer surplus as well as who the upgrades can be integrated with dynamic pricing. Reallocation can potentially alleviate inefficiencies that arise from misallocation due to intertemporal and intratemporal uncertainty. The estimated model allows us to calculate counterfactual profits, consumer surplus, and total welfare for changes to the design of the upgrade mechanisms and the allocation mechanism more generally. The counterfactual experiments we preform allow us to compare the split of welfare obtained under each to different welfare benchmarks.

Figure 8: Estimated Arrival Process by Market



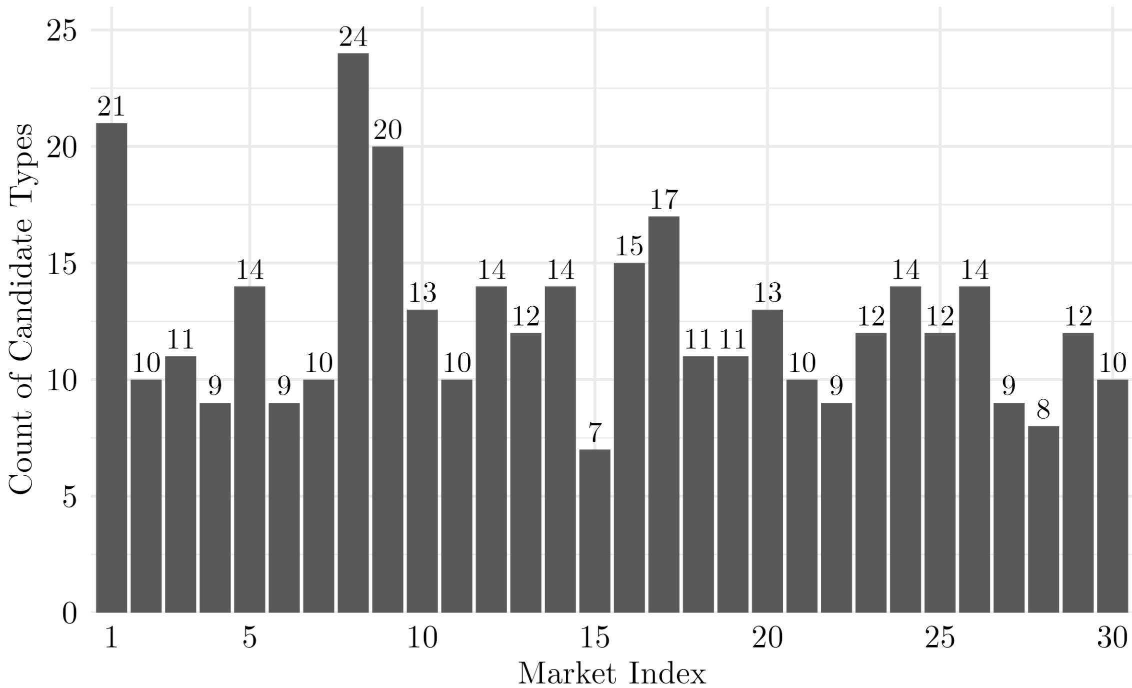
(a) Estimated Arrival Rates (b) Estimated Mixing Probabilities

Notes: The above plots are the estimated arrival processes at the daily level. When used for solving the model, the arrival rates and mixing probabilities are appropriately aggregated by the grouped time periods in the parameterization. Both the arrival rates and mixing probabilities are smoothed with a kernel regression. The mixing probabilities end at 150 days before departure because the search data does not cover the entire span that tickets were sold.

The estimated market arrival processes are shown in Figure 8. The shape and magnitude of the arrival rates differ across markets, with arrivals peaking between 50 to 100 days before departure. The mixing probabilities also differ substantially across market with some increasing approaching departure while others decrease. These differences allow us to separate changes in arrivals from changes in preferences that are unique to a market, which aids in the identification of the distribution of demand preferences for flights in each market.

Relatively few of the 2,500 candidate types are responsible 99.99% of the weight within each market. Figure 9 shows the number of types in each marketed that receive a weight of 0.01% or greater (i.e. the fewest number of types required to obtain 99.99% of the total

Figure 9: Number of Surviving Candidate Types by Market



Notes: The count of candidate types with the largest estimated weight which contribute to 99.99% of the probability mass. The market index is simply an index from 1 to 30 for each market used in estimation.

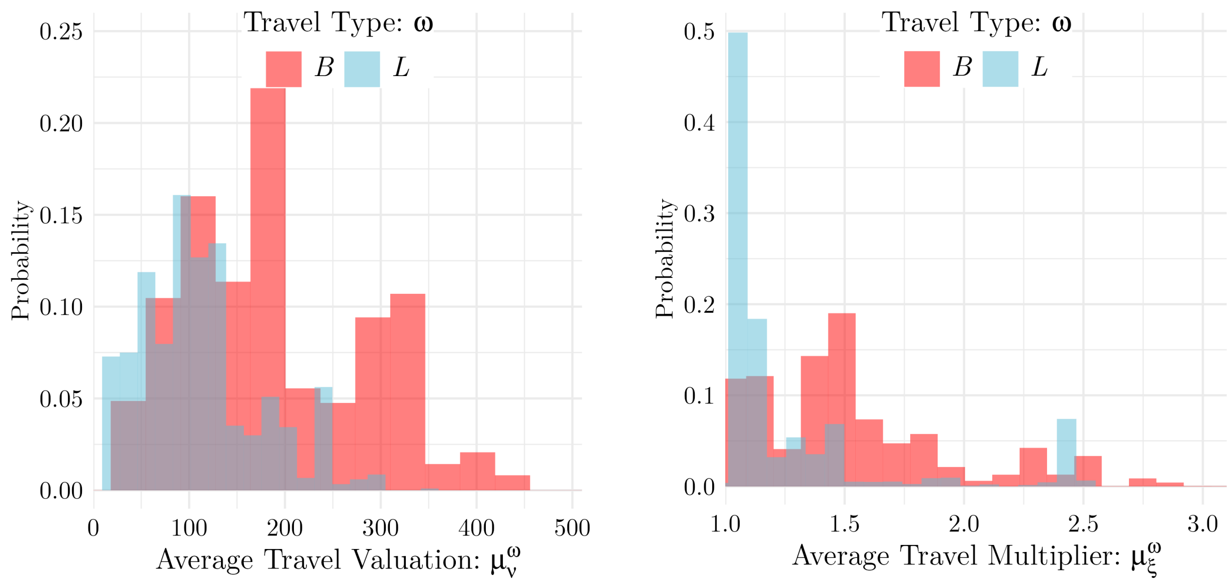
mass in a market). There are 375 total types across all markets that receive a weight of at least 0.01%. Markets 8 and 9 require the most types at 21 and market 15 requires the fewest at only 7.

5.1 Consumer and Market Heterogeneity

Figure 10 shows the estimated marginal distribution of consumer preference parameters for both leisure and business travelers. These parameters are interpreted as the mean willingness-to-pay for a flight (μ_ν^ω) and the mean quality multiplier for the premium cabin (μ_ξ^ω). The distributions are aggregated across markets by integrating the weights across markets using the market probabilities from Equation 11.

The modal mean flight valuation for leisure travelers is around \$100 with a mean of \$111.67 compared to a mode of around \$200 and mean of \$188.97 for business travelers. The average value of δ_ν^B is 0.774, which means that business travelers value flights 77.4% more than leisure travelers, on average. Furthermore, the distribution of means for leisure travelers is more concentrated around the mode, whereas the distribution for business travelers is more varied. The modal mean quality multiplier for leisure travelers is close to 1 and the mean is 1.265, and the probability mass quickly drops toward zero as the mean increases. However,

Figure 10: Estimated Marginal Distributions of Demand Parameters



(a) Mean Willingness-To-Pay for Flight

(b) Mean Quality Multiplier

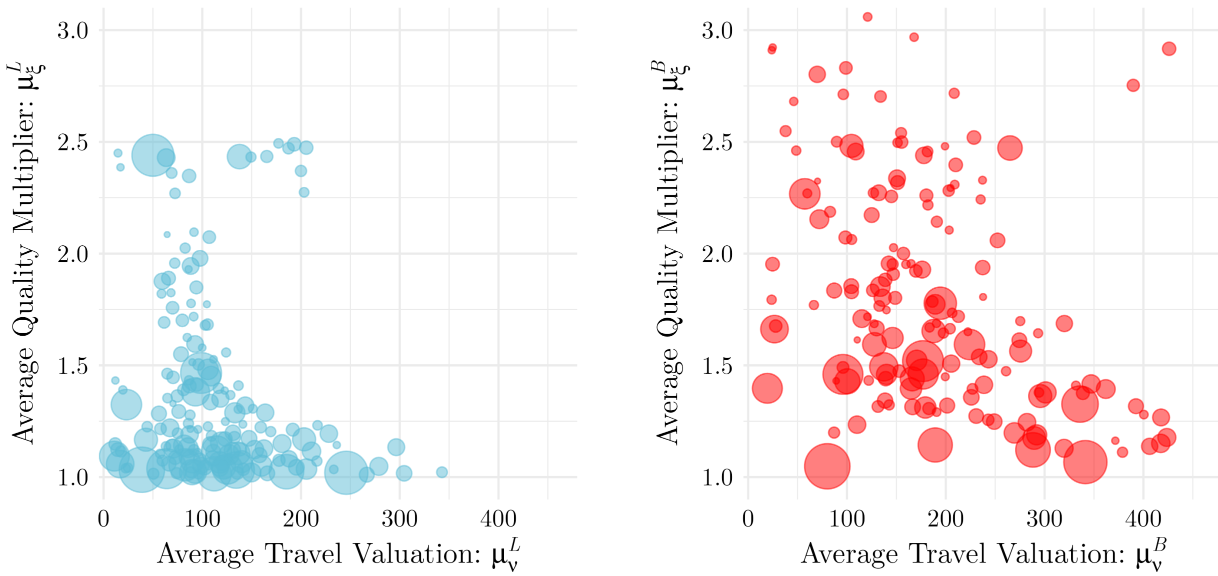
Notes: The probability of a candidate type is the total probability of that type across all markets. These are histograms where the probability of a type is used as a weight when creating the figures.

the modal mean quality multiplier for business travelers is round 1.5 with a mean of 1.654, and the probability mass is much more equally distributed around the mode. The mean value of δ_ξ^B is 0.389, meaning that business travelers value the premium cabin 38.9% more than leisure travelers. This suggests that most leisure travelers have very low willingness-to-pay for quality, and there is much more heterogeneity in the willingness-to-pay for quality among business travelers.

All together, business travelers have larger and more varied distributions for both preference parameters. While this is to be expected as we assume first-order stochastic dominance for the business traveler distributions by imposing $\mu_\nu^L \leq \mu_\nu^B$ and $\mu_\xi^L \leq \mu_\xi^B$ during estimation, the clear separation between both types in each distribution highlights the importance of capturing the mixture of higher and lower valuation consumers. This highlights the trade-off the airline faces: keeping enough premium seats to sell to high-value consumers while using upgrades to reallocate capacity and collect upgrade revenue if not enough premium seats have been purchased.

Figure 11 shows the joint distribution of the estimated parameters by leisure travelers and business travelers, respectively. These figures further highlight that the distribution of parameters for leisure travelers is smaller and much less varied than that of business travelers. There is a negative correlation between μ_ν^ω and μ_ξ^ω and the candidate types in the top right quadrant receive very little weight for both leisure and business travelers even though regions

Figure 11: Estimated Joint Distributions of Demand Parameters

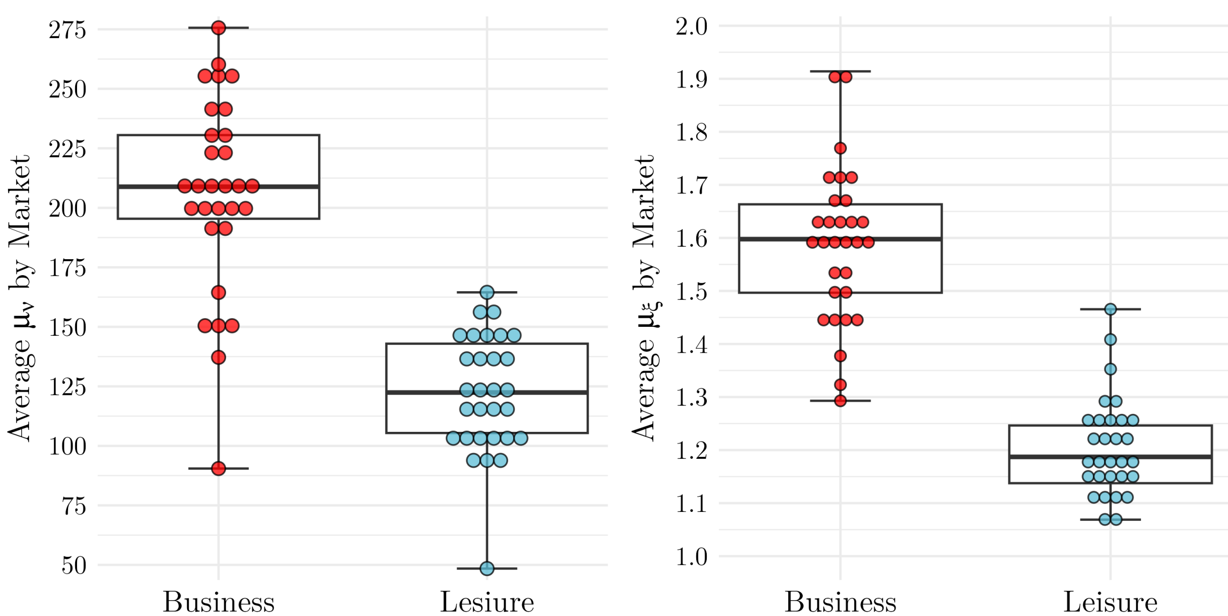


(a) Joint Distribution: Leisure

(b) Joint Distribution: Business

Notes: A random sample of size 1,000,000 is drawn from the estimated joint distribution for each travel type aggregated across markets. The points are scaled by the probability of that candidate type. The points are semi-transparent in order to observe the density of overlapping points in a region.

Figure 12: Heterogeneity in Average Parameters by Market



(a) Mean Willingness-To-Pay for Flight

(b) Mean Quality Multiplier

Notes: Each box plot is composed of thirty points where each point is the average parameter estimate for each of the thirty markets.

have plenty of candidate types in those regions (see Appendix D.3). There is also significant overlap in the scaled points meaning that near by points get relatively different weight.

This is likely due to market heterogeneity as different types may be more prevalent in some markets and not others, emphasizing the importance of performing estimation at the market level to capture this heterogeneity. This is supported by Figure 12, which shows box plots for the distribution of average parameter estimates within each market. There is substantial heterogeneity across markets, and the distribution of parameters for business travelers is more varied than that of leisure travelers for both types of parameters.

To better understand the overall demand curve, we calculate the matrix of price elasticities for all consumers as well as by travel type. For simplicity, we average the demand parameters and arrival processes across markets and solve the model for this average market. We then simulate the model and obtain an average price and capacity state per-period. Per-period elasticities are calculated by deviating the average cabin period price by 1% in each direction. These elasticities are then aggregated across period using the probability of arriving in each period. The resulting elasticities are in Table 3.

Table 3: Elasticities of the Average Market

	All Travelers		Leisure Travelers		Business Travelers	
	Premium	Economy	Premium	Economy	Premium	Economy
Premium	-3.005	1.202	-20.066	11.250	-2.772	1.073
Economy	1.133	-3.288	0.408	-4.341	1.483	-2.757
No Purchase	0.072	0.370	0.001	0.345	0.164	0.402
Market Share	0.063	0.116	0.002	0.077	0.125	0.154
Overall Elasticity	-1.486		-4.185		-1.060	

Notes: Each elasticity η_{jk} is read as the percent change in quantity demanded of j given a 1% increase in price k where j is the row and k is the column. The market share is the probability of purchase. To calculate the overall price elasticity demand for both premium and economy, define $s_{k|m} = \frac{s_k}{s_f + s_e}$ to be the market share of k conditional on being in the market, and $\varepsilon_{k|m} = \varepsilon_{kk} s_{k|m} + \varepsilon_{-kk} s_{-k|m}$ to be the price elasticity of demand for k of those in the market. The overall elasticity in the market ε^m can be calculated using $\varepsilon^m = s_{f|m}(\varepsilon_{f|m} - \varepsilon_{of}) + s_{e|m}(\varepsilon_{e|m} - \varepsilon_{oe})$, where ε_{ok} is the cross-price elasticity of the outside option from an increase in the price of k. ε_{ok} is subtracted off to account for consumers who would have bought k but left the market due to an increase in its price.

To better understand the overall demand curve, we calculate the matrix of price elasticities for all consumers as well as by travel type. For simplicity, we average the demand parameters and arrival processes across markets and solve the model for this average market. We then simulate the model and obtain an average transaction price for each cabin, and then calculate the price elasticities for each cabin assuming no capacity constraints and upgrades. The resulting elasticities are in Table 3.

The overall price elasticity of all consumers is -1.486 with the own price elasticities being -3.288 for the economy cabin and -3.005 for premium cabin. The overall price elasticity matches what is typically found in the literature The economy and premium price elasticities of leisure travelers are, respectively, -4.341 and -20.066 compared to the price elasticities of business travelers of -2.757 and -2.772. As business travelers have higher valuations for quality and are responsible for a disproportionate share of premium purchases, it is unsurprising that they are much less price sensitive to the price of premium when compared to leisure travelers who are extremely price sensitive. The overall elasticities for leisure and business travelers are -4.185 and -1.060, respectively. Qualitatively, these match the elasticities in Aryal et al. [2023] and Berry and Jia [2010] with leisure travelers having rather elastic demand and business travelers having less elastic demand.[24] The difference between the own price elasticities, cross price elasticities, and overall price elasticities also highlights that business travelers are more likely to substitute to other options in the market when faced with a price increase. Whereas, leisure travelers are mostly considering economy seats and likely to leave the market when faced with a price increase. However, business travelers are more likely to leave the market entirely when faced with an increase in price of premium as their cross-price elasticity to the outside option is 0.164 compared to 0.001 for leisure travelers. This happens when preferences satisfy $\xi_i \geq p_t^f/p_t^e$ as the consumer will always prefer premium to economy no matter their ν_i.[25] While their mass is small, these types of consumers do exist in reality and cannot be captured in models with only heterogeneity in ν but not ξ [e.g. Mussa and Rosen, 1978, Gershkov and Moldovanu, 2009, Loertscher and Muir, 2022].

5.2 Model Fit

To evaluate the fit of the model, we compare the estimated moments from the model to the equivalent moments in the data. Overall, the model fits the data quite well and qualitatively captures various features of the data. The mean squared error of all the moments aggregated across markets is 0.0259. This translates into a mean absolute deviation of 0.1168. Figure 13a breaks out the mean absolute deviation by market. The market with the best fit is market 3 with a mean absolute deviation of 0.080 and the market with the worst fit is market 28 with a mean absolute deviation of 0.222.

[24] In both Aryal et al. [2023] and Berry and Jia [2010], the estimates of the price elasticity imply that business travelers have inelastic demand. Although our estimate is essentially unit elastic, our definition of business and leisure is slightly different from the other two papers. Because our business type are more analogous to high loyalty customers, it is unsurprising that they are slightly more elastic than a traditional business traveler.

[25] To see this, suppose $p_t^f/p_t^e \leq \xi_i$ or $p_t^f \leq p_t^e\xi_i \iff -p_t^f \geq -p_t^e\xi_i \iff \nu_i\xi_i - p_t^f \geq \nu_i\xi_i - p_t^e\xi_i = (\nu_i - p_t^e)\xi_i$. Because $\xi_i \geq 1$, we have $\nu_i\xi_i - p_t^f \geq \nu_i - p_t^e$, which means $f \succsim e$. So a passenger with $\xi_i \geq p_t^f/p_t^e$ will always prefer premium to economy no matter the value of ν_i.

Table 4 compares selected summary statistics from the data discussed in Section 2.4 to the equivalent statistics from the model. Overall, the model matches the data quite well even though it misses a few statistics. The average fares are rather similar with model fares being larger for economy and lower for premium. Similarly, the average submitted bid is similar with the model producing average bids only $15 smaller that the data. The model matches the average number of bids and accepted bids per flight as well as the probability a bid is accepted. The upgrade probability is particularly noteworthy as it is not used in estimation in any way.

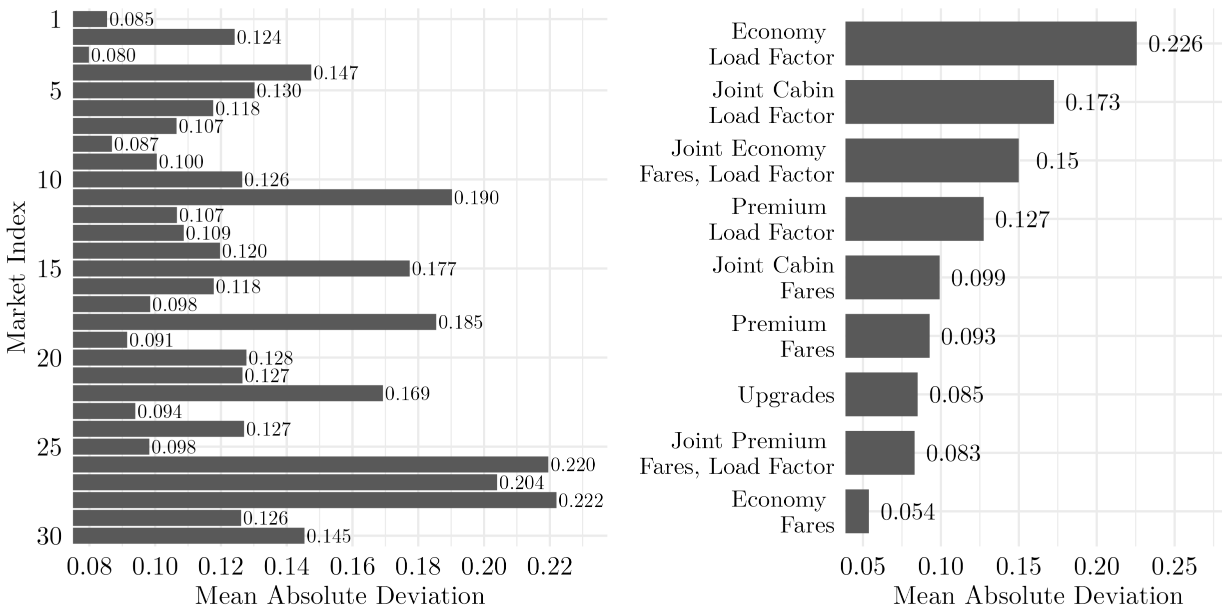
Table 4: Descriptive Statistics from Data and Model

Statistic	**Data**	**Model**
Average Paid Economy Fare	199.49	221.04
Average Paid Premium Fare	457.97	405.70
Average Submitted Bid	220.71	204.93
Average Submitted Bid: Normalized	0.23	0.06
Probability Bid Accepted	0.45	0.46
Average Bids Per Flight	1.54	1.91
Average Accepted Bids Per Flight	0.69	0.71
Average Check-In Upgrades Per Flight	1.86	0.03
Average Economy Departure Load Factor	0.81	0.99
Average Premium Departure Load Factor	0.80	0.96

Notes: The above statistics from the data are identical to those from Section 2.4 in Table 1 and Table 2. The equivalent statistics are computed in each model solution and averaged across solutions using the joint probability of that flight type and market. This is obtained by multiply the estimated flight type weight within a market with the probability of that market.

The statistics the model fails to match are the average departure load factors in both cabins, the average number of check-in upgrades per flight, and the average normalized submitted bid. These are all likely caused by a current limitation of this model: the airline can only control capacity through prices (and upgrades closer to departure). This is supported by Figure 13b which shows the load factor variables are the ones with the highest mean absolute deviation. In reality, airlines often set a fixed number of seats available at each price. We plan to add the seat releases policies from Aryal et al. [2023] in the future to better match load factors as they will give the airline more control over capacity. While this will increase the computational complexity of solving the airline's dynamic program as the addition of the seat release policies turns the dynamic program into a mixed-integer non-linear program, the conceptual complexity of the model hardly changes: the airline simply sets a state-dependent maximum number of premium and economy seats to sell in each period along with the state-dependent cabin prices in each period.

Figure 13: Decomposition of Model Fit



(a) Market

(b) Variable Groups

Notes: In a, the absolute difference between data and model moments are averaged across all 15,400 moments by market. For b, the groups on the y axis are groups of the moments discussed in Section 4.4 used to estimate the model. The economy and premium fares and load factor groups are the respective cabin specific moments from items (1) and (3). Similarly, the cabin specific joint fare and load factor groups are from the cabin moments in item (5). The joint cabin fares and load factor groups are the moments from the joint distribution of fares and load factors across cabins from items (2) and (4). The joint cabin fare group also includes the moments from item (6). Lastly, moments from items (7), (8), (9), and (10) are all grouped into the upgrades group.

5.3 Counterfactuals

The estimated structural model allows us to run various counterfactuals to examine the implications changing the design of the allocation mechanisms on profits and consumer surplus. The complexity of the environment leads to little intuition from economic theory for the optimal design of these auctions. Furthermore, because the dynamic pricing and upgrade mechanisms are not integrated, it is not clear how integration would affect profits and consumer surplus. We chose counterfactuals that would illuminate how to optimally award upgrades in this environment. The total welfare achieved in each counterfactual is compared to the first-best welfare from allocating seats to those with the highest values of (ν, ξ) regardless of arrival.[26]

Upgrade Mechanisms Introduction Simulation. The first counterfactual mimics the introduction of both upgrade mechanisms by simulating the model with and without the

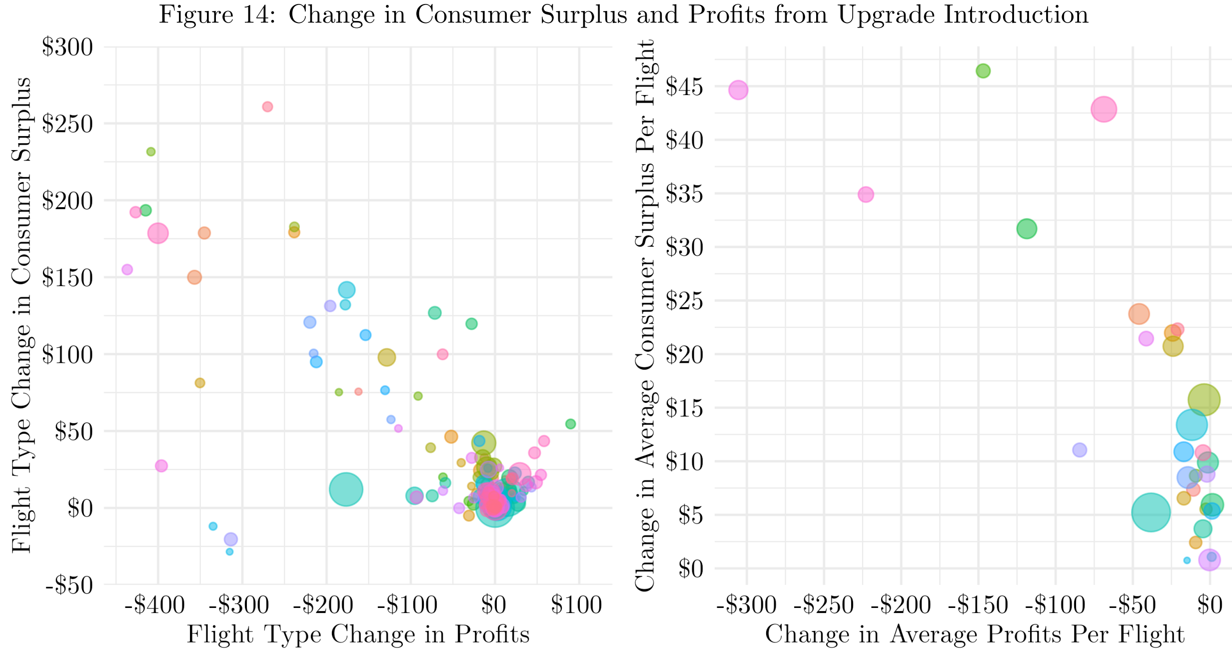
[26]See Appendix E to see a formal description of how we calculate first-best welfare.

upgrade mechanisms present. The upgrade mechanisms are removed by setting all beliefs to zero. Comparing the outcomes with and without the upgrade mechanism simulates what happened after the airline introduced both upgrade mechanisms.

Figure 14 shows the change in flight profits and consumer surplus at the flight type and market levels. Consumers almost always benefit with consumer surplus increasing 93% of the time with an average increase of $15.10 whereas profits only increased about half of the time with an average decrease of $37.52. Figure 14a highlights that there are some specifications where both consumer surplus and profits increase, happening about 49.1% of the time. However, Figure 14b shows when flights are aggregated within a market, average profits decrease in almost all markets and consumer surplus increases.

Total welfare increased about 73% of the time; however, large decreases to profit actually result in an average decrease in total welfare of about $22.42. Dynamic pricing alone achieves around 86.5% of the first-best welfare with profits and welfare achieving 76.0% and 10.4% of it, respectively. The introduction of the upgrades decreased total welfare to achieving 86.4% of the first best welfare, which is a decrease of 0.049%. After introduction, profits decreased by 0.094% and consumer surplus increased by 0.276%.

Figure 14: Change in Consumer Surplus and Profits from Upgrade Introduction



(a) Flight Type

(b) Market Average

Notes: Points in a are the flight type change in profits and consumer surplus of the types that make up 99.99% of the weight for each market and points in b are the corresponding market averages. Points in a are sized by their total type probability i.e. the probability of that market and flight type. Points in b are sized by the market probability. All points are colored by market.

To explore how the demand parameters affect the change in the split of surplus between the airline and consumers from the upgrade introduction, we calculate the mean parameter values and average flight outcomes in all four regions based on the sign of the change to profits and consumer surplus. These averages are in Table 5 and 6, respectively. For cases where consumer surplus increased, the difference between the business and leisure parameters is larger in the cases where profits decreased rather than increased. However, the values of μ_ξ^L and μ_ξ^B are larger relative to the unconditional mean in the cases where profits increased whereas in cases where profits decreased, μ_ξ^L is smaller and μ_ξ^B is only modestly larger that the unconditional means. This suggests that upgrades increase both profits and consumer surplus when both travel types have a high average willingness-to-pay for quality. However, when only business travelers have significantly higher average willingness-to-pay for travel and quality, the upgrades increase consumer surplus of business travelers and decreased profits. Cases where consumer surplus decreased happen only 7.05% of the time. These appear to be when the average willingness-to-pay is very high for the flight itself but low for quality. This is true for both travel types.

For the average arrival process, it appears that in cases where profits decreased after introducing upgrades, the flights had more arrivals on average. This is intuitive because there are more opportunities for allocative inefficiencies when there are fewer arrivals. More arrivals increases the order statistic distribution of preferences, resulting in higher prices which are paid by those who value seats the most. The total number of arrivals appears to be a major driver of the sign of the change to consumer surplus. In cases where it is positive, there were fewer expected arrivals. When there are many arrivals, the airline can set prices higher to ensure that only the people with the highest valuations are purchasing seats. Once the upgrades are introduced to these flights, there will already be less consumer surplus and little ex-post allocation inefficiencies. This is supported by the fact that before the upgrades were introduced, these flights were already achieving a larger share of first-best welfare with a larger share going towards profits and smaller share towards consumer surplus. The share of business travelers is slightly lower in markets where profits decreased. Because business travelers are more likely to buy premium seats through prices rather than upgrades, having fewer business travelers leads to fewer opportunities to recoup lost premium revenue from the upgrades with full price premium ticket purchases.

Changing Slider Minimum and Auction Timing. In this counterfactual, we change the timing of the auction as well as the minimum possible bid (i.e. the slider minimum) to examine the effect of the slider minimum and auction timing on profits and consumer surplus. Changing the auction timing is straightforward and only involves modifying $\tilde{t}$ and resolving equilibrium beliefs. To change the slider minimum, shift all bids in the bid space $\boldsymbol{B}$ up and down by the uniform bid increment used to create the bid space.

Table 5: Average Parameters by Sign of Profit and Consumer Surplus Change

	Mean	Conditional Mean by Case			
	(1)	(2)	(3)	(4)	(5)
Demand Parameters					
μ_ν^L	111.17	95.14	113.43	228.76	207.03
μ_ν^B	188.97	165.87	197.87	321.83	292.09
μ_ξ^L	1.27	1.44	1.11	1.02	1.02
μ_ξ^B	1.65	1.78	1.60	1.08	1.09
$\mu_\nu^B - \mu_\nu^L$	77.80	70.73	84.44	93.07	85.06
$\mu_\xi^B - \mu_\xi^L$	0.39	0.36	0.49	0.06	0.07
Arrival Parameters					
λ	1963.0	1890.0	1992.0	1958.6	2318.5
γ	0.047	0.046	0.047	0.047	0.0491
Cases					
Consumer Surplus		+	+	−	−
Profits		+	−	+	−
Share of Flights	1.0000	0.4906	0.4389	0.0054	0.0651

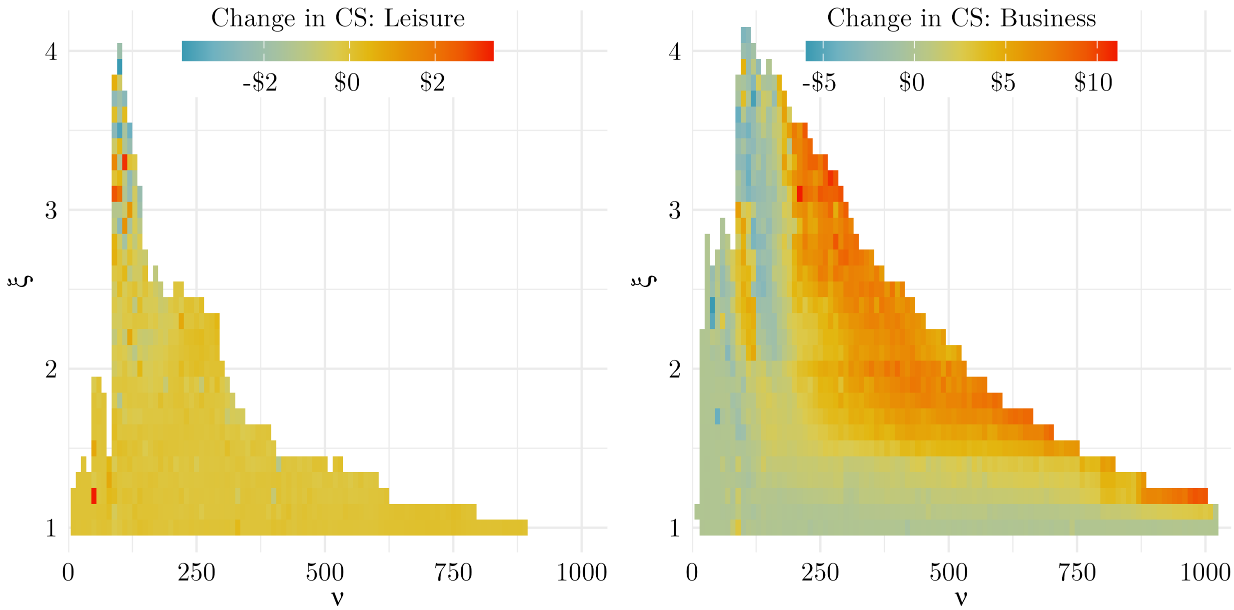
Notes: The means are calculated using the total probability of a DGP and market as a weight. λ is the total expected arrivals: $\lambda = \sum_{t=1}^{T} \lambda_t$. γ is share of business arrivals to total arrivals: $\gamma = \sum_{t=1}^{T} \gamma_t \lambda_t / \lambda$.

Table 6: Average Flight Outcomes by Changes to Profits and Consumer Surplus

	Mean	Conditional Mean by Case			
	(1)	(2)	(3)	(4)	(5)
Flight Variables					
Distance	1698.4	1601.0	1727.0	1295.3	2272.7
Paid Economy Fare	223.93	188.23	228.62	464.80	441.45
Paid Premium Fare	417.57	425.31	398.15	504.79	482.97
Number of Bids	1.91	1.21	2.97	0.01	0.11
Bid Accepted	0.464	0.385	0.521	0.811	0.783
Share of First-Best Welfare					
Total Welfare: Before	0.844	0.836	0.846	0.889	0.879
Profits: Before	0.733	0.724	0.734	0.798	0.792
Consumer Surplus: Before	0.110	0.112	0.112	0.091	0.087
Cases					
Consumer Surplus		+	+	−	−
Profits		+	−	+	−
Share of Flights	1.0000	0.4906	0.4389	0.0054	0.0651

Notes: The means are calculated using the total probability of a DGP and market as a weight. The shares of first-best welfare are from before the upgrades are introduced.

Figure 15: Change in Consumer Surplus from Upgrade Introduction in Preference Space



(a) Leisure Travelers

(b) Business Travelers

Notes: For a candidate DGP, the simulated values of ν and ξ for all consumers who purchased a ticket are binned by rounding to the nearest 10 and 0.1, respectively. After binning, the average consumer surplus with and without the upgrades is calculated by bin combination within the DGP. The grid of conditional means by bin group are the aggregated across candidate DGPs.

Allowing Bids to Influence Prices. The main counterfactual of interest is allowing the airline to update their pricing policy given the presence of the auction. The model currently assumes that the airline only upgrades consumers using their bids at time $\tilde{t}$ and does not change its policy function from before the auction was introduced. This assumption is motivated by the fact that our data from the airline reflects this behavior. However, in the absence of technology and implementation constraints, the airline would likely want to allow the pricing and auction mechanisms to work together. The goal of this paper is to analyze the difference in revenues and welfare when the airline can reoptimize prices to work with the auction mechanism. In the next subsection, we lay out the notation for allowing the auction and pricing mechanisms to work together given how the auction already works.

6 Conclusion

We study the introduction of mechanisms for allocating upgrades into the revenue-management systems of airlines. Our model captures how strategic consumers and the siloed objectives of the auction and dynamic-pricing groups within an airline can impact profitability. Consistent the intuition and predictions from the model, we find that the introduction of an upgrade auction leads to a small transfer of surplus from the airline to consumers with no improvement in overall welfare.

In counterfactual calculations, we show that there is substantial opportunity for profit and welfare improvement because current practice is well short of the first-best frontier of outcomes. We then demonstrate two potential ways in which reallocation can be improved through better integration of upgrade and pricing policies with strategic consumers. Profits and welfare increase if the auction uses state-specific reservation values to provide commitment to less accommodative bid-acceptance policies or information from the auction is shared so that pricing policies can be conditioned on bids.

In a broader context, our work highlights the challenges inherent in outsourcing and automation of decisions to third-party vendors. High among these challenges is ensuring that these algorithmic decision makers complement rather than compete with one another. In our setting, the auction's objective of making the most from unfortunate circumstances undermines the screening intention of the dynamic-pricing practices and makes those unfortunate circumstances more likely. By not accounting for this loss, the auction appears to perform admirably by collecting meaningful bid revenue in adverse demand conditions that actually arise due to the response of strategic consumers.

Our paper leaves a number of issues to be addressed in future research. While we provide a framework for studying reallocative mechanisms in a monopolistic setting, consistent with the setting in our empirical application, competition may alter some of the implications and conclusions. Computational limits also place bounds on how strategic consumers are in their decision making in our model. Equilibrium bidding accounts for dynamic selection into the auction, but we do not allow consumers to delay their purchase. Given that airline prices are expected to increase approaching departure due to a changing composition of demand, this is likely less of a problem in our application but may be in other settings. Finally, as regulatory and policy concerns grow over more sophisticated discriminatory strategies by firms, a fruitful area for future research will be understanding whether factors like strategic consumers and competition can mitigate any harm or policy and regulatory intervention is necessary.

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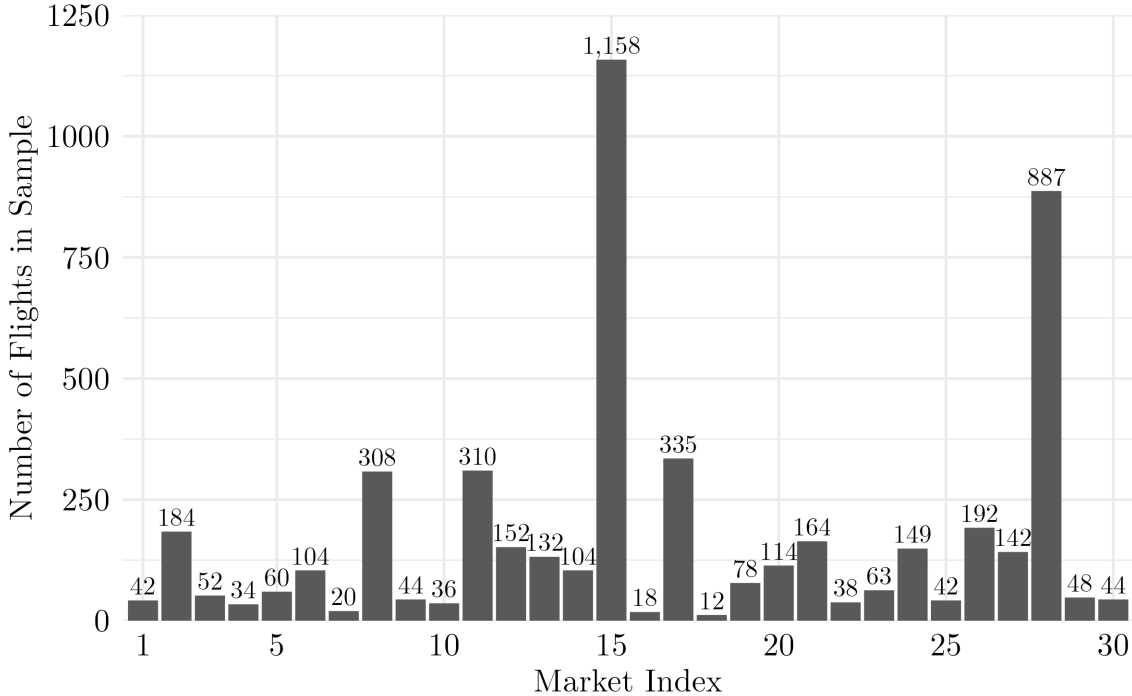
Appendices

A Sample Selection Descriptives

The sample selection process results in 30 markets and 5,066 flights used for estimation. Figure A.1 shows the number of flights in each of the 30 markets. There are two markets (markets 15 and 27) that have significantly more flights than the other 28 markets. Most markets have around 100 flights with a few markets having only a few flights.

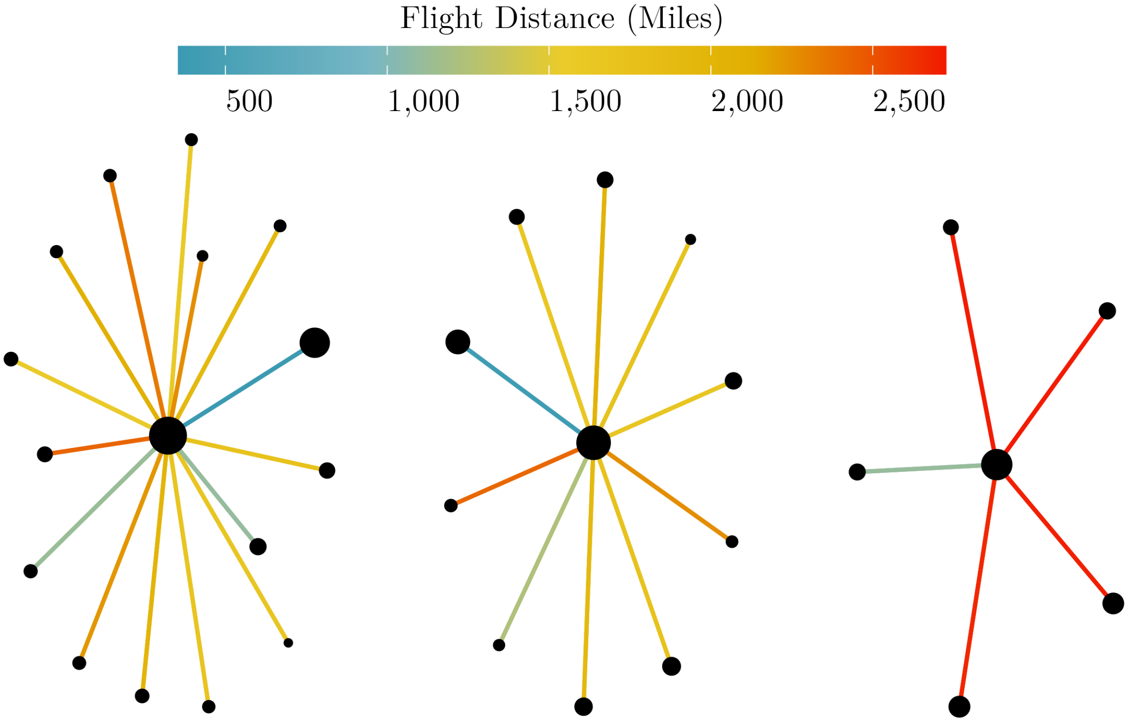
Figure A.1: Number of Flights in Sample by Market



Notes: The number of flights in the sample for each market.

The flight network of selected markets is depicted in Figure A.2. The requirement that markets must have at least one airport that enplanes and receives passengers from only one other airport results in three disconnected hub-and-spoke networks. The networks are sorted from left to right in descending order by the number of markets in that network. While the right most network has the fewest markets, the flight distance these markets is significantly further than the other two.

Figure A.2: Network of Selected Markets



Notes: Nodes represent airports and edges represent segments flown by the airline. The size of the nodes are scaled by how many passengers travel through that airport during the sample. The edges are colored by the flight distance between the airports in miles.

B Additional Model Definitions

In this section of the appendix, we formally define objects not defined in Section 3 due to ease of exposition.

Upgrade Value. Let $\mathcal{I}_t$ be the airline's information set at time t. As well, let $\mathcal{E}_{T+1}$ be the indices of those who have an economy seat at the beginning of the check-in period. Lastly, let $N^u_{T+1}(r)$ be the random variable for the number of economy passengers at the beginning of the check-in period willing to upgrade at fee r. Then the check-in value of $\boldsymbol{k}$ at t, denoted $U_t(\boldsymbol{k})$, is r multiplied by the expected number of economy passengers willing to pay check-in fee r conditional on the airline's information set at t, i.e.

$$\begin{aligned} U_t(\boldsymbol{k}) &= r\,\mathbb{E}_t[N^u_{T+1}(r)] = r\,\mathbb{E}[N^u_{T+1}(r)|\mathcal{I}_t] \\ &= r\sum_{k=0}^{k^f_t} k \times \Pr\Big(\sum_{i\,\in\,\mathcal{E}_{T+1}} \mathbb{1}[\nu_i(\xi_i - 1) \geq r] = k|\mathcal{I}_t\Big). \end{aligned} \tag{12}$$

Auction Upgrades With Non-Monotonic Marginal Costs. If $\Delta TV_t(n, \boldsymbol{k})$ is not increasing in n, then the stopping condition in Equation 7 is not guaranteed to result in the optimal number of auction upgrades. Instead, the optimal auction upgrades $n^u_{\tilde{t}}(\boldsymbol{k}, \boldsymbol{\mathcal{b}})$ can be found by solving

$$n^u_{\tilde{t}}(\boldsymbol{k}, \boldsymbol{\mathcal{b}}) = \underset{n \in \{0,1,\ldots,k^f\}}{\arg\max} \sum_{l=0}^{n} b^{(l)} - (TV_t(\boldsymbol{k} + n\boldsymbol{i}^{\boldsymbol{u}}) - TV_t(\boldsymbol{k})), \tag{13}$$

where $b^{(l)}$ is the l^{th} highest bid in $\boldsymbol{\mathcal{b}}$ and k^f is the number of remaining premium seats. The intuition is that for n upgrades, $\sum_{l=0}^{n} b^{(l)}$ is the total revenue and $TV_t(\boldsymbol{k} + n\boldsymbol{i}^{\boldsymbol{u}}) - TV_t(\boldsymbol{k})$ is the total cost.

C Solving Equilibrium Beliefs

The most difficult aspect of solving the model is computing equilibrium beliefs $\boldsymbol{\varrho}_t(\boldsymbol{k})$ and $\varphi_t(\boldsymbol{k})$. This section of the appendix explains the details of how we do it. Section C.1 explains how we compute beliefs for bid types not placed within a single simulated demand path, and Section C.2 explains the details of how we implement our iterative procedure to solve for equilibrium beliefs.

C.1 Computing Beliefs for Bid Types Not Placed in a Simulation

If a bid type is not placed within a simulation (i.e. $\mathcal{b}^j_{\tilde{t}} = 0$ for some $b^j \in \boldsymbol{B}$), the probability $\Pr(b^j \in \boldsymbol{b}^a_{\tilde{t}} | \boldsymbol{k}_{\tilde{t}}, \boldsymbol{\mathcal{b}}_{\tilde{t}})$ in Equation 9 is not well defined. In order to update beliefs, we must compute these probabilities for bid types not placed. We do this by allowing passengers to lower their bid if it does not risk altering the allocation and by using information from the airline's total value function $TV_{\tilde{t}}(\boldsymbol{k})$ to inform passengers what the airline would have accepted. This results in three cases.

Let simulation r have bid portfolio $\boldsymbol{\mathcal{b}}^r_{\tilde{t}}$ and remaining capacity $\boldsymbol{k}^r_{\tilde{t}}$ at the time of the auction, and the airline accepted $n^u_{\tilde{t}}(\boldsymbol{k}^r, \boldsymbol{\mathcal{b}}^r)$ bids with $\boldsymbol{b}^a_{\tilde{t}}$ being the set of accepted bids. Suppose $\mathcal{b}^j_{\tilde{t}} = 0$ for some $b^j \in \boldsymbol{B}$.

1. Bid types not placed but either large enough for the airline to accept given the simulation's $\boldsymbol{\mathcal{b}}_{\tilde{t}}$ and $\boldsymbol{k}_{\tilde{t}}$ or larger than a bid type that was placed and accepted. Beliefs for these bid types are set to 1.

2. Bid types not placed but would unambiguously be rejected given the simulation's $\boldsymbol{\mathcal{b}}_{\tilde{t}}$ and $\boldsymbol{k}_{\tilde{t}}$. Beliefs for these bid types are set to 0.

3. Bid types not placed but acceptance is contingent upon which consumers change their bids. To compute these beliefs, we find the closest bid type that was accepted and larger than the bid type whose belief is being computed. We allow one bidder who placed this larger bid type to decrease her bid until it is no longer accepted or the next smallest bid type was a type placed in the simulation. Beliefs are set to 1 for all bid types that were accepted by lowering the bid and 0 for the first bid type rejected and all smaller bid types.

These steps are motivated by the notion of Nash Equilibrium as they allow consumers to change their bid if profitable deviations exist as long as doing so does not risk changing their allocation nor their ranking among the other consumers. That is, consumers will increase their bid if doing so results in strictly higher $\mathcal{U}_{it}^{e}$ and will decrease their bid to not leave money on the table as long as doing so does not put them among consumers who previously placed lower bids. Lowering a bid increases expected utility if it does decrease the probability of winning. Furthermore, bids should not be lowered too much if the optimal bid is no longer weakly increasing in a consumer's valuation (the gross utility of the upgrade). This guides consumer behavior towards Nash Equilibrium and forces the equilibrium bids at a state to be increasing in the valuation, a common assumption in bidding models.

The steps in the procedure guide consumers to their optimal choice in equilibrium even if the current beliefs are far from their equilibrium values. Step (1) encourages those who were already buying economy to increase their bid, allowing beliefs to be more optimistic if they were previously too pessimistic. Conversely, (2) sets beliefs to zero for bid types that have no chance of winning given how consumers are currently behaving. This removes some of the option value associated with an economy ticket by lowering $\mathcal{U}_{it}^{e}$ and encourages those who really value premium to buy it outright. Lastly, (3) moves successful bidders towards lower bid types without the risk of losing the upgrade, increasing their value of $\mathcal{U}_{it}^{e}$. Each step guides consumers towards their optimal bid given current behavior of other consumers. This moves beliefs towards their equilibrium values and reduces the chance of multiple equilibria (see the last paragraph of the next section, C.2).

C.2 Implementation Details of Procedure to Solve Beliefs

For an initial guess of beliefs $\boldsymbol{\varrho}_t^0(\boldsymbol{k})$ and $\varphi_t^0(\boldsymbol{k})$, we forward simulate the model and obtain updated beliefs $\boldsymbol{\varrho}_t^1(\boldsymbol{k})$ and $\varphi_t^1(\boldsymbol{k})$ as described in Section 4.2. Because the procedure for updating the beliefs is not guaranteed to be a contraction mapping, we use dampening as described in Judd [1998] to ensure the beliefs are not updated too quickly. Let $\delta \in (0, 1]$ be the dampening parameter. The new, dampened beliefs are then a convex combination of the initial and updated beliefs i.e. $\boldsymbol{\varrho}_t^{1,\delta}(\boldsymbol{k}) = \delta\boldsymbol{\varrho}_t^1(\boldsymbol{k}) + (1-\delta)\boldsymbol{\varrho}_t^0(\boldsymbol{k})$ and $\varphi_t^{1,\delta}(\boldsymbol{k}) = \delta\varphi_t^1(\boldsymbol{k}) + (1-\delta)\varphi_t^0(\boldsymbol{k})$. The dampening parameter δ can be thought of as the share that the fully updated beliefs contribute to the dampened beliefs, with no dampening when $\delta = 1$. These dampened beliefs, $\boldsymbol{\varrho}_t^{1,\delta}(\boldsymbol{k})$ and $\varphi_t^{1,\delta}(\boldsymbol{k})$, are used to forward simulate the model again. We continue alternating between forward simulation and obtaining new, dampened beliefs until $\max\{d(\boldsymbol{\varrho}^{l+1}, \boldsymbol{\varrho}^{l,\delta}), d(\varphi^{l+1}, \varphi^{l,\delta})\} < \varepsilon$ where $d : \boldsymbol{X} \times \boldsymbol{X} \longrightarrow \mathbb{R}_+$ is a distance metric between two vectors of beliefs for every point in the state space $\boldsymbol{\Omega}$ and ε is a stopping tolerance. After terminating, $\boldsymbol{\varrho}_t^{l+1}(\boldsymbol{k})$ and $\varphi_t^{l+1}(\boldsymbol{k})$ are the equilibrium beliefs i.e. these are the beliefs from the final update and are not dampened.

To implement this procedure, we need values for $\boldsymbol{\varrho}_t^0(\boldsymbol{k})$, $\varphi_t^0(\boldsymbol{k})$, δ, and ε as well as a distance metric d. We set $\delta = 0.5$, $\varepsilon = 0.005$, and $\boldsymbol{\varrho}_t^0(\boldsymbol{k}) = \boldsymbol{0}$, $\varphi_t^0(\boldsymbol{k}) = 0$, $\forall \boldsymbol{k}_t \in \mathcal{K} \times \mathcal{T}_\tau$ where τ is the last time period where the beliefs are relevant.[27] The distance metric we use is a "weighted mean absolute deviation" metric. To explain this metric, we need a little more notation. Let $\boldsymbol{K}_t^{l,R}$ be a set of ordered pairs $(\boldsymbol{k}_t, r)$ of states reached during iteration l with R simulations and the simulation index $r \in \{1, ..., R\}$ in which $\boldsymbol{k}_t$ was reached. $N(\boldsymbol{k}_t, \boldsymbol{K}_t^{l,R})$ is the number of times $\boldsymbol{k}_t$ appears in $\boldsymbol{K}_t^{l,R}$. Lastly, let $\boldsymbol{x} : \boldsymbol{\Omega} \longrightarrow [0,1]^J$ and $\boldsymbol{y} : \boldsymbol{\Omega} \longrightarrow [0,1]^J$ be functions that map all points in the state space $\boldsymbol{\Omega} = \mathcal{K} \times \mathcal{T}_\tau$ to a belief vector with J probabilities.[28] Then the weighted mean absolute deviation $d(\boldsymbol{x}, \boldsymbol{y})$ between belief vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ is defined as

$$d(\boldsymbol{x}, \boldsymbol{y}) \;=\; \sum_{\boldsymbol{k}_t \in \mathcal{K} \times \mathcal{T}_\tau} \frac{N(\boldsymbol{k}_t, \boldsymbol{K}_t^{l,R})}{R \times |\mathcal{T}_\tau| \times J} \big|\big|\boldsymbol{x}_t(\boldsymbol{k}) - \boldsymbol{y}_t(\boldsymbol{k})\big|\big|_1 \tag{14}$$

where $||\boldsymbol{x}_t(\boldsymbol{k}) - \boldsymbol{y}_t(\boldsymbol{k})||_1$ is the l-1 norm and $|\mathcal{T}_\tau|$ is the size of set $\mathcal{T}_\tau$.

We weight the l-1 norm by how many simulations reach state $\boldsymbol{k}_t$ because we are using a frequency simulator to approximate the probabilities that form consumer beliefs. Deviations maybe be large when only a small number of simulations reach a state because the approximated probabilities will be noisy.[29] This metric will give less weight to deviations from noisier approximates and more weight to deviations in states that are reached with higher frequency. Note that states not reached within a simulation will not affect the metric as defined in Equation 14. This is ideal because deviations will be zero for states not reached as beliefs are not updated as described in Section 4.2, but these null deviations are not informative of convergence. The distant metric has a nice interpretation, that being the distance between $\boldsymbol{x}$ and $\boldsymbol{y}$ is the average absolute deviation between all the probabilities in the two

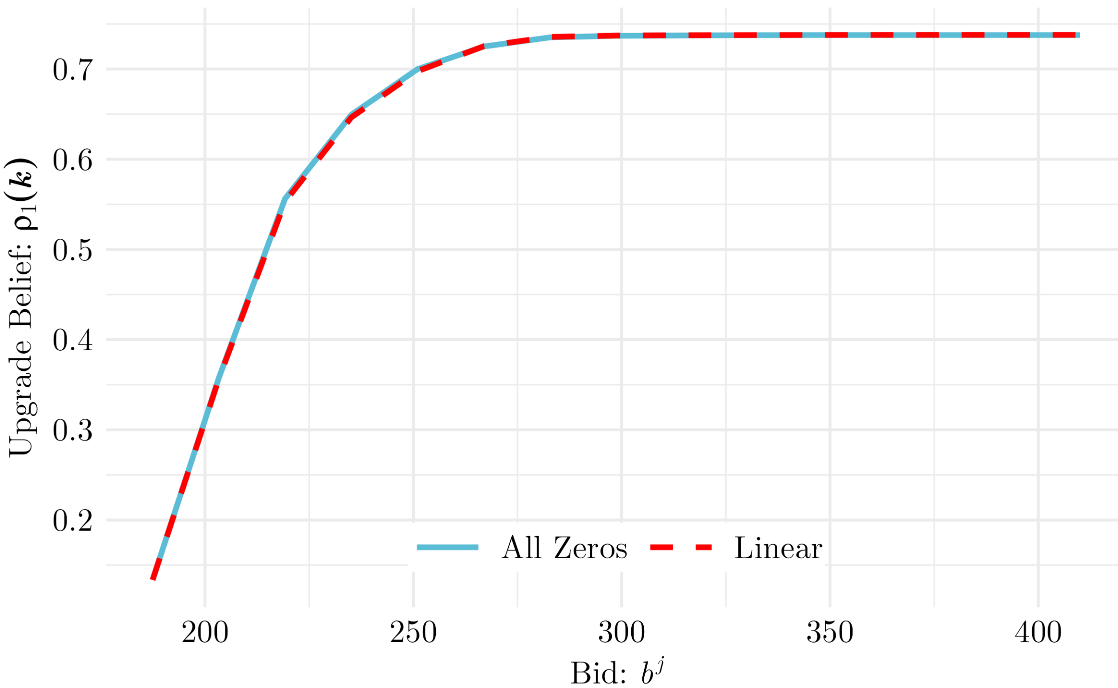
[27] $\tau = \tilde{t} - 1$ for the auction beliefs, and $\tau = T$ for the check-in beliefs.

[28] If $\boldsymbol{x}_t(\boldsymbol{k}) = \varphi_t(\boldsymbol{k})$, then $J = 1$. If $\boldsymbol{x}_t(\boldsymbol{k}) = \boldsymbol{\varrho}_t(\boldsymbol{k})$, then $J = |\boldsymbol{B}|$ i.e. the number of bids in the bid space.

[29] To see this, consider the approximated probability if only one simulation reaches a state.

belief vectors where the average is weighted by how often a state occurs in a simulation.

Figure C.1: Equilibrium Beliefs for the Average Market in the First Period with Different Initial Guesses



Notes: The equilibrium beliefs displayed are for all consumers arriving in the first period who all face the same state i.e. no seats sold. Two different guesses of initial beliefs are used: all zeros and linearly increasing beliefs between 0 and 1.

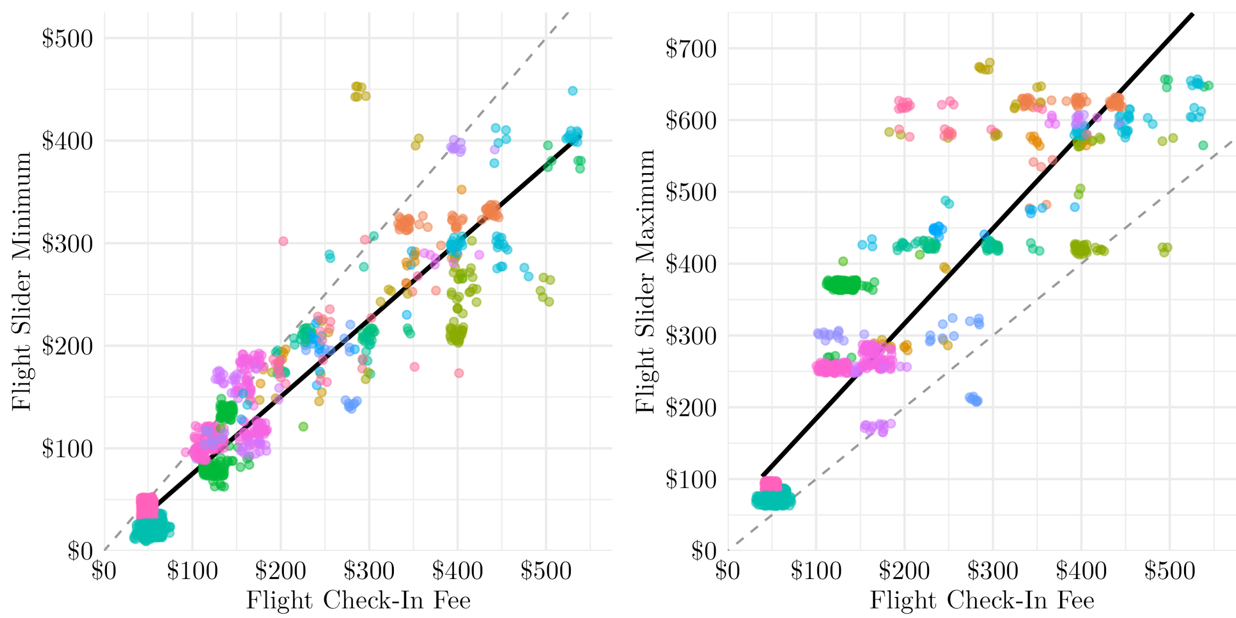
Lastly, we use initial beliefs of all zeros because this mimics an environment with only prices and no upgrades. Because we cannot guarantee that the updating procedure forms a contraction mapping, there may be multiple equilibria which makes the choice of beliefs important. First, the updating procedure guides consumers away from equilibria supported by self-fulling beliefs e.g. without the use of the airline's upgrade policy function to inform consumers what bid types the airline would have accepted, initial beliefs of all zeros would result in no bidding and the implied upgrade probabilities would also be zero. Second, to examine how much of a concern the initial guess of beliefs is, we solve equilibrium beliefs for the average market used to calculate the elasticities in Section 5 with two different initial guesses of beliefs. The first guess being all zeros and the second being linearly increasing beliefs where the belief of the lowest bid is 0 and the belief of the largest bid is 1. The resulting beliefs in the first period (where all consumers arrive in the same state) can be seen in Figure C.1. Although these equilibrium beliefs do differ from those with initial beliefs of all zero, this is likely due to simulation error because the resulting behavior of consumers is virtually identical. This eases concerns over multiple equilibria.

D Estimation Details

D.1 Calibrating Slider Features

Because the airline uses a third party software to run the auction, we do not know exactly how the slider minimum and maximum are set for a flight. Conversations with the airline suggested that the slider features were roughly based on the check-in fee for that flight. Figure D.1 suggests that there is strong clustering within a market that determines the relationship between the check-in fee and the slider features. While there is some variation in the check-in fee within market, Table D.1 shows that market fixed-effects explains 97.5% of the variation in the slider check-in fee.

Figure D.1: Relationship Between Check-In Fee and Slider Features



(a) Slider Minimum

(b) Slider Maximum

Notes: The gray dashed line is the 45 degree line and the solid black line is the line of best fit. Points are colored by market, and a "jitter" transformation has been applied to the points which adds random noise vertically and horizontally so that points on top of each other are moved slightly.

The figure also suggests that the slope of the relationship between the check-in fee and the slider minimum is less than one and the slope of the relationship between the check-in fee and the slider maximum is greater than one. Table D.1 shows these slopes are 0.752 and 1.324, respectively. If the regressions exclude a constant, the slopes are 0.752 and 1.521.

Table D.1: Check-In Fee and Slider Features Regressions

	Check-In Fee	Slider Min	Slider Max	Slider Min	Slider Max
	(1)	(2)	(3)	(4)	(5)
Variables					
Constant		-0.156	51.847		
		(11.623)	(36.545)		
Check-In Fee		0.752***	1.324***	0.752***	1.521***
		(0.062)	(0.143)	(0.037)	(0.106)
Fixed-effects					
Market	Yes	No	No	No	No
Fit statistics					
Observations	1,333	1,333	1,333	1,333	1,333
R^2	0.975	0.882	0.803	0.882	0.772

*Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

D.2 Arrival Process

To estimate λ_t, we use the search data to estimate an average daily booking rate in each market approaching departure. Then we use the inventory data to estimate the average daily number of bookings in each market approaching departure. Because the search data covers a different sample period than the rest of the data, we make the assumption that the average daily booking rate in each market is the same in the two periods in order to recover the implied λ_t by dividing the average daily bookings by the average daily booking rate in each market. For γ_t, we estimate the daily probability that a search is conducted by a high loyalty tier customer (a loyalty tier of 2 or greater) because we do not have anything in our data to credibly identify the business traveler mixing probability γ_t. This choice is motivated by results from Marsh et al. [2024a] which uses the same search data and shows evidence that these high tier customers likely have higher willingness-to-pay than lower tier customers. The details of how we estimate $\mathbf{\Lambda}$ are in Appendix D.2. The resulting arrival processes can be seen in Figure 8. Because we estimate $\mathbf{\Lambda}$ for each market, we obtain $M = 30$ different arrival processes $\mathbf{\Lambda}_m$ indexed by market $m \in \{1, ..., M\}$.

D.3 Grid of Candidate Types of Flights

Figure D.2 depicts the 2,500 candidate types formed from Halton sets by travel type. The candidate types cover the type space fairly well. Figure D.3 shows the implied averages for consumer preferences after mapping the δ version of the parameters back to the μ version of the parameters. The grid is relatively sparse in the top and bottom right-hand corners for the business parameters. However, because the estimates do not hit the boundary, this is not a concern.

E Welfare Benchmark Models

Let $N = \sum_{t=1}^{T} N_t$ be the total number of arrivals and the vectors $\boldsymbol{\nu}^f = (\nu_1\xi_1, ..., \nu_N\xi_N)'$, $\boldsymbol{\nu}^e = (\nu_1, ..., \nu_N)'$ be the corresponding gross utilities for the premium and economy cabins for all arriving passengers. Similarly, let $\boldsymbol{q}^f = (q_1^f, ..., q_N^f)$ and $\boldsymbol{q}^e = (q_1^e, ..., q_N^e)$ be the allocations of seats in the premium and economy cabins where $q_i^j \in [0, 1]$ is the probability that consumer i is allocated a seat in cabin j.

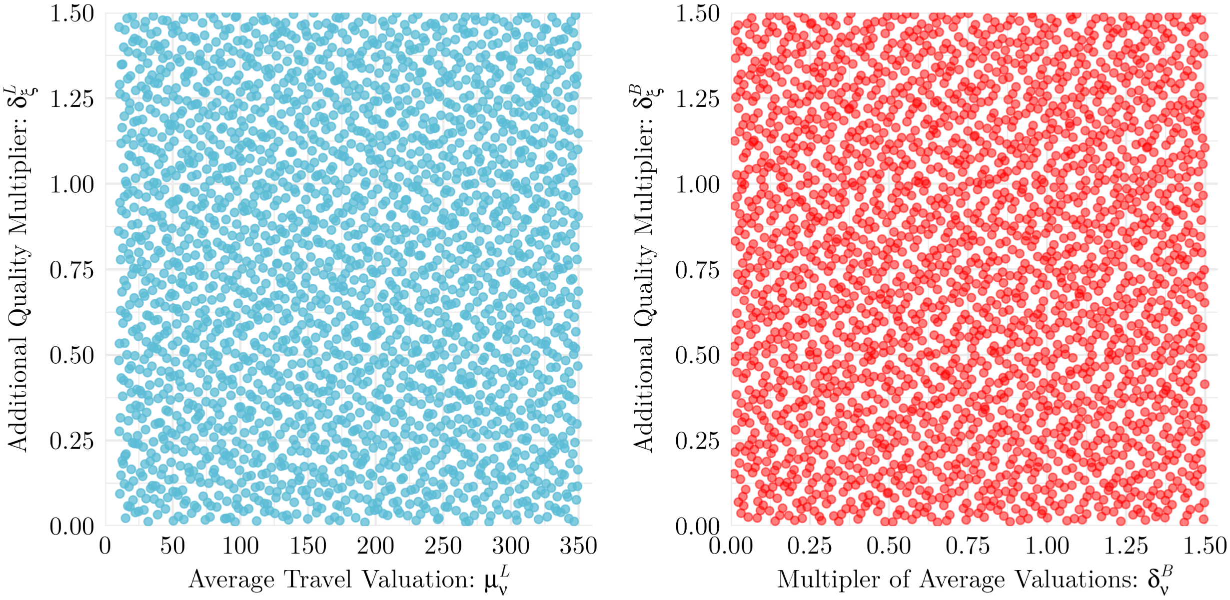
E.1 The Social Planner's Problem with Perfect Foresight

A social planner with perfect foresight can allocate seats in such a way that maximizes social welfare conditional upon those who arrived and their preferences. The efficient allocation under a deterministic allocation rule resulting in optimal social welfare W^* solves the following integer linear program,

$$\begin{gathered} W^* = \mathbb{E}\big[\max_{\boldsymbol{q}^f, \boldsymbol{q}^e} \boldsymbol{\nu}^f \cdot \boldsymbol{q}^f + \boldsymbol{\nu}^e \cdot \boldsymbol{q}^e\big] \\ \text{subject to } q_i^f \in \{0,1\},\ q_i^e \in \{0,1\},\ q_i^f + q_i^e \leq 1,\ \forall i \in \{1, ..., N\} \\ \sum_{i=1}^{N} q_i^f \leq k_1^f, \text{ and } \sum_{i=1}^{N} q_i^e \leq k_1^e. \end{gathered} \tag{15}$$

Because the solution to Equation 15 only requires model primates, W^* can easily be calculated using simulation.

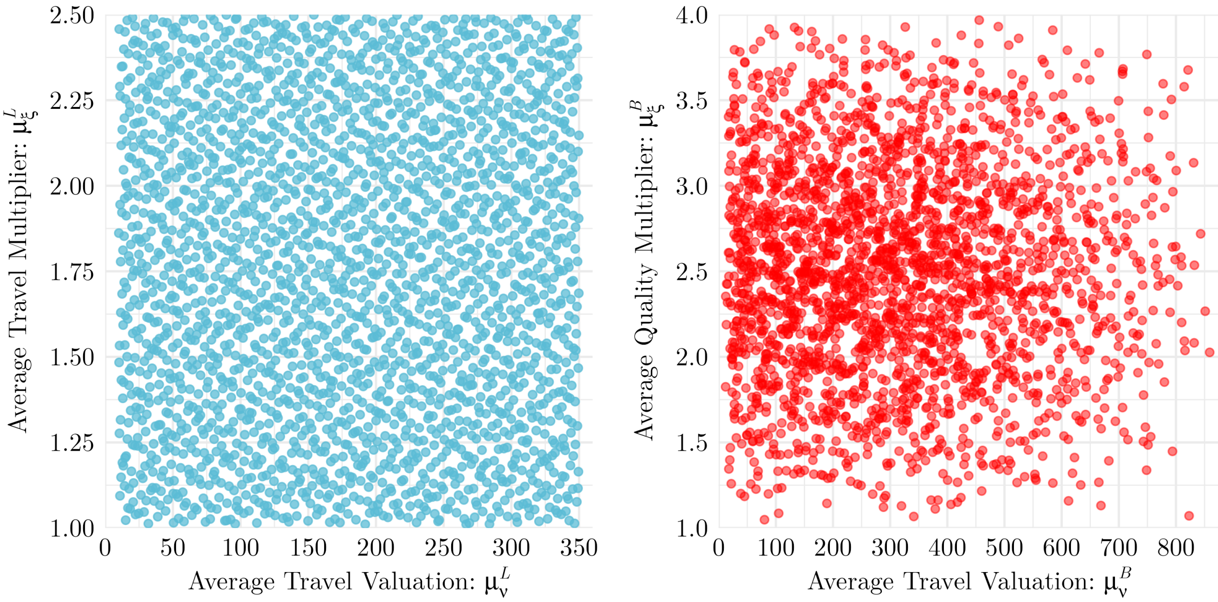
Figure D.2: Grid of Demand Parameters for Estimation



(a) Leisure Parameters

(b) Business Parameters

Figure D.3: Grid of Implied Means for Demand Preferences



(a) Leisure Parameters

(b) Business Parameters